

Investigating the effect of stress ...

S/123/61/000/020/014/035
A004/A101

obtained during the testing of specimens with keyways of different shapes. The most expedient shape with a minimum value of $K_{\sigma} = 1.0$ proved to be keyways with rectangular or semicircular cross section having smooth junctions with the specimen surface. There are 6 figures and 2 references. ✓

I. Bernshteyn

[Abstracter's note: Complete translation]

Card 2/2

S/137/60/000/03/10/013

Translation from: Referativnyy zhurnal, Metallurgiya, 1960, No 3, p 251,
6238

AUTHORS: Petergerya, D.M., Oleynik, N.V.

TITLE: The Scale Effect in Overloading Alloyed Steel

PERIODICAL: Nauchn. zap. Odessk. politekhn. in-t, 1959, Vol 16, pp 252-265

TEXT: The authors investigated the effect of preliminary cyclic load on metal creep depending on the absolute dimensions of the specimen. The investigation was carried out on preliminary normalized 30XH3A (30XhN3A) alloyed steel specimens of 7.52; 12.5; 20 and 27 mm in diameter. The results were compared to those obtained previously from 45 steel tests. It was established that the absolute dimensions represented a factor which affected strongly the fatigue limit of the material and its overload strength. It is shown that the overload resistance of steel grows with increased absolute dimensions: the cyclic durability, under similar relative overload, increases with a larger diameter of the specimens. It is assumed that the regularities determined may in principle also be applied to other structural steel grades.

Card 1/1

Z.P. ✓

OLEYNIK, N.V.; PETERGERYA, D.M.

Most suitable shape of cylindrical specimens for fatigue testing
in pure bending. Zav.lab. 26 no.2:210 '60. (MIRA 13:5)

1. Odesskiy politekhnichesk y institut.
(Steel--Testing)

OLEYNIK, N.Y., SHAYGER, Ye.V.

Determining the weight of fatigue testing machinery. Zav.lab.
26 no.5:609-610 '60. (MIRA 13:7)

1. Odesskiy politekhnicheskii institut.
(Fatigue-testing machines)

10 74 00

29339

S/122/61/000/010/005/011
D221/D304

AUTHORS: Petergerya, D.M., Engineer, and Oleynik, N.V. Candidate of Technical Sciences, Docent

TITLE: On the influence of absolute sizes of a specimen section on the magnitude of effective stress concentrations during loads above the fatigue limit

PERIODICAL: Vestnik mashinostroyeniya, no. 10, 1961, 32 - 34

TEXT: The authors quote results of investigations concerning the effect of overloads on fatigue resistance related to stress concentrations derived from experiments with structural steels. Specimens were made in steels 45 and 30XH3A (30KhN3A). Effective coefficients of stress concentration, corresponding to the horizontal portion of fatigue curve are designated by K_{σ} and in the region of limited strength by K'_{σ} . The present article tabulates data which demonstrate that the inclination of fatigue curves for undercut and large specimens is greater than in the case of small and plain samples. The latter exhibit a tendency to rise with an increase of diameter.
Card 1/4

29339

S/122/61/000/010/005/0
D221/D304

On the influence of absolute ...

confirming, therefore, the law which expresses the drop of K'_0 at higher overloads. It also reveals the effect of absolute sizes of a specimen section on the character of this reduction. Variations of K'_0 with length of ligament and diameter of specimen were investigated. The abscissae of bends for plain samples, N_0 , were smaller than the corresponding abscissae N'_0 of curves for undercut specimens, but at $N = N'_0$, $K'_0 = K_0$. Results of calculating K'_0 plotted in semi-log coordinates are shown in Fig. 1. Further analysis allows the law of changes of K'_0 with $\log N$ in the ranges $0-N$ and $N_0 - N'_0$ to be represented by approximation of linear equations. Consequently, the full curves $K'_0 = f(\log N)$ can be assumed as three broken sections. In ranges of $0-N$ and $N_0 - N'_0$ they follow

$$10^{2-K_0} \cdot \left(\frac{N_1}{N_2}\right)^q = 1$$

where K_{02} is the coefficient of stress concentration in the case of
Card 2/4

On the influence of absolute ...

2339

S/122/61/000/010/005/011
D221/D304

life N_2 ; K_{σ_1} - ditto for life of N_1 ; q is the angular coefficient, which characterizes the inclination of the curve of the relationship between the effective stress concentration coefficient and \log

of life, $q = \frac{K_{\sigma_2} - K_{\sigma_1}}{\lg N_2 - \lg N_1}$. Computation of the latter indicates that K' varies more sharply between $N_0 - N'_0$, than in the region $10^5 - N_0$, and that inclination of curves $K'_\sigma - N$ increases with greater diameters of specimens. The expression for K'_σ which is quoted

below, was based on the step law of the left hand branch of the fatigue curve as given by D.N. Reshetov (Ref. 6: Sb. "Povysheniye prochnosti detaley mashin" Izd. AN SSSR, 1949). The deduction can be understood from arbitrary curves of fatigue for plain and undercut specimens. There are 2 figures, 2 tables and 6 references: 5 Soviet-bloc and 1 non-Soviet-bloc. The reference to the English-language publication reads as follows: H. Moore and D. Markovin, Proceedings of Amer.Soc.for Testing Mater., v.42-44, 1942-1944.

Card 3/4

On the influence of absolute ...

29339

S/122/61/000/010/005/011
D221/D301

Fig. 1. Curves giving the relationship between K' and $\lg N$:

Legend: 1 - Specimens of steel 45 with diameter of 7.52 mm; 2 - specimens of steel 45 with diameter of 30 mm; 3 - specimens of steel 30KhN3A with diameter of 7.52 mm; 4 - specimens of steel 30KhN3A with a diameter of 27 mm.

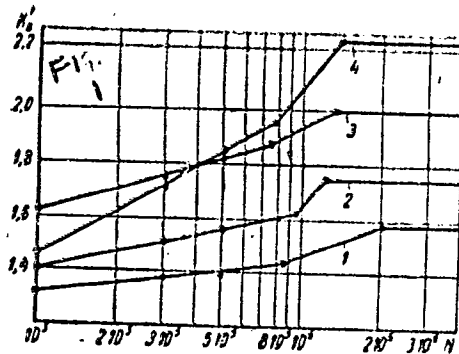


Рис. 1. Кривые зависимости K' от $\lg N$:

1 - образцы из стали 45 диаметром 7,52 мм; 2 - образцы из стали 45 диаметром 30 мм; 3 - образцы из стали 30ХН3А диаметром 7,52 мм; 4 - образцы из стали 30ХН3А диаметром 27 мм.

Card 4/4

S/032/61/027/001/022/037
B017/B054

AUTHORS: Oleynik, N. V. and Si'vanskiy, N. A.

TITLE: Reconstruction of the HY(NU) Machine for Program Tests

PERIODICAL: Zavodskaya laboratorija, 1961, Vol. 27, No. 1, pp. 84-85

TEXT: By reconstruction of the HY(NU) machine it is possible to determine the balance of stress in samples during testing. Figures show longitudinal and cross sections of the machine, and the load mechanism is schematically represented. The mechanism is simple, and can be successfully used for investigations taking account of the conditions $\sigma_2 - \sigma_1 = \text{const.}$ and

$\sigma_3 - \sigma_2 = - \text{const.}$, as well as for investigations under the conditions

$\sigma_2 / \sigma_1 = \text{const.}$ and $\sigma_3 / \sigma_2 = \text{const.}$ There are 4 figures.

ASSOCIATION: Odesskiy polytekhnicheskiy institut (Odessa Polytechnic Institute)

Card 1/1

OLEYNIK, N.V.

*277
A*

Sovetskoye po unobrazovaniyu. 1951, Moscow, 31 p.

Tolklichebnye poshchobki k stat'kam po unobrazovaniyu, 1951 - 1952. Sbornik: "Sovetskoye po unobrazovaniyu". Moscow, 1951, 1952. 353 p. Poshchobki k stat'kam po unobrazovaniyu.

Resp. Ed.: I. A. Gerasimov. Release of the USSR: A. M. Gerasimov. Tech. Ed.: A. P. Gerasimov.

NOTE: This collection of research work is available.

COMMENTS: The collection is at the Institute of Education at the Institute of Education with the network of...

Card 1/3

Cyclic Metal Strength (Cont.)

SOV/6025

and growth of fatigue cracks, the role of plastic deformation in fatigue fracture, an accelerated method of determining fatigue strength, the plotting of fatigue diagrams, and various fatigue test methods. New data are presented on the sensitivity of high-strength steel to stress concentration, the effect of stress concentration on the criterion of fatigue failure, the effect of the size factor on the strength of metal under cyclic loads, and results of endurance tests of various machine parts. Problems connected with cyclic metal toughness, internal friction, and the effect of corrosion media and temperature on the fatigue strength of metals are also discussed. No personalities are mentioned. Each article is accompanied by references, mostly Soviet.

TABLE OF CONTENTS:

NATURE OF FATIGUE FRACTURE

Oding, I. A. Diffusionless Mechanism of Formation and Growth of a Fatigue Crack
Card 2/7

3

Cyclic Metal Strength (Cont.)

SOV/6025

EFFECT OF THE STRESS CONCENTRATION
AND THE SIZE FACTOR ON THE FATIGUE
STRENGTH

Oding, I. A., and S. Ye. Gurevich. Notch Sensitivity of
High-Strength Steels Under Cyclic Load

169

Oleynik, N. V., and I. S. Mezentsev. Effect of Stress
Concentration on Characteristics of the Summation of
Fatigue Damage

177

Glikman, L. A., and Ye. N. Kostrov. Effect of the Size
Factor on Resistance of Metals to Corrosion Fatigue

187

Markovets, M. P. Technological Theory of the Size Factor
in Fatigue Tests

199

CYCLIC TOUGHNESS AND INTERNAL
FRICTION

Postnikov, V. S. Internal Friction and Strength
Card 6/9

207

14964

S/124/63/000/001/065/080
D234/D308

18 8200

AUTHORS: Oleynik, N.V. and Mezentsev, I.S.

TITLE: Effect of stress concentration on the summation characteristic of fatigue damage

PERIODICAL: Referativnyy zhurnal, Mekhanika, no. 1, 1963, 74, abstract 1V575 (In collection: Tsiklich. prochnost' metallov. M., AN SSSR, 1962, 177-186)

TEXT: In repeated single-step overloadings with sign-changing bending with rotation of specimens made of 40X (40Kh) steel, smooth, with shallow ring-shaped groove, with three such grooves, with a transverse hole, with deep ring-shaped groove, with three such grooves and with a deep and an unloading groove, the authors studied the effect of stress concentration on the inclination of secondary fatigue curves of special kind, and eventually on the magnitude of accumulated cycle ratio a , determined from

Card 1/3

Effect of stress concentration ...

