LEONTOVYC, Roman, inz.

Notes on the brown slime flux of poplars in Slovakia. Les cas 9 no.1:61-70 Ja '63.

1. Vyskumny ustav lesneho hospodarstva, Banska Stiavnica.

LEONT'YEV, A.

Fiftieth anniversary of Soviet trade unions. Grazhd. av. 14 no.8: 1-3 Ag '47. (MLRA 10:9)

1. Sekretar' TSentral'nogo komiteta profsoyuza aviarabotnikov SSSR.
(Aeronautics, Commercial)

LEONT'YEV, A.

Struggle of trade unions and the economic laws of capitalism. Vsem. prof.dvizh. no.9:13-15 S'55. (MLRA 8:11)
(Trade unions)

LEONT'YEV, A.

What are relations between labor and capital based on. Vsem.prof. dvizh. no.11:43-3 of cover N '55. (MLRA 9:1)
(Industrial relations)

LEONT'YEV, A.

Development of capitalism and conditions of the laboring classes. Vsem.prof.dvizh. no.12:43-3 of cover D '55. (MLRA 9:4)
(Labor and laboring classes)(Capitalism)

LEONT'YEV, A.

Technical progress under capitalism and interests of the laboring class. Vsem.prof.dvizh. no.1:42-44 Ja '56. (MLRA 9:5)
(Machinery in industry) (Labor and laboring classes)

LEONT'YEV, A.

Labor and technology under capitalism. Sots.trud no.1:110-117 Ja '56. (Technology) (Capitalism) (MLRA 9:7)

LEONT'YEV, A.

[redacted]

Productivity and intensity of work under capitalism. Vses.prof. dvizh. no.2:42-45 F '56. (MLRA 9:5)

(Labor productivity)

LEONT'YEV, A

84-8-2/36

AUTHOR: Leont'yev, A., Secretary of the TsK of the Aviation Workers' Trade Union of the USSR

TITLE: Fiftieth Anniversary of Soviet Trade Unions (Pyatidesyatiletiye sovetskikh profsoyuzov)

PERIODICAL: Grazhdanskaya Aviatsiya, 1957, Nr 8, pp. 1-3 (USSR)

ABSTRACT: The article commemorates the fiftieth anniversary of Soviet trade unions and their role in labor movements. The aviation workers' trade union was established in 1934 and comprises workers and employees of the GVF, the VVS and the hydrometeorological service. The length of internal air routes grew from 6,000 km in 1928 to 93,300 km in 1937. The leading pilots of this period were N. P. Shebanov, N. I. Novikov and V. I. Monakhov. During the German-Soviet war, all attention of the Soviet trade unions was fixed on achieving victory. Hence a large scale expansion began only after the war. The LERM's of the Novosibirsk and Vnukovo airports propounded a new speed method of replacing and reconditioning engines. Flow methods in repair shops allowed a 30 to 50 per cent increase in efficiency. Due to numerous innovations, 4,000,000 rubles were saved in 1956. Hundreds of thousands of square meters of housing were opened for occupancy. During the sixth Five-Year plan, the GVF has received jet planes. A large construction program is under way for the GVF personnel

Card 1/2

84-58-6-50/59

AUTHOR: Leont'yev, A., Secretary, Central Committee of the Aviation Workers' Trade Unions

TITLE: Seventeen Days in France (Semnadtsat' dney vo Frantsii)

PERIODICAL: Grazhdanskaya aviatsiya, 1958, Nr 6, p 38 (USSR)

ABSTRACT: The author, head of a delegation of representatives of Soviet aviation trade unions to France, relates his impressions and observations of French civil aviation and its trade union organization.

1. Civil aviation--France

Card 1/1

LEONT'YEV, A.

SOV/84-58-10-31/54

AUTHORS: Mishinkin, V., Chairman of Central Committee of USSR Trade Union of Aviation Workers; ~~Leont'yev, A.,~~ Secretary of Trade Union Central Committee; Lapeyre, Roger, General Secretary of the Federation of Public Works and Transportation, and Dault [?], Jean, General Secretary of Trade Union Cartel of Commercial Aviation

TITLE: Joint Statement of Central Committee of USSR Trade Union of Aviation Workers and of the Representatives of the Federation of Public Works and Transportation "Force Ouvrière" (France). (Sovmestnoye zayavleniye Tsentral'nogo Komiteta profsoyuza aviarabotnikov SSSR i predstaviteley Federatsii Obshchestvennykh rabot i Transporta "Fors Uvrier" (Frantsiya)

PERIODICAL: Grazhdanskaya aviatsiya, 1958, Nr 10, p 18 (USSR)

ABSTRACT: A jointly signed statement was issued 14 September 1958 in Moscow by the Central Committee of the USSR Trade Union of Aviation Workers and the French delegation of Trade Unions of Civil Aviation, members of the Federation of Public Works and

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SOV/84-58-10-31/54

Joint Statement of Central Committee (Cont.)

Transportation "Force Ouvriere", headed by Roger Lapeyre, General Secretary of the Federation, and Jean Danit [?], General Secretary of the Cartel of Commercial Aviation. The delegates visited Moscow, Leningrad, Kiyev, Stalingrad, and Sochi. They interviewed leaders of Aeroflot (Air Fleet), members of the Central Committee of Trade Unions, called at airfields, inspected their service quarters and aircraft repair shops, and visited the Kiyevskiy institut inzhenerov Grazhdanskogo vozdushnogo flota (Kiyev Institute of Engineers of Civil Air Fleet). The delegation secured data on working conditions, the wages of workers in different brackets, and rest and medical facilities available at Sochi sanatorium. The delegates had arrived in Moscow in a Super-Constellation plane of Air France and returned home in a Tu-104 plane of Aeroflot (Air Fleet).

ASSOCIATION: Central Committee of USSR Trade Union of Aviation Workers (TsK profsoyuza aviarabotnikov SSSR)

Card 2/2

LEONT'YEV, A., inzh.

Anemometer. Bezop.truda v prom. 3 no.9:31 S '59.
(MIRA 13:2)

1. Gosgortekhnadzor Kazakhskoy SSR.
(Anemometer)

LEONT'YEV, A., imzh. (g.Sverdlovsk)

Reinforced concrete bridge in Sverdlovsk. Zhil.-kom. khoz. 10 no.8:25-26 '60. (MIRA 13:9)
(Sverdlovsk--Bridges, Concrete)

APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000929310005-2"

GUSE., V., general-mayor; BIBA, G., polkovnik veterinarnoy sluzhby;
LEONT'YEV, A., podpolkovnik veterinarnoy sluzhby

For efficient management of farms attached to army messes. Tyl
i snab.Sov.Voor.Sil 21 no.1:61-63 Ja '61. (MIRA 14:61.
(Russia--Army--Commissariat)

KARPOV, A.A., inzh.; KUSTOBAYEV, G.G., inzh.; LAUSHKIN, N.P., inzh.; LANGE, Z.I., inzh.; NOSYREVA, M.D., inzh.; SAVEL'YEV, G.V., inzh.; SHCHULEPNIKOV, I.S., inzh.; Prinimali uchastiye: SYCHKOV, B.A., inzh.; MILIKHIN, A.Ye., inzh.; ZAYTSEV, R.A., inzh.; ZARZHITSKIY, Yu.A., inzh.; LEONT'YEV, A.L., inzh.; VIKTOROVA, T.Ye., inzh.; SERIKOV, A.A., inzh.

Operation of recuperator soaking pits in the 1150 MMK rolling mill. Stal' 22 no.8:753-758 Ag '62. (MIRA 15:7)

1. Magnitogorskiy metallurgicheskiy kombinat.
(Furnaces, Heating) (Rolling mills)

LEONT'YEV, A.

Economic problems of the communist labor movement. Vop. ekon. no.7:157-160 Jl '63. (MIRA 16:8)
(Socialist competition--Congresses)

L 02419-67 EWT(1) RO

ACC NR: AP6005046 (A) SOURCE CODE: UR/0350/65/000/011/0009/0010

AUTHOR: Leont'yev, A. (Agricultural chemistry department assistant)

ORG: Agricultural Chemistry Department of the Voronezh Agricultural Institute (Kafedra agrokhimii Voronezhskogo sel'skokhozyastvennogo instituta)

TITLE: Potassium phosphate fertilizers for peas

SOURCE: Zernobobovyye kul'tury, no. 11, 1965, 9-10

TOPIC TAGS: agriculture crop, fertilizer, potassium compound, phosphate

ABSTRACT: From 1962 to 1964 the effect of potassium phosphate fertilizers on peas of the Ramon 77 variety was investigated in a series of experiments conducted at the Voronezh Agricultural Institute in leached chernozem soil. In the first experimental series, superphosphate and potassium salt were introduced into the soil during fall fallowing and superphosphate was introduced again at time of sowing nitragin treated pea seeds in the spring. In the second series, nitragin treated pea seeds were planted in the spring at which time superphospate was introduced into the soil. Control pea seeds were not treated or fertilized. Study data indicate that potassium phosphate fertilizers significantly increase pea crop yields. This was observed not only in the wet years, but also in the relatively dry year of 1963. Application of superphosphate and potassium salt to soil during fallowing followed by application of

 UDC: 635.656:631.85

L 09449-67

ACC NR: AP6005046

superphosphate during sowing of nitragin treated pea seeds increases crop yields to 35 centners/hectare and enriches soil fertility. Orig. art. has: 1 table and 2 figures.

SUB CODE: 02/ 06/ SUBM DATE: none

Card 2/2

LEONT'YEV, A.A.

Adding an additional staff device to the wires of switch signal systems.
Avtom., telem. i sviaz' no.12:25 D '57. (MIRA 10:12)

1. Starchiy elektromekhanik Ostashevskoy distantsii signalizatsii i svyazi Kalininskoy dorogi.

(Railroads--Signaling)

38207. LEONT'YEV, A. A.

Opyt aeroseva saksaula v turkmenii. Les i step', 1949, No 8, s. 88-91 [illegible], A. A.

Afforestation

Evaluation of growing forests on sand Les i step' No. 4 April 1952

Monthly List of Russian Accessions, Library of Congress, August, 1952. UNCLASSIFIED.

LEONT'EV, A. A.

USSR/Forestry - Forest Culture. J-4

Abs Jour : Referat Zhur - Biologiya, No 16, 25 Aug 1957, 69133

Author : Leont'ev, A.A., Stepanov, A.M., Neborak, A.N., Koksharova, N.E., Kukorekina, E.A.

Inst :

Title : Most Effective Methods of Bind and Afforesting Shifting Sands.

Orig Pub : Byul. nauchn.-tekhn. inform. Sredneaz. n.-i. in-ta lesn. kh-va, 1955, No 1, 6-16

Abstract : Based on experiments conducted on sands of Turkmen and Uzbek SSR, recommendations are suggested on rationalization of sand consolidation measures. Instead of mechanical protection with plantings of shoots and seedlings, especially in districts with comparatively light winds, the use of a lightened spread of mechanical protection is recommended: yantak, reed, mace and wormwood in conjunction with combined sowings and plantings. In furrowed

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APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000929310005-2"

USSR/Forestry - Forest Culture. J-4

Abs Jour : Referat Zhur - Biologiya, No 16, 25 Aug 1957, 69133

grooves a mechanized sowing of haloxylon is suggested without mechanical protection. Data are given on protective construction, agrotechnique of cultivations and assortment of species.

Card 2/2 - 58 -

LEONT'YEV, Anatoliy Aleksandrovich; TIKHONOVA, I.S., red.;
BABAKHANOV, A., tekhn. red.

