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GEOPHYSICS, ASTRONOMY AND SPACE (FOUO 4/79)

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USSR Report

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USSR REPORT

GEOPHYSICS, ASTRONOMY AND SPACE

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UDC 528.716.2:621.375.826

NONCONTACT LASER SOUNDING OF SEA WATER

Moscow IVUZ, GEODEZIYA I AEROFOTOS"YEMKA in Russian No 1, 1979 pp 99-107

[Article by Candidate of Technical Sciences V. V. Polovinko, Moscow Institute of Geodetic, Aerial Mapping and Cartographic Engineers, submitted for publication 24 April 1978]

[Text] In order to exploit marine resources it is necessary to carry out comprehensive investigations over major water areas with the use of routine measurement methods. These include optical noncontact methods which are intensively developing at the present time [1-5], among which the measurement methods based on noncontact laser sounding of sea water constitute an individual group. In these methods sea waters are sounded by pulses of laser radiation with a small angular divergence, the brightness of backscattering of sea water (BSSW) is measured and on the basis of the spectral and temporal characteristics of BSSW it is possible to determine the characteristics of sea water.

This article is devoted to the creation of a mathematical model of noncontact laser sounding of sea water. The investigations were made with allowance for elastic scattering and nonlinear phenomena arising during the interaction of laser radiation with sea water.

Radiation Brightness of Sea Water During Laser Sounding

Now we will investigate the dependence of the brightness of BSSW during laser sounding on the characteristics of the sounding radiation and the characteristics of sea waters taking into account the changes in hydrooptical characteristics in the case of nonlinear phenomena in sea water.

Assume that a laser sounds sea waters with a light pulse in the direction of the normal to the surface of the water-air discontinuity at a point with the coordinates x', y' (Fig. 1). The angular divergence of laser radiation in sea water is $2W'_1$, the wavelength of the radiation is λ; the cross-sectional area of the light beam of rays with z = 0 is equal to S_0, and the intensity of the radiation is F_0; z is the distance traversed by the laser radiation in water.

1. We will find the dependence of attenuation of diverging laser radiation in the case of nonlinear phenomena in sea water. We will assume that the sea water scattering index σ does not change its value from the value of the sounding radiation and the absorption index changes in conformity to the law $\varkappa(1 - a'E^N)$, where $\varkappa$ is the absorption index of sea water in the case of a low density of the sounding radiation; a' is a nonlinearity parameter which is related to the nonlinearity parameter a (in the notations of study [6]) by the equation $a' = k'a^N$ (k' is a coefficient); E is illumination at the depth z ($E = F \cdot S$); N is a coefficient dependent on the initial density of the sounding radiation ($0 < N \leqslant 1$); $\varkappa + \sigma = \varepsilon$ is the index of sea water attenuation.

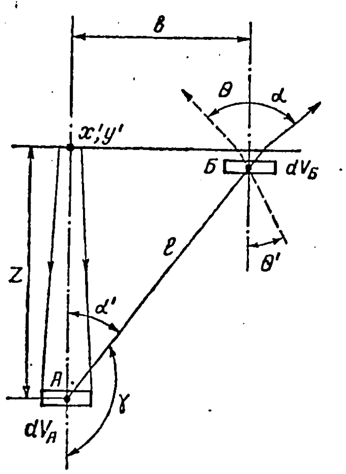

Fig. 1.

At the depth z we will define the elementary volume dV' and we will write the differential equation (1) for the attenuation of laser radiation by this volume:

$$\frac{dF}{dz} = -F\left[\sigma + \varkappa\left(1 - \frac{a'F^N}{S_0^N(1+\beta z)^{2N}}\right)\right], \qquad (1)$$

where $S = S_0(1 + \beta z)^2$ is the cross-sectional area of the light beam at the distance z,

$$\beta = \left(\frac{\omega_1'}{S_0}\right)^{1/2};$$

ω'_1 is a solid angle within whose limits the laser radiation is propagated in sea water.

Assuming $u = F^{-N}$, we find

$$\frac{dU}{dz} - N\varepsilon U + \frac{N\varkappa a'}{S_0^N (1+\beta z)^{2N}} = 0. \tag{2}$$

Solving the differential equation (2), we obtain

$$U = \left[\int \frac{(-N\varkappa a') e^{-N\varepsilon z} dz}{S_0^N (1+\beta z)^{2N}} + C_1 \right] e^{N\varepsilon z}. \tag{3}$$

We will transform equation (3) to the following expression:

$$F = F_0 \left[1 + E_0^N \int_0^z \frac{(-N\varkappa a') e^{-N\varepsilon x}}{(1+\beta x)^{2N}} dx \right]^{-\frac{1}{N}} e^{-\varepsilon z}. \tag{4}$$

We will solve the integral equation (4) with N = 1/2. For this we will find the value of the integral:

$$G_1 = \int \frac{e^{-\frac{1}{2}\varepsilon x} dx}{(1+\beta x)}. \tag{5}$$

Using the notation $r = -1/2\,\varepsilon$, $s = 1 + \beta z$, we will represent the integral (5) in the following form:

$$G_1 = \frac{e^{\frac{r}{\beta}}}{\beta} \int \frac{e^{\frac{r}{\beta} s}}{s} ds. \tag{6}$$

We will find the solution of this integral in the form of a series

$$G_1 = \frac{e^{-\frac{r}{\beta}}}{\beta} \left[\ln s + \frac{\frac{r}{\beta} s}{1!} + \frac{\left(\frac{r}{\beta} s\right)^2}{2 \cdot 2!} + \frac{\left(\frac{r}{\beta} s\right)^3}{3 \cdot 3!} + \dots \right]. \tag{7}$$

We will transform equation (7) to the form

$$G_1 = \frac{e^{\frac{\varepsilon}{2\beta}}}{\beta} \left[-c - \ln \frac{\varepsilon}{2\beta} + c + \ln\left(\frac{\varepsilon}{2\beta} s\right) - \frac{\left(\frac{\varepsilon}{2\beta} s\right)}{1!} + \frac{\left(\frac{\varepsilon}{2\beta} s\right)^2}{2 \cdot 2!} - \frac{\left(\frac{\varepsilon}{2\beta} s\right)^3}{3 \cdot 3!} + \dots \right], \tag{8}$$

where c = 0.5772 is the Euler-Masceroni constant.

Using the formulation of the tabulated integral exponential functions E_0 and E_1 in (8), we have

$$G_1 = \frac{2}{3} E_0^{-1}\left(\frac{\varepsilon}{2\beta}\right)\left[-c - \ln\left(\frac{\varepsilon}{2\beta}\right) - E_1\left(\frac{\varepsilon}{2\beta} s\right)\right]. \tag{9}$$

From equation (4), taking (9) into account, we obtain an expression for the intensity of radiation at the depth z, taking into account nonlinear phenomena when sounding sea water by divergent laser radiation:

$$F=F_0\left\{1-\frac{\varkappa}{\varepsilon}a'E_0^{1/2}\left[\frac{E_1\left(\frac{\varepsilon}{2\beta}\right)-E_1\left(\frac{\varepsilon}{2\beta}+\frac{1}{2}\varepsilon z\right)}{E_0\left(\frac{\varepsilon}{2\beta}\right)}\right]\right\}^{-2}e^{-\varepsilon z}. \qquad (10)$$

When $z\to\infty$,

$$F=F_0\left\{1-\frac{\varkappa}{\varepsilon}a'E_0^{1/2}\frac{E_1\left(\frac{\varepsilon}{2\beta}\right)}{E_0\left(\frac{\varepsilon}{2\beta}\right)}\right\}^{-2}e^{-\varepsilon z}. \qquad (11)$$

In this equation the ratio

$$\frac{E_1\left(\frac{\varepsilon}{2\beta}\right)}{E_0\left(\frac{\varepsilon}{2\beta}\right)}<1$$

and with $\varepsilon/2\beta\to\infty$ tends to 1.

We will solve equation (4) with N = 1/4. For this we introduce the formulation

$$e=\sqrt{\frac{\varepsilon}{2\beta}+\frac{1}{2}\varepsilon z'},$$

then

$$G_2=\int\frac{e^{-1/4\varepsilon x}\,dx}{(1+\beta x)^{1/2}}=\frac{4}{\varepsilon}\frac{\left(\frac{\varepsilon}{2\beta}\right)^{1/2}}{\frac{1}{\sqrt{2\pi}}e^{-1/4\varepsilon/\beta}}\cdot\frac{1}{\sqrt{2\pi}}\int e^{-e^2/2}\,de. \qquad (12)$$

Using the tabulated functions of the probability integral Φ and the densities of the normal distribution φ, we have

$$G_2=\frac{4}{\varepsilon}\frac{\Phi(e)}{\varphi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)/\sqrt{\frac{\varepsilon}{2\beta}}}. \qquad (13)$$

Substituting (13) into (4) with N = 1/4, we obtain the following expression for F:

$$F=F_0\left\{1-\frac{\varkappa}{\varepsilon}a'E_0^{1/4}\left[\frac{\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}+\frac{1}{2}\varepsilon z'}\right)-\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)}{\varphi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)/\sqrt{\frac{\varepsilon}{2\beta}}}\right]\right\}^{-4}e^{-\varepsilon z}. \qquad (14)$$

When $z\to\infty$,

$$F=F_0\left\{1-\frac{\varkappa}{\varepsilon}a'E_0^{1/4}\left[\frac{0{,}5-\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)}{\varphi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)/\sqrt{\frac{\varepsilon}{2\beta}}}\right]\right\}^{-4}e^{-\varepsilon z}. \qquad (15)$$

The ratio

$$\frac{0,5-\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)}{\varphi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)\Big/\sqrt{\frac{\varepsilon}{2\beta}}}<1$$

and when $\varepsilon/2\beta\to\infty$ tends to 1.

