

L 1986-66 EWT(1)/EWT(m)/FCC/I/EJA(h) LJP(c) GS/GH
ACCESSION NR: AT5022827 UR/0000/65/000/000/0087/0102

AUTHOR: Nikol'skiy, S. I.

TITLE: Extensive air showers of cosmic radiation

SOURCE: ¹⁴ Vsesoyuznoye soveshchaniye po kosmofizicheskomu napravleniyu
issledovaniy kosmicheskikh luchey. Ist, Yakutsk, 1962. Kosmicheskiye luchy i
problemy kosmofiziki (Cosmic rays and problems in cosmophysics); trudy sovesh-
chaniya. Novosibirsk, Redizdat Sib. otd. AN SSSR, 1965, 87-102

TOPIC TAGS: extensive air shower, primary cosmic ray, mu meson, cosmic
radiation energy, cosmic radiation composition

ABSTRACT: This survey of experimental data on extensive air showers includes
a discussion of: (1) space distribution and total number of charged particles
in extensive air showers at the observational level; (2) spectrum of extensive
air showers relative to the number of particles; (3) mu-mesons in extensive air
showers; (4) fluctuations in the relative number of mu-mesons at the measure-
ment level; (5) energy flux carried by the various components of extensive air
showers; (6) energy spectrum of primary cosmic radiation; (7) search for the
anisotropy of primary cosmic radiation. In conclusion, the author states the
basic objectives of future experiments: (1) study of the energy spectrum and

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search for the anisotropy in the spatial distribution of primary cosmic radiation at energies $E_0 > 10^{19}$ ev, and (2) study of the chemical composition of primary cosmic radiation. Orig. art. has: 8 figures and 1 table.

ASSOCIATION: Fizicheskii institut im. P. N. Lebedeva, AN SSSR (Physics Institute, AN SSSR)

SUBMITTED: 29Oct64

ENCL: 00

SUB CODE: AA

NO REF SOV: 028

OTHER: 020

2/2

L 00342-66 EWT(m)/FCC/T IJP(c)
ACCESSION NR: AP5017950

UR/0367/65/001/006/1079/1092

AUTHOR: Murzina, Ye. A.; Nikol'skiy, B. I.; Tukish, Ye. I.; Yakovlev, V. I.

TITLE: Nuclear-active high-energy particles and the accompanying cosmic ray ex-
tensive air showers

SOURCE: Yadernaya fizika, v. 1, no. 6, 1965, 1079-1092

TOPIC TAGS: cosmic ray measurement, cosmic radiation composition, cosmic ray
shower, cosmic ray telescope, ionization hodoscope, spectrum analysis

ABSTRACT: The authors report the experimental results on the energy spectrum of nuclear-active particles in the region 3×10^{12} to 10^{14} eV at an elevation of 3860 m above sea level, and on the extensive air showers accompanying these particles. The apparatus is shown schematically in Fig. 1 of the Enclosure and consists of two trays of ionization chambers placed under a thick layer of carbon in a cavity surrounded by lead shielding. These chambers were used to detect the high-energy nuclear-active particles. Two additional trays of ionization chambers, under a relatively thin layer of lead, were placed above the carbon to measure the energy of the electron-photon component of the shower cord. The number of particles in the extensive showers was determined with hodoscopic counters placed both immediately above the block of ionization chambers and at a distance of about 30 meters from the

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ACCESSION NR: AP5017950

center of the apparatus. The measured energy spectrum cannot be described by a power law with a single exponent over the entire energy range. The mean free paths were determined for absorption and for nucleon interaction in the atmosphere, and found to be 120 and 83 g/cm² respectively, for particle energies above 10¹³ eV. An analysis of the distribution of the total number of particles of extensive air showers accompanying nuclear-active particles of a given energy in the region $\geq 3 \times 10^{18}$ eV leads to the assumption that a change in the picture of the collision of a nucleon and the air nuclei takes place at an incident-nucleon energy above 10¹³ eV. This change explains the features of the photon energy spectrum in the upper atmosphere and the published data on extensive air showers with 10⁴--10⁶ particles. Although the spectra of the air showers could also be attributed to a sharp change in the composition of the primary cosmic radiation near 10¹³ eV, the latter assumption is not borne out by direct balloon and rocket data on the composition of the primary radiation. Orig. art. has: 9 figures, 31 formulas, and 3 tables.

ASSOCIATION: Fizicheskii institut im. P. N. Lebedeva Akademii nauk SSSR (Physics Institute, Academy of Sciences, SSSR)

SUBMITTED: 028sep64

ENCL: 01

SUB CODE: NP, OP

NR REF BOV: C10

OTHER: 006

Card 2/3

L 00542-56
ACCESSION NR: AP5017950

ENCLOSURE: 01

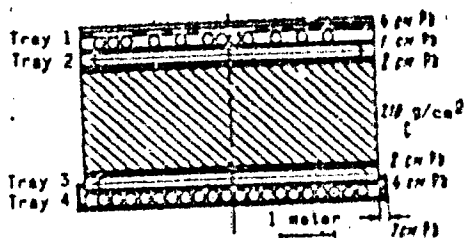


Fig. 1. Detector of nuclear-active particles and of electron-photon showers of high energy.

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VERNOV, S.N.; YEGOPOV, T.A.; YEFIMOV, N.N.; KRASIL'NIKOV, E.D.; KUZ'MIN,
A.I.; MAKSIMOV, S.V.; NESTEPOVA, N.M.; NIKOL'SKIY, S.I.;
SLEPTSOV, Ye.I.; SHAFFER, Yu.G.

Project of a large setup for studying extensive air showers
in Yakutsk. Izv. AN SSSR. Ser. fiz. 29 no.9:1690-1692 S '65.
(MIRA 18:9)

NIKOL'SKIY, S.I.

Prospects of studying cosmic rays in the ultrahigh-energy region.
Izv. AN SSSR.Ser.fiz. 29 no.10:1927-1934 0 '65.

(MIRA 18:10)

MURINA, Y. A.: 1952, 1953, 1954, 1955, 1956.

Nuclear reaction of the type α, n and α, p in
the deuterium nucleus. *Ann. N.Y. Acad. Sci.* 49 no. 10: 19-20-
1952. P. 195. (MIRA 18:10)

V. DANILOVA, S. L. NIKOLSKIY

Nuclear active particles in showers with various number of particles

Report submitted for the 8th Intl. Conf. on Cosmic Rays (IUPAP), Jaipur India,
2-14 Dec 1963

L 4481-66 ENT(L)/ENT(N)/ECC/T/ERA(N) IJE(C) GR

ACC NO. AP0044636 SOURCE CODE: UR/004/06/089/000/1690/1692

AUTHOR: Yermov, S.N.; Yegorov, T.A.; Yegorov, N.N.; Kravtchenko, S.P.; Kuznetsov, A.L.;
Makalov, S.V.; Nesterova, N.N.; Pribludny, S.I.; Slobozov, Ye. I.; Ginter, Ye. S.

ORG: none

TITLE: Plan for a large installation at Yakutsk for study of extensive air showers
/Report, All-Union Conference on Cosmic Ray Physics held at Aptity 24-31 August 1964/

SOURCE: AN SSSR. Izvestiya. Seriya fizicheskaya, v. 20, no. 9, 1968, 1690-1692

TOPIC TAGS: primary cosmic ray, secondary cosmic ray, extensive air shower, spectral
energy distribution, cosmic radiation composition, cosmic radiation anisotropy

ABSTRACT: After a discussion of the significance of extensive air showers for the investigation of ultrahigh energy primary cosmic rays, the authors briefly describe an installation to be completed in the next two or three years near sea level at latitude 62° N in the Yakutsk region; it is anticipated that the installation will yield information concerning the energy spectrum, composition, and anisotropy of primary cosmic rays with energies up to 10²⁰ eV. The installation, intended for investigation of extensive air showers, will comprise 65 stations spread over an area of 25 km². Each station will be equipped with scintillation counters with a total sensitive area of 1 m² or 4 m², and at the central station - 10 m². The total sensitive area of scintillation

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ACC NO: AP0024038

Each station will be equipped with photomultipliers (total cathode area 120 cm² at each station) for recording the Cherenkov flash accompanying a shower. In addition, there will be mesh detectors with a total sensitive area of 22 m². Pulses will be transmitted from the more remote stations to the central laboratory by radio. It is anticipated that this installation will record 2×10^6 showers per year with energies exceeding 10¹⁵ eV and 2 showers per year with energies exceeding 10¹⁶ eV. Orig. art. has 1 figure and 1 table.

PUB CODE: 17/ DATE DATE: 02/- CASE NO: 000/ GEN NO: 000

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 Cont 2/2

REF ID: A6009710

SOURCE CODE: UR/0001/06/001/08/1137/0150

AUTHOR: Nikol'skiy, S. I.

ORG: Institute of Physics Im. P. N. Lebedev, Academy of Sciences,
USSR (Fizicheskiy Institut Akademii nauk SSSR)

TITLE: On processes of inelastic interactions of nucleons and nuclei
at high energy

SOURCE: Zhurnal eksperimental'noy i teoreticheskoy fiziki. Pis'ma
v redaktsiyu. Prilozheniye, v. 3, no. 4, 1966, 143-150

TOPIC TAGS: nucleon interaction, cosmic ray shower, cosmic ray
effect, pion, *nucleon*, *nucleus*

ABSTRACT: In view of the disparity between calculation and experiment
concerning the collision and interaction of nucleons and pions of
energy greater than 10^{13} ev in extensive air showers, the author shows
that the presently used model (two-fireball model) cannot explain the
extremely rapid radiation of the particle-number spectrum of showers
with both large and small number of particles, and there is no explan-

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ACC NR: AF6009710

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ation at all for the shift in the location of the 'break' in the shower particle number spectrum in the region of small showers on going from sea level to mountain altitudes. However, calculations and estimates show that the entire aggregate of the experimental data on the phenomena occurring in cosmic rays of energy higher than 10^{13} ev, including 'break' and 'strange' results, can be explained by assuming that a large fraction ($\sim 70\%$) of the nuclear energy is transferred to the electron-photon component, bypassing pionization, and that the exponent of the energy spectrum of the primary cosmic radiation changes by an amount ~ 0.5 . The manner in which inclusion of these assumptions simplifies the interpretation of the many strange observational results is briefly described. The presence, besides ordinary pionization, of acts with predominant energy dissipation in the electron-proton component increases, on the one hand, the total flux of particles in extensive air showers in the stratosphere, and on the other hand decreases the relative number of muons in the shower. The most characteristic detail of the new process is a rapid

increase in the effective cross section in the energy interval 10^{13} --

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ACC NR: AP6009710

10^{14} . The process cannot be verified until enough direct data are
obtained in the properties of the elementary interactions at energies
 10^{14} ev.

