JPRS L/9569

24 February 1981

Translation

ANALYTICAL DESIGN OF SHIPS

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FBIS FOREIGN BROADCAST INFORMATION SERVICE

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JPRS L/9569

24 February 1981

ANALYTICAL DESIGN OF SHIPS

Leningrad ISSLEDOVATEL'SKOYE PROYEKTIROVANIYE KORABLEY (Analytical Design of Ships) in Russian 1980 signed to press 26 Mar 80 pp 1-240

[Book by L.Yu. Khudyakov, Sudostroyeniye, 2,200 copies, 240 pages, UDC 629.12.001.2]

CONTENTS

ANNOTATION

[Text] A study is made of the basic content and methods of solving problems in the theory of analytical design of ships -- a new area in general design theory connected with scientific substantiation of design assignments for ships of various classes and types. Methods of evaluating the technical characteristics of ships in the analytical design phase, their effectiveness indexes, economic indexes, in particular, the cost of building and maintaining the ship as part of the fleet, the military-economic optimization of the tactical-technical characteristics of the ship are discussed. The book is intended for ship design specialists.

There are 8 tables, 61 illustrations and 67 references.

SCIENTIFIC EDITOR'S PREFACE

The scientific substantiation of the development of the naval fleet, the studies of the laws and principles of its construction touch on a broad class of problems pertaining to determining the expedient direction of development of the fleet, the nature of its balance, the deadlines and sequence for building ships, weapons, accessory equipment and engineering devices, and their operation and maintenance. The solution of the indicated problems is encountering serious difficulties in connection with the unprecedented technical complication, rise in cost, and short deadlines for building ships, their weapons, accessory equipment and equipment.

Accordingly, generation of the assignment to build a new ship under modern conditions requires profound, multivariant scientific research developments, comprehensive analysis of possible alternative solutions and, on the basis of this analysis, synthesis of the design of a ship with optimal tactical-technical characteristics. This approach to the process of building a ship has taken the shape of a special phase called analytical design.

This book by L. Yu. Khudyakov is one of the first attempts to present a systematic discussion of the procedural problems of the theory of the analytical design of ships. Along with three basic aspects (operative-tactical, technical and economic) which provide the basis for analytical design, significant attention is given to the problem of optimizing the tactical-technical characteristics of the designed ship. The book has been written on a very high scientific level using the corresponding mathematical apparatus and illustrative examples.

The circle of problems considered during the analytical design of ships is quite broad, and it is very difficult to discuss each of them in sufficient detail.

Traditional courses in ship design theory are devoted to the problems of determining displacement and the principal dimensions and also the planned provision for the basic shipbuilding properties of the ship as a floating facility. At the same time the approximate solution of these problems is only one of the steps (blocks in the terminology of the author) of analytical design -- the technical development phase of the versions of the ship. In the plan for writing the book there is a more complex problem of the development of the methodology of analytical design encompassing analysis of the broad spectrum of combat and engineering characteristics of a ship interrelated with the indexes of military-economic effectiveness. Therefore from the point of view of completeness and, above all, detail of the discussion of all problems considered in the analytical design of ships, the book by L. Yu. Khudyakov cannot claim to be the "last word" in solving the stated problem. For example, the book does not consider the specific problems of planned provision for various characteristics of the ships in the analytical design phase, the complete algorithms for calculating the effectiveness indexes, qualitative conclusions with respect to the optimal combination of tactical-technical characteristics of ships of different classes, and so on.

However, the purpose of this book is only to introduce the reader to a group of basic ideas and methods of the theory of analytical design of ships, and from this point of view it has been written entirely satisfactorily and will be useful for a broad class of specialists involved with the analytical design of ships. The book is especially useful for specialists who have begun to study analytical design inasmuch as after reading it they will be able to delve more purposefully into their field.

The value of the book also lies in the fact that it is a useful scientific start for further development and improvement of analytical design using the achievements of computer engineering and its capabilities for the creation of automated design.

INTRODUCTION

The military potential of the naval fleet is determined by many factors, and one of them is the makeup of the fleet containing various types and classes of submarines, surface ships, and support ships in the number and combinations corresponding to the missions facing the fleet and the balance of it [15].

This paper has as its purpose to offer a brief systematic discussion of the problems pertaining to the method of developing ship design assignments and also design research connected with substantiating various solutions in shipbuilding.

The effectiveness of a ship arises from a number of its properties such as accessory equipment, shielding, survival probability, reliability, seakeeping, and so on. These qualities are quantitatively characterized by values called tactical-technical elements (TTE). In turn, the TTE depend on a number of characteristics of the weapons, accessory equipment and technical means -- the technical design parameters (TDP).

In the consolidated plan, the insurance of high effectiveness of a ship is connected with solving two basic problems: the development and the creation of improved types of weapons, accessory equipment and technical means -- the "bricks" from which the "building" of the ship is constructed -- and the substantiation of an efficient combination of primary characteristics of the ship, that is, its intent and the specific values of the TTE and the TDP.

The solution of the first of the indicated problems involves machine building, electrical engineering, instrument making, material sciences, ship's theory and structural mechanics and also a number of other areas of science and engineering. The second problem is within the competence of ship design theory. At the same time, it must be noted that the two indicated problems cannot be solved separately from a third pertaining to the conditions of combat use of the ship as an element of the system of mixed forces of the fleet called on to solve one combat problem or another. This problem pertains to the operative-tactical area, but it must be considered in its interrelation to the formation of the engineering design of the ship.

The creation and the introduction of nuclear missiles, nuclear power engineering, radio electronics and other achievements of scientific and technical progress on board ships have led to a qualitative jump in improvement of the effectiveness of the ships. Under these conditions and in connection with sharp complication of ships from the tactical point of view, increased cost of building and maintaining them and also the more complex nature of the combat use of ships, the problem of optimizing the TTE and the TDP has acquired special urgency. At the present time it is insufficient to consider only the operative-tactical aspects in the earliest stages of compiling the assignments for building ships. A complex study is needed which takes into account the technical and economic factors.

This problem is solved by design developments with broad variation of the entire set of tactical-technical characteristics (TTC) of the ships. Such developments aimed at the discovery of the optimal combination of TTC of a designed ship have come to be called analytical design (AD). Its basic practical "output" is the scientifically substantiated data used to develop shipbuilding plans and also the ship design assignment.

In this book a study is made of the basic content and methods of solving the problems of AD theory as one of the basic areas of general ship design theory. The class of problems arising during the process of the analytical design of ships is very broad. The statement and the methods of solving a number of AD problems, for example, estimating the effectiveness and optimization of the TTE and TDP do not enter into the traditional courses in ship design theory [4, 26, 35]. The book by V. V. Ashik [1] which contains a chapter on the optimization of the characteristics of the designed ship, published in 1975, constitutes an exception. However, the problems of the methodology of the development of ship design assignments are not considered in this book. It is also necessary to note the book by A. A. Narusbayev [34] which is very similar with respect to content to the problematics of the theory of analytical design, but it does not fully encompass the class of analytical design problems. In contrast to the paper by A. A. Narusbayev, the present book contains a more detailed investigation, for example, of the problems of constructing and calculating the effectiveness indexes, evaluating the economic indexes and certain other problems. Therefore, the author hopes that the material of this book will help the reader to have a more complete view of the analytical design problem.

Analytical design as a system and an independent step in the multistep process of ship design has now found its place in the work practice of specialized organizations. The given book pursues the goal of familiarizing the reader only with the fundamentals of methodology of analytical design inasmuch as this methodology and the theory corresponding to it are continuing to be developed and improved intensely. Accordingly, the author has tried to give attention to a number of procedural difficulties and problems and also to say more about the problems which previously were not included in traditional courses on ship design theory.

When discussing the material of the book, the author used the SI system. In particular, the condition of equilibrium of a floating ship without way is expressed not in the form of a direct equality of the weight of the ship (the pressure of the ship in the water caused by gravity) to the buoyancy force (the total force of hydrostatic pressure of the water against the hull of the ship), but in the form of equality of the mass of the ship to the mass of the water displaced by it.

Many comrades gave the author a great deal of assistance in working on the book. Among them it is necessary especially to mention honored scientists and engineers of the RSFSR, Doctors of Technical Sciences, Professors V. N. Burov and L. A. Korshunov, Candidates of Technical Sciences B. A. Kolyzayev,

A. P. Makkaveyev and Ya. V. Gakh. The author expresses deep appreciation to them for their assistance and support. The improvement of the book has been greatly promoted by the counsel and advice of the reviewers: Honored Scientist and Engineer of the RSFSR, Doctor of Naval Sciences, Professor S. K. Svirin and Doctor of Technical Sciences, Professor V. T. Tomashevskiy. The author is also grateful to N. M. Ivanova, M. G. Tepina and G. D. Karanatova for assistance in laying out the manuscript of this book.

CHAPTER 1. BASIC PROBLEMS OF THE THEORY OF ANALYTICAL DESIGN OF SHIPS

§1.1. Subject of Analytical Design Theory

The subject of AD theory is the set of TTC[1] of a ship investigated from the operative-tactical, technical and economic points of view to select the version (or versions) of the ship most preferable in the sense of effectiveness of the solution of the stated problems and the possibilities of building and maintaining the ship as part of the fleet.

This definition can be extended also to civilian ships if by the TTC we mean the operating-technical characteristics of the ships, and by the operative-tactical point of view, the investigation of the effectiveness of their use for the corresponding purposes (transport, fishing, scientific research, and so on).

At the present time AD is conducted to solve the following problems facing shipbuilding: the substantiation of the directions of development of ships for the future considering the achievements of scientific and technical progress; the development of ship design assignments; the discovery of areas in which the general requirements on ship design and substantiation of these requirements must be developed; substantiation of the directions of development of ship equipment -- and for warships also weapons and accessory equipment -- in the future.

The necessity for AD in the interests of the two last-mentioned problems arises from the fact that in the general case the deficiency of individual technical solutions can be established only by estimating their effect on the overall complex of TTC of the ship and through this complex, on the effectiveness and the possibility of building and maintaining the ship. This evaluation of partial technical solutions sometimes is called "evaluation in terms of the ship." Although in practice "evaluation in terms of the ship" frequently encounters significant difficulties, it is the most objective.

Thus, AD is characterized by the general procedural solution of a number of problems connected with building ships, by weapons, accessory equipment and technical means. The basis of this methodology is all around investigation of the TTC of the ship as a complex technical system and search for the optimal version considering the use of the ship within the soyedineniye and taken together with

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[1] Hereafter the TTC will refer to the set of TTE and TDP.

the combat materiel of other types of forces. Apparently, the term "analytical design" was introduced for the first time in Soviet literature by V. L. Pozdyunin, who defined it as follows [38]:

"Analytical or experimental design takes into account the existing production and operating conditions from the point of view of replacement of them by new conditions or alteration of them to create new structural models of the ships or their component elements corresponding either to entirely new goals of water transportation and defense of the country or certain alterations of the goals."

What are the objective causes of isolation of AD theory in an independent area of general ship design theory and broad application of AD in the activity of scientific and design organizations?

A brief answer to this question is that the objective requirement for a scientific solution to the basic AD problems has begun to correspond to the general scientific possibilities of the creation of the corresponding theory.

The objective requirement of a scientific solution to the problems of developing the requirements on designed ships, their weapons, accessory equipment and technical means has existed for a long time, but during the periods of basically evolutionary development of shipbuilding when for long time intervals the TTC of ships of different classes and types changed comparatively little, the problem of substantiation of the assignments for the design of new ships was not very urgent. The solution of this problem on the basis of intuition and experience of the designer considering information about the TTC of ships already built and information about their practical use, as a rule, did not lead to serious errors although in a number of cases errors did occur.

The demands for objective scientific methods of substantiating the requirements on the designed ships has increased as the ships have become more complicated and various alternative technical conditions have appeared, none of which could be recognized as clearly preferable without the corresponding quantitative analysis.

In 1908, A. N. Krylov proposed [25] a quantitative method of comparative evaluation of different designs of battleships developed for competition. This method was based on the intuitive construction of a criterion which depends on the TTC of the ships. At that time there were no general scientific possibilities for stricter solution of the problems of optimizing the TTC of ships. V. L. Pozdyunin also indicated the clearly intuitive method of solving the problem of optimizing the TTC of ships, talking about the necessity of "collective investigation of both assignments and designs in various design stages at specially organized scientific and engineering conferences..." [38]. Although at this time the so-called expert evaluations is used, the basis for it is the special formal-logical methods of processing opinions and estimates of the experts which were developed comparatively recently.

The necessity for careful development of assignments for design of modern ships, optimization of their TTC and the directions of development is obvious at the present time. This arises from the complication of ships from the technical point of view, a sharp increase in the number of possible alternative technical solutions and versions of operative-tactical use and also an increase in cost of building and maintaining the ships. There are examples where the ships have become obsolete during the design process or they have turned out to be ineffective as a result not only of the high rates of scientific and technical progress, but also the insufficiently complete and comprehensive substantiation of the ship assignments. Generally speaking under the conditions of acceleration of the rates of scientific and technical progress, the rate at which the ships become obsolete is increasing and, consequently, the optimal times for them to be part of the fleet are decreasing. This fact, along with an increase in cost of building and maintaining the ships, is leading to an increase in the price of errors and miscalculations when designing them.

It is clear that for sufficiently accurate optimization of the TTC of a ship it is already necessary in the design stage to estimate the effectiveness of solving the missions assigned to the ship. At the same time the ship is a complex engineering dynamic system, the characteristics of which can, generally speaking, vary randomly with time as a result of interaction with the external environment and the enemy. The mathematical expectation of the processes of the functioning of such systems is encountering serious difficulties, including difficulties of a computational nature. Similar difficulties also exist when solving the problems of optimization occurring when designing complex engineering objects. Here, first of all, it is necessary to note the complexity of the optimizable functionals which, as a rule, can be given only by sufficiently complex algorithms, ambiguity of the optimization criteria, complexity and variety of the limiting conditions, and so on.

The above-indicated difficulties in solving the basic AD problem -- optimization of the TTC of the ship -- were essentially, although not finally, overcome after World War II when high speed computer engineering came into being, and a number of new mathematical methods of optimization and also mathematical simulation of the structure and functioning of complex objects and processes were developed. One of the first studies by the new mathematical methods of optimization must be considered the works of L. V. Kantrovich with respect to statement and solution of the problems of linear programming [22].

Thus, the increasing demands for scientific solution of the basic problems of the analytical design of ships has to a known degree begun to correspond to the general scientific possibilities of the theoretical formation of AD as a new applied scientific area. This is also an objective cause for scientific formulation of AD theory as an independent branch of general design theory.

§1.2. Basic Problems of AD Theory and the General Characteristic of the Methods of Solving Them

In accordance with the final goal of AD -- optimization of the TTC of the ship on the basis of analysis of the operative-tactical, technical and economic aspects connected with building and using it in the fleets -- it is possible to isolate the following procedural problems subject to solution in the AD process.

1. Determination of the sets of TTC of the versions of the ship subject to comparison for optimization -- the so-called engineering design block.[1]

2. Determination of the effectivenesss index of the ship with different sets of TTC when solving the problems invested in it -- the effectiveness evaluation block.

3. Estimation of the service indexes, that is, the expenditures of different types of reserves,[2] including economic, required for the building and maintenance of the ship -- the reserve block.

4. The optimization of the TTC of a ship, including the selection and construction of the optimization criteria -- the optimization block.

These problems, the block diagram of the solution of which is shown in Figure 1.1[3] are characteristic of the AD process not only for ships, but also any accessory equipment system [51], civilian vessels and, in general, any technical systems with the corresponding interpretation of the concept of effectiveness.

In the subsequent chapters a study will be made of the content, and a characteristic will be presented for the methods of solving the problems of each of the above-indicated AD blocks, and meanwhile we shall make a few preliminary remarks.

In essence, the problem of the engineering design block which will be called the technical block for brevity, coincides with the basic problem of ship design theory in the understanding established up to now. Actually, we are talking about determining the set of TTC of a ship which satisfies certain values of variable characteristics. The basic content of design theory is defined in this way in the existing literature. The methods of solving different problems connected with technical development of the designed ship are discussed in a number of courses and monographs on design theory [1, 4, 26, 35].

The necessity for investigating a large number of versions of the designed ship with limited information with respect to a number of initial and intermediate data, including those pertaining to the geometric shape of the ship and its

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[1] Here and hereafter the term "engineering design" reflects the essence of the problem solved in the given stage -- determination of technically compatible sets of TTC of the ship (in contrast to the engineering design stage in the general process of building the ship).

[2] In the given case the term "reserve" is understood in the broader sense in contrast to the application of this term, for example, for denoting the endurance index in reliability theory.

[3] Hereafter the step of developing the intent of the ship will not be considered inasmuch as basically unformalized methods and approaches are used in this step.

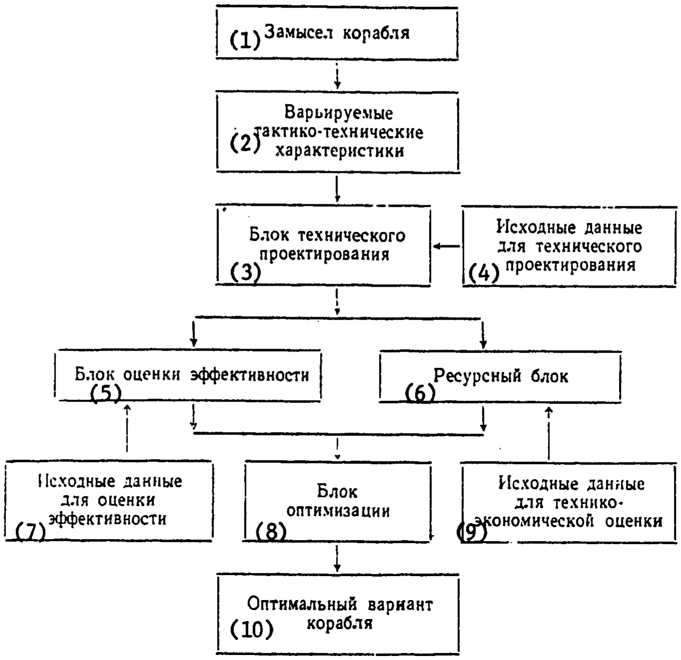

Figure 1.1. Block diagram of the process of analytical design of a ship

Key:

1. Intent of the ship
2. Variable tactical-technical characteristics
3. Engineering design block
4. Initial data for engineering design
5. Effectiveness evaluation block
6. Reserve block
7. Initial data for estimating effectiveness
8. Optimization block
9. Initial data for technical-economic evaluation
10. Optimal version of the ship

component elements is characteristic for AD. Therefore in the AD stage broad use is made of the so-called analytical methods of design (see Chapter 2). Inasmuch as during the AD process quite frequently it is necessary to consider versions of the ship not having designed or constructed analog, it is necessary for the specialists in AD operatively to develop relatively simple approximate procedures and approaches for quantitative evaluation of a number of characteristics. Here, just as when solving other AD problems, much depends on the skill, experience and intuition of the researcher, although the decisive role unconditionally belongs to knowing the basic principles and achievements of the corresponding special sciences. An important methodological procedure of AD theory is also the application of the methods of similarity and mathematical statistics which permit determination of the characteristics of the designed ship by analogous characteristics of analogs already built or designed.

From the point of view of the technical development of a ship, the design process usually is divided into the following steps: technical developments when preparing and substantiating the operative-tactical intent of building the ship (as a rule, they are satisfied within the framework of the research with respect to shaping the prospective shipbuilding programs); technical developments for the preparation and substantiation of the tactical-technical assignment for ship design (the conceptual design), the preliminary design, contact design, development of working drawings (detail design). The technical development of the versions of the ship during AD is carried out on the level of the first two of the above-indicated steps.

In order to have the possibility of quantitatively evaluating the effectiveness of the designed ship, it is necessary first of all to select the index or indexes of effectiveness and, secondly, to construct the algorithm making it possible to calculate their values as a function of the TTC of the ship.

The theoretical basis for solving the indicated problems is a comparatively new applied science formed after World War II -- operations research -- which studies the methods of quantitative estimation and optimization of various purposeful effects. The basis for this science is mathematical simulation of the functioning of the investigated system and optimization of various parameters on which this functioning depends. The theory of operations research uses a large arsenal of mathematical methods connected with probability theory, random process theory, mathematical statistics, mathematical methods of optimization, and so on. The solution of the majority of practical problems in the field of operations research requires the application of a computer. There is a broad literature on the theory of operations research and its practical applications [12, 13, 19, 24, 31, 45, 59].

It must be noted that individual problems of operations research, in particular, shooting theory, have already been resolved for a long time both in the Soviet Union and abroad. For example, the general principles of estimating effectiveness as applied to the problems of shooting theory are investigated by A. N. Kolmogorov [22].

Very frequently, during AD, cost is used as a united measure of the expenditures of different types of resources required for building ships and maintaining them in the fleet. In order to estimate this index in individual design stages, the methods of similarity and statistics are used permitting evaluation of the cost of the designed ship with respect to given analogs. If cost is used as the basic service index, then optimization of the TTC of a ship in the AD process is called military-economic optimization, and the research aimed at solving this problem is called military-economic analysis.

The military-economic approach when optimizing the TTC of ships and other accessory equipment systems, combat complexes, and so on has found broad application at the present time [18, 51]. In the United States this approach is called "cost-effectiveness analysis." The optimization of the operating-technical characteristics by cost-effectiveness analysis is the basic problem of technical-economic analysis for civilian ships [34].

At the same time, the military-economic approach is not the only one for optimizing the TTC and individual technical systems of the ships. The nature of the reserves which must be considered in each specific case can be highly varied (displacement and dimensions of the ship, the conditions of its combat and base support, the deadlines for building, the production capabilities of industry, and so on). The problems of optimizing the geometric composition of a ship are of a specific nature, that is, the placement of a weapons, accessory equipment, technical means and personnel on the ship. Such problems sometimes must be solved in the AD stage.

What has been stated above gives rise to the different forms and hierarchical levels of the optimization problems arising during analytical design of ships. As for the purely mathematical nature of these problems, they most frequently are formulated as mathematical program problems. At the same time it must be noted that many of the problems of design optimization of ships' characteristics still do not have strict mathematical statement.

In connection with the complexity of adequate mathematical models of the blocks of the problem of technical development of versions and evaluation of their effectiveness, the problems of optimizing the TTC of designed ships frequently lose visibility and the correct statement, analysis and interpretation of the optimization results require a great deal of experience and intuition on the part of the researcher. The solution of the design problems of optimization is also impossible without great mathematical skill inasmuch as very frequently it is necessary to use individual mathematical methods in different combinations.

It is necessary to consider that AD theory with respect to the optimization of the TTC of ships only generates recommendations and the necessary information for adopting design solutions. The final adoption of these solutions remains within the competence of the directing agents and people responsible for the solutions. Such people can be, in particular, the customer representative confirming the design assignment and the chief designer heading up the entire subsequent process of developing the ship's design.

From what has been discussed it is obvious that AD theory is an example of the scientific area at the junction of a number of scientific fields, in particular, ship-building sciences, operations research, applied economics, mathematical methods of optimization, and so on. The occurrence of such areas, as is known, is one of the characteristic features of the modern phase of development of basic and applied sciences.

In AD theory, just as in general ship design theory, there is still no united and sufficiently stable terminology with respect to a number of problems and concepts. Therefore the terminology used in the given book to a great extent reflects the personal point of view of the author and also the views of specialists known to him, and it does not claim to final editing. In particular, the following terms will be used many times in the text of the book: "weapons," "accessory equipment," "means of combat," "combat supplies on hand," "technical means." As applied to the ship, unless specially stipulated, these terms can be understood as follows:

Means of combat are means (rockets, torpedoes,mines, bombs, missiles, and so on) having a direct damaging effect on the enemy;

Combat supplies on hand are the means of combat at storage locations and in the installations for use;

Weapons are the set of means of combat and devices for the direct use of them (rocket launchers and artillery mounts, torpedo tubes, mortar tubes, and so on plus the above-indicated means of combat and combat supplies on hand);

Accessory equipment is the set of means provided for ship handling, communications and also the use of weapons for target detection, target indication, determination of location coordinates and the plane of the true meridian, and so on;

Technical means are various types of machinery, devices, systems and other equipment giving the ship maneuverability, its seakeeping qualities, stealth, invulnerability and habitability.

It is also necessary to consider that the terminology of AD theory is highly fluid inasmuch as it must take into account the terminological changes in a number of scientific disciplines and branches of industry, new classes and types of ships, new architectural and structural features, and so on. Therefore the author hopes that when evaluating this book from the point of view of the terminology used the reader will consider the above-mentioned facts.

§1.3. General Statement of the Problem of Optimizing a Ship's Tactical-Technical Characteristics as a Problem of Mathematical Programming

With respect to the nature of their influence on combat effectiveness, the entire set of TTC of a ship can be broken only into already known groups: TTE and TDP.

By TTE we mean the characteristics which directly influence the combat effectiveness of the ship, that is, directly enter into the algorithms for calculating the combat effectiveness indexes as initial data. For example, the TTE include the following: the composition of the weapons and accessory equipment considering their natural characteristics, maximum speed of the ship, its sea endurance, cruising range, principal dimensions, handling characteristics, stealth characteristics, protection, reliability, invulnerability, and so on.

By the TDP we mean the characteristics indirectly influencing the effectiveness indexes through the TTE which depend on the TDP. For example, the TDP include the following: the structural design and architectural class of the hull, the class and characteristics of the power plant, the electric power system, the type and characteristics of the ship's hull material, and so on.

When optimizing the set of TTE and TDP of a ship, a set of scalar characteristics are always isolated which are subjected to optimization, and during the process of this optimization they can vary independently of each other. Let us designate these characteristics by the vector X with the components $x_1, \ldots, x_n$. This means that each set of values of the variable characteristics can be placed in correspondence to some point of an n-dimensional euclidian space. The coordinates of this point are the values of the individual variable characteristics.

With respect to its mathematical nature, the optimizable characteristics of x_i, $i=1,\ldots, n$ can be continuous and discrete values, that is, in the general case the variables x_i can be divided into two groups: continuously variable and discretely variable. This separation is connected with the fact that for optimization with respect to continuous and discrete values in the general case different mathematical methods are used. Sometimes certain discrete characteristics can be approximately considered as continuous or vice versa.

Along with the characteristics of the ship defined by the vector X, the effectiveness indexes and resource indexes can depend on a number of other parameters which do not vary in each specific investigation. The values of these parameters are fixed by the designer either in accordance with the general and special requirements on the design of the ship, its mechanisms and systems or on the basis of analysis of prototypes or, finally, intuitive arguments. Let us designate these characteristics by the vector Θ. For example, frequently the parameters of the shape of the ship's hull, the reserve buoyancy, the habitability standards, the general layout, and so on are fixed in the AD stage. In special cases, individual characteristics mentioned above as components of the vector X can also be fixed.

In addition to the characteristics defined by the vectors X and Θ, there is a third group of so-called dependent characteristics, the values of which are completely defined by giving the vectors X and Θ. Let us denote these characteristics by the vector Y with the components $y_1,\ldots, y_m$. This group includes displacement and principal dimensions of the ship, its load components, seakeeping and handling characteristics, the parameters of the provisional laws of damage under the effect of various forms of weapons and ammunition, stealth and protection characteristics.

The establishment of the relation between the vectors X, Θ and Y in the form

$$Y=f(X,\ \Theta) \tag{1.1}$$

is the primary goal of the engineering design block.

In the general case the relation between the vectors X, Θ and Y is not fully formalized, and it is established by the graphoanalytical method, for the determination of a number of ship's characteristics requires graphical development of the design solutions. However, inasmuch as in AD it is necessary to consider a large number of versions, an effort is made to use approximate analytical methods with a minimum of highly tedious graphical operations.

Recently special automated design computer systems (SAPR) for ships and other engineering projects have begun to be developed and used [18, 30, 67]. The basis for these systems is computers and specialized electronic devices for graphical data input and output and also the performance of certain operations of making drawings. The SAPR combine the great computing capabilities of the computer with the possibility for the human designer operatively to influence the design process, including making and evaluating a number of the construction and other decisions which at the present time cannot be fully algorithmized. For example, such decisions include the principles of the general placement of accessory equipment, machinery, equipment and personnel on the ship, architectural features of the hull, and so on. On the whole, the development and improvement of SAPR is a prospective area for the improvement of AD methodology.

In the effectiveness evaluation block relations are established for the effectiveness indexes $Э_l$, $l = 1, \ldots, L$, as a function of vectors X, Θ and Y:

$$Э_l = \tilde{\varphi}_l(X, \Theta, Y), \quad l = 1, \ldots, L, \tag{1.2}$$

where L is the number of investigated effectiveness indexes.

Inasmuch as the vectors X, Θ and Y are related to each other by the expression (1.1), the relations (1.2) can be represented in the form

$$Э_l = \varphi_l(X, \Theta), \quad l = 1, \ldots, L. \tag{1.3}$$

In accordance with (1.3), the functional ϕ_ℓ must be given in the set of possible values of the vectors X and Θ.

In the block for estimating the service indexes, the relation is established between the service indexes S_k, k=1, ..., K and the vectors X, Θ and Y. Considering expression (1.1) these relations have the form

$$S_k = \psi_k(X, \Theta), \quad k = 1, \ldots, K, \tag{1.4}$$

where K is the number of investigated service indexes.

The functionals ψ_k must be given in the set of possible values of the vectors X and Θ.

An optimalness criterion including the effectiveness indexes (1.3), service indexes (1.4) and in the general case, the X and Y vector components is primarily constructed in the optimization block. The criterion is a system of conditions imposed on the above-indicated indexes and components. These conditions must be satisfied by the optimal vector X or the optimal set of vectors X if the solution of the problem is not unique.

Most frequently the optimalness criterion is formulated as an extremal problem in a closed bounded set of vectors X which is called the admissible set $\mathfrak{X}$. The optimal set of vectors X is a subset of the set $\mathfrak{X}$, and in case of a unique solution the optimal vector X is a point of the set $\mathfrak{X}$.

The procedure for giving the set $\mathfrak{X}$ has great significance. Two basic cases are of the greatest interest here: the set $\mathfrak{X}$ is given in the form of a discrete finite set of vectors X, that is, a finite number of versions of the ship characterized by the vectors $X_1, \ldots, X_{\mathcal{N}}$ is given, where $\mathcal{N}$ is the number of investigated versions; the set $\mathfrak{X}$ is given in the form of a set of continuous closed bounded sets of values of continuous variable components of the vector X for each of the possible sets of values of discretely variable characteristics which are also components of the vector X.

In the first case in the n-dimensional euclidian space (n is the number of components of the vector X) the set $\mathfrak{X}$ is represented in the form of the set of discrete points (Figure 1.2, a). In the second case the set $\mathfrak{X}$ can be represented in the form of a combination of $(n-n_1)$-dimensional sets $\mathfrak{X}_p$, $p = 1, \ldots, \mathcal{P}$ where $\mathcal{P}$ is the number of possible sets of values of discretely variable components oı the vector X, and n_1 is the number of these components ($n_1 \leq n$). Figure 1.2, b gives the form of the sets $\mathfrak{X}_p$ schematically for the three-dimensional case when there is one discrete component x_3 which assumes two values x_3^1 and x_3^2 and two continuous components x_1 and x_2.

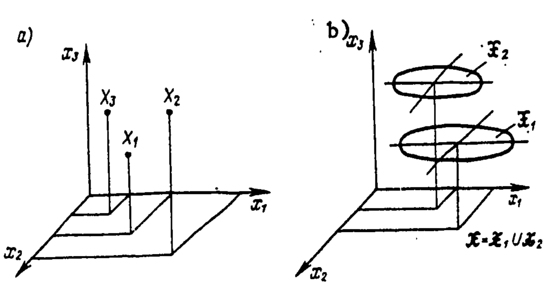

Figure 1.2. Methods of giving the set $\mathfrak{X}$

For the discrete method of giving the set $\mathfrak{X}$ the optimization of the ship's characteristics is called comparative evaluation of versions; in the second case it is called optimization in space of the TTE and the TDP of the ship.

CHAPTER 2. SOLUTION OF THE PROBLEMS OF THE SHIP ENGINEERING DESIGN BLOCK

§2.1. Characteristics of the Problems of the Engineering Design Block

The basic problem of the engineering design block in the general AD scheme is establishment of the relation between the independently variable characteristics of the ship subject to optimization and the dependent characteristics which influence the effectiveness and service indexes. Mathematically this problem reduces to expansion of the function (1.1). Here the central problem is determination of the displacement, principal dimensions and load elements of the ship.

As is known, the load is the set of data on the distribution of all loads on the ship. For convenience of the calculations it is divided into a number of sections and items combining uniform loads. However, the load breakdown is to a defined degree arbitrary.

Along with displacement and the principal dimensions, a number of other characteristics which influence the effectiveness and service indexes are also determined during the AD process. For example, it is possible to select the engine power as the optimized characteristic instead of the speed of the ship. In this case after determining the displacement and the principal dimensions it is necessary to calculate the maximum speed.

The basic characteristics of the problem of expansion of the function (1.1) consist in the following.

1. In the general case the function (1.1) is algorithmic inasmuch as the relation between the majority of the ship's characteristics cannot be expressed in explicit analytical form.

2. The determination of the number of the ship's characteristics requires graphical development of the design solutions connected with the general placement of the machinery, equipment and accessory equipment and the placement of personnel and also the shape of the hull and its architectural features. The greater part of these problems cannot be formalized at the present time; therefore creative participation of the designer is necessary.

3. The expansion of the function (1.1) usually requires adoption of compromise solutions (partial optimization with respect to individual indexes) inasmuch as insurance of individual qualities of the ship, as a rule, is connected with satisfying contradictory requirements, and consideration of the latter in the general optimization model greatly complicates the problem.

4. From the mathematical point of view the function (1.1) is given in implicit form, for individual components of the vector Y can depend not only on the components of the vectors X and Θ, but also each other. In particular, the masses of some of the structural elements of the ship, its technical means and accessory equipment depend on the displacement and the principal dimensions which, in turn, depend on the above-indicated masses of individual elements. For example, the engine power required to insure given maximum speed of the ship depends on the displacement and the principal dimensions of the ship at the same time as the displacement and the principal dimensions themselves depend on the weight and volume of the power plant and, consequently, its power. Analogously, the weights of the hull, certain systems and devices, the equipment of the compartments depend on displacement and directions of the ship, and the displacement and dimensions depend on the above-indicated weights. Accordingly, the expansion of the function (1.1) is connected with the necessity for compiling and solving the system of equations given algorithmically in the general case.[1]

5. The problem of the expansion of the function (1.1), as a rule, is undefined inasmuch as the number of equations relating the individual components of the vector Y to the components of the vectors X and Θ is usually less than the number of desired characteristics (components of the vector Y). In particular, this fact gives rise to the possibility of using compromise solutions mentioned above.

6. The expansion of the function (1.1) requires an iterative approach (successive approximations) both because of the algorithmic assignment of the corresponding equations and in connection with the possibility of compiling the complete system of equations in which all requirements on the ship would be considered sufficiently exactly. Therefore the satisfaction of a significant number of the requirements is established by check calculations after developing the drawings of individual structural elements of the ship and determining a number of its elements.

The check calculations usually reveal defined lack of correspondence of the obtained characteristics of the ship to the required characteristics. In order to eliminate these divergences the designer takes the corresponding measures after which the check calculations are repeated. The process continues until all discrepancies are eliminated (Figure 2.1).

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[1] By algorithmic we mean the giving of equations in which their individual parts are defined by the algorithms for calculating the values of the corresponding functions of the desired and given (fixed) arguments.

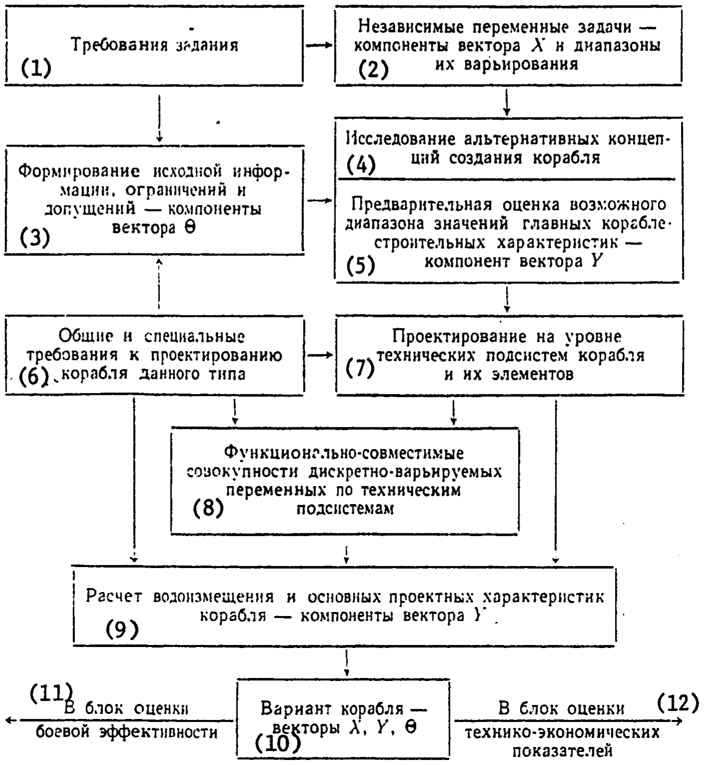

Figure 2.1. Schematic diagram of the solution of the problems of the engineering design block during the AD process

Key:
1. Assignment requirements
2. Independent variable of the problem -- components of the vector X and their ranges of variation
3. Formation of the initial data, restrictions and assumptions -- components of the vector Θ
4. Investigation of alternative concepts of the construction of the ship
5. Preliminary estimation of the possible range of values of the principal shipbuilding characteristics -- components of the vector Y
6. General and special requirements on the design of a ship of the given class
7. Design on the level of the engineering subsystems of the ship and their elements
8. Functionally compatible sets of discrete variables with respect to engineering subsystems
9. Calculation of displacement and the basic design characteristics of the ship -- components of the vector Y
10. Version of the ship -- vectors X, Y, Θ
11. To the combat effectiveness evaluation block
12. To the technical-economic indexes evaluation block

§2.2. Three Basic Methods of Determining Displacement and Principal Dimensions of a Ship

At present, usually three basic methods of solving the problem connected with determining displacement and the principal dimensions of a ship are isolated as applied to AD: the graphical method, analytical and graphoanalytical.

In the graphical method the drawings of the general placement of weapons, accessory equipment, machinery, fittings, personnel, storage and other objects on the ship are made first. The shape of the ship's hull is given graphically in the form of a theoretical drawing. The degree of detail of the drawings depends on the stage of development of the design. The design is guided by the general qualitative arguments and some local indexes connected with insuring various properties of the ship. On the basis of the qualitative nature of the above-indicated arguments and local nature of the partial indexes, as a rule, the ship corresponding to the developed drawings does not meet a number of the requirements and conditions. This fact is established by a set of test calculations which are performed using the above-indicated drawings. For example, the weight of the ship can fail to correspond to the floating volume. The trimming conditions, stability, unsinkability, and so on may fail to be satisfied. The designer eliminates these discrepancies by altering the dimensions and shape of the hull, the location of the weapons, accessory equipment, technical means, and so on. The corresponding changes are introduced into the drawings, the test calculations are again performed, and the matching process continues.

The nomenclature of the check calculations depends on the stage of development of the design, and it continues to expand as we go deeper into the developed stages. Thus, in the earliest stages (conceptual design) the load, buoyancy, initial stability, trim, propulsive performance and cruising range are calculated. Then the strength, unsinkability stability at large angles of inclination, steerability, roll, and so on are calculated. As a result of several approximations it is possible to satisfy all of the requirements and obtain the desired characteristics (components of the vector Y). Here the number of approximations depends on the experience of the designer. The intuition and experience of the designer to a significant degree also determine the optimalness of the obtained version of the ship with respect to certain partial criteria, for example, minimum displacement. It must be noted that the solution of the problem of determining the set of TTC of a ship satisfying the given requirements does not always exist for limited ranges of possible values of the TDP.

The highly tedious nature of the graphical method has led to the development of methods of determining the elements of a designed ship based on analytical or algorithmic representation of the relations between the desired and the given characteristics. These relations can have the form of equations, formulas, algorithms and so on. In spite of the fact that until recently it was not possible to develop a sufficiently accurate analytical representation of many of the relations connected with the graphical representation of a ship, analytical methods play an important role in the early stages of design when a large number of versions are developed and it is necessary to lower the requirements on accuracy of determining the desired characteristics. Here it is necessary to consider that the accuracy of the method of determining the developments of a ship in the early design stages must correspond to the relatively low accuracy of the initial data for these stages. (The basic equations of the analytical method of determining displacement and the principal dimensions of a ship will be considered in §2.3.)

By the graphical analytical method frequently we mean the method which is referred to as graphical above. Here we must consider that in the graphical method check calculations are made which are taken into account by the analytical part of this method. However, this representation must be recognized as not completely accurate inasmuch as the basic problem -- determination of the displacement and the principal dimensions of the ship -- is solved graphically in the graphical method.

More strictly, it is necessary to consider the graphoanalytical method to be the method in which the ship's characteristics are determined by solving the corresponding system of equations but graphical drawings of the ship and individual structural assemblies of it are used to compile these equations. At the present time this method is the most typical of the typical AD stage.

Among the above-investigated methods the most exact is the graphical method -- essentially the only one in the final design stages. At the same time, the graphical method is the most labor-consuming. Even when a computer is used a significant amount of time is spent on developing the drawings, preparing the required initial data based on them and input of this data to the computer. Therefore one of the prospective areas has come to be automation of the graphical operations and input of the required initial data from the drawings to the computer for performing the corresponding calculations. The accuracy of the development of the versions of the ship is improved as a result of broader use of graphical procedures..

As for the analytical method, it retains its value as the first approximation and as a method of recalculating the ship's characteristics with few versions of them.

As is known, the analytical method is connected with using a large number of initial data in accordance with the previously constructed and designed prototypes; therefore usually it is impossible to determine the elements of ships with new structural solutions by the purely analytical method. Actually, when compiling the equations of the analytical method for submarines it is necessary (see §2.3) to know the weight of a unit volume of the pressure hull as a function of such parameters as the maximum depth of submersion, the type of hull material, the material strength characteristics, and so on. In solving this problem the methods of similarity statistics are used which permit the problem to be solved on the basis of data on previously constructed and designed hulls. However, if the hull of the designed submarine has essentially new structural features, this approach leads to a large error and can turn out to be unacceptable. In this case it is first necessary to develop a new hull design, determine the mass characteristics of it as a function of the above-indicated variables and then use this function in the equations of the analytical method. However, with this approach the designer is already using the graphoanalytical method.

As an example of the structural features of the pressure hulls of submarines it is possible to present the so-called "figure eight" design which was used in the vicinities of compartments II and IV on the XXI series German submarines built at the end of World War II. The design, which resembled a figure eight in transverse cross section was two horizontal cylinders joined by a common spacer platform which made it possible to increase the usable volume of the storage battery compartments (battery tanks) on the small and medium displacement diesel electric submarines, that is, with relatively small hull diameters. At the same time, the application of the "figure eight," generally speaking, increased the weight of the hull (by comparison with a hull made up of one circular cylinder) as a result of installation of the spacer platform and fastenings in the transition areas from the "figure eight" to the circular cylinder.

The structural features of the hulls of heavy surface ships include armor and underwater protection (UWP) systems. In particular, the on-board UWP systems of battleships, heavy cruisers and aircraft carriers built and designed before World War II and afterward were distinguished by great variety of specific realization. The leading shipbuilding powers proposed and realized their own UWP systems (Fig 2.2). Although a detailed discussion of the advantages and disadvantages of the various UWP systems is beyond the scope of this book, let us make a few comments.

Among the foreign countries probably Germany achieved the greatest success in improving the UWP before World War II. The UWP systems of the German battleships of the "Bismarck" class and "N" design (not built) had comparatively high effectiveness. The underwater protection of the "Bismarck" class battleships had a two-chamber design with one basic longitudinal armored bulkhead as far as possible from the outer hull of the ship. This system (with one basic longitudinal armored bulkhead) was used on the Japanese "Yamato" class ships.[1]

On the heavy Soviet ships designed and laid up before World War II and afterward, the UWP systems, just as a number of other basic elements, received significant development and were greatly improved [15]. It is possible to state with certainty that our ships were the best from the point of view of the UWP system. The design of these ships was supported by broad experimental and theoretical research of various UWP systems considering their effectiveness, the mass-dimensional characteristic, and so on.

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[1] E. Ye. Gulyayev proposed underwater protection for a ship by installing a number of longitudinal watertight bulkheads connected to each other by framing which, as they collapsed, gradually absorbed the energy of the blast, in 1900. This principle formed the basis for all of the UWP systems proposed and built later.

The idea and the first structural proposal to introduce cylindrical bulkheads into the UWP system were advanced by the prominent Russian shipbuilder I. G. Bubnov. Later similar bulkheads (cylindrical and curved) appeared in the Italian Pualese design (1920) and in the American Hovgard design (1940).

The proposal to introduce an internal armored bulkhead into the UWP was made by N. Ye. Kuteynikov on the basis of experience in the Russian-Japanese War.

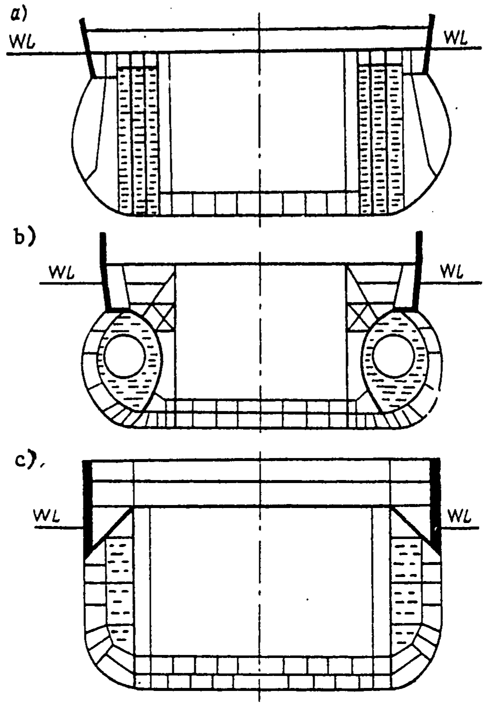

Figure 2.2. UWP systems for surface ships: a -- Russian system (Gulyayev type); b -- Italian "cylindrical" system; c -- German system with filtration bulkhead

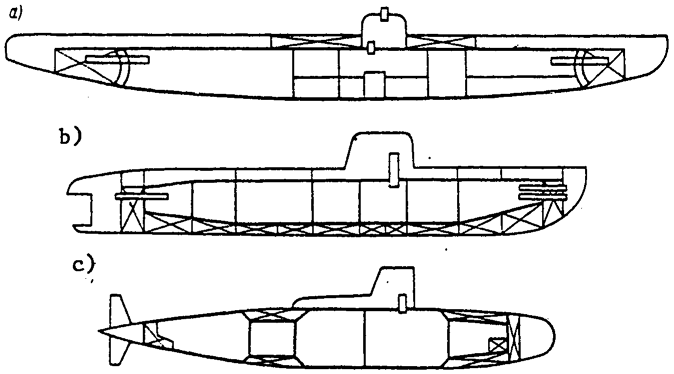

Figure 2.3. Architectural types of submarines

Finally, it is necessary to mention the original architectural class of submarine ("Minoga," "Bars") proposed in his time by I. G. Bubnov and called the Russian class (Figure 2.3, a). On these single-hull submarines the main ballast tanks were located only in the extremities outside the pressure hull and in the midsection of the superstructure (the deck tank). Subsequently the diesel submarines were developed and improved, the double-hull architectural class with reserve buoyancy became predominant. It insured the condition of "single-compartment" surface unsinkability (Figure 2.3, b). The series XXI, XXIII and XXVI German submarines built at the end of World War II constituted an exception. Surface unsinkability was not provided for on these submarines.

The architectural class of modern American atomic submarines is very close to the single-hull class. Surface unsinkability is not structurally provided for in these submarines and, consequently, the possibilities of rescuing the submarine from the submerged position when water gets into the pressure hull are extremely limited (Figure 2.3, c). It can be stated that theoretically the architectural class of American atomic submarines is very similar to the Russian class of I. G. Bubnov (single hull with location of the main ballast tanks outside the pressure hull), although this theoretical similarity is concealed by the absence of a developed superstructure and external hull design of the submarines insuring the best hydrodynamic qualities for underwater cruising. (Let us note that the outside lines and the developed superstructure on the Bubnov submarines were the result of the necessity for insuring satisfactory seakeeping qualities on the surface and placement of outside lattice torpedo tubes in the superstructure.)

After the loss of the "Thresher" (1963) and "Scorpion" (1968) atomic submarines and also considering the fact that from 1900 to 1971, 21 American submarines had been lost as a result of various accidents and 431 people had died, the problem of improving the degree of insurance of unsinkability and rescue of personnel from sunken submarines was subjected to further investigation and discussion in the United States. In the opinion of the specialists, the navy could equip its nuclear-powered submarines with all of the necessary devices insuring complete safety of them, but then the submarines would not be suitable for anything. This statement indicates that significant reduction of the requirements on insuring invulnerability of American atomic submarines was in the interests of improving other combat characteristics.[1]

Ending this brief investigation of the three basic methods of determining the engineering characteristics of a designed ship from the point of view of their use in analytical design, let us again emphasize that the purely analytical method, as a rule, is connected with carryover of the majority of structural and technical solutions realized on previously built and designed ships to the ship being designed. In this sense the purely analytical method has highly limited possibilities, and when designing ships with new engineering solutions, sometimes it turns out, in general, to be useless. Another serious deficiency of the purely analytical method, as was already noted previously, is the difficulty of

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[1] J. Gorz, POD"YEM ZATONUVSHIKH KORABLEY [Raising Sunken Ships], Leningrad, Sudostroyeniye, 1978.

sufficiently accurate consideration within the framework of this method of such factors as the shape of the ship's hull and the placement of weapons, accessory equipment, technical means and personnel on it and also the stability conditions, unsinkability, trimming, invulnerability and so on connected with these factors.

§2.3. Equations of the Analytical Method of Ship Design

In this section a study is made of the principles of compiling the basic equations of the analytical method of ship design. The basic equations include the equations of mass, buoyancy, volume, initial transverse stability and power (speed) of the ship.

1. Mass Equation. The mass equation is an analytical expression of the condition of equality of the mass of the ship to the sum of the masses entering into the load, that is, the sum of the masses of the component elements of the ship.

Inasmuch as, according to Archimedes law, the mass of a ship floating without way must always be equal to the mass displacement, that is, the mass of water displaced by the ship, we have the equality

$$D_m = \sum_i m_i, \tag{2.1}$$

where D_m is the mass displacement, m_i is the mass of the individual load components.

Expression (2.1) is an equation on the basis of the fact that the individual mass m_i can depend on the displacement D_m.

The expansion of the mass equation consists in establishing the dependence of m_i on TTE and TDP of the ship (the components of the vectors X, Y and Θ) including the displacement and the principal dimensions. The number of terms in the righthand side of equation (2.1) depends on the adopted load breakdown. The expedient load breakdown can be determined from arguments of accuracy of calculating the displacement [2].

Frequently in the initial design stages the mass equation is used expressed as a function of displacement, that is, the relative mass in the righthand side of equation (2.1) is considered dependent on the displacement D_m. Here equation (2.1) is represented in the form

$$D_m = \sum_i m_i(D_m) + m_\Sigma, \tag{2.2}$$

where m_Σ is the sum of the masses which do not depend on the displacement.

For establishment of the dependence of the masses m_i on displacement D_m and the other investigated characteristics of the ship, the methods of similarity and mathematical statistics are used.. The application of these methods arises from the complexity of an exact mathematical description of the investigated relations and also the absence of a number of initial data in the early design stages. For these conditions recalculation of various characteristics and relations from prototype ships already designed or has important significance. If various characteristics of the designed ship are recalculated from different prototypes, the latter are called partial prototypes. By the arguments of calculation accuracy, as a rule, it is necessary to select a ship that has been developed no less than the contract design level as only a designed, but not built prototype.

When using prototypes the similarity methods establish the dimensionless relations (similarity criteria) serving for recalculation of the corresponding characteristics, and the methods of mathematical statistics are used to find the values of the parameters of the functional relations between the masses m_i and the components of the vectors X, Y and Θ, including the displacement D_m as one of the components of the vector Y (see §2.4, 2.5).

The simplest form of the mass equation expressed as a function of displacement is the so-called three-term mass equation. When compiling this equation the masses of the hull, the number of the ship's systems and devices, decks, hatch covers, ladders, hull fittings, and so on are considered proportional to displacement to the first power, that is, the mass of the ship. The masses of the power plant, the fuel, water and oil reserves required for operation of the power plant are taken proportional to the displacement to the 2/3 power, that is, the surface of the loaded part of the ship's hull.[1] The masses of the weapons, accessory equipment, payload (for transport ships), systems and devices servicing the weapons and accessory equipment or providing for storage, loading and unloading the payload, are considered independent of the displacement. Thus, all of the loads making up the ship's load are divided into three groups, and the mass equation acquires the form

$$AD_m + BD_m^{2/3} + C = D_m, \tag{2.3}$$

where AD_m, $BD_m^{2/3}$ are the sums of the masses proportional to D_m to the first power and to the 2/3 power; C is the sum of the masses which do not depend on the displacement.

In some cases a term proportional to $D_m^{1/3}$, that is, the linear dimension of the ship, for example, the length, is introduced into the mass equation. It is possible to include the mass of the electric power lines, the pipes of certain systems, and so on in this group. For surface ships sometimes it is necessary to isolate a term proportional to $D_m^{4/3}$ characterizing the mass of the longitudinal couplings of the ship's hull [2]. In general, as S. A. Bazilevskiy demonstrated [2], the form of the mass equation (the necessity for considering the terms proportional to different powers of D_m) can be substantiated beginning with arguments of calculation accuracy. The coefficients A, B and C depend on the TTE and TDP of the ship (the vectors X, Y and Θ), and, consequently, solving this equation, we obtain the displacement D_m as a function of the TTE and TDP.

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[1] The mass displacement is proportional to the volume of the ship submerged in the water, and the surface of the loaded part of the hull is proportional to this volume to the 2/3 power.

There are several procedures for solving equation (2.3). For example, by substitution of $D_m^{1/3}=z$, this equation is reduced to the cubic equation

$$-(1-A)z^3+Bz^2+C=0, \qquad (2.4)$$

which is solved by the numerical method or by the Cardan formulas.[1] The numerical method of solving the direct equation (2.3) can also be used.

The simplest solution is the solution of equation (2.3) by the method of successive approximations based on the known principle of a stationary point from functional analysis [60]. Let us denote the lefthand side of equation (2.3) by f(D) (for simplicity of notation the index on D_m will be omitted). The function f(D) can be considered as the special case of the operator given in the set of values D with values again in this set. It is obvious that for C>0 the displacement found from the mass equation is always greater than $D_*=(B/(1-A))^3$. Consequently, the desired displacement lies in the range (D_*, ∞). It is easy to check that for $D \in (D_*, \infty)$ the values of the function f(D) are in the same interval. In addition for $D \in (D_*, \infty)$ the inequality f'(D)<1 is valid, where f'(D) is the derivative of the function f(D).

Thus, in the interval (D_*, ∞) the function f(D) is the compression operator [60], and the solution of equation (2.3) can be obtained as the limit of the series D_0, D_1, D_2, ..., that is

$$D=\lim_{n\to\infty} D_n,$$

where $D_n=f(D_{n-1})$, n=1, 2, ... is the number of the approximation, and D_0 is an arbitrary value greater than D_*.

It is obvious that in the absence of any additional arguments with respect to the expected value of D it is possible to take the maximum[2] of the values of D_* or C as D_0. For the n-th approximation we have the estimate

$$|D_n-D| \leqslant \frac{|f(D_0)-D_0|}{1-f'(D_0)}|f'(D_0)|^n,$$

where

$$f'(D_0)=A+\frac{2}{3}\frac{B}{\sqrt[3]{D_0}}.$$

---

[1] The substitution proposed by A. N. Krylov $z=\frac{D_m^{1/3}}{B}(1-A)$ is more convenient, after which the solution can be obtained using the tables of squares and cubes of numbers. Appendix 1 contains a discussion of the procedure for solving equation (2.3) using the table of values of an auxiliary function.

[2] Usually $C>D_*$; therefore it is possible to set D_0 = C.

If the error in calculating D is measured with respect to D_0, then the following inequality is valid:

$$\frac{|D_n - D|}{D_0} \leqslant \frac{1}{D_0} \frac{|f(D_0) - D_0|}{1 - f'(D_0)} [f'(D_0)]^n.$$

From the condition $\frac{|D_n - D|}{D_0} \leqslant \varepsilon$ the required number of approximations can be obtained for determining the displacement with given accuracy

$$n \geqslant \frac{\ln \left\{ \varepsilon [1 - f'(D_0)] \frac{D_0}{|f(D_0) - D_0|} \right\}}{\ln f'(D_0)}.$$

As an example let us present elementary arguments permitting discovery of the coefficient B of the mass equation as a function of TTE and TDP.

The mass of the power plants of many types and the mass of the fuel, water and oil reserves are proportional to the power which, in turn, is proportional to the speed of the ship and the resistance of water to its movement. As is known [52], the resistance of the water to the movement of the ship can be represented in the form

$$r = \zeta \frac{\rho v^2}{2} \Omega, \tag{2.5}$$

where r is the resistance, v is the speed of the ship, Ω is the wetted surface of the hull [for geometrically similar hulls $\Omega \sim (D_m/\rho)^{2/3}$], ζ is the drag factor, ρ is the density of the water.

Considering (2.5) the following formula is valid for the required engine power (N):

$$N = \frac{r v_{\max}}{\eta} = \frac{1}{2} \rho^{1/3} \frac{\zeta \omega}{\eta} v_{\max}^3 D_m^{2/3},$$

where $v_{\max}$ is the maximum speed of the ship, $\omega = \rho^{2/3} \Omega / D_m^{2/3}$ is the relative wetted surface of the hull, η is the propulsive coefficient.

It is possible to write the expression for N in the form of the so-called admiralty formula

$$N = \frac{v_{\max}^3 D_m^{2/3}}{C_w}, \tag{2.6}$$

where C_w is the dimensional coefficient called the admiralty coefficient:

$$C_w = \frac{2}{\rho^{1/3}} \frac{\eta}{\zeta \omega}. \tag{2.7}$$

If we set $\rho=1$ ton/m^3 and assume the following units of magnitudes: [N]=kilowatt, [D_m]=tons, [v_{max}]=knots, then formula (2.7) assumes the form

$$C_w = 14{,}7\frac{\eta}{\zeta\omega}. \qquad (2.8)$$

For submarines under water the drag usually is reduced to the wetted surface of the outer hull and the normal displacement is taken as D_m. In this case

$$C_w = 14{,}7\frac{1}{k^{2/3}}\frac{\eta}{\zeta\omega},$$

where k is the coefficient of transition from normal displacement to the volume of the outer hull; ω is the wetted surface of the outer hull reduced to its volume to a power of 2/3.

Considering formula (2.6), the mass of the power plant $m_{\text{эу}}$ can be represented in the form

$$m_{\text{эу}} = q_{\text{эу}}\frac{v^3_{\max}D_m^{2/3}}{C_w}, \qquad (2.9)$$

where $q_{\text{эу}}$ is the specific mass of the power plant, that is, the mass per unit power.

If the cruising range (R) at an economical speed (v_{ec}) is given when designing the ship, then the mass of the fuel, water and oil reserves m_T can be defined by the formula

$$m_{\text{т}} = q_{\text{т}}\frac{v^3_{\text{эк}}D_m^{2/3}}{C_{w\,\text{эк}}}\frac{R}{\underset{(1)}{v_{\text{эк}}}} = q_{\text{т}}\frac{v^2_{\text{эк}}D_m^{2/3}}{C_{w\,\text{эк}}}R, \qquad (2.10)$$

Key: 1. ec

where q_T is the total specific consumption of fuel, water and oil at economical speed; $C_{w\ ec}$ is the admiralty coefficient for economical speed.

Usually only part of the total fuel, water and oil reserve, for example, half, is included in the normal displacement of the ship. In this case the corresponding coefficient is introduced into formulas (2.10).

Considering formulas (2.9) and (2.10), the mass equation (2.3) assumes the form

$$AD_m + \left(q_{\text{эу}}\frac{v^3_{\max}}{C_w} + q_{\text{т}}\frac{v^2_{\text{эк}}}{C_{w\,\text{эк}}}R\right)D_m^{2/3} + C = D_m. \qquad (2.11)$$

The values of the admiralty coefficients C_w and $C_{w\ ec}$ and also the individual terms of the coefficient A are selected in the first approximation by the prototype or by several prototypes after statistical processing.

Very frequently the variation of the displacement D_m given by the equation (2.3) for small variations of the coefficients A, B and C is of interest. Using the complete differential formula, it is possible to write the approximate equality

$$\Delta D_m = \frac{\partial D_m}{\partial A} \Delta A + \frac{\partial D_m}{\partial B} \Delta B + \frac{\partial D_m}{\partial C} \Delta C,$$

where ΔD_m, ΔA, ΔB, ΔC are small increments of the displacements and the coefficients A, B and C.

The partial derivatives $\partial D_m/\partial A$, $\partial D_m/\partial B$, $\partial D_m/\partial C$ are calculated on the basis of equation (2.3) by the differentiation rule of implicit functions. Performing the corresponding calculations, we find

$$\begin{aligned} \frac{\partial D_m}{\partial A} &= \frac{D_m}{1 - A - \frac{2}{3}\frac{BD_m^{2/3}}{D_m}}, \\ \frac{\partial D_m}{\partial B} &= \frac{D_m^{2/3}}{1 - A - \frac{2}{3}\frac{BD_m^{2/3}}{D_m}}, \\ \frac{\partial D_m}{\partial C} &= \frac{1}{1 - A - \frac{2}{3}\frac{BD_m^{2/3}}{D_m}}. \end{aligned} \tag{2.12}$$

The derivative $\partial D_m/\partial C$ called the Norman coefficient is equal to the variation in displacement of the ship with variation of the mass of the constant loads (the group C) per unit. In the more general case where the mass equation has the form

$$D_m = \sum_i A_i D_m^{k_i} + C, \tag{2.13}$$

The derivatives $\partial D_m/\partial C$ and $\partial D_m/\partial A$ are defined by the expressions

$$\frac{\partial D_m}{\partial C} = \frac{1}{1 - \sum\limits_i k_i A_i D_m^{k_i - 1}}, \quad \frac{\partial D_m}{\partial A_i} = \frac{D_m^{k_i}}{1 - \sum\limits_i k_i A_i D_m^{k_i - 1}}. \tag{2.14}$$

The variation of the displacement ΔD_m with small variations of the ship characteristics (Δx_j) on which the coefficients A_i and C depend, can be found by the formula

$$\Delta D_m = \sum_j \left[\sum_i \frac{\partial D_m}{\partial A_i} \frac{\partial A_i}{\partial x_j} + \frac{\partial D_m}{\partial C} \frac{\partial C}{\partial x_j} \right] \Delta x_j. \tag{2.15}$$

A study was made above of the mass equation expressed as a function of the displacement. At the same time for surface ships the masses of the hull and the power plant depend to a significant degree on the choice of the relations of the principal dimensions and the coefficients of the lines coefficients. For example, the mass of the hull with respect to the general strength conditions depends on the bending moments at the apex and foot of the wave proportional to the product of the displacement and length of the ship. The drag of the water with respect to movement of the ship and, consequently, the required engine power and its mass depend on the Froude and Reynolds numbers, the shape of the hull, that is, the geometric characteristics of the hull.

For the above-indicated region in a number of cases the mass equation expressed as a function of the principal dimensions of the ship and its lines coefficients is used for surface ships. Here the masses of all load items are represented by functions of length L, breadth B, draught T, hull height H of the ship and its lines coefficients: block coefficient δ, longitudinal prismatic coefficient ϕ, vertical prismatic coefficient χ, the coefficient of fineness of the construction water line α, and the midship section coefficient β. The enumerated coefficients are related to each other by the expressions $\alpha\chi=\delta$ and $\beta\phi=\delta$ as a result of which it is sufficient to give only three coefficients, for example, α, β, and δ.

For submarines under water, the length and breadth of the hull are taken as L and B, and instead of T, the hull height.

Inasmuch as the condition $D_m=\rho\delta LBT$ must be satisfied from the buoyancy condition, the mass equation can be written in the form

$$\rho\delta LBT = \sum_i m_i(\delta,\ L,\ B,\ H,\ T) + m_\Sigma, \tag{2.16}$$

where m_Σ is the sum of the masses which do not depend on the displacement and the principal dimensions; H is the sheer height.

In equation (2.16), the values of L, B, T, H and δ are unknown. For determination of them it is necessary to give additional relations between the principal dimensions. Usually the value of δ and the ratios L/B, B/T and H/T are given. The values of δ and L/B have significant influence on the propulsive performance of the ship, B/T, on the stability, H/T on the reserve buoyancy and, consequently, the unsinkability. Giving the above-indicated ratios permits only one unknown variable to be left in equation (2.16), for example, the length of the ship L. Considering that for geometric similarity the coefficients $\ell=\rho^{1/3}L/D_m^{1/3}$, $b=\rho^{1/3}B/D_m^{1/3}$, $t=\rho^{1/3}T/D_m^{1/3}$ remain invariant and, in addition, $H=TH/T=t(H/T)D_m^{1/3}/\rho^{1/3}$, it is possible to represent equation (2.16) in the form

$$\sum_i m_i\left(D_m,\ \delta,\ l,\ b,\ t,\ \frac{H}{T}\right) + m_\Sigma = D_m. \tag{2.17}$$

The values of ℓ, b and t are expressed in terms of δ, L/B and B/T by the formulas

$$l = \left[\frac{1}{\delta}\left(\frac{L}{B}\right)^2 \frac{B}{T}\right]^{1/3},$$

$$b = \left[\frac{1}{\delta}\left(\frac{L}{B}\right)^{-1} \frac{B}{T}\right]^{1/3},$$

$$t = \left[\frac{1}{\delta}\left(\frac{L}{B}\right)^{-1} \left(\frac{B}{T}\right)^{-2}\right]^{1/3}.$$

Considering these ratios and equation (2.17), we obtain the mass equation in the form

$$\sum_i m_i\left(D_m,\ \delta,\ \frac{L}{B},\ \frac{B}{T},\ \frac{H}{T}\right)+m_\Sigma=D_m. \tag{2.18}$$

From equation (2.18) the displacement D_m can be found as a function of the ratios of the principal dimensions L/B, B/T, H/T and the block coefficient δ for fixed mass m_Σ, the basic part of which is the mass of the weapons and accessory equipment or the payload for transport ships.

If we assume that the mass of the ship's hull is defined by the so-called cubic modulus coefficient $m_k=q_k LBH$, where q_k is the hull mass reduced to the value of LBH, and the mass of the power plant with reserves is calculated by the admiralty formula, then the three-term mass equation can be represented in the form

$$\left(\frac{q_к}{\rho\delta}\frac{H}{T}+p_к\right)D_m+\left(q_{эу}\frac{v^3_{\max}}{C_ш}+q_т\frac{v^2_{эк}}{C_{ш\,эк}}R\right)D_m^{2/3}+m_\Sigma=D_m, \tag{2.19}$$

where p_k is the relative mass of the load items proportional to the displacement to the first power with the exception of the weight of the hull.

When solving equation (2.19) for surface ships and submarines on the surface, it is necessary to consider the complex dependence of the admiralty coefficients (C_w and $C_{w\ ec}$) on the speeds v_{max} and v_{ec} and the shape of the submerged part of the hull which, as was pointed out previously, is approximately characterized by the ratios L/B, B/T and the lines coefficients α, β and δ. The complexity of the dependence of the admiralty coefficients on the above-indicated parameters basically arises from the complexity of the corresponding function for the wave-drag coefficient. As for the propulsive coefficient, its value is primarily determined by the number and the diameter of the screws, their rpm, and depends the least on the speed of the ship.

For geometrically similar hulls, while keeping the propulsive properties identical, the admiralty coefficients depend primarily only on the values of the Froude numbers (Fr), and if a prototype exists which is sufficiently close with respect to hull shape and characteristics of the propulsion system, then when solving equation (2.19) the function $C_w=f(Fr)$ taken by the prototype can be used. Let us note that the standard function $C_w=f(Fr)$ is characterized by the presence of zones of values of Froude numbers ($Fr \simeq 0.10$ to 0.25 and $Fr \simeq 0.40$ to 0.60), in which the admiralty coefficient remains approximately constant.

The admiralty coefficient C_w can be represented in the form $C_w=C_e\eta$, where $C_e=2/\rho^{1/3}\cdot 1/(\zeta\omega)$ is the admiralty coefficient with respect to towrope or effective power, η is the propulsive coefficient.

The coefficient C_e characterizes the hydrodynamic perfection only of the ship's hull without the propulsion system and its interaction with the hull. Inasmuch as the Froude number and the hull shape influence primarily the value of Ce and to a much lesser degree, the propulsive coefficient, when determining these coefficients it is possible to use partial prototypes. The function $C_e=f(Fr)$ is selected by the prototype, the hull shape of which is similar to the designed ship, and the value of η, by another prototype having the corresponding characteristics of the propulsion system.

For submarines under water the values of the admiralty coefficients C_W and C_e can be considered in practice independent of the speed, inasmuch as in this case the total drag coefficient depends very weakly on the Reynolds number (Re) and, consequently, on the speed of the submarine.

The problem of selecting the optimal parameters of the shape of the ship's hull, including the ratios of the principal dimensions and the lines coefficients is a highly complex problem although urgent for the design development phases that follow AD. The complexity of this problem arises both from the large number of conditions and requirements which must be considered (seakeeping qualities, general layout, trim, and so on) and the nature of the effect of the hull shape parameters on various qualities of the ship. Accordingly, in the AD stage it is necessary, as a rule, to select the parameters of the hull shape by prototypes and by especially approximate empirical relations [35]. The final choice of the principal dimensions of the ship and the hull shape parameters can be realized after development of the general arrangement plan and the lines drawing.

If small variations of the ship's characteristics are considered, then the mass equation in differential form is used. Let us write the mass equation in the form

$$m_\Sigma = D_m - F(\delta,\ L,\ B,\ T,\ H,\ x_1,\ \ldots,\ x_n), \tag{2.20}$$

where $x_1, \ldots, x_n$ are the investigated independent variable TTE and TDP of the ship.

