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OF THE 6TH ALL-UNION SEMINAR ON STATISTICAL
HYDROACOUSTICS (FOUO 2/79)

31 JULY 1979

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JPRS L/8597

31 July 1979

USSR Report

GEOPHYSICS, ASTRONOMY AND SPACE

(FOUO 2/79)

Proceedings of the 6th All-Union Seminar on Statistical Hydroacoustics

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USSR REPORT

GEOPHYSICS, ASTRONOMY AND SPACE

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PROCEEDINGS OF THE 6TH ALL-UNION SEMINAR ON STATISTICAL HYDROACOUSTICS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO GIDROAKUSTIKE in Russian 1975 pp 3-24, 33-44, 54-73, 91-96, 109-136, 140-153, 220-269, 277-294, 298-365, 372-378

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MODELING IN STATISTICAL HYDROACOUSTICS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 3-24

[Article by V. V. Ol'shevskiy]

[Text] 1. Preliminary Comments. At the present time in different fields of science and technology extensive use is already being made of physical and electronic modeling of different kinds of processes, fields and systems (for example, see the books [1-4], as well as the bibliographies to these works). The basic tendency in this direction is the increasingly broader use of digital computers in solving modeling problems. This is related to the considerable progress both in the creation of computers and in their mathematical support.

Relatively few studies have been published in the field of hydroacoustics directly concerned with modeling problems. However, in the transactions of the five All-Union Seminar Schools on Statistical Hydroacoustics (1969-1973) a scientific basis was essentially laid for a serious and intensive development of this direction [5-59], and all the moreso in adjacent fields, in particular, in statistical radioengineering, in the creation of methods for the modeling of processes and systems, in which considerable progress has been attained.

Before discussing the peculiarities of digital modeling in statistical hydroacoustics, the subject to which this study is for the most part devoted, we will examine possible methods for modeling in this field (see Fig. 1), and specifically, hydrophysical and electronic modeling.

Hydrophysical modeling. This is a method for reproducing real research objects and the hydroacoustic conditions for their observation at a considerably lesser scale with adherence to the principle of similarity of physical phenomena.

Thus, in hydrophysical modeling use is made of the similarity of physical phenomena transpiring under natural conditions and in the model. An important positive property of physical modeling is the possibility of obtaining a greater volume of experimental data with monitoring of the conditions for carrying out experiments. In hydrophysical modeling there are two possibilities of carrying out investigations, to wit:

-- modeling under artificially created and monitorable conditions (bath, basin);

-- modeling under monitorable conditions (fluvial water body, lake, coastal marine region).

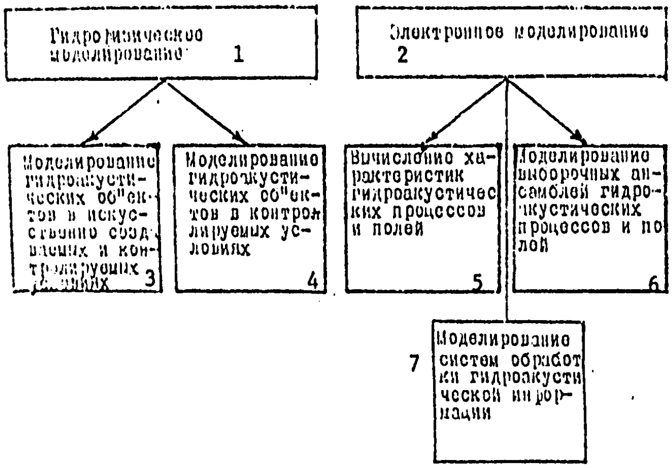

Fig. 1. Different modeling methods in statistical hydroacoustics

KEY:

1. Hydrophysical modeling
2. Electronic modeling
3. Modeling of hydroacoustic objects under artificially created and monitorable conditions
4. Modeling of hydroacoustic objects under monitorable conditions
5. Computation of characteristics of hydroacoustic processes and fields
6. Modeling of sample sets of hydroacoustic processes and fields
7. Modeling of systems for the processing of hydroacoustic data

The principal difficulty in interpreting the results of hydrophysical modeling of hydroacoustic objects is related to the influence of numerous scale effects which can substantially change the physical picture of the distribution, absorption and scattering of acoustic waves in comparison with natural conditions.

Electronic modeling. This is a method for the reproduction of real research objects, the hydroacoustic conditions for their observation and systems for the processing of hydroacoustic information using electronic devices (analog, digital and combined). Thus, in electronic modeling use is made of the mathematical similarity of description of phenomena transpiring under natural conditions and duplicated in models. In electronic modeling there are three research directions, to wit:

-- computation of the characteristics of hydroacoustic processes and fields (numerical methods for solution of problems);
-- modeling of sample sets of records of hydroacoustic processes and fields;
-- modeling of systems for the processing of hydroacoustic information.

An important advantage of electronic modeling is the possibility of repeated reproduction of the investigated objects in a broad range of their observation conditions and also the possibility of checking different hypotheses. The principal difficulty in interpreting the results of electronic modeling is related to the possibility of an incomplete allowance for the physical properties of real research objects and the possible ambiguity of the choice of their mathematical models.

Now we will examine some peculiarities of digital modeling of hydroacoustic processes and systems for the processing of hydroacoustic information, leaving outside our attention the following unconditionally important problems:
-- the relationship of hydrophysical and electronic modeling;
-- the role of analog, digital and combined modeling methods;
-- the relationship of numerical and analytical methods for constructing models of hydroacoustic processes and fields.

These problems are of considerable theoretical and practical interest and merit individual discussion.

2. The Problem of Choice of a Mathematical Model.

The choice of a stochastic model of a hydroacoustic process is the central problem in the digital modeling of hydroacoustic processes and fields. The success of the modeling as a whole is dependent on the adequacy of the adopted model to the real investigated objects and on how constructive it is, that is, how simple and productive.

First we will give a review of studies [5-59] in relation to the formulation of such models of hydroacoustic processes and fields.

We will begin with some definitions of basic concepts.

In accordance with the established concepts [5, 29, 30, 46, 50], by a stochastic model of a hydroacoustic process is meant its mathematical representation, which makes it possible to compute (or postulate) the stochastic characteristics of the process, important in the problem to be solved. A stochastic model is an idealized real research object and is formulated on the basis of the following representation:

$$X(t) = m_{\xi}\{\xi_i(t)\}, \qquad i = \overline{1,R}, \tag{1}$$

where X(t) is the investigated process, m_{ξ} is the formation operator, $_i(t)$ are elementary random processes whose stochastic characteristics are stipulated and can be physically interpreted.

Assume that $\theta(\vec{\ell}/m)$ is the stochastic characteristic of the process X(t) under the condition of adoption of its model m; then the representation (1) must make it possible to derive the equation:

$$\theta(\vec{\ell}/m) = \mu_m\{\pi_i(\vec{\lambda})\}, \tag{2}$$

where μ_m is the formation operator for the stochastic characteristic $\theta(\ell/m)$ from the characteristics $\pi_i(\vec{\lambda})$ describing the properties of the elementary processes $\xi_i(t)$, $\vec{e}$ and $\vec{\lambda}$ are the corresponding arguments (time, frequency, level, etc.).

The set of models

$$m \in M, \tag{3}$$

which the researcher has forms a thesaurus of models M and each of these models is such that

$$m = \{\theta_p(\vec{\ell}/m)\} = \{\theta(\vec{\ell}, \vec{a}_p/m), \vec{a}_p \in A_p, P(p, \vec{a}_p), p = \overline{1,N}\}, \tag{4}$$

where $\theta_p(\vec{\ell}/m)$ is a stochastic characteristic corresponding to the model m; $\vec{a}_p$ are parameters which are determined by hydroacoustic conditions (conditions of propagation, attenuation, scattering, etc.); A_p is parameter space a_p; $P(P, \vec{a}_p)$ is the joint distribution of probabilities of the number p of classes of characteristics $\theta_p(\vec{\ell}/m)$ and their parameters $\vec{a}_p$; N is the number of classes.

Thus, we assume that each model m corresponds to a parametric set of stochastic characteristics $\theta(\vec{\ell}, \vec{a}_p/m)$, $p = \overline{1,N}$.

Among the studies which we are reviewing, we can note the following two groups.

The first group includes studies [11-15, 19, 20, 24, 28, 37, 42, 52-54] in which the authors develop wave models of hydroacoustic processes and fields; the level of stochastic description is limited by the determination of the correlation and spectral characteristics (except for studies [14-48], in which an attempt is made to seek the characteristic functionals of the hydroacoustic fields).

The second group includes studies [5-10, 16-18, 21, 22, 23, 25-27, 29-36, 38-41, 43-47, 49, 50, 55-59], in which the authors develop phenomenological models of hydroacoustic processes and fields; in many cases the level of the stochastic description here is not limited to the correlation-spectral characteristics, but attains the level of determination of the distributions for probabilities, moment functions and semi-invariants of higher orders than the correlation characteristics.

Our comparison of the methods for constructing wave and phenomenological stochastic models shows that wave models, being physically adequately sound, make it possible to attain only the correlation-spectral description of the processes and fields, whereas the phenomenological models, although they are more formal than physical, in principle make it possible to obtain the distribution of probabilities, but in not all cases are they adequately well physically interpretable.

The thesaurus of models -- the set M -- can be divided into two sets M_1 and M_2, such that

$$M = M_1 \dot{U} M_2 , \tag{5}$$

and

$$m_1 \in M_1 , \quad m_2 \in M_2 , \tag{6}$$

M_1 is a set of so-called ideal models for which a full stochastic description was given, M_2 is a set of so-called working models for which a partial stochastic description was given. Naturally, the researcher never knows the ideal models m_1; they correspond to an exhaustive determination of the properties of the investigated object. At the same time, the working models are incomplete, only partially describing the objects, but they must be sufficiently adequate to the real investigated objects, and what is especially important, adequately simple with respect to solution of the modeling problem.

We introduce, much the same as was done in [29, 60] , the two characteristics:

-- measure of adequacy of the working model relative to the m_1 model:

$$\rho_{\theta} (m_1, m_2) = \rho[\theta(\vec{e}/m_1), \theta(\vec{e}/m_2)]; \tag{7}$$

-- cost function

$$C_{\theta} (m_2) = c[\theta(\vec{e}/\bar{m}_2)], \tag{8}$$

which characterizes the expenditures associated with modeling of the random process in accordance with the model m_2. In typical cases of examination of models of hydroacoustic processes there is the following tendency: with an increase in ρ_{θ} (m_1, m_2) the $C(m_2)$ value usually decreases.

As already noted, the researcher never knows the ideal (true) model. Accordingly, it is natural to replace the model m_1 by another -- the model $m_2^* \in M_2^* C M_2$, which would satisfy the condition

$$\tag{9}$$

$$\rho_\theta (m_1, m_2^*) \leqslant \Delta \rho^*, \tag{9}$$

$\Delta \rho^*$ is a sufficiently small value. According to the terminology in [60], the model which replaces the true model m_1 with an accuracy adequate for practical requirements is called the meta model. The meta model, to be sure, must be modelable and easily interpreted.

In the light of the introduced functions (7) and (8) and the meta model (9) the following two types of problems arise in the choice of the best (optimum) working models opt m_2 for modeling purposes.

The first type of problems is formulated in the following way:
-- restrictions on the cost of modeling are introduced

$$C_\theta (m_2^c) < \Delta C, \tag{10}$$

here

$$m_1^c \in M_2^c \subset M_2 ; \tag{11}$$

the optimum model opt m_2 is selected proceeding from the condition:

$$\text{opt } m_2 = \arg \inf_{m_2 \in M_2^c} \rho_\theta (m_2^*, m_2) \tag{12}$$

Thus, the optimum model opt m_2, in accordance with (10)-(12), is a model which corresponds to the maximum adequacy of the meta model m_2^* and takes into account limitations on cost (complexity) of modeling.

The second type of problems is formulated in the following way:
-- limitations are introduced on the degree of adequacy of the working models m_2 relative to the meta model m_2^*

$$\rho_\theta (m_2^*, m_2^\rho) < \Delta \rho, \tag{13}$$

here

$$m_2^\rho \in M_2^\rho \subset M_2 ; \tag{14}$$

The optimum model opt m_2 is selected proceeding from the condition

$$\text{opt } m_2 = \arg \inf_{m_2 \in M_2^*} C_\theta (m_2). \tag{15}$$

Thus, the optimum model opt m_2, in accordance with (13)-(15), is such a model which corresponds to the minimum cost (complexity) of modeling and takes into account limitations on the degree of its adequacy to the meta model.

Figure 2 gives a geometrical interpretation of the choice of the optimum models corresponding to the two considered types of problems.

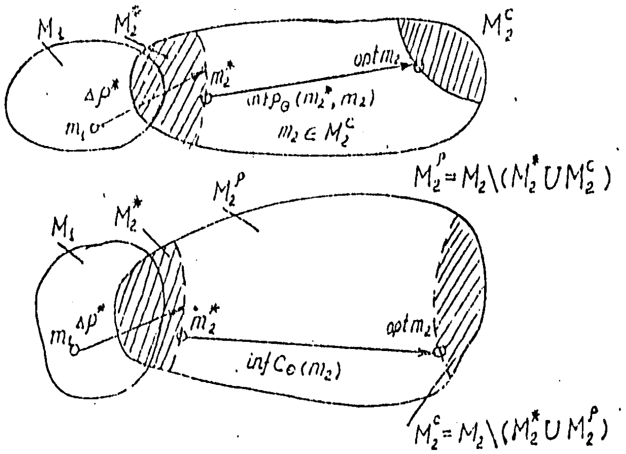

Fig. 2. Geometrical interpretation of choice of optimum model in modeling of sample records of hydroacoustic processes.
  a) case of limitation on cost of modeling;
  b) case of limitation on degree of adequacy of model.

3. Review of Specific Types of Models of Hydroacoustic Processes

Now we will examine some phenomenological stochastic models typical for the representation of hydroacoustic processes [5, 29] . Assume that X(t) is the investigated random process $\xi_i(t)$, $i = \overline{1,N}$ are elementary processes (determined, quasidetermined or random) with known characteristics.

Parametric models. These can describe echo signals in sonar and signals in hydroacoustic communication. These models have the form:

$$X(t) = \xi(t, \vec{\alpha}), \qquad (16)$$

where $\vec{\alpha}$ is the totality of random parameters with stipulated distributions of probabilities.

Constructive models. These can describe the totality of signals and noise, signals with additive and multiplicative interactions, and for these models the following representation is correct:

$$X(t) = \{\xi_i(t) \otimes \xi_i(t)\}, \qquad (17)$$

where $\otimes$ is the symbol of interaction of elementary processes.

Discrete canonical models. These can describe signals with multiray propagation, sea reverberation and some types of underwater noise. These models have the form:

$$X(t) = \sum_{k=0}^{N} a_k \xi_k(t), \qquad (18)$$

where a_k and N are random values.

Integral canonical models. These can describe signals propagating in a water medium, scattered signals and echo signals. These models have the form:

$$X(t) = \int_{-\infty}^{\infty} a(t, t')\xi(t')dt', \qquad (19)$$

where a(t, t') is some random, or determined, or mixed function.

Differential models. These can describe signals propagating in a water medium or echo signals. These models can be of the two following types:

$$X(t) = \sum_{\ell=0}^{L} b_\ell \frac{d^\ell}{dt^\ell} \xi(t), \qquad (20)$$

$$\xi(t) = \sum_{\gamma=0}^{\ell} c_\gamma \frac{d^\gamma}{dt^\gamma} X(t), \qquad (21)$$

where β_ℓ and C_γ are random values.

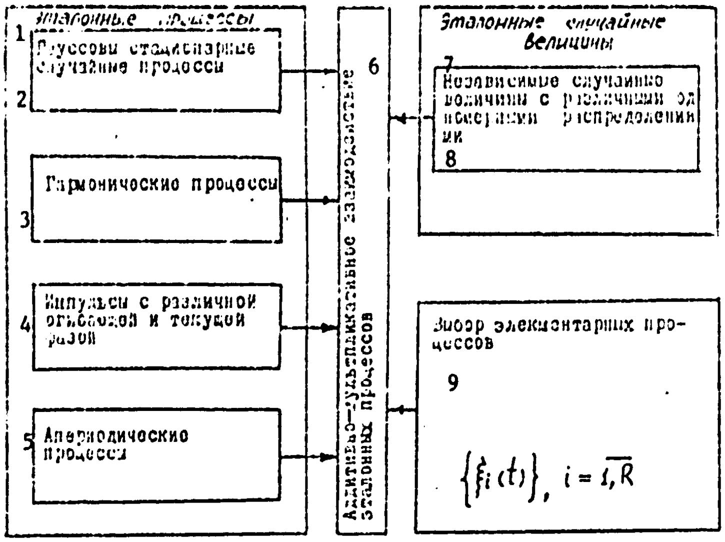

Fig. 3. Classification of elementary processes $\xi_i(t)$ for construction of models of investigated processes.

Key on following page

KEY:
1. Standard processes
2. Gaussian stationary random processes
3. Harmonic processes
4. Pulses with different envelope and current phase
5. Aperiodic processes
6. Additive-multiplicative interaction of standard processes
7. Standard random values
8. Independent random values with different one-dimensional distributions
9. Choice of elementary processes

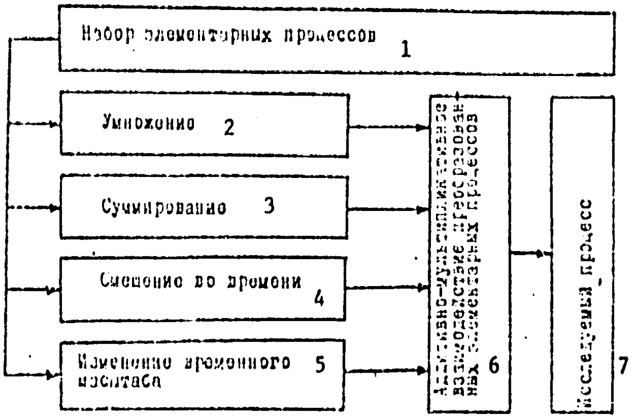

Fig. 4. Classification of operators m_ξ of formation of investigated processes.

KEY:
1. Set of elementary processes
2. Multiplication
3. Summation
4. Time shift
5. Change in time scale
6. Additive-multiplicative interaction of transformed elementary processes
7. Investigated process

We examined very simple models of random processes typical for hydroacoustics. To be sure, the need can arise for constructing combined models. In this case an analysis of the stochastic characteristics is more complex than in a study of very simple models of the type (16)-(21).

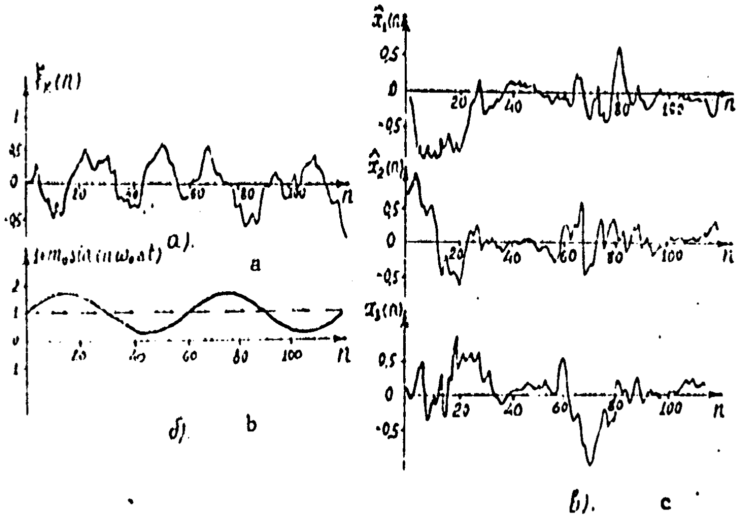

Fig. 5. Sample records of nonstationary random process.

KEY:
a. Gaussian stationary process;
b. Modulating function;
c. Sample ensemble of nonstationary random process.

4. Digital Modeling of Hydroacoustic Random Processes

The next step after developing and adopting a stochastic model of a random process is the choice of a method (algorithm) for its modeling. As follows from the preceding section, the stipulation of a stochastic model X(t) in accordance with (1) means to stipulate the stochastic characteristics of elementary random processes -- $\xi_i(t)$, $i = \overline{1,N}$, and also the operator m_ξ of formation of X(t) from $\xi_i(t)$. Very simple phenomenological models were determined by expressions (16) and (21). Thus, the problem first arises of the choice of types of elementary processes $\xi_i(t)$ which in accordance with (2) are described by the stochastic characteristics $\pi_i(\bar{\lambda})$. Figure 3 shows the classification of the considered elementary processes, which are formed from standard processes and standard random values.

We note that the elementary processes must have the property of physical and systemic interpretability [64] . The elementary processes are very simple processes which have the mentioned properties. However, the standard processes have only a very simple stochastic structure, their properties must be completely known, and finally, their modeling with any stipulated accuracy must be constructive.

Proceeding from the representation of the models (16) and (21), the operators for formation of m_ξ of the investigated processes X(t) can be classified in accordance with the diagram in Fig. 4.

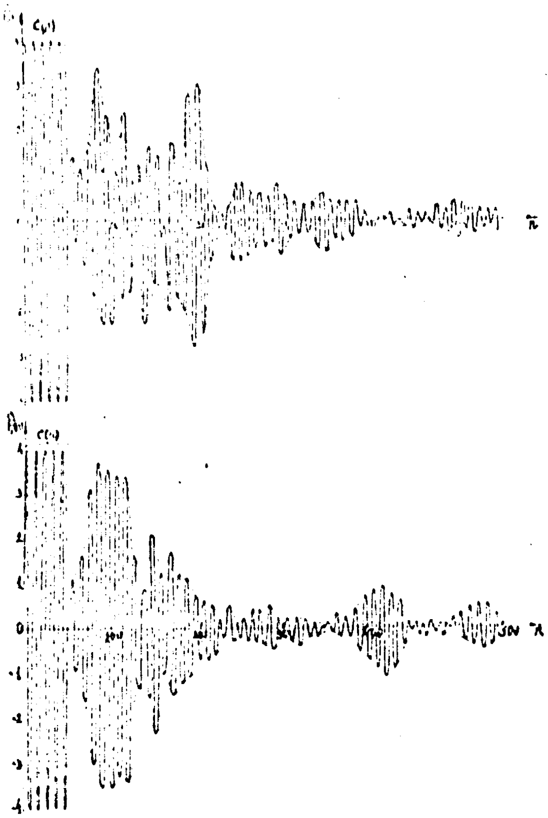

Fig. 6. Sample records of sea reverberation.

The principal operators here correspond to the physical transformations of hydroacoustic processes during radiation, propagation, scattering and reflection of acoustic waves in the water medium.

There is basis for assuming that using the elementary processes and their transformation operators, considered above, it is possible to model the following hydroacoustic processes:
-- direct signals, propagating under conditions of influence of refraction, reflection and scattering from the boundaries of the water medium;
-- echo signals from different sounded objects;
-- sea reverberation;
-- acoustic noise of the carriers of hydroacoustic systems;
-- acoustic noise of the sea.

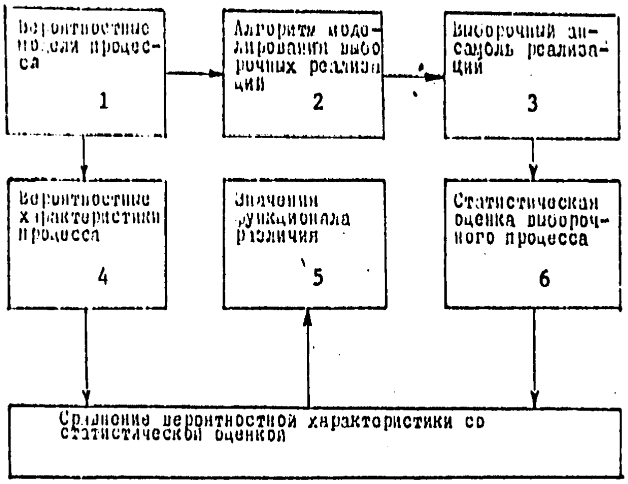

Fig. 7. Structural diagram of modeling and statistical measurements.

KEY:
1. Stochastic models of process
2. Algorithm for modeling of sample records
3. Sample ensemble of records
4. Stochastic characteristics of process
5. Values of difference functional
6. Statistical evaluation of sample process
7. Comparison of stochastic characteristic with statistical evaluation

Now we will examine two examples of the modeling of random processes specific for hydroacoustics.

The first example relates to so-called periodically modulated noise. A noise model of such a type relates to the constructive type (17) and has the following form [10, 61] :

$$X(t)=(1+m_0\sin\omega_0 t)\xi(t), \qquad (21')$$

where ξ is a Gaussian stationary random process, m_0 is the coefficient of intensity of amplitude modulation, ω_0 is modulation frequency. Such a model is characteristic, for example, for the noise observed in the course of acoustic cavitation [10]. In a discrete form, suitable for digital modeling, model (21') is written in the following way:

$$X(n)=[1+m_0\sin(n\omega_0\Delta t)]\xi(n), \qquad (22)$$

where Δt is the interval of discrete readings, n is the current number of the reading of the process with time. Figure 5 shows sample records of a process of type (22), modeled on an electronic computer of the BESM-6 type using the method described in [61].

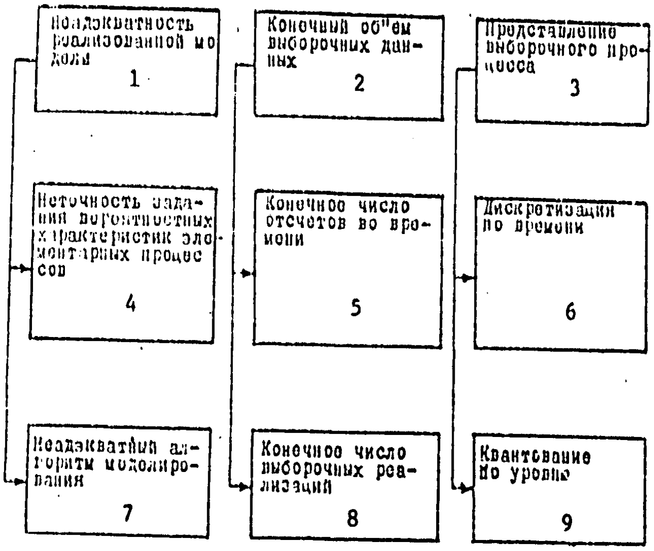

Fig. 8. Classification of possible reasons for the appearance of errors in modeling and statistical measurements.

KEY:

1. Inadequacy of realized model
2. Finite volume of sample data
3. Representation of sample process
4. Inaccuracy in stipulating stochastic characteristics of elementary processes
5. Finite number of readings in time
6. Time discretization
7. Inadequate modeling algorithm
8. Finite number of sample records
9. Level quantization

The second example is related to the modeling of sea reverberation. The reverberation model relates to discrete canonical models of type (18) and has the following form [5, 50, 62] :

$$\xi(t)=\sum_{i=0}^{N(t)} a_i f(t_i)\, c(t-t_i), \qquad (23)$$

where a_i are the random amplitudes of the elementary scattered signals, N(t) is their random number, $f(t_i)$ is a function characterizing the decrease in signals with distance during their propagation, C(t) is the emitted signal, t_i is the random moment of arrival of the i-th signal at the observation point.

The discrete form of the record of the model (23) has the following form:

$$F(n) = \sum_{i=0}^{N(n)} a_i f(n_i) C(n-n_i), \tag{24}$$

where n is the number of the reading, n_i is a random value.

On the assumption that N(n) is a Poisson random value, the mean value being dependent on time (that is, on n), a_i is also a random value, and n_i is distributed in some interval, a BESM-6 computer was used in modeling sample records (the modeling method was described in [62] . An example of such records is shown in Fig. 6.

Thus, digital modeling makes it possible to obtain sample records for a broad class of hydroacoustic random processes stipulated in the form of stochastic models, the basis for which is the totality of standard and elementary processes.

5. Statistical Measurements in Modeling

After the optimum model opt m_2 has been adopted and the sample records $\{\hat{x}_k(t)\}$ of the random process X(t), corresponding to this model, have been modeled, it is necessary to determine the adequacy of the realized model $\hat{m}_2$ and the model opt m_2.

Figure 7 is a structural diagram of modeling and determination of the adequacy of the characteristic of the modeled process and its stochastic model (scheme of statistical measurements). In accordance with this scheme, on the one hand we have the model opt m_2 of the random process X(t) and its stochastic characteristic $\theta(\vec{\ell}/\text{opt } m_2)$, and on the other hand, the modeled sample process $X(t) = \{\hat{x}_k(t)\}$ and the statistical evaluation $\hat{\theta}(\vec{\ell})$, which is obtained by the processing of its sample records $\hat{x}_k(t)$.

In order to be able to make sound decisions concerning the effectiveness of the algorithms and the quality of modeling of the sample ensemble X(t) of the random processes X(t) it is necessary to introduce the quantitative measure of the difference between the stochastic characteristic $\theta(\vec{\ell}/\text{opt } m_2)$ and its statistical evaluation $\hat{\theta}(\ell)$. We will denote this difference function in the form

$$\rho_\theta(\text{opt } m_2, \hat{m}_2) = \rho[\theta(\vec{\ell}/\text{opt } m_2), \hat{\theta}(\vec{\ell})], \tag{25}$$

where ρ is the operator for formation of the difference function, in general having the same sense as in formula (7); opt m_2 is the optimum model which is adopted in modeling (see section 2, $\hat{m}_2$ is the evaluation of the working model using the results of modeling, that is, using the statistical evaluation $\hat{\theta}(\vec{\ell})$ of the stochastic characteristic $\theta(\vec{\ell}/\text{opt } m_2)$.

The reasons for the appearance of errors in statistical measurements can be classified as (see Fig. 8): inadequacy of the stochastic model realized in modeling, the finite volume of sample data, and finally, the discrete representation of the sample process.

A quantitative analysis of the mentioned reasons for the appearance of modeling errors is an important theoretical and practical problem which still must be solved.

Together with the value of the error in statistical measurements $\rho_0(\text{opt } m_2, \hat{m}_2)$ it is natural to introduce the function

$$C_\theta(\hat{m}_2) = C[\hat{\theta}(\vec{\ell})], \tag{26}$$

which characterizes the expenditures in statistical measurements and in determining the evaluation of the realized working model $\hat{m}_2$.

Much as was done in section 2, we note two types of problems in choosing the optimum evaluation of the model $\hat{m}_2$ on the basis of examination of the procedure of statistical measurements.