In general form the equation for the intensity of sounding radiation at the depth z, with nonlinear phenomena taken into account, can be written as follows:

$$F=F_0Ae^{-\varepsilon z}. \quad (16)$$

In this expression

$$A=\left[1-\frac{\varkappa}{\varepsilon}a'E_0^N\Psi_N(z,\beta)\right]^{-1/N}, \quad (17)$$

where

$$\Psi_{1/2}=\frac{E_1\left(\frac{\varepsilon}{2\beta}\right)-E_1\left(\frac{\varepsilon}{2\beta}+\frac{1}{2}\varepsilon z\right)}{E_0\left(\frac{\varepsilon}{2\beta}\right)}; \quad (18)$$

$$\Psi_{1/4}=\frac{\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}+\frac{1}{2}\varepsilon z}\right)-\Phi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)}{\varphi\left(\sqrt{\frac{\varepsilon}{2\beta}}\right)\Big/\sqrt{\frac{\varepsilon}{2\beta}}}. \quad (19)$$

2. We will write the equation for illumination E' at the point A, situated at the distance z' from the water surface:

$$E=\frac{F_0}{S}Ae^{-\varepsilon z}. \quad (20)$$

The intensity of radiation $I(\alpha')$ in the direction α' (Fig. 1) will be described by the expression

$$dI(\alpha')=\frac{1}{4\pi}x(\gamma_1)\sigma F_0Ae^{-\varepsilon z}dz, \quad (21)$$

where $x(\gamma)$ is the scattering indicatrix, $\alpha'+\gamma_1=\pi$.

Taking into account refraction at the water-air discontinuity, it is possible to obtain the following expression for brightness $B(\alpha)$ of the singly scattered component of radiation emanating from sea water:

$$\frac{dB(\alpha)}{dz}=\frac{x(\gamma_1)\sigma T_2F_0}{4\pi n^2S\cos\alpha}Ae^{-\varepsilon(z+l)}, \quad (22)$$

where $\sin \alpha' = n \sin\alpha$; n is the sea water refractive index; T_2 is the coefficient of refraction of radiation at the water-air discontinuity, l is the distance between two elementary volumes of sea water dV at the points A and B.

The illumination of the elementary volume dV_B, situated at the water surface, by singly scattered radiation will be equal to:

$$dE(\alpha')=\frac{x(\gamma_1)\sigma F_0}{4\pi l^2} A e^{-\varepsilon(z+l)}\, dz. \tag{23}$$

The intensity of the radiation scattered by this volume in the direction θ' is written in the following form:

$$d^2 I(\theta')=\frac{x(\gamma_1)\, x(\gamma_2)\sigma^2 F_0}{(4\pi l)^2} A e^{-\varepsilon(z+l)}\, dz dV\ , \tag{24}$$

where $\gamma_2 = \alpha' + \theta'$; $dV = dz\cdot S$.

Transforming equation (24), we obtain an expression for the brightness of laser radiation doubly scattered along the path x', y', A, B after emanating from the water in the direction θ:

$$\frac{d^2B}{d^2z}=\frac{x(\gamma_1)\, x(\gamma_2)\sigma^2 T_2 F_0}{(4\pi nl)^2\cos\theta} A e^{-\varepsilon(z+l)}. \tag{25}$$

Equations for Noncontact Laser Sounding of Sea Water

Now we will examine two generalized sounding schemes (Fig. 2), where 1 is the laser; 2 is the transmitting optical system; 3 is the receiving optical system; 4 is the radiation detector.

In a general case the intensity of the radiation flux incident on the photodetector of the optical-electronic receiver, can be determined from expression [7]:

$$F=\pi\tau BS\sin^2 U, \tag{26}$$

where B is the brightness of the radiation source; S is the area of the radiation source; τ is the coefficient of transmission of the atmosphere and receiving optical system; U is the forward aperture angle of the receiving system; $\pi \sin^2 U = \omega$ is a solid angle within whose limits the radiation is propagated.

By analogy with (26), for the sounding scheme (Fig. 2,a) we have

$$\frac{d^2F}{d^2z}=\tau(\pi\sin W_2\cdot R_{\text{вх.зр}})^2\frac{d^2B(\theta)}{d^2z}, \tag{27}$$

[BX. ЗP = entrance pupil] where $R_{\text{ent pupil}}$ is the radius of the entrance pupil of the receiving optical system; $2W_2$ is the field of the receiving system 3 (Fig. 2).

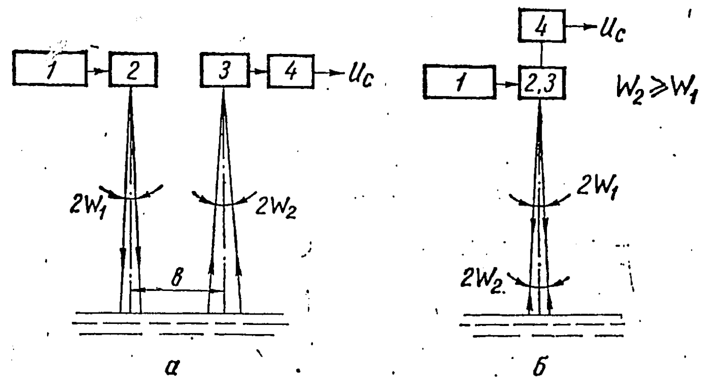

Fig. 2.

We obtain an equation relating the intensity of the radiation flux incident on the photodetector to the radiation brightness of the sea during the sounding of sea waters using the scheme in Fig. 2,b. Figure 3 is a diagram of the path of rays from an elementary volume dV to the entrance pupil of the receiving optical system. In accordance with Fig. 3 we will write the following equation:

$$\omega \cdot \Delta z^2 = \omega' z^2,$$

from which

$$\Delta z = \frac{z}{n}. \qquad (28)$$

Using (28),

$$\omega = \frac{S_{\text{вх.зр}}}{(z^* + z/n)^2}, \qquad (29)$$

where z^* is the distance between the receiving-transmitting system and sea level in the direction of sounding.

Then

$$\frac{dF}{dz} = \tau \frac{S_{\text{вх.зр}} \cdot S}{(z^* + z/n)^2} \cdot \frac{dB(a)}{dz} + \tau m c F_{л} \frac{c(z)}{dz}, \qquad (30)$$

[л = laser; a = atmo(sphere); O. C. = op sys = optical system] where $m = \tau_a \tau_{0.S.1} \rho k$; τ_a is the coefficient of atmospheric transmission; $\tau_{0.S.1}$ is the transmission coefficient for the transmitting optical system; F_{laser} is the intensity of laser radiation; ρ is the coefficient of sea surface reflection;

$$c(z) = \begin{cases} 1, & z = 0 \\ 0, & z \neq 0 \end{cases}; \qquad (31)$$

$$k = \begin{cases} 1, & k^* > 1 \\ 0, & k^* \leqslant 1. \end{cases} \qquad (32)$$

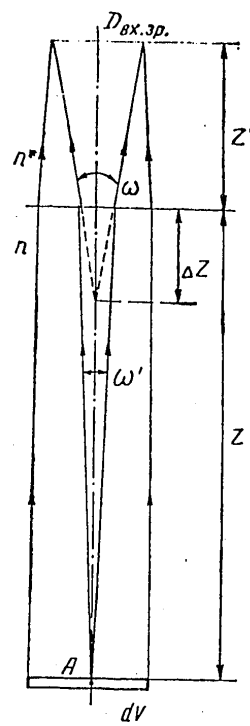

Fig. 3.

The k^* coefficient is determined from the expression

$$k^* = \frac{S_{вх.зр.}}{\left(S^{1/2}_{вых.зр} + 2\pi^{1/2} z^* \sin W_1\right)^2}, \tag{33}$$

[BX. ЗР. = entrance pupil; ВЫХ. ЗР = exit pupil]

where $S_{exit\ pupil}$ is the area of the exit pupil of the transmitting optical system; $2W_1$ is the field of the transmitting system.

In equations (27), (30) we convert from the independent variable z to the independent variable t, where t is the time of radiation of sea waters. For this purpose we write the following system of equations:

$$\begin{cases} z + l = c't + b; \\ z^2 + b^2 = l^2, \end{cases}$$

from which

$$z = \frac{(c't+b)^2 - b^2}{2(c't+b)}; \tag{34}$$

$$l = \frac{(c't+b)^2 + b^2}{2(c't+b)}; \tag{35}$$

where c' is the speed of light in water.

The intensity of the laser radiation entering the water is determined as

$$F_0 = \tau_a \cdot \tau_{o.c.1} T_1 F_Л; \tag{36}$$

[Л = laser; O. C. = O.S.]

$$F_Л = W \cdot f(t), \tag{37}$$

where W is the radiation energy of the laser; f(t) is a function describing the form of the radiation impulse

$$\left(\int_{-\infty}^{\infty} f(t)\, dt = 1 \right).$$

T_1 is the coefficient of refraction of laser radiation at the air-water discontinuity.

The refraction coefficient T_1 is determined using the Fresnel formulas with

$$n^2 = \mu + \nu [1 - a' E_0^N], \tag{38}$$

where μ and ν are parameters dependent on the properties of sea water and the wavelength of the sounding radiation [6].