S/124/63/000/001/065/080
D234/D308

$$a = \frac{L_2 \left(\frac{\sigma_2}{\sigma_1}\right)^q \left[\beta + \left(\frac{\sigma_2}{\sigma_1}\right)^m \right]}{N_2 \left(\frac{\sigma_2}{\sigma_1}\right)^m (1+\beta)}$$

where m and q are cotangents of inclination angles (inclination indices) of the primary and secondary fatigue curves, σ_1 and σ_2 are levels of two-step loading, $\sigma_1 > \sigma_2$; $\beta = n_1/n_2$ is the programming factor, n_1 and n_2 are the total numbers of cycles corresponding to σ_1 and σ_2 respectively, N_2 and L_2 are normal durabilities on the σ_2 level for the primary and secondary fatigue curve respectively. With increase of the effective coefficients of stress concentration determined with respect to the bases of limited (10^5 , 3×10^5 and 7×10^5) and long-term (5×10^9) durability, the accumulated cycle ratio decreases from approx. 1.8 to 0.97. The decrease of a is the sharper the higher the overload level. Stress

Card 2/3

Effect of stress concentration ...

S/124/63/000/001/065/080
D234/D308

concentration decreases a in comparison with smooth specimens. With large overloads and strong stress concentrations a becomes smaller than 1.

[Abstracter's note: Complete translation]

Card 3/3

OLEYNIK, N.V., kand.tekhn.nauk, dotsent; PETERGERYA, D.M., inzh.

Role of the scale effect in overload testing of metal. Vest.-
mashinostr. 42 no.6:20-23 Je '62. (MIRA 1510)
(Steel--Testing)

OLEYNIK, N.V., kand.tekhn.nauk, dotsent

Role of stress concentration in case of programmed loading.
Vest.mashinostr. 45 no.2:13-16 F '65.

(MIRA 18:4)

ZIL'BERSHTEYN, I.M.; OLEYNIK, N.Ye. (Irkutsk)

Case of skin lesions caused by the influence of chromium in
the printing industry. Gig.tzuda i prof.zab. no.11:54-55 '61.
(MIRA 14:11)

1. Irkutskiy meditsinskiy institut, Irkutskiy oblastnoy kozhno-
venereologicheskiy dispanser.

(PRINTERS--DISEASES AND HYGIENE) (CHROMIUM--TOXICOLOGY)

OLEYNIK, O.A.

G. Fichera's problem. Dokl. AN SSSR 157 no.6:1297-1300
Ag '64. (MIRA 17:9)

1. Moskovskiy gosudarstvennyy universitet im. Lomonosova.
Predstavleno akademikom I.G. Petrovskim.

OLEYNIK, O. A.

Moscow, 1947

Moscow State Univ im Lomonosov, 1947

"On Dirichlet's problem for equations of elliptical type," Matemat Sbor, 24, No. 1, 1949
(Rec'd 29 Apr 47)

OLZYNIK, O. A.

Cand. Physicomath Sci.

Dissertation: "Concerning the Topology of Algebraic Curves on an Algebraic Surface."

1/3/50

Sci. Res. Inst. of Mathematics, Moscow Order of Lenin State U. imeni.

M. V. Lomonosov

SO Vecheryaya Moskva
Sum 71

OLEYNIK, O. A.

"Algebraic Curves on an Algebraic Surface," Usp. Mat. Nauk Vol. 6 No. 4 (144),
pp 193-220, 1951.

U-1635, 16 Jan 52

USSR/Mathematics - Mathematical Societies Nov/Dec 51

"Sessions (11 and 16 September 1951) of the Moscow Mathematical Society"

"Uspekhi Matemat Nauk" Vol VI, No 6 (46), pp. 155-157

P. S. Aleksandrov, Pres of the Society, noted that 14 Sep 51 was the 60th birthday of I. M. Vinogradov, the great mathematician, and urged the members to write to him. Ye. B. Dynkin reported on "Semisimple Subgroups of Simple Groups of Lie." O. A. Oleynik, "Second Boundary-Value Problem for the Elliptic

196T77

USSR/Mathematics - Mathematical Societies (Contd 1) Nov/Dec 51

Type Equation With Small Parameters in Its Either Derivatives." I. M. Vinogradov was chosen as honorary member of the Society. Vice-Pres A. G. Kurosh read the note of the absent Pres Aleksandrov urging all members to undersign the

was peace Council for Conclusion of the Peace Pact. I. S. Gradshteyn gave his report "Application of the Theory of Stability by Liapunoff to the Theory of Differential Equations With Small Multipliers in the Derivatives" [extensive abstract is given]. V. A. Rokhlin, "Homotopical Classification of Continuous Reflections of a (n+3)-Dimensional Sphere onto a n-Dimensional Sphere" [contents of this lecture published in "Dok Ak Nauk SSSR" Vol LXXX, No 4, 1951, and Vol LXXXI, No 1 1951.]

196T77

PA 196T77

OLEYNIK, O. A.

OLEYNIK, O. A.

USSR/Mathematics - Small Parameter

11 Aug 51

"Second Boundary-Value Problem for an Elliptic-Type Equation With Small Parameter in the Highest Derivatives," O. A. Oleynik

"Dok Ak Nauk SSSR" Vol LXXIX, No 5, pp 735-737

Considers the soln $U_\epsilon(x,y)$ of the eq $L_\epsilon(U) = f(x,y)$ satisfying the condition $dU/dn = F(P)$ on the boundary Γ of the region G . Investigates the behavior of U_ϵ when the small parameter ϵ (greater than zero) tends to zero. Submitted by Acad I. G. Pefrovskiy 19 Jun 51.

210157

1. CLEYNIK, O. A.
2. USSR (600)
4. Differential Equations
7. "Differential equations in mathematical physics." V. I. Levin, YU. I. Grosberg.
Reviewed by O. A. CleyNIK, Sov. kniga no. 11 1952.

9. Monthly List of Russian Accessions, Library of Congress, February 1953. Unclassified.

OLBYNIK, O.A.

One class of discontinuous solutions to a quasi-linear equation of the first order. Nauch.dokl.vys.shkoly; fiz.-mat.nauki no.3:91-98 '58. (MIRA 12:7)

1. Moskovskiy gosudarstvennyy universitet im. M.V. Lomonosova. (Differential equations Partial)

May/June 52

USSR/Mathematics - Boundary-Value Problems

"Properties of the Solutions of Certain Boundary-Value Problems for Equations of the Elliptic Type,"

O. A. Oleynik, Moscow

"Matemat Sbor" Vol XXX (72), No 3, pp 695-702

Demonstrates the uniqueness and continuous dependence of the solns of certain boundary-value problems for eqs of the elliptic type upon the coeffs of the eq, its right part and boundary

217T62

function. Considers differential eqs of the form:
 $L(u) = \sum_{i,k} a_{ik} \cdot u_{x_i x_k} + \sum b_i u_{x_i} + cu = f$ (summed over repeated indices), where a, b, c and f are functions of the n independent variables x .
Submitted 18 Jan 52.

OLEYNIK, O.A., MOSCOW

217T62

USSR/Mathematics - Small Parameter, Jul/Aug 52
Elliptic Type

"Elliptic-Type Equations With Small Parameter in the Highest Derivatives," O.A. Oleynik, Moscow, Math Inst imeni Steklov, Acad Sci USSR

"Matemat Sbor" Vol XXXI (73), No 1, pp 104-117

Studies the behavior of the solns of the regular elliptic differential eq $\epsilon(u_{xx} + u_{yy}) + Au_x + Bu_y + Cu = f(x,y)$, with the condition $u_n + a(P)u = F(P)$ (u_n is the deriv in direction of normal n ; P is a

220177

point of boundary S), in the case where small parameter ϵ tends to zero. The behavior of solns of the Dirichlet problem for this eq in small parameter ϵ has been investigated by N. Levinson (Annals of Math, 51, No 2, 1950, 428-445). Submitted 1 Feb 52.

220177

OLEYNIK, O. A.

OLEYNIK, O. A.

PA 237T92

USSR/Mathematics - Small Parameter

Nov/Dec 52

"Boundary-Value Problem for the Equation $ey'' = F(x,y,y')$ for Small e ," O. A. Oleynik, Moscow, and A. I. Zhizhina, Moscow

"Matemat Sbor" Vol 31 (73), No 3, pp 701-717

Study the behavior of the solution $y_e(x)$ of subject eq as the small parameter e tends to 0. Rewrite subject eq in the form: $ey'' + A(x,y)y' = f(x,y,y')$, along with appropriate boundary conditions. The simpler case where $f(x,y,y')$ equals $B(x,y)$ was considered by Coddington and Levinson ("A boundary value problem for a nonlinear differential eq with a small parameter," Bull Amer Math Soc, 58, No 1 (1952)).

237T92

OLEYNIK, O. A.

235T73

USSR/Mathematics - Small-Parameter Problem 21 Jul 52

"Boundary-Value Problems for Equations With a Small Parameter in the Highest Derivatives,"
O. A. Oleynik

"Dok Ak Nauk SSSR" Vol 85, No 3, pp 493-495

Studies the behavior of the solns of boundary-value problems of the elliptic and parabolic type for the case where there is a small parameter epsilon in the highest derivs: $\epsilon(u_{xx} + u_y) + A(x,y)u_x + B(x,y)u_y + C(x,y)u = f(x,y)$, with condition $u_n + au = F$. Submitted by Acad I. G. Petrovskiy 3 May 52.

235T73

OLEYNIK, O. A.

PA 240T87

USSR/Mathematics - Elliptic Type 11 Dec 52

"Equations of the Elliptic Type Which Degenerate on the Boundary of the Region," O. A. Oleynik, Math Inst imeni Steklov, Acad Sci USSR

"DAN SSSR" Vol 87, No 6, pp 885-888

Studies the problem of the existence and uniqueness of the soln $u(x,y)$ bounded in region D of the following eq $L(u) \equiv u_{xx} + y^m u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = 0$ satisfying on the boundary the condition $u_g \cdot Au = f$, where derivative u_g is in direction g at an acute angle to normal n of the boundary. Presented by Acad I. G. Petrovskiy 24 Oct 52.

240T87

OLEYNIK, Olga A.

PHASE I

TREASURY ISLAND BIBLIOGRAPHICAL REPORT

AID 425 - I

BOOK

Call No.: 1P689576

Author: PETROVSKI, I. P.

Full Title: LECTURES ON PARTIAL DIFFERENTIAL EQUATIONS. 2nd rev. ed.