SUB CODE: 20/ SUEM DATE: 25Dec65/ ORIG REF: 005/

Card 3/3 BLS

ACC NR: A97005443

SOURCE CODE: UR/0367/66/000/002/0070/0000

AUTHOR: Amineva, T. P. Dedenko, L. G.; Nikol'skiy, S. I.

ORG: Physics Institute im. P. N. Lebedev, AN SSSR (Fizicheskiy institut AN SSSR)

TITLE: Comparison of the mean characteristics of extensive atmospheric showers with nuclear cascade avalanches, calculated under various assumptions regarding nucleon-nucleus interactions

SOURCE: Yadernaya fizika, v. 4, no. 3, 1966, 578-584

TOPIC TAGS: nucleon interaction, cosmic radiation

ABSTRACT: Experimental data on extensive atmospheric showers are compared with calculations of nuclear cascade avalanches for three different assumptions concerning the character of the elementary interaction between a nucleon and a nucleus of an atom in the atmosphere. Fluctuations of the shower production height and the complex composition of the primary cosmic radiation are taken into account in calculations of the mean characteristics. Among the considered models for interaction of the nucleon - nucleus of atmospheric atoms the two-fireball model results in the least disagreement with experiments. Orig. art. has: 4 figures and 3 tables. [JPRS: 38,764]

SUB CODE: 20 / SUBM DATE: 15Jan66 / ORIG REF: 005 / OTH REF: 002

Card 1/1

0956 2318

NIKOL'SKIY, S.M. (Moskva)

On embedding theorems for classes of differential functions of many variables. Studia math Ser spec no.1:79-89 '63.

NIKOLSKII, S. M.

"Concerning the Asymptotic Evaluation of the Remainder of the Approximation by
Means of Fourier's Sums," Dok. Akad. Nauk, Vol. 32, No. 6, 1941. (Stekloff
Inst. of Math., Acad. of Sci. 1941.

NIKOLSKY, S. P.

"Approximation by Polynomials of Functions Verifying the Lipschitz Condition,"
Dok. Ak. Nauk, Vol. 42, No. 3, 1943. (Stekloff Math. Inst., USSR Acad. of
Sci., c1943)

1. NIKOL'SKIY, S.N.

2. USSR (600)

"Works of the Mathematical Institute imeni V.A. Steklov." Uspekhi Matemat
Nauk. 1, Nos. 5-6, 1946

9. Report U-1493. 27 Sep 1951

The following information selected out of many indicates the
Source: Mathematical Reviews; Vol. 8, No. 3

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Nikol'skiy, S. M.

Nikol'skiy, S. On the best approximation of functions satisfying Lipschitz's conditions by polynomials. *Bull. Acad. Sci. USSR, Ser. Math.* [Izvestia Akad. Nauk SSSR, Ser. Mat.] 29-32 (1946). (Russian). English summary.
It is well known [Favard, *Bull. Soc. Math.* 27, no. 24 (1931); Akhiezer and Korovkin, *Dokl. Akad. Nauk SSSR* (N.S.) 18, 103-104 (1946)] that for a function $f(x)$ of period 2π satisfying the condition $|f(x) - f(y)| \leq A|x - y|^\alpha$, the error of approximation by a trigonometric polynomial of order n is bounded and the result is best possible. The author considers the corresponding problem of the best approximation of functions f defined in the interval $[0, 1]$ by polynomials of degree n . He obtains that $R_n(f) \sim M_n(\alpha, n)$, where $M_n(\alpha, n)$ is a certain function of n and α , and that the point in the best approximation is, for $\alpha > 1/2$, the point $x = 1/2$. For $\alpha = 1/2$ the point is not uniquely determined. For $\alpha < 1/2$ the point is not determined. The author also obtains the asymptotic behavior of $M_n(\alpha, n)$ as $n \rightarrow \infty$. He also obtains the asymptotic behavior of $R_n(f)$ as $n \rightarrow \infty$. The author also obtains the asymptotic behavior of $R_n(f)$ as $n \rightarrow \infty$.

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The point in the best approximation... found on the left being taken... with respect to all functions f satisfying (1). Similarly, if...
The Journal of the American Mathematical Society
L'Institut de Mathématique de l'Université de Paris

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... the ... shows that there is an ... which ...

$$\limsup (n/\log n) f(x) = \sum_{p \leq x} f(p) / (2/x) A(x)$$

where $A(x) = \limsup_{n \leq x} \dots$ for every x there is ...

Source: Mathematical Review

~~Nikolsky, S.~~ Sur la meilleure approximation au moyen des polynômes des fonctions vérifiant la condition de Lipschitz. C. R. (Doklady) Acad. Sci. URSS (N.S.) 52, 7-9 (1946).

Soit (M) la classe des fonctions continues, définies pour $-1 \leq x \leq 1$, et satisfaisant à la condition de Lipschitz $|f(x') - f(x'')| \leq M|x' - x''|$ et $E_n f$ la meilleure approximation de f par des polynômes en x de degré $n - 1$ au plus. Réponse à un problème posé par le rapporteur [Bull. Sci. Math. (2) 62, 338-351 (1918)] en particulier, p. 344. L'auteur étend les résultats au cas des courbes

(1) $\sup_{-1 \leq x \leq 1} E_n f = O(n^{-\alpha})$, $0 < \alpha < 1$, $\ln \log n$

(2) il y a dans (M) des fonctions telles que $\lim_{n \rightarrow \infty} n^\alpha E_n f = 0$ et (3) une méthode de sommation asymptotiquement la meilleure pour les fonctions de (M) consiste à prendre le développement de f en série de polynômes trigonométriques et à lui appliquer la méthode de sommation indéfinie par le rapporteur pour les fonctions périodiques satisfaisant à la condition de Lipschitz α -ième. Pour les sommes locales de ce développement l'auteur résout un problème analogue à celui posé par Kolmogoroff [Ann. Sci. Math. (2) 34, 421-526 (1913)] / Fourier Series

Journal Mathematical Review

NIKOLSKIY, S. N.

"Concerning the Best Approximation by Means of Polynomials of Functions Verifying the Conditions of Lipschitz," Dok. Akad. Nauk, Vol. 52, No. 1, 1946 (Stekloff Inst. Math., Acad. Sci. 1946)

"Fourier's Series of the Function of which the Module of Continuity is Given," Dok. Akad. Nauk, Vol. 52, No. 3, 1946

Nikol'sky, S. On the best approximation in the mean to
 the function $|x - a|^r$ by polynomials. *Bull. Acad. Sci.
 USSR Ser. Math. [Izv. Akad. Nauk SSSR]* 11, 139-
 146 (1947) (Russian; English summary)
 Bernstein [G. P. (Dobinski) Acad. Sci. USSR (N.S.) 11,
 379-384 (1948)] has reported the best approximation of the
 function $|x - a|^r$ by polynomials. In the present paper the
 author studies the best approximation of $|x - a|^r$ in the
 mean. Let

$$E_n(|x - a|^r) = \min_{P_n} \int_{-1}^1 |f(x) - P_n(x)|^2 w(x) dx,$$

where the minimum is taken for all the polynomials P_n of
 degree n , and $w(x)$ is the weight function. Let $E_n(|x - a|^r) = E_n$.
 It is shown, among other things, that (i) if $-1 < a < 1$ and
 if r exceeds -1 and is not an even integer, then

$$E_n(|x - a|^r) = M_n^{-1} n^{-r} (1 + O(n^{-1} \log n))$$

with M_n depending on a only. (ii) if $-1 < a < 1$ then
 asymptotically for $n \rightarrow \infty$

$$E_n(|x - a|^r) \sim \frac{1}{2} \int_{-1}^1 |x - a|^r w(x) dx$$

(iii) if $r > -1$, $r \neq 2k + 1$, $a = 1$, $f(x) = \sum_{k=0}^{\infty} A_k (x - a)^k$
 then

$$E_n(|x - a|^r) \sim M_n^{-1} \sum_{k=0}^{\infty} A_k (1 - a)^{k+r} (k+r)$$

A. Zygmund, Chicago, Ill.

Mathematical Reviews,

Vol. 11, p. 10.

NIKOLSKY, S.A.

1000

Nikolsky, S. Sur la meilleure approximation en moyenne par polynômes des fonctions ayant des singularités de la forme $|a - x|^{\alpha}$. C. R. (Doklady) Acad. Sci. URSS (N.S.) 55, 191-194 (1947).

On appelle meilleure approximation en moyenne, avec le poids $w(x)$, de la fonction $f(x)$ définie dans $-1 \leq x \leq 1$ par un polynôme de degré n , l'expression

$$E_{n,w}(f) = \min \int_{-1}^1 |f(x) - P_n(x)| w(x) dx$$

(ou pour $E_{n,w}(f) = E_n(f)$, le minimum étant à prendre parmi tous les polynômes P_n de degré n au plus. Lorsque le poids satisfait à certaines conditions l'auteur donne l'expression asymptotique de $E_{n,w}(f)$ pour $f(x) = \sum_{k=0}^{\infty} A_k |x - a_k|^{\alpha}$ ($-1 < a_k < 1$, $\alpha > -1$ mais différent d'un nombre pair). Cette expression reste valable pour $\alpha = -1$ lorsque $\sum |A_k| < \infty$. Lorsque α est un entier impair, et lorsque $f(x)$ possède dans $-1 \leq x \leq 1$ une dérivée d'ordre $\alpha - 1$ qui est la primitive d'une fonction $\phi(x)$ de variation bornée, où $\phi(x)$ est la fonction des sauts et où $\phi(x)$ est absolument continue, on en déduit également l'expression asymptotique de $E_{n,w}(f)$ lorsque $n \rightarrow \infty$ sous une condition dans $-1 < \alpha < 1$.

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Mathematical Reviews 1948, Vol. 9, No. 2

Nikol'skiy, S

PA 2157

Mathematics - Integrals

Jan 1947

"The Best Approach to Multi-members in the Middle
Function with Particular Form $|a - x|^s$," S Nikol'skiy,
4 pp

"Dok Ak Nauk SSSR" Vol LV, No 3

Submitted by S N Bernshteyn, Mathematics Institute
imeni V A Steklova, Academy of Sciences of the USSR,
22 Jul 47. Mathematical explanation of the function
 $|a - x|^s$ ($s > 0$). Bernshteyn is credited with work
on this function.

