

VISHENKOV, S.A., kand.tekhn.nauk; GOSTENINA, V.M., inzh.; YEKATOVA, V.S.,  
inzh.; PAYKINA, L.A., inzh.; FILIMONOVA, L.V., inzh.

Chemical nickel coating of welded aluminum parts. Metalloved.  
i term. obr. met. no.12:33-36 D '62. (MIRA 16:1)  
(Aluminum--Welding) (Diffusion coatings) (Nickel)

25453

S/137/61/000/006/092/092  
A006/A101

18.8200

AUTHORS: Borisov, V.S., Vishenkov, S.A.

TITLE: The effect of a chemical nickel coating on fatigue strength of machine parts

PERIODICAL: Referativnyy zhurnal. Metallurgiya, no. 6, 1961, 59, abstract 61473  
(V sb. "Povysheniye iznosostoykosti i sroka sluzhby mashin", v. 2, Kiyev, AN UkrSSR, 1960, 214 - 219)

TEXT: The chemical nickel plating of steel specimens without subsequent heat treatment does practically not reduce their  $\sigma_w$ , but the limited durability of nickel-plated specimens is strongly impaired. As a result of heat treatment of the Ni-coating, applied by chemical means,  $\sigma_w$  of steel decreases (to 45%). The chemical Ni-coating raises  $\sigma_w$  of an Al-4 aluminum alloy (up to 38%) at a 0.03 mm thick layer on the side and on the basis of tests at  $20 \cdot 10^6$  loading cycles. After precipitation of Ni, the specimens are subjected to tempering at 230°C for 1 hour.

Ye. Layner

[Abstracter's note: Complete translation]

Card 1/1

VISHENKOV, S.A. AND BORISOV, V.S.

The Effect of Chemical Nickel Plating on the Fatigue Resistance of Parts.

Povysheniye iznosostoykosti i sroka sluzhby mashin. t. 2 (Increasing the Wear Resistance and Extending the Service Life of Machines. v. 2) Kiyev, Izd-vo AN UkrSSR, 1960. 290 p. 3,000 copies printed. (Series: Its: Trudy, t. 2)

Sponsoring Agency: Vsesoyuznoye nauchno-tekhnicheskoye obshchestvo mashinostroitel'noy promyshlennosti. Tsentral'noye i Kiyevskoye oblastnoye pravleniya. Institut mekhaniki AN UkrSSR.

Editorial Board: Resp. Ed: B.D. Grozin; Deputy Resp. Ed.: D.A. Draygor; M.P. Braun, I.D. Faynerman, I.V. Kragel'skiy; Scientific Secretary: M.L. Barabash; Ed. of v. 2: Ya. A. Samokhvalov; Tech. Ed.: N.P. Rakhlina.

COVERAGE: The collection contains paper presented at the Third Scientific Technical Conference held in Kiyev in September 1957 on problems of increasing the wear resistance and extending the service life of machines. The conference was sponsored by the Institut stroitel'noy mekhaniki AN UkrSSR (Institute of Structural Mechanics of the Academy of Sciences Ukrainian SSR), and by the Kiyevskaya oblastnaya organizatsiya nauchno-tekhnicheskogo obshchestva mashinostroitel'noy promyshlennosti (Kiyev Regional Building Industry).

VISHENKOV, S. A., AND GARKUNOV, D. N.

Antifriction Properties of the Nickel-Phosphorus Coating

Povysheniye iznosostoykosti i sroka sluzhby mashin. t. 2 (Increasing the Wear Resistance and Extending the Service Life of Machines. v. 2) Kiyev, Izd-vo AN UkrSSR, 1960  
290 p. 3,000 copies printed. (Series: Its: Trudy, t. 2)

Sponsoring Agency: Vsesoyuznoye nauchno-tekhnicheskoye obshchestvo mashinostroitel'noy promyshlennosti. Tsentral'noye i Kiyevskoye oblastnoye pravleniye. Institut mekhaniki AN UkrSSR.

Editorial Board: Resp. Ed.: B. D. Grozin; Deputy Resp. Ed.: D. A. Draygor; M. P. Braun, I. D. Faynerman, I. V. Dragel'skiy; Scientific Secretary: M. L. Barabash; ED. of v. 2: Ya. A. Samokhvalov; Tech. Ed.: N. P. Rakhlina;

COVERAGE: The collection contains papers presented at the Third Scientific Technical Conference held in Kiyev in September 1957 on problems of increasing the wear resistance and extending the service life of machines. The conference was sponsored by the Institut stroitel'noy mekhaniki AN UkrSSR (Institute of Structural Mechanics of the Academy of Sciences Ukrainian SSR), and by the Kiyevskaya oblastnaya organizatsiya nauchno-tekhnicheskogo obshchestva mashinostroitel'noy promyshlennosti (Kiyev Regional Organization of the Scientific Technical Society of the Machine-Building Industry).

S/514/61/000/005/000/014  
100./1207

**AUTHORS:** Vischenkov, S.A., and Selezneva, V.I.

**TITLE:** Surface hardening of machine components by chemical nickel-plating

**SOURCE:** Akademiya Nauk SSSR. Komissiya po tekhnologii mashinostroyeniya. Seminar po kachestvu poverkhnosti. Trudy. no.5, 1961. Kachstvo poverkhnosti detalay mashin; metody i pribory, uprochneniye metallov, tekhnologiya mashinostroyeniya, 146-145

**TEXT:** A brief description is given of methods for chemical nickel-plating. Investigations and tests were carried out to study both the properties of components subjected to chemical nickel-plating, and the mechanism of this process. Diffusion processes improved the adhesion of coatings to components by affecting their internal structure. Heat treatment considerably influenced the durability and adhesion of surface coatings. The wear resistance and anti-seizing properties of nickel-phosphate coatings were markedly improved by heat treatment. Maximum specific load on nickel-phosphate coated components prior to heat treatment was 45 kg/cm<sup>2</sup>, it increased after

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S/5,14/61/000/005/006/014  
1001/1207

Surface hardening of...

heat treatment of  $250^{\circ}\text{C}$ , to  $80 \text{ kg/cm}^2$ , and at  $300-700^{\circ}$  to  $420 \text{ kg/cm}^2$ . Nickel-phosphate coatings if not subjected to heat treatment do not improve fatigue strength, due to low adhesion of the coating. However they ensure good corrosion resistance. Microhardness measurements of nickel-phosphate coated components showed this characteristic to be greatly improved. Chemical-nickel and phosphate plating is an advanced process ensuring improved surface hardness, wear and corrosion resistance. The process permits components of various shape, size and composition to be efficiently coated, thus markedly increasing the production range of components. The service life of components subjected to chemical nickel-plating, increased as much as 2-3 times. There are 4 figures and 3 tables.

Card 2/2

ACC NR: AP6032487 SOURCE CODE: UR/0413/66/000/017/0025/0025

INVENTOR: Vishenkov, S. A.

ORG: none

TITLE: Method of heat treatment of electroless coatings. Class 18,  
No. 185356 <sup>4</sup><sub>6</sub>

SOURCE: Izobreteniya, promyshlennyye obraztsy, tovarnyye znaki, no. 17,  
1966, 25

TOPIC TAGS: metal coating, electroless coated metal heat treatment,  
adhesion, hardness, wear resistance, induction hardening

ABSTRACT: This Author Certificate introduces a method for heat treat-  
ment of electroless coatings such as nickel-phosphorus, cobalt-  
phosphorus, or nickel-cobalt-phosphorus coatings. To increase the  
hardness and wear-resistance of coatings and their adhesion to the base  
metal and to prevent distortion and softening of coated parts, high-  
frequency induction heating is used.

SUB CODE: 11, 13/ SUBM DATE: 30Mar65/

Card 1/1

UDC: 621.785.545.4:621.357.77

VISHENKOV, Semen Arkad'yevich; MEL'NIKOVA, M.M., red.; TEMKINA, B.Ya.,  
otv. za vypusk; SUKHAREVA, R.A., tekhn.red.

[Increasing the wear resistance of parts by chemical nickel  
coating] Povyshenie iznosostoikosti detalei khimicheskim nikeli-  
rovaniem. Moskva, 1959. 59 p. (Moskovskii Dom nauchno-tekhnicheskoi  
propagandy. Peredovoi opyt proizvodstva. Seriya: Progressivnaia  
tekhnologiiia mashinostroeniia, vyp.5) 59 p. (MIRA 13:9)  
(Protective coatings) (Nickel plating)



S/137/62/000/006/142/163  
A057/A101

AUTHORS: Vishenkov, S. A., Velemitsina, V. I.

TITLE: Strengthening of the surface of machine parts by the method of chemical nickel plating

PERIODICAL: Referativnyy zhurnal, Metallurgiya, no. 6, 1962, 94 - 95, abstract 6I599 (V sb. "Kachestvo poverkhnosti detaley mashin. Sb. 5", Moscow, AN SSSR, 1961, 146 - 155)

TEXT: The coatings were applied on the parts in an acidic solution of the composition (in g/l):  $\text{NiCl}_2$  21, Na-hypophosphite 24, Na-acetate 10, pH 5.0 - 5.3, temperature of the bath 90 - 92°C, or in an alkaline solution of the composition (in g/l):  $\text{NiCl}_2$  21, Na-hypophosphite 24,  $\text{NH}_4\text{Cl}$  30, Na-citrate 45 and 25% solution of ammonia 50 - 60 ml/l; pH 8.3 - 8.5, temperature of the bath 85 - 88°C. Coatings obtained from the acidic solution contained 5% P, and from the alkaline solution 9% P. The coatings were tested on resistance to wear, antifriction properties, resistance to galling, and resistance to gas corrosion at high temperatures. Chemical nickel plating yields coatings which strengthen considerably

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Strengthening of...

S/137/62/000/006/142/163  
A057/A101

steel and Al-articles. The life of the articles increases 2-3 times. Ni-P-coatings can be applied to articles of any shape.

Ye. Layner

[Abstracter's note: Complete translation]



Card 2/2

VISHENPOL'SKIY, A.B.

Significance of pressure receptors of the nasal cavity in birds  
in the regulation of respiration [with summary in English].  
Biul.eksp.biol. i med. 45 no.3:27-31 Mr'58 (MIRA 11:5)

1. Iz kafedry normal'noy fiziologii (zav. - prof. G.Ya. Khvoles)  
Karagandinskogo meditsinskogo instituta (dir. - dotsent P.M. Pospelov).  
Predstavlena deystvitel'nyu chlenom AMN SSSR V.N. Chernigovskim.