[Sandy deserts in Central Asia and their improvement by afforestation] Peschanye pustyni Srednei Azii i ikh lesomeliorativnoe osvoenie. Tashkent, Gos.izd-vo UzSSR, 1962. 158 p.
(MIRA 16:4)

(Soviet Central Asia--Sandy soils)
(Soviet Central Asia--Afforestation)

ACCESSION NR: AT4042313 S/0000/63/003/000/0357/0361

AUTHOR: Leont'yev, A.A., Smirnov, A.G.

TITLE: Mixing of an electrolyte by means of a transverse magnetic field in electrolytic processes

SOURCE: Soveshchaniye po teoreticheskoy i prikladnoy magnitnoy gidrodinamike. 3d, Riga, 1962. Voprosy* magnitnoy gidrodinamiki (Problems in magnetic hydrodynamics); doklady* soveshchaniya, v. 3. Riga, Izd-vo AN LatSSR, 1963, 357-361

TOPIC TAGS: electromagnetic mixing, electromagnetic stirrer, electrolysis, electroplating, hydromagnetics, ponderomotor force

ABSTRACT: The authors point out that electrolyte mixing by means of an external magnetic field constitutes an area in which there is a total lack of published research, despite the fact that this form of forced hydrodynamics may well be of practical as well as theoretical interest from the point of view of electrochemical technology. In the present article, the authors demonstrate the satisfactory mixing of an electrolyte by means of a transverse magnetic field during the electrolytic precipitation of metals (plating). The general applicability of the system of equations employed in hydrodynamics and electrodynamics to the specific problem of the electrolyte is demonstrated, with

ACCESSION NR: AT4042313

practically identical. Visual observations of the movement of the liquid in the bath during electroplating of iron and cobalt indicate that, in the transverse magnetic field, the electrolyte at the cathode noticeably rises and directional mixing increases in intensity both as the density of the elctric current is increased and as the strength of the magnetic field is decreased. It was also discovered that the imposition of the transverse magnetic field produced no changes in the texture of the electro-precipitated metal, with no predominant orientation of the crystals. Orig. art. has: 2 formulas and 2 figures.

ASSOCIATION: none

SUBMITTED: 04Dec63 ENCL: 00 SUB CODE: EM, MM

NO REF SOV: 004 OTHER: 001

LEONT'YEV, A. A.

"Nekotorye voprosy glottogeneza v svete sovremennykh paleontologicheskikh dannykh."

report submitted for 7th Intl Cong, Anthropological & Ethnological Sciences, Moscow, 3-10 Aug 64.

Leont'ev, A. F. On a class of functions defined by series of Dirichlet polynomials. Uspehi Matem. Nauk (N.S.) 3, no. 4(26), 176–180 (1948). (Russian)

This is a summary, without proofs, of the results of the author's thesis. Let $\{\lambda_n\}$ denote a sequence of distinct complex numbers of nondecreasing modulus. The author quotes [without reference] the result that the condition $\limsup n/|\lambda_n| = \infty$ is both necessary and sufficient for the set S of functions $\exp(-\lambda_n s)$ to be complete in every bounded region of the complex plane, i.e., for any holomorphic function to be represented by a uniformly convergent sequence of "Dirichlet polynomials" (1) $P_n(s) = \sum_{k=1}^{n} a_{nk} \exp(-\lambda_k s)$. If the set S is not complete and a sequence of type (1) converges uniformly in some region D, either the region D or the limit function must have special properties. The author states a number of theorems regarding these properties, with the hypotheses that the numbers λ_n are positive, that the density $\epsilon = \lim n/\lambda_n$ exists, that the region D contains in its interior a closed vertical segment of length $2\pi\epsilon$ on some line $s = x_0 + iy$, and that (1) converges to $f(s)$ uniformly in D. Under these conditions (1) converges in some half-plane $x > \beta$ $(\beta < x_0)$. The quantities $\lim_{n\to\infty} a_{ni} = \alpha_i$ exist; a second sequence of polynomials P_n' converges to $f(s)$ if and only if $\lim a_{ni}' = \lim a_{ni}$ for each i, and in particular the Dirichlet series (2) $\sum \alpha_i \exp(-\lambda_i s)$ converges to $f(s)$, if it converges. The sequence of polynomials $R_n(s) = \sum_{i=1}^{n} \alpha_i \exp(-\lambda_i s) L_n(\lambda_i)$, where $L_n(s) = \prod_{j=n+1}^{\infty}[1-(s/\lambda_j)^2]$ converges to $f(s)$ in some half-plane $x > \alpha$ $(\alpha \leqq \beta)$. If the function $f(s)$ is not entire, every closed segment of length $2\pi\epsilon$ on its axis of holomorphism contains a singularity of $f(s)$. There exists a sequence $\{m_k\}$ of positive integers, depending only on the sequence $\{\lambda_n\}$, such that the sequence of partial sums of order m_k of the series (2) converges to $f(s)$ in the half-plane $x > \alpha$. If the quantity $\delta = \limsup \lambda_n^{-1} \log |1/L_0'(\lambda_n)|$ is finite, the series (2) converges in the half-plane $x > \alpha + \delta$. If $\delta = \infty$, there exists a function for which the series (2) diverges everywhere. A necessary and sufficient condition for (2) to converge in the half-plane $x > a$ is that the function $f(s)$ be bounded in every half-plane $x > a + \epsilon$ $(\epsilon > 0)$. The author states that his results depend on the fact that, if the numbers α_j are determined by the identity $L_0(x) = \sum \alpha_j x^j$, the limit function of any uniformly convergent sequence of type (1) is a solution of the differential equation $\sum_{j=0}^{\infty} \alpha_j f^{(j)}(s) = 0$.

The author states that his results and methods can be applied to obtain theorems on interpolation of entire functions of exponential type. For example, a necessary and sufficient condition for the existence of an entire function $\omega(s)$ of exponential type with $\omega(\pm\lambda_n) = a_{\pm n}$, $|a_{\pm n}| < \exp\{C|\lambda_n|\}$, is that $\limsup n/|\lambda_n| < \infty$ and $\limsup |\lambda_n|^{-1} \log |1/L_0'(\lambda_n)| < \infty$.

R. P. Boas, Jr., and G. Piranian.

Source: Mathematical Reviews, Vol 10 No. 6

LEONT'YEV, A.F.

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Leont'ev, A. F. On interpolation in the class of entire functions of finite order. Doklady Akad. Nauk SSSR (N.S.) 61, 785-787 (1948). (Russian)

Let $\{\lambda_n\}$ be a sequence of complex numbers of non-decreasing absolute value and let $\{a_n\}$ be a sequence such that $\limsup \log\log|a_n|/\log|\lambda_n| \leq \rho$, $0<\rho<\infty$. The author gives the following set of necessary and sufficient conditions for the existence of an entire function $\omega(z)$, of order not exceeding ρ, such that $\omega(\lambda_n)=a_n$. (A) The exponent of convergence of $\{\lambda_n\}$ does not exceed ρ. (B) Let $F(z)$ be the canonical product with zeros at λ_n. Then

$$\limsup\,[\log|\lambda_n|]^{-1}\log\log|1/F'(\lambda_n)|\leq\rho.$$

R. P. Boas, Jr. (Providence, R. I.).

Source: Mathematical Reviews

LEONT'YEV, A. F.

Leont'ev, A. F. On entire functions of exponential type assuming given values at given points. Izvestiya Akad. Nauk SSSR. Ser. Mat. 13, 33–44 (1949). (Russian)

The author seeks conditions on a sequence $\{\lambda_n\}$ of complex numbers under which, for every sequence $\{a_{\pm n}\}$ satisfying $|a_{\pm n}| < e^{a|\lambda_n|}$, there will exist at least one entire function $\omega(z)$ of exponential type such that (*) $\omega(\pm\lambda_n) = a_{\pm n}$. His principal results are as follows. Assume the obviously necessary conditions that $\{\lambda_n\}$ has no finite limit point and $\limsup n/\lambda_n < \infty$, and introduce $F(z) = \prod(1 - z^2/\lambda_n^2)$. (1) The additional condition

$$\limsup |\lambda_n|^{-1} \log |1/F'(\lambda_n)| = \delta < \infty$$

is necessary and sufficient. (2) Let $F(z)$ be of type h and let $\limsup |\lambda_n|^{-1} \log |a_{\pm n}/F'(\lambda_n)| = \gamma$. Then for any positive ϵ there is at least one entire function $\omega(z)$ satisfying (*), of type less than $h + \gamma^+ + \epsilon$, where $\gamma^+ = \gamma$ if $\gamma \geq 0$, $\gamma^+ = 0$ if $\gamma < 0$. This result is sharp in the sense that sequences $\{\lambda_n\}$ and $\{a_{\pm n}\}$ can be found for which there is no $\omega(z)$ of type less than $h + \gamma^+$, satisfying (*). If the λ_n are real and positive and $\lim n/\lambda_n = \sigma < \infty$, then $h + \gamma^+ + \epsilon$ can be replaced by $(\pi^2\sigma^2 + (\gamma^+)^2 + \epsilon)^{\frac{1}{2}}$. *R. P. Boas, Jr.* (Providence, R. I.)

Source: Mathematical Reviews, Vol. 10, No. 9

LEONT'YEV, A.F.

Leont'ev, A. F. On functions represented by series of Dirichlet polynomials. Izvestiya Akad. Nauk SSSR. Ser. Mat. 13, 221–230 (1949). (Russian)

Let $\{\lambda_n\}$ be a sequence of complex numbers, with $\sigma = \limsup n/|\lambda_n| < \infty$. The author considers properties of a function $f(z)$ which is represented by a uniformly convergent series of functions $P_n(z) = \sum_{j=1}^{n} a_{n,j} e^{\lambda_j z}$ in a set D containing a circle of radius $r > \pi\sigma$. [He has previously considered the problem in more detail for $\lambda_n \geqq 0$ in the case when $\lim n/\lambda_n$ exists; his results were outlined in Uspehi Matem. Nauk (N.S.) 3, no. 4(26), 176–180 (1948); these Rev. 10, 364, but proofs have not been made available in any publication circulated outside the USSR.] (1) The region consisting of those regular points of $f(z)$ which can be reached from the circle mentioned above without going within $\pi\sigma$ of a singular point is simply connected and one-sheeted. (2) If $\sigma = 0$, the domain of regularity of $f(z)$ is convex. (3) If the sequences $P_n(z)$ and $P_n'(z) = \sum_{j=1}^{n} a'_{n,j} e^{\lambda_j z}$ both converge uniformly to $f(z)$ in D, then

$$\lim_{n\to\infty} a_{n,j} = \lim_{n\to\infty} a'_{n,j} = a_j.$$

Hence the Dirichlet series $\sum a_j e^{\lambda_j z}$ converges to $f(z)$ if it converges uniformly in D. A final theorem assures the convergence of this Dirichlet series if $f(z)$ is regular in a certain region. [Cf. Mandelbrojt, Trans. Amer. Math. Soc. 55, 96–131 (1944); these Rev. 6, 176.]