2157

Nikol' Nikol's. Best approximation in the mean of a class of functions by arbitrary polynomials. Doklady Akad. Nauk SSSR (N.S.) 23:23 (1947) (Russian)

Let $\phi_0(x), \phi_1(x), \dots, \phi_n(x)$ be fixed functions from $L(a, b)$ and for any $f \in L(a, b)$ let

$$E_n(f) = \min_{\sum_{i=0}^n \lambda_i \phi_i(x)} \int_a^b |f(x) - \sum_{i=0}^n \lambda_i \phi_i(x)| dx$$

Let $K_n(x)$ be defined for $a \leq x \leq b$, $a < x < b$, and let it satisfy the conditions $K_n(x) \geq 0$, $\int_a^b K_n(x) dx = 1$.

$$\lim_{n \rightarrow \infty} \int_a^b K_n(x) dx = 1, \lim_{n \rightarrow \infty} \int_a^b K_n(x) dx = 0$$

in every sense. Let H denote the class of functions $f(x) = \sum_{i=0}^n \lambda_i \phi_i(x)$ where λ_i is a bounded variation in (a, b) . The author obtains these results by using the method of δ -functions. For all $f \in H$, $E_n(f) \leq \delta$ where δ is a function of n which tends to zero as $n \rightarrow \infty$. The author also obtains the results $E_n(f) \leq \delta$ where δ is a function of n which tends to zero as $n \rightarrow \infty$. Then

$$E_n(f) = \int_a^b f(x) K_n(x) dx$$

is a linear combination of the ϕ_i and thus it reaches an ϵ -maximum in L . The expression $E_n(f)$ will be the best method of approximating f in

$$(1) \quad \sup_{\sum_{i=0}^n \lambda_i \phi_i(x)} \int_a^b |f(x) - \sum_{i=0}^n \lambda_i \phi_i(x)| dx = E_n(f)$$

Let μ be the value of ϵ for which the maximum in (1) is reached. The author shows that a necessary and sufficient condition for (2) is

$$\int_a^b |K_n(x) - P_n(x)| dx \leq E_n(K_n)$$

Of interest is the special case when $(a, b) = (0, 2\pi)$, $K_n(x) = K_n(x)$ and $E_n(f)$ is the approximation in L of f by trigonometric polynomials of order $n-1$.

A. Zygmund (Chicago, Ill.)

Mathematical Reviews.

Vol 1 86

W. J. Karwowski, S. M. On the best linear method of approxi-
mation to the class of functions of degree n by
polynomials of degree m in the case of bounded variation
(1947) (Russian)

Let $f(x)$ be the class of functions $f(x)$ having the $(s-1)$ th derivative $f^{(s-1)}(x)$ of bounded variation and such that the total variation of $f^{(s-1)}$ is at most 1. Let $E_n(f)$ be the minimum of $\int_{-1}^1 |f(x) - P_n(x)| dx$ for all polynomials $P_n(x)$ of degree n , and let $E_n(f)$ be the minimum of $\int_{-1}^1 |f(x) - P_n(x)| dx$ for all polynomials $P_n(x)$ of degree n . The author shows that $E_n(f) \sim E_n(f)$ remains unchanged if we require that the $(s-1)$ th derivative of f is absolutely continuous and of total variation at most 1. Furthermore, he shows that $E_n(f) \sim E_n(f)$ is either

$$E_n(f) \sim \frac{1}{n^{s-1}} \max_{-1 \leq x \leq 1} |f^{(s-1)}(x)|$$

$$E_n(f) \sim \frac{1}{n^{s-1}} \int_{-1}^1 |f^{(s-1)}(x)| dx$$

or

$$E_n(f) \sim \frac{1}{n^{s-1}} \max_{-1 \leq x \leq 1} |f^{(s-1)}(x)|$$

$$E_n(f) \sim \frac{1}{n^{s-1}} \int_{-1}^1 |f^{(s-1)}(x)| dx$$

depending on whether s is even or odd (see also [1]).

For $s=2$, $E_n(f) \sim \frac{1}{n} \int_{-1}^1 |f'(x)| dx$

by the polynomial of degree n giving the best approximation in the L_1 metric over $[-1, 1]$ to the function $f(x)$ is given by the polynomial of degree n which is odd. Then the linear method

$$L_n(f, x) = \frac{1}{n} \int_{-1}^1 |f'(t)| dt$$

is the best method for the class $H_{\infty}^{(s)}$.

$$L_n(f, x) = \frac{1}{n} \int_{-1}^1 |f'(t)| dt$$

of approximation of f (in L_1) by polynomials of degree n is the best method for the class $H_{\infty}^{(s)}$. A. Zygmund.

Mathematical Reviews.

Vol 7 No. 6

SKIV, S.M.

On linear methods of summation of series. *Izvestiya Akad. Nauk SSSR Ser. Mat.* 1948, 12, 218-219 (1948). (Russian)

The paper is concerned with a general theory of summation of trigonometric series. If

$$f(x) = f_0 + \sum_{k=1}^{\infty} \lambda_k^{(n)} (a_k \cos kx + b_k \sin kx)$$

is a necessary and sufficient condition that

$$\lim_{n \rightarrow \infty} U_n(f, x) = f(x)$$

for any continuous f (for any Lebesgue point of an integrable function) is $\lim_{n \rightarrow \infty} \lambda_k^{(n)} = 1$ and

$$\int_0^{2\pi} (1 + \sum_{k=1}^{\infty} \lambda_k^{(n)} \cos kx) dx = M$$

follows from classical theorems of Hahn and Zygmund in B -spaces. The paper is devoted to finding conditions which $\lambda_k^{(n)}$ have to satisfy in order that (A) be satisfied. It is shown that (B) implies the existence of a B -space.

that $\lambda_k^{(n)} \rightarrow 1$ and $\sum_{k=1}^{\infty} \lambda_k^{(n)} \cos kx = M$. An example shows that (A) and (B) are not equivalent to (C). But if, for every n , $\lambda_k^{(n)}$ is convex (concave) this equivalence holds. *Ernest W. Weir (Berkeley, Calif.)*

WAW
EWT

Source: Mathematical Reviews.

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NIKOLSKI, S. M.

Nikol'ski, S. A generalization of the theorem of S. M.

on the theory of differential equations of the type

$$y'' + p(x)y' + q(x)y = r(x)$$

 where $p(x)$ and $q(x)$ are polynomials of degree n and m respectively, and $r(x)$ is a polynomial of degree k . The author proves that if the equation has a solution of the form $y = e^{\lambda x}$, then λ is a root of the characteristic equation $\lambda^2 + p_0\lambda + q_0 = 0$, where p_0 and q_0 are the constant terms of $p(x)$ and $q(x)$ respectively. The author also discusses the case where $p(x)$ and $q(x)$ are not polynomials but functions satisfying certain conditions.

Mathematical Reviews.

Vol. 10, No. 1, p. 115.

1970

NIKOL'SKIY, S. M.

1. Nikolskiy, S. M. A generalization of some inequalities of ...
 Doklady Akad. Nauk SSSR (N.S.) 60, 1949, 1114 (1948). (Russian)

2. Nikolskiy, S. M. A generalization of an inequality of S. N. ...
 Doklady Akad. Nauk SSSR (N.S.) 60, 1949, 1110 (1948). (Russian)

3. Nikolskiy, S. M. A generalization of an inequality of ...
 to entire functions of finite degree.
 Doklady Akad. Nauk SSSR (N.S.) 60, 1949, 1100 (1948). (Russian)

4. Nikolskiy, S. M. A trigonometric polynomial of degree n and ...
 $\sum_{k=0}^n |a_k| \leq 1$. The first paper proves the ...
 $\sup_{|x| \leq 1} |f(x)| \leq \sup_{|x| \leq 1} |f'(x)|$.

Mathematical Reviews.

Vol. 9 No. 10

Copy

The second paper extends (*) to entire functions of expo-
 nential type λ (no longer necessarily an integer) in the case
 of $\lambda > 0$ and then to functions of more than one variable.

The third paper gives a different proof of (*) for $\lambda = 1$ and
 to the functions of exponential type, and adds the inequality

0 < λ < ∞ .
 The reviewer remarks that (*) and (**) for finite Fourier-
 Stieltjes integrals follow at once from a theorem of P. Civin
 [Ann. Math. (2) 48, 265 (1941); Abstr. Rev. 3, 1941].
 For the more general case of entire functions of exponential
 type they may then be deduced by a simple limiting process.
 S. C. BENTON, Jr. (Providence, R. I.)

NEROL'SKIY, S. K.

"Fourier Series of Functions Possessing a Derivative of Bounded Variation," Iz. Ak. Nauk SSR, Ser. Matemat., Vol. 13, No. 6, 1949. (Math. Inst. Leningrad V. A. Steklov, USSR Acad. of Sci., 1949) Article also published in Dok. Ak. Nauk, Vol. 65, No. 1, 1949.

"Asymptotically Optimum Linear Method of Approximating Differential Functions by Polynomials," Dok. Ak. Nauk, 69, No. 2, 1949

1958/19 3/24

Nikol'skii, S. M. The Fourier series of functions with derivatives of bounded variation. (Doklady Akad. Nauk SSSR (N.S.) 65, 13-15 (1949) (Russian))

If $f(x)$ is of period 2π and has an n th derivative $f^{(n)}(x)$ of bounded variation with discontinuities at x_1, x_2, \dots , then the n th partial sum $s_n(x)$ of the Fourier series of f satisfies the relation

$$|s_n(x) - f(x)| \leq \frac{C}{n} \sum_{k=1}^n \omega_k(x) \quad (1 \leq n \leq +\infty)$$

where $\omega_k(x)$ is the modulus of continuity of $f^{(k)}(x)$ and C is a constant given by a rather complicated double integral. An inequality is given for $|s_n(x) - f(x)|$ if f is continuous and has a modulus of continuity of order α . Corresponding results are obtained for f' when the right hand sides of the relations acquire the factor $\log n$. A. Zygmund

Source: Mathematical Reviews.

v-1 10 No. 8

(Handwritten initials)

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USSR/Mathematics - Approximations 11 Nov 49
Applied Mathematics

"Asymptotically Optimum Linear Method of Approximating Differential Functions by Polynomials," S. N. Nikol'skiy, 4 pp

"Dok Ak Nauk SSSR" Vol LXIX, No 2

Demonstrates one of the possible methods, asymptotically optimum and linear, for approximating functions $f(x)$ of class W^r , defined on interval $-1,1$ and possessing on it the derivative $f'(x)$ always less than one in absolute value, for any r , by polynomials. Submitted 16 Sep 49 by Acad S. N. Bernshteyn.

157142

Mathematics, Analysis

Mathematical Reviews
Vol. 12 No. 11
Dec. 1952
Analysis

8-10-51
LL

✓
Nikol'skii, S. M. Some questions of the approximation
of differentiable functions. Comptes Rendus du Premier
Congrès des Mathématiciens Hongrois, 27 Août - 2 Sept-
embre 1950, pp. 113-124. Akadémiai Kiadó, Budapest,
1952. (Russian. Hungarian summary)

Math
3

The author describes the results which he has published
over the last five years of polynomial approximation of
differentiable functions and its applications.