(RESPIRATION, physiology,  
nasal baroreceptors in birds (Rus))  
(BIRDS,

resp.significance of nasal baroreceptors in birds (Rus))  
(ATMOSPHERIC PRESSURE,

nasal baroreceptors in birds, role in resp. (Rus))  
(NASAL CAVITY, physiology,

baroreceptors, role in resp. in birds (Rus))

KOZHEVNIKOV, S.N.; SKICHKO, P.Ya., kand.tekhn.nauk; SKUMS, V.A., inzh.;  
VISHENSKIY, I.I., inzh.

Experimental investigation of scale cars. Trudy Inst.chern.met.  
AN URSR 16:9-14 '62. (MIRA 15:12)  
(Weighing machines)

VISHENSKIY, I.I., inzh.

Some remarks on the operation of the weighing mechanism of  
scale-cars. Shor. nauch. trud. KGRI no.13:112-114 '62.  
(MIRA 16:8)

(Scales(Weighing instruments))

VISHENSKIY, I. I.

"Prolonging the Life of the Type Wheels of Baudot Receivers," Vest. svyazi,  
No.8, p. 25, 1953

Engineer of the Tyumen Telegraph

Translation No. 544, 30 Apr 56

KOZHEVNIKOV, S.N.; SKICHKO, P.Ya.; VISHENSKIY, I.I.

Investigating the propulsive resistance of weighing cars. Izv.  
vys. ucheb. zav.; chern. met. no.10:163-166 '60. (MIRA 13:11)

1. Dnepropetrovskiy metallurgicheskiy institut.  
(Blast furnaces--Equipment and supplies)

VISHEV, A.I., Cand Med Sci—(disc) "Experimental data on the effect of  
a specific immune-serum on the course of acute infection <sup>ic</sup> from ~~the~~ <sup>costly</sup> seeds." ~~of~~  
~~control plants~~ Ufa, 1958. 18 pp (Bashkir State Med Inst in 15<sup>th</sup>  
Anniversary of the <sup>100<sup>th</sup> Anniversary of the</sup> ~~100<sup>th</sup> Anniversary of the~~), 210 copies (K1,26-52,115)



CHERNOMORDIK, P.M.; VISHEVNIK, B.Z.; VOLKOVA, A.G.; MOSKVINA, R.I.;  
KUGARO, YU.V.; BAVAL'SKAYA, N.M.

Clinical treatment with proserine of chronic diseases of the nervous system. Nevropat.psikhiat., Moskva 20 no.1:68-70 Jan-Feb 51.  
(CLML 20:6)

1. Of the Nerve Division of the Hospital for Chronic Cases imeni Karl Marks (in consultation with S.N.Davidenkova, Active Member of the Academy of Medical Sciences USSR).

IVANOV, A.Ye.; VISHEVNIK, I.Z.

Surgical complications in acute pneumonias. Trudy LSGMI  
39:195-202 '58. (MIRA 12:8)

1. Kafedra obshchey khirurgii Leningradskogo sanitarno-  
gigiyenicheskogo meditsinskogo instituta (zav.kafedroy -  
prof.I.M.Tal'man).

(PNEUMONIA, compl.  
surg. (Rus))

VISHEVNIK, V. K.

137-1957-12-23412

Translation from: Referativnyy zhurnal, Metallurgiya, 1957, Nr 12, p 82 (USSR)

AUTHORS: Madyanov, A. M., Permitin, Ye. S., Miller, M. R., Lyutov, A. I.,  
Vishevnik, V. K., Kaznevskaya, V. A.

TITLE: An Experiment in Casting an Eight-ton Ingot With Small Height-  
diameter Ratio (H/D=0.5) [Opyt otlivki vos'mitonnogo slitka  
s malym otnosheniyem vysoty k diametru (H/D=0.5)]

PERIODICAL: V sb.: Novoye v liteyn. proiz-ve. Nr 2. Gor'kiy, Knigoizdat,  
1957, pp 222-232

ABSTRACT: An experimental ingot of the 40-A type was cast. The small ratio  $H/D=0.5$  was dictated by the conditions of forging. In order to achieve horizontal orientation of the crystallization plane, the following steps were taken: the exterior of the mold (M) was covered with heat-insulating slag-wool, the bottom of the M was cooled by air-water jets, and the shrinkage head was heated by an electric arc of a capacity of 1500 A. The pouring of the body of the ingot required 300 seconds, and the pouring of the shrinkage head (12 percent of the weight of the ingot) 210 seconds. The solidification time was 7 hrs. The horizontal orientation of the principal crystallization plane was not achieved. A study of the

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137-1957-12-23412

An Experiment in Casting an Eight-ton Ingot (cont.)

longitudinal templets showed a lack of axial sponginess, and a satisfactory macrostructure, with the shrinkage cavity open on top. Liquation beyond the axial zone was observed. In the cross-sectional templets the zone of small crystals occupied 20-30 mm, that of acicular crystals 50-60 mm, the remainder being non-oriented crystals of medium magnitude. On the cross-sectional templets taken from the center area and from the area below the sinkhead, large liquation-spots were discovered. The heat-insulating layer around the walls of the M proved to be detrimental, since it placed the liquation zones further away from the area of the arc's action. The employment of electrical heating improved the quality of the axial portion of the ingot. Plans for the cooling of the lower section of the ingot and for the design of a mold are presented.

G. S.

1. Castings-Development
2. Castings-Test methods
3. Castings-Test results

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VISHEVSKIY, A. K.

"Razvitie Priemnikov Infrazrasmnoi Radiacii" ("Development of Receptors of Infrared Radiation"), Priroda 1948 No. 4, pp 7-11.

L 31084-66

ACC NR: AT6022822

SOURCE CODE: HU/2505/65/028/003/0265/0272

AUTHOR: Balint, Peter; Visy, Maria--Vishi, M.

32  
B+1

ORG: Institute of Physiology, Medical University, Budapest (Orvostudományi Egyetem Előtti Intézet)

TITLE: Presence of "true creatinine" and "pseudocreatinine" in the blood plasma of the dog

22

SOURCE: Academia scientiarum hungaricae. Acta physiologica, v. 28, no. 3, 1965, 265-272

TOPIC TAGS: dog, blood plasma, biochemistry, animal physiology

ABSTRACT: The use of Lloyd's reagent has been added to the Popper, Mandel and Mayer (1937) method of creatinine determination in order to fractionate the chromogen materials present in body fluids which give a positive Jaffe reaction. The following conclusions were drawn. 1) In the normal dog, not more than 56 per cent of the total chromogen in blood plasma is true creatinine, the remaining 44 per cent consists of pseudocreatinine. 2) The renal clearance of true creatinine is equal to the inulin clearance and can be used for the estimation of the glomerular filtration rate. The chromogen clearance, on the other hand, is not suited for even an approximate evaluation of the GFR in a normal dog. 3) After nephrectomy, the chromogen concentration in

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ACC NR: AT6022822

plasma rises concomitantly with the non-protein nitrogen. In the azotemic animal, a consistently smaller fraction of the total chromogen is made up of pseudocreatinine and the elevation of chromogen concentration, under such conditions, is almost entirely due to an increase in the true creatinine level. Orig. art. has: 3 tables. [Orig. art. in Eng.] [JPRS]

SUB CODE: 06 / SUBM DATE: 07Jan65 / ORIG REF: 004 / OTH REF: 027

Card 2/2 CC

VISHIK, M.I.

Boundary value problems for quasi-linear strongly elliptic systems  
of equations having a divergent form. Dokl. AN SSSR 138 no.3:518-  
521 My '61. (MIRA 14:5)

1. Predstavleno akademikom S.L. Sobolevym.  
(Boundary value problems) (Differential equations, Linear)  
(Functional, Elliptic)



VISHIK, M. I. Cand. Physicomath. Sci.

Dissertation: "Method of Orthogonal Projections for Linear Elliptic Self-Conjugate Differential Equations." Mathematics Inst. imeni V. A. Steklov, Acad. Sci. USSR, 24 Apr. 1947.

SO: Vechernyaya Moskva, Apr. 1947 (Project #17836)

VISHEVNIK, I.Z.

Date for the improvement of radioscope diagnosis of cancer of the vault and paracardial part of the stomach. Trudy LSCMI 53:129-142 '59. (MIRA 13:10)

1. Kafedra rentgenologii s meditsinskoy radiologiyey Leningradskogo sanitarno-gigiyenicheskogo meditsinskogo instituta (zav. kafedroy - prof. B.M. Shtern).

(STOMACH--CANCER) (STOMACH--RADIOGRAPHY)

VISHIK, M.I.

Solution to a system of quasi-linear equations having a divergent form under periodic boundary conditions. Dokl. AN SSSR 137 no.3: 502-505 Mr '61. (MIRA 14:2)

1. Moskovskiy energeticheskiy institut. Predstavleno akademikom I.G.Petrovskim.

(Differential equations)

24032

S/020/61/138/003/004/017

C111/C333

16.3500

AUTHOR: Vishik, M. I.

TITLE: Boundary value problems for quasilinear strongly elliptic simultaneous equations having a divergent form

PERIODICAL: Akademiya nauk SSSR. Doklady, v. 138, no. 3, 1961, 518-521

TEXT: The author proves solubility and uniqueness theorems for the solutions of boundary value problems for a certain class of quasilinear systems of differential equations; these systems are characterized by the fact that the corresponding equations in variations from a strongly elliptic operator. In the domain  $D \subset R^n$  with boundary  $\Gamma$  let the system

$$L(u) \equiv \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D_{\alpha} A_{\alpha}(x, D_{\beta} u) = h(x) \quad (!)$$

be given, where  $x = (x_1, x_2, \dots, x_n)$ ,  $D_{\alpha} = \partial^{|\alpha|} / \partial x_1^{\alpha_1} \dots \partial x_n^{\alpha_n}$ ,  $\alpha = (\alpha_1, \dots, \alpha_n)$ ,  $|\alpha| = \alpha_1 + \dots + \alpha_n$ ,  $D_0 = E$ . The  $u = (u^1, \dots, u^N)$ , and  $h = (h^1, \dots, h^N)$ ,  $A_{\alpha}(\ ) = (A_{\alpha}^1(\ ), \dots, A_{\alpha}^N(\ ))$  attain vector  
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S/020/61/138/003/004/017