Put

$$L_{k,n}(z) = \prod_{j=k}^{n} (1 - z^2/\lambda_j^2) = \sum (a_{n,k,j}/j!) z^j,$$

$$L_{k,\infty}(z) = \prod_{j=k}^{\infty} (1 - z^2/\lambda_j^2) = \sum (a_{k,j}/j!) z^j$$

and define operators $M_{k,n}$ and $M_{k,\infty}$ by replacing z^j by $f^{(j)}(z)$ in $L_{k,n}$, $L_{k,\infty}$. Then

(a) $$M_{k,\infty}(e^{\lambda z}) = e^{\lambda z} L_{k,\infty}(\lambda);$$

(b) $$M_{k,\infty}(f) = M_{k,n}\{M_{n+1,\infty}(f)\};$$

(c) if $f_n(z) \to f(z)$ uniformly in $|z - z_0| < \rho$, where $\rho > \pi\sigma$, then $M_{k,\infty}(f_n) \to M_{k,\infty}(f)$ in a neighborhood of z_0; (d) in the component of D which contains z_0, $\lim_{k\to\infty} M_{k,\infty}(f) = f(z)$. Proof of (1). We have $M_{1,\infty}(P_n) = 0$ by (a), hence $M_{1,\infty}(f) = 0$ near z_0 by (d). Hence by (b) $M_{1,n}\{M_{n+1,\infty}(f)\} = 0$ and $M_{n+1,\infty}(f)$, as a solution of the linear differential equation with constant coefficients $M_{1,n}(y) = 0$, is an exponential polynomial and so by (d) $f(z)$ is the limit of a sequence of entire functions.

Proof of (2). If the domain of regularity of $f(z)$ is not convex we can find a singular point ξ of $f(z)$ through which an arc of a circle C can be drawn lying in D except for ξ. We represent $f(z)$ as $f_1(z) + f_2(z)$, where $f_i(z)$ is the Cauchy integral of $f(z)$ along an arc L_i such that $L_1 + L_2$ is a simple

1/2

Source: Mathematical Reviews, Vol. 10, No. 10

A. F. L.

contour on which $f(z)$ is regular and L_2 separates L_1 from ξ. Then $f_1(z)$ is regular at ξ and so $f_2(z)$ is singular at ξ. We can represent $f_2(z)$ as $\int_0^{\infty e^{i\varphi}} \Phi(t)e^{-zt}dt$, where $\Phi(t)$ is entire, of order 1 and finite type ρ, the radius of a circle C_1 passing through ξ and containing C, on which $f_2(z)$ has no singularity except ξ. It follows easily that ξ is a singular point of $M_{1,\infty}(f_2)$: in fact $M_{1,\infty}(f_2)=\int_0^{\infty e^{i\varphi}} \Phi(t)L_{1,\infty}(t)e^{-zt}dt$. Since $\sigma=0$, $L_{1,\infty}(t)$ is of growth at most order 1, minimum type. Hence $M_{1,\infty}(f_2)$ has a singular point on C_1, and this can only be ξ. Since $M_{1,\infty}(f_1)+M_{1,\infty}(f_2)=M_{1,\infty}(f)=0$ and $M_{1,\infty}(f_1)$ is regular at ξ we obtain a contradiction. Proof of (3). We have $M_{n+1,\infty}(f)=\lim_{k\to\infty} M_{n+1,\infty}(P_k)=\lim_{k\to\infty} \sum_{j=1}^{n} \alpha_{k,j}L_{n+1,\infty}(\lambda_j)e^{\lambda_j z}$ and the same relation with primed letters, whence $\alpha_{k,j}$ and $\alpha'_{k,j}$ both approach the same limit as $k\to\infty$.

R. P. Boas, Jr. (Providence, R. I.)

Source: Mathematical Reviews, Vol 10, No. 10

LEONT'YEV, A.F.

Leont'ev, A. F. Differential-difference equations. Mat. Sbornik N.S. 24(66), 347–374 (1949). (Russian)

The author investigates, in the complex plane, the form and general analytic properties of the solutions of equation (i) $M(f) \equiv \sum_{m=0}^{p} \sum_{n=1}^{q} a_{m,n} d^m f(x+h_n)/dx^m = 0$ $(0=h_1<h_2<\cdots<h_q)$, where the $a_{m,n}$ are constants. Let $L(x)$ be the characteristic function

$$L(x) = \sum_{m=0}^{p} \sum_{n=1}^{q} a_{m,n} x^m \exp(h_n x),$$

and let the set $\{\lambda_n\}$ of zeros of $L(x)$ be divided into the two classes $\{\lambda_n'\}: |\lambda_1'| \leqq |\lambda_2'| \leqq \cdots$; $\{\lambda_n''\}: |\lambda_1''| \leqq |\lambda_2''| \leqq \cdots$, where $\{\lambda_n'\}$ consists of those λ_n for which either $\Im(\lambda_n)>0$ or $\Im(\lambda_n)=0$, $\Re(\lambda_n)>0$. Further, let $y_n'(x)$, $y_n''(x)$ be those solutions of (i), corresponding respectively to λ_n' and λ_n'', given by $y_n'(x) = x^j \exp(\lambda_n' x)$, $y_n''(x) = x^k \exp(\lambda_n'' x)$, where λ_n', λ_n'' are of respective multiplicity $j+1$, $k+1$. Let D denote an infinite strip $-\infty \leqq \alpha < \Im(x) < \beta \leqq \infty$.

The following results are established. (1) If $f(x)$ is regular in a strip D and satisfies (i), then in D it has the representation $f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} \{\alpha_{n,i} y_i'(x) + \beta_{n,i} y_i''(x)\}$, the convergence being uniform on every closed bounded set in D. (2) There is the decomposition $f(x) = \varphi(x) + \psi(x)$, where the functions

$$\varphi(x) = \lim_{n\to\infty} \sum_{i=1}^{n} \alpha_{n,i} y_i'(x), \quad \psi(x) = \lim_{n\to\infty} \sum_{i=1}^{n} \beta_{n,i} y_i''(x)$$

are regular in the respective half-planes $\Im(x)>\alpha$, $\Im(x)<\beta$, and are solutions of (i). (3) The limits $\alpha_i = \lim_{n\to\infty} \alpha_{n,i}$ exist, $i=1, 2, \cdots$, and if series (*) $\sum_{i=1}^{\infty} \alpha_i y_i'(x)$ converges in some half-plane $\Im(x)>c$ its sum in this half-plane is $\varphi(x)$; correspondingly for $\psi(x)$. (4) If $\lambda_{n_k-1}' \neq \lambda_{n_k}'$, $\lambda_{n_k+p_k}' \neq \lambda_{n_k}'$, $\lambda_{n_k}' = \lambda_{n_k+r}'$ $(r=1, 2, \cdots, p_k-1)$, and $n=n_k+j$ $(j<p_k)$, and if $\delta = \limsup |\lambda_n'|^{-1} \log |\gamma_n|$, where

$$|\gamma_n| = |\{(z-\lambda_n')^{p_k}/L(z)\}^{(p_k-j-1)}|$$

evaluated at $z=\lambda_n'$ (the superscript p_k-j-1 indicating order of differentiation with respect to z), then series (*) converges in the half-plane $\Im(x)>\alpha+\delta$. (And functions $\varphi(x)$ exist for which the abscissa of convergence $\alpha+\delta$ cannot be lowered.) (5) For each ρ in $0\leqq\rho\leqq\infty$ there are equations of form (i) for which $\delta=\rho$. (If the differences $h_2, \cdots, h_q$ are commensurable, then $\delta=0$.) (6) There exists a sequence of partial sums obtained from series (*) by a permutation of its terms that converges to $\varphi(x)$ in every half-plane $\Im(x)>\alpha$ where $\varphi(x)$ is regular.

Source: Mathematical Reviews, 1950 Vol 11 No. 2

LEONT'YEV, A.F.

Leont'ev, A. F. On interpolation in the class of entire functions of finite order and normal type. Doklady Akad. Nauk SSSR (N.S.) 66, 153–156 (1949). (Russian)

The author extends his earlier results [Izvestiya Akad. Nauk SSSR. Ser. Mat. 13, 33–44 (1949); same Doklady (N.S.) 61, 785–787 (1948); these Rev. 10, 602, 289]. Let m be the smallest integer exceeding ρ, and $\epsilon = e^{2\pi i/m}$. Let $\{\lambda_n\}$ be a sequence of complex numbers arranged according to increasing modulus. In order that, for every system $\{a_{n,p}\}$, $p=0, 1, \cdots, m-1$, such that $\limsup \log |a_{n,p}| \cdot |\lambda_n|^{-\rho} < \infty$, there should exist an entire function $\omega(z)$ of growth not exceeding order ρ, finite type, with $\omega(\epsilon^p \lambda_n) = a_{n,p}$, it is necessary and sufficient that $\{\lambda_n\}$ satisfy the conditions

$$\text{(A)} \qquad \limsup n/|\lambda_n|^\rho = \sigma < \infty,$$

$$\text{(B)} \qquad \limsup |\lambda_n|^{-\rho} \log |1/F'(\lambda_n)| = \delta < \infty,$$

$$F(z) = \prod_{n=1}^{\infty} (1 - z^m/\lambda_n^m).$$

Some supplementary results are deduced as by-products.

R. P. Boas, Jr. (Providence, R. I.).

Source: Mathematical Reviews, Vol 10, No. 10

LEONT'YEV, A.F.

Leont'ev, A. F. On an interpolation problem. Doklady Akad. Nauk SSSR (N.S.) 66, 331–334 (1949). (Russian)

Let $\{\lambda_n\}$ be an increasing sequence of positive real numbers, such that $\limsup n/\lambda_n^\rho < \infty$, $\rho > 0$. If there is a function $f(z)$ regular in some angle $|\arg z| < \mu$, $\mu > 0$, and satisfying $|f(z)| < \exp(c|z|^\rho)$ for large $|z|$ in the angle, with the property $f(\lambda_n) = (-1)^n$, then for every set $\{a_n\}$ such that $\limsup \lambda_n^{-\rho} \log |a_n| < \infty$ there is an entire function $\omega(z)$ of growth not exceeding order ρ, finite type, such that $\omega(\lambda_n) = a_n$. Hence the author deduces a necessary and sufficient condition on $\{\lambda_n\}$ for such an entire function to exist.

R. P. Boas, Jr. (Providence, R. I.).

Source: Mathematical Reviews, Vol 10, No. 10

LEONT'YEV, A.F.

Mathematical Reviews
Vol. 14 No. 11
Dec. 1953
Analysis

Leont'ev, A. F. Ryady polinomov Dirihlet i ih obobščeniya. [Series of Dirichlet polynomials and their generalizations.] Trudy Mat. Inst. Steklov., vol. 39. Izdat. Akad. Nauk SSSR, Moscow, 1951. 214 pp. 8.80 rubles.