B. F. Egan, Jr. (Evanston, Ill.).

NIKOLAIK M. 319

V. I. Krasovskiy. Concerning estimation for approximate quadrature formulas. Uspehi Matem. Nauk (N.S.) 5, no. 2(36), 163-177 (1950). (Russian)

CP 800

Let $f(x)$ be a function defined in (a, b) and x_1, x_2, \dots, x_n and $\theta_1, \theta_2, \dots, \theta_n$ be given numbers. In the comparison of the integral $\int_a^b f(x) dx$ by the approximate quadratures $\sum_{k=1}^n \theta_k f(x_k)$ we may be interested not only in the error for individual functions f but also in the maximum of the error for all f belonging to a certain class H . Correspondingly, we set

$$E(H) = \max_{f \in H} \left| \int_a^b f(x) dx - \sum_{k=1}^n \theta_k f(x_k) \right|$$

The author investigates the cases when H is one of the classes $KW^r, W^r, K, W^r, M, K, W^r, M, K, W^r, M$. The two classes are those with the least error. The author gives times and sets of quadrature nodes and weights. The author adds the conditions $f(x) \in H$ and $f(x) \in H$ and gives the data for $r=1, 2, 3, 4$ are given when the x_k and θ_k are those of the quadrature formula. The author also gives the reason for the choice of the nodes and weights. The author also obtains a result that $E(H) \leq E(H)$ for H is a minimum. The case $r=1$ is investigated in greater detail and e.g., for $H=W^1$ the author obtains that the error of the approximation is 1.5 times better than given by the formula for the same number of nodes and weights.

A. Zyrenov (Chicago, Ill.)

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USSR/Mathematics - Approximations

21 Jul 51

"Approximation of Continuous Periodic Functions of Two Variables Satisfying the Lipschitz Condition by Means of Interpolational Trigonometrical Polynomials," P. I. Bugayets

"Dokl Ak Nauk SSSR" Vol LXXXII, No 3, pp 381-384

Current article is devoted to an extension of S. M. Nikol'skiy's results (cf. ibid. 31, Vol XXXI, No 3, 1941; "Trudy Matemat Inst imeni Steklov," 15, 1945) to the 2-dimensional case. A. S. Beslyudov (cf.

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"Dokl Ak Nauk SSSR" Vol LXX, No 3, 1949) extended his results to the 2-dimensional case for a special class of functions $f(x,y)$ satisfying certain Lipschitz inequalities. Bugayets demonstrates a certain theorem concerning $f(x,y)$. Submitted by Acad A. N. Kolmogorov 19 May 51.

211761

NIKOL'SKIY

NIKOL'SKIY, S. M.

IN 24279

USSR/Mathematics - Mathematician Sep/Oct 52

"The Works of Sh. Ye. Mikeladze," S. M. Nikol'skiy

"Usp Matemat Nauk" Vol 7, No 5(51), pp 237-8

Mikeladze, professor and corr mem, Acad Sci Georgian BSR, was awarded 13 Mar 52 a Stalin prize for his work on approximate methods of mathematical analysis. He is a prominent Soviet mathematician and outstanding representative of the Tbilisi school, direct successor to the late Acad A. N. Krylov. Numerous works are devoted by him to applied computational mathematical analysis, such as approximate integration, interpolation, solutions, etc.

24279

NIKOL'SKIY, S. M.

Mathematics - Quadratic Formulas, Mar/Apr 52
Approximation

"Quadratic Formulas," S. M. Nikol'skiy

"Is Ak Nauk SSSR, Ser Matemat" Vol XVI, No 2, pp
181-196

Considers the quadratic formulas which represent
the ordinary ones employed in the practice of com-
bination of classical quadratic formulas, exact
for polynomials of given power. Studies the eval-
uation of approximations by such formulas. Submit-
ted by Acad S. N. Bernshteyn 4 Oct 51.

20873

NIKOL'SKIY, S.M.

Nikol'skiy, S. M. On the continuation of differentiable functions of several variables. Doklady Akad. Nauk SSSR (N.S.) 62, 521-524 (1982) (Russian)

The author proves the existence of a function $f(x_1, x_2, \dots, x_n)$ of n real variables $(x_i \in \mathbb{R}, i=1, \dots, n)$ satisfying certain regularity conditions on its partial derivatives and such that $f(x) = 0$ if and only if x is equal to an assigned function $\phi(x_1, \dots, x_{n-1})$ which satisfies suitable regularity requirements. Details are too long for reproduction.

W. H. Fleming, Illinois, U.S.A.

[Handwritten signature]

Mathematical Reviews.

for $\epsilon=0$. Theorem 2.1. Let D be a domain in \mathbb{R}^n . Then the function f is continuous and the solution of the Dirichlet problem on D for the boundary function f has a finite Dirichlet integral. Further, given f and g satisfying (*) are denoted and g is a function on D satisfying (*) are denoted and g is a function on D .

Mathematical Reviews,

NIKOL'SKIY, PROF S. M.

USSR/Mathematics - Approximation

Aug 53

"Approximate Representation of Functions," Prof S. M. Nikol'skiy

Priroda, No 8, pp 12-20

States that a large school of mathematicians is working on the theory of interpolation in Moscow, Leningrad, Kiev, and Tbilisi (from Tbilisi comes Sh. Ye. Mikeladze, winner of a Stalin prize in 1952 for his approximate integration of differential equations). Another related school is working on the new constructive approximation theory of functions, which

276790

was largely developed by S. N. Bernshteyn, winner of a Stalin prize in 1941 for works on approximation; namely in Moscow, Leningrad, Khar'kov, Yerevan, Kiev, and Dnepropetrovsk. Further states that M. A. Lavrent'yev, M. V. Keldysh, and S. N. Margelyan (of Armenia) are conducting new studies on approximations of complex-variable functions.

NIKOL'SKIY, S.M. (Moscow).

Properties of certain classes of functions of several variables on differentiable manifolds. Mat.sbor. 33 no.2:261-326 S-O '53. (MIRA 6:9)
(Functions of several variables)

NIKOL'SKIY, S.M.

NITSEN, A.V.; FERROVSKIY, I.G., akademik, otvetstvennyy redaktor;
NIKOL'SKIY, S.M., zamestitel' otvetstvennogo redaktora.

Equations of the mixed type. A.V.Nitsen. Trudy Mat.inst. 41
58 p. '53.

(Differential equations, Partial)

MANUSCRIPTS, S. N.

17 1952

USSR/Mathematics - Functions

1 Jan 53

"Second Note on the Extension of Differentiable Functions of Many Variables," S. M. Nikol'skiy, Math Inst imeni Steklov, Acad Sci USSR

Dokl SSSR, Vol 88, No 1, pp 17-19

**Demonstrates several theorems which permit one to study exhaustively the properties of differentiable functions in any linear spaces of a basic space that are parallel to the coordinate axes. Presented by Acad S. N. Bernsheyn
13 Oct 1952.**

24712

НИИ МАТЕМАТИКИ С. С.

249T12

USSR/Mathematics - Differentiable Functions 11 Jan 53

"The Properties of Differentiable Functions of Many Variables on Closed Smooth Manifolds," S. M. Nikol'skiy, Math Inst Imeni Steklov, Acad Sci USSR

ISSN 0013-788X, Vol 88, No 2, pp 213-6

Continues a previous work (ibid. 88, No 1 (1953)). Considers functions of certain classes in various n -dimensional manifolds S . Presented by Acad S. N. Bernstein 13 Oct 52.

249T12

SECRET, S. S.

249731

USSR/Mathematics - Variational Method 21 Jun 53

"Problem of the Solution of the Polyharmonic Equation by the Variational Method," S. N. Nikol'skiy, Math Inst Imeni Steklov, Acad Sci USSR

DAN 2888, Vol 88, No 3, pp 409-411

States that the problem of finding the function u satisfying in a certain region omega the polyharmonic function $\Delta^m u = 0$ for suitable boundary conditions reduces to finding the minimum of a certain multiple integral $D_m(u)$ among all possible functions u having the same boundary conditions as u .

249731

Discusses here the problem of what the boundary conditions must be so that the construction is possible. Presented by Acad S. N. Bernsheyn
21 Oct 52.

249731

259T62

NIKOL'SKIY, S. M.

USSR/Mathematics - Approximations

21 Apr 53

"Best Approximation of a Function Whose s-th Derivative Possesses a Discontinuity of the First Kind," I. I. Ibragimov, Azerbaydzhan State Pedagogic Inst in Lenin, Baku

DAN SSSR, Vol 89, No 6, pp 973-975

Derivation of the asymptotic value of the best approximation of function $f(x)$ whose s-th derivative $f^{(s)}(x)$ possesses in the interval $-1 \leq x \leq 1$ discontinuities of the first kind (at least in one interior point), where s is any real positive number not

259T62

necessarily an integer (the case of s= odd integer was considered by S. M. Nikol'skiy, DAN SSSR, Vol 55, No 2 (1947)), which derivation is connected with finding the best approximation of $\sqrt[s]{a(x-c)+b/x-c} / x-c^{s-1}(c/c < 1)$, where a, b are reals. Presented by Acad S. N. Bernshteyn
25 Feb 53.

NIKOL'SKIY, S. M.

USSR/Mathematics - Approximation,
Series

21 Jun 53

"Generalization of One Result of S. M. Nikol'skiy,"
T. I. Amanov, Math Inst im Steklov, Acad Sci USSR

DAN SSSR, Vol 90, No 6, pp 949-952

Demonstrates several theorems, proved for the case
 $n=1$ by S. M. Nikol'skiy (DAN SSSR, 83, No 1 (1952)),
that govern the Fourier-series expansion of certain
harmonic functions and the convergence of the series.
Presented by Academician S. N. Bernshteyn 13 Apr 53.

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NIKOL'SKIY, S. M.

Doklady Akademii Nauk SSSR / Mathematics - Approximation,
Series

21 Jun 53

"Generalization of One Result of S. M. Nikol'skiy,"
T. I. Amanov, Math Inst in Steklov, Acad Sci USSR

DAN SSSR, Vol 90, No 6, pp 949-952

Demonstrates several theorems, proved for the case
 $n=1$ by S. M. Nikol'skiy (DAN SSSR, 83, No 1 (1952)),
that govern the Fourier-series expansion of certain
harmonic functions and the convergence of the series.
Presented by Academician S. N. Bernshteyn 13 Apr 53.