It is obvious that the relation is valid for the differentials

$$dm_\Sigma = dD_m - dF. \tag{2.21}$$

Considering the independent variables δ, L, B, T, H, $x_1, \ldots, x_n$ as independent, we have

$$dD_m = \frac{D_m}{\delta}d\delta + \frac{D_m}{L}dL + \frac{D_m}{B}dB + \frac{D_m}{T}dT,$$

$$dF = \frac{\partial F}{\partial \delta}d\delta + \frac{\partial F}{\partial L}dL + \frac{\partial F}{\partial B}dB + \frac{\partial F}{\partial T}dT + \frac{\partial F}{\partial H}dH + \sum_{i=1}^{n}\frac{\partial F}{\partial x_i}dx_i,$$

where the expression for dD_m follows from the equality $D_m = \rho\delta LBT$.

Considering these expressions, equation (2.21) can be represented in the form

$$dm_\Sigma = \left(\frac{D_m}{\delta} - \frac{\partial F}{\partial \delta}\right)d\delta + \left(\frac{D_m}{L} - \frac{\partial F}{\partial L}\right)dL + \left(\frac{D_m}{B} - \frac{\partial F}{\partial B}\right)dB + \\ + \left(\frac{D_m}{T} - \frac{\partial F}{\partial T}\right)dT - \frac{\partial F}{\partial H}dH - \sum_{i=1}^{n}\frac{\partial F}{\partial x_i}dx_i. \tag{2.22}$$

In equation (2.22), dδ, dL, dB, dT and dH can be considered as unknowns, and dm_Σ and dx_i as given, defined by small variations of the TTE and the TDP of the ship. Being given certain additional relations for the above-indicated unknowns, it is possible to determine their values.

The differential method was first investigated by Norman and I. G. Bubnov [8]. In particular, I. G. Bubnov stated the idea of the possibility of optimizing the values of dδ, dL, dB, dT and dH by some criterion under the condition of satisfying expression (2.22). This criterion, for example, can be the condition $\min(dD_m)$.

If in addition to expression (2.22) additional linear restrictions on the values of dδ, dL, dB, dT and dH are used where these restrictions together with expression (2.22) define some complex region in the five-dimensional euclidian space, the coordinates of which are the indicated variables, then optimization by the criterion min (dD_m) reduces to solving the problem of linear programming. Let us note that minimizing the value of dD_m is equivalent to minimizing the expression $d\delta/\delta+dL/L+dB/B=dT/T$, that is, the sum of the relative variations of length, breadth, draught and the block coefficient of the ship.

As an additional linear restriction we can use, for example, the condition of invariability or given variation of the initial transverse metacentric height. This restriction can be obtained from the stability equation (see item 4 of this section) presented in differential form.

From expression (2.42) for dδ=0 and dα=0, that is, for invariant block coefficient and coefficient of fineness of the construction water line, comes the equation for the variation of the initial transverse metacentric height

$$dh = 2\,\frac{B^2}{T}\,\varphi_2\,\frac{dB}{B} + T\left[\varphi_1 - \left(\frac{B}{T}\right)^2 \varphi_2\right]\frac{dT}{T} - \xi H\,\frac{dH}{H}, \tag{2.23}$$

in which ϕ_1, ϕ_2, ξ are constants defined by the values of δ, α and the relative position of the center of gravity of the ship with respect to height.[1]

The equation (2.23) is the stability equation in differential form. In this equation setting dh=0, we obtain the linear relation between the relative variations of breadth, draught and hull height of the ship, insuring invariability of the initial transverse stability.

Equations (2.22), (2.23) and other similar differential expressions are valid for sufficiently small values of dδ, dL, dB, dT and dH. Therefore when stating the problem of optimizing these values it is necessary to consider restrictions of the type

$$-\Delta \leqslant \frac{dz}{z} \leqslant \Delta, \tag{2.24}$$

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[1] In accordance with the SI system the center of gravity should be more strictly called the center of masses, but we retain the more usual term center of gravity for this point.

where z is each of the values of $d\delta$, dL, dB, dT and dH, and Δ is a given, quite small value (usually Δ=0.1).

Let us propose that when minimizing the value of dD_m, only the conditions (2.22), (2.23) are considered for dh=0 and the restrictions (2.24), and the displacement D_m is determined from the mass equation of the type

$$q_{\text{к}}LBH + q_{\text{эу}}\frac{v^3}{C_w}\rho^{2/3}(\delta LBT)^{2/3} + m_\Sigma = \rho\delta LBT. \tag{2.25}$$

In this case for $d\delta=0$, $dx_i=0$, $i=1, \ldots, n$, and $dm_\Sigma \neq 0$ we obtain the problem of linear programming

$$\begin{gathered}\min(\overline{dL}+\overline{dB}+\overline{dT}),\\ \left(1-\frac{L}{D_m}\frac{\partial F}{\partial L}\right)\overline{dL}+\left(1-\frac{B}{D_m}\frac{\partial F}{\partial B}\right)\overline{dB}+\\ +\left(1-\frac{T}{D_m}\frac{\partial F}{\partial T}\right)\overline{dT}-\frac{H}{D_m}\frac{\partial F}{\partial H}\overline{dH}=\overline{d}m_\Sigma,\\ 2\left(\frac{B}{T}\right)^2\varphi_2\overline{dB}+\left[\varphi_1-\left(\frac{B}{T}\right)^2\varphi_2\right]\overline{dT}-\xi\frac{H}{T}\overline{dH}=0,\\ -\Delta\leqslant\overline{dL}\leqslant\Delta,\quad -\Delta\leqslant\overline{dB}\leqslant\Delta,\\ -\Delta\leqslant\overline{dT}\leqslant\Delta,\quad -\Delta\leqslant\overline{dH}\leqslant\Delta,\end{gathered}\tag{2.26}$$

where $\overline{dL}=dL/L$, $\overline{dB}=dB/B$, $\overline{dT}=dT/T$, $\overline{dH}=dH/H$ and $\overline{dm_\Sigma}=dm_\Sigma/D_m$.

In accordance with equation (2.25) and considering the equality $D_m=\rho\delta LBT$ we have

$$\begin{gathered}1-\frac{L}{D_m}\frac{\partial F}{\partial L}=1-\frac{q_{\text{к}}}{\rho\delta}\frac{H}{T}-\frac{2}{3}q_{\text{эу}}\frac{v^3}{C_w}D_m^{-1/3},\\ 1-\frac{B}{D_m}\frac{\partial F}{\partial B}=1-\frac{q_{\text{к}}}{\rho\delta}\frac{H}{T}-\frac{2}{3}q_{\text{эу}}\frac{v^3}{C_w}D_m^{-1/3},\\ 1-\frac{T}{D_m}\frac{\partial F}{\partial T}=1-\frac{2}{3}q_{\text{эу}}\frac{v^3}{C_w}D_m^{-1/3},\\ \frac{H}{D_m}\frac{\partial F}{\partial H}=\frac{q_{\text{к}}}{\rho\delta}\frac{H}{T}.\end{gathered}$$

Let us note that the righthand side of the formulas written above are expressed in terms of the relative masses of the hull and the power plant, inasmuch as

$$\frac{m_{\text{к}}}{D_m}=\frac{q_{\text{к}}}{\rho\delta}\frac{H}{T} \quad\text{and}\quad \frac{m_{\text{эу}}}{D_m}=q_{\text{эу}}\frac{v^3}{C_w}D_m^{-1/3}.$$

The physical meaning of problem (2.26) consists in optimization [from the point of view of the condition min (dD_m)] of the variations of the principal dimensions of the ship for some given variation of the mass of the constant loads and maintenance of invariant transverse metacentric height.

By introducing the new variable $z_1=\overline{dL}+\Delta$, $z_2=\overline{dB}+\Delta$, $z_3=\overline{dT}+\Delta$, $z_4=\overline{dH}+\Delta$ and four additional fictitious nonnegative variables $z_{4+j}\geq 0$, j=1, ..., 4, problem (2.26) reduces to the problem of linear programming in standard form:

$$\begin{gathered}\min(z_1+z_2+z_3), \qquad (2.27)\\ a_1z_1+a_2z_2+a_3z_3+a_4z_4=C_1,\\ b_2z_2+b_3z_3+b_4z_4=C_2,\\ z_1+z_5=2\Delta,\quad z_2+z_6=2\Delta,\quad z_3+z_7=2\Delta,\\ z_4+z_8=2\Delta,\quad z_j\geqslant 0,\quad j=1,\ \ldots,\ 8,\end{gathered}$$

where

$$\begin{gathered}a_1=a_2=1-\frac{m_{\text{к}}}{D_m}-\frac{2}{3}\frac{m_{\text{эу}}}{D_m},\quad a_3=1-\frac{2}{3}\frac{m_{\text{эу}}}{D_m},\quad a_4=-\frac{m_{\text{к}}}{D_m},\\ b_2=2\left(\frac{B}{T}\right)^2\varphi_2,\quad b_3=\varphi_1-\left(\frac{B}{T}\right)^2\varphi_2,\quad b_4=-\xi\frac{H}{T},\\ C_1=\overline{dm}_{\Sigma}+3\left(1-\frac{m_{\text{к}}}{D_m}-\frac{2}{3}\frac{m_{\text{эу}}}{D_m}\right)\Delta,\\ C_2=\left[\varphi_1+\left(\frac{B}{T}\right)^2\varphi_2-\xi\frac{H}{T}\right]\Delta.\end{gathered}$$

In problem (2.27), the number of unknowns exceeds the number of restrictions by only 2, which permits us to find the solution by the descriptive graphical method. For this purpose, it is necessary to select any two linearly independent variables, for example, in problem (2.27) it is possible to take z_1 and z_2 as such variables and, using the restrictions, to express all the remaining variables in terms of z_1 and z_2. It is also necessary to express the purpose function in terms of the variables z_1 and z_2. Then, using the conditions of nonnegativity of all the variables, it is easy to obtain a system of linear inequalities with respect to the variables z_1 and z_2.

If the obtained system of inequalities defines in the plane (z_1, z_2) a region (this regionwill always be convex) of admissible values of z_1 and z_2, then one or several extreme (boundary) points of this region define the optimal values of $z_{1\ \text{opt}}$ and $z_{2\ \text{opt}}$. In the problems of linear programming, the admissible region always has the form of a convex polyhedron (in the two-dimensional case, a polygon) and, consequently, the optimal point is one of the apexes of this polyhedron or all the points of any face are optimal.

If the system of inequalities obtained above is incompatible, that is, no point of the investigated space satisfies all inequalities simultaneously, the solution of the stated problem of linear programming does not exist.

When the values of $z_{1\ \text{opt}}$ and $z_{2\ \text{opt}}$ are defined in (2.27), from the equalities entering into the formulation of this problem the optimal values of the remaining variables z_j, j=3, ..., 8 are calculated, and then, the optimal values of the initial variables $\overline{dL}$, $\overline{dB}$, $\overline{dT}$ and $\overline{dH}$. In the general case the solution of the problems of the type of (2.26) for any number of restrictions can be found by well-developed numerical methods [61].

Let us demonstrate the graphical method of solving the problem (2.26) in a numerical example with the following initial data: $B/T=2.6$; $H/T=1.5$; $\xi=0.6$; $\delta=0.65$; $\alpha=0.75$; $m_k/D_m=0.30$; $m_{ec}/D_m=0.15$; $dm_\Sigma=0.10$; $\Delta=0.10$.

In order to determine the value of the coefficient ϕ_1 let us use the formula of L. M. Nogid, and for the coefficient ϕ_2, the formula of A. P. Van der Fleet (see item 4 of this section).

For the above-indicated initial data the problem (2.27) assumes the form

$$\begin{gathered}\min(z_1+z_2+z_3),\\ 0{,}60z_1+0{,}60z_2+0{,}90z_3-0{,}30z_4=0{,}28,\\ 1{,}05z_2+0{,}014z_3-0{,}90z_4=0{,}0168,\\ z_1+z_5=0{,}20;\quad z_2+z_6=0{,}20;\quad z_3+z_7=0{,}20,\\ z_4+z_8=0{,}20;\ z_j\geqslant 0,\ j=1,\ldots,8.\end{gathered}\tag{2.28}$$

Expressing all of the variables in terms of z_1 and z_2, we obtain the system of linear inequalities

$$\begin{gathered}z_1\geqslant 0,\quad z_2\geqslant 0,\\ 0{,}306-0{,}669z_1-0{,}279z_2\geqslant 0,\\ -0{,}0139-0{,}010z_1+1{,}16z_2\geqslant 0,\\ 0{,}200-z_1\geqslant 0,\quad 0{,}200-z_2\geqslant 0,\\ -0{,}106+0{,}669z_1+0{,}279z_2\geqslant 0,\\ 0{,}201+0{,}010z_1-1{,}16z_2\geqslant 0.\end{gathered}$$

The purpose function of the problem expressed in terms of z_1 and z_2 has the form $f=0.306+0.331z_1+0.721z_2$. The graphical solution of the problem is illustrated in Figure 2.4 where the restriction $z_3\geqslant 0$ is not shown in the figure, for all points of the square defined by the conditions $0<z_1\leqslant 0.200$ and $0\leqslant z_2\leqslant 0.200$ satisfy the indicated restriction on z_3. The admissible region of values of z_1 and z_2 is crosshatched in Figure 2.4, and the crosshatching on the individual lines indicates the part of the plane in which the corresponding restrictions (inequalities) are satisfied.

The conditions of constancy of the purpose function f define the family of straight lines, two of which (for $f=0.406$ and $f=0.356$) are shown in the figure. The minimum value of f in the admissible region of values of z_1 and z_2 is achieved at the point with the coordinates $z_{1\ opt}=0.152$; $z_{2\ opt}=0.013$; and the values of z_3, z_4 and f corresponding to this point will be $z_{3\ opt}=0.200$; $z_{4\ opt}=0$; $f_{opt}=0.356$.

Returning to the initial variables, we find $(\overline{dL})_{opt}=0.052$; $(\overline{dB})_{opt}=-0.087$; $(\overline{dT})_{opt}=0.10$; $(\overline{dH})_{opt}=-0.10$.

Thus, in the investigated example with an increase in relative mass of the constant loads by 10% it is necessary to increase the length by 5.2% and the draught of the ship by 10%, to decrease the breadth by 8.7%, and the hull height by 10%.

Here the increase in displacement will be minimal and equal to ~6.5%[1].

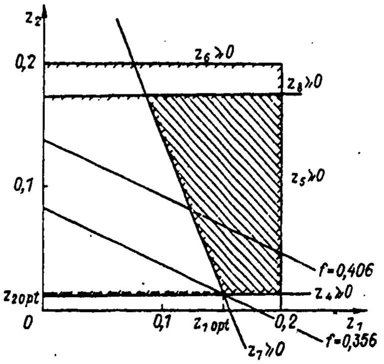

Figure 2.4. Graphical solution of the problem (2.28)

2. Buoyancy Equation. When investigating the mass equation expressed as a function of the principal dimensions, the expression

$$D_m = \rho\delta LBT \tag{2.29}$$

was used which can be considered as the buoyancy equation. For this reason, the above-presented different forms of the mass equation expressed as a function of the principal dimensions can be interpreted as a result of the joint investigation of the mass and buoyancy equations. The buoyancy equation in differential form is written in the form

$$dD_m = \frac{D_m}{\delta} d\delta + \frac{D_m}{L} dL + \frac{D_m}{B} dB + \frac{D_m}{T} dT. \tag{2.30}$$

The buoyancy equation expresses the fact that the mass of the water in the body of the submerged part of the ship's hull must be equal to the mass of the ship, that is, its mass displacement.

In the general case the necessity for using the buoyancy equation is connected with the fact that the masses of the individual load items can depend on different components of the floating volume of the ship, where these components themselves depend on the unknown load items and displacement. Let us explain this fact as applied to submarines.

As will be demonstrated in the following section, under certain conditions the mass of the pressure hull of the submarine is considered proportional to its volume, which is a basic, but not unique component part of the constant floating volume -- the normal displacement. Along with the normal displacement D_m the

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[1] In the given case $\overline{dD_m} < \overline{dm_\Sigma}$, inasmuch as a decrease in the hull height is permitted.

volume of the pressure hull $V_{\pi k}$ can be considered as the desired unknown. Knowing $V_{\pi k}$ is highly significant inasmuch as the basic technical means, accessory equipment and personnel are placed in the pressure hull.

Isolating the mass of the pressure hull in the three-term mass equation expressed as a function of the normal displacement, we obtain the equation

$$q_{\text{пк}} V_{\text{пк}} + A' D_m + B D_m^{2/3} + C = D_m, \tag{2.31}$$

where $q_{\pi k}$ is the mass of a unit volume of the pressure hull; $A'D_m$ is the sum of the masses proportional to the normal displacement.

Very frequently the value of $V_{\pi k}$ is considered proportional to the displacement:

$$V_{\text{пк}} = \bar{\mathfrak{w}}_{\text{пк}} \frac{D_m}{\rho}, \tag{2.32}$$

where $\bar{\mathfrak{w}}_{\text{пк}}$ is a dimensionless constant called the relative volume of the pressure hull.

In this case the equation (2.31) is reduced to the ordinary three-term mass equation, and expression (2.32) will be the simplest form of the buoyancy equation. After substitution of expression (2.32) in equation (2.31) we obtain

$$\left(\frac{q_{\text{пк}} \bar{\mathfrak{w}}_{\text{пк}}}{\rho} + A'\right) D_m + B D_m^{2/3} + C = D_m. \tag{2.33}$$

With this representation the equality of $\bar{\mathfrak{w}}_{\text{пк}}$ to the given value can also be considered as one of the simplest forms of the buoyancy equation.

In the more general case the buoyancy equation for a submarine can be represented in the form

$$\rho V_{\text{пк}} + \rho \sum_i V_i = D_m, \tag{2.34}$$

where $\sum_i V_i$ is the total impermeable volume with the exception of the pressure hull.

The equations (2.31) and (2.34) can be considered as a system with respect to the two unknowns D_m and $V_{\pi k}$. After substitution of the value of $V_{\pi k}$ from (2.34) in equation (2.31) we obtain

$$\left(\frac{q_{\text{пк}}}{\rho} + A'\right) D_m + B D_m^{2/3} + C - q_{\text{пк}} \sum_i V_i = D_m.$$

This equation is already the result of joint investigation of the mass and buoyancy equations. After its solution with respect to D_m the value of $V_{\pi k}$ is found by the formula

$$V_{\text{пк}} = \frac{D_m}{\rho} - \sum_i V_i.$$

3. Volume Equation. The displacement and the principal dimensions of the ship found by joint solution of the mass and buoyancy equations cannot in the general case be considered final inasmuch as the corresponding volume of the hull can be insufficient for placement of the weapons, accessory equipment, technical means and personnel. The most exact testing of the conditions of the general plan is realized by the graphical method by compiling the corresponding general arrangement plans. One of the approximate analytical methods of considering the general arrangement conditions is compiling and solving the volume equation proposed by V. L. Pozdyunin [38].

This equation expresses the fact that the volume of the ship's hull and superstructures (for surface ships) must be equal to the sum of the volumes required for placement of the weapons, accessory equipment, technical means and personnel, and for transport vessels, the hauled cargo and passengers. Thus, the volume equation has the general form

$$V_{\text{к}} + w_{\text{н}} = \sum_i V_i, \tag{2.35}$$

where V_k is the volume of the ship's hull with respect to the upper deck; w_H is the volume of the superstructures; V_i are the volumes required for placement of technical means, weapons, accessory equipment and personnel.

The expansion of equation (2.35) consists in establishing the dependence of the left and righthand sides on the hull volume, principal dimensions, displacement and other investigated characteristics. In order to compile the volume equation the same methods are used as for the mass equation, that is, the methods of similarity and statistics. In the first approximation it is possible to propose that part of the volumes is proportional to the displacement, part is proportional to the hull volume V_k, and part does not depend on the displacement or the hull volume.

The volumes V_i required for placement of the power plant, fuel, oil and water, just as the masses of these components are expressed in the first approximation using the admiralty formula

$$\begin{aligned} V'_{\text{эу}} &= \bar{w}_{\text{эу}} \frac{v^3_{\max}}{C_w} \rho^{2/3} D_V^{2/3}, \\ V'_{\text{т}} &= \frac{q^*_{\text{т}}}{\rho_{\text{т}}} \frac{v^2_{\text{эк}}}{C_{w\,\text{эк}}} R\rho^{2/3} D_V^{2/3}, \\ V'_{\text{м}} &= \frac{q^*_{\text{м}}}{\rho_{\text{м}}} \frac{v^2_{\text{эк}}}{C_{w\,\text{эк}}} R\rho^{2/3} D_V^{2/3}, \\ V_{\text{п}} &= \frac{q^*_{\text{в}}}{\rho_{\text{в}}} \frac{v^2_{\text{эк}}}{C_{w\,\text{эк}}} R\rho^{2/3} D_V^{2/3}, \end{aligned} \tag{2.36}$$

where q^*_T, q^*_M, Q^*_B are the specific consumption of the fuel, oil and water respectively, tons/(kilowatt-hour); $\bar{w}_{\text{эу}}$ is the specific volume of the power plant, m^3/kilowatt; D_V is the volumetric displacement, m^3; ρ_T, ρ_M, ρ_B are the specific mass of the fuel, oil and water. (The specific mass ρ_B of water used for operation of the power plant can differ from the specific mass of seawater for which the ship's displacement is determined.)

If the hull volume and displacement are represented in the form $V_k = \delta_V$ LBH, $D_V = \delta$LBT, where H is the hull height of the ship; δ_V is the coefficient of fullness of displacement with respect to the upper deck, then

$$V_к = \frac{\delta_V}{\delta}\frac{H}{T}D_V.$$

Considering this relation and the above-presented expressions for $V_{эу}$, $V_т$, $V_м$ and V_B, we obtain the equation (2.35) in more expanded form:

$$\frac{\delta_V}{\delta}\frac{H}{T}(1+\bar{w})D_V = A_{V1}\frac{\delta_V}{\delta}\frac{H}{T}D_V + A_{V2}D_V + \\ + \rho^{1/3}\left(\bar{w}_{эу}\frac{v^3_{max}}{C_w} + \sum_{i=т, м, в}\frac{q_i^*}{C_{w\,эк}}\frac{v^2_{эк}}{\rho_i}\right)D_V^{2/3} + C_V, \qquad (2.37)$$

where $\bar{w} = w_H/V_k$ is the relative volume of the superstructures; A_{V1} is the coefficient of volumes proportional to V_k; A_{V2} is the coefficient of volumes proportional to D_V; C_V is the sum of the volumes not dependent on V_k and D_V.

The displacement D_V found from the volume equation must be compared with the displacement D_m obtained by the method of joint solution of the mass and buoyancy equations. If $D_m \geq \rho D_V$, the final displacement of the ship will be D_m. In this case, from the buoyancy conditions the volume of the ship's hull will be greater than the volume required by the general plan conditions or equal to it. For $D_m < \rho D_V$ the buoyancy and general plan conditions must be matched by variation of certain component loads and the floating volume of the ship. The variation of the characteristics which will increase the mass of the ship without significantly changing the required hull volume is the most favorable. Such characteristics, for example, are the mass of the hull, solid ballast, and so on. The final displacement will in this case be close to ρD_V, but in practice it will always exceed this amount somewhat. Let us note that in the case of $D_m > \rho D_V$ the hull volume remains "free," equal to $(1/\rho)(D_m - \rho D_V)$, can be used for improvement of some of the ship's characteristics connected with the necessity for increasing the volumes without a significant increase in mass. (For example, there is a possibility of disposition of additional personnel, certain forms of stores, improvement of the habitability conditions, and so on.)

For submarines the volume equation is represented in three-term form (entirely analogous to the three-term mass equation):

$$A_V D_V + B_V D_V^{2/3} + C_V = D_V, \qquad (2.38)$$

where A_V, B_V, C_V are the corresponding coefficients.

It is possible to consider the volume equations expressed as a function of the principal dimensions analogously to how this was done for the mass equation.

The volume equation is appreciably more approximate by comparison with the mass equation. First, it does not take into account the specific geometric configuration of the arranged objects and numerous, basically unformalized requirements on their individual and mutual arrangements, and it operates only with their volumes. Secondly, finding the coefficients of the volume equation by processing the characteristics of prototypes is very difficult as a result of complexity of isolating the volume not connected with placement conditions and caused by requirements on buoyancy, stability, other seakeeping qualities, trim, and so on.

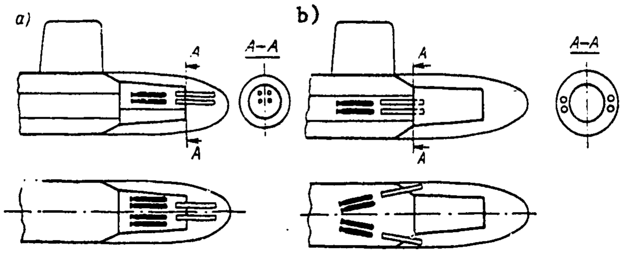

Figure 2.5. Diagrams of the placement of torpedo tubes and spare torpedoes on submarines

The use of the volume equation, just as other equations of the analytical design method is connected with carrying over the structural solution used on prototypes to the designed ship; therefore when designing ships with new structural solutions compiling the volume equation requires graphical developments in the general case of at least basic new structural assemblies.

For example, it is known [10] that on foreign submarines two arrangements of the torpedo tubes are used: in the bow, horizontal and parallel to the diametral plane, with cutting of the forward bulkhead of the pressure hull (Figure 2.5, a) and in the midsection along the length of the ship, horizontally at an angle to the diametral plane, with cutting of the pressure hull performed in this area in the form of a "circular" cone (Figure 2.5, b).

The first arrangement is traditional; it has been used in practice on all submarines since World War I. The second arrangement is realized on the series XXVI German submarines[1] built at the end of World War II, and at the present time it is used on the modern American atomic submarines [10]. Each of these arrangements are characterized by defined advantages and disadvantages, the discussion of which is beyond the scope of this book. From the point of view of the problem investigated here of compiling the volume equation it is important to note that for the two indicated arrangements of the torpedo tubes (considering placement of the spare torpedoes) the volumetric characteristics, for example, the volume per spare torpedo, turn out to be different.

It is clear that the designer having prototypes only for the first arrangement of the torpedo tubes at his disposal and deciding to use the second arrangement, cannot obtain sufficiently exact initial data for compiling the volume equation without graphical development. In this case it is necessary first to develop the

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[1] On the series XXVI submarines the torpedo tubes were aimed aft in accordance with the tactic of "diving" under the target.

structural assembly of the placement of the torpedo tubes and spare torpedoes graphically (for different numbers of tubes and spare torpedoes), to define the corresponding volumetric characteristics on the basis of this development and then use them when compiling the volume equation.

It must be noted that in some cases, even with graphical method of design, first the most important original structural assembly is developed, and then the entire ship is developed. This approach has come to be called "assembly design."

Now let us propose that the designer has all of the data required for compiling the volume equation. In this case the cause of occurrence of errors in determining the volumetric displacement can be the above-mentioned failure to consider the configuration of the placed objects and the requirements on their individual and mutual arrangement on the ship. For example, when designing submarines armed with ballistic missiles (submarine missile carriers), it is highly desirable (by stability and propulsive performance arguments) to select the diameter of the pressure hull in the vicinity of the missile compartment so as to insure minimum possible protrusions of the missile tubes beyond the pressure hull considering restrictions with respect to draught, technological possibilities, and so on. This structural solution is applicable, in particular, on the American nuclear-powered submarine missile carriers [10]. The diameter of the pressure hull selected in this way frequently becomes defining for all other compartments, that is, the minimum required diameters of the other compartments turn out to be smaller than the missile compartment diameter.

Selecting the diameter and length (considering the number of missile tubes) of the missile compartment, the designer proceeds with laying out the other compartments, in particular, the primary control station (PCS) compartment located forward of the missile compartment. The minimum required length of the PCS compartment in some cases is determined by the conditions of placement of the telescopic devices (periscopes, radio and radio technical antennas, and so on). For the adopted length and diameter equal to the diameter of the missile compartment, the volume of the PCS compartment can turn out to be extremely large by comparison with the volume required for the placement of equipment and facilities which must be located in the PCS compartment. As a result, "free" space appears in this compartment. The elimination of this "free" space by decreasing the diameter of the PCS compartment is undesirable inasmuch as a sharp change in diameter leads to an increase in hull weight, volume of the interside space, and so on. Theoretically the designer can try to fill the "free" space with equipment and facilities from other compartments, but restrictions on the individual and mutual arrangement of the objects on the ship come into play. For example, it is undesirable to put main power plant elements in the PCS compartment, and so on [10].

The above-indicated peculiarities leading to the appearance of "free" spaces when composing the general arrangement of the weapons, accessory equipment, technical means and personnel on the ship are not taken into account by the volume equation inasmuch as it considers only the sum of the volumes required for placement of each of the objects individually without considering their configuration and requirements on arrangement by compartments.

Analogous examples can also be presented for surface ships. However, here the problem of considering the volumes required by the placement conditions is not so urgent as for submarines inasmuch as by the conditions insuring unsinkability of surface ships it is necessary to have a comparatively large (usually more than 100%) reserve buoyancy which can be used for combat supplies on hand, elements of accessory equipment, technical means and personnel. The volume equation has more significance for transport ships. Sometimes the so-called capacity equation is used for transport ships instead of the volume equation [35].

The author has considered it necessary to present a special discussion of the above-indicated peculiarities of compiling and using the volume equation so as to emphasize once again the importance of graphical development in the process of designing ships, including during analytical design.

4. Initial Transverse Stability Equation. The initial transverse stability of a ship is characterized by the metacentric height

$$h = z_C + \rho - z_G, \tag{2.39}$$

where z_C, z_G are the y-coordinates of the center of buoyancy and center of gravity; ρ is the transverse metacentric radius.

The value of z_G is determined using the general arrangement plans, and the values of z_C and ρ, using the lines plan. The expansion of equation (2.39) consists in establishing the analytical relations of the variables entering into it as a function of the principal dimensions of the ship, the lines coefficients and other characteristics.

In order to find the value of z_C in the initial design stages when the lines plan is unavailable, analytical representations are used for the curve of waterplane areas. As applied to surface ships these are most frequently parabolic curves. The expression for z_C is represented in general form:

$$z_C = \varphi_1\left(\frac{\delta}{\alpha}\right) T, \tag{2.40}$$

where $\phi_1(\delta/\alpha)$ is a function of the ratio δ/α. In reference [35] the following expressions are presented for $\phi_1(\delta/\alpha)$ obtained on the basis of processing the statistical data by V. V. Ashik

$$\varphi_1\left(\frac{\delta}{\alpha}\right) = 0{,}858 - 0{,}370\frac{\delta}{\alpha},$$

V. G. Vlasov

$$\varphi_1\left(\frac{\delta}{\alpha}\right) = 0{,}372 + 0{,}168\frac{\alpha}{\delta},$$

and L. M. Nogid

$$\varphi_1\left(\frac{\delta}{\alpha}\right) = \frac{1}{2}\left(\frac{\alpha}{\delta}\right)^{1/3}.$$

The approximate analytical expression for the transverse metacentric radius ρ can be obtained under the condition of analytical assignment of the shape of the construction waterline. For surface ships frequently the parabolic representation of the waterline shape is used. The general expression for ρ is assumed in the form

$$\rho = \varphi_2\left(\frac{\delta}{\alpha}, \alpha\right)\frac{B^2}{T}, \qquad (2.41)$$

where, according to A. P. Van der Fleet [1, 35],

$$\varphi_2\left(\frac{\delta}{\alpha}, \alpha\right) = \frac{1}{k}\frac{\alpha^2}{\delta}, \quad k = 11.4 \text{ to } 11.7.$$

If we express the y-axis of the center of gravity z_G in fractions of a hull height, then considering (2.40) and (2.41) we obtain the formula for h in the form

$$h = \varphi_1\left(\frac{\delta}{\alpha}\right)T + \varphi_2\left(\frac{\delta}{\alpha}, \alpha\right)\frac{B^2}{T} - \xi H, \qquad (2.42)$$

where ξ is a statistical coefficient.

Formula (2.42) is also written in the following equivalent forms:

$$\begin{aligned} \frac{h}{B} &= \varphi_1\left(\frac{\delta}{\alpha}\right)\frac{T}{B} + \varphi_2\left(\frac{\delta}{\alpha}, \alpha\right)\frac{B}{T} - \xi\frac{H}{T}\frac{T}{B}, \\ \frac{h}{T} &= \varphi_1\left(\frac{\delta}{\alpha}\right) + \varphi_2\left(\frac{\delta}{\alpha}, \alpha\right)\frac{B^2}{T^2} - \xi\frac{H}{T}. \end{aligned} \qquad (2.43)$$

Expressions (2.42) or (2.43) can be considered as the equations with respect to the principal dimensions B, H and T. The stability equation must be considered jointly with the equations of masses, volumes and buoyancy. If the displacement and the principal dimensions of the ship are defined in advance, then using expressions (2.42) or (2.43), it is possible to find the metacentric height.

When considering the stability conditions in the initial design stages it is necessary to consider the possible simplifications of the problem connected with the peculiarities of the ship's hull shape, the general arrangement system, and so on. Let us illustrate this principle in the example of submarines.

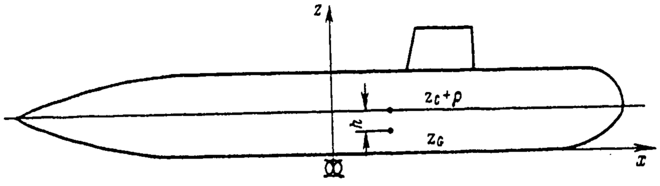

Figure 2.6. Mutual arrangement of the transverse metacenter and center of gravity of a submarine with hull in the shape of a solid of revolution

For atomic submarines of the majority of traditional classes and types, the shape of the outer hull is very close to a solid of revolution [10]. With this hull shape, for any draught, including underwater, the metacenter is approximately on the longitudinal axis of symmetry of the hull. In this case the insurance of initial stability reduces to the placement of the basic loads for which the center of gravity of the ship will be below the longitudinal axis of symmetry of the hull (Figure 2.6). Here it is necessary to consider that the center of gravity of the hull of investigated shape can be considered approximately on the axis of rotation (the axis of symmetry). Considering the above-indicated peculiarities, the initial stability of a submarine both on the surface and under water basically is determined by the values (masses) and arrangement with respect to height of only the following loads (Figure 2.7): the power plant (MPP), the missile and torpedo weaponry (MW and TW), and the solid ballast (SB).

For fixed displacement D=const the metacentric height will be

$$h=-\frac{\underset{(1)}{m_{\text{эу}}\Delta z_{\text{эу}}}+\underset{(2)}{m_{\text{то}}\Delta z_{\text{то}}}+\underset{(3)}{m_{1\text{р}}n}\,\underset{(4)}{\Delta z_{\text{ро}}}+\underset{(5)}{m_{\text{тб}}\Delta z_{\text{тб}}}}{D}, \qquad (2.44)$$

Key: 1. power plant; 2. torpedoes; 3. missile; 4. missiles; 5. solid ballast where m_{MPP}, m_{TW}, m_{SB}, m_{1m} are the masses of the power plant, the torpedo weaponry, the solid ballast and one missile tube with missile; Δz_{MPP}, Δz_{TW}, Δz_{MW}, Δz_{SB} are the distances of the centers of gravity of the corresponding loads from the axis of symmetry of the hull; n is the number of missile tubes; D is the normal displacement.

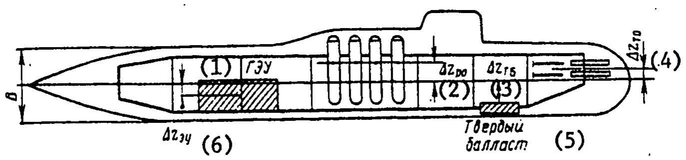

Figure 2.7. Diagram of the arrangement of basic loads determining the transverse stability of a missile submarine

Key:
1. main power plant
2. Δz_{MW}
3. Δz_{SB}
4. Δz_{TW}
5. solid ballast
6. Δz_{MPP}

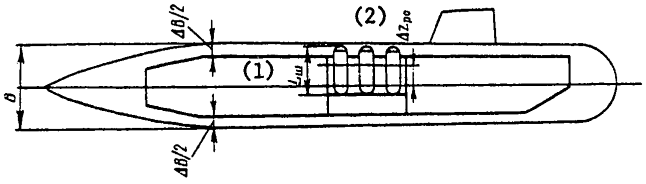

Figure 2.8. Diagram of the arrangement of rocket tubes for $L_{tube}<B'$

Key: 1. L_{tube}; 2. Δz_{WP}

If z_{Gi} are the x-coordinates of the centers of gravity of the loads with respect to the base line, B is the diameter of the hull, then $\Delta z_i = z_{Gi} - B/2$. Usually the inequalities exist: $\Delta z_{MPP} < 0$, $\Delta z_{TW} > 0$, $\Delta z_{MW} > 0$, $\Delta z_{SB} < 0$. Considering these inequalities from expression (2.44) it is possible to find the maximum admissible (with respect to stability conditions) number of missile tubes for fixed stability coefficient (Dh=const)

$$n_{\max} = \frac{m_{эу}|\Delta z_{эу}| + m_{тб}|\Delta z_{тб}| - m_{то}|\Delta z_{то}| - Dh}{m_{1р}|\Delta z_{ро}|}, \qquad (2.45)$$

where $|\Delta z_i|$ are the absolute values of the corresponding variables.

If we consider the displacement as a function of the number of missile tubes, then the maximum number of tubes will be found from the equation

$$n + \frac{D(n)}{m_{1р}} \frac{h}{|\Delta z_{ро}|} = \frac{m_{эу}|\Delta z_{эу}| + m_{тб}|\Delta z_{тб}| - m_{то}|\Delta z_{то}|}{m_{1р}|\Delta z_{ро}|}.$$

Let the length of the missile tube be L_{tube}, and the distance of its center of gravity (with missile) from the lower edge of the tube to be αL_{tube}, where α is a given value ($0<\alpha<1$). Let us propose that for $L_{tube}<B-\Delta B$ the tube is "hung" on the upper edge of the pressure hull (Figure 2.8). Then

$$\Delta z_{ро} = \begin{cases} \alpha L_{ш} - \frac{B'}{2} & \text{for } B' \leqslant L_{ш}, \\ \frac{B'}{2} - (1-\alpha) L_{ш} & \text{for } B' > L_{ш}, \end{cases} \qquad (2.46)$$

$$2\frac{\Delta z_{ро}}{L_{ш}} = \begin{cases} 2\alpha - \bar{B}' & \text{for } \bar{B}' \leqslant 1, \\ \bar{B}' - 2(1-\alpha) & \text{for } \bar{B}' > 1, \end{cases}$$

where $\bar{B}' = \frac{B - \Delta B}{L_{ш}} = \frac{B'}{L_{ш}}$ and $\Delta B/2$ is the width of the interside space.

For fixed values of Δz_{MPP}, Δz_{TW} and Δz_{SB} the maximum admissible number of tubes is reached under the condition of the minimum value of Δz_{MW}. From the formulas (2.46) it follows that the minimum of Δz_{MW} is reached for $\bar{B}'=1$ when the length of the tube is approximately equal to the hull diameter. Figure 2.9 shows the values of $2\,\Delta z_{MW}/L_{tube}$ as a function of $\bar{B}'$ for $\alpha=0$, 0.5 and 1.0. In practice the value of $|\Delta z_{SB}|$ increases with an increase in the hull diameter, but inasmuch as usually $m_{1m}n > m_{SB}$, the conclusion of the expediency of insuring the equality $B' \simeq L_{tube}$ is maintained also considering this fact.

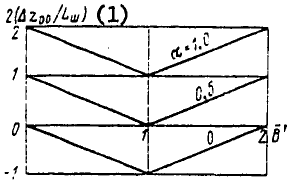

Figure 2.9. Dependence of $2\,\Delta z_{MW}/L_{tube}$ on $\bar{B}'$

Key: 1. $2(\Delta z_{MW}/L_{tube})$

In the investigated example the stability conditions can be taken into account directly in the mass equation. Let us propose that the stability will always be insured with an increase in the mass of the solid ballast. From the formula (2.44) it follows (for $\Delta z_{TW} \simeq 0$) that

$$m_{\text{тб}} = -\frac{Dh + m_{\text{эу}}\Delta z_{\text{эу}} + m_{1p}\, n\Delta z_{po}}{\Delta z_{\text{тб}}}$$

or considering $\Delta z_{SB} < 0$

$$m_{\text{тб}} = \frac{Dh + m_{\text{эу}}\Delta z_{\text{эу}} + m_{1p} n\, \Delta z_{po}}{|\Delta z_{\text{тб}}|}.$$

Assuming that $|\Delta z_{SB}| \simeq B/2$, we obtain

$$m_{\text{тб}} = 2D\frac{h}{B} + 2m_{\text{эу}}\frac{\Delta z_{\text{эу}}}{B} + 2m_{1p} n\frac{\Delta z_{po}}{B}. \tag{2.47}$$

For a hull in the shape of a solid of revolution the ratio $L/V^{1/3}$, where V is the total geometric volume of the hull, is a function of the ratio L/B, that is, $\frac{L}{V^{1/3}} = f\left(\frac{L}{B}\right)$. Considering this function and the relation between the value of V and the displacement D, it is possible to represent formula (2.47) in the form

$$m_{\text{тб}} = 2\rho^{1/3}D^{2/3}h\frac{L}{Bf\left(\frac{L}{B}\right)}\frac{1}{k^{1/3}} + 2m_{\text{эу}}\frac{\Delta z_{\text{эу}}}{B} + 2m_{1p} n\frac{\Delta z_{po}}{B},$$

where $k = \rho V/D$.

Let us denote the righthand side of the last expression in terms of A, and let us define the relative mass of the solid ballast by the formula

$$\overline{m}_{\text{тб}} = \frac{m_{\text{тб}}}{D} = \begin{cases} \frac{A}{D} & \text{for } \frac{A}{D} > \overline{m}^{(0)}_{\text{тб}}, \\ m^{(0)}_{\text{тб}} & \text{for } \frac{A}{D} \leqslant \overline{m}^{(0)}_{\text{тб}}, \end{cases} \tag{2.48}$$

where $\overline{m}^{(0)}_{SB}$ is the given minimum admissible value of the relative mass of the solid ballast determined by the requirements of trim, modification reserve, and so on.

Substituting expression (2.48) in the mass equation, we obtain an equation in which the conditions of initial (transverse) stability will be approximately taken into account.

It must be noted that when compiling the basic equations of the analytical method of design it is expedient to strive, at least approximately, to consider the largest possible number of requirements and conditions which the ship must satisfy.

For example, the requirements on unsinkability can in a number of cases be considered by the appropriate selection of the reserve buoyancy as a function of displacement and the number of compartments. This relation is established by the statistical method or on the basis of investigation of the simplest system for calculating the unsinkability. For submarines it is possible approximately to consider the condition of trim by the introduction of a special volume providing for trim into the volume equation. The magnitude of this volume will be connected with certain characteristics of the ship, for example, the mass and volume, and, consequently, the engine power, inasmuch as, as a rule, for nuclear-powered submarines the power plant, which is placed in the minimum required volume, is the basic load shifting the center of gravity of the ship aft with respect to its center of buoyancy.

5. Ship Power (Speed) Equation. The power equation relates the required power of the ship's engine to its maximum speed, displacement, principal dimensions and characteristics of the hull shape. The above-mentioned admiralty formula is the simplest form of power equation.

In the general case the power equation has the form

$$N = \frac{\rho v_{max}^{3}}{2} \frac{\zeta\omega}{\eta} V^{2/3}, \tag{2.49}$$

where V is the volume of the part of the ship's hull submerged in the water.

The expansion of equation (2.49) consists in considering the dependence of the values of ω, ζ and η on the displacement, the principal dimensions, speed, hull shape parameters, number of screws, their rpm, and so on. As is known, this problem does not at the present time have a sufficiently exact analytical solution, especially for surface ships and submarines on the surface. In this case the power equation is given using the tables and graphs compiled on the basis of the results of systematic tests of models of ships in model testing basins [52].

The problem of expansion of the power equation for submarines under water is solved more simply inasmuch as in this case there is no wave drag and, in addition, for well streamlined bodies such as the hulls of modern submarines, the ratio of the eddying drag to the friction drag is a value independent of the speed of the submarine determined by the hull elongation (L/B), its ellipticity (H/B) and the shape of the ends which, as a rule, are either made spindle shaped or stem [10]. This relation presented in Figure 2.10 can be approximated, which together with the formulas for the friction drag coefficient permits representation of the drag analytically. On the basis of the fact that the drag coefficient decreases with an increase in L/B, and the wetted surface of the hull increases, the problem of determining the optimal value of the ratio L/B can be solved beginning with the condition $\min(\zeta\omega)$.

If we use the data of Figure 2.10 and formula $\omega=0.26L/B+5.60$ which, according to [10], is valid for hulls in the shape of well streamlined solids of revolution, then from the point of view of propulsive performance under water we obtain the estimate $(L/B)_{opt} \simeq 5$. This estimate is valid for $\zeta_T=1.85\cdot10^{-3}$, $\Delta\zeta_{tube}=0.4\cdot10^{-3}$,

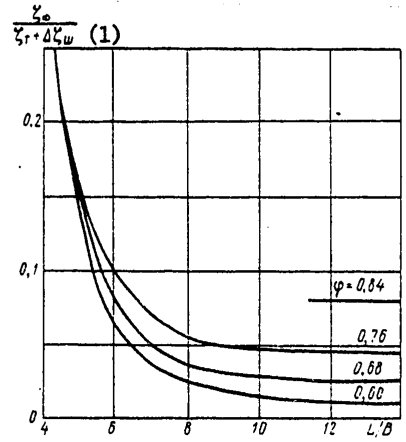

Figure 2.10. Value of $\zeta_\phi/(\zeta_T+\Delta\zeta_{tube})$ as a function of the ratio L/B and the prismatic coefficient ϕ for the hulls of submarines executed in the form of solids of revolution [10]

Key:

1. $\Delta\zeta_{tube}$

the drag coefficient of the protruding parts $\zeta_{pro}=0.80\cdot10^{-3}$ and $\phi=0.68$. According to the data of [10], the above-indicated values of the drag coefficients and the prismatic coefficient occur for American submarines of the "Skipjack" class. Inasmuch as the optimum with respect to L/B is very gently sloping, in the existing submarines the values of L/B usually exceed the indicated optimum.

During the AD process the power equation is used to determine the power and then the mass of the power plant if the maximum speed of the ship is taken as one of the independently variable characteristics. When the power of the power plant is taken as the independent variable, the formula (2.49) is used to determine the maximum speed of the ship after finding the displacement and the principal dimensions from other equations. Here the mass and the volume of the power plant enter into the mass and volume equations as a power function.

The latter case of using the power equation which in the given case is more correctly called the speed equation, appears to be expedient primarily for surface ships, inasmuch as the introduction of a complex power equation having no analytical representation into the mass and volume equations greatly complicates the latter and complicates their solution.

Let us consider some problems connected with expansion of the speed equation for surface displacement ships. The problem consists in representation of the right-hand side of equation (2.49) in the form of the speed function v_{max} considering the dependence on it of the drag coefficient and, generally speaking, the propulsive coefficient. Let us represent equation (2.49) in the form $N=N_e/\eta$, where N_e (or EPS) is the towrope or effective power

$$N_e=\frac{1}{2}\rho v^3_{max}\zeta\omega V^{2/3}.$$

For approximations the dependence of the drag coefficient and, consequently, the towrope power on the speed and the basic parameters of the ship's hull shape, is given using tables, graphs and diagrams (the Dwyer-Girs diagram, the Taylor diagram, Papmel' diagram, and the Girs graph) permitting determination of the residual drag, that is, the sum of the wave and eddying components of the total drag or the towrope power directly. In particular, the diagram and the corresponding Papmel' empirical formula offer the possibility of finding the towrope power directly. The Papmel' empirical formula has the form

$$N_e = 0{,}736 \frac{D}{L} \frac{\Delta}{\lambda} \sqrt{\psi} \frac{1}{C} v^3.$$

Here N_e is the towrope power, kilowatts; D is the displacement, m^3; v is the speed, knots; L is the construction waterline length of the ship, meters; λ is the correction coefficient to the length; Δ is a coefficient taking into account the effect of the protruding parts; ψ is a coefficient characterizing the longitudinal fineness of the hull; C is a coefficient defined by the Papmel' diagram as a function of the coefficient ψ and the relative speed $v'=v\sqrt{\psi/L}$. The coefficients λ and ψ are defined by the expressions

$$\lambda = \begin{cases} 0{,}7 + 0{,}3 \sqrt{\frac{L}{100}} & \text{for } L < 100, \\ 1 & \text{for } L \geqslant 100, \end{cases}$$

$$\psi = 10\delta \left(\frac{L}{B}\right)^{-1},$$

where δ is the block coefficient. The coefficient Δ depends on the number of screws z. For z=1, 2, 3, for Δ=1, 1.05, 1.075, and 1.1, respectively.

The Papmel' diagram is applicable for ships with the characteristics ψ=0.35 to 1.10; B/T=1.5 to 3.5, L/B=4 to 11; δ=0.35 to 0.80. If we use the expression

$$L = lD^{1/3} = \left[\frac{1}{\delta}\left(\frac{L}{B}\right)^2 \frac{B}{T}\right]^{1/3} D^{1/3},$$

all the values entering into the Papmel' formula can be expressed in terms of L/B, B/T, δ, v and D. If we also use the empirical function for selecting the relative length as a function of the Froude number, then the towrope power will depend only on B/T, δ, v and D.

When solving the speed equation by computer, the table of values of the coefficient C will be input to the computer memory or the solution will be realized in the dialogue mode where the designer inputs new values of C to the computer as the successive approximations are made, using the current values of the parameters v' and ψ for definition of them (by the Papmel' diagram).

Previously, the important significance of the graphical dialogue was noted for automated design of ship using a computer. At the same time, insurance of the dialogue mode of operation of the computer with operative and graphical output of the intermediate and final results and also input of new additional initial data has important significance when solving purely computational problems of ship design.

The author has considered it appropriate to make a comment regarding the use of the dialogue capabilities of computers when solving design problems in connection with the fact that modern computers and computer systems based on using computer engineering are equipped with a quite large arsenal of means of insuring the dialogue mode of operation and it is necessary to make skillful use of these means.

Turning to the problem of discovering the type of equation (2.49), we shall not present arguments pertaining to the determination of the towrope power for submarines under water, inasmuch as this problem is solved significantly more simply than for surface ships, and for the approximate calculation it is possible to obtain an analytical expression of the towrope power.

Let us briefly consider the problem of approximate calculation of the propulsive coefficient, considering obtaining the corresponding analytical expression. The value of η usually is represented in the form

$$\eta = \eta_p \frac{1-t}{1-w} i,$$

where η_p is the efficiency of the screws in open water; w, t are the wake and thrust deduction coefficients; i is the coefficient of effect of nonuniformity of flow. The value of $i \frac{1-t}{1-w} = \eta_K$ is called the hull-efficiency factor. The value of i is very close to one (usually $i \gtrsim 0.95$), and the values of the coefficients w and t are selected in the initial design stages by prototypes.

At the present time for approximate determination of the hydrodynamic characteristics of the screws in open water, including determination of their efficiency, special diagrams which were constructed on the basis of processing the results of series tests of screw models. In Soviet practice the diagrams of E. E. Papmel' have found the greatest application. They are a graphical representation of the thrust coefficient (K_1) and torque coefficient (K_2) as a function of the relative advance coefficient of the screw $\lambda_p = \frac{v(1-w)}{nd}$ for different pitch ratios of the zero thrust H_1/d. Here $v(1-w)=v_p$ is the speed at the propeller disc; d is the screw diameter; n is the rpm; H_1 is the zero thrust step.

From the point of view of the initial data when solving the problem of selecting the screws (the number of screws z is given) the following four cases are the most difficult:

1) The speed of the ship v, the diameter of the screws d and the resistance r are given, and it is necessary to determine the rpm n and the efficiency η_p;

2) v, n and r are given, and d and η_p are determined;

3) v, d and the available power on the screws N are given, and n and η_p are determined;

4) v, n and N are given, and d and η_p are defined.

In order to facilitate a solution of the problem of selecting the screws, lines of equal values of the four coefficients which are presented in the series corresponding to the four cases of assignment of the initial data are plotted on the Papmel' diagrams[1]

$$K'_d = \frac{\lambda_p}{\sqrt{K_1}} = dv(1-w)\sqrt{\frac{\rho}{r}(1-t)z},$$

$$K'_n = \frac{\lambda_p}{\sqrt[4]{K_1}} = \frac{v(1-w)}{\sqrt{n}}\sqrt[4]{\frac{\rho}{r}(1-t)z},$$

$$K''_d = \sqrt{\frac{75\lambda_p^3}{2\pi K_2}} = dv(1-w)\sqrt{\frac{\rho v}{N}(1-w)z},$$

$$K''_n = \sqrt[4]{\frac{75\lambda_p^5}{2\pi K_2}} = \frac{v(1-w)}{\sqrt{n}}\sqrt[4]{\frac{\rho v}{N}(1-w)z}.$$

The fixed values of the above-indicated conditions correspond to certain sets of screws with different values of the desired characteristics. Therefore, for a unique solution of the problem (for fixed geometric parameters of the screw, except the pitch ratio), it is necessary to introduce an additional condition, for example, maximizing the efficiency.[2] In order to solve the problem of selecting the screws in this statement the lined $K'_{d\ opt}(\lambda_p)$, $K'_{n\ opt}(\lambda_p)$, $K''_{d\ opt}(\lambda_p)$ and $K''_{n\ opt}(\lambda_p)$ are plotted on the Papmel' diagrams, at each point of which the efficiency reaches a maximum.

The relations for the optimal values of the efficiency (η_p^{opt}) as a function of the corresponding coefficient K_d and K_n can be constructed by the Papmel' diagram. Approximating these functions, the analytical expressions for the efficiency are obtained in terms of the coefficients K_d and K_n, and, consequently, in terms of the speed of the ship, the screw diameter (or rpm), the drag of the ship (or available power), and the number of screws. For example, the function $\eta_p^{opt}(K'_n)$ for three-bladed screws[3] is approximated well by the expression (2.11).

$$\eta_p^{opt}(K'_n) = 1{,}60\,(K'_n)^{0{,}145} - 1 \quad \text{for} \quad K'_n = 0{,}5 \text{ to } 1.7.$$

---

[1] It is necessary to consider that the Papmel' diagrams are constructed for the system of units kilogram-force, meter, seconds, where the power is measured in horsepower. When using the SI system this must be taken into account by recalculating the values of the corresponding variables.

[2] It is also possible to use other versions of the additional condition, for example, along with the efficiency, to consider the acoustic qualities in the screws, and so on.

[3] The Papmel' diagram for these screws is presented in Appendix 1 of the book by V. I. Solov'yev and A. D. Chumak KORABEL'NYYE DVIZHITELI [Ship Propulsion Units], Moscow, Voyenizdat, 1948).

Similar approximation relations can be obtained for the efficiency also as a function of the remaining coefficients K_d and K_n.

Inasmuch as K''_d and K''_n do not depend on the drag, when selecting the screws in the case of the given available power it is possible to obtain an approximate analytical expression for the propulsive coefficient of both surface ships and submarines above and below water. In this case the equation (2.49) is used to determine the speed of the ship. Its analytical representation is possible for submarines under water, for in this case the towrope power is expressed analytically.

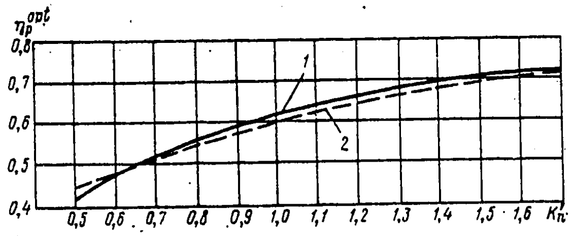

Figure 2.11. Optimal value of the efficiency (η_p^{opt}) of the screw (A/A_d=0.55) as a function of the coefficient K'_n.
1 -- by the Papmel' diagram, 2 -- approximation

When equation (2.49) is used as the power equation for substitution of it in the mass and volume equation and subsequent determination of the displacement, the analytical expression of the propulsive coefficient is obtained only for submarines inasmuch as in this case the drag on which the coefficients K'_d and K'_n depend is expressed analytically. At the same time from the formula for K'_n it is obvious that the dependence of this coefficient on the drag is very weak (inversely proportional to $\sqrt[4]{r}$. In addition, the value of η_p^{opt} for the values of the screw efficiency used in practice varies also very slowly with variation of K'_n. Defining the values of K'_n this makes it possible to use an approximate value of the resistance found, for example, using the admiralty formula after solving the three-term mass equation (the admiralty coefficient is taken by the prototype). By this procedure it is possible to obtain the analytical expression for the propulsive coefficient in the power equation of surface ships when selecting screws that are optimal with respect to diameter for the given rpm. However, the general problem of analytical assignment of the power equation of surface ships in this case has not been completely solved as a result of absence of the analytical expression for the towrope power.

From the Papmel' diagrams, in particular, from the above-presented approximation function η_p^{opt} (K'_n), it follows that the screw efficiency increases with an increase in K'_n. In turn, the value of K'_n depends primarily on the ratio $v/\sqrt{n}$, and it increases with an increase in this ratio. Hence, we have the conclusion of expediency of decreasing the propeller rpm for any speed of the ship. However, this conclusion is valid to a defined limit, for when some critical value of the ratio $v/\sqrt{n}$ is reached, the screw efficiency begins to decrease as a result of an increase in resistance to turning of the screw caused by the frictional forces of the water. From the practical point of view the limiting decrease in the screw rpm is limited, as a rule, by the maximum admissible diameter of the screws with respect to the conditions of replacement, strength, possibility of manufacture, and so on.

The effort to insure maximum screw efficiency as a result of decreasing their rpm and, correspondingly, placement of the screws of the greatest diameter gives rise to preference for the single-screw propulsion system for submarines on which the most favorable conditions for placement of maximum diameter screws exists with the single-screw system. In practice all modern foreign submarines, including all of the basic classes and types of nuclear-powered American submarines have a single-screw propulsion system with relatively low propeller rpm [10]. The single-screw propulsion system with low propeller rpm is also favorable from the point of view of decreasing the noise of the screws, which is highly significant for submarines [10]. A more detailed and comprehensive investigation of this problem is beyond the scope of this book which is basically of a procedural nature.

§2.4. Use of Similarity and Dimensionality Theories when Compiling the Equations of the Analytical Method of Ship Design

As is known, when compiling the equations of the analytical method of ship design, the similarity and dimensionality methods are used, which arises from the complexity of the relations entering into the equations and the difficulty, and sometimes practical impossibility, of the development of exact mathematical models of the corresponding physical phenomena.

The basic idea of the similarity method consists in the following basic theorem of dimensionality theory.

Let the dimensional variable x of interest to us be a function of n dimensional variables $x_1, \ldots, x_n$, where the units of the values of $x_1, \ldots, x_k$, where $k<n$, are independent,[1] that is the corresponding dimensionalities cannot be obtained from each other in the form of a power monomial. It is possible to show [46] that the dimensionality formula of any physical variable has the form of a power monomial. In dimensionality theory the statement is proved that the initial relation between variables x and $x_1, \ldots, x_n$

$$x = f_1(x_1, \ldots, x_n), \tag{2.50}$$

expressing a physical law independent of the choice of units of measure, can be represented in the form of a relation between dimensionless variables Π, $\Pi_1, \ldots, \Pi_{n-k}$

$$\Pi = f_2(1, \ldots, 1, \Pi_1, \ldots, \Pi_{n-k}), \tag{2.51}$$

---

[1] The number of basic units of measure must be greater than or equal to k.

where
$$\Pi=\frac{x}{x_1^{m_1}\ldots x_k^{m_k}},\quad \Pi_1=\frac{x_{k+1}}{x_1^{p_1}\ldots x_k^{p_k}},\quad\ldots,\quad \Pi_{n-k}=\frac{x_n}{x_1^{q_1}\ldots x_k^{q_k}},$$

m_i, p_i, q_i, i=1, ..., k are the values that define the expressions for the units of measure of the variables x, x_{k+1}, ..., x_n in terms of the independent units of x_1, ..., x_k.

On the basis of the single-term power type expression of the units of any value the following relations exist:

$$[x]=[x_1]^{m_1}[x_2]^{m_2}\ldots[x_k]^{m_k},$$
$$[x_{k+1}]=[x_1]^{p_1}[x_2]^{p_2}\ldots[x_k]^{p_k},$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$[x_n]=[x_1]^{q_1}[x_2]^{q_2}\ldots[x_k]^{q_k},$$

where the brackets denote the units of the corresponding values.

The above-presented fact of the possibility of going from expression (2.50) to expression (2.51) is called the Π-theorem. According to this theorem, the initial relation between n+1 dimensional variables, x, x_1, ..., x_n can be replaced by the relation between (n+1-k) dimensionless combinations Π, Π_1, ..., Π_{n-k}, as a result of which the number of arguments in the investigated function decreases by the amount k, that is, by the number of independent units of the initial arguments x_1, ..., x_n. This fact theoretically simplifies the investigation of the initial function the more so, the smaller the value of n-k. In the special case where n=k, the theorem permits establishment of the form of the initial function with accuracy to an arbitrary constant. In this case $\Pi=f_2(1, \ldots, 1)$, that is

$$\frac{x}{x_1^{m_1}\ldots x_k^{m_k}}=f_2(1,\ \ldots,\ 1)=C=\text{const},\qquad (2.52)$$
$$x=Cx_1^{m_1}\ldots x_k^{m_k},$$

where the exponents m_1, ..., m_k are easily found from the dimensionality formula for x.

From the Π-theorem, we have another important corollary: if the dimensionless variable depends on a number of dimensional arguments, it can depend only on the set of independent dimensionless combinations made up of dimensional arguments. These dimensionless combinations have the form of power monomials.

In the general case the Π-theorem does not permit the form of the investigated function to be found. This must be done either experimentally or on the basis of investigation of exact mathematical models. The use of prototypes can be considered as an experiment. In cases where the experiment is accompanied by errors, we resort to statistical processing of the results. The statistical methods are used when processing the data of several prototypes, considering that each prototype is only approximately similar to the designed ship. Sometimes in order to obtain the type of investigated function, we refer to approximate mathematical models. In these cases for determination of the series of coefficients and parameters of the investigated function statistical methods are also used.

Dimensionality theory shows that out of n variables, k of which have independent units of measure, it is possible to compile no more than n-k independent dimensionless combinations. It is obvious that the effect of applying dimensionality theory will be greater the smaller the number of initial parameters (n) and the more basic units of measures selected.

Generally speaking, the number of basic units of measure is selected arbitrarily, but increasing the number of basic units of measure is connected with introducing additional physical constants which must figure among the defining parameters (arguments of the function f_1). Increasing the number of units of measure, we increase the number of dimensional constants. Here the difference (n+1-k) equal to the number of independent dimensionless combinations remains constant. Therefore it is possible to consider that in the general case the number of independent units of measure will not exceed 3 (for example, kilograms, meters, seconds).

The Π-theorem is essentially the basic theorem of mechanical similarity, a special case of which is the geometric and dynamic forms of similarity. The similarity condition is the equalities -- similarity criteria -- independent of the dimensionless combinations $\Pi_1, \ldots, \Pi_{n-k}$ compiled from the defining parameters $x_1, \ldots, x_n$. Obviously if the similarity conditions are satisfied, then according to the data of the dimensionless characteristics of one object or phenomenon the corresponding characteristics of another object or phenomenon can be found with variable values of the defining parameters. Actually, if for the two objects we have $\Pi_i^{(1)}=\Pi_i^{(2)}$, i=1, ..., n-k, where the superscript refers to the number of the object, then the equality $\Pi^{(1)}=\Pi^{(2)}$ permits the desired value of $x^{(2)}$ to be found by the given values of $x^{(1)}$, $x_1^{(1)}, \ldots, x_k^{(1)}$ and $x_1^{(2)}, \ldots, x_k^{(2)}$

$$x^{(2)} = x^{(1)} \frac{(x_1^{(2)})^{m_1} \ldots (x_k^{(2)})^{m_k}}{(x_1^{(1)})^{m_1} \ldots (x_k^{(1)})^{m_k}}.$$

Below, two examples of using the methods of similarity for determining the functions entering into the mass, volume and power equation are investigated.

1. Determination of the mass of the pressure hull of submarines in the initial design stages. The cylindrical pressure is characterized by the following parameters on which the skin thickness t depends: d -- diameter; ℓ -- characteristic linear dimension of the index contour -- frame spacing; p -- design pressure; σ_D -- admissible stresses in the skin material; E -- modulus of elasticity of the skin material; ω_{frame} -- cross sectional area of the index contour means (frames).

From these seven parameters (considering the value of t) it is possible to compile five independent dimensionless combinations, for the number of variables with independent units of measure is equal to two. The following can be taken as such combinations: t/d -- the relative thickness of the skin; $\Pi=p/\sigma_D$ -- strength indexes; $Ж = p/E$ -- rigidity index; $\Gamma_1=\ell/d$, $\Gamma_2=\ell d/\omega_{frame}$ -- geometric indexes.

On the basis of the Π-theorem we have the expression

$$\frac{t}{d} = f(\Pi, Ж, \Gamma_1, \Gamma_2). \tag{2.53}$$

Considering (2.53) for the mass of the skin we obtain

$$\underset{(1)}{m_{\text{обш}}} = \pi d l t \rho_{\text{м}} = \pi d^2 l \rho_{\text{м}} f(\Pi,\ \text{Ж},\ \Gamma_1,\ \Gamma_2), \tag{2.54}$$

Key: 1. skin

where ρ_M is the density of the pressure hull material.

The specific mass of the skin of the pressure hull (PH) q_{PH}, by which we mean the mass per unit volume of the shell, is defined by the expression

$$\underset{(1)}{q_{\text{обш}}} = 4\frac{m_{\text{обш}}}{\pi d^2 l} = 4\rho_{\text{м}} f(\Pi,\ \text{Ж},\ \Gamma_1,\ \Gamma_2). \tag{2.55}$$

Key: 1. skin

The form of the function f must be found either experimentally (by prototypes) or by mathematical models of structural mechanics.

Actually, the minimum value of the PH mass is of greatest interest, which permits us to exclude the indexes Γ_1 and Γ_2 from the investigation as a result of selecting the optimal geometric relations of the shell. In addition for sufficiently large values of p/σ_D corresponding to efficient use of the material with respect to stresses, the rigidity criterion turns out to be insignificant.

Under the assumption made, instead of (2.55) we obtain

$$\underset{(1)}{q_{\text{обш}}} = \rho_{\text{м}} f(\Pi) = \rho_{\text{м}} f\left(\frac{p}{\sigma_{\text{д}}}\right), \tag{2.56}$$

Key: 1. skin

where the coefficient 4 is taken into account in the function f.

Let us establish the general form of the function f in expression (2.56), beginning with investigation of the simplified mathematical model of calculation of the strength of the skin of the unreinforced cylindrical shell. Stresses in the longitudinal cross sections of an unreinforced cylindrical shell are twice the stresses in the transverse cross sections (Figure 2.12), and they are determined by the so-called "boiler" formula $\sigma = pd/2t$. Consequently, by the strength condition $(\sigma = \sigma_D)$ the thickness and the specific mass of such a shell have the form $t = pd/2\sigma_D$; $q_{skin} = 2\rho_M p/\sigma_D$.

From investigation of the simplified mathematical model we have the linear form of the function f. At the same time, from the practical calculations and the data on the manufactured hulls it is possible to establish that the mass of the skin usually is the main part of the total PH mass. Accordingly, the similarity law for the skin mass of an unreinforced cylindrical shell can be approximately extended to the mass of the entire PH. It is also known that the load item of the "pressure hull" includes certain parts (welds, local foundations under small equipment in the interside space, and so on), the mass of which does not depend on the design pressure or the strength characteristics of the PH material.

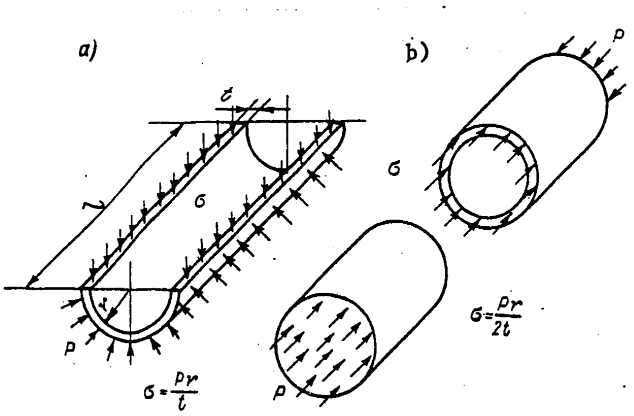

Figure 2.12. Schematic of the determination of stresses acting in the longitudal (a) and transverse (b) cross sections of the circular cylindrical shell

Thus, the formula for the specific mass of the PH can be proposed:

$$\underset{(1)}{q_{\text{пк}}} = \rho_{\text{м}}\left(a\frac{H}{\sigma_{\text{т}}}+b\right), \qquad (2.57)$$

Key: 1. PH

where H is the specific depth of submersion of the submarine; σ_T is the yield point of the PH material; a, b are the coefficients defined by statistical means.

In formula (2.57) it is considered that the designed pressure p is selected proportional to H, and the admissible stresses, proportional to σ_T.

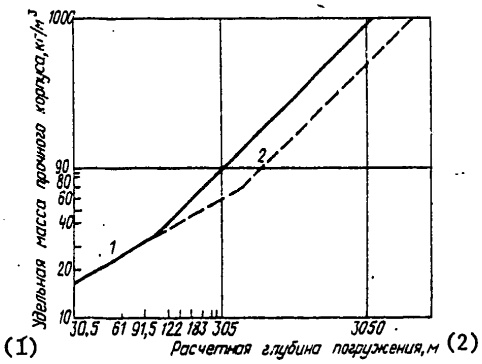

Figure 2.13. Specific mass of a steel pressure hull of a submarine as a function of the designed depth of submersion.
1 -- σ_T=519 newtons/mm^2; 2 -- σ_T=1029 newtons/mm^2

Key:
1. Specific mass of the pressure hull, kg/m^3
2. Designed depth of submersion, meters

If the volume of the PH is proportional to displacement D_m, then the mass of the pressure hull is introduced into the mass equation by the relation

$$m_{пк}=\frac{\rho_м}{\rho}\bar{w}_{пк}\left(a\frac{H}{\sigma_т}+b\right)D_m, \tag{2.58}$$

where $\bar{w}_{пк}$ is the relative volume of the PH; ρ is the density of water.