Soit $f(z)$ une fonction entière, et soit $\{\lambda_n\}$ une suite de nombres complexes, $0<|\lambda_1|\leqq|\lambda_2|\leqq\cdots\leqq|\lambda_n|\leqq\cdots$. Si la suite $\{f(\lambda_n z)\}$ n'est pas complète dans un domaine D, la convergence uniforme d'une combinaison linéaire (1) $P_n(z)=\sum_1^n a_{ni}f(\lambda_i z)$ dans D (c'est-à-dire, la convergence uniforme dans tout compact intérieur à D) implique des propriétés de la limite (2) $P(z)=\lim P_n(z)$, qu'il s'agit de mettre en évidence. Ainsi, on peut définir un opérateur, dépendant de f et de $\{\lambda_n\}$, annulant toute fonction P définie par (2). L'équation fonctionnelle ainsi établie permet de fournir des propriétés importantes aussi bien de la fonction P que de la suite (1) elle-même. On peut, par exemple, étendre le domaine où (2) a lieu, et où, par conséquent, P est encore holomorphe. Si $f(z)=\sum a_n z^n$, avec $a_n\neq 0$ $(n\geqq 0)$, est d'ordre $\rho>0$, du type $\sigma\neq 0, \infty$, $\liminf|a_n|^{1/n}n^{1/\rho}=(\sigma e\rho)^{1/\rho}>0$, à toute fonction entière $L(z)$ du même ordre ρ et du type $\sigma_1<\infty$ correspond une fonction $\gamma(z,t)$ holomorphe en t, lorsque $|z|\leqq R$, dans un cercle $|t|>\mu(R)$, et qui permet d'écrire $2\pi i L(\zeta)f(\zeta z)=\int\gamma(z,t)f(\zeta t)dt$, l'intégrale étant étendue sur un cercle autour de l'origine contenant à l'intérieur toutes les singularités de γ par rapport à t. A partir de L on forme l'opérateur $M(F)=(2\pi i)^{-1}\int\gamma(z,t)F(t)dt$; lorsque F est holomorphe dans $|z|<R_1$, l'intégrale est étendue sur

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largeur supérieure à $2\pi r$) où P peut être prolongée est simplement connexe et a un seul feuillet. Dans $D(z_0)$, $M_{k,\infty}(P)$ a un sens et lim $M_{k,\infty}(P) = P$. L'auteur énonce aussi le résultat suivant : si $r = 0$, $\rho = 1$, le domaine d'existence de P est convexe, sans citer à cette occasion Pólya, qui a démontré ce théorème antérieurement [Nachr. Ges. Wiss.

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LEONT'YEV, A.

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USSR/Mathematics - Functions, Analytical May/Jun 51

"Review of A. I. Markushevich's Book 'Theory of Analytical Functions,'" A. Leont'yev

"Uspekh Matemat Nauk" Vol VI, No 3 (43), pp 181-187

Subject book was published 1950 in Moscow and Leningrad by the State Tech Press, for 29 rubles; 702 pp. Authorized by the Ministry of Higher Educ of USSR as textbook for institutes of higher learning. Generally favorable review. Considered valuable addn to the literature on the theory of functions of complex variable.

188T65

LEONT'YEV, A.F.

Gel'fond, A. O., and Leont'ev, A. F. On a generalization of Fourier series. Mat. Sbornik N.S. 29(71), 477–500 (1951). (Russian)

Let $f(z)=\sum_0^\infty a_n z^n$ ($a_n\neq 0$, $n=0,1,\cdots$) be an entire function of order ρ and finite, non-zero type σ, with (a) $\lim_{n\to\infty} n^{1/\rho}|a_n|^{1/n}=(\sigma e\rho)^{1/\rho}$; and let $F(z)=\sum_0^\infty b_n z^n$ be an arbitrary analytic function, regular in $|z|<R$ ($R\leqq\infty$). Define the operator (b) $D^nF=D^n(F,f)=\sum_{k=n}^\infty b_k(a_{k-n}/a_k)z^{k-n}$. This series for D^nF converges in $|z|<R$. (The authors remark that condition (a) can be weakened, but that it is assumed for simplicity in formulas, etc.) The operator D^nF is a generalization of differentiation in the sense that for the particular choice $f(z)=e^z$ we have $D^n(F,e^z)=d^nF(z)/dz^n$. Let $\varphi(t)=\sum_0^\infty c_n t^n$ be an entire function of order $\leqq\rho$ and finite type σ_1 (where if the order is less than ρ then take $\sigma_1=0$). The principal aim of the paper is to consider the equation of infinite order in the generalized derivative:

$$\text{(c)}\qquad L[F]=\sum_0^\infty c_n D^nF=0.$$

Since $L[f(\lambda z)]=\sum_0^\infty c_n\lambda^n f(\lambda z)=\varphi(\lambda)f(\lambda z)$ (where f was given above) the function $\varphi(\lambda)$ is called the characteristic function of equation (c). [The reference to Fourier series in the title is explained as follows: The analytic function $F(z)$ is expressible in a Fourier series (in multiples of $2\pi z$) if it is periodic with a period 1; i.e., if $F(z+1)-F(z)=0$. Now this last relation is representable as an equation of form (c) if we take $f(z)=e^z$, $\varphi(t)=e^t-1$.]

To determine the convergence properties of series (c), set (d) $\varphi(\lambda)f(\lambda z)=\sum_0^\infty B_n(z)\lambda^n$, (e) $A_n(z)=B_n(z)/a_n$, $n=0,1,\cdots$, and (f) $\psi(z,t)=\sum_0^\infty A_n(z)/t^{n+1}$. It is shown that for $r>0$, series (f) converges in $|z|<r$, $|t|>\mu(r)=(\sigma_1/\sigma+r^\rho)^{1/\rho}$, so that ψ is regular for z,t in this region; and that for $F(z)$ regular in $|z|<R=\mu(r)$, then series (c) converges in $|z|<r$, and in every circle $|z|<r-\epsilon$ ($\epsilon>0$), $L[F]$ has the representation (g) $L[F]=(2\pi i)^{-1}\int_C\psi(z,t)F(t)dt$. (Here C is the circle $|t|=R_1$ with $\mu(r-\epsilon)<R_1<R$.) If λ is a zero of $\varphi(\lambda)$, and is of multiplicity p, then the p functions $z^m f^{(m)}(\lambda z)$ ($m=0,1,\cdots,p-1$) are particular solutions of $L[F]=0$.

Now take a function $f_1(z)=\sum_0^\infty \alpha_n z^n$ satisfying the same conditions as $f(z)$, and, keeping the same characteristic

Source: Mathematical Reviews, Vol 13 No. 7

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function $\varphi(t)$, form an operator L_1: $L_1[F]=\sum_0^\infty c_n D^n(F, f_1)$. The functions $\eta(z)=\sum_0^\infty (a_n/\alpha_n)z^n$, $\eta_1(z)=\sum_0^\infty (\alpha_n/a_n)z^n$ are analytic in $|z|<1$. Form with these the operators

(h)
$$M[F]=\sum_0^\infty \frac{a_n}{\alpha_n} b_n z^n = \frac{1}{2\pi i}\int_C \eta\left(\frac{z}{t}\right) F(t)\frac{dt}{t},$$
$$M_1[F]=\sum_0^\infty \frac{\alpha_n}{a_n} b_n z^n = \frac{1}{2\pi i}\int_C \eta_1\left(\frac{z}{t}\right) F(t)\frac{dt}{t},$$

where $F(z)=\sum_0^\infty b_n z^n$ is regular in $|z|<R$ and C is the circle $|t|=R_1$ with $|z|<R_1<R$. Then $M[F]$ and $M_1[F]$ are regular functions in $|z|<R$; and $M_1[F]$ is inverse to $M[F]$: (i) $MM_1[F]=M_1M[F]=F(z)$. Also,

(j) $$D^n(F, f_1)=M^{-1}\{D^n(M[F], f)\}.$$

We thus have (k) $L_1[F]=M^{-1}LM[F]$; so that if F satisfies $L_1[y]=\Phi(z)$ then $M[F]$ satisfies $L[y]=M[\Phi]$, and if F_1 satisfies $L[y]=\Phi_1(z)$ then $M^{-1}[F_1]$ satisfies $L_1[y]=M^{-1}[\Phi_1]$. As seen earlier, if λ_n $(n=1, 2, \cdots)$ are the zeros of $\varphi(t)$, and p_n the corresponding multiplicities, then $L[y]=0$ has as particular solutions $y_j=z^m f^{(m)}(\lambda_n z)$, with $m=0, 1, \cdots, p_n-1$; $n=1, 2, \cdots$; $j=p_1+p_2+\cdots+p_{n-1}+m+1$; and similarly, $\bar{y}_j=z^m f_1^{(m)}(\lambda_n z)$ are solutions of $L_1[y]=0$. It then follows that if $F(z)$, satisfying equation (c) in $|z|<R$, has there the uniformly convergent representation

$$F(z)=\lim_{k\to\infty}\sum_{j=1}^{m_k}\beta_j y_j(z)$$

then the solution $M^{-1}[F]$ of $L_1[y]=0$ will have in $|z|<R$ the representation $M^{-1}[F]=\lim_{k\to\infty}\sum_{j=1}^{m_k}\beta_j\bar{y}_j(z)$.

Consider the equation (l) $L[F]=0$ where $L[F]$ is now given in the integral form (g). Moreover, take

$$f(z)=\sum_0^\infty a_n z^n=\sum_0^\infty [\sigma^{n/\rho}/\Gamma(n/\rho+1)]z^n,$$

so that $1/a_n=\int_0^\infty x^n d\tau(x)$, $\tau(x)=-e^{-\sigma x^\rho}$. (From what was stated earlier about operators L and L_1, it suffices to discuss the case of one representative $f(z)$.) The function $\psi(z, t)$ of (g) is now shown to have the form

(m) $$\psi(z, t)=\frac{1}{t}\int_0^\infty \varphi\left(\frac{x}{t}\right) f\left(\frac{zx}{t}\right) d\tau(x).$$

Moreover, if $F(z)$ is regular in $|z|<\mu(r)$ and satisfies equation (l), then for $|z|<r-\epsilon$ $(\epsilon>0)$ the relation

Source: Mathematical Reviews, Vol 13 No. 7

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$$\text{(n)}\quad \int_{|t|=r_1} \frac{F(t)}{t}dt \int_{|u|=q} \frac{du}{\varphi(u)} \int_0^{\infty} \left\{ \varphi\left(\frac{x}{t}\right) - \varphi(u) \right\} \times \frac{f(zx/t) - f(zu)}{\frac{x}{t} - u} d\gamma(x) = 0$$

holds, where $\mu(r-\epsilon) < \omega_1 < \mu(r)$, and $\varphi(u) \neq 0$ on $|u| = q$. Also for such F, if we set

$$\text{(o)}\quad Q(z,q) = \frac{1}{(2\pi i)^2} \int_{|t|=r_1} \frac{F(t)}{t} dt \int_{|u|=q} \frac{f(zu)}{\varphi(u)} du \times \int_0^{\infty} \frac{\varphi(x/t) - \varphi(u)}{\frac{x}{t} - u} d\gamma(x),$$

then $Q(z,q)$ is a linear combination of the solutions $y_j(z)$ corresponding to those zeros of $\varphi(u)$ lying in $|u| < q$: $Q(z,q) = \sum_{j=1}^{s} \beta_j y_j(z)$. The coefficients β_j are independent of q, so $Q(z,q)$ represents a partial sum of order s of the formal series (p) $\sum_{j=1}^{\infty} \beta_j y_j(z)$. It is then shown that the remainder term $R(z,q) = F(z) - Q(z,q)$ is given by

$$\text{(q)}\quad R(z,q) = -\frac{1}{(2\pi i)^2} \int_{|t|=r_1} \frac{F(t)}{t^{n+1}} dt \int_{|u|=q} \frac{du}{u^n \varphi(u)} \times \int_0^{\infty} \frac{\varphi(x/t) - \varphi(u)}{\frac{x}{t} - u} f\left(\frac{zx}{t}\right) d\gamma(x)$$

in $|z| < r - \epsilon$ (n being an arbitrary non-negative integer).