The first type of problems is formulated in the following way:
-- limitations are introduced on the cost of the statistical measurements

$$C_\theta(\hat{m}_2^c) < \Delta\hat{C}, \tag{27}$$

here

$$\hat{m}_2^c \in M_2^{\hat{c}} \subset M_2 ; \tag{28}$$

the optimum realized working model is selected on the basis of the condition

$$\text{opt } \hat{m}_2 = \arg \inf_{\hat{m}_2 \in M_2^{\delta}} \rho_\theta(\text{opt } m_2, \hat{m}_2). \tag{29}$$

Thus, the optimum model opt $\hat{m}_2$, in accordance with (27)-(29), is a model which corresponds to the maximum adequacy of the model opt m_2 and takes into account limitations on the cost (complexity) of statistical measurements.

The second type of problems is formulated in the following way:
-- limitations are introduced on the degree of adequacy of evaluations of the working models $\hat{m}_2$ relative to the model opt m_2

$$\rho_\theta(\mathrm{opt}\, m_2, \hat{m}_2^\rho) < \Delta\hat{\rho}, \tag{30}$$

where

$$\hat{m}_2^\rho \in M_2^{\hat{\rho}} \subset M_2; \tag{31}$$

the optimum model opt m_2 is selected from the condition

$$\mathrm{opt}\, \hat{m}_2 = \arg \inf_{\hat{m}_2 \in M_2^{\hat{\rho}}} C_\theta(m_2). \tag{32}$$

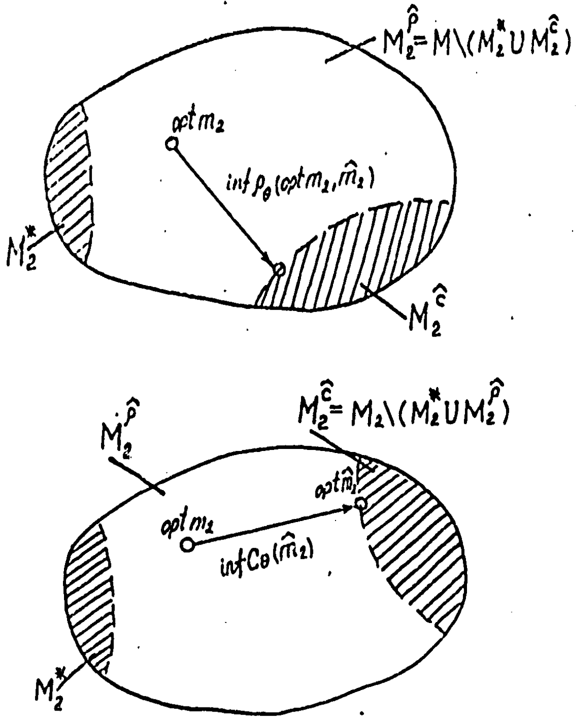

Fig. 9. Geometrical interpretation of choice of evaluation of realized working model in statistical measurements.
a) in case of limits on cost of statistical measurements
b) in case of limits on degree of adequacy of model

Thus, the optimum model opt m_2 in this case, in accordance with (30)-(32), is a model which corresponds to the minimum cost (complexity) of the statistical measurements and takes into account limits on the degree of its adequacy to the model opt m_2.

Figure 9 gives a geometrical interpretation of the choice of the optimum realized models corresponding to the two considered types of problems.

In conclusion, we note that the problems of optimization in modeling must, naturally, be considered in their full range -- both in a stochastic model and in statistical measurements, in this case taking into account the errors and cost of the modeling itself and statistical measurements. Some mathematical aspects of these problems are discussed in [64].

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SOME MATHEMATICAL ASPECTS OF MODELING IN STATISTICAL HYDROACOUSTICS USING AN ELECTRONIC COMPUTER

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[Article by A. A. Kaptyug and V. V. Ol'shevskiy]

[Text] General problems. Formulation of problem of classification of modeling problems. In [1] the author examined the stages in modeling in statistical hydroacoustics and the peculiarities of problems in the choice of stochastic models. In [2, 3] the authors described a specific method for the modeling of records of typical hydroacoustic processes. An analysis of the conclusions in these and other studies (see [4]) indicates that it is of interest to give a separate formal examination of modeling in statistical hydroacoustics and a formulation of its problems. Using [1] as a point of departure, in digital modeling in statistical hydroacoustics it is possible to define the following stages:
-- selection of a heuristic model of the process subject to formal modeling;
-- formal modeling of sample records on a digital computer;
-- statistical measurements on a digital computer.

These three stages form modeling in a broad sense, that is, the entire set of operations in choice of models of processes, their analysis by computer and evaluation of the adequacy of the adopted and employed models. In this connection, formal modeling on a digital computer in statistical hydroacoustics can be defined as a modification of programming, whose operators and objects have a definite physical and systemic interpretation. Henceforth in all cases by "modeling" we will mean specifically this stage of modeling in a broad sense.

Modeling involves use of a well-defined formal language Γ. Each object f_j of the principal significant objects to be interpreted $f_0, f_1, \ldots, f_k$ in this language forms a form which is sematically independent of the other objects, having numerical values and definite representations. Despite the semantic independence, the objects f_j can be related by a formal theoretical-multiple relationship (see [5]). One of the formal objects f_j is time. Assume this to be f_0. Then for any $j \neq 0$ the $\sigma(f_j)$ value of the object at the time $\mathfrak{S}(f_0)$ is $\mathfrak{S}(f_j, \mathfrak{S}(f_0)$.

The model can be considered as an operator determined by the means Γ in some of its objects and reflecting them in other objects of the same language.

Modeling can be regarded as a time-dependent procedure of construction and use of the model.

From the point of view of meaningful interpretation the model is a system of programs ensuring a definite structure and behavior of the modeling scheme.

The structure of some formally described system is the totality of all its relationships with other formal objects, nondependent on time, and the behavior of the system is the totality of all such relationships, dependent on time. Some of the principal objects $f_{i1},\ldots,f_{ie}$ can be regarded as the controlling parameters of the model, external relative to the model of situations. A general contradiction to the purpose of modeling in a broad sense is the attainment of extremal values. Thus, the general problem of modeling in a broad sense arises; this, in general, is incorrect. For its correction it is either necessary to change the formal description of the model or change the nature of the requirements imposed on it.

Now we will examine the formal criteria for modeling in statistical hydroacoustics.

First, modeling is a computation procedure, retaining in the new (formal) model the defined formal properties of the earlier adopted (heuristic) model. In this sense, in the modeling of functions a restoration of their form with a definite accuracy is not, generally speaking, necessary, that is, reference is to retention of formal properties, that is, equivalence, a special case of which is coincidence with a definite accuracy.

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Second, in the modeling of random processes use is made of flows of random numbers and on their basis ensembles of sample records of random processes. This leads to an a priori uncertainty of the modeling result and a formal noncontrollability of the procedure itself. We emphasize that noncontrollability in both the mentioned senses (conditions and computation procedure) is of fundamental importance, that is, in the opposite case modeling would not be necessary.

Third, in modeling in statistical hydroacoustics the stochastic models change in dependence on time and space coordinates of the observation region. The principal reasons for this are related to the comparability (in value) of the mean speed of sound and the mean velocities of the significant changes in observation conditions, to the considerable manifestations of the Doppler effect, to the considerable influence of refraction, reflections from discontinuities, etc.

Fourth, in accordance with the logic of this type of modeling the modeled processes as a result are described using the stochastic characteristics and their statistical evaluations. Accordingly, a specific characteristic is the formation at some stage of the integral functionals of the flows of random numbers and ensembles of sample records.

Fifth, modeling should allow a physical interpretation in all stages and for all the principal objects and operators. The initial models, the models, realized in the electronic computer, the stochastic characteristics and their statistical evaluations must be described in the language of the physical objects considered, such as radiation, propagation, scattering and reflection of acoustic waves.

Sixth, the modeling must allow a systemic interpretation for both the initial and resultant models.

In this connection, it is important that systemic interpretability of models is related to the purposeful examination of the modeled objects, specifically, with solution of the problems involved in data processing. We emphasize that systemic interpretability assumes the organization of the investigated objects and the purposes of functioning of the system related to these objects (physical interpretability assumes the presence only of the organization of the investigated objects).

The following conclusions can be drawn from the above considerations.

1. The problems to be solved in the case of correct modeling can be of two types. First, problems arising when constructing the model (initial construction or correction). Second, problems arising in functioning of the model (optimization or correction).

Since any model is constructed using other models, the process of its construction can be described as a result of the functioning of some model and it is sufficient to examine problems of the second type. The capacity of the model to solve different classes of problems is closely related to the effectiveness of modeling, which is interpreted in the sense adopted above. The "broader" the class of problems to be solved, the more effective is the modeling.

2. The presence of formally described special modeling criteria in statistical hydroacoustics leads to the necessity for formally taking them into account in constructing the class of modeling problems in statistical hydroacoustics.

3. The following problem makes sense. After defining the formal language for description in modeling, formulate a description of the class of all problems to be solved. Since in constructive modeling this class is finite and the class of all conceivable modeling problems in statistical hydroacoustics is unlimited, it is necessary to indicate the mechanism of choice of the problems, taking into account the specifics of the investigated region.

By analogy with the formal languages of programming, we will examine objects and operators in the modeling of random processes.

Determination of Some Basic Objects

We will call the representation $\tilde{L}$ of the object L (element or set) in the defined language Γ a formal expression in which symbols of variables, considered together with the regions of their change, are included. An arithmetical formula is a special case of representation.

Each problem can be described by two sets of parameters. The set $\{B_i\}_1^m$ represents the stipulated parameters and the set $\{B_j\}_1^n$ represents the parameters to be determined. Each parameter must have a unique representation. The type of problem (generated by a particular problem) can describe the considered $\{B_{is}\}_1^{m}$, $m_1 < m$, $\{B_{js}\}_1^n$, $n_1 \leq n$ are incomplete sets of parameters. Those parameters which must be joined to $\{B_{is}\}_1^{m_1}$ and $\{B_{js}\}_1^{n_1}$ in order to obtain a correctly formulated problem in the considered type of problems can be considered described implicitly. The problem is considered correctly formulated if it has a unique solution with a representation being a refinement of an a priori representation which is stipulated prior to solution of the problem. The structure of the problem can be described by examining the sets of parameters $\{B_i^1\}_i^m$, $\{B_j^1\}_1^n$, where each parameter B_i^1 or B_j^1 has a not less broad set of values than B_i or B_j respectively. Thus, the problem is a special case of the type of problem or structure of the problem. We will call the type of problem described by two single-element sets of the type $\{B_{i_1}\}$ and $\{B_{j_1}\}$ an "elementary problem."

To "solve a problem" means to find the representation of its solution in the terms of the defined descriptive language. In this sense the type of problem or the structure of the problem can be considered as a problem within the framework of a more general descriptive language interpreting more general objects than in the case of solution of a special problem.

Against the background of what has been said, it makes sense to examine the concept of "structure of the type of problem," being an obvious generalization of the concept "structure of the problem." We will use the term "semantic structure of the type of problem" for the most general (from the operational point of view) of structures of the type of problem, if it exists.

The statistically conjugate semantic structure for a particular semantic structure of the type of problem $\{D_i\}_1^p, \{d_i\}_1^q$ is designated by the formal expression $\{\hat{D}_i\}_1^p, \{\hat{d}_i\}_1^q$, having the following meaningful interpretation. "$\wedge$" is an operator related to the change in the sense of the parameter to which it is applied. For parameters having the sense of objects in the theory of probabilities the operator "$\wedge$" has the sense of a statistical evaluation. For parameters having the sense of statistical evaluations the operator "$\wedge$" has the sense either of the evaluation itself or the object to be evaluated. The properties of the "$\wedge$" operator can be expressed formally in the following way:

1. $\hat{(\hat{s})} = s$ or $\hat{s}$.

2. $(\widehat{s_1, \ldots, s_m}) = (\hat{s}_1, \ldots, \hat{s}_m)$.

3. $(\widehat{F(s_1, \ldots, s_m}))= \hat{F}(\hat{s}_1, \ldots, \hat{s}_m)$.

4. For each examination there are formally defined "fundamental parameters" $x, y, z, t, \ldots$, for which $x = \hat{x}, \ldots, t = \hat{t}, \ldots$ If $u = \hat{u}$, then u is the fundamental parameter.

Since any parameter can be regarded as the fundamental parameter, the set of fundamental parameters is included among the parameters whose values must be determined prior to examination of modeling problems.

Once again we emphasize that the parameters differ with respect to the type of the description in the examination, to wit: 1. explicitly described parameters (determined or not); 2. implicitly described parameters (determined or not).

For example (see Table 1), the parameters h_i, e_j, ξ_i((parameters to be determined) can be described explicitly; H_i, E_j, Ξ_i--determined parameters can also be described explicitly. Then P_ξ, R_i,...stand out as implicitly described parameters. Assume that some "optimization problem" is stipulated. That is, in some set A it is necessary to find the elements $a \in A$; this gives the maximum of the functional $f(x)$. The optimization problem is in essence a method for examining any problem. If it is remembered that in a correct formulation of the problem there must be a unique solution, we have two optimization problems: first, when $x \in A$ and the unique point a of the extremum is sought; second, when $x \in A$, the unique set $\{a\}$ of extremum points is sought. The second case is reduced to the first if we examine the defined class $A' \ni \{a\}$ of subsets of the set A for which $f(x) = \text{const}$, as a new set. Below, in all cases the optimization problems are represented in the first form.

In order to obtain a finite classification of problems it is possible to discriminate symmetric (in the sense of structure) types of problems, taking advantage of the fact that in any optimization problem there are fixed parameters stipulating the set A, the parameters to be determined, stipulating its current element x, the functional f(x), which determines the choice of elements $a \in A$.

The parameters fixed in this problem give (the known) region of change of the current element. This region, being unknown, can in turn be determined in other problems.

The parameters to be determined in this problem are determined from their general form. This general form can be fixed in other problems.

Thus, each class of optimization problems, to be examined with an accuracy to the functional to be optimized, is compared with a dual class of problems. These classes of problems can be called "mutually reciprocal."

Direct problems are those for which the parameters to be determined have the sense of a general form of the current elements of some sets.

Inverse problems are those for which the parameters to be determined have the sense of a general form of the sets or limits of the sets.

On the basis of the definitions introduced earlier we will examine a formal construction in which the direct and inverse problems are symmetric.

We note that in an examination of optimization problems there are three fixed objects:
-- representation $\tilde{x}$ of the current element x;
-- representation $\tilde{A}$ of the region of change of A;
-- the functional f(x) to be optimized.

The direct problem is formulated in the following way:

"Determine $\max_{x \in A} f(x)$ with stipulated f, $\tilde{x}$, $\tilde{A}$ and the variable x."

The inverse problem is formulated as follows:

"Determine $\max_{x \in A} f(x)$ with stipulated f, $\tilde{x}$, $\tilde{A}$ and the variable A."

Since the type of problem or the structure of the problem can be regarded as problems in themselves, direct or inverse types of problems or structures of problems are clearly definable. It is obvious that these concepts are formally equivalent to the concepts of "types" or "structures" of direct or inverse problems.

We note that the "semantic" structure of any type of problems can be organized on the basis of the semantic structure of direct and inverse elementary problems. For any type of problem, regarded as a problem of the defined type, that is, with the defined language of the solution, the mentioned elementary problems are types of direct or inverse problems.

In actuality, assume that there is stipulation of the representation $\tilde{x}$ of the general form of the elements x of some set Λ. Since $\tilde{x}$ is some formula in the corresponding language and the set of values $\tilde{x}$ is not empty, the set of interpretations of the set Λ and its representation $\tilde{\Lambda}$ are not empty, and this means that there is a maximum class of sets having the sense of the set Λ and its representation $\tilde{\Lambda}$. Thus, the stipulation of $\tilde{x}$ gives rise to semantic structures Λ and $\tilde{\Lambda}$. The semantic structure x is generated of necessity. Now assume that there is stipulation of the representation $\tilde{\Lambda}$ of the set Λ of elements x. Since $\tilde{\Lambda}$ is some formula in the corresponding language and the set of values $\tilde{\Lambda}$ is not empty, the set of interpretations of the element $\tilde{x}$ is not empty, and this means that there is a maximum class of values of the element x and its representation $\tilde{x}$. Thus, the stipulation of $\tilde{\Lambda}$ generates the semantic structures x and $\tilde{x}$. The semantic structure Λ is generated of necessity. Since the representation of any considered object can be interpreted either as $\tilde{x}$ or as $\tilde{\Lambda}$ $(x \in \Lambda)$, any problem generates either some type of direct problem or some type of inverse problem. In addition, the semantic structure of any inverse problem can be generated by the direct problem. It follows from what has been said that for generating the semantic structure of any problem it is sufficient to use the semantic structures of direct problems. Accordingly, each of the semantic structures cited in Table 1 is a semantic structure of the direct problem.

Description of Table 1 and Rules for Its Use

The table gives the principal semantic structures which can be used in modeling in statistical hydroacoustics.

Each of the objects indicated in one of the last two columns of the table (we will call it the object of the table) is dependent on time. In addition it is assumed that each of the objects in the table can be dependent on the parameters obtained using the following rules:

1. Each object in the table is a parameter.
2. Each of the results of application of the rules for use of the table to the objects in the table is a parameter.
3. Each parameter is explicitly or implicitly dependent on the parameters.
4. The result of application of the operator "$\wedge$" to a parameter is a parameter.

As a result, we obtain a computation class of semantic parametric constructions. Using the limitations on the complexity of the representation, it is possible to define its finite subclass.

In the generation of the semantic structures of direct elementary optimization problems, from the column "stipulated" we select one parameter having the sense of a set; in this same line, in the column "is determined," we select a parameter having the sense of an element.

Table 1

Objects of Modeling, Their Relationships and Semantic Structures of the Elementary Optimization Problems Generated by Them

№	Объекты исследования	Представления и отношения объектов	Оптимизационные задачи	
			задано	определ.
1	2	3	4	5
1.	Оператор h_i формирования элементарных процессов ξ_i из эталонных процессов e_j	$\xi_i = h_i(e_j)$, $\xi_i \in \Xi_i \subseteq \Xi$, $h_i \in H_i \subseteq H$, $e_j \in E_j \subseteq E$	Ξ_i H_i E_j	ξ_i h_i e_j
2.	Алгоритмы r_i формирования элементарных процессов ξ_i	$\xi_i = r_i(h_i)(e_j)$, $r_i \in R_i \subseteq R$	R_i	r_i
3.	Оператор m_t формирования случайного процесса x из элементарных процессов ξ_j	$x = m_t(\xi_j)$, $x \in \mathcal{X} \subseteq X$, $m_t \in M_t \subseteq M$, $\xi_j \in \Xi_j \subseteq \Xi$	x M_t Ξ_j	x m_t ξ_j
4.	Алгоритмы P_t формирования случайного процесса x	$x = p_t(m_t)(\xi_j)$, $p_t \in P_t \subseteq P$	P_t	p_t
5.	Операторы μ_{mp} формирования вероятностных характеристик $\theta_p(\bar{\ell}/m)$ процесса x из характеристик $\pi_j(\bar{\lambda})$ элементарных процессов. Здесь $m, m_t, \bar{\ell}, \bar{\lambda}, p$ - системные	$\theta_p(\bar{\ell}/m) = \mu_{mp}(\pi_j(\bar{\lambda}))$, $\theta_p(\bar{\ell}/m) = \theta(\bar{\ell}, \vec{a}_p/m)$, $\theta_p(\bar{\ell}/m) \in \mathcal{T}_p \subseteq \mathcal{T}$, $\vec{a}_p \in A_p \subseteq A$,	$\mathcal{T}_p$ A_p $\mathcal{M}_{mp}$	$\theta_p(\bar{\ell}/m$ $\vec{a}_p$ μ_{mp}
5	метры; $\vec{a}_p$ - физические метры.	$\mu_{mp} \in \mathcal{M}_{mp} \subseteq \mathcal{M}$ $\pi_j(\lambda) \in \Pi_j \subseteq \Pi$	Π_j	$\pi_j(\lambda)$
6	горитмы $q_{\mu m}$ формирования ятностных характеристик . $m)$ процесса x из характеристик $\pi_j(\bar{\lambda})$ элементарных цессов ξ_j	$\theta_p(\bar{\ell}/m) = q_{\mu m}(\pi_j(\bar{\lambda}))$, $q_{\mu m} \in Q_{\mu m} \subseteq Q$	$Q_{\mu m}$	$q_{\mu m}$
7	$v_x(u, w)$ памяти $w \in u$ представления x ; u - ктура памяти моделирующего ройства, а w - структура мяти, отводимой для представления x ; промежуток рассмотрения T_0 фиксирован	$u \in U_x \subseteq U$, $v_x(u, w) \in V_x \subseteq V$, $w \in W_x \subseteq W$	U_x V_x W_x	u $v_x(u, w)$ w

8	Объем $t_x(s,z)$, времени $z \subseteq s$ для представления x; s – поведение памяти моделирующего устройства; z – поведение памяти, отводимой для представления x; промежуток рассмотрения T_0 фиксирован	$s \in S_x \subseteq S$, $t_x(s,z) \in T_x \subseteq T$, $z \in Z_x \subseteq Z$	S_x T_x Z_x	s $t_x(s,z)$ z
9	Функционал $\rho_x(x, \hat{x})$ качества моделирования процесса x	$\rho_x(x,\hat{x}) = \rho_x(\hat{x},x) = \rho_x$, $\rho_x \in F_x \subseteq F$	F_x	ρ_x
	Критерий (порог ρ_{x0}) по процессу	$\rho_{x0} \in F_{x0} \subseteq F_0$	F_{x0}	ρ_{x0}
	Функционал $\varepsilon(\theta_x, \hat{\theta}_x)$ качества моделирования вероятностной характеристики θ_x.	$\varepsilon_\theta(\theta_x, \hat{\theta}_x) = \varepsilon_\theta'(\hat{\theta}_x, \theta_x) = \varepsilon_\theta$, $\varepsilon_\theta \in \varphi_\theta \subseteq \varphi$	φ_θ	ε_θ
	Критерий (порог $\varepsilon_{\theta 0}$) по характеристике θ_x	$\varepsilon_{\theta 0} \in \varphi_{\theta 0} \subseteq \varphi_0$	$\varphi_{\theta 0}$	$\varepsilon_{\theta 0}$

KEY:

Horizontal

1. Objects of investigation
2. Representations and relationships of objects
3. Optimization problems
4. Stipulated
5. To be determined

Vertical

1. Operator h_i of formation of elementary processes ξ_i from standard processes e_j
2. Algorithms ∂_i of formation of elementary processes ξ_i
3. Operator m_ξ of formation of random process x from elementary processes ξ_j
4. Algorithms P_ξ of formation of random process x
5. Operators μ_{mp} of formation of stochastic characteristics $\theta_p(\vec{l}/m)$ of process x from characteristics $\pi_j(\vec{\lambda})$ of elementary processes. Here m/n_ξ, $\vec{e}$, $\vec{\lambda}$, p are systemic parameters; $\vec{a}_p$ are physical parameters
6. The algorithms $q_{\mu m}$ of formation of the stochastic characteristics $\theta_p(\vec{l}/m)$ of the process x from the characteristics $\pi_j(\vec{\lambda})$ of the elementary processes ξ_j
7. The volume $v_x(u, w)$ of the memory $w \in u$ for the representation x; u is the structure of the memory in the modeling unit, and w is the the structure of the memory allocated for the representation x; the interval of consideration T_0 is fixed

8. The volume $t_x(s, z)$ of time $z \leq s$ for the representation x; s is the behavior of the memory of the modeling device; z is the behavior of the memory allocated for the representation x; the interval T_0 of the examination is fixed
9. The functional $\rho_x(x, \hat{x})$ of the quality of modeling of the x process
10. The criterion (threshold ρ_{x0}) for the process
11. The functional $z(\theta_x, \hat{\theta}_{\hat{x}})$ of the quality of modeling of the stochastic characteristic θ_x
12. The criterion (threshold $z_{\theta 0}$) for the characteristic θ_x

The semantic structures of inverse problems are obtained by a corresponding transpositioning of the parameters. We will use the term "semantic structure" of the parameter for the maximum set of its values with a fixed representation of the parameter admissible from the operational point of view.

We will say that two parameters have an identical semantic structure if their semantic structures are characterized only by the names of the parameters.

In the generation of the semantic structures of arbitrary problems some sequence of the following operations is carried out:

1. Substitution of implicitly described parameters by their names.
2. Substitution of explicitly described parameters by a different semantic structure. The following condition is satisfied. Assume $x \in A$, where x and A are the "names" of the current element and set respectively, identified with sets of its values $\{\sigma(x)\}$ and $\{\sigma(A)\}$. Then the substitutions $A \leftarrow A(\lambda)$ and $x \leftarrow x(\lambda)$, where $x(\lambda) \in A(\lambda)$ are the new possible current element and set, are carried out, always together. If one of the parameters x, A is implicitly stipulated, the substitution in it can be accomplished implicitly.

Each substitution into a specific semantic structure of the problem can be accompanied by the inverse operation -- discarding of the structural elements. It is possible to examine the space C of different semantic structures of the problems. Then each substitution and each rejection is an inverse image of one part of C on the other. All possible substitutions and rejections form a group which characterizes the space of semantic structures of problems.

We emphasize that each semantic structure of the problem on the basis of definition of the concept "parameter" can be regarded as a parameter.

We also note that when selecting from the class of all modeling problems in statistical hydroacoustics of those problems having a limited complexity of representation of the description or solution procedure, a significant role is played not only by the language Γ, but also the class A_Γ of methods for coding problems and the procedures for their solution.

In a meaningful interpretation Γ is an analog of computer logic, A_Γ is the analog of a system of programs understood as apparatus or algorithmic units.

Assume that the semantic structures, to be solved in modeling, that is, the space C, have been formulated. In an examination of the problem it is necessary to select a structure of the type of problem corresponding to the defined formal language Λ_α for representation of solution of the problem. Strictly speaking, precisely this structure of the type of the problem should be regarded as the "problem." This means that both the formulation and the solution of the problem must be represented in the same formal language. However, those parameters which do not enter into the representation of solution of the problem must be stipulated implicitly. It can be assumed that $\Lambda \subset \Gamma$ with any α. It is obvious that a new problem is considered with each new α.

Thus, in constructing a formal description of a class of modeling problems it is necessary that for each considered problem, in addition to the language of its solution, there be a set of representations of its solution, nonequivalent from the point of view of the language Γ.

Also correct are more general requirements: in forming a class of modeling problems only those problems whose solution exists in some formal sense must be considered. Among such requirements is a limitation on the complexity of representation of solution of problems or limitation on the complexity of representation of procedures for solution of the problem. In a meaningful interpretation this corresponds to a limitation on the computer memory and the time for obtaining a solution.

By virtue of the comments made, all the restrictions on representation and the field of values of the parameters are divided into two groups:
-- limitations changing the class of problems to be solved, that is, conditions either on the nature of existence or on the region of determination of a priori representation of the solution;
-- limitations not changing this class, that is, conditions refining the a priori representation of the solution.

The nature of interpretation of limitations is dependent on the language Γ, and this means, in the last analysis, on the purposes of the modeling.

It follows from what has been said above that for forming a class of modeling problems, taking into account the limitations not changing the class of problems, it is first possible to form a class of such problems in the absence of limitations; then in place of the parameters having a defined sense and an undetermined set of values use is made of formulas or algorithms represented in the Γ language. The class of formulas, like the class of algorithms, is not greater than calculated, and each formula or algorithm represents admissible a priori representations of the parameters or construction procedures. The class of modeling problems which is obtained from the preceding on the basis of significant limitations is determined from an examination of the parametric forms of the theorem of existence of a solution. It should also be mentioned what is meant by the values of the principal objects, special cases of which are the objects in the table. Assume that f is the principal object. Then $\mathcal{O}(f)$ is either a number or a vector-function or an operator represented therefore partially by ordered sets of numbers or the space of operators of the mentioned type, or a logical variable.

In a general case the value of the principal objects is determined by the interpretation of the Γ language.

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Summary

This paper is only an approach to precise formulations of the problems involved in evaluating the effectiveness of modeling, but already from the considerations presented here the following conclusions can be drawn:

1. The description of the class of modeling problems can be formalized using the proposed procedures and objects. However, it is necessary to define formally a set of parameters regulating completeness of description of the class of modeling problems in the stipulated constructive sense.

2. The specifics of description of the space of modeling problems in statistical hydroacoustics must be expressed in standard form in the language Γ, for which in turn it is necessary to describe the principal formal criteria of this language.

3. If the class of modeling problems to be solved is finite, it is possible to formulate rigorously the problem of determining the number of its elements, which in a general case can be regarded as a nontrivial "combination" problem. It must be noted that for a comparison of effectiveness it may be necessary to have only a sufficiently precise evaluation of its solution.

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DIGITAL MODELING OF THE RESPONSE FUNCTION FOR COMPLEX SIGNALS USING A FAST FOURIER TRANSFORM ALGORITHM

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 54-58

[Article by I. B. Vaysman and K. P. L'vov]

[Text] In the theory and practice of complex signals great attention is devoted to an analysis of the response functions of a processing system. This is accomplished using a matched filter or cross-correlation processing. In the case of matched filtering (the weighting function of the filter is a mirror reflection of the sounding signal with time) the output signal can be written as

$$S_{вых}(t) = \int_{-\infty}^{\infty} S_{пр}(\tau) g(t-\tau) d\tau. \tag{1}$$

For finding the response function we introduce into consideration a matched filter with a weighting function conjugate in the Hilbert sense with the weighting function g(t), that is

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} d\tau. \tag{2}$$

Then the response function (envelope of the processing system output signal) can be represented in the form

$$e(t) = \sqrt{S_{вых}(t) + \hat{S}^2_{вых}(t)}, \tag{3}$$

[βыX = out(put)] where

$$\hat{S}_{вых}(t) = \int_{-\infty}^{\infty} S_{пр}(\tau) \hat{g}(t-\tau) d\tau. \tag{4}$$

[πp = rec(eived)]

The response function e(t) is of great importance in sonar for analysis and use of complex sounding signals. In some cases it is difficult to obtain the response functions analytically due to the complexity of integration; this sometimes leads to the need for using the approximate stationary phase principle [1]. When using classical numerical integration methods and electronic computers considerable computation time is required because the integrands in (1) and (4) are rapidly oscillating. [For example, see the studies by S. V. Pasechnyy, et al., O. D. Mrachkovskiy, Ye. B. Libenson, Ye. I. Bovbel', et al., published in this collection of articles.]