1. On the basis of expression (27), with (25), (34), (35) taken into account, we obtain an equation for noncontact laser sounding of sea water in accordance with the scheme (Fig. 2,a):

$$\frac{dF(t)}{dz} = \int_{-\infty}^{\infty} \varphi_1\, c' W \cdot f(t-t') \frac{A(t-t') x(\theta', t') e^{-\varepsilon (c't'+b)}}{(c't'+b)^2 + b^2} dt'. \tag{39}$$

In this equation $x(\gamma_1)$ in the range of angles $\gamma_1 = \pi/2 - \pi$ is assumed to be constant, and the terms in it are equal to:

$$\varphi_1 = \frac{x(\gamma_1) a^2}{8 n^2 \cos\theta} \tau_a^2 \tau_{o.c.1} \tau_{o.c.2} T_1 T_2 \sin^2 W_2 R^2_{вх.зр}; \tag{40}$$

$$A(t-t') = \left[1 + \frac{\varkappa}{\varepsilon} a' H_0^N f^N (t-t') \Psi_N (t', \beta) \right]^{-1/N}; \tag{41}$$

$$x(\theta', t') = x\left(\theta' + \arccos \frac{(c't'+b)^2 - b^2}{(c't'+b)^2 + b^2} \right), \tag{42}$$

where H_0 is the quantity of illumination at the distance $z = 0$.

With a duration of the radiation pulse $\tau_{pulse} \to 0$ the equation (39) can be reduced to the form

$$\frac{dF(t)}{dz} = \varphi_1 c' W \frac{A(t) x(\theta', t) e^{-\varepsilon(c't+b)}}{(c't+b)^2+b^2}, \quad (43)$$

where

$$A(t) = \left[1 - \frac{x}{\varepsilon} a' E^N_{0\,(max)} \Psi_N(t, \beta)\right]^{-1/N}.$$

2. Taking (22) and (30) into account, we will write an integral equation for a noncontact laser sounding of sea waters using the scheme (Fig. 2,b):

$$F(t) = \int_{-\infty}^{\infty} \left\{\varphi_2 c' W \frac{A(t-t') e^{-\varepsilon c't'}}{(2nz^* + c't')^2} + \varphi_3 \delta(t') \cdot W\right\} f(t-t')\,dt', \quad (44)$$

in which $\delta(t')$ is the Dirac delta-function;

$$\varphi_2 = \frac{1}{2} x(\gamma_1) \sigma \tau_a^2 \tau_{o.c.1} \tau_{o.c.2} T_1 T_2 R^2_{вх.зр}; \quad (45)$$

$$\varphi_3 = \tau_a^2 \tau_{o.c.1} \tau_{o.c.2} \rho k. \quad (46)$$

With $\tau_{pulse} \to 0$ we have

$$F(t) = \varphi_2 c' W \frac{A(t) e^{-\varepsilon c't}}{(2nz^* + c't)^2} + \varphi_3 c(t) f_{max} W. \quad (47)$$

Taking into account the strong anisotropy of the scattering indicatrix $x(\gamma)$ of sea and ocean waters it is possible to use a transport approximation in the expressions derived above. For this ε is replaced by

$$\left[1 - \frac{\Lambda}{1 + 2/(K-1)}\right] \varepsilon, \text{ a } x(\gamma) = \frac{2}{K+1} [8],$$

where Λ is the probability of survival of a photon; K is the asymmetry coefficient for the scattering indicatrix.

The derived equations (39), (44) of noncontact sounding of sea waters, taking into account elastic scattering and nonlinear phenomena arising during the interaction of powerful laser radiation with sea water, can be used for synthesizing noncontact laser measurement methods and for engineering computations in the development and designing of optical-electronic instruments for investigating sea resources.

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TRANSFORMATION OF SPECTRUM OF SURFACE WAVES UNDER THE INFLUENCE OF AN INTERNAL WAVE

Moscow IZVESTIYA AKADEMII NAUK SSSR, FIZIKA ATMOSFERY I OKEANA in Russian Vol 15, No 6, 1979 pp 655-661

[Article by A. Ya. Basovich, Institute of Applied Geophysics, submitted for publication 22 May 1978, resubmitted 11 July 1978]

Abstract: A study is made of the evolution of surface gravitational waves under the influence of a current created at the surface by an internal wave. The spectrum of surface waves is described by a kinetic equation. By analogy with the problem of the motion of charged particles in a variable electric field it is possible to determine the trajectories of wave packets in coordinate space and wave vectors in the spectrum of waves. It is shown that as a result of the reflection of surface waves in the variable current and their capture by an internal wave considerable changes arise in the wave spectrum. Cases of periodic and solitary internal waves are considered.

[Text] The influence of an internal wave on surface waves has been repeatedly observed under natural conditions [1, 2] and has been investigated theoretically [3-7] and experimentally [8]. It was demonstrated in [3] that as a result of the blocking effect surface waves with a group velocity close to the phase velocity of the internal wave change most strongly. Source [9] has a nonuniform parameter, the wave spectrum, stationary in the reference system, related to the wave. However, in [3, 4] no allowance was made for the reflection of surface waves during blocking, noted in [5] and investigated in detail in [6]. In this study, in a WKB approximation, a study was made of the temporal evolution of the wave spectrum and it is shown that its most significant changes are associated with reflection during blocking and the capture of surface waves by an internal wave. [After sending this study to press the author learned of article [14] devoted to an investigation of similar problems. However, in contrast to [14], here the emphasis is on a detailed analysis of evolution of the spectrum of surface waves with time, carried out differently than in [14]].

1. Equation for the Spectrum of Surface Waves

We will investigate the evolution of the spectrum of surface waves in the field of an internal wave arising at some moment in time and creating a variable current at the surface. Such an idealization is acceptable if the time for setting in of the internal wave is small in comparison with the characteristic time of transformation of the wave spectrum. The internal wave can be considered fixed as a result of the smallness of the influence of surface waves on it [7].

For describing waves in a slowly changing current, created by a long internal wave, we will use the concept of a dynamic amplitude spectrum with the density:

$$W(\mathbf{x},\mathbf{k},t)=\frac{1}{2}\rho g\frac{1}{(2\pi)^2}\int B(\mathbf{r},\mathbf{x},t)\,e^{-i\mathbf{kr}}d\mathbf{r},$$
$$B(\mathbf{r},\mathbf{x},t)=\left\langle \eta\left(\mathbf{x}-\frac{\mathbf{r}}{2},t\right)\eta\left(\mathbf{x}+\frac{\mathbf{r}}{2},t\right)\right\rangle, \qquad (1.1)$$

where B(r, x, t) is the correlation function for displacement of the water surface; ρ is fluid density; g is the acceleration of gravity. The wavelengths of all the spectral components and the correlation radius are assumed to be much less than the characteristic scale of the current nonuniformity (for example, the lengths of the internal wave). In a moving medium the change in spectral density is determined by the kinetic equation for the density of the wave effect (number of quasiparticles) [7, 9].

$$\frac{\partial N}{\partial t}+\dot{\mathbf{x}}\frac{\partial N}{\partial \mathbf{x}}+\dot{\mathbf{k}}\frac{\partial N}{\partial \mathbf{k}}=S, \qquad (1.2)$$

$$N(\mathbf{x},\mathbf{k},t)=\frac{W(\mathbf{x},\mathbf{k},t)}{\omega_c(\mathbf{x},\mathbf{k},t)}, \qquad (1.3)$$

where ω_c(x, k, t) is the frequency corresponding to the particular spectral component k at the point x at the time t in the reference system moving with the velocity of the current U(x,t), created at the surface of an internal wave; S is a term corresponding to the nonlinear interaction of individual spectral components and the effect of wind and viscosity on waves. Henceforth, assuming the effect of the wind and viscosity, and also the intensity of waves and accordingly the nonlinear interaction of the spectral components to be weak, the S value will be neglected. With S = 0, from equation (1.2) it is necessary to retain the values N(x, k, t) along the trajectory of movement of wave groups (wave packets) in space (x, k), corresponding to solutions of the WKB equations [The kinetic equation is also applicable in the presence of caustics for waves (blocking points), despite a deviation of the approximation of geometrical optics in near-caustic regions [10]:

$$\dot{\mathbf{x}}=\frac{\partial\omega(\mathbf{x},\mathbf{k},t)}{\partial\mathbf{k}},\quad \dot{\mathbf{k}}=-\frac{\partial\omega(\mathbf{x},\mathbf{k},t)}{\partial\mathbf{x}}, \qquad (1.4)$$

where $\omega(x, k, t)$ is the frequency in a laboratory reference system. In order to determine N(x, k, t) it is sufficient to integrate equations (1.4) and to determine the initial position (when t = 0) of the wave packet x_0 and k_0 arriving at the point x at the moment in time t with the wave vector k. Assume that the initial spectrum N(x, k, 0) is stipulated, and its small disturbance at the time of appearance of a current can be neglected. Then N(x, k, t) is determined by the expression

$$N(\mathbf{x}, \mathbf{k}, t) = N(\mathbf{x}_0(\mathbf{x}, \mathbf{k}, t), \mathbf{k}_0(\mathbf{x}, \mathbf{k}, t), 0), \tag{1.5}$$

and the amplitude spectrum of the waves, with (1.3) and the dispersion expression $\omega_c^2 = g\,|k|$ taken into account, has the form

$$W(\mathbf{x}, \mathbf{k}, t) = N\omega_c = \left(\frac{|\mathbf{k}|}{|\mathbf{k}_0(\mathbf{x}, \mathbf{k}, t)|}\right)^{1/2} W(\mathbf{x}_0(\mathbf{x}, \mathbf{k}, t), \mathbf{k}_0(\mathbf{x}, \mathbf{k}, t), 0). \tag{1.6}$$

Thus, the problem of finding the spectrum of waves at the surface is reduced to an investigation of the equations of motion of the wave packets.