The basic theorem of the paper is now established: Let there exist an infinite sequence of circles $|u| = q_m$ ($m = 1, 2, \cdots$) on which

$$\text{(r)}\quad |\varphi(u)| > e^{-(\theta+\epsilon)|u|^{\rho}}$$

($\theta \geq 0$) for $m > N(\epsilon)$, $\epsilon > 0$ arbitrary. If $F(z)$ is regular and satisfies equation (l) in $|z| < R$, then

$$\text{(s)}\quad |F(z) - Q(z,q_m)| < \exp\{(\theta + \sigma_1 + \sigma r^{\rho} - \sigma R^{\rho} + \epsilon) q_m^{\rho}\}$$

for $|z| < r$ and $m > N_1(\epsilon)$. As a corollary, if $\varphi(u)$ satisfies the conditions just stated, and if $F(z)$ is regular and satisfies (l) in $|z| < R = \mu_1(r) = [(\theta + \sigma_1)/\sigma + r^{\rho}]^{1/\rho}$, then for $|z| < r$, $F(z)$ is given by $F(z) = \lim_{m\to\infty} Q(z,q_m)$, uniformly in $|z| \leq r - \epsilon$ ($\epsilon > 0$ arbitrary). It is shown moreover that $\varphi(z)$, being of order ρ and type σ_1, a possible choice of θ in (r) is $\theta = [2\rho]\sigma_1$, where $[2\rho]$ = greatest integer not exceeding 2ρ.

I. M. Sheffer (State College, Pa.)

Source: Mathematical Reviews, Vol 13 No. 7

3/3 AOG; AFL

LEONT'YEV, A. F.

Polynomials

Dirichlet's polynomial series and their generalizations. Trudy Mat. inst. no. 39 1951.

MONTHLY LIST OF RUSSIAN ACCESSIONS. Library of Congress, August 1952. UNCLASSIFIED.

USSR/Mathematics - Complex Variable, Liouville Theorem — Jul/Aug 52

"Generalization of Liouville's Theorem," A.F. Leont'yev, Gor'kiy
"Matemat Sbor" Vol XXXI (73), No 1, pp 201-208

The familiar Liouville theorem states that if an entire (integral) function $f(z) = A_n z^n$ (n-summed, 0 to ∞) is bounded in the entire plane then it is const. A more general Liouville theorem states that f(z) is a polynomial of deg not higher than m if /f(z)/ does not increase faster than $/z/^m$ for sufficiently large /z/. In current article the

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author demonstrates a still more general theorem, similar to the more general Liouville theorem. Submitted 31 Mar 52.

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[illegible], A. F.

LEONT'YEV, A. F.

USSR/Mathematics - Analytical Functions, Completeness Sep/Oct 52

"Completeness of One Systems of Analytical Functions," A. F. Leont'yev, Gor'kiy

"Matemat Sbor" Vol 31 (73), No 2, pp 381-414

Demonstrates that in general a region in which the familiar properties of a sequence of linear arguments does not begin right after a region of completeness for system $(f(\lambda_n z))$. Also shows that the radius R of completeness of system $(f(\lambda_n z))$ depends upon the characteristic sigma. Submitted 31 Mar 52.

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LEONT'YEV, A. F.

Mathematical Reviews
Vol. 15 No. 4
Apr. 1954
Analysis

8-24-54
LL

Leont'ev, A. F. On representation of entire functions by sequences of linear aggregates. Mat. Sbornik N.S. 33(75), 453–462 (1953). (Russian)

If $\{f_n(z)\}$ is a sequence of entire functions of finite order, the author says [following Pólya] that the sequence is of finite order if $|f_n(z)| \leq \exp\{|z|^\mu\}$ for all n and $|z| > r_0$ (independent of n); the infimum of such μ is called the order of the sequence. Let $f(z)$ be an entire function of order ρ with no $f^{(n)}(0)=0$, and let $\{\lambda_n\}$ be a sequence of distinct complex numbers of increasing absolute value with exponent of convergence $\rho_1 > 0$. Let $F(z)$ be an entire function of order ν. The author proves that there is a sequence of finite sums $f_n(z) = \sum a_{nk} f(\lambda_k z)$ converging everywhere to $F(z)$ and having order at most $\max\{\nu, \rho\rho_1/(\rho_1-\rho)\}$. The novelty of the result consists in the fact that a bound is obtained for the order of the approximating sums. That the bound cannot generally be reduced follows from the complementary result that if $f(z) = \sum a_n z^n$ and $\lim (n \log n)/\log|1/a_n| = \rho$, and the other conditions of the first theorem are satisfied, then every entire function $F(z)$, of order less than ρ, which is a limit of $f_n(z)$ of the indicated form, with the order of $\{f_n(z)\}$ less than $\rho\rho_1/(\rho_1-\rho)$, is identically 0.

R. P. Boas, Jr. (Evanston, Ill.).

LEONT'YEV, A. F.

[illegible] and inventions [illegible] the following [illegible] books and textbooks have been [illegible] years [illegible] (Sovetskaya Rossiya, Moscow, No. [illegible] Apr 1954)

[illegible]	[illegible]	[illegible]
Leont'yev, A. F.	"Series of Dirichlet Polynomial and Their Generalizations" "The Completeness of One System of Analytical Functions" "The Problem of Representation of Entire Functions by Sequences of Linear Sets"	Gor'kiy State University

LEONT'YEV, A.F.

USSR/Mathematics - Dirichlet polynomials FD-1028

Card 1/1 Pub. 64 - 8/9

Author : Leont'6ev, A. F. (Gor'kiy)

Title : Region of boundedness of a sequence of Dirichlet polynomials

Periodical : Mat. sbor., 35(77), No 1, 175-186, Jul-Aug 1954

Abstract : The author's purpose in the present article is to demonstrate that in the case where the limit (n/t_n) equals 0 as n tends to infinity the region D is convex; here, (t_n) is a sequence of complex numbers, $P_n(z)$ is a sequence of Dirichlet polynomials formed by means of (t_n), G is the set of all points in a sufficiently small neighborhood of each which the sequence $P_n(z)$ is bounded, and D is one of the connected components of G. Six references, 3 USSR (e.g. A. O. Gel'fond, Ischisleniye konechnykh raznostey [Calculus of finite differences], 1952) and 3 Western (1917- 1929).

Institution : --

Submitted : 30 October 1953

LEONT'YEV, A. F.

Leont'ev, A. F. On overconvergence of a series. Doklady Akad. Nauk SSSR (N.S.) 94, 381–384 (1954). (Russian)

Ostrowski proved [Abh. Math. Sem. Hamburg. Univ. 1, 327–350 (1922), see p. 335] that if a power series $\sum a_n z^{\lambda_n}$ has very large gaps ($\lambda_{n_k+1}/\lambda_{n_k} \to \infty$), it overconverges uniformly throughout the domain of regularity of the function which it represents. Under certain hypotheses which cannot be stated here, the author proves an analogous proposition concerning series of the form $\sum a_n f(\lambda_n z)$, where $f(z) = \sum a_n z^n$ is an entire function of finite order.

G. Piranian.

APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000929310005-2"

LEONT'YEV, A.F. (Moscow)

Completeness of a system of exponential functions in a curvilinear strip. Mat.sbor. 36 no.3:555-568 My-Je '55. (MLRA 8:6)
(Functions, Exponential)

LEONT'YEV, A.F.

Transactions of the Third All-union Mathematical Congress (Cont.) Moscow, Jun-Jul '56, Trudy '56, V. 1, Sect. Rpts., Izdatel'stvo AN SSSR, Moscow, 1956, 237 pp.
Mention is made of Goluzin, G. M. Kuvayev, M. R., Semukhina, N. V., and Chistyakov, Yu. V.

There is 1 USSR reference.

Leont'yev, A. F. (Moscow). On interpolation of Entire Functions of a Finite Order. 86-87

There is 1 USSR reference.

Mandzhavidze, G. F. (Tbilisi). On the Approximate Solution of Boundary Problems of the Theory of Analytic Functions. 88

Melentsov, A. A. (Sverdlovsk). On the Hausdorff Transformations Theory. 88

There are 2 references, 1 of which is German, and the other a translation into Russian.

Card 27/80

LEONT'YEV, A.F.

1-FW

3

Leont'ev, A. F. On properties of sequences of linear aggregates that converge in a region in which the system of functions generating the linear aggregates is not complete. Uspehi Mat. Nauk (N.S.) 11 (1956), no. 5(71), 26–37. (Russian)

The situation described in the title is that in which a set $\{f_n(z)\}$ of analytic functions is not complete in a region and one investigates the properties of the linear manifold spanned by $\{f_n(z)\}$, i.e. the properties of limits of sums $\sum_{k=1}^{n} a_{kn} f_k(z)$. The author reviews, without proofs, the literature of this (fairly young) field. *R. P. Boas, Jr.*

LEONT'YEV, A.F.

SUBJECT USSR/MATHEMATICS/Theory of functions CARD 1/2 PG - 527
AUTHOR LEONT'EV A.F.
TITLE On the region of regularity of the limit function of an analytic function sequence.
PERIODICAL Mat.Sbornik, n. Ser. 39, 403-422 (1956)
reviewed 1/1957

The author proves the following theorem: Let

$$f(z) = \sum_{n=0}^{\infty} a_n z^n = 1 + \sum_{n=1}^{\infty} \frac{z^n}{P(1)P(2)\ldots P(n)} ; \quad p(x) = \alpha_p x^p + \ldots + \alpha_1 x ,$$

where $P(k) \neq 0$ $(k=1,2,..)$ (the function $f(z)$ then has the order $\varrho = \frac{1}{p}$ and the type $\sigma = \frac{p}{\sqrt[p]{|\alpha_p|}}$). Let the sequence of the complex numbers $\{\lambda_n\}$ satisfy the condition

$$\lim_{n \to \infty} \frac{n}{|\lambda_n|^{1/p}} = 0.$$

If the sequence

$$P_n(z) = \sum_{k=1}^{p_n} a_{nk} f(\lambda_k z) \qquad (n=1,2,\ldots)$$

converges uniformly in a region, then the region of existence of the limit

LEONT'YEV, A.F.

2

Leont'ev, A. F. On convergence of a sequence of Dirichlet polynomials. Dokl. Akad. Nauk SSSR (N.S.) 108 (1956), 23–26. (Russian)

Soit $\{\lambda_n\}$ une suite positive telle que lim sup $n^{-1}\lambda_n=\sigma$, et $P_n(z)$ une suite de polynômes de Dirichlet, dont les exposants appartiennent à $\{\lambda_n\}$, et qui converge uniformément dans un cercle $|z-z_0|<R$, $R>\pi\sigma$. Alors $P_n(z)$ converge uniformément dans un domaine G, precisé par l'auteur; on peut prendre pour G le domaine $|y|<\exp(\alpha x)$, $x>x_0$ assez grand, avec $\alpha\sigma<1$; mais on ne le peut pas si $\alpha\sigma>1$. L'auteur cherche ensuite des conditions supplementaires, portant sur $\{\lambda_n\}$, permettant de prendre pour G un demi-plan $x>\alpha$. (Remarque: le référent [Ann. Inst. Fourier, Grenoble 5 (1953–1954), 39–130; voir pp. 92–97; MR 17, 732] a établi des théorèmes analogues à ceux de l'auteur, quoique distincts; en particulier, il suffit que $\lambda_{n+1}-\lambda_n>h>0$ pour qu'on puisse prendre G: $\{x>\alpha\}$.)

J. P. Kahane (Montpellier).

Math

Power. Eng. Inst. im KRZHIZHANOVSKIY, Acad. Sci. USSR

DOUBTFUL

LEONT'YEV, A.F.