Formulas (2.57) and (2.58) are valid for quite large values of H/σ_T when the strength conditions are defining, and not the stability conditions of the shell. For small H/σ_T when the stability conditions are limited, the values of q_{PH} and m_{PH} will be greater than those defined by relations (2.57) and (2.58). What has been stated is illustrated by Figure 2.13 which is borrowed from [10].

The presented example illustrates the joint use of the similarity method (discovery of the dimensionless parameter H/σ_T), approximate mathematical models (establishment of the linear form of the function f) and statistical methods (determination of the numerical values of the coefficients a and b).

2. Determination of the required power and mass of the power plant of a ship in the initial design stages. The mass of many types of power plants of ships can be considered in a quite wide range of powers directly proportional to the power:

$$\underset{(1)}{m_{эу}}=q_{эу}N, \tag{2.59}$$

Key: 1. power plant = PP

where q_{PP} is the specific mass of the power plant.

The power of the power plant required to insure the given maximum speed of the ship v_{max}) is defined by the formula

$$N=\frac{rv_{max}}{\eta}. \tag{2.60}$$

Let us show how the similarity methods are used to obtain the drag as a function of the ship's characteristics.

In the case of geometric similarity of hulls, including similarity with respect to trim of the ship with respect to the water, the defining parameters for r are the following: L -- length of the ship; v_{max} -- its maximum speed; ρ -- water density; g -- gravitational acceleration; μ -- coefficient of dynamic viscosity of water. From these five parameters and the value of r, three independent dimensionless combinations can be compiled (here the number of variables with independent units of measure is 3). It is possible to select the following of these combinations:

$$\frac{r}{\rho L^2 v_{max}^2},$$

the Froude number

$$Fr=\frac{v_{max}}{\sqrt{gL}},$$

the Reynolds number

$$\mathrm{Re} = \frac{\rho v_{\max} L}{\mu}.$$

Thus, on the basis of the Π-theorem the drag can be represented in the form

$$r = \rho L^2 v_{\max}^2 f(\mathrm{Fr},\ \mathrm{Re}) = \rho S v_{\max}^2 f(\mathrm{Fr},\ \mathrm{Re}), \tag{2.61}$$

where S is a characteristic area, using the wetted surface of the ship Ω without protruding parts (the wetted surface of the bare hull). For geometrically similar hulls we have the expression $\Omega = \omega V_{BH}^{2/3}$, where ω is the relative (dimensionless) wetted surface, V_{BH} is the volume of the bare hull. Considering this expression formula (2.61) can be written as follows:

$$r = \frac{1}{2} \rho v_{\max}^2 \omega \underset{(1)}{V_{\text{гк}}^{2/3}} f(\mathrm{Fr},\ \mathrm{Re}). \tag{2.62}$$

Key: 1. bare hull = BH

The value of $\zeta = f(\mathrm{Fr},\ \mathrm{Re})$ in formula (2.62) is called the drag coefficient.

According to formulas (2.59), (2.60) and (2.62) the mass of the power plant is defined by the function

$$m_{\text{эу}} = q_{\text{эу}} \frac{\rho v_{\max}^2}{2} \frac{\zeta\omega}{\eta} \frac{k^{2/3}}{\rho^{2/3}} D_m^{2/3} \tag{2.63}$$

(for surface ships k=1).

As was demonstrated above, the establishment of the analytical form of the function f(Fr, Re) for surface ships is impossible at the present time. This problem can be approximately solved only for submarines under water when the similarity with respect to the Froude number becomes insignificant.

The determination of the resistance of surface ships by model testing is based on the practical possibilities of representing formula (2.61) in the form

$$r = \zeta_{\text{т}}(\mathrm{Re}) \frac{\rho v_{\max}^2}{2} \Omega + \zeta_0'(l,\ \mathrm{Fr}) \rho g V_{\text{гк}},$$

where $\zeta_T(\mathrm{Re})$ is the frictional drag; $\zeta'_0(\ell,\ \mathrm{Fr})$ is the residual drag coefficient reduced to the displacement; ℓ is the relative length of the ship ($\ell = L/V_{BH}^{1/3}$).

The coefficient ζ_T for the full scale ship and the model is calculated by semi-empirical relations used for flat plates. The coefficient ζ'_0 for the full scale ship is determined experimentally by testing geometrically some of the models in model testing basins with observation of the Froude similarity law (that is, with observation of the conditions ℓ=idem, Fr=idem for the model and the full scale ship) with subsequent classification and formulation as graphs, diagrams or tables. For example, the Dwyer graph defines the value of ζ'_0 as a function of ℓ and the Froude number with respect to displacement $\mathrm{Fr}_D = \mathrm{Fr}\sqrt{\ell}$.

If the value of $\zeta'_0(\ell, \mathrm{Fr})$ is known, then the residual drag coefficient reduced to the wetted surface of the bare hull is defined by the formula

$$\zeta_0(l,\ \mathrm{Fr}) = 2\zeta_0'(l,\ \mathrm{Fr})\frac{1}{l\omega}\mathrm{Fr}^{-2}.$$

Let us consider the problem of using the methods of similarity and dimensionality when determining the basic hydrodynamic characteristics of the screws to which the thrust P, the moment of resistance of the water to rotation of the screw M and the screw efficiency η_p are reduced. In this example we shall demonstrate in more detail the transition from the general relation (2.50) to the relation (2.51) based on using the Π-theorem.

If we fix such geometric parameters of the screws as the number of blades, the disc-area ratio, the shape of the generatrix of the blades, the relative thickness and width of the blades (besides the pitch of the screw), then the values of P, M and η_p will be defined by the following parameters: d -- screw diameter; n -- rpm; H -- screw pitch; v_p -- speed of the water in the propeller disc; ρ -- water density.

Thus, there are three initial functions

$$P = f_{1P}(d,\ n,\ \rho,\ H,\ v_p),$$
$$M = f_{1M}(d,\ n,\ \rho,\ H,\ v_p),$$
$$\eta_p = f_{1\eta}(d,\ n,\ \rho,\ H,\ v_p).$$

Let us consider the first of these relations in detail. Out of the six dimensional variables P, d, n, ρ, H and v_p only three have independent units of measure. Therefore in the given case only three independent dimensionless combinations Π, Π_1 and Π_2 can be compiled.

These combinations are obtained as follows:

Any three having independent units of measure, for example, d, n, and ρ are selected from the parameters d, n, ρ, H and v_p (the result does not change if any other three variables are selected, for example, H, n, ρ or d, v_p, ρ and so on);

The units of the remaining values of P, H and v_p are expressed in the form of power monomials in terms of the units of measure d, n and ρ, that is,

$$[P] = [\rho]^{x_1}[n]^{y_1}[d]^{z_1},$$
$$[H] = [\rho]^{x_2}[n]^{y_2}[d]^{z_2},$$
$$[v_p] = [\rho]^{x_3}[n]^{y_3}[d]^{z_3};$$

Defining the values of x_i, y_i and z_i, i=1,2,3, the dimensionless combinations are compiled (in accordance with the Π-theorem)

$$\Pi = \frac{P}{\rho^{x_1}n^{y_1}d^{z_1}},\quad \Pi_1 = \frac{H}{\rho^{x_2}n^{y_2}d^{z_2}},\quad \Pi_2 = \frac{v_p}{\rho^{x_3}n^{y_3}d^{z_3}}$$

and the initial function is represented in the form

$$P = \rho^{x_1}n^{y_1}d^{z_1}f(\Pi_1,\ \Pi_2).$$

The values of x_i, y_i, z_i, $i=1,2,3$ are easily found by selection. In the general case for determination of them it is possible to compile the corresponding systems of equations. Let us demonstrate the means of compiling such equations in the example of values of x_1, y_1 and z_1.

The relation between the units of measure of the values of P, d, n and ρ has the form $kg\text{-}m/sec^2=(kg/m^3)^{x_1}\ (1/sec)^{y_1}(m)^{z_1}$ or $kg\text{-}m\text{-}sec^{-2}=(kg)^{x_1}(sec)^{-y_1}(m)^{z_1-3x_1}$. In the latter equality, equating the exponents of the units in the lefthand and righthand sides, we obtain the system of equations $x_1=1$, $y_1=2$, $z_1-3x_1=1$, from which we have $x_1=1$, $y_1=2$, $z_1=4$. We find $x_2=0$, $y_2=0$, $z_2=1$, $x_3=0$, $y_3=1$, $z_3=1$ entirely aanalogously.

Consequently, the expression for the thrust can be represented in the form

$$P=\rho n^2 d^4 f_{2P}\left(\frac{H}{d},\ \frac{v_p}{nd}\right).$$

Considering the expressions for the torque and efficiency similarly, we obtain

$$M=\rho n^2 d^5 f_{2M}\left(\frac{H}{d},\ \frac{v_p}{nd}\right),$$

$$\eta_p=f_{2\eta}\left(\frac{H}{d},\ \frac{v_p}{nd}\right).$$

The form of the functions f_{2P}, f_{2M} and $f_{2\eta}$ is established either experimentally or theoretically on the basis of calculating the hydrodynamic characteristics of the screw by eddy theory. The arguments of the functions f_{2P}, f_{2M} and $f_{2\eta}$ are the relative advance of the screw $\lambda_p=v_p/nd$ and the pitch ratio H/d. Inasmuch as we have the expression

$$\eta_p=\frac{Pv_p}{2\pi nM}=\frac{f_{2P}}{f_{2M}}\,\frac{\lambda_p}{2\pi},$$

only the functions f_{2P} and f_{2M} are subject to experimental or theoretical determination. For various λ_p and H/d the values of these functions are the thrust coefficient (K_1) and the torque factor (K_2).

If we fix the pitch ratio as one of the geometric characteristics of the screw, then the thrust coefficient and the torque factor will depend only on the relative advance λ_p; these relations (the curves for the effect of the screws with different geometric characteristics) are depicted graphically on special diagrams used for the design calculations of screws. As has already been mentioned, in Soviet practice the Papmel' diagrams have become the most widespread (see §2.3).

§2.5. Application of Mathematical Statistics when Compiling the Equations of the Analytical Method of Ship Design

As is known, the problem of designing a ship that is entirely original in all of its parts arises only in exceptional cases where the general principles of building a ship are changed. Usually we are talking about the development or alteration of ships that have already been built by adopting individual new solutions.

At the same time, design is a creative process including a number of decisions of an intuitive nature. Therefore the characteristics of the individual structural elements, assemblies, and so on designed according to identical assignments and by the same general principles, but by different designers, can differ somewhat from each other. In general when designing each specific ship a number of individual peculiarities are encountered which cannot in practice be taken into account in the early design stages and which can be considered random factors. For consideration of the above-indicated facts, statistical data processing with respect to several prototypes is used. This processing is done by the methods of correlation and regression analysis permitting the relations to be found between the characteristics of the investigated phenomenon, object or process considering the effect of random factors. In ship design problems most frequently it is necessary to establish the dependence of the masses and volumes of individual engineering subsystems on certain parameters: displacement, principal dimensions, speed, cruising range, sea endurance, and so on.

In the general statement the problem consists in establishing the dependence of the investigated variable y on the variables $x_1, \ldots, x_n$. Inasmuch as y is a random variable, it is proposed that this function can be described by the function

$$E(y/X) = f(X), \tag{2.64}$$

where X is the vector of the variables $x_1, \ldots, x_n$; E(y/X) is the mathematical expectation of the value of y as a function of the vector X.

The determinate function f(X) is called the response function or regression equation of y in X in statistical literature. It can depend on the unknown parameters $\vartheta_1, \ldots, \vartheta_m$, and in the general case the form of f(X) itself can be unknown. Let us designate the vector of the parameters $\vartheta_1, \ldots, \vartheta_m$ by Θ. Three basic cases of a priori information with respect to the function f(X, Θ) exist.

1. The form of the function f(X, Θ) is given. It is necessary to determine or more precisely define the vector of the parameters Θ. The procedure used to find the statistical estimates for unknown parameters Θ is called regression analysis. In §2.4, for example, a formula was found for the specific mass of the pressure hull of a submarine

$$\underset{(1)}{q_{\text{пк}}} = \rho_{\text{м}}\left(a\frac{H}{\sigma_{\text{т}}} + b\right).$$

Key: 1. PH

In this relation the parameters a and b are established by the methods of regression analysis using prototypes. The role of the vector X here is played by the scalar value H/σ_T, and the role of the components of the vector Θ, the parameters a and b.

2. Several possible forms of the function f(X, Θ); $f_1(X, \Theta_1)$, $f_2(X, \Theta_2)$, ..., $f_M(X, \Theta_M)$ are given. The dimensionality of the physical meaning of the vectors Θ_i, i=1, ..., M can be different. It is necessary to determine which of the given functions best corresponds to the statistical data and find the unknown parameters. For example, for determination of the mass with respect to the "Hull" section for surface ships various formulas are proposed [14], in particular the percentage formula

$$m_к = \bar{p}_1 D_m,$$

the cubic modulus formula

$$m_к = \bar{p}_2 \frac{1}{\rho\delta} \frac{H_{пр}}{T} D_m,$$

the formula for the cubic capacity of the hull

$$m_к = \bar{p}_3 \frac{H_{пр}}{T} \frac{\delta_V}{\rho\delta} D_m,$$

the A E. Tsukshverdt formula

$$m_к = \bar{p}_4 \frac{L}{H} D_m,$$

where $\bar{p}_i$, i=1, ..., 4, are coefficients subject to determination by the prototype data.

It is possible to state the problem of discovering the most preferable of the indicated formulas and determining the value of the corresponding coefficient $\bar{p}_i$.

3. The form of the function x(X, Θ) is in general unknown. It is only assumed that this function is approximated quite well by a finite series with respect to some system of given functions. It is necessary to find the best description of the function f(X, Θ) considering the statistical material and the selected system of approximating functions.

In ship design practice, the latter statement of the problem is encountered comparatively rarely inasmuch as it, as a rule, leads to complex analytical expressions for the investigated functions. In addition, this statement of the problem frequently turns out to be simply impossible as a result of restriction of the statistical material and impossibility as a result of this of representation of f(X, Θ) by a series, for example, a polynomial with a sufficiently large number of terms. In general, considering the problem of establishing the form of the function f(X, Θ) in the third information situation, it is necessary to consider that the statistical methods theoretically permit only selection of one of the a priori advanced hypotheses with respect to the desired function and not establishment of the true form of this function.

For regression analysis most frequently linear models are used in which the function f(X, Θ) is assumed to be linear with respect to the parameters Θ and the variables X, that is,

$$f(X, \Theta) = \Theta^* X, \tag{2.65}$$

where * is the sign of transposition.

In addition to the variables $x_1, \ldots, x_n$, the component $x_0 \equiv 1$ is included in the vector X (in this section by vectors we mean the column vectors of the components). The component $x_0 \equiv 1$ corresponds to the free term in the linear representation of the function f(X, Θ) with respect to the variables $x_1, \ldots, x_n$.

However, hereafter, as before, we shall assume that the vectors X and Θ have dimensionality equal to n. This does not limit the generality of the arguments.

The representation of (2.65) is justified first by the fact that any smooth function in a sufficiently small range of variation of the arguments can be approximately represented in linear form, and in a broad interval, approximated by a piecewise-linear function. Secondly, if f(X, Θ) is represented by the polynomial with respect to powers of the arguments $x_1, \ldots, x_n$, then by introducing the fictitious variable $x_0 \equiv 1$ and new variables for different powers and products of the arguments $x_1, \ldots, x_n$, the polynomial representation f(X, Θ) reduces to linear.

Let us assume that in the linear regression model for certain fixed vectors $X_1, \ldots, X_N$ (N>n) values $y_1, \ldots, y_N$ of the investigated variable y are obtained, where y_i, i=1, ..., N, are values of the independent random variables with identical, but known dispersions equal to σ^2. Under this assumption it is proved [41, 54] that the best (justifiable,unbiased and having minimum dispersions) estimates for the parameters can be found by the least squares method from the condition

$$\min_{\Theta} \sum_{i=1}^{N} (y_i - \Theta^* X_i)^2,$$

which leads to the system of linear algebraic equations with respect to Θ:

$$\sum_{i=1}^{N} X_i (y_i - \Theta^* X_i) = \bar{0}, \tag{2.66}$$

where $\bar{0}$ is the zero n-dimensional vector.

The system of equations (2.66) follows from equality of the partial derivatives to zero

$$\frac{\partial}{\partial \vartheta_j} \sum_{i=1}^{N} (y_i - \Theta^* X_i)^2 = 0, \quad j = 1, \ldots, n.$$

The solution of this system of equations (2.66) has the form

$$\hat{\Theta} = \left(\sum_{i=1}^{N} X_i X_i^* \right)^{-1} \sum_{i=1}^{N} y_i X_i, \tag{2.67}$$

where $\hat{\Theta}$ is the estimate of the vector Θ.

If we introduce the matrix of experimental results into the investigation $\mathfrak{X} = \{x_{ij}\}$, $i = 1, \ldots, N$, $j = 1, \ldots, n$, where x_{ij} is the value of the j-th variable in the i-th observation, and the vector Y with the compents $y_1, \ldots, y_N$, then the formula (2.67) can be written in the form

$$\hat{\Theta} = (\mathfrak{X}^* \mathfrak{X})^{-1} \mathfrak{X}^* Y. \tag{2.68}$$

The estimate for the mean square deviation of σ^2 will be defined by the formula

$$\hat{\sigma}^2 = \frac{1}{N-n}\sum_{i=1}^{N}(y_i - \hat{\Theta}^* X_i)^2 = \frac{1}{N-n}(Y^*Y - \hat{\Theta}^*\mathfrak{X}^*Y). \tag{2.69}$$

It is assumed that the columns of the matrix $\mathfrak{X}$, that is, the vectors $\tilde{X}_j^*=(x_{1j}, \ldots, x_{Nj})$, $j=1, \ldots, n$, are linearly independent. This means that the matrix $\mathfrak{X}^*\mathfrak{X} = \sum_{i=1} X_i X_i^*$ is nonsingular, that is, its rank is equal to n.

If the expression (2.67) or (2.68) is substituted in (2.65), then we obtain the estimate $\hat{f}(X, \hat{\Theta})$ for the mean value of the predicted value of y. Inasmuch as the values of $\hat{\Theta}$ and, correspondingly, $\hat{f}$ depend on the random vector of the observations Y, they are themselves random.

The estimates of $\hat{\Theta}$ and $\hat{f}$ found by the least squares method are so-called point estimates, which do not permit determination of the possible deviations of the true function from the estimate $\hat{f}$. For the interval estimate (using confidence intervals) it is necessary to know the provisional distribution function of the variable y for fixed values of the vector X. Usually this function is unknown. However, very frequently from the general argument it is possible to assume that the random fluctuations of the variable y for each fixed X are normally distributed with a dispersion σ^2 which is identical for all x, but unknown. Here the true mathematical expectation y for each X is also unknown to us, and it is estimated by observation results. Under these conditions the confidence interval for estimating $\hat{f}$ can be found using the Student distribution [48]:

$$\hat{f}(X, \hat{\Theta}) \pm t_{q,N-n}\hat{\sigma}(X^*\mathcal{D}^{-1}X)^{1/2}, \tag{2.70}$$

where

$$\mathcal{D}^{-1} = (\mathfrak{X}^*\mathfrak{X})^{-1} = \left(\sum_{i=1}^{N} X_i X_i^*\right)^{-1},$$

$t_{q,\ N-n}$ is the quantile of the Student distribution with N-n degrees of freedom corresponding to the q% confidence probability. The value of $t_{q,\ N-n}$ is found from the condition $S_{N-n}(t_q)-S_{N-n}(-t_q)=q/100$, where $S_{N-n}(t)$ is the Student distribution function with N-n degrees of freedom.

The error estimate for the mean value of $\hat{f}$ at the point N has the form

$$\hat{\sigma}(\hat{f}) = (X^*\mathcal{D}^{-1}X)^{1/2}\hat{\sigma}. \tag{2.71}$$

From the expressions (2.70) and (2.71) we have an important corollary: expansion of the confidence interval going away from the point $\overline{X}=\frac{1}{N}\sum_{i=1}^{N}$ in the space of the variables $x_1, \ldots, x_n$, that is, from the center of grouping of the points, for which the observations were made. This means that it is necessary very carefully to extrapolate the equation found $f(X,\Theta)$ beyond the limits of the experimental points.

The minimum dispersion of the estimate $\hat{f}$ equal to $\hat{\sigma}^2/N$ is reached for $X=\bar{X}$.

The least squares method is also frequently used for the nonlinear function $f(X,\Theta)$ with respect to the parameters Θ and the variables X, finding these parameters from the solution of the system of equations

$$\frac{\partial}{\partial \vartheta_j}\sum_{i=1}^{N}[y_i - f(X_i, \Theta)]^2 = 0, \quad j = 1, \ldots, m.$$

In the given case the least squares method guarantees justifiable, effective estimates only for the normal distribution law of the values of y and their independence. When satisfying this equation the least squares method follows from the method of maximum plausibility [48, 54].

As an example of statistical dependence it is possible to present the formula recommended by L. M. Nogid [35] for determining the mass of a steel hull of ships more than 70 meters long:

$$m_{\text{к}} = \bar{p}\delta^{0,334}L^{1,25}B^{0,75}H^{0,50},$$

where $\bar{p}$ is a dimensional coefficient, δ is the block coefficient.

A similar function can be obtained using the linear regression model if we assume that a formula of the following type is appropriate for determining the mass of the hull

$$m_{\text{к}} = \bar{p}\delta^{\vartheta_1}L^{\vartheta_2}B^{\vartheta_3}H^{\vartheta_4}, \qquad (2.72)$$

where $\bar{p}, \vartheta_1, \ldots, \vartheta_4$ are unknown parameters.

Proceeding to the new variables $f=\log m_k$, $x_1=\log \delta$, $x_2=\log L$, $x_3=\log B$, $x_4=\log H$ and $\alpha=\log \bar{p}$, we obtain the linear function

$$f(x_1, \ldots, x_4) = \alpha + \sum_{j=1}^{4}\vartheta_j x_j.$$

The parameters can be found by the least squares method with respect to the statistical data on prototype ships. Let us also note that in the given case the estimation of the confidence interval for log m_k does not, generally speaking, permit us to obtain a confidence interval for the value of m_k itself.

In order to represent the process of establishing the statistical function of the type (2.72), let us refer the reader, for example, to the book by S. R. Rao [41, p 240].

The statistical method to a known degree levels some of the individual peculiarities of the prototype ships which sometimes it is expedient to consider. In this case the method of joint use of statistical data and a specific prototype can be proposed. The idea of this method [29] consists in the fact that the regression line is drawn through some point X_0, the coordinates of which correspond to the characteristics of the prototype. For the linear regression model the estimate Θ of the regression parameters is found, solving the problem of quadratic programming

$$\min_{\Theta} \sum_{i=1}^{N} (y_i - \Theta^* X_i)^2, \qquad (2.73)$$
$$\Theta^* X_0 = y_0,$$

where y_0 is the value of y corresponding to the vector X_0.

The problem (2.73) can be solved by the method of undefined Lagrange factors [29].

As an illustration let us consider the use of regression analysis for establishment of the dependence of the specific mass (q_{PH}) of the pressure hull of submarines on the parameter H/σ_T. If the strength conditions are defining and not the stability, then the value of q_{PH} can be determined by the approximate formula (2.57). For the fixed characteristics of the PH material (with the exception of the yield point σ_T) the function (2.57) can be represented in the form

$$q_{\text{пк}} = \vartheta_0 + \vartheta_1 \frac{\rho g H}{\sigma_т},$$

where ρ is the density of water; g is the gravitational acceleration; ϑ_0 and ϑ_1 are the coefficients which depend on the specific mass of the material and the structural peculiarities of the PH.

We shall consider that the units of measure of the values of H, σ_T, ρ and g are selected so that the parameter $\rho gH/\sigma_T$ will be dimensionless. Then the values of ϑ_0 and ϑ_1 will have units identical with q_{PH} (t/m^3, kg/m^3). The numerical values of the coefficients ϑ_0 and ϑ_1 will be found by statistical means based on the data on prototype submarines with similar structural designs of the pressure hull.

Let us assume that the following information is available with respect to five prototypes (the numbers are selected arbitrarily):

	1	2	3	4	5
$\rho gH/\sigma_T$	0.004	0.006	0.008	0.010	0.015
q_{PH}, t/m^3	0.08	0.10	0.17	0.25	0.32

Hereafter, for simplification of the notation the value of q_{PH} will be designated when necessary by y, and $\rho gH/\sigma_T$, by x. Generally speaking, the value of $\rho gH/\sigma_T$ should be denoted by x_1, the variable $x_0 \equiv 1$ should be introduced and the expression presented for q_{PH} in the form $y = \vartheta_0 x_0 + \vartheta_1 x_1$. With this notation the vectors X_i, i=1, ..., 5, characterizing the values of x_0 and x_1 from the prototypes, will be two-dimensional. The first components of these vectors are equal to one. The following calculations can be performed by the formulas and functions previously presented in vector form. However, in the investigated simplest case where only one variable x_1 is significant, the calculations can be performed by simpler formulas written in scalar form [48].

From the system of equations of the type (2.66) which in the given case consists of two equations, it is easy to find the estimates for the parameters ϑ_0 and ϑ_1 and : $\hat{\vartheta}_0 = 0{,}002$ т/м^3 & $\hat{\vartheta}_1 = 22{,}4$ т/м^3.

The equation of the approximate regression line will have the form $\hat{y}(x)=0.002+22.4x$ or $\hat{q}_{PH}=22.4\frac{\rho gH}{\sigma_T}+0.002$, where $\hat{q}_{PH}$ is the point statistical estimate for the mathematical expectation of the value of q_{PH} for different values of $\rho gH/\sigma_T$. By formula (2.69) in which it is necessary to set N=5 and n=2, we find the estimate for the mean square deviation of $\hat{q}_{PH}$ from its true mathematical expectation

$$\hat{\sigma}^2 = \frac{1}{3}\sum_{i=1}^{5}(y_i - 22{,}4x_i - 0{,}002)^2 = 8{,}04\cdot 10^{-4}.$$

The confidence intervals for $\hat{q}_{PH}$ (under the assumption of normalness of the distribution law q_{PH} for each fixed $\rho gH/\sigma_T$) are defined by the expression [48]

$$\hat{y}(x) \pm \frac{t_{q,3}}{\sqrt{N}}\hat{\sigma}\sqrt{1+\frac{N^2(x-\bar{x})^2}{N\sum\limits_{i=1}^{N}x_i^2-\left(\sum\limits_{i=1}^{N}x_i\right)^2}},$$

where $\bar{x} = \frac{1}{5}\sum\limits_{i=1}^{5}x_i = 0{,}008\,6$ is the mean value of the parameter $\rho gH/\sigma_T$ for the prototypes used.

In the investigated example with 90% confidence probability the expression for the confidence intervals will assume the form

$$(22{,}4x + 0{,}002) \pm 0{,}029\,8\sqrt{1 + 7{,}10\cdot 10^4(x - 0{,}008\,6)^2}.$$

The approximate regression line and also the confidence intervals for q_{PH} are shown in Figure 2.14 (the lines for the upper and lower bounds of the confidence intervals are depicted by a dotted line). In the figure the expansion of the confidence intervals on going away from the mean value of $\rho gH/\sigma_T$ equal to 0.0086 is quite noticeable. In particular, the confidence intervals very quickly increase during extrapolation of the regression function obtained to values of $\rho gH/\sigma_T$ exceeding the value of the fifth prototype. In addition, it is obvious that the values of q_{PH} for the second and fourth prototypes lie in practice on the boundary of the confidence interval corresponding to a sufficiently high (90%) confidence probability. This serves as the basis for more careful investigation of the indicated prototypes for the presence in them of certain structural differences or peculiarities not permitting consideration of the second and fourth prototypes similar to the rest.

If differences are detected which are of a regular nature and at the same time permit consideration of them in the specifically solved problem of ship design as a whole, the second and fourth prototypes are excluded from investigation, and after using additional statistical material, the regression relations are constructed for q_{PH} as a function of $\rho gH/\sigma_T$ for hulls similar to the second and fourth prototypes, each individually. Here the confidence intervals decrease.

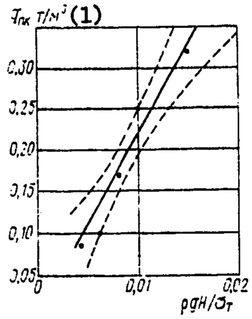

Figure 2.14. Regression dependence of q_{PH} on the parameter $\rho gH/\sigma_T$

Key:

1. q_{PH}, t/m^3

It is necessary, however, to consider that the extraordinary detail and the splitting of the individual factors and calculated relations can have a negative effect from the point of view of accuracy of determining the TTC of the ship. This especially pertains to future design which is of a prediction nature when the investigation and quite rigid fixing of a number of the design solutions are impossible. In this case the use of less-rigid functions can give a more accurate result in the probability sense.

Completing this brief survey of the use of statistical methods for analytical design of the ships, it is necessary to warn the reader against overestimation of their role and significance. First, the statistical methods, just as similarity methods, are only a tool (although a quite effective tool) for using the prototype data; therefore it is necessary to remember all of the deficiencies, and sometimes the inadequacies of this approach when designing ships with new engineering solutions. Secondly, very frequently studies are encountered in which a detailed analysis is made of the accuracy (the confidence intervals are estimated by the Student number) and adequacy (screening out the insignificant factors by the Fisher number) of the statistical relations and mathematical models used, at the same time as the basis for such an analysis itself frequently is absent or insufficiently well founded.

As is known, the basis for solving the above-indicated problems is knowing the provisional distribution laws of the investigated random variable (or system of such variables) for different values of the defining factors. Unfortunately, the solution of the latter problem and problems connected with ship design very frequently is impossible as a result of limited volume of statistical material. Therefore the estimation of the accuracy and adequacy of the relations and models used usually is preceded by the phrase that from the "general arguments" it is possible to assume normalness of the provisional distribution laws of the investigated variables, and this, in turn, gives the right to apply the Student and Fisher criteria. Here not only certain quantitative data pertaining to the above-indicated general arguments, but also sufficiently convincing qualitative arguments with respect to this question fail to be presented. This situation resembles building a house on sand.

The above-presented remarks were made not to discredit the statistical methods, but only for the proper understanding of their role and possibilities.

In conclusion, the author considers it necessary to mention that the methods of solving the problems connected with the technical development of a designed ship investigated in Chapter 2 were also used before the occurrence of analytical design as an independent design step. In the given book these methods, as the most characteristic for AD, are isolated from the general set of ship design methods and procedures. It is necessary to note that in addition to determining displacement and principal dimensions of the ship which was basically discussed in Chapter 2, During technical development of the ship it is necessary to define its other characteristics. In Appendix 3 a number of relations are introduced in the initial design stages. The general procedural approach here remains as before, that is, the methods of similarity and statistics are used in combination with approximate methods and models of special sciences (ship theory, construction mechanics, the theory of acoustic, electromechanical and other physical fields of the ship, and so on).

CHAPTER 3. METHODS OF ESTIMATING THE EFFECTIVENESS OF SHIPS

Combat operations in the sea are characterized by exceptional variety, in particular, with respect to nature and type of operations (control of the groupings of surface ships, operations against shore targets, submarine-chasing operations, and landing operations); with respect to the forces participating in these operations (joint operations of submarines, surface ships and aviation in various combinations) and the combat means used (missiles, torpedoes, mines, bombs, artillery shells and so on).

It does not appear possible to give a sufficiently complete and detailed description of the methods of estimating effectiveness for the entire variety of types and classes of ships and methods of their operations within the scope of this book. Therefore in the given chapter the methods of estimating effectiveness will be investigated as examples only for certain special cases of combat operations. The corresponding mathematical models of estimating the effectiveness are not, generally speaking, models that explicitly contain TTE and TDP, inasmuch as the creation of such models requires detailed specification of the class and type of ships, the combat means, the conditions of solving the problem, and so on. However, the examples presented below, in the opinion of the author, permit us to obtain an idea about the methods used when constructing mathematical models of combat operations in the stage of AD of ships. A description will be presented of the mathematical models of two levels: a simplified model of the combat operations of ships on two sides in the marine theater (§3.3) and the model of combat between single ships on two sides (§3.4). These models give an idea of the simulation of the basic steps of the combat operations: transfer, search and, finally, combat, which is the crowning step and the most complex step of the combat operations.

In §3.5 the mathematical model of combat operations of ships investigating a given part of the sea (for example, in the interests of a new type of reconnaissance) under conditions of counteraction by the enemy is described. Consideration of the counteraction is demonstrated by determining the required fleet of ships.

The actual models of estimation of effectiveness of ships are various interrelated combinations of partial models for different steps of combat operations. In §3.3-3.5, some of the examples of such partial and more general models are discussed.

§3.1. Basic Concepts. General View of the Effectiveness Indexes

Ships are built for carrying out defined missions. These missions are carried out during the process of performing operations[1] by which in the most general case we mean any purposeful acts.

The degree of correspondence of the results of the operations to the stated goals is called the effectiveness [12]. Inasmuch as the course of an operation and, consequently, its results, depend, in particular, on the characteristics of the ship (or ships) performing the operation, the effectiveness of the operation characterizes the effectiveness of the ship. Quantitative measures of the correspondence of the results of an operation to the stated goals are called effectiveness indexes [12].

Hereafter, we shall make use of the above-presented and generally adopted concepts of effectiveness, understanding that we are talking about the degree of achievement of the stated goals with no consideration of the magnitude of the resources expended for this.[2]

The general means of constructing effectiveness indexes consists in the following.

A parameter η is selected which quantitatively characterizes the outcome of an operation. For combat operations, for example, this parameter can be the losses imposed by the enemy or the prevented losses to one's own forces, the time characteristics of the operation, the characteristics indirectly connected with the losses (the number of missiles launched, aircraft that took off, delivered loads, and so on). For transport ships, frequently the so-called carrying capacity is taken as the parameter η [1, 3], which is the product of the amount of transported load, the speed of transporting it and the time during which the ship is directly involved with transporting the load.

When selecting the parameter η it is necessary that its values actually characterize the outcome of the operation from the point of view of the stated goal. Morse and Kimbell present [31] an example where the number of enemy aircraft knocked down was incorrectly selected as the parameter η for estimating the effectiveness of the antiaircraft weapons on the transport convoys, at the same as the purpose of installing the weapons consisted in increasing the invulnerability of the transports. In the former case the antiaircraft weapons produced no effect, and in the latter, turned out to be expedient inasmuch as they complicated precision bombing.

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[1] In this chapter no distinction is made between marine operations and systematic combat operations inasmuch as from the point of view of quantitative methods of estimating the effectiveness of ships this has no theoretical significance.

[2] A. A. Narusbayev proposed [34] that the term "useful effect" be used in the technical-economic studies, and that the military-economic or effectiveness-reserve meaning be included in the concept of effectiveness, designating the relation between the useful effect and the expenditures of resources on obtaining it as the effectiveness of the solution. This concept of effectiveness can be called military-economic effectiveness or effectiveness-reserve quality.

On the basis of the random nature of the process of performance of the operations the parameter η is also random. As is known, the most complete characteristic of a random variable is its distribution function. Let us denote the distribution function of the variable η by $F(y)$, where y are the values assumed by η. The parameter η is a function of the TTE and the number of ships performing the operations (the qualitative and quantitative composition of the active forces). Therefore we shall consider the distribution functions of the type $F'(y/X, \mathcal{N})$, which depend on $\mathcal{N}$ -- the number of active ships, the so-called fleet of ships, and the vector X, that is, the set of TTE of the ships.

If the operations of dissimilar or mixed-type ships are considered, then the distribution functions of the type $F(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k)$ must be taken into account, where $\mathcal{N}_i$ is the number of operating ships of the i-th type, X_i are the TTE of the i-th type ship.

Let us denote by $\mathfrak{F}$ the set of all possible distribution functions $F(y)$. The problem of constructing the effectiveness index reduces to establishing some order of preference in the set $\mathfrak{F}$, that is, the construction in this set of the functional $\text{Э}(F)$ having the property $\text{Э}(F_1) \geqslant \text{Э}(F_2)$ when and only when the distribution function F_1 is preferable to or equivalent to the function F_2.

Generally speaking, the functional $\text{Э}(F)$ can also be constructed from the condition $\text{Э}(F_1) \leqslant \text{Э}(F_2)$ where F_1 is preferable to or equivalent to F_2. Everything depends on the specific meaning of the parameter η. For example, it is desirable to maximize the loss imposed on the enemy, that is, for construction of $\text{Э}(F)$ it is necessary to use the first condition, and the problem solution time must be minimized, and the second condition used in this case.

Hereafter we shall use the notation $F_1 \succsim F_2$ where F_1 is preferable to or equivalent to F_2; $F_1 \succ F_2$, where F_1 is strictly preferable to F_2; $F_1 \sim F_2$ where F_1 is equivalent to F_2.

For the preference ratio for $\mathfrak{F}$ the following properties are assumed to be valid:

1. For any $F_1 \in \mathfrak{F}$ and $F_2 \in \mathfrak{F}$ or $F_1 \succ F_2$, or $F_2 \succ F_1$. If both relations are valid, then $F_1 \sim F_2$.

2. If $F_1 \succsim F_2$ and $F_2 \succsim F_3$, then $F_1 \succsim F_3$. This property is called transitiveness.

3. The equivalence ratio is reflexive, symmetric and transitive, that is, $F \sim F$; if $F_1 \sim F_2$, then $F_2 \sim F_1$; if $F_1 \sim F_2$ and $F_2 \sim F_3$, then $F_1 \sim F_3$.

Assuming that in the set $\mathfrak{F}$ there is a limited functional $\text{Э}(F)$, having the properties: $\text{Э}\left(\sum_{k=1}^{\infty} \lambda_k F_k\right) = \sum_{k=1}^{\infty} \lambda_k \text{Э}(F_k)$ for any series of numbers $\lambda_k \geq 0$, $\sum_{k=1}^{\infty} \lambda_k = 1$ and $\text{Э}(F_1) \geqslant \text{Э}(F_2)$ when and only when $F_1 \succsim F_2$, it is possible to show [6] that $\text{Э}(F)$ has the form

$$\text{Э}(F) = \int_y u(y)\, dF(y). \tag{3.1}$$

In expression (3.1) the function u(y) is defined as a value of the functional $\text{Э}(F)$ for the distribution function $F_{\delta y}$ according to which the probability of the value y of the variable η is equal to one, and the probability of the remaining values, zero, that is, P{η=y}=1, P{η≠y}=0. Thus,

$$u(y) = \text{Э}(F_{\delta y}). \qquad (3.2)$$

The function u(y) sometimes called [64] the utility function, gives the preference ratio in the set of nonrandom outcomes of the value of η. In accordance with the expression (3.1) the functional $\text{Э}(F)$ is the mathematical expectation of the utility function with respect to the probability distribution F(y). For determination of the specific form of $\text{Э}(F)$ the function u(y) and F(y) must be given.

Let us consider the conditions of existence of the integral in the expression (3.1) following from the properties of the Stiltjes integral. For the existence of the integral (3.1), as is known [60], monotonicity and limited nature of the function F(y) and continuity of the limited nature of the function u(y) are sufficient. Monotonicity and limitedness are the basic properties of any distribution function and, consequently, the first part of the above-indicated conditions is always satisfied.

The function u(y) in the general case cannot be continuous. However, in cases encountered in practice it has only a finite number of first type discontinuities. In view of the fact that such functions are approximated as exactly as one might like by continuous functions, the basic sufficient condition of existence of the integral (3.1) is limitedness of u(y). This condition is not necessary, for the integral (3.1) in special cases can exist also for unlimited functions u(y). In many practical problems the limitedness of u(y) is a corollary of the "saturation effect" where achievement of the additional results of an operation becomes inconceivable, for example, on the basis of the physical limitedness of the total volume of the solved problem (insurance of the number of destroyed targets exceeding the total number, does not lead to an increase in effect, and so on).

From the point of view of estimating the effectiveness of ships, it is of interest to consider the distribution functions of the type $F(y/X, \mathcal{N})$ or $F(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k)$. Here, in accordance with expression (3.1) we have

$$\text{Э}(F) = \text{Э}(X, \mathcal{N}) = \int_y u(y)\,dF(y/X, \mathcal{N}), \qquad (3.3)$$

$$\text{Э}(F) = \text{Э}(X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k) = \int_y u(y)\,dF(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k). \qquad (3.4)$$

Hence, it is obvious that the functional $\text{Э}(F)$ defines the corresponding preference ratio in the set of possible values of the TTE of the ships (vectors X) and their fleet numbers (N) simultaneously with the preference ratio in the set $\mathfrak{G}$.

If we return to the general definition of the concept of the effectiveness index given at the beginning of the section, then it is possible to consider the function u(y) as a quantitative characteristic of the goals set for the operation, and the functions $F(y/X, \mathcal{N})$ or $F(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k)$, as the quantitative characteristics of the achieved results.

Let us note that the general expressions (3.3) and (3.4) are also valid when the parameter η defining the outcome of the operation is not a random variable, but deterministic. In this case in expressions (3.3) and (3.4) it is necessary to use the distribution function for the deterministic variable considered as a special form of random variable. When investigating the uniform detail of forces the distribution function will have the form

$$F(y/X,\ \mathcal{N})=\begin{cases}0 \text{ for } y < y^*(X,\ \mathcal{N}),\\ 1 \text{ for } y \geqslant y^*(X,\ \mathcal{N}),\end{cases}$$

where $y^*(X,\ \mathcal{N})$ is the deterministic value of η under the condition that $\mathcal{N}$ ships with TTE characterized by the vector X are in operation.

In accordance with the expression (3.3), the effectiveness index will be defined by the expression $\text{Э}(X,\mathcal{N}) = u\,(y^*(X,\ \mathcal{N}))$.

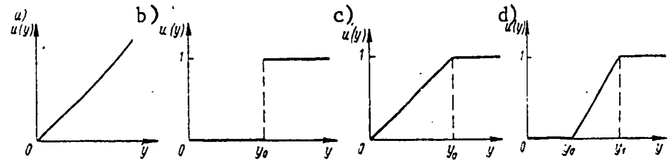

Figure 3.1. Some typical utility functions

Let us consider some special cases of effectiveness indexes following from the general form. Here let us limit ourselves only to a uniform fleet, since for a dissimilar fleet the corresponding expressions are obtained analogously.

1. Let u(y)=y for all y, that is, u(y) be a linear function (Figure 3.1, a) for which the effect of the operation increases uniformly as the achieved value of η increases. From expression (3.3) it follows that

$$\text{Э}(X,\ \mathcal{N})=\int\limits_{y} y\,dF(y/X,\ \mathcal{N})=E_\eta(X,\ \mathcal{N}), \tag{3.5}$$

where $E_\eta(X,\ \mathcal{N})$ is the mathematical expectation of the value of η with the distribution function $F(y/X,\ \mathcal{N})$.

Thus, the effectiveness index is the mathematical expectation of the random variable η -- the parameter determining the outcome of the operation.

Inasmuch as for $y\to\infty$ the linear function u(y) is not valid, for this utility function the effectiveness index in the form of (3.3) does not always exist. For example, for the random variables distributed by Pascal's law there is no mathematical expectation.

2. Let u(y) have the form of a Heviside unit function or step function (Figure 3.1, b). Then

$$u(y)=\begin{cases}0 & \text{for } y<y_0,\\ 1 & \text{for } y\geqslant y_0,\end{cases}$$

where y_0 is a given value.

For this function u(y) for solution of the stated problem it is necessary to insure that the value of η no less than y_0 is achieved (exceeding this value does not give an additional effect). Substituting the expression for u(y) in (3.3), we obtain

$$\text{Э}(X, \mathcal{N})=\int\limits_{y\geqslant y_0} dF(y/X, \mathcal{N})=P\{\eta\geqslant y_0/X, N\}. \tag{3.6}$$

Consequently, here the effectiveness index will be the probability of the event $\eta\geqslant y_0$, and the value of $P\{\eta\geqslant y_0\}$ is the probability of solving the stated problem.

Let us note that the two indicated cases were investigated by A. N. Kolmogorov [23], and at the present time they are the most frequently used in practical research.

3. Let the function u(y) have the form shown in Figure 3.1, c. The analytical expression for this function called the linear function with saturation will be

$$u(y)=\begin{cases}0 & \text{for } y<0,\\ y/y_0 & \text{for } 0\leqslant y<y_0,\\ 1 & \text{for } y\geqslant y_0,\end{cases}$$

where y_0 is the value of y for which "saturation" occurs.

Substituting this expression in (3.3), we obtain

$$\begin{aligned}\text{Э}(X, \mathcal{N})&=\frac{1}{y_0}\int\limits_0^{y_0} y\,dF(y/X, \mathcal{N})+\int\limits_{y_0}^{\infty} dF(y/X, \mathcal{N})=\\&=\frac{\overset{(1)}{E_\eta^{\text{усл}}}(X, \mathcal{N})}{y_0}P\{\eta<y_0/X, \mathcal{N}\}+P\{\eta\geqslant y_0/X, \mathcal{N}\},\end{aligned} \tag{3.7}$$

Key: 1. provisional

where $E_\eta^{\text{усл}}(X, \mathcal{N})=\frac{1}{F(y_0/X, \mathcal{N})}\int\limits^{y_0} y\,dF(y/X, \mathcal{N})$ is the provisional mathematical expectation of the value of ${}^0\eta$ under the condition $\eta<y_0$;

$$P\{\eta<y_0/X, \mathcal{N}\}=F(y_0/X, \mathcal{N}),$$
$$P\{\eta\geqslant y_0/X, \mathcal{N}\}=1-F(y_0/X, \mathcal{N}).$$

Thus, in the investigated case the effectiveness index cannot be interpreted so simply as occurred for the linear and step utility functions.

4. The function u(y) shown in Figure 3.1, d will be called a linear function with saturation and delay of growth. For this function the presence of the minimum required level y_0 to which the effect of the operation is equal to zero, and the level y_1 after which "saturation" occurs, is characteristic. On variation of y from y_0 to y_1 the effect increases according to a linear law from zero to one.

The analytical expression for u(y) in this case has the form

$$u(y)=\begin{cases} 0 & \text{for } y<y_0, \\ \frac{y-y_0}{y_1-y_0} & \text{for } y_0\leq y<y_1, \\ 1 & \text{for } y\geq y_1. \end{cases}$$

After substitution of this expression in (3.3) we find

$$Э(X, \mathcal{N})=\frac{E_{\eta}^{\text{усл}}(X, \mathcal{N})-y_0}{y_1-y_0}P\{y_0\leq\eta<y_1/X, \mathcal{N}\}+ \\ +P\{\eta\geq y_1/X, \mathcal{N}\}, \qquad (3.8)$$

where $E_{\eta}^{\text{усл}}(X, \mathcal{N})=\frac{1}{F(y_1/X, \mathcal{N})-F(y_0/X, \mathcal{N})}\int\limits_{y_0}^{y_1} y\,dF(y/X, \mathcal{N})$ is the provisional mathematical expectation of the value of η under the condition $y_0\leq\eta<y_1$;

$$P\{y_0\leq\eta<y_1/X, \mathcal{N}\}=F(y_1/X, \mathcal{N})-F(y_0/X, \mathcal{N}), \\ P\{\eta\geq y_1/X, \mathcal{N}\}=1-F(y_1/X, \mathcal{N}).$$

The investigated special cases of the form of the utility functions and the corresponding effectiveness indexes do not exist their possible variety, but they represent the most typical ones. In particular, the use of the step utility functions is characteristic when solving problems of combat with nuclear missile carriers, and the use of linear functions, for systematic operations against shipping and shore facilities, finally, linear functions with saturation (including with growth delay), for operations against convoys. Thus, in an operation of stopping a nuclear strike by a carrier striking group (CSG) the problem will be solved only on destruction of the striking aircraft carrier. Here the parameter y is the number of destroyed aircraft carriers with step utility function

$$u(y)\begin{cases} 0 & \text{for } y<1, \\ 1 & \text{for } y\geq 1. \end{cases}$$

The effectiveness index will be the probability of destruction of an aircraft carrier from the CSG, respectively.

There are certain types of combat operations for which the above-indicated sharp boundary characterizing the condition of solution of the stated problem, does not exist, or establishment of it is difficult. This situation occurs, for example, for systematic combat operations of submarines with respect to the destruction of the marine communications of the enemy. Here it is possible to take the number of destroyed enemy ships as the parameter y. If in this case the total number of ships is quite large, then the utility function usually can be considered linear. The effectiveness index in this case will be the mathematical expectation of the number of destroyed ships.

When investigating the operations with respect to destroying convoys which do not have especially important strategic significance, the utility function frequently has linear form with saturation. In the given case the value of y_1 corresponds to the number of destroyed ships for which the convoy ceases to exist as an integral combat unit. Here a delay in the growth of the utility function characterizing the minimum loss beginning with which the effect of the strike on the convoy is manifested, can be observed.

Inasmuch as the effectiveness index is a decreasing function of the fleet number $\mathcal{N}$ during operations of the uniform fleet as an indirect index of effectiveness it is possible to use the fleet solving the problem with given effectiveness. The size of this fleet, which is a function of the vector X, that is, the vector of the TTE of the ships, can be found from the equation $\text{Э}(X, \mathcal{N}) = \text{Э}_0$, where Э_0 is the given value of the effectiveness index.

It is also obvious that the fleet number $\mathcal{N}(X)$ depends on the value of Э_0, that is, we have the function $\mathcal{N}(X, \text{Э}_0)$.

General requirements exist which must be satisfied by the effectiveness indexes in addition to the requirements of being a quantitative measure of the correspondence of the operation results to the stated goals. The basic ones of these requirements are the following [12, 19]: criticalness with respect to the investigated characteristics and parameters of the active forces and means and also the methods of using them; calculatability and, insofar as possible, simplicity and descriptiveness; the completeness of consideration of the factors significantly influencing the outcome of the operation.

As applied to the problems of estimating the effectiveness of ships in the AD stage, all three of the above-indicated requirements are highly significant. Here special attention must be given to the criticalness of the effectiveness indexes with respect to the TTE and the TDP of the ship, for only in this case is subsequent optimization of these characteristics possible.

§3.2. Calculation of Effectiveness Indexes

In accordance with the general expressions (3.3) and (3.4), the specific form of the effectiveness index is determined by giving the utility function u(y) and the distribution functions $F(y/X, \mathcal{N})$ or $F(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k)$. The specific form of the utility function that quantitatively characterizes the goals set for the operation can theoretically be obtained, considering the operation of hierarchical level. However, this problem is very complicated. In practice the form of the utility function is given from the general arguments or it is determined by the method of expert estimates. In the problems connected with optimization frequently solutions are found which correspond to several given utility functions. If the optimal solutions for the different utility functions differ from each other, the method of generating the solutions are used which take into account several effectiveness indexes and under indeterminacy conditions (see Chapter 5).

The basic and most labor-consuming problem when calculating the effectiveness indexes is the development and execution of the algorithm permitting calculation of the parameters of the distribution functions $F_k(y/X, \mathcal{N})$ or $F(y/X_1, \ldots, X_k, \mathcal{N}_1, \ldots, \mathcal{N}_k)$, , and in the general case permitting these functions themselves to be found. This problem is solved by developing a mathematical model of the corresponding operation and functioning of the ships in this operation. The development of the mathematical model is preceded by a meaningful description of the conditions of performing the operation, the active forces, the methods of their action, and so on. When estimating the combat effectiveness for ships this description is called the operative-tactical model, and for civilian ships, the use model.

When developing the operative-tactical model, the operation is usually divided into a number of phases, for example, the transition to the combat operations zone, finding the target and destroying them in the combat operations zone, transfer from the combat operations zone to the base after the sea endurance has expired or the combat supplies on hand have been exhausted, and location of the ship in base preparing for the next voyage. The operative-tactical model usually is represented graphically on maps and diagrams, for which special notation is used for the active forces and means and also nature of their operations (search, combat engagement, and so on).

For AD, that is, for the early stages of substantiating the TTE of ships, the use of the so-called typical operative-tactical models which are constructed on the basis of generalizing the various versions of the conditions of performing the operations is characteristic. (A number of secondary factors are not considered.)

The most universal method of mathematical simulation of an operation to calculate the effectiveness indexes is computer-aided statistical simulation [11]. Here random events and processes describing the course of the operation are realized. However, this approach usually requires significant time, especially when it is necessary to investigate a large number of versions of active forces and conditions of performing operations. Therefore when investigating the effectiveness of the AD stage it is necessary to make a number of assumptions and simplifications permitting description of the process of performing the operations by analytical models. In this case the statistical simulation can be used in a limited volume to simulate special situations.

In addition to the subdivision of the mathematical models of evaluation of effectiveness from the computational point of view into statistical, analytical and combined, they are distinguished with respect to degree of completeness of consideration of the random factors. Three classes of models are isolated by this attribute: regular, quasiregular and random (stochastic).

In purely regular models the outcomes of all events and time characteristics for all the processes are considered strictly deterministic. For example in a regular model of the functioning of a transport ship, the latter arrives in ports and leaves them at strictly fixed points in time, it takes a strictly defined amount of cargo on board, although some ports do not have this amount of cargo; the duration of the trip can fluctuate randomly depending on the external and internal causes with respect to the ship, and so on. The regular models in pure form are used very rarely as a result of inadequacy with respect to actual conditions, and in general, the processes of the functioning of complex objects such as ships.

In the random (stochastic) models, in contrast to regular models, all of the characteristics of random events and processes accompanying the functioning of the ship during the operation are taken into account. Theoretically such models are the most adequate. In the simplest versions the stochastic model can be constructed on the basis of a system of random events, but, as a rule, it is necessary to consider random time processes characterizing the functioning of the ship. Here the theory of Markov processes is widely used as an analytical apparatus [5, 55], the basic property of which is dependence of the behavior in the future only on the current (present) state of the process and independence of the future behavior with respect to history. Although these conditions frequently are not strictly satisfied, Markov models in many cases are acceptable for estimating the effectiveness of ships in their design stage.

The purely random models unfortunately, as a rule, turn out to be extraordinarily awkward for the description of the functioning of complex objects (as a result of the large number of possible versions of outcomes of random events and the occurrence of the corresponding random processes). Therefore quasiregular models are used where it is assumed that in each step of the process it is not the random outcome, but the mean expected outcome or result that is realized. The quasiregular models are less adequate than the random models, but they are less awkward.

Let, for example, a series of combat engagements (duels) with the enemy be considered where in each engagement the ship expends a random amount of the combat supplies on hand. If the condition of the ship is characterized by the number of duels it has been engaged in and the quantity of combat supplies left on hand, then in the random model it is necessary to consider all possible states of the ship with respect to the indicated parameters. In the quasiregular model it is considered that in each duel, a deterministic quantity of combat supplies on hand is expended equal to the expected mean expenditure of these supplies in one duel instead of a random quantity. Here, the number of possible states of the ship is reduced significantly.

Let us consider the example of constructing the mathematical model of estimating effectiveness on the basis of a system of random events.

Let a ship be designed for the performance of some single mission which is performed with the possibility $p_1(X)$ which depends on its TTE (the vector X). Let us assume that the ships of uniform composition act independently of each other, and the parameter η defining the outcome of their actions is the random number of ships carrying out the mission.

As follows from probability theory, for the assumptions made above, the value of η is distributed with respect to a binomial law; therefore, the distribution function $F(y/X, \mathcal{N})$ in the given case will have the form

$$F(y/X,\mathcal{N}) = \begin{cases} 0 & \text{for } y \leq 0, \\ \sum\limits_{k=0}^{[y]} C_{\mathcal{N}}^{k} p_1^k(x)[1-p_1(X)]^{\mathcal{N}-k} & \text{for } 0<y<\mathcal{N}, \\ 1 & \text{for } y > \mathcal{N}, \end{cases}$$

where $C_{\mathcal{N}}^{k}$ is the number of combinations of $\mathcal{N}$ with respect to k; [y] is the integral part of the number y; $\mathcal{N}$ is the number of operating ships (the fleet).

With a sufficiently large $\mathcal{N}$ the distribution η approaches the normal distribution, that is, the following approximate equality occurs

$$F(y/X,\mathcal{N}) \approx \frac{1}{\sqrt{2\pi}\sigma(X,\mathcal{N})} \int\limits_{-\infty}^{y} \exp\left\{-\frac{[z-M(X,\mathcal{N})]^2}{2\sigma^2(X,\mathcal{N})}\right\} dz, \tag{3.9}$$

where

$$M(X,\mathcal{N}) = p_1(X)\mathcal{N}; \quad \sigma^2(X,\mathcal{N}) = p_1(X)[1-p_1(X)]\mathcal{N}.$$

For large $\mathcal{N}$ the expression (3.9) defines the distribution function $F(y/X,\mathcal{N})$ entering into the general expression (3.3) for the effectiveness index. After giving the utility function u(y) it is possible to find a specific expression for the effectiveness index. For example, if u(y) is linear, then $\text{Э}(X,\mathcal{N}) = p_1(X)\mathcal{N}$. In the case of the step function u(y) for $y_0 < \mathcal{N}$

$$\text{Э}(X,\mathcal{N}) = \sum_{k=y_0}^{\mathcal{N}} C_{\mathcal{N}}^{k} p_1^k(X)[1-p_1(X)]^{\mathcal{N}-k}.$$

When using the asymptotic representation of (3.9) we correspondingly obtain (for the step utility function)

$$\text{Э}(X,\mathcal{N}) = 1 - \frac{1}{\sqrt{2\pi}\sigma(X,\mathcal{N})} \int\limits_{-\infty}^{y_0} \exp\left\{-\frac{[z-M(X,\mathcal{N})]^2}{2\sigma^2(X,\mathcal{N})}\right\} dz.$$

For final expansion of the above-presented expressions it is necessary to have the specific function $p_1(X)$, which can be done when specifying the essence of the mission carried out by the ship. Let us assume that the problem solved by each ship consists in detection of a stationary target in a given time T in a vicinity of the sea of area Ω. If we assume that the search processes Poisson [19], the probability $p_1(X)$ of target detection by each ship will be defined by the formula $p_1(X)=1-e^{-\gamma T}$ where $\gamma=2dv/\Omega$ is the search intensity; v is the speed of the ship during search; d is the range of deteccion of the target by the ship. For detection means in which d does not denote v, it is expedient to take the search speed equal to the maximum possible speed of the ship [if there are no other arguments, for example, economic arguments, except the desired to maximize $p_1(X)$]. If d depends on v, as occurs for sonar equipment, then the speed of the ship during the search must be selected from the conditions $\max\limits_{v} p_1(X)$ or $\max\limits_{v}[d(v)v]$, where d(v) is the dependence of d on v.

Thus, the probability $p_1(X)$ depends on the maximum speed of the ship (this is a continuously variable component of the vector X. Here it is assumed that for each search means there is a defined function d(v) which can be considered as a discretely variable component of the vector X.

In the most general case where the wave action of the sea can have an influence on the speed of the ship during search, the probability $p_1(X)$ will depend on the displacement and the principal dimensions of the ship (these are the components of the vector Y). The time T and the search area Ω in the given case are external tactical parameters.

In the above-presented example, the number of ships operating directly in the combat zone was taken as $\mathcal{N}$. This fleet is called the polygon fleet ($\mathcal{N}_{\text{п}}$).

For systematic combat operations, the full fleet which takes into account the necessity for part of the ships to be in the bases (for repair, preventive technical maintenance, taking on supplies) and also en route to the combat zone and back, has important significance. For consideration of the above-indicated factors, two coefficients are introduced.

The operative use coefficient (KOI) -- the ratio of the time the ship spends in the combat zone to the total time at sea and at the base.

If R is the distance of the combat zone from the base, A is the sea endurance of the ship (the time at sea), v_{trans} is the speed for the ship in transit to the combat zone and back, $T_{M\pi}$ is the time between voyages, that is, the time the ship spends in base between successive voyages, then the operative use coefficient k_{ou} is defined by the expression

$$\underset{(1)}{k_{\text{ои}}} = \begin{cases} \dfrac{A}{A+T_{\text{мп}}}\left(1-\dfrac{2R}{\underset{(2)}{v_{\text{пер}}A}}\right) & \text{for } \dfrac{2R}{v_{\text{пер}}A} < 1, \\ 0 & \text{for } \dfrac{2R}{v_{\text{пер}}A} > 1. \end{cases} \qquad (3.10)$$

Key: 1. ou
2. trans

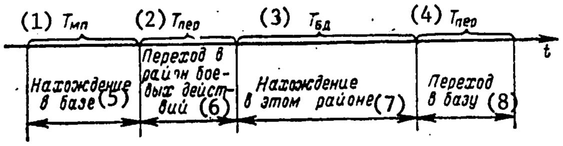

Figure 3.2. Time graph of the use of a ship between successive voyages

Key:
1. between voyages; 2. in transit to the combat zone and back;
3. time in the combat zone; 4. transit time; 5. in base; 6. transit to the combat zone; 7. in the combat zone; 8. transit to base

Formula (3.10) for $2R/v_{trans}A<1$ follows from the transformation

$$\underset{(1)}{k_{\text{ои}}} = \frac{T_{\text{бд}}}{\underset{(3)}{T_{\text{бд}}} + 2\underset{(2)}{T_{\text{пер}}} + \underset{(4)}{T_{\text{мп}}}} = \frac{A - 2T_{\text{пер}}}{A + T_{\text{мп}}} = \frac{A}{A + T_{\text{мп}}}\left(1 - \frac{2T_{\text{пер}}}{A}\right) = \frac{A}{A + T_{\text{мп}}}\left(1 - \frac{2R}{v_{\text{пер}}A}\right),$$

Key: 1. ou; 2. trans; 3. combat; 4. between

where T_{trans}, T_{combat} are the times spend in transit to the combat operations zone (or back) and the time spent in the combat zone.

The time graph (Figure 3.2) characterizes the use cycle of the ship between successive voyages considering one intervoyage period.

The ratio $A/(A+T_{between})$, that is, the ratio of the time the ship spends at sea to the total time at sea and at the base is called the operative stress coefficient (OSC).[1] From formula (3.10) we have the relation between the OUC [operative use coefficient] and the OSC [operative stress coefficient]:

$$\underset{(1)}{k_{\text{ои}}} = \begin{cases} k_{\text{он}}\left(1 - \dfrac{2R}{\underset{(2)}{v_{\text{пер}}}A}\right) & \text{for } \dfrac{2R}{v_{\text{пер}}A} < 1, \\ 0 & \text{for } \dfrac{2R}{v_{\text{пер}}A} \geqslant 1. \end{cases}$$

Key: 1. ou; 2. trans

It is obvious that the values of k_{ou} and k_{os} can vary from zero to one.

For the known values of k_{ou} and the polygon fleet $\mathcal{N}_{\text{п}}$ the total fleet $\mathcal{N}$ is defined by the formula

$$\mathcal{N} = \frac{\mathcal{N}_{\text{п}}}{\underset{(1)}{k_{\text{ои}}}}. \qquad (3.11)$$

Key: 1. ou

The values of OUC and OSC can be given a probability interpretation on the basis of the mathematical description of the process of functioning of the ship by a Markov random process.

Let us assume that during the process of functioning of a ship it can be in the following states (Figure 3.3): 1 -- in the combat operations zone; 2 -- in base between voyages; 3 -- in transit from the base to the combat zone; 4 -- in transit from the combat zone to the base. Let us also assume that the transition

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[1] The time the ship spends at sea is made up of the time it is in the combat zone and the time spent in transit to the combat zone and back to base.

of the ship from state to state depends on a number of random factors. The time the ship is in each of the states is subject to exponential laws $F_1(t)=1-e^{-\alpha_i t}$, $i=1,2,3,4$, where $F_i(t)$ are the corresponding distribution functions of the time the ship spends in each of the states; $\alpha_1=1/T_{combat}$, $\alpha_2=1/T_{between}$, $\alpha_3=\alpha_4=1/T_{trans}$.

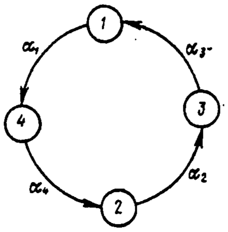

Figure 3.3. Graph of the process of the functioning of a ship during systematic operations considering the periods between voyages

In Markov process theory the values of α_i are called the intensities of transitions of the system from state to state. For visual representation of the state of the system, the possible transitions and intensities of the transitions are depicted using an oriented graph [36], which in the investigated case has the form shown in Figure 3.3.

From the theory of Markov random processes it follows that for a sufficiently large operating time of the system required by the graph in Figure 3.3, the probabilities that the system will be in each of the states assume steady-state values which do not depend on the initial state of the system, that is, its state at the beginning of operations. These probabilities P_i, $i=1, 2, 3, 4$, satisfy the system of linear algebraic equations (it is considered that in the investigated case $\alpha_3=\alpha_4$):

$$\begin{gathered}
-\alpha_1 P_1+\alpha_3 P_3=0,\\
-\alpha_2 P_2+\alpha_3 P_4=0,\\
\alpha_2 P_2-\alpha_3 P_3=0,\\
\alpha_1 P_1-\alpha_3 P_4=0,\\
P_1+P_2+P_3+P_4=1.
\end{gathered}$$

Solving this system of equations, we find

$$P_1=\frac{1}{1+2\frac{\alpha_1}{\alpha_3}+\frac{\alpha_1}{\alpha_2}}=\frac{T_{\text{бд}}}{T_{\text{бд}}+2T_{\text{пер}}+T_{\text{мп}}}=k_{\text{он}},$$

$$P_1+P_3+P_4=\frac{1+2\frac{\alpha_1}{\alpha_3}}{1+2\frac{\alpha_1}{\alpha_3}+\frac{\alpha_1}{\alpha_2}}=\frac{T_{\text{бд}}+2T_{\text{пер}}}{T_{\text{бд}}+2T_{\text{пер}}+T_{\text{мп}}}=k_{\text{он}}.$$

Thus, the OUC and the OSC can be interpreted as the probabilities that the ship will be in the corresponding states for the steady-state operating mode.

Formula (3.11) also obtains the corresponding interpretation. It defines the mean value of the total fleet as a function of the fixed polygon fleet. Here the individual ships operate independently of each other in accordance with the graph in Figure 3.3, and each of them is in the combat zone with a probability of k_{ou} at an arbitrarily selected point in time (the time of solution of the problem).

It must be noted that the above-presented system of combat operations or use of the ship (Figure 3.2), although it is the most typical, is not the only one. For example, a harbor fire-fighter or rescue tug solves the stated problem by being in base ready to go out on a mission. In this case the values of k_{ou} and k_{os} coincide, inasmuch as $R\approx0$, and it is necessary to take the time during which the ship is ready to solve the problem as A.[1]

The characteristics not only of the ship (sea endurance, reliability, and so on), but also the base service and support means influence the value of OSC. The value of the latter factor is so important that in a number of cases analytical design is performed not for an isolated ship, but a system of ships -- a set of base support means.[2]

The engineering characteristics of the ships and the base support system determine the maximum possible value of OSC which sometimes is called the technical use coefficient of the ship (k_{tu}).

In the general expression for the effectiveness index $\text{Э}(X, \mathcal{N})$ the total fleet must be taken as the argument $\mathcal{N}$. Therefore the model used to define the distribution function $F(y/X, \mathcal{N})$ must take into account the functioning of the base service system. Let us consider an illustrative example of a model of estimating the effectiveness of a ship considering the effectiveness of the functioning of the base service system. For simplicity of notation the argument X in th corresponding distribution functions and their parameters will be omitted.

Let us assume that the following basic assumptions are valid:

a) During peace time ships are systematically at sea to carry out a defined combat mission with the beginning of war. If there are ζ ships in the combat use

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[1] Generally, $k_{ou}=k_{os}$ for all ships which solve their problem directly on going to sea.

[2] In addition to the engineering characteristics of the ship and the base support system, the value of OSC also depends on the arguments on the operative and economic levels. For example, considering the arguments of economy of reserves (in the sense of reliability) during defined periods of peace time the ships can be used at less than maximum possible stress.

zones, then the random parameter η defining the outcome of the operation can be represented in the form of the sum

$$\eta = \sum_{i=1}^{\zeta} \eta_i,$$

where η_i, $i=1, \ldots, \zeta$ are independent random variables defining the results of the operations of each of the ships;

b) The value of ζ is a discrete random variable which assumes $\mathcal{N}$ values $z = 1, \ldots, \mathcal{N}$. The probabilities corresponding to these values will be denoted by $p_z=P\{\zeta=z\}$, $z=1, \ldots, \mathcal{N}$. The probabilities p_z characterize the distribution of the number of ships which at the beginning of the war turn out to be in the appropriate combat zones. Consequently, the probabilities p_z take into account the effectiveness of the base support system;

c) All the values of η_i, $i=1, \ldots, \zeta$ have the same distribution function $F_1(y/z)$ which can depend on z, that is, the value assumed by the variable ζ.

The distribution function of interest to us $F(y/\mathcal{N})$ of the value of η is defined by the general expression [55]

$$F(y/\mathcal{N}) = \sum_{z=1}^{\mathcal{N}} p_z F^*(y/z), \tag{3.12}$$

where

$$F^*(y/z) = \underbrace{F_1(y/z) * \cdots * F_1(y/z)}_{z \text{ times}};$$

* is the operation of convolution of distributions.

It is relatively simple to find the expressions for the mathematical expectation and the dispersion η as a function of the parameters of the distribution functions η_i and ζ:

$$E\eta = \sum_{z=1}^{\mathcal{N}} z p_z m_z,$$

$$\mathcal{D}\eta = \sum_{z=1}^{\mathcal{N}} z^2 p_z m_z^2 + \sum_{z=1}^{\mathcal{N}} z p_z \sigma_z^2 - \left(\sum_{z=1}^{\mathcal{N}} z p_z m_z\right)^2, \tag{3.13}$$

where $E\eta$, $\mathcal{D}\eta$ are the mathematical expectation and dispersion of the value of η; m_z, σ_z^2 are the mathematical expectation and dispersion of the value of η_i with distribution functions $F_1(y/z)$.

In the special case where m_z and σ_z^2 do not depend on z, that is, the effectiveness of the operations of each ship does not depend on the total number of ships in the combat zone, the formulas (3.13) are simplified:

$$\begin{aligned} E\eta &= mE\zeta, \\ \mathcal{D}\eta &= m^2\mathcal{D}\zeta + \sigma^2 E\zeta, \end{aligned} \tag{3.14}$$

where $E\zeta$, $\mathcal{D}\zeta$ are the mathematical expectation and dispersion of the value of ζ:

$$E\zeta = \sum_{z=1}^{\mathcal{N}} zp_z,$$

$$\mathcal{D}\zeta = \sum_{z=1}^{\mathcal{N}} z^2 p_z - \left(\sum_{z=1}^{\mathcal{N}} zp_z\right)^2;$$

m, σ^2 are the mathematical expectation and the dispersion of the values of η_i, the distribution function of which $F_1(y)$ does not depend on z in the given case.