2. Equations of Motion of a Wave Packet

Now we will examine a wave packet moving in the field of an internal wave, which at the fluid surface creates a variable current U = U(x - Ct); C is the phase of the wave. With transformation to a reference system moving with the velocity C the form of the equations (1.4) does not change, and in place of ω we substitute Ω -- frequency of the wave in the system C, determined from the dispersion equation [6]:

$$\Omega = -kC(1-\beta(\tilde{x})) + (gk)^{1/2}, \tag{2.1}$$

where $\beta = U(\tilde{x})/C$; $\tilde{x} = x - Ct$.

As a simplification we will assume that $k \parallel C \parallel U$. In (2.1) we selected a branch corresponding to surface waves which can be captured as an internal wave ($k > 0$).

Equations (1.4) are similar to the Hamiltonian equations for a particle with the Hamiltonian ω, determined by (2.1), where x is the coordinate of the particle and k is its momentum. The integration of these equations is an unwieldy problem which can be simplified by taking advantage of the smallness of β.

An internal wave exerts a substantial influence only on surface waves having group velocities close to C and accordingly wave numbers close to $k_* = g/4C^2$. The corresponding frequency in the C system is equal to $\Omega^*(k_*, \beta = 0) = Ck_*$. In the case of small β the width of the interval of wave numbers in which the spectrum is highly transformed is small in comparison with k_*. Representing k in the form $k = k_* + \tilde{k}$ ($|\tilde{k}| \ll k_*$), we expand expression (2.1) into a series of powers of $\tilde{k}$:

$$\Omega = \Omega^{\cdot} + \Omega^{\cdot}\beta(\tilde{x}) - \frac{1}{8}\frac{\sqrt{g}}{k_{\cdot}^{3/2}}\tilde{k}^2 + O(\tilde{k}^3). \quad (2.2)$$

In (2.2) we have omitted the term $\beta C\tilde{k}$. As follows from the text, the width of the interval of significant influence of the internal wave is $\Delta\tilde{k} \sim \beta_m^{1/2}$ ($\beta_m = U_m/C$, where U_m is the maximum current velocity at the surface, so that $\beta k \sim \beta^{3/2} \sim \tilde{k}^3$. Limiting ourselves in (2.2) to terms of the order of $\tilde{k}^2$ and substituting (2.2) into (1.4), we obtain the following equations:

$$\dot{\tilde{x}} = -\frac{1}{4}\frac{\sqrt{g}}{k_{\cdot}^{3/2}}\tilde{k}, \quad \dot{\tilde{k}} = -k_{\cdot}C\beta', \quad (2.3)$$

where β' is the derivative β of $\tilde{x}$. The equations (2.3) are completely similar to the equations of motion of an electron in the field of a plasma wave [11]:

$$\dot{x} = p/m, \quad \dot{p} = e\varphi', \quad (2.4)$$

where p is particle momentum; m is mass; e is charge (e > 0); φ is electric field potential. The motion of an electron is described by the Hamiltonian

$$H = -e\varphi + p^2/2m = -e\varphi + {}^1/_2 m\dot{x}^2. \quad (2.5)$$

Expressions (2.4), (2.5) and (2.3), (2.2) are equivalent under the condition:

$$H = \Omega^{\cdot} - \Omega, \quad p = -\tilde{k}, \quad m = 4k_{\cdot}^{3/2}/\sqrt{g}, \quad e\varphi = \Omega^{\cdot}\beta. \quad (2.6)$$

The problem of the motion of a particle in the electric field of a wave is well studied and equations (2.4) are integrated in the most intensive cases of periodic and solitary waves [11, 12]. In particular, in the wave field $\varphi = \varphi_0 \cos(\omega_1 t - qx)$ the law of conservation of energy gives the first integral of the system (2.4)

$$^1/_2 m\dot{x}^2 = \mathscr{E} + e\varphi_0 \cos qx, \quad (2.7)$$

where $\mathscr{E}$ is the total mechanical energy of the particle. Depending on the value of the parameter $\varkappa^2 = 2e\varphi_0/(\mathscr{E} + e\varphi_0)$ the particles will be captured ($\varkappa^2 > 1$) or fly by ($\varkappa^2 < 1$). Using [11] it is easy to obtain the following expressions for particle momentum at the initial moment in time (t = 0) in dependence on its momentum and position at the moment in time t:

$$\varkappa^2 > 1: \; p_0(x, p, t) = \pm\frac{2m}{q\tau\varkappa}\,\mathrm{cn}\left\{F\left[\frac{1}{\varkappa}, \arcsin\left(\varkappa \sin\frac{qx}{2}\right)\right] - \frac{t}{\tau}, \frac{1}{\varkappa}\right\}, \quad (2.8)$$

$$\varkappa^2 < 1: \; p_0(x, p, t) = \pm\frac{2m}{q\tau\varkappa}\,\mathrm{dn}\left\{F\left[\varkappa, \frac{qx}{2}\right] - \frac{t}{\varkappa\tau}, \varkappa\right\}, \quad (2.9)$$

$$\varkappa^2 = \left[\left(\frac{qp}{2m}\right)^2 \tau^2 + \sin^2\left(\frac{qx}{2}\right)\right]^{-1/2}, \quad (2.10)$$

where cn and dn are elliptical functions; F is an elliptical integral of the first kind; $\tau = q^{-1}(m/e\varphi_0)^{1/2}$ is the characteristic time of particle motion.

By total analogy with what has been said above, surface waves may be untrapped or trapped as a result of blocking. The capture of surface waves occurs between the minima of the current velocity created at the surface by an internal wave $U = U_0 \cos (\omega_1 t - qx)$.

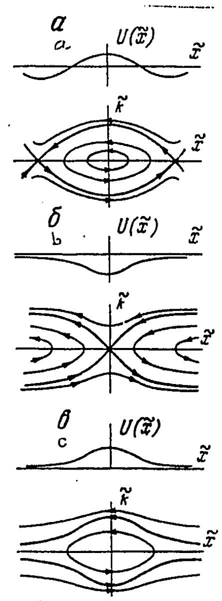

Fig. 1. Trajectories of packets of surface waves on plane $(\tilde{x}, \tilde{k})$ in the field of periodic (a) and solitary (b -- $U_0 < 0$ and c -- $U_0 > 0$) internal waves.

In the case $\varkappa^2 = 2\Omega^*\beta_0/[\Omega^*(1+\beta_0) - \Omega] > 1$ the waves are trapped, whereas when $\varkappa^2 < 1$ they are not trapped. Figure 1,a shows the form of the trajectories of wave packets on the plane $(\tilde{x}, \tilde{k})$. The trapped waves are indicated by closed trajectories and the untrapped waves are indicated by unclosed trajectories.

With (2.6) taken into account, from (2.8), (2.9) and (2.10) it is easy to obtain expressions determining the initial wave number of the packet at the time t having the wave number $\tilde{k}$ and situated at the point $\tilde{x}$

$$\varkappa^2 > 1: \ \tilde{k}_0(\tilde{x}, \tilde{k}, t) = \pm \frac{2k.\sqrt{2\beta_0}}{\varkappa} \times \times \operatorname{cn}\left\{F\left[\frac{1}{\varkappa}, \arcsin\left(\varkappa \sin\frac{q\tilde{x}}{2}\right)\right] - \frac{t}{\tau}, \frac{1}{\varkappa}\right\}, \qquad (2.11)$$

$$\varkappa^2 < 1: \ \tilde{k}_0(\tilde{x}, \tilde{k}, t) =$$

$$=\pm\frac{2k_*\sqrt{2\beta_0}}{\varkappa}\,\mathrm{dn}\left\{F\left(\varkappa,\frac{q\tilde{x}}{2}\right)-\frac{t}{\varkappa\tau},\varkappa\right\}, \tag{2.12}$$

$$\varkappa^2=\left[\frac{1}{8\beta_0}\left(\frac{\tilde{k}}{k_*}\right)^2+\sin^2\left(\frac{q\tilde{x}}{2}\right)\right]^{-1/2}, \tag{2.13}$$

where $\tau = T/(\pi\sqrt{2\beta_0})$; T is the period of the internal wave.