New proof of a theorem on convergence of the sequence of Dirichlet's polynomials. Usp.mat.nauk 12 no.3:165-170 My-Je '57.
(MIRA 10:10)

(Polynomials) (Series)

SUBJECT	USSR/MATHEMATICS/Theory of functions	CARD 1/2	PG - 735
AUTHOR	LEONT'EV A.F.		
TITLE	On the question of interpolation in the class of entire functions of finite order.		
PERIODICAL	Mat.Sbornik,n.Ser. 41, 81-96 (1957) reviewed 5/1957		

Let be given the system of numbers $\{a_n\}$, where

$$|a_n| < e^{K|\lambda_n|^{\varsigma}} .$$

In order that on $[\varsigma,\infty]$ there exists at least one function which attains the values a_n in the points λ_n, it is necessary and sufficient that

1) $\overline{\lim\limits_{n\to\infty}} \frac{n}{|\lambda_n|^{\varsigma}} < \infty$,

2) $\overline{\lim\limits_{n\to\infty}} \frac{\ln\left|\frac{1}{\eta_n}\right|}{|\lambda_n|^{\varsigma}} < \infty$.

Here $\eta_n = \prod (1 - \frac{\lambda_n}{\lambda_s})$, the product is extended over all $\lambda_s \neq \lambda_n$ for which $(1 - \delta_1)|\lambda_n| < |\lambda_s| < (1 + \delta)|\lambda_n|$, (δ and δ_1 arbitrary positive numbers being smaller than 1).
The present paper completes the papers of the author in Doklady Akad.Nauk 66, 153-156 (1949) and Doklady Akad.Nauk 66, 331-334 (1949).

INSTITUTION: Moscow.

AUTHORS: Gel'fond, A.O., Leont'yev, A.F. and Shabat, B.V. SOV/42-13-6-28/33

TITLE: Aleksey Ivanovich Markushevich (on the Occasion of his 50th Birthday) (Aleksey Ivanovich Markushevich (K pyatidesyatiletiyu so dnya rozhdeniya))

PERIODICAL: Uspekhi matematicheskikh nauk, 1958, Vol 13, Nr 6, pp 213-220 (USSR)

ABSTRACT: This is a brief account of the life of A.I.Markushevich: born in 1908 at Petrozavodsk, studied till 1930 under Romanovskiy at Tashkent; aspirant under Lavrent'yev at Moscow. Candidate dissertation on polynomial approximation of analytic functions in 1934. Since 1938 docent at the Moscow State University. Doctor dissertation in 1944 on approximations and expansions of functions in series. 1950 vice-president of the Academy of Pedagogical Sciences. 1958 first deputy of the minister of education of the RSFSR (Russian Soviet Federated Socialist Republic). His pupils are: N.A.Davydov, G.Ts.Tumarkin, S.Ya. Khavinson. There follows a list of 83 publications (1928-1957) and a photo of Markushevich.

Card 1/1

AUTHOR: Leont'yev, A.F. 38-22-2-4/8

TITLE: On Sequences of Linear Aggregates Which are Formed of the Solutions of Differential Equations (O posledovatel'nostyakh lineynykh agregatov, obrazovannykh iz resheniy differentsial'nykh uravneniy)

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya Matematicheskaya, 1958, Vol 22, Nr 2, pp 201-242 (USSR)

ABSTRACT: The author considers the sequences of linear aggregates which are formed of the solutions $y_k(z, \lambda_j)$ (k=1,2,...,s; j=1,2,..) of the equations

$$Dy \equiv \sum_{k=0}^{s} Q_k(z)\, y^{(s-k)}(z) = \lambda_j y \quad , \quad \overline{\lim_{n\to\infty}} \frac{n}{|\lambda_n|^{1/s}} = \tau < \infty$$

where the $Q_0(z), \dots, Q_s(z)$ are arbitrary analytic functions and the λ_j in general complex numbers. A series of well-known properties of the Taylor series, of the Dirichlet series and of the Dirichlet polynomial sequences is transferred to the considered sequences. The author's investigations are essen-

Card 1/2

On Sequences of Linear Aggregates Which are Formed of the Solutions of Differential Equations 38-22-2-4/8

tially based on the consideration of the equation

$$M(y) \equiv \sum_{k=0}^{\infty} a_k D^k y = 0 \text{ , where } a_k = \text{const}, \; D^0 = 1, \; D^k = D(D^{k-1}).$$

Under suitable choice of the a_k then the functions $y_k(z, \lambda_j)$ are solutions of $M(y) = 0$ for all k and j . An estimation of Klimczak [Ref 6] is essentially used. The sequences of linear aggregates are particularly considered which are formed of Jacobi, Chebyshev-Laguerre, Chebyshev-Hermite polynomials and of cylinder functions. On the whole 28 theorems, several consequences and lemmata are proved. In the last of the 16 paragraphs of the paper those questions are formulated the author did not solve. There are 14 references, 7 of which are Soviet, and 7 American.

PRESENTED: by M.A. Lavrent'yev, Academician

SUBMITTED: May 27, 1957

AVAILABLE: Library of Congress

Card 2/2 1. Differential equations 2. Analytic functions 3. Polynomials

AUTHOR: Leont'yev, A.F. SOV/38-22-3-6/9

TITLE: The Values of an Entire Function of Finite Order in Given Points (O znacheniyakh tseloy funktsii konechnogo poryadka v zadannykh tochkakh)

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1958, Vol 22, Nr 3, pp 387-394 (USSR)

ABSTRACT: Let the sequence $\{\lambda_n\}$ satisfy the condition

(1) $$\overline{\lim_{n\to\infty}} \frac{\ln n}{\ln |\lambda_n|} \leqslant \varrho$$

or

(2) $$\overline{\lim_{n\to\infty}} \frac{n}{|\lambda_n|^{\varrho}} < \infty$$

The author asks for the conditions the a_n have to satisfy in order that there is at least one function $\omega(z)$ of the order $\leq \varrho$ for which $\omega(\lambda_n) = a_n$.

Card 1/3

The Values of an Entire Function of Finite Order in Given Points SOV/38-22-3-6/9

Let $\mu_1^{(n)} (\mu_1^{(n)} = \lambda_n)$, $\mu_2^{(n)}, \ldots, \mu_{q_n}^{(n)}$ be the points λ_m lying in the circle $|z - \lambda_n| < |\lambda_n|^{-h}$, $h \geqslant 0$. Let $\alpha_1^{(n)} (\alpha_1^{(n)} = a_n), \alpha_2^{(n)}, \ldots, \alpha_{q_n}^{(n)}$ the numbers a_m which correspond to the points λ_m.

Let

$$A_1^{(n)} = \mu_1^{(n)} \alpha_1^{(n)}, \ldots\ldots, A_k^{(n)} = \mu_1^{(n)} \ldots \mu_k^{(n)} \sum_{p=1}^{k} \frac{\alpha_p^{(n)}}{\prod\limits_{\substack{j=1 \\ j \neq p}}^{k} \left(\mu_j^{(n)} - \mu_p^{(n)}\right)}$$

$(k=2,\ldots,q_n)$ and let B_n denote the maximum of the absolute values of these magnitudes.

Theorem: Let $\{\lambda_n\}$ satisfy (1). In order that there exists a function $\omega(z)$ of order $\leq \varrho$ for which $\omega(\lambda_n) = a_n$, it is

Card 2/3

The Values of an Entire Function of Finite Order in Given Points SOV/38-22-3-6/9

necessary and sufficient that

$$\overline{\lim_{n\to\infty}} \frac{\ln \ln B_n}{\ln |\lambda_n|} \leqslant \rho$$

Theorem: Let $\{\lambda_n\}$ satisfy (2). In order that there exists an interpolation function of order $\leqslant \rho$, it is necessary and sufficient that

$$\overline{\lim_{n\to\infty}} \frac{\ln B_n}{|\lambda_n|^{\rho}} < \infty .$$

There are 12 references, 9 of which are Soviet, 2 English, and 1 French.

PRESENTED: by S.L. Sobolev, Academician
SUBMITTED: January 21, 1957

1. Functions--Theory

Card 3/3

AUTHOR: Leont'yev, A.F. (Moscow) SOV/39-45-3-2/7

TITLE: On Some Solutions of the Linear Difference Equation With Linear Coefficients (O nekotorykh resheniyakh lineynogo raznostnogo uravneniya s lineynymi koeffitsiyentami)

PERIODICAL: Matematicheskiy sbornik, 1958, Vol 45, Nr 3, pp 323-332 (USSR)

ABSTRACT: Let the general linear difference equation be given with linear coefficients

$$(1)\quad L(y) = \sum_{k=0}^{n} (a_k z + b_k) y(z+h_k) = 0 \qquad h_0 < h_1 < \ldots < h_n$$

When does (1) possess solutions which are analytic in the half plane Re $z > \alpha$ and possessing poles (so-called solutions of the type Γ)?
Theorem: (1) possesses no solutions of the type Γ if and only if $a_0 = 0$ or if there exists an operator of the type

$$M(y) = \sum_{m=0}^{r} \beta_m y(z+p_m),$$ so that

Card 1/4

$$M\left\{L(y)\right\} = z\sum_{j=1}^{\nu} d_j y(z+\mu_j) \quad , \quad d_j = \text{const}$$

Here β_m and p_m are determined by the following equations:

$$\vartheta(t) = \sum_{k=o}^{n} a_k e^{h_k t} \; ; \quad \vartheta_1(t) = \sum_{k=o}^{n} b_k e^{h_k t} \; ;$$

$$\exp\left(-\int_{t_o}^{t} \frac{\vartheta_1(t)}{\vartheta(t)}\,dt\right) = \sum_{m=o}^{r} \beta_m e^{p_m t}$$

Theorem: If (1) possesses solutions of the type Γ, then one of them can always be represented in one of the following forms :

$$(2) \quad y(z) = \int_{-\infty+id}^{\infty+id} \gamma(t)\, e^{tz} dt \; , \quad (3) \quad y(z) = \int_{-\infty}^{(\pm a)} \gamma(t)\, e^{tz}\, dt \; ,$$

$$(4) \quad y(z) = \int_{-\infty}^{(a)} \left[\gamma(t)\ln(t-a) + \psi(t)\right] e^{tz}\, dt$$

where

$\gamma(t) = \frac{A}{\vartheta(t)} \exp\left(\int_{t_0}^{t} \frac{\vartheta_1(t)}{\vartheta(t)} dt \right)$ and $\psi(t)$ for $t \neq a$ is unique and regular; (2) corresponds to the case $a_n = 0$; (3) corresponds to the case that $\gamma(t)$ is multivalent in the neighborhood of the zero a of $\vartheta(t)$, and (4) corresponds to the case that $\gamma(t)$ possesses no pole of order $\geqslant s$ in the s-fold zero a of $\vartheta(t)$.

A third theorem says that, if $\sum (a_k z + c_k) f(z + h_k) = K$ possesses a solution with finitely many singular points, it holds the relation $[\vartheta(t)\eta(t)]' - \vartheta_2(t)\eta(t) \equiv 0$, where

On Some Solutions of the Linear Difference Equation With Linear Coefficients SO7/39-45-3-2/7

$$\vartheta_2(t) = \sum_{k=o}^{n} c_k e^{h_k t}$$ and $\eta(t)$ is a quasipolynomial.

The inversion holds too (for a certain K).
There is 1 Soviet reference.