For sufficiently large $\mathcal{N}$ the distribution function $F(y/\mathcal{N})$ will be approximately normal:

$$F(y/\mathcal{N}) = \frac{1}{\sqrt{2\pi\mathcal{D}\eta}} \int_{-\infty}^{y} e^{-\frac{(\xi - E\eta)^2}{2\mathcal{D}\eta}} d\xi.$$

Let us assume that each ship in the system made up of the ship and the base surface means functions independently of the others, and at any point in time it will be in the combat zone with the probability k_{ou}. In this case $E\zeta = k_{\text{он}}\mathcal{N}$, $\mathcal{D}\zeta = k_{\text{он}} \times \times (1 - k_{\text{он}}) \mathcal{N}$ and, consequently,

$$E\eta = k_{\text{он}} m \mathcal{N},$$
$$\mathcal{D}\eta = k_{\text{он}} (1 - k_{\text{он}}) m^2 \mathcal{N} + k_{\text{он}} \sigma^2 \mathcal{N}. \qquad (3.15)$$

If in the combat zone the ship carries out a mission with probability p, and the number of ships carrying out the mission is taken as the parameter η, then the formulas (3.15) assume the form

$$E\eta = k_{\text{он}} p \mathcal{N},$$
$$\mathcal{D}\eta = k_{\text{он}} p (1 - k_{\text{он}} p) \mathcal{N}, \qquad (3.16)$$

for in the given case m=p and σ^2=p(1-p). Here the exact distribution of the parameter η will be binomial:

$$P\{\eta = y/\mathcal{N}\} = C_{\mathcal{N}}^{y} p_1^{y} (1 - p_1)^{\mathcal{N} - y}, \quad y = 0, 1, \ldots, \mathcal{N},$$

where $$p_1 = k_{\text{он}} p, \quad C_{\mathcal{N}}^{y} = \frac{\mathcal{N}!}{y!(\mathcal{N} - y)!}.$$

Now let each ship in the combat zone perform some mission with probability p, the volume of which is characterized by a deterministic value Q_1, and with a probability 1-p the volume of the solved problem is zero. In this case (Fig 3.4)

$$F_1(y/z) = \begin{cases} 1 - p & \text{for } y < Q_1, \\ 1 & \text{for } y \geqslant Q_1. \end{cases}$$

Understanding the parameter η as the total volume of the problems solved by $\mathcal{N}$ ships, for Eη and $\mathcal{D}\eta$ we obtain

$$E\eta = k_{\text{он}} p Q_1 \mathcal{N}, \qquad (3.17)$$
$$\mathcal{D}\eta = k_{\text{он}} p (1 - k_{\text{он}} p) Q_1^2 \mathcal{N}.$$

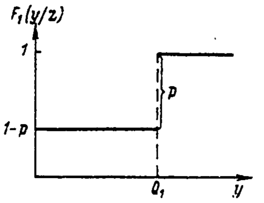

Figure 3.4. Graph of the function $F_1(y/z)$

The distribution of η here will also be binomial:

$$P\{\eta = kQ_1/\mathcal{N}\} = C_{\mathcal{N}}^k p_1^k (1 - p_1)^{\mathcal{N}-k},$$
$$k = 0,\ 1,\ \ldots,\ \mathcal{N}, \qquad (3.18)$$

where $p_1 = k_{ou}p$.

When using the asymptotically normal distribution for $F(y/\mathcal{N})$, when η can assume only positive values with respect to physical meaning, it is necessary that the condition $E\eta - 3\sqrt{\mathcal{D}\eta} \geqslant 0$ be satisfied.

The condition $E\eta - 3\sqrt{\mathcal{D}\eta} \geqslant 0$ defines the corresponding minimum value of $\mathcal{N}$. For the formulas (3.15) we have

$$\mathcal{N} \geqslant 9\,\frac{1 - k_{\text{он}} + \left(\frac{\sigma}{m}\right)^2}{k_{\text{он}}},$$

and for the formulas (3.16) and (3.17), respectively,

$$\mathcal{N} \geqslant 9\,\frac{1 - k_{\text{он}} p}{k_{\text{он}} p}.$$

Concluding this general survey of methods of calculating the effectiveness indexes, let us note some peculiarities of constructing the operative-tactical and the corresponding mathematical models as applied to the problems of analytical design of ships.

1. The process of constructing the mathematical model of estimating the effectiveness of a ship in the AD stages can be provisionally divided into two steps: construction of the model containing complex operative-tactical characteristics (handling, intensity of detection, range of detection means, target indication, communication; probability of detection and destruction, and so on); construction of the model containing explicitly the TTE and TDP of the ship by

expansion of the relations for the above-indicated complex characteristics as functions of TTE and TDP.

2. The individual TTE and TDP of a ship are manifested to a different degree at different hierarchical levels of investigation of the combat operations. For example, the sea endurance and cruising range are purely operative TTE. In the majority of cases it is also possible to include the combat supplies on hand in this group. At the same time the speed of the ships is exhibited simultaneously on the operative level (deployment in the combat zones) and the tactical level (attacking, avoiding the enemy weapons, after salvo maneuvering). The characteristics of steerability and maneuverability are among the purely tactical qualities. The construction of models which are critical with respect to the entire set of TTE and TDP is very difficult and, as a rule, leads to in practice unexecutable algorithms.

In the models of different hierarchical levels, different mathematical methods are used. Thus, on the operative level a more important role is played by the analytical methods, and on the tactical level, frequently it is necessary to use the method of statistical simulation. It is highly significant that many of the tactical characteristics are weakly manifested in the models on the operative level. The most typical methods of overcoming the indicated difficulties are the following: selection of the series of TTE and TDP by optimizing them on the basis of the investigation of special tactical situations and statistical simulation of the special tactical situations with subsequent approximation and use of the analytical relations for the individual tactical characteristics as a function of the TTE and TDP of the ship in models of a higher hierarchical level. (The second of the indicated approaches sometimes is called the principle of generalized information use [51]).

3. When developing the algorithms for estimating the effectiveness of various versions of the ship very frequently it is necessary to optimize the procedures for using the forces and means. For example, it is necessary to optimize the speed of the submarines during target search. In the problems of estimating the effectiveness of using weapons it is necessary to optimize the firing method as a function of accuracy of the target indication means, the weapons dispersion characteristics, and so on.[1]

Thus, in the model designed for optimizing the TTE and the TDP of a ship, and this is the primary goal of AD, "internal" optimization problems appear. This complicates the algorithm, the more so in that in the general case for different versions of the ship the optimal tactical operating procedures are different. Theoretically it is necessary to perform the above-indicated "internal" optimization in each step of the movement toward the optimum with respect to the TTE

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[1] In the general case the optimization of the methods of using forces and means must be two-way, that is, it is carried out both for our own forces and for the forces of the enemy. This leads to the necessity for solving the corresponding game problems. There is an applied discussion of the game theory as applied to naval thematics in the book by V. G. Suzdal' TEORIYA IGR DLYA FLOTA [Game Theory for the Fleet], Moscow, Voyenizdat, 1976.

and the TDP. However, this method is frequently impossible from the computational point of view, and the majority of tactical procedures (except the most significant ones) must be rigidly fixed for all versions of the ship in the AD models.

4. From the general point of view, the TTE and the TDP of a ship can be optimized on the basis of simulating an entire war as a whole and even postwar problems. The construction of such a model, as a rule, is impossible. Only a single typical operation or sequence of typical operations is simulated, as a result of which it is necessary to make the achieved effect commensurate with the losses of ships. It is only admissible not to consider the losses for ships meant to be used one time. At the same time, none of the ships from the classes or subclasses can be considered as ships meant for one-time use.

Thus, in practice, it is always necessary to consider at least two indexes: the achieved effect and the losses, which in time lead to the optimization problems considering several effectiveness indexes. As will be demonstrated in Chapter 5, at the present time such problems are not subject to strict formalization.

§3.3. Mathematical Model of Estimating the Effectiveness of Independent Combat Operations of Ships of Side A Against Ships of Side B

Let us assume that targets (ships of side B) are distributed in an unlimited part of the ocean. The destruction of these ships is the mission of the ships of side A. The number of ships of side B is considered unlimited. The ships on both sides operate individually, independently of each other. (This system can be characteristic for certain types of combat operations of submarines.) The ships of side A search for the ships of side B. The search process is considered Poisson [19].

After detection there is a duel between the ships which lasts until one of the sides is destroyed. The duration of the duels will be neglected by comparison with the total time the ships of side A are in the combat zone. The mutual destruction of both sides in one duel is considered impossible (see §3.4.). The consumption of the combat supplies on hand in each duel, according to the quasiregularity principle, is taken equal to its mathematical expectation until destruction of the target.

The process of combat operations of each ship of side A on one voyage halts either on expiration of a given time (the residual sea endurance P_{combat}) or after a given quantity of combat supplies on hand (the supplies allocated for operations in the zone) are expended or as a result of destruction by the enemy. Under the assumptions that have been made, the functioning of each ship of side A can be described by a Markov process (Figure 3.5).

Let us introduce the following notation: γ -- the intensity of detections of ships on side B by ships on side A; $P_{des\ A}$ -- the probability of destruction of a ship of side A in one duel; $P_{des\ B}$ -- the same for a ship of side B; M is the number of ships of side B which can be destroyed by one ship of side A with respect to the quantity of combat supplies on hand. The value of M is equal to the total quantity of combat supplies on hand allocated for operations in the zone divided by the mean consumption of combat supplies in one duel.

A ship of side A functions in the states with the numbers 0, 1, ..., M-1, destroying 0, 1, ..., M-1 ships of side B, respectively. In the state M a ship of side A stops combat operations as a result of expending the combat supplies on hand, destroying M ships of side B at this time. In the states M+1, M+2,..., 2M a ship of side A was destroyed by the enemy after destruction of 0, 1, 2,..., M-1 ships of side B, respectively.

As the parameter defining the outcome of the combat operations of the ship of side A, it is possible to take the random number of destroyed targets on one voyage (without considering counteraction in transit to the combat zone and back). Let us designate this value by η_1. The distribution function of this value is completely defined by the probabilities

$$\begin{gathered} P\{\eta_1 = 0\} = P_0(T_{бд}) + P_{M+1}(T_{бд}), \\ P\{\eta_1 = 1\} = P_1(T_{бд}) + P_{M+2}(T_{бд}), \qquad (3.19) \\ \text{(1)} \\ \cdots\cdots\cdots\cdots\cdots \\ P\{\eta_1 = k\} = P_k(T_{бд}) + P_{M+k+1}(T_{бд}), \quad k = 0, 1, \ldots, M-1, \\ \cdots\cdots\cdots\cdots\cdots \\ P\{\eta_1 = M\} = P_м(T_{бд}), \end{gathered}$$

Key: (1) combat

where the values entering into the righthand side are the probabilities that the introduced Markov process will be in the corresponding states at the time $t=T_{combat}$.

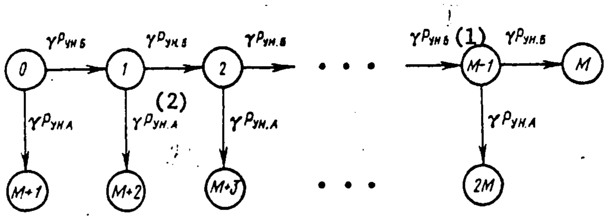

Figure 3.5. Graph of the process of functioning of a ship on side A.

Key: (1) des A

(2) des B

The probability that a ship of side A would be lost in the combat zone is defined by the formula

$$\underset{(1)}{P_{\text{ун. бд}}} = \sum_{k=1}^{M} P_{M+k}(T_{\text{бд}}) \qquad (3.20)$$

Key: 1. des. combat

or

$$P_{\text{ун. бд}} = 1 - \sum_{k=0}^{M} P_k(T_{\text{бд}}).$$

The probability distribution (3.19) makes it possible to find any numerical characteristics of the random variable η_1. For example, the mathematical expectation η_1 will be

$$E\eta_1 = MP_M(T_{\text{бд}}) + \sum_{k=0}^{M-1} k[P_k(T_{\text{бд}}) + P_{M+k+1}(T_{\text{бд}})].$$

The mathematical expectation of the number of targets destroyed by one ship of side A before its own destruction considering counteraction of the enemy on the crossings, can be found by the formula

$$E\eta = E\eta_1 \frac{(1-P_{\text{ун 1}})}{1-(1-P_{\text{ун. бд}})(1-P_{\text{ун 1}})(1-P_{\text{ун 2}})},$$

where η is a random number of targets destroyed by one ship of side A before its own destruction on a series of voyages; $P_{\text{des 1}}$, $P_{\text{des 2}}$ are the probabilities of destruction of a ship of side A, respectively, in transit to the combat zone and back.

The value of $\frac{1}{1-(1-P_{\text{ун. бд}})(1-P_{\text{ун 1}})(1-P_{\text{ун 2}})}$ is the mathematical expectation of the number of voyages of a ship of side A before its destruction.

If a study is made of the combat operations of a fleet of $\mathcal{N}$ ships of side A, then the average number of targets destroyed by the fleet before its own destruction will be $E\eta_{\mathcal{N}} = E\eta\mathcal{N}$. Here the value of $E\eta$ depends on the vector X characterizing the TTE of the ships of side A.

Thus, for complete description of the investigated model it is necessary to have the expressions for the probabilities $P_k(T_{\text{combat}})$ and $P_{M+k}(T_{\text{combat}})$, $k=0, 1, \ldots, M$ or the algorithm for calculating them.

Differential Equations for Finding the Probabilities $P_k(T_{\text{combat}})$ and $P_{M+k}(T_{\text{combat}})$. In accordance with the theory of Markov random processes the probability of the states of the process characterized by the graph in Fig 3.5 satisfies the system of ordinary differential equations

$$\begin{gathered}\frac{dP_0(t)}{dt}=-\gamma P_0(t),\\ \frac{dP_k(t)}{dt}=\gamma P_{\text{ун}\,Б}P_{k-1}(t)-\gamma P_k(t),\quad k=1,\ldots,M-1,\\ \frac{dP_M(t)}{dt}=\gamma P_{\text{ун}\,Б}P_{M-1}(t),\\ \frac{dP_{M+k}(t)}{dt}=\gamma P_{\text{ун}\,A}P_{k-1}(t),\quad k=1,\ldots,M,\end{gathered}\tag{3.21}$$

where t is the current time of the process.

When writing equations (3.21) it is considered that $P_{\text{des A}}+P_{\text{des B}}=1$, for by assumption the duel lasts until one of the sides is destroyed and the mutual destruction of the sides is possible.[1]

As the initial conditions for the system (3.21) it is exepdient to take

$$P_0(0)=1,\quad P_k(0)=0,\quad P_{M+k}(0)=0,\quad k=1,2,\ldots,M.\tag{3.22}$$

After introduction of the dimensionless time $\tau=\gamma t$ the system (3.21) assumes the form

$$\begin{gathered}\frac{dP_0(\tau)}{d\tau}=-P_0(\tau),\\ \frac{dP_k(\tau)}{d\tau}=P_{\text{ун}\,Б}P_{k-1}(\tau)-P_k(\tau),\quad k=1,\ldots,M-1\\ \frac{dP_M(\tau)}{d\tau}=P_{\text{ун}\,Б}P_{M-1}(\tau),\\ \frac{dP_{M+k}(\tau)}{d\tau}=P_{\text{ун}\,A}P_{k-1}(\tau),\quad k=1,\ldots,M.\end{gathered}\tag{3.23}$$

The probabilities $P_k(t)$ and $P_{M+k}(t)$, k=0, 1, ..., M, found from the system (3.21) for $t=T_{\text{combat}}$ are equal to the corresponding probabilities found from the system (3.23) for $\tau=\gamma T_{\text{combat}}$.

In the system (3.23) let us replace the variables, setting $P_k(\tau)=e^{-\tau}u_k(\tau)$, k=0, 1, ..., 2M, where $u_k(\tau)$ are new variables. As a result, we obtain the system of equations with respect to $u_k(\tau)$:

$$\begin{gathered}\frac{du_0(\tau)}{d\tau}=0,\\ \frac{du_k(\tau)}{d\tau}=P_{\text{ун}\,Б}u_{k-1}(\tau),\quad k=1,\ldots,M-1,\\ \frac{du_M(\tau)}{d\tau}=u_M(\tau)+P_{\text{ун}\,Б}u_{M-1}(\tau),\\ \frac{du_{M+k}(\tau)}{d\tau}=u_{M+k}(\tau)+P_{\text{ун}\,A}u_{k-1}(\tau),\quad k=1,\ldots,M\end{gathered}\tag{3.24}$$

under the initial conditions

$$u_0(0)=1,\quad u_k(0)=0,\quad u_{M+k}(0)=0,\quad k=1,\ldots,M.\tag{3.25}$$

---

[1] This is an example of fixing the method of operations of sides in a given model. In the more general statement it is possible to find the optimal duration of the duel taking into account the fact that in a number of cases the targets can stop the duel by their discretion.

Performing the corresponding calculations, it is possible to see that the solution of (3.24) under the conditions (3.25) has the form

$$u_0(\tau)=1,$$

$$u_k(\tau)=\frac{P^k_{\text{ун Б}}}{k!}\tau^k, \quad k=1,\ \dots,\ M-1,$$

$$u_M(\tau)=P^M_{\text{ун Б}}\left[1-e^{-\tau}\sum_{\nu=0}^{M-1}\frac{\tau^\nu}{\nu!}\right]e^\tau,$$

$$u_{M+k}(\tau)=P_{\text{ун А}}P^{k-1}_{\text{ун Б}}\left[1-e^{-\tau}\sum_{\nu=0}^{k-1}\frac{\tau^\nu}{\nu!}\right]e^\tau, \quad k=1,\ \dots,\ M.$$

Returning to the variables P_k and dimensionless time t, we obtain the solution of the system (3.21) under the conditions (3.22):

$$P_k(t)=\frac{(\gamma t)^k}{k!}P^k_{\text{ун Б}}e^{-\gamma t}, \quad k=0,\ \dots,\ M-1,$$

$$P_M(t)=P^M_{\text{ун Б}}\left[1-e^{-\gamma t}\sum_{\nu=0}^{M-1}\frac{(\gamma t)^\nu}{\nu!}\right], \tag{3.26}$$

$$P_{M+k}(t)=P_{\text{ун А}}P^{k-1}_{\text{ун Б}}\left[1-e^{-\gamma t}\sum_{\nu=0}^{k-1}\frac{(\gamma t)^\nu}{\nu!}\right], \quad k=1,\ \dots,\ M.$$

The probabilities $P_k(T_{\text{combat}})$, k=0, 1, ..., 2M corresponding to the point in time $t=T_{\text{combat}}$, which permit the effectiveness indexes $E\eta_1$ and $E\eta$ to be found, are found by formulas (3.26).

Asymptotic Solutions of the System of Equations (3.21). In a number of cases the asymptotic solutions of the system of equations (3.21) for $\gamma T_{\text{combat}}\to\infty$ and $M\to\infty$ are of interest. This corresponds to unlimited residual sea endurance and unlimited combat supplies on hand.

1. $\gamma T_{\text{combat}}\to\infty$, M is finite. From formula (3.26) it follows that:

$$P_k=\lim_{\gamma t\to\infty}P_k(t)=0, \quad k=0,\ \dots,\ M-1,$$

$$P_{\text{м}}=\lim_{\gamma t\to\infty}P_{\text{м}}(t)=P^M_{\text{ун Б}},$$

$$P_{M+k}=\lim_{\gamma t\to\infty}P_{M+k}(t)=P_{\text{ун А}}P^{k-1}_{\text{ун Б}}, \quad k=1,\ \dots,\ M.$$

Hence, we obtain

$$\lim_{\gamma T_{\text{бд}}\to\infty}E\eta_1=MP^M_{\text{ун Б}}+P_{\text{ун А}}\sum_{k=0}^{M-1}kP^k_{\text{ун Б}}=$$

$$=MP^M_{\text{ун Б}}+\frac{P_{\text{ун Б}}}{P_{\text{ун А}}}(1-P^{M-1}_{\text{ун Б}})-(M-1)P^M_{\text{ун Б}}=\frac{P_{\text{ун Б}}}{P_{\text{ун А}}}(1-P^M_{\text{ун Б}}). \tag{3.27}$$

When deriving formulas (3.27) the condition $P_{\text{des A}} + P_{\text{des B}} = 1$ and the formula for the sum of a decrease in arithmetic-geometric progression are used [43]:

$$\sum_{k=0}^{M-1} kq^k = -\frac{(M-1)\,q^M}{1-q} + \frac{q\,(1-q^{M-1})}{(1-q)^2}, \quad q < 1.$$

Just as should be expected, the condition

$$\sum_{k=0}^{2M} P_k = P_{\text{ун Б}}^M + P_{\text{ун А}} \sum_{k=1}^{M} P_{\text{ун Б}}^{k-1} = P_{\text{ун Б}}^M + P_{\text{ун А}} \frac{1-P_{\text{ун Б}}^M}{1-P_{\text{ун Б}}} = 1$$

is satisfied (let us remember that here $P_{\text{des A}}=1-P_{\text{des B}}$).

It is also easy to obtain the formulas

$$\lim_{\gamma T_{\text{бд}}\to\infty} P_{\text{ун бд}} = P_{\text{ун А}} \sum_{k=1}^{M} P_{\text{ун Б}}^{k-1} = 1 - P_{\text{ун Б}}^M, \tag{3.28}$$

$$\lim_{\gamma T_{\text{бд}}\to\infty} E\eta = \frac{P_{\text{ун Б}}}{P_{\text{ун А}}} (1 - P_{\text{ун Б}}^M) \frac{1-P_{\text{ун 1}}}{1-P_{\text{ун Б}}^M (1-P_{\text{ун 1}})(1-P_{\text{ун 2}})}. \tag{3.29}$$

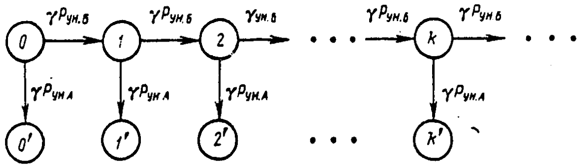

Figure 3.6. Graph of the process of functioning of a ship of side A with unlimited combat supplies on hand

If in formulas (3.27)-(3.29) we also to the limit for M→∞, then

$$\lim E\eta_1 = \frac{P_{\text{ун Б}}}{P_{\text{ун А}}},$$

$$\lim E\eta = \frac{P_{\text{ун Б}}}{P_{\text{ун А}}} (1 - P_{\text{ун1}}),$$

$$\lim P_{\text{ун бд}} = 1, \tag{3.30}$$

$$(\gamma T_{\text{бд}} \to \infty,\ M \to \infty).$$

If the counteraction in transit to the combat zone is absent ($P_{\text{des 1}}=0$), we have

$$\lim_{\substack{\gamma T_{\text{бд}}\to\infty \\ M\to\infty}} E\eta = \frac{P_{\text{ун Б}}}{P_{\text{ун А}}}, \tag{3.31}$$

that is, the average number of destroyed targets is equal to the probability ratio of destruction of the ships of the sides in one duel.

2. $M\to\infty$, γT_{combat} are infinite. In this case the functioning of the ship of side A can be described by a Markov process with an infinite denumerable number of states (Figure 3.6). Here in the states 0, 1, ..., k, ... a ship of side A functions, destroying 0, 1, ..., k, ... targets, respectively. In the states 0', 1', ..., k', ... a ship of side A is destroyed, destroying 0, 1, ..., k, ... targets, respectively, before this.

The distribution of η_1 is given by the probabilities

$$\begin{gathered} P\{\eta_1=0\}=P_0(T_{бд})+P_0^*(T_{бд}), \\ P\{\eta_1=1\}=P_1(T_{бд})+P_1^*(T_{бд}), \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ P\{\eta_1=k\}=P_k(T_{бд})+P_k^*(T_{бд}), \quad k=0,\ 1,\ \dots, \end{gathered}$$

where $P_k(T_{combat})$, k=0, 1, ..., are the probabilities of the states 0, 1, ..., k,... for $t=T_{combat}$; $P^*_k(T_{combat})$, k=0, 1, ..., are the probabilities of the states 0', 1', ..., k', ...

Considering the system of equations similar to (3.21) and the initial conditions $P_0(0)=1$, $P^*_0(0)=0$, $P_k(0)=P^*_k(0)=0$, k=1, 2, ..., for the probabilities $P_k(t)$ and $P^*_k(t)$, it is possible to obtain the formulas

$$\begin{gathered} P_k(t)=\frac{(\gamma t)^k}{k!}P^k_{ун\ Б}e^{-\gamma t}, \quad k=0,\ 1,\ \dots, \\ P_k^*(t)=P_{ун\ А}P^k_{ун\ Б}\left[1-e^{-\gamma t}\sum_{\nu=0}^{k}\frac{(\gamma t)^\nu}{\nu!}\right], \quad k=0,\ 1,\ . \end{gathered}$$

In the given case for the value of $\lim\limits_{M\to\infty} E\eta$ we have

$$\lim_{M\to\infty}E\eta_1=\sum_{k=0}^{\infty}k\left[P_k(T_{бд})+P_k^*(T_{бд})\right].$$

After the corresponding calculations, it is possible to obtain

$$\lim_{M\to\infty}E\eta_1=\frac{P_{ун\ Б}}{P_{ун\ А}}\left[1-e^{-P_{ун\ А}\gamma T_{бд}}\right]. \tag{3.32}$$

It is natural that for $\gamma T_{combat}\to\infty$ the formula (3.32) coincides with the corresponding formula (3.30).

Analogously, it is possible to find

$$\lim_{M\to\infty}P_{ун\ бд}=1-e^{-P_{ун\ А}\gamma T_{бд}}.$$

Hence, it follows that without considering counteraction in transit

$$\lim_{\substack{M\to\infty\\ \gamma T_{\text{бд}}\to\infty}} E\eta = \frac{P_{\text{уи Б}}}{P_{\text{уи А}}}, \tag{3.33}$$

that is, we again obtain formula (3.31).

Concluding Remarks. In order that the above-described model of combat operations be able to be used during AD of the ships of side A, it is necessary to express the operative-tactical values of γ, $P_{des\ A}$, $P_{des\ B}$ and M in terms of the TTE of the ships of both sides and also the characteristics of their detection, target indication means and weapons. This problem requires specific definition of class and type of ships of the sides and also the type and characteristics of their weapons and accessory equipment. Here it is necessary also to fix the method of using the weapons (for example, firing a salvo of a fixed number of combat means at the present location of the target, and so on). Here, as has already been pointed out above, the problems of "internal" optimization, for example, of the speed of the ships during search (if the range of the detection means depends on the speed of the ship), the number of combat means in one salvo, the firing method, and so on, arise.

§3.4. Markov Mathematical Models of Two-Way Duels

When constructing the mathematical models of the combat operations of different weapon carriers, including ships, frequently it is necessary to consider the results of two-way duels with the enemy. In the general case the models of duel situations are special modules of more general models of the combat operations. One of the examples of such a general model was investigated in §3.3. Models of duel situations can also have independent significance when estimating measures to improve the characteristics of the weapons (for example, firing rate), target indicating and laying means, protection of the ships, and so on.

One of the most effective analytical methods of describing duel situations is the use of the theory of Markov random processes with a discrete number of states. Below the two simplest models of two-way duels between single targets are considered as examples.

The duel always presupposes the action taken by the enemies on each other by a weapon. Therefore in models of duels the effectiveness of the application of the weapon must be taken into account. The problems of estimating the effectiveness of the firing and also the effectiveness of applying a weapon are an independent division of operations research [12, 19].

The estimate of the firing effectiveness is based on knowledge of two groups of characteristics: the characteristics defining the probability that the weapon[1] will hit a vulnerable part of the target, and the characteristics of the

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[1] In this section the terms "weapons" and "combat means" are identical (see Chapter 1), that is, by a weapon we mean combat means.

damaging effect of the weapon, including vulnerability of the target with respect to various types of ammunition.

For example, when estimating the effectiveness of firing at a single target [12], the A. N. Kolmogorov formula is used:

$$\underset{(1)}{P_{\text{пор}}}(n) = \sum_{m=0}^{n} P_{n,\,m} G(m), \tag{3.34}$$

Key: 1. damage

where $P_{damage}(n)$ is the probability of damaging the target in n rounds fired; $P_{m,\ n}$ is the probability of m hits for n rounds; G(m) is the probability of damaging the target in m hits. The function G(m) characterizing the vulnerability of the target is called the provisional damage law (PDL).

The probabilities $P_{n,\ m}$ depend on the method and the accuracy of firing (the scattering characteristics of the shells, the accuracy of target indication, and so on), the damaged space of the target determined by its dimensions, and for proximity weapons, the characteristics of the physical field to which the sighting system of the weapon and the proximity fuse react. The probabilities $P_{n,\ m}$ also depend on the nature of the maneuvering of the target, that is, on its maneuvering characteristics, the use of means of counteracting the weapon, and so on.

The function G(m) has the simplest form in the absence of the effect of accumulation of loss by the target where each shell, hitting the damaged space, damages the target independently of the others. In this case $G(m)=1-(1-1/\omega)^m$, where ω is the average number of required hits to damage the target.

If in this case the individual rounds are independent in the sense of the probability of hits, then from the A. N. Kolmogorov formula we have the expression [23]

$$\underset{(1)}{P_{\text{пор}}}(n) = 1 - \prod_{i=1}^{n} \left(1 - \frac{p_i}{\omega}\right), \tag{3.35}$$

Key: 1. damage

where P_i is the probability of a hit on the i-th round.

Let us note that establishment of the PDL is a very complex problem, especially for large surface ships having relatively high invulnerability. This arises, first of all, from the complexity and ambiguity (depending on the nature of the solved problem) of determining the event equivalent to damaging the ship. Secondly, when using nonnuclear weapons large surface ships have the effect of accumulation of loss. At the present time the basic method of calculating the PDL is the method of statistical simulation in which it is necessary to know the location of the basic damage of all targets on the ship and also their protection characteristics. As a rule, this does not permit the use of the method of statistical simulation in the AD stage; therefore the problem of developing the analytical methods of calculating the PDL is highly urgent.

By investigating duel situations, the problem is complicated also by the fact that during the process of a duel the ship can go from state to state in which the possibility of the enemy's influencing it changes, as it is hit by the shells.

In the examples of duels presented below, the PDL assumes the simplest form, and it is called the unit PDL. This law is given by the expression

$$G(m)=\begin{cases}0 \text{ for } m=0,\\ 1 \text{ for } m\geqslant 1.\end{cases}$$

For the unit PDL, the target is damaged on the first hit. This situation occurs for targets with low invulnerability or high-power shells.

We shall also assume that during the process of the duels the targets exchange independent rounds (salvos) with respect to the probability of hitting the target.

Duel with Successive Exchange of Salvos. Considering the assumptions made above, a two-way duel with successive exchange of salvos can be described by a Markov process with a discrete number of states and discrete time.

The graph of the states and the probabilities of the transitions in one salvo (step) of the process is illustrated in Figure 3.7. In states 1 and 2 both targets are functioning; in state 1 the next salvo is by target A, and in state 2, target B. In state 3 target B is destroyed; in state 4, target A. The probability of damaging target B by one salvo of target A is p_A, and target A by one salvo of target B, p_B respectively.

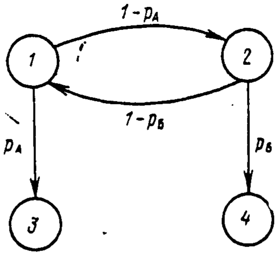

Figure 3.7. Graph of the process of a two-way duel with successive exchange of salvos

Let us denote by $P_k(n)$, k=1, ..., 4, the probabilities of the states of the process after n salvos, and by P(n), the column vector of these probabilities. The duel is fully characterized by the initial vector P(0) and the matrix of probabilities of transitions in one salvo which in the given case has the form

$$Q_1=\begin{pmatrix} 0 & q_A & p_A & 0 \\ q_Б & 0 & 0 & p_Б \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(here $q_A=1-p_A$ and $q_B=1-p_B$).

The vectors $P(n)$, $n=1, 2, \ldots$, satisfy the recurrent expression $P(n)=Q_1{}^*P(n-1$, $n=1, 2, \ldots$, from which we have

$$P(n)=(Q_1^n)^* P(0), \tag{3.36}$$

where Q_1^n is the n-th degree of the matrix Q_1, * is the transposition sign.

In the general case the probabilities p_A and p_B can vary during the duel process, for example, as a result of variation of the distance between targets or as a result of the accumulation of damage in the targets. Then we have

$$P(n)=\left(\prod_{i=1}^{n} Q_{1i}\right)^* P(0), \tag{3.37}$$

where Q_{1i} is the matrix of transition probabilities for the i-th salvo.

The most typical initial conditions are

$$P^*(0)=(\alpha,\ 1-\alpha,\ 0,\ 0), \tag{3.38}$$

where $0\leq\alpha\leq 1$ is the probability that the duel begins with a salvo of target A.

The probability α frequently is identified with the probability of leading in detection. This is possible, for example, when firing range by the weapon is greater than the detection range.

Let us propose that the ranges of the detection means of the targets are independent random normally distributed variables with mathematical expectation d_A and d_B and dispersions $\sigma 2_A$ and $\sigma 2_B$. Under these conditions the probability α is calculated by the formula

$$\alpha=\frac{1}{2}\left[1+\Phi\left(\frac{d_A-d_Б}{\sqrt{2}\sqrt{\sigma_A^2+\sigma_Б^2}}\right)\right],$$

where

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

In some cases it is possible to consider that $\sigma_A/d_A=\sigma_B/d_B=k=\text{const}$, that is, the ratio of the mean square deviation of the detection range to its mathematical expectation is a constant which is identical for both targets. Under this assumption α depends only on d_A/d_B and k (Figure 3.8).

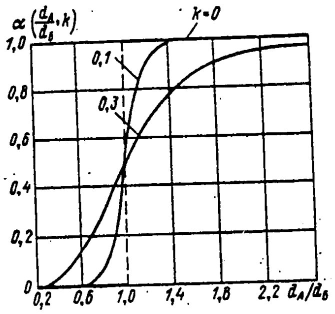

Figure 3.8. Probability of leading in detection α as a function of the values of d_A/d_B and k

The formulas (3.36) or (3.37) permit calculation of the probabilities of destruction of each of the targets after any given number of salvos. Sometimes the outcome of the duel is of interest for an infinite number of salvos, that is, vector $P(\infty) = \lim_{n\to\infty} P(n)$. It is obvious that in such a duel $P_1(\infty)=P_2(\infty)=0$, and $P_3(\infty)$ and $P_4(\infty)$ are the probabilities of destruction of each of the sides.

For calculation of $P(\infty)$ we shall consider a duel with successive exchange of salvos, but with continuous time, assuming that the time between successive salvos is distributed by exponential law with parameter λ. The value of 1/λ is the average time between successive salvos. In such a duel, the probabilities of states $P_i(t)$, i=1, ..., 4, as a function of time t satisfy the system of differential equations

$$\begin{aligned}\frac{dP_1(t)}{dt} &= -\lambda P_1(t) + \lambda q_Б P_2(t),\\ \frac{dP_2(t)}{dt} &= \lambda q_A P_1(t) - \lambda P_2(t),\\ \frac{dP_3(t)}{dt} &= \lambda p_A P_1(t),\\ \frac{dP_4(t)}{dt} &= \lambda p_Б P_2(t)\end{aligned} \tag{3.39}$$

under the initial conditions

$$P_1(0)=\alpha,\ P_2(0)=1-\alpha,\ P_3(0)=P_4(0)=0. \tag{3.40}$$

It is obvious that we have the equality $P(\infty) = \lim_{n\to\infty} P(n) = \lim_{t\to\infty} P(t)$, where $P(t)$ is the probability vector $P_i(t)$, i=1, ..., 4.

Using the integral Laplace transformation

$$\tilde{P}_i(s) = \int_0^\infty P_i(t)\, e^{-st}\, dt,\ i=1, \ldots, 4$$

it is possible to reduce the system of differential equations (3.39) to a system of linear algebraic equations with respect to $\tilde{P}_i(s)$:

$$\begin{aligned} s\tilde{P}_1(s)-P_1(0)&=-\lambda\tilde{P}_1(s)+\lambda q_{\text{Б}}\tilde{P}_2(s),\\ s\tilde{P}_2(s)-P_2(0)&=\lambda q_A\tilde{P}_1(s)-\lambda\tilde{P}_2(s),\\ s\tilde{P}_3(s)-P_3(0)&=\lambda p_A\tilde{P}_1(s),\\ s\tilde{P}_4(s)-P_4(0)&=\lambda p_{\text{Б}}\tilde{P}_2(s). \end{aligned}$$

Solving this system, we find

$$\begin{aligned} \tilde{P}_1(s)&=\frac{(s+\lambda)P_1(0)+\lambda q_{\text{Б}}P_2(0)}{(s+\lambda)^2-\lambda^2q_Aq_{\text{Б}}},\\ \tilde{P}_2(s)&=\frac{\lambda q_AP_1(0)+(s+\lambda)P_2(0)}{(s+\lambda)^2-\lambda^2q_Aq_{\text{Б}}},\\ \tilde{P}_3(s)&=\frac{1}{s}P_3(0)+\frac{\lambda p_A}{s}\frac{(s+\lambda)P_1(0)+\lambda q_{\text{Б}}P_2(0)}{(s+\lambda)^2-\lambda^2q_Aq_{\text{Б}}},\\ \tilde{P}_4(s)&=\frac{1}{s}P_4(0)+\frac{\lambda p_{\text{Б}}}{s}\frac{\lambda q_AP_1(0)+(s+\lambda)P_2(0)}{(s+\lambda)^2-\lambda^2q_Aq_{\text{Б}}}. \end{aligned}$$

Using the property $\lim\limits_{t\to\infty}P_i(t)=\lim\limits_{s\to0}s\tilde{P}_i(s),\quad i=1,\ldots,4,$

considering the initial conditions (3.40), we obtain the formulas for the components of the desired vector $\mathbf{P}(\infty)$

$$\begin{aligned} P_1(\infty)&=0,\\ P_2(\infty)&=0,\\ P_3(\infty)&=\alpha\frac{p_A}{1-q_Aq_{\text{Б}}}+(1-\alpha)\frac{p_Aq_{\text{Б}}}{1-q_Aq_{\text{Б}}},\\ P_4(\infty)&=\alpha\frac{p_{\text{Б}}q_A}{1-q_Aq_{\text{Б}}}+(1-\alpha)\frac{p_{\text{Б}}}{1-q_Aq_{\text{Б}}}. \end{aligned} \tag{3.41}$$

In §3.3 it was demonstrated that the ratio $\frac{P_3(\infty)}{P_4(\infty)}=\frac{P_{\text{ун Б}}(\infty)}{P_{\text{ун А}}(\infty)}$ is the average number of targets B destroyed by one target A before its own destruction. If we set $p_A=p_B=p$, that is, consider the effectiveness of the weapon of each of the sides identical, we obtain

$$\frac{P_3(\infty)}{P_4(\infty)}=\frac{1-(1-\alpha)p}{1-\alpha p}, \tag{3.42}$$

where

$$P_3(\infty)=\frac{1-(1-\alpha)p}{2-p}\text{ and }P_4(\infty)=\frac{1-\alpha p}{2-p}.$$

Graphically, this function is represented in Figure 3.9.

The problem can arise of the required relation between p_A, p_B and α insuring predominance of one of the sides, for example, side A, which corresponds to the condition $P_3(\infty)>P_4(\infty)$. Considering the formulas (3.41), this condition assumes the form

$$p_A > p_Б \frac{1}{1+(2\alpha-1)p_Б}. \quad (3.43)$$

The condition (3.43) is presented graphically in Figure 3.10 in which the regions of values of p_A and p_B satisfying (3.43) are located above the corresponding curves for each of the fixed values of α. From Figure 3.10 it is obvious, in particular, that for $p_B>0.5$ reliable lead at the beginning of the duel by the target B over the target A ($\alpha=0$) cannot be compensated for by any increase in effectiveness of the weapon of side A, that is, the probability p_A.

The investigated model of a duel makes it possible to consider the lead not only in the first salvo, but also in several first salvos. Let, for example, the target A lead the target B with a probability one in the first n_0 salvos. Then when calculating $\bar{P}(\infty)$ it is only necessary to vary the initial conditions, setting $P1(0)=(1-p_A)^{n_0-1}$, $P_3(0)=1-(1-p_A)^{n_0-1}$, $P_2(0)=P_4(0)=0$. Let us set $\alpha=1$ and calculate the salvos for target A from the value of n_0-1.

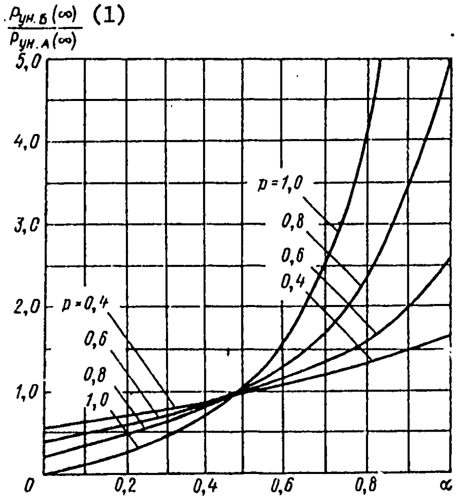

Figure 3.9. Ratio of the probabilities of destroying the targets as a function of the values of α and p in a duel of unlimited duration with successive exchange of salvos ($n\to\infty$, $p_A=p_B=p$).

Key:

1. $P_{des\ B}(\infty)/P_{des\ A}(\infty)$

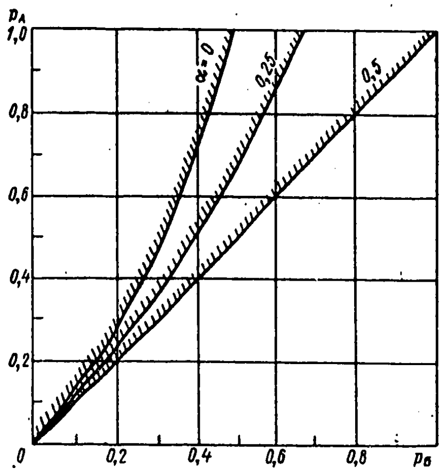

Figure 3.10. Regions of values of the probabilities p_A and p_B insuring predominance of side A in the duel of unlimited duration with successive exchange of salvos

The more general problem can also be considered where the number of leading salvos is a random variable with distribution function $F(n_0)$. If $P_i(n, n_0)$, $i=1, \ldots, 4$ are the probabilities of states for the number of leading salvos equal to n_0, then the total probabilities defined by the expressions

$$P_i(n) = \int_{n_0} P_i(n, n_0)\, dF(n_0), \quad i = 1, \ldots, 4.$$

Duel with Random Exchange of Salvos and Continuous Time. Let us assume that each of the targets A and B produces salvos independently of each other at random points in time where the time between the next salvos of each of the sides is distributed by an exponential law with the parameters $\lambda_A(t)$ and $\lambda_B(t)$ which can, generally speaking, depend on the time reckoned from the beginning of the duel. The time of movement of the weapons of both targets toward the target will be neglected by comparison with the time between the next salvos. Parameters $\lambda_A(t)$ and $\lambda_B(t)$ are the firing rates (the number of salvos per unit time) of targets A and B, respectively. The probabilities of damaging the targets by one salvo, just as before, will be designated by $p_A(t)$ and $p_B(t)$, considering their possible dependence on time.

The investigated duel can be described by a Markov process (Figure 3.11). The targets A and B are destroyed in states 1 and 2, respectively. In state 3 both targets function.

Let us denote by $\overline{\lambda}_A(t)$ and $\overline{\lambda}_B(t)$ the intensities of the flows of so-called damaging rounds:

$$\overline{\lambda}_A(t) = \lambda_A(t)\, p_A(t), \ \overline{\lambda}_B(t) = \lambda_B(t)\, p_B(t).$$

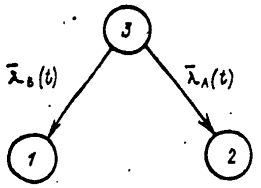

Figure 3.11. Graph of the process of a two-way duel with random exchange of salvos

The weapon reserve of each of the sides will be considered unlimited.

The probability $P_i(t)$, $i=1, 2, 3$, of the states of the process in the given case satisfies the system of differential equations

$$\frac{dP_1(t)}{dt} = \bar{\lambda}_Б(t)\,P_3(t),$$

$$\frac{dP_2(t)}{dt} = \bar{\lambda}_A(t)\,P_3(t), \qquad (3.44)$$

$$\frac{dP_3(t)}{dt} = -[\bar{\lambda}_A(t) + \bar{\lambda}_Б(t)]\,P_3(t).$$

The solution of the system of equations (3.44) gives

$$P_1(t) = P_1(0) + P_3(0)\int_0^t \bar{\lambda}_Б(\tau)\exp\left\{-\int_0^\tau [\bar{\lambda}_A(\xi) + \bar{\lambda}_Б(\xi)]\,d\xi\right\}d\tau,$$

$$P_2(t) = P_2(0) + P_3(0)\int_0^t \bar{\lambda}_A(\tau)\exp\left\{-\int_0^\tau [\bar{\lambda}_A(\xi) + \bar{\lambda}_Б(\xi)]\,d\xi\right\}d\tau,$$

$$P_3(t) = P_3(0)\exp\left\{-\int_0^t [\bar{\lambda}_A(\xi) + \bar{\lambda}_Б(\xi)]\,d\xi\right\},$$

where $P_i(0)$, $i=1, 2, 3$, are the initial values of the probabilities of states.

In the case where $\bar{\lambda}_A(t)=\bar{\lambda}_A$ and $\bar{\lambda}_B(t)=\bar{\lambda}_B$, that is, when the intensities of the flows of damaging salvos do not depend on time, we obtain

$$P_1(t) = P_1(0) + P_3(0)\frac{\bar{\lambda}_Б}{\bar{\lambda}_A + \bar{\lambda}_Б}\{1 - \exp[-(\bar{\lambda}_A + \bar{\lambda}_Б)t]\},$$

$$P_2(t) = P_2(0) + P_3(0)\frac{\bar{\lambda}_A}{\bar{\lambda}_A + \bar{\lambda}_Б}\{1 - \exp[-(\bar{\lambda}_A + \bar{\lambda}_Б)t]\}, \qquad (3.45)$$

$$P_3(t) = P_3(0)\exp[-(\bar{\lambda}_A + \bar{\lambda}_Б)t].$$

For a duel of infinite duration ($t\to\infty$) from the formulas (3.45) we have

$$P_1(\infty) = P_1(0) + P_3(0)\frac{\bar{\lambda}_Б}{\bar{\lambda}_A + \bar{\lambda}_Б},$$

$$P_2(\infty) = P_2(0) + P_3(0)\frac{\bar{\lambda}_A}{\bar{\lambda}_A + \bar{\lambda}_Б}; \qquad (3.46)$$

$$P_3(\infty) = 0.$$

On derivation of the formulas (3.45) and (3.46) it was proposed that directly at the time of beginning of the duel (for $t=0$) the targets do not produce salvos, and therefore damage to the targets is impossible.

Using the initial conditions $P_i(0)$, $i=1, 2, 3$, various versions of the beginning of the duel can be taken into account. [Hereafter it is assumed everywhere that the probabilities $p_A(t)$ and $p_B(t)$ and also $\lambda_A(t)$ and $\lambda_B(t)$ do not depend on time.]

1. The duel begins with simultaneous exchange of salvos at the initial point in time ($t=0$). In this case the damage at the time $t=0$ to one of the targets or their mutual destruction is possible. The probabilities of destroying the targets in such a duel are calculated by the formulas

$$\begin{aligned} P_{\text{ун}\,A}(t) &= p_A p_Б + (1 - p_A p_Б) P_1(t), \\ P_{\text{ун}\,Б}(t) &= p_A p_Б + (1 - p_A p_Б) P_2(t), \end{aligned} \qquad (3.47)$$

where the probabilities $P_1(t)$ and $P_2(t)$ are found from the solution of the system of equations (3.44) with the initial conditions

$$P_1(0) = \frac{p_Б(1-p_A)}{1-p_A p_Б}, \quad P_2(0) = \frac{p_A(1-p_Б)}{1-p_A p_Б}, \quad P_3(0) = \frac{(1-p_A)(1-p_Б)}{1-p_A p_Б}.$$

For $t \to \infty$ the formulas (3.47) assume the form

$$P_{\text{ун}\,A}(\infty) = p_Б + (1 - p_A)(1 - p_Б)\frac{\bar{\lambda}_Б}{\bar{\lambda}_A + \bar{\lambda}_Б},$$

$$P_{\text{ун}\,Б}(\infty) = p_A + (1 - p_A)(1 - p_Б)\frac{\bar{\lambda}_A}{\bar{\lambda}_A + \bar{\lambda}_Б}.$$

In the given case $P_{\text{des}\,A}(\infty) + P_{\text{des}\,B}(\infty) = 1 + p_A p_B > 1$, for the probabilities $P_{\text{des}\,A}(\infty)$ and $P_{\text{des}\,B}(\infty)$ take into account the possibility of mutual damage of the targets at the time $t=0$.

2. The duel begins with successive exchange of salvos at the initial point in time $t=0$. Here the probabilities of destruction of the targets are found by solution of equations (3.44) under the initial conditions

$$\begin{aligned} P_1(0) &= \alpha p_Б(1 - p_A) + (1 - \alpha) p_Б, \\ P_2(0) &= \alpha p_A + (1 - \alpha)(1 - p_Б) p_A, \\ P_3(0) &= 1 - p_A - p_Б + p_A p_Б, \end{aligned}$$

where α is the probability that at the time $t=0$ the salvo of target A will be made first.

After simple calculations for t→∞ we obtain

$$P_{\text{ун}\,A}(\infty) = P_1(\infty) = p_Б - \alpha p_A p_Б + (1 - p_A - p_Б + p_A p_Б)\frac{\bar{\lambda}_Б}{\bar{\lambda}_A + \bar{\lambda}_Б},$$

$$P_{\text{ун}\,Б}(\infty) = P_2(\infty) = p_A - (1 - \alpha) p_A p_Б +$$

$$+ (1 - p_A - p_Б + p_A p_Б)\frac{\bar{\lambda}_A}{\bar{\lambda}_A + \bar{\lambda}_Б}.$$

It is natural that in the given case $P_{\text{ун}\,A}(\infty) + P_{\text{ун}\,Б}(\infty) = 1$.

3. The duel with probability α begins with the salvo of target A, and with probability 1-α, with the salvo of target B. Here at the time t=0 the salvo is produced only by one of the targets, after which the salvos of the targets follow after random time intervals distributed by exponential laws.

The probabilities of destruction of the sides in the given case are found by solving the equations (3.44) with the initial conditions

$$P_1(0) = (1 - \alpha)\, p_Б, \quad P_2(0) = \alpha p_A, \quad P_3(0) = \alpha (1 - p_A) + (1 - \alpha)(1 - p_Б).$$

For t→∞ we obtain

$$P_{\text{ун}\,A}(\infty) = P_1(\infty) = (1 - \alpha) P_Б +$$

$$+ [\alpha (1 - p_A) + (1 - \alpha)(1 - p_Б)]\frac{\bar{\lambda}_Б}{\bar{\lambda}_A + \bar{\lambda}_Б},$$

$$P_{\text{ун}\,Б}(\infty) = P_2(\infty) = \alpha P_A +$$

$$+ [\alpha (1 - p_A) + (1 - \alpha)(1 - p_Б)]\frac{\bar{\lambda}_A}{\bar{\lambda}_A + \bar{\lambda}_Б}.$$

For $\lambda_A = \lambda_B = \lambda$, that is, when the firing rates of the targets are identical, the formulas for $P_{\text{des}\,A}(\infty)$ and $P_{\text{des}\,B}(\infty)$ assume the form

$$P_{\text{ун}\,A}(\infty) = (1 - \alpha) p_Б + [\alpha q_A + (1 - \alpha) q_Б]\frac{p_Б}{p_A + p_Б}, \tag{3.48}$$

$$P_{\text{ун}\,Б}(\infty) = \alpha p_A + [\alpha q_A + (1 - \alpha) q_Б]\frac{p_A}{p_A + p_Б}.$$

Considering these formulas we obtain the condition insuring predominance of the target A in a duel of infinite duration

$$p_A > p_Б \frac{1}{1 + 2p_Б (2\alpha - 1)}. \tag{3.49}$$

Graphically, the condition (3.49) is illustrated in Figure 3.12. Comparing the conditions (3.43) and (3.49), it is possible to discover that in duels of infinite duration with random exchange of salvos predominance of the first salvo is more significant than duels with successive exchange of salvos. Thus, from Figure 3.12 it is obvious that compensation of the lead by the enemy (the target B) at the beginning of the duel as a result of effectiveness of the weapon itself (the target A) becomes impossible for $p_B > 0.33$. In a duel with successive exchange of salvos, this limit was equal to 0.5.

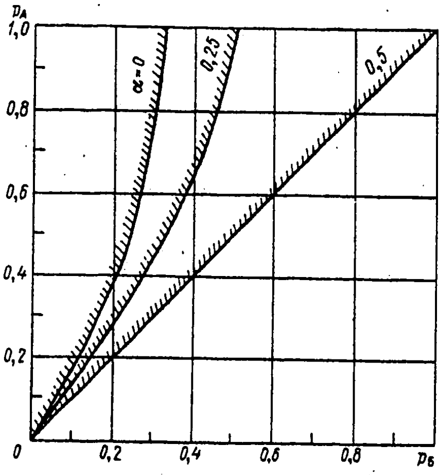

Figure 3.12. Regions of values of the probabilities p_A and p_B insuring predominance of side A in a duel of unlimited duration with random exchange of salvos ($t \to \infty$, $\lambda_A = \lambda_B = \lambda$).

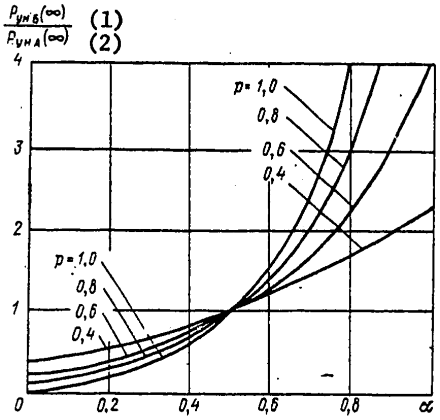

Figure 3.13. Ratio of the probabilities of destruction of targets as a function of the values of α and p in a duel of unlimited duration with random exchange of salvos ($t \to \infty$, $\lambda_B = \lambda_A = \lambda$, $p_B = p_A = p$)

Key:

1. des.B
2. des.A

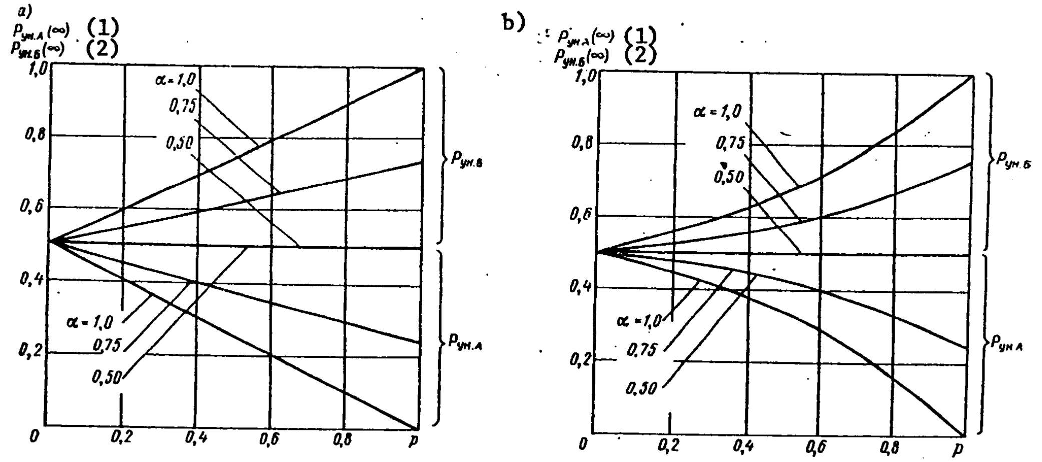

Figure 3.14. Probabilities of destruction of the sides as functions of the values of α and p in duels of unlimited duration: a -- with random exchange of salvos; b -- with successive exchange of salvos

Key:
1. des.A
2. des.B

If we set $p_A=p_B=p$ in formulas (3.48), then

$$P_{\text{ук}\,A}(\infty)=(1-\alpha)p+\frac{1}{2}(1-p),$$

$$P_{\text{уп}\,\text{Б}}(\infty)=\alpha p+\frac{1}{2}(1-p),$$

$$\frac{P_{\text{уп}\,\text{Б}}(\infty)}{P_{\text{уч}\,A}(\infty)}=\frac{1-(1-2\alpha)p}{1+(1-2\alpha)p}.$$

The graphically presented relations are presented in Figure 3.13 and 3.14. The last figure illustrates the obvious conclusion that the role of the lead at the beginning of the duel, for example, as a result of lead in detection, increases with an increase in weapon effectiveness.

The models of duels described in this section require expansion of the relations for the tactical characteristics (λ_A, λ_B, p_A, p_B,α) as a function of the TTE of the ships and their weapons in order to use them in the AD process. Thus, the probabilities p_A and p_B depend on the scattering parameters of the weapons, the accuracy of target indication, the reaction radii of the guidance systems and the proximity fuses, the maneuvering characteristics of the targets, and so on. The reaction radii of the guidance systems and the proximity fuses, in turn, are determined by the characteristics of the physical fields to which the systems react. The characteristics of the physical fields are functions of the TTE and the TDP of the ship and in some cases (for example, the hydroacoustic field) they depend on the conditions of motion (speed, operating conditions of the power plant, and so on).

The probability of lead in detection α is influenced by the targets and characteristics of the corresponding detection means. The ranges of the detection means and, consequently, the probability α also depend on the characteristics of the corresponding physical fields of the ships.

§3.5. Mathematical Model of Estimating the Effectiveness of Operations of Ships Patrolling a Given Part of the Sea in the Presence of Enemy Counteraction

Statement of the Problem and General Scheme for Its Solution. Let us assume that it is necessary to examine a region of the sea of area S_0 under the condition of sufficiency of single coverage of the region by the range of some detection means and in the presence of the counteraction. We shall also consider that the range d of the detection means is a determinant factor, and the flow of damaging effects of the enemy on each ship participating in the operation is Poisson with intensity λ.

The problem consists in constructing the mathematical model permitting determination of the polygon fleet $\mathcal{N}_n$ of ships providing for examination of the region in the given time T with the given probability P_3.

Let us consider the general scheme for solving the stated problem. The area which is examined by $\mathcal{N}_n$ ships in the time T is a random variable in connection with the random nature of destruction of the ships by the enemy. Let us denote this random variable by $\tilde{S}(\mathcal{N}_n, T)$. The desired fleet of ships can be found from the equation

$$P\{\tilde{S}(\mathcal{N}_n, T) \geqslant S_0\} = P_3. \qquad (3.50)$$

For the solution of this equation it is necessary to find the distribution function of the variable $\tilde{S}(\mathcal{N}_n, T)$.

From the point of view of the general procedure for constructing the effectiveness indexes discussed in §3.1, the parameter defining the outcome of the combat operations in the given example will be the examined area in the given time T. The utility function is assumed to be depth function, and it is characterized by the value of S_0 -- the required value of the random area equal to the area of the region. Accordingly, the effectiveness index is the probability of the solution of the problem. An indirect index -- the fleet of ships insuring solution of the problem with given probability P_3 -- is used.

It is possible to represent the random variable $\tilde{S}(\mathcal{N}_n, T)$ in the form

$$\tilde{S}(\mathcal{N}_n, T) = \underbrace{s_1\tau_1(T) + s_1\tau_1(T) + \cdots + s_1\tau_1(T)}_{\mathcal{N}_n \text{ times}}, \qquad (3.51)$$

where $s_1=2dv$ is the area examined by one ship per unit time; d is the range of the detection means; v is the speed of the ship when examining the region; $\tau_1(T)$ is the random "life" time of the ship under the condition that the examination of the region lasts a time T.

It is possible to introduce the random variable $\tilde{S}_1(T)=s_1\tau_1(T)$ which represents the area examined by one ship in the time T considering the probability of destruction of the ship, into the investigation.

If we assume that the ships operate independently of each other, then on the basis of (3.51) the distribution function $\tilde{S}(\mathcal{N}_n, T)$ is the $\mathcal{N}_n$-fold convolution of the distribution functions $\tilde{S}_1(T)$. Here the mathematical expectation and the dispersion $\tilde{S}(\mathcal{N}_n, T)$ are defined by the formulas

$$\begin{aligned} &E_s(\mathcal{N}_n, T) = \mathcal{N}_n s_1 E_{\tau 1}(T), \\ &\sigma_s^2(\mathcal{N}_n, T) = \mathcal{N}_n s_1^2 \sigma_{\tau_1}^2(T), \end{aligned} \qquad (3.52)$$

where $E_s(\mathcal{N}_n, T)$, $\sigma_s^2(\mathcal{N}_n, T)$ are the mathematical expectation and dispersion $\tilde{S}(\mathcal{N}_n, T)$; $E_{\tau_1}(T)$; $\sigma_{\tau_1}^2(T)$ are the same for the value of $\tau_1(T)$.

For sufficiently large values of $\mathcal{N}_n$ (in practice for $\mathcal{N}_n \geqslant$ to 6) the distribution $\tilde{S}(\mathcal{N}_n, T)$ will approximately be normal with parameters defined by the formulas (3.52). In this case the equation (3.50) assumes the form

$$\frac{1}{2}\left[1 - \Phi\left(\frac{S_0 - E_s(\mathcal{N}_n, T)}{\sqrt{2}\,\sigma_s(\mathcal{N}_n, T)}\right)\right] = P_3, \qquad (3.53)$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

For $P_3 \geq 0.5$ the equation (3.53) can be written as follows:

$$E_s(\mathcal{N}_n, T) - \nu\sigma_s(\mathcal{N}_n, T) = S_0. \qquad (3.54)$$

The values of the coefficient ν defined by the given probability P_3 will be:

P_3	0.50	0.75	0.80	0.85	0.90	0.95	0.98
ν^3	0	0.675	0.840	1.04	1.28	1.65	2.05

For expansion of the equation (3.54) it is necessary to find expressions that define $E_{\tau 1}(T)$ and $\sigma^2_{\tau 1}(T)$. These values depend only on the time T and the flow intensity of the damaging effects λ. Consequently, the values of $\lambda E_{\tau 1}(T)$ and $\lambda^2\sigma^2_{\tau 1}(T)$, being dimensionless, depend only on the dimensionless parameter λT.

Let us introduce the notation $\lambda E_{\tau_1}(T) = \varphi(\lambda T)$, $\lambda^2\sigma^2_{\tau_1}(T) = \psi(\lambda T)$. Considering this notation and formulas (3.52) equation (3.54) assumes the form

$$\mathcal{N}_n\varphi(\lambda T) - \nu\sqrt{\mathcal{N}_n}\sqrt{\psi(\lambda T)} = \frac{\lambda S_0}{s_1}. \qquad (3.55)$$

This equation defines $\mathcal{N}_n$ as a function of the dimensionless parameters $\lambda S_0/s_1$, T and P_3 where the dependence on P_3 is defined by the function $\nu(P_3)$. The solution of equation (3.55) gives

$$\mathcal{N}_n = \frac{\nu^2}{4}\frac{\psi(\lambda T)}{\varphi^2(\lambda T)}\left[1 + \sqrt{1 + \frac{4}{\nu^2}\frac{\lambda S_0}{s_1}\frac{\varphi(\lambda T)}{\psi(\lambda T)}}\right]^2. \qquad (3.56)$$

Hereafter, we shall not specially stipulate that the magnitude of the fleet in practice must be integral. If necessary this can be insured by rounding the corresponding nonintegral value to the next largest whole number.

Thus, in order to find $\mathcal{N}_n$ by formula (3.56) it is necessary to find the form of the functions $\phi(\lambda T)$ and $\psi(\lambda T)$.

Determining the Type of Functions $\phi(\lambda T)$ and $\psi(\lambda T)$. For Poisson flow of the damaging effects the distribution function $F_{\tau 1}(t/T)$ of the variable $\tau_1(T)$ has the form

$$F_{\tau_1}(t/T) = \begin{cases} 1 - e^{-\lambda t} & \text{for } 0 < t < T, \\ 1 & \text{for } t > T. \end{cases}$$

In accordance with the general expressions

$$E_{\tau_1}(T) = \int_0^\infty t\,dF_{\tau_1}(t/T),$$

$$\sigma^2_{\tau_1}(T) = \int_0^\infty t^2\,dF_{\tau_1}(t/T) - [E_{\tau_1}(T)]^2$$

and considering the specific form of $T_{\tau 1}(t/T)$ we find

$$E_{\tau_1}(T)=\frac{1}{\lambda}(1-e^{-\lambda T}),$$

$$\sigma^2_{\tau_1}(T)=\frac{1}{\lambda^2}(1-e^{-2\lambda T})-2\,\frac{T}{\lambda}e^{-\lambda T}.$$

Thus, the functions $\phi(\lambda T)$ and $\psi(\lambda T)$ have the form

$$\varphi(\lambda T)=1-e^{-\lambda T},$$

$$\psi(\lambda T)=1-e^{-2\lambda T}-2\lambda Te^{-\lambda T}. \tag{3.57}$$

Graphically, these functions are illustrated in Figure 3.15.

Some Special Cases of Solving the Equation (3.54). In accordance with formulas (3.52

$$\begin{aligned} E_s(\mathcal{N}_{\text{п}},\ T) &= \mathcal{N}_{\text{п}}\frac{s_1}{\lambda}\varphi(\lambda T),\\ \sigma^2_s(\mathcal{N}_{\text{п}},\ T) &= \mathcal{N}_{\text{п}}\left(\frac{s_1}{\lambda}\right)^2\psi(\lambda T). \end{aligned} \tag{3.58}$$

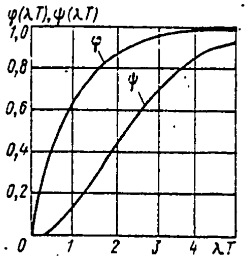

Figure 3.15. Graph of the functions $\phi(\lambda T)$ and $\psi(\lambda T)$

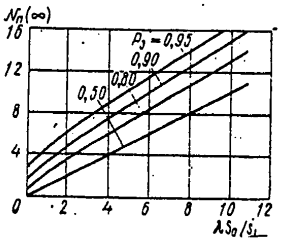

Figure 3.16. $\mathcal{N}_{\text{п}}(\infty)$ as a function of $\lambda S_0/S_1$ and P_3

From the formulas (3.57) and (3.58) for $\lambda\to0$, that is, in the absence of an enemy counteraction, we obtain the obvious result

$$\lim_{\lambda\to0}E_s(\mathcal{N}_{\text{п}},\ T)=\mathcal{N}_{\text{п}}s_1T,$$

$$\lim_{\lambda\to0}\sigma^2_s(\mathcal{N}_{\text{п}},\ T)=0.$$

The fleet $\mathcal{N}_{\text{п}}$ in this case is $S_0/(s_1T)$, and the area examined by it is a determinate value.

Now let us consider the case $T\to\infty$ for finite value of λ where the given time T significantly exceeds the average lifetime of the ship ($1/\lambda$) with unlimited examination time of the region. In this case, from the formulas (3.57) and (3.58) we have

$$E_s(\mathcal{N}_n, \infty) = \lim_{T\to\infty} E_s(\mathcal{N}_n, T) = \mathcal{N}_n \frac{s_1}{\lambda},$$

$$\sigma_s^2(\mathcal{N}_n, \infty) = \lim_{T\to\infty} \sigma_s^2(\mathcal{N}_n, T) = \mathcal{N}_n \left(\frac{s_1}{\lambda}\right)^2.$$

For $T\to\infty$, and in practice for quite large values of λT the fleet $\mathcal{N}_n$ can be found by the formula

$$\mathcal{N}_n(\infty) = \left(\frac{\nu}{2} + \sqrt{\frac{\nu^2}{4} + \frac{\lambda S_0}{s_1}}\right)^2, \tag{3.59}$$

which follows from (3.56) for $\phi(\infty)=\psi(\infty)=1$ and it is valid for $\mathcal{N}_n(\infty)\geqslant 5$ to 6.

As is obvious from formula (3.59), the value of $\mathcal{N}_n(\infty)$ depends only on two dimensionless parameters $\lambda S_0/s_1$ (Figure 3.16) and ν(or P_3).

Here $P_3=0.98$, which corresponds to $\nu\simeq 2$, the formula (3.59) is simplified:

$$\mathcal{N}_n(\infty) = \left(1 + \sqrt{1 + \frac{\lambda S_0}{s_1}}\right)^2.$$

Let us also note that for finite T and for $P_3=0.5$ ($\nu=0$)

$$\mathcal{N}_n = \frac{\lambda S_0}{s_1} \frac{1}{1 - e^{-\lambda T}}.$$

Let us assume that the flow of damaging effects begins with certain stationary objects (combat means) of the enemy uniformly distributed in the region. The ship is damaged when it falls into a circular zone with radius d_{damage} around each enemy object.[1] The enemy objects (for example, mines) are not detected by the ship. In this case the parameter $\lambda S_0/s_1$ can be represented in the form

$$\frac{\lambda S_0}{s_1} = S_0 \frac{2 d_{\text{пор}} v \rho}{2 d v} = \rho S_0 \frac{\overset{(1)}{d_{\text{пор}}}}{d} = M \frac{d_{\text{пор}}}{d},$$

Key: 1. damage

where ρ and M are the distribution density and the total number of enemy objects in the region, respectively.

For any value of M and ν the fleet $\mathcal{N}_n$ decreases with a decrease in d_{damage}/d. Therefore the effectiveness of the ships can be estimated by the criteria of min d_{damage}/d or max d/d_{damage}.