Transliterated Title: Lektsii ob uravneniyakh s chastnyimi proizvedennymi

Publishing Data

Originating Agency: None

Publishing House: State Publishing House of Technical and Theoretical Literature

Date: 1953

No. pp.: 360

No. of copies: 10,000

Editorial Staff

Contributors to this 2nd edition: Oleynik, Olga A. wrote a new four sections (23, 28, 42 and 43) and added exercises; Acad. SMIRNOV, V. I., Michkiss, A. D., Ladyzhenskaya, O. A. and Chudov, L. A. gave valuable comments.

Text Data

Coverage: This is a course of lectures given for several years by the author to students in mathematics of the Department of Mechanics and Mathematics of the Moscow State University. The book was awarded the Stalin Prize of second class for 1951. The text is well covered by the table of contents.

1/4

OLEYNIK, O. A.

"Concerning Equations with Partial Derivatives which Contain a Small Parameter with Antecedent Derivatives," report given at the All-University Scientific Conference "Lomonosov Lectures", Mosk. Un. Vest., No.8, 1953.

Translation U-7895, 1 Mar 56

OLENIK, O. A.

"Boundary-Value Problems for Partial Differential Equations
With Small Parameters in the Largest Derivatives, and the Cauchy
Problem for Nonlinear Equations in the Large." Dr Phys-Math Sci,
Moscow Order of Lenin State U imeni M. V. Lomonosov, 19 Nov 54.
(VM, 9 Nov 54)

Survey of Scientific and Technical Dissertations Defended at USSR Higher
Educational Institutions (11)

SO: Sum. No. 521, 2 Jun 55

OLEYNIK, G.A.

[Cauchy's problem for non-linear differential equations with
discontinuous initial conditions] Zadacha koshi dlia neli-
nainykh differentsial'nykh uravnenii s razryvnyimi nachal'nymi
usloviami. Moskva, Izd-vo Akademii nauk SSSR, 1954. 19 p.
(Differential equations) (ALRA 8:11)

OLEYNIK, O. A.

"Elliptic-Type Equations Degenerating on the Boundary of the Region,"
Uspekhi Matematicheskikh Nauk, Vol 8, No 2 (54), pp 159-167.

OLEYNIK, O. A.

"On the Cauchy Problem for Nonlinear Equations in a Class of Discontinuous Functions," Dokl. AN SSSR, 95, pp. 451-454, 1954

The author is one of the outstanding mathematicians of the USSR in the field of partial differential equations. It is claimed that she is the only woman in the USSR ever to have received the degree of Dr. of Physico-Mathematical Sciences, and the fact that she did so at the age of 29 is considered extraordinary.

Translation of paper by Rand V 6929, in Library

OLEYNIK, O.A.; VENTSEL', T.D.

Cauchy problem and the first boundary value problem for a quasilinear equation of the parabolic type. Dokl. Akad. Nauk SSSR 97 no.4:605-608 (MIRA 7:9)
Ag '54.

1. Predstavleno akademikom I.G.Petrovskim.
(Differential equations, Partial)

OLEYNIK, O.A.

Stability of the Neumann problem. Usp. mat. nauk 11 no.1:223-225
Ja-F '56. (MIRA 9:6)
(Differential equations, Partial)

OLEYNIK, O.A.

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/2 PG - 496
 AUTHOR OLEJNIK O.A.
 TITLE On discontinuous solutions of non-linear differential equations.
 PERIODICAL Doklady Akad.Nauk 109, 1098-1' 01 (1956)
 reviewed 1/1957

A bounded measurable function $u(t,x)$ is denoted as a solution of the Cauchy problem

$$(1) \quad \frac{\partial u}{\partial t} + \frac{\partial \varphi(t,x,u)}{\partial x} = 0$$

$$u(0,x) = u_0(x), \quad t \geq 0 - \text{boundedly measurable,}$$

if

1) for every continuously differentiable function $f(t,x)$ which vanishes outside of a finite region, the relation holds:

$$\int_{\geq 0} \int \left[\frac{\partial f}{\partial t} u(t,x) + \frac{\partial f}{\partial x} \varphi(t,x,u(t,x)) \right] dx dt + \int_{-\infty}^{+\infty} f(0,x) u_0(x) dx = 0,$$

where the first integral is extended over the halfplane $t \geq 0$;

2) for every finite domain D there exists a monotonely decreasing function $K(t)$, ($t > 0$), where for two arbitrary points (t,x_1) and (t,x_2) of D for $t > 0$ the inequation

Doklady Akad.Nauk 109, 1098-1101 (1956)

CARD 2/2

PG - 496

$$\frac{u(t, x_1) - u(t, x_2)}{x_1 - x_2} \leq K(t)$$

is valid.

The author proves that the solution defined in this way exists for (1) and is unique. For the function φ here it is assumed that it is defined for $t, x, u \geq 0$, that it has continuous derivatives of second order, where $\varphi''_{uu} \geq 0$ and that $\varphi'_u(t, x, u)$ for all x and $t \geq 0$ is bounded if u changes in a finite interval.

INSTITUTION: Mathematical Institute of the Academy of Sciences.

Oleynik, O. A.

25-7-6/51

AUTHOR: Oleynik, O.A., Doctor of Physico-Mathematical Sciences, Professor
at the Moscow University

TITLE: Let Us Be Friends, Say the Youth of All Continents (Budem
druzhit', govorit mlodezh' vsekh kontinentov)

PERIODICAL: Nauka i Zhizn', 1957, # 7, p 4 (USSR)

ABSTRACT: Young students from many foreign countries attend schools in
the Soviet capital. When their studies are completed they re-
turn to their native countries, but they do not forget their
former friends and teachers. When taking part in conferences
and meetings abroad, the author noticed that representatives
from all corners of the world got along fine. They love their
own countries and at the same time show sympathy toward the
others. To everyone, peace is dear. It is also a necessity for
the development and growth of science.
The article contains one photo.

AVAILABLE: Library of Congress

Card 1/1

OLEYNIK, O. A.

OLEYNIK, O.A.

Discontinuous solutions of nonlinear differential equations. Usp.
mat.nauk 12 no.3:3-73 My-Je '57. (MIRA 10:10)
(Differential equations)

Oleynik, O.A.

42-6-11/17

AUTHOR: OLEYNIK, O.A.

TITLE: ~~On the Uniqueness of the Generalized Solution of the Cauchy Problem for a Nonlinear System of Equations Appearing in the Mechanics (O yedinstvennosti obobshchennogo resheniya zadachi Koshi dlya odnoy nelineynoy sistemy uravneniy, vstrechayushcheyasya v mekhanike)~~

PERIODICAL: Uspekhi Matematicheskikh Nauk, 1957, Vol. 12, No. 6, pp. 169-176 (USSR)

ABSTRACT: The same method which the author has already used in several papers [Ref. 3, 4] for the consideration of equations of first order, here is used for the investigation of the equation

$$\frac{\partial u}{\partial t} + \frac{\partial \varphi(t, x, v)}{\partial x} = 0, \quad \frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = c,$$

where $\varphi'_v < 0$ in all arguments shall have continuous derivatives of second order and $\varphi''_{vv} > 0$. In the class of piecewise continuous functions which besides satisfy the condition $u(t, x+0) \leq u(t, x-0)$, and in the class of bounded measurable functions (with the same condition) the author proves the uniqueness of the solution of the Cauchy problem.

Card 1/2

On the Uniqueness of the Generalized Solution of the Cauchy
Problem for a Nonlinear System of Equations Appearing in the
Mechanics

42-6-11/17

Five Soviet references are quoted.

SUBMITTED: November 2, 1956
AVAILABLE: Library of Congress

Card 2/2

06/15/77 2 11

SUBJECT USSR/MATHEMATICS/Differential equations CARD 1/1 PG - 734
AUTHOR OLEJNIK O.A., VENTZELJ T.D.
TITLE The first boundary value problem and the Cauchy problem for
quasilinear equations of parabolic type.
PERIODICAL Mat.Sbornik, n.Ser. 41, 105-128 (1957)
reviewed 5/1957

The present paper contains the proofs, the elaboration and some generalizations
of the author's announcements in Doklady Akad.Nauk 97, 605-608 (1954).

INSTITUTION: Moscow.

AUTHOR
TITLE

OLEYNIK O.A., VVEDENSKAYA N.D.

PA - 3126

The Solution of the Cauchy Problem and the Boundary Value Problem For the Nonlinear Equations In A Class of Unsteady Functions.

(Resheniye zadachi Koshi i krayevoy zadachi dlya nelineynykh uravneniy v

klasse razryvnykh funktsiy -Russian)

Doklady Akademii Nauk SSSR, 1957, Vol 113, Nr 3, pp 503-506 (U.S.S.R.)

Reviewed 7/1957

PERIODICAL

Received 6/1957

ABSTRACT

The present paper furnishes the correct formulation of the Cauchy Problem and the boundary value problem for the equation $\partial_t u + \partial_x(\varphi(t,x,u(t,x)))/\partial x + \psi(t,x,u(t,x)) = 0$ within a large domain with unsteady initial- and boundary condition. The general solution is determined here in accordance with the paper by O.A. OLEYNIK, Dokl. Akad. Nauk, Vol 109, Nr 6 (1956). This process is equivalent to the determination of the general solution by the introduction of a "vanishing viscosity", i.e. the boundary value (if the parameter ε tends toward zero), of the solutions of the corresponding problems is sought for the parabolic equation $\varepsilon \partial_x^2 u / \partial x^2 = \partial u / \partial t + \partial \varphi(t,x,u) / \partial x + \psi(t,x,u)$.

1.) CAUCHY'S Problem: $\varphi(t,x,u)$ and $\psi(t,x,u)$ have steady derivations of second order. $\varphi_{uu} > 0$ is assumed to apply and, $u_0(x)$ is assumed to be a limited function measurable at all x . At first the generalized solution of CAUCHY'S problem is given for the equation written down above. This generalized solution exist and is unique. A further theorem is given and proved.

2.) The Boundary Value Problems: The authors examine the boundary problem for the equation given above with the conditions $v(0,x) = u_0(x)$, $u(t,0) = u_1(t)$,

Card 1/2

The Solution of the Cauchy Problem And the Boundary Value Problem For the Nonlinear Equations In A Class of Unsteady Functions. PA - 3126

$u(t,1) = u_1(t)$ in the rectangle $R \{0 \leq t \leq T, 0 \leq x \leq 1\}$.
Next, the conditions are given for the case that the limited measurable function $u(t,x)$ is a generalized solution of the boundary problem to be solved. This solution is unique for certain classes of functions given here. In conclusion two theorems are given and proved.
(No illustrations)

ASSOCIATION Mathematical Institute V.A. STEKLOV of the Academy of science of the USSR
PRESENTED BY I.G. PETROV, Member of the Academy
SUBMITTED 16.10.1956
AVAILABLE Library of Congress
Card 2/2

20-6-6/59

AUTHOR
TITLE

OLEYNIK, O.A.