SUBMITTED: January 21, 1957

1. Differential equations 2. Mathematics--Theory 3. Operators (Mathematics)--Applications

Card 4/4

AUTHOR: Leont'yev,A.F. SOV/20-121-5-8/50

TITLE: On the Completeness of the System $\{z^{\lambda_k}\}$ on Curves in the Complex Plane (O polnote sistemy $\{z^{\lambda_k}\}$ na krivykh v kompleksnoy ploskosti)

PERIODICAL: Doklady Akademii nauk SSSR, 1958,Vol 121,Nr 5, pp 797-800 (USSR)

ABSTRACT: Let the curve L free of loops intersect the plane into the simply connected domains $G_1, G_2, \ldots, G_m$, $m < \infty$. Let L be rectifiable in the finite plane and let the arc length $\sigma(z)$ satisfy the relation $d\,\sigma(z) \leq M d|z|$, M-const. On L let be defined a continuous real function p(z), where for large $|z|$

$$p(z) \geqslant p_0(|z|) - p_0(a) + \int_a^{|z|} \frac{\omega(t)}{t}\,dt,$$

where $\omega(t) \geqslant 0$ and $\omega(t) \uparrow +\infty$. Let every domain G_i contain an angle Δ_i with the opening $\frac{\pi}{\alpha_i}$, $\frac{1}{2} < \alpha_i < \infty$, the vertex of which has not necessarily to lie in the origin. Let the class $L_2[p(z)]$ consist of functions f(z) defined on L and measurable, for which $\int_L e^{-p(z)} |f(z)|^2\, d\sigma < \infty$. Let $\{\lambda_i\}$ be a sequence of

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given non-negative integers.
Theorem: Let $\{\mu_n\}$ be the sequence of all positive integers being missing in $\{\lambda_n\}$; Let $\lim\limits_{n\to\infty} \frac{n}{\mu_n} = \sigma < 1$. Let $\frac{\pi}{\alpha_1} > 2\pi\sigma$; let G_1 contain the curvilinear angle P (in a great distance from the origin P and Δ_1 are identical) with the vertex in the origin. Let P intersect every circle $|z| = r$, $0 < r < \infty$ along an arc $> 2\pi\sigma r$ (in this sense the opening of P is greater than $2\pi\sigma$). For an $\varepsilon_0 > 0$ let

$$\int\limits_r^{\infty} \frac{p_0(r)\,dr}{1+\omega_1+\varepsilon_0} = \infty, \quad \omega_1 = \max(\beta_1, \beta_2, \ldots, \beta_m), \quad \frac{\pi}{\beta_j} = \frac{\pi}{\alpha_j} - 2\pi\sigma.$$

Then the system $\{z^{\lambda_k}\}$ is complete on L, i.e.

$$\inf_{\{Q\}} \int\limits_L e^{-p(z)} |f(z)-Q(z)|^2 \, d\sigma = 0,$$

On the Completeness of the Systems $\{z^{\lambda_k}\}$ on Curves in the Complex Plane SOV/20-121-5-8/50

where Q(z) denotes various polynomials.
The proof bases on the application of the integral transformation of Cauchy (according to Dzhrbashyan [Ref 2]) and the differential equations of infinite order.
There are 6 Soviet references.

ASSOCIATION: Moskovskiy energeticheskiy institut (Moscow Power Institute)

PRESENTED: April 10, 1958, by I.N.Vekua, Academician

SUBMITTED: April 7, 1958

Card 3/3

16(1) SOV/38-23-4-5/8

AUTHOR: Leont'yev, A.F.

TITLE: On Properties of Sequences of Linear Aggregates Which are Formed From the Polynomials of Jacobi, and on Their Application in the Question Concerning the Completeness of the System of the Jacobi Polynomials

PERIODICAL: Izvestiya Akademii nauk SSSR, Seriya matematicheskaya, 1959, Vol 23, Nr 4, pp 565-594 (USSR)

ABSTRACT: § 1. Let

(1) $$Dy = y'' + Q_1(z)y' + Q_2(z)y \, .$$

If $y(z)$ and $Q_j(z)$ are regular in $|z - a| \leqslant R$ and if there it is $|y(z)| \leqslant M(R,y)$, $Q_j(z) \leqslant N$, then in the point a for every $r < R$ there holds the estimation

(5) $$|D^k y(a)| \leqslant \frac{(2k)!}{r^{2k}} \cdot \frac{M(R,y)}{(1 - \frac{r}{R})^{NR+1}}$$

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On Properties of Sequences of Linear Aggregates Which are Formed From the Polynomials of Jacobi, and on Their Application in the Question Concerning the Completeness of the System of the Jacobi Polynomials SOV/38-23-4-5/8

§ 2. Let the operator

(1) $$M(y) = \sum_{0}^{\infty} a_k D^k y$$

be considered. The characteristic function $L(z) = \sum_{0}^{\infty} a_k z^k$ is assumed to be an entire function of order 1/2 and of the finite type σ. Then $M(y)$ is defined in every point possessing the property : $y(z)$ and $Q_j(z)$ are regular in a circle with the center in this point and with radius $> \sigma$.

§ 3. Let the operator $N(y) = \sum_{0}^{\infty} a_k D^k y$ be considered, where $Dy \equiv (1-z^2)y'' + [\beta - \alpha - (\alpha + \beta + 2)z]\, y'$. A domain of definition for $N(y)$ (domain of convergence of the series) is given.

§ 4.5. For $N(y)$ the integral representation

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$$(2) \quad N(y) = \frac{1}{2\pi i} \int_C y(\xi)\, N_z\left(\frac{1}{\xi - z}\right) d\xi$$

is given, where N_z the result of the action of the operator N on the function $\frac{1}{\xi - z}$ is understood as a function of z. In order that this representation makes a sense, $y(\xi)$ must be analytic in a certain domain. The domain of definition of $N(y)$ is extended by the integral representation.
§ 6. It is stated that the solution of the equation $N(y) = 0$ is representable as a sum of two series $\sum_1^\infty a_j P_{\nu_j}(z)$ and $\sum_1^\infty b_j Q_{\nu_j}(z)$ and that it is regular in a certain ring between two ellipses. § 7. Properties of the se-

On Properties of Sequences of Linear Aggregates Which are Formed From the Polynomials of Jacobi, and on Their Application in the Question Concerning the Completeness of the System of the Jacobi Polynomials SOV/38-23-4-5/8

quence $f_m(z) = \sum_{j=1}^{p_m} \left[a_{mj} P_{\nu_j}(z) + b_{mj} Q_{\nu_j}(z) \right]$, $\lim_{k \to \infty} \frac{k}{\nu_k} = \tau$.

§ 8.9. contains the fundamental theorem on the completeness of the system of the Jacobi polynomials $\left\{ P_{\mu_n}^{(\alpha,\beta)}(z) \right\}$ on a curve L of the complex plane. The author uses methods of M.M. Dzhrbashyan (the Cauchy integral transformation) and the results of the present and the former paper [Ref 5] of the author. - I.O. Khachatryan is mentioned in the paper. - There are 11 references, 8 of which are Soviet, 2 American, and 1 French.

PRESENTED: by I.N. Vekua, Academician

SUBMITTED: October 11, 1958

Card 4/4

16(1)

AUTHOR: Leont'yev, A.F.

SOV/39-48-2-1/9

TITLE: The Problem of [illegible] Sequences of Linear Aggregates Formed out of the Solutions of Differential Equations

PERIODICAL: Matematicheskiy sbornik, 1959, Vol 48, Nr 2, pp 129-136 (USSR)

ABSTRACT:

By the operator $F(z) = A(\phi) = \phi(z) + \int_{-z}^{z} K(z,t)\,\phi(t)dt$, to every function analytic in a domain star-shaped with respect to the origin, there is adjoint a function $F(z)$ being analytic in the domain. Let $B(F)$ be the operator inverse to $A(\phi)$. The author considers the sequence of the aggregates

$$(1) \qquad Q_n(z) = \sum_{j=1}^{p_n} \left[\alpha_j^{(n)} y_j(z) + \beta_j^{(n)} y_{-j}(z) \right], \quad n=1,2,\ldots,$$

where $y_n(z)$ and $y_{-n}(z)$ are two linearly independent solutions of $y'' + q(z)y = \lambda_n^2 y$ ($q(z)$ is an entire function). Theorem 1 asserts that from the uniform convergence of (1) in a domain G symmetric and star-shaped with respect to z_0 there follows the uniform

convergence of the sequence of Dirichlet polynomials

$$f_n(z) = \sum_{j=1}^{p_n} \left[(\alpha_j^{(n)} a_j + \beta_j^{(n)} c_j) e^{-\lambda_j z_0} e^{\lambda_j z} + (\alpha_j^{(n)} b_j + \beta_j^{(n)} d_j) e^{\lambda_j z_0} e^{-\lambda_j z} \right]$$

in the same domain G, and reversely. The $a_j, b_j, \ldots$ are defined by

$$y_j(z + z_0) = a_j \psi_j(z) + b_j \psi_{-j}(z)$$

$$y_{-j}(z + z_0) = c_j \psi_j(z) + d_j \psi_{-j}(z)$$

and $\psi_j(z) = A(e^{\lambda_j z})$, $\psi_{-j}(z) = A(e^{-\lambda_j z})$.

Theorem: 2: Let $0 < \lambda_1 < \lambda_2 < \ldots$, $\lim_{n \to \infty} \frac{n}{\lambda_n} = \sigma$ and let the sequence (1) converge uniformly inside of a domain D containing the vertical closed straight line of the length $2\pi\sigma$. Then (1) converges uniformly inside the vertical strip $\alpha < R(z) < \beta$. Theorem 3 specializes this result for the Chebyshev-Hermitean functions $h_{n_k}(z)$.

The Problem of ... Sequences of Linear Aggregates Formed out of the Solutions of Differential Equations SOV/39-48-2-1/9

Finally it is shown that with the aid of the mentioned theorems the solution of the equation $\sum_{k=0}^{\infty} \frac{a_{2k}}{(2k)!} D^k F = f(z)$, where $DF = F'' + q(z)F$, $D^k = D(D^{k-1})$, can be reduced to the solution of $\sum_{k=0}^{\infty} \frac{a_{2k}}{(2k)!} \phi^{(2k)}(z) = B(f)$. The author mentions B.Ya.Levin, and V.A.Marchenko.

There are 11 references, 6 of which are Soviet, 2 American, 2 French, and 1 Swiss.

SUBMITTED: September 30, 1957

Card 3/3

LEONT'YEV, A.F.

Lacunary Taylor's series of functions of several complex variables. Uch.zap.MOPI 77:99-110 '59. (MIRA 13:5)
(Series, Taylor's)
(Functions of complex variables)

16(1)

AUTHOR: Leont'yev, A.F. SOV/20-126-5-6/69

TITLE: Completeness of Certain Systems of Polynomials in Some Regions of the Complex Plane

PERIODICAL: Doklady Akademii nauk SSSR, 1959, Vol 126, Nr 5, pp 939-942 (USSR)

ABSTRACT: Let K be a bounded continuum, G_{∞} that adjacent domain containing $z = \infty$. Assume that $w = \Phi(z) = z + \alpha_0 + \frac{\alpha_{-1}}{z} + \ldots$ maps conformally the domain G_{∞} onto the exterior of the circle Γ with the center $w = 0$, where $\Phi(\infty) = \infty$, $\lim_{z \to \infty} \frac{\Phi(z)}{z} = 1$.

Let be $[\Phi(z)]^n = z^n + \alpha_{n-1}^{(n)} z^{n-1} + \ldots + \alpha_0^{(n)} + \frac{\alpha_{-1}^{(n)}}{z} + \ldots = \Phi_n(z) + \frac{\alpha_{-1}^{(n)}}{z} + \ldots$.

Let $\{\lambda_n\}$ be a sequence of positive integers with density τ:

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Completeness of Certain Systems of Polynomials in Some Regions of the Complex Plane SOV/20-126-5-6/69

$\lim_{n\to\infty} \frac{n}{\lambda_n} = \tau < 1$.