We will also cite the result of integration of the equations (2.3) in a case when the current at the surface is created by the wave

$$U=U_0\,\mathrm{ch}^{-2}\left[\frac{1}{\Delta}(x-Ct)\right].$$

Depending on the stratification of fluid density with depth a solitary wave can create at the surface a current directed both against ($U_0 < 0$) and along ($U_0 > 0$) the direction of its propagation. When $U_0 < 0$ the problem of the movement of the wave packet in the field of the internal wave is similar to the problem of the motion of an electron in the field of a Langmuir solitone [12]. The trajectories of the wave packets on the plane ($\tilde{x}$, $\tilde{k}$) in dependence on the parameter $\delta^2 = \Omega^* \beta_0/(\Omega - \Omega^*)$ are shown in Fig. 1,b. The trajectories within the separatrix correspond to waves reflected on the front and rear slopes of the internal wave ($\delta^2 < 1$), outside the separatrix -- untrapped waves ($\delta^2 < 1$). The expressions for $\tilde{k}_0(\tilde{x}, \tilde{k}, t)$ have the form:

$$\delta^2>1:\ \tilde{k}_0(\tilde{x},\tilde{k},t)=\pm\frac{2k_*\sqrt{\delta^2-1}\,|\beta_0|^{1/2}}{\delta}\times$$

$$\times\frac{\mathrm{sh}\left[\mathrm{Arch}\left(\frac{\mathrm{sh}\,\xi}{\sqrt{\delta^2-1}}\right)-\frac{t}{\tau_0\delta}\right]}{\left\{1+(\delta^2-1)\,\mathrm{ch}^2\left[\mathrm{Arch}\frac{\mathrm{sh}\,\xi}{\sqrt{\delta^2-1}}-\frac{t}{\tau_0\delta}\right]\right\}^{1/2}}, \tag{2.14}$$

$$\delta^2<1:\ \tilde{k}_0(\tilde{x},\tilde{k},t)=\pm\frac{2k_*\sqrt{1-\delta^2}\,|\beta_0|^{1/2}}{\delta}\times$$

$$\times\frac{\mathrm{ch}\left[\mathrm{Arsh}\left(\frac{\mathrm{sh}\,\xi}{\sqrt{1-\delta^2}}\right)-\frac{t}{\tau_0\delta}\right]}{\left\{1+(1-\delta^2)\,\mathrm{sh}^2\left[\mathrm{Arsh}\left(\frac{\mathrm{sh}\,\xi}{\sqrt{1-\delta^2}}\right)-\frac{t}{\tau_0\delta}\right]\right\}^{1/2}}, \tag{2.15}$$

$$\delta^2=\mathrm{ch}^2(\xi)\left[1-\frac{1}{4\beta_0}\left(\frac{\tilde{k}}{k_*}\right)^2\mathrm{ch}^2(\xi)\right]^{-1}, \tag{2.16}$$

where $\tau_0^2 = \Delta^2/(2|\beta_0|c^2)$, $\xi = \tilde{x}/\Delta$.

When $U_0 > 0$, as in the case of a periodic internal wave, there are trapped ($\delta^2 > 1$) and untrapped ($\delta^2 < 0$) surface waves (Fig. 1,c). The $\tilde{k}_0(\tilde{x}, \tilde{k}, t)$ value is determined by the following expressions:

$$\delta^2>1:\ \tilde{k}_0(\tilde{x},\tilde{k},t)=\pm\frac{2k_*\sqrt{\delta^2-1}\beta_0^{1/2}}{\delta}\times$$

$$\times\frac{\cos\left[\arcsin\left(\frac{\mathrm{sh}(\xi)}{\sqrt{\delta^2-1}}\right)-\frac{t}{\tau_0\delta}\right]}{\left\{1+(\delta^2-1)\sin^2\left[\arcsin\left(\frac{\mathrm{sh}(\xi)}{\sqrt{\delta^2-1}}\right)-\frac{t}{\tau_0\delta}\right]\right\}^{1/2}},\qquad(2.17)$$

$$\delta^2<0:\ \tilde{k}_0(\tilde{x},\tilde{k},t)=\pm\frac{2k_*\sqrt{1-\delta^2}\beta_0^{1/2}}{|\delta|}\times$$

$$\times\frac{\mathrm{ch}\left[\mathrm{Arsh}\left(\frac{\mathrm{sh}(\xi)}{\sqrt{1-\delta^2}}\right)-\frac{t}{\tau_0|\delta|}\right]}{\left\{1+(1-\delta^2)\mathrm{sh}^2\left[\mathrm{Arsh}\left(\frac{\mathrm{sh}(\xi)}{\sqrt{1-\delta^2}}\right)-\frac{t}{\tau_0|\delta|}\right]\right\}^{1/2}},\qquad(2.18)$$

where δ^2 is determined by expression (2.16), with the difference in the sign on β_0 taken into account.

3. Change in Spectrum of Surface Waves

Assume that in the absence of an internal wave there is stipulation of a uniform stationary wave spectrum W(k). Evolution of the spectrum after the appearance of a wave at the time t = 0 is described by expression (1.6) with the substitution of the expressions for $\tilde{k}_0(\tilde{x}, \tilde{k}, \tilde{t})$ found above. In order to obtain some idea concerning the magnitude of the transformation arising in the spectrum under the influence of an internal wave, as the unperturbed spectrum of weak turbulence we select $W(k) = Ak^{-9/2}$ [13]. Figures 2 and 3 show a series of curves showing the change in this spectrum with time for solitary waves when $U_0 > 0$ and when $U_0 < 0$. (The cited curves have an illustrative character since the spectrum of weak turbulence is evidently not observed in nature. Similar curves are obtained for other initial spectra decreasing relative to k.) The case of solitary internal waves reflects the principal processes characteristic for a periodic wave (capture of surface waves when $U_0 > 0$), but also has peculiarities (reflection of surface waves when $U_0 < 0$). The curves were constructed for the value $\beta_0 = U_0/C = 0.0083$, and the spectral density and the wave number are normalized: $W = W/W_0(k_*)$, $\alpha = \tilde{k}/k_*$. In particular, the selected β_0 value with C = 60 cm/sec corresponds to a velocity at the surface $U_0 = 0.5$ cm. Assuming that for such a wave $\Delta = 100$ m, we have $\tau_0 \approx 20$ min. (Estimates using the formulas in [7], where a study was made of the interaction of waves without allowance for the capture of surface waves by an internal wave, give a characteristic time for change in the parameters of the internal wave τ_i of the order of several hours in the case of strong surface

With the considered amplitudes of the internal wave $\tau_0 \ll \tau_i$; therefore, for solution of the self-consistent problem there must be an approach different from [7], taking the blocking effect into account. In the first approximation it is possible to consider the evolution of the wave spectrum in a stipulated field to be a "strong" internal wave.) In this case there is capture of surface waves with a length of about a meter ($k_* = 6.82\ m^{-1}$). For such waves it is possible to neglect the effect of viscosity ($\tau_v \sim 2 \cdot 10^2$ minutes) and a weak wind during the time of change in the wave spectrum under the influence of the internal wave.

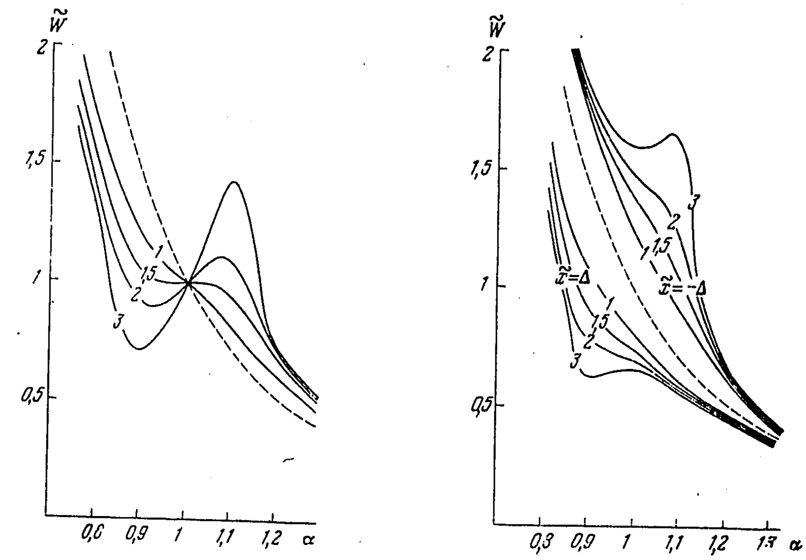

Fig. 2. (left) Form of wave spectra at times $t/\tau_0 = 1, 1.5, 2, 3$ in field of solitary internal wave ($U_0 > 0$) when $x = 0$. Fig. 3 (right). Form of wave spectra at times $t/\tau_0 = 1, 1.5, 2, 3$ in field of solitary internal wave ($U_0 < 0$) when $\tilde{x} = -\Delta$ and when $\tilde{x} = \Delta$.

Figure 2 shows curves of the wave spectrum at the times $t/\tau_0 = 1, 1.5, 2, 3$ at the peak ($\tilde{x} = 0$) of a solitary wave when $U_0 > 0$. The dashed curve corresponds to an undisturbed spectrum. From the moment of appearance of the internal wave the spectral density begins to increase in the region $k > k_*$ ($\alpha > 1$) in connection with the arrival of wave packets from the long-wave region. The spectral density with $k < k_*$ ($\alpha < 1$) accordingly decreases. With $t > \tau_0$ the change in the spectrum is already significant. The relative change in the spectral density by this time in the short-wave region is about 15-20%. As time passes the spectral changes increase and when $t > \tau_0$ the greatest increase is associated with trapped waves and occurs in the interval $|\alpha - 1| \lesssim 0.15$. The greatest changes are attained when $t = \pi\tau_0$ when the wave packets have made half the "revolutions" in trajectories close to the center. Thereafter with an increase in t a complex dissected spectrum arises; this is associated with the difference in the periods of movement of the wave packets in closed trajectories corresponding to different δ^2 values.

Figure 3 shows the form of the spectra in the field of a solitary wave when $U_0 < 0$ at the same moments in time as in the preceding case. The curves below the dashed line correspond to the point $\tilde{x} = \Delta$ on the front slope, and above -- to the point $\tilde{x} = -\Delta$ on the rear slope of the wave. The suppression of the spectrum on the front slope of the wave is attributable to the fact that the internal wave "overtakes" the short surface waves and transforms them to longer waves. The reverse process transpires on the rear slope and the spectral density increases. The changes in the spectrum with identical β_0 when $U_0 < 0$ are more significant than when $U_0 > 0$. For example, when $t > \tau_0$ the relative change in the spectral density in the short-wave part is about 15-25%. With times $t > \tau_0$ the main change in the spectrum occurs in the range of wave numbers corresponding to the reflected waves. With an increase in t the spectrum asymptotically approaches stationary and the spectral density at all points in space is determined by wave packets arriving from the region $U = 0$.