C111/C333

Boundary value problems for ... values in  $R^N$ , where  $A_{\alpha\beta}(x,0) = 0$ . Assume that the operator  $L(u)$  has the same order  $2m$  in ...  $u^N$ . The solution of (1) and

$$u|_{\Gamma} = \varphi_0(x), \dots, \partial^{\alpha} u / \partial x^{\alpha} |_{\Gamma} = \varphi_{m-1}(x), \quad x \in \Gamma \quad (2)$$

is defined as a function  $u$  which satisfies (1) in  $D$  and (2) on  $\Gamma$  and for which it holds

$$[A_{\alpha\beta}(x, D_{\beta} u), D_{\alpha} v] = [h, v] \quad (3)$$

for an arbitrary function  $v$  which satisfies the vanishing conditions (2) on  $\Gamma$ ;  $[ \cdot, \cdot ]$  is the ordinary scalar product. Assume that  $V_0 \subset U$ , where  $V_0$  is the space of the  $v$  and  $U$  is the space in which the solution is sought. Let

$$A(\pi; v, v) \equiv \sum_{|\alpha|, |\beta| \leq m} [A_{\alpha\beta}(x, D_{\beta} w) D_{\beta} v, D_{\alpha} v] \quad (4)$$

The following conditions are formulated

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Boundary value problems for . . .

$$A(w; v, v) \gg c^2 \left( \sum_{j, |\alpha|=m} [ |D_\alpha w^j|^{d_j} D_\alpha v^j, D_\alpha v^j ] + \dots \right), \quad d_j \gg 0^*, (I_1)$$

where the summation need not be carried out over all  $\alpha$ , with  $|\alpha| \geq m$ ,  
or

$$A(w; v, v) \gg c^2 \sum_{|\beta|=m} \left[ \left( 1 + \sum_{|\alpha| \leq m} |D_\alpha w|^2 \right)^{-\delta/2} D_\beta v \cdot D_\beta v \right] \quad (I_2)$$

where  $-\delta > -1$ .

The condition II says that under differentiation of  $A_\alpha(\cdot)$  with respect to  $D_\gamma v$  the order of growth decreases by 1, under partial differentiation with respect to  $x$  it does not increase. The most important part of condition II is

$$\left| \left[ \sum_{\alpha, \beta} A_{\alpha\beta}(x, D_x(f+z)) D_\alpha v, D_\beta z \right] \right| + \left| \left[ \sum_{\alpha, \beta} A_{\alpha\beta}(\cdot) D_\alpha z, D_\beta v \right] \right| \leq \leq \xi A(f+z; v, v) + MI(z) + C \quad (|\alpha| \leq m, |\beta| \leq m) \quad (7)$$

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C111/C333

Boundary value problems for . . .

where  $\epsilon > 0$  is arbitrarily small;  $M = M(\epsilon)$  and  $C$  are constants which do not depend on  $v$  and  $z$ ,  $f$  -- fixed,  $\xi$  is a function which vanishes on  $\Gamma$  together with the derivatives. Let  $w = f + tv$  and  $J(v) \equiv$

$[A_{\alpha\beta}(x, D_\beta(f+v)) - A_{\alpha\beta}(x, D_\beta f), D_\alpha v]$ . The third condition is: there exists a  $p_2 > 1$  such that

$$\|A_{\alpha\beta}(x, D_\beta(f+v))\|_{p_2} \leq CI(v) + M \tag{III}$$

holds for  $v \in V_0$ .

The system (1) is called strongly elliptic and definite for boundary conditions of the first boundary value problem, if (I), (II), (III) is satisfied.

Theorem 1: If the quasilinear system (1) is strongly elliptic and definite, then the problem (1), (2) is soluble for an arbitrary right side  $h \in H$ , i. e. there exists a  $u \in U$  which satisfies (2), (3).

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0111/0333

'Boundary value problems for ...  
 $W_q^{(1)}$  can be taken as  $H$ , sometimes  $H = L_q$  or  $H = L_q^{(-s)}$ ; see M. J. Vishik (Ref.2: DAN, 137, No. 3 (1961)).

Theorem 2: If the condition  $(I_2)$  or  $(I_1)$ , in the right part of which there occurs the summand  $[v, v^2]$ , is satisfied, then the solution of the strongly elliptic system (1) is unique in  $U$ . Assume that

$A_\alpha(x, D_\nu u) \in L_{p_\alpha}$ ,  $D_\alpha u \in L_{q_\alpha}$ ,  $1/p_\alpha + 1/q_\alpha = 1$  for  $u \in U$ ; see (Ref.2).

Similar statements can be made for the problem (1) with  $m = 1$  and

$$\sum_{|\alpha| \leq n} A_\alpha(x, D_\nu u) \cos(n, x_\alpha) |_{\Gamma} = \varphi = (\varphi^1, \dots, \varphi^N) \quad (12)$$

The solution of the problem (1), (12) is determined by the relation

$$[A_\alpha(x, D_\nu u) - A_\alpha(x, D_\nu f), D_\alpha v] = [h, v] - [A_\alpha(x, D_\nu f) D_\alpha v] \quad (13)$$

where  $f(x)$ ,  $x \in D - \Gamma$ , satisfies the condition (12) on  $\Gamma$ ,  $v$  is an arbitrary element of the space  $V$  of the type  $W_p^{(1)}$  which is not  
Card 5/6

X



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S/020/61/138/003/004/017  
C111/C333

Boundary value problems for ...  
subject to any conditions on  $\Gamma$ .

Finally, for a number of examples the author gives sufficient algebraic conditions for the solubility and uniqueness of the first and second boundary value problem in an arbitrary domain D. Let e. g.

$$A_{\alpha} (x, D_{\gamma} u) = \sum a_{\alpha \gamma \delta} (x) (D_{\gamma_1} u)^{\delta_1} \dots (D_{\gamma_N} u)^{\delta_N},$$

$$\gamma_1 = (\gamma_{11}, \dots, \gamma_{1n}), |\gamma_1| = m, \delta_1 + \dots + \delta_N = 2l + 1.$$

If the coefficients  $a_{\alpha \gamma \delta} (x)$  are bounded in D and if they possess bounded derivatives, where

$$c^2(\gamma)(\delta) \geq (A_{\alpha \beta} (x, \eta_{\gamma}) \xi_{\beta} \cdot \xi_{\alpha}) \geq c^2(\sum |\eta_{\gamma}|^{2l}) (\sum |\xi_{\alpha}|^2) \quad (14)$$

is satisfied, then the system (1) with conditions (2) is strongly elliptic, it holds theorem 1.  $W_{p, m}^l$ ,  $p=2l+2$ , can be taken as U. There are 7 Soviet-bloc references.

PRESENTED: January 7, 1961, by S. L. Sobolev, Academician  
SUBMITTED: January 4, 1961

Card 6/6

"APPROVED FOR RELEASE: 09/01/2001

CIA-RDP86-00513R001860020020-2

*11158 HIK M V*

APPROVED FOR RELEASE: 09/01/2001

CIA-RDP86-00513R001860020020-2"

VISHIK, M. I.

PA 36<sup>I</sup>27

USSR/Mathematics - Equations, Differential Nov 1947  
Mathematics - Equations, Linear

"Methods for Orthogonal Projection for General Linear  
Self-coupled Elliptical Differential Equations," M. I.  
Vishik, 4 pp

"Dok Ak Nauk" Vol LVIII, No 6

Author limits himself to explaining the methods for  
obtaining results for the bi-harmonic equation  
 $\Delta\Delta u(x_1, \dots, x_n) = 0$  in an arbitrary limited  
area G. Author makes references to Gilbert's space.  
Submitted by Academician S. L. Sobolev 4 Jun 1947.

~~36I27~~  
36I27

VISHIK, M. I.

"Linear Boundary Problems for Differential Equations," <sup>Dok Ak Nauk SSSR</sup> ibid., 65,  
NO. 6, 1949

Dok Ak Nauk SSSR

VISHIK applies results he obtained earlier (Dok Ak Nauk SSSR Vol LXV, No 4, 1949) to generalized Laplacian operator of n-dimensions, operating in bounded flat space. Give usual theorms of Green, Dirichlet, Hilbert, etc., in their generalized form for n-dimensions. Submitted by Acad S.L. Sobolev 25 Feb '49

**"APPROVED FOR RELEASE: 09/01/2001**

**CIA-RDP86-00513R001860020020-2**

**APPROVED FOR RELEASE: 09/01/2001**

**CIA-RDP86-00513R001860020020-2"**

VISHIK, M. I.

VISHIK, M. I.

"Systems of Elliptic Differential Equations, and General Boundary Problems."  
Sub 29 Nov 51, Mathematics Inst imeni V. A. Steklov, Acad Sci USSR.

Dissertations presented for science and engineering degrees in Moscow  
during 1951.

SO: Sum. No. 480, 9 May 55.

**"APPROVED FOR RELEASE: 09/01/2001**

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VISHIK, M. I.

USSR/Mathematics - Differential Equations, Elliptic      Nov/Dec 51

"Strongly Elliptic Systems of Differential Equations," M. I. Vishik, Moscow

"Matemat Sbor" Vol XXIX (71), No 3, pp 615-676

Considers the 1st boundary-value problem for a certain class of systems of elliptic differential eqs of order  $2m$ , which are called strongly elliptic. Demonstrates that the 1st boundary-value problem for these systems possesses the fundamental properties known for this problem in the case of one  $2d$ -order. Submitted 17 May 51.

198T43

VISHIK, M. I.

Author: Vishik, M.I.

Title: On the general form of linear boundary problems for an elliptical differential equation.

Journal: Doklady Akademii Nauk SSSR, 1951, Vol. 77, No. 3, p. 373

Subject: Mathematics

From: D.S.I.R. Oct 51

VISHIK, M. I.

Author: Vishik, M.I.

Title: Certain Boundary Problems for elliptic differential equations.

Journal: Doklady Akademii Nauk SSSR, Vol.77, 1951, No.4, p. 553

Subject: Mathematics

From: D.S.I.R., Oct. 1951

VISHIK M-1

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VISHIK, M. I.

*Differential Equations*

General boundary problems for elliptic differential equation. Trudy Mosk. mat. ob., No. 1, 1952

Monthly List of Russian Accessions, Library of Congress, November 1952. UNCLASSIFIED.

<sup>H</sup>  
VISIK, M.I.

On the First Boundary Value Example for Elliptic Differential Equations with  
Operator Coefficients. Soobseniya Akad. Nauk. Gruzinskoj SSR 13, 129-130 (1952).

Mathematical Reviews  
 Vol. 14 No. 7  
 July - August, 1953  
 Analysis.