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(as $M^2 = 10(u)$ and $\epsilon = 0$) Theorem 1. Let $p \in C^1$ on $[0, 2\pi]$ and satisfy (*) for some $M, \epsilon > 0$ then there exists a harmonic function u on D having finite Dirichlet integral $D(u) < M^2 \epsilon^{2\alpha} (2\pi - 1)$ and some radial limit u almost everywhere. The Dirichlet space \mathcal{D} of all spaces in a dimension n is studied and some interesting results that the behavior of the parameters M and ϵ in terms of the Dirichlet integral are shown. 17. 11. 1988

10-
229

MARKOV, A.A.; PETROVSKIY, I.O., akademik, redaktor; NIKOL'SKIY, S.M., professor; SAZONOV, L.S., redaktor; ABOM, R.A., ~~VSEKHOGOVORNIY~~ redaktor.

Theory of algorithms. Trudy Mat.inst. 42:3-374 '54. (NIMA 8:5)
(Algebra)

POSTNIKOV, N.M.; PETROVSKIY, I.G., akademik, redaktor; NIKOL'SKIY, S.M.,
professor, redaktor; RYVKIN, A.E., redaktor; GRAKOVA, Ye.D., tehni-
cheskiy redaktor

[Investigation on the homotopic theory of continuous mappings.
Part 1. Algebraic theory of systems. Part 2. Natural systems
and the homotopic type.] Issledovaniya po geometricheskoj teorii
neprotivnykh otobrazhenii. I. Algebraicheskaia teoriia sistem. II.
Natural'naia sistema i geometricheskii tip. Moskva, Izd-vo Akade-
mii nauk SSSR, 1955. 158 p. (Akademiia nauk SSSR. Matematicheskii
institut. Trudy, vol. 46) (MIRA 8:9)
(Conformal mapping)

NOVIKOV, P.S.; PETEROVSKIY, I.O., akademik, redaktor; NIKOL'SKIY, S.M., pre-
fessor; GUREV, K.F., redaktor; POLYANOVA, T.V., ~~vedushchaya~~ re-
daktor.

Algorithmic undecidability of the word identity problem in the theory
groups. *Trudy Mat. Inst.* 44:3-140 '55. (NIRA 8:5)
(Groups, Theory of)

ALEXANDROV, P.S.; PETROVSKIY, I.G., akademik; NIKOL'SKIY, S.M., professor

Topological theorems of duality. Part 1. Closed sets. Trudy Mat.
inst. no.48:5-110 '55. (MIRA 8:11)

(Topology)

NIEDO'SKIY, S.M.

Inequality for periodic functions. *Usp.mat.nauk.* 11 no.1:219-222
Ja-F '56. (MLA 9:6)
(Inequalities (Mathematics)) (Functions, Periodic)

NIKOL'SKIY, S.M.

One family of functional spaces. *Usp.mat. nauk* 11 no.6:203-212
M-B '56. (MIRA 18:3)

(Spaces, Generalized)

SUBJECT USSR/MATHEMATICS/Theory of functions CARD 1/1 PG - 777
AUTHOR NIKOL'SKI S.M.
TITLE Compactness of the classes $H_p^{(r_1, \dots, r_n)}$ of functions of
 several variables.
PERIODICAL Izvestija Akad.Nauk 20, 611-622 (1956)
 reviewed 5/1957

The author investigates the compactness of the classes $H_p^{(r_1, \dots, r_n)}$ of
 differentiable functions of several variables which were already treated by
 the author in other papers (Mat.Sbornik, n.Ser. 33, 261-326 (1953); ibid. 40,
 243-268 (1956)). In the class being defined for arbitrary positive numbers
 r_1, \dots, r_n a metric is introduced. In the sense of this metric of $H_p^{(r_1, \dots, r_n)}$ a set
 E of functions f is bounded by the constant M , then this set for arbitrary
 $r'_1 < r_1$ is compact in the sense of the metric $H_p^{(r'_1, \dots, r'_n)}$. Here it is stated
 that that function f to which the corresponding subset of E converges in the
 arbitrary metric $H_p^{(r'_1, \dots, r'_n)}$, itself belongs to the initial class $H_p^{(r_1, \dots, r_n)}$
 and has a norm which is not greater than M .

Mat.Sbornik, n.Ser. 40, 243-268 (1956)

CARD 3/3

PG - 760

Then the following theorem is valid:

If $f \in W_p^{(\xi)}(\alpha)(G, \bar{M})$, then a small $\delta > 0$ can be determined such that the function which is continued to the domain $G + \Delta_\delta^n$ with respect to (2) belongs to $W_p^{(\xi)}(\alpha)(G + \Delta_\delta^n, \bar{M})$. Here

$$\left\| \frac{\partial^l f}{\partial x_1^{l_1} \dots \partial x_n^{l_n}} \right\|_{L_p(G + \Delta_\delta^n)} + \bar{M} < c (\|f\|_{L_p(G)} + M)$$

for all $l = \sum_{k=1}^n l_k \leq \xi$, where c is a constant being independent of $\|f\|_{L_p(G)} + M$.

The theorem remains true of the class $W_p^{(\xi)}$ if $l \leq \xi - 1$.

Some further special cases are considered.

SUBJECT USSR/MATHEMATICS/Theory of functions CARD 1/3 PG - 765
 AUTHOR NIKOL'SKI S.M.
 TITLE Limit properties of the functions which are defined on a domain
 with corners.I.
 PERIODICAL Mat.Sbornik,n.Ser. 40, 303-318 (1956)
 reviewed 5/1957

The author considers the function classes $H_p^{(r)}(G)$ with $r > 0$, $1 < p < \infty$ being defined in two-dimensional domains G . (The definition of $H_p^{(r)}(G)$ is given e.g. by Nikol'ski in Mat.Sbornik,n.Ser. 33, 261-326 (1953).) In the open domain G let be given a piecewise smooth curve Γ , where the smooth pieces have a continuous second derivative and form with each other the angles α which satisfy the inequation $0 < \alpha < 2\pi$. The function f belonging to $H_p^{(r)}(G)$ induces a periodic (period = length of Γ) limit function

$$f|_{\Gamma} = \varphi(s)$$

on Γ which can belong to a periodic class $H_{p^*}^{(\lambda)}$ (see Mat.Sbornik,n.Ser. 40, 243-268 (1956)). Then the following theorems are valid:

1. If $0 < r - \frac{1}{p} < 1$ and $f(x,y) \in H_p^{(r)}(G,M)$, then the limit function is

Mat.Sbornik, n.ser. 40, 305-310 (1956)

CARD 2/5

PG - 765

$\varphi \in H_{p^*}^{(r-\frac{1}{p})}(\bar{M})$, where

$$\bar{M} < c \|r\|_{L_p(Q)}^{(r)}$$

and c is independent of the norm $\|r\|$.

2. If $0 < r - \frac{1}{p} < 1$, $r \neq \frac{2}{p}$ and on Γ a function $\varphi(s)$ is given which belongs to the class $H_{p^*}^{(r-\frac{1}{p})}$ of functions being periodic with the period ω , then there exists a function f being defined in G and belonging to the class $H_p^{(r)}(G)$ such that

$$f|_{\Gamma} = \varphi(s),$$

$$\|r\|_{L_p(G)}^{(r)} < c \|\varphi\|_{L_p(\Gamma)}^{(r-\frac{1}{p})}$$

$$\bar{M} \leq cM.$$

NIKOL'SKIY, S.M.

SUBJECT USSR/MATHEMATICS/Theory of functions CARD 1/4 PG - 375
 AUTHOR NIKOL'SKIY S.M.
 TITLE On the Dirichlet problem for regions with corners.
 PERIODICAL Doklady Akad. Nauk 109, 33-35 (1956)
 reviewed 11/1956

Let G be a region of the n -dimensional space, $1 \leq p \leq \infty$, $r = \bar{r} + \alpha$, where \bar{r} is an integer and $0 < \alpha < 1$. Let the function f belong to the class $H_p^{(r)}(G)$ if it and its non-mixed partial derivatives $\frac{\partial^k f}{\partial x_1^k} = \psi_1^{(k)}$ are defined on G and there

are integrable up to the order \bar{r} inclusive ($k=0, 1, \dots, \bar{r}$; $i=1, \dots, n$) with p -th power and if there the inequation

$$(1) \quad \|\psi_1^{(\bar{r})}(\bar{q} + \bar{h}_1) - \psi_1^{(\bar{r})}(q)\|_{L_p(G')} < M|\bar{h}_1|^\alpha$$

is satisfied for all $G' \subset G$, a constant M not depending on M and all vectors \bar{h}_1 which translate G' in the direction of the x_1 -axis. Let here

$$\|\varphi\|_{L_p(G)} = \left(\int_G |\varphi|^p d\alpha \right)^{1/p} \quad \text{and} \quad \|\varepsilon\|_{L_p(G)} = \sqrt{\|\varepsilon\|_{L_p(G)}^2 + M^2}$$

Doklady Akad. Nauk 109, 33-35 (1956)

CARD 2/4

PG - 375

where M denotes the smallest constant for which (1) is satisfied.

If now G is a two-dimensional region which is bounded by a piecewise smooth curve Γ with the length 2π and the angles ω ($0 < \omega < 2\pi$), then a function of the arc length $\varphi(s)$ defined on Γ belongs to the class of the τ -periodic functions $H_p^{(\tau)}(\Gamma)$ if it and its derivatives are integrable over the period up to the order $\bar{\tau}$ and if

$$\|\varphi(\bar{s})(s+h) - \varphi(\bar{s})(s)\|_{L_p(\Gamma)} \leq M |h|^\alpha \quad (0 < \alpha < 1).$$

The author formulates the following theorems without proof:

1. If $0 < \tau - \frac{1}{p} < 1$ and the function $f \in H_p^{(\tau)}(R_2)$ (R_2 is the whole two-dimensional plane), then the function

$$\varphi(s) = f|_{\Gamma}$$

belongs to the class $H_p^{(\tau-1/p)}(\Gamma)$, where $\|\varphi\|_{L_p(\Gamma)}^{(\tau-1/p)} \leq c \|f\|_{L_p(G)}^{(\tau)}$.

2. If $0 < r-1/p < 1$, $r-2/p \neq 0$ and $\varphi \in H_p^{(r-1/p)}(\Gamma)$, then on R_2 a function $f(x,y) \in H_p^{(r)}(R_2)$ can be determined such that (1) is satisfied.

The last result relates only to harmonic functions which on a rectangle Δ ($G = \Delta$) are defined in the metric L_2 .