The influence of the wind, viscosity and nonlinearity can lead to a significant change in the determined wave spectra. In particular, nonlinear effects can lead to the collapsing of the surface waves as a result of a considerable increase in spectral density in the short-wave region. The turbulent viscosity arising in this case can cause an attenuation of waves and the formation of slicks [1]. A satisfactory description of this phenomenon is evidently possible only by taking into account all the factors enumerated above.

The author expresses appreciation to V. I. Talanov for constant interest in the work and Ye. N. Pelinovskiy for discussion of the results.

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[481-5303]

II. UPPER ATMOSPHERE AND SPACE RESEARCH

Translations

MONOGRAPH ON THE COMMAND-MEASUREMENT COMPLEX

Moscow KOMANDNO-IZMERITEL'NYY KOMPLEKS (The Command and Measurement Complex) in Russian 1979 p 2

[Annotation and table of contents from book by P. A. Agadzhanov, Professor, Doctor of Technical Sciences and Winner of the Lenin Prize, Izdatel'stvo "Znaniye," 32,250 copies, 64 pages]

[Text] Annotation. This brochure describes the command and measurement complex, the group of technical means and ground services that aid in the control of space flights. Information on the basic components of this complex is given and the operation of its ground and on-board equipment is described.

The brochure is written for engineers, teachers and students of the higher schools as well as for a wider circle of readers interested in problems of modern cosmonautics.

Contents

[475-5303]

UDC 629.783/.785

SELECTING A FAMILY OF SATELLITE-LAUNCHING ROCKETS FOR A SPACE RESEARCH PROGRAM INVOLVING REPEATED LAUNCHINGS

Moscow KOSMICHESKIYE ISSLEDOVANIYA in Russian Vol 16, No 4, 1978 pp 514-521

[Article by A. V. Sollugub and V. N. Ofitserov]

Abstract: The article examines methods for solving the problems arising in validating the choice of a family of carriers for implementing a space research program and a program for the construction of spacecraft taking into account the possibility of repeated use of the carriers and vehicles. The authors give formulations of these problems in terms of whole-number programming and give algorithms for their solution by the dynamic programming method. An approach described in [1] is used. The algorithms can be used in automated planning systems.

[Text] We will assume that the space research program is determined by the three-element cortege $\langle G, m, k \rangle$, where $G = (G_1, G_2, \ldots, G_k)$ is the vector of weights of the spacecraft which must be put (at different times) into computed orbits, $m = (m_1, m_2, \ldots, m_k)$ is the vector of the planned number of launchings of spacecraft, k is the nomenclature (number of types) of spacecraft. The elements of the G vector are arranged in increasing order. The problem involves determination of the number of times of use of each spacecraft and the programs for their construction, that is, the vectors: $p = (p_1, p_2, \ldots, p_k)$ is the vector of "number of times of use of a spacecraft" and $q = (q_1, q_2, \ldots, q_k)$ is the vector of the "program for construction of a spacecraft." Here p_i is the maximum number of launchings for which one specific spacecraft of the i-th type is rated, q_i is the program for the construction of a spacecraft of the i-th type.

In determining the program for a spacecraft of the i-th type we will use as a point of departure the condition of a minimum of expenditures on its realization.

We will assume that the following representations are known:

$$\varphi_1\colon (G_a, p) \to \Lambda', \quad \varphi_2\colon (G_a, p, q) \to \Lambda'', \quad \varphi_3\colon (G_a, p) \to \Lambda''',$$

where Λ' is the cost of a unit weight of the spacecraft, obtained taking into account the expenditures on scientific research, experimental-design developments and the mastery of production of a particular type of spacecraft; Λ'' is the cost of a unit weight of the spacecraft, determined by the expenditures on construction of a specific spacecraft; Λ''' is the cost of a unit weight of a spacecraft, determined by expenditures in carrying out a specific spacecraft launching.

All three types of expenditures are determined by the weight G_a and the number p of spacecraft launchings.

Two extreme cases can be discriminated:
a) $p_i = 1$, $q_i = m_i$ $(i = 1,\ldots,k)$,
b) $p_i = m_i$, $q_i = 1$ $(i = 1,\ldots,k)$.

The first case corresponds to the single use of a spacecraft, the second -- to repeated use. It is entirely obvious that these solutions in a general case are not optimum.

We note that the possible solutions for the i-th type of spacecraft are described by the following pairs of numbers:

$$p_i \in P_i, \quad q_i = \overline{E}(m_i/p_i) \tag{1}$$

where p_i runs through all the values from the set P_i;

$$P_i;\ P_i = \bigcup_{j=1}^{m_i} \{\overline{E}(m_i/j)\};\ \overline{E}(\alpha) = E(\alpha)+\beta,\ E(\alpha)$$

is the whole part of the number α,

$$\beta = \begin{cases} 0, & E(\alpha) = \sigma, \\ 1, & E(\alpha) \neq \alpha. \end{cases}$$

The set P_i is completely determined by the number m_i; therefore, henceforth, we will designate it $P(m_i)$.

In expression (1) the q_i program is determined by the multiplicity p_i. If we vary the program q_i, then we obtain

$$q_i \in Q_i \quad p_i = \overline{E}(m_i/q_i), \quad Q_i = \bigcup_{j=1}^{m_i} \{\overline{E}(m_i/j)\} \tag{2}$$

It follows from a determination of P_i and Q_i that $P(m_i) = Q(m_i)$. Such a selection of p_i and q_i values excludes what is known to be nonoptimum solutions. In actuality, for all $\overline{E}(m_i/2) \leqslant q_i \leqslant m_i$ with the multiplicity $p_i = 2$ the program of launching of spacecraft of the i-th type can be carried out with a minimum number of vehicles $q_i = \overline{E}(m_i/2)$. Accordingly, it is infeasible to examine programs for the construction of a great number of spacecraft $\overline{E}(m_i/2) \leqslant q_i < m_i$ when $p_i = 2$.

Solutions with $\overline{E}(m_i/3) < q_i < E(m_i/2)$ and $p_i = 3$, etc. are considered nonoptimum.

Example. Assume that $m_i = 25$. We will confirm that expression (2) excludes knowingly nonoptimum solutions.

For possible q_i values and the p_i values corresponding to them see below:

q_i	1	2	3	4	5	6	7	8	9...12	13...24	25
p_i	25	13	9	7	5	5	4	4	3...3	2...2	1

By virtue of the requirement of a minimum of expenditures on the research program

$$Q_i=\{1,2,3,4,5,7,9,13,25\}=\bigcup_{j=1}^{25}\{\overline{E}(25/j)\} \quad \text{and} \quad \forall q_i \in Q_i \; p_i=\overline{E}(25/q_i).$$

The optimum solution $\langle p^{opt}, q^{opt} \rangle$ can be found from the condition

$$Z(p_i^{opt}, q_i^{opt}) = \min_{q_i \in Q_i} \{[\Lambda'(G_i,p_i)+\Lambda''(G_i,p_i,q_i)q_i+ +\Lambda'''(G_i,p_i)m_i]G_i\}, \quad (i=1,\ldots,k)$$

where $p_i = \overline{E}(m_i/q_i)$.

For any research programs which are practical to implement $\langle G, m, k \rangle$ the minimum can be found by the method of analysis of all $q_i \in Q_i$ $(i = 1,\ldots,k)$. In this case the number of examined variants does not exceed

$$\sum_{i=1}^{k} |Q_i|.$$

2. The space research program is stipulated here as in the first section. The problem involves finding the family of carriers $G_{car} = (G_{car1}, G_{car2},\ldots, G_{car\,\ell})$, the construction program $r = (r_1, r_2,\ldots,r_\ell)$ and the multiplicity of use of the carriers $n = (n_1, n_2,\ldots,n_\ell)$, in their totality ensuring satisfaction of the entire research program with minimum expenditures. Here G_{car_i}, r_i, n_i are the maximum payload of a carrier of the i-th type, the number of carriers and the number of times that a carrier of the i-th type is used, whereas ℓ is the nomenclature of the carriers.

With allowance for the expenditures we will assume the following representations to be known:

$$F_1\colon (G_\pi, n) \to \lambda', \quad F_2\colon (G_\pi, n, r) \to \lambda'', \quad F_3\colon (G_\pi, n) \to \lambda''',$$

[π = car] where λ' is the cost of a unit weight of the carrier payload, obtained taking into account the expenditures on scientific research, experimental design work and the mastery of production of a particular type of carrier; λ'' is the cost of a unit weight of the carrier payload, determined by the expenditures on the construction of one carrier of a particular type; λ''' is the cost of a unit weight of carrier payload, obtained taking into account the expenditures on launching (also taken into account here are the expenditures on restoring the carrier for carrying out a repeated launching and also the expenditures in seeking and transporting the salvaged parts of the carriers).

In the cited expressions we have not taken into account the expenditures going directly for the development and production of the payload. It is also assumed that each carrier can in one launching put only one spacecraft into orbit.

Now we will examine the problem of seeking the optimum solution

$$\langle G_{\text{п}}^{opt}, r^{opt}, n^{opt}, l^{opt}\rangle \qquad (3)$$

[П = car; opt = optimum] from the condition of ensuring the minimum expenditures on the program

$$Z(G_{\text{п}}, r, n, l) \to \min.$$

We will assume arbitrarily that for each type of spacecraft provision is made for developing its carrier. However, each of the carriers can be developed in different variants. For example, for a carrier of the first type, which can launch a spacecraft only with the weight $G_1 (G_{car1} = G_1)$, it is possible to represent

$$|N_1| = |\bigcup_{j=1}^{m_1} \{\bar{E}(m_1/j)\}|$$

variants of the development.