If on each engagement with the enemy there is a duel between sides, then $\lambda=2\rho d_{\text{damage}} v P_{\text{des.k}}$, where $P_{\text{des.k}}$ is the probability of destruction of the ship in

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[1] It is proposed that $d_{\text{damage}}<d$.

the duel. Then $\lambda S_0/s_1 = MP_{des,k} d_{damage}/d$. The probability $P_{des,k}$ is calculated, for example, in accordance with the models investigated in §3.4.

Determination of Losses of Ships when Solving the Investigated Problem. The fleet of ships required to examine a region in the given time with given probability does not completely characterize the effectiveness of the ships when solving the investigated combat problem. The losses of ships have great significance. In particular, the problem of in what way the ships are compared to each other for different fleets and different losses accompanying the combat operations have great significance (see Chapter 5).

It is obvious that for $\lambda = 0$ the number of destroyed ships is zero. Therefore hereafter only the case $\lambda > 0$ will be considered.

Let us denote by $\tilde{\mathcal{N}}_{\text{пор}}$ the random variable -- the number of destroyed ships. The general means of finding the distribution function $F(\mathcal{N}_{\text{пор}})$ of this random variable consists in the following.[1]

1. Let us find the distribution function $F_\tau(T)$ of random duration τ of actual examination of the region by a fleet of $\mathcal{N}_{\text{п}}$ ships which will be calculated in advance by formula (3.56) as a function of the parameters $\lambda S_0/s_1$, λT and P_3. The function $F_\tau(t)$ is obtained in terms of the distribution function $F_S(S)$ of the random variable $\tilde{S}(\mathcal{N}_{\text{п}}, T)$:

$$F_\tau(t) = P\{\tau < t\} = P\{\tilde{S}(\mathcal{N}_{\text{п}}, t) \geqslant S_0\} = 1 - F_s(S_0).$$

The function $F_S(S)$ with sufficiently large $\mathcal{N}_{\text{п}}$ is approximately normal with the parameters defined by the formula (3.58). Consequently, for sufficient large $\mathcal{N}_{\text{п}}$

$$F_\tau(t) = \frac{1}{2}\left[1 - \Phi\left(\frac{S_0 - \mathcal{N}_{\text{п}}\frac{s_1}{\lambda}\varphi(\lambda t)}{\sqrt{2}\sqrt{\mathcal{N}_{\text{п}}}\frac{s_1}{\lambda}\sqrt{\psi(\lambda t)}}\right)\right].$$

2. Let us find the distribution function $F(\mathcal{N}_{\text{пор}}/t)$ of the number of sunken ships under the condition that examination of the region loss a determinate time t. The algorithm for finding this function will be considered below. Let us note that the function $F(\mathcal{N}_{\text{пор}}/t)$ is completely defined by the set of probabilities $P\{\mathcal{N}_{\text{пор}} = k/t\}$, $k = 0, 1, \ldots, \mathcal{N}_{\text{п}}$, that is, as the probabilities of loss of a different number of ships with an examination time of the region equal to t.

3. The desired distribution function $F(\mathcal{N}_{\text{пор}})$ in correspondence to the total probability formula is defined by the expression $F(\mathcal{N}_{\text{пор}}) = \int_0^\infty F(\mathcal{N}_{\text{пор}}/t)\,dF_\tau(t)$.

This function is fully defined also by the probabilities

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[1] The case is considered where the given probability of solving the problem P_3 is so great that it is possible to neglect the probability $P\{\tilde{S}(\mathcal{N}_{\text{п}}, \infty) < S_0\}$.

$$P\{\mathcal{N}_{\text{nor}} = k\} = \int_0^\infty P\{\tilde{\mathcal{N}}_{\text{nor}} = k/t\}\, dF_\tau(t), \quad k = 0, 1, \ldots, \mathcal{N}_{\text{n}}.$$
(1)

Key: 1. destroyed

Let us consider the problem of calculating the probability $P\{\tilde{\mathcal{N}}_{\text{nor}} = k/t\}$, $k = 0, 1, \ldots, \mathcal{N}_{\text{n}}$. Under the assumption made at the beginning of this section the process of the variation of the number of operating ships with time can be considered as a Markov process of pure destruction [36]. If the states of the process are numbered by the number of sunken ships, then the graphical process will have the form shown in Figure 3.17. The probabilities of the states of the process satisfy the system of differential equations

$$\begin{gathered} \frac{dP_0(\tau)}{d\tau} = -\mathcal{N}_{\text{n}} P_0(\tau), \\ \frac{dP_k(\tau)}{d\tau} = [\mathcal{N}_{\text{n}} - (k-1)]\, P_{k-1}(\tau) - (\mathcal{N}_{\text{n}} - k)\, P_k(\tau), \\ k = 1, \ldots, \mathcal{N}_{\text{n}}, \end{gathered} \tag{3.60}$$

where $\tau=\lambda t$ is a dimensionless time.

This system of equations must be integrated under the initial conditions

$$P_0(0) = 1, \; P_k(0) = 0, \; k = 1, \ldots, \mathcal{N}_{\text{n}}. \tag{3.61}$$

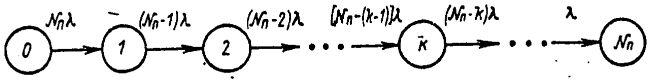

Figure 3.17. Graph of the process of destruction of ships

The solution of the system (3.60) under the conditions (3.61) has the form [36]

$$P_0(\tau) = e^{-\mathcal{N}_{\text{n}}\tau},$$

$$P_k(\tau) = (-1)^k \prod_{i=0}^{k-1} \delta_i \sum_{j=0}^{k} \frac{e^{-\delta_j \tau}}{\prod\limits_{\substack{l=0 \\ l \neq j}}^{k} (\delta_j - \delta_l)}, \quad k = 1, \ldots, \mathcal{N}_{\text{n}} - 1,$$

$$P_{\mathcal{N}_{\text{n}}}(\tau) = 1 - \sum_{k=0}^{\mathcal{N}_{\text{n}}-1} P_k(\tau),$$

where $\delta_k = \mathcal{N}_{\text{n}} - k$, $k = 0, 1, \ldots, \mathcal{N}_{\text{n}}$. The index $\ell \neq j$ in formulas (3.62) means that the product of the differences $(\delta_j - \delta_\ell)$ is considered only for $\ell \neq j$ (for $\ell = j$ the corresponding differences are taken equal to one).

The desired probabilities $P\{\tilde{\mathcal{N}}_{\text{nor}} = k/t\}$ are determined by the equalities $P\{\tilde{\mathcal{N}}_{\text{nor}} = k/t\} = P_k(\lambda t)$, $k = 0, 1, \ldots, \mathcal{N}_{\text{n}}$. Here $P_k(\lambda t)$, $k=0, 1, \ldots, \mathcal{N}_{\text{n}}$, is the solution of the system of equations (3.60) under the conditions (3.61) for $\tau=\lambda t$.

The above-described general approach permits us to find the distribution function of the number of sunken ships, that is, the most complete characteristic of this random variable. At the same time the principle of quasiregularity permits us to obtain simple approximate formulas for the mathematical expectation and dispersion of the random variable $\tilde{\mathcal{N}}_{\text{nor}}$ -- the number of sunken ships.

Using the principle of quasiregularity, we assume that the actual time of examination of the region is a determinate value and can be found from the equation $E_s(\mathcal{N}_{\text{п}}, t) = S_0,$ in which $\mathcal{N}_{\text{п}}$ is considered defined by formula (3.56).

Thus, it is assumed that the examination time of the region is determined by the equality of mathematical expectation of the examined area and the area of the entire region.

Considering the results of the given section, the above-presented equations for t assume the form

$$\mathcal{N}_{\text{п}} \frac{s_1}{\lambda}(1 - e^{-\lambda t}) = S_0.$$

Hence, we find

$$t = -\frac{1}{\lambda} \ln\left(1 - \frac{\lambda S_0}{s_1} \frac{1}{\mathcal{N}_{\text{п}}}\right).$$

Let us note that for $P_3 > 0.5$ ($\nu > 0$) the inequality $\mathcal{N}_{\text{п}} > \lambda S_0 / s_1$ is satisfied and consequently, the value of t is positive.

If the examination of the region lasts a determinate time t, then the mathematical expectation $(E\tilde{\mathcal{N}}_{\text{nor}})$ and the dispersion $(\mathcal{D}\tilde{\mathcal{N}}_{\text{nor}})$ of the number of destroyed ships are equal

$$E\tilde{\mathcal{N}}_{\text{nor}} = \mathcal{N}_{\text{п}}(1 - e^{-\lambda t}), \quad \mathcal{D}\tilde{\mathcal{N}}_{\text{nor}} = \mathcal{N}_{\text{п}} e^{-\lambda t}(1 - e^{-\lambda t}).$$

Substituting the expression for t, we obtain

$$E\tilde{\mathcal{N}}_{\text{nor}} = \frac{\lambda S_0}{s_1}, \quad \mathcal{D}\tilde{\mathcal{N}}_{\text{nor}} = \frac{\lambda S_0}{s_1}\left(1 - \frac{\lambda S_0}{s_1} \frac{1}{\mathcal{N}_{\text{п}}}\right).$$

We again emphasize that the value of $\mathcal{N}_{\text{п}}$ in the formula for the dispersion is defined by the expression (3.56).[1]

Just as in all of the examples of the given chapter, for use of the above-described model during analytical design it is necessary to express the tactical parameters λ, d, v (in the special case d_{damage} and $P_{\text{des.k}}$) in terms of the TTE and the TDP of the ships performing the operation of reconnaissance of the region.

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[1] It is interesting that the mathematical expectation of the number of destroyed ships does not depend on the polygon fleet of ships $\mathcal{N}_{\text{п}}$.

CHAPTER 4. METHODS OF ESTIMATING ECONOMIC INDEXES

During analytical design the expenditures of various types of resources connected with building and maintaining ships are considered. Quantitatively, these expenditures are characterized by the service indexes which, together with the effectiveness indexes, are used to construct the optimization criteria of the TTE and the TDP of ships.

The nature of the service indexes is always varied. For example, if the displacement and the principal dimensions of the ship have great significance (the possibility of building, maintaining, transporting), then these values are considered as service indexes. The service indexes include the design and building time, the number of personnel, and so on. Just as the effectiveness indexes, the service indexes can be of a random nature. However, in contrast to the effectiveness indexes where the random factors constitute the essence of the combat operations and the functioning of the ship, the random nature of the service indexes basically is connected with unreliability of the initial data and imperfection of the calculation techniques. These problems have still been insufficiently investigated; therefore the service indexes at the present time in practice are always considered as determinate values. Here the researchers know that the determinate values of the service indexes are only the mathematical expectations of the corresponding random variables.. The above-indicated determinate approach is understood in the sense that when constructing the optimalness criteria, the distribution functions of the service indexes are not used.

One of the most important forms of the service indexes is the economic index. The cost of building and maintaining the ship is the generalized economic index. In this chapter the methods of determining only the cost service indexes, in particular, the cost of building and maintaining the ships in the fleet are considered.

The cost of building any product is made up of three basic parts: the cost of the consumed production means, that is, the cost of raw materials, fuel, materials and the fraction of the cost of the means of labor corresponding to their wear during the production process; the cost of the personal use fund of the workers performing the labor in the form of wages; the cost of the social use fund connected with expansion of production, the maintenance of the state administrative agencies, public health, education, and so on.

Inasmuch as the determination of the third component of the cost is very difficult, the cost including only the first two cost components is used as the generalized economic index. Thus, only the cost of building and maintaining ships will be considered although instead of the term "complete cost" we shall use the term "cost."

The cost of building a ship in the general case includes the cost of construction and the cost of the planning and design work. However, considering the design cost is connected with defined labor arising first of all from the inadequate study of the relations between the TTE and the TDP and the cost of designing the ship and, secondly, by the fact that the design cost must be distributed to the entire series of ships of a given design being built at the same time as in the AD stage the volume of the series usually is unknown. (The methods of determining the ship design cost are not considered in this book.)

The same general requirements can be imposed on the service indexes as on the effectiveness indexes, that is, criticalness with respect to the TTE and the TDP, calculatability and simplicity. Here criticalness also has important significance, for in subsequent optimization it is necessary that the service indexes simultaneously be critical with respect to the entire set of TTE and TDP selected for optimization.

§4.1. Determining the Construction Cost of Ships

The cost of building a ship is determined by the sum of the expenditures with respect to the following costing items: materials, intermediate products and finished products; the wages of the production workers; the overhead (shop and general plant); deliveries and operations by other contracting parties; other direct expenditures (individual expenditures).

The expenditures on materials, wages, deliveries by other contracting parties and also individual expenditures are direct expenses which pertain to each ship being built. The shop and the general plant overhead are distributed to all of the ships built proportionally to the wages. These are the indirect expenses.

There are [27] three basic methods of calculating the cost of building a ship corresponding to the various design stages: by the consolidated mass[1] normatives (for the conceptual design and the preliminary design); with respect to the consolidated statistical normatives (for the engineering design); with respect to the costing normatives (for the detailed design).

It is natural that in AD it is expedient to use the first of the above-indicated methods. Here the initial data are the mass load items of the ship compiled by the structural breakdown groups. Inasmuch as in the engineering design block (see Chapter 2) the relation is established for the individual load items as a function of the TTE and the TDP of the ship, the method of calculating the cost by the consolidated mass indexes permits establishment of the relation between the cost of building the ship and its TTE and TDP. Let us remember that the cost of the combat supplies on hand (missiles, torpedoes, artillery supplies) and variable loads (the fuel and oil reserves, the ZIP[spare parts, tools and accessories] and other equipment) is not considered. The cost of equipment is

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[1] The mass normative (or index) is the ratio of the labor consumption of manufacturing the product to its mass.

taken into account in the cost of maintaining the ship, and the necessity for considering the cost of the combat supplies on hand arises in cases where ships with different types and forms of weapons are considered during the AD process, that is, the type and form of weapon enter into the optimizable parameters.

In essence the method of calculating the construction cost by the consolidated mass indexes is based on similarity arguments, in accordance with which single-type structural assemblies of the ships are considered similar to each other, and the cost of creating them, proportional to their mass. Just as when calculating the load elements, the similarity methods can be used jointly with statistical methods.

In accordance with the costing items presented at the beginning of this section, the cost of building the ship is represented in the form

$$S = (1 + k_2)(S_{\text{рв}} + S_{\text{м}} + S_{\text{кп}} + S_{\text{пр}}), \qquad (4.1)$$

where S is the cost of building the ship; S_{pB}, S_M, $S_{K\pi}$, $S_{\pi p}$ are the costs of the shipyard operations, materials, deliveries by other contracting agents and other direct expenditures; k_2 is the coefficient taking into account the trading overheads and planned deductions.

The determination of the labor consumption with respect to individual types of operations can be used as the basis for calculating the values of S_{pB} and $S_{\pi p}$: building the hull, installing accessory equipment, machinery and equipment at the shipyard. The labor consumption of the operations is determined as a function of the masses of the individual load items, the type of materials, and so on. In order to establish the type of dependence of the labor consumption on the masses, prototype data or the data from statistical processing of several prototypes are used. Thus,

$$S_{\text{рв}} = (1 + k_1)\sum_{i=1}^{n} s_{\text{нч}\,i} T_i = (1 + k_1)\sum_{i=1}^{n} s_{\text{нч}\,i} f_i(m_i), \qquad (4.2)$$

Key: 1. overhead

where T_i, i=1, ..., n, is the labor consumption with respect to individual structural-technological assemblies; $T_i=f_i(m_i)$, i=1, ..., n, is the dependence of the labor consumption on the corresponding masses; $s_{\text{overhead } i}$, i=1, ..., n, is the cost of a unit of labor consumption (norm-hour) for individual assemblies; k_1 is the overhead coefficient; n is the number of structural-technological assemblies.

Frequently formula (4.2) is simplified by introducing the average cost of a norm-hour with respect to all assemblies

$$S_{\text{рв}} = (1 + k_1)\, s_{\text{нч}} \sum_{i=1}^{n} f_i(m_i), \qquad (4.3)$$

where s_{overhead} is the mean cost of a norm-hour for the defined builder.

In order to find the functions $f_i(m_i)$, as a rule, statistical processing of the data with respect to several prototypes by the methods of regression analysis is used. Here, usually linear models are considered, in accordance with which the functions $f_i(m_i)$ are represented in the form

$$f_i(m_i) = a_i m_i + T_{oi}, \quad i = 1, \ldots, n, \tag{4.4}$$

where a_i, T_{0i}, $i=1, \ldots, n$, are statistical coefficients which depend on the TDP of the ship, for example, the peculiarities of the architecture and the material of the hull, the type of power plant, and so on. These parameters can be optimizable discrete variables.

If the data from one prototype are used, then the regression line passes through the origin of coordinates and the point corresponding to the prototype, then

$$T_i = \frac{T_i^{(\text{пр})}}{m_i^{(\text{пр})}} m_i, \quad i = 1, \ldots, n,$$

where $m_i^{(\text{пр})}$, $T_i^{(\text{пр})}$, $i = 1, \ldots, n$, are the variables pertaining to the prototypes; $T_i/m_i=t_i$ are the specific labor consumptions defined by the prototype, or the statistical data of several prototypes.

When using the specific labor consumptions, the formula for S_{pB} can be written as follows:

$$S_{\text{рв}} = (1 + k_1) \sum_{i=1}^{n} s_{\text{нч}i}\, t_i m_i. \tag{4.5}$$

The cost of other direct expenses $S_{\text{пр}}$ usually is found by the statistical data as a function of the total labor consumption $\sum_{i=1}^{n} T_i$, considering $S_{\text{пр}}$ proportional to the total labor consumption.

The cost of materials depends on the mass of the individual assemblies $S_M=\sum_{i=1}^{n} s_{Mi} m_i$, where s_{Mi} is the specific cost of materials with respect to the i-th assembly (the cost of materials per ton).

In the AD stage the most complex is determination of the cost of the deliveries by other contracting agents $S_{k\pi}$ which is a significant part of the total cost of building ships. According to the data of [47], the deliveries and operations by other contracting agents for civilian maritime ships amount to 25 to 35% of the total cost of the ship. In warships this proportion increases as a result of greater saturation with radio electronics and automation means, the presence of weapons use systems, the application of more complex power plants, and so on. Some idea of the cost of deliveries and operations by other contracting agents for warships can be obtained on the basis of the data on the American nuclear-powered submarines (Tables 4.1, 4.2).

Table 4.1

Cost of Shipyard Operations for the American Nuclear-Powered Submarines (according to the data of [65])

Cost,	Torpedo submarines			Missile submarines	
millions of dollars	"Lipscomb"	"Sturgeon"	"Los Angeles"	"Lafayette"	"Trident"
Total (construction)	178	81.3	233	127	780
Shipyard operations	80	44	83	22	285
The same, %	45	54.2	35	17.3	36.5

Table 4.2

Components (%) of the Construction Cost of American Nuclear-Powered Submarines (according to the data of [10])

Type of submarine	Shipyard operations	Materials	Operations and deliveries by other contracting agents
Torpedo	43-46	14-15	40-42
Missile	33038	15-16	48-50

In the engineering and detailed design stages the problem of determining the cost of deliveries by other contracting agents does not come up for the ship designer inasmuch as in these stages, as a rule, there are already technical specifications for the delivery of products by other contracting agents with the application of the cost determined by the suppliers. During AD, when it is necessary to know the cost as a function of a number of characteristics of the products of other contracting agents, the problem is more complicated, for the products delivered by other contracting agents are created by many branches of industry, each of which has its own characteristic features arising from the specific nature of the built products and production facilities. These peculiarities are not always known to the ship designers or they cannot be taken into account sufficiently completely and with sufficiently high quality. Although the method of determining the construction cost by the consolidated mass indexes is generally accepted in the metals industry, it usually cannot be used for radio electronic equipment, navigation and communications system, information control systems, and so on.

Beginning with what has been said, the cost of the enumerated products and systems is calculated by the data of the other contracting agents considering the composition and the type of accessory equipment. With this approach the optimizable (during the AD process) characteristics can turn out to be only the type and quantitative composition of the accessory equipment, where the type is a discrete variable, and the quantitative composition can be approximately considered a continuous variable.

For all the remaining products of other contracting agents (except the accessory equipment), the overwhelming majority of which is made up of mechanical and electrotechnical products, theoretically it is possible to use the method of calculating the cost by the consolidated mass indexes in terms of labor consumption with respect to the known specific labor consumption of the structural-technological assemblies of the corresponding products and also the data on the cost of a norm-hour at the manufacturing plants. Obtaining the indicated data is difficult as a result of the large nomenclature of products, their structural differences and the large number of manufacturing plants. Under these conditions it is possible to recommend the method of determining the cost directly by the mass indexes without intermediate determination of the labor consumption. With this approach the cost ($S'_{k\pi}$) of the products of other contracting agents in the investigated group is defined by the formula

$$S'_{\text{кп}} = \sum_{j=1}^{l} s_j m_j, \tag{4.6}$$

where m_j is the mass of the product of the j-th type of the other contracting agents; s_j is the specific cost (cost per ton of mass) of the j-th type product; ℓ is the number of types of products of other contracting agents in the investigated group.

The values of s_j are found by the data from prototypes considering the type and the characteristic features of the corresponding products. For example, the specific cost of various types of power plants (nuclear steam turbine, non-nuclear steam turbine, diesel, gas turbine, and so on) must be determined separately. Here the type and layout of the power plant can be discrete optimizable variables. Inasmuch as the mass m_j depends on the characteristics of the corresponding products and the ship as a whole, the function (4.6) determines the relation of $S'_{k\pi}$ to the TTE and the TDP of the ship.

Thus, the cost of building the ship is defined by the general expression

$$S = (1 + k_2)\Big[(1 + k_1)\sum_{i=1}^{n} s_{\text{нч}\,i} f_i(m_i) + \sum_{i=1}^{n} s_{\text{м}i} m_i + {} + k_{\text{пр}} \sum_{i=1}^{n} f_i(m_i) + \sum_{j=1}^{l} s_j m_j + \underset{(1)}{S_{\text{воор}}}\Big], \tag{4.7}$$

Key: 1. accessory equipment = acc

where S_{acc} is the cost of the accessory equipment; $k_{\pi p}$ is the coefficient of other direct expenses.

Correspondingly, when using the average cost of a norm-hour and the specific labor consumption we have

$$S = (1 + k_2)\left[k \sum_{i=1}^{n} t_i m_i + \sum_{i=1}^{n} s_{\text{м}i} m_i + \sum_{j=1}^{l} s_j m_j + S_{\text{воор}}\right], \tag{4.8}$$

where $k = (1 + k_1)\, s_{\text{нч}} + k_{\text{пр}}$.

The specific expansion of expressions (4.7) or (4.8) as functions of TTE and TDP depends on the type and structural characteristics of the designed ship.

Expression (4.8) can be represented in the form

$$S = (1 + k_2)\left[k\sum_{i=1}^{n} t_i\overline{m}_i + \sum_{i=1}^{n} s_{\text{м}i}\overline{m}_i + \sum_{j=1}^{l} s_j\overline{m}_j\right]D + (1 + k_2)\,S_{\text{воор}}, \tag{4.9}$$

where D is the displacement of the ship; $\overline{m}_i$ are the relative masses ($\overline{m}_i$=mi/D) of the structural-technological assemblies.

For ships that are similar to each other, the values entering into the coefficient in front of the displacement in formula (4.9) are approximately identical. This makes it possible to propose another more approximate formula for calculating the cost of building the ship

$$S = s_D D + (1 + k_2)\,S_{\text{воор}}, \tag{4.10}$$

where s_D is the cost of all expenses (in addition to the cost of accessory equipment) per ton of displacement.

For civilian ships the value of S_{acc} basically including the navigational and communications means is relatively low and, according to the data of B. M. Smirnov [47], it amounts to 2-3% of the total cost of the ship. Therefore formula (4.10) for civilian ships can be represented in the form

$$S = s_D D. \tag{4.11}$$

The cost (in thousands of dollars) of a ton of standard displacement of U.S. Navy ships according to the data of [10] is:

Submarines:	
Diesel electric	12-14
Nuclear-powered torpedo submarines of the	
"Skipjack" type	16.5-21.5
"Thresher" type	13.5-16
Nuclear-powered missile submarines	
"George Washington" type	16-20
"Lafayette" type	16-17
Trawlers	4-5
Destroyers, frigates	7-11
Aircraft carriers	3-4
Nuclear-powered frigates	15-19
Nuclear-powered cruisers	23
Nuclear-powered aircraft carriers	5

It is possible to more precisely define formulas (4.10) and (4.11) somewhat by dividing the load into two basic component parts: the hull with equipment and the power plant. In this case, instead of (4.10) and (4.11) we obtain

$$S = (s_{\text{ко}}\bar{m}_{\text{ко}} + s_{\text{эу}}\bar{m}_{\text{эу}})\,D, \tag{4.12}$$

$$S = (s_{\text{ко}}\bar{m}_{\text{ко}} + s_{\text{эу}}\bar{m}_{\text{эу}})\,D + S_{\text{воор}}, \tag{4.13}$$

where $\bar{m}_{\text{ко}}, \bar{m}_{\text{эу}}$ are the relative masses of the hull with equipment and power plants; $s_{\text{ко}}, s_{\text{эу}}$ are the specific cost per unit mass of the hull with equipment and power plant.[1]

§4.2. Determining the Construction Cost of a Series of Ships. Consideration of the Size of the Series

In some cases it is necessary to consider the so-called "series factor" in the calculations of determining the cost of building a fleet of ships. This factor is connected with a decrease in construction cost with an increase in the order number of the ship in the series. As is known, with an increase in the number of ships in series the materials are used more efficiently, the labor consumption goes down as a result of improvement of technology, introduction of specialization and so on. These factors also lead to a decrease in cost of deliveries by other contracting agents.

In the general case the cost $S(\mathcal{N})$ of construction of $\mathcal{N}$ ships is equal to the sum of the cost of the individual ships:

$$S(\mathcal{N}) = \sum_{i=1}^{\mathcal{N}} S_i, \tag{4.14}$$

where S_i is the cost of a ship with the i-th order number.

The dependence of the cost S_i on the order number i is characterized by the following properties: for i=1, that is, for the prototype in the series, the cost is maximal; for $i \to \infty$ the values of S_i approach some limit -- the cost of the series ship. The exact establishment of the dependence of S_i on i requires a detailed analysis of the reduction in expenditures with respect to each of the component parts of the total cost. In the initial design stages this analysis is difficult; therefore the statistical data with respect to the ship as a whole are used. The following dependence of S_i on i satisfying the above-indicated general properties can be proposed for statistical analysis:

$$S_i = S_\infty\left(b + \frac{a}{i}\right), \tag{4.15}$$

where S_∞, a, b are statistically determined values.

From expression (4.15) it follows that

$$\frac{S_i}{S_1} = \frac{1}{a+b}\left(b + \frac{a}{i}\right).$$

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[1] V. L. Pozdyunin [38] indicated the possibility of approximate determination of the construction cost by formula (4.12).

The last expression permits determination of the coefficients a and b on the basis of statistical data on the values of S_i/S_1.

The algorithm for calculating the cost of the ship usually is constructed as applied to some fixed order number n. In this case, it is necessary in formula (4.15) to set

$$S_\infty = \frac{S_n}{b + a/n}.$$

Thus, for the cost of building $\mathcal{N}$ ships, we obtain

$$S(\mathcal{N}) = aS_\infty \varphi(\mathcal{N}), \tag{4.16}$$

where

$$\varphi(\mathcal{N}) = \sum_{i=1}^{\mathcal{N}} \frac{1 + \bar{b}i}{i}, \quad \bar{b} = \frac{b}{a}.$$

The function $\varphi(\mathcal{N})$ satisfies the required relations

$$\varphi(\mathcal{N}) = \varphi(\mathcal{N} - 1) + \frac{1}{\mathcal{N}} + \bar{b}, \tag{4.17}$$

which permits quite simple calculation of $\varphi(\mathcal{N})$ for small values of $\mathcal{N}$. For sufficiently large $\mathcal{N}$ from (4.17) we obtain equation

$$\frac{d\varphi(\mathcal{N})}{d\mathcal{N}} = \bar{b} + \frac{1}{\mathcal{N}},$$

the solution of which gives

$$\varphi(\mathcal{N}) = \bar{b}\mathcal{N} + \ln \mathcal{N} + \varepsilon,$$

where ε=0.577 is the Euler constant. The last formula facilitates the calculation of $\varphi(\mathcal{N})$ for large $\mathcal{N}$.[1]

It is necessary to turn attention to the correctness of considering the series factor on the procedural level inasmuch as the fleets of ships considered during military and economic studies frequently are only provisional values and do not have direct bearing on the actual volume of the construction series of the ships. This situation usually occurs when optimizing the TTE and the TDP of the ship by the criterion of minimum cost of the fleet solving a defined problem with given level of effectiveness. Considering the increase in cost for the first ship of the series becomes meaningless in this case, and it is necessary to take the cost of the series ship in the calculation. The cost of building the fleet in this case is a linear function of the number of ships.

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[1] For example, for $\bar{b}$=1 this formula gives an error of 3.2% for $\mathcal{N}=3$, and for $\mathcal{N}=5$ the error does not exceed 4% for any values of b.

At the same time using the criterion of maximum effectiveness for the given budget for building and maintaining the fleet the series factor must be considered if the budget corresponds to its actual value.

§4.3. Determination of the Cost of Maintaining Ships

The cost of maintaining a ship in the fleet is made up of two parts: the cost of operation and maintenance and the cost of building and maintaining the base support means (the basing cost).[1]

The problems of determining the basing cost are considered in this book. The base support system in the general case contains shore and floating facilities. In order to determine the cost of building and maintaining the floating facilities the same procedure can be used as for ships. Calculating the cost of shore facilities requires the application of special methods.

It is necessary to note the complexity of the problem of the basing cost distribution for individual ships or forces of ships. Similar difficulties arise also when considering the cost of other types of shore and floating forces and means of supporting the fleet operations such as communications, navigation, and so on. For these reasons it is expedient to consider the cost only of that part of the specific means which are needed to support the activity of the class and type of ships considered in the AD process. For example, if ships with their own detection and target indication means are compared with ships using external sources of information, the cost of the latter means must be taken into account. Analogously, when comparing ships with nuclear and nonnuclear power plants, the cost of the special means of shore support of the ships with nuclear power plants is taken into account.

The cost of operating and maintaining a ship is made up of the basic components connected with repairs and planned replacement of equipment; material and technical supply, including fuels and lubricants and also consummable training combat supplies; maintenance of personnel.

The operating and maintenance cost usually pertains to one year of service life of the ship. For calculation of it a time graph is constructed (see Figure 4.1) for the use of the ship between successive deliveries of it to the repair yard (considering the duration of one yard repair).

For ships with nuclear power plants, periodic recharging of the reactive cores must be taken into account, which can be combined with yard repair.

If the time parameters A, $T_{M\pi}$ and $T_{\text{yard repair}}$ are known, where A is the sea endurance, $T_{M\pi}$ is the time between voyages, $T_{\text{yard repair}}$ is the yard repair

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[1] The cost of maintaining a ship includes a number of other expenses (training of personnel, maintenance of central administration and so on), but the expenditures on operation, maintenance and basing are primary and direct.

time, then the mean annual operating maintenance cost ($s'_\text{э}$) can be found by the formula

$$s'_\text{э} = s'_\text{лс} n_\text{лс} + \frac{\frac{R}{A} S^{(1)}_\text{мто} + \left(\frac{R}{A} - 1\right) S^{(1)}_\text{мпр} + S^{(1)}_\text{зр}}{R + \left(\frac{R}{A} - 1\right) T_\text{мп} + T_\text{зр}}, \tag{4.18}$$

where R is the reserve of the basic ship's equipment before the yard repair; $S^{(1)}_\text{мпр}$ is the cost of one repair between voyages; $S^{(1)}_\text{зр}$ is the cost of one yard repair; $S^{(1)}_\text{мто}$ is the cost of material and technical supply for one voyage; $s'_\text{лс}$ is the average cost of maintaining one crewman per unit time (for a year); $n_\text{лс}$ is the number of crew.

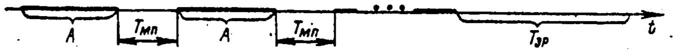

Figure 4.1. Time graph of the use of ships between yard repairs

The parameters $T_\text{мп}$, $T_\text{зр}$ and costs $S^{(1)}_\text{мпр}$, $S^{(1)}_\text{мто}$, $S^{(1)}_\text{зр}$ depend on the TTE and the TDP of the ship.

Theoretically the cost of the annual operation and maintenance of a ship must increase with time, at least for the subsequent periods between successive yard repairs. The reason for this is the physical wear of the machinery, equipment and hull, which increases the frequency and volume of repairs. In cases where obtaining the indicated function is difficult, the annual operating and maintenance cost can be assumed invariant for the entire service life of the ship.

In spite of the external simplicity of the formula (4.18), the expansion of the relations for the variables entering into it as a function of the TTE and TDP of the ship presents defined difficulties, especially in the initial design stages. Accordingly, the annual operating and maintenance cost of the ships sometimes is expressed as a fraction of the construction cost. For example, for civilian ships, according to the data of [47], the depreciation deductions are taken equal to 5-6% of the construction cost, the cost of each current and medium repair, 3-3.5%, respectively, and the expenditures on material and technical support (except fuel), 0.5%. The indirect expenditures (general operation and administrative-technical expenditures on steamship lines), are taken equal to 25% of the cost of the crew maintenance according to the statistical data.

Thus, for approximate calculation it is possible to propose the formula

$$s'_\text{э} = \bar{c}_s S, \tag{4.19}$$

where S is the construction cost; $\bar{c}_s$ is the coefficient defined by the prototype (or by several prototypes).

In §4.2.it was demonstrated that in a number of cases the construction cost can be approximately considered proportional to the ship displacement. Consequently, when using formula (4.19) it is also possible to consider the annual operating and maintenance cost proportional to the displacement. For this reason, for a long time during the period of evolutionary development of shipbuilding the displacement was used as the economic service index. At the present time when ships are equipped with expensive accessory systems, power plants and special systems, the use of displacement as the economic service index is insufficient inasmuch as a more detailed expansion of the dependence of the cost of building and maintaining the ship on its TTE and the TDP is required.

During the military-economic studies it is necessary to know the cost of operating and maintaining a fleet of ships for the time of its actual operation in the fleet. When determining this cost usually a function is used that is linear with respect to the number of ships $\mathcal{N}$

$$S_{\text{э}}(\mathcal{N}, T) = S_{\text{э}1}(T)\,\mathcal{N}, \tag{4.20}$$

where $S_{\text{э}}(\mathcal{N}, T)$ is the cost of operating and maintaining $\mathcal{N}$ ships for the time T (the time the ship is in the fleet); $S_{\text{э}1}(T)$ is the operating and maintenance cost of one ship for the time T.

Let us consider the peculiarities of calculating $S_{\text{э}1}(T)$ beginning with the assumption of invariability of the annual operating and maintenance cost $s'_{\text{э}}$ for the entire service life of the ship.

One of the peculiarities of determining the total operating and maintenance cost of a ship for the entire time it is in the sea consists in the necessity for reducing the expenditures at different times to a single point in time, for example, the time of completion of construction, inasmuch as the expenditures which will be made t years after the current point in time must be decreased by $(1+\alpha)^t$ times, where α is the so-called discount rate or the normative investment effectiveness coefficient [27]. Introduction of the discount rate is explained by the fact that one ruble invested in the national economy today will bring a profit after t years and become equivalent to $(1+\alpha)^t$ rubles. Therefore the expenditures of 1 ruble today are more significant than the expenditures of the same ruble after t years.

Thus, under the condition of constancy in time of the value of $s'_{\text{э}}$ the operating and maintenance cost of a ship for T years is calculated by the formula

$$S_{\text{э}1}(T) = s'_{\text{э}} \sum_{t=1}^{T} \frac{1}{(1+\alpha)^t} = s'_{\text{э}} \frac{1}{\alpha}\left[1 - \frac{1}{(1+\alpha)^T}\right]. \tag{4.21}$$

Formula (4.21) assumes that the annual expenditures pertain to the end of each year; therefore during the first year of operation the reduced expenditures will be not $s'_{\text{э}}$, , but $s'_{\text{э}}\ 1 + \alpha.$. If the annual expenditures are reduced to the beginning of the year, then

$$S_{\text{э}1}(T) = s'_{\text{э}} \sum_{t=1}^{T} \frac{1}{(1+\alpha)^{t-1}} = s'_{\text{э}} \frac{1+\alpha}{\alpha}\left[1 - \frac{1}{(1+\alpha)^T}\right]. \tag{4.22}$$

For sufficiently small values of α formulas (4.21) and (4.22) give close results. In practice α does not exceed 0.08-0.10.

It is characteristic that for $T \to \infty$ the formulas (4.21) and (4.22) give finite values of $s'_э \alpha$ and $s'_э (1 + \alpha) \alpha$, respectively. Generally speaking, this is an unexpected result.

For the fixed time T and sufficiently small values of α the formula (4.21) has the form

$$S_{э1}(T) \approx s'_э T, \tag{4.23}$$

inasmuch as for small α the approximate equality $(1+\alpha)^{-T} \simeq 1-\alpha T$ is valid. In this case the discount rate in practice is not considered.

For final determination of the operating and maintenance cost of the ship it is necessary to establish the length of time for which the annual operating and maintenance cost must be summed. For civilian ships this period can be considered to coincide with the service life of the ship, and the latter, in turn, is determined by the nature of variation in time of the obsolescence and physical wear of the ship. (In Chapter 6 some approaches to solving the problem of substantiating the service life of the ships will be considered.)

The operating time of warships in the fleet is equal to the sum of the operating time in peacetime and the lifetime (before destruction) during a war. Both of these values are random variables which depend on many factors.

Let us consider one of the approaches to the determination of the operating time of warships.

The random time τ of operation of the ship can be represented in the form $\tau = \tau_{MB} + \tau_{BB}$, where τ_{MB}, τ_{BB} are the random operating time of the ship during peacetime and during wartime, respectively.

For a fixed service life $T_{service}$ the average operating time of the ship during peacetime can be found by the formula

$$T_{мв} = T^{у}_{мв} F(T_{сл}) + T_{сл} [1 - \underset{(1)}{F(T_{сл})}], \tag{4.24}$$

Key: 1. service

where F(t) is the time distribution function before the beginning of the war after construction of the ship; $T^{у}_{мв}$ is the provisional mathematical expectation of the value of τ_{MB} under the condition $\tau_{MB} < T_{service}$ (the war begins during the service life).

In order to find the distribution function F(t) let us use the data from article [66] in which the results of statistical studies of the estimation of the duration and scale of wars (log M) are presented for the period from 1820 to 1949, where M is the human casualties in the war.

According to the data in this article the flow of "critical incidents" capable of growing into a war can be considered Poisson with some constant for each historical period with an intensity δ_{ci}. The value of δ_{ci} for the period from

1900 to 1949 (the last of those investigated in the paper) with casualties of more than 10^3 men was estimated as 2 to 3 cases per year. However, each such incident does not lead to a "big" war. The probability that the war will reach a scale of log $M \geqslant 6$ for the period from 1900 to 1949 is estimated at $3.2 \cdot 10^{-3}$. The general trend consists in the fact that the intensity of "critical incidents" decreases with time with a simultaneous increase in probability of growth of the incident into a "big" war.

Thus, the intensity of the occurrence of a "big" war according to the experience from 1900 to 1949 can be estimated by a value of $\delta \simeq 3 \cdot 10^{-3} \delta_{ci} \simeq 10^{-2}$ 1/year. For a Poisson flow of occurrence of war we have

$$F(t) = 1 - e^{-\delta t}. \tag{4.25}$$

On the basis of the absence of the aftereffect in the Poisson flow formula (4.25) is valid for any point of reckoning the time.

In accordance with (4.24) and (4.25) we find

$$T_{\text{мв}} = \frac{1}{\underset{(1)}{\delta T_{\text{сл}}}} \left(1 - e^{-\delta T_{\text{сл}}}\right) T_{\text{сл}} = \varphi(\delta T_{\text{сл}}) T_{\text{сл}},$$

Key: 1. service

where

$$\varphi(\delta T_{\text{сл}}) = \frac{1 - e^{-\delta T_{\text{сл}}}}{\delta T_{\text{сл}}}.$$

From Table 4.3 it follows that for real values of the service life it is possible to consider $T \simeq T_{MB} \simeq T_{service}$, that is, the average operating time (T) of the ship in the fleet is approximately equal to its service life. Accordingly, during the military-economic studies the operating and maintenance cost must be considered for the entire service life of the ship.

Table 4.3

Values of the Coefficient $\phi(\delta T_{service}) = T_{MB}/T_{service}$

δ $T_{service}$	0.10	0.15	0.20	0.25	0.30
$T_{service}$ (years) for $\delta = 10^{-2}$ 1/year	10	15	20	25	30
$\phi(\delta T_{service})$	1.00	0.94	0.90	0.88	0.87

§4.4. General Formula for Determining the Cost of Building and Maintaining a Fleet of Ships. The Cost of Solving the Problem as One of the Military-Economic Indexes

The total cost of building and maintaining a fleet of $\mathcal{N}$ ships can be represented in the form [1]

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[1] In the general case the cost of building a ship and, consequently, a fleet of ships, can also depend on the service life, but this fact is not reflected in Formula (4.26).

$$S_\Sigma(\mathcal{N},\ \underset{(1)}{T}) = S(\mathcal{N}) + \underset{(3)}{S_{\text{э}}}(\mathcal{N},\ T_{\text{сл}}) + \underset{(2)}{S_{\text{баз}}}(\mathcal{N},\ T_{\text{сл}}), \qquad (4.26)$$

Key: 1. service; 2. basing; 3. operation and maintenance

where $S(\mathcal{N})$ is the cost of building the fleet (S_1 is the cost of building the prototype ship):

$$S(\mathcal{N}) = aS_\infty \sum_{i=1}^{\mathcal{N}} \frac{1+\bar{b}i}{i} = \frac{S_1}{1+\bar{b}} \sum_{i=1}^{\mathcal{N}} \frac{1+\bar{b}i}{i};$$

$\bar{S}_{\text{э}}(\mathcal{N},\ T_{\text{сл}}) = s'_{\text{э}}\mathcal{N}\frac{1}{\alpha}\left[1 - \frac{1}{(1+\alpha)^{T_{\text{сл}}}}\right]$ is the cost of operating and maintaining the fleet; $S_{\text{base}}(\mathcal{N},\ T_{\text{сл}})$ is the cost of basing the fleet; $S_\Sigma(\mathcal{N},\ T_{\text{сл}})$ is the total cost of building and maintaining the fleet of ships.

In order to use the formula (4.26) it is necessary to express the individual components of the righthand side in terms of the TTE and the TDP of the ship subject to optimization. As the economic index it is possible to consider the fleet of ships which will be built and will be maintained in the fleet for some fixed sum of expenditures $S_{\Sigma 3}$, that is, under the condition $S_\Sigma(X,\ \mathcal{N}) \doteq S_{\Sigma з}$. The fleet number found from this equation or the equation $Э(X,\ \mathcal{N}) \doteq Э_з$, is considered as an indirect index of effectiveness or economic index in cases where optimal correspondence between the quantitative $(\mathcal{N})$ and qualitative (the vector X) characteristic of the fleet of ships is meaningful. This statement of the problem is highly typical, but not unique. When designing ships of certain classes each individual ship has a defined mission which does not permit by statement (or theoretically) compensation for negative effects on the qualitative characteristic of the ships by increasing their number. For example, when designing large surface ships (battleships and cruisers) frequently the "duel principle" has been used, in accordance with which it was necessary to insure superiority or equality during combat engagement with an analogous ship of the enemy.

The cost of building and maintaining the fleet of ships is the complete economic characteristic in the case where the model of the effectiveness estimate takes into account the entire volume of missions carried out by the ship during its entire service life. If the model of estimating the effectiveness is constructed considering only a single type of operation, it is necessary to consider the losses of the ship, including the cost of the lost ships. It is natural that ships having identical level of effectiveness of solving the stated problem, identical cost of construction and maintenance but different losses cannot be considered equivalent.

In connection with the above-indicated fact, the necessity arises for calculating the cost of the losses. The latter, and more precisely, its mathematical expectation, for a uniform fleet can be represented in the form

$$S_{\text{п}}(X,\ \mathcal{N}) = \sum_{i=1}^{\mathcal{N}} S_{0i}(X,\ t_{\text{к}})\, p_i(X,\ \mathcal{N}),$$

where $S_{0i}(X, t_k)$ is the residual cost of the i-th ship at the time t_k of completion of the operation; $p_i(X, \mathcal{N})$ is the probability of loss of the i-th ship; X is the vector of the TTE and the TDP of the ship.

If the residual cost and probabilities of loss of each ship are identical, we obtain the formula

$$S_п(X, \mathcal{N}) = S_0(X, t_к)\overline{\mathcal{N}}_п(X, \mathcal{N}), \tag{4.27}$$

where $\overline{\mathcal{N}}_п(X, \mathcal{N})$ is the average number of lost ships.

The exact calculation of the residual cost is a very complex problem. This cost depends on the obsolescence and the physical wear of the ship at the investigated point in time. (Certain arguments with respect to the calculation of the residual cost considering obsolescence and physical wear will be presented in Chapter 6.) It is frequently approximately assumed that the residual cost varies linearly in time from the construction cost to zero for $t = T_{service}$, that is, the residual cost of the ship serving its entire service life is neglected (Figure 4.2). In this case we have

$$S_0(X, t_к) = S(X)\left(1 - \frac{t_к}{T_{сл}}\right),$$

where S(X) is the construction cost.

For civilian ships the value of $S_0(X, t_k)$ is called the balance cost, and the residual cost is the cost corresponding to the end of the service life, assuming that it is not zero.

Thus, formulas (4.27) can be represented in the form

$$S_п(X, \mathcal{N}) = S(X)\left(1 - \frac{t_к}{T_{сл}}\right)\overline{\mathcal{N}}_п(X, \mathcal{N}). \tag{4.28}$$

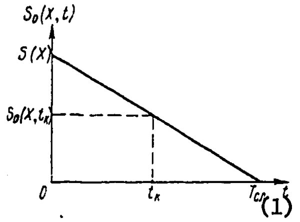

Figure 4.2. Linear dependence of the residual cost of a ship on its operating time

Key:

1. $T_{service}$

For multiple-use ships, the so-called cost of solving the problem is used as the economic index in certain cases. In addition to the cost of losses this includes expenditures on operation and maintenance of the ships during performance of the operation, the cost of the consumed combat supplies and the change in residual cost during this time period. (The change in residual cost plays the role of depreciation deductions here.)

The cost of solving the problem is calculated by the formula

$$\underset{(1)}{S_{\text{реш. з}}}(X,\ \mathcal{N}) = S(X)\left(1 - \frac{t_{\text{к}}}{T_{\text{сл}}}\right)\overline{\mathcal{N}}_{\text{п}}(X,\ \mathcal{N}) + \\ + s_{\text{э}}'(X)\,\mathcal{N}\,\Delta t + S(X)\frac{\Delta t}{T_{\text{сл}}}[\mathcal{N} - \overline{\mathcal{N}}_{\text{п}}(X,\ \mathcal{N})] + \underset{(2)}{S_{\text{бз}}}, \qquad (4.29)$$

Key: 1. solving problem; 2. combat supplies

where Δt is the duration of the operation; $S_{\text{combat supplies}}$ is the cost of the consumed combat supplies on hand; $\mathcal{N}$ is the fleet of ships providing for solution of the stated combat problem with given effectiveness.

§4.5. Power Correlation-Regression Models of Estimating the Cost Service Indexes

The methods of determining the cost of building and maintaining ships discussed above are based on physical representation of the total cost in the form of the sum of the individual components. Here provision is made for using the statistical method to determine a number of values and functions. For example, statistical methods can be used to establish the dependence of the labor consumption of the individual types of operations of building the ship on the masses of the corresponding structural assemblies.

Recently the purely statistical power correlation-regression models of determining the cost of building and maintaining ships have become widespread. In these models the cost is represented in the form of the product of the defining variables (most frequently the masses of the individual load items) raised to some powers. For example, in [34] the following relation is presented for the cost of constructing cargo motor vessels of the "river-sea" class

$$S = 1{,}83 m_{\text{к}}^{0.82} m_{\text{м}}^{0.093} m_{\text{э}}^{0.074}, \qquad (4.30)$$

where m_k is the mass of the hull with equipment and the general ship's systems; m_M is the mass of the main engine and the engine room equipment, $m_{\text{э}}$ is the mass of the electrical equipment, radio and radar equipment.

The coefficient and the exponents in these relations are determined by the least squares method as applied to the linear regression model which is obtained by logarithmizing the initial function.

The basic deficiency of the power models of estimating cost consists in the fact that with respect to structure they do not correspond to the physical representation of the total cost of building and maintaining the ship as the sum of individual components. At the same time the statistical method must, as a rule, be applied not to arbitrarily selected functions, but for correction (by statistical determination of individual parameters) of the models at least approximately corresponding to the physical essence of the investigated phenomenon. This approach was illustrated in Chapter 2 in the example of determining the mass of the pressure hull of a submarine.

In connection with what has been discussed above, it must be recognized that a function of the type of (4.13) is preferable to the power function (4.30) with consideration of the fact that in the function of the type (4.13) it is possible to represent the cost of the accessory equipment (S_{acc}) in the form of a function of its mass, including the cost of electrical equipment in this mass.

CHAPTER 5. OPTIMIZATION OF TACTICAL-TECHNICAL CHARACTERISTICS OF SHIPS

Ships are made up of a set of interrelated subsystems with different purposes, different nature of use and a large number of possible states. This gives rise to the complexity of the processes of the functioning of ships both with respect to variation and state of the individual subsystems and from the point of view of interaction with the external environment, including the enemy. Therefore the structure of the criteria for optimizing the TTC of ships is highly complex and requires a significant volume of calculations to find the optimum. As a rule, ships are multipurpose, and this means that the problems of organizing their TTC are organically characterized by a multicriterial nature. The methods of solving such problems are less formalized than when solving the problems of single-criterial optimization. Finally, the optimization of the TTC of ships is also complicated by the undefined nature of much of the initial data on the operative-tactical, technical and economic levels.

Everything that has been stated above emphasizes the problematic nature of the problems of optimizing the TTC of ships when designing them. A brief characteristic is presented below of the methods of solving these problems beginning with the construction and analysis of the military-economic optimization criteria to the methods of finding the optimal solutions under the conditions of multicriterialness and indeterminacy. In order to keep the discussion clear and understandable, we shall illustrate the theoretical principles in simple examples.

§ 5.1. "Cost-Effectiveness" Type Criteria and Their General Properties

The first step in solving the problems of optimizing the TTE and TDP of ships is selection of the optimalness criterian -- the set of conditions which must be satisfied by the optimal vector X of the variable TTE and TDP. In the AD stage the military-economic criteria of the "cost-effectiveness" type are most frequently used. Compiled from the conditions imposed on the effectiveness indexes, economic and other indexes, they can also contain restrictions of the individual TTE and TDP of the ship. For example, the limits can be given for the maximum speed of the ship, its displacement, and so on.

Two basic types of criteria are used for military-economic optimization (MEO).

1. The optimization of the index or in the general case, indexes of effectivness with restrictions on the cost service index (indexes). In the case of one effectiveness index and one economic index this criterion has the form

$$\max_{X \in \mathfrak{X}} \text{Э}(X, \mathcal{N}), \qquad S(X, \mathcal{N}) < S_0, \tag{5.1}$$

where S_0 is the given value of the cost service index; $\mathfrak{X}$ is the set of admissible values of the vector X without considering the service restriction; $\mathcal{N}$ is the fleet of ships.

2. Optimizing the cost service index (indexes) with limitation of the effectiveness index.

Mathematically, this is written as follows:

$$\min_{X \in \mathfrak{X}} S(X, \mathcal{N}), \qquad \text{Э}(X, \mathcal{N}) > \text{Э}_0, \tag{5.2}$$

where Э_0 is the given effectiveness level; $\mathfrak{X}$ is the set of admissible values of the vector X without considering the condition $\text{Э}(X, \mathcal{N}) > \text{Э}_0$.

In criteria (5.1) and (5.2) it is proposed that an increase in the effectiveness index corresponds to an increase in the effectiveness which obviously does not limit the generality of the arguments. Hereafter by $S(X, \mathcal{N})$ we shall mean any of the cost indexes investigated in Chapter 4.

In the simplest case the set $\mathfrak{X}$, in which max Э or min S is found can be given in the form of a parallelepiped which corresponds to the system of inequalities

$$x_{i\,\min} < x_i < x_{i\,\max}, \quad i = 1, \ldots, n, \tag{5.3}$$

where x_i, i = 1, ..., n are the components of the vector X -- variable TTE and TDP.

In the more complex case the restrictions on the dependent TTE (components of the vector Y), that is, some of the functions of the vector X, are added to the restrictions of the type of (5.3). Finally, in the still more general case the restrictions on certain indexes which are functions of the vectors X and Y can participate in the formation of the set $\mathfrak{X}$. For example, if the engine power appears as one of the variable characteristics, then the assignment of the minimum admissible maximum speed of the ship reduces to third type restrictions inasmuch as the maximum speed is a function of the engine power (a component of the vector X) and displacement (a component of the vector Y).

Let us consider the course of the solution of the optimization problem in the case of the criterion (5.2).

From the general arguments it is possible to assume that the minimum cost $S(X, \mathcal{N})$, corresponding to the optimal solution for each fixed value of Э_0 does not decrease monotonically with an increase in Э_0. This is entirely natural. The higher the effectivness level, the more expensive it is. It follows from this proposition that the optimal solution will be reached under the condition $\text{Э}(X, \mathcal{N}) = \text{Э}_0$ (the restriction in the form of an inequality is replaced by an equality). Entirely analogously, in the criterion (5.1) it is possible to assume that the maximum effectiveness is a nondecreasing function of the value of S_0 and, accordingly, the inequality $S(X, \mathcal{N}) \leq S_0$ is replaced by an equality.

From the equation $Э(X, \mathcal{N}) = Э_0$ the fleet number $\mathcal{N}$ can at least theoretically be found as a function of the vector X and the value of $Э_0$, that is, the function $\mathcal{N}(X, Э_0)$ is found. Subsituting this function in the purpose function $(X, \mathcal{N})$, we arrive at the problem

$$\min_{X \in \mathfrak{X}} S(X, Э_0), \tag{5.4}$$

which is the general problem of mathematical programming.

As a result of solving problem (5.4), the optimal vector $X_{opt}(Э_0)$ is found which in the general case depends on the given effectivness level $Э_0$. If this function is substituted in the function $\mathcal{N}(X, Э_0)$, we obtain the fleet number $\mathcal{N}_{opt}(Э_0)$, corresponding to the optimal version of the ship (the vector X_{opt}). Finally, substituting the functions $X_{opt}(Э_0)$ and $\mathcal{N}_{opt}(Э_0)$ in the purpose function $S(X, \mathcal{N})$, we obtain the optimal cost of the fleet of ships solving the problem with the given effectiveness level. This cost $S_{opt}(Э_0)$ depends only on the value of $Э_0$. Repeating the analogous arguments in the case of criterion (5.1), it is possible to establish that the results of the solution will be the functions $X_{opt}(S_0)$, $\mathcal{N}_{opt}(S_0)$ and $Э_{opt}(S_0)$.

For the assumptions made above with respect to the general properties of the functions $S_{opt}(Э_0)$ and $Э_{opt}(S_0)$, and namely S_{opt} does not decrease with an increase in $Э_0$ and $Э_{opt}$ does not decrease with an increase in S_0, the functions $S_{opt}(Э_0)$ and $Э_{opt}(S_0)$ define each other mutually uniquely. The function $Э_{opt}(S_0)$ can be found from $S_{opt}(Э_0)$ by expansion of the implicit expression $S_{opt}(Э_{opt}) = S_0$. Analogously, $S_{opt}(Э_0)$ is found from $Э_{opt}(S_0)$ by expansion of the expression $Э_{opt}(S_{opt}) = Э_0$.

Let, for example, $S_{opt}(Э_0) = -k \ln(1 - Э_0)$, where $Э_0 < 1$ [this function $S_{opt}(Э_0)$ is encountered later in one of the examples]. In this case $Э_{opt}(S_0)$ is found from the equation $-k \ln(1 - Э_{opt}) = S_0$, which leads to the expression $Э_{opt}(S_0) = 1 - e^{-\frac{1}{k} S_0}$.

The indicated relation between the functions $S_{opt}(Э_0)$ and $Э_{opt}(S_0)$ is highly significant, for it permits consideration of only one (any) criterion (5.1) or (5.2) for construction of these functions.

The functions $S_{opt}(Э_0)$ and $Э_{opt}(S_0)$ play an important role when substantiating the optimal characteristics of the accessory equipment systems consisting of various means. Before we consider this problem, let us present an elementary example of the optimization of a continuously variable element of a ship by the criteria (5.1) and 5.2). Let us note that the optimization examples presented below, which are based on analytical representation of the functions $Э(X, \mathcal{N})$ and $S(X, \mathcal{N})$ are of an illustrative nature. In practical problems the indicated functions are given by highly complex algorithms permitting investigation of the functions $Э(X, \mathcal{N})$ and $S(X, \mathcal{N})$ only locally by calculating the values of these functions for discrete values of X and $\mathcal{N}$.

Let us propose that the problem of finding a stationary target in a section of the sea of area Ω in the time T is considered. As the effectiveness index let us take the probability of detection of the target which in the given case corresponds both to step and linear usefulness functions (see Chapter 3). If the problem is solved by a fleet of $\mathcal{N}$ ships, the operations of the individual ships are independent, and the search process is Poisson, then for the effectiveness index (detection probability) we have the expression

$$P(v, \mathcal{N}) = 1 - e^{-\frac{2dv}{\Omega} T \mathcal{N}}, \qquad (5.5)$$

where $P(v, \mathcal{N})$ is the probability of target detection by a fleet of $\mathcal{N}$ ships; v is the speed of the ship; d is the range of the detection means.

We shall consider that the search speed is equal to the maximum speed of the ship although in the general case, for example, considering hydrometeorological factors, the search speed can be a function of the maximum speed of the ship, its displacement and principal dimensions. We shall also assume that the range of the detection means is a defined value which does not depend on the speed of the ship during the search or its other characteristics. The speed of the ship (under the given assumptions (the maximum speed) must be optimized.

As the economic index let us take the cost of building the fleet of ships. Let us propose that this cost is expressed by the formula

$$S(v, \mathcal{N}) = (A + bv^n)\,\mathcal{N}, \qquad (5.6)$$

where A, b, n are values defined by approximating the corresponding function obtained by direct calculation of the cost of building one ship.

The criterion (5.2) for the investigated example has the form

$$\min_v (A + bv^n)\,\mathcal{N},$$

$$1 - e^{-\frac{2dv}{\Omega} T \mathcal{N}} = P_0. \qquad (5.7)$$

Here P_0 is the given probability of target detection.

From the limiting condition we find the required fleet of ships

$$\mathcal{N}(v, P_0) = \left[-\frac{\Omega}{2dT}\ln(1 - P_0)\right]\frac{1}{v}.$$

Substituting this function in the purpose function, we arrive at the problem [the coefficient $-\frac{\Omega}{2dT}\ln(1 - P_0)$ is omitted as independent of v]

$$\min_v \frac{A + bv^n}{v},$$

the solution of which gives the optimal value of the maximum speed of the ship:

$$v_{opt} = \sqrt[n]{\frac{A}{(n-1)b}}. \qquad (5.8)$$

As is obvious from formula (5.8), v_{opt} exists only for $n > 1$, that is, when with an increase in v the cost increases faster than by a linear law, which occurs in practice. In the given example, it turned out that v_{opt} does not depend on the probability P_0 which plays the role of the value of $Э_0$ in the criterion (5.2). However, generally speaking, this is only a special case.

Substituting (5.8) in the expression for $\mathcal{N}(v, P_0)$ and then in expression (5.6), we find the function

$$S_{opt}(P_0) = -k \ln(1 - P_0), \qquad (5.9)$$

$$k = \frac{A + bv^n_{opt}}{v_{opt}} \frac{\Omega}{2Td} = A \frac{\Omega}{2Td} \frac{n}{n-1} \sqrt[n]{\frac{(n-1)b}{A}}.$$

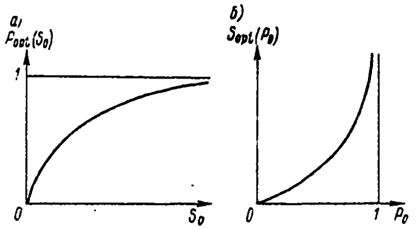

Figure 5.1. The nature of the functions $P_{opt}(S_0)$ and $S_{opt}(P_0)$ given by the formulas (5.10) and (5.9).

It is easy to see that also for the criterion of the type of (5.1) the optimal speed v_{opt} is defined by the expression (5.8). Here the function $P_{opt}(S_0)$ has the form

$$P_{opt}(S_0) = 1 - e^{-\frac{1}{k} S_0}, \qquad (5.10)$$

where k is the coefficient in (5.9).

From the expression (5.10) it is obvious that the derivative $dP_{opt}(S_0)/dS_0$ decreases monotonically with an increase in S_0, that is, the effectiveness increment ΔP_{opt} decreases with an increase in S_0 for a sufficiently small value of ΔS_0 of the additionally allocated budget (Figure 5.1,a). Analogously, from expression (5.9) it follows that $dS_{opt}(P_0)/dP_0$ increases monotonically with an increase in P_0, that is, the required increment of the budget ΔS_{opt} for obtaining a small increment of effectiveness ΔP_0 increases (Figure 5.1,b).

In addition to the above-indicated peculiarities, quite frequently the functions $Э_{opt}(S_0)$ and $S_{opt}(Э_0)$ have the form presented in Figure 5.2. In these cases the budget level S_0^* and effectiveness level $Э_0^*$, exist for which the derivatives $dЭ_{opt}/dS_0$ and $dS_{opt}/dЭ_0$ reach a maximum and minimum, respectively. If we find the optimal vectors X_{opt} by the criteria (5.1) and (5.2), setting $\text{Э}_0 = \text{Э}_0^*$ and $S_0 = S_0^*$ in them, these solutions will correspond to the versions of the ship where a maximum increase in effectiveness per additionally spent unit of budget and minimum increase in cost per additionally obtained unit of effectiveness index will be insured.

In some cases the versions of the ship corresponding to the following conditions are of interest:

$$\max_{S_0} \frac{\text{Э}_{\text{opt}}(S_0)}{S_0}, \quad \min_{\text{Э}_0} \frac{S_{\text{opt}}(\text{Э}_0)}{\text{Э}_0}, \tag{5.11}$$

that is, maximum effectiveness per unit of expended budget and, correspondingly, minimum cost per unit of obtained effectiveness are achieved. The values of S_0^{**} and Э_0^{**}, satisfying the conditions (5.11) are formed from the equations

$$\frac{d\text{Э}_{\text{opt}}(S_0)}{dS_0} = \frac{\text{Э}_{\text{opt}}(S_0)}{S_0}, \quad \frac{dS_{\text{opt}}(\text{Э}_0)}{d\text{Э}_0} = \frac{S_{\text{opt}}(\text{Э}_0)}{\text{Э}_0}. \tag{5.12}$$

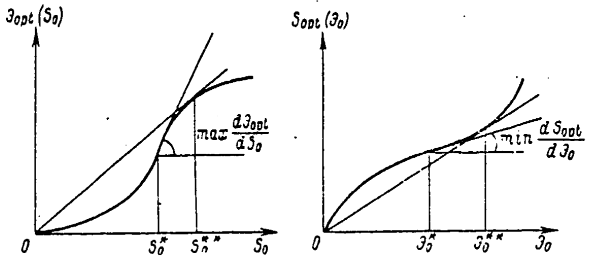

Figure 5.2. Nature of the functions $\text{Э}_{\text{opt}}(S_0)$ and $S_{\text{opt}}(\text{Э}_0)$ in the presence of the inflection points S_0^* and Э_0^*.

Geometrically, as is obvious from Figure 5.2, the solutions of equations (5.12) correspond to points on the curves $\text{Э}_{\text{opt}}(S_0)$ and $S_{\text{opt}}(\text{Э}_0)$, in which the tangents to these curves coincide with the rays drawn from the origin of the coordinates to these points. Inasmuch as the versions of the ships satisfying the criteria (5.11) have maximum specific effectiveness and minimum specific cost, the values of $\text{Э}_{\text{opt}}(S_0)/S_0$ and $S_{\text{opt}}(\text{Э}_0)/\text{Э}_0$ are sometimes called the indexes of combat economicalness, and the conditions (5.11) are called the criteria of combat economicalness.[1]

The combat economicalness criteria are convenient in that they do not require assignment of specific values of S_0 and Э_0. As will be demonstrated below, the assignment of these variables for investigations on the level of designing an individual ship is very difficult and requires investigation of a system of higher hierarchical level. At the same time, the optimal solutions by the combat economicalness criteria do not always exist. Thus, in the example investigated above with optimization of the speed of the ship the solution by the combat economicalness

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[1] The term "combat economicalness," which is not entirely correct, has not been generally recognized among specialists.

criteria does not exist inasmuch as $S_0^{**} \to 0$ and $Э_0^{**} \to 0$, that is, "the best solution of the problem is no solution." This paradox sometimes is called the "Chinese junk paradox": the version of the ship having infinitely low effectiveness and infinitely low cost turns out to be optimal.

Let the indexes $Э(X, \mathcal{N})$ and $S(X, \mathcal{N})$ are directly proportional to the fleet number $\mathcal{N}$, that is, $Э(X, \mathcal{N}) = Э_1(X)\mathcal{N}$, $S(X, \mathcal{N}) = S_1(X)N$, where $Э_1(X)$, $S_1(X)$ are the effectiveness and cost indexes of one ship. In this case the criteria (5.11) have the form

$$\max_X \frac{Э_1(X)}{S_1(X)} \text{ and } \min_X \frac{S_1(X)}{Э_1(X)}. \tag{5.13}$$

Let us propose that $Э_1(X)$ and $S_1(X)$ increase monotonically with an increase in each of the components of the vector X, and the set $\mathfrak{X}$ of admissible vectors X is defined by the condition $X \geq 0$ where $Э_1(0) = 0$ and $S_1(0) = 0$. If in this case when $X \to 0$, $Э_1(X)$ decreases more slowly than $S_1(X)$, then the criteria (5.13) give $X_{opt} \to 0$. This is also the "Chinese junk paradox," according to which it is advantageous to compensate infinitely for the low effectiveness of a single ship by increasing the number of ships (the fleet). Such solutions and, in general, the solutions according to which the optimal version simultaneously has the lowest effectiveness and the lowest cost, require careful checking with respect to completeness and correctness of the initial data and also the adopted models of estimating effectivness and cost, especially with respect to considering the various types of limiting conditions. Such solutions also require caution and the most effective and most expensive ships turn out to be optimal. This is also a type of paradox, but now the "superbattleship paradox."

In the general case for optimization of TTE and TDP of ships by the criteria (5.1) and (5.2) if we do not reduce them to the combat economicalness criteria (5.11), it is necessary to assign specific values of S_0 and $Э_0$ -- the values of the available budget and the required effectiveness level. These values can be determined on the level of optimization of the system in which the ships of the investigated class are only one of the subsystems. At the same time optimization of the system, generally speaking, is impossible without optimizing the subsystems. Thus, a closed circle arises. It is possible to resolve this indeterminacy by two methods.

First, the optimal characteristics of a ship defined by criteria (5.1) and (5.2) cannot depend on S_0 and $Э_0$. Such a case occurred in the above-investigated example with optimization of the maximum speed of the ship when solving the search problem. In such cases a closed circle does not actually exist. In order to check this fact, it is necessary to carry out parametric optimization for various values of S_0 and $Э_0$ and see that the optimal vectors X_{opt} do not depend on the values of these variables.

Secondly, if the optimal TTE and TDP of a ship depend on S_0 and $Э_0$, it is possible to use the method of successive approximations. Initially the functions $S_{opt}(Э_0)$ or $Э_{opt}(S_0)$ are found by parametric optimization. In this step it is expedient to use approximate functions and models in the engineering design, effectiveness index evaluation and economic index evaluation blocks. The functions $S_{opt}(Э_0)$ and $Э_{opt}(S_0)$ "fed" to the level of investigation of the corresponding accessory equipment system in which the investigated class of ship is one of the subsystems. Analogous functions must be "entered" with respect to the other subsystems. If on the system level the total budget S_Σ is distributed in the budgets S_ℓ, $\ell = 1, \ldots, L$ allocated for each of L subsystems, then the functions $\vartheta_{\text{opt}}^{(l)}(S_l)$, $l = 1, \ldots, L$. are isolated for the system level. In the case of distribution of the total effectiveness on the system level between the subsystems, the functions $S_{\text{opt}}^{(l)}(\vartheta_l)$, $l = 1, \ldots, L$ are isolated for the system level.

After optimization and the system level, that is, determination of S_0 and ϑ_0 for the investigated class or type of ship, the problems (5.1) or (5.2) are again solved, but now on the basis of more complete and exact models and AD algorithms for fixed values of S_0 or ϑ_0. The optimal vector X_{opt} of the TTE and the TDP of the ship is found as a result.

Let us again refer to the example connected with optimization of the maximum speed of a ship when solving the search problem, but let us propose that the problem can be solved by a different fleet of ships characterized by different values of the coefficient k in the formula (5.10). If the distribution of the budget S_Σ with respect to individual types of ships is given by the values of $S_1, \ldots, S_L$ where $\sum_{l=1}^{L} S_l = S_\Sigma$ and S_ℓ is the budget allocated for the ℓth type of ship, then the total effectiveness will be

$$\vartheta_\Sigma = 1 - e^{-\sum_{l=1}^{L} \frac{S_l}{k_l}},$$

where k_ℓ are the same values as in formula (5.9), but defined by each type of ship.

On the ship system level it is natural to find the budgets S_ℓ, $\ell = 1, \ldots, L$ by the criterion

$$\max_{S_l} \left(1 - e^{-\sum_{l=1}^{L} \frac{S_l}{k_l}}\right), \quad \sum_{l=1}^{L} S_l = S_\Sigma, \quad S_l \geqslant 0, \quad l = 1, \ldots, L. \tag{5.14}$$

The solution of problem (5.14) is equivalent to the solution of the problem of linear programming

$$\max_{\bar{S}_l} \sum_{l=1}^{L} \frac{1}{k_l} \bar{S}_l, \quad \sum_{l=1}^{L} \bar{S}_l = 1, \quad \bar{S}_l \geqslant 0, \quad l = 1, \ldots, L, \tag{5.15}$$

where $\bar{S}_\ell = S_\ell / S_\Sigma$ is the relative size of the budget allocated for the ℓth class of ship.