3. Let r and $r-1/2$ be non-integral numbers and the harmonic function $u \in H_2^{(r)}(\Delta)$. Then the function $\varphi(\sigma) = u|_{\Gamma}$ satisfies the following conditions:

a) $\varphi \in H_2^{(r-1/2)}([s_k, s_{k+1}])$, where s_0, s_1, s_2, \dots are the σ -values which correspond to the corners of Γ ($s_{k+1} = s_k + \nu_k$) and $\|\varphi\|_{L_2[s_k, s_{k+1}]}^{(r-1/p)} < c \|u\|_{L_2(\Delta)}^{(r)}$

b) $\varphi^{(2l)}(s_{k+0}) - (-1)^l \varphi^{(2l)}(s_{k-0}) = 0$ for all $l=0, 1, 2, \dots$ for which $2l < \bar{\xi}$, where $r - \frac{1}{2} = \xi - \bar{\xi} + \beta$, $\bar{\xi}$ - integral and $0 < \beta < 1$.

c) If $\bar{\xi} = 2l_0$ is even, then additionally the inequation

$$\left(\int_0^h |\varphi^{(2l_0)}(s_k+u) - (-1)^{l_0} \varphi^{(2l_0)}(s_k-u)|^2 du \right)^{1/2} \leq c \|u\|_{L_2(\Delta)}^{(r)} |h|^\alpha$$

SUBJECT USSR/MATHEMATICS/Theory of functions CARD 1/2 PG - 766
 AUTHOR NIKOL'SKI S.M.
 TITLE Limit properties of functions in domains with angles.
 PERIODICAL Doklady Akad.Nauk 111, 26-28 (1956)
 reviewed 5/1957

Let G be a bounded two-dimensional domain the boundary Γ of which consists of a finite number of smooth pieces which form with each other the angles ω , $0 < \omega < 2\pi$. If a function $f(x,y) \in H_p^{(\tau)}$ is given (this class of functions was introduced by the author, see Doklady Akad.Nauk 88, 17 (1953)), then for all λ , which satisfy the condition $\tau - \lambda - \frac{1}{p} > 0$, the limit functions

$$(1) \quad \frac{\partial^\lambda f}{\partial n^\lambda} \Big|_{\Gamma} = \varphi_\lambda(s)$$

can be defined. Here s is the arc of Γ , n denotes the normal. It is shown that from $f(x,y) \in H_p^{(\tau)}$ there follows $\varphi_\lambda(s) \in H_p^{(\tau - \lambda - \frac{1}{p})}(\Gamma_k)$ for all smooth pieces Γ_k of Γ , where

Doklady Akad.Nauk 111, 26-28 (1956)

CARD 2/2

PG - 766

$$\|\varphi_\lambda\|_{L_p(\Gamma_k)}^{(r-\lambda-\frac{1}{p})} \leq c \|r\|_{L_p}^{(r)}.$$

Relative to the behavior in the corner points it is shown that the numbers $\varphi_\lambda(s_k + 0)$, $\varphi_\lambda(s_k - 0)$ (s_k - arc coordinate of the corner point) satisfy certain conditions which are so as if in the x - y -plane there would exist a continuously differentiable function $f(x,y)$ which satisfies (1). It is stated that (1) represents a continuously reversible operator in the Banach space.

INSTITUTION: Math.Inst., Acad.Sci.

NIKOL'SKIY, S.M.

NIKOL'SKIY, S.M.

Letter to the editors of "Uspekhi matematicheskikh nauk."

Uspekhi matematicheskikh nauk 12 no. 3: 264 Ny-Je '57.

(NINA 10:10)

(Spaces, Generalized)

AUTHOR: NIKOL'SKIY, S.M. (Moscow) 39-1-8/6
 TITLE: Boundary Properties of the Functions Which are Defined on Domains With Joints. II. Harmonic Functions on Rectangular Domains (Granichnyye svoystva funktsiy, opredelennykh na oblasti s uglovymi tochkami. II. Garmoniicheskiye funktsii na pryamougol'nykh oblastyakh).
 PERIODICAL: Matematicheskii Sbornik, 1957, Vol. 43, Nr 1, pp. 127-144 (USSR)
 ABSTRACT: The present paper is a continuation of [Ref.15]. By specializing the consideration to harmonic functions the author obtains more extensive results as in the first part of the paper. The notations of [Ref.12] and [Ref.15] are used. The author considers the determination of the solution of the differential equation $\Delta u = 0$ in the rectangle $G: 0 < x < \tau, 0 < y < a$ for the boundary conditions $u(0,y) = u(\tau,y) = u(x,a) = 0; u(x,0) = \varphi(x)$, where $\varphi(x)$ is a function given on $(0,\tau)$.
 Theorem: In order that the solution of the considered boundary value problem belongs to the class $H_2(r + \frac{1}{2})(G;M)$,
 it is necessary and sufficient that φ belongs to the class $H_2^c(r)$ (cM) after odd periodical continuation, where c is a

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Boundary Properties of the Functions Which are Defined on Domains With Joints. II. Harmonic Functions on Rectangular Domains. 39-1-6/8

constant independent of M .

Let Ω be a domain which is bounded by a finite number of intervals parallel with the axes, Γ the boundary of Ω .

Let $0 = s_0, s_1, \dots, s_{N-1}$ be the joints of Γ . Let the arc s be counted from $s_0 = 0$.

Theorem: Let on Ω a harmonic function $u(x,y) \in H_p^{(r)}(\Omega)$ be defined, where r and $\bar{q} = r - 1/p > 0$ are non-integers. Furthermore it is assumed that $\bar{q} = \bar{q} + \alpha$, where \bar{q} is an integer and $0 < \alpha < 1$. Then the boundary function $u|_{\Gamma} = f(s)$ satisfies the following conditions:

1.) $f \in H_p^{(\bar{q})}([s_k, s_{k+1}])$ on each section $[s_k, s_{k+1}]$ of Γ ,

$$\text{where } \|f\|_{L_p(s_k, s_{k+1})}^{(\bar{q})} \leq c \|u\|_{L_p(\Omega)}^{(r)}$$

2.) For the derivatives $f^{(2l)}(s)$, where $2l \leq \bar{q} - 1$, it

$$\text{holds: } f^{(2l)}(s_{k+0}) = (-1)^l f^{(2l)}(s_{k-0})$$

3.) For $\bar{q} = 2l$ it holds

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Boundary Properties of the Functions Which are Defined on Domains With Joints. II. Harmonic Functions on Rectangular Domains. 39-1-8/8

$$\left(\int_0^h \left| \varphi^{(2l)}(s_k+s) - (-1)^{2l} \varphi^{(2l)}(s_k-s) \right|^p ds \right)^{\frac{1}{p}} \leq c \|u\|_{L_p(\Omega)}^{(r)h^\alpha}$$

Theorem: Let $p = 2, \varphi = r - \frac{1}{2} > 0, \varphi = \bar{\varphi} + \alpha$ ($\bar{\varphi}$ integer, $0 < \alpha < 1, \alpha \neq \frac{1}{2}$). Let a function $\varphi(s)$ of the arc length be defined on the contour Γ of the rectangle $G: 0 \leq x \leq a, 0 \leq y \leq b$, whereby
 1.) $\varphi \in H_2^{(\varphi)}([s_k, s_{k+1}]), k=0, 1, 2, 3$ 2.) $\varphi^{(2l)}(s_k + 0) = (-1)^{2l} \varphi^{(2l)}(s_k - 0)$ where $2 \leq \bar{\varphi} - 1$ 3.) For $\bar{\varphi} = 2 \pm 1$ it is assumed to hold

$$\left(\int_0^h \left| \varphi^{(2l)}(s_k+s) - (-1)^{2l} \varphi^{(2l)}(s_k-s) \right|^2 ds \right)^{1/2} \leq Mh^\alpha$$

Then the function $u(x,y)$, harmonic on G , which satisfies the condition $u|_\Gamma = \varphi(s)$ belongs to the class $H_2^{(r)}(G)$, where

$$\|u\|_{L_2(G)}^{(r)} \leq c \sum_{k=0}^3 \|\varphi\|_{L_2(s_k, s_{k+1})}^{(\varphi)}$$

and where c is a constant independent of the second factor. 16 Soviet and 2 foreign references are quoted.

Card 3/4

AUTHOR: NIKOL'SKIY, S.M.

20-4-7/52

TITLE: On a Variation Problem of Hilbert (Ob odnoy variatsionnoy zadache Gil'berta)

PERIODICAL: ^{SSSR/} Doklady Akademii Nauk, 1957, Vol. 117, Nr. 4, pp. 573-575 (USSR)

ABSTRACT: A variation problem considered by Hilbert [Ref. 17] on the Riemannian surface is investigated by the author in the n -dimensional space. Let G be a domain of the real E_n being bounded by a two times differentiable surface \mathcal{A} . On $E_n - \mathcal{A}$ let be defined a function f having generalized derivatives of first order and satisfying the condition

$$(1) \quad D[f] = \int_G \sum_1^n \left(\frac{\partial f}{\partial x_i} \right)^2 dG < \infty.$$

In almost all points $Q \in \mathcal{A}$ there exist $\lim_{P \rightarrow +Q} f(P) = f_+(Q)$ and

$\lim_{P \rightarrow -Q} f(P) = f_-(Q)$, where $+$ and $-$ denote the direction of the

Card 1/2

approximation to \mathcal{A} . Let H be the set of functions $\varphi = f_+$, where

On a Variation Problem of Hilbert

20-4-7/52

f is an arbitrary function defined on $E_n - \Lambda$ which satisfies (1).
 On Λ let be given continuously differentiable functions $a(Q) \neq 0$
 and $b(Q)$, besides let $\mathcal{K}(Q) \in H$. Let γ be an $(n-1)$ -dimensional
 parallel in E_n but not in Λ . Let $af_+ - bf_- = \mathcal{K}$ on Λ and
 $\int_{\gamma} f d\gamma = 0$. Among the functions f with these properties there
 exists one uniquely determined function u for which $D[f]$
 reaches a minimum. The function u is harmonic on $E_n - \Lambda$.
 1 Soviet and 1 foreign references are quoted.

ASSOCIATION: Mathematical Institute im. V.A. Steklov, Academy of Sciences USSR
 (Matematicheskii institut im. V.A. Steklova Akademii nauk SSSR)
PRESENTED: By M.A. Lavrent'yev, Academician, 6 June 1957
SUBMITTED: 22 May 1957
AVAILABLE: Library of Congress
 Card 2/2

PHASE I BOOK EXPLOITATION

708

Nikol'skiy, Sergey Mikhaylovich

Kvadraturnyye formuly (Quadrature Formulas) Moscow, Gos. izd-vo fiziko-mat. lit-ry, 1958. 122 p. (Series: Biblioteka prikladnogo analiza i vychislitel'noy matematiki) 8,000 copies printed.

Ed.: Solntsev, Yu.K.; Tech. Ed.: Yermakov, Ye.A.; Ed. of Series: Sobolev, S.L., Academician.

PURPOSE: This book is intended for engineers, scientific workers and senior students acquainted with mathematical analysis on the vtuz level.