The sounding signals used in sonar have a finite duration. Then the output signal of a physically embodied matched filter with the weighting function g(t) [2] is represented in the following form

$$S_{\text{вых}}(t)=\begin{cases}\int^{t} S_{\text{пр}}(\tau)g(t-\tau)d\tau & \text{when } 0\leq t\leq T,\\ \int_{t-T} S_{\text{пр}}(\tau)g(t-\tau)d\tau & \text{when } T\leq t\leq 2T,\\ & \text{when } t>2T,\end{cases}\qquad(5)$$

[вых = out(put)] and with the weighting function $\hat{g}(t)$ -- in the form

$$\hat{S}_{\text{вых}}(t)=\begin{cases}\int^{t} S_{\text{пр}}(\tau)\hat{g}(t-\tau)d\tau & \text{when } 0\leq t\leq T,\\ \int_{t-T} S_{\text{пр}}(\tau)\hat{g}(t-\tau)d\tau & \text{when } T< t\leq 2T,\\ 0 & \text{when } t>2T.\end{cases}\qquad(6)$$

Digital modeling of the response functions involves use of the following discrete expressions of corresponding (5), (6) and (3)

$$\tilde{S}_{\text{вых}}[k]=\begin{cases}\Delta t\sum_{i=0}^{k} S_{\text{пр}}[i]g[k-i] & \text{when } 0\leq k\leq N-1,\\ \Delta t\sum_{i=k-N}^{N-1} S_{\text{пр}}[i]g[k-i] & \text{when } N\leq k\leq 2N-1,\\ 0 & \text{when } k\geq 2N,\end{cases}\qquad(7)$$

$$\hat{S}_{\text{вых}}[k]=\begin{cases}\Delta t\sum_{i=0}^{k} S_{\text{пр}}[i]\hat{g}[k-i] & \text{when } 0\leq k\leq N-1,\\ \Delta t\sum_{i=k-N}^{N-1} S_{\text{пр}}[i]\hat{g}[k-i] & \text{when } N\leq k\leq 2N-1,\\ 0 & \text{when } k\geq 2N,\end{cases}\qquad(8)$$

$$\tilde{e}[k]=\sqrt{\tilde{S}^{2}_{\text{вых}}[k]+\hat{S}^{2}_{\text{вых}}[k]},\quad k=0,1,2,3,\ldots,2N-1,\qquad(9)$$

where Δt is the discretization interval, $N = T/\Delta t$, and $\tilde{S}_{out}[k]$, $\dot{S}_{rec}[k]$, $g[k]$, $\hat{S}_{out}[k]$, $\hat{g}[k]$ and $\tilde{e}[k]$ are "grid" functions [3] corresponding to the continuous functions $S_{out}(t)$, $S_{rec}(t)$,

For the purpose of generality, in expressions (5), (6), (7) and (8) no allowance has been made for the influence of the Doppler effect on the received signal S_{rec} (t). This makes it possible to use the results for different models of the received signal S_{rec} (t) and the weighting function g(t).

A discrete model (7) of an analog matched filter (5) is based on the rule of rectangles and is approximate. The systematic error in the discrete model (7) can be determined as

$$\delta[k] = \tilde{S}_{out}[k] - S_{out}[k], \tag{10}$$

where $\delta[k]$ is the "grid" function of errors, $S_{out}[k]$ is the reading of the output signal of the analog matched filter (5) at the moment in time $t_k = k\Delta t$,

$$S_{out}[k] = S_{out}(k\Delta t).$$

The received signal $S_{rec}(t)$ and the weighting function g(t) have a virtually finite extent along the frequencies axis of the modulus of the integral Fourier transform. If the spectra are concentrated in some frequency band $\omega_{min} \leqslant \omega \leqslant \omega_{max}$, it can be shown that when selecting the discretization interval (integration interval) satisfying the inequality [5]

$$\frac{2\omega_{max}}{g} \leqslant \frac{2\pi}{\Delta t} \leqslant \frac{2(\omega_{max} - \Omega)}{g-1} \tag{11}$$

where

$$g = 1, 2, 3, \ldots, entier[\frac{\omega_{max}}{\Omega}],$$
$$\Omega = \omega_{max} - \omega_{min}$$

the systematic error $\delta[k]$ is equal to zero.

The first region (zone) of virtual absence of systematic error $\delta[k]$ is determined by the choice of the discretization interval

$$\Delta t \leqslant \frac{1}{2f_{max}}, \tag{12}$$

and the last region gives the value

$$\Delta t \simeq \frac{\pi}{\Omega}. \tag{13}$$

In the case of choice of the maximum admissible value of the discretization interval ($g = entier\ [\omega_{max}/\Omega]$) the resulting width of the region of virtually zero errors is extremely small.

In the digital modeling of response functions in the model of the received signal $S_{rec}(t)$ it is necessary to take into account the Doppler effect and a case of practical importance is the model of the Doppler effect as a time lengthening or shortening of the received signal $S_{rec}(t)$ with a model of the weighting function g(t) displaced relative to its central frequency ω_0.

All this leads to a need for broadening the values of the frequency band $\omega_{min} \leqslant \omega \leqslant \omega_{max}$ and the choice of a non-maximum value of the discretization interval value.

Since the Fourier transforms of the weighting functions g(t) and $\hat{g}$(t) differ only with respect to the phase shift of all the frequency components by $\pi/2$, one and the same discretization intervals give a virtual absence of the systematic error of the discrete models (7), (8), (9) for determining $S_{out}(t)$, $\hat{S}_{out}(t)$ and e(t).

The computation time required for modeling the readings of the envelope of an output signal of a matched filter e(t) on a 2N-1 digital computer can be estimated as

$$T_{\text{пр}} \simeq 2N[2N(\tau_y + \tau_c) + 2\tau_y + \tau_c + \tau_r], \qquad (14)$$

[Пр = rec]

where τ_y, τ_c, τ_r are the times required for the multiplication, addition and subtraction of the square root operations respectively.

For complex signals with a great value of the base $T\Omega/2\pi$ the N value considerably exceeds unity and it can be said that the computation time (time required for modeling the response function) is proportional to N^2. Therefore, the direct programming of expressions (7), (8) and (9) requires a considerable computation time.

At the present time for computing expressions of the type (7) and (8) use is made of the so-called fast faltung method based on the fast Fourier transform algorithm [5]. Depending on the N value it is possible to use two modifications of the fast faltung method.

If the value of the signal base $T\Omega/2$ and the selected discretization interval gives an N value which does not require use of external memory units of a digital computer, it is necessary to use a modification of the fast faltung method based on supplementation of the "grid" functions $S_{rec}[k]$, g[k] and $\hat{g}$[k] by zeroes in a number equal to N [5].

With N values not making it possible to get by only with the operating memory unit of the digital computer, it is necessary to use a modification of the fast faltung method based on sectioning [5].

The computation time required for obtaining 2N - 1 values of the response function e (t) by means of the fast faltung method is approximately equal to [5]

$$T_{\text{БПФ}} \simeq 4N(2\tau_y + \tau_c)(3\log_2 N + 4). \qquad (15)$$

[БПФ = FFT = fast Fourier transform]

A comparison of the T_{rec} and T_{FFT} values reveals a considerable advantage of use of the fast faltung method for modeling the response functions in the case of a large signal base.

On the basis of the above we developed a program for the digital modeling of the response function. The bases for the program was a standard FFT program, concise information on which was published in [6].

The programs were prepared in the input language of a "Signal" translator, being a specific version of the ALGOL-60 algorithmic language.

The results of the modeling are graphically registered using a FAK-P phototelegraphic apparatus.

The set of programs makes it possible to model the response function with allowance for the Doppler effect in the received signal $S_{rec}(t)$ as a frequency shift or as a lengthening or shortening in time. The model of the weighting function g(t) can be shifted in frequency for investigating the possibility of use of a set of matched filters differing only with respect to the central frequency, for processing the received signal $S_{rec}(t)$, being lengthened or shortened in time.

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DIGITAL MODELING OF SEA REVERBERATION

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 59-65

[Article by V. V. Ol'shevskiy and V. A. Panfilov]

[Text] The development of methods for measuring the stochastic characteristics of sea reverbearation in its different models, investigation of the properties of reverberation at the output of data-processing systems, and also solution of a number of other statistical problems, leads to a need for modeling reverberation. As a result of such modeling we should have sample sets of reverberation records and then it is possible to measure its characteristics and ascertain the adequacy of the adopted stochastic models and their statistical evaluations [1]. The number of $\hat{F}_k(t)$ records of modeled reverberation in the sample set

$$\hat{F}(t) = \{\hat{F}_k(t)\}, \quad k = 1, \ldots, N \tag{1}$$

by virtue of the nonstationary nature of this process must be quite great (tens or even hundreds of independent sample records) in order to ensure a good accuracy. At the same time, stochastic models of reverberation (for example, see [2, 3]), corresponding to different conditions of propagation and scattering of acoustic waves, and also to the movement of scattering inhomogeneities and acoustic antennas, are quite complex. Accordingly, the algorithms for modeling reverberation are also complex. All this leads to a desirability of digital modeling of sample sets of reverberation using modern electronic computers.

The simplest reverberation model F(t), observed at the reception point, is a discrete canonical model [1-3] in the form:

$$F(t) = \sum_{i=1}^{N(t)} a_i f(t_i) c(t - t_i), \tag{2}$$

where N(t) is the random number of elementary scattered signals arriving at the reception point at the moment in time t; a_i is a random value characterizing the scattering properties of the i-th scatterer; $f(t_i)$ is a function of the change in signal levelduring propagation; c(t) is a function determined by the type of emitted signal; t_i is a random moment of arrival of the i-th scattered signal. In digital modeling the representation of (2) should be written in discrete form

$$F(n) = \sum_{i=1}^{N(n)} a_i f(n - \alpha_i \frac{T}{\Delta t}) c(n - n_i - \alpha_i \frac{T}{\Delta t}), \tag{3}$$

here

$$t = n \Delta t, \quad t_i = n_i \Delta t + \alpha_i T, \tag{4}$$

where n is the number of the reverberation process reading; Δt is the discretization time interval; α_i is a random value, distributed in the interval (0, 1); T is the duration of the emitted signal; n_i is the number of the reading n corresponding to the appearance of the i-th scattered signal at the observation point.

On the basis of the discrete model (3) it is possible to write the following representation for the k-th record $F_k(n)$ of the reverberation:

$$F_k(n) = \sum_{i=1}^{N_k(n)} a_{ki} f(n - \alpha_{ki} \frac{T}{\Delta t}) c(n - n_{ki} - \alpha_{ki} \frac{T}{\Delta t}), \tag{5}$$

where the subscript k on the random parameters $N_k(n)$, a_{ki}, α_{ki} and n_{ki} corresponds to the number of the modeled record.

Each reverberation record was modeled using an algorithm represented in the form of the structural diagram in Fig. 1, which corresponds to the model (5).

The dependence of the reverberation record was ensured by the choice of independent flows of the random numbers N_k, a_{ki} and α_{ki} for any pair of records.

As distributions of the probabilities of random parameters for each n-th reading we assumed:

-- for N_k -- the Poisson law

$$P(N/n) = \frac{\langle N(n) \rangle^N}{N!} \exp(-\langle N(n) \rangle), \tag{6}$$

where $\langle N(n) \rangle$ is the mean value for the number of scattered signals forming the reverberations at the n-th moment of the reading, that is, in the interval $(n\Delta t \; (n + 1)\Delta t)$;

-- for the a_{ki} seminormal law with the probability density

$$W(a/n) = \frac{\sqrt{2}}{\sqrt{\pi d_a(n)}} \exp\left(-\frac{a^2}{2d_a(n)}\right), \quad a \in [0, \infty], \tag{7}$$

where $d_a(n)$ is the dispersion;

-- for α_{kl} is a universal law with the probability density

$$W(\alpha) = 1, \quad \alpha \in [0, 1]. \tag{8}$$

The nonstationary properties of reverberation were modeled using definite laws of change of the parameters $\langle N(n) \rangle$, $d_a(n)$ and the function f(n).

As the emitted signal we used a segment of a sine curve with a rectangular envelope

$$C(n) = \sin(n\omega_0 \Delta t), \quad n \in [1, \frac{T}{\Delta t}], \tag{9}$$

and in this computer experiment it was assumed that $\omega_0 T = 10\pi$.

Figure 2 shows typical sample records of reverberation obtained using a BESM-6 computer.

The use of the modeled reverberation, taking into account its "carrier" frequency, as shown in Fig. 2, is inconvenient due to the necessity for making a great number of readings in time (in this case each record consisted of 500 readings). A considerably more economical representation of reverberation is obtained through its quadrature components:

$$F_k(n) = F_{ck}(n) \cos(n\omega_0 \Delta t) - F_{sk}(n) \sin(n\omega_0 \Delta t), \tag{10}$$

where

$$\left. \begin{aligned} F_{ck}(n) &= E_k(n) \cos \Psi_k(n), \\ F_{sk}(n) &= E_k(n) \sin \Psi_k(n) \end{aligned} \right\} \tag{11}$$

are the "cosine" (CK) and "sine"(SK) quadrature components of the k-th reverberation record, $E_k(n)$ and $\Psi_k(n)$ are its envelope and current phase.

The quadrature components $\hat{F}_{CK}(n)$ and $\hat{F}_{SK}(n)$ of the initial process $\hat{F}_k(n)$ were obtained by taking only those readings which differed from one another by the value

$$m = \frac{2\pi}{\Delta t \omega_0}. \tag{12}$$

We note that the conversion from reverberation records to the quadrature components reduces the total number of readings in each record by a factor of 10.

Figure 3 gives the quadrature components of reverberation corresponding to its sample records, represented in Fig. 2.

In the next stage of modeling after obtaining the sample records it is necessary to confirm the correspondence between the adopted stochastic model of reverberation to the statistical evaluations of its characteristics [1]. We will examine one of such stochastic characteristics -- the autocorrelation coefficient

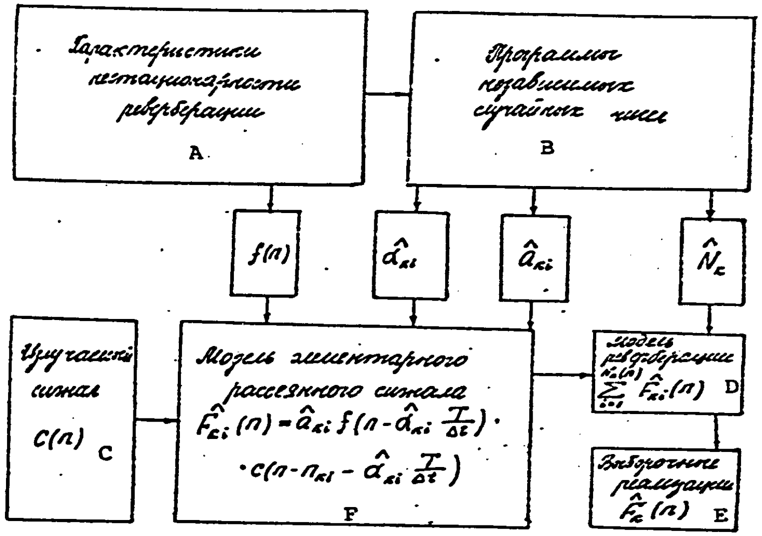

Fig. 1. Structural diagram of modeling of sample reverberation records.

KEY:
- A) Characteristics of nonstationarity of reverberation
- B) Program of independent random numbers
- C) Emitted signal
- D) Model of reverberation
- E) Sample records $\hat{F}_k(n)$
- F) Model of elementary scattered signal

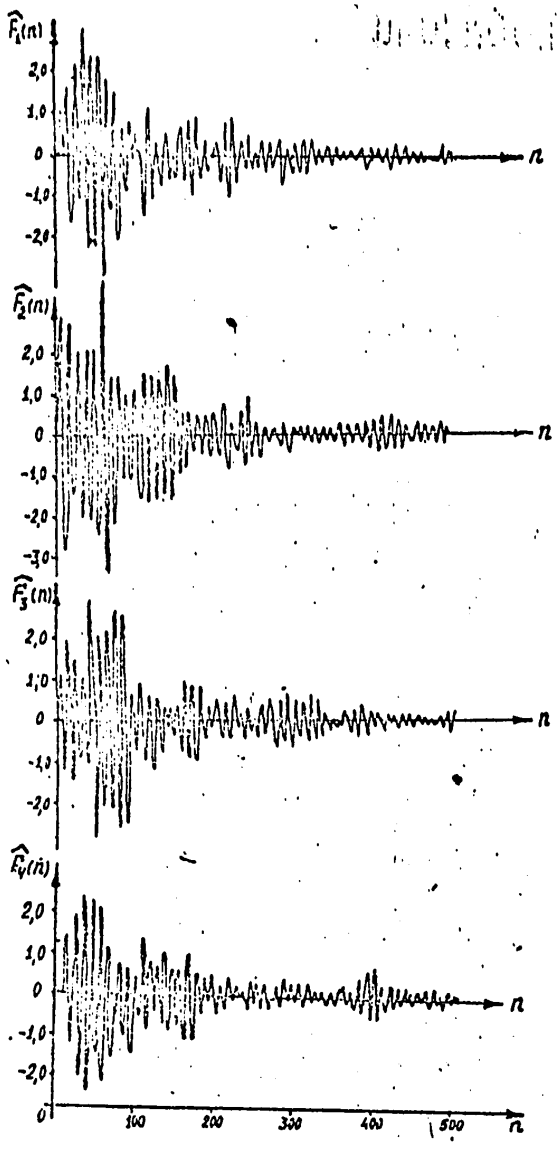

Fig. 2. Sample reverberation records $\hat{F}_k(n)$ with carrier frequency taken into account.

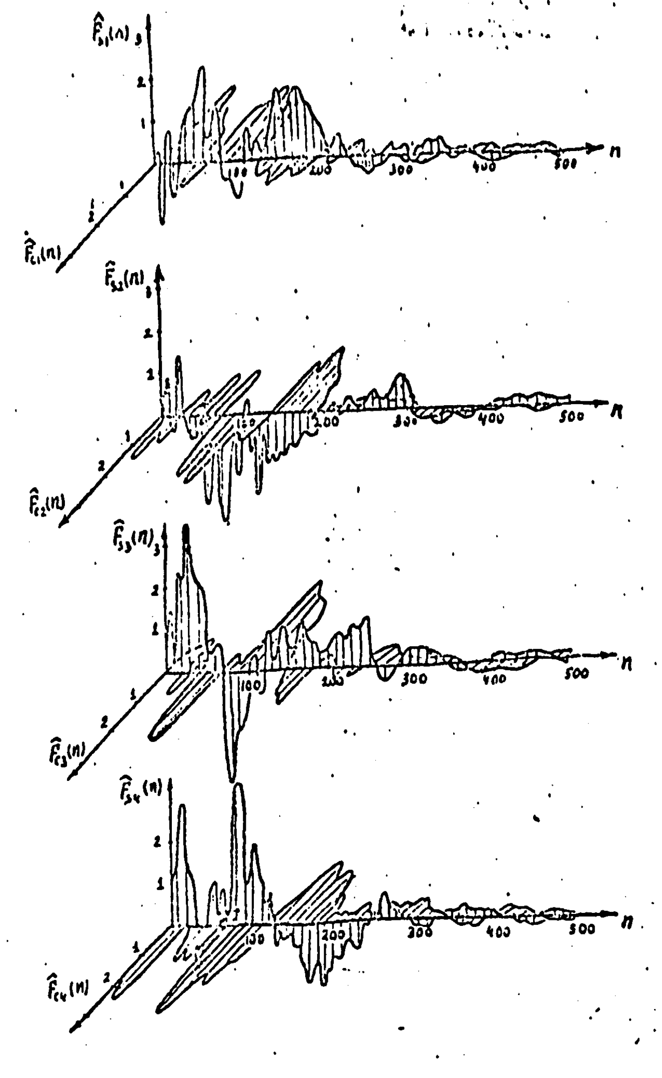

Fig. 3. Sample records $\hat{F}_{CK}(n)$ and $\hat{F}_{SK}(n)$ of quadrature components of reverberation.

$$R_F(\tau) = \frac{\langle F(n) F(n+\tau) \rangle}{\langle F^2(n) \rangle}, \tag{13}$$

where n is the initial number of the reading, $\tau = 0, \ldots, T/\Delta t$ is the correlation shift. It is known [2] that for the adopted reverberation model.

$$R_F(\tau) = \frac{\sum\limits_{n=1}^{(\frac{T}{\Delta t}-\tau)} C(n)\, C(n+\tau)}{\sum\limits_{n=0}^{T/\Delta t} C^2(n)}. \tag{14}$$

For an emitted signal of type (9) on the basis of (14) we find that

$$R_F(\tau) = \left(1-|\tau|\frac{\Delta t}{T}\right)\cos(\tau\omega_0 \Delta t). \tag{15}$$

As a statistical evaluation of the correlation coefficient $\hat{R}_F(\tau)$ we used the expression:

$$\hat{R}_F(\tau) = \frac{\sum\limits_{k=1}^{M} \hat{F}_k(n)\, \hat{F}_k(n+\tau)}{\sum\limits_{k=1}^{M} \hat{F}_k^2(n)}, \tag{16}$$

where M is the number of sample records of the modeled reverberation.

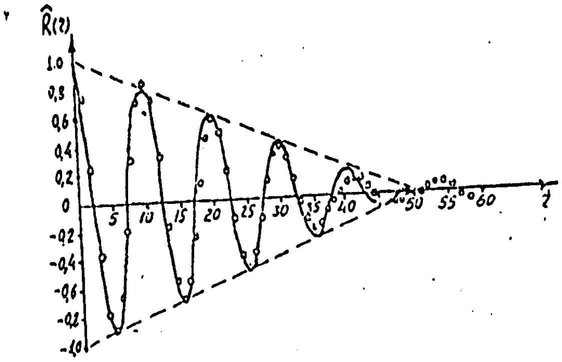

Fig. 4. Computed reverberation correlation coefficient (solid curve) and its statistical evaluation (circles) for sample set with M = 60 and M = 50.

Figure 4 shows the theoretical values of the correlation coefficient $R_F(\tau)$ computed using formula (15) and the statistical evaluations $\hat{R}_F(\tau)$ obtained by the processing of a sample reverberation set in accordance with (16). It follows from the comparison that at the correlation level the characteristics of the sample set correspond to the adopted mathematical model of reverberation.

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DIGITAL MODELING OF A SAMPLE SET OF A NONSTATIONARY RANDOM PROCESS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 66-73

[Article by Ye. V. Kirillov, V. V. Ol'shevskiy and Ye. A. Savinov]

[Text] Many types of random processes in hydroacoustics are nonstationary. Such processes include, for example, cavitation noise, sea reverberation, some types of direct hydroacoustic signals, echo signals, etc. (for example, see 1-3). One of the simplest models of a nonstationary random process $\chi(t)$ is a multiplicative constructive model of the type [3, 4]

$$\chi(t)=f(t)\xi(t) \tag{1}$$

where $\xi(t)$ is a stationary random process, f(t) is a known (determined) function.

In this case any k-th record of the $\chi_x(t)$ process $\chi(t)$ is determined as

$$\chi_k(t)=f(t)\xi_k(t) \tag{2}$$

where $\xi_x(t)$ is a record of the stationary component of the process

$$\chi(t)=\{\chi_k(t)\}$$

In modeling on an electronic computer we deal with discrete models, each record of which $\chi_x(n)$ is represented in the form:

$$\chi_k(n)=f(n)\xi_k(n)\,,\quad n=\frac{t}{\Delta t} \tag{3}$$

where n = 1,2,..., is the number of the current reading, the number of readings is a power of the number 2. Δt is the time discretization interval. The algorithm for the modeling of sample records

$$\hat{\chi}_k(n)=f(n)\hat{\xi}_k(n), \tag{4}$$

is represented in the structural diagram in Fig. 1.

In accordance with this model, the initial mass of data used is the mass of independent numbers $\{\hat{y}_i\}$, which is then subjected to so-called current weight summation [5]

$$\hat{\xi}_k(n) = \sum_{i=1}^{N} a_i \hat{y}_{n-i} \tag{5}$$

where a_i are weighting coefficients whose values are selected on the basis of the required stochastic model of the process $\xi(n)$, N is the number of the summed independent values

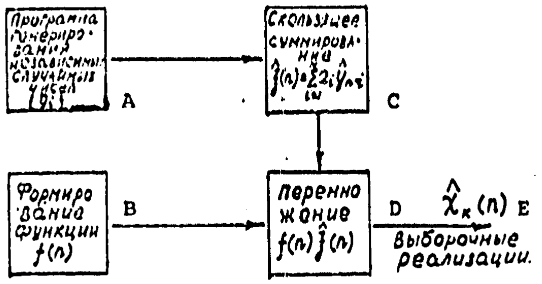

Fig. 1. Structural diagram of modeling of sample records $\hat{x}_k(n)$ of nonstationary random process

KEY:
- A) Program for generation of independent random numbers (g_i)
- B) Formation of function f(n)
- C) Moving summation...
- D) Multiplication...
- E) Sample records

Then an electronic computer is used in modeling the function f(n), and then, in accordance with (4) and (5), we obtain sample records of the nonstationary process

$$\hat{x}_k(n) = f(n)\sum_{i=1}^{N} a_i \hat{y}_{n-i} \cdot \tag{6}$$

In the described computer experiment a BESM-6 computer was used in modeling the g_i numbers conforming to a Gaussian distribution with a zero mean value and a unitary dispersion and the values of all the a_i coefficients were assumed equal to unity. Such $a_i = 1$ values correspond to the following autocorrelation function of the ergodic process $\xi(n)$

$$B_\xi(\tau) = d_\xi\left(1-\frac{|\tau|}{N}\right), \qquad |\tau| \le N, \tag{7}$$

where d_ξ is the dispersion of the process.

The distribution of the probabilities $\xi(n)$ should be Gaussian.

As the f(n) functions we used such

$$f(n) = 1 + m\sin\left(\frac{\pi\cdot 2}{N_0}n\right), \tag{8}$$

where m is the modulation intensity coefficient, N_0 is the number of readings of the function in the period of the modulating harmonic oscillation, and also

$$f(n) = exp(-\frac{n}{N_f}), \tag{9}$$

where N_f is the number of readings in the time interval in which f(n) decreases by a factor of θ.

The characteristic sample records $\hat{\chi}_k(n)$ of the nonstationary process $\chi(n)$ are represented in Figures 2-4. Figures 2 and 3 give the sample records of a periodically nonstationary random process

$$\hat{\chi}_k(n) = [1+m sin(n\frac{2\pi}{N_0})]\sum_{i=1}^{N} a_i \hat{y}_{n-i} \tag{10}$$

and Figure 4 shows a record of an aperiodically nonstationary process

$$\hat{\chi}_k(n) = exp(-\frac{n}{N_f})\sum_{i=1}^{N} a_i \hat{y}_{n-i} \tag{11}$$

Now we will examine the correlation characteristics of the modeled random process $\chi(n)$, whose records are determined in accordance with (3) and (8)

$$\chi_k(n) = [1+m sin(n\frac{2\pi}{N_0})]\xi_k(n), \tag{12}$$

where $\xi_x(n)$ is a record of the ergodic process $\xi(n)$ with the correlation function (7). According to the classification of the stochastic characteristics of hydroacoustic random processes 3 we should consider three types of correlation functions, to wit: t is the current (in this case n-current) correlation function

$$B_x(n,\tau) = \langle x_k(n)x_k(n+\tau)\rangle = \lim_{Q\to\infty}\frac{1}{Q}\sum_{k=-q}^{q} x_k(n)x(n+\tau), \quad Q=2q+1 \tag{13}$$

and k is the current correlation function

$$B_x(k,\tau) = \overline{x_k(n)x_k(n+\tau)} = \lim_{M\to\infty}\frac{1}{M}\sum_{n=-p}^{p} x_k(n)x_k(n+\tau), \qquad M=2p+1 \tag{14}$$

is the mean correlation function

$$B_x(\tau) = \langle\overline{x_k(n)x_k(n+\tau)}\rangle = \lim_{Q\to\infty}\lim_{M\to\infty}\frac{1}{QM}\sum_{k=-q}^{q}\sum_{n=-p}^{p} x_k(n)x_k(n+\tau) \tag{15}$$

Here $B_x(n,\tau)$ characterizes the nonstationary properties of the random process, $B_x(k,\tau)$ characterizes its inhomogeneous properties, and $B_x(\tau)$ characterizes the properties of the process as a whole.

First we will examine the n-current correlation function. According to (12) and (13)

$$B_x(n,\tau) = \lim_{Q\to\infty}\frac{1}{Q}\sum_{k=-q}^{q}[1+m sin(n\frac{2\pi}{N_0})\times[1+m sin(n+\tau)\frac{2\pi}{N_0}]]\xi_k(n)\xi_k(n+\tau) \tag{16}$$

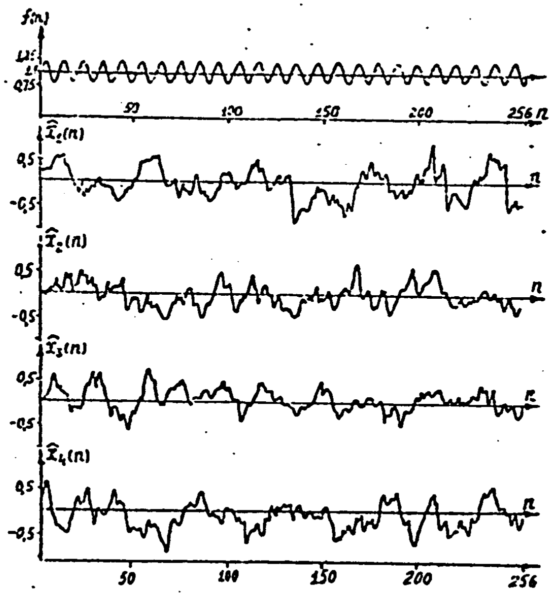

Fig. 2. Modulating function f(n); m = 0.25; N_0 = 10 and sample record $\hat{x}_k(n)$ of periodically nonstationary random process.