In accordance with the criterion (5.15) the entire budget must be allocated for one class of ship, for which the condition $\max\limits_{\ell} \frac{\ell}{k_{\ell}}$ occurs, that is, a uniform fleet will be optimal in the given case. It is natural that this "pure" (and not "mixed") strategy in the total budget distribution is, generally speaking, a special case.

Let us present an example where the optimal budget distribution leads to a "mixed" strategy -- a nonuniform fleet of forces. Let us propose that it is necessary to distribute the budget S_{Σ} with respect to nonuniform or different means[1] solving different combat problems. Each type of means solves only its problem and cannot be used to solve other problems. The probabilities of the solution of the problems met by each of the investigated types of means are given by the functions $P_{\ell}(S_{\ell})$, $\ell = 1, \ldots, L$, where S_{ℓ}, just as before, is the size of the budget allocated for the ℓth type of means, and L is the number of investigated types of means.

If the total effectivness of the system of means is characterized by the probability of the performance of all of the stated missions, and the events included in the performance of each of the missions are independent, the effectiveness index has the form $\partial_{\Sigma} = \prod\limits_{l=1}^{L} P_l(S_l)$. Here the importance of all the problems is considered the same.

The optimal values of S_{ℓ} are found by solving the problem of nonlinear programming

$$\max_{S_l} \sum_{l=1}^{L} \ln P_l(S_l),$$
$$\sum_{l=1}^{L} S_l \leqslant S_{\Sigma},$$
$$S_l \geqslant 0, \quad l = 1, \ldots, L.$$

In this problem the purpose function is obtained by logarithmizing the expression for ∂_{Σ}, which has no effect on the results of optimizing S_{ℓ}, $\ell = 1, \ldots, L$.

In many practical cases the following conditions are satisfied for all $\ell = 1, \ldots, L$: the functions $P_{\ell}(S_{\ell})$ do not decrease with an increase in S_{ℓ}; the probability of each of the missions is equal to zero if the corresponding budget S_{ℓ} is equal to zero, that is, P (0) = 0; the identities P (S) ≡ 0 can occur only for $0 \leq S_{\ell} \leq S_{\ell}^{*}$, where $\sum\limits_{l=1}^{L} S_l^{*} \leqslant S_{\Sigma}$. The last condition that the total budget S_{Σ} is sufficient for performance of each of the combat missions with nonzero effectiveness.

It is obvious that under the above-indicated conditions the optimal distribution of the budget S_{Σ} must provide for allocation of a defined portion of it for each of the investigated types of means, and, namely, with optimal distribution $S_{\ell} > 0$ for $\ell = 1, \ldots, L$. Actually, if $S_{\ell} = 0$ for at least one of the means (the budget is not allocated for this type of means and the corresponding mission), the effectiveness index vanishes, which corresponds not to the maximum, but the minimum value of ∂_{Σ}.

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[1] Here by means we mean different combat units of the fleet (surface ships, submarines, aircraft, and so on).

It is known that for concave functions $P_\ell(S_\ell)$ the solution of the above-formulated problem of nonlinear programming is unique. This solution can be found by the method of undefined Lagrange factors, the methods of dynamic programming or numerical methods. When using the Lagrange method it is necessary to consider that on the basis of the previously indicated properties of the function $P_\ell(S_\ell)$ the inequality $\sum_{l=1}^{L} S_l \leqslant S_\Sigma$ can be replaced by the equality, and the restrictions $S_\ell \geq 0$, $\ell = 1, \ldots, L$ are insignificant.

Considering the stated problem as a problem of dynamic programming, we have a system of functional equations

$$F_1(x) = \max_{0 \leqslant S_1 \leqslant x} \varphi_1(S_1),$$

$$F_l(x) = \max_{0 \leqslant S_l \leqslant x} [\varphi_l(S_l) + F_{l-1}(x - S_l)], \quad l = 2, \ldots, L,$$

where $\varphi_l(S_l) = \ln P_l(S_l)$, $F_l(x) = \max\limits_{S_1, \ldots, S_l} \sum_{k=1}^{l} \varphi_k(S_k)$, x is the variable value of the budget S_Σ.

From this system, by successive solution of the problems of uniform optimization we find the functions $F_\ell(x)$ and $f_{\ell,\ell}(x)$, $\ell = 1, \ldots, L$, where $f_{\ell,\ell}(x)$ are the optimal values of the budget (as functions of x) allocated for the ℓth type of means under the condition that the total budget x is distributed for ℓ types of means (the number of types of means is ℓ).

Then, moving in the opposite direction from $\ell = L$ to $\ell = 1$, we find the optimal values of $S_\ell^{opt}(x)$, $\ell = 1, \ldots, L$ for distribution of the budget x to L types of means

$$S_L^{\text{opt}}(x) = f_{L,L}(x),$$

$$\cdot \; \cdot$$

$$S_l^{\text{opt}}(x) = f_{l,l}\left(x - \sum_{k=l+1}^{L} S_k^{\text{opt}}\right), \quad l = L - 1, \ldots, 1$$

$$\cdot \; \cdot$$

Finally, setting $x = S_\Sigma$, we obtain the solution of the initial problem of optimal distribution of the budget S_Σ with respect to L types of means.

Now let us consider the problem of coincidence of the optimal solutions with respect to the criteria (5.1) and (5.2). Let us propose that the vector (X) of TTE and TDP contains only continuously variable characteristics. Comparison of the solutions for linearity of the functions $\text{Э}(\dot{X}, N)$ and $S(X, \mathcal{N})$ with respect to the variable $\mathcal{N}$ follows from identicalness of the systems of equations for determination of X_{opt}: in the case of the criterion (5.1)

$$\frac{\partial \text{Э}_1(X)}{\partial x_i} S_1(X) - \frac{\partial S_1(X)}{\partial x_i} \text{Э}_1(X) = 0, \quad i = 1, \ldots, n,$$

in the case of the criterion (5.2)

$$\frac{\partial S_1(X)}{\partial x_i}\text{Э}_1(X)-\frac{\partial \text{Э}_1(X)}{\partial x_i}S_1(X)=0,\quad i=1,\ \ldots,\ n,$$

where x_i, $i = 1, \ldots, n$ are components of the vector X.

In the general case, using the method of undefined Lagrange factors it is possible to obtain the following equations for determining the vector X_{opt}: for the criterion (5.1)

$$\begin{gathered}\frac{\partial \text{Э}(X,\ \mathcal{N})}{\partial x_i}-\lambda_1\frac{\partial S(X,\ \mathcal{N})}{\partial x_i}=0,\quad i=1,\ \ldots,\ n,\\ \frac{\partial \text{Э}(X,\ \mathcal{N})}{\partial \mathcal{N}}-\lambda_1\frac{\partial S(X,\ \mathcal{N})}{\partial \mathcal{N}}=0,\\ S(X,\ \mathcal{N})-S_0=0;\end{gathered}\tag{5.16}$$

for the criterion (5.2)

$$\begin{gathered}\frac{\partial S(X,\ \mathcal{N})}{\partial x_i}+\lambda_2\frac{\partial \text{Э}(X,\ \mathcal{N})}{\partial x_i}=0,\quad i=1,\ \ldots,\ n,\\ \frac{\partial S(X,\ \mathcal{N})}{\partial \mathcal{N}}+\lambda_2\frac{\partial \text{Э}(X,\ \mathcal{N})}{\partial \mathcal{N}}=0,\\ \text{Э}(X,\ \mathcal{N})-\text{Э}_0=0.\end{gathered}\tag{5.17}$$

The solutions of the systems of equations (5.16) and (5.17) do not coincide generally speaking. Only certain pairs of values of S_0, Э_0, can exist for which the indicated comparison occurs. It is obvious that these values lie on the curves given by the equations $\text{Э}_{opt}=\text{Э}_{opt}(S_0)$ or $S_{opt}=S_{opt}(\text{Э}_0)$.

In connection with noncoincidence of the solutions in the general case with respect to criteria (5.1) and (5.2) the question arises of preferableness of using one of the indicated criteria or another. Obviously there is no precise answer to this question, although a number of researchers give preference to the criterion (5.2) beginning with the following arguments.

Ships, as a rule, are multipurpose combat complexes for which the construction of a single effectiveness index is frequently difficult. Optimization with respect to several indexes without reduction of them to one generalized index is also connected with defined difficulties. Under these conditions optimization by the criterion (5.2), that is, from the condition of minimizing the cost of the fleet solving an entire spectrum of problems with given effectiveness, appears to be a simpler problem, and this is correct. However, the problem of establishing the required levels of effectiveness of the solution of each of the problems remains open.

It is appropriate to mention that for parametric optimization in order to obtain the functions $\text{Э}_{opt}(S_0)$ and $S_{opt}(\text{Э}_0)$ any of the criteria (5.1) or (5.2) can be used.

§ 5.2. Example of Optimizing the Characteristics of Ship when the Service Index is the Displacement

The effectiveness of transport ships frequently is characterized [3] by the capacity $Q = m_0vt$, where m_0 is the weight of the hauled cargo, tons; v is the average speed of the ship on a trip, knots; t is the travel time per year, hours. Considering that the speed of the ship when transporting the cargo is proportional to the maximum speed of the ship v_{max}, we obtain

$$Q = km_0 v_{\max}, \tag{5.18}$$

where k is a constant coefficient taking into account the relation of v to v_{max} and the travel time t.

Let us consider the problem of finding optimal values of m_0 and v_{max} from the condition of maximum capacity with limited displacement of the ship. Here we propose that the displacement can be found from the three-term mass equation (see Chapter 2)[1]

$$AD + \frac{q_{\text{эу}} + q_{\text{т}}T}{C_w} v_{\max}^3 D^{2/3} + m_0 + \sum_i m_i = D.$$

The stated optimization problem has the form

$$\max_{m_0,\, v_{\max}} (m_0 v_{\max}),$$
$$AD_0 + \frac{\bar{q}_{\text{эу}}}{C_w} v_{\max}^3 D_0^{2/3} + m_0 + \sum_i m_i = D_0, \tag{5.19}$$
$$m_0 > 0, \quad v_{\max} > 0,$$

where $\bar{q}_{\text{эу}} = q_{\text{эу}} + q_{\text{т}}T$ and D_0 is the given displacement.

Expressing m_0 in terms of v_{max} using the mass equation and substituting this expression in the purpose function of the criterion (5.19), we find v_{max}^{opt} from the condition $\max\limits_{v_{\max}} [m_0(v_{\max}) v_{\max}]$. As a result, we obtain

$$v_{\max}^{\text{opt}} = 0{,}63 D_0^{1/9} \left(1 - A - \frac{\sum_i m_i}{D_0}\right)^{1/3} \left(\frac{C_w}{\bar{q}_{\text{эу}}}\right)^{1/3}. \tag{5.20}$$

Correspondingly,

$$m_0^{\text{opt}} = \frac{3}{4} D_0 \left(1 - A - \frac{\sum_i m_i}{D_0}\right). \tag{5.21}$$

The mass of the power plant with fuel, water and oil reserves in the optimal case will be

$$m_{\text{эу}}^{\text{opt}} = \frac{1}{4} D_0 \left(1 - A - \frac{\sum_i m_i}{D_0}\right).$$

The function $Q_{opt}(D_0)$ has the form

$$Q_{\text{opt}}(D_0) = 0{,}472 D_0^{10/9} \left(1 - A - \frac{\sum_i m_i}{D_0}\right)^{4/3} \left(\frac{C_w}{\bar{q}_{\text{эу}}}\right)^{1/3} k. \tag{5.22}$$

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[1] It is proposed that the fuel, oil and water reserves are selected calculating the movement of the ship with maximum velocity from the time T. If the distance is given, an equation of the type of (2.11) is used.

In order to solve the problem of minimizing the displacement of the ship with given capacity, that is,

$$\begin{gathered}\min_{m_0,\, v_{\max}} D(m_0,\ v_{\max}),\\ Q(m_0,\ v_{\max}) \geqslant Q_0,\\ m_0 \geqslant 0,\ v_{\max} \geqslant 0,\end{gathered} \tag{5.23}$$

it is necessary to equate the right-hand side of equation (5.22) to the value of Q_0 and from the equation obtained, find the value of D_0 which will be equal to the minimum D in the problem (5.23). Substituting the value of D in formulas (5.20) and (5.21), we find the optimal values of m_0 and $v_{\max}$ for the problem (5.23).

The results of solving the problem (5.19) obtained above will permit the following qualitative conclusions to be drawn: the optimal speed of the ship depends slightly on the given displacement D_0; the optimal mass of the hauled cargo is directly proportional to the given displacement and sum of the relative masses of payload and power plant, that is, the value of $1 - A - \frac{\sum_i m_i}{D_0}$; in the optimal case the masses of the power plant and the payload are in the ratio of 1/4:3/4.

These results are valid under the condition that the admiralty coefficient C_w does not depend on the speed and displacement (dimensions) of the ship. This assumption is acceptable in the initial design stages only for submarines. Designs for transport submarines have been developed abroad [10]. For surface ships, the above-indicated function must be considered, which does not permit simple analytical results to be obtained.

In the general case, the displacement of the ship must be determined, solving the mass equations and other equations of design theory jointly. For submarines (both military and civilian) it is especially important to consider the volume equation. In the opinion of foreign specialists [10], the displacement of modern nuclear-powered submarines is determined by the volumes required for placement of weapons, accessory equipment, technical means and personnel. The mass can become defining only for deep-sea submarines with depths of submersion of more than 1000 meters [10].

For joint investigation of the mass and volume equations, the displacement of the ship D in the first approximation can be taken equal to the maximum value of the mass and volume displacement:

$$D = \max\{D_m,\ \rho D_V\}, \tag{5.24}$$

where the mass displacement (D_m) and volume displacement (D_V) are found from the equations

$$\begin{gathered}A_m D_m + \frac{\bar{q}_{\text{эу}}}{C_w} v_{\max}^3 D_m^{2/3} + m_0 + \sum_i m_i = D_m,\\ A_V D_V + \rho^{2/3}\, \frac{\bar{w}_{\text{эу}} + \bar{w}_{\text{т}} T}{C_w} v_{\max}^3 D_V^{2/3} + V_0 + \sum_i V_i = D_V.\end{gathered} \tag{5.25}$$

Here $\bar{w}_{\text{т}}$ is the specific volumetric fuel consumption, m^3/horsepower-hour, and V_0 is the volume occupied by the payload (transported cargo). The remaining notation is explained in Chapter 2.

Expressing V_0 in terms of m_0 by using the specific mass of the hauled cargo, the relation is found for the displacement as a function of the speed and mass of the hauled cargo, and then the problem of optimizing the capacity is solved under the condition $D = D_0$. It is possible to obtain the graphical solution of this problem (Figure 5.3). First the functions $D(m_0)$ are constructed for several values of $v_{max}^{(i)}$, $i = 1, 2, \ldots, n$. Inasmuch as the optimal value of v_{max}^{opt} depends comparatively weakly on the given displacement D_0, the range of variation of v_{max} will be small, and it is easy to predict. Then from the condition $D = D_0$ the corresponding pairs of values $(v_{max}^{(i)}, m_0^{(i)})$ are found for which the values of the function $F(v_{max}, m_0) = m_0 v_{max}$ are calculated. Constructing the graph of this function, the optimal values of v_{max}^{opt} and m_0^{opt} are found (interpolation is carried out for finding v_{max}^{opt} in the general case).

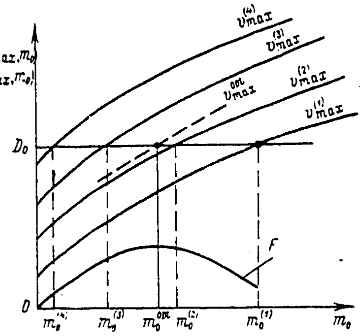

Figure 5.3. Schematic of the graphical solution of the problem of optimizing the weight of the hauled cargo and the transport speed of the ship.

In conclusion let us note that for transport submarines, for example, submarine tankers in which the hauled cargo is placed outside the pressure hull, together with the mass and volume equations it is necessary to also consider the buoyancy equation.

§ 5.3. Optimization of TTE and TDP of a Ship by the Method of Comparative Evaluation of Versions

The simplest method of optimizing the TTE and the TDP of a ship in the design stage is selection of the optimal version from a finite given number of them. In this case the finite set of vectors X: $X_1, X_2, \ldots, X_\ell, \ldots, X_L$ is given. Each of the vectors $X_\ell = (x_{1\ell}), \ldots, x_{n\ell})$, $\ell = 1, \ldots, L$ defines the set of values of the optimized TTE and TDP of the corresponding versions of the ship. The set of AD problems (technical, effectiveness, service blocks) is solved for each of the versions, after which the optimal version is selected in accordance with the value of the purpose function of the adopted optimalness criterion.

The optimization of the TTE and the TDP of a ship by comparative evaluation of versions given by the designer on the basis of intuitive arguments is the oldest, traditional method which has essentially been used since the time designers began to solve the problems of quantitative evaluation of various alternative solutions.

The method of comparative evaluation of versions has to a great extent given rise to the so-called successive scheme of solution of the basic AD problems. In this scheme initially the engineering design block is realized for all of the investigated versions, then the economic index evaluation block, effectiveness index evaluation block and finally the optimal version selection block successively.

The basic deficiencies of the method of comparative evaluation of versions and the successive AD scheme consist in the following.

1. There is no ordered sorting of the investigated versions based on the results of the calculations by the preceding versions, that is, there is no feedback between the "input" and "output" of the optimization algorithm. The process of searching for the optimum in this case is highly uneconomical inasmuch as it is necessary to investigate a large number of "bad" versions.

2. The fact of the presence of an actually optimal version in the discrete set of them selected for investigation is based exclusively on the designer's intuition. Improvement of the probability of finding an exact optimum as a result of increasing the number of points in the space of the TTE and TDP is limited by the computation possibilities.

3. The method of comparative evaluation of versions considering limited computation possibilities does not correspond to the trend toward a continuous increase in the number of versions potentially subject to investigation in the AD process.

Let us explain the last-mentioned fact in more detail. If each version of the ship is characterized by n variable TTE and TDP, and each of these characteristics is assigned j_k, k = 1, ..., n values, respectively, then the number of versions subject to investigation is defined by the formula

$$L = \prod_{k=1}^{n} j_k. \quad (5.26)$$

For j_k = idem, that is, when each of the characteristics is assigned an identical number of values equal to j, we obtain $L = j^n$ (Figure 5.4). If we approximately assume that for the successive AD scheme about 1000 versions can be examined (by computer) in acceptable time, then as is obvious from Figure 5.4, for j = 3 to 4, the number of variable TTE and TDP must be no more than six. These values of j and n cannot be considered acceptable.

In spite of the above-indicated deficiencies, the method of comparative evaluation of versions remains the only one which leads for sure to the goal by the method of finding the optimum with respect to discretely variable TTE and TDP. In the general case only complete sorting of all possible versions insures that the optimum will be found with respect to such TTE and TDP which cannot be represented approximately as values with a continuous set of values. The type and layout of the power plant, the type of hull material, the type of weapons and means of placement of them, the hull structure, and so on are examples of such characteristics.

§ 5.4. Search for the Optimal Version of a Ship in the Space of the TTE and the TDP

The division of the TTE and the TDP of the ship into continuously variable $X_{\sim} = (x_{1\sim}, \ldots, x_{\mu\sim})$ and discretely variable $X_{.} = (x_{1.}, \ldots, x_{\nu .})$ groups where μ and ν

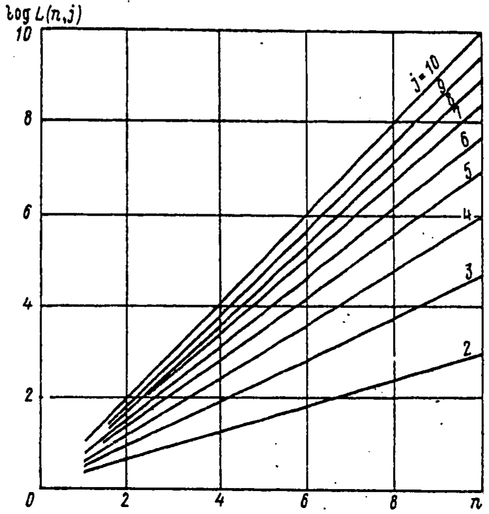

Figure 5.4. Number of versions of a ship (L) as a function of the number of variable TTC (n) and the number of values assigned to them (j).

are the numbers of continuously and discretely variable characteristics, respectively) arises from the possibility of using the methods of directional search with optimization with respect to continuously variable characteristics. Here the scheme for the solution of the basic AD problems will become parallel when the military-economic studies are performed during the process of searching for the optimum after technical development and evaluation of the technical-economic indexes (the service block) of each current version. Accordingly, the most expedient direction of variation of the continuously variable TTE and TDP will be discovered for the fastest movement toward the optimum.

Let us denote by $\Phi(X_\sim, X_\cdot)$ the purpose function of the adopted optimalness criterion after substitution of the expression for the set of limiting conditions in this function (see § 5.1). The process of searching for the optimum consists in the following:

For each set $X_\cdot^{(j)}$ of values of discretely variable characteristics the problem of finding the optimal set of continuously variable characteristics is solved, that is, the problems

$$\underset{X_\sim}{\operatorname{extr}}\, \Phi(X_\sim, X_\cdot^{(j)}), \quad j = 1, \ldots, J, \qquad (5.27)$$

are solved where J is the number of possible sets of values of discretely variable characteristics. Let us denote the solutions of these problems in terms of $X_{\sim opt}^{(j)}$, j = 1, ..., J;

The optimal set of continuously and discretely variable characteristics is found by solving the problem

$$\underset{j=1,\ldots,J}{\text{extr}}\ \Phi\left(X_{\sim\,\text{opt}}^{(j)},\ X_{\cdot}^{(j)}\right), \qquad (5.28)$$

that is, a comparison is made between the purpose function of the optimalness criterion for all possible sets of values of discretely variable characteristics with consideration of the optimal values of continuously variable characteristics.

Thus, the sorting with respect to possible values of the discretely variable TTE and TDP is retained, which greatly limits the possible number of them for optimization. For this reason the values of a large number of discretely variable TDP must be substantiated immediately for the entire set of ships of the investigated class and type. This substantiation is formulated as the general requirements on the design of ships of a defined class and type. For example, general requirements on stability, unsinkability, protection, habitability, power plant and propulsion complex systems, and so on, can exist.

In the above-presented general system for finding optimal values of TTE and TDP the basic problem is (5.27). From the mathematical point of view, this is the general problem of nonlinear programming, and its basic feature is absence of an analytical expression for the function $\Phi(X_{\sim}, X_{\cdot})$ given by the set of AD algorithms. The study of this function for each fixed $X_{\cdot}^{j}$ is possible only by discrete methods — by calculating its values (responses) at discrete points of the space of the vectors $X_{\sim}$. The space, the coordinates of which are components of the vector $X_{\sim}$ is called the phase space, and the components themselves, the phase coordinates.

Let us consider the essence of the basic methods of solving the problem (5.27). Inasmuch as we are talking only about continuously variable TTE and TDP, the corresponding index on the vector $X_{\sim}$ and its components can be omitted.

Gradient Methods. When selecting the direction of movement toward the optimum, the use of the gradient vector of the purpose function is common to the methods having a large number of different versions:

$$\operatorname{grad}\Phi(X) = \left(\frac{\partial\Phi}{\partial x_1},\ \ldots,\ \frac{\partial\Phi}{\partial x_\mu}\right),$$

where $x_1, \ldots, x_\mu$ are components of the vector X. (For brevity of notation the fixed argument $X_{\cdot}^{(j)}$ is omitted in the purpose function.)

It is known that the direction of the gradient vector at each point of the phase space corresponds to the greatest growth of the values of the purpose function. The search for the extremum of the function $\Phi(X)$ is made by successive approximations. The process of movement toward the extremum in the general case consists in solving the following problems.

1. An arbitrary initial vector $X^{(0)} \in \mathfrak{X}$ is selected where $\mathfrak{X}$ is the set of possible values of the vector X defined by the system of limiting conditions. In the simplest case $\mathfrak{X}$ is a parallelepiped characterized by two-way restrictions imposed on the components of the vector X. The value of the function Φ which we shall denote by $\Phi^{(0)}$ is calculated at the point $X^{(0)}$.

2. In a sufficiently small vicinity of the point $X^{(0)}$, μ points $X^{(01)}, \ldots, X^{(0\mu)}$ are selected, and the values of the purpose function $\Phi^{(01)}, \ldots, \Phi^{(0\mu)}$ are calculated for them. Inasmuch as the TTE and the TDP of the versions of the ship corresponding to the vectors $X^{(0i)}$, $i = 1, \ldots, \mu$, differ relatively little from the characteristics of the version $X^{(0)}$, for calculation of $\Phi^{(01)}, \ldots, \Phi^{(0\mu)}$ it is possible to use the approximate procedures and models. For example, in the engineering design block it is possible to use purely analytical methods, including the differential forms of the basic equations of determining the displacement and the principal dimensions of the ship [56].

3. The components of the vector grad Φ are calculated approximately at the point $X^{(0)}$. The calculation is based on a linear representation of the function $\Phi(X)$ in the vicinity of the point $X^{(0)}$:

$$\Phi(X) = \Phi(X^{(0)}) + \sum_{k=1}^{\mu} (x_k - x_k^{(0)}) \frac{\partial\Phi}{\partial x_k}\Big|_{X=X^{(0)}},$$

where $\frac{\partial\Phi}{\partial x_k}\Big|_{X=X^{(0)}}$, $k = 1, \ldots, \mu$ are components of the vector grad Φ at the point $X = X^{(0)}$.

In order to find μ unknowns $\frac{\partial\Phi}{\partial x_k}\Big|_{X=X^{(0)}}$, $k = 1, \ldots, \mu$, we have μ linear algebraic equations

$$\sum_{k=1}^{\mu} (x_k^{(0i)} - x_k^{(0)}) \frac{\partial\Phi}{\partial x_k}\Big|_{X=X^{(0)}} = \Phi(X^{(0i)}) - \Phi(X^{(0)}).$$

In the special case the points $X^{(0i)}$, $i = 1, \ldots, \mu$ are selected so that each of them will differ from $X^{(0)}$ by only one of the coordinates. This means that the versions of the ship corresponding to the points $X^{(0i)}$ differ from the version $X^{(0)}$ by the value of only one of the variable TTE and TDP. Let, for example, $X^{(0i)}$ differ from $X^{(0)}$ only by the coordinate x_i. Then

$$\frac{\partial\Phi}{\partial x_i}\Big|_{X=X^{(0)}} = \frac{\Phi(X^{(0i)}) - \Phi(X^{(0)})}{x_i^{(0i)} - x_i^{(0)}}, \quad i = 1, \ldots, \mu,$$

where $x_i^{(0i)}$ are components of the vector $X^{(0i)}$.

Interesting problems with respect to the optimal location of the points $X^{(0i)}$ in the vicinity of the initial point $X^{(0)}$ occur when the calculation of the responses of the purpose function Φ is connected with the appearance of random errors caused, for example, by the presence of unalgorithmized operations of decision making by the designer or specialists with respect to estimating the effectiveness and the service indexes in the individual AD blocks. In this case errors will appear in the gradient vector components. They can be influenced by the location of the points $X^{(0i)}$. Thus, it is possible to demonstrate [33] that the arrangement of the points $X^{(0i)}$ will not be optimal if they differ from the initial point $X^{(0)}$ by only coordinate.

The problems of studying and optimizing various processes and systems, the internal structure of which is unknown or is quite complex, and obtaining data by experimental (physical or numerical) means is connected with random errors, are the subject of a new area of mathematical statistics -- experiment planning theory [32, 33, 54].

4. After calculating the vector grad $\Phi(X^{(0)})$ along this direction a transition is made to the new point $X^{(1)}$. The direction of movement along the gradient vector is determined by the meaning of the operator extr Φ (maximum or minimum). In the general case the vector $X^{(1)}$ is defined by the expression:

$$X^{(1)} = X^{(0)} + t \operatorname{grad} \Phi(X^{(0)}).$$

where t is a scalar value defining the step along the gradient.

If max Φ is found, then $t > 0$, and if min Φ, then $t < 0$.

5. For the point $X^{(1)}$ and subsequent points, the operations described in items 1-4 are repeated. If during the process of the calculations the points $X^{(1)}$, $X^{(2)}$... always belong to the set $\mathfrak{X}$, then the process continues to the point X_{opt} for which the condition grad $\Phi(X) = 0$ is satisfied -- the necessary and sufficient condition for the minimum of the convex and maximum of the concave function $\Phi(X)$. The latter is convex if the following inequality is satisfied for any $X^{(1)}$ and $X^{(2)}$

$$\Phi[\alpha X^{(1)} + (1-\alpha) X^{(2)}] < \alpha\Phi(X^{(1)}) + (1-\alpha)\Phi(X^{(2)}),$$

where α is any number from the interval [0, 1].

In the case of the inverse inequality, the function is concave.

Figure 5.5,a illustrates movement toward the extremal point (maximum) using the gradient vector (the direction of the vector is shown by the arrows). During the process of movement toward the extremum the step t must vary for acceleration of convergence and an increase in accuracy of determination of the vector X_{opt}. Thus, at the points where the modulus of the gradient vector is large, it is expedient to increase the step and, vice versa, with a decrease in modulus of the gradient vector the step must be decreased.

There are a number of methods of variation of t as a function of the results of the preceding calculations. For example, movement along the gradient vector can be realized to the point at which the purpose function reaches a maximum in this direction. A new gradient vector is defined at this point, and movement is realized until the maximum of the purpose function is reached in the new direction (Figure 5.5, b).

The problem of finding the extremum by gradient methods is complicated significantly when we go beyond the limits of the admissible set $\mathfrak{X}$ during the process of movement toward the extremum. This is especially significant in the absence of analytical expressions for the conditions defining the set $\mathfrak{X}$. It is also very difficult to check the presence of convexity or concavity of the purpose function, and without this it is possible only to count on achievement of the local extremum or saddle point.

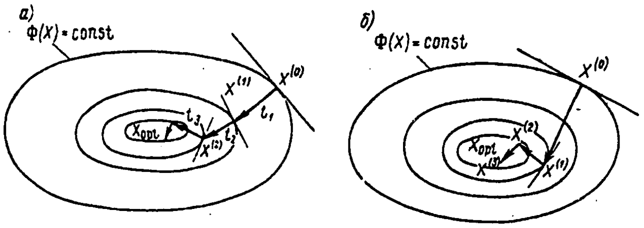

Figure 5.5. Geometric illustration of the gradient method; a — for arbitrary variable step size; b — for step defined by reaching the extremum along the direction of the gradient vector.

There are also a number of other deficiencies of the gradient methods. For example, the presence of so-called "peaks" and "valleys" in the purpose function significantly reduces the speed of movement toward the extremum. In addition, the speed of movement toward the extremum depends on the choice of the scales of the variables inasmuch as this determines the configuration of the response surface of the purpose function [see 53]. The author of this paper considers some of the procedures for finding the extremum of a function having peculiarities of the "peak" and "valley" type.

For the indicated reasons the purely gradient methods can rarely be used to solve practical problems of optimizing the TTE and TDP of a ship in the AD process. Sufficiently complete reviews of the various versions of the gradient methods are presented in special mathematical literature [21, 53].

Let us present one of the versions of the gradient method which can be called the provisional gradient method [16, 17]. The essence of the method consists in the following (a column vector of components is taken as X):

An arbitrary point $X^{(1)} \in \mathfrak{X}$ is selected, and the vector grad Φ $(X^{(1)})$ is found;

from the condition

$$\underset{X \in \mathfrak{X}}{\operatorname{extr}} [X^* \operatorname{grad} \Phi (X^{(1)})], \tag{5.29}$$

where * is the transposition symbol, the vector $\bar{X}^{(1)}$ is found. In the problem (5.29) the purpose function, which is the scalar product of the vectors X and grad Φ $(X^{(1)})$, is linear with respect to X, which in special cases simplifies the solution of the problem. For example, if the set $\mathfrak{X}$ is defined by a system of linear equalities or inequalities, then we obtain the problem of linear programming. The methods of solving such problems have been well developed [61]. However, when the set $\mathfrak{X}$ is given by nonlinear relations and, the more so, the algorithms, it is complicated, as before, to find the solution of the problem (5.29);

the solution of the problem

$$\underset{\alpha \in [0,1]}{\operatorname{extr}} \Phi (X_\alpha^{(1)}), \tag{5.30}$$

is found where $X_\alpha^{(1)} = \alpha X^{(1)} + (1-\alpha) \bar{X}^{(1)}$, $\alpha \in [0, \ 1]$. The vectors $X_\alpha^{(1)}$ are linear combinations of the vectors $X^{(1)}$ and $\bar{X}^{(1)}$. The set of all possible vectors $X_\alpha^{(1)}$

for $\alpha \in [0, 1]$ coincides to a segment of the straight line joining the vectors $X^{(1)}$ and $\bar{X}^{(1)}$. The vector, which is the solution to the problem (5.30), is taken as the second approximation, after which problems (5.29) and (5.30) are solved again. The process of successive approximations is continued until the condition $\Phi(X^{l+1}) \approx \Phi(X^{(l)})$ is satisfied, where ℓ is the number of the approximation.

In the described method the vectors $X^{(k)}$, k = 1, 2, ..., can be found not from solution of the extremal problem (5.29), but from the condition $(\bar{X}^{(k)})^{*} \operatorname{grad} \Phi(X^{(k)}) < (X^{(k)})^{*} \operatorname{grad} \Phi(X^{(k)})$ in the case of minimizing Φ (X) and the inverse inequality, in the case of maximizing Φ (X). Analogously, instead of the solution of the extremal problem (5.30), the vector of the next approximation is found from the conditions $\Phi(X_{\alpha}^{(k)}) < \Phi(X^{(k)})$ or $\Phi(X_{\alpha}^{(k)}) > \Phi(X^{(k)})$.

The convergence of the provisional gradient method is insured for convex (in the case of min) and concave (in the case of max) functions Φ (X) and the convex set $\mathfrak{X}$.

The process of movement toward the minimum of the function Φ (X) is illustrated in Figure 5.6. The solution of the problem (5.29) gemetrically consists in finding the vector (point of the set $\mathfrak{X}$) having minimum projection on the gradient vector for the corresponding approximation step. The solution of the problem (5.30) consists in finding the extremal point on the straight line joining the points $X^{(\ell)}$ and $\bar{X}^{(\ell)}$. Actually, this is the problem of finding the extremum of a function of one variable α. For such problems effective numerical methods of solution have been developed [53].

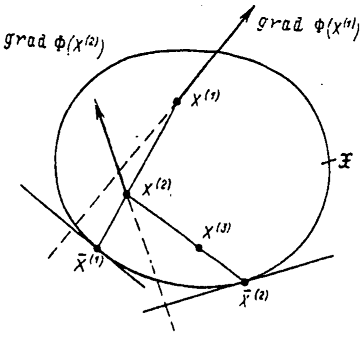

Figure 5.6. Geometric illustration of the provisional gradient method.

The geometric illustration presented in Figure 5.6 to some degree explains the meaning of the term "provisional gradient method," where movement toward the optimum is realized in each step not along the gradient vector of the purpose function, but in a direction deviating somewhat from the gradient vector.

Random Search Methods. These methods are the most universal with respect to type of purpose function and the set giving it. The essence of the random search methods consists in the following:

The initial point $X^{(1)} \in \mathfrak{X}$ is selected, and the value of the purpose function $\Phi(X^{(1)})$ is calculated;

The coordinates of the point $X^{(1)}$ vary in accordance with a defined random mechanism. In the problem of minimizing the purpose function Φ (X), the obtained point $X^{(2)}$ is taken as the second approximation with simultaneous satisfaction of the conditions $\Phi(X)X^{(2)} \in \mathfrak{X}$ and $\Phi(X^{(2)}) < \Phi(X^{(1)})$. In the case of maximizing Φ (X) in the second condition the inequality sign changes to the opposite. If at least one of these conditions is not satisfied, a new effort is made to vary the coordinates of the initial point $X^{(1)}$;

The same operations are performed for the point of the second approximation, $X^{(2)}$ as for the point $X^{(1)}$. The third approximation is found as a result;

The process ends after obtaining a quite long series of unsuccessful efforts to vary the coordinates of the point of the preceding approximation. This point is considered optimal.

Different versions of the random search method are related to the random law of variation of variables and variation of this law on moving toward the extremum considering the previously obtained values of the purpose function. The universality of the random search method is not achieved for "free" from the point of view of volume of calculations. There is a special literature on the random search methods [40, 42]. The random search methods are used when optimizing the characteristics of ships abroad [62].

Various combinations of the deterministic directional search methods, in particular, gradient methods and random search methods deserve special attention. For example, after obtaining the solution by the gradient methods for checking globalness of the extremum, the researcher can use random search, taking the solution obtained by the gradient methods of the initial point[1].

As an example let us consider one of the versions of the random search method -- the so-called method of exponential random search discussed in reference [62]. Let us have the extremal problem

$$\underset{X}{\operatorname{extr}}\, \Phi(X),$$

$$x_{i\,\min} \leqslant x_i \leqslant x_{i\,\max}, \quad i = 1, \ldots, n, \tag{5.31}$$

where x_i is a component of the vector X; $x_{i\,\min}$, $x_{i\,\max}$ are the minimum and maximum values of the ith component.

The following law is adopted as the random transformation of the coordinates of the vector X:

$$\xi_i = x_i^{*} + (x_{i\,\max} - x_{i\,\min})\,\eta^{m}, \quad i = 1, \ldots, n, \tag{5.32}$$

where ξ_i is a random value of the ith component of the vector X; x_i^* is the value of the ith component for the best of the preceding approximations; η is a random

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[1] At the present time the so-called dialog optimization systems are being intensely developed (see, for example, N. N. Moiseyev, Yu. P. Ivanilov, Ye. M. Stolyarova, METODY OPTIMIZATSII (Optimization Methods), Moscow, Nauka, 1978).

variable uniformly distributed in the interval [-1, 1]; m is a parameter characterizing the degree of randomness of search.

In accordance with expression (5.32) for odd values of m the probability density of the random variables ξ_i has the form

$$f(x_i) = \begin{cases} 0 & \text{for } x_i < x_{i\,\min}, \\ \frac{1}{2m\,\Delta x_i}\left(\frac{x_i - x_i^*}{\Delta x_i}\right)^{\frac{1-m}{m}} & \text{for } x_{i\,\min} \leqslant x_i \leqslant x_{i\,\max}, \\ 0 & \text{for } x_i > x_{i\,\max}, \end{cases} \tag{5.33}$$

where $\Delta x_i = x_{i\ \max} - x_{i\ \min}$, m is an odd positive number.

From expression (5.33) it is obvious that by using the parameter m it is possible to regulate the dispersion of the values of ξ_i with respect to the point x_i^*, that is, to regulate the degree of randomness of the search. For example, for m = 1 the values of ξ_i are uniformly distributed in the interval $[x_i^* - \Delta x_i, x_i^* + \Delta x_i]$ of length $2\Delta x_i$. With an increase in m the values of ξ_i in the probability sense are concentrated more and more around the point x_i^* — the degree of randomness of the search decreases (Figure 5.7). In reference [62] the function $f(x_i)$ is called the search intensity function.

For the model of optimization of the characteristics of a cargo ship adopted in [62] the following recommendations are presented with respect to variation of the parameter m in the search process:

k/K	0—0,6	0,6—0,8	0,8—0,9	0,9—1,0
m	1	3	5	7

Here k is the current number of steps, K is the total number of steps in the search process.

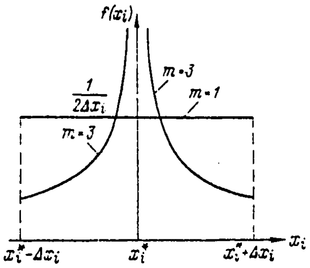

Figure 5.7. Distribution density of the ith variable for m = 1 and m = 3.

The transformation of (5.32) does not guarantee mandatory incidence of ξ_i in the interval $[x_{i\ \min}, x_{i\ \max}]$ for any values of η. Accordingly, in each step it is proposed that only one of the components of the vector X be varied. With simultaneous variation of all of the components the probability of incidence of each of them in the corresponding interval can be very low, which leads to an increase in the number of unsuccessful attempts at random variation of the vector X. If, for example, for m = 1 the probability of the event $\xi_i \in [x_{i\,\min}, x_{i\,\max}]$ is equal to 0.5, then on variation of only four components the probability of a successful attempt to vary the vector X, that is, the probability of incidence of all four components in the admissible intervals will be equal to only $0.5^4 = 0.0625$. On variation of only the ith component of the vector X in the kth step it is proposed that the remaining components will remain the same as in the k - 1 step. This increases the degree of variability of the vector X, which is highly desirable, especially at the beginning of the search process.

It is possible to propose a transformation of the type of (5.32) for which the condition $\xi_i \in [x_{i\,\min}, x_{i\,\max}]$ will be insured for any values of $\eta \in [-1, 1]$:

$$\xi_i = \begin{cases} x_i^{*} + (x_{i\,\max} - x_i^{*})\,\eta^m & \text{for } \eta > 0, \\ x_i^{*} + (x_i^{*} - x_{i\,\min})\,\eta^m & \text{for } \eta < 0. \end{cases}$$

The problem of estimating the convergence of the random search method from the point of view of selecting the required number of steps is most complex. For the answer to this problem in the general case it is necessary to perform a numerical experiment as applied to the investigated class of problems. In reference [62] a sufficiently good convergence is insured as applied to the model of optimization of the characteristics of ships investigated in it and the adopted version of the random search method for a number of steps $K \geq 1000n$, where n is the number of components of the vector X, that is, the number of optimizable characteristics of the ship.

§ 5.5. Example of Optimizing the Mass of Hauled Cargo and the Ship's Speed by the Provisional Gradient Method

Let us consider the problem of optimizing the mass of hauled cargo m_0 and speed v of a ship by the criterion

$$\max \frac{m_0 v}{S_э}, \quad m_{0\,\min} \leqslant m_0 \leqslant m_{0\,\max}, \quad v_{\min} \leqslant v \leqslant v_{\max}, \tag{5.34}$$

where $S_э$ is the cost of hauling; $m_{0\,\min}$ and $m_{0\,\max}$ are the minimum and maximum admissible values of the mass of the hauled cargo; $v_{\min}$ and $v_{\max}$ are minimum and maximum admissible values of the velocity.

Let us propose that the cost of hauling is proportional to the displacement

$$S_э = s_D D, \tag{5.35}$$

where D is the displacement of the ship; s_D is a constant coefficient which is the cost of operating the ship per ton of displacement.

Considering (5.35) as the purpose function in the criterion (5.34) it is possible to set $m_0 v/D$ -- the value proportional to the capacity per ton of displacement. Just as in the example in § 5.2, we shall assume that the displacement of the ship can be found from the three-term mass equation

$$AD + \frac{\bar{q}_{эу}}{C_w} v^3 D^{2/3} + m_0 + \sum_i m_i = D. \qquad (5.36)$$

Thus, in the investigated example the components of the vector X are the values of m_0 and v, and the function Φ (X) has the form

$$\Phi(m_0, v) = \frac{m_0 v}{D(m_0, v)},$$

where $D(m_0, v)$ is the solution of equation (5.36).

The set $\mathfrak{X}$ in the given example appears to be a rectangle given by the inequalities in the criterion (5.34).

For the numerical example let us take the following initial data:

$\bar{q}_{эу} = 0{,}04$ tons/kilowatt; C_w = 100 $\text{knots}^3\cdot\text{ton}^{2/3}$/kilowatt; A = 0.3;

v_{min} = 10 knots; v_{max} = 30 knots; $m_{0\ min}$ = 5000 tons; $m_{0\ max}$ = 20,000 tons.

Let us consider the solution of the stated problem by the provisional gradient method (see § 5.4).

1. Let us take the vector $X^{(1)} = \begin{pmatrix}10\\15\end{pmatrix}$[1] as the first approximation. From solution of the mass equation we find $D^{(1)}$ = 15,900 tons, $\Phi^{(1)}$ = 9.45 knots.

2. Let us estimate the vector grad Φ(X) for $X = X^{(1)}$, for which we calculate the values of the function Φ(X) at two points $X^{(11)}$ and $X^{(12)}$ close to $X^{(1)}$. As $X^{(11)}$ and $X^{(12)}$ we take $X^{(11)} = \begin{pmatrix}11\\15\end{pmatrix}$ and $X^{(12)} = \begin{pmatrix}10\\17\end{pmatrix}$.

In order to find $D^{(11)}$ and $D^{(12)}$ let us use the approximate differential formulas

$$D^{(11)} = D^{(1)} + \left.\frac{\partial D}{\partial m_0}\right|_{X=X^{(1)}} \Delta m_0,$$

$$D^{(12)} = D^{(1)} + \left.\frac{\partial D}{\partial v}\right|_{X=X^{(1)}} \Delta v,$$

where, as was demonstrated in Chapter 2, the derivatives $\frac{\partial D}{\partial m_0}$, $\frac{\partial D}{\partial v}$ are defined by the expressions

$$\frac{\partial D}{\partial m_0} = \frac{1}{1 - A - \frac{2}{3}\frac{\bar{q}_{эу}}{C_w}\frac{v^3}{D^{1/3}}},$$

$$\frac{\partial D}{\partial v} = \frac{3D^{2/3}v^2\frac{\bar{q}_{эу}}{C_w}}{1 - A - \frac{2}{3}\frac{\bar{q}_{эу}}{C_w}\frac{v^3}{D^{1/3}}}.$$

---

[1] The first component of the vector X is the carrying capacity of the ship, and the second, its speed.

Performing the corresponding calculations, we find $D^{(11)} = 17{,}400$ tons, $D^{(12)} = 16{,}400$ tons, $\Phi^{(11)} = 9{,}48$ knots; $\Phi^{(12)} = 10{,}35$ knots.

On the basis of these data we obtain the approximate values of the components of the gradient vector at the point $X^{(1)}$

$$\frac{\partial\Phi}{\partial m_0}\Big|_{X=X^{(1)}} \approx \frac{9{,}48-9{,}45}{1} = 0{,}03 \text{ knots/thousand tons,}$$

$$\frac{\partial\Phi}{\partial v}\Big|_{X=X^{(1)}} \approx \frac{10{,}35-9{,}45}{2} = 0{,}45.$$

3. Let us solve the linear problem

$$\max_{m_0,\, v}\left[m_0\frac{\partial\Phi}{\partial m_0}\Big|_{X=X^{(1)}} + v\frac{\partial\Phi}{\partial v}\Big|_{X=X^{(1)}}\right],$$

$$5 \leqslant m_0 \leqslant 20,$$

$$10 \leqslant v \leqslant 30.$$

As a result, we obtain the vector $\overline{X}^{(1)} = \begin{pmatrix}20\\30\end{pmatrix}$.

4. Let us find the value of α_1 from the condition

$$\Phi(X^{(1)}_{\alpha_1}) = \max_{\alpha}\Phi(X^{(1)}_{\alpha}),$$

where

$$X^{(1)}_{\alpha} = \begin{pmatrix}10\alpha + 20(1-\alpha)\\15\alpha + 30(1-\alpha)\end{pmatrix}.$$

The solution of this problem gives $\alpha_1 = 0.3$, and, consequently, as the second approximation we take the vector $X^{(2)} = X^{(1)}_{\alpha_1} = \begin{pmatrix}17\\25{,}5\end{pmatrix}$. For the vector $X^{(2)}$ we find $D^{(2)} = 34{,}800$ tons, $\Phi^{(2)} = 12.45$ knots.

5. Let us estimate the vector grad Φ (X) at the point $X = X^{(2)}$, selecting the vectors $X^{(21)} = \begin{pmatrix}18\\25{,}5\end{pmatrix}$, $X^{(22)} = \begin{pmatrix}17\\27{,}5\end{pmatrix}$.

Performing the corresponding calculations, we obtain

$$\frac{\partial\Phi}{\partial m_0}\Big|_{X=X^{(2)}} = 0{,}1 \text{ knot/thousand tons,} \quad \frac{\partial\Phi}{\partial v}\Big|_{X=X^{(2)}} = -0{,}025.$$

6. The solution of the linear problem $\max\limits_{m_0,\, v}(X\cdot\operatorname{grad}\Phi^{(2)})$ gives the vector

$$\overline{X}^{(2)} = \begin{pmatrix}20\\10\end{pmatrix} \text{ and } D^{(2)} = 35{,}100 \text{ tons, } \Phi^{(2)} = 5{,}7 \text{ knots.}$$

Now it is necessary to solve the problem

$$\max_{\alpha} \Phi [\alpha X^{(2)} + (1-\alpha) \overline{X}^{(2)}].$$

However, further movement toward the optimum by the provisional gradient method becomes difficult, for the points corresponding to the next approximations have coordinates that are close to each other, and advancement toward the optimum takes place only slowly. The reason for this is the presence on the response surface of the investigated purpose function of peculiarities in the form of an ascending peak. On hitting the peak, the provisional gradient method is not effective, as is obvious from Figure 5.8, where the geometric illustration of two steps of movement toward the extremum by the provisional gradient method is shown.

The equation which is satisfied by the peak points can be obtained using formulas (5.20) and (5.21), excluding D_0 from them. If for the investigated example we neglect the value of $\sum_i m_i . D_0$, then the equation of the peak has the form

$$v = 0,65 (1 - A)^{2/9} \left(\frac{C_w}{\bar{q}_{эу}}\right)^{1/3} m_0^{1/9},$$

where v, m_0 are coordinates of the peak points.

Figure 5.8 shows lines that correspond to the condition of constancy of the displacement $D(m_0, v)$ = const. The equations of these lines obtained from the mass equation considering that we neglect the value of $\sum_i m_i/D_0$ have the form

$$m_0 = (1 - A) D_0 - \frac{\bar{q}_{эу}}{C_w} v^3 D_0^{2/3}.$$

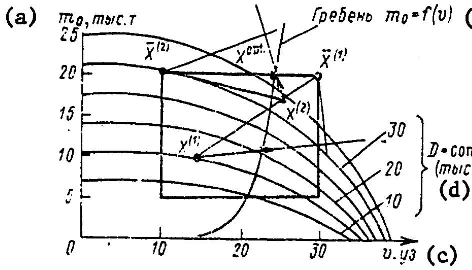

Figure 5.8. Geometric illustration of optimization of the mass of hauled cargo and speed of the ship by the provisional gradient method.

Key:
- a. m_0, thousands of tons
- b. peak
- c. v, knots
- d. (thousands of tons)

The peak points are the highest (in the sense of the values of the purpose function) points of the lines D = const where the height of the peak increases with an increase in the values of m_0 and v.

The solution of the problem investigated in this section is at the highest point of the peak belonging to the admissible range of values of m_0 and v. This point is at the intersection of the peak with the upper side of the rectangle (see Figure 5.8).

For the optimal point X^{opt} we have m_0^{opt} = 20,000 tons and v^{opt} = 24.5 knots. The displacement of the ship corresponding to the optimal values of m_0 and v will be 38,400 tons.

The investigated example, which is of an illustrative nature, indicates the effect of the peculiarities of the purpose function of the criterion on the effectiveness of the mathematical methods of optimization used.

§ 5.6. Optimization of TTE and TDP of a Ship Considering Several Effectiveness Indexes

The functioning of a ship carrying out the mission with which it is charged is a complex multistage process, each stage of which is distinguished by its own effectiveness indexes. Although these partial indexes theoretically determine the overall effectiveness index, they sometimes are of independent interest. This occurs in cases where, for example, in the adopted model it is not possible sufficiently completely to consider the effect of any partial indexes or when difficulties arise in constructing the general index.

The majority of ships are multipurpose, designed to solve several problems. The effectiveness of the solution of each problem, as a rule, is characterized by its own indexes. Under these conditions frequently it is difficult to isolate some generalized problem defined by one effectiveness index. This is one of the reasons for the fact that some researchers prefer to use the criteria of the type of (5.2), and not (5.1), for it is possible to include all of the effectiveness indexes in the system of limiting conditions in them. When the criteria of the type of (5.1) are used, in the presence of several effectiveness indexes it is not always easy to isolate the main index subject to optimization with restrictions on the other (auxiliary) indexes. In addition, it is unclear at what price strict optimization of one index with respect to the others will be achieved.

Let us consider some approaches to the solution of the optimization problem in the presence of several effectiveness indexes. (We shall not include the case where one of the indexes is optimized and the others are transferred to the category of limiting conditions, in this problem.)

Optimization considering several indexes is sometimes called vector optimization [7], emphasizing that in the given case the general effectiveness index can be interpreted as a vector, the components of which are the partial indexes.

The formulation of the problem of vector optimization consists in the following [7].

There is a set of scalar variables $\eta_1, \ldots, \eta_n$ characterizing the outcome or outcomes of the ship operations when carrying out one or several missions. The values of $\eta_1, \ldots, \eta_n$, beging, generally speaking, random, form the random vector η. The values of this sector will be denoted by y, and the values of its components, by $y, \ldots, y_n$. The components $y_1, \ldots, y_n$ are values of $\eta_1, \ldots, \eta_n$. On the basis of of the parameters $\eta_1, \ldots, \eta_n$, the partial effectiveness indexes $Э_1, \ldots, Э_n$ can be constructed (see Chapter 3), which form the nonrandom vector $Э$. The values of the indexes $Э_1, \ldots, Э_n$ depend on the vector of the TTE and the TDP of the ship. Just as before, we shall denote it by x, and the set of its admissible values, by $\mathfrak{X}$.

The problem consists in finding X_{opt} from the condition

$$\underset{X \in \mathfrak{X}}{\mathrm{opt}}\, Э(X). \tag{5.37}$$

The basic problem is resolution of the meaning of the operator opt in the criterion (5.37). The solution reduces to constructing a functional $\Phi(Э)$ having the property $\Phi(Э_1) > \Phi(Э_2)$ when and only when the vector $Э_1$ is preferable to the vector $Э_2$. If the functional Φ is defined, then the problem (5.37) reduces to the problem of mathematical programming

$$\max_{X \in \mathfrak{X}} \Phi[Э(X)]. \tag{5.38}$$

When constructing the functional Φ from the condition $\Phi(Э_1) < \Phi(Э_2)$, then $Э_1$ is preferable $Э_2$, in the criterion (5.38) it is necessary to minimize $\Phi(Э(X))$. Hereafter, for determinacy we shall consider that the functional Φ must be maximized and that growth of individual partial indexes corresponds to an increase in Φ. In connection with the fact that the functional Φ maps the set of possible values of the partial indexes $Э_i$, $i = 1, \ldots, n$, onto the set of real numbers, the problem of constructing the functional Φ is called the scalarization problem. The functional Φ defines the meaning of the optimization operator in the problem (5.37).

Before proceeding with solution of the problem (5.37), it is always necessary to isolate the so-called region of compromises -- the subset of the set $\mathfrak{X}$ in which simultaneous improvement of all partial indexes $Э_i$ is impossible -- from the set $\mathfrak{X}$. The optimal solution of X_{opt} always lies in the region of compromises. Hereafter, we shall assume that $\mathfrak{X}$ already is the region of compromises.

When constructing the functional Φ in the general case it is necessary to consider the relative importance of the partial indexes $Э_i$. As a rule, when constructing the functional Φ normalization of the partial indexes is always necessary, that is, reduction of them to a single measurement scale. This normalization is based on introduction of the ideal vector $Э^{(и)}$ and the transition from the values on $Э_i$ to the relative values of $e_i = Э_i/Э_i^{(и)}$ where $Э_i^{(и)}$ are components of the vector $Э^{(и)}$.

As the vector $Э^{(и)}$ let us take $Э^{(и)} = [\sup_X Э_1(X), \ldots, \sup_X Э_n(X)]$, where $\sup_X Э_i(X)$, $i = 1, \ldots, n$, are the exact upper bounds of the partial indexes in the set $\mathfrak{X}$. If $\sup_X Э_i(X) \to \infty$, then it is possible to take sufficiently high levels of partial indexes approximately as the components of $Э_i^{(и)}$.

The selection of the ideal vector $Э^{(и)}$ reduces the problem of constructing Φ to the problem of quantitative evaluation of the degree of approximation to the ideal vector $Э^{(и)}$. The principles of this approximation can be different. The basic ones of them are the following [7]; the principle of uniform (Chebyshev) optimization

$$\Phi(Э) = \min_i e_i;$$

the principle of the integral (average) optimization

$$\Phi(\text{Э}) = \sum_{i=1}^{n} e_i;$$

the principle of proper compromise

$$\Phi(\text{Э}) = \prod_{i=1}^{n} e_i \quad \text{or} \quad \Phi(\text{Э}) = \sum_{i=1}^{n} \log e_i.$$

Let us note that the principle of proper compromise does not require normalization of the partial indexes.

All of the above-enumerated principles are valid for identical importance of the partial indexes. However, in the problems of optimizing the TTE and the TDP of ships this case is very rarely encountered. Generally speaking, the importance of the partial indexes can be considered an additional normalization of the vector e (with the components $e_1, \ldots, e_n$) by the vector $\alpha = (\alpha_1, \ldots, \alpha_n)$, $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i > 0$, $i = 1, \ldots, n$, that is, the transition to the vector $e_\alpha = (\alpha_1 e_1, \ldots, \alpha_n e_n)$. Here the theoretical difficulty lies in selecting the vector α, the components of which are called the "weight" coefficients of the partial indexes.

Very frequently when constructing the functional Φ, the integral optimization principle is used considering the vector of the "weight" coefficients $\Phi(\text{Э}) = \sum_{i=1}^{n} \alpha_i e_i$.

In this expression it is necessary to take the sum in the algebraic sense, putting the signs in front of the terms in accordance with the nature of the effect of the corresponding indexes on the total effectivness. If the index e_i increases the effectivenss, it enters into the sum with a positive sign and vice versa.

Sometimes when designing ships efforts are made to formulate the components of the effectiveness index directly on the basis of the components of the vector of the TTE and the TDP of the ship without calculating the partial indexes e_i, $i = 1, \ldots, n$, considering the heuristic nature of the effect of the individual TTE and TDP of the ship on its effectiveness. In particular, this approach was proposed as applied to ships in 1908 by A. N. Krylov [25][1]. However, this estimate is of a highly subjective nature.

The component indexes are also used at the present time. For example, in article [62] when optimizing the characteristics of a cargo ship, the displacement, the prismatic coefficient, the ratio of the length of the ship to the side height, the ratio of the width to the draft and the Froude number are considered as components of the vector X. The expression

$$\alpha_1 S^2 + \alpha_2 (m_{\text{тр}} - m)^2 + \alpha_3 (V_{\text{тр}} - V)^2,$$

(1)

Key: 1. req

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[1] Analogous proposals were also advanced by other authors (see, for example, MORSKOY SBORNIK (Maritime Collection), Vol CCCXII, No 9, 1902, pages 41-70).

is taken as the purpose function of the optimization criterion, where S is the cost of the ship; m_{req}, m are the required and obtained masses of the hauled cargo; V_{req}, V are the required and obtained cargo capacity; α_1, α_2, α_3 are the "weight" coefficients.

Another example of the component index is the weighted sum of the cost of building and maintaining a fleet of ships and the cost of solving the standard problem (see § 4.5): $\alpha_1 S(X, \mathcal{N}) + \alpha_2 \times S_{\text{problem solution}}(X, \mathcal{N})$, where $S(X, \mathcal{N})$ is the cost of building and maintaining the fleet; $S_{\text{problem solution}}(X, \mathcal{N})$ is the cost of solving the problem. For ships to be used once it is possible to take $\alpha_1 = 1$ and $\alpha_2 = 0$. However, in the general case the assignment of specific values to the coefficients α_1 and α_2 is a highly complex problem.

It is possible to avoid difficulties in defining the vectors $\text{Э}^{(\text{н})}$ and α by using the principle of rigid priority of the partial indexes. According to this principle the partial indexes are arranged in order of preference (according to their importance) $\text{Э}_1 \succ \text{Э}_2 \succ \ldots \succ \text{Э}_n$. The numbering of the indexes in this series can differ from the initial numbering. The optimization is carried out successively, beginning with the most important index Э_1, optimized in the initial set $\mathfrak{X} \equiv \mathfrak{X}_1$. The second index with respect to importance Э_2 is optimized in the set

$$\mathfrak{X}_2 = \{X / \text{Э}_1(X) = \text{Э}_1^{\text{opt}}\} \cap \mathfrak{X}_1.$$

This means that the index Э_2 is optimized in the subset $\mathfrak{X}_2 \subset \mathfrak{X}$, in which the index Э_1 retains it optimal value. The optimization of the subsequent indexes is carried out analogously: the ith index is optimized in the set

$$\mathfrak{X}_i = \{X / \text{Э}_{i-1}(X) = \text{Э}_{i-1}^{\text{opt}}\} \cap \mathfrak{X}_{i-1}.$$

In the set $\mathfrak{X}_i$ all of the higher indexes with respect to importance retain their optimal values. As a result of this procedure it is possible to find the optimal vector X_{opt} or a set of these vectors.

The deficiency of this method consists in the fact that very frequently the optimization with respect to the first index or with respect to the first several indexes gives a unique optimal vector X_{opt}, and optimization of the following indexes becomes impossible, that is, the series of indexes is actually not considered. In order to eliminate the indicated deficiency, the principle of successive steps is used [12]. Here when optimizing the ith index the set $\mathfrak{X}_i$ contains the vectors X, for which the value of the index Э_{i-1} is not exactly equal to the optimal value, and it differs from it by an amount $\Delta\text{Э}_{i-1}$. The modulus of this value is called a step. The latter characterizes the "price" of improving the indexes of lower rank with respect to the indexes of higher rank. The values of the steps can be given in relative units with respect to the optimal values of the indexes Э_i^{opt}. There are no strict mathematical methods of defining the steps (just as when determining the "weight" coefficients in the component indexes), but the step method is more graphic and does not require normalization of the partial indexes.

The application of the step method is highly effective inasmuch as in the problems of optimizing the TTC of ships the purpose functions very frequently are gently sloping in the vicinity of the extrema. In reference [62] this property was given the special name — flat-laxity. As a result of the property of flat-laxity of the purpose functions with relatively small values of the steps, we obtain quite broad sets of values of the vector X for optimizing the indexes of lower rank.

For illustration of this method let us again consider the problem of § 5.2 connected with optimizing the mass of the hauled cargo and the speed of the ship with limited displacement. In addition, let us propose that along with maximizing the hauling capacity it is of interest to maximize the mass of the hauled cargo directly. The primary index here will be considered to be the hauling capacity, and the auxiliary index, the mass of the hauled cargo. In accordance with the results of § 5.2, optimization with respect to hauling capacity gives

$$m_0^{\text{opt}} = \frac{3}{4} a, \quad v^{\text{opt}} = \sqrt[3]{\frac{a}{4b}}, \quad \text{Э}_1^{\text{opt}} = \frac{3}{4} a \sqrt[3]{\frac{a}{4b}} k,$$

where $a = D_0 \left(1 - A - \frac{\sum_i m_i}{D_0}\right)$, $b = \frac{\bar{q}_{\text{эу}}}{C_w} D_0^{2/3}$, Э_1^{opt} is the optimal value of the hauling capacity.

Let us state the problem of maximizing the mass of the hauled cargo (the index Э_2) as a result of the relative step $\overline{\Delta \text{Э}}_1$ with respect to hauling capacity. The set of values m_0 satisfying the step is given by the condition

$$\frac{\text{Э}_1^{\text{opt}} - \text{Э}_1(m_0, v)}{\text{Э}_1^{\text{opt}}} \leqslant \overline{\Delta \text{Э}}_1 \tag{5.39}$$

or

$$\text{Э}_1^{\text{opt}} (1 - \overline{\Delta \text{Э}}_1) \leqslant \text{Э}_1 (m_0, v),$$

where $0 < \overline{\Delta \text{Э}}_1 < 1$.

The expression for the index Э_1 after substitution of the formula $v = \sqrt[3]{\frac{a - m_0}{b}}$, in it which follows from the mass equation, has the form

$$\text{Э}_1 (m_0) = m_0 \sqrt[3]{\frac{a - m_0}{b}} k.$$

Correspondingly, the condition (5.39) can be represented as follows:

$$b [\text{Э}_1^{\text{opt}} (1 - \overline{\Delta \text{Э}}_1)]^3 \leqslant (a m_0^3 - m_0^4) k^3$$

or after substitution of the expression for Э_1^{opt}

$$0{,}105 (1 - \overline{\Delta \text{Э}}_1)^3 a^4 \leqslant a m_0^3 - m_0^4. \tag{5.40}$$

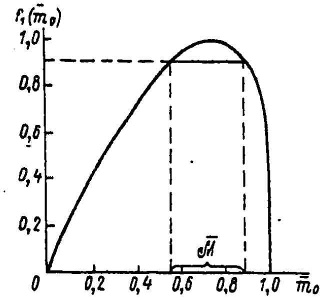

Figure 5.9. Graph of the function $f_1(\bar{m}_0) = 2.12\,\bar{m}_0\sqrt[3]{1-\bar{m}_0}$.

Introducing the relative value of $\bar{m}_0 = m_0/a$, we arrive at the following problem:

$$\max_{\bar{m}_0\in\bar{\mathscr{M}}} \bar{m}_0, \tag{5.41}$$

where

$$\bar{\mathscr{M}} = \{\bar{m}_0/\bar{m}_0^3 - \bar{m}_0^4 \geqslant 0{,}105\,(1-\overline{\Delta Э}_1)^3\}.$$

The set $\bar{\mathscr{M}}$ is the interval $[\bar{m}_{0\,\min}, \bar{m}_{0\,\max}]$, the limits of which are defined by the size of the step $\Delta Э_1$. It is obvious that the solution of the problem (5.41) gives $\bar{m}_0^{\,opt} = \bar{m}_{0\,\max}$.

The value of $\bar{m}_{0\,\max}$ can be found graphically, using the graph of the function $f(\bar{m}_0) = \bar{m}_0^3 - \bar{m}_0^4$. It is also possible to use the graph of the function $f_1(\bar{m}_0) = 2{,}12\bar{m}_0\sqrt[3]{1-\bar{m}_0}$ (Figure 5.9) inasmuch as the set $\bar{\mathscr{M}}$ is given by the condition

$$1-\overline{\Delta Э}_1 \leqslant \frac{4}{3}\sqrt[3]{4}\,\bar{m}_0\sqrt[3]{1-\bar{m}_0} = 2{,}12\bar{m}_0\sqrt[3]{1-\bar{m}_0}.$$

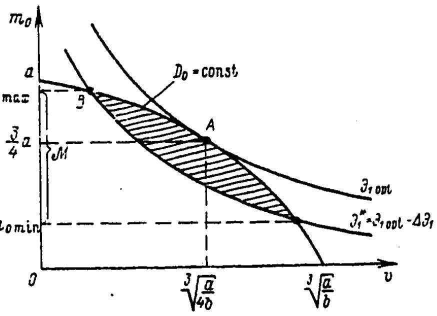

Figure 5.10. Geometric illustration of the method of successive steps when optimizing the mass of hauled cargo and speed of the ship.

From Figure 5.9 it is obvious that for $\overline{\Delta Э}_1 = 0{,}1$ $\bar{m}_{0\,\max} = 0{,}88$. Consequently, in this case the optimization of the indexes $Э_1$ and $Э_2$ by the method of successive steps leads to the solution

$$m_0^{\mathrm{opt}} = 0{,}88a, \quad v^{\mathrm{opt}} = 0{,}494\sqrt[3]{\frac{a}{b}}.$$

The expression for v^{opt} follows from the mass equation after substitution of m_0 = 0.88a in it.

Let us give a more general geometric illustration of the solution of the investigated problem by the step method.

In the plane (m_0, v) the lines $Э_1(m_0, v) = \mathrm{idem}$, that is, the constant hauling capacity lines, are hyperbolas (Figure 5.10). The lines D_0 = idem are given by the equation $m_0 = a - bv^3$ following from the mass equation. The optimal solution with respect to the index $Э_1$ corresponds to the point A of tangency of the hyperbola $Э_1 = \mathrm{idem}$ with the line D_0 = const corresponding to the given value of D_0. Let for a step equal to $\Delta Э_1$, the lines $Э_1^{\mathrm{opt}} - \Delta Э_1 = Э_1 = \mathrm{const}$ run as shown in Figure 5.10. Then the crosshatching indicates the region of values of m_0 and v in which the index $Э_2 = m_0$ must be optimized (in the given case, maximized). The maximum m_0 in this regions corresponds to the point B, the coordinates of which also give the optimal solution by the step method.

§ 5.7. Optimization of the TTE and the TDP of a Ship Under Conditions of Indeterminacy with Respect to Some Initial Data

When solving the basic problems of ship AD, a number of external values of an operative-tactical and technical-economic nature with respect to the ship, for example, the quantitative and qualitative composition of the active forces of the enemy, the methods of their operations, hydrometeorological conditions in the combat areas and on the routes to these areas, cost of a norm-hour of labor consumption, overhead coefficient, and so on are used.

The following three basic levels of indeterminacy in knowing the indicated external parameters are possible: the parameters are determinate, and their values are given; the parameters are not determinate (random), but their probability characteristics are completely known; the parameters are indeterminate and their probability characteristics are unknown. The problems of optimization under conditions of indeterminacy belong to the last level. In this case the choice of specific values of TTE and TDP (the vector X) of the ship leads to a set of values of the purpose function of the optimalness criterion. The probability characteristics of these outcomes are entirely unknown.

At this time there is only one sufficiently substantiated principle of resolving the indeterminacy to select the optimal solution under the indicated conditions. This principle presupposes that the undefined parameters are controlled (completely or partially) by a conscious enemy, the purposes of which in a defined sense are opposite to the purposes of the side making the decision. The methods of optimizing decisions in this case have come to be called game theory. If a conscious enemy with opposite goals is absent, then the optimization methods with random external parameters are called statistical solution theory or the theory of games with nature.

At the present time a number of optimization criteria have been proposed [28] in games with nature under conditions of complete indeterminacy. For example, it is proposed that the principle of insufficient substantiation be used according to which the values of the undefined parameters are considered to be equiprobable in the given range. In the minimax and maximin criteria the nature of the variables controlled by a conscious enemy with antagonistic goals is assigned to the undefined parameters, although in reality this may not follow from the essence of the investigated phenomenon.