Višik, M. I' On boundary problems for systems of elliptic differential equations and on the stability of their solutions. Doklady Akad. Nauk SSSR (N.S.) 86, 645-648 (1952). (Russian)

Let  $L$  be a differential operator

$$Lu = -\sum \frac{\partial}{\partial x_i} \left( A_{ij}(x) \frac{\partial u}{\partial x_j} \right) + \sum B_i(x) \frac{\partial u}{\partial x_i} + C(x)u(x),$$

where  $u$  is a function defined in real  $n$ -space with values in  $N$ -dimensional unitary space (scalar product  $(u(x), v(x))$ ) and all the coefficients are  $N \times N$  continuous matrices. The following mixed boundary value problem (1) is considered: (1)  $Lu = h$  in a region  $D$  with boundary  $\Gamma = \Gamma_1 \cup \Gamma_2$ ; (2)  $u = \varphi_1$  on  $\Gamma_1$ ; (3)  $\sum E_{ij} \cos(\nu, x_j) \partial u / \partial x_j + Qu = \varphi_2$  on  $\Gamma_2$ . (Here  $E_{ij} + E_{ji} = 2A_{ij}$ ;  $\nu$  the interior normal;  $Q$  a continuous linear operator from  $L_p(\Gamma_2)$  to  $L_q(\Gamma_2)$ ,  $p < 2(n-1)/(n-2)$ ,  $q > 2(n-1)/n$ .) The operator  $L$  is supposed to be elliptic in the sense that the bilinear form

$$E(f, g) = \int_D \sum (E_{ij}(x) \partial f / \partial x_i \partial g / \partial x_j) dx$$

is supposed to satisfy  $\Re E(f, f) + (f, f) \geq c(\Delta(f, f) + (f, f))$  ( $c > 0$ ), where  $\Delta(f, f)$  is Dirichlet's integral and

$$(f, f) = \int_D (f(x), f(x)) dx.$$

Then  $L$  and the adjoint problem  $L^*$  constitute a Fredholm pair and when it exists the solution  $u$  depends in a certain sense continuously on  $Q$ , the coefficients of  $L$  and the functions  $h$ ,  $\varphi_1$  and  $\varphi_2$ . The corresponding eigenvalue problem has discrete eigenvalues whose real parts are bounded from below. No proofs. *L. Garding (Lund).*

VIŠIK, M. I.

Mathematical Reviews  
Vol. 14 No. 11  
Nov 1953  
Analysis

✓ Višik, M. I. On systems of elliptic differential equations and their general boundary problems. *Uspchi Matem. Nauk (N.S.)* 8, no. 1(53), 181-187 (1953). (Russian)  
Author's summary of a thesis consisting of two articles: *Mat. Sbornik N.S.* 29(71), 615-676 (1951); *Trudy Mosk. Mat. Obšč.* 1, 187-246 (1952); these Rev. 14, 474, 473.  
*L. Garding (Lund).*



VISHIK, M. I.

259T54

USSR/Mathematics - Elliptic Operators 1 Mar 53

"Strongly Elliptical Differential Operators,"  
L. N. Slobodentskiy

DAN SSSR, Vol 89, No 1, pp 13-15

Discussion of the general linear differential operator of the type described as strongly elliptical by M. I. Vishik (Mat Sbornik, 29 (71), No 3, 615 (1951)). Presented by Acad V. I. Smirnov 2 Jan 53.

259T54

VISHIK, P. I.

USSR/Mathematics - Elliptic  
Equations

1 Nov 53

"First Boundary Problem for Elliptic Equations That  
Degenerate on the Boundary of the Region," M. I.  
Vishik

DAN SSSR, Vol 93, No 1, pp 9-12

States that this work was influenced by M. V.  
Keldysh's investigation (DAN, 77, No 2, 1951) of  
the first boundary problem for  $u_{xx} + y^m u_{yy}$  in a re-  
gion part of whose boundary lies on  $y=0$ . Discusses  
certain problems for the general case. Employs

275T70

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functional methods, which enables one to clarify  
the structure of the resolving operator and to  
find the conditions sufficient for unique resolv-  
ability and resolvents. Presented by Acad S. L.  
Sobolev 22 Aug 53.

VISHIK, M.I.; SOBOLEV, S.L., akademik.

Boundary problems for elliptic equations degenerating at the limit of a domain. Dokl.AN SSSR 93 no.2:225-228 N '53. (MLBA 6:10)

1. Akademiya nauk SSSR (for Sobolev). (Differential equations)

VISHIK, M. I.

"Certain Problems of Theories of Boundary-Value Problems for Elliptic  
Differential Equations," Uspekhi Matematicheskikh Nauk, Vol 8, No 2 (54), pp 159-167.

**"APPROVED FOR RELEASE: 09/01/2001**

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VISHIK, M. I.

USSR/Mathematics - Topology

Card : 1/1

Authors : Vishik, M. I.

Title : Mixed boundary problems and an approximate method of their solutions

Periodical : Dokl. AN SSSR, 97, Ed. 2, 193 - 196, July 1954

Abstract : A solution of a mixed boundary problem, having a strong elliptical spatial part, is presented. Galerkin's method of approximate solutions of mixed (with the time) boundary problems is outlined and suggested for application to the problem mentioned. Eight references: 1-USSR since 1938.

Institution : The V. M. Molotov Energetics Institute, Moscow

Presented by : Academician S. L. Sobolev, April 24, 1954

Vishik, M. I.

USCR/ Mathematics - Boundary problems

Card 1/1 : Pub. 22 - 1/40

Authors : Vishik, M. I.

Title : Mixed boundary problems for equations with the first derivative with respect to time and an approximate method for their solution

Periodical : Dok. An SSSR 99/2, 189-192, Nov 11, 1954

Abstract : An approximate method is described for the solution of mixed boundary problems expressed by differential equations whose first derivatives are taken with respect to time. A few cases of boundary problems (with homogeneous and heterogeneous boundaries) are analyzed. The analysis is accomplished in the form of theorems, proofs of which are presented. Four USSR references (1950-1954).

Institution : Moscow Power Institute im. V. M. Molotov

Presented by: Academician S. L. Sobolev, June 7, 1954



Vishik, M. I.

USSR/Mathematics - Mixed boundary problems

Card 1/1 Pub. 22 - 2/54

Authors : Vishik, M. I.

Title : Mixed boundary problems for the systems of differential equations containing the second derivative with respect to time, and an approximate method of their solutions

Periodical : Dok. AN SSSR 100/3, 409-412, Jan. 21, 1955

Abstract : A system of differential equations  $Lu \equiv A_m'(x, t, \frac{\partial}{\partial x}) \frac{\partial u}{\partial t} + B_r'(x, t, \frac{\partial}{\partial x}) \frac{\partial u}{\partial t} + C_s'(x, t, \frac{\partial}{\partial x}) u = h(x, t)$ , is considered in connection with a solution of the boundary problem for a cylinder  $Q = D \times (0 < t < 1)$ ;  $D$  is the bounded region in the  $E^{(n)}$  with the border  $\Gamma$ . The solution is found by the method of approximations for various cases determined by conditions of the differential operators  $A$ ,  $B$ , and  $C$  (some of them are strongly elliptical operators). Nine references: 8 USSR and 1 USA (1949-1953).

Institution : Moscow, V. M. Molotov Power Institute

Presented by: Academician S. L. Sobolev, October 25, 1954

VISHIK, M.I.; LADYZHENSKAYA, O.A.

Boundary problems for equations with partial derivatives and  
certain classes of operator equations. Usp.mat.nauk 11 no.6:41-  
97 N-D '56. (MIRA 10:3)  
(Differential equations, Partial) (Operators (Mathematics))

VIŠIK, M.I.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/1 PG - 487  
 AUTHOR BARENBLATT G.I., VIŠIK M.I.  
 TITLE On the finite velocity of propagation for nonsteady filtration of fluids and gases.  
 PERIODICAL Priklad.Mat.Mech. 20, 411-417 (1956)  
 reviewed 1/1957

Let the ground water propagate by plane waves. Then the pressure head satisfies the equation

$$\frac{\partial H}{\partial t} = a^2 \frac{\partial^2 H}{\partial x^2},$$

where  $a$  is a constant which depends on the properties of the porous medium and the fluid. Let the initial and boundary conditions be

$$H(x,0) = \phi(x), \quad H(0,t) = \psi(t).$$

Then the following is proved: The functions  $\phi^{(1)}(x)$ ,  $\psi^{(1)}(t)$  are assumed to correspond to the solution  $H^{(1)}(x,t)$  and the functions  $\phi^{(2)}(x)$ ,  $\psi^{(2)}(t)$  are to correspond to the solution  $H^{(2)}(x,t)$ . If then

$$\phi^{(2)}(x) \geq \phi^{(1)}(x), \quad \psi^{(2)}(t) \geq \psi^{(1)}(t)$$

it follows:  $H^{(2)}(x,t) \geq H^{(1)}(x,t)$ .

This monotone dependence of the solution on the initial and boundary conditions is applied to prove the well-known fact that in this case the velocity of propagation of the "disturbance" (wave zone) is finite.

INSTITUTION: Moscow.

19-23, 273-275; NK 19, 277

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**CIA-RDP86-00513R001860020020-2"**

VIŠIK, M.I.

CARD 1/3 PG - 303

SUBJECT

USSR/MATHEMATICS/Functional analysis

AUTHOR

VIŠIK M.I.

TITLE

On the first boundary problem for elliptic equations without limitations on the finiteness of the Dirichlet integral.