In actuality, this carrier is intended for the launching of m_1 spacecraft of the first type. Therefore, passing over the reasonings similar to those in section 1, we obtain

$$\forall n_1 \in N_1, \quad r_1 = \bar{E}(m_1/n_1), \quad N_1 = \bigcup_{j=1}^{m_1} \{\bar{E}(m_1/j)\} = N(m_1).$$

We will designate by $C_{\nu, \alpha, s}$ the expenditures on the development, construction and launching of carriers of the ν-th type putting α types of spacecraft into orbit and having the multiplicity s. Then for carriers of the first type

$$C_{1,1,s} = [\lambda'(G_{\text{п}1}, s) + \lambda''(G_{\text{п}1}, s, r_{1,1}) r_{1,1} + \lambda'''(G_{\text{п}1}, s) m_1] G_{\text{п}1}, \ \forall s \in N(m_1),$$

where $r_{1,1} = \bar{E}(m_1/s)$.

A carrier of the second type $(G_{car2} = G_2)$ can be used for the launchings of spacecraft of the second type and also spacecraft of the second and first types.

Accordingly, there can be $|N(m_2)| + N(m_1+m_2)$ variants of development of this carrier.

In the case of launchings of spacecraft only of the second type

$$C_{2,1,s}=[\lambda'(G_{n_2},s)+\lambda''(G_{n_2},s,r_{2,1})r_{2,1}+\lambda'''(G_{n_2},s)m_2]G_{n_2},$$
$$\forall s\in N(m_2),\quad r_{2,1}=\bar{E}(m_2/s).$$

[π= car]

In the launching of spacecraft of the second and first types

$$C_{2,2,s}=[\lambda'(G_{n_2},s)+\lambda''(G_{n_2},s,r_{2,2})r_{2,2}+\lambda'''(G_{n_2},s)(m_1+m_2)]G_{n_2},$$
$$\forall s\in N(m_1+m_2),\quad r_{2,2}=\bar{E}((m_1+m_2)/s).$$

In a general case the expenditures on the development, construction and launching of carriers of the ν-th type, on the assumption of their use for the launching of α types of vehicles (and specifically vehicles with the weights $G_\nu, G_{\nu-1},\ldots,G_{\nu-\alpha+1}$; α for the ν-th type of carrier assumes values from 1 to ν), and rated for s multiple uses (s for the ν-th type of carriers) can assume all values from the set

$$\left(\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega\right)),\quad \text{and}$$

can be written in the form

$$C_{\nu,\alpha,s}=[\lambda'(G_{n_\nu},s)+\lambda''(G_{n_\nu},s,r_{\nu,\alpha})r_{\nu,\alpha}+\lambda'''(C_{n_\nu},s)\times$$
$$\times\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega]G_{n_\nu},\ \forall s\in N\left(\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega\right)\ r_{\nu,\alpha}=\bar{E}\left(\frac{1}{s}\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega\right),\ G_{n_\nu}=G_\nu.$$

For the ν-th type of carrier we have

$$B_\nu=\sum_{\alpha=1}^{\nu}\left|N\left(\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega\right)\right|$$

variants for development and construction. The total number of variants for the considered program is equal to

$$\sum_{\nu=1}^{k}B_\nu=\sum_{\nu=1}^{k}\sum_{\alpha=1}^{\nu}\left|N\left(\sum_{\omega=\nu-\alpha+1}^{\nu} m_\omega\right)\right|.$$

For example, with k = 5. m_i = 10 (i = 1,...,k), $\sum_{\nu=1}^{5} b_\nu=130$,

with k = 10 $\sum_{\nu=1}^{10} B_\nu=500$,

with k = 5, $m_i=20$ $\sum_{\nu=1}^{5} B_\nu=200$.

For each possible variant of construction of carriers we accordingly have a variable value of the Boolean type:

$$y_j=\begin{cases}1 \text{ -- the variant } j \text{ enters into an optimum combination of variants}\\ 0 \text{ -- the variant } j \text{ does not enter into the optimum combination of variants}\end{cases}$$

$$\left(j = 1, 2, \ldots, \sum_{\nu=1}^{k} B_\nu\right)$$

We introduce into consideration the "correspondence matrix" of variants of types of spacecraft:

$$a_{ij} = \begin{cases} 1 \text{ -- a spacecraft of the i-th type is included in the j-th variant,} \\ 0 \text{ -- a spacecraft of the i-th type is included in the j-th variant,} \end{cases}$$

$$\left(i = 1, 2, \ldots, k; \quad j = 1, 2, \ldots, \sum_{\nu=1}^{k} B_\nu\right)$$

For k = 2 the correspondence matrix assumes the form:

	j		
i	$y_{1,1,s}$ $\forall s \in N(m_1)$	$y_{2,1,s}$ $\forall s \in N(m_2)$	$y_{2,2,s}$ $\forall s \in N(m_1 + m_2)$
	$y_1 \ldots y_{\|N(m_1)\|}$	$y_{\|N(m_1)\|+1} \ldots y_{\|N(m_1)\|+N(m_2)\|}$	$y_{\|N(m_1)\|+\|N(m_2)\|+1} \ldots y_{\sum_{\nu=1}^{2} B_\nu}$
m_1	1...1	0 ... 0	1 ... 1
m_2	0...0	1 ... 1	1 ... 1

Here we have given two systems of notations for the Boolean variables, y is the "through" numeration and notations similar to those introduced earlier for $C_{\gamma,\alpha,s}$. Depending on circumstances, it is convenient to use either one or the other of them.

If for each variant j the corresponding expenditures are denoted by C_j, the problem of seeking the optimum solution is reduced to the problem of whole-number linear programming:

$$\min f = \sum_j C_j y_j \qquad (4)$$

with

$$\sum_j a_{ij} y_j \geq 1, \quad i = 1, \ldots, k, \quad y_j = \begin{cases} 1 \\ 0 \end{cases}, \left(j = 1, 2, \ldots, \sum_{\nu=1}^{k} B_\nu\right).$$

For solving the problem it is possible to use the dynamic programming method. We will examine hypothetical programs for launching the first, first two, ..., and finally, k types of spacecraft.

The hypothetical program for launching a spacecraft of the first type has $|N(m_1)|$ possible realizations. The optimum variant is determined from the expression

$$f_1^{opt}(y_{1,1,s_{opt}^{(1)}}) = \min_{s \in N(m_1)} C_{1,1,s}. \qquad (5)$$

For a hypothetical program for launching the first two types of spacecraft the optimum solution is determined as follows:

1) $\alpha = 1$ (case of launching of one type of spacecraft, specifically G_2)

$$f_{2,1}^{opt}(y_{2,1,s_1}) = \min_{s \in N(m_2)} C_{2,1,s} + f_1^{opt}(y_{1,1,s_{opt}^{(1)}}),$$

2) $\alpha = 2$ (case of launching of two types of spacecraft -- G_2 and G_1)

$$f_{2,2}^{opt}(y_{2,2,s_2}) = \min_{s \in N(m_1+m_2)} C_{2,2,s}.$$

Hence

$$f_2^{opt}(y_{2,\alpha_{opt}^{(2)},s_{opt}^{(2)}}) = \min_{\alpha=1,2} f_{2,\alpha}^{opt}(y_{2,\alpha,s_\alpha}). \tag{6}$$

For a hypothetical program for the launching of the first ν types of spacecraft we have:

1) $\alpha = 1$ (launching of spacecraft G_ν)

$$f_{\nu,1}^{opt}(y_{\nu,1,s_1}) = \min_{s \in N(m_\nu)} C_{\nu,1,s} + f_{\nu-1}^{opt}(y_{\nu-1,\alpha_{opt}^{(\nu-1)},s_{opt}^{(\nu-1)}})$$

2) $\alpha = 2$ (launching of the spacecraft G_ν and $G_{\nu-1}$)

$$f_{\nu,2}^{opt}(y_{\nu,2,s_2}) = \min_{s \in N(m_\nu+m_{\nu-1})} C_{\nu,2,s} + f_{\nu-2}^{opt}(y_{\nu-2,\alpha_{opt}^{(\nu-2)},s_{opt}^{(\nu-2)}})$$

....

v) $\alpha = v$ (launching of the spacecraft $G_\nu, G_{\nu-1}, \ldots, G_{\nu-v+1}$)

$$f_{\nu,v}^{opt}(y_{\nu,v,s_v}) = \min_{s \in N\left(\sum_{\omega=\nu-v+1}^{\nu} m_\omega\right)} C_{\nu,v,s} + f_{\nu-v}^{opt}(y_{\nu-v,\alpha_{opt}^{(\nu-v)},s_{opt}^{(\nu-v)}})$$

....

ν) $\alpha = \nu$ (launching of the spacecraft $G_\nu, G_{\nu-1}, \ldots, G_1$)

$$f_{\nu,\nu}^{opt}(y_{\nu,\nu,s_\nu}) = \min_{s \in N\left(\sum_{\omega=1}^{\nu} m_\omega\right)} C_{\nu,\nu,s}.$$

As a result

$$f_\nu^{opt}(y_{\nu,\alpha_{opt}^{(\nu)},s_{opt}^{(\nu)}}) = \min_{\alpha=1,\ldots,\nu} f_{\nu,\alpha}^{opt}(y_{\nu,\alpha,s_\alpha}). \tag{7}$$

Similarly we find an optimum solution for a hypothetical program for the launching of all k spacecraft included in the initial program:

$$f_k^{opt}(y_{k,\alpha_{opt}^{(k)},s_{opt}^{(k)}}) = \min_{\alpha=1,\ldots,k} f_{k,\alpha}^{opt}(y_{k,\alpha,s_\alpha}). \tag{8}$$

The volume of the computations can be reduced substantially by the use in each interval of solutions obtained in the preceding intervals. In each succeeding interval we use the results of all preceding intervals. Therefore, for carrying the computations through to an end in each interval ν it is necessary to store the values f_{ν}^{opt}.