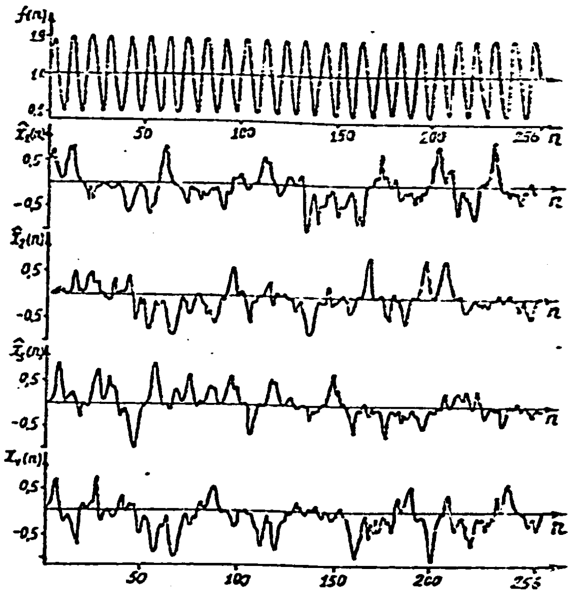

Fig. 3. Modulating function f(n); m = 0.9; N_0 = 10 and sample records $\hat{x}_k(n)$ of periodically nonstationary random process.

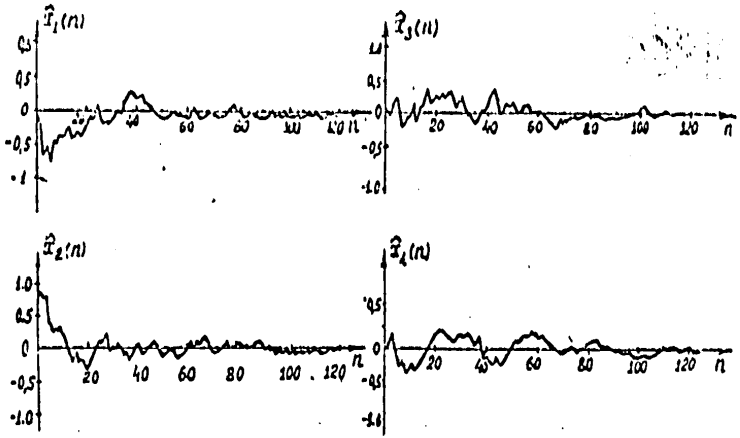

Fig. 4. Sample records $\hat{x}_k(n)$ of aperiodically nonstationary random process with modulating function $f_n = \exp(-n/N_f)$; $N_f = 120$.

Bearing in mind that

$$B_f(\tau) = \langle f_K(n) f(n+\tau) \rangle = \lim_{Q \to \infty} \frac{1}{Q} \sum_{K=-q}^{q} f_K(n) f_K(n+\tau) \quad (17)$$

is the correlation function of the ergodic process $\xi(n)$ of the type (7), for $B_x(n, \tau)$ in accordance with (16) and (17) we obtain

$$B_x(n,\tau) = d_\xi \left[1 + m \sin\left\{(n+\tau)\frac{2\pi}{N_0}\right\}\right]\left[1 + m \sin\left(n\frac{2\pi}{N_0}\right)\right]\left(1 - \frac{|\tau|}{N}\right) \quad (18)$$

in accordance with the definition

$$B_x(n, \tau) = \sqrt{d_x(n) d_x(n+\tau)}\; R_x(n, \tau), \quad (19)$$

where dx(n) and dx(n + τ) is the dispersion of the process $\chi(n)$ at the times of the reading n in n + τ, and R_x (n, τ) is the correlation coefficient. Comparing (18) and (19) we have:

$$d_x(n) = d_\xi \left[1 + m \sin\left(n \frac{2\pi}{N_0}\right)\right]^2 \quad (20)$$

$$d_x(n+\tau) = d_\xi \left[1 + m \sin\left((n+\tau) \frac{2\pi}{N_0}\right)\right]^2 \qquad R_x(n;\tau) = \left(1 - \frac{|\tau|}{N}\right) \quad (21)$$

It follows from (20) and (21) that only the dispersions are dependent on the current time reading and the correlation coefficient is not dependent on n.

Now we will examine the k-current correlation function. According to (12) and (14)

$$B_x(k,\tau) = \lim_{M \to \infty} \frac{1}{M} \sum_{n=-p}^{p} \left[1 + m \sin\left(n \frac{2\pi}{N_0}\right)\right]\left[1 + m \sin\left((n+\tau) \frac{2\pi}{N_0}\right)\right] \times \xi_k(n) \xi_k(n+\tau) \quad (22)$$

After the corresponding transformations expression (22) is reduced to the following form:

$$B_x(K,\tau)=\left[1+\frac{m^2}{2}\cos\left(\tau\frac{2\pi}{N_0}\right)\right]B_\xi(\tau), \tag{23}$$

where

$$B_\xi(\tau)=\overline{\xi_K(n)\xi_K(n+\tau)}=\lim_{M\to\infty}\frac{1}{M}\sum_{n=-p}^{p}\xi_K(n)\xi_K(n+\tau) \tag{24}$$

Taking (7), (23) and (24) into account, we find that

$$B_x(K,\tau)=d_\xi\left[1+\frac{m^2}{2}\cos\left(\tau\frac{2\pi}{N_0}\right)\right]\left(1-\frac{|\tau|}{N}\right), \tag{25}$$

It follows from (25) that the dispersion dx and the correlation coefficient $R_x(k, \tau)$ will be equal, respectively, to

$$d_x=d_\xi\left(1+\frac{m^2}{2}\right) \tag{26}$$

$$R_x(K,\tau)=\frac{1+\frac{m^2}{2}\cos\left(\tau\frac{2\pi}{N_0}\right)}{1+\frac{m^2}{2}}\left(1-\frac{|\tau|}{N}\right) \tag{27}$$

An analysis of the derived expressions (20), (21), (26) and (27) shows that the considered random process $\chi(n)$ is nonstationary with respect to dispersion (t -- the current correlation coefficient -- is not dependent on the reading moment and the modulation intensity coefficient) and is homogeneous; however, k -- the current correlation coefficient -- is dependent on the modulation intensity.

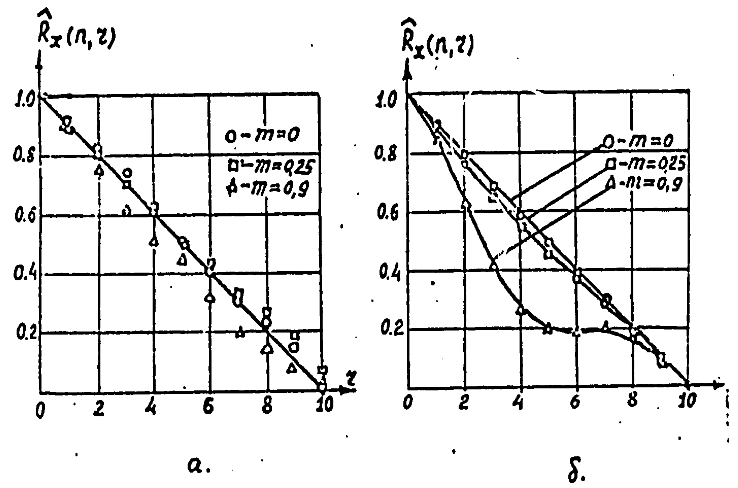

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Fig. 5. Statistical evaluations of correlation coefficients. a) t -- current correlation coefficient Q = 205; the solid line represents the computed values $R_x(n, \tau)$ using formula (21); b) k -- current correlation coefficient, M = 2048; the curves represent the computed values $R_x(k, \tau)$ using formula (27).

Figure 5 represents statistical evaluations of the correlation coefficients $R_x(\tau)$ corresponding to expressions (21) and (27) on the assumption that $N = 10$, $N_0 = 10$, that is, the correlation interval of the $\xi(n)$ process coincided with the period of the modulating function.

As indicated by the cited curves, at the correlation level of the description the random process was modeled satisfactorily.

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USE OF MULTIVARIATE STATISTICAL ANALYSIS METHODS IN HYDROACOUSTIC DIAGNOSIS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 91-96

[Article by V. M. Levin, V. P. Lesunovskiy and V. K. Maslov]

[Text] The construction or repair of modern hydroacoustic complexes in some cases involves the necessity for working under water with power or mechanical equipment. Deep-water drilling, deep pumps of oil wells [1], the underwater grader of the "bed" under hydraulic structures, etc. operate under water.

An increase in the reliability of systems and apparatus, the need for routine monitoring of their technical condition, and also the considerable expenditures arising during lowering and raising, make it desirable to carry out the monitoring of operating regimes and technical conditions during normal operation. However, the specifics of underwater operation frequently make it necessary to check the apparatus without direct contact with them, for example, on the basis of the noise emitted into the water. Examples of acoustic diagnosis of engines and mechanisms are known [2, 3].

In this paper we give the results of an experimental investigation of the possibility of automatic monitoring of mechanical systems and diagnosis of their malfunctions on the basis of the emitted hydroacoustic noise. The complexities involved in ensuring water-tightness and underwater descent led to the following experimental scheme: the apparatus to be monitored was mounted on a light barge, the receiving base was lowered under water to a depth as great as 50 m. The investigated objects were three diesels of the 2MCh make (two-cylinder, four-stroke), mounted in turn on the barge. The experiments were carried out periodically over a period of two years. Two "typical" regimes were selected for control purposes: P_1 -- a normal operating regime, P_2 -- the engine operated with one "damaged" cylinder. The received hydroacoustic signals were subjected to 1/3-octave spectral analysis in the infrasonic (25 cps + 160 cps) and sonic (160 Hz + 1.0 KHz) frequency ranges. Each measurement was represented by 19 spectral parameters (the 18th and 19th parameters are the integral levels in the indicated ranges) or according to the geometrical interpretation [4] -- the vector (point) x in 19-dimensional Euclidean space of the spectral description X. The total volume of measurements is 119 records in the first regime and 133 records in the second regime.

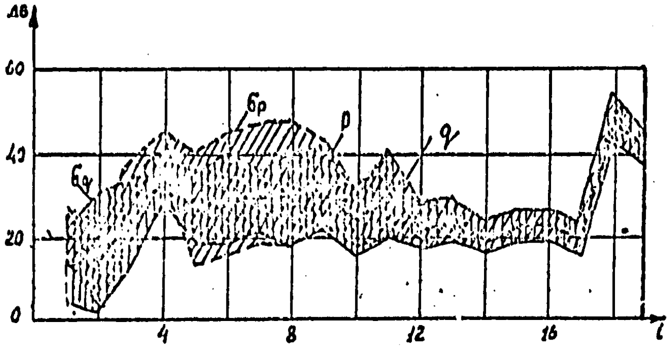

Fig. 1.

The random nature of the appearance of vibroacoustic signals, the complexity of the processes of their transmission into the water and propagation in it and the individual peculiarities of the engines to be monitored for all practical purposes do not make it possible to describe a "spectral model" of the malfunction in definite terms. In addition, the received signals were encumbered by noise by the background of other operating mechanisms and the measurements themselves were displaced in time and carried out under different weather conditions. Therefore, for detecting the diagnostic characteristics of condition of the engine on the basis of the characteristics of underwater noise we used a statistical model of the theory of image recognition [4, 5].

Figure 1 shows the mean sample spectrograms of "properly operating" and "unfit" (p) classes, the standard deviations $\pm\sigma_i$ of levels in each filter and the extremal (maximum and minimum) sample values of these levels.

The figure shows that on the average the noise level for the improperly operating engines was somewhat higher than for the properly operating engines. In the Euclidean distance between the sample means, equal to ρ = 6.75, the greatest contribution was made by filters 1, 3 and 4 (25 Hz, 40 Hz, 50 Hz), which reflects well the appearance of a sound series with Δ f = 12.5 Hz (25 Hz, 37.5 Hz, 50 Hz...).

However, the differences between the classes, noted on the basis of the most stable characteristics (mean spectra), completely level out the variability of individual records (see Fig. 1). At the same time, in an analysis of the correlation structure of the spectral makeup of underwater noise we discovered quite high levels of statistical correlation between individual parts of the spectrum reflecting the noise of both the well-functioning and malfunctioning engines. For example, the correlation coefficients between "distant" (nonadjacent) filters attain 0.8-0.95, which can be attributed to the "sound series" structure of the vibrosignals. This fact was used essentially in seeking diagnostic criteria.

Table 1

Номер фильтра 1	Величина K_{qpi} 2	номер фильтра 1	Величина K_{qpi} 2	Номер фильтра 1	Величина K_{qpi} 2
I	0,67	7	0,I6	I3	0,05
2	0,08	8	0,08	I4	0,00
3	0,30	9	0,0I	I5	0,0I
4	0,32	I0	0,02	I6	0,09
5	0,0I	II	0,0I	I7	0,I8
6	0,02	I2	0,07	I8	0,20
				I9	0,03

KEY:

1. Number of filter
2. Value K_{qpi}

In the first approximation the information content of individual spectral parameters for identification of properly operating and malfunctioning engines can be evaluated using the one-dimensional separability criterion K_{qpi} [4]:

$$K_{qpi} = \frac{|M_{qi} - M_{pi}|}{\sigma_{qi} + \sigma_{pi}} ,$$

where M_{qi}, M_{pi}, σ_{qi}, σ_{pi} are the mean values of the investigated regimes and the standard deviations in the i-th filter.

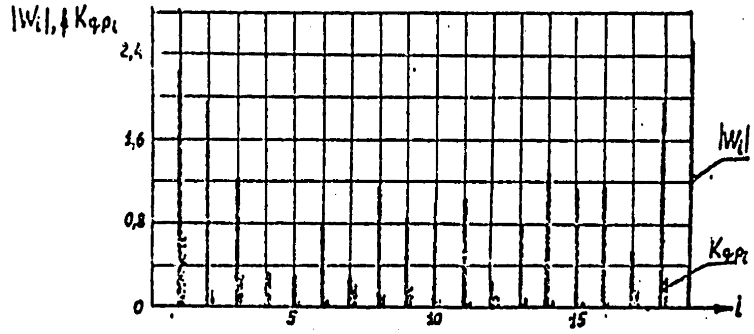

Fig. 2.

The values of this criterion cited in Table 1 show that the initial spectral characteristics are poorly adapted for differentiating the investigated conditions of the engines (all $K_{qpi} < 1$).

For forming of the "adapted" description (discriminant criteria) and "concentration" of the separating information we used the methods set forth in [4, 5]. The algorithmic criteria were formed from the "instructional" material formed from the 19th and 13th records of the first and second classes respectively.

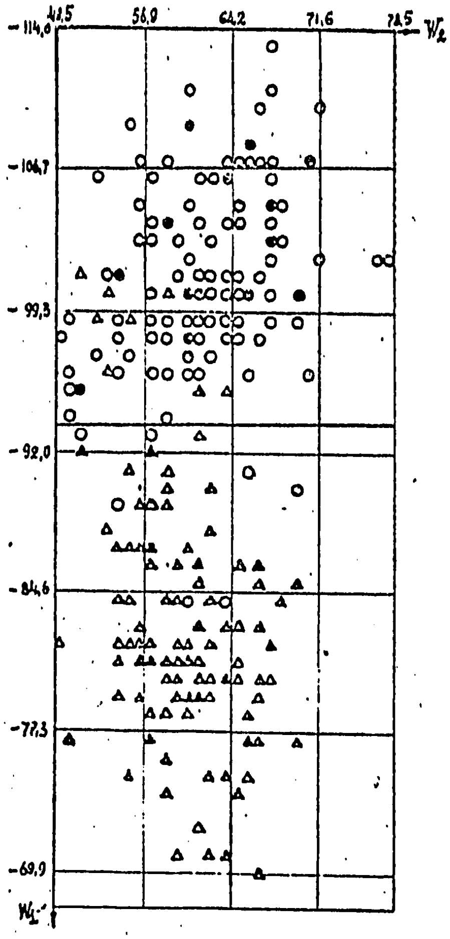

Fig. 3.

The criteria subspace base was determined by solution of the system

$$(\tilde{K} - \lambda_i \bar{R}) W_i = 0 \qquad (i = 1,2),$$

where W_i is the base spectrum (algorithmic criterion); $\tilde{K}$ is the weighted interclass matrix of covariations of the spectral makeup of engine noise; $\bar{R}$ is the averaged intraclass covariation matrix, λ_i is the Lagrange factor.

The system of base spectra W_i can be regarded as a system of linear filters in the frequency region for which the base spectra play the role of the "pulse responses" of these filters and the signal is the spectral makeup of engine noise.

The experimentally determined algorithmic criteria can be interpreted as some diagnostic combinations of spectral parameters. An analysis indicated that the first diagnostic combination (W_1 criterion) carried the most information. The value of the one-dimensional separability criterion according to this W_1 value is equal to 1.846, that is, exceeds by a factor of almost three the similar criterion of the most informative first filter ($K_{qpx1} = 0.67$).

Figure 2 shows the information content of the spectral characteristics computed independently of one another (K_{qpi}), and with their statistical correlation ($|W_i|$) taken into account.

Figure 3 shows the spectral distributions in the two most informative (with respect to K_{qpi}) coordinates (algorithmic criteria) W_1 and W_2. The blackened symbols represent the instructional sample.

Figure 3 shows that the investigated conditions of the engines are quite reliably differentiated using a very simple threshold rule:

$$y = \begin{cases} \sum_{i=1}^{19} W_{1i}\, x_i < \delta & \text{well-operating engine;} \\ \sum_{i=1}^{19} W_{1i}\, x_i > \delta + \Delta & \text{damaged cylinder.} \end{cases}$$

The distribution of the test records makes it possible to conclude that the W_i criterion has high extrapolation properties.

Thus, using the methods of multivariate statistical analysis the difference in structure of the spectra for well-functioning and improperly functioning engines could be described by the one algorithmic criterion W_i. The latter has a low "sensitivity" to the measurement conditions and individual peculiarities of the specific engine.

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FFT EQUIVALENT OF THE WIENER-KHINCHIN THEOREM FOR A NONHOMOGENEOUS NONSTATIONARY RANDOM WAVE FIELD

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 109-111

[Article by V. A. Geranin]

[Text] The spectral-correlation analysis of spatial-temporal information systems with use of a fast Fourier transform is possible if the characteristics of the effect are represented in the form of a finite Fourier transform (FFT).

The FFT described below is an equivalent of the Wiener-Khinchin theorem for a nonhomogeneous nonstationary random wave field. We examine the acoustic field $\varphi(x, y, z, t)$ in the water $(z > 0)$ excited by a "noise spot" situated on the plane surface $z = 0$ of a deep sea.

A pair of FFT, relating the field and its spectral function $S(k_x, k_y, k_z, \omega)$, has the form:

$$\varphi^{***}(n\Delta_x, m\Delta_y, z\Delta_z, r\Delta_t) = \frac{\Delta_x \Delta_y \Delta_\omega}{(2\pi)^3} \sum_{p=0}^{N_x-1} \sum_{s=0}^{N_y-1} \sum_{q=0}^{N-1} S^{***}(p\nabla_x, s\nabla_y, q\nabla_\omega) \times W_N^{qr} W_{N_x}^{-pn} W_{N_y}^{-sm} W_{N_z}^{-\ell z}, \tag{1}$$

$$S^{***}(p\nabla_x, s\nabla_y, q\nabla_\omega) = \Delta_x \Delta_y \Delta_t \sum_{n=0}^{N_x-1} \sum_{m=0}^{N_y-1} \sum_{r=0}^{N-1} \varphi^{***}(n\Delta_x, m\Delta_y, 0, r\Delta_t) \times W_N^{-qr} W_{N_x}^{pn} W_{N_y}^{sm}, \tag{2}$$

where Δ_x, Δ_y, Δ_z, Δ_t are the quantization intervals in space coordinates and time respectively, ∇x, ∇y, ∇z, $\nabla\omega$ are the quantization intervals for the space frequencies k_x, k_y, k_z and the frequency ω,

$$\Delta_x \nabla_x = \frac{2\pi}{N_x}, \quad \Delta_y \nabla_y = \frac{2\pi}{N_y}, \quad \Delta_z \nabla_z = \frac{2\pi}{N_z}, \quad \Delta_t \nabla_t = \frac{2\pi}{N}, \tag{3}$$

$$\ell \nabla_z = \sqrt{\left(\frac{q\nabla_\omega}{c}\right)^2 - (p^2 \nabla_x^2 + s^2 \nabla_y^2)}, \tag{4}$$

$$\Gamma^*(\lambda) = \sum_{i=-\infty}^{\infty} \Gamma(\lambda + i\Lambda) \tag{5}$$

The Wiener-Khinchin theorem applicable to a nonhomogeneous nonstationary random wave field in FFT language is represented as follows:

$$B^{********}(n_1\Delta_x, m_1\Delta_y, z_1\Delta_z, \gamma_1\Delta_t; n_2\Delta_x, m_2\Delta_y, z_2\Delta_z, \gamma_2\Delta_t) = \nabla_x^2\nabla_y^2\nabla_\omega^2 \sum_{p_1=0}^{N_x-1}\sum_{s_1=0}^{N_y-1}\sum_{q_1=0}^{N-1}\sum_{p_2=0}^{N_x-1}\sum_{s_2=0}^{N_y-1}\sum_{q_2=0}^{N-1} \psi^{******}(p_1\nabla_x, s_1\nabla_y, q_1\nabla_\omega; p_2\nabla_x, s_2\nabla_y, q_2\nabla_\omega)\times \times W^{(q_2 n - q_1 n)} \times W_{N_x}^{-(p_1 n_1 - p_2 n_2)} W_{N_y}^{-(s_1 m_1 - s_2 m_2)} W_{N_z}^{-(\ell_1 z_1 - \ell_2 z_2)} \tag{6}$$

$$\psi^{******}(p_1\nabla x, s_1\nabla y, q_1\nabla\omega; p_2\nabla x, s_2\nabla y, q_2\nabla\omega) = \frac{\Delta_x^2\Delta_y^2\Delta_t^2}{(2\pi)^6} \sum_{n_1=0}^{N_x-1}\sum_{m_1=0}^{N_y-1}\sum_{t_1=0}^{N-1}\sum_{n_2=0}^{N_x-1}\sum_{m_2=0}^{N_y-1}\sum_{t_2=0}^{N-1} B^{******}(n_1\Delta x, m_1\Delta y, 0, t_1\Delta t; n_2\Delta x, m_2\Delta y, 0, t_2\Delta t)\, W_N^{-(q_1 t_1 - q_2 t_2)} W_{N_x}^{(p_1 n_1 - p_2 n_2)} W_{N_y}^{(s_1 m_1 - s_2 m_2)} \tag{7}$$

Here

$B(x_1, y_1, z_1, t_1, x_2, y_2, z_2, t_2)$

$\psi(k_{x_1}, k_{y_1}, \omega_1; k_{x_2}, k_{y_2}, \omega_2)$

are the space-time correlation function and the frequency and direction correlation function respectively.

The spectrum φ (k_{x1}, k_{y1}, ω_1; x_2, y_2, z_2, t_2) [1] and the field spatial-temporal correlation function form a FFT pair in the form:

$$B^{********}(n_1\Delta x, m_1\Delta y, z_1\Delta z, t_1\Delta t; n_2\Delta x, m_2\Delta y, z_2\Delta z, t_2\Delta t) = \nabla x\nabla y\nabla\omega \sum_{p_1=0}^{N_x-1}\sum_{s_1=0}^{N_y-1}\sum_{q_1=0}^{N-1} \varphi^{*******}(p_1\nabla x, s_1\nabla y, q_1\nabla\omega; n_2\Delta x, m_2\Delta y, z_2\Delta x, t_2\Delta t)\, W_N^{q_1(t_1-t_2)} W_{N_x}^{-p_1(n_1-n_2)} W_{N_y}^{-s_1(m_1-m_2)} W_{N_z}^{-\ell_1(z_1-z_2)}, \tag{8}$$

$$\varphi^{******}(p_1\nabla x, s_1\nabla y, q_1\nabla\omega; n_2\Delta x, m_2\Delta y, 0, t_2\Delta t) = \frac{\Delta x\Delta y\Delta t}{(2\pi)^3} \sum_{n_1=0}^{N_x-1}\sum_{m_1=0}^{N_y-1}\sum_{t_1=0}^{N-1} B^{******}(n_1\Delta x, m_1\Delta y, 0, t_1\Delta t; n_2\Delta x, m_2\Delta y, 0, t_2\Delta t)\, W_N^{-q_1(t_1-t_2)} W_{N_x}^{p_1(n_1-n_2)} W_{N_y}^{s_1(m_1-m_2)} \tag{9}$$

The field spectrum and the frequency and direction correlation function are related by the expressions

$$\varphi^{*******}(p_1\nabla x, s_1\nabla y, q_1\nabla\omega; n_2\Delta x, m_2\Delta y, z_2\Delta z, t_2\Delta t) = \nabla x\nabla y\nabla\omega \sum_{p_2=0}^{N_x-1}\sum_{s_2=0}^{N_y-1}\sum_{q_2=0}^{N-1} \psi^{******}(p_1\nabla x, s_1\nabla y, q_1\nabla\omega; p_2\nabla x, s_2\nabla y, q_2\nabla\omega)\times \times W_N^{-(q_2-q_1)t_2} W_{N_x}^{(p_2-p_1)n_2} W_{N_y}^{(s_2-s_1)m_2} W_{N_z}^{(\ell_2-\ell_1)z_2}, \tag{10}$$

$$\psi^{''''''}(p_1\nabla_x, s_1\nabla_y, q_1\nabla_\omega; p_2\nabla_x, s_2\nabla_y, q_2\nabla_\omega) = \frac{\Delta x\Delta y\Delta t}{(2\pi)^3}\sum_{n_1=0}^{N-1}\sum_{m_1=0}^{M-1}\sum_{t_1=0}^{T-1}\Phi^{******}(p_1\nabla_x, s_1\nabla_y, q_1\nabla_\omega; n_1\Delta x, m_1\Delta y, 0, t_1\Delta t)\times W_N^{(q_2-q_1)t_1} W_{Nx}^{-(p_2-p_1)n_1} W_{Ny}^{-(s_2-s_1)m_1} \tag{11}$$

The FFT pair relating the function of the difference arguments for time, coordinates, frequency and directions with the spatial-temporal correlation function reads as follows:

$$\varkappa^{******}(f\nabla_x, h\nabla_y, \mu\nabla_\omega, b\Delta x, d\Delta y, 0, \rho\Delta t) = \frac{\Delta x\Delta y\Delta t}{(2\pi)^3}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\sum_{t=0}^{T-1} B^{******}[(n+b)\Delta x, (m+d)\Delta y, 0, (\delta+\rho)\Delta t; n\Delta x, m\Delta y, 0, \delta\Delta t]\, W_N^{\mu\delta} W_{Nx}^{-fn} W_{Ny}^{-hm}, \tag{12}$$

$$B^{********}[(n+b)\Delta x, (m+d)\Delta y, (z+a)\Delta z, (\delta+\rho)\Delta t; n\Delta x, m\Delta y, z\Delta z, \delta\Delta t] = \nabla_x\nabla_y\nabla_\omega\sum_{f=0}^{N_x-1}\sum_{h=0}^{N_y-1}\sum_{\mu=0}^{N-1}\varkappa^{********}(f\nabla_x, h\nabla_y, \mu\nabla_\omega; b\Delta x, d\Delta y, a\Delta z, \rho\Delta t)\, W_N^{-\mu\delta} W_{Nx}^{fn} W_{Ny}^{hm} W_{Nz}^{(\nu\cdot\ell\cdot z)z}, \tag{13}$$

The relationship between the function of the difference arguments for time, coordinates, frequency and directions and the frequency and directions correlation function is expressed as follows:

$$\varkappa^{*******}(f\nabla_x, h\nabla_y, \mu\nabla_\omega; b\Delta x, d\Delta y, a\Delta z, \rho\Delta t) = \nabla_x\nabla_y\nabla_\omega\sum_{p=0}^{N_x-1}\sum_{s=0}^{N_y-1}\sum_{q=0}^{N-1} Q^{******}[p\nabla_x; s\nabla_y, q\nabla_\omega; (p+f)\nabla_x, (s+h)\nabla_y, (q+\mu)\Delta z]\, W_N^{q\rho} W_{Nx}^{-pb} W_{Ny}^{-sd} W_{Nz}^{-\ell a}, \tag{14}$$

$$Q^{******}[p\nabla_x, s\nabla_y, q\nabla_\omega; (p+f)\nabla_x, (s+h)\nabla_y, (q+\mu)\nabla_z] = \frac{\Delta x\Delta y\Delta t}{(2\pi)^3}\sum_{b=0}^{N-1}\sum_{d=0}^{N-1}\sum_{\rho=0}^{N-1}\varkappa^{******}(f\nabla_x, h\nabla_y, \mu\nabla_\omega; b\Delta x, d\Delta y, 0, \rho\Delta t)\times W_N^{-q\rho} W_{Nx}^{pb} W_{Ny}^{sd} \tag{15}$$

Finally, the relationship between the function of the difference arguments and field spectrum in FFT form is as follows:

$$\varkappa^{********}(f\nabla_x, h\nabla_y, \mu\nabla_\omega; b\Delta x; d\Delta y, a\Delta z, \rho\Delta t) = \frac{\Delta x\Delta y\Delta t\nabla_x\nabla_y\nabla_\omega}{(2\pi)^3}\sum_{p=0}^{N_x-1}\sum_{s=0}^{N_y-1}\sum_{q=0}^{N-1}\sum_{n=0}^{N_x-1}\sum_{m=0}^{N_y-1}\sum_{t=0}^{N-1}\Phi^{******}(p\nabla_x, s\nabla_y, q\nabla_\omega; n\Delta x, m\Delta y, 0, \delta\Delta t)\, W_N^{(q\rho+\mu\delta)} W_{Nx}^{-(pb+fn)} W_{Ny}^{-(sd+hm)} W_{Nz}^{-\ell a}, \tag{16}$$

$$\Phi^{******}(p\nabla_x, s\nabla_y, q\nabla_\omega; n\Delta x, z\Delta y, \delta\Delta t) = \frac{\Delta x\Delta y\Delta t\nabla_x\nabla_\omega\nabla_y}{(2\pi)^3}\sum_{f=0}^{N_x-1}\sum_{h=0}^{N_y-1}\sum_{\mu=0}^{N-1}\sum_{b=0}^{N_x-1}\sum_{d=0}^{N_y-1}\sum_{\rho=0}^{N-1}\varkappa^{******}(f\nabla_x, h\nabla_y, \mu\nabla_\omega; b\Delta x, d\Delta y, 0, \rho\Delta t)\, W_N^{-(q\rho+\mu\delta)} W_{Nx}^{(pb+fn)} W_{Ny}^{(sd+hm)} W_{Nz}^{(\nu-\ell+z)z} \tag{17}$$

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INFLUENCE OF DISCRETIZATION AND QUANTIZATION ON THE CHARACTERISTICS OF A DIGITAL QUADRATURE-CORRELATION DETECTOR

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 112-116

[Article by K. B. Krukovskiy-Sinevich and V. V. Mikhaylovskiy]

[Text] The use of complex sounding signals in echo sounding systems makes it possible not only to increase considerably the reliability achieved in detection, but also to increase resolution with respect to both range and velocity. In this connection, recently studies have been carried out for creating quite reliable and small processing devices for investigating the fine structure of the echo signal. One of the most promising directions in the creation of such devices is the use of digital devices, and especially quadrature-correlation detectors.