PERIODICAL

Doklady Akad. Nauk 107, 781-784 (1956)  
reviewed 10/1956

The author solves the Dirichlet problem for the equation  $\Delta u = h$  for very weak assumptions on the right hand side  $h$  and on the boundary function. The general scheme is free of the usual restriction to the solutions with a finite Dirichlet integral. Definitions:  $(a, b) = \int_{\Omega} a(x)b(x)dx$ ,  $x = (x_1, \dots, x_n)$ .

$$Gu = (\partial u / \partial x_1, \dots, \partial u / \partial x_n); \quad B(Gu, Gv) = \int_{\Omega} \sum_{i=1}^n (\partial u / \partial x_i \cdot \partial v / \partial x_i) dx.$$

$C_0$  is the set of the regular functions with compact carriers in  $\Omega_n$ .  $R_0 \stackrel{\text{def}}{=} GC_0$ .  $R_0(\pi)$  is the space  $R_0$  with the topology  $\pi$ .  $R_0^*(\pi)$  is the space being dual to  $R_0(\pi)$ .  $\overline{R_0(\pi)}$  is the closure of the set  $R_0$  in the topology  $\pi$ . The following topologies are introduced: 1.  $\pi_1$  - the Dirichlet topology induced by the norm  $\|u\|_1^2 \stackrel{\text{def}}{=} B(Gu, Gu)$ ; 2.  $\pi_2$  that which is stronger than  $\pi_1$  ( $\pi_2 > \pi_1$ );

Doklady Akad. Nauk 107, 781-784 (1956)

CARD 2/3

PG - 303

3. the local convex topology  $\pi_3$  given by the countable sequence of the seminorms

$$\|Gu\|_{1, \Omega_n}^2 \stackrel{\text{def}}{=} \int_{\Omega_n} \sum (\partial u / \partial x_1)^2 dx, \text{ where } \Omega_n^v, v = 1, 2, \dots$$

is the sequence of the subdomains of  $\Omega_n$  which fill the domain  $\Omega_n$ ; 4.  $\pi_3 < \pi_4 < \pi_1$

The element  $Gu_0$  defines the linear functional  $l_{Gu_0}$ :

$$l_{Gu_0}(Gv_0) \stackrel{\text{def}}{=} B(Gu_0, Gv_0) = (u_0, v_0)_1. \text{ Let } K \text{ be the map } R_0(\pi) \xrightarrow{K} R_0^*(\pi)$$

defined by  $K(Gu_0) = l_{Gu_0} \in R_0^*(\pi_2)$ . For  $R_0(\pi_2)$  it is assumed that the closure

$\bar{K}$  of  $K$  has the inversion  $\bar{K}^{-1}$  with  $D(\bar{K}^{-1}) = R_0^*(\pi_2)$ . For the boundary function it is assumed that it is continuable to such a function  $F$  that (1)  $GF \in R_0(\pi_3)$

and that  $l_{GF}$  is continuous on  $R_0(\pi_2)$ . It is assumed that (2)  $(h, -)$  is continuous on  $R_0(\pi_2)$ . The solution  $u$  of the first boundary problem may satisfy

(ex definitions) the conditions

$$(Rd) \quad B(Gu, Gv_0) = (h, v_0) \text{ identical for } v_0 \in C_0.$$

$$(GF - Gu) \in D(\bar{K}).$$

The following theorem is proved: The boundary problem (Rd) is uniquely solvable for (1), (2). The following example shows that the above arrangement of the problem is very general: For (2) it is sufficient that  $(\zeta h, h) < \infty$ , where



Doklady Akad. Nauk 107, 781-784 (1956)

CARD 3/3

PG - 303

$\zeta = O(\sigma^3 |\lg \sigma|^{2+\varepsilon})$ ,  $\varepsilon > 0$ , ( $\sigma = O(n)$ ,  $n$  - the normal of  $\partial \Omega_n$ ). Thus the right hand side of the equation  $\Delta u = h$  may increase arbitrarily quick at the bound  $\partial \Omega_n$  of  $\Omega_n$ . (1) is satisfied if  $\varphi \in L^1(\partial \Omega_n)$  and  $\varphi$  satisfies an integral Hölder condition of logarithmic order. The above considerations can easily be generalized to elliptic operators with a positive principal part.

~~VISHIK, M.I.~~ VISHIK, M.I.

SUBJECT USSR/MATHEMATICS/Functional analysis      CARD 1/2      PG - 671  
 AUTHOR VISHIK M.I., LJUSTERNIK L.A.  
 TITLE Stabilization of the solutions of certain differential equations  
 in the Hilbert space.  
 PERIODICAL Doklady Akad.Nauk 111, 12-15 (1956)  
 reviewed 4/1957

The family of trajectories  $\{u = u(t)\}$  ( $t_0 \leq t < \infty$ ) stabilizes to the curve  $v(t)$  if  $\lim_{t \rightarrow +\infty} \rho(u(t), v(t)) = 0$  is valid for all  $u(t)$ .

In the Hilbert space the authors consider the differential equation

$$(1) \quad \frac{du(t)}{dt} + A(t)u(t) = f(t) \quad u|_{t=t_0} = u_0$$

and the corresponding family of equations (depending on  $t$ )

$$(2) \quad A(t)v(t) = f(t).$$

It is assumed that  $\{A(t)\}$  is a family of linear operators which possess a region of definition  $\Omega$  being dense in  $H$  and independent of  $t$ , where besides

$$(A(t)u, u) \geq \chi(t)(u, u) \quad \chi(t) > 0.$$

If besides

$$\|A'_t(t)v\| \leq \delta(t) \|A(t)v\|,$$

then one of the following conditions is sufficient that the solutions  $u(t)$  of (1)

Doklady Akad.Nauk 111, 12-15 (1956)

CARD 2/2

PG - 671

stabilize with respect to the solution  $v(t)$  of (2): either

$$\gamma(t) \geq c^2 > 0, \quad \varepsilon(t) = \frac{1}{\gamma(t)} \|f'(t)\| + \frac{\delta(t)}{\gamma(t)} \|f(t)\| = o(t^{-r}), \quad r > 0$$

or

$$\gamma(t) = o(t^{-r_1}), \quad 0 < r_1 < 1, \quad \varepsilon(t) = o(t^{-r}), \quad r > 0$$

or

$$\gamma(t) = o(t^{-1}), \quad \varepsilon(t) = o(t^{-r}), \quad r > 1.$$

For establishing the criteria of stabilization by aid of these conditions one often necessitates an estimation for  $\frac{du(t)}{dt}$ . The authors propose:

$$\left\| \frac{du(t)}{dt} \right\| \leq \left\| \frac{du}{dt} \right\|_{t=t_0} \exp\left(-\int_{t_0}^t \gamma_1(\tau) d\tau\right) + \psi(\gamma_1(t), \varepsilon(t)),$$

$$\text{where } \gamma_1(t) = \gamma(t) - \delta(t) > 0 \text{ and } \psi(\gamma(t), \varepsilon(t)) = \int_{t_0}^t \varepsilon(\tau) \exp\left(-\int_{\tau}^t \gamma(\sigma) d\sigma\right) d\tau.$$

~~VISHIK, M.I.~~ VISHIK, M.I.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/1 PG - 661  
AUTHOR VISHIK M.I., LJUSTERNIK L.A.:  
TITLE Stabilization of the solutions of parabolic equations.  
PERIODICAL Doklady Akad.Nauk 111, 273-275 (1956)  
reviewed 3/1957

The criteria obtained by the authors for the stabilization of solutions of non-stationary equations into corresponding solutions of stationary equations are applied in order to investigate the stabilization of the solutions of mixed problems for parabolic equations into solutions of corresponding boundary value problems for elliptic equations.  
At first sufficient conditions for the convergence in the mean of the initial solution to the other one are set up, and then sufficient conditions for the uniform convergence are given.

VISHIK, M.I.; SOBOLEV, S.L., akademik.

General formulation of certain boundary problems for  
elliptical differential equations with partial derivatives.  
Dokl. AN SSSR 111 no.3:521-523 N '56. (MLRA 10:2)

(Differential equations, Partial) (Functional analysis)

VISHIK M. I.

42-5-1/17

AUTHOR: VISHIK M. I., LYUSTERNIK L. A.  
 TITLE: Regular Degeneration and Boundary Layer for Linear Differential Equations With a Small Parameter (Regulyarnoye vyrozhdeniye i pogranchnyy sloy dlya lineynykh differentsial'nykh uravneniy s malym parametrom)

PERIODICAL: Uspekhi Mat.Nauk, 1957, Vol.12, Nr.5, pp.3-122 (USSR)

ABSTRACT: In the domain  $Q$  of the  $n$ -dimensional space ( $n \geq 1$ ) let be given the linear differential equations

$$(1) \quad L_{\varepsilon} u_{\varepsilon} = h$$

and on the boundary  $\Gamma$  of  $Q$  let be given certain boundary conditions  $\mathcal{L}_{\varepsilon}$ . Here let the coefficients of  $L_{\varepsilon}$  depend on  $\varepsilon$  such

that for  $\varepsilon = 0$  the coefficients vanish for the highest derivatives. This boundary value problem is called the problem  $A_{\varepsilon}$ . For  $\varepsilon = 0$  it changes to the problem  $A_0$ : solution of the equation

$$(2) \quad L_0 w_0 = h$$

for the boundary conditions  $\mathcal{L}_0$ , where  $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ . The solution

Card 1/2

of  $u_0$  of  $A_0$  in general does not satisfy the conditions  $\mathcal{L}_1$ ,

## Regular Degeneration and Boundary Layer for Linear Differential Equations With a Small Parameter

42-5-1/17

besides often it is less smooth than the solution of  $A_\epsilon$ . But in a number of boundary value problems for small  $\epsilon$ ,  $u_\epsilon - u_0$  differs only noticeably from zero in the neighborhood of  $\Gamma$ , here the principal part of the difference has the so-called character of boundary layers which compensates the non-satisfaction of the conditions  $\mathcal{L}_1$ . In the present paper the authors prove the existence of a great class of problems  $A_\epsilon$  with the described boundary layer effect ("problems with a regular degeneration"), they give a method of construction of the boundary layer, obtain an asymptotic expansion for the solution  $u_\epsilon$  of  $A_\epsilon$  and estimate the remainder terms of the obtained approximate solutions and their derivatives. The detailed paper consists of an introduction, ten paragraphs, the formulation of some questions being in connection with the present investigations and a bibliography of 53 numbers. The first three paragraphs treat ordinary differential equations and contain already all essential methods of the authors, in the paragraphs 4-10 then these results are extended to the elliptic partial equations of second and higher order and to parabolic equations with a degenerating elliptic part. 35 Soviet and 18 foreign references are quoted.