On the Equations of the Type of the Equations of the In-
stationary Filtration. (Ob uravneniyakh tipa uravneniy
nestatsionarnoy fil'tratsii.- Russian)

PERIODICAL

Doklady Akademii Nauk SSSR 1957, Vol 111, Nr 6, pp 1210-1213
(USSR)

ABSTRACT

The equations of the instationary filtration of a liquid and of a gas, the equations of the thermal diffusion at heat conduction which depends on the temperature, and the equations of the boundary layer have already been investigated by several authors. These equations are nonlinear equations of the parabolic type which degenerate at certain values of the sought solution. The author investigates in the paper under review the Cauchy problem for the equation $(\partial u / \partial t) = \partial^2 \varphi(t, x, u) / \partial x^2$ with the initial condition

$u|_{t=0} = u_0(x)$. In this context, $u_0(x)$ is limited at all x ,

and we have $u_0(x) \gg 0$. We furthermore have $\varphi'_u \gg 0$ at $u > 0$,

and $\varphi'_u > 0$ at $u > 0$. If we have $\varphi = u^2$, then the above equation is the equation of filtration. The methods employed

CARD 1/2

20-6-6/59

On the Equations of the Type of the Equations of the In-
stationary Filtration.

by the paper under review for the solution of the Cauchy problem can also be utilized for the investigation of the boundary value problems and of the equations of this type with an arbitrary number of independent variables (in particular, of the equations of the instationary filtration in a nonuniform medium). The author of the present paper first of all gives a definition of the generalized solution of the Cauchy problem for the equation $(\partial u / \partial t) = \partial^2 \varphi(t, x, u) / \partial x^2$ with the condition $u|_{t=0} = u(x)$, and then he proves the existence and the unambiguosness of such a solution. Then the following is demonstrated: The generalized solution $u(t, x)$ has continuous derivations which enter the above equation, and it satisfies this equation in all points where we have $u(t, x) \neq 0$. These proofs of unambiguosness and of existence are discussed step by step in great detail.

ASSOCIATION: Moscow State University
PRESENTED BY: I.G. PETROVSKIY, Member of the Academy, 10.11. 1956
SUBMITTED: 10.11.1956
AVAILABLE: Library of Congress.

CARD 2/2

OLEYNIK, O. A.

"On Cauchy's Problem for Quasi-Linear Hyperbolic Equations."

paper submitted at International Congress Mathematicians, Edinburgh, 14 - 21 Aug
58.

O. A. OLEJNIK

16(1)
AUTHORS:

Shorov, I.A., University Lecturer, and Kopylov, V.D., Scientific Assistant

TITLE:

Lomonosov - Lectures 1957 at the Mechanical-Mathematical Faculty of Moscow State University (Doklady Akad. Nauk SSSR, 1958, No. 4, pp. 241-246 (USSR))

PERIODICAL:

Vestnik Moskovskogo Universiteta, Seriya Matematika, Mekhanika, Astronomiya, Fizika, Khimiya, 1958, No. 4, pp. 241-246 (USSR)

ABSTRACT:

The Lomonosov lectures 1957 took place from October 17 - October 21, 1957 and were dedicated to the 40-th anniversary of the October revolution. In the general meeting A.N. Kolmogorov, Academician spoke "On Approximative Representation of Functions of Several Variables by Superposition of Functions with Less Variables and ϵ -Entropy of Classes of Functions". The lecture generalizes the results of Kolmogorov, A.G. Vitushkin, V.I. Arnold and V.M. Tikhonov. The contents has been already published (Doklady Akademi nauk SSSR, 194,5). Professor S.A. Akhmatullin, Member of the Academy of Sciences of the U.S.S.R., spoke on "Investigation of the Boundary Layer of the Motion of a Two-Component Liquid".

The other lectures were given separately in the sections Mechanics and Mathematical Geology. The titles of the lectures are given below.

8. A.G. Kulikovskiy, Aspirant, Shock against a Flexible Thread.

9. A.G. Kulikovskiy, Aspirant, Flow around Magnetized Bodies by Conducting Liquid.

10. N.V. Yarsneyev, Lecturer, Instruments for the Analysis and Synthesis of Mechanisms.

11. V.S. Lanakly, Lecturer, Some General Laws in the Behavior of Multiply Loaded Metals.

12. Y.B. Elyashnikov, Aspirant, A Variant of the Theory of the Increase of Deformation and Elasto-Plastic Stability.

13. Professor M.I. Meshik and Professor L.A. Lyubimov, Asymptotic Behavior of the Solutions of Linear Equations with Small Parameter in the Derivatives.

14. Professor V.M. Shukh-Bura and P.S. Trifonov, Senior Scientific Assistant, Automation and Programming.

Card 5/5

16(1)

SOV/155-58-3-18/37

AUTHOR:

Oloynik, O.A.

TITLE:

On a Class of Discontinuous Solutions of the Cauchy Problem of the First Order (Ob odnom klasse razryvnykh resheniy nezavisimogo uravneniya pervogo porjadka)

PERIODICAL: Nauchnyye doklady vysshey shkoly. Fiziko-matematicheskiye nauki, 1958, Nr 3, pp 91-98 (USSR)

ABSTRACT: The author considers the equation

$$\frac{\partial u}{\partial t} + \frac{\partial \varphi(u, t, x)}{\partial x} = 0,$$

where φ is an arbitrary, in general not convex function. He gives a class of discontinuous solutions in which the Cauchy problem is solvable uniquely and the solution is stable for a change of the initial function. Solutions for piecewise continuous or piecewise smooth initial functions are constructed. Along the characteristics these solutions can be discontinuous and may have properties not possible in the case $\varphi_{u_1} \neq 0$. For the equation

Card 1/2

On a Class of Discontinuous Solutions of a Quasilinear SOV/155-58-3-18/37
Equation of the First Order

$$\varepsilon \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} + \frac{\partial \varphi(u, t, x)}{\partial x}$$

the Cauchy problem with the initial condition for $t = 0$ is considered. It is shown that the solutions converge weakly for $\varepsilon \rightarrow 0$. The author mentions I.M.Gel'fand, V.F.I'yachenko, and N.D.Vvedenskaya.

There is 1 Soviet reference.

ASSOCIATION: Moskovskiy gosudarstvennyy universitet imeni M.V.Lomonosova
(Moscow State University imeni M.V.Lomonosov)

SUBMITTED: April 28, 1958

Card 2/2

OLEYNIK, O.A.; KIRILLOV, A.A. (Moskva)

Twentieth mathematics contest in Moscow schools. Mat. pros.
no.3:221-227 '58. (MIRA 11:9)
(Moscow--Mathematics--Competitions)

OLEYNIK, O.A.; KALASHNIKOV, A.S.; CHZHOU YUY-LIN' [Chou Yu-lin].

Cauchy's problem and boundary problems for nonstationary
filtration equations. Izv.AN SSSR.Ser.mat. 22 no.5:667-704
3-D '58. (MIRA 11:12)

1. Predstavleno akademikom S.L.Sobolevym.
(Differential equations, Partial)

AUTHOR: Il'in, A.M. and Oleynik, O.A. SCV/20-120-1-5/6
 TITLE: On the Behavior of the Solutions of Cauchy's Problem for Some
 Quasi-Linear Equations for Unlimited Increase of the Time (O po-
 vedenii resheniy zadachi Koshi dlya nekotorykh kvazilineynykh
 uravneniy pri neogranichennom vozrastanii vreneni)
 PERIODICAL: Doklady Akademii nauk ^{SSSR}, 1958, Vol 120, Nr 1, pp 25-28 (USSR)

ABSTRACT: The authors investigate the solutions of Cauchy's problem for the
 equations

$$(1) \quad \frac{\partial u}{\partial t} + \frac{\partial \psi(u)}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2} \quad (2) \quad \frac{\partial u}{\partial t} + \frac{\partial \psi(u)}{\partial x} = 0, \quad \varepsilon > 0$$

under the initial conditions

$$(3) \quad u|_{t=0} = u_0(x), \quad -\infty < x < +\infty \quad \text{for } t \geq 0$$

I. The case (1),(3). Let $u_0(x)$ be bounded and measurable; $\psi(u)$
 is assumed to have continuous derivatives of fourth order,
 $\psi'' \geq \mu > 0$, $\lim_{x \rightarrow -\infty} u_0(x) = u_-$, $\lim_{x \rightarrow +\infty} u_0(x) = u_+$. Existence and
 uniqueness of the solutions were proved by Oleynik in [Ref 3].
 Theorem: Let be $u_- > u_+$ and assume that the integrals

Card 1/3

$$\int_{-\infty}^0 (u_0(x) - u_-) dx, \quad \int_0^{+\infty} (u_0(x) - u_+) dx$$

On the Behavior of the Solutions of Cauchy's Problem for SOV/20-120-1-5/63
Some Quasi-Linear Equations for Unlimited Increase of the Time

exist, their sum being equal to A. The equation (1) possesses only one solution $\tilde{u}_\xi(x-Kt)$ which only depends on $x-Kt$, where

$$K = \frac{\varphi(u_+) - \varphi(u_-)}{u_+ - u_-} \quad \text{and satisfies the condition}$$

$$\int_{-\infty}^0 (\tilde{u}_\xi(x) - u_-) dx + \int_0^{+\infty} (\tilde{u}_\xi(x) - u_+) dx = A$$

For $t \rightarrow \infty$ $|\tilde{u}_\xi(x-Kt) - u_\xi(t, x)|$ tends uniformly to zero with respect to x . If for certain constants $\alpha_1 > 0$ and $M_1 > 0$ the function $u_0(x)$ satisfies the additional conditions

$$\left| \int_{-\infty}^x (u_0(x) - u_-) dx \right| \leq M_1 e^{\alpha_1 x}, \quad \left| \int_x^{\infty} (u_0(x) - u_+) dx \right| \leq M_1 e^{-\alpha_1 x}$$

then it holds $|\tilde{u}_\xi(x-Kt) - u_\xi(t, x)| \leq M_2 e^{-\beta t}$ for all x and t , where $\beta > 0$ and $M_2 > 0$ are certain constants.

Theorem: Let be $u_+ = u_- = a$. $|u_\xi(t, x) - a|$ tends uniformly in x

Card 2/3

On the Behavior of the Solutions of Cauchy's Problem for SGV /20-120-1-5/63
Some Quasi-Linear Equations for Unlimited Increase of the Time

to zero for $t \rightarrow \infty$. If $|u_0(x) - a| \leq M_1 e^{-\lambda_1 |x|}$, then it holds

$|u_\varepsilon(t, x) - a| \leq M t^{1/2} |\ln t|^B$ for all x and $t \geq 0$.