COVERAGE: This small monograph on the numerical integration formulas should be considered a supplement to existing monographs in this field. The monograph deals with certain general theoretical problems which are connected with any numerical integration formula, without regard to its individual properties. In the first place, general

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Quadrature Formulas

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methods of determining the errors of numerical integration formulas for various classes of functions are studied. On the basis of these methods constants are calculated for many basic numerical integration formulas of certain classes of functions, some of them for the first time in mathematical literature. Rigorous mathematical substantiation of the characteristic properties of numerical integration formulas which depend on the degree of the polynomials and for which a given formula is exact, is given. The problem of the best numerical integration formulas for certain classes of functions and under certain conditions is presented. The book deals mostly with one-dimensional numerical integration formulas. Multi-dimensional numerical integration formulas are investigated only in cases where they can be reduced to one-dimensional ones. Soviet personalities mentioned include: P. Plikh, who took part in the calculation of constants; Yu.Ya.Doronin, who also took part in the calculation of constants and in writing Chapter 5; A.I.Kiselev; K.M.Kardashevskiy and T.A.Shaydayeva, authors of unpublished works on the best numerical integration formulas; and A.N.Kolmogorov, who first stated the problem of the best numerical integration

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Quadrature Formulas

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formulas. The author thanks S.B.Stechkin, Professor V.I.Krylov, graduate student V.V.Fufayev and Ya.K. Sointsev for help in preparing the book. There are 16 references, of which 14 are Soviet (including 2 translations), and 2 English.

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AVAILABLE: Library of Congress (QA41.N5)

Card 4/4

LK/hcr
11-10-58

AUTHOR: Nikol'skiy, S.M. SOV/38-22-3-1/9
 TITLE: ~~Embedding Theorem for Functions~~ the Partial Derivatives of which are Considered in Different Metrics (Teorema vlozheniya dlya funktsiy s chastnymi proizvodnymi, rassmatrivayemyimi v razlichnykh metrikakh)

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1958, Vol 22, Nr 3, pp 321-336 (USSR)

ABSTRACT: The paper is a further contribution of the author to the theory of differentiable functions of several variables. Generalizing the functional classes formerly considered the author introduces the functional class

$$H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G; M). \text{ It is defined as the}$$

intersection of the classes $H_{p_1, \dots, p_n}^{(r_1)}(G; M)$ which were investigated by the author in [Ref 11, 12]. Accordingly, the partial derivatives of $f \in H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G; M)$ with respect to the dif-

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Embedding Theorem for Functions the Partial Derivatives of Which are Considered in Different Metrics SOV/58-22-5-1/9

ferent variables x_i are considered in different metrics L_{p_i} .

At first the author proves an embedding theorem for the case that G is identical with the whole R_n , $r_i > 0$, $1 \leq p_i \leq q < \infty$,
Let's

$$1 \leq i \leq n, \varphi_i = \frac{r_i}{r_i} > 0 \quad (i=1, \dots, n), \kappa_i = 1 - \sum_{l=1}^n \frac{1/p_l - 1/q}{r_l}$$

$$\begin{array}{l} \varphi_i \\ \kappa_i \end{array} \left| \begin{array}{l} 1 - \sum_{l=1}^n \frac{1/p_l - 1/q}{r_l} \\ - \sum_{l=1}^n \frac{1/p_l - 1/q}{r_l} \end{array} \right| \begin{array}{l} - \frac{1}{q} \sum_{l=1}^n \frac{1}{r_l} \\ 1 - \frac{1}{q} \sum_{l=1}^n \frac{1}{r_l} \end{array} \right|$$

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Embedding Theorem for Functions the Partial Derivatives SOV/58-22-5-1/9
of Which are Considered in Different Metrics

For fixed (x_{m+1}, \dots, x_n) then $f(x_1, \dots, x_n) \in H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(\mathbb{M})$
belongs as a function of x_1, \dots, x_m to the class $H_q^{(r_1, \dots, r_m)}(\mathbb{M})$
Here it is

$$\|f\|_{L_q^{(n)}} + M < c \left(\min_{1 \leq i \leq n} \|f\|_{L_{p_i}^{(n)}} + M \right) .$$

where c is independent of the brackets.
The proof is based on the approximation of f by entire functions of finite degree (of exponential type). A theorem of Jackson is generalized, and the inequalities for norms due to the author [Ref 10,11] are used. It is shown that the given embedding theorem cannot be improved in a certain sense.

Then a theorem on the compactness of the class $H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}$

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Embedding Theorem for Functions the Partial Derivatives of Which are Considered in Different Metrics SOV/58-22-3-1/9

is proved. Finally it is shown how to obtain from the proved embedding theorem analogous theorems in other cases (another G, periodic case) by continuation of the functions. There are 17 references, 15 of which are Soviet, 1 American, and 1 Italian.

PRESENTED: by S.L. Sobolev, Academician

SUBMITTED: June 17, 1957

1. Functions--Theory

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SOV/38-22-5-5/10

AUTHOR: Nikol'skiy, S.M.

TITLE: The Variation Problem of Hilbert (Variatsionnaya zadacha Gil'berta)PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1958,
Vol 22, Nr 5, pp 599-630 (USSR)ABSTRACT: The variation problem considered by Hilbert in [Ref 17] is
generalized.

In the n -dimensional space $R_n = R$ of the points (x_1, \dots, x_n) let
be given a closed two times continuously differentiable surface Λ
and outside of it an $(n-1)$ -dimensional parallelepiped \bar{G} . The
continuously differentiable functions $a(Q) \neq 0$, $b(Q)$ and a
function $\kappa(Q)$ are defined on Λ ; $\kappa(Q)$ satisfies weaker con-
ditions which are fulfilled e.g. if $\kappa(Q)$ in the metric $L_2(\Lambda)$
satisfies the Lipschitz condition with the exponent $\frac{1}{2} + \epsilon$ ($\epsilon > 0$).

The author considers a class \bar{M} of functions defined on R , for

which $D[f] = \int_R \sum_1^n \left(\frac{\partial f}{\partial x_i} \right)^2 dR < \infty$, $af_+ - bf_- = \kappa$ on Λ , where

besides $\int_{\bar{G}} f d\bar{G} = 0$. He proves the existence and uniqueness of

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The Variation Problem of Hilbert

SOV/58-22-5-3/10

the solution of the variation problem

$$\min_{f \in \mathcal{U}} D[f] = D[u];$$

furthermore it is shown that u is harmonic on $R-\Lambda$. The

derivatives $\frac{\partial f}{\partial x_i}$ are comprehended in the generalized sense.

There are 18 references, 15 of which are Soviet, 2 German, and 1 Hungarian.

SUBMITTED: December 12, 1957

PRESENTED: by S. L. Sobolev, Academician

Card 2/2

AUTHOR: Nikol'skiy, G. H., Doctor of Physical and Mathematical Sciences. -30-1-14/39

TITLE: **Mathematical Conference in Szeged**
(*Matematicheskaya Konferentsiya v Segede*).

PERIODICAL: Vestnik AN SSSR, 1958, Vol. 23, Nr 1, pp. 84-85 (USSR)

ABSTRACT: The conference took place in connection with the 10 years' celebrations of the Hungarian Mathematical Society from September 21 to September 23, 1957, and was organized jointly with the Hungarian Academy of Science. The Society is named after the great Hungarian mathematician Yanosh Boya, who, independent of Lobachevskiy, discovered non-Euclidian geometry. The names of F. Ris, A. Khar, and B. Nad' are connected with the university of Szeged. At present the professors I. Kalmar (logics), L. Redei (geometry, algebra), and B. Nad' (analysis of functions, theory of functions) are leading in their respective fields. The periodical "Acta Mathematica" is being published since 1922. The conference was attended by foreign guests from Bulgaria, the German Democratic Republic, Poland, Romania, USSR, Czechoslovakia, and Yugoslavia. L. Kalmar's report was about the theoretical description of an electron machine planned by him. G. Khaydash, in his report on

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Mathematical Conference at Szeged

"Accuracy in Geometry", spoke about the character of the representation of geometric problems in the textbook for students. G. Alekshich spoke about the convergence of orthogonal series, and B. Had' on some new results obtained with the theory of operators in Gilbert's space. L. Redei spoke about remarkable points of the triangle, O. Varga on new results obtained by the theory of differential-geometric spaces, L. Fuks on the extension of the Khayeski theorem to infinite groups. Among the foreign guests it was the author (USSR), who spoke about the theorems of the enclosure (vlozheniye) for functions the different partial derivatives of which are differently normed; Kh. Grell (German Democratic Republic) spoke about regular prime ideals; T. Popovichi (Roumania) spoke about a problem connected with the theory of ordinary differential equations. In the sections for algebra and the theory of numbers, analytics, geometry, logics, the theory of mathematical machines, the theory of probabilities, and pedagogics special reports were heard and discussed. The Collective of Hungarian Mathematicians successfully combines the development of a number of general theoretical fields of mathematics with the development of the applied fields, especially of statistics and

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Mathematical Conference in Szeged

30-2-1, '33

Machine Mathematics.

AVAILABLE:

Library of Congress

1. Mathematics-Conference

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59-45-2-4/7

AUTHOR: Nikel'skiy, S.K. (Moscow)

TITLE: The Boundary Properties of Functions Defined on a Domain With Vertices. III. Connection With the Polyharmonic Problem (Granichnyye svoystva funktsiy, opredelennykh na oblasti s uglovymi tochkami, III. Svyaz' s poligarmonicheskoy zadachey)

PERIODICAL: Matematicheskiy sbornik, 1958, Vol 45, Nr 2, pp 181-194 (USSR)

ABSTRACT: The author considers the following problem: In the plane let be given a function $f(x,y)$ belonging to the class $H_p(\Gamma)$. Let Γ be a piecewise smooth curve, let the angles at the vertices be different from 0 and 2π . Which necessary and sufficient conditions have to be satisfied by the functions $\psi_0(s), \psi_1(s), \dots$ in order that

$$f|_{\Gamma} = \psi_0(s), \quad \frac{\partial f}{\partial n}|_{\Gamma} = \psi_1(s), \dots$$

where n denotes the normal to Γ ? In the special case $r - \frac{1}{p} < 1$

this problem was treated by the author in an earlier paper [Ref 4]. Now these results are used in order to answer the question in the general case (with the exception of some r -values). The principal result asserts that if $f \in H_p(\Gamma)$, then $\psi_\lambda \in H_p(\Gamma - \lambda - \frac{1}{p})$.

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The Boundary Properties of Functions Defined on a Domain With Vertices. III. Connection With the Polyharmonic Problem 59-45-2-4/7

and reversely. The author discusses the possibility to extend the result to piecewise smooth surfaces in an n -dimensional space. Finally the author uses the results for obtaining (with the aid of the variation principle) a theorem of existence for the solution of the polyharmonic problem: $\Delta^l u = 0$ on G ,

$$\frac{\partial^\lambda u}{\partial n^\lambda} \Big|_\Lambda = \psi_\lambda \quad (\lambda=0,1,\dots,l-1), \text{ where } \Lambda \text{ is the piecewise}$$

smooth boundary of G .