1. Differential equations-Applications    2. Boundary layer

Card 2/2

AUTHOR VISHIK M.I., Corresponding Member of the Academy, PA - 3030  
 LYUSTERNIK L.A.,

TITLE On Elliptical Equations Which Contain Small Parameters in the Higher Derivations.  
 (Ob ellipticheskikh uravneniykh, sodержashchiye maliye parametry pri starshikh proizvodnykh -Russian)

PERIODICAL Doklady Akademii Nauk SSSR, 1957, Vol 113, Nr 4, pp 734-737 (U.S.S.R.)  
 Received 6/1957 Reviewed 7/1957

ABSTRACT In the linear case the following problem, among others, arises for such equations: A family of operators is assumed which depend upon the parameter  $\epsilon$  and are defined within the domain  $Q$  of the space  $(x_1, \dots, x_n)$ :  

$$L_{r,\epsilon} u = \sum_{s=p}^s \epsilon^{s-p} L_s u$$
 Here  $L_s u$  denotes a differential operator of the order  $\leq s$ , which, for reasons of simplicity, is in this case not assumed to depend on  $\epsilon$ . The solution  $u_\epsilon(x)$  of the equations  $L_{r,\epsilon} u = h$ , the asymptotic behavior of the solutions  $u_\epsilon(x)$  for small  $\epsilon$ , and their connection with a certain solution  $w(x)$  of the equation  $L_p w = h$ , are investigated for the corresponding boundary conditions. The present paper examines the case in which  $r$  and  $p$  are even numbers:  $p = 2k$ ,  $r = 2(k+1)$ . The operators  $L_{r,\epsilon}$  and  $L_p$  are assumed to be elliptical. The solution  $u_\epsilon(x)$  of the equation  $L_{r,\epsilon} u = h$  is here investigated within the domain  $Q$  with the boundary  $q$  and the homogeneous boundary conditions of the first boundary value problem are assumed on  $q$ . Also inhomogeneous boundary conditions, by the way, present no dif-

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On Elliptical Equations Which Contain Small Parameters in PA - 3030  
the Higher Derivations.

facilities. The conditions imposed here upon the operators  $L_{T,\epsilon}$  and  $L_p$  are given. The difference  $v_\epsilon(x)$  between  $u_\epsilon(x)$  and  $w(x)$  here has the character of a boundary layer of  $k$ -th order. It is one of the aims of this paper to obtain, if possible, the elementary construction of the boundary layer  $v_\epsilon(x)$ , which can be extended also to other problems with small parameters. This construction of  $v_\epsilon$  is here reduced to the solution of an ordinary differential equation with constant coefficients. The course of the computations is followed. Eventually following theorem is obtained: the solution  $u$  of the problem  $L_{T,\epsilon} u = h(1)$

$$u|_q = \dots = \frac{\partial^{k-1} u}{\partial n^{k-1}} \Big|_q = 0, \frac{\partial^k u}{\partial n^k} \Big|_q = \dots = \frac{\partial^{k+1} u}{\partial n^{k+1}} \Big|_q = 0$$

can be represented by the formula  $u_\epsilon = (w_0 + v_\epsilon^* + \epsilon \alpha_0^*) + \epsilon (w_1 + v_{1\epsilon}^* + \epsilon \alpha_1^*) + \epsilon^2 (w_2 + v_{2\epsilon}^* + \epsilon \alpha_2^*) + \beta_3$  where  $w_0$  denotes the solution of the problem (2), (3), and  $w_1$  the solution of the problem (2), (3) with replacement of the right side  $h$  by  $\sum_{j=1}^{2k+1} L_{2k+j}(w_{j-1}) - \sum_{j=1}^{2k+1} L_{2k+j-1}(\alpha_j^*)$ . Further  $[i] = \min(i, 2l)$   $v_{j\epsilon}^*$  is a function of the type of a boundary layer and  $\alpha_j^*$  is a limited function. The remainder  $\beta_3$  is of a corresponding order and small. (No ill...)

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1.11.1956  
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VISHIK, M.I.

20-5-4/67

AUTHOR VISHIK M.I. and LYUSTERNIK L.A., Corresponding Member of the Academy of Sciences of the USSR.

TITLE On Several Elliptic Equations of Even Order Which Contain Small Parameters at the Higher Derivations and Which Degenerate into Equations of First (and in general odd) Order.  
(O nekotorykh ellipticheskikh uravneniyakh chetnogo porjadka, sodержashchikh малыe parametry pri starshykh proizvodnykh i vyrozhdayushchikhsya v uravneniya pervogo (i voobshche nechetnogo) porjadka.- Russian)

PERIODICAL Doklady Akademii Nauk SSSR 1957, Vol 113, Nr 5, pp 962-965 (USSR)

ABSTRACT In a preliminary paper the authors of the paper under review investigated the asymptotic behaviour of the solution of the first boundary value problem for the elliptical equations (of even order, with small parameters at the higher derivations). At  $\xi = 0$  this solution degenerated into the solution of the first boundary value problem for the elliptical equation of the even lowest order. The paper under review now contains the following: It is possible to apply this method for the investigation of asymptotic behaviour with the corresponding complications also to that case in which the degenerated equation is of odd order.

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On Several Elliptic Equations of Even Order Which Contain Small Parameters at the Higher Derivations and Which Degenerate into Equations of First (and in general odd) Order.

The DIRICHLET Problem for the equation of second order which degenerates into a CAUCHY Problem for an equation of first order.

Because the transition to the case with  $n$  dimensions offers no difficulties, the authors of the paper under review limit themselves to the plane case. The elliptical equation under consideration is given explicitly. The paper also defines the "smoothness"  $p$  of the parameters of the problem. Also the CAUCHY Problem  $L_1 w = h_1 \omega \mid_{\Gamma^+} = 0$  is investigated. The authors construct in the paper under review two recurrence processes under the assumption that the parameters of the problem have the smoothness  $2n$ . The first recurrence process consists in the construction of the functions  $w_0 = w, w_1, \dots, w_{n-1}$ , so that at  $i > 0$  we have  $L_1 w_i = -L_2 w_{i-1}, w_i \mid_{\Gamma^+} = 0$ . The second recurrence process (which is closely related to the first process) serves the construction of the "boundary layers" which compensate the differences (unevennesses?) in the boundary conditions of the solutions  $u$  and  $w_i$  on  $\Gamma^-$ . The paper gives the relevant computations and theorems. Finally the paper investigates the equations of higher order. In analogy to above, two recurrence processes are constructed

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Equations of First (and in general odd) Order.

also here. The boundary layer is adjusted (?) already at  $n = 1$ .  
(No reproduction)

ASSOCIATION: not given.

PRESENTED BY: -

SUBMITTED: 3.2. 1957

AVAILABLE: Library of Congress.

CARD 3/3

М.И. ВЕРБА

16(1)

AUTHORS:

TITLE:

PERIODICAL:

ABSTRACT:

- Cherny, I.A., University Lecturer, and Kopylov, V.B., Scientific Assistant  
 Lomonosov - Lectures 1957 at the Mechanical-Mathematical Faculty of Moscow State University (Lomonosovskiyeheniya 1957 goda na mekhaniko-matematicheskoy fakul'tete MSU)  
 Vestnik Moskorskogo Universiteta, Seriya matematiki, mekhaniki, astronomii, fiziki, khimii, 1958, No. 4, pp. 241-246 (USSR)  
 The Lomonosov lectures 1957 took place from October 17 - October 21, 1957 and were dedicated to the 40-th anniversary of the October revolution.  
 On Appreciative Representation of Functions of Several Variables by Superposition of Functions with Less Variables and  $\epsilon$ -Entropy of Classes of Functions. The lecture generalizes the results of Kolmogorov, A.G. and others. (cf. Arnold's and V.M. Tikhonov's. The contents has been published in Doklady Akademi nauk SSSR, 194, 5). Professor I.A. Cherny is a Member of the Academy of Sciences of the USSR, spoke on "Investigation of the Boundary Layer of the Motion of a Two-Component Liquid".  
 The other lectures were given separately in the sections Mechanics and Mathematics, including following lectures: 8. I.L. Pavlenko, Lecturer - Generalization of the Theory of the Transverse Shock Against a Flexible Thread.  
 9. A.C. Kulikovsky, Aspirant - Flow Around Magnetized Bodies by Conducting Liquid.  
 10. V.S. Gerasimov, Lecturer - Instruments for the Analysis and Synthesis of Mechanisms.  
 11. V.S. Gerasimov, Lecturer - Some General Laws in the Behavior of Multiply Loaded Metals.  
 12. V.B. Klyamov, Aspirant - A Variant of the Theory of the IC Dynamics of Information and Elastic-Plastic Stability.  
 13. Professor M.I. Babin and Professor L.A. Lyubskiy, Aspirant - The Theory of the Solutions of Linear Equations with Small Parameters in the Derivatives.  
 14. Professor O.A. Diernik, Senior Researcher, Institute of Physics, Chabon Yul'ina, S.S. Fedorovskiy, A.S. Palashnikov, Ye.S. Zakharov, S.L. Kazanovskiy, A.S. Palashnikov, Professor M.B. Zhuk-Bura and P.S. Zifonov, Senior Scientific Assistant - Autozation and Programming.

Card 3/5

VISHIK, M.I.; SHILOV, G.Ye.

I. M. Gel'fand's seminar on functional analysis and mathematical  
physics at the Moscow State University. Usp.mat.nauk 13 no.2:253-263  
Mr-Apr '58. (MIRA 11:4)

(Functional analysis)  
(Mathematical physics)

AUTHOR: Vishik, M.I. and Lyusternik, L.A., 20-119-4-3/59  
Corresponding Member of the Academy of Sciences of the USSR

TITLE: On the Asymptotic Behavior of the Solutions of Partial Differential Equations for Quickly Oscillating Boundary Conditions (Ob asimptotike resheniy zadach s bystro otsilliruyushchimi granichnymi usloviyami dlya uravneniy s chastnymi proizvodnymi) SSSR,

PERIODICAL: Doklady Akademii Nauk. / 1958, Vol 119, Nr 4, pp 636-639 (USSR)

ABSTRACT: The authors consider the first boundary value problem for arbitrary elliptic equations for quickly oscillating boundary conditions. With the aid of the methods formerly published by the authors [Ref 1,2] they do not only obtain the asymptotic behavior of the solutions in the interior of the domain but also in the near of the boundary. The notion "quickly oscillating" is defined in different ways, e.g.: Let a family of functions  $\{f_\xi\}$  be given on  $\Gamma$  which depend on the parameter  $\xi$ . Let this family be called  $\frac{1}{\xi}$  - oscillating in the interval  $\mu(\varphi_0 \leq \varphi \leq \varphi_1), \mu \subset \Gamma$ , if for each  $\varphi$  of this interval it is

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20-119-4-3/59

On the Asymptotic Behavior of the Solutions of Partial Differential  
Equations for Quickly Oscillating Boundary Conditions

$$\left| \int_{\varphi_0}^{\varphi} f_{\varepsilon}(\varphi) d\varphi \right| < K\varepsilon$$

The family  $\{f_{\varepsilon}\}$  is  $\frac{1}{\varepsilon}$  - oscillating on the whole  $\Gamma$ , if  $\Gamma$   
can be covered with a finite number of intervals  $\mu_i$ , in  
each  $\{f_{\varepsilon}\}$  of which it is  $\frac{1}{\varepsilon}$  - oscillating. There are  
3 Soviet references.