By their very nature digital processing systems require preliminary time discretization and level quantization of both input and reference signals. In this process inevitable losses arise in the form of a deterioration of noise immunity and distortion of the output signals. For a number of problems it is necessary to use such digital processing methods for which the discretization procedure with an insignificant deterioration of noise immunity would lead to minimum distortions of the shape of the processed signal. At present there have been no systematic investigations in this field whose results would make it possible to formulate requirements on the number and distribution of the time- and level-quantization intervals with stipulated distortions of the shape of the processed signal. It is of interest to investigate the errors of a uniform approximation. This field of mathematics does not have such a powerful and quite simple basis as the theory of a mean square approximation [1]. As a result, use here is made of methods for modeling on an electronic computer. The authors selected precisely such an approach. The modeling was carried out for a quadrature-correlation detector for which the reference signals with an accuracy to the initial phase coincide with the expected signals.

As is well known, the signal at the output of such a detector is a function of the relative lag (τ) of the input and reference signals and is described by the expression:

$$U_2(\tau)=\{(\int_{-\infty}^{\infty}U_1(t-\tau)A(t)\cos[\omega_0 t+\varphi(t)]dt)^2+(\int_{-\infty}^{\infty}U_1(t-\tau)A(t)\sin[\omega_0 t+\varphi(t)]dt)^2\}^{1/2} \qquad (1)$$

where $U_1(t)$ is the input signal, $U_2(\tau)$ is the output signal, $A(t)$ and $\varphi(t)$ are the envelope and modulating function of the echo signal.

The computations were made both in the case of a fixed and a uniformly moving target for two types of signals:

-- signal with linear frequency modulation

$$U_1(t)=\cos[\omega_0 t+\beta t^2+\varphi_0] \qquad 0\leq t\leq T$$

-- signal with quadratic frequency modulation

$$U_1(t)=\cos[\omega_0 t+\gamma t^3+\varphi_0] \qquad 0\leq t\leq T$$

In addition, we studied a detector of a linear frequency modulated signal with weight processing by the Hemming method [1], where

$$A(t)=0,08+0,92\cos^2[\pi(\frac{t-\frac{T}{2}}{T})] \qquad 0\leq t\leq T$$

After substitution into (1)

$$U_1(t-\tau)=\cos[\omega_0(t_0-\tau)+\varphi(t-\tau)+\varphi_0]$$

and simple trigonometric transformations we obtain:

$$\begin{aligned}U_2(\tau)=\{(&\int_{\tau}^{T}A(t)\cos[\varphi(t)+\varphi_0]\cos[\varphi(t-\tau)]dt+\\&+\int_{\tau}^{T}A(t)\sin[\varphi(t)+\varphi_0]\cdot\sin[\varphi(t-\tau)]dt)^2+\\&+(\int_{\tau}^{T}A(t)\sin[\varphi(t)+\varphi_0]\cos[\varphi(t-\tau)]dt-\\&-\int_{\tau}^{T}A(t)\cos[\varphi(t)+\varphi_0]\cdot\sin[\varphi(t-\tau)]dt)^2\}^{1/2}\end{aligned} \qquad (2)$$

In accordance with expression (2) the model for the digital detector was formulated using the following algorithm:

$$\begin{aligned}U_2(\tau_i)=\{(&\sum_{k=1}^{N-i+1}A(\tfrac{T}{N}k)\cos[\varphi(\tfrac{T}{N}k)+\varphi_0]\cdot\cos[\varphi(\tfrac{T}{N}k-\tfrac{T}{N}i)]+\\&+\sum_{k=1}^{N-i+1}A(\tfrac{T}{N}k)\cdot\sin[\varphi(\tfrac{T}{N}k)+\varphi_0]\cdot\sin[\varphi(\tfrac{T}{N}k-\tfrac{T}{N}i)])^2+\\&+(\sum_{k=1}^{N-i+1}A(\tfrac{T}{N}k)\cdot\sin[\varphi(\tfrac{T}{N}k)+\varphi_0]\cdot\cos[\varphi(\tfrac{T}{N}k-\tfrac{T}{N}i)]-\\&-\sum_{k=1}^{N-i+1}A(\tfrac{T}{N}k)\cos[\varphi(\tfrac{T}{N}k)+\varphi_0]\cdot\sin[\varphi(\tfrac{T}{N}k-\tfrac{T}{N}i)])^2\}^{1/2}\end{aligned}$$

where $U_2(\tau_i)$ is the discrete value of the detector output effect; N is the number of signal discretization intervals, i = 1, 2,...N.

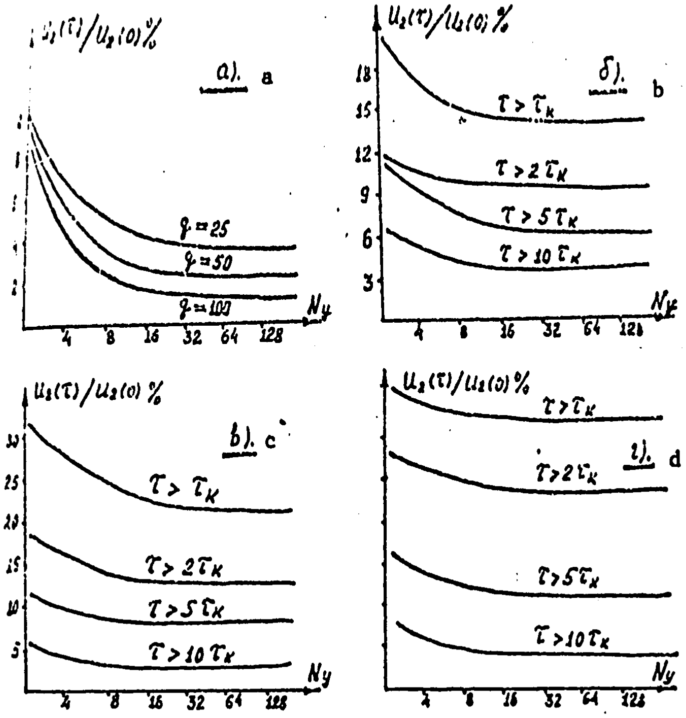

Fig. 1. Influence of quantization procedure on magnitude of side lobes of processed signal: a) LFM-detector with weight processing by Hemming method in case of a fixed target; b) LFM-detector with weight processing by the Hemming method with $\Delta f_{add} = 0.25f_{deviation}$; c) LFM-detector without weight processing; d) detector with quadratic frequency modulation.

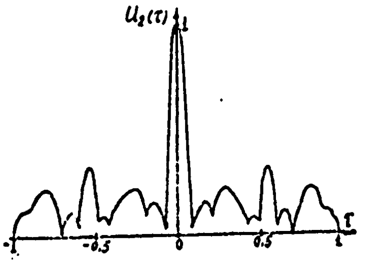

Fig. 2. Output effect of PRIS filter.

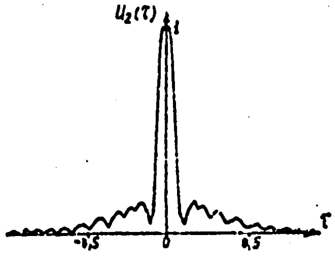

Fig. 3. Output effect of digital analog of PRIS filter with inversely proportional weight processing.

N is the number of signal discretization intervals, $i = 1, 2, \ldots N$.

The program for determining the input and reference signals provided for the use of a floating decimal point with six decimal places. For modeling the level quantization procedure the reference signals were rounded off, taking into account the required quantization interval. The results of the modeling, presented in Fig. 1, make it possible to evaluate the dependence of the increase in the relative level of the secondary side lobes with a decrease in the number of the quantization intervals. Figure 1a shows the characteristics of a detector for a LFM signal with weighting processing by the Hemming method for a fixed target for different coefficients of signal complexity q; Figure 1b shows similar dependences for a moving target, where $\Delta f_{\text{Doppler}} = 0.25$ f of the deviation. Figures 1c and 1d make it possible to evaluate the influence of the quantization procedure for a LFM signal without weighting processing and a signal with quadratic FM with the complexity q = 15. In an investigation of the influence of level quantization for quadratic FM and for FM of signals with Doppler distortions (Figures 1b-1d) there are definite methodological difficulties attributable to the fact that quantization leads to a different change in the levels $U_2(\tau)$ with different τ. In these cases an evaluation was made of the maximum level of the side lobes for $\tau > M\tau_k$, where τ is the interval of strong correlation of the undistorted signal, M is a natural number. As can be seen from the curves shown in Fig. 1, a decrease in the number of quantization intervals to 16/8 for positive values of the reference signals and 8 for negative values does not lead to significant distortions of the processed signal. It must be taken into account that the number of quantization intervals for the input signal is selected on the basis of the required dynamic range.

The distortions in the shape of the output signal caused by uniform time discretization were examined in considerable detail in the periodic literature. For example, the author of [2] determined the mean square error in distortion of form for digital detectors, and in [3] -- for discrete-analog detectors. Considerably lesser attention has been devoted to the errors caused by discretization, the frequency of which is also a function of time, although the use of such a procedure makes it possible to simplify the digital processor [4]. For example, the author of [5] proposed the analog device PRIS, a filter constituting a delay line with "branches." In order to compensate the energy losses caused by a decrease in the number of "branches," the author proposes that there be a corresponding weighting processing of the signal before the summator. The value of each particular weighting coefficient is directly proportional to the lag introduced by the delay line between a particular and adjacent branches.

A discrete variant of such a device would be a quadrature-correlation detector in which the input oscillation would experience discretization with a constant frequency and all the samples of the reference signal, other than the maximum values for each half-period of the modulating function, would be equal to zero. The amplitudes of the remaining samples should be directly proportional to the time interval between the corresponding zeroes of the modulating function. It is evident that for such a detector of the LFM signal the number of multiplication operations in each quadrature channel is reduced by a factor of approximately 3. Modeling indicated that the output effect of such a detector has a considerable increase in the secondary side lobes (Fig. 2). In actuality, an increase in some samples at the expense of others seemingly leads to a "constriction" of the reference signal energy to individual discrete intervals and accordingly to an increase in the cross-correlation interval in the case of sufficiently great delay times.

Proceeding on this basis, in order to decrease the level of the side lobes it is desirable to decrease the remaining samples, not increase them. Figure 3 shows the output effect for the detector of a LFM signal for which the levels of the remaining samples are inversely proportional to the time interval between the corresponding zeroes of the modulating function. A comparison of Figures 2 and 3 shows that such a representation of the reference signal makes possible a considerable decrease in the level of the side lobes of the processed signal. However, it must be taken into account that such weighting processing leads to a decrease in the noise immunity of the detector. Therefore in the designing of devices of such a type the optimum technical system solution must be sought as a compromise between the mentioned factors.

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SYNTHESIS OF HYDROACOUSTIC SIGNALS IN THE REGION OF STRONG CORRELATION OF THE VELOCITY-LAG UNCERTAINTY FUNCTION

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 117-136

[Article by K. B. Krukovskiy-Sinevich]

[Text] #1. Introduction. In connection with the complication of the tactical problems to be solved by modern echo sounding systems, the problem of the synthesis of complex signals with a stipulated uncertainty (ambiguity) function during recent years has been devoted considerable attention. In particular, we should mention studies [1-5]. However, the mentioned studies are characterized by definite limitations which are caused to a considerable degree by their radar directivity. Radioelectronic sonar methods are characterized by the following relationships:
-- the velocities of the targets are extremely small in comparison with the velocity of signal propagation;
-- the required time resolution (lag) is considerably less than $1/f_{Dop}$, where f_{Dop} is the Doppler shift of the carrier.

In these cases, as a rule, it is possible to use a simplified form of the two-dimensional velocity-lag uncertainty function proposed by Woodward:

$$R(\tau, f_{\text{д}}) = \int_{-\infty}^{\infty} s(t-\tfrac{\tau}{2})\, s^{*}(t+\tfrac{\tau}{2})\, exp\, j 2\pi f_{\text{доп}} t\, dt, \qquad (1)$$

[доп= Dop] where s(t) is a complex signal (complex envelope).

For an uncertainty function of the type (1) it is possible to develop methods for signal synthesis in a quite general form. Use is made of the concept of distance in some generalized space and a signal having the uncertainty function closest to the stipulated function is determined [2, 5].

In addition, if it is assumed that the uncertainty function is described by (1), it is also possible to formulate the conditions under which it can be applied (2), (5). However, it appears that known methods of signal synthesis using a simplified uncertainty function are still far from that which could be used extensively in the planning of echo sounding systems.

We will note only the two most important reasons for such a situation.

If it is unknown whether the stipulated uncertainty function is practicable, it is impossible to guarantee a high accuracy in the approximation to it; existing methods make it possible to evaluate only the mean square approximation (2). For practical purposes, however, it is important to ensure a uniform approximation, which, in particular, guarantees an admissible level of the secondary maxima of the uncertainty function.

On the other hand, the evaluation of the practicability of the uncertainty function involves solution of an essentially nonlinear integral equation (2), (5). At the present time we know of no effective methods for solution of equations of such a type, which puts in doubt the practical possibility of checking the uncertainty function using the practicality criterion.

In the synthesis of sonar signals the situation is aggravated to a still greater degree. In this case these methods are essentially inapplicable since in the case of sonar detection of targets:

-- there are relatively great ratios of target velocity to the velocity of propagation of an acoustic signal in the water (up to 0.01-0.02);
-- the extent of the targets is great, as a result of which the required time resolution (lag) is considerably less than $1/f_{Dop}$.

Therefore, the synthesis of sonar signals must be based on the concept of a generalized uncertainty function. At the present time there are no general methods for the synthesis of signals for the generalized uncertainty function. There has been adequate solution only for the problem of synthesis of a signal invariant to the Doppler transform of the spectrum (6), (7), (8), (9), (10). Such a signal ensures the worst velocity resolution with a stipulated time (lag) resolution.

We will show that there is a definite possibility of applying the Sussman method to the generalized uncertainty function.

Adhering to Sussman, we will write the sought-for signal in the form of the sum

$$S(t) = \sum_{k} S_k f_k(t) , \qquad (2)$$

where $f_k(t)$ form a full orthogonal system of base functions.

We will assume that series (2) converges uniformly. Then the generalized uncertainty function for a signal with a unit energy will be equal to the sum of an also convergent double series

$$\chi(\tau,\alpha) = \int_{-\infty}^{\infty} S(t+\tau) S^*[t(1+\alpha)]dt = \sum_{k} \sum_{m} S_k S_m \varphi_{km} (\tau, \alpha) . \qquad (3)$$

Here

$$\varphi_{km}(\tau,\alpha) = \int_{-\infty}^{\infty} f_k(t+\tau) f_m[t(1+\alpha)]\, dt, \quad \alpha = \frac{2V}{C}, \tag{4}$$

V is the radial component of target velocity relative to the sonar; C is the velocity of propagation of a signal in the medium.

Assuming that there is some signal $\gamma(t)$ which corresponds to a stipulated uncertainty function $R_0(\tau, \alpha)$ and that for this signal the series

$$\gamma(t) = \sum_k \gamma_k f_k(t), \tag{5}$$

converges uniformly, we will represent $R_0(\tau, \alpha)$ in the form

$$R_0(\tau,\alpha) = \sum_k \sum_m R_{km} \varphi_{km}(\tau,\alpha). \tag{6}$$

In accordance with (1.3) and (1.6) the closeness coefficient will be equal to

$$c(\tau,\alpha) = Re \sum_k \sum_m \sum_n \sum_p A_{kmnp} s_k s_m R_{np}, \tag{7}$$

where

$$A_{kmnp} = \int\int_{-\infty}^{\infty} \varphi_{km}(\tau,\alpha) \varphi_{np}(\tau,\alpha)\, d\tau\, d\alpha. \tag{8}$$

Then the synthesis problem, according to the ideas of Sussman, is reduced to the selection of the coefficients S_k, maximizing $c(\tau,\alpha)$ under the condition

$$\sum_k |s_k|^2 = 1. \tag{9}$$

[Such normalization makes sense if with a definite degree of approximation we consider the volume of a body of uncertainty to be independent of the signal.]

Since expression (1.7) can be reduced to the form

$$C(\tau,\alpha) = Re \sum_k \sum_m B_{km} s_k s_m, \tag{10}$$

where

$$B_{km} = \sum_n \sum_p R_{np} A_{kmnp}, \tag{11}$$

it forms a quadratic form relative to [symbol omitted]. Thus, synthesis of a signal on the basis of the generalized uncertainty function, the same as on the basis of the simplified uncertainty function, is reduced to the problem of maximizing the quadratic form under normalization conditions. However, synthesis of the generalized uncertainty function on the general basis of its envelope has not been considered at all in the known literature.

Thus, the solution of the problem of signal synthesis on the basis of the generalized uncertainty function is extremely complex and at the present time we do not know any means which could lead to constructive results. Nevertheless, in developing new sonar systems it is an urgent matter to synthesize complex signals having a definite (and not the worst or best) accuracy in measuring range and velocity. It is well known that this resolution is almost entirely determined by the extent of the region of strong correlation of the uncertainty function. Therefore, in this study the requirement on correspondence between the stipulated and synthesized uncertainty function is limited to the region close to the main maximum. In addition, it was assumed that the synthesizable signal is related to a class of signals of the type

$$U(t) = \begin{cases} U_0 \cos[\varphi_m(t)], & |t| < T, \\ 0, & |t| \geqslant T, \end{cases} \qquad (12)$$

where

$$\varphi_m(t) = \sum_{i=1}^{m} a_i t^i.$$

The following argumentation can be cited in support of choice of precisely this class.

On the one hand, the advantages of phase-modulated signals are well known. On the other hand, U(t) of type (12) is a quite general form of registry of a signal with intrapulse FM. In actuality, a continuous function, in this case the FM law, in accordance with the Weierstrass theorem, can with an accuracy as great as desired, be approximated by a power-law polynomial.

Here it should only be added that for all practical purposes in real electric circuits there cannot be signals with a discontinuity of the instantaneous phase, despite the fact that in a number of cases it is convenient to examine precisely such a model.

Taking into account the cited restrictions, we solve the problem of synthesis of complex signals on the basis of the uncertainty function.

The selected approach makes it possible to carry the formulated problem to a successful solution and develop a method acceptable for engineering practice and necessary in the planning of optimized sonar systems using complex signals. This result seems ill-suited for practical application because the difficulties arising here of a computational and fundamental character are substantially greater than in the case of a simplified uncertainty function.

In particular, we note the complexity in computing the expansion coefficients (6). This difficulty is attributable to the fact that the base $\varphi_{km}(\tau, \alpha)$ is not orthogonal, since during movement of the target there is a change in the time scale of the echo signal. It is known that in the case of nonorthogonality the determination of the coefficients is reduced to solution of a system of linear equations.

If the signal is of great complexity, as indicated by an additional analysis, the number of equations is great and in a number of cases the resulting system is slightly conditional. The slight conditionality leads to considerable errors in computing R_{km}, and accordingly, to a low accuracy in approximating $R_0(\tau, \alpha)$.

Moreover, as a result of the extremely prolonged computations in connection with the unsuccessful choice of $R_0(\tau, \alpha)$ the synthesized signal can be unsuccessful in practical use. [For example, the peak power of such a signal will considerably exceed the mean.] In this case the Sussman method does not give an answer to the question as how to change $R_0(\tau, \alpha)$ so that the signal properties will change in the necessary direction.

Finally, we will mention what in our opinion is the greatest weakness of such a method. Since the system of functions $\varphi_{km}(\tau, \alpha)$ for the generalized uncertainty function is not orthogonal, in this case the assertion of completeness of the system $\varphi_{km}(\tau, \alpha)$ loses sense.

Moreover, it follows from the demonstration of the completeness of the base functions, cited in the study by Sussman, that evidently for the generalized uncertainty function the system $\varphi_{km}(\tau, \alpha)$ in a general case is incomplete. Our attempts to demonstrate the reverse were not crowned with success. If the system $\varphi_{km}(\tau, \alpha)$ is incomplete, the errors in approximation of $R_0(\tau, \alpha)$ by the series (6) can be so great that the closeness coefficient ceases to be a criterion of the smallness of the mean square error in approximation. [The latter comment also pertains to extremely wide-band signals when the volume of the body of uncertainty is essentially dependent on the signal energy spectrum [14]].

As correctly pointed out by D. Ye. Vakman [5], equal difficulties are encountered in the synthesis of the Woodward uncertainty function, stipulated only in absolute value. In particular, it is unclear how to solve the nonlinear integral equation determining the optimum phase of the uncertainty function.

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#2. Synthesis of Signal With Unambiguous Reading of Lag and Velocity

Among the signals having a stipulated extent of the region of strong correlation in velocity-range coordinates it is of particular interest to examine signals allowing an unambiguity of both range and velocity in the reading. As is well known, for such signals the maximum value of the uncertainty function or its envelope, regardless of the velocity of motion of the target, corresponds to the relative lag of the reference and echo signal, equal to zero. Here as a simplification we will limit ourselves to an examination of a very simple correlation detector when the dependence of the useful output signal on velocity and range is determined by the corresponding section of the body of uncertainty.

Proceeding on the basis of the requirements on unambiguity, using (12) we we find the maximum value of the useful signal at the output [as a simplification the attenuation of an echo signal during propagation and reflection was assumed equal to 0.]

$$U_1(\alpha) = \frac{U_0^2}{2}\int_{-T}^{T}\cos[2\omega_0 t + \alpha\omega_0 t + \sum_{i=2}^{m} a_i t^i + \sum_{i=2}^{m}(1+\alpha)^i a_i t^i]\,dt + \\ + \frac{U_0^2}{2}\int_{-T}^{T}\cos[\alpha\omega_0 t - \sum_{i=2}^{m}(1+\alpha)^i a_i t^i]\,dt, \qquad (13)$$

where

$$\alpha = 2\frac{V}{C}$$

V is the radial component of velocity of a target relative to the sonar; C is is the velocity of signal propagation; $a_1 = \omega_0$ is the carrier frequency, which is assumed to be stipulated.

The first term with $\omega_0 T > 1$ can be neglected. Then for $|\alpha| \ll 1$

$$U_1(\alpha) = \frac{U_0^2}{2}\int_{-T}^{T}\cos[\alpha\omega_0 t + \alpha\sum_{i=2}^{m} i a_i t^i]\,dt. \qquad (14)$$

We will examine how it is possible to ensure the necessary constancy $U_1(\alpha)$ during movement of the target. A trivial solution of this problem is choice of the signal parameters from the condition

$$|\alpha\sum_{i=2}^{m} i a_i t^i| \ll \frac{\pi}{2}, \qquad (15)$$

$$|\alpha|\omega_0 T \ll \frac{\pi}{2} \qquad (16)$$

for

$$0 < |\alpha| < |\alpha|_{max}$$

In this case the cosine under the integral will be close to zero and $U_1(\alpha)$ is close to $U_1(0)$.

Such a choice of signal parameters is not always acceptable, since it follows from (16) that

$$T \ll \frac{\pi}{2|\alpha|_{max}\,\omega_0} \,. \tag{17}$$

Therefore, in the case of large Doppler shifts of the carrier the signal duration is short and a high level of the energy potential of the sounding signal can be ensured only by means of considerable pulse power.

However, there can also be another approach to choice of the modulation parameters making it possible to synthesize signals of a considerably greater duration with the same velocity resolution. In order to demonstrate such a possibility, the argument of cosine in expression (14) is written in the form

$$\begin{aligned}\alpha\omega_0 t + \alpha\sum_{i=2}^{m} i a_i t^i = \alpha\,\Delta\omega_1 T\Big[\frac{\omega_0}{\Delta\omega_1}\,\frac{t}{T} +\\ + \sum_{i=2}^{m}\frac{i a_i T^i}{\Delta\omega_1}\,\frac{t^i}{T^i}\Big] = \alpha\,\Delta\omega_1 T\Big[\omega_0 t_1 + \sum_{i=2}^{m}\frac{i a_i T^i}{\Delta\omega_1}\Big],\end{aligned} \tag{18}$$

where $t_1 = t/T$.

It is known [11] that the polynomial in parentheses in expression (18) with an odd value can be transformed into a Chebyshev polynomial by means of a corresponding choice of the coefficients a_i. In the considered case the free parameters are $\Delta\omega_1$, $a_2, \ldots a_m$. For optimized parameters, when the polynomial has Chebyshev coefficients, the following expression is correct

$$\left|\frac{\omega_0}{\Delta\omega_1}t_1 + \sum_{i=2}^{m}\frac{i a_i T^i}{\Delta\omega_1}t^i\right| < \frac{|C_m|}{2^{m-1}} \,, \tag{19}$$

where C_m is the leading coefficient.

We have [11]

$$\frac{\omega_0}{\Delta\omega_1} = \frac{mC_m}{2^{m-1}} \,. \tag{20}$$

On the basis of (19) and (20) we obtain

$$\left|\alpha\omega_0 t + \alpha\sum_{i=2}^{m} i a_i t^i\right| < \frac{|\alpha\omega_0 T|}{m} \,. \tag{21}$$

Stipulating the required constancy of a useful signal in the limits of the working range of velocities (that is, the reading level of the region of strong correlation along the velocity axis), we find the required degree of the polynomial

$$m = \frac{\delta\pi}{|\alpha\omega_0 T|} \,, \tag{22}$$

where $0 < \delta\pi < \pi/2$ is the maximum admissible value of the cosine argument determined by the reading level.

It follows from (22) that by means of an increase in the degree of the polynomial it is possible to increase the signal duration by a factor of m in comparison with a very simple signal without intrapulse frequency modulation.

Now we will evaluate the properties of a synthesized signal and in particular its wide-band nature. The wide-band nature of the signal, to be more precise, the relationship between the effective signal band and its mean frequency, is dependent on the relationship of the phase leads in the course of pulse duration due to the linear term and due to the nonlinear terms, in this case $\sum a_i t^i$.* The mentioned evaluation is made easily on the basis of expression (18). In actuality, the difference in the leads due to the linear and nonlinear terms is determined by the difference between

$$\frac{\omega_0}{\Delta\omega_1} t_1 = \frac{m C_m}{2^{m-1}} t_1 \qquad \text{and} \qquad \sum_{i=1}^{m} \frac{i a_i T^i}{\Delta\omega_1} t_1^i$$ [*illegible]

The maximum lead of the linear part is equal to

$$\Delta\varphi_{lin} = m\,|C_m|/2^{m-1}.$$

The maximum value of the polynomial in the braces in (18) is limited by expression (19). Therefore, the maximum phase lead due to the nonlinear term is approximately equal to

$$|\Delta\varphi_{nonlin}| \approx \left|\frac{C_m}{2^{m-1}} - \frac{m C_m}{2^{m-1}}\right| = \frac{C_m}{2^{m-1}}(m-1).$$

With $m \gg 1$ $$|\Delta\varphi_{nonlin}| \approx \frac{m|C_m|}{2^{m-1}} \approx |\Delta\varphi_{лин}|. \qquad (23)$$

Thus, it is demonstrated that the contribution of the nonlinear part is of the same order of magnitude as the contribution of the linear part, and therefore the synthesized signal is wide-band with an effective spectral width of the order of the carrier frequency. In the first approximation the range resolution is determined by the effective width of the signal spectrum and in this case is approximately $1/\omega_0$, that is, with a stipulated carrier is a constant value which cannot be changed by a particular choice of the modulation law. Accordingly, in the class of signals with a rectangular envelope and continuous modulation there are no signals making it possible within definite limits to change the relationship between velocity and range resolution in the case of unambiguous measurement of these parameters. There is only a relatively narrow-band signal with a range resolution of the order of T, with a Doppler shift resolution of the carrier 1/T, and a super-wide-band signal with a relative band of the order of 1 and a range resolution of the order of f/f_0, with a Doppler shift resolution of the carrier of approximately m/T, where m is the degree of the polynomial. Additional investigations of the quadrature-correlation detector, which are omitted here due to unwieldiness, lead to similar conclusions.

#3. Synthesis of Signals of Maximum Complexity With Ambiguous Measurement of Velocity and Lag

In the preceding section it was demonstrated that in the considered class of signals there are not those which would ensure a definite freedom of selection of resolution with respect to velocity and lag with an unambiguous measurement of these parameters. Therefore, in this section we will investigate problems involved in the synthesis of signals in the form (12) on the assumption that unambiguous measurement of velocity and lag is absent.

We will limit ourselves to a case which is of the greatest practical interest when the detector is a quadrature-correlation device with a reference signal coinciding with the anticipated signal. The useful signal at the output of such a detector in the case of relatively narrow-band modulation and the presence of a relative lag of the reference and echo signals is determined by expression (12)

$$\begin{aligned} U_1^2(\alpha) = \frac{1}{4}\Big(\int_{-\infty}^{\infty} U_a[x(1+\alpha)]\, U_a(x+\tau) \times \\ \times \cos\{\varphi_m[x(1+\alpha)] - \varphi_m(x+\tau)\}\,dx\Big)^2 + \\ + \frac{1}{4}\Big(\int_{-\infty}^{\infty} U_a[x(1+\alpha)]\, U_a(x+\tau) \sin\{\varphi_m[x(1+\alpha)] - \varphi_m(x+\tau)\}\,dx\Big)^2, \end{aligned} \tag{24}$$

where $U_a(x)$ is the envelope, τ is the relative lag of the reference and reflected signals with which the useful signal at the output is maximum.

It is evident, despite the movement of the target, that $U_1^2(\alpha)$ will be equal to the maximum corresponding to $\alpha = 0$ if

$$\cos\{\varphi_m[t(1+\alpha)] - \varphi_m(t+\tau)\} = 1, \tag{25}$$

which is equivalent to

$$\varphi_m\,[t(1+\alpha)] - \varphi_m(t+\tau) - 2\pi n = 0. \tag{26}$$

It is impossible to ensure the equality (26) whatever may be the choice of the parameters T, $a,\ldots,a_m$ in any finite interval of α changes. However, it is sufficient to require that

$$|\varphi_m[t(1+\alpha)] - \varphi_m(t+\tau) - 2\pi n| \leqslant \pi\delta, \tag{27}$$
$$0 < \delta < \frac{1}{2}$$

where $0 < \delta < 1/2$.