There are 7 references, 6 of which are Soviet and 1 English.

SUBMITTED: December 22, 1956

1. Topology 2. Functions--Theory

Card 2/2

20-118-1-9/58

AUTHOR: NIKOL'SKIY, S.M.
TITLE: Embedding Theorems for Functions With Partial Derivatives Which are Considered in Different Metrics (Teoremy vlozheniya dlya funktsiy s chastnymi proizvodnymi, rasstrivayemyi 7 razlichnykh metrikakh)

PERIODICAL: Doklady Akademii Nauk/1958, Vol 118, Nr 1, pp 35-37 (USSR)

ABSTRACT: In R_n there are considered functions $f = f(x_1, \dots, x_n)$. The norm is defined by

$$\|f\|_{L_p^{(m)}} = \left(\int_{R_m} |f(x_1, \dots, x_m, x_{m+1}, \dots, x_n)|^p dx_1 \dots dx_m \right)^{1/p}$$

$m = 1, \dots, n$

Furthermore let the positive numbers M, r_1, p_1 be given, where $1 \leq p_1 \leq \infty$. Let be $f \in H_{p, r_1}^{(r)}(M)$, if f and its

derivatives $\frac{\partial^k f}{\partial x_1^k}$ ($k = 0, 1, \dots, \bar{r}$), where $r = \bar{r} + 1$,

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Embedding Theorems for Functions With Partial Derivatives 20-118-1-9/58
Which are Considered in Different Metrics

\bar{r} integer and $0 < \alpha \leq 1$, are integrable on R_n in p -th power
and furthermore

$$\|f_{x_1}^{(\bar{r})}(x_1+h, x_2, \dots, x_n) - f_{x_1}^{(\bar{r})}(x_1, \dots, x_n)\|_{L_p(n)} \leq M|h|^\alpha \text{ for } \alpha = 1 \text{ and}$$
$$\|f_{x_1}^{(\bar{r})}(x_1+h, x_2, \dots, x_n) - 2f_{x_1}^{(\bar{r})}(x_1, \dots, x_n) + f_{x_1}^{(\bar{r})}(x_1-h, x_2, \dots, x_n)\|_{L_p(n)} \leq$$

$M|h|^\alpha$ for $\alpha = 1$. Let be $f \in H_{(p_1, \dots, p_n)}^{(r_1, \dots, r_n)}(M)$ if f simul-

taneously belongs to all the classes $H_{p_1 x_1}^{(r_1)}(M)$. If $p_1 = p_2 = \dots = p_n = p$, let be $H_{p_1 \dots p_n}^{(r_1, \dots, r_n)} = H_p^{(r_1, \dots, r_n)}$.

Theorem: Let be $r_i > 0$, $1 \leq p_i \leq q \leq \infty$; n, n natural numbers, $1 \leq m \leq n$. Furthermore let be $s_i = \frac{r_i}{p_i} > 0$ ($i = 1, \dots, n$) where

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APPROVED FOR RELEASE: Tuesday, August 01, 2000
Embedding Theorems for Functions With Partial Derivatives 20-118-1-9/58
Which are Considered in Different Metrics

$$\alpha = \begin{vmatrix} 1 - \sum_{i=1}^n \frac{1}{p_i} - \frac{1}{q} & -\frac{1}{q} \sum_{i=1}^n \frac{1}{r_i} \\ -\sum_{m=1}^n \frac{1}{p_m} - \frac{1}{q} & 1 - \frac{1}{q} \sum_{m=1}^n \frac{1}{r_m} \end{vmatrix}, \quad \alpha_i = 1 - \sum_{l=1}^n \frac{1}{p_l} - \frac{1}{p_i}$$

Let be $f \in H_{p_1 \dots p_n}^{(r_1, \dots, r_n)}(M)$. For every fixed (x_{m+1}, \dots, x_n)

f belongs as a function of x_1, \dots, x_m to the class

$H_q^{(s_1, \dots, s_m)}(M)$. Here it is

$$\|f\|_{L_q(m)} + M < c (\min_i \|f\|_{L_{p_i}(n)} + M)$$

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• Embedding Theorems for Functions With Partial Derivatives 20-118-1-9/58
Which are Considered in Different Metrics

.. ASSOCIATION: Matematicheskiy institut imeni V.A. Steklova Akademii nauk
SSSR (Mathematical Institute imeni V.A. Steklov, Academy of
Sciences, USSR)

PRESENTED: June 25, 1957, by S.L. Sobolev, Academician

SUBMITTED: June 20, 1957

AVAILABLE: Library of Congress

Card 5/5

16(1)

AUTHOR:

Nikol'skiy, S.M.

SOV/38-23-2-5/10

TITLE:

Some Properties of Differentiable Functions Defined on an Open
 n - Dimensional Set (Nekotoryye svoystva differentsiruyemykh
 funktsiy, zadannykh na n - mernom otkrytom mnozhestve)

PERIODICAL:

Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1959,
 Vol 23, Nr 2, pp 213 - 242 (USSR)

ABSTRACT:

The former results of the author [Ref 5,6] are generalized :

Formerly the author considered the classes $\bar{W}_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G)$

and $H_{p_1 \dots p_n}^{(r_1, \dots, r_n)}(G)$ under the assumption that G is the n -di-

mensional R_n or special parts of it; now G can be an arbitrary
 open set of the R_n . In a parallel epipedon \mathcal{G} , the edges of
 which are parallel with the axes, at first the author considers

the function $f(\bar{x}) \in \bar{W}_{p_1 \dots p_n}^{(r_1, \dots, r_n)}(\mathcal{G})$ or $\in H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(\mathcal{G})$.

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Some Properties of Differentiable Functions Defined
on an Open n - Dimensional Set

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He gives an effective representation $f = P + F$, where P is a polynomial of the order \bar{r}_1 in x_1 , while F depends linearly

on the derivatives $\frac{\partial^{\bar{r}_1} f}{\partial x_1^{\bar{r}_1}}$ (\bar{r}_1 is the integer part of r_1).

Then some theorems on the continuation of f beyond $\tilde{\sigma}$ are proved, from which there are obtained embedding theorems for the $f \in \mathcal{H}(\tilde{\sigma})$, $H(\tilde{\sigma})$. Then the author defines new classes

$\bar{h}_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G)$ and $\bar{H}_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G)$ being little different

from $H_{p_1, \dots, p_n}^{(r_1, \dots, r_n)}(G)$. For functions belonging to these classes

\bar{h} , \bar{H} a general embedding theorem is proved, where G can be

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6

Some Properties of Differentiable Functions Defined on an Open n - Dimensional Set 307/38-23-2-5/10

an arbitrary open set of R_n . The theorem is analogous to the result in [Ref 5,6] for $n = m$; its assumptions cannot be weakened. The author mentions I.A. Ezrokhi, V.K. Dzyadyk, Yu.D. Kashchenko, S.L. Sobolev. There are 9 references, 6 of which are Soviet, 1 Italian, 1 English, and 1 Swedish.

PRESENTED: by S.L. Sobolev, Academician.

SUBMITTED: October 9, 1958

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16.3500

~~16(1)~~

AUTHOR:

Nikol'skii, S.M.

05700

SOV/58-23-5-4/8

TITLE:

On the Estimation of a Function, the Dirichlet Integral of Which is Finite, and Application to Boundary Value Problems

PERIODICAL:

Investiya Akademii nauk SSSR, Seriya matematicheskaya, 1959, Vol 23, Nr 5, pp 677 - 696 (USSR)

ABSTRACT:

Theorem : Let a function f with the finite Dirichlet integral

$$(1) \quad D_{\Delta} [f] < \infty$$

be given on the region $\Delta = \{a_i \leq x_i \leq b_i, i = 1, \dots, n\}$.Then it exists a constant c depending on Δ so that

$$(2) \quad \int_{x_1}^{x_1+h} \int_{a_2}^b \dots \int_{a_{n-1}}^b f(x_1, \dots, x_n)^2 dx_1 \dots dx_{n-1} <$$

$$< c \left(\|f\|_{L_2(\Delta)}^2 + D_{\Delta} [f] \right) h \ln \frac{1}{h}, \quad (h > 0)$$

holds for all $x_n \in [a_n, b_n]$ and all x_1 and h for which

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On the Estimation of a Function, the Dirichlet Integral of $SCV/58-23-5-4/8$
Which is Finite, and Application to Boundary Value Problems

$a_1 \leq x_1 < x_2 + h \leq b_1$. (2) is strongly exact in the sense of
the order concerning h .

The estimation (2) is exclusively used in order to solve the
Hilbert and Dirichlet problem in the space with the variation
method, if the boundary conditions are given on a boundary
surface. The author especially shows that the results obtained
in [Ref 7] in the investigation of the Hilbert problem for
closed sufficiently smooth surfaces are also valid for parts
of these surfaces, if the boundary is sufficiently smooth.
The author gives some further completions for [Ref 7].
He mentions N.I. Muskhelishvili, V.A. Khvedelidze and A.V.
Bitsadze.

There are 11 references, 10 of which are Soviet, and 1 Swedish.

PRESENTED: by S.L. Sobolev, Academician

SUBMITTED: December 8, 1959

Card 2/2

NIKOL'SKIY, S.N.

Boundary estimation for a function which is harmonic in an
n-dimensional region. Sib.mat.natur. 1 no.1:76-87 Nov-Dec '60.
(NIMA 13:11)

(Functional analysis)

(Mathematic physics)

NIKOL'SKIY, S. M.

Cycle of reports on the theorems of insertion, extension, and approximation of the differential multivariable functions. Rev math pures 6 no.2:209-256 '61.

NIKOL'SKIY, S.S.

Imbedding, continuation, and approximation theorems of differentiable functions of several variables. Usp. mat. nauk
16 no.5:63-114 8-0 '61. (MIRA 14:10)
(Functions of several variables)

16.5700

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S/517/61/064/000/003/006
D299/D301

AUTHOR: Nikol'skiy, S. M.

TITLE: On inequalities between partial derivatives

SOURCE: Akademiya nauk SSSR. Matematicheskiy institut. Trudy.
v. 64, 1961, 147-164

TEXT: Natural (or positive) numbers r_1, \dots, r_n are given; the class $W(r_1, \dots, r_n)(G)$ is defined as consisting of functions f with finite norm

$$\|f\|_{W_p(\vec{r})(G)} = \|f\|_{L_p(G)} + \sum_1^n \left\| \frac{\partial^{r_k} f}{\partial x_k^{r_k}} \right\|_{L_p(G)} \quad (3)$$

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