All of the above-indicated criteria are of a subjective nature. Although subjective solutions are also used in practice, at the present time exact quantitative methods cannot be proposed for substantiation of them. It is no accident, for example, that in the book by Luce and Reif [28] examples of paradoxical solutions contradicting common sense are presented for each of the proposed optimization criteria under conditions of complete indeterminacy. According to the views held by many specialists, the basic efforts must be aimed not at finding objective optimization criteria under conditions of complete indeterminacy, inasmuch as this is essentially the search for an "information perpetual motion machine," but at the methods of decreasing the initial indeterminacy so that subsequently the minimum required information will be obtained for objective substantiation of the optimal solution. Here, if possible, the indeterminacy must be completely removed.

The above indicated point of view was shared, in particular, by I. Ya. Diner [19], who proposed a procedure for decreasing the initial indeterminacy in knowing the parameters influencing the optimal solution. The essence of this procedure called the method of regionalization in space of undefined parameters [19, 20] consists in the following.

Let a set of undefined parameters exist characterized by the vector $T = (t_1, \dots, t_n)$, where t_k, $k = 1, \dots, n$ are undefined parameters. The sets $\Theta_1, \dots, \Theta_n$ of possible values of individual parameters will be considered given.

In the space with the coordinates $t_1, \dots, t_n$, let us consider the set Θ_T of possible values of the vector T formed by the direct product of sets $\Theta_1, \dots, \Theta_n$: $\Theta_T = \Theta_1 \times \Theta_2 \times \dots \times \Theta_n$. It is obvious that each point of the set Θ_T (each vector $T \in \Theta_T$) corresponds to an optimal vector X_T. In practice very frequently the same vector X_T is optimal for a subset of the set Θ_T (for several different vectors T), that is, the set Θ_T can be broken down into a system nonintersecting subsets $\Theta_T^{(i)}$ such that for each $T \in \Theta_T^{(i)}$, the same vector X_i is optimal.

Consequently, if the above-indicated procedure is performed for breaking down the set Θ_T into the subsets $\Theta_T^{(i)}$, then in order to find the optimal vector X it is necessary to establish in which of the subsets the vector T fell in the specific problem of optimization or to determine the probabilities of incidence of the vector T in the individual subsets $\Theta_T^{(i)}$. The latter problem is frequently solved more simply than the definition of the probabilities in all elements of the initial set Θ_T.

If in the breakdown of the set Θ_T all of the subsets $\Theta_T^{(i)}$, except one, turned out to be empty, then the indeterminacy is completely removed, for independently of the specific values of the undefined parameters the same vector X will be optimal. Let us remember the example from § 5.1 where the speed of the search ship was optimized. If in this example we consider the search area and the given search time undefined parameters, then this indeterminacy is insignificant inasmuch as the optimal value of the speed does not depend on the specific values of these parameters.

It must be noted that the regionalization method is very labor-consuming and obviously reduces to parametric optimization. However, at the present time there are no other reasonable approaches to solving the optimization problem under conditions of complete indeterminacy.

Let us consider an example which illustrates the application of the regionalization method. Let us propose that the structure of a fire-fighting system designed to extinguish fires in m compartments of the ship is investigated. The system has a different number n = 1, ..., m of pumps, the total output capacity of which will be considered constant, selected from the condition of simultaneous extinguishing of fires in all m compartments. Let the reliability of each unit be characterized by the probability p of fail-safe operation. We shall consider this probability to be independent of the number of pumps in the system and their output capacity.

Let us consider the problem of finding the optimal number of pumps in the system, that is, let us define the optimal degree of centralization or decentralization of the system with respect to pumps. An entirely centralized system has one "large" pump serving all compartments, and a completely decentralized system, m "small" pumps, each of which services only one compartment.

In accordance with the general scheme for constructing the effectiveness indexes (§ 3.1), let us take a random number ξ of compartments provided with the output capacity of fail-safe pumps as the functioning characteristic of the system. If the system consists of n pumps functioning independently of each other, the distribution function F(x/n) of ξ has the form (binomial distribution)

$$F(x n) = \begin{cases} \sum\limits_{i=0}^{\left[\frac{xn}{m}\right]} C_n^i p^i (1-p)^{n-i} & \text{for} \quad x < m, \\ 1 & \text{for} \quad x \geqslant m, \end{cases}$$

where [xn/m] is the integral part of the number xn/m.

For a linear utility function $u_1(x) = x$ the effectiveness index is the mathematical expectation of the number of serviced compartments $\text{Э}_1(n) = pnm/n = pm$. This index, as is obvious from the formula, does not depend on n, that is, on the number of pumps in the system. Consequently, the number of pumps in the system can be selected as any number in the interval [1, m].

Now let the utility function have the step form:

$$u_2(x) = \begin{cases} 1 & \text{for} \quad x < m, \\ 0 & \text{for} \quad x \geqslant m. \end{cases}$$

This means that the system carries out its mission only when supplying all compartments. In this case the effectiveness index will be $\text{Э}_2(n) = P\{\xi = m/n\} = p^n$. Inasmuch as $0 < p < 1$, the entirely centralized system with one "large" pump is optimal with respect to the criterion $\max \text{Э}_2$.

Now let us propose that the system with probability ω_1 must service m/k compartments (k is a positive integer, a multiple of m), with a probability ω_2, 2m/k compartments, and so on, and finally with a probability ω_k it is required to serve as km/k = m compartments (all m compartments). Let all of the enumerated cases form the total group of events, that is, $\sum_{i=1}^{k} \omega_i = 1$, and the utility functions for each case have the step form

$$u_i(x) = \begin{cases} 1 & \text{for } x \geqslant i\frac{m}{k}, \\ 0 & \text{for } x < i\frac{m}{k}, \quad i = 1, \ldots, k. \end{cases}$$

Considering performance of the mission in all of the indicated cases identically important and considering that the functions $u_i(x)$, i = 1, ..., k are already normalized, it is possible to define the weighted mean utility function

$$u(x, \omega) = \begin{cases} 0 & \text{for } x < \frac{m}{k}; \\ \sum_{j=1}^{i} \omega_j & \text{for } i\frac{m}{k} \leqslant x < (i+1)\frac{m}{k}, \quad i = 1, \ldots, k-1; \\ 1 & \text{for } x \geqslant m; \end{cases}$$

where $\omega = (\omega_1, \ldots, \omega_k)$ is the vector giving the probability distribution of different cases of use of the system.

In this case the effectiveness index depends on the vector ω: $Э_3(n, \omega) = \int_x u(x, \omega)\, dF(x\ n)$. Consequently, the optimal number of pumps also depends on ω.

Let us propose that the probability distribution $\omega_1, \ldots, \omega_k$ is unknown to us. Here the regionalization method consists in the following.

The conditions $\sum_{i=1}^{k} \omega_i = 1,\ \omega_i \geqslant 0,\ i = 1, \ldots, k$ define the set Ω_k, which is a side of a k-dimensional simplex.[1] The criterion $\max Э_3(n, \omega)$ defines the function $n_{opt}(\omega)$ given in the set Ω_k. The function $n_{opt}(\omega)$, in turn, gives the breakdown of Ω_k into subsets $\Omega_k^{(n)}$, n = 1, ..., k such that $\bigcup_{n=1}^{k} \Omega_k^{(n)} = \Omega_k$, $\Omega_k^{(\nu)} \cap \Omega_k^{(\mu)} = \Lambda$ for $\nu \neq \mu$ and $n_{opt}(\omega) = n$ for all $\omega \in \Omega_k^{(n)}$.

If a breakdown of the set Ω_k of this sort is performed, then for determination of n_{opt} it is necessary to establish in which the subsets $\Omega_k^{(n)}$ the vector ω fell in the specific problem.

It is natural that in the general case this is done more simply the more exactly we

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[1] A simplex is a set of points, the coordinates of which in n-dimensional space satisfies the conditions $\sum_{i=1}^{n} x_i = 1,\ x_i \geqslant 0,$ where $x_i,\ i = 1, \ldots, n$ are the coordinates of the point.

know all the probabilities $\omega_1, \ldots, \omega_k$. For example, let k = 2 and $\omega_1 = \omega$, $\omega_2 = 1 - \omega$, that is, let it be necessary with a probability ω that half of the compartments be serviced, and with a probability $(1 - \omega)$, all m compartments. The optimal number of pumps in this case can be 1 or 2, depending on the value of ω. For n = 1 (a single-pump system) $\text{Э}_3(1, \omega) = p$. For a two-pump system $(n = 2)$ $\text{Э}_3(2, \omega) = \omega [1 - (1 - p)^2] + (1 - \omega) p^2 = p [p + 2\omega(1 - p)]$. The condition of preference for the two-pump system defined by the inequality $\text{Э}_3(2, \omega) > \text{Э}_3(1, \omega)$, has the form $p + 2\omega (1 - p) > 1$, which is equivalent to the condition $\omega > 1/2$.

Thus, the interval [0, 1], which is the initial set of values of the undefined parameter ω is broken down into two subintervals [0, 1/2] and [1/2, 1]. If $\omega \in [0, 1/2]$, then $n_{opt} = 1$, and if $\omega \in [1/2, 1]$, then $n_{opt} = 2$. It is natural that it is simpler to obtain the data to answer the question as to which of the inequalities $(\omega \leq 1/2$ or $\omega > 1/2)$ occurs in the specific problem than to find the exact value of ω.

§ 5.8. Example of Optimizing the TTC of a Ship by the Method of Comparative Evaluation of Versions

As is known, the optimization of the TTC of a ship must be carried out on the basis of investigation of a system of usually mixed forces designed to solve the given problem. Cases are also possible where a different-type fleet of ships of one class and type of forces of the fleet turns out to be optimal when solving the same problem.

Below, an example of optimizing the TTC of a ship by the method of comparative evaluation of versions is considered which takes into account the effect of a fleet of ships of one class, but with different TTC. Hereafter such a fleet will be called mixed-type. (The basis for this example was borrowed from reference [64].)

Let us propose that for the solution of some problem ships of n types (versions) are used. The application of mixed-type fleets is permitted. The ships operate independently of each other, and the volumes of the solution of the general problem by one ship of each of the types are characterized by random variables η_{1i}, i = 1, ..., n, the distribution functions of which have the form

$$F(y_{1i}) = \begin{cases} 0 & \text{for } y_{1i} < 0, \\ 1 - k_{\text{он}}^{(i)} p_i & \text{for } 0 \leqslant y_{1i} < Q_{1i}, \\ 1 & \text{for } y_{1i} \geqslant Q_{1i}, \end{cases} \tag{5.42}$$

where y_{1i} are the values of the random variables η_{1i}; Q_{1i}, $k_{\text{он}}^{(i)}$; p_i are given values which depend on the TTC of the ships of the investigated types.

Thus, it is proposed that each ship of the ith type solves a problem in the volume Q_{1i} with the probability $\bar{p}_i = k_{\text{он}}^{(i)} p_i$, and with a probability $1 - \bar{p}_i$ the volume of solution of the problem is zero. The value of $k_{\text{он}}^{(i)}$ is the operative use coefficient of the ith type ship and, as is demonstrated in § 3.2, characterizes the probability that the ship will be in the combat zone at the time of solving the problem. The value of p_i is the probability of solving the problem in the volume Q_{1i} under the condition that the ship is in the zone. (This system of combat operations was investigated in § 3.2).

Let us assume that during operations of the fleet of ships, including mixed-type, the total volume of solution of the problem defined by the random variable η is equal to the sum of the volumes of solution of the problem by each of the ships of the fleet:

$$\eta = \sum_{i=1}^{n} \eta_{1i} \mathcal{N}_i, \tag{5.43}$$

where $\mathcal{N}_i$ is the number of ith type ships in the fleet.

Let us consider the problem of selecting the optimal fleet of ships beginning with the military-economic arguments considering the assumptions made above. Here we shall adhere to the previously discussed general optimization scheme including the construction of the effectiveness and cost indexes, the selection of the optimalness criterion and, especially, finding the optimal solution.

The parameter characterizing the outcome of the combat operations in the given example is the random variable η. As was demonstrated in Chapter 3, for construction of the effectiveness index first of all it is necessary to have the utility function u(y), where y is the values of η, and, secondly, to define the distribution function $F(y/\mathcal{N})$ of the value η for solution of the problem by the fleet of ships defined by the vector $\mathcal{N} = (\mathcal{N}_1, \ldots, \mathcal{N}_n)$. Let us consider two forms of the function u(y): linear u(y) = y and step

$$u(y) = \begin{cases} 0 & \text{for } y < Q_\Sigma, \\ 1 & \text{for } y \geqslant Q_\Sigma, \end{cases}$$

where Q_Σ is a given value of the total volume of solution of the problem.

For a linear utility function the effectiveness index is the mathematical expectation $M_\eta(\mathcal{N})$ of the variable η:

$$Э_1(\mathcal{N}) = \sum_{i=1}^{n} \bar{p}_i Q_{1i} \mathcal{N}_i = M_\eta(\mathcal{N}). \tag{5.44}$$

It is not necessary to find the specific form of the distribution function $F(y/\mathcal{N})$ here.

With the step utility function the effectiveness index is the probability of solving the problem in the volume equal to Q_Σ or exceeding it:

$$Э_2(\mathcal{N}) = \int_{Q_\Sigma}^{\infty} dF(y/\mathcal{N}) = P\{\eta \geqslant Q_\Sigma/\mathcal{N}\}. \tag{5.45}$$

In the given case it is necessary to define the specific form of

If the total number of ships in the operating fleet is quite large (in practice it is sufficient that $\sum_{i=1}^{n} \mathcal{N}_i > 5$ to 6), then under the assumptions made regarding independence of the operations of the individual ships and correctness of the formula (5.43) the function $F(y/\mathcal{N})$ will approximately have the form of a normal probability distribution law

$$F(y/\mathcal{N}) = \frac{1}{\sqrt{2\pi}\sigma_\eta}\int_{-\infty}^{y} e^{-\frac{(\xi-M_\eta)^2}{2\sigma_\eta^2}}\, d\xi, \tag{5.46}$$

where $M_\eta = \sum_{i=1}^{n} \bar{p}_i Q_{1i}\mathcal{N}_i$, $\sigma_\eta^2 = \sum_{i=1}^{n} \bar{p}_i \bar{q}_i Q_{1i}^2 \mathcal{N}_i$, $\bar{q}_i = 1 - \bar{p}_i$.

Substituting (5.46) in (5.45) and replacing variables $\frac{y-M_\eta}{\sigma_\eta} = t$, we obtain the expression for $Э_2(\mathcal{N})$:

$$Э_2(\mathcal{N}) = \int_{-\infty}^{\frac{M_\eta - Q_\Sigma}{\sigma_\eta}} \varphi(t)\, dt, \tag{5.47}$$

where $\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$.

As the economic index let us take the cost $S(\mathcal{N})$ of building and maintaining the fleet of ships, and we shall consider the function $S(\mathcal{N})$ linear with respect to $\mathcal{N}$:

$$S(\mathcal{N}) = \sum_{i=1}^{n} s_i \mathcal{N}_i; \tag{5.48}$$

here s_i is the cost of building and maintaining one ship of the ith type.

The next step is selection of the optimalness criterion for finding the optimal vector $\mathcal{N}_{\text{opt}}$, defining the optimal composition of the forces in quantitative and qualitative respects. The versions of the ship will be optimal which correspond to the nonzero components in the vector $\mathcal{N}_{\text{opt}}$; but the optimalness of these versions in the general case is retained only when they are used in the mixed-type fleet.

As the military-economic optimalness criterion let us take the condition of maximizing the effective index with limited cost of building and maintaining the fleet of forces

$$\max_{\mathcal{N}} Э(\mathcal{N}),$$
$$S(\mathcal{N}) \leqslant S_\Sigma, \tag{5.49}$$
$$\mathcal{N}_i \geqslant 0, \quad i = 1, \ldots, n.$$

where S_Σ is the given allocated budget.

Let us consider the solution of the problem (5.49) for the adopted two forms of the utility function: linear and step. Here we use the fact that the maximizing of the index $Э_2(\mathcal{N})$ in accordance with expression (5.47) is equivalent to maximizing the value of $\omega = (M_\eta - Q_\Sigma)/\sigma_\eta$.

In the case of the linear utility function the problem (5.49) has the form

$$\max_{\mathcal{N}} \sum_{i=1}^{n} \bar{p}_i Q_{1i} \mathcal{N}_i,$$
$$\sum_{i=1}^{n} s_i \mathcal{N}_i \leqslant S_\Sigma, \tag{5.50}$$
$$\mathcal{N}_i \geqslant 0, \quad i = 1, \ldots, n.$$

The solution of this type of problem of linear programming has already been considered in § 5.1. The optimal vector $\mathcal{N}_{\text{opt}}$ will contain only one nonzero component $\mathcal{N}_k$, for which the condition

$$\max_i \frac{\bar{p}_i Q_{1i}}{s_i}, \tag{5.51}$$

is satisfied, that is, the optimal fleet is single-type and the optimal version is found by the criterion of combat economicalness (5.51).

For a step utility function the problem (5.49) is equivalent to the problem

$$\begin{gathered} \max_{\mathcal{N}} \frac{\sum_{i=1}^{n} \bar{p}_i Q_{1i} \mathcal{N}_i - \bar{Q}_\Sigma}{\sqrt{\sum_{i=1}^{n} \bar{p}_i \bar{q}_i Q_{1i}^2 \mathcal{N}_i}}, \\ \sum_{i=1}^{n} s_i \mathcal{N}_i \leqslant S_\Sigma, \\ \mathcal{N}_i \geqslant 0, \; i = 1, \; \ldots, \; n. \end{gathered} \tag{5.52}$$

The cost restriction in the form of an inequality can be replaced here by the equality inasmuch as from the physical arguments it is clear that the optimal value of the index $\partial_2(\mathcal{N})$ does not decrease with an increase in S_Σ.

After replacement of the variables $z_i = (s_i \, s_\Sigma) \mathcal{N}_i, \quad i = 1, \ldots, n,$ and introduction of the relative values of $\bar{Q}_{1i} = Q_{1i}/Q_\Sigma, \; \bar{s}_i = s_i/S_\Sigma, \; i = 1, \ldots, n,$ we obtain

$$\max_z \frac{\sum_{i=1}^{n} \bar{a}_i z_i - 1}{\sqrt{\sum_{i=1}^{n} \bar{b}_i z_i}}, \tag{5.53}$$

$$\sum_{i=1}^{n} z_i = 1, \quad z_i \geqslant 0, \quad i = 1, \; \ldots, \; n,$$

where $Z = (z_1, \ldots, z_n)$ is the vector of the new variables, and $\bar{a}_i = \bar{p}_i \bar{Q}_{1i}/\bar{s}_i$, $\bar{b}_i = \bar{p}_i \bar{q}_i \bar{Q}_{1i}^2/\bar{s}_i, \; i = 1, \ldots, n.$

Solving problem (5.53), that is, finding the optimal vector Z_{opt}, by going to the old variables we find the optimal vector $\mathcal{N}_{\text{opt}}$.

Let us note that $\frac{M_\eta}{Q_\Sigma} = \sum_{i=1}^{n} \bar{a}_i z_i, \quad \frac{\sigma_\eta^2}{Q_\Sigma^2} = \sum_{i=1}^{n} \bar{b}_i z_i.$

The problem (5.53) can be solved by numerical methods, for example, the provisional gradient method investigated in § 5.4. Here the auxiliary problem (5.29) will be the elementary problem of linear programming.

At the same time the problem (5.53) can be solved also by sorting using the fact that the optimal vector $\mathcal{N}_{\text{opt}}$ will contain no more than two nonzero components. The correctness of the given statement follows from the following arguments. The purpose function in the problem (5.53) has the form $\omega(\alpha, \beta) = \frac{\alpha - 1}{\sqrt{\beta}}$, where $\alpha = \sum_{i=1}^{n} \bar{a}_i z_i, \quad \beta = \sum_{i=1}^{n} \bar{b}_i z_i$.

This function increases with respect to α for each fixed value of β. Consequently, for any fixed β the optimal vector Z is the solution of the problem of linear programming

$$\max_{Z} \sum_{i=1}^{n} \bar{a}_i z_i, \quad \sum_{i=1}^{n} \bar{b}_i z_i = \beta, \quad \sum_{i=1}^{n} z_i = 1, \qquad (5.54)$$
$$z_i \geqslant 0, \quad i = 1, \ldots, n.$$

Inasmuch as in any problem of linear programming the number of nonzero components of the optimal vector does not exceed the number of restrictions (besides the conditions $z_i \geq 0$, i = 1, ..., n), the optimal vector in the problem (5.54) can contain no more than two nonzero components. Hence, it follows that also in the initial problem (5.53) the number of nonzero components of the optimal vector will not exceed two.

The above-presented result is valid not only for the step functions, but also for any nondecreasing bounded utility function u(y). Actually, for an arbitrary function u(y) and normal distribution law of the value of η, the effectiveness index $\mathcal{N}_{\text{opt}}$ has the form

$$Э_2(M_\eta, \sigma_\eta) = \int_{-\infty}^{\infty} u(M_\eta + \sigma_\eta t)\, \varphi(t)\, dt. \qquad (5.55)$$

If u(y) is a nondecreasing and bounded function of y, then the integral in the expression (5.55) converges and $Э_2(M_\eta, \sigma_\eta)$ is a nondecreasing function of M_η for each fixed value of σ_η. The latter means that $Э_2(M_\eta, \sigma_\eta)$ has the same property as the function ω(α, β) in the problem (5.53). Consequently, for any nondecreasing bounded utility function the optimal fleet of forces in the investigated example can consist of ships of no more than two different types (versions).

Finally, it is possible to state [64] that this conclusion remains effective not only for the linear, but also for any concave cost function $S(\mathcal{N})$. Let us remember that for a concave function $S(\mathcal{N})$ for any two vectors $\mathcal{N}_1$ and $\mathcal{N}_2$ and any number $0 \leq \delta \leq 1$, the equality $S(\delta\mathcal{N}_1 + (1-\delta)\mathcal{N}_2) > \delta S(\mathcal{N}_1) + (1-\delta) S(\mathcal{N}_2)$ is satisfied. In particular, the cost function $S(\mathcal{N})$ will be concave when considering the series factor (see § 4.3). Let us return to this division of the problem (5.53). From what has been discussed above it follows that it is sufficient to solve this problem for all possible pairs of investigated versions of the ship and then to compare the corresponding values of the purpose function ω(Z). The pair for which the value of the purpose function turns out to be the highest will give the optimal vector of the problem (5.53). For n = 2, that is, for two versions of the ship characterized by the parameters $(\bar{a}_1, \bar{b}_1)$ and $(\bar{a}_2, \bar{b}_2)$, the problem (5.53) reduces to optimizing a function of one variable

$$\max_{0 \leqslant z_1 \leqslant 1} \frac{\bar{a}_2 + (\bar{a}_1 - \bar{a}_2) z_1 - 1}{\sqrt{\bar{b}_2 + (\bar{b}_1 - \bar{b}_2) z_1}}. \qquad (5.56)$$

In the given case the vector Z has the form $Z = (z_1, 1 - z_1)$.

The problem (5.56) can be solved by elementary methods. For the presence of a maximum inside the interval [0, 1], and not at its boundaries (for $z_1 = 0$ or $z_1 = 1$) the conditions

$$\left.\frac{d\omega(z_1)}{dz_1}\right|_{z_1=0} > 0, \quad \left.\frac{d\omega(z_1)}{dz_1}\right|_{z_1=1} < 0; \tag{5.57}$$

should be satisfied, where $\omega(z_1)$ is the purpose function in the problem (5.56).

The maximum inside the interval [0, 1] means that on investigation of the given pair of versions of the ship the fleet including both of these versions is optimal. If the maximum is reached at $z_1 = 1$, then the optimal fleet consists of ships of only the first version; for $z_1 = 0$ the optimal fleet consists of ships of only the second version.

It is possible to demonstrate [64] that for the step utility function the mixed-type optimal fleet of ships will occur only in the case of relatively small values of the allocated budget (S_Σ) by comparison with the required volume (Q_Σ) of solution of the combat problem when the probability of its solution (the event $\eta \geq Q_\Sigma$) by a single-type fleet of each of the versions is less than 0.5. Here the mathematical expectation of the volume of solution of the problem by the single-type fleet of each of the versions is less than Q_Σ. In practice the situation must be recognized as more an exception than the rule inasmuch as the values of the probability of solution of the combat less than 0.5 usually are considered unacceptable.

Consequently, for real values of Q_Σ and S_Σ the optimal version will be, as a rule, single-type. The optimal version of the ship does not necessarily insure maximum mathematical expectation of the volume of the solution of the stated combat problem.

The physical meaning of the theoretical possibility of the appearance of optimal mixed-type fleets in the investigated example is very simple. For small budgets where $M_\eta < Q_\Sigma$ sometimes it turns out to be more advantageous to use the mixed-type fleet insuring a smaller value of M_η (by comparison with the best single-type fleet), but having a larger value of the dispersion σ_η^2 which, in turn, can give a larger value of the probability $P\{\eta \geq Q_\Sigma\}$. For $M_\eta < Q_\Sigma$ from the two (or more) fleets with identical values of the mathematical expectations of the volume of solution of the problem the best is the fleet with the largest value of the dispersion σ_η^2. On the contrary, in the case of $M_\eta > Q_\Sigma$ and equality of the mathematical expectations the best fleet will be the one insuring the minimum dispersion σ_η^2.

§ 5.9. Use of the LP-Search Method When Optimizing the TTC of Ships

When solving the complex problems of optimization where the purpose functions and the admissible regions of optimizable parameters do not have such properties as unimodality, convexity, concavity, connectedness, and so on or when checking the satisfaction of the indicated conditions is difficult (the corresponding functions are given by complex algorithms), the necessity arises for investigating the space of the optimized parameters at a finite number of discrete points. For example, in pure form this approach is used when optimizing the TTC of ships by the method of comparative estimation of versions. As was pointed out in § 1.3 and 5.3, this method consists in calculating the values of the purpose function of the optimalness criterion for a given number of versions clarifying the limiting conditions, with subsequent selection of the optimal versions in accordance with the value of the purpose function.

The discrete method of investigating the space of optimizable parameters at individual points located in the entire admissible region or some part of it can be used (in combination with the methods of directional search) for selection of the initial point, and falling into a valley or falling on a peak of the purpose function, and so on.

The following problem arises: how are the points (sets of TTC of the versions of the ship) selected for the discrete method of examining the space of optimizable parameters?

In the case of practical design, this problem frequently is solved on the basis of the experience and intuition of the designer designating both the ranges of variation of the optimizable TTC and the specific TTC of the individual investigated versions of the ship. If the solution of the indicated problem on the basis of experience and intuition is difficult, the uniform coverage of the region of values of the continuously variable TTC by random or nonrandom points, from which the best is selected with respect to the value of the purpose function, is used. This coverage must theoretically be accomplished for each of the possible sets of discretely variable TTC which in the final analysis permits the optimal version to be approximately found by the variables of both types.

The LP-search method [50] is based on covering the admissible region of optimizable parameters by a nonrandom grid of points. The latter is constructed using the so-called LP_τ-series of points uniformly distributed in a unit n-dimensional cube where n is the number of continuously variable parameters. This series was constructed by I. M. Sobol' [49].

The LP-search method sometimes turns out to be quite effective when searching for extremal values of complex functions of many variables in the case of complex nature of the admissible regions from the assignment. The LP-search method can be used also for the statement and solution of the problems of vector optimization (see § 5.6) although the method of analyzing the tables of values of the purpose and the bounding functions discussed below is applicable for any discrete examination of the admissible region of values of optimizable parameters.

Let us first consider the basic principles of the LP-search method when solving optimization problems with respect to one index. The general form of these problems is:

$$\begin{gathered}\operatorname*{extr}_{X} F(X),\\ C_l^{*} \leqslant f_l(X) \leqslant C_l^{**}, \quad l=1, \ldots, L,\\ x_i^{*} \leqslant x_i \leqslant x_i^{**}, \quad i=1, \ldots, n,\end{gathered} \tag{5.58}$$

where F(X) is the optimiable index; $f_\ell(X)$, $\ell = 1, \ldots, L$, are the indexes on which the restrictions are imposed in the form of two-way inequalities; C_l^{*}, C_l^{**}, x_i^{*}, x_i^{**},

$l = 1, \ldots, L,\ \ i = 1, \ldots, n,$ are given values, where $C_l^{**} > C_l^*$ and $x_i^{**} > x_i^*$ for all ℓ and i; $X = (x_1, \ldots, x_n)$ is the vector of continuously variable parameters.

Usually the restrictions directly on the values of x_i are called parametric, and on $f_\ell(X)$, functional. For properness of the problem (5.58) from the mathematical point of view the above-indicated restrictions must define some nonempty, bounded and closed set $\mathfrak{X}$ in the n-dimensional space of vectors X. Geometrically this set is the intersection of an n-dimensional parallelepiped (π_n) defined by the parametric restrictions with the set defined by the functional restrictions. The functions F(X) and $f_\ell(X)$, $\ell = 1, \ldots, L$ in practical problems, as a rule, are piecewise-continuous and bounded at all points of the parallelepiped π_n. However, the set $\mathfrak{X}$ can have a highly complex form. Thus, it can be not collected, convex, and so on. These peculiarities greatly complicate the direct application of the methods of directional search, for example, the gradient methods, for solution of the problem (5.58).

When optimizing the TTC of a ship byn the method of comparative evaluation of versions in the parallelepiped π_n, $\mathcal{N}$ points $X_0, \ldots, X_{\mathcal{N}-1}$ are selected. Let $\mathcal{N}'$ points $(\mathcal{N}' \leqslant \mathcal{N})$ satisfy the functional restrictions. The point out of the $\mathcal{N}'$ points for which extr F(X) is achieved is considered optimal.

If there is no a priori information about the nature of the set $\mathfrak{X}$ and the behavior of the function F(X) in it, then it is necessary to strive for more uniform probing by the points $X_j,\ j = 0, \ldots, \mathcal{N} - 1,$ of the entire parallelepiped π_n. It is also obvious that the points X_j must be selected so that for $\mathcal{N} \to \infty$ the probability of incidence of at least one point in the vicinity as small as one might like of the unknown optimal point X_{opt} will approach one.

For random search the indicated requirements are satified if independent random points uniformly distributed in π_n are selected as X_j. The probing of the parallelepiped π_n can also be carried out by deterministic, uniformly distributed points. However, not all of the uniform distributions of points in π_n have identical uniformity in the defined sense. for a quantitative evaluation of this property it is necessary to introduce the corresponding measure.

Let us note above all that in order to introduce the new variables

$$\tilde{x}_i = \frac{x_i - x_i^*}{x_i^{**} - x_i^*}, \quad i = 1, \ldots, n,$$

the parallelepiped π_n can be reduced to a unit n-dimensional cube K_n and, correspondingly, it is possible to consider the degree of uniformity of arrangement of the points in K_n. As the measure of nonuniformity of arrangement of the points $\tilde{X}_0, \ldots, \tilde{X}_{\mathcal{N}-1},$ belonging to the cube K_n, the value

$$\Delta = \sup_{\tilde{X}_P \in K_n} | \xi_{\mathcal{N}}(\pi_P) - \mathcal{N} v_P |, \tag{5.59}$$

can be selected which is called the deviation.

In formula (5.59) π_p is a parallelepiped with sides parallel to the coordinate axes, the diagonal 0P and located in the cube K_n; v_P is the volume of the parallelepiped π_p; $\xi_{\mathcal{N}}(\pi_P)$ is the number of points (out of the total number $\mathcal{N}$), falling in π_p (Figure 5.11).

The series of points is called uniformly distributed in K_n if the limiting relation is satisfied

$$\lim_{\mathcal{N} \to \infty} \frac{\Delta}{\mathcal{N}} = 0. \tag{5.60}$$

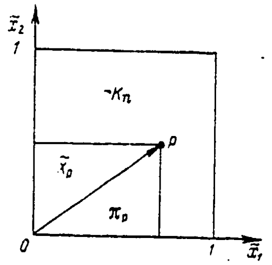

Figure 5.11. Position of the parallelepiped π_p in the cube K_n for the two-dimensional case (n = 2).

Most frequently the points of cubic grids are selected as uniformly distributed points, for which

$$\tilde{x}_1 = \frac{m_1 - 0,5}{M}, \ \ldots, \ \tilde{x}_n = \frac{m_n - 0,5}{M},$$
$$1 \leqslant m_1, \ \ldots, \ m_n \leqslant M.$$

Figure 5.12, a shows such a grid for M = 4. The total number of points in the uniform cubic grid is equal to M^n.

For the cubic grid sup in the formula (5.59) is achieved for the selection of points $\tilde{X}_P = \left(\frac{1,5}{M} - \varepsilon, 1, \ldots, 1\right)$, where ε is a small value. In this case $\xi_{\mathcal{N}}(\pi_P) = M^{n-1}$, $v_P = \frac{1,5}{M} - \varepsilon$, $M = \mathcal{N}^{\frac{1}{n}}$ and for $\varepsilon \to 0$ the deviation will be $\Delta = 0,5\mathcal{N}^{1-\frac{1}{n}}$.

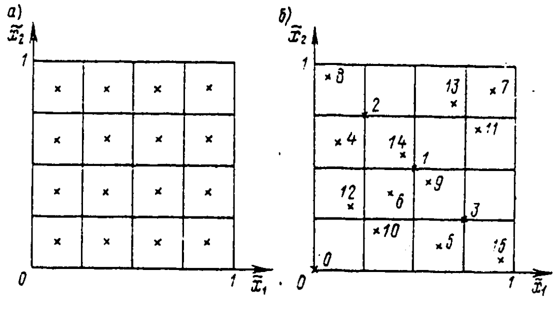

Figure 5.12. Arrangement of 16 uniformly distributed points in a two-dimensional unit cube K_2: a — for the cubic grid; b — for points of the Sobol' series.

Correspondingly, for the ratio $\Delta/\mathcal{N}$ we obtain the estimate $\frac{\Delta}{\mathcal{N}} \sim \frac{1}{\sqrt[n]{\mathcal{N}}}$.

When investigating the random uniform arrangement of points in the cube K_n the random variable

$$\frac{\xi_{\mathcal{N}}(\pi_P) - \mathcal{N}v_P}{\sqrt{\mathcal{N}v_P(1 - v_P)}}$$

for sufficiently large $\mathcal{N}$ is distributed by a normal law with mathematical expectation equal to zero and dispersion equal to one. Hence, it follows that

$$P\{|\xi_{\mathcal{N}}(\pi_P) - \mathcal{N}v_P| \geqslant 2\eta\sqrt{\mathcal{N}v_P(1 - v_P)}\} = 1 - \Phi(2\eta), \qquad (5.61)$$

where $\Phi(z) = \frac{2}{\sqrt{2\pi}}\int_0^z e^{\frac{t^2}{2}}dt$ is the probability integral; η is a given positive number.

The expression under the square root sign in (5.61) reaches a maximum for $v_P = 1/2$. Consequently, selecting the point $X_P = (1/2, 1, \ldots, 1)$, we obtain

$$P\{\Delta \geqslant \eta\sqrt{\mathcal{N}}\} \geqslant 1 - \Phi(2\eta).$$

For a sufficiently small value of $\eta = \eta_1$ with probability close to one the inequality $\Delta \geqslant \eta_1\sqrt{\mathcal{N}}$ is valid. On the other hand, for a sufficiently large value of $\eta = \eta_2$ $\Delta \leqslant \eta_2\sqrt{\mathcal{N}}$. Hence, it follows that $\eta_1\sqrt{\mathcal{N}} \leqslant \Delta \leqslant \eta_2\sqrt{\mathcal{N}}$ and, consequently,

$$\frac{\Delta}{\mathcal{N}} \sim \frac{1}{\sqrt{\mathcal{N}}}.$$

Thus, the random uniform grid for $n \geq 3$ has an advantage over the cubic grid in the sense of asymptotic (for $\mathcal{N} \to \infty$) uniformity of arrangement of the points in the n-dimensional unit cube.

However, there are series with better indexes of uniformity of arrangement of the points for $\mathcal{N} \to \infty$. These series include the Sobol' LP_τ-series. The Sobol' points are found by the following algorithm.

Let the binary representation of the number of the jth Sobol' point $\tilde{X}_j$ have the form $j = e_m 2^{m-1} + \ldots + e_2 2^1 + e_1 2^0$, where e_s, $s = 1, \ldots, m$ are the binary numbers assuming values of 0 or 1. Then the coordinates $\tilde{x}_{ij}$ $(i = 1, \ldots, n;\ j = 0, \ldots, \mathcal{N} - 1)$ of the n-dimensional Sobol' points are calculated by the formulas $\tilde{x}_{i0} = 0$, $i = 1, \ldots, n$, $\tilde{x}_{ij} = e_1 V_i^{(1)} * e_2 V_i^{(2)} * \ldots * e_m V_i^m$, $i = 1, \ldots, n$, $j = 1, 2, \ldots$ Here $V_i^{(s)}$ are socalled directional numbers, and the operation * denotes bit-by-bit mod 2 addition in the binary system. The numbers $V_i^{(s)}$ are fractions, the numerators of which are in the tables[1], and the denominators of all numbers are 2^s. The table of directional numbers permits calculation of the coordinates of the

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[1] I. M. Sobol', Yu. L. Levitan, "Obtaining Points Uniformly Arranged in a Multidimensional Cube," Institute of Applied Mathematics of the USSR Academy of Sciences, Moscow, 1976, preprint No 40.

n-dimensional points with the numbers $j < 2^{21}$ for dimensionality $n \leq 51$. In this paper for $\mathcal{N} = 2^{\nu}$, $\nu = 1, 2, \ldots,$ the deviation is estimated as $\Delta \leqslant C(n) \ln^{n-1} \mathcal{N}$, where C(n) is a constant which depends on n.

In appendix 2 a table of coordinates of the first 33 ten-dimensional Sobol' points is presented. In order to obtain points of lower dimensionality it is necessary to select the corresponding number of first coordinates of each point (Figure 5.12, b).

Thus, for $\mathcal{N} \to \infty$ the series of Sobol' points has the best uniformity of arrangement in a unit cube by comparison with the cubic and random uniform grids. The series with the best uniformity are unknown. However, it is necessary to consider that this conclusion is of an asymptotic nature $(\mathcal{N} \to \infty)$.

It does not appear possible to give a unique answer to the problem of the required number of Sobol' points for the solution of problem (5.58) inasmuch as this depends on the nature of the function F(X), the set $\mathfrak{X}$ and the required accuracy of finding the optimum. I. M. Sobol' presents the following estimate for the required number of points:

$$\mathcal{N} = 2^{2+[\sqrt{n}]} \qquad (5.62)$$

Here $[\sqrt{n}]$ is the integral part of the number $\sqrt{n}$ (n, just as before, is the dimensionality of the vector X).

The estimate of (5.62) was obtained on the basis of solving simple problems for $n \leq 12$. In the above-cited paper [50] it is pointed out that when solving practical problems most frequently a value of $\mathcal{N} = 256$ is used.

Thus, the basic idea of the LP-search method consists in examination of the admissible region of values of optimized parameters at discrete points corresponding to the Sobol' points uniformly distributed in a unit cube. It is obvious that even when selecting the number of points by the recommendation of (5.62) this number turns out to be quite large. The possibility of practical use of the LP-search method, just as the other methods, based on discrete nondirectional sorting, essentially depends on the time for calculation of the values of the functions f(X) and $f_{\ell}(X)$, $\ell = 1, \ldots, L$. Therefore the application of the LP-search method in pure form for optimizing the characteristics of such complex objects as a ship as a whole is hardly expedient. Obviously it is more efficient to use the LP-search in combination with the directional search methods, considering that the series of Sobob' points has quite good uniformity and with small numbers of probing points. In particular, if we break down the series of Sobol' points into sections of length 2^n (that is, into groups of 2^n points following each other) and correspondingly break up the cube K_n into 2^n "octants" by a uniform cubic grid, then for each $n \leq 16$ the points of each of the sections of the series are distributed with respect to the different "octants." An analogous property, but already only for each $n \leq 6$, occurs also in the case of breakdown of the series into sections of length 4^n and, correspondingly, the cube into 4^n "octants.

Now let us consider how the properness of the statement of the problem (5.58) is analyzed using the LP-search method. Selecting a number $\mathcal{N}$ of uniformly distributed points in the parallelepiped π_n, it is possible to expect that only $\mathcal{N}'$

points will fall into the region $\mathfrak{X}$, in which the functional restrictions are satisfied. The value $\mathcal{N}'$, and, more precisely, its mathematical expectation, can be found by the formula $\mathcal{N}' = \mathcal{N}\frac{v_{\mathfrak{X}}}{v_{\pi}}$, where $v_{\pi} = \prod_{i=1}^{n} (x_i^{**} - x_i^{*})$ is the volume of the parallelepiped π_n; $v_{\mathfrak{X}}$ is the volume of the region $\mathfrak{X}$.

If $v_{\mathfrak{X}} = 0$, the solution of problem (5.58) does not exist. However, direct establishment of this fact for the complex form of the functions $f_{\ell}(X)$, for example without rhythmic assignment of them, is very difficult. In addition, in essence the parameters C_{ℓ}^{*} and C_{ℓ}^{**} in the functional restrictions cannot have the nature of absolutely rigidly given values. In a number of cases the researcher can relax the restrictions or, on the contrary, make them more rigid. This means that some functions $f_{\ell}(X)$ are of a criterial nature and, consequently, the problem (5.58) can be considered as a vector optimization problem which is reduced to the problem of optimization with respect to one index by imposing restrictions on the other indexes $f_{\ell}(X)$.

The idea of analyzing properness and more precise definition of the statement of the problem (5.58) consists in investigation of it as a vector optimization problem as a result of conversion of the nonrigid functional restrictions to optimizable indexes. After this transformation of the problem (5.58) for all of the selected uniformly distributed points in π_n, a table of values of the functions $F(X)$ and $f_{\ell}(X)$, $\ell = 1, \ldots, L$, is constructed. For sufficiently large $\mathcal{N}$ the analysis of such tables makes it possible to judge whether at least one of the points falls into the admissible region $\mathfrak{X}$ and how variation of the nonrigid restrictions influences the admissible region $\mathfrak{X}$ and the choice of the optimal point.

Let two nonrigid restrictions on the functions $f_1(X)$ and $f_2(X)$ and one rigid restriction on the function $f_3(X)$ exist in the problem (5.58). Let us assume that the table of values of the functions $F(X)$ and $f_{\ell}(X)$, $\ell = 1, 2, 3$ has the form of Table 5.1 in which the admissible values of the function $f_3(X)$ are noted. Let us consider that the values of the functions are arranged in increasing order, and the index $F(X)$ must be maximized.

Table 5.1. Values of the functions $F(X)$ and $f_{\ell}(X)$ at eight points

$F(X)$	F_0	F_5	F_3	F_6	F_1	F_4	F_2	F_7
$f_1(X)$	f_{10}	f_{14}	f_{12}	f_{16}	f_{11}	f_{15}	f_{13}	f_{17}
$f_2(X)$	f_{20}	f_{26}	f_{25}	f_{24}	f_{21}	f_{23}	f_{22}	f_{27}
$f_3(X)$	f_{37}	f_{32}	f_{31}	f_{33}	f_{34}	f_{35}	f_{36}	f_{30}

Note. F_j is the value of the function $F(X)$ at the point X_j, $f_{\ell j}$ is the value of the ℓth function $f_{\ell}(X)$ at the point X_j.

From Table 5.1 it is obvious that beginning with the rigid requirement of $f_3(X)$ the points X_1, X_2, X_3 and X_4 will fall in the admissible region of $\mathfrak{X}$. The best point is X_2.

If, for example, the one-way restrictions $f_1(X) \leq C_1^{**}$ and $f_2(X) \leq C_2^{**}$ are imposed on the functions $f_1(X)$ and $f_2(X)$, where $f_{12} > C_1^{**} > f_{14}$ and $C_2^{**} > f_{24}$, then the optimal point will be the point X_4 which in the given case will be the only admissible point. At the same time if the nonrigid restrictions are relaxed somewhat, setting $C_1^{**} = f_{12}$ and $C_2^{**} = f_{22}$, then 2 points X_2 and X_4 will fall into the admissible region, of which the better will be X_2. Analogously, if $C_1^{**} < f_{14}$, then no point will fall into the admissible region. In this case it is necessary to try to increase the number of points or relax the restriction on $f_1(X)$.

The correction of the nonrigid restrictions is made not only in the direction of relaxation. For example, if all of the trial points turn out to be admissible, then the researcher can narrow the admissible region (make the restrictions more rigid), on grounds following from the essence of the solved problem.

The above-described analysis of the tables of trial points in the space of optimizable parameters is of an informal nature and requires the use of certain heuristic arguments of the researcher. However, this analysis frequently turns out to be highly useful for proper formulation and solution of complex problems of optimization. When realizing a computer-aided LP-search, the step in which the table of trial points is analyzed is expediently performed using the researcher-computer dialog mode.

An analysis of the tables of trial points is highly useful for the formulation and solution of the problems of vector optimization. Such an analysis permits, for example, detection of indexes that vary insignificantly little, the discovery of dependent indexes simultaneously increasing or decreasing, facilitation of the solution of the scalarization problem (see § 5.6), and so on.

It must be considered that on the basis of the LP-search it is possible to construct iterative procedures in which the trial points are selected by series with gradual narrowing of the search region with respect to the best of the points of the preceding series. This approach is considered as a deterministic analog of the directional random search (see § 5.4).

Let us consider the problem (5.34) of optimizing the mass of hauled cargo and the speed of a transport ship as a numerical example of using the LP-search method. This problem was solved in § 5.5 by the provisional gradient method. Let us remember that in § 5.5 the solution of this problem was not obtained as a result of the presence of a singularity in the purpose function in the form of an ascending peak.

As the first series of trial points in the LP-search method let us take 16 points in a rectangle defined by the parametric restrictions [functional restrictions are absent in problem (5.34)]. The coordinates of the points and the values of the purpose function for the initial data adopted in § 5.5 are presented in Table 5.2. In this table $\tilde{x}_{1j}$ and $\tilde{x}_{2j}$ will be coordinates of the Sobol' points in a unit two-dimensional cube. From Table 5.2 it follows that the best points are those with the numbers 7, 11 and 13 which are in the upper right-hand corner of the initial rectangle π_2.

Now it is possible to narrow the initial search region and select a second series of trial points. As was stated earlier, joint application of the methods of directional search and LP-search can turn out to be effective. In § 5.5, the point m_0 = 17,000 tons, v = 25.5 knots for which D = 34,800 tons and Φ = 12.45 knots, was obtained as a result of two steps by the provisional gradient method. Here $\partial\Phi/\partial m_0 > 0$ and $\partial\Phi/\partial v < 0$. Using these data, let us apply the LP-search method in a rectangle defined by the conditions $17 \leq m_0 \leq 20$ and $20 \leq v \leq 25$.

Table 5.2. Probing the initial region of optimizable parameters m_0 and v at 15 Sobol' points.

j	$\tilde{x}_{1j}$	$\tilde{x}_{2j}$	m_{0j} тыс. т(a)	v_j уз(b)	D_j тыс. т(a)	Φ_j уз (b)
0	0	0	5,00	10.0	7,80	6,40
1	0.500	0,500	12,5	20.0	21,9	11,4
2	0.250	0,750	8,75	25.0	19,5	11,2
3	0.750	0,250	16,2	15.0	25,2	9,65
4	0,125	0,625	6,90	22,5	14,0	11,1
5	0.625	0,125	14,4	12.5	21,8	8,27
6	0.375	0,375	10,4	17,5	17,3	10,5
7	0.875	0,875	18,0	27,5	40,0	12,4
8	0,062	0,937	5,90	28,7	18,2	9,30
9	0,562	0.437	13,5	18,7	22,6	11,1
10	0.312	0,187	9,70	13.7	15,9	8,35
11	0.812	0,687	17,2	23,7	32,8	12.4
12	0,187	0,312	7.80	16.2	12,9	9,80
13	0,687	0,812	16,3	26.2	34,6	12,35
14	0,437	0,562	11,5	21,2	21,0	11,6
15	0,937	0,062	19,0	11,2	28,3	7,45

Key: a. thousands of tons b. knots

Table 5.3 Probing a restricted region of optimizable parameters m_0 and v at eight Sobol' points

j	$\tilde{x}_{1j}$	$\tilde{x}_{2j}$	m_{0j} тыс. т(a)	v_j уз (b)	D_j тыс. т (a)	Φ_j уз(b)
0	0	0	17,0	20,0	29,0	11,7
1	0,500	0,500	18,5	22,5	33,5	12,4
2	0,250	0,750	17,8	23,8	34,0	12,5
3	0,750	0,250	19,3	21,3	33,7	12,2
4	0,125	0,625	17,4	23,0	32,4	12,35
5	0,625	0,125	18,9	20,6	32.5	12,0
6	0,375	0,375	18,1	21,9	[illegible]	12,2
7	0,875	0,875	19,6	24,4	37,9	12,6

Key: a. thousands of tons b. knots

In Table 5.3 the results are presented from probing the indicated region by eight Sobol' points. From this table it is obvious that the eighth point turns out to be the best, which considering rounding of the coordinates coincides with the optimal point (see § 5.5).

CHAPTER 6. SOME SPECIAL PROBLEMS CONNECTED WITH THE ANALYTICAL DESIGN OF SHIPS

One of the peculiarities of the procedural approach of ship design theory, including AD theory, consists in the necessity for partial optimization of some design solutions in order to decrease the number of variables optimized "in terms of the ship" as a whole as much as possible. The reason for this is complexity of the construction of the mathematical models of optimization which take into account all of the variety of characteristics of the ship simultaneously which are manifested on different hierarchical levels of investigation of its combat use. It is very complicated, for example, to perform a quantitative evaluation and optimize the technical solutions in the general model connected with insuring certain operating qualities of the ships. At the same time, neglecting the latter can lead to very serious consequences. As has already been pointed out, the TDP of a ship influence its effectiveness indirectly, through the TTE, and sometimes it is very difficult to trace this effect to the end (to indexes of a sufficiently high hierarchical level).

It is expedient to perform a partial optimization with respect to the TTC which are relatively stable on variation of the conditions of combat use of the ship or to a known degree secondary with respect to their influence on effectiveness and expenditures of resources.

In the given chapter some problems connected with the above-indicated partial optimization are investigated as examples.

§ 6.1. Consideration of the Reliability of Technical Means When Estimating the Effectiveness of Ships[1]

When estimating the effectiveness of a ship in the general case it is necessary to consider the process of functioning of its individual technical subsystems which "insure" different TTE of the ship. The possible operating failures, combat and emergency damage to the subsystems lead to the fact that the TTE vector of the ship can vary randomly during the performance of the stated mission by the ship. In addition, in the case of systematic operations the volume and nature of the failures and damage have an influence on the repair time between voyages and, consequently, the operative stress coefficient of the ship.

Let us remember that the capacity of the ship (or its individual technical subsystems) to retain the properties required for a given purpose in time, for given

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[1] In this section results are presented which were obtained by the author jointly with V. A. Usachev [37].

operating conditions is called the reliability, and the capacity of the ship (or its individual subsystems) to retain the properties required for the given purpose, in the presence of effects (explosions, fire, flooding, and so on) not provided for by normal operating conditions is called the invulnerability or survival probability [44].

From the presented definitions it follows that the difference of the concepts of reliability and invulnerability is highly provisional and is connected with the nature of external operating conditions. Consideration of the properties of reliability and invulnerability requires investigation of the process (or processes) of transition of individual technical subsystems of the ship from state to state as a result of possible appearance of operating failures and also combat and emergency damage[1]. This process will be called the process of internal functioning defining the set of values of the TTE of the ship which can be insured by the technical subsystems at each point in time.

By analogy with the process of internal functioning it is possible to introduce the process of external functioning into the investigation which at each point in time determines the set of values of TTE of the ship which are required for the performance of the corresponding stage of the operation.

The joint investigation of the processes of external and internal functioning permits estimation of the effectiveness of the ship considering its reliability or, on the contrary, estimation of the reliability using the effectiveness indexes. Here two basic cases are possible: estimation of the effectiveness of the ship considering the reliability of its technical subsystems and estimation of the reliability of the ship or its individual subsystems with a fixed external functioning process. In the given section, a study is made of the two indicated cases in the example of the power plant (PP).

Estimation of the reliability of the ship as a whole or estimation of its effectiveness considering the reliability of individual technical subsystems in the general case leads to highly complex and awkward mathematical models which basically is caused by a large number of states in which the technical subsystems can be and the presence of a mutual relation between the functioning processes of the individual subsystems. Accordingly, it is necessary to make a series of additional simplifying assumptions. Frequently it is assumed [9] that each of the TTE of the ship is insured by only one subsystem (each subsystem insures only one of the TTE). For example, the speed is insured only by the PP, and so on. In addition, the assumption is made that individual subsystems function independently of each other. Inasmuch as the methods of estimating the reliability of the ship's technical subsystems and the ships as a whole are discussed in a number of special sources [77, 34], only examples illustrating the methods of considering reliability and Markov models of estimating the effectiveness of ships will be presented below.

Let us consider the problem of estimating the reliability of a single ship when solving the problem of finding a target in a given part of the sea. We shall consider the following assumptions valid: the search process is Poisson [19]; in the

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[1] Hereafter, we shall consider only the evaluation of reliability; the investigated examples are of a procedural nature.

search process the target does not go outside the given region, and it does not avoid detection; the range of the target detection means by the ship does not depend on the speed of the ship; the speed of the ship is determined during the search only by the condition of the PP [power plant], which can vary with time as a result of failures of the elements of the power plant; the PP is considered a system that is not repairable at sea. The run time of the individual elements per failure is a random variable with exponential distribution law; the elements of the power plant are the following: the nuclear reactor (P), the main turbogear assembly (T) and the line of shafting (B).

A study is made of two power plant systems under the condition of insuring identical maximum speed of the ship: the power plant (1, 1, 1) consisting of one reactor, one main turbogear assembly GTZA and one line of shafting (Figure 6.1,a) and the power plant (2, 2, 2) consisting of two reactors, two GTZA and two lines of shafting (Figure 6.1,b). The power plant (2, 2, 2) is, according to the diagram, a redundant power plant (1, 1, 1). A study is made of two subversions of the power plant (2, 2, 2): without a connector between the reactors (Figure 6.1,b) and with a connector (Figure 6.1, c). The reliability of the connector is considered ideal. Each of the power plants of the (1, 1, 1) type entering into the power plant (2, 2, 2) is provisionally called the power plant of one side. The reliability characteristics of the individual elements for the above-indicated two power systems are assumed to be identical and independent of the power at which the elements operate.

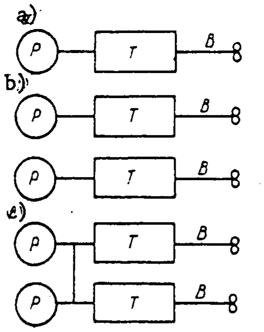

Figure 6.1. Schematics of power plants.

The probability of target detection with unlimited search time is taken as the effectiveness index.

Power Plant (1, 1, 1). Under the assumptions that are made, the functioning of the ship with power plant (1, 1, 1) can be described by a Markov random process, the graph of states and transitions of which is presented in Figure 6.2,a. The states of this process are the following: 1, 0 -- the power plant is in a state of good repair, and the target is not detected; 1, 1 -- the power plant is in a state of good repair, and the target is detected; 0, 0 -- the power plant has failed, and the target is not detected.

The value of λ in Figure 6.2,a is the total intensity of failures of the reactor, the turbine and the line of shafting, and γ_1 is the detection intensity. On the

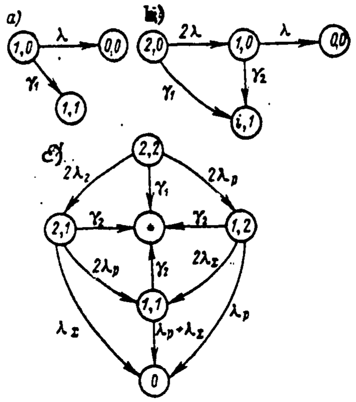

Figure 6.2. Graphs of the processes of the functioning of a ship: a -- with power plant (1, 1, 1), b -- with power plant (2, 2, 2) without connector in the ready-reserve mode, c - the same, but in the presence of a connector.

basis of independence of the detection range with respect to speed of the ship and also λ with respect to power, the search can be performed with maximum speed of the ship v_{max}. Here γ_1 is calculated by the formula $\gamma_1 = 2d\bar{v}_{\rho\ max}/\Omega$, where d is the detection range; $\bar{v}_{\rho\ max}$ is the mean relative search speed; Ω is the area of the search region. If the target is stationary, then $\bar{v}_{\rho\ max} = v_{max}$. If the speed of the target is v_{target}, then $\bar{v}_{\rho\ max}$ can be calculated by the approximate formula [19]

$$\bar{v}_{\rho\ \max} = \frac{v_{\max}^2 + v_{u}^2}{v_{\max} + \underset{(1)}{v_{u}}} + 0{,}3\sqrt{v_{\max} v_{u}}.$$

Key: 1. target

The probabilities of the states of the process satisfy the system of differential equations

$$\begin{aligned} \frac{dP_{10}(t)}{dt} &= -(\lambda + \gamma_1) P_{10}(t), \\ \frac{dP_{00}(t)}{dt} &= \lambda P_{10}(t), \\ \frac{dP_{11}(t)}{dt} &= \gamma_1 P_{10}(t), \end{aligned} \qquad (6.1)$$

where $P_{ij}(t)$ are the probabilities of states; t is the time reckoned from the beginning of the search process. Solving the system of equations (6.1) under the initial conditions $P_{10}(0) = 1$, $P_{00}(0) = P_{11}(0) = 0$, for the target detection probability in the time t we obtain

$$P^{(1)}_{обн}(t) = P_{11}(t) = \frac{\gamma_1}{\gamma_1+\lambda}\left[1 - e^{-(\lambda+\gamma_1)t}\right].$$

(1)

Key: 1. detection

With unlimited search time ($t \to \infty$) the detection probability will be

$$P^{(1)}_{обн}(\infty) = \frac{\gamma_1}{\gamma_1+\lambda}. \tag{6.2}$$

(1)

Key: 1. detection

Thus, for nonideal ($\lambda \neq 0$) reliability of the power plant (1, 1, 1) the search process is diverging [19], inasmuch as $\lim\limits_{t\to\infty} P^{(1)}_{detection}(t) < 1$. For converging search processes the target detection probability with unlimited search time appoaches one.

Power Plant (2, 2, 2) without Connector. Let us first consider the power plant (2, 2, 2) without connector (Figure 6.1,b) under the condition that in a fully repaired state the power plants of both sides operate at full power, and the search is conducted at maximum speed. The functioning of the ship in this case is described by a Markov process with graph of states and transitions presented in Figure 6.2,b.

The states of the process are as follows: 2, 0 -- the power plants of both sides are in a state of good repair, the target is not detected; 1, 0 -- only the power plant on one of the sides is on good repair, the target is not detected; i, 1 -- the target is detected with the power plant in full or partial repair; 0, 0 -- the power plant has failed, the target is not detected.

The value of γ_2 in Figure 6.2,b is the search intensity for operation of the power plant on one side when the power plant of the other side has failed.

Let us denote $\gamma_2/\gamma_1 = k$, where $0 < k < 1$ is a coefficient defined by the search intensity as a function of the speed of the ship and the speed of the ship as a function of the engine power. If we consider that the target is stationary ($v_{target} = 0$), and the required power of the power plant is directed proportional to the cube of the speed, then

$$k = \frac{v_1}{v_{max}} = \sqrt[3]{\frac{N_1}{N_{max}}} = \sqrt[3]{0,5} \approx 0,8,$$

where N_1, v_1 are the power of the engine on one side and the speed provided by it.

For probabilities of states of the ship's functioning process, we have the system of equations

$$\frac{dP_{20}(t)}{dt} = -(2\lambda + \gamma_1) P_{20}(t),$$

$$\frac{dP_{10}(t)}{dt} = -(\lambda + \gamma_2) P_{10}(t) + 2\lambda P_{20}(t),$$
$$\frac{dP_{00}(t)}{dt} = \lambda P_{10}(t), \qquad (6.3)$$
$$\frac{dP_{i1}(t)}{dt} = \gamma_1 P_{20}(t) + \gamma_2 P_{10}(t)$$

under the initial conditions

$$P_{20}(0) = 1, \quad P_{10}(0) = P_{00}(0) = P_{i1}(0) = 0. \qquad (6.4)$$

Applying the Laplace transformation

$$\tilde{P}(s) = \int_0^\infty P(t) e^{-st} dt,$$

under conditions (6.4) it is possible to reduce the system (6.3) to a system of linear algebraic equations with respect to the representations $\tilde{P}(s)$ of the probabilities P(t)

$$s\tilde{P}_{20}(s) - 1 = -(2\lambda + \gamma_1)\tilde{P}_{20}(s),$$
$$s\tilde{P}_{10}(s) = -(\lambda + \gamma_2)\tilde{P}_{10}(s) + 2\lambda\tilde{P}_{20}(s),$$
$$s\tilde{P}_{00}(s) = \lambda\tilde{P}_{10}(s),$$
$$s\tilde{P}_{i1}(s) = \gamma_1\tilde{P}_{20}(s) + \gamma_2\tilde{P}_{10}(s).$$

From this system of equations we find

$$\tilde{P}_{i1}(s) = \frac{\gamma_1(s + \lambda + \gamma_2) + 2\lambda\gamma_2}{s(s + 2\lambda + \gamma_1)(s + \lambda + \gamma_2)}.$$

Hence, using the property

$$\lim_{t\to\infty} P_{i1}(t) = \lim_{s\to 0} s\tilde{P}_{i1}(s),$$

we obtain the formula for the detection probability at t → ∞

$$\underset{(1)}{P^{(2)}_{\text{обн}}}(\infty) = P_{i1}(\infty) = \frac{\gamma_1(\lambda + \gamma_2) + 2\lambda\gamma_2}{(2\lambda + \gamma_1)(\lambda + \gamma_2)}. \qquad (6.5)$$

Key: 1. detection

It is possible to represent the formula (6.5) in the form

$$\underset{(1)}{P^{(2)}_{\text{обн}}}(\infty) = P^{(1)}_{\text{обн}}(\infty)\,\varphi_1(k, \gamma_1/\lambda),$$

Key: 1. detection

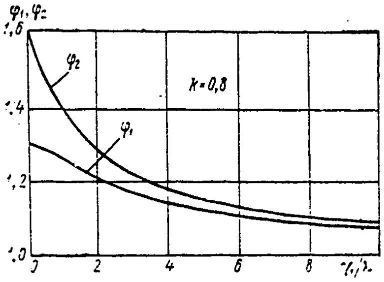

Figure 6.3. Graphs of the functions $\phi_1(\gamma_1/\lambda)$ and $\phi_2(\gamma_1/\lambda)$.

where

$$\varphi_1(k,\ \gamma_1/\lambda)=\frac{1+\gamma_1/\lambda}{1+k\gamma_1/\lambda}\,\frac{1+2k-k\gamma_1/\lambda}{2+\gamma_1/\lambda},$$

and $P^{(1)}_{\text{detection}}(\infty)$ is the probability of detection for the power plant (1, 1, 1) defined by the formula (6.2).

The coefficient $\phi_1(k,\ \gamma_1/\lambda)$ characterizes the advantage of the power plant (2, 2, 2) without connector over the power plant (1, 1, 1) with respect to the limiting value at t → ∞ of the probability of target detection (Figure 6.3).

Let us now consider the mode of use of the power plant (2, 2, 2) without a connector in which the search is always conducted with operation of the power plant on one side, and the power plant on the other side is in unloaded reserve, that is, it goes into operation after failure of the operating power plant. Let us assume that the reserve power plant does not fail during the waiting period. In the terms of reliability theory [39], the above investigated mode of simultaneous use of the power plants on both sides is called the ready reserve mode.

The functioning of power plant (2, 2, 2) using it in the unloaded reserve mode of one of the sides is described by the same process as in the case of the ready reserve, but the intensity of transitions from the state (2, 0) to the state (1, 0) is equal to λ instead of 2λ, and the intensity of transitions from the state (2, 0) to the state (1, 1) is equal to γ_2 instead of γ_1.

Performing the corresponding calculations, we find

$$\underset{(1)}{P^{(2)}_{\text{обн}}}(\infty)=\frac{k\gamma_1(\lambda+k\gamma_1)+k\gamma_1\lambda}{(\lambda+k\gamma_1)^2} \tag{6.6}$$

Key: 1. detection

or

$$P^{(2)}_{\text{обн}}(\infty)=P^{(1)}_{\text{обн}}(\infty)\,\varphi_2(k,\ \gamma_1/\lambda),$$

where

$$\varphi_2(k,\ \gamma_1/\lambda) = k\frac{1+\gamma_1/\lambda}{1+k\gamma_1/\lambda}\frac{2+k\gamma_1/\lambda}{1+k\gamma_1/\lambda}.$$

The coefficient $\phi_2(k, \gamma_1/\lambda)$ characterizes the advantage of power plant (2, 2, 2) operating in the unloaded reserve mode of one of the sides over the power plant (1, 1, 1). Correspondingly, the difference $\phi_2 - \phi_1$ characterizes the effect of the transition to the unloaded reserve mode for power plant (2, 2, 2) without connector (see Figure 6.3).

The value of γ_1/λ is the ratio of the average time before a failure of power plant type (1, 1, 1) to the average time to detection of the target when searching at maximum speed.

The limiting equations exist

$$\lim_{\gamma_1/\lambda\to\infty}\varphi_1(k,\ \gamma_1/\lambda) = \lim_{\gamma_1/\lambda\to\infty}\varphi_2(k,\ \gamma_1/\lambda) = 1,$$

that is, with high reliability of the elements which corresponds to $\lambda \to 0$ and $\gamma_1/\lambda \to \infty$, none of the investigated power plants has advantages, which is entirely natural.

The advantage of power plant (2, 2, 2) by comparison with power plant (1, 1, 1) increases with a decrease in the reliability of the elements. At the limit when $\gamma_1/\lambda \to 0$, this gain is 1.3 and 1.6 times for the ready and unloaded reserve modes, respectively.

Let us assume that the probability of fail-safe operation of power plant (1, 1, 1) for the time $1/\gamma_1$ (for a time equal to the average time until detection of the target when searching at maximum speed) is equal to no less than the given value of P_3. In this case $e^{-\lambda/\gamma_1} \geq P_3$, hence $\gamma_1/\lambda \geq -1/\ln P_3$. For $P_3 = 0.9$, we obtain $\gamma_1/\lambda \geq 10$ and, consequently, $\phi_1 \leq 1.07$ and $\phi_2 \leq 1.08$. This means that for the above-indicated reliability of the power plant (1, 1, 1) the application of the power plant (2, 2, 2) provides a gain of approximately 7% in the value of $P_{detection}(\infty)$. The effect of using the unloaded reserve mode is insignificant.

Let us note that if by a failure of the power plants we mean a decrease in the available power to zero, then power plant (2, 2, 2) has a significant advantage with respect to the average time of fail-safe operations. The average times of fail-safe operation of the power plants will be: $1/\lambda$ for power plant (1, 1, 1); $3/2\lambda$ for power plant (2, 2, 2) in the ready reserve mode and $2/\lambda$ for power plant (2, 2, 2) in the unloaded reserve mode.

Sometimes the range of the detection means depends on the speed of the ship (for example, as a result of interference in operation of the sonar equipment). In this case the speed of the ship during search must be optimized from the condition $\max_v \gamma(v)$, where $\gamma(v)$ is the search intensity as a function of speed. Here the intensity of the failures λ is considered to not depend on the engine power and, correspondingly, the speed of the ship.

If the optimal speed of the ship during search turns out to be greater than the speed provided by the power plant on one side, then all of the above presented relations remain in force. It is only necessary to take the search intensity with optimal speed of the ship as γ_1 and determine the corresponding value of the coefficient k. If the optimal speed turns out to be less than the speed provided by the power plant on one side, then it is necessary to set $k = 1$ and $\gamma_1 = \gamma_2 = \gamma$ in all of the formulas, where γ is the search intensity at optimal speed of the ship.

Then the formulas for the coefficients ϕ_1 and ϕ_2 assume the form

$$\varphi_1(\gamma/\lambda) = \frac{3 + \gamma/\lambda}{2 + \gamma/\lambda}, \quad \varphi_2'(\gamma/\lambda) = \frac{2 + \gamma/\lambda}{1 + \gamma/\lambda}.$$

Power Plant (2, 2, 2) with Connector. The connector between the reactors in power plant (2, 2, 2) permits the main turbogear assembly of each of the sides to be provided with steam from the reactor of the opposite side. The functioning of the power plant (2, 2, 2) with connector in the ready reserve mode is described by the process, the graph of the states and the transitions of which is shown in Figure 6.2,c, where the following states are introduced: 2, 2 -- all elements of the power plants of both sides are in a state of good repair, the target is not detected; 2, 1 -- the main turbogear assembly or line of shafting of one of the sides has failed, the target is not detected; 1, 2 -- the reactor of one side has failed, the target is not detected; 1, 1 -- one of the reactors has failed and also the main turbogear assembly or line of shafting of one side has failed, the target is not detected; 0 -- the power plant has failed; the target is not detected; * -- the target is detected with the power plant fully in working order or partially in working order.

The meaning of the values of γ_1 and γ_2 is the same as in the case of power plant (2, 2, 2) without connector, and for the intensities of failures of the elements of the reactor, the main turbogear assembly and line of shafting, the notation λ_p, λ_T, λ_B is introduced, respectively. The notation $\lambda_\Sigma = \lambda_T + \lambda_B$ is also used, where in the given case $\lambda = \lambda_p + \lambda_T + \lambda_B = \lambda_p + \lambda_\Sigma$.