Under the condition (27) the value of the output signal is limited by the inequality

$$U_0^2 T(1 - \tfrac{1}{2}|F|) \geqslant U_1(\alpha) \geqslant U_0^2 T(1 - \tfrac{1}{2}|F|)\cos\delta\pi, \tag{28}$$
$$F = \frac{\tau}{T}.$$

On the basis of (28) it is possible to determine the requirements on the synthesized signal, proceeding on the basis of the stipulated velocity distribution. A second requirement is the accuracy in measuring lag (range).

However, range accuracy is usually unambiguously related to the effective width of the sounding pulse spectrum (6), (3), (12). However, such an approach is possible only in a case if expansion into a Taylor series is applied to the autocorrelation function (or its envelope).

For the signal considered here the second derivative of the autocorrelation function becomes equal to infinity when $\tau = 0$, which does not make it possible to use a Taylor series.

In order to avoid this difficulty, we will use a somewhat different evaluation of the accuracy of range measurement. For this we will turn to an expression of the signal function in the case of quadrature-correlation reception. This function with an accuracy to an insignificant factor and a fixed target has the form [12]

$$S_1(\tau) = \left(\int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) \cos[\varphi(t+\tau) - \varphi(t)]\, dt\right)^2 + \\ + \left(\int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) \sin[\varphi(t+\tau) - \varphi(t)]\, dt\right)^2 \qquad (29)$$

where

$$\varphi(t) = \sum_{i=2}^{m} a_i t^i .$$

In the region of strong correlation (small $/\tau/$) it can be assumed that

$$\cos[\varphi(x+\tau) - \varphi(x)] \simeq 1 - \frac{\tau^2}{2}[\varphi'(x)]^2, \qquad (30)$$

$$\sin[\varphi(x+\tau) - \varphi(x)] \simeq \tau \varphi'(x). \qquad (31)$$

Substituting (30) and (31) into (29), we obtain

$$S_1(\tau) \simeq \left(\int_{-\infty}^{\infty} U_a(x) U_a(x+\tau)\, dx - \right. \\ \left. - \frac{\tau^2}{2} \int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) [\varphi'(x)]^2 dx\right)^2 + \\ + \tau^2 \left(\int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) \varphi'(x)\, dx\right)^2 . \qquad (32)$$

Since a region of small $/\tau/$ is considered is considered, then

$$\frac{\tau^2}{2} \int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) [\varphi'(x)]^2 dx \simeq \\ \simeq \frac{\tau^2}{2} \int_{-\infty}^{\infty} U_a^2(x) [\varphi'(x)]^2 dx , \qquad (33)$$

$$\tau^2 \int_{-\infty}^{\infty} U_a(x) U_a(x+\tau) \varphi'(x)\, dx = \tau^2 \int_{-\infty}^{\infty} U_a^2(x) \varphi'(x)\, dx . \qquad (34)$$

The autocorrelation function of the envelope has the form

$$\int_{-\infty}^{\infty} U_a(x) U_a(x+\tau)\, dx = 2U_0^2 T \left(1 - \frac{|\tau_0|}{2T}\right). \qquad (35)$$

Substituting (33), (34) and (35) into (32) and omitting the power of τ above the second, we arrive at the expression

$$S_1(\tau) = 2U_0^4T^4\left(1-\frac{|\tau|}{2T}\right)^2 + U_0^4T^2\left(\frac{\tau}{T}\right)^2 \times \left[\int_{-T}^{T}\varphi'(x)dx\right]^2 - \frac{1}{2}U_0^4T^2\left(\frac{\tau}{T}\right)^2 T\int_{-T}^{T}[\varphi'(x)]^2dx, \tag{36}$$

hence the normalized signal function

$$S(\tau) = \frac{S_1(\tau)}{S_1(0)} = \left(1-\frac{|\tau|}{2T}\right)^2 - \frac{1}{2}\left(\frac{\tau}{T}\right)^2\left\{T\int_{-T}^{T}[\varphi'(x)]^2dx - \frac{1}{2}\left[\int_{-T}^{T}\varphi'(x)dx\right]^2\right\}. \tag{37}$$

Stipulating a weakening of the signal function with $|\tau| = |\tau_0|$, we obtain an equation limiting the values of the signal parameters with the accuracy in measuring the lag

$$S(\tau_0) = \left(1 - \frac{1}{2}\frac{|\tau_0|}{T}\right)^2 - \frac{\tau_0^2}{T^2}A_1^2(a_1\ldots a_m, T) + \frac{\tau_0^2}{T^2}B_1^2(a_1\ldots a_m, T), \tag{38}$$

where

$$2A_1^2(a_1\ldots a_m, T) = T\int_{-T}^{T}[\varphi'(x)]^2dx, \tag{39}$$

$$4B_1^2(a_1\ldots a_m, T) = \int_{-T}^{T}\varphi'(x)dx. \tag{40}$$

The use of inequality (28) and the expressions (38), (39), (40) makes it possible to solve the problem of choosing the optimum signal parameters on the basis of the theory of orthogonal Chebyshev polynomials (polynomials of the best approximation to zero). In actuality, substituting (23) into (27) and grouping similar terms, we obtain

$$\delta\mathcal{I} \leqslant \left|\sum_{i=0}^{m} \beta_i t^i\right|, \tag{41}$$

where β_i are coefficients dependent on the coefficients of the initial polynomial $\varphi_m(t)$, τ and n.

Normalizing time, after obvious transformations we have

$$\delta\mathcal{I} \leqslant \left|\sum_{i=0}^{m} C_i \cdot t_1^i\right|, \tag{42}$$

where $t_1 = t/T$, $C_i = \beta_i T^i$.

There is a single possibility of selecting the coefficients $C_0 \ldots C_{m-1}$ with which the absolute maximum deviation of the polynomial on the right-hand side of (42) will be minimum (11). The magnitude of this deviation is dependent on the leading coefficient and is equal to (11)

$$\left|\sum_{i=0}^{m} C_i t_1^i\right|_{max} = \frac{|C_m|}{2^{m-1}}. \tag{43}$$

The leading coefficient with $|\alpha| \ll 1$ is equal to

$$C_m = a_m \alpha m T^m. \tag{44}$$

Then

$$\delta\pi \leqslant \frac{m|\alpha a_m|T^m}{2^{m-1}}. \tag{45}$$

Equating the coefficients in (42) to the corresponding coefficient of the Chebyshev polynomial, we obtain a system of m equations

$$C_{m-1} = C^0_{m-1},$$
$$\cdots$$
$$C_0 = C^0_0,$$

where C^0_i is the Chebyshev coefficient.

Still another equation is related to the admissible change in the useful signal at the output during movement of the target

$$U_1(0) > U_1(\alpha) \geqslant U_1(0)\Delta, \quad 0 < \Delta < 1, \tag{46}$$

where

$$1 - \tfrac{1}{2}|\xi| > \Delta \geqslant (1 - \tfrac{1}{2}|\xi|)\cos\delta\pi. \tag{47}$$

The latter equation follows from the requirements on range accuracy (38). Accordingly, for determining m-1 parameters of modulation ($\omega_0 = \alpha_1$ -- the carrier is stipulated), the signal duration T, the relative optimum lag between the reference and echo signal $\tau/T = \xi$, δ and n, we derived a system of m + 3 equations and the inequalities (38), (46) and (47). The total number of equations and inequalities is equal to the number of unknowns. Since the polynomial (43) is a Chebyshev polynomial, its leading term with stipulated m and δ can be selected as the maximum value. Then it follows from (34) that the proposed method makes it possible to synthesize a signal of maximum duration. Due to the fact that the range resolution is stipulated, this leads to synthesis of the maximum complexity (base) signal.

Now we will examine the problem of choice of the α value with which it is necessary to seek a solution of the mentioned system. We will show that for the parameters of a signal optimized with $\alpha = |\alpha|_{max}$ it is possible with great accuracy to guarantee the relationship (47) with any $|\alpha| < |\alpha|_{max}$. When $|\alpha| \neq |\alpha|_{max}$ we have

$$\varphi_m(t+\tau) - \varphi_m[(1+\alpha)t]2\pi n = a_m|\alpha|_{max} \times mT^m \sum_i C^0_i t_1^i \left(\frac{1+a}{1+|\alpha|_{max}}\right)^i = \Delta\varphi. \tag{48}$$

It follows from (45) that

$$m|\alpha|_{max}|a_m|T^m = 2^{m-1}\delta\pi. \tag{49}$$

Since $|\alpha| \leqslant 1$, then

$$\left(\frac{1+\alpha}{1+|\alpha|_{max}}\right)^{i} \simeq 1 + i\Delta\alpha , \qquad (50)$$

where $\Delta\alpha = \alpha - |\alpha|_{\max}$

On the basis of (49) and (50), from (48) we obtain

$$\Delta\varphi \approx \delta\pi + 2^{m-1}\delta\pi\Delta\alpha \sum_{i} C_i^0 t_1^i i . \qquad (51)$$

It is easy to note that

$$\sum_{i} C_i^0 t_1^i i = t_1 \frac{d}{dt_1} \sum_{i} C_i^0 t_1^i . \qquad (52)$$

Using a representation of the Chebyshev polynomial in the form

$$T_m(x) = \frac{1}{2^{m-1}} \cos (m \arccos x) , \qquad (53)$$

we find

$$\frac{d}{dt_1} \sum_{i} C_i^0 t_i^i = \frac{m}{2^{m-1}} \frac{\sin(m \arccos x)}{\sqrt{1-x^2}} \qquad (54)$$

The right-hand side of (54) assumes a maximum value at the point x closest to 1 when sin(m arccos x) becomes equal in absolute value to 1.

Accordingly x_0 is the greatest root of the equation

$$\cos(m \arccos x) = 0 \qquad (55)$$

It is known II that (55) has the maximum root

$$x_0 = \cos \frac{\pi}{2^m}$$

Thus

$$\left|\sum_{i} C_i^0 t_1^i i\right| < m \left(\sin \frac{\pi}{2m}\right)^{-1} \qquad (56)$$

In any case $\left|\sum_{i} C_0^i t_1^i i\right| < \frac{2m^2}{\pi} ,$

hence $\left|\sum_{i} C_i^0 t_1^i i \left(\frac{1+\alpha}{1+|\alpha|_{max}}\right)^{i}\right| < |\delta|\pi + 2|\delta\Delta\alpha| m^2 .$ (57)

Accordingly $U_1(\alpha) > (1 - \frac{1}{2}|\xi|)[\cos\delta\pi \cos 2\delta\Delta\alpha m^2 - \sin\delta\pi \sin 2|\delta\Delta\alpha| m^2] U_1$

As indicated by additional computations, there is no practical need to use polynomials with a degree higher than the fourth. Therefore, we will limit ourselves to a case when $m \leqslant 4$. The maximum relative velocities of the target are observed in sonar and do not exceed $2 \cdot 10^{-2}$. Then with m = 4

$$\sin 2\delta|\Delta\alpha|m^2 \simeq \sin 0{,}1\delta\pi, \tag{58}$$

$$\cos 2\delta\Delta\alpha m^2 \simeq \cos 0{,}1\delta\pi. \tag{59}$$

Since

$$|\delta| < \frac{1}{2},$$

then

$$\cos 0{,}1\delta\pi \simeq 1,$$
$$\cos 0{,}1\delta\pi \gg \sin 0{,}1\delta\pi \simeq 0$$

The cited demonstration makes it possible in (47) to leave only the right-hand side, and replacing the inequality sign by an equality sign, seek a solution of the system determining the optimum parameters of the signal for the maximum (in absolute value) target velocity. In this case it can be guaranteed that at any velocity in absolute value less than the maximum velocity, the useful signal at the output exceeds

$$U_0^2 T(1 - \frac{1}{2}|\xi|)\cos\delta\pi.$$

#4. Synthesis of Signals of an Arbitrary Complexity in Case of Ambiguous Measurement of Velocity and Lag

In #3 we developed a method for the synthesis of signals of the type (12) under the condition of obtaining a maximum complexity for selected m. An analysis of such signals indicated that a conversion from linear (m = 2) to nonlinear types of frequency modulation leads to a substantial increase in complexity (an order of magnitude or more). In this case it may seem, for example, that the complexity of the signal for m = 2 is small, whereas with $m \geqslant 3$ it is inadmissibly high. Therefore, it is of considerable interest to consider a method which would ensure the possibility of synthesis of a signal of stipulated complexity. On the basis of the results in #3 it is possible to establish those limiting complexity values which correspond to different degrees of the polynomial $\varphi_m(t)$. It should be noted that the formulated problem is considerably more complex than that which was solved above, at least for the following reasons.

In the synthesis of signals of limiting complexity the problem has a unique solution (in any case for $m \leqslant 4$). The introduction of an additional requirement on signal complexity leads to a nonambiguity of the solution. For example, it is quite obvious that by retaining constant the effective width of the signal spectrum, whose value is determined by the requirement on range resolution, and decreasing the phase lead in comparison with the optimum value at the expense of the cubic term, it is possible to achieve a smooth conversion from the third-degree polynomial to a second-degree polynomial. Such an approach will inevitably be accompanied by a complexity decrease. However, such a complexity decrease can also be achieved by an increase in $|a_3T^3|$ in comparison with the optimum value. Similar methods are also applicable to polynomials of higher degrees. Therefore, we will restrict the synthesis problem to such signals as are obtained with conversion from a higher degree polynomial to the next lower degre due to a decrease in the phase lead of the leading term.

This approach was then used applicable to a third-degree polynomial since at the present time in such a sonar region where the target velocities are especially great no need arises for complexities exceeding the limiting value with m = 3. A similar approach in case of necessity can also be applied to a fourth-degree polynomial.

Due to the fact that the complexity of the synthesized signal is less than the limiting value, the coefficients of the polynomial (42) do not coincide with the Chebyshev coefficients. This difference can be written in the form

$$\frac{2\alpha a_2 + 3a_3\tau}{3\alpha a_3 T} = \Delta C_2\ , \tag{60}$$

$$\frac{\alpha\omega_0 + 2a_2\tau + 3a_3\tau^2}{3\alpha a_3 T^2} = \Delta C_1 - \frac{3}{4}\ , \tag{61}$$

$$\frac{\omega_0\tau - a_2\tau^2 - a_3\tau^3 - 2\pi n}{3\alpha a_3 T^3} = \Delta C_0 \tag{62}$$

Then

$$P_{n1} = 3\alpha a_3 T^3[t_1^3 - \tfrac{3}{4}t_1] + 3\alpha a_3 T^3[t_1^2\Delta C_2 + t_1\Delta C_1 + \Delta C_0] \tag{63}$$

Substituting (63) into (24), we obtain an inequality similar to (28).

$$U_1(\alpha) \geqslant U_2^2 T(1 - \tfrac{1}{2}|\xi|)\cos\delta_0\pi\cos\delta\pi, \tag{64}$$

where

$$\delta_0\pi = \left|\frac{3\alpha a_3 T^3}{2}\right|\ ,$$

$$\delta\pi = \left|3\alpha a_3 T^3(\Delta C_2 t_1^2 + \Delta C_1 t_1 + \Delta C_0)\right|_{max}. \tag{65}$$

(66)

Since here reference is to signals with a reduced contribution of the cubic term, then

$$\delta_0\pi \ll \frac{\pi}{2}\ ,$$

hence

$$U_1(\alpha) \geqslant U_0^2 T(1 - \tfrac{1}{2}|\xi|)\cos\delta\pi. \tag{67}$$

In computing $\delta\pi$ it must be taken into account that this parameter corresponds to those τ and n values for which the expression in parentheses (66) is minimum. In this case for stipulated a_2, a_3 and TU_1 (α) will be maximum, since it will be demonstrated below that the ξ parameter is virtually not dependent on τ and n. The $\delta\pi$ evaluation is a quite unwieldy problem, related to analysis of the dependence of the maximum deviation of a quadratic form

$$\Delta C_2 t_1^2 + \Delta C_1 t_1 + \Delta C_0 = \theta_2(t) \tag{68}$$

on the parameter τ.

This analysis, which is not presented here in connection with the unwieldiness mentioned above, shows that the minimum of the deviation $\theta_2(t)$ from zero, maximum in absolute value, is obtained with such a choice of τ and n when

$$\Delta C_1 = \Delta C_2 \, . \tag{69}$$

In this case

$$2\delta\pi = 3\alpha a_3 T^3 / \Delta C_2 / \, . \tag{70}$$

Omitting intermediate computations, we cite only the final expressions for the parameters of the synthesized signal:

$$T = \frac{2}{\sqrt{3}} \tau_0 q_c \frac{1}{\sqrt{1 - S(\tau_0)}} , \tag{71}$$

$$a_2 = \frac{3}{4} \frac{[1 - S(\tau_0)]}{q_c \tau_0^2} , \tag{72}$$

$$|\xi| = \frac{|\alpha \omega_0 \tau_0|}{\sqrt{3}\sqrt{1 - S(\tau_0)}} = \left| \frac{\alpha \omega_0}{2 a_2 T} \right| . \tag{73}$$

$$a_3 = \frac{3[1 - S(\tau_0)]^2}{4} \frac{q_c - \frac{\delta\pi}{\alpha}}{\omega_0 \tau_0^4 q_c^3} , \tag{74}$$

$$\delta\pi = \arccos \frac{\Delta}{1 - \frac{\delta\pi}{2\sqrt{3}\sqrt{1 - S(\tau_0)}}} \tag{75}$$

Thus, we have obtained expressions which make possible an unambiguous determination of all the unknown parameters of the synthesized signal. It follows from the form of the expression that a_2, a_3 and T should be determined for $\alpha = |\alpha|_{max}$.

#5. Brief Conclusions

We examined the problems involved in synthesis of an FM signal of quite general form

$$u(t) = U_0 \cos[\varphi_m(t)] , \quad |t| < T, \tag{76}$$

$$\varphi_m(t) = \sum_{i=1}^{m} a_i t^i , \tag{77}$$

for a stipulated region of strong correlation of a two-dimensional velocity-lag uncertainty function and obtained the following results:

1) The problem of synthesis with a stipulated carrier $\omega_0 = a_1$ is reduced to the choice of the optimum values of signal duration T and the modulation parameters $a_3 \ldots a_m$.

2. It is impossible to synthesize a signal with an ambiguous (independent) measurement of the time lag and the velocity of the target allowing a definite freedom of choice of extent of the region of strong correlation along the lag and velocity axes. If in the synthesis it is necessary to take into account the requirement on the mentioned ambiguity, the synthesized signal is obtained with a relative effective spectral width of about unity. Thus, its range resolution is virtually not dependent on the duration and parameters of modulation $a_2 \ldots a_m$. The duration of a signal synthesized in such a way is approximately $m/\Delta f_{Dop}$, where Δf_{Dop} is the maximum Doppler shift of the carrier.

3. If the requirements of unambiguity are dispensed with, there is a possibility of synthesis of signals with a definite freedom of choice of extent of the region of strong correlation of the two-dimensional uncertainty function.

The possibilities of synthesis are limited to definite maximum dimensions of this region along the lag and velocity axes. For a level of reading of the region of strong correlation equal to 0.7 the compatability of requirements on the extent of the indicated region is determined by the inequality

$$K_0 = |\alpha|_{max}\, \omega_0 \tau_0 < 0{,}73$$

Here

$$|\alpha|_{max} = 2\frac{|U|_{max}}{C},$$

$-|U|_{max} - |U|_{max}$ is the extent of the region of strong correlation along the velocity axis, C is the velocity of signal propagation, $-\tau_0 - \tau_0$ is the extent of the region of strong correlation along the time lag axis.

4. In the case of ambiguous measurement of velocity-range for each degree of the polynomial there is a signal of limiting duration and complexity. For m = 2 (linear FM) the limiting complexity is approximately $1/|\alpha|_{max}$ and the limiting duration is $1/|\alpha|^2_{max}\omega_0$. With an increase in the degree of the polynomial the limiting complexity and duration increase in comparison with the corresponding parameters for linear FM by a factor of approximately

$$\frac{K_0^{m-2}}{|\alpha|^{m-2}_{max}}$$

An analysis of the limiting values of the durations and complexities for a number of characteristic situations indicated that the use of polynomials with a degree above the fourth is not caused by practical necessity.

5. The law of change of the instantaneous phase of signals of limiting complexity coincides with the logarithmic phase within the limits of accuracy in approximating the expansion of $\ln(1 + x)$ into a series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

Accordingly, for m = 2 -- by two terms, for m = 3 -- by three terms, for m = 4 -- by four terms. Signals of an unlimited complexity and duration do not have such a property.

6. For signals of limiting complexity the conversion from linear FM ($m = 2$) to nonlinear FM ($m \geqslant 3$) causes an increase in the ambiguity of range measurement by a factor of approximately $1/|\alpha|^{m-2}_{\max}$.

7. The autocorrelation function for signals with an ambiguous reading is described by the expression

$$R(\tau) = R_0 \frac{\sin 2a_2 T\tau}{2a_2 T\tau} \cos\omega_0\tau .$$

8. The section of the two-dimensional uncertainty function for $U \neq 0$, parallel to the lag axis, is little distorted within the limits of the computed interval of target velocities. Thus, small distortions are also experienced by a signal at the output of a matched filter during motion of the target if its velocity U falls in the range $-|U|_{\max} \leqslant U \leqslant |U|_{\max}$, where $U_{\max}$ is the computed velocity.

9. The expression for the optimum modulation parameters ($a_2 \ldots a_m$) and signal duration with an ambiguous measurement of velocity and range was obtained in a quite simple form, allowing their use in engineering computations.

In conclusion we note that control computations indicated a sufficient accuracy of the proposed method for the synthesis of complex signals in a limited region of a two-dimensional body of uncertainty.

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UNCERTAINTY FUNCTIONS OF SOME TYPES OF COMPLEX SIGNALS

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[Article by A. A. Belousov, V. M. Vol'f, V. B. Galanenko, N. G. Gatkin, L. N. Kovalenko, L. S. Kovalenko and S. V. Pasechnyy]

[Text] The uncertainty function is used for describing the possibilities of echo sounding systems. The uncertainty function characterizes the "sharpness" in tuning the system in the parameter space of the echo signal and can be expressed as the dependence of the normalized response of the system (detector) on the vector

$$\vec{\xi} = \{\vec{\xi}_0, \vec{\xi}'\},$$

where $\vec{\xi}_0$ is the vector of the parameters characterizing the echo signal, $\vec{\xi}'$ is the vector of the corresponding evaluation parameters of the system. By $\vec{\xi}_0$ we will understand the space coordinates, velocity, acceleration, etc.

The uncertainty function is dependent not only on the parameters of the object but also on the type of detector (optimum, nonoptimum), interference (through the structure of the detector), propagation conditions, etc.

The known Woodward uncertainty function characterizes an optimum reception system in the time region of a signal reflected from a point target moving in a homogeneous unbounded medium with a relatively small constant velocity in the presence of stationary white Gaussian noise and an extremely narrowband signal.

Later generalizations were made of the uncertainty function for nonwhite nonstationary noise [6], for accelerated movement of a target [7], with space coordinates taken into account [8,9,10], etc.

The increase in relative velocities of sounded objects and the broadening of the frequency spectrum of the sounding signals was taken into account by the introduction of a wide-band uncertainty function [11].

For the case of a detector structure different from a matched filter we have introduced the term "relative uncertainty [ambiguity] function" [12].

The properties of the sounding signals described by different modifications of the uncertainty function can differ substantially. Nevertheless, most studies [1-3] are limited to study of the properties of signals using the Woodward uncertainty function.

In this paper we examine the properties of a series of known sonar signals on the basis of study of a wide-band relative uncertainty function for a correlation detector in which there is compensation only of the Doppler shift of the carrier frequency of an echo signal from a point source moving with a constant velocity in a homogeneous medium.

With processing in such a channel the uncertainty function is characterized by the expression:

$$\Psi(\tau,\alpha)=\frac{\left|\int\limits_{-\infty}^{\infty}\dot{S}_{\text{пр}}(t)\,\dot{S}_0^*(t)\,dt\right|}{\sqrt{\int\limits_{-\infty}^{\infty}|\dot{S}_{\text{пр}}(t)|^2dt\int\limits_{-\infty}^{\infty}|\dot{S}_0(t)|^2dt}}, \qquad (1)$$

[πp = rec(eived)] where

$$\dot{S}_{\text{пр}}(t)=\dot{S}[(1+\alpha_0)t-\tau_0]\,e^{j\omega[(1+\alpha_0)t-\tau_0]} \qquad (2)$$

is the received echo signal

$$\dot{S}_0(t)=\dot{S}(t-\tau')\,e^{j\omega[(1+\alpha')t-\tau']} \qquad (3)$$

is the reference signal, τ_0 and α_0 are the true parameters of the target; τ' and α' are reference signal evaluation parameters; $\tau = \tau' - \tau_0$; $\alpha = \alpha' - \alpha_0$.

With the substitution of (2), (3) into (1) we obtain the following expression for the uncertainty function:

$$\Psi(\tau,\alpha)=\frac{\sqrt{1+\alpha_0}\left|\int\limits_{-\infty}^{\infty}\dot{S}[(1+\alpha_0)t]\,\dot{S}^*(t-\tau)\,e^{j\omega\alpha t}dt\right|}{\int\limits_{-\infty}^{\infty}|\dot{S}(t)|^2dt}, \qquad (4)$$

When $\alpha_0 \ll 1$ the coefficient $\sqrt{1+\alpha_0}$ can be omitted.

The uncertainty function (4) makes it possible to ascertain the limits of "tolerance" of different signals to dispersion distortions of the modulation law due to movement of the object. An analysis of expression (4) makes it possible to evaluate the desirability of using different signals for detection, resolution and evaluation of parameters of objects creating an appreciable Doppler shortening (lengthening) of the signal during processing in a multichannel Doppler system in which the reference signals differ only with respect to the central frequencies. Below we give computation formulas for the uncertainty functions of pulsed signals of a rectangular shape with linear frequency modulation (LFM), quadratic frequency

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modulation (QFM), hyperbolic frequency modulation (HFM), and also with noise amplitude and phase modulation, obtained in accordance with (4) on the assumption that $\alpha_0 \ll 1$.

Some results of such computations are illustrated in the figures, which show the uncertainty functions and their sections.

We note that expression (4) with $\alpha_0 = 0$ formally is transformed into the Woodward uncertainty function, whereas when $\alpha' = 0$ -- into a wide-band uncertainty function.

Signal With Linear Frequency Modulation (LFM)

The complex envelope of the signal is

$$\dot{S}(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)e^{j\pi\rho t^2} \tag{5}$$

The substitution of (5) into (4) leads to an uncertainty function in the form

$$\Psi(u,v) = \left|\int_{-\frac{1}{2}+u}^{\frac{1}{2}} \exp\{j2\pi\Delta FT[\alpha_0 x^2 + (u+v)x]\}dx\right|, \tag{6}$$

where

$$x = \frac{t}{T};\quad u = \frac{\tau}{T};\quad v = \frac{\alpha f}{\Delta F};\quad \rho = \frac{\Delta F}{T}\quad ;$$

ΔF is the frequency deviation, T is signal duration.

Signal With Quadratic Frequency Modulation (QFM)

The complex envelope of the signal is described by the expression

$$\dot{S}(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)\exp\left\{j\frac{4}{3}\pi\Delta FT\left(\frac{t}{T}\right)^3\right\}, \tag{7}$$

In accordance with (4) and (7) the QFM uncertainty function is described by the expression

$$\Psi(u,v) = \left|\int_{-\frac{1}{2}+u}^{\frac{1}{2}} \exp\{j(mx^3 + nx^2 - px)\}dx\right|. \tag{8}$$

$$m = 8\pi\Delta FT\alpha_0;\quad n = 8\pi\Delta FTu;\quad p = 8\pi\Delta FT\left(u^2 - \frac{v}{u}\right).$$

Signal With Hyperbolic Frequency Modulation (HFM)

The complex envelope of the signal is

$$\dot{S}(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)\exp\left\{-j\frac{2\pi F}{k}\ln(1-kt)\right\} \tag{9}$$

The uncertainty function of such a signal can be represented in the form

$$\Psi(u,v) = \left|\int_{-\frac{1}{2}+u}^{\frac{1}{2}} \exp j2\pi\left\{\frac{F'}{k'}\ln\frac{1-k'\Delta FT(x+u)}{1-(1+\alpha_0)k'\Delta FTx} + v\Delta FTx\right\}dx\right|, \tag{10}$$

where
$$F' = \frac{F}{\Delta F}\ ;\ K' = \frac{K}{\Delta F}\ .$$

The K' coefficient is related to the signal parameters by the expression

$$K' = \frac{2}{\Delta FT}\left(\sqrt{(F')^2+1} - F'\right)$$

In order to retain information on the Doppler shift in the HFM signal it is shaped by transfer into the region of the carrier frequency of oscillations with an instantaneous frequency changing in conformity to the law

$$f(t) = \frac{F}{1-Kt};$$

In the shaping of a signal in the time interval from -T/2 to T/2, F corresponds to the mean frequency of the shaped oscillations.

Noise Signal and Noise Phase Modulation

As the sounding signals in principle it is possible to use a segment of a record of narrow-band noise with a spectral density constant in the working band -- a signal with noise amplitude modulation (NAM)

$$\dot{S}(t) = \mathrm{rect}\left(\frac{t}{T}\right)\dot{N}(t) \tag{11}$$

For the correlation function $K(\tau) = \langle\dot{N}(t)n(t-\tau)\rangle$

$$K(\tau) = \frac{\sin \pi\Delta F\tau}{\pi\Delta F\tau}$$

the uncertainty function of such a signal is described by the expression

$$\Psi(u,v) = \left|\int_{-1+u}^{1} \frac{\sin[(\pi\Delta FT)(u-\alpha,x)]}{\pi\Delta FT(u-\alpha,x)} \exp\{j2\pi\Delta FTvx\}dx\right| \tag{12}$$

The radiation of such a signal is infeasible from the energy point of view. This shortcoming is lacking in a signal with noise phase modulation (NFM)

$$\dot{S}(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)e^{j\beta n(t)}, \tag{13}$$

where n(t) is a segment of a record of Gaussian noise with a zero mean dispersion $\sigma_n^2 = 1$; β is a coefficient characterizing the modulation intensity.