Theorem: Let be $u_+ > u_-$. Let the function $H(s)$ be defined by:
 $s = \varphi'(H(s))$ for $\varphi'(u_-) \leq s \leq \varphi'(u_+)$, $H(s) = u_-$ for $s \leq \varphi'(u_-)$
and $H(s) = u_+$ for $s \geq \varphi'(u_+)$. Then the solution $u_\varepsilon(t, x)$ of
(1), (3) in x and ε uniformly tends to $H(\frac{x}{t})$ for $t \rightarrow \infty$.

II. The case (2), (3). Analogous theorems as in the case (1), (3)
are proved; e.g. Theorem: Let $u_+ = u_- = a$; $u(t, x)$ be the
solution of (2), (3). $|u(t, x) - a|$ tends uniformly to zero in x
for $t \rightarrow \infty$. There are 5 references, 4 of which are Soviet, and
1 American.

PRESENTED: January 10, 1958, by I.G. Petrovskiy, Academician

SUBMITTED: January 10, 1958

Card 3/3 1. Integral equations 2. Linear equations 3. Functions 4. Time
--Applications

OLEYNIK, O.A.

16(0) PRAISE I BOOK EXPLOITATION SOV/3177

Matematika v SSSR za posled 10 let, 1917-1957. tom 1: Obzor'nye stat'i (Mathematics in the USSR for forty years, 1917-1957). Vol. 1: Review Articles) Moscow, Fizmatgiz, 1959. 1002 p. 5,500 copies printed.

Eds: A. G. Kurosh, (Chief Ed.), V. I. Ityutskov, V. G. Bol'yanskii, Ye. B. Dynkin, G. Ye. Shilovs, and A. P. Yuzhnevich; Ed. (Inside book): A. P. Lapko; Tech. Ed.: S. N. Akhizov.

PURPOSE: This book is intended for mathematicians and historians of mathematics interested in Soviet contributions to the field.

COVERAGE: This book is Volume I of a major 2-volume work on the history of Soviet mathematics. Volume I surveys the chief contributions and Soviet mathematicians during the period 1917-1957; Volume II will contain a bibliography of major works since 1917 and biographic sketches of some of the leading mathematicians. This work follows the same of the leading mathematicians: Matematika v SSSR za poslednie 10 let (Mathematics in the USSR for 10 Years) and Matematika v SSSR za triadtsat' let (Mathematics in the USSR for 30 Years). The book is divided into the major divisions of the field: algebra, topology, theory of probabilities, functional analysis, groups, topology, and combinatorics and outstanding problems in each discipline, and containing of some 1100 Soviet mathematicians is included with references to their contributions in the field.

Vishnik: M. Y. A. D. Kuzhik, and O. A. Oleynik, Partial Differential Equations 563

Ch. I. Elliptic-type Equations 566

1. Classical equations of mathematical physics 566
2. Linear elliptic equations of the second order 572
3. Elliptic equations of the plane 572
4. Solution of boundary value problems by means of integral equations 580
5. Vessing theorems 582
6. Variational methods of solving boundary value problems 586
7. Max-min methods for elliptic type equations 589
8. Maximum-minimum methods for elliptic type equations 594
9. Nonlinear elliptic-type equations 597
10. Degenerate cases 599

Ch. II. Hyperbolic and Parabolic-type Equations 604

1. Classical equations of mathematical physics 604
2. Cauchy's problem for linear equations 605
3. Mixed boundary value problems for linear equations 613
4. Variation methods for nonstationary equations 619
5. Degenerate cases 622
6. Nonstationary cases 622
7. Obstacles to classical types. Various studies 629

Ch. III. Other Problems 631

Lyubimik, L. A. Variational Calculus

1. Introduction 637
2. One-dimensional problems 638
3. Multi-dimensional problems 639
4. Variational theory of general nonlinear operators 641
5. Topological methods of the theory of critical points 643
6. Variational calculus in the large and the topology of functional spaces 645
7. Variational methods of solving problems in physics and engineering 647

16(1)

AUTHOR: Oleynik, O.A.

SOV/4.1-14-2-3/19

TITLE: On the Construction of the Generalized Solution of the Cauchy Problem for a Quasilinear Equation of first Order by Introduction of a "Vanishing Tenacity"

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 1, pp 159-164 (USSR)

ABSTRACT: The present paper is a completion of the preceding paper of I.M. Gel'fand and contains the proof of the existence of the

generalized solution of $\frac{\partial u}{\partial t} + \frac{\partial \varphi(u, t, x)}{\partial x} = 0$, $u|_{t=0} = u_0(x)$. The generalized solution is understood as a limit value of the

corresponding Cauchy problem for $\varepsilon \frac{\partial^2 u_\varepsilon}{\partial x^2} = \frac{\partial u_\varepsilon}{\partial t} + \frac{\partial \varphi(u_\varepsilon, t, x)}{\partial x}$, $\varepsilon > 0$,

for $\varepsilon \rightarrow 0$. The same problem was already treated by the author [Ref 2, 3].

There are 6 Soviet references.

SUBMITTED: December 7, 1958

Card 1/1

16(1)

AUTHOR: Oleynik, O.A.

SOV/42-14-2-4/19

TITLE: On the Uniqueness and Stability of the Generalized Solution of the Cauchy Problem for a Quasilinear Equation

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 2, pp 165-170 (USSR)

ABSTRACT: This is a second completion of the preceding paper of Gel'fand.

The author investigates discontinuous solutions of $\frac{\partial u}{\partial t} + \frac{\partial \varphi(u, t, x)}{\partial x} = 0$, where φ in general has not to be convex. The author proves the existence and stability of these solutions, while the question of existence was already treated by A.S. Kalashnikov.

SUBMITTED: December 8, 1958

Card 1/1

13

16(1)

AUTHORS: Oleynik, O.A., and Sobolev, S.L.

SOV/42-14-2-14/19

TITLE: Partial Differential Equations at the International Congress in Edinburgh

PERIODICAL: Uspekhi matematicheskikh nauk, 1959, Vol 14, Nr 2, pp 247-250 (USSR)

ABSTRACT: This is a report on the deliveries and opinions of western mathematicians on the subject of "partial differential equations". Soviet deliveries are not mentioned. Incidentally the authors mention I.G.Petrovskiy, A.D.Myskhis, M.I.Vishik, Landis, and Pliss.

Card 1/1

16(1)

AUTHOR: Oleynik, O.A.

SOV/20-124-6-10/55

TITLE: The Solution of the Fundamental Boundary Value Problems for Equations of Second Order With Discontinuous Coefficients (Resheniye osnovnykh krayevykh zadach dlya uravneniy vtorogo poryadka s razryvnymi koeffitsiyentami)

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 124, Nr 6, pp 219-1222(USSR)

ABSTRACT: The solutions of the most important boundary value problems (generalized solution of the Dirichlet problem, classical solution of it, first boundary value problem, Cauchy's problem) for partial differential equations of elliptic and parabolic type with discontinuous coefficients are obtained as limit values of solutions of corresponding problems for equations with smooth coefficients. The statements of existence are the most simple ones. The method is also applicable to hyperbolic equations. Former results of S.N.Bernshteyn are essentially used.

Card 1/2

The Solution of the Fundamental Boundary Value Problems for Equations of Second Order With Discontinuous Coefficients SOV/20-124-6-10/55

There are 9 references, 7 of which are Soviet, 1 French, and 1 American.

ASSOCIATION: Matematicheskiy institut imeni V.A. Steklova AN SSSR
(Mathematical Institute imeni V.A. Steklov, AS USSR)

PRESENTED: November 5, 1958, by I.G. Petrovskiy, Academician

SUBMITTED: November 4, 1958

Card 2/2

OLEYNIK, O. A.

"On some problems in mechanics of continuous media."

report presented at the First All-Union Congress on Theoretical and Applied
Mechanics, Moscow, 27 Jan - 3 Feb 60

32448

S/044/61/000/010/1112/051
G111/C222

16.3500

AUTHORS: Il'in, A.M., and Oleynik, O.A.

TITLE: On the asymptotic behavior of the solution of the Cauchy problem for $t \rightarrow \infty$ for some quasilinear equation:

PERIODICAL: Referativnyy zhurnal. Matematika, no. 10, 1961. 33-34, abstract 10 B 153. ("Tr. Vses. soveshchaniya po differentsial'n. uravneniyam, 1958". Yerevan, AN Arm SSR, 1960, 98-101)

TEXT: The authors consider the quasilinear equations

$$\frac{\partial u}{\partial t} + \frac{\partial \varphi(u)}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2}, \quad \varepsilon > 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{\partial \varphi(u)}{\partial x} = 0, \quad (2)$$

and the initial conditions

$$u|_{t=0} = u_0(x), \quad -\infty < x < \infty \quad (3)$$

Card 1/4

32448

S/044/61/000/010/012/051
C111/C222

On the asymptotic behavior of the ...

where $u_0(x)$ is a bounded measurable function, $\varphi(u)$ has continuous derivatives of fourth order, and

$$u_0(x) \rightarrow u_- \text{ for } x \rightarrow -\infty, u_0(x) \rightarrow u_+ \text{ for } x \rightarrow \infty. \quad (4)$$

Let $\varphi(u)$ be so that there exists a solution $\tilde{u}_\epsilon(x-kt)$ of (1) so that

$$\tilde{u}_\epsilon(\xi) \rightarrow u_- \text{ for } \xi \rightarrow -\infty, \tilde{u}_\epsilon(\xi) \rightarrow u_+ \text{ for } \xi \rightarrow +\infty.$$

Let exist the integrals

$$\int_{-\infty}^x [u_0(\xi) - u_-] d\xi \text{ and } \int_x^{\infty} [u_0(\xi) - u_+] d\xi.$$

Then there exists a solution $\tilde{u}_\epsilon(x-kt)$ of (1) so that

$$\int_{-\infty}^{\infty} [\tilde{u}(\xi) - u_0(\xi)] d\xi = 0 \quad (5)$$

Card 2/4

32448

On the asymptotic behavior of the ... S/044/61/000/010/012/051
C111/C222

and there holds the

Theorem 1 : If the initial function $u_0(x)$ satisfies the above mentioned postulates then, uniformly in x , the solution of the Cauchy problem for (1) with $t \rightarrow \infty$ tends to the solution $\tilde{u}_\varepsilon(x-kt)$ which is determined by (5). If for certain constants $\alpha > 0$ and $M_1 > 0$ it holds additionally

$$\left| \int_{-\infty}^x [u_0(\xi) - u_-] d\xi \leq M_1 e^{-\alpha x}, \right.$$

$$\left| \int_x^{\infty} [u_0(\xi) - u_+] ds \leq M_1 e^{-\alpha x}, \right.$$

then it holds

$$|\tilde{u}_\varepsilon(x-kt) - u_\varepsilon(t,x)| \leq M_2 e^{-Bt}$$

for all x and t , where $B > 0$, $M_2 > 0$ - const.