The system of differential equations for the probabilities of states has the form

$$\frac{dP_{22}(t)}{dt} = -(2\lambda_p + 2\lambda_\Sigma + \gamma_1) P_{22}(t),$$

$$\frac{dP_{21}(t)}{dt} = -(2\lambda_p + \lambda_\Sigma + \gamma_2) P_{21}(t) + 2\lambda_\Sigma P_{22}(t),$$

$$\frac{dP_{12}(t)}{dt} = -(\lambda_p + 2\lambda_\Sigma + \gamma_2) P_{12}(t) + 2\lambda_p P_{22}(t),$$

$$\frac{dP_{11}(t)}{dt} = -(\lambda_p - \lambda_\Sigma + \gamma_2) P_{11}(t) + 2\lambda_p P_{21}(t) + 2\lambda_\Sigma P_{12}(t),$$

$$\frac{dP_0(t)}{dt} = \lambda_p P_{12}(t) + \lambda_\Sigma P_{21}(t) + (\lambda_p + \lambda_\Sigma) P_{11}(t),$$

$$\frac{dP_*(t)}{dt} = \gamma_1 P_{22}(t) + \gamma_2 [P_{11}(t) - P_{12}(t) + P_{21}(t)]$$

under the initial conditions $P_{22}(0) = 1,\ P_*(0) = P_0(0) = P_{11}(0) = P_{21}(0) = 0.$

Performing the corresponding calculations, it is possible to obtain the expressions for the limiting (for t → ∞) value of the detection probability

$$P^{(2)}_{\text{обн}}(\infty) = P^{(1)}_{\text{обн}}(\infty)\,\varphi_3(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda), \tag{6.7}$$

where

$$\varphi_3(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda) = \frac{1+\gamma_1/\lambda}{2+\gamma_1/\lambda}\left[1 + \frac{2k(1-\bar\lambda_{\text{p}})}{1+\bar\lambda_{\text{p}}+k\gamma_1/\lambda} + \frac{2k\bar\lambda_{\text{p}}}{2-\bar\lambda_{\text{p}}+k\gamma_1/\lambda} + \right.$$
$$\left. + \frac{2k\bar\lambda_{\text{p}}(1-\bar\lambda_{\text{p}})}{(1+k\gamma_1/\lambda)(1+\bar\lambda_{\text{p}}+k\gamma_1/\lambda)} + \frac{2k\bar\lambda_{\text{p}}(1-\bar\lambda_{\text{p}})}{(1+k\gamma_1/\lambda)(2-\bar\lambda_{\text{p}}+k\gamma_1/\lambda)}\right],$$
$$\bar\lambda_{\text{p}} = \frac{\lambda_{\text{p}}}{\lambda_{\text{p}}+\lambda_{\text{т}}+\lambda_{\text{в}}}.$$

In formula (6.7) $P^{(1)}_{\text{detection}}(\infty)$ is, just as before, the probability of detection when using a power plant of the (1, 1, 1) type. The difference $\varphi_3(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda) - \varphi_1(k,\ \gamma_1/\lambda)$ characterizes the effect of the presence of a connector in the power plant (2, 2, 2), operating in the ready reserve mode. From the formula for ϕ_3 it is obvious that

$$\lim_{\bar\lambda_{\text{p}}\to 0}\varphi_3(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda) = \lim_{\bar\lambda_{\text{p}}\to 1}\varphi_3(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda) = \varphi_1(k,\ \gamma_1/\lambda).$$

For $\bar\lambda_p = 0$, the reactors are absolutely reliable, and for $\bar\lambda_p = 1$ the main turbogear assembly and lines of shafting are absolutely reliable, inasmuch as for $\bar\lambda_p = 1$ $\lambda_T + \lambda_B = 0$. It is clear that in both cases the connector between the reactors gives no gain in reliability.

The graph of the process of functioning of the ship with the power plant (2, 2, 2) and the connector in the case of unloaded reserve has the same form as in Figure 6.2,c. Only the following intensities of the transitions vary: from the state 2, 2 to the state 2, 1 and from the state 1, 2 to the state 1, 1 -- from $2\lambda_\Sigma$ to λ_Σ; from state 2, 2 to state 1, 2 and from state 2, 1 to state 1, 1, from $2\lambda_p$ to λ_p; from state 2, 2 to state *, from γ_1 to γ_2.

After the corresponding calculations we find

$$P^{(2)}_{\text{обн}}(\infty) = P^{(1)}_{\text{обн}}(\infty)\,\varphi_1(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda), \tag{6.8}$$

where

$$\varphi_4(k,\ \bar\lambda_{\text{p}},\ \gamma_1/\lambda) = k\,\frac{1+\gamma_1/\lambda}{1+k\gamma_1/\lambda}\left[\frac{2+k\gamma_1/\lambda}{1+k\gamma_1/\lambda} - 2\,\frac{\bar\lambda_{\text{p}}(1-\bar\lambda_{\text{p}})}{(1+k\gamma_1/\lambda)^2}\right].$$

Here, just as for the ready reserve, the limiting relations exist

$$\lim_{\bar{\lambda}_p \to 0} \varphi_4(k,\ \bar{\lambda}_p,\ \gamma_1, \lambda) = \lim_{\bar{\lambda}_p \to 1} \varphi_4(k,\ \bar{\lambda}_p,\ \gamma_1, \lambda) = \varphi_2(k,\ \gamma_1, \lambda).$$

The effectiveness of a single ship was estimated above. The case of using a fleet of ships can also be considered.

Let us use the principle of quasiregularity [19], assuming that at any point in time the search is made by an average number of ships with engine in a complete or partial state of good repair. Under this assumption the average number of ships with power plant (1, 1, 1) searching at the time τ is equal to $\mathcal{N}e^{-\lambda\tau}$ and, consequently, the probability of detection of the target in the time t will be

$$P^{(1)}_{\text{обн}}(\mathcal{N},\ t) = 1 - \exp\left\{-\mathcal{N}\frac{\gamma_1}{\lambda}(1 - e^{-\lambda t})\right\}. \tag{6.9}$$

Analogously, for power plant (2, 2, 2) without connector for the ready reserve mode we have

$$P^{(2)}_{\text{обн}}(\mathcal{N},\ t) = 1 - \exp\left\{-\gamma_1\int_0^t \mathcal{N}_{20}(\tau)\,d\tau - \gamma_2\int_0^t \mathcal{N}_{10}(\tau)\,d\tau\right\}, \tag{6.10}$$

where $\mathcal{N}_{20}(\tau)$, $\mathcal{N}_{10}(\tau)$ are the average numbers of ships at the time τ in the states 2, 0 and 1, 0 (Figure 6.2, b).

The functions $\mathcal{N}_{20}(\tau)$ and $\mathcal{N}_{10}(\tau)$ satisfy the system of equations

$$\frac{d\mathcal{N}_{20}(\tau)}{d\tau} = -2\lambda\mathcal{N}_{20}(\tau), \quad \frac{d\mathcal{N}_{10}(\tau)}{d\tau} = -\lambda\mathcal{N}_{10}\tau + 2\lambda\mathcal{N}_{20}(\tau),$$

the solution of which under the conditions $\mathcal{N}_{20}(0) = \mathcal{N}$ and $\mathcal{N}_{10}(0) = 0$ gives

$$\mathcal{N}_{20}(\tau) = \mathcal{N}e^{-2\lambda\tau},\ \mathcal{N}_{10}(\tau) = 2\mathcal{N}e^{-\lambda\tau}(1 - e^{-\lambda\tau}). \tag{6.11}$$

Substituting (6.11) in (6.10), we find

$$P^{(2)}_{\text{обн}}(\mathcal{N},\ t) = 1 - \exp\left\{-\mathcal{N}\frac{\gamma_1}{\lambda}\left(\frac{1+2k}{2} - \frac{1-2k}{2}e^{-2\lambda t} - 2ke^{-\lambda t}\right)\right\}.$$

If the required value P_3 of the probability of detection of the target and the admissible search time T are given, then the ratio of the required fleets of ships $\mathcal{N}^{(1)}_T$ for the power plant (1, 1, 1) and $\mathcal{N}^{(2)}_T$ for the power plant (2, 2, 2) will be

$$\frac{\mathcal{N}^1_T}{\mathcal{N}^2_T} = \frac{0{,}5 + k - (0{,}5 - k)e^{-2\lambda T} - 2ke^{-\lambda T}}{1 - e^{-\lambda T}}.$$

Hence, it follows that for $\lambda T \to 0$, that is, for sufficiently high reliability of the power plant (1, 1, 1), $\mathcal{N}^{(1)}_T \doteq \mathcal{N}^{(2)}_T$, and for $\lambda T \to \infty$, that is, for unlimited search time or low reliability, $\mathcal{N}^{(1)}_\infty \doteq (0{,}5 + k)\mathcal{N}^{(2)}_\infty$. For k = 0.8 in the last case we have $\mathcal{N}^{(1)}_\infty = 1{,}3\mathcal{N}^{(2)}_\infty$.

The above-investigated consideration of the reliability of the technical subsystems in the general model of estimation of the ship's effectiveness is the most complete. At the same time, when solving the large class of special problems of comparative estimation of various versions of individual technical subsystems sometimes it is expedient to use the approximate approach for which the process of external functioning, that is, the admissible states of the system as a function of time, is fixed. Here the external functioning process can be a deterministic or random function of time.

Let X be not by the TTE vector of the ship, and let $\mathfrak{X}$ be the set of possible values of the vector X defined by the structural characteristics of the ship and its technical subsystems. Let us denote that $\mathfrak{X}_{\text{в}}(t)$ the set of values of the vector X which can be insured at an arbitrary point in time t considering the state of the technical subsystems (considering the possibility of the appearance of failures in these subsystems).

The function $\mathfrak{X}_{\text{в}}(t)$ is the mathematical description of the process of internal functioning of the ship. It is obvious that for any t we have $\mathfrak{X}_{\text{в}}(t) \subset \mathfrak{X}$, that is, $\mathfrak{X}_{\text{в}}(t)$ is a subset of the set $\mathfrak{X}$.

Let us introduce the set $\mathfrak{X}_{\text{п}}(t)$, into the investigation which defines the values of the vector X admissible at the time t, beginning with the performance of the mission set for the ship when [$\mathfrak{X}_{\text{п}}(t)$ is the mathematical description of the process of external functioning of the ship]. We shall assume that for any t the condition $\mathfrak{X}_{\text{п}}(t) \subset \mathfrak{X}$ exists.

Failure of the ship can be defined as the event

$$\exists t \in [0, T] : \mathfrak{X}_{\text{п}}(t) \cap \mathfrak{X}_{\text{в}}(t) = \Lambda, \tag{6.12}$$

where Λ is an empty set; $\exists$ is the symbol of existence; $\cap$ is the symbol of intersection of sets; : is the symbol of properties; [0, T] is the time interval in which the functioning of the ship is considered.

The symbolic expression (6.12) is the abbreviation for the condition: in the interval [0, T] there is a (at least one) point in time t when the intersection of the sets of the required and provided values of the vector X is an empty set, that is, the sets $\mathfrak{X}_{\text{п}}(t)$ and $\mathfrak{X}_{\text{в}}(t)$ o not have common elements. In turn, the latter means that at the indicated point in time (points in time) the required value of at least one of the TTE of the ship (component of the vector X) cannot be insured.

In accordance with the introduced concept of failure, the probability of fail-safe operation of the ship is designed with the probability of the event opposite to (6.12):

$$\begin{aligned} P(T) = P\{\forall t \in [0, T] : \mathfrak{X}_{\text{п}}(t) \cap \mathfrak{X}_{\text{в}}(t) \neq \Lambda\} = \\ = 1 - P\{\exists t \in [0, T] : \mathfrak{X}_{\text{п}}(t) \cap \mathfrak{X}_{\text{в}}(t) = \Lambda\}, \end{aligned} \tag{6.13}$$

where $\forall$ is the symbol of universality.

As was pointed out above, the expansion of expression (6.13) is connected with significant difficulties. If we assume that the individual technical subsystems function independently of each other, each subsystem insures only one TTE, then expression (6.13) assumes the form

$$P(T)=\prod_{i=1}^{n} P\{\forall t\in[0,\ T]:\mathfrak{X}_{\text{п}}^{(i)}(t)\cap\mathfrak{X}_{\text{в}}^{(i)}(t)\neq\Lambda\}, \tag{6.14}$$

where $\mathfrak{X}_{\text{п}}^{(i)}$, $\mathfrak{X}_{\text{в}}^{(i)}$ are sets of required and provided values of the ith TTE; n is the number of TTE and, correspondingly, subsystems of the ship.

Each of the cofactors of the formula (6.14) serves as the reliability index of the corresponding technical subsystem of the ship. Considering that the TTE of the ship are related to the technical characteristics of the subsystems, the concept of failure of each subsystem can be formulated for values of the corresponding technical characteristics of this subsystem. For example, if we assume that the speed of the ship, as one of the TTE, is insured only by the PP [power plant], then failure of this subsystem can be defined as impossibility of insuring the required shaft power. The required power is determined by the required speed.

Let us illustrate the application of the above-discussed approach in the example of the power plant of a ship considering that the power is a unique characteristic of the external and internal functioning of this subsystem. We shall assume that the power plant is not repairable at sea, and the operating time of the elements before failure is distributed according to an exponential law.

Let us consider the power plant of the type (2, 2, 2) without connector operating in the ready reserve mode. Let us assume that the process of external functioning is given by the deterministic relation for the required power $N_{\pi}(t)$ as a function of time t. Let us consider two versions of this function:

$$\bar{N}_{\text{п}}^{(1)}(t)\in\begin{cases}(0;\ 0{,}5] & \text{for}\quad 0\leqslant t<0{,}5T,\\ (0{,}5;\ 1] & \text{for}\quad 0{,}5T<t\leqslant T,\end{cases} \tag{6.15}$$

$$\bar{N}_{\text{п}}^{(2)}(t)\in\begin{cases}(0{,}5;\ 1] & \text{for}\quad 0\leqslant t<0{,}5T,\\ (0;\ 0{,}5] & \text{for}\quad 0{,}5T<t\leqslant T,\end{cases} \tag{6.16}$$

where $\bar{N}_{\pi}(t) = N_{\pi}(t)/N_{max}$ is the relative power of the power plant; N_{max} is the maximum power; T is the time of operation of the power plant.

In accordance with formulas (6.15) and (6.16) for 50% of the time a power of less than $0.5N_{max}$ is required, and for 50% of the time, more than $0.5N_{max}$. For the function $\bar{N}_{\pi}^{(1)}(t)$ a power of less than 50% is required in the first half of the operating period, and for the function $\bar{N}_{\pi}^{(2)}$, in the second period. In Figure 6.4, the regions of values of these functions are denoted by crosshatching.

In the given example we shall assume that during operation of the power plant at less than 50% power the failure intensity of the power plant of one side is λ_{50}, and at more than 50% power, λ_{100}.

Under the assumptions made above, the probability of fail-safe operation of the power plant in the case of the function $\bar{N}_{\pi}^{(2)}(t)$ is defined by the formula

$$P_2^{(2)}(T)=e^{-\lambda_{100}T}\left(2e^{-0{,}5\lambda_{50}T}-e^{-\lambda_{50}T}\right). \tag{6.17}$$

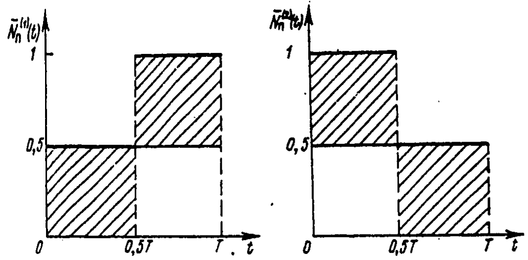

Figure 6.4. Graphs of the required power functions.

In the case of function $\bar{N}_{\pi}^{(1)}(t)$, correspondingly,

$$P_1^{(2)}(T) = e^{-(\lambda_{100}+\lambda_{50})T}. \tag{6.18}$$

From formulas (6.17) and (6.18) we have

$$P_2^{(2)}(T) - P_1^{(2)}(T) = 2e^{-\lambda_{100}T}\left(e^{-0,5\lambda_{50}T} - e^{-\lambda_{50}T}\right) > 0,$$

that is, the reliability (the probability of fail-safe operation) of the power plant with required power function $\bar{N}_{\pi}^{(2)}(t)$ is higher than for the function $\bar{N}_{\pi}^{(1)}(t)$. This confirms the dependence of the reliability indexes on the nature of the process of external functioning of the system.

Let us assume that in the above-investigated example the transitions from the power $\bar{N}_{\pi} \leq 0.5$ to thepower $\bar{N}_{\pi} > 0.5$ and vice versa take place at random points in time, forming the simplest flows of events with transition intensities ν_{12} and ν_{21}, respectively. This means that the external functioning process is a Markov process.

Let us consider the power plants of the types (1, 1, 1) and (2, 2, 2), where the power plant (2, 2, 2) does not have a connector, and on intermediate powers it operates in the ready reserve mode. The failure intensity of the power plant (1, 1, 1) when operating at more than 50% power will be denoted by λ_{100}, and when operating at less than 50% power, λ_{50}. Let us assume that these failure intensities will occur also for each of the sides in the power plant (2, 2, 2).

Power Plant (1, 1, 1). Under the assumptions made, the functioning of the power plant (1, 1, 1) considering the processes of its external and internal functioning is described by a Markov random process (Figure 6.5, a). The states of this process are denoted as follows (the first figure indicates the number of the internal state, and the second figure, the external state): 2, 2 -- all elements of the power plant are in a state of good repair; more than 50% power is required; 2, 1 -- all of the elements of the power plant are in a state of good repair, a power of less than 50% is required; 0 -- the power plant has failed.

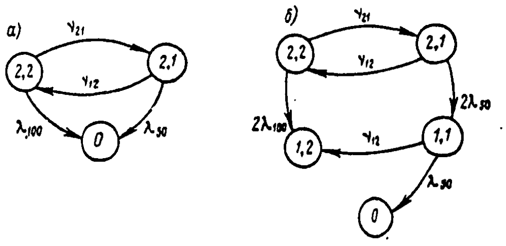

Figure 6.5. Graphs of the processes of joint internal and external functioning: a -- for power plant (1, 1, 1); b -- for power plant (2, 2, 2) without connector for the ready reserve mode.

The probabilities of states of the process satisfy the system of differential equations

$$\frac{dP_{22}(t)}{dt} = -(\nu_{21} + \lambda_{100}) P_{22}(t) + \nu_{12} P_{21}(t),$$
$$\frac{dP_{21}(t)}{dt} = -(\nu_{12} + \lambda_{50}) P_{21}(t) + \nu_{21} P_{22}(t),$$
$$\frac{dP_0(t)}{dt} = \lambda_{50} P_{21}(t) + \lambda_{100} P_{22}(t) \qquad (6.19)$$

under the initial conditions

$$P_{21}(0) = 1, \quad P_{22}(0) = P_0(0) = 0 \qquad (6.20)$$

or

$$P_{22}(0) = 1, \quad P_{21}(0) = P_0(0) = 0 \qquad (6.21)$$

Solving the system of equations (6.19) under the conditions (6.20) or (6.21), it is possible to obtain the formula for the probability $P_0(t)$ of failure of the powe plant in the time t. If $\lambda_{100} = \lambda_{50} = \lambda$, then independently of ν_{12}, ν_{21} and the initial conditions

$$\underset{(1)}{P^{(1)}_{\text{отк}}}(t) = P_0(t) = 1 - e^{-\lambda t}. \qquad (6.22)$$

Key: 1. failure

The average time before failure is defined by the expression

$$T_1 = \int_0^\infty t\, dP_0(t) = \int_0^\infty [1 - P_0(t)]\, dt.$$

For $\lambda_{100} = \lambda_{50} = \lambda$

$$T_1 = 1/\lambda \text{ or } \tau_1 = 1, \qquad (6.23)$$

where $\tau_1 = \lambda T_1$ is the dimensionless magnitude of the average time of fail-safe operation.

Power Plant (2, 2, 2). The functioning of the power plant (2, 2, 2) is described by a Markov process (Figure 6.5, b). The states 2, 2 and 2, 1 have the same meaning as for the power plant (1, 1, 1), and the other states are denoted as follows: 1, 1 -- the power plant of one side is in a state of good repair, less than 50% power is required; 1, 2 -- the power plant of one side is in a state of good repair, a power of more than 50% is required; 0 -- the power plants on both sides have failed.

The probabilities of the states of the process satisfy the system of differential equations

$$\begin{aligned}
\frac{dP_{22}(t)}{dt} &= -(\nu_{21} + 2\lambda_{100}) P_{22}(t) + \nu_{12} P_{21}(t),\\
\frac{dP_{21}(t)}{dt} &= -(\nu_{12} + 2\lambda_{50}) P_{21}(t) + \nu_{21} P_{22}(t),\\
\frac{dP_{11}(t)}{dt} &= -(\nu_{12} + \lambda_{50}) P_{11}(t) + 2\lambda_{50} P_{21}(t),\\
\frac{dP_{12}(t)}{dt} &= \nu_{12} P_{11}(t) + 2\lambda_{100} P_{22}(t),\\
\frac{dP_0(t)}{dt} &= \lambda_{50} P_{11}(t)
\end{aligned} \tag{6.24}$$

under the initial conditions

$$P_{21}(0) = 1, \quad P_{22}(0) = P_{11}(0) = P_{12}(0) = P_0(0) = 0 \tag{6.25}$$

or

$$P_{22}(0) = 1, \quad P_{21}(0) = P_{11}(0) = P_{12}(0) = P_0(0) = 0. \tag{6.26}$$

The probability of failure of the power plant in the time t is equal to the sum of the probabilities $P_0(t)$ and $P_{12}(t)$. These probabilities correspond to the states for which the available power of the power plant is less than required.

Let us consider the special case where $\nu_{12} = \nu_{21} = \nu$ and $\lambda_{100} = \lambda_{50} = \lambda$. The solution of the system of equations (6.24) under the conditions (6.25) gives

$$P_{12}(t) = \frac{2}{1-\bar{\nu}^2} e^{-(1+\bar{\nu})\lambda t} - \frac{1}{1-\bar{\nu}} e^{-2\lambda t} - \frac{1}{1+\bar{\nu}} e^{-2(1+\bar{\nu})\lambda t},$$

$$P_0(t) = \frac{2}{(1-\bar{\nu}^2)(1+\bar{\nu})}\left[1 - e^{-(1+\bar{\nu})\lambda t}\right] - \frac{1}{2(1-\bar{\nu})}\left(1 - e^{-2\lambda t}\right) -$$

$$- \frac{1}{2(1+\bar{\nu})^2}\left[1 - e^{-2(1+\nu)\lambda t}\right],$$

where $\bar{\nu} = \nu/\lambda$.

The probability of failure of the power plant will be

$$P^{(2)}_{\underset{(1)}{\text{отк}}}(t) = 1 - \left[\frac{2}{1-\bar{\nu}^2} e^{-(1+\bar{\nu})\lambda t} - \frac{\bar{\nu}}{1-\bar{\nu}} e^{-2\lambda t} - \frac{1}{1+\bar{\nu}} e^{-2(1+\bar{\nu})\lambda t}\right].$$

Key: 1. failure

Calculating the integral $\int\limits_0^\infty t\,dP^{(2)}_{\text{отк}}(t)$, we find the average time of fail-safe operation

$$T_2 = \frac{3-2\bar{\nu}^2-\bar{\nu}^3}{2(1-\bar{\nu})(1+\bar{\nu})^2}\,\frac{1}{\lambda} \tag{6.27}$$

or in dimensionless form

$$\tau_2 = \frac{3-2\bar{\nu}^2-\bar{\nu}^3}{2(1-\bar{\nu})(1+\bar{\nu})^2}. \tag{6.28}$$

Thus, the average operating time before failure of the power plant (2, 2, 2) depends on ν, that is, the switching frequency from the mode $\bar{N}_\pi \le 0.5$ to themode $\bar{N}_\pi > 0.5$, and vice versa. Let us note that for $\nu_{12} = \nu_{21} = \nu$ the power plant must operate on the average an identical amount of time in the modes $\bar{N}_\pi \le 0.5$ and $\bar{N}_\pi > 0.5$, but the operating time T_2 decreases with an increase in ν for λ = const. From Figure 6.6 it is obvious that for $0 \le \nu < 0.6$ the power plant (2, 2, 2) has the advantage with respect to operating time before failure, for in this region $\tau_2 > 1$. Correspondingly, for $\bar{\nu} > 0.6$ the power plant (1, 1, 1) for which $\tau_1(\bar{\nu}) \equiv 1$ has the advantage. The maximum advantage of the power plant (2, 2, 2) over power plant (1, 1, 1) is $\tau_2/\tau_1 = 1.5$, and it is reached for $\bar{\nu} = 0$. In this case the required power remains at all times less than 50% inasmuch as this power is required for t = 0, and the switching frequency from mode-to-mode is zero. The maximum advantage of the power plant (1, 1, 1) is $\tau_1/\tau_2 = 2$, and it is reached for $\bar{\nu} \to \infty$, which corresponds to the case where the required power is always greater than 50%.

From the investigated example, in particular, it follows that the assignment of only the stationary probability distribution of different levels of required power is insufficient for estimation of the reliability of the power plant inasmuch as this distribution does not uniquely define the intensities of the transitions between the required operating modes of the power plant with respect to power. In the above-investigated example the stationary probability distribution of the required power has the form

$$h_{100} = \frac{\nu_{12}}{\nu_{12}+\nu_{21}}, \quad h_{50} = 1 - h_{100} = \frac{\nu_{21}}{\nu_{12}+\nu_{21}}.$$

However, assignment of the probabilities h_{100} and h_{50} permits determination only of the relation between the intensities ν_{12} and ν_{21}, and not the values of these intensities which determine the reliability characteristics of the power plant.

In the given section a study was made of the simplest method of estimating the reliability of complex engineering systems of the ship based on using the theory of Markov random processes with a discrete set of states and continuous time. This method is connected with a quite strong assumption with respect to the nature of the flows of failures and repairs of the system elements (the flows must be ordinary and without aftereffect). At the present time the methods of theoretical

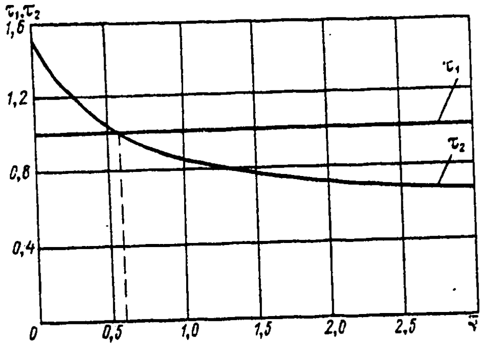

Figure 6.6. Graphs of the functions $\tau_1(\bar{\nu})$ and $\tau_2(\bar{\nu})$.

evaluation of the system reliability have been developed for less rigid requirements on the nature of the flows of failures and repairs of the elements. These methods are based on the theory of semimarkov processes and the use of logical-probability and logical-statistical models. In the latter case the structure of the system and the peculiarities of its functioning are described by the methods of mathematical logic, and the quantitative estimate of the reliability is made by the methods of probability theory or statistical simulation [44]. The logical-probability and logical-statistical methods find broad application when estimating the structural reliability of engineering systems.

§ 6.2. Optimization of the Service Life of Ships

Efficient service life of the ship as part of a fleet depends on the **rates** of its physical wear and obsolescence. At the present time the defining factor is obsolescence by which we mean a reduction in effectiveness of the ship as a result of an increase in the effectiveness of the forces and means of the enemy. As a result of sharp acceleration of the rates of scientific and technical progress this form of wear has become basic for warships.

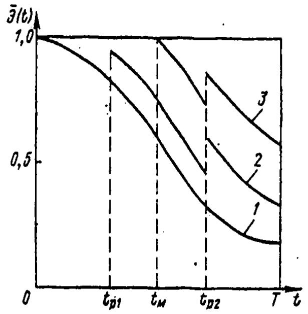

Figure 6.7. Nature of the variation of $Э(t)$ in time. 1 — without repairs and modifications; 2 — with two repairs at the times t_{p1} and t_{p2}; 3 — for two repairs and one modification at the time t_M under the condition that the modification completely compensates for physical wear and obsolescence.

In the problems of substantiating optimal service lives, ships are considered as potential means of waging war. Therefore the effect of finding the ship in the fleet can be characterized by an integral index with respect to time which depends on the effectiveness of the ship at each current point in time. A value called the combat potential is used as this index:

$$\Pi(T)=\int_0^T \text{Э}(t)\,dt, \qquad (6.29)$$

where $\text{Э}(t)$ is the current value of the effectiveness index under the condition that the combat operations begin at the time t; T is the service life of the ship.

The indexes of the type (6.29) are also used for civilian ships [34, 37]. In the latter case the value of $\Pi(T)$ is called the service effect of the ship [34].

Because of obsolescence and physical wear the function $\text{Э}(t)$, generally speaking decreases with an increase in t. An increase in $\text{Э}(t)$ can occur only after repairs and modifications. (Here it is considered that the repairs are primarily aimed at elimination of physical wear, and modifications paratially compensate also for obsolescence).

Instead of $\text{Э}(t)$ it is possible to consider the relative effectiveness $\bar{\text{Э}}(t)=\text{Э}(t)/\text{Э}(0)$, where $\text{Э}(0)$ is the effectiveness corresponding to the time the ship becomes part of the fleet (Figure 6.7). When considering modification the values of the function $\bar{\text{Э}}(t)$ do not necessarily fall in the interval [0, 1], as occurs when considering only certain repairs. Theoretically after modification the effectiveness of the ship can exceed its initial value $\text{Э}(0)$. When using the function $\bar{\text{Э}}(t)$ the expression for the combat potential will be $\bar{\Pi}(T)=\int_0^T \bar{\text{Э}}(t)\,dt$. For convenience of calculations the discontinuous function $\bar{\text{Э}}(t)$ in the presence of repairs and modifications (including consideration of their duration) is replaced by a continuous function from the condition of equality of combat potentials [34].

In a number of cases the so-called residual cost of the ship serving for t years is of interest (see § 4.4). It is natural to assume that the residual cost $S_0(t)$ is proportional to the residual combat potential:

$$S_0(t)=S\frac{\int_t^T \bar{\text{Э}}(\tau)\,d\tau}{\int_0^T \bar{\text{Э}}(\tau)\,d\tau}=S\,\frac{\bar{\Pi}(T)-\bar{\Pi}(t)}{\bar{\Pi}(T)}, \qquad (6.30)$$

where S is the initial cost (the building cost) of the ship;

$$\bar{\Pi}(t)=\int_0^t \bar{\text{Э}}(\tau)\,d\tau.$$

Here it is assumed that S(T) = 0, that is, the residual cost of the ship serving the entire service life is zero. In accordance with expression (6.30) the combat potential plays the role of the "production" to which the cost of the ship is carried over during its service.

The above-presented expressions for the combat potential, which are integrals of the functions $\text{Э}(t)$ or $\bar{\text{Э}}(t)$, are valid in the case where the importance of insuring some level of effectiveness at various points in time is identical. If the effectiveness insured at various points in time is not equivalent, then the combat potential can be defined as $\bar{\Pi}(T) = \int_0^T \bar{\text{Э}}(t)\,w(t)\,dt$, where w(t) is the "weight" function characterizing the relative significance of the effectiveness at various points in time. Later we shall consider the case $w(t) \equiv 1$.

The simplest statement of the problem of optimizing the service life consists in maximizing the combat potential per unit expenditure connected with the building and maintenance of the ship during its service life. Let us represent the cost of building and maintaining the ship in the form $S_\Sigma(T) = S + \int_0^T s'_{\text{э}}(t)\,dt$, where $S_\Sigma(T)$ is the total cost of building and maintenance during the service life; S is the building cost; $s'_{\text{э}}(t)$ is the cost of maintenance per unit time at the time t(T).

The above-stated problem of optimizing the service life reduces to maximization of the function

$$\frac{\bar{\Pi}(T)}{S_\Sigma(T)} = \frac{\int_0^T \bar{\text{Э}}(t)\,dt}{S + \int_0^T s'_{\text{э}}(t)\,dt}$$

with respect to T.

Differentiating this function with respect to T and equating the derivative to zero, we obtain the equation for finding the optimal service life

$$\frac{\bar{\text{Э}}(T)}{s'_{\text{э}}(T)} = \frac{\int_0^T \bar{\text{Э}}(t)\,dt}{S + \int_0^T s'_{\text{э}}(t)\,dt}. \tag{6.31}$$

In accordance with this equation the optimal service life is characterized by the fact that at the time $t = T_{opt}$ the ratio of the current value of the effectiveness to the expenditures on maintaining the ship per unit time is equal to the ratio of the values of the combat potential and the total expenditures that accumulated at this time.

As an illustration let us consider the solution of the equation (6.31) under the following assumptions: the cost of maintaining the ship per unit time ($s'_{\text{э}}$) does not depend on time and, correspondingly, the value fo $S_\Sigma(T)$ is calculated by the formula

$$S_\Sigma(T) = S + s'_{\text{э}}T, \tag{6.32}$$

the function $\bar{\text{Э}}(t)$ is exponential:

$$\bar{\text{Э}}(t) = \overline{a^t}, \tag{6.33}$$

where $0 < a < 1$ is a given value.

Introducing the notation $\nu = -\ln a$, let us represent the exponential function by the exponent

$$\bar{\text{Э}}(t)=e^{-\nu t},\quad \nu>0. \tag{6.34}$$

The exponential form of the function $\bar{\text{Э}}(t)$ does not always correspond well to the actual process of obsolescence of ships. With this form of $\bar{\text{Э}}(t)$ the obsolescence rate is proportional to the current value of the effectiveness and, consequently, the entirely "new" ship has the greatest obsolescence rate. At the same time, from the general arguments it is possible to conclude that during the process of obsolescence there must be a time delay when the "new" ship becomes obsolescent comparatively slowly. After this period, an increase in the obsolescence rate takes place as a result of the appearance of new scientific and technical possibilities for improving the characteristics of the ships and gradual introduction of the ships into operation on which the indicated possibilities have been implemented. Finally, in the third period the obsolescence rate again decreases, and the effectiveness stabilizes at some, as a rule, very low level.

The above indicated general properties are satisfied by the so-called logistic law of aging (obsolescence) [34] given by the function

$$\bar{\text{Э}}(t)=\frac{(1+\beta)\,e^{-\nu t}}{1+\beta e^{-\nu t}},$$

where ν, β are the given parameters.

Hereafter for simplicity we shall consider only the exponential function $\bar{\text{Э}}(t)$.

Under the given assumptions with respect to the form of the functions $\bar{\text{Э}}(t)$ and $S_\Sigma(T)$, equation (6.31) reduces to the form

$$e^{\Theta}=\Theta+B, \tag{6.35}$$

where $B=(1+b)/b,\ b=s'_{\text{э}}/(\nu S),\ \Theta=\nu T.$

The solution of equation (6.35), that is, the value of Θ_{opt} depends on the dimensionless parameter $s'_{\text{э}}/(\nu S)$. The optimal service life (T_{opt}) is found from the expression $T_{opt} = \Theta_{opt}/\nu$. Let, for example, $\nu = 0{,}05$ 1/год, $s'_{\text{э}} = 1\cdot10^6$ rubles/year, $S = 50\cdot10^6$ rubles. For these initial data we obtain $b = 0.40$, $\Theta_{opt} = 1.64$ and $T_{opt} = 32.8$ years.

Let us show that introducing the discount rate when determining the operating cost (see Chapter 4) increases the value of T_{opt}. On introducing the discount rate the cost of building and maintaining the ship is calculated by the formula

$$S_\Sigma(T)=S+\frac{s'_{\text{э}}}{\alpha}\left[1-\frac{1}{(1+\alpha)^T}\right],$$

where α is the discount rate.

Setting (on the basis of smallness of α) $\ln(1+\alpha) \approx \alpha$, it is possible to obtain the equation for finding Θ_{opt}

$$e^{\Theta} = A\frac{\nu}{\alpha}\left[(1+\alpha)^{\frac{\Theta}{\nu}} - \frac{1}{A}\left(1-\frac{\alpha}{\nu}\right)\right], \tag{6.36}$$

where $A = \frac{\alpha S}{s'_{\text{э}}}\left(1+\frac{s'_{\text{э}}}{\alpha S}\right)$.

Let us propose the same values of the initial data $(S, s'_{\text{э}}, \nu)$, exist as in the preceding example, and $\alpha = 0.03$ 1/year. For these initial data it is possible to reduce equation (6.36) to the form $\ln(e^{\Theta} + 0{,}67) = 1{,}64 + 0{,}6\Theta$. Its solution gives $\Theta_{opt} = 3.5$, and, consequently, $T_{opt} = 70$ years instead of 32.8 years as occurred without introducing the discount rate.

It is obvious that modifications also increase the optimal service life, for they decrease the average obsolescence rate (for an exponential law of obsolescence — the mean value of ν).

The most complex problem for the above-investigated approach to determining the optimal service life consists in establishing the form of the function $\bar{\text{Э}}(t)$ or the parameter ν when approximating $\bar{\text{Э}}(t)$ by an exponential function. The solution of this problem requires prediction of the development of forces and means of the enemy in qualitative and quantitative respects for comparatively long time intervals. The forecasting problem on the methodological and practical levels is in the initial stage of development. Finding the function $\bar{\text{Э}}(t)$ is also connected with cementing significant computational difficulties. Actually, finding the values of $\bar{\text{Э}}(t)$ for each fixed t requires execution of a block of the effectiveness estimation algorithms. In Chapter 3 it is pointed out that even a single execution of the algorithms for estimating the effectiveness of a ship frequently turns out to be a problem of quite large computational volume. In these cases it is expedient not to include the service life among the optimizable variables during AD of the ships but to substantiate them individually. This approach is based on the following arguments.

Let us represent the effective index $\text{Э}(X, t)$ which depends on the vector X (optimizable TTE and TDP) and time t in the form $\text{Э}(X, t) = \text{Э}(X, 0)\,\bar{\text{Э}}(X, t)$, where $\bar{\text{Э}}(X, 0)$ is the value of the effectiveness index on completion of construction of the ship (t = 0); $\bar{\text{Э}}(X, t)$ is the function characterizing the obsolescence of the ship.

In this case for the combat potential we have the expression

$$\Pi(X, T) = \text{Э}(X, 0)\int_0^T \bar{\text{Э}}(X, t)\,dt.$$

Let us assume that the function $\bar{\text{Э}}(X, t)$ does not depend on the vector X. For the exponential representation where $\bar{\text{Э}}(X, t) = e^{-\nu t}$, this means that the parameter ν does not depend on X. As was demonstrated above, the optimal service life found by the criterion (6.31) depends only on the ratio $s'_{\text{э}}/S$ and the parameter ν. If the ratio

$s'_э/S$ does not depend on the vector X or depends weakly on it, then optimization of the service life can be brought about independently of optimization of the vector X. In this case the optimal service life found by the above-indicated method must hereafter be considered when determining the cost of building and maintaining the ship in the optimization criteria for the vector X.

If the function $\bar{Э}(X, t)$ depends on X, then optimization of the service life must be carried out jointly with optimization of the vector X. The service life in this case is included in the vector X as one of the components, and the function $П$ (X) is considered as the effectiveness index.

The optimal values of X_{opt} and T_{opt} can also be found successively. Initially, the problem

$$\max_T \frac{\int_0^T \bar{Э}(X, t)\,dt}{S(X) + s'_э(X)\,T}.$$

is considered.

The solution of this problem for each fixed X defines the function $T_{opt}(X)$ which subsequently must be used when constructing the criteria of optimization of the vector X. It is obvious that in the procedures for searching for the optimal version of the ship (the vector X_{opt}) by the method of successive approximations there is no necessity for finding the function $T_{opt}(X)$ in advance. In this case, it is necessary to include the solution of the partial problem of optimization of the service life for fixed vector X in the set of algorithms for the calculations performed in each step of the process. This service life is assumed for calculation of the cost of building and maintaining the version of the ship corresponding to the given vector X.

Now with certain alterations let us consider the approach to optimization of the service life of technical means contained in the Manne article [63].

It is proposed that the fixed budget can be spent in three areas: the development of new, improved types of ships of a given class, the building of ships, maintenance of ships as part of the fleet.

Let us make the following assumptions:

1. The expenditures on maintaining ships per unit time are proportional to the expenditures on building them. If the expenditures on building in the time t are equal to xt, where x are the expenditures on building per unit time, then the expenditures on maintenance per unit time of all the ships in operation will be $\bar{c}$xt, where $\bar{c}$ is a constant having dimensionality the inverse of time;

2. The introduction of each new type of ship into construction requires certain constant expenditures S_p connected with the development and introduction of a new series into production;

3. The total effectiveness $\bar{Э}_\Sigma$ of all ships in operation at each point in time is an additive function of the effectiveness of individual ships, that is, $\bar{Э}_\Sigma$ is the sum of the effectivenesses of individual ships;

4. The real obsolescence function $\bar{Э}(t)$ considering repairs and possible modifications is approximated by a monotonically nonincreasing function of time, that is, $d\bar{Э}(t)/dt \leqslant 0$ for all t $\geq$ 0. The time t = 0 is considered to be the present (current) point in time, and the time t = τ is τ years in the past from t = 0 (the reckoning of time goes in the opposite direction).

If new types of ships are introduced every v years, and the old types are taken out of the fleet after nz years, where n is an integer, then $\bar{Э}_\Sigma$ can be represented in the form

$$\bar{Э}_\Sigma(x, n, z) = \sum_{k=1}^{n} \mathcal{N}_k \bar{Э}(kz), \tag{6.37}$$

where $\mathcal{N}_k$ is the number of ships constructed in the time intervals [kz, (k − 1) z], k = 1, ..., n. The lengths of these intervals are identical and equal to z years.

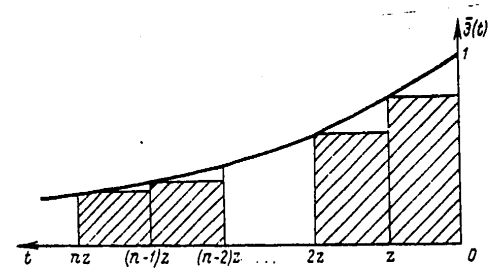

Figure 6.8. Geometric meaning of the index (6.38).

If independently of the type of ship the expenditures on building it are equal to S, then $\mathcal{N}_k = xz/S,\ k = 1, \ldots, n$. The expression (6.37) is written as follows in this case:

$$\bar{Э}_\Sigma(x, n, z) = \frac{1}{S} xz \sum_{k=1}^{n} \bar{Э}(kz).$$

Inasmuch as S = const, during optimization it is possible to use the index

$$xz \sum_{k=1}^{n} \bar{Э}(kz), \tag{6.38}$$

which must be maximized with respect to the variables x, n and z. Geometrically the index (6.38) is a value proportional to the area of the rectangles crosshatched in Figure 6.8.

Let us denote the total expenditures on development, building and maintenance of ships in the time z by zs_Σ, where s_Σ is the total expenditures per unit time, for example, for a year.

Obviously the equality

$$xz + \bar{c}xnz^2 + S_p = zs_\Sigma. \tag{6.39}$$

The problem of maximizing the total effectiveness of the ships in operation

$$\max_{x,\,n,\,z}\left[xz\sum_{k=1}^{n}\bar{\text{Э}}(kz)\right], \quad x+\bar{c}xnz+\frac{S_p}{z}=s_\Sigma, \quad x>0,\ n>0,\ z>0. \tag{6.40}$$

Let us assume that the function $\bar{\text{Э}}(t)$ is exponential, that is $\bar{\text{Э}}$ (t) = a^t, $0<\alpha<1$.) In this case

$$\sum_{k=1}^{n}\bar{\text{Э}}(kz)=\sum_{k=1}^{n}a^{kz}=a^z\frac{1-a^{nz}}{1-a^z},$$

$$xz\sum_{k=1}^{n}\bar{\text{Э}}(kz)=x(1-a^{nz})f(z),$$

where $f(z)=\frac{za^z}{1-a^z}$.

Introducing the notation nz = T, where T is the service life of the ship, we arrive at the problem

$$\max_{x,z,T}[xf(z)(1-a^T)], \quad x+\bar{c}xT+\frac{S_p}{z}=s_\Sigma, \quad x>0,\ z>0,\ T>0. \tag{6.41}$$

In accordance with the method of indeterminant Lagrange factors, the optimal values of x, z and T are found from the system of algebraic equations

$$\begin{aligned} &f(z)(1-a^T)-\lambda(1+\bar{c}T)=0,\\ &-xf(z)a^T\ln a-\lambda\bar{c}x=0,\\ &x(1-a^T)f'(z)+\lambda\frac{S_p}{z}=0,\\ &x+\bar{c}xT+\frac{S_p}{z}-s_\Sigma=0, \end{aligned} \tag{6.42}$$

where λ is the Lagrange factor, and $f'(z) = df(z)/(dz)$.

Substituting the expression for λ found from the second equation in the first equation of system (6.42), we obtain the equation for finding the optimal value of the service life T_{opt}

$$\frac{1}{\bar{c}}+T=\frac{1}{\ln a}(1-a^{-T}). \tag{6.43}$$

After introduction of the notation $a = e^{-\nu}$ and $\Theta = \nu T$ the equation (6.43) assumes the form

$$\theta+\left(1+\frac{\nu}{\bar{c}}\right)=e^{\theta}. \tag{6.44}$$

Thus, if we set $B = 1 + \nu/\bar{c}$, we again obtain the equation (6.35).

Let us assume that $\nu = 0.05$ 1/year and $\bar{c} = 0.02$ 1/year as occurred above. The solution of equation (6.44) for these initial data gives $\Theta_{opt} = 1.64$, $T_{opt} = 32.8$ years. This coincides with solution of equation (6.35) considering the fact that $c = s'_з/S$.

From equation (6.44) it is directly obvious that T_{opt} depends only on the coefficient $\bar{c}$ and the obsolescence parameter ν. Neither the size of the budget S_Σ nor the expenditures on development S_p influence the optimal service life. It is possible to demonstrate [63] that the optimal time interval between introducing new types of ships (z_{opt}) depends on $\bar{c}$, ν and S_p/s_Σ, where z_{opt} decreases with a decrease in S_p/s_Σ. Consequently, with an increase in the budget (s_Σ) the optimal time interval between introducing new types of ships decreases.

It is possible to show that T_{opt} increases monotonically with a decrease in ν. This is also an entirely natural result: the more slowly the ship ages (the smaller ν) the longer the optimal service life and vice versa.

In the above-investigated statement of the problem when obtaining the expression for $Э_\Sigma$ it was assumed that either the cost of building the ships (S) does not change with time, including ships of new types, or the total effectiveness of the ships built in each of the time intervals [kz, (k - 1) z] does not depend on the number of ships, but is proportional only to the total expenditures on construction during these periods.

Let us change the statement of the problem somewhat, and let us consider the increase in cost of building ships with time. The practice of naval shipbuilding and civilian ship building demonstrates that along with improvement of the characteristics of the ships with time and, as a consequence, improvement of the effectiveness, the cost of building and maintaining them increases. Here the variation in effectiveness and cost takes place in such a way that the new ships turn out to be preferable with respect to the "cost-effectiveness" type criteria. According to the data of B. M. Smirnov [47], the cost of building transport ships increased approximately 25-35% in the 20 years preceding 1961. According to the data of A. A. Narusbayev [34], the cost of building ships in American shipyards increased by 88-100% from 1962 to 1966 by comparison with 1949-1954. The increase in cost of building warships on which the latest achievements of science and engineering are introduced the most quickly takes place still faster (Figure 6.9).

It is necessary to consider that the increase in cost of building ships takes place not only because of their technical complication and improvement of elements, but also as a result of the so-called growth of the price index [34] -- the rise in prices for industrial production. In some economic analysis problems the above-indicated causes must be broken down to obtain the costs in comparable prices. In the investigated problem there is no need for this breakdown inasmuch as the budget allocated for building and maintaining ships is fixed and does not change with time. In this case the specific causes of an increase in cost of building and maintaining ships have no significance.

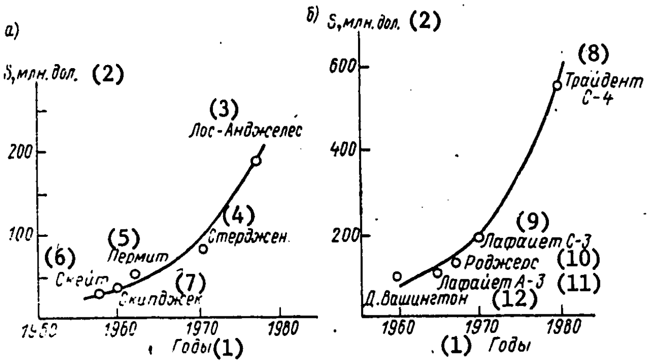

Figure 6.9. Increase in cost of building American nuclear-powered submarines: a -- torpedo, b -- missile [65].

Key:
1. years
2. S, millions of dollars
3. Los Angeles
4. Sturgeon
5. Permit
6. Skate
7. Skip jack
8. Trident C-4
9. Lafayette C-3
10. Rogers
11. Lafayette A-3
12. George Washington

Along with the aging functions $\bar{\text{Э}}(t)$ let us introduce the function of relative increase in cost of building ships $\bar{S}(t) = S(t)/S(0)$ into the investigation, where $S(0)$ is the cost of building a ship at the current point in time; $S(t)$ is the same cost for the ship built t years ago.

Considering the variation in cost, the number of ships built in the time interval [kz, (k - 1) z], will be

$$\mathcal{N}_k = \frac{1}{S(0)} \frac{xz}{\bar{S}(kz)},$$

and instead of the expression (6.38), we obtain

$$xz \sum_{k=1}^{n} \frac{\bar{\text{Э}}(kz)}{\bar{S}(kz)}. \qquad (6.45)$$

Let us assume that the function $\bar{S}(t)$ is (or can be approximated by) an exponential function, that is, $\bar{S}(t) = b^t$, where $0 < b < 1$ is the given parameter. Introducing the notation $\mu = -\ln b$, we find $\bar{S}(t) = e^{-\mu t}$, $\mu > 0$. In the case of approximation of $\bar{S}(t)$ by an exponential function the values of the parameter b or μ can be obtained by the least squares method.

Let the values of the function $S(t)$ exist for certain given values of t_i, $i = 1, \ldots, m$. The corresponding values of $\bar{S}(t)$ will be denoted by $\bar{S}(t_i)$. In accordance with the least squares method the value of μ will be determined from the condition

$$\min_{\mu}\left\{\sum_{i=1}^{m}[e^{-\mu t_i}-\overline{S}(t_i)]^2\right\},$$

which leads to a transcendental equation with respect to the parameter μ:

$$\sum_{i=1}^{m} t_i e^{-\mu t_i}=\sum_{i=1}^{m}\overline{S}(t_i)t_i.$$

For simplification of the problem, the least squares method can be applied to the function $\psi(t) = (-\ln\overline{S}(t)/t$. In this case the value of μ is found by the formula

$$\mu=-\frac{1}{m}\sum_{i=1}^{m}\frac{\ln\overline{S}(t_i)}{t_i}.$$

In this formula values of μ were calculated for torpedo and missile submarines of the United States according to data taken from reference [65][1]. For torpedo submarines, the year 1977 was taken as the time $t = 0$, and the cost of the submarine "Los Angeles" equal to 190 million dollars was taken as $S(0)$. For missile submarines the time $t = 0$ corresponds to the year 1980, and $S(0) = 552$ million dollars -- the submarine "Trident." Let us remember that in the investigated problem the reckoning of time goes into the past. In both cases (for torpedo and missile submarines) it turned out that $\mu \approx 0.1$. (See Figure 6.9 where the curves are shown for the variation of cost with exponential approximation of $\mu = 0.1$. It is obvious that the individual points corresponding to the investigated submarines fit these curves well.)

For the exponential form of the function $\overline{S}(t)$ the problem of optimizing the values of x, z and T considering the increase in cost of building the ships has the form (6.41) if the ratio a/d which is equal to $e^{-(\nu-\mu)}$ is taken as a. Consequently, for $\nu > \mu$ the optimal service life T_{opt} can be found from the equation

$$\Theta+\left(1+\frac{\nu-\mu}{c}\right)=e^{\Theta}, \tag{6.46}$$

where $\Theta = (\nu - \mu)T$ and $\nu > \mu$.

Inasmuch as for $\nu > \mu$ the value of $\nu - \mu$ is always less than ν, the service life found from equation (6.46), that is, considering the increase in cost will always be greater than the service life found from equation (6.44), on derivation of which the cost of building the ships was considered invariant with respect to time.

The condition $\nu > \mu$ means that the increase in building cost proceeds more slowly than the increase in effectiveness. This is entirely natural for new ships (we have in mind ships of new types); otherwise building them would be expedient. For this reason the case $\nu < \mu$ does not need to be considerd. Mathematically for $\nu > \mu$ equation (6.46) has no real positive roots.

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[1] When constructing the graphs (Figure 6.9) and calculating μ the data from article [65] for the submarines "Los Angeles" and "Trident" were altered somewhat considering the series factor.

CONCLUSION

The theory of analytical design of ships is in a stage of intensive development. This theory is constructed on the basis of a large number of applied scientific disciplines, including the shipbuilding sciences, the theory of effectiveness evaluation and operations research, reliability theory, invulnerability theory, and so on. Accordingly, it is very difficult to write a book in which the methods of solving the basic problems of analytical design are discussed identically completely and with identical detail, especially considering the specific nature of ships of different classes and types. However, the author hopes that the given book will be useful from the procedural point of view as an introduction to the theory of analytical design of ships.

The basic problems of the theory of analytical design of ships can at the present time include:

Mathematical simulation of the set of design calculations considering the process of functioning of a ship as a complex dynamic system;

Optimization of the TTE and the TDP of the ship with respect to several effectiveness indexes and under the conditions of indeterminacy caused by incompleteness and insufficient reliability of the initial data;

The development of computer-aided automated analytical ship design systems using computer and electronic means of displaying the graphical data permitting the designer to influence the solution of the design problems operatively.

For example, the first group of problems includes the creation of mathematical models of ships considering the entire variety of cause-effect relations between individual engineering subsystems. The search is continuing for improved basic equations of the analytical method of design. One of the versions of the approach to the construction of a mathematical model of a ship is discussed in [58]. The basis for this approach is separation of all engineering systems of the ship into two groups. The first group includes the systems directly providing various characteristics of the ship, that is, its "output" characteristics. The second group is formed of systems providing for the functioning of the systems of the first group on the part of their placement, power supply, control, servicing, operating conditions, and so on.

At the present time the area connected with automation of the ship design processes [30, 67] and creating automated design systems (SAPR) is developing very intensely.

Such systems are especially effective for the analytical design stage inasmuch as they permit significant expansion of the research connected with optimizing the TTC of ships.

The primary goal of the SAPR systems in the analytical design stage is the directional search for the optimal version of the ship based on using the graphical or graphoanaltyical method and operation of the system in the designer-computer dialog mode. Here the entire set of calculation and graphical operations with respect to the problems indicated in Figure 1.1 must be realized in connected form.

The application of the SAPR in the analytical design stage will permit a number of new qualities to be obtained: namely, an increase in the possibilities for finding the optimal version as a result of broadly encompassing the space of the TTC of the ship and great depth of investigation of each version; reduction of the design times as a result of profound substantiation of the assignments for the ship design; insurance of greater purposefulness of the scientific research and experimental design work when building the ship as a result of more detailed selection of the means of technical implementation of the ship in the analytical design stage.

On the whole, the creation and the use of ship SAPR systems is a large step in the improvement of the methodology of ship design.

In conclusion, the author considers it his duty again to emphasize that for the above-indicated reason the given book does not pretend to completeness or to be the final word in the discussion of the investigated topic. Critical remarks and suggestions will be gratefully accepted by the author.

APPENDIX 1. SOLUTION OF THE THREE-TERM MASS AND VOLUME EQUATIONS USING THE AUXILIARY FUNCTION TABLE AND OBTAINING AN APPROXIMATE SOLUTION IN EXPLICIT FORM

The general form of the three-term mass and volume equation

$$AD + BD^{2/3} + C = D, \tag{Π.1.1}$$

where D is the desired mass or volume displacemen; A, B and C are the given coefficients of the mass or volume equations ($0 < A < 1$; $B > 0$; $C > 0$).

By introducing a new variable $x = D/\bar{C}$ and the notation $a = \bar{B}/C^{1/3}$, $\bar{B} = B/(1 - A)$, $\bar{C} = C/(1 - A)$ the equation (Π.1.1) is reduced to the form

$$ax^{2/3} + 1 = x. \tag{Π.1.2}$$

Table Π.1. Values of the function f(a)

a	f(a)	a	f (a)	a	f (a)	a	f (a)
0,10	1,107	0,60	1,930	1,10	3,57	1,60	6,67
0,15	1,166	0,65	2,05	1,15	3,80	1,65	7,09
0,20	1,229	0,70	2,18	1,20	4,05	1,70	7,53
0,25	1,297	0,75	2,31	1,25	4,31	1,75	8.00
0,30	1,370	0,80	2.46	1,30	4,59	1.80	8,49
0,35	1,448	0,85	2,61	1,35	4,89	1,85	9,01
0,40	1,531	0,90	2,78	1,40	5,20	1,90	9,56
0,45	1,621	0,95	2,96	1,45	5,54	1,95	10,1
0,50	1,717	1,00	3,15	1,50	5,89	2,00	10,7
0,55	1,820	1,05	3,35	1,55	6,27	2,05	11,4

If the solution of the equation (Π.1.2) is found as a function of the parameter a, that is, the function $x = f(a)$ is defined, then the solution of the initial equation (Π. 1. 1) will be $D = \bar{C}f(a)$ or, in expanded form

$$D = \frac{C}{1-A} f\left(\frac{B}{(1-A)^{2/3} C^{1/3}}\right). \tag{Π.1.3}$$

The values of the function f(a) for $0.1 \le a \le 2.05$ with step $\Delta a = 0.05$ are presented in Table Π.1. If it is necessary to obtain the intermediate values of f(a) the linear interpolation is carried out[1]. The graphs of the function f(a) for $0 \le a \le 2$ and the function

$$\tilde{f}(a) = 1 + 2{,}3a^2, \qquad (\text{П}.1.4)$$

which approximates f(a) well in the range of $0.6 \le a \le 2$ are illustrated in Figure П. 1.

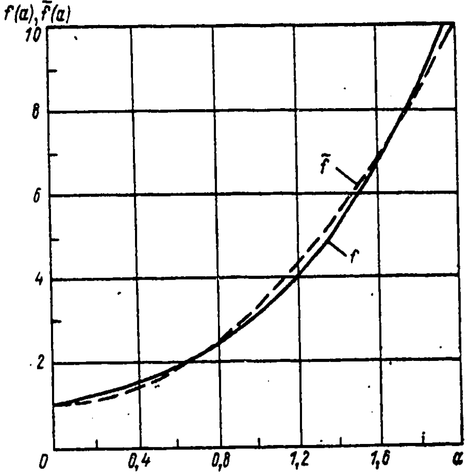

Figure П.1. The graphs of the functions f(a) and f̃(a).

In accordance with the function (П.1.4) the solution of equation (П.1.1) can be found by the approximate formula

$$D = \frac{C}{1 - A}\left\{1 + 2{,}3\left[\frac{B}{(1 - A)^{2/3}C^{1/3}}\right]^2\right\}. \qquad (\text{П}.1.5)$$

In the range of values

$$0{,}6 < \frac{B}{(1 - A)^{2/3}C^{1/3}} < 2{,}0$$

the relative error of formula (П.1.5) does not exceed 5-6% by comparison with the exact solution of equation (П.1.1).

The method of solving the three-term mass and volume equations using the table of values of the auxiliary function f(a) and also by formula (П.1.5) is convenient for calculating estimates without using a computer. The given method of solving the mass and volume equations, including formula (П.1.5) are applicable only for $C \ne 0$ ($C > 0$).

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[1] The table of values of the function f(a) was calculated by A. M. Ivanov.

APPENDIX 2. COORDINATES OF THE TEN-DIMENSIONAL SOBOL' POINTS (WITH ROUNDING TO THE THIRD PLACE)

l	i									
	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0
1	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500	0,500
2	0,250	0,750	0,250	0,750	0,250	0,750	0,250	0,750	0,250	0,750
3	0.750	0,250	0,750	0,250	0,750	0,250	0,750	0,250	0,750	0,250
4	0,125	0,625	0,875	0,875	0,625	0,125	0.375	0,375	0,875	0,625
5	0,625	0,125	0,375	0,375	0,125	0,625	0,875	0,875	0,375	0,125
6	0,375	0,375	0,625	0,125	0,875	0,875	0,125	0,625	0,125	0,875
7	0,875	0,875	0,125	0,625	0,375	0,375	0,625	0,125	0,625	0,375
8	0,063	0,938	0,688	0,313	0,188	0,063	0,438	0,563	0,813	0,688
9	0,563	0,438	0,188	0,813	0,688	0,563	0,938	0,063	0,313	0,188
10	0,313	0,188	0,938	0,563	0,438	0,813	0,188	0,313	0,063	0,938
11	0,813	0,688	0,438	0,063	0,938	0,313	0,688	0,813	0,563	0,438
12	0,188	0,313	0,313	0,688	0,563	0,188	0,063	0,938	0,188	0,063
13	0,438	0,563	0,063	0,438	0,813	0,938	0,313	0,188	0,938	0,313
14	0,938	0,063	0,563	0,938	0,313	0,438	0,813	0,688	0,438	0,813
15	0,031	0,531	0,406	0,219	0,469	0,281	0,969	0,281	0,094	0,844
16	0,531	0,031	0,906	0,719	0,969	0,781	0,469	0,781	0,594	0,344
17	0,281	0,281	0,156	0,969	0,219	0,531	0,719	0,531	0,844	0,594
18	0,781	0,781	0,656	0,469	0,719	0,031	0,219	0,031	0,344	0,094
19	0,156	0,156	0,531	0,844	0,844	0,406	0,594	0,156	0,969	0,469
20	0,656	0,656	0,031	0,344	0,344	0,906	0,094	0,656	0,469	0,969
21	0,406	0,906	0,781	0,094	0,594	0,656	0,844	0,906	0,219	0,219
22	0,906	0,406	0,281	0,594	0,094	0,156	0,344	0,406	0,719	0,719
23	0.094	0,469	0,844	0,406	0,281	0,344	0,531	0,844	0,781	0,406
24	0,594	0,969	0,344	0,906	0,781	0,844	0,031	0,344	0,281	0,906
25	0,844	0,219	0,094	0,156	0,531	0,094	0,281	0,594	0,531	0,656
26	0,219	0,844	0,219	0,531	0,906	0,469	0,906	0,719	0,156	0,781
27	0,719	0,344	0,719	0,031	0,406	0,969	0,406	0,219	0,656	0,281
28	0,469	0,094	0,469	0,281	0,656	0,719	0,656	0,469	0,906	0,531
29	0,969	0,594	0,969	0,781	0,156	0,219	0,156	0,969	0,406	0,031
30	0,016	0,797	0,953	0,673	0,797	0,922	0,734	0,890	0,547	0,828
31	0,516	0,297	0,453	0,172	0,297	0,422	0,234	0,390	0,047	0,328
32	0,266	0,047	0,703	0,422	0,547	0,172	0,984	0,141	0,297	0,578

APPENDIX 3. SOME EXPRESSIONS USED IN THE INITIAL STAGES OF SHIP DESIGN

1. For invariant values of the engine power (N) and the admiralty coefficient (C_w) the maximum speed (v) of a ship is inversely proportional to the displacement D:

$$v = (NC_w)^{1/3} D^{-2/9} \sim D^{-2/9}. \qquad (\text{П}.3.1)$$

2. For invariant values of the specific mass of the power plant (q_{PP}), the coefficients C_w and A (a is the coefficient of the mass or volume equation) the maximum speed of the ship increases with an increase in the given displacement (D_0). According to the mass equation we have

$$v = (C_w / \underset{(1)}{q_{\text{эу}}})^{1/3} \left(1 - A - \sum_i m_i / D_0\right)^{1/3} D_0^{1/9}. \qquad (\text{П}.3.2)$$

Key: 1. PP = power plant

When using the volume equation instead of A it is necessary to substitute A_V, and instead of $\sum_i m_i$ the value of $\sum_i V_i$ (the sum of the volumes which do not depend on displacement). If, in addition $\sum_i m_i / D_0 = \text{idem}$, then $v \sim D_0^{1/9}$, that is, the speed increases very slowly.

3. The variation of the maximum speed of a ship with an increase in power as a result of an increase in the number of echelons of the power plant (the power of each echelon is fixed, the number of screws is equal to the number of echelons) is:

$$\frac{v_n}{v_1} = n^{1/3} k_{cn}^{1/3} \left[\frac{1 - A}{1 - A + (k_{mn} - 1)\, \bar{m}_{\text{эу}}^{(1)}} \right]^{2/9}, \qquad (\text{П}.3.3)$$

where $k_{cn} = C_{wn}/C_1$, $k_{mn} = m^n_{\text{эу}} / m^1_{\text{эу}}$, $\bar{m}^1_{\text{эу}} = m^1_{\text{эу}} / D_1$ and the index n pertains to the n-echelon system, and the index 1, to the single-echelon system.

The formula (П.3.3) corresponds to the case where the displacement is defined by the mass equation and buoyancy ($D_m \geq \rho D_V$). If the displacement is defined by the volume equation, then instead of A it is necessary to substitute A_V, and instead of $\bar{m}_{PP}^{(1)}$, the value of $\bar{V}_{PP}^{(1)} = V_{PP}^{(1)} / D_1$. Let us consider the example of using expression (П.3.3) for submarines. Let $n = 2$, $k_{m2} = 2$, $k_{c2} = 0.7$, $A = 0.59$, $\bar{m}_{PP}^{(1)} = 0.16$ to 0.30

[10]. For these initial data we have: $v_2/v_1 = 0.99$ to 1.04, that is, the application of the two-echelon power plant of double power in the given case in practice does not give a gain in speed. Inasmuch as the cofactor in brackets in expression (П.3.3) increases with a decrease in A, setting $A_V \approx 0$ with a margin and approximately setting $\bar{m}_{PP}^{(1)} \approx \bar{V}_{PP}^{(1)}$, we obtain $v_2/v_1 < 1.03$ to 1.09. It is possible to obtain a noticeable increase in the speed (by 12-20%) only for $k_{c2} \approx 1$, that is, when insuring identical propulsive qualities of the single-shaft and two-shaft propulsion systems. However, here it is necessary to consider that the gain in speed is connected with doubling the power of the power plant and increasing the displacement by 1.4 to 1.7 times[1].

4. The formulas for recalculating the lines plan curves for linear transformation of a prototype: $D = D_0 k_L k_B k_T$ (displacement), $z = z_0 k_T$ (the y-coordinate of the center of buoyancy), $\rho = \rho_0 k_B^2/k_T$ (the transverse metacentric radius), $x = x_0 k_L$ (the x-axis of the center of buoyancy and the center of flotation), $R = R_0 k_L^2/k_T$ (the longitudinal metacentric radius). Here k_L, k_B, k_T are the coefficients of linear transformation of the principal dimensions L, B, T.

5. The limits of variation of the lines coefficients and the ratios L/B and B/T for surface ships[2]:

Class of ships	δ	α	β	L/B	B/T
Cruisers	0,45—0,60	0,69—0,73	0,76—0,90	8,5—11,3	2,6—4,2
Destroyers	0,44—0,53	0,68—0,73	0,75—0,86	9,2—11,9	2,5—4,1
Patrol vessels	0,40—0,55	0,75—0,85	0,75—0,85	8,3—10,1	2,6—4,0
Trawlers	0,50—0,60	0,65—0,80	0,75—0,95	6,4—7,5	3,5—4,3

6. The relative wetted surface of submarines underwater [10]: with "stem" lines $\omega = 6.05 + 0.26L/B$; with hulls in the form of solids of revolution $\omega = 5.60 + 0.26L/B$.

7. Values of the total drag coefficient and the propulsive coefficient of modern American submarines underwater [10]: single-shaft $\zeta = (3.0 \text{ to } 3.2)\cdot 10^{-3}$, $\eta = 0.80$ to 0.85; two-shaft $\zeta = (3.2 \text{ to } 3.5)\cdot 10^{-3}$, $\eta = 0.65$ to 0.75.

8. The heel period (τ_θ) and pitch period (τ_ψ) of ships in calm water

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[1] When selecting the number of echelons of the power plant it is also necessary to consider the reliability factor which can turn out to be significant when making the final decision.

[2] A. V. Gerasimov, A. I. Pastukhov, V. I. Solov'yev, OSNOVY TEORII KORABLYA (Fundamentals of Ship Theory), Moscow, Voyenizdat, 1958.

$$\tau_\theta = \frac{2\pi}{\sqrt{g}} k_\theta \frac{B}{\sqrt{h_\theta}}, \quad \tau_\psi = \frac{2\pi}{\sqrt{g}} k_\psi \sqrt{T}, \qquad (\Pi.3.4)$$

where B, T are the beam and draft of the ship, h_θ is the transverse metacentric height, g is the gravitational acceleration, k_θ, k_ψ are dimensionless coefficients.

9. The steady turning radius of a surface ship

$$\underset{(1)}{R_ц} = (k_1/k_2)(D/S_p), \qquad (\Pi.3.5)$$

Key: 1. turning

where D is the displacement, m^3; S_p is the rudder area, m^2; k_1, k_2 are dimensionless coefficients.

The coefficient k_1 is defined by the value of $(\delta/\varepsilon)(B/L)$ (L, B are the length and beam of the ship, δ is the block coefficient, ε is the coefficient of fineness of the submerged part of the longitudinal center plane):

$(\delta/\varepsilon)(B/L)$	0,05	0,075	0,10	0,125	0,15
k_1	1,5	0,71	0,46	0,33	0,29

The coefficient k_2 depends on the angle of helm α_p:

α_p, degrees	0	10	20	30
k_2	0	0,61	0,89	1,0

The speed drop in the turning circle can be approximately defined by the formula of G. A. Firsov

$$\frac{v}{\underset{(1)}{v_ц}} = \operatorname{th} \frac{R_ц}{2,45L}. \qquad (\Pi.3.6)$$

Key: 1. turn

10. The distribution (in percentage of the normal displacement) of the mass load of nonnuclear surface ships with artillery armament [35]:

Class of ships	Load[1]					
	$m_к$	$m_б$	$m_в$	$m_м$	$m_т$	$m_с$
Battleships	28—40	22—40	12—22	6—12	3—12	1—2
Heavy cruisers	28—34	14—33	13—15	10—20	11—15	2—3
Light cruisers	34—38	15—22	10—15	12—28	7—10	2—3
Destroyers	32—36	0—2	10—15	32—38	10—16	3—4

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[1] $m_к$; $m_б$; $m_в$; $m_м$; $m_т$; $m_с$ — masses of the hull; armor; armament; machinery, fuel, water and oil; equipment, consumable materials and crew.

11. Distribution (in percentage of normal displacement) of the mass load of American nuclear-power submarines [10]:

Load divisions	Subclass of submarine	
	Torpedo	Missile
Hull	38-42	40-44
Machinery	26-40	14-18
Electrical equipment	4-6	3-5
Radioelectronic equipment	2-3	1-2
Systems and devices	7-8	5-7
Equipment	2-3	2-3
Armament	2-4	10-14
Fuel and drinking water	3-4	2-3
Crew and supplies	3-4	2-3
Reserve displacement	2-3	2-3
Solid ballast	2-3	2-3

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CSO: 8144/0419

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