The uncertainty function of the signal (13) is described by the expression

$$\Psi(u,v) = \left|\int_{-\frac{1}{2}+u}^{1} \exp\{j2\pi\Delta FTvx - \beta^2[1-RT(u+\alpha,x)]\}dx\right|. \tag{14}$$

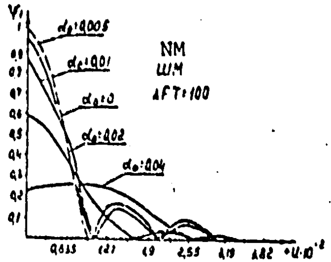
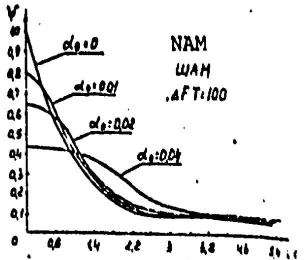


РИС. 1



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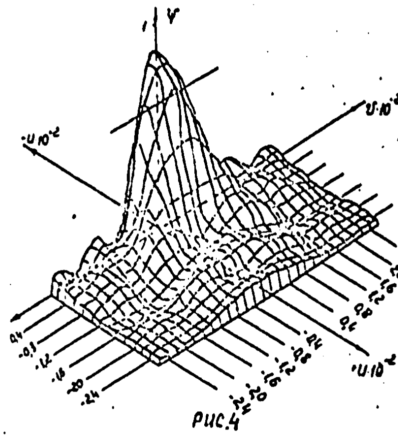

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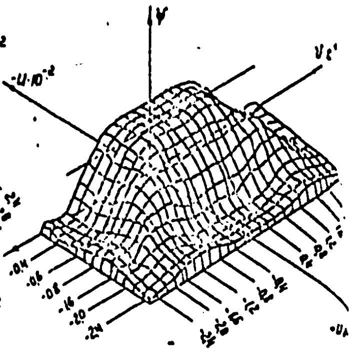

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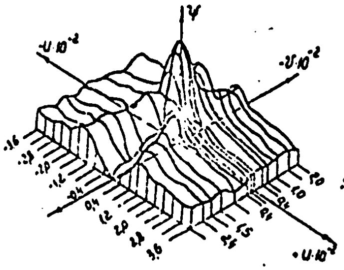

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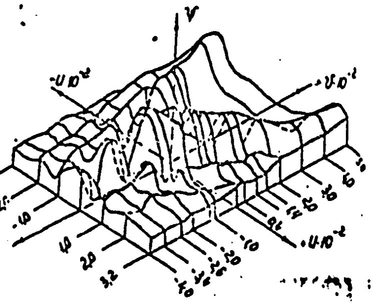


РИС.6

Figures 1, 2, 3, 4, 5, 6.

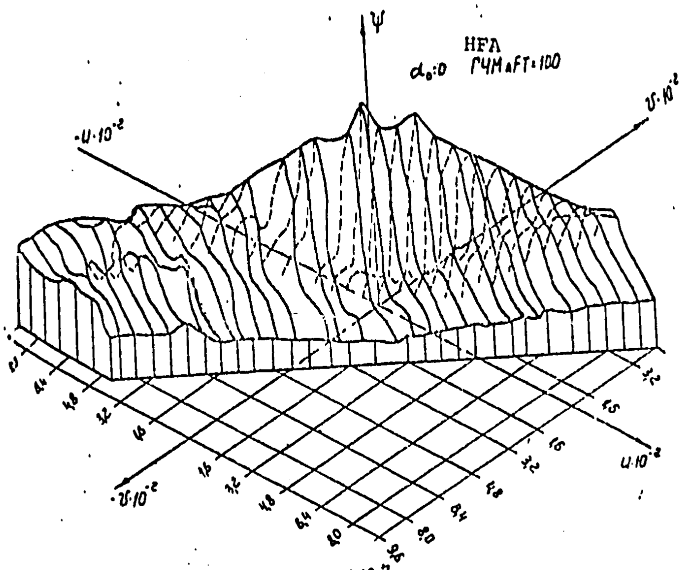


рис.7

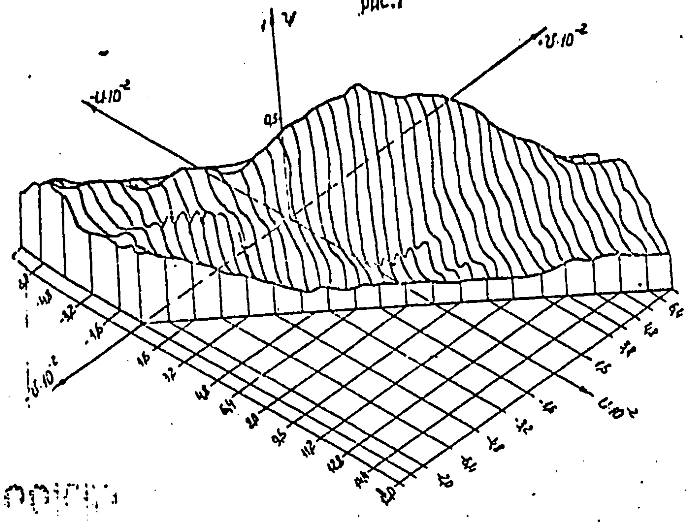


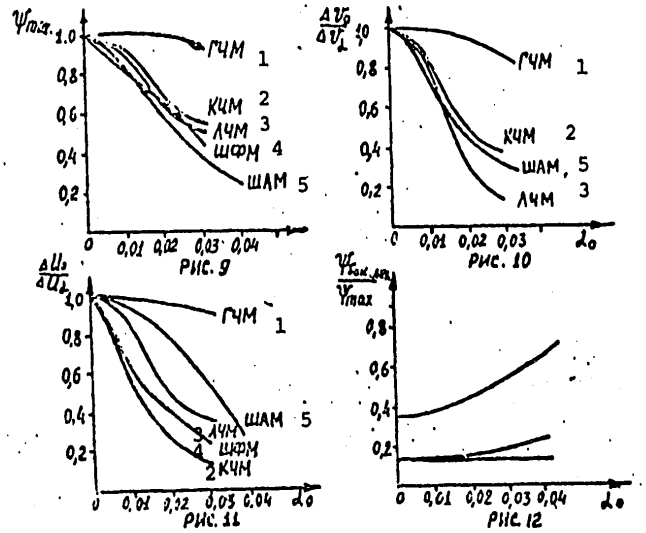
рис.8

Figures 7 and 8.

For the correlation function of a modeling process in the form

$$R(\tau)=\begin{cases}1-\frac{|\tau|}{\tau_k} & \text{when } |\tau|\leq\tau_k\\ 0 & \text{when } |\tau|>\tau_k,\end{cases}$$

where τ_k is the correlation interval, Fig. 1 shows sections of the uncertainty function (14) of the coordinate plane v = 0; Fig. 2 shows similar sections of the uncertainty function (12).



Figures 9, 10, 11, 12

KEY:
1. HFM
2. QFM
3. LFM
4. NPM
5. NAM

A comparison of the indicated sections shows that the difference in the properties of signals with noise, amplitude and phase modulation is insignificant.

On the basis of the derived expressions we computed the uncertainty functions of signals of the enumerated types for different values of the coefficients u and v, being a measure of the error in evaluating the signal parameters τ_0,

α_0. The results of such computations are given in [5].

Figures 3-8 show the typical form of uncertainty functions of different complex signals when processing in a multichannel receiving channel with compensation of the Doppler distortions by means of displacement of the carrier frequency only. These are given as illustrations.

Figures 9-12 show curves characterizing the effect at the output of the processing channel and the potential resolution of signals with respect to frequency and time shifts in dependence on the velocity of movement of the sounded objects.

On the basis of the data in [5] it is possible to construct different sections of the uncertainty function. A study was made of the regularities of change in the peak values of its main and side lobes in dependence on the parameter $\alpha_0 = 2v\tau/c$. The number of channels in which it is sufficient to carry out only compensation of the carrier frequency of the oscillations was determined.

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EVALUATION OF THE PULSE CHARACTERISTIC CURVE OF A HYDROACOUSTIC CHANNEL

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 148-153

[Article by V. I. Paderno and I. R. Romanovskaya]

[Text] During recent years the characteristics of a hydroacoustic channel and its parameters have been an object of close attention from both experimenters and theoreticians who have formulated pertinent models.

This report examines the problems involved in evaluating the pulse reaction of a channel during its testing by known signals. We will limit ourselves to the case of a channel with constant parameters in an interval of solution time. If the process of signal propagation in the medium is equivalent in output effect to the use of a linear operator for the emitted signal (this assumption is correct in most real cases), the pulse reaction of the channel is its total characteristic.

Now we will formalize the formulated problem. Assume that x(t) is an emitted determined signal which is propagated in the hydroacoustic channel with the pulse reaction $h\varphi(t)$. The input of the processing system receives a signal y(t) in combination with additive noise interference n(t). The problem is to find some such a system for processing a received signal, which at its output would give the best (according to the adopted criterion) evaluation of the channel pulse reaction $h\varphi(t)$. We will seek a solution of the problem in the class of linear systems [1]. Such a system can be unambiguously described by the pulse reaction h(t).

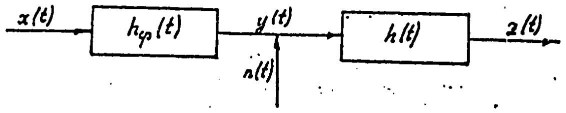

Fig. 1. cp = mean

As the optimality criterion for the system we will use the minimum of the mean square error in evaluation of the pulse reaction of the medium in the observation interval [-T/2, T/2]. Then the sought-for pulse reaction is a function minimizing

$$F = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \overline{[z(t) - h_{cp}(t)]^2}\, dt \tag{1}$$

where z(t) is the evaluation of the pulse reaction of the hydroacoustic channel and the line at top denotes averaging for the set of observations.

We will express the output process z(t) through the input process x(t) and the characteristics of the filters (see Fig. 1)

$$z(t) = \int_{-\infty}^{+\infty} h(t_0)[y(t - t_0) + n(t - t_0)]\, dt_0 \tag{2}$$

$$y(t) = \int_{-\infty}^{+\infty} h_{cp}(t_1)\, x(t - t_1)\, dt_1 \tag{3}$$

Substituting (3) into (2), we obtain

$$z(t) = \iint_{-\infty}^{+\infty} h(t_0) h_{cp}(t_1)\, x(t - t_0 - t_1)\, dt_1\, dt_0 + \int_{-\infty}^{+\infty} h(t_0)\, n(t - t_0)\, dt_0 \tag{4}$$

We will determine the mean square error in evaluating the pulse reaction in the form

$$F = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \overline{[h_{cp}(t) - \iint_{-\infty}^{+\infty} h(t_0) h_{cp}(t_1)\, x(t - t_0 - t_1)\, dt_1\, dt_0 -} \\ \overline{- \int_{-\infty}^{+\infty} n(t - t_0)\, h(t_0)\, dt_0]^2}\, dt \tag{5}$$

Now we will simplify expression (5), assuming that additive noise is a stationary process with a zero mean. Then after squaring the expression in the parentheses, we obtain

$$F = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} h_{cp}^2(t)\, dt - \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} h_{cp}(t) \iint_{-\infty}^{+\infty} h(t_0) h_{cp}(t)\, x(t - t_0 - t_1)\, dt_1\, dt_0\, dt + \\ + \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \iint_{-\infty}^{+\infty} h(t_0) h(t_2)\, n(t - t_0)\, n(t - t_2)\, dt_0\, dt_2\, dt + \\ + \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \iiiint_{-\infty}^{+\infty} h(t_0) h(t_2) h_{cp}(t_1)\, x(t - t_0 - t_1)\, x(t - t_2 - t_3)\, dt_0\, dt_1\, dt_2\, dt_3\, dt \tag{6}$$

We will seek a solution of the problem with the use of the methods of the calculus of variations and in particular the method of undetermined Lagrange factors [2]. We will represent h(t) in the form

$$h(t) = h_0(t) + \lambda \gamma(t)$$

where $h_0(t)$ is an optimum solution, $\gamma(t)$ is an arbitrary function, λ is an arbitrary factor.

We will find $h_0(t)$ as a solution of the equation

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda=0}=0$$

Then

$$\begin{aligned}&-\frac{2}{T}\int_{-T/2}^{T/2} h_{\varphi}(t)\iint_{-\infty}^{+\infty}\gamma(t_0)h_{\varphi}(t_1)x(t-t_0-t_1)\,dt_0\,dt_1\,dt\;+\\&+\frac{2}{T}\int_{-T/2}^{T/2}\iint_{-\infty}^{+\infty}\gamma(t_0)h_0(t_2)n(t-t_0)n(t-t_2)\,dt_0\,dt_2\,dt\;+\\&+\frac{1}{T}\int_{-T/2}^{T/2}\iiiint_{-\infty}^{+\infty}\gamma(t_0)h_0(t_2)h_{\varphi}(t_3)h_{\varphi}(t_1)x(t-t_0-t_1)x(t-t_2-t_3)\,dt_0\,dt_1\,dt_2\,dt_3\,dt\\&+\frac{1}{T}\int_{-T/2}^{T/2}\iiiint_{-\infty}^{+\infty}\gamma(t_0)h_0(t_2)h_{\varphi}(t_3)h_{\varphi}(t_1)x(t-t_2-t_3)x(t-t_0-t_1)\,dt_0\,dt_1\,dt_2\,dt_3\,dt=0\end{aligned}\tag{7}$$

We transform (7) to the form

$$\int_{-\infty}^{+\infty}\Big[\gamma(t_0)\sum_{i=1}^{n}\mathcal{J}_i\Big]dt_0=0\tag{8}$$

Since $\gamma[t_0]$ is an arbitrary function, (8) is realized in a case when

$$\sum_{i=1}^{n}\mathcal{J}_i=0$$

We take into account that $\overline{n(t)n(t+\tau)}=R_n(\tau)$ is the correlation function of noise. We will make the assumption that the signal x(t), the pulse reaction of the medium $h_{\varphi}(t)$ and the filter $h_0(t)$ are finite functions of time, and

$$\begin{aligned}&x(t)=0 &&\text{when} && t>\Delta,\; t<0\\&h_{\varphi}(t)=0 &&\text{when} && t>\delta,\; t<0\end{aligned}\tag{9}$$

Then

$$\begin{aligned}&\frac{1}{\Delta}\int_0^{\Delta}x(t)x(t+\tau)\,dt=R_x(\tau)\\&\frac{1}{\delta}\int_0^{\delta}h_{\varphi}(t)h_{\varphi}(t+\tau)\,dt=R_{\varphi}(\tau)\end{aligned}\tag{10}$$

where $R_x(\tau)$ is the correlation function of the signal, $R\varphi(\tau)$ is the correlation function of the pulse reaction of the medium.

Selecting a sufficiently large T value, after simple transformations we reduce the equation to the form

$$-2\frac{\Delta}{T}\int x(-\tau)R_{\varphi}(t_0-\tau)\,d\tau+2\int h_0(t_2)R_n(t_2-t_0)\,dt_2+\tag{11}$$

$$+T\frac{\Delta}{T}\frac{\delta}{T}\iint h_0(t_1)R_x(t_1-\tau)R_\varphi(\tau-t_0)\,dt_1\,d\tau+$$
$$+T\frac{\Delta}{T}\frac{\delta}{T}\iint h_0(t_1)R_x(\tau-t_1)R_\varphi(t_0-\tau)\,dt_1\,d\tau=0 \tag{11}$$

We will apply the Fourier transform to (11) and will make use of the fact that the Fourier transform of the faltung of two functions is a product of their Fourier transforms.

We use the notation

$$F\{h_0(t)\}=\dot{S}_{h_0}(\omega)$$
$$F\{x(t)\}=\dot{S}_x(\omega)$$
$$F\{h_\varphi(t)\}=\dot{S}_\varphi(\omega)$$

-- the amplitude-phase spectral characteristics of the filter, signal and medium respectively.

$$F\{R_x(\tau)\}=\frac{|\dot{S}_x(\omega)|^2}{\Delta}$$
$$F\{R_\varphi(\tau)\}=\frac{|\dot{S}_\varphi(\omega)|^2}{\delta}$$

$F\{R_n(\tau)\}=H_n(\omega)$ is the spectrum of noise intensity.

With this taken into account, from (11) we obtain

$$-2\dot{S}_x^*(\omega)\frac{|\dot{S}_\varphi(\omega)|^2}{T}+2\dot{S}_{h_0}(\omega)H_n(\omega)+2T\dot{S}_{h_0}(\omega)\frac{|\dot{S}_x(\omega)|^2}{T}\frac{|\dot{S}_\varphi(\omega)|^2}{T}=0 \tag{12}$$

Here the asterisk denotes complex conjugation.
It follows from (12) that

$$\dot{S}_{h_0}(\omega)=\frac{\dot{S}_x^*(\omega)}{|\dot{S}_x(\omega)|^2+\frac{T\cdot H_n(\omega)}{|\dot{S}_\varphi(\omega)|^2}} \tag{13}$$

The channel pulse reaction can be found as a Fourier transform (13).

It follows from an analysis of (13) that in order to obtain the best evaluation of the pulse reaction of a hydroacoustic channel $h\varphi(t)$ in general it is necessary to have some idea concerning the characteristics of this channel. In this case reference is to the square of the modulus of the frequency characteristic of the medium $|S\varphi(\omega)|^2$ or the autocorrelation function $R\varphi(\tau)$ of its pulse reaction, a knowledge of which is necessary in constructing the filter.

We will determine the mean square error in evaluating the pulse reaction of the hydroacoustic channel when using the proposed filter. Now we will examine the frequency representation of each of the four integrals entering into the sum (6).

$$A_1 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} h_{\text{cp}}^2(t)\,dt = \frac{1}{2\pi T}\int_{-\infty}^{+\infty} |\dot{S}_{\text{cp}}(\omega)|^2\,d\omega$$

according to the Parseval theorem.

$$A_2 = -\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\iint h_{\text{cp}}(t)\,h(t_0)\,h_{\text{cp}}(t_1)\,x(t-t_0-t_1)\,dt_1\,dt_0\,dt$$

Using the faltung theorem, we obtain

$$A_2 = -\frac{1}{\pi T}\int_{-\infty}^{+\infty} |\dot{S}_{\text{cp}}(\omega)|^2\,\dot{S}_{h0}(\omega)\,\dot{S}_x(\omega)\,d\omega$$

$$A_3 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\iint h(t_0)\,h(t_2)\,\overline{n(t-t_0)\,n(t-t_2)}\,dt_0\,dt_2\,dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_n(\omega)\,|\dot{S}_h(\omega)|^2\,d\omega \tag{14}$$

$$A_4 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\iiiint h(t_0)\,h(t_2)\,h_{\text{cp}}(t_1)\,h_{\text{cp}}(t_3)\,x(t-t_0-t_1)\,x(t-t_2-t_3)\,dt_0\,dt_2\,dt_1\,dt_3 =$$

$$= \frac{1}{2\pi T}\int_{-\infty}^{+\infty} |\dot{S}_{\text{cp}}(\omega)|^2\,|\dot{S}_h(\omega)|^2\,|\dot{S}_x(\omega)|^2\,d\omega$$

$$F = \sum_{i=1}^{4} A_i = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\Big\{\frac{|\dot{S}_{\text{cp}}(\omega)|^2}{T} - 2\frac{|\dot{S}_{\text{cp}}(\omega)|^2}{T}\dot{S}_h(\omega)\,\dot{S}_x(\omega) +$$

$$+ |\dot{S}_h(\omega)|^2\,\frac{|\dot{S}_{\text{cp}}(\omega)|^2}{T}\,|\dot{S}_x(\omega)|^2 + |\dot{S}_h(\omega)|^2 H_n(\omega)\Big\}\,d\omega\,.$$

We substitute into (14) the optimum solution for $\dot{S}_h(\omega)$ found in (13). After some transformations we obtain

$$F_0 = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{H_n(\omega)}{|\dot{S}_x(\omega)|^2 + \frac{T\cdot H_n(\omega)}{|\dot{S}_{\text{cp}}(\omega)|^2}}\,d\omega \tag{15}$$

It can be seen from (15) that in the absence of noise the mean square error in evaluating the pulse characteristic of the medium is equal to 0.

Now we will examine some statistical properties of the process x(t) in selecting a filter with a frequency characteristic in the form (13), taking into account that in this case z(t) is the best evaluation of the pulse reaction of the medium.

Taking into account that the mean noise value is equal to zero, we will write

$$M\{z(t)\} = \iint h(t_0)\,h_{\text{cp}}(t_1)\,x(t-t_0-t_1)\,dt_1\,dt_0 =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\dot{S}_h(\omega)\,\dot{S}_{\text{cp}}(\omega)\,\dot{S}_x(\omega)\,e^{j\omega t}\,d\omega = \tag{16}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{|\dot{S}_x(\omega)|^2\,\dot{S}_{\text{cp}}(\omega)\,e^{j\omega t}}{|\dot{S}_x(\omega)|^2 + \frac{T\,H_n(\omega)}{|\dot{S}_{\text{cp}}(\omega)|^2}}\,d\omega$$

$$\mathcal{D}\{z(t)\} = M\{z^2(t)\} - M^2\{z(t)\}\,.$$

Using (16), from (4), after similar transformations of the process, employing its representation in the frequency region, we obtain

$$\mathcal{D}\{z(t)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\dot{S}_n(\omega)|^2 H_n(\omega)\, e^{j\omega t} dt = \\ = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{}{\left[|\dot{S}_x(\omega)|^2 + \frac{T H_n(\omega)}{|S_{cp}(\omega)|^2}\right]^2} d\omega \qquad (17)$$

from (16) and (17) it can be seen that when $H_n(\omega) \to 0$

$$M\{z(t)\} \longrightarrow \frac{1}{2\pi}\int_{-\infty}^{+\infty} \dot{S}_{cp}(\omega)\, e^{j\omega t} d\omega = h_{cp}(t)$$

$$\mathcal{D}\{z(t)\} \longrightarrow 0$$

that is, with the spectral density of the noise being equal to zero, the evaluation of the pulse reaction of the medium is unbiased with a zero dispersion. With an increase in the spectral density of the noise there is an increase in the bias of the evaluation and its dispersion.

We note in conclusion that in the solution of the formulated problem we employed the so-called structural approach in which it was necessary to select the best processing system in a particular class. By varying the structure of the system within the framework of a given class of linear systems we found a discrete system optimum from the point of view of the selected criterion.

An obvious advantage of the structural approach is that in its use it is usually sufficient to have only a partial description (stipulation) of the processes.

An obvious shortcoming of this method is that frequently it is impossible to say whether the structure has been correctly selected. At first glance it may seem that one of the ways to choose the most suitable structure is to assume that the sought-for structure is an arbitrary nonlinear system with time-variable parameters. In other words, the class of structures selected is so broad that it takes in all the possible systems. The difficulty here is that there is no convenient mathematical approach, for example, such as the faltung integral (Duhamel integral) for expressing the output voltage of a nonlinear system through the voltage across its input. Precisely for that reason we have limited ourselves to an examination of a class of linear systems. However, we feel that in the future it will be possible to apply the results to the case of the pulse characteristic of a channel with time-variable parameters.

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SPECTRUM OF SEA REVERBERATION AS A NONSTATIONARY RANDOM PROCESS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 220-224

[Article by V. A. Geranin, A. N. Prodeus and B. I. Shotskiy]

[Text] An investigation of the correlation properties of sea reverberation shows that in the case of an arbitrary signal duration and observation time [1] the reverberation at the output of an acoustic antenna does not belong to a class of random processes reducible to a stationary class.

The author of [1] investigated a symmetrized reverberation correlation function and found its approximate spectrum. The author used a determination of the spectrum of a nonstationary random process from Bendat and Pirsol. A weak side of this determination of the spectrum is the impossibility of its measurement and the difficulty of a physical interpretation. Both these weaknesses are attributable to the fact that the ω argument of the Bendat-Pirsol spectrum is not identical to the argument of the spectral function of the analyzed process.

In this connection it is of theoretical and practical interest to seek an analytical expression of the reverberation spectrum in the S. Ya. Rayevskiy definition [2].

This study is the next step in the plan for investigating the spectral-correlation structure of sea reverberation as a nonstationary random process. The conditions under which the assumptions adopted in computing the spectrum in [1] are justified were formulated. A general expression for the spectrum in the S. Ya. Rayevskiy determination of volume reverberation was rigorously derived. The spectra in tonal and f-m signals were found.

The complex envelope of the symmetrized correlation function of reverberation

$$R(\tau,t)=\langle F(t+\tfrac{\tau}{2})F(t-\tfrac{\tau}{2})\rangle \qquad (1)$$

in the simplest case when there is no relative movement of the antenna and scatterers has the form [1]:

$$R_0(\tau,t)=N\int_{-\infty}^{\infty}z^2A^2(z)S\left(t-\frac{2z}{C_0}+\frac{\tau}{2}\right)S^*\left(t-\frac{2z}{C_0}-\frac{\tau}{2}\right)dz \tag{2}$$

where A(z) is the law of change in the amplitude of an acoustic oscillation on the path "source-scatterer-receiver"; S(t) is the complex envelope of a signal with the duration T.

Carrying out replacement of the variables t - 2z /C_0 = t_1 and using the notation

$$\left(\frac{C_0}{2}t_1\right)A\left(\frac{C_0}{2}t_1\right)S(t_2)=S_1(t_1+t_2,\ t_2) \tag{3}$$

expression (2) can be reduced to the form

$$R_0(\tau,t)=\frac{C_0}{2}N\int_{-\infty}^{\infty}S_1\left(t+\frac{\tau}{2},\ t_1+\frac{\tau}{2}\right)S_1^*\left(t-\frac{\tau}{2},\ t_1-\frac{\tau}{2}\right)dt_1 \tag{4}$$

It is easy to show that for the moments t $\gg$ T expression (4) can be replaced by the approximate expression

$$R_0(\tau;t)\approx\frac{C_0}{2}N\int_{-\infty}^{\infty}S_1\left(t,\ t_1+\frac{\tau}{2}\right)S_1^*\left(t,\ t_1-\frac{\tau}{2}\right)dt_1, \tag{5}$$

which (with an accuracy to the notation) coincides with (21) in [1]. In the case of an exponential model A(z) formula (5) is precise.

Within the framework of correctness of expression (5) the expression of the Bendat-Pirsol reverberation envelope looks exceedingly simple (see formula (19) in [1]). Incidentally, computations show that the approximation (5) is also satisfactory when t$\geqslant$T.

The complex envelope of a nonsymmetrized correlation function of volume reverberation

$$K(\tau,t)=\langle F(t+\tau)F(t)\rangle \tag{6}$$

looks as follows

$$K_0(\tau,t)=N\int_{-\infty}^{\infty}z^2A^2(z)S\left(t-\frac{2z}{C_0}+\tau\right)\dot{S}\left(t-\frac{2z}{C_0}\right)dz \tag{7}$$

The S. Ya. Rayevskiy spectrum of the reverberation envelope is a result of the effect on (7) of the direct Fourier transform operator for τ and replacement of the variable v = 2z /C_0

$$\varphi_0(\omega,t)=\frac{NC_0}{4\pi}e^{j\omega t}Z(\omega)\int_{-\infty}^{\infty}\frac{C_0^2}{4}v^2A^2\left(\frac{C_0}{2}v\right)\dot{S}(t-v)e^{-j\omega v}dv; \tag{8}$$

where

$$Z(\omega)=\mathcal{F}\{S(t)\} \tag{9}$$

Below it is assumed that

$$H(z) = z^{-1} e^{-\beta z} \tag{10}$$

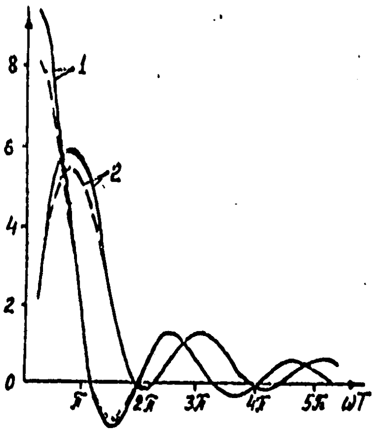

Fig. 1.

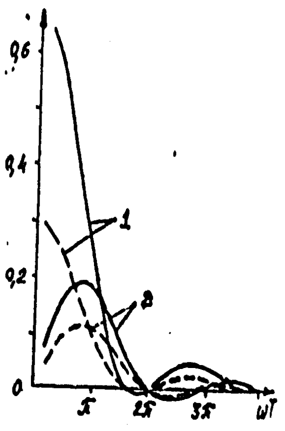

Fig. 2.

With the radiation of a pulse with tonal "filling," with (10) taken into account, the general expression assumes the form

$$\varphi_0(\omega,t) = 2NS^2 \cdot e^{j\omega t}\left\{\frac{\sin\frac{\omega T}{2}}{\omega}\left[\frac{e^{-(\beta c_0+j\omega)(t-\frac{T}{2})}}{t-\frac{T}{2}} - \frac{e^{-(\beta c_0+j\omega)(t+\frac{T}{2})}}{t+\frac{T}{2}} - \right.\right. \tag{11}$$

$$\left.\left. -(\beta c_0+j\omega)\{E_1[(\beta c_0+j\omega)(t-\frac{T}{2})]-E_1[(\beta c_0+j\omega)(t+\frac{T}{2})]\}\right]\right\},$$

where

$$E_1(z) = \int_z^\infty \frac{e^{-u}}{u}\,du \tag{12}$$

is the integral exponential function of the complex argument tabulated in [3].

In Figures 1 and 2, for t' = t/T = 0.55, t' = 1.0 respectively, we have shown the real (1) and fictitious (2) parts of the product

$$\left(\frac{\pi N S^2}{2c_0}\right)^{-1} \varphi_0(\omega,t)$$

The dashed curves correspond to $\beta' = 2\beta c_0 T = 0.5$; the solid curves correspond to $\beta' = 0$.