Theorem: The system $\left\{\phi_{\lambda_n}(z)\right\}$ is complete in the domain $D \subset G_\infty$ which passes over into the curvilinear angle E with the aperture angle $2\pi\tau$ for the mapping $w = \phi(z)$.
Let the curve L_1 of the w-plane run from Γ to ∞, L_2 is assumed to arise from L_1 by rotation around $w = 0$ by the angle φ ; the domain bounded by L_1, L_2 and the Γ - arc is called curvilinear angle with the aperture angle φ) .
Theorem: The system of the Jacobi polynomials $\left\{P_{\lambda_n}^{(\alpha,\beta)}(z)\right\}$ is complete in a domain D which does not contain the interval $[-1, +1]$ of the real axis and which passes over into a curvilinear angle with the aperture angle $2\pi\tau$ for the mapping of the ex-

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Completeness of Certain Systems of Polynomials in Some Regions of the Complex Plane

SOV/20-124-5-6

terior of the interval [-1, + 1] of the z-plane onto the exterior of the unit circle of the w-plane.
There are 4 references, 3 of which are Soviet, and 1 American.

PRESENTED: March 9, 1959, by I.M. Vinogradov, Academician

SUBMITTED: March 2, 1959

Card 3/3

PHASE I BOOK EXPLOITATION [illegible]

Issledovaniya po sovremennym problemam teorii [illegible] sbornik statey (Investigation of [illegible] Variables; Collection of Articles) [illegible] 3,000 copies printed.

Ed. (Title page): A. I. [illegible] & Ya. [illegible]; Tech. Ed.: [illegible]

PURPOSE: This book is [illegible] complex variable. [illegible] scientific workers, and [illegible]

COVERAGE: The book contains [illegible] Conference on the Theory of [illegible] University from [illegible] in the modern theory of [illegible] into 7 parts. The first part [illegible] series, boundary and [illegible] functions and inter[illegible] discusses functions of many [illegible] conformal mappings and [illegible] Riemann surfaces and the theory of [illegible] discusses generalized analytic functions, [illegible] miscellaneous problems. No personalities are [illegible] accompany each article.

TABLE OF CONTENTS:

LEONT'YEV, A.F.

Evaluating the growth of a solution of a differential equation in cases of large parameter values with respect to the modulus and its application in some questions of the theory of function. Sib. mat. zhur. 1 no.3:456-487 S-O '60. (MIRA 14:?)
(Differential equations)

LEONT'YEV, A.F.

Convexity of the region in which the solution of a differential equation of infinite order is regular. Dokl.AN SSSR 132 no.5:1019-1022 Je '60. (MIRA 13:6)

1. Moskovskiy energeticheskiy institut. Predstavleno akademikom I.M. Vinogradovym.
(Differential equations)

LEONT'YEV, A.F.

Applicability of properties of entire functions of the order of less than one half to some other functions. Trudy Mat.inst. 64: 126-146 '61. (MIRA 15:3)
(Functions, Entire) (Functional analysis)

LEONT'YEV, A. F.

"On incomplete systems of Faber polynomials and some other functions"

report submitted at the Intl Conf of Mathematics, Stockholm, Sweden, 15-22 Aug 62

ALEKSANDROV, P.S., red.; BOL'SHEV, L.N., red.; VLADIMIROV, V.S., red.; KUDRYAVTSEV, L.D., red.; LEONT'YEV, A.F., red.; NIKOL'SKIY, S.N., red.; POSTNIKOV, M.M., red.; SOLOMENTSEV, Ye.D., red.; SHAFAREVICH, I.R., red.; GRIBOVA, M.P., tekhn. red.

[English-Russian mathematical dictionary]Anglo-russkii slovar' matematicheskikh terminov. Red. kollegiia; P.S.Aleksandrov (predsedatel') i dr. Moskva, Izd-vo inostr. lit-ry, 1962. 369 p. (MIRA 15:11)

1. Akademiya nauk SSSR. Matematicheskiy institut.
(English language--Dictionaries--Russian)
(Mathematics--Dictionaries)

LEONT'YEV, A. F.

On the closure of a system of functions on the semiaxis.
Izv. AN SSSR. Ser. mat. 26 no.5:781-792 S-O '62.
(MIRA 15:10)

(Sequences(Mathematics))

LEONT'YEV, A.F.

A property of a subsequence of Faber polynomials. Trudy MEI no.42:75-97 '62. (MIRA 16:7)

(Sequences (Mathematics)) (Polynomials)

S/038/63/027/002/002/002
B112/B242

AUTHOR: Leont'yev, A. F.

TITLE: Properties of functions which can be approximated by linear combinations of given functions close to power functions

PERIODICAL: Akademiya nauk SSSR. Izvestiya. Seriya matematicheskaya, v. 27, no. 2, 1963, 397 - 434

TEXT: A sequence $\{\omega_n(z)\}$ of analytic functions $\omega_n(z) = [\varphi(z)]^n [1 + \psi_n(z)]$, $|\psi_n(z)| < q^n$, $q < 1$ is considered. Problems of approximation of analytic functions by linear combinations of the functions of a partial sequence $\{\omega_{\lambda_n}(z)\}$ are investigated. The indices λ_n have to fulfill a condition $\lim\limits_{n \to \infty} (n/\lambda_n) = \sigma < 1$. The domain of completeness of the system $\{\omega_{\lambda_n}(z)\}$ is determined.

SUBMITTED: March 5, 1962
Card 1/1

LEONT'YEV, A.F.

Phragmen-Lindelof type theorems for harmonic functions in a cylinder. Izv. AN SSSR Ser. mat. 27 no.3:661-676 My-Je '63. (MIRA 16:6)

(Harmonic functions)

LEONT'YEV, A.F. (Moskva)

Analyticity of the kernel of an integral transformation. Mat. sbor. 62 no.1:31-38 S '63. (MIRA 16:10)

(Transformations (Mathematics)) (Functions, Analytic)

LEONT'YEV, A.F.

Completeness of the system $\{e^{\lambda_i z}\}$ in a closed strip. Dokl. AN SSSR 152 no.2:266-268 S '63. (MIRA 16:11)

1. Predstavleno akademikom P.S. Novikovym.

LEONT'YEV, A.F. (Moskva)

Growth of functions defined by Dirichlet series and some other more general series. Mat. sbor. 63 no. 2:227-237 F '64.
(MIRA 17:5)

L 20687-66 EWT(d)/EWP(1) IJP(c)

ACC NR: AP6012003 SOURCE CODE: UR/0199/65/006/002/0364/0382

AUTHOR: Leont'yev, A. F.

20
B

ORG: none

TITLE: Growth comparison of linear combinations of functions from two similar systems

SOURCE: Sibirskiy matematicheskiy zhurnal, v. 6, no. 2, 1965, 364-382

TOPIC TAGS: analytic function, mathematics

ABSTRACT: In an earlier work the author considered the sequence of analytic functions

$$\omega_n(z) = [\varphi(z)]^n [1 + \psi_n(z)] \quad (n = 1, 2, \ldots)$$

and studied questions of the approximation of the functions by means of the system

$$\{\omega_{\lambda_n}(z)\}, \ \lim_{n\to\infty} \frac{n}{\lambda_n} = \sigma < 1.$$

The present article considers a more general system of functions with less restrictive conditions. Let $\{\lambda_n\}$ be a sequence of positive numbers (generally speaking, nonintegers) with density

$$\sigma = \lim_{n\to\infty} \frac{n}{\lambda_n},$$

it being given that $\lim_{n\to\infty} \frac{1}{\lambda_n} \ln \left| \frac{1}{L'(\lambda_n)} \right| = 0, \quad L(z) = \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{\lambda_n^2}\right).$

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The functions of the system here dealt with have the form

$$f_n(z) = e^{\lambda_n z}[1 + \psi_n(z)] \quad (n = 1, 2, \ldots).$$

It is assumed that the functions $\psi_n(z)$ are regular in region S and satisfy the following condition: on any bounded closed set $F \subset S$

$$|\psi_n(z)| < Aq^{\lambda_n} \quad (n = 1, 2, \ldots),$$

with $q < 1$ and A and q not depending on z and n (generally speaking, they depend on F). It is further assumed that if in some region $E \subset S$

$$\sum_{n=1}^{\infty} a_n f_n(z) = 0,$$

then all $a_n = 0$. For simplicity let S be the vertical zone: $x_1 < \mathrm{Re}(z) < x_2$. It is assumed that

$$P_n(z) = \sum_{\nu=1}^{p_n} a_{n\nu} f_\nu(z) \quad (n = 1, 2, \ldots)$$

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and $D \subset S$ is used to denote any region which contains a closed vertical segment of length $2\pi\sigma$; z_0 is the center of this segment. The article examines the behavior of the sequence $\{P_n(z)\}$ in the zone T: $x_1 < \mathrm{Re}(z) < x_0$ ($x_0 = \mathrm{Re}(z_0)$) if its behavior in region D is known. It is shown that if the sequence $\{P_n(z)\}$ converges uniformly in region D, it then converges in zone T. The author outlines the proof of the following fundamental theorem: Let the sequence $\{\lambda_n\}$ possess the additional property that its condensation index

$$\delta = \overline{\lim_{n\to\infty}} \frac{1}{\lambda_n} \ln \left| \frac{1}{L'(\lambda_n)} \right| = 0, \quad L(z) = \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{\lambda_n^2}\right).$$

Then there exists a constant K which does not depend on $\{P_n(z)\}$ and satisfies the inequality

$$\sum_{\nu=1}^{p_n} |a_{n\nu} e^{\lambda_\nu z}| < K \max_{t \in \overline{D}} |P_n(t)|, \quad \mathrm{Re}(z) < \mathrm{Re}(z_0).$$

Orig. art. has: 44 formulas. [JPRS]

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Card 3/3 BK

LEONT'YEV, A.F.

Properties of sequences of Dirichlet polynomials converging on the interval of an imaginary axis. Izv. AN SSSR. Ser. mat. 29 no.2:269-328 '65. (MIRA 18:5)

APPROVED FOR RELEASE: 08/23/2000 CIA-RDP86-00513R000929310005-2"

LEONT'YEV, A.F.

Derivation of a functional equation with a given system of particular solutions. Dokl. AN SSSR 160 no.1:36-39 Ja '65. (MIRA 18:2)

1. Matematicheskiy institut im. V.A. Steklova AN SSSR. Submitted June 22, 1964.

L 1671-66 EWT(d)/T IJP(c)

ACCESSION NR: AP5023355 UR/0020/65/164/001/0040/0042

AUTHOR: Leont'yev, A. F. 44,55

TITLE: Representation of arbitrary functions by Dirichlet series 16,44,55 20 B

SOURCE: AN SSSR. Dokaldy, v. 164, no. 1, 1965, 40-42

TOPIC TAGS: infinite series

ABSTRACT: The author considers the Dirichlet series

$$f(z) = \sum_{n=1}^{\infty} a_n e^{\lambda_n z}, \quad \lambda_1 \neq 0, \quad |\lambda_n| \uparrow \infty, \qquad (1)$$

He shows that for certain sequences $\{\lambda_n\}$ one can find a region D in which an arbitrary analytic function f(z) allows representation (1). In particular, the region D can be found when λ_n is chosen along three rays in a prescribed manner. Orig. art. has: 6 formulas.

ASSOCIATION: Matematicheskiy institut im. V. A. Steklova, Akademii nauk SSSR (Mathematical Institute, Academy of Sciences SSSR) 44,55

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