SUBMITTED: January 29, 1958

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SOV/20-120-1-2/63

AUTHOR:

Vishik, M.I. and Lyusternik, L.A., Corresponding Member of the Academy of Sciences, USSR

TITLE:

The Asymptotic Behavior of the Solutions of Some Boundary Value Problems With Oscillating Boundary Conditions (Asimptotika resheniy nekotorykh krayevykh zadach s ostsilliruyushchimi granichnymi usloviyami) USSR

PERIODICAL:

Doklady Akademii nauk, 1958, Vol 120, Nr 1, pp 13-16 (USSR)

ABSTRACT:

The authors consider the parabolic equation

$$\frac{\partial u}{\partial t} + L_{2k} u = 0,$$

where  $L_{2k}$  is an elliptic operator, the hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2} - L_2 u = 0$$

and similar ones for boundary conditions of the type

$$\left. \frac{\partial^s u}{\partial n^s} \right|_{\Gamma} = A_s(\varphi, t) e^{i(\omega t + \delta \varphi)} \quad s=0, 1, \dots, k-1.$$

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The asymptotic behavior of the solutions is investigated with

The Asymptotic Behavior of the Solutions of Some Boundary Value Problems With Oscillating Boundary Conditions SOV

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the same methods as in [Ref 1,2]. The consideration is carried out in the coordinates  $\varphi, \varrho$ , where  $\varphi$  is the running coordinate on the boundary  $\Gamma$  and  $\varrho$  is the distance from the boundary. It is stated that a strong oscillation of the boundary conditions in  $t$  or  $\varphi$  leads to a boundary layer effect. There are 3 Soviet references.

SUBMITTED: February 10, 1958

1. Hyperbolic functions
2. Topology

Card 2/2

**AUTHOR:** Vishik M.I., Lyusternik, L.A., Corresponding Member SOV/20-121-5-2/50  
of the Academy of Sciences of the USSR

**TITLE:** On the Asymptotic Behavior of the Solutions of Boundary Value Problems for Quasilinear Differential Equations (Ob asimptotike resheniya krayevykh zadach dlya kvazilineynykh differentsial'nykh uravneniy)

**PERIODICAL:** Doklady Akademii nauk SSSR, 1958, Vol 121, Nr 5, pp 778-781 (USSR)

**ABSTRACT:** The methods combined with boundary layer considerations used by the authors in earlier papers [Ref 1,2] for the establishment of asymptotic developments of solutions of linear boundary value problems now are applied to simplest nonlinear cases. Beside of

$$(1) \quad L_{\varepsilon} y \equiv \varepsilon y'' + \varphi(x, y)y' - \psi(x, y) = 0, \quad y(0) = A, \quad y(1) = B$$

the authors consider

$$(2) \quad L_0 w \equiv \varphi(x, w)w' - \psi(x, w) = 0.$$

It is shown that for  $\varphi(x, y) \geq \gamma > 0$  and a sufficient smoothness of  $\varphi$  and  $\psi$  the solution of (1) has the following development in a certain subdomain M of the strip  $0 < x < 1$ :

$$\tilde{y}_{\varepsilon}(x) = w_0(x) + v_0(x) + \tilde{R}_0(x), \quad \tilde{R}_0(x) = O(\varepsilon |\ln \varepsilon|),$$

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On the Asymptotic Behavior of the Solutions of Boundary Value Problems for Quasilinear Differential Equations

SOV/20-12-5-246

$$y_\varepsilon(x) = \left[ w_0(x) + \sum_{s=1}^n \varepsilon^s w_s(x) \right] + \left[ v_0(x) + \sum_{s=1}^{n+1} \varepsilon^s v_s \right] + R_n(x)$$

$$R_n(x) = O(\varepsilon^{n+1})$$

Here  $v_0(x)$  is the principal part of the difference  $v(x) = \tilde{y}_\varepsilon(x) - w(x)$  and satisfies the equation

$$\varepsilon v_0'' + \varphi(v_0 + a)v_0' = 0 \quad (v_0(0) = 1 - a, \quad \varepsilon = w(0), \quad \varphi(y) = \varphi(0, y))$$

The  $w_k$  are determined successively by the solution of certain linear equations. The  $v_k$  are of the boundary layer type ( $v|_{\infty} = 0$ ) and are obtained successively too.

If (1) has two solutions  $\tilde{y}(x)$  and  $\tilde{y}^*(x)$ , then

$$\tilde{y}(x) - \tilde{y}^*(x) = O(\exp(-\gamma \varepsilon^{-k}))$$

where  $k$  may be an arbitrary fixed number between 0 and 1.

Sufficient for the uniqueness in  $M$  is:  $\varphi > \gamma > 0$ ,  $\frac{\partial \varphi}{\partial y} > 0$ ,

$$(A-a)\varphi_y' \geq 0.$$

There are 4 references, 2 of which are Soviet, 1 German, and 1 American.

SUBMITTED: May 10, 1958

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VISHI-K, M. I.

16(1)

PHASE I BOOK EXPLOITATION

SOV/2660

Vsesoyuzny matematicheskiy s'ezhd. 3rd, Moscow, 1956  
Trudy. t. 4: Kratkoye soderzhanie sektsionnykh dokladov. Doklady inostrannykh uchenykh (Transactions of the 3rd Mathematical Conference in Moscow. vol. 4: Summary of Sectional Reports. Reports of Foreign Scientists) Moscow, Izd-vo AN SSSR, 1959. 247 p. 2,200 copies printed.

Sponsoring Agency: Akademiya nauk SSSR. Matematicheskiy Institut.  
Tech. Ed.: G.M. Shcherbako; Editorial Board: A.A. Abramov, V.G. Bolozhki, A.M. Vasil'yev, B.V. Medvedev, A.D. Myshkis, S.M. Nikolskiy (Resp. Ed.), A.G. Postnikov, Yu. V. Prokhorov, K.A. Eshnikov, P. L. Ul'yanov, V.A. Uspenskiy, M.G. Chetaev, G. Ye. Shilov, and A.I. Shirshov.

PURPOSE: This book is intended for mathematicians and physicists.  
COVERAGE: The book is Volume IV of the Transactions of the Third All-Union Mathematical Conference, held in June and July 1956. The book is divided into two main parts. The first part contains summaries of the papers presented by the scientists at the Conference that were not included in reports submitted to the editor. The second part contains the text of those papers when the non-Soviet scientists submit a copy of his paper to the editor, the title of the paper is cited and, if the paper was printed in a previous volume, reference is made to the appropriate volume. The papers, both Soviet and non-Soviet, cover various topics in number theory, algebra, differential and integral equations, function theory, functional analysis, probability theory, topology, mathematical problems of mechanics and physics, computational mathematics, mathematical logic and the foundations of mathematics, and the history of mathematics.

Karpelevich, P.I. (Moscow). Semisimple subgroups of real groups	10
Kurbatov, Ya.A. (Sverdlovsk). Solvable equations of prime degree	11
Mukhamedshah, Kh. Kh. (Sverdlovsk). On the theory of invariant solvable groups	12
Soykin, Yu. I. (Moscow). Rings as sets with one operation subjected to a single identity	13
Section on Differential and Integral Equations	
Andrisanov, G.M. (Kazan'). Integral equations of inverse boundary value problems	14
Vinograd, B.K. (Moscow). On the upper bound of characteristic indices in small perturbations	14
Vishik, M.I. (Moscow). Solution of boundary value problems for elliptic equations in certain functional spaces	14

16(1)

AUTHORS: Lyusternik, L.A., Vishik, M.I. SOV/42-14-3-18/22

TITLE: Sergey L'vovich Sobolev (On the Occasion of his 50-th Birthday)

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 3,  
pp 203 - 214 (USSR)

ABSTRACT: The paper contains a short biography and a survey of the scientific merits of S.L. Sobolev. He was born in 1908 in Leningrad, matriculation there in 1925, his teachers were Professor V.I. Smirnov and N.M. Gyunter; in 1933 he was elected Corresponding Member of the Academy of Sciences of the USSR. Since 1935 S.L. Sobolev is Professor of the Moscow State University. In 1939 he became Member of the Academy of Sciences. Since 1940 he is member of the Communist Party. A.O. Gel'fond, I.A. Lappo-Danilevskiy and R.A. Aleksandryan are mentioned. A list of the publications from 1929 to 1957 with 86 titles and a photograph of S.L. Sobolev are given.

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16(1)

AUTHORS: Vishik, M.I., and Lyusternik, L.A.,  
Corresponding Member of the AS USSR

307/20-125-2-1/64

TITLE: Asymptotic Behavior of the Solutions of Differential Equations With Large and Quickly Variable Coefficients (Asimptoticheskoye povedeniye resheniy differentsial'nykh uravneniy s bol'shimi i bystro izmenyayushchimisya koeffitsiyentami)

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 125, Nr 2, pp 247-250 (USSR)

ABSTRACT: Several problems of mathematical physics lead to boundary value problems for equations with large and quickly variable coefficients. Here the solutions mostly change quicker in the direction cross to the boundary than in the direction parallel to the boundary. Therefore, for the approximate investigation, the appearing operator can be split up into two parts, the principal part of which corresponds to the change perpendicular to the boundary and is essentially simpler than the original operator. This method was applied by the authors already for several times [Ref 2,3,4,5] for equations with small coefficients for highest derivatives, problems with oscillating boundary values,

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Asymptotic Behavior of the Solutions of Differential Equations With Large and Quickly Variable Coefficients SO7/20-125-2-1/64-

etc. In the present paper the authors show by the example of equations of second order how to apply this method for large and quickly variable coefficients of equations. There are 6 Soviet references.

SUBMITTED: January 10, 1959

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68139

T6(1) 16.3500

AUTHORS: M.I. Vishik, Lyusternik, L.A., SOV/20-129-6-2/69  
Corresponding Member, Academy of Sciences

TITLE: Certain Questions Concerning Perturbations in Boundary Value Problems for Partial Differential Equations

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 129, Nr 6, pp 1203 - 1206 (USSR)

ABSTRACT: In the domain D with the boundary  $\Gamma$  the problem

$$(3) L_{\varepsilon} u = (L_0 + \varepsilon L_1 + \dots + \varepsilon^s L_s)u(x) = h(x) \quad x=(x_1, \dots, x_r) \in D$$

$$(5) \quad \nu_0 u = u|_{\Gamma} = 0$$

$$(6) \quad \nu_i u = \frac{\partial u}{\partial n_i} = 0 \quad i = 1, 2, \dots, l$$

is considered, where  $L_0$  is an elliptic operator of second order,  $s = 2l$ ,  $L_i$  are operators of order  $\leq i + 2$ .