Card 3/4

3244B

On the asymptotic behavior of the ...

S/044/61/000/010/0 2/051
C111/C222

Let

$$H(\xi) = \begin{cases} u_- & \text{for } \xi \leq \psi'(u_-) \\ u_+ & \text{for } \xi \geq \psi'(u_+) \end{cases}$$

Let $u_\epsilon(t, x)$ be the solution of the problem (1), (3), (4) $u_- < u_+$, $\psi''(u) > 0$. Then for $t \rightarrow \infty$, $u_\epsilon(t, x)$ tends uniformly in x and t tends to $H(x/t)$. And finally: If $u_\epsilon(t, x)$ is the solution of (1), (3), $u(x, t)$ is the solution of (2), (3), if u_0 satisfies the conditions (4) and if $u_- = u_+ = a$, $\psi''(u) > 0$ then it holds uniformly in x $u_\epsilon(t, x) \rightarrow a$, $u(t, x) \rightarrow a$ for $t \rightarrow \infty$. Proofs are missing.

[Abstracter's note : Complete translation.]

Card 4/4

32450

S/044/61/000/010/0 4/051
C111/C222

16.3500

AUTHOR: Oleynik, O.A.

TITLE: The solution of basic boundary value problems for equations of second order with discontinuous coefficients

PERIODICAL: Referativnyy zhurnal. Matematika, no. 10, 1961, 18-39, abstract 10 B 170. ("Tr. Vses. soveshchaniya po differentsial'n. uravneniyam, 1958". Yerevan, AN Arm SSR, 1960, 113-114)

TEXT: The author considers the elliptic equation

$$\sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left(a_{ij}(x) \frac{\partial u}{\partial x_j} \right) = 0, \quad a_{ij} = a_{ji}, \quad (1)$$

$x = (x_1, \dots, x_n)$, $a_{ij}(x)$ -- sufficiently smooth everywhere in the region Ω with a probable exception of points of certain smooth $(n-1)$ -dimensional manifolds in which the a_{ij} may be discontinuous. These manifolds decompose Ω into a finite number of regions $\Omega_i, i = 1, \dots, m$.

Card 1/4

32450

S/044/61/000/010/011/051
C111/C222

The solution of basic boundary ...

Γ_{kl} denotes the boundary of the regions Ω_k and Ω_l . It is assumed that a_{ij} at both sides of Γ_{kl} has the boundary values a_{ij}^k and a_{ij}^l . The boundary S of Ω is smooth. The author seeks a function $u(x)$ (classical solution of the Dirichlet problem) which is continuous in $\Omega + S$ and which in all points of $\Omega - \{\Gamma_{kl}\}$ satisfies the equation (1) and the conditions

$$u|_S = f \tag{2}$$

$$a_k \frac{du}{dN_k} = a_l \frac{du}{dN_l} \text{ on } \Gamma_{kl} ; \tag{3}$$

here f is a function given on S , $a_j > 0$ const; du/dN_k , du/dN_l are derivatives in the directions of the conormals of Γ_{kl} . Besides, the generalized solution of the problem is considered, i.e. a $u(x) \in W_2^{(1)}(\Omega)$ which satisfies (2) and which is so that for every $F(x) \in W_2^{(1)}(\Omega)$ which

Card 2/4

32450

S/044/61/000/010/01./051
C111/C222

The solution of basic boundary ...

vanishes on S the relation

$$\iint_{\Omega} \sum_{i,j=1}^n a_{ij} \frac{\partial u}{\partial x_i} \frac{\partial F}{\partial x_j} d\Omega = 0,$$

is satisfied, where a is a function in Ω which in the points of Ω_j is equal to the constant a_j . It is proved that the solution of the Dirichlet problem (classical and generalized) is unique. Under the same assumptions for $a_0(x)$ and $a_{ij}(x)$ the author considers the first boundary value problem and the Cauchy problem for the parabolic equation

$$a_0(x) \frac{\partial u}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_j} \left(a_{ij}(x) \frac{\partial u}{\partial x_j} \right). \quad (4)$$

Likewise generalized solutions of the Cauchy problem and the mixed problem can be obtained for a hyperbolic equation with discontinuous coefficients; furthermore the solution of the first boundary value

Card 3/4

32450

The solution of basic boundary ...

S/044/61/000/010/01./051
C111/C222

problem and the Cauchy problem for equations (4) in the case where a_0 and χ
 a_{ij} depend on t and x and have discontinuities on surfaces of the space
 x being variable with the time (cf. R Zh Mat 1961, 7 B 205).

[Abstracter's note : Complete translation.]

Card 4/4

IL'IN, A.M.; OLEYNIK, O.A. (Moskva)

Asymptotic behavior of solutions of the Cauchy problem for
some quasilinear equations with large time values. Mat. sbor.
51 no.2:191-216 Je '60. (MIRA 13:9)
(Differential equations, Partial)

86821

S/020/60/135/005/006/043
C111/G22216.3.50⁹AUTHOR: Oleynik, O.A.

TITLE: A Method for Solving the Generalized Stephan's Problem

PERIODICAL: Doklady Akademii nauk SSSR, 1960, Vol.135, No.5, pp.1054-1057

TEXT: Let S_i ($i=1, \dots, N$) be given constants (values of the temperature u for which the state of aggregation of the medium changes). The points S_i ($S_i < S_{i+1}$) divide the u -axis into $N+1$ intervals. Let l_i be the i -th interval. A continuous function $u(t, x)$ defined in $Q \{0 \leq t \leq T, 0 \leq x \leq 1\}$ is called the solution of the Stephan's problem if 1) In the interior of Q , where $u(x, t) \in l_i$, $u(x, t)$ satisfies the equation

$$(1) \quad a_i(t, x) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + b_i(t, x) \frac{\partial u}{\partial x} + c_i(t, x)u + f_i(t, x), \quad i=1, \dots, N,$$

where a_i, b_i, c_i, f_i are smooth functions, $a_i \geq \bar{a} > 0$, $\bar{a} = \text{const}$. 2) On the curves $x = x_i(t)$, where $u(t, x) = S_i$, it holds

Card 1/5

86821

S/020/60/135/005'006/043
C111/C222

A Method for Solving the Generalized Stephan's Problem

$$(2) \quad k_i(t,x) \frac{\partial u}{\partial x} \Big|_{u=S_i-0} - k_{i+1}(t,x) \frac{\partial u}{\partial x} \Big|_{u=S_i+0} = \lambda_i(t,x) \frac{dx_i}{dt},$$

where $k_i(t,x) \geq \bar{k} > 0$, $\lambda_i(t,x) \geq 0$ are smooth functions given in Q , $i=1, \dots, N$.

3) It holds

$$(3) \quad u(0,x) = u_0(x), \quad u(t,0) = u_1(t), \quad u(t,1) = u_2(t),$$

where $u_0(x)$, $u_1(t)$, $u_2(t)$ are given. If $u(x,t)$ satisfies these conditions, then it is called the classical solution of the Stephan's problem. It is shown that the classical solution satisfies the identity

Card 2/5

86821

S/020/60/135/005/006/043
G111/G222

A Method for Solving the Generalized Stephan's Problem

$$(6) \int_Q \left[\psi(t,x,u) \frac{\partial F}{\partial t} + v(t,x,u) \frac{\partial^2 F}{\partial x^2} - (\psi(t,x,u) - v_x(t,x,u)) \frac{\partial F}{\partial x} + H(t,x,u)F \right] dQ + \int_0^1 F(0,x) \varphi(0,x,u_0(x)) dx - \int_0^T v(t,0,u_1(t)) \frac{\partial F}{\partial x} dt + \int_0^T v(t,1,u_2(t)) \frac{\partial F}{\partial x} dt = 0 ;$$

here $F(t,x)$ is a smooth function, $F(T,x) = F(t,0) = F(t,1) = 0$. Furthermore:

$$\psi(t,x,u) = \int_{c_0}^u a(t,x,s)k(t,x,s)ds + \gamma_1 \text{ for } u \text{ from } l_1, \text{ where } \gamma_1(t,x) = \sum_{j=1}^{i=1} \lambda_j(t,x),$$

$$c_0 \leq S_1 \text{ is a constant; } \psi(t,x,u) = \int_{c_0}^u (kb - k_x) ds; \quad v(t,x,u) = \int_{c_0}^u k(t,x,s) ds;$$

Card 3/5

8682.

S/020/60/135/005/006/043
C111/C222

A Method for Solving the Generalized Stephan's Problem

$H(t,x,u) = \varphi_t - \varphi_x + kcu + kf$; h_x is $\frac{\partial h(t,x,u)}{\partial x}$; $k(t,x,u)$, $a(t,x,u)$, $b(t,x,u)$, $c(t,x,u)$, $f(t,x,u)$ are functions which for u from l_1 are equal to the functions $k_1(t,x)$, $a_1(t,x)$ etc. A bounded summable function $u(t,x)$

satisfying the mentioned class of the identity (6) for every $F(t,x)$, is called a generalized solution of (1)-(3). Here for every generalized solution $u(t,x)$, to the generally discontinuous functions $\varphi(t,x,u)$ and $H(t,x,u)$ certain bounded values are adjoined for $u(t,x) = S_1$, where

$$\Psi(t,x,S_1-0) \leq \varphi(t,x,S_1) \leq \Psi(t,x,S_1+0).$$

The author proves the existence and uniqueness of the generalized solutions defined in this way. The proof of existence is given by the construction of a quasilinear parabolic auxiliary equation whose solution depending on an index n , for $n \rightarrow \infty$ converges uniformly to the generalized solution of (1) - (3).

Card 4/5

86821

S/020/60/135/001/006/043
C111/C222

A Method for Solving the Generalized Stephan's Problem

It is stated that the generalized solution satisfies the condition (2) in the integral sense, while (1) is satisfied in the ordinary sense there where $u(t,x) \neq S_i$.

There are 6 Soviet references.