For seeking the optimum solution (3) it is also necessary to retain the numbers of the Boolean variables $y_{\nu, \alpha_{opt}^{(\nu)}, s_{opt}^{(\nu)}}$,

to be more precise, the three numbers $\langle \nu, \alpha_{opt}^{(\nu)}, s_{opt}^{(\nu)} \rangle$,

providing the optimum solutions for the corresponding hypothetical programs.

As a result of computations of the components of some auxiliary vectors on the basis of the recurrent expressions:

$$G_{\text{п}}^{(\nu)} = G_{\mu_\nu},\ r^{(\nu)} = E\left(\left(\sum_{j=\eta_\nu}^{\mu_\nu} m_j\right) \Big/ s_{opt}^{(\mu_\nu)}\right),\ n^{(\nu)} = s_{opt}^{(\mu_\nu)} \tag{9}$$

$$(\nu = 1, 2, \ldots, l),$$

where

$$\mu_1 = k,\ \eta_\nu = \mu_\nu - \alpha_{opt}^{\mu_\nu} + 1,\ \mu_\nu = \eta_{\nu-1} - 1,\ l = \nu|_{\eta_\nu = 1},$$

it is possible easily to obtain the components of the vectors

$$G_{\text{п}}^{opt} = (G_{\text{п}_1}, \ldots, G_{\text{п}_l}),\quad r^{opt} = (r_1, \ldots, r_l),\ n^{opt} = (n_1, \ldots, n_l)$$

constituting the optimum solution:

$$G_{\text{п}_j} = G_{\text{п}}^{(l-j+1)},\quad n_j = n^{(l-j+1)},\quad r_j = r^{(l-j+1)}\ (j = 1, 2, \ldots, l) \qquad \text{and} \qquad \nu = l.$$

It must be emphasized that not every optimum solution for intermediate hypothetical programs mandatorily enters into the optimum solution for the program as a whole.

3. We will assume that by the time of development of the space research program there are a number of carriers which can be used in a future program.

Henceforth we will be interested in carriers satisfying the condition $G_{car}' \geqslant G_1$, that is, those carriers by means of which it is possible to launch some spacecraft from the program G.

It is assumed that the following vectors are known:
$G_{car}' = \{G_{car1}', G_{car2}', \ldots, G_{car\gamma}'\}$ is the vector of payloads of existing carriers;
$r' = \{r_1', r_2', \ldots, r_\gamma'\}$ is the vector of the number of constructed carriers;
$n' = \{n_1', n_2', \ldots, n_\gamma'\}$ is the vector of multiplicity of use of existing carriers.

We will assume that the already constructed carriers are used in launching a number of spacecraft with the maximum possible total weight. Either all the constructed carriers will be used or all the spacecraft launched by these carriers will be exhausted.

As before, we will denote the remaining part of the program $\langle G, m, k \rangle$, and for this part we will solve the problem of selecting a family of carriers, taking into account existing (but still not prepared) carriers.

The computation scheme presented in the second section can with some changes be used for the particular case.

In a general case G'_{car_i} ($i = 1,\ldots,\gamma$) may not coincide with the components of the G vector. Further reasonings will be related to the vector G'', obtained by combining and arranging in increasing order the components of the vectors G and G_{car}'. The G'' vector includes the weights of the payloads of hypothetical and existing carriers.

A case is possible when there is not one, but several components of the vector G_{car}' between the pairs of adjacent components of the vector G in the vector G''. This is reflected in the notation of the corresponding Boolean variables.

We use the following notation of the Boolean variables $y^{(\nu)}_{i,j,s}$: i is the type of the closest (to the left) hypothetical carrier in the vector G''; ν is the sequence number of an existing carrier among carriers situated between a pair of adjacent hypothetical carriers; the variables j, s have the same sense as before.

The Boolean variables corresponding to variants for hypothetical carriers will be denoted $y^{(0)}_{i,j,s}$.

Example:

$$G=\{G_1, G_2, G_3\}, \quad m=\{m_1, m_2, m_3\},$$
$$G_n'=\{G_{n_1}', G_{n_2}', G_{n_3}'\}, \quad r'=\{r_1', r_2', r_3'\},$$
$$n'=\{n_1', n_2', n_3'\}$$

and assume

$$G_2<G_{n_1}'<G_{n_2}'<G_3, \quad G_{n_3}'>G_3. \qquad [\pi = car]$$

Then

$$G''=\{G_1, G_2, G_{n_1}', G_{n_2}', G_3, G_{n_3}'\}$$

The fragments of the matrix of correspondence of variants for the cases $G_{car} = G_{car_1}'$, $G_{car} = G_{car_2}'$ and $G_{car} = G_{car_3}'$ have the form

	$G_n = G'_{n_1}$		$G_n = G'_{n_2}$	
	$y^{(1)}_{2,1,n_1'}$	$y^{(1)}_{2,2,n_1'}$	$y^{(2)}_{2,1,n_2'}$	$y^{(2)}_{2,2,n_2'}$
m_1	0	1	0	1
m_2	1	1	1	1
m_3	0	0	0	0

	$G_{\text{п}} = G'_{\text{п}_3}$		
	$y^{(1)}_{3,1,n_3'}$	$y^{(1)}_{3,2,n_3'}$	$y^{(1)}_{3,3,n_3'}$
m_1	0	0	1
m_2	0	1	1
m_3	1	1	1

[π = car]

Now we will examine one of the fragments of the G'' vector:

$$\ldots, G_i, G_{\text{п}_1}', G_{\text{п}_2}', \ldots, G_{\text{п}_\nu}', G_{i+1}, \ldots,$$

including the pair of adjacent components G_i, G_{i+1} of the vector G and ν components of the vector G_{car}' satisfying the condition

$$G_i \leqslant G_{\text{п}_1}' \leqslant G_{\text{п}_2}' \leqslant \ldots \leqslant G_{\text{п}_\nu}' \leqslant G_{i+1}.$$

Carriers with the maximum payloads $G_{car} = G_i$, G_{car1}', G_{car2}',..., G_{car_ν}' can be used for launching one and the same group of spacecraft G_1, G_2,..., G_i; therefore, among the hypothetical programs ensured by each carrier from the mentioned group it is necessary to find the program with the minimum expenditures and use it as the optimum program in further computations.

Thus, the optimum program is determined from the expression

$$f_i^{\text{opt}}(y_{i,\alpha^{(i)}_{\text{opt}},s^{(i)}_{\text{opt}}}) = \min_{j=0,1,\ldots,\nu} \tilde{f}_i^{(j)}$$

Here $\tilde{f}_i^{(0)}$ are the expenditures on an optimum variant of a hypothetical program with the participation of a hypothetical carrier $G_{car} = G_i$:

$$\tilde{f}_i^{(0)}(y_i^{(0)}{}_{,\alpha^{(i)}_{\text{opt}},s^{(i)}_{\text{opt}}}) = \min_{\alpha=1,\ldots,i} \tilde{f}_{i,\alpha}^{(0)}(y^{(0)}_{i,\alpha,s_\alpha}),$$

where

$$\tilde{f}^{(0)}_{i,\alpha}(y^{(0)}_{i,\alpha,s_\alpha}) = \min_{s \in N\left(\sum\limits_{l=i-\alpha+1}^{i} m_l\right)} C^{(0)}_{i;\alpha,s} + f^{\text{opt}}_{i-\alpha}(y_{i-\alpha,\alpha^{(i-\alpha)}_{\text{opt}},s^{(i-\alpha)}_{\text{opt}}}).$$

$f_i^{(j)}$, $j > 0$ are the expenditures on an optimum variant of a hypothetical program with the participation of an existing carrier G'_{carj} and a maximum payload G_i:

$$\tilde{f}_i^{(j)}(y^{(j)}_{i,\alpha^{(i)}_{\text{opt}},n_j'}) = \min_{\alpha=1,\ldots,i} f^{(j)}_{i,\alpha}(y^{(j)}_{i,\alpha,n_j'}),$$

where

$$f^{(j)}_{i,\alpha}(y^{(j)}_{i,\alpha,n_j'}) = C^{(j)}_{i,\alpha,n_j'} + f^{\text{opt}}_{i-\alpha}(y_{i-\alpha,\alpha^{(i-\alpha)}_{\text{opt}},s^{(i-\alpha)}_{\text{opt}}}),$$

$$C^{(j)}_{i,\alpha,n_j'} = \left[\lambda''(G'_{\text{п}_j}, n_j', r^{(j)}_{i,\alpha}) r^{(j)}_{i,\alpha} + \lambda'''(G'_{\text{п}_j}, n_j') \sum_{\omega=i-\alpha+1}^{i} n_\omega\right] G'_{\text{п}_j},$$

$$r_{i,\alpha}^{(j)} = \bar{E}\left(\frac{1}{n_j'} \sum_{\omega=i-\alpha+1}^{i} m_\omega\right).$$

In computing $C_{i}^{(j)}{}_{\alpha\ \ j}$, $(j>0)$ no allowance is made for the expenditures associated with scientific research, experimental-design work and the mastery of production (λ').

The proposed algorithms are not sensitive to the dimensionalities of the problems to be solved, are effective with respect to computation time on an electronic computer and can be used both independently and in the interests of an automated planning system.

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