Main result: If (3)-(5)-(6) is solvable for arbitrary, sufficiently small  $\varepsilon$ , if the condition

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Certain Questions Concerning Perturbations in  
Boundary Value Problems for Partial Differential Equations

$$(9) \quad \|L_\epsilon^{-1}\| \leq \frac{A}{\epsilon^{n_1}}$$

is satisfied, and if the algebraic condition for the regularity of the degeneration of  $L_\epsilon$  in  $L_0$  (see [Ref 1]) is fulfilled, then the solution  $u_\epsilon$  has the following asymptotic expansion:

$$(11) \quad u_\epsilon = \frac{C_0 u_0}{\epsilon^n} + \frac{C_0 u_1 + C_1 u_0}{\epsilon^{n-1}} + \dots + \frac{C_0 u_{n-1} + \dots + C_{n-1} u_0}{\epsilon} +$$

$$+ \epsilon^0 [w_0 + (C_1 u_{n-1} + \dots + C_n u_0)] + \dots$$

$$\dots + \epsilon^p [w_p + (C_{p+1} u_{n-1} + \dots + C_{n+p} u_0)] + \dots$$

$$\dots + \left\{ \frac{C_0 v_0}{\epsilon^n} + \dots + \frac{C_0 v_{n-1} + \dots + C_{n-1} v_0}{\epsilon} + \right.$$

$n \leq n_1$



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Boundary Value Problems for Partial Differential Equations

$$+ \sum_{p=0}^{\infty} \dot{v}_p + (C_{p+1} v_{n-1} + \dots + C_{n+p} u_0)$$

The so-called adjoint functions of the problem  $u_0, u_1, \dots, u_{n-1}$  and the so-called adjoint boundary layers  $v_0, v_1, \dots, v_{n-1}$  are obtained from a recurrence system, the constants  $C_i$  and the functions  $w_i$  are obtained by substituting (11) into (3), (5), (6). The authors mention A.L. Gol'denveyzer. There are 2 Soviet references. ✓

SUBMITTED: September 30, 1959

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80210

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S/042/60/015/03/01/002

AUTHORS: Vishik, M.I., and Lyustarnik, L.A.TITLE: The Solution of Some Problems on Perturbations in the Case of  
Matrices and of Selfadjointed and non-Selfadjointed Differential  
Equations. 16

PERIODICAL: Uspekhi matematicheskikh nauk, 1960, Vol.15, No.3, pp.3-80

TEXT: The present paper consists of two chapters. The most essential results are already announced in (Ref.12,13). Chapter I: Perturbations of symmetric matrices. The authors consider linear algebraic problems, where they try to give the proofs so that a transfer to analytic problems considered later is possible. § 1. Introduction, § 2. Perturbation of the solutions of linear algebraic equations. The asymptotic behavior of the solution of the inhomogeneous linear system of equations  $A_\xi \bar{y}_\xi = h$  is given. If  $\det A_0 \neq 0$ , then the problem is elementary. In the other case  $\bar{y}_\xi$  as a function of  $\xi$  has a pole of n-th order for  $\xi = 0$ , where n is the maximal length of the Jordan chains of the adjoined vectors of the problem. § 3. Perturbations of the eigenvalues and eigenvectors. The asymptotic behavior of the eigenvalues  $\lambda_\xi$  and the eigenvectors  $v_\xi$  of the matrix  $A_\xi = A_0 + \xi A_1$  is considered, if to the eigenvalue  $\lambda_0$

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of the limit matrix  $A_0$  there corresponds a Jordan chain;  $\lambda_\xi$  and  $v_\xi$  are developed with respect to powers of  $\xi^{1/2}$ ,  $n$  - length of the Jordan chains of  $A_0$ ; the general case is treated in the appendix I. Chapter II. Perturbations of selfadjointed and non-selfadjointed boundary value problems for the equation  $L_\xi u_\xi = h$ . § 4. If the order of the equation is not elevated by the perturbations, the results of § 2 can be transferred to this problem. § 5. (partially contained already in (Ref,1)). If the order of  $L_\xi$  is greater than the order of  $L_0$  and if  $L_0 u = h$  is solvable for all  $h$ , then the asymptotic behavior of the solution is obtained with the aid of two iteration processes: for a regular degeneration that leads to functions of the type of boundary layers. § 6. If there exists an eigenfunction  $u_0$ ,  $L_0 u_0 = 0$ , then the boundary layer methods of § 5 and the methods of § 2 are combined. In § 7 the perturbation of eigenvectors and eigenfunctions is considered in the general non-adjointed case; correspondingly the methods of § 2 are replaced by the

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methods of § 3. The case where the order of  $L_{\xi}$  is not greater than the  
order of  $L_0$  is solved completely.

The author mentions M.A. Leontovich, A.L. Gol'denveyzer, Yu.L. Daletskiy,  
Slobodetskiy, A.B. Shabat and N.M. Leontovich. There are 25 references:  
22 Soviet and 3 American.

SUBMITTED: December 1, 1959

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10(1)

367/70-130-1-2/02

AUTHORS: VISHIK, M. I., and IL'IN, L. A., Corresponding Member of Academy of Sciences, USSR

TITLE: Perturbation of Eigenvalues and Eigenelements for Some Non-Self-Adjoint Operators

PERIODICAL: Doklady Akademi nauk SSSR, 1969, Vol 130, Nr 2, pp 251-253 (USSR)

ABSTRACT: In their previous papers the authors investigated the perturbation of solutions of some algebraic and differential equations. In this paper same methods are applied to the question of perturbation of eigenvalues and eigenelements in the case of non-self-adjoint operators. 1. The Algebraic Case. Let  $A_0$ ,  $A_1$ , and  $A_\epsilon = A_0 + \epsilon A_1$  are the non-Hermitian matrices of the n-th dimension, and let  $\lambda = 0$  be eigenvalue for  $A_0$ . To the latter corresponds the invariant space  $S_0$  of dimension  $\Pi$ . The matrix  $A_0$  has a Jordan basis  $x_{ij}$  corresponding to the eigenvalue  $\lambda = 0$ .

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Perturbation of Eigen Values and Eigen Elements for Some Non-selfadjoint Operators

Let  $E_{i0}$  be the linear hull of all eigen vectors  $x_{i0}$ , to which there correspond Jordan chains of the same length  $n_i$ ;  $n_1 > n_2 > \dots > n_1 - 1$ ;  $p_i$  dimension of  $E_{i0}$ ; let  $E_{ij}$  ( $j=1, 2, \dots, n_i-1$ ) be the linear hull of the adjoint vectors  $x_{ij}$ . It is

$$N = \sum_{i=1}^1 p_i n_i, \quad S_0 = \sum_{i=1}^1 \sum_{j=0}^{n_i-1} E_{ij} .$$

Theorem : Under the described structure of the invariant space  $S_0$  corresponding to the eigen value  $\lambda = 0$  of the matrix  $A_0$  the matrix  $A_\epsilon$  has

$N = \sum_{i=1}^1 n_i p_i$  eigen values  $\lambda_i$  tending to zero for  $\epsilon \rightarrow 0$ ;  $n_i p_i$  of these eigen values are represented by power series

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Perturbation of Eigen Values and Eigen Elements for Some Non-selfadjoint Operators

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$$(1) \quad \lambda_{\epsilon} = \sum_{k=1}^{\infty} \frac{\epsilon^{k/n_1}}{k}$$

where  $\lambda_k = \lambda_k(q)$  ( $q = 1, 2, \dots, n_1 p_1$ ), and to them there correspond the eigen vectors

$$(2) \quad v_{\epsilon} = \sum_{k=0}^{\infty} \frac{\epsilon^{k/n_1}}{k} v_k$$

Here it is  $v_0 = \sum_{j=1}^i v_{j0}$ ,  $v_{j0} \in E_{j0}$ ;  $v_{i0}$  and  $\lambda_1^{n_1}$  are the eigen vector and value corresponding to each other of an operator  $C_i$  which acts from  $E_{i0}$  into  $E_{i0}$ , while  $v_{j0}$  is defined by the  $v_{i0}$  by means of a fixed linear operator  $B_{ji}$  (from  $E_{i0}$  into  $E_{j0}$ ).

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4

Perturbation of Eigen Values and Eigen Elements  
for Some Non-selfadjoint Operators

SOV/20-130-2-2/69

The theorem is transferred to differential operators  
 $L = L_0 + L_1$ , where  $L_0$  is an elliptic operator of second  
order and  $L_1$  an operator of at most second order.  
There are 2 Soviet references.

SUBMITTED: October 26, 1959

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AUTHOR: Vishik, M.I., Corresponding Member of the AS USSR, and  
Lyusternik, L.A.TITLE: Initial Jump for Nonlinear Differential Equations Containing a  
Small Parameter <sup>16</sup>

PERIODICAL: Doklady Akademii nauk SSSR, 1960, Vol. 132, No. 6, pp. 1242-1245

TEXT: The authors consider the Cauchy problem

$$(1) \quad L_{\varepsilon} y = \varepsilon y'' + \varphi(x, y, y') = 0, \quad y|_{x=0} = y_0, \quad y'|_{x=0} = \frac{C}{\varepsilon B} \quad (B > 0).$$

They seek the geometrical limit value for  $\varepsilon \rightarrow 0$  of the integral curves  $y_{\varepsilon}$  which may contain the straight line  $[y_0, y + B]$  of the  $y$ -axis. The phenomenon is called an initial jump and is considered for a quasilinear equation in (Ref. 1). With the aid of an asymptotic development of the solution it is stated: If  $\varphi(x, y, y')$  for  $y' \rightarrow \infty$  increases as  $|y'|^1$ ,  $0 < 1 \leq 2$ , then there is an initial jump for  $0 < \alpha < 1$  for  $B = (1 - \alpha)^{-1}$ ,  
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Initial Jump for Nonlinear Differential Equations Containing a Small Parameter

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where  $1 = 1 + \alpha$  ; the length B can be computed by :

$$(3) \quad C^{1-\alpha} = (1 - \alpha) \int_{y_0}^{y_0+B} \psi_{00}(0,y)dy ,$$

where  $\psi_{00}$  is taken from the development of  $\psi$ , e.g.  $\psi(x,y,y') = y^{1+\alpha} [\psi_{00}(x,y) + O(1)y'^{-r}]$ . If  $\alpha = 1$ , i.e.  $\psi(\cdot, y, y') = O(y'^2)$ , then the initial jump appears if  $y'|_{x=0} = e^{C/\epsilon}$ .

The authors mention S.N. Bernshteyn. There are 5 Soviet references.

SUBMITTED: March 31, 1960



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