ASSOCIATION: Moskovskiy gosudarstvennyy universitet imeni M.V.Lomonosova
(Moscow State University imeni M.V.Lomonosov)

PRESENTED: July 1, 1960, by I.G.Petrovskiy, Academician

SUBMITTED: June 30, 1960

Card 5/5

BILASHEVSKIY, M.M. [Bilashivs'kyi, M.M.]; PIVOVAR, M.G. [Pyvovar, M.H.];
OLEYNIK, O.Ya. [Oliinyk, O.Ya.]; PISHKIN, B.A. [Pyshkin, B.A.], otv.
red.; PYECHKOVSKAYA, O.M. [Piechkovs'ka, O.M.], red.izd-va;
YEFTIMOVA, M.I. [IEfimova, M.I.], tekhn. red.

[Calculating the conjugation of head and tail waters and the
strengthening of the bottom below spillway dams] Rozrakhunky
spriazhennia b'iefiv i kriplen' dna za vodozlyvnyy hreblamy.
Kyiv, Vyd-vo Akad. nauk URSR, 1961. 166 p. (MIR: 15:2)

1. Chlen-korrespondent Akademii nauk URSR (for Pishkin).
(Spillways)

GAL'PERN, S.A.G OLEYNIK, O.A.

8 Ivan Georgievich Petrovskii (on the occasion of his 60th birthday).
Vest.Mosk.un. Ser.1: Mat., mekh. 16 no.1:3-7 Ja-F '61. (MIRA 14:3)
(Petrovskii, Ivan Georgievich, 1901-)

OLEYNIK, O.A.; RYBNIKOV, K.A.

Second Hungarian Mathematics Congress. Usp. mat. nauk
16 no.1:219-220 Ja-F '61. (MIRA 14:6)
(Mathematics--Congresses)

S/042/61/016/001/005/005
0111/0444

AUTHORS: Aleksandrov P. S., Myshkas A. D., Oleynik O. I.,
TITLE: Ivan Georgiyevich Petrovskiy (on the occasion of his
sixtieth birthday).
PERIODICAL: Uspekhi matematicheskikh nauk, v.16, no.3, 1961, 219-238.

TEXT: The authors give a survey on the most important scientific re-
sults of the jubilar and some short biographical dates. I. G. Petrovs-
kiy dedicated himself to the following questions:
1.) correctness of the Cauchy problem.

$$\frac{\partial^k u_i}{\partial t^k} = \sum_{j=1}^N \sum_{k_1+\dots+k_p=M} A_{ij}^{k_1, \dots, k_p} (1) \frac{\partial^{k_1+\dots+k_p} u_j}{\partial x_1^{k_1} \partial x_2^{k_2} \dots \partial x_p^{k_p}} = f_i(t, x_1, \dots, x_p)$$

$$\frac{\partial^k u_i}{\partial t^k} \Big|_{t=0} = \varphi_{ij}^k(x_1, \dots, x_p) \quad (2)$$

(i = 1, ..., N; k = 0, ..., k_p - 1),

Card 1/4

S/042/61/016/003/005/005
011/0444

Ivan Georgiyevich ...

where $A(x)$, $f(t,x)$ and $\varphi(x)$ is sufficiently smooth, but usually not analytic. The problem was treated by aid of Fourier transformation of (1) - (2) and led to a statement of sufficient and necessary conditions ("condition A" of Petrovskiy) for the uniform correctness of the problem (1) - (2). It was especially stated, that the condition A is satisfied for hyperbolic systems. Further the conception of parabolic (in the Petrovskiy sense) systems was established, and it was shown that the Cauchy problem is always correct for them. On the elliptic (in the Petrovskiy sense) system it was proved that all sufficiently smooth solutions are analytic, if the equations are analytic in all arguments.

2.) The dependence of the solution of the Cauchy problem for hyperbolic systems on initial conditions (in connection with the so-called wave-diffusion). For linear homogenous systems with constant coefficients necessary and sufficient conditions for the existence of "stable" (i.e. not vanishing under small changes of the coefficients) gaps were given.

3.) Conditions for the solubility of the first boundary value problem for the heat equation (by aid of the method of "superfunctions" of Card 2/4

Ivan Georgiyevich ...

S/042/61/C16/003/005/005
C111/C444

Perron).

4.) Behaviour of the solutions of the Cauchy problem for

$$\frac{\partial u}{\partial t} = \sum_{k=1}^n \frac{\partial^2 u}{\partial x_k^2} + F(x) \text{ for } t \rightarrow \infty.$$

5.) Sufficient conditions for the fact that near the origin the integral curves of

$$\frac{dx_i}{dt} = \sum_{k=1}^n a_{ik} x_k + O\left(\sum_{k=1}^n |x_k|\right) \quad (i = 1, \dots, n; a_{ik} = \text{const}), (9)$$

behave like the integral curves of the corresponding linear system (without $O(\sum)$).

6.) Research of the number of limit cycles of

$$\frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)}, \quad (9)$$

where P and Q are polynomials. By extension of the complex and application of the methods of algebraic geometry, an upper bound for the Card 3/4

3/042/61/016/001/005/001
0111/0444

Ivan Georgiyevich ...

number of limit-cycles is obtained, e.g. for P and Q of second degree there exist at most three limit cycles.

7.) A few papers in the field of real functions, probability theory, algebraic geometry.

After the description of his scientific merits, his accomplishments as a pedagogue and organizer are honored. Petrovskiy was dean of the Mechanical - Mathematical Faculty of the M G U (Moscow State University) during 1940 - 44 and is now rector since 1951. Since 1953 he is a member of the Presidency of the Academy of Sciences.

Biographic notes: He was born January 18, 1901, passed the high school in Sevsk, studied mathematics in Moscow since 1922, since 1925 in the seminary of D. F. Yegorov, since 1929 he taught at the Moscow University.

There is a portrait of the jubilar added to the article as well as a list of his publications (1929-1959) with 47 titles.

The author mentions: I. M. Gelfand, G. Ye. Shilov, S. N. Bernshteyn, L. A. Lynsternik, A. N. Kolmogorov, N. S. Piskunov, L. A. Zhudov, Ye. M. Landis, N. N. Bautin, O. A. Oleynik, A. Ya. Khinchin, S. L. Sobolev, A. N. Nikhonorov.

Card 4/4

29830
S/042/61/016/005/002/005
C111/C444

16.3500

AUTHORS:

Oleynik, O. A., Kruzhkov, S. N.

TITLE:

Quasilinear parabolic equations of second order with several independent variables

PERIODICAL:

Uspekhi matematicheskikh nauk, v. 16, no. 5, 1961.
115 - 155

TEXT:

One considers the existence of solutions for Cauchy problems and for boundary value problems for quasilinear parabolic equations of second order with several independent variables. Considered are solutions "in the large", i. e. solutions for an arbitrary previously given t-interval. In the introduction it is stated first of all that for an arbitrary parabolic equation

$$u_t - a_{ij}(t, x, u, u_x)u_{x_i x_j} = f(t, x, u, u_x)$$

with sufficiently smooth a_{ij} and f , there always exists a "local" solution of the considered problems, i. e. for a sufficiently small t-interval. For the existence of solutions "in the large" it proves to be necessary that the growth of the a_{ij} and of f satisfies certain restrictions.

Card 1/16

λ

Quasilinear parabolic equations...

29830
S/042/61/016/005/002/005
C111/C444

§2 contains a priori estimations of the solutions of linear parabolic equations. There are mentioned results of the following references: (Ref 31: I. Nash, Continuity of solutions of parabolic and elliptic equations, Amer. Journ. Math. 80, no. 4(1958), 931 - 954; Ref 8: A. Friedman, On quasi-linear parabolic equations of the second order II, Journ. Math. and Mech. 9, no. 4(1960), 539 - 556; Ref 26: A. Friedman, Boundary estimates for second order parabolic equations and their applications, Journ. Math. and Mech. 7, no. 5(1958), 771 - 797; Ref 34: A. Friedman, Interior estimates for parabolic systems of partial differential equations, Journ. Math. and Mech. 7, no. 3, (1958), 393 - 417) as well as the following generalisation of the theorem of I. Nash: Let Ω be a domain of \mathbb{R}_n , Ω^{δ} be the largest subdomain of Ω , its distance from the boundary of Ω being $\delta > 0$. Let $Q^{\delta} = \{\Omega^{\delta} \times (0, T)\}$, $Q = \{\Omega \times (0, T)\}$; let T be the lower base and S the face of Q ; in Q one considers bounded solutions of the parabolic equation

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(t,x) \frac{\partial u}{\partial x_j}) + \sum_{i=1}^n b_i(t,x) \frac{\partial u}{\partial x_i} + f(t,x) \quad (2.3)$$

Card 2/16

X

29830

S/042/61/016/005, 002/005
C111/C444

Quasilinear parabolic equations...

the coefficients being sufficiently smooth, $a_{ij}(t, x) = a_{ji}(t, x)$

and
$$\mu_1 \sum_{i=1}^n \xi_i^2 \leq \sum_{i,j=1}^n a_{ij}(t, x) \xi_i \xi_j \leq \mu_2 \sum_{i=1}^n \xi_i^2, \quad 0 < \mu_1 \leq \mu_2, \quad (2.4)$$

$$|b_i| \leq B, \quad i = 1, \dots, n, \quad |f| \leq N. \quad (2.5)$$

being satisfied. Then

Theorem 2: Let $u(t, x)$ be a solution of (2.3) in Q ; $|u(t, x)| \leq M$. Then for $(t_1, x_1), (t_2, x_2) \in Q^d, 0 < t_1 \leq t_2, 0 < \delta \leq 1$ holds the inequality

$$|u(t_2, x_2) - u(t_1, x_1)| \leq A \max \left[\frac{M+N}{\delta^\alpha}, (M+N)B^\alpha, \frac{M}{\min(\sqrt{t_1}, 1)} \right] |x_2 - x_1|^\alpha + A \max \left[\frac{M+N}{\delta^{2\beta}}, (M+N)B^{2\beta}, \frac{M}{\min(\sqrt{t_1}, 1)} \right] (t_2 - t_1)^\beta \quad (2.6)$$

for a certain $\alpha \in (0, \frac{1}{2}), \beta \in (0, \frac{1}{4})$.

A, α, β are constants, only depending on μ_1, μ_2 and n .

The following later used signs are introduced in §2: $u \in C^q, q =$

Card 3/16

✓

29830

S/042/61/016/005/002/005

C111/C444

Quasilinear parabolic equations...

$= 0, \alpha, 1+\alpha, 2+\alpha$) means that $|u|_Q$ is finite. Let $P_1(t', x')$,

$P_2(t'', x'') \in Q\{\Omega \times (0, T)\}$; then

$$d(P_1, P_2) = (|x' - x''|^2 + |t' - t''|)^{\frac{1}{2}}, \quad (2.20)$$

$$|u|_0 = \sup_Q |u|, \quad |u|_\alpha = |u|_0 + \sup_{P_1, P_2 \in Q} \frac{|u(P_1) - u(P_2)|}{d(P_1, P_2)^\alpha}, \quad 0 < \alpha < 1, \quad (2.21)$$

$$|u|_{1+\alpha} = |u|_\alpha + \sum_{i=1}^n \left| \frac{\partial u}{\partial x_i} \right|_\alpha, \quad (2.22)$$

$$|u|_{2+\alpha} = |u|_{1+\alpha} + \sum_{i=1}^n \left| \frac{\partial u}{\partial x_i} \right|_{1+\alpha} + \left| \frac{\partial u}{\partial t} \right|_\alpha. \quad (2.23)$$

In §3 there are given apriori estimations for the solutions of quasilinear parabolic equations. Let $u(t, x)$ be the solution of the equation:

Card 4/16

X

29830
S/042/61/016/005/002/005
C111/C444

Quasilinear parabolic equations...

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left(a_{ij}(t, x, u) \frac{\partial u}{\partial x_j} \right) + \sum_{i=1}^n b_i(t, x, u, u_x) \frac{\partial u}{\partial x_i} + c(t, x, u, u_x), \quad u_x = (u_{x_1}, \dots, u_{x_n}). \quad (3.1)$$

in the cylinder $Q \{ \Omega \times (0, T) \}$ which corresponds to the boundary condition

$$u|_{\Gamma} = \psi|_{\Gamma}, \quad (3.3)$$

$\psi(t, x)$ being defined in \bar{Q} and $|\psi| \leq M_0$.

The following conditions be satisfied:

A. For $(t, x) \in \bar{Q}$ and arbitrary u let $c_u(t, x, u, 0) \leq c_0, |c(t, x, 0, 0)| \leq c_1$

B. For $(t, x) \in \bar{Q}$ and arbitrary u let $a_{ij} = a_{ji}$ and

$$\mu_1(|u|) \sum_{i=1}^n \sum_{j=1}^n a_{ij}^2(t, x, u) \leq \mu_2(|u|) \sum_{i=1}^n a_{ii}^2$$

Card 5/16

X

29830

S/042/61/016/005/002/005
C111/C444

Quasilinear parabolic equations...

μ_1 being a non-increasing and μ_2 being a non-decreasing positive function.

C. For $(t, x) \in \bar{Q}$, $|u| \leq \bar{M} = \max \left\{ M_0 e^{\gamma T}, \frac{c_1 e^{\gamma T}}{\gamma - c_0} \right\}$, where $\gamma > c_0$ and arbitrary u_{x_k} be $|b_i| \leq B_0(|\text{grad}_x u| + 1)$, $|(b_i)_{x_k}| + |(b_i)_u| \leq B_1(|\text{grad}_x u| + 1)$, $|(b_i)_{u_{x_k}}| \leq B_2$, $|c| + |c_{x_k}| + |c_u| + |c_{u_{x_k}}| \leq N$.

Under these suppositions one proves six lemmata, e. g.

Lemma 2: There exists an M_1 such that

$$|u(t, x) - \psi(t, x)| \leq M_1 t$$

Relying on these lemmata one proves:

Theorem 10: The b_i in (3.1) are supposed to satisfy instead of C the more strict condition:

D. For $(t, x) \in \bar{Q}$, $|u| \leq \bar{M}$ and arbitrary u_{x_k} holds:
Card 6/16

✓

29830

S/042/61/016/C05/002/005
C111/C444

Quasilinear parabolic equations...

$|b_i| \leq [\lambda(p) + \kappa](p + 1)$, where $p = |\text{grad}_x u|$, $[\lambda(p) + \kappa](p + 1)$,
being a positive increasing function for $p \geq 0$, $\lambda(p)$ being bounded,
 $\lim_{p \rightarrow +\infty} \lambda(p) = 0$, and $0 \leq \kappa \leq M$;

$$|(b_i)_{x_k}| + |(b_i)_u| \leq B_1(p + 1); |(b_i)_{u_{x_k}}| \leq B_2(|c| + |c_{x_k}| + |c_u| + |c_{u_{x_k}}|) \leq N.$$

Then in \bar{Q} holds:

$$|\text{grad}_x u| \leq M_{15}. \tag{3.13}$$

The function is said to belong to the class $C_{q+\alpha}$ in the domain G , with respect to certain arguments, if f and all its derivatives with respect to these arguments up to the q -th order are bounded in G , satisfying in G the Hölder condition with the exponent α with respect to all arguments.

Theorem 11: Let $u(t, x) \in C^{2+\nu}$ be the solution of (3.1), (3.3) in the cylinder $Q\{\Omega \times (0, T)\}$; let $\Omega \in A^{2+\nu}$ and the coefficients of (3.1)
Card 7/16

29830
S/042/61/016/005/002/005
C111/C444

Quasilinear parabolic equations...

satisfying the conditions A, B, D. In every finite part of the domain

$\{(t, x) \in \bar{Q}, |u| \leq \bar{M}, -\infty < u_{x_i} < +\infty\}$ be the coefficients $a_{ij} \in C_{2+\nu}$

with respect to x_k and u , the function c and $b_i \in C_{1+\nu}$ with respect to x_k, u, u_{x_k} ; let $\psi \in C^{2+\nu}$. Then in \bar{Q} holds a priori:

$$|u|_{2+\nu} \leq M, \tag{3.19}$$

where $M = \text{const}$ is determined by the data of the problem (3.1), (3.3).

$\Omega \in A^{2+\alpha}$ means that the boundary of Ω may be cut up into a finite number of pieces with the equations $x_i = h(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$, where $h \in C^{2+\alpha}$.

Let $u(t, x) \in C^{2+\nu}$ be in the layer $H\{0 \leq t \leq T\}$ and let

$$u(0, x) = \psi_0(x), \quad -\infty < x_1 < \infty; \tag{3.20}$$

be the solution of the Cauchy problem for (3.1); if then the coefficients of (3.1) satisfy the conditions A, B, D, belong to the same

Card 8/16

4

27830

S/042/61/016/005/002/005

C111/C444

Quasilinear parabolic equations...

classes as in theorem 11, and if $\psi_0 \in C^{2+\nu}$ in H, then $|u|_{2+\nu} \leq k$,

k only depending on the data of the problems (3.1), (3.20). (Theorem 12)

Relying on the a priori estimations of §3, the authors prove in §4: Theorem 13: Under the suppositions of theorem 11 with respect to the coefficients of (3.1) and to the boundary of Ω in the cylinder

$Q \{ \Omega \times (0, T) \}$ there exists in \bar{Q} a unique solution $u(t, x) \in C^{2+\nu}$ of the problems (3.1), (3.3), if $\psi(t, x) \in C^{2+\nu}$ in \bar{Q} , and if on the boundary of the lower base of Q the following condition is satisfied:

$$\frac{\partial \psi}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(t, x, \psi) \frac{\partial \psi}{\partial x_j}) + \sum_{i=1}^n b_i(t, x, \psi, \psi_x) \frac{\partial \psi}{\partial x_i} + c(t, x, \psi, \psi_x).$$

Theorem 14: Under the suppositions of theorem 12 with respect to the coefficients of (3.1) in the layer H $\{ 0 \leq t \leq T \}$ there exists in H a unique solution $u(t, x) \in C^{2+\nu}$ of the problem (3.1), (3.20), if

$$\psi_0(x) \in C^{2+\nu} \text{ in H.}$$

Card 9/16

X

29830

S/042/61/016/005/002/005
C111/C444

Quasilinear parabolic equations...

In §5 the parabolic equation

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(t, x, u) \frac{\partial u}{\partial x_j}) + \sum_{i=1}^n \frac{\partial b_i(t, x, u)}{\partial x_i} + a(t, x, u), \quad (5.1)$$

is considered. A function $u(t, x)$, bounded in Q is called a generalised solution of the first boundary value problem

$$u|_S = 0, \quad u(0, x) = u_0(x), \quad |u_0(x)| \leq M \quad (5.2)$$

in Q for (5.1), if

- 1.) $u(t, x)$ satisfies the Hölder condition inside of \bar{Q} .
- 2.) $u(t, x)$ possesses generalised derivatives $\frac{\partial u}{\partial x_i}$ in Q , which are square summable.

- 3.) for every smooth $F(t, x)$, where $F|_S = 0$, $F|_{t=T} = 0$, the identity

Card 10/16

4

Quasilinear parabolic equations...

29730
S/042/61/016/003/002/005
C111/G444

$$\int_Q \left[u \frac{\partial F}{\partial t} + \sum_{i,j=1}^n u \frac{\partial}{\partial x_j} \left(a_{ij}(t, x, u) \frac{\partial F}{\partial x_i} \right) - \sum_{i=1}^n b_i(t, x, u) \frac{\partial F}{\partial x_i} + a(t, x, u) F \right] dt + \int_{\Omega} u_0(x) F(0, x) dx \quad (5.3)$$

is satisfied.

4.) "u(t, x) takes the value zero on S in the mean".

Let the a_{ij} be continuous in $R_N \{ (t, x) \in \bar{Q}, |u| \leq N \}$, possessing bounded derivatives with respect to x and u for every N; let $b_i, (b_i)_{x_i}$ and a be bounded in R_N , satisfying the Lipschitz condition with respect to u; for a_{ij} let the condition B of §3 be satisfied.

Then:

Theorem 15: The generalised solution of (5.1), (5.2) is unique. If (5.1) is written in the following form:

Card 11/16

X

29830

S/042/61/016/005/002/005
G111/G444

Quasilinear parabolic equations...

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(t, x, u) \frac{\partial u}{\partial x_j}) + \sum_{i=1}^n \frac{\partial b_i(t, x, u)}{\partial u} \frac{\partial u}{\partial x_i} + \sum_{i=1}^n (b_i)_{x_i} + a(t, x, u) \tag{5.11}$$

supposing

$$\left(\sum_{i=1}^n (b_i)_{x_i} + a \right)_u \leq c_0. \tag{5.12}$$

then holds

Theorem 16: The generalised solution of (5.1), (5.2) exists and is limit (for $\tau \rightarrow 0$) of the sequence $u^\tau(t, x)$, $0 < \tau \leq 1$. The function u^τ is defined in \bar{Q} as a solution of

$$\frac{\partial u^\tau}{\partial t} = \sum_{i,j=1}^n \frac{\partial}{\partial x_i} \left[a_{ij}^\tau(t, x, u^\tau(t-\tau, x)) \frac{\partial u^\tau}{\partial x_j} \right] + \sum_{i=1}^n B_i^\tau(t, x, u^\tau(t-\tau, x)) \frac{\partial u^\tau}{\partial x_i} + c^\tau(t, x, u^\tau(t-\tau, x)) u^\tau + f^\tau(t, x) \tag{5.13}$$

Card 12/16

✕