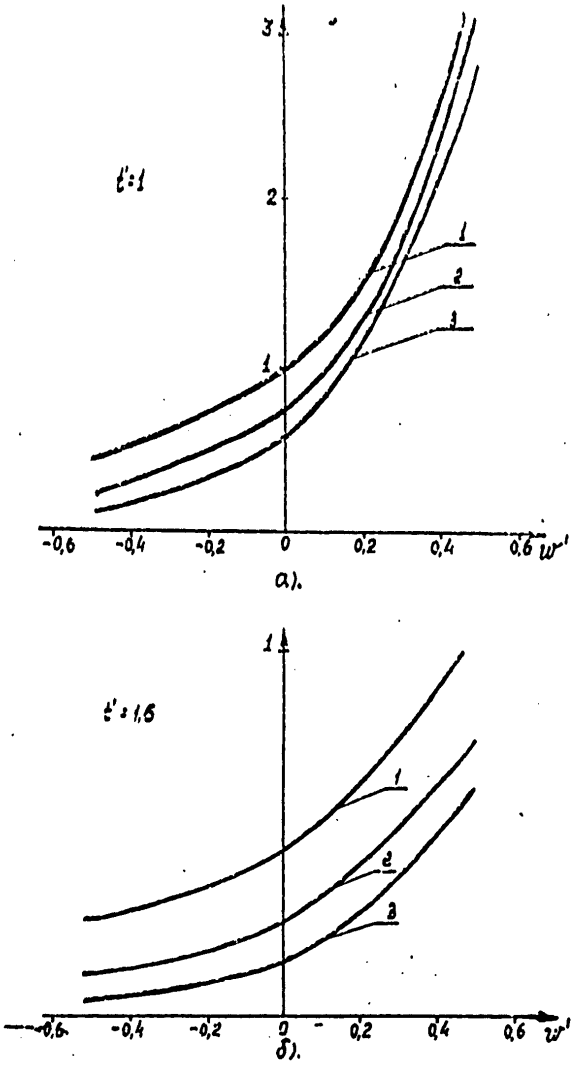

Fig. 3.

We note that with an increase in t' the spectrum $\varphi_0(\omega_1,t)$ is ever-closer to the function

$$S_0^2(\frac{\omega T}{2})$$

is the Fourier transform of the sounding signal correlation integral.

In the case of a rectangular video pulse with linear frequency modulation

$$s(t) = S_0\,\mathrm{rect}(\frac{t}{T})e^{j\pi\gamma t^2}, \tag{13}$$

where $\gamma = \Delta F/T$, ΔF is the frequency deviation, the S. Ya. Rayevskiy reverberation envelope has the form

$$\varphi_0(\omega,t) = \frac{NS_0^2}{2c_0\pi\sqrt{\pi\gamma}} e^{-j(\frac{\omega^2}{4\pi\gamma} - \omega t + \pi\gamma t^2)} \cdot [C(-\frac{T\sqrt{\pi\gamma}}{2} - \frac{\omega}{2\sqrt{\pi\gamma}}, -\frac{T\sqrt{\pi\gamma}}{2} - \frac{\omega}{2\sqrt{\pi\gamma}}) + jS(-\frac{T\sqrt{\pi\gamma}}{2} - \frac{\omega}{2\sqrt{\pi\gamma}}, -\frac{T\sqrt{\pi\gamma}}{2} - \frac{\omega}{2\sqrt{\pi\gamma}})] \cdot \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{e^{-(\beta c_0 - j(2\pi\gamma t - \omega))t'} e^{-j\pi\gamma t'^2}}{t'^2}\, dt'. \tag{14}$$

In a case of practical importance $\Delta FT \gg 1$

$$\varphi_0(\omega,t) = \frac{NS_0^2}{2c_0\pi\sqrt{\gamma}} e^{-j(\frac{\omega^2}{4\pi\gamma} - \omega t + \pi\gamma t^2)} \mathrm{rect}(\frac{\omega}{\Delta\omega}) \cdot \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{e^{-(\beta c_0 - j(2\pi\gamma t - \omega))t'} e^{-j\pi\gamma t'^2}}{t'^2}\, dt'. \tag{15}$$

When using the stationary phase method expression (15) is reduced to the form

$$\varphi_0(\omega,t) = \frac{N\Pi_0^2}{c_0} \cdot \frac{e^{-\beta c_0 \pi(\frac{t}{T} - \frac{\omega}{\Delta\omega})^2}}{(\frac{t}{T} - \frac{\omega}{\Delta\omega})^2} e^{-j\frac{\pi}{4}} \mathrm{rect}(\frac{\omega}{\Delta\omega}). \tag{16}$$

Figure 3, for t' = 1 and t' = 1.5 gives the modulus of the fraction

$$\varphi_0(\omega_1 t)/\frac{N\pi\delta}{c_0}.$$

Curve 1 corresponds to $\beta' = 0, 2$, $\beta' = 0.5$, $\beta' = 1$.

The emphasizing of the high frequencies in the reverberation spectrum is attributable to the fact that by the end of the sounding pulse the instantaneous frequency of linear frequency modulation filling is increased.

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UDC 621.391.2

OPTIMUM DETECTION OF MULTIRAY SIGNALS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 225-233

[Article by N. G. Gatkin, L. N. Kovalenko, L. G. Krasnyy and S. V. Pasechnyy]

[Text] In the propagation of sound in an inhomogeneous medium at the reception point there is interference of the sound oscillations propagating along different paths and this leads to distortions of the spatial (angular) and temporal structure of the signals. This circumstance must be taken into account in the synthesis of optimum detectors. However, the realization of optimum spatial-temporal processing involves serious technical difficulties. Accordingly, it is of interest to determine the structure and noise immunity of the receiving channels, taking into account only the temporal structure of the multiray signals.

As a model of a multiray signal it is customary to use a set of plane waves arriving at the reception point with different time lags τ_i:

$$S(t) = \sum_{i=1}^{n} \varepsilon_i A(t-\tau_i) \cos[\omega_0 (t-\tau_i) + \varphi(t-\tau_i) + \varphi_i], \quad (1)$$

where ε_i and φ_i are the fluctuating amplitudes and phases, n is the number of components in the total signal.

We will assume that ε_i fluctuates independently in conformity to the Rayleigh law and φ_i are uniformly distributed in the interval [0, 2π]. If the number of rays and the lags between them are determined, the signal S(t) has a Gaussian distribution with the correlation function

$$K_s(t,t_1) = Re\left\{\sum_{i=1}^{n} \dot{S}(t-\tau_i)\dot{S}^*(t-\tau_i) e^{j\omega_0 (t-t_1)}\right\}; \quad (2)$$

where

$$\dot{S}(t) = A(t) e^{j\psi(t)}$$

The correlation function of narrow-band noise is

$$K_N(t,t_1) = Re\left\{z(t,t_1) e^{j\omega_0 (t-t_1)}\right\} \quad (3)$$

We will use expressions (2) and (3) for the synthesis of an optimum detector.

Structure of Optimum Detector

The optimum detection of a signal (1) can be reduced to checking the hypothesis of correlation functions: the hypothesis H_0 -- the adopted realization of a u(t) Gaussian process with a zero mean and correlation function $K_H(t, t_1)$, the hypothesis H_S - u(t) -- a Gaussian process with a zero mean and correlation function $K_{SH}(t, t_1) = K_S(t, t_1) + K_H(t, t_1)$.

In accordance with the results in [1], the optimum processing algorithm has the form:

$$u_0 = \iint u(t_1)u(t_2)H(t_1,t_2)\,dt_1\,dt_2, \tag{4}$$

where $H(t_1, t_2)$ is the solution of the integral equation:

$$\iint_T K_{SN}(t,t_1)H(t_1,t_2)K_N(t_2,t_3)\,dt_1\,dt_2 = K_S(t,t_3) \tag{5}$$

The solution of equation (5) with the kernels (2), (3) will be sought in the form:

$$H(t_1,t_2) = 4\mathrm{Re}\left\{\sum_{i=1}^{n}\sum_{k=1}^{n} h_{ik}\dot{f}_{ik}(t_1,t_2)e^{j\omega_0(t_1-t_2)}\right\} \tag{6}$$

Substitution of (2), (3) and (6) into (5) gives:

$$\sum_{k=1}^{n} h_{ik}\iint_T\left\{\hat{z}(t,t_1)+\sum_{p=1}^{n}\dot{S}(t-\tau_p)\cdot\dot{S}^*(t_1-\tau_i)\right\}\times \tag{7}$$
$$\times f_{ik}(t_1,t_2)\cdot z(t_2,t_3)\,dt_1\,dt_2 = \dot{S}(t-\tau_i)\cdot\dot{S}^*(t_3-\tau_i)$$

If it is assumed that

$$f_{ik}(t_1,t_2) = \dot{B}(t_1-\tau_k)\dot{B}^*(t_2-\tau_i), \tag{8}$$

where B(t) is a solution of the integral equation

$$\int_T z(t,t_1)\dot{B}(t_1)\,dt_1 = \dot{S}(t), \quad t\in[0,T], \tag{9}$$

then in place of (7) we have:

$$\sum_{k=1}^{n} h_{ik}\left\{\dot{S}(t-\tau_k)+\sum_{p=1}^{n}\dot{S}(t-\tau_p)\Psi_{kp}\right\} = \dot{S}(t-\tau_i), \tag{10}$$

where

$$\Psi_{kp} = \int_T \dot{S}^*(t-\tau_p)\dot{B}(t-\tau_k)\,dt \; -$$

is the uncertainty function in the case of nonwhite noise introduced in [2].

Transforming equation (10) to the form

$$\sum_{p=1}^{n}\dot{S}(t-\tau_p)\left\{\sum_{k=1}^{n} h_{ik}(\delta_{kp}+\Psi_{kp})\right\} = \dot{S}(t-\tau_i),$$

we obtain a system of equations for determining the coefficients h_{ik}:

$$\sum_{k=1}^{n} h_{ik}(\delta_{kp}+\Psi_{kp}) = \delta_{ip}; \quad i,p = 1,\dots,n, \tag{11}$$

where δ_{Kr} is the Kronecker symbol.

With expression (8) taken into account the solution of equation (5) has the form:

$$H(t_1,t_2) = 4\,\mathrm{Re}\left\{\sum_{i=1}^{n}\sum_{k=1}^{n} h_{ik}\,\dot{B}(t_1-\tau_k)\,\dot{B}^*(t_2-\tau_i)\,e^{j\omega_0(t_1-t_2)}\right\},$$

hence the algorithm for optimum processing of a multiray signal (1)

$$u_0 = \mathrm{Re}\left\{\sum_{i=1}^{n}\sum_{k=1}^{n} h_{ik}\,e^{j\omega_0(\tau_k-\tau_i)}\left[\int_0^T \dot{B}(t)e^{j\omega_0 t}u(t+\tau_i)dt\right]\times \times\left[\int_0^T \dot{B}^*(t)e^{-j\omega_0 t}u(t+\tau_k)dt\right]\right\} \tag{12}$$

It follows from expression (12) that an optimum detector consists of n partial channels (in accordance with the number of components of the total signal), each of which contains a correlator with a reference signal operative from the moment of arrival of the corresponding component. The channel outputs were combined in a cross-correlation processing scheme with the coefficients $h_{ik}e^{j\omega 0(\varepsilon_k-\varepsilon_i)}$ Fig. 1a).

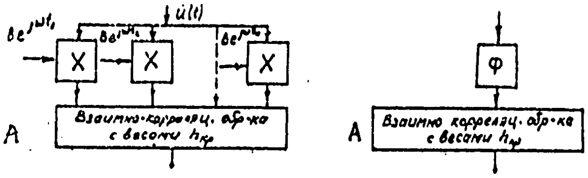

Fig. 1. Structural diagram of optimum detector. A) Cross-correlation processing with weights h_{Kr}.

There is also another possible construction of the processing channel (12). The channel contains one filter with a frequency-phase characteristic complexly conjugate with the spectrum of an individual component of the total signal. The effects at the filter output, read at the moments in time corresponding to the delays $\tau_1,\dots,\tau_n$, correlate with the coefficients $h_{ik}e^{j\omega s(\tau_k-\tau_i)}$ (Fig. 1b).

The structure of the optimum processing channel is substantially simplified if the components of the total signal after optimum filtering do not overlap in time. In this case $\Psi_{Kr} = \Psi_{kk}\delta_{Kr}$ and

$$h_{ik} = \frac{\delta_{ik}}{1+\Psi_{ii}} \tag{13}$$

With the substitution of this expression into (12) we obtain:

$$u_0 = \sum_{i=1}^{n} \frac{1}{1+\psi_{ii}} \left| \int_0^T \dot{B}(t) e^{j\omega_0 t} \cdot u(t+\tau_i)\, dt \right|^2 \qquad (14)$$

Accordingly, in this case the optimum for combined "partial" channels is the incoherent accumulation of their output effects.

In problems of communication and range measurement the need may arise for discriminating only one k-th component of the total signal. All the remaining components, other than the discriminated component, must be classified as noise. In such a case the procedure for detecting the k-th component is described by the functional transformation

$$\lambda(u) = \frac{\mu_n(du)}{\mu_{n-1}(du)}, \qquad (14')$$

where $\lambda(u)$ is the probability ratio, μ_n is a measure corresponding to the sum of all "n" components of the total signal, μ_{n-1} is a measure corresponding to the sum of "n-1" signal components (without the "k-th" component).

The expression (14') for $\lambda(u)$ can be easily transformed to the form

$$\log \lambda(u) = \log \lambda_n(u) - \log \lambda_{n-1}(u)$$

where $\lambda_n(u)$, $\lambda_{n-1}(u)$ is the probability ratio for the "n" and "n - 1" (except the "k-th" components of the total signal (1)).

Taking into account this expression and (12), the algorithm for discriminating the "n-th" component is described by the expression

$$u_0 = \mathrm{Re}\left\{ \sum_{i=1}^{n} \sum_{p=1}^{n} \left(h_{ip}^{(n)} - h_{ip}^{(n-1)}\right) \left[\int \dot{B}(t) e^{j\omega_0 t} u(t+\tau_p)\, dt\right] \times \right. \qquad (15)$$
$$\left. \times \left[\int_0^T \dot{B}^*(t) e^{-j\omega_0 t} u^*(t+\tau_i)\, dt\right] \right\},$$

in which the notation $h_{ip}^{(n)}$ and $h_{ip}^{(n-1)}$ indicates the order of the h_{ip} matrices (n x n and (n - 1)(n - 1), respectively).

Taking into account the properties of the inverse matrices h_{ip} expression (15) is transformed to the form

$$u_0 = \left| \int_0^T \dot{B}(t) e^{j\omega_0 t} \left\{ \sum_{p=1}^{n} \frac{h_{kp}^{(n)}}{\sqrt{h_{kk}^{(n)}}} \cdot u(t+\tau_p) \right\} dt \right|^2 \qquad (16)$$

Thus, the channel for discriminating the "k-th" component contains an optimum filter with a preliminary weighted accumulation of the components delayed by $\tau_1, \ldots, \tau_n$.

In the special case (13) expression (16) with an accuracy to constant factors assumes the form

$$u_0 = \left| \int_0^T \dot{B}(t) e^{j\omega_0 t} u(t+\tau_k)\, dt \right|^2, \qquad (17)$$

that is, in this case the channel for discriminating the "k-th" component contains only one channel optimum for this component and ignores the presence of other components.

Expressions (12) and (16) describe the algorithms for the optimum processing of a multiray signal (1) with independently fluctuating components. It is of interest to compare the case of independent fluctuations of signals with the case of harmonious fluctuations ($\mathcal{E}_1 \equiv \mathcal{E}$) and coherent rays ($\varphi_1 \equiv \varphi$).

The solution of equation (5) with the signal (1), for which $\mathcal{E}_1 \equiv \mathcal{E}$, and $\varphi_1 \equiv \varphi$ leads to the following algorithm for the optimum detection of a multiray signal:

$$u_0 = \Big|\sum_{i=1}^{n} \int_T \dot{B}(t) e^{j\omega_0 t} u(t+\varepsilon_i)\, dt\Big|^2 \cdot \tag{18}$$

The channel consists of "n" "partial" channels ensuring the optimum filtering of individual components, after which the effects should be coherently summed.

In problems of discriminating the "k-th" ray from "n" harmoniously fluctuating components, the optimum processing algorithm is described by expression (17), that is, in the case of harmonious fluctuations of signals the optimum discrimination of the "k-th" component is reduced to the optimum reception of a signal, propagating only along this ray, whereas the presence of other components in this case is ignored. We recall that in the case of independent fluctuations the same result occurred only in the special case (13).

Noise Immunity of Optimum Detectors

For analysis of the noise immunity of algorithm (12) it is necessary to compute the distribution of the quadratic form

$$u_0 = Re\Big\{\sum_{i=1}^{n}\sum_{k=1}^{n} h_{ik}\, u_i\, u_k^*\Big\}, \tag{19}$$

For the coefficients h_{ik}, determined from the system of equations (11), the following representation is correct:

$$h_{ik} = \sum_{j=1}^{n} \frac{1}{1+\lambda_i} \dot{\varphi}_{ji} \dot{\varphi}_{jk}^* \tag{20}$$

where λ_i and φ_{ii} are the eigenvalues and eigenvectors of the matrix $\{\Psi_{pk}\}$, which are found from the equation:

$$\sum_{k=1}^{n} \Psi_{pk}\, \varphi_{jk} = \lambda_j\, \varphi_{jp}$$

In actuality, with the substitution of (20) into the right-hand side of (11) we obtain

$$\sum_{k=1}^{n} h_{ik}(\delta_{kp} + \Psi_{kp}) = \sum_{j=1}^{n} \frac{1}{1+\lambda_j} \varphi_{ji} \Big(\varphi_{jp}^* + \sum_{k=1}^{n} \Psi_{kp}\, \varphi_{jk}^*\Big)$$

$$\sum_{j=1}^{n} \frac{1}{1+\lambda_j} \varphi_{ji} (\varphi_{jp}^* + \lambda_j \varphi_{jp}^*) = \sum_{j=1}^{n} \varphi_{ji} \varphi_{jp}^*,$$

which coincides with the expansion δ_{ip}.

Expression (20) makes it possible to write (19) in canonical form:

where

$$u_0 = \sum_{j=1}^{n} \frac{1}{1+\lambda_j} |z_j|^2,$$

$$z_j = \sum_{i=1}^{K} \varphi_{ji} u_i$$

It is easy to show that the random values z_i are uncorrelated both in the absence and in the presence of a signal. They are also distributed like u_i, in conformity to a Gaussian law, with a zero mean and the dispersion:

$$\sigma_i^2 = \begin{cases} \lambda_i & \text{in the absence of a signal} \\ \lambda_i(1+\lambda_i) & \text{in the presence of a signal} \end{cases}$$

The distribution of the sum of the squares of Rayleigh random values with different dispersions can be computed easily. Hence we obtain expressions for the probability of correct detection D and a false alarm F:

$$D = \prod_{i=1}^{n} \frac{1}{2\lambda_i} \sum_{k=1}^{n} \frac{2\lambda_k \cdot e^{-\frac{U_0}{2\lambda_k}}}{\prod_{\substack{j=1 \\ j\neq k}}^{n} \left(\frac{1}{2\lambda_j} - \frac{1}{2\lambda_k}\right)}, \qquad (21)$$

$$F = \prod_{i=1}^{n} \frac{(1+\lambda_i)}{2\lambda_i} \sum_{k=1}^{n} \frac{\frac{2\lambda_k}{1+\lambda_k} e^{-\frac{U_0(1+\lambda_k)}{2\lambda_k}}}{\prod_{\substack{j=1 \\ j\neq k}}^{n} \left(\frac{1+\lambda_j}{2\lambda_j} - \frac{1+\lambda_j}{2\lambda_k}\right)}, \qquad (22)$$

where U_0 is the threshold, and λ_i are the eigenvalues determined from the equation

$$\det \| \Psi_{ik} - \lambda_i \delta_{ik} \| = 0$$

In the case of harmonious fluctuations, the distribution (19) is exponential and therefore

$$D = F^{\frac{1}{1+d}}$$

where

$$d = \sum_{i=1}^{n} \sum_{k=1}^{n} \Psi_{ik}$$

is the signal-to-noise ratio after the coherent accumulation of voltages u_i from the outputs of the "partial" detectors.

In the channel for discriminating the "k-th" component from "n" components in the case of both independent and harmonious fluctuations the distribution (16) is also exponential. Therefore, D is determined by expression (23), in which

$$d = \begin{cases} \sum_{i=1}^{M} |\psi_{ki}|^2 & \text{in the case of harmonious fluctuations} \\ \dfrac{1-2h_{kk}+\sum_{i=1}^{n} |h_{ki}|^2}{h_{kk}-\sum_{i=1}^{n} |h_{ki}|^2} & \text{in the case of independent fluctuations} \end{cases}$$

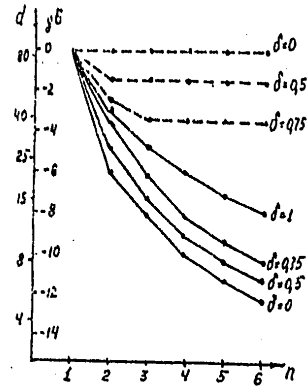

Fig. 2

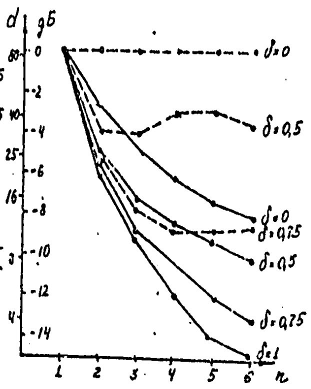

Fig. 3

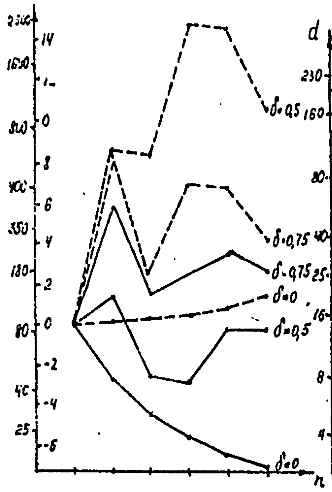

Fig. 4

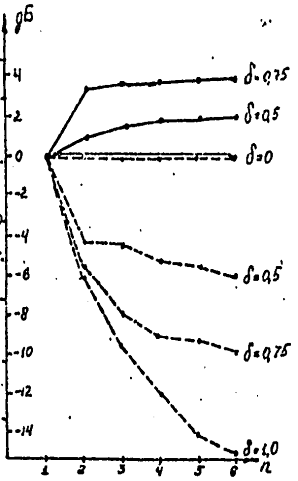

Fig. 5

For comparison of the noise immunity of channels for the optimum processing of multiray signals we will determine the dependence of the threshold signal-to-noise ratio $\psi_{ii} = \alpha \int \dot{S}^*(t)\dot{B}(t)$ dt on the number "n" of components and the relative delay between them δ = 1 $T_{out}/\Delta\tau$; ($\tau_i = i\Delta\tau$; T_{out} is the duration of the signal component after correlation processing).

The results of the computations of noise immunity of the algorithms (12) and (18) are represented in Figures 2-3 by solid curves.

The figures show that in the case of independent fluctuations of the components the noise immunity of the optimum detector increases with a decrease in the degree of overlapping of the rays. For example, with n = 6 this gain attains 4.5 db. In the case of harmonious fluctuations of the components an inverse effect is observed in coherent rays. For example, with n = 6 the gain due to overlapping of the signals is 7 db. However, this effect exists only with definite relationships between the period of the carrier frequency of the components $*/\omega_0$ and their lag τ_i. Situations are possible when the relative overlapping of the harmoniously fluctuating components is such that with definite relationships between the pertinent parameters there is a marked decrease in noise immunity (see Fig. 4, solid curves). [*illegible]

A similar character of the dependence of the d parameter on the relative delay δ is also manifested for channels for discriminating one of the "n" components (Fig. 5).

Now we will compare the noise immunity of optimum processing of multiray signals with a processing channel optimized for reception of the signal s(t) in the absence of a multiray situation. The structure of such a channel is described by expression (17), whereas noise immunity is determined by expression (23), in which

$$d = \begin{cases} \sum_{i=1}^{n} |\psi_{ix}|^2 & \text{in the case of independent fluctuations} \\ |\sum_{i=1}^{n} \psi_{ix}|^2 & \text{in the case of harmonious fluctuations.} \end{cases}$$

The results of computations of noise immunity of such a channel are represented in Figures 2-4 by dashed lines. A comparison of these results with the noise immunity of optimum detectors shows that ignoring signal structure can lead to losses of about 5-10 db, which indicates a need for optimizing the signal detection algorithms under multiray propagation conditions.

It is characteristic for the resulting processing algorithms that the highest noise immunity (for a model of a signal with independent fluctuations of components) and absence of a dependence of noise immunity on the relationship between the period of the carrier frequency of the components and their delays (for a model of a signal with harmonious fluctuations in coherent rays) is attained in the case of separation of the components in time after correlation processing. Accordingly, under conditions of multiray propagation it is desirable to use complex signals, whose optimum processing can ensure a high resolution of the components. It is obvious that the same result can also be obtained in ensuring the spatial separation of the rays along which the signals arrive at the reception point with equal or close delays.

We note in conclusion that with a priori unknown conditions of signal propagation (the number of components of the multiray signal components and the delays between them are unknown) the optimum detector must have a multichannel structure ensuring "sorting" for all the anticipated parameters and a system for sampling the maximum at the channel output.

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SPECTRAL-CORRELATION ANALYSIS OF AN ANTENNA SITUATED IN A NONHOMOGENEOUS NONSTATIONARY HYDROACOUSTIC FIELD

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 234-241

[Article by E. A. Artemenko, V. A. Geranin, M. I. Karnovskiy, A. N. Prodeus and G. D. Simonova]

[Text] The principles of spectral-correlation analysis of systems embodying a transformation of the "wave field - process" type have never been set forth in the literature. In this article this gap in the theory of information systems is partially filled.

We examine two models of a linear spatial-temporal system: continuous and discrete one-dimensional hydroacoustic antenna systems.

The model of the phenomenon is a field in water excited by noise sources situated on the plane surface of a deep sea.

Continuous antenna. We will examine first a discrete antenna with electric scanning of the directional diagram, the distance between whose elements is small in comparison with the wavelength. We will call such an antenna a continuous antenna and we will describe it by integral expressions.

Assume that the continuous antenna is situated on the x-axis of a Cartesian coordinate system. The weighting function of an antenna element W(x, t - τ, t) is the response to an effect in the form:

$$\varphi(x, 0, 0, t) = \delta(x - x_1)\delta(t - \tau) \qquad (1)$$

where δ (u) - δ is the Dirac function.

The function W(x, t - τ , t) characterizes the distribution of sensitivity and inertial properties along the antenna. The latter take into account both the characteristic inertia of the converters and the presence of a variable time lag in the circuit of each converter, ensuring electric scanning of the directional diagram.

The complex frequency characteristic of the antenna is

$$H(\omega\alpha_x, \omega, t) = \iint_{-\infty} W(x, v, t)\exp\{-j\omega(v - \alpha_x x)\}\,dx\,dv \tag{2}$$

If no provision is made for scanning and the inertia of the converters is identical, then

$$W(x, t-\tau, t) = I(x)\,h(t-\tau) \tag{3}$$

and we have

$$H(\omega\alpha_x, \omega, t) = R(\omega\alpha_x)\,K(\omega) \tag{4}$$

where

$$R(\omega\alpha_x) = \int_{-\infty}^{\infty} I(x)\exp\{j\omega\alpha_x x\}\,dx \tag{5}$$

$K(\omega)$ is the complex frequency characteristic of the converter. If the antenna is oriented along an arbitrary straight line in space,

$$\frac{x - x_0}{n_{11}} = \frac{y - y_0}{n_{21}} = \frac{z - z_0}{n_{31}} \tag{6}$$

then the complex frequency characteristic is

$$H_0(\omega\alpha_x, \omega\alpha_y, \omega, t) = \frac{1}{n_{11}} \iint_{-\infty}^{\infty} W(x, y, z, v, t) \times \\ \times \exp\{-j\omega[v - (\alpha_x x + \alpha_y y + \alpha_z z)]\}\,dx\,dv \tag{7}$$

where y and z are related to x by expression (6); W(x, y, z, v, t) is the weighting function of an antenna element corresponding to its new spatial position.

It is easy to confirm that

$$H_0(\omega\alpha_x, \omega\alpha_y, \omega, t) = H[\omega(\alpha_x n_{11} + \alpha_y n_{21} + \alpha_z n_{31}), \omega, t] \times \\ \times \exp\{-j\omega(\alpha_x x_0 + \alpha_y y_0 + \alpha_z z_0)\} \tag{8}$$

We note that $H_0(\omega\alpha_x, \omega\alpha_y, \omega, t)$ is the complex envelope of antenna response to the effect of a plane monochromatic wave

$$\varphi(x, y, z, t) = \exp\{j\omega[t - (\alpha_x x + \alpha_y y + \alpha_z z)]\} \tag{9}$$

In a general case W(x, y, z, v, t) is a nonstationary random process. Therefore, it is desirable to introduce the following antenna characteristics

$$B_W(x_1, y_1, z_1, v_1, t_1; x_2, y_2, z_2, v_2, t_2) = \langle W(x_1, y_1, z_1, v_1, t_1)W(x_2, y_2, z_2, v_2, t_2)\rangle \tag{10}$$

$$\varphi_W(\omega_1\alpha_x, \omega_1\alpha_y, \omega_1, \omega; x, y, z, v, t_2) = \\ = \frac{1}{(2\pi)} \langle W(x, y, z, v, t_2)\Gamma(\omega_1\alpha_x, \omega\alpha_y, \omega_1, \omega_1 + \omega)\rangle \times \\ \times \exp\{j[\omega_1 v + \omega t_1 - \omega_1(\alpha_x x + \alpha_y y + \alpha_z z)]\} \tag{11}$$

$$\varphi_W(\omega'\alpha'_x, \omega'\alpha'_y, \omega', \omega_1; \omega''\alpha''_x, \omega''\alpha''_y, \omega'', \omega_2) = \frac{1}{(2\pi)^2} \langle \Gamma(\omega'\alpha'_x, \omega'\alpha'_y, \omega', \omega'+\omega_1) \times \bar{\Gamma}(\omega''\alpha''_x, \omega''\alpha''_y, \omega'', \omega''+\omega_2) \rangle \tag{12}$$

$$\varkappa_W(\chi_x, \chi_y, \Omega; u_x, u_y, u_z, \tau) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} K_W(u_x, u_y, 0, \tau; x, y, 0, t) \times \exp\{j[\Omega t - (\chi_x x + \chi_y y)]\}\, dx\, dt = \iiint_{-\infty}^{\infty} Q_W(\chi_x, \chi_y, \Omega; \kappa_x, \kappa_y, \omega) \times \exp\{j[\omega\tau - (\kappa_x u_x + \kappa_y u_y + \kappa_z u_z)]\}\, d\kappa_x\, d\kappa_y\, d\omega \tag{13}$$

They can be interpreted as the autocorrelation function, spectrum, frequency correlation function and the function of the difference arguments for time and frequency respectively.

Here

$$\Gamma(\omega'\alpha'_x, \omega'\alpha'_y, \omega'; \omega_1) = \int_{-\infty}^{\infty} H_0(\omega'\alpha'_x, \omega'\alpha'_y, \omega'; t) \times \exp\{-j(\omega_1 - \omega')t\}\, dt \tag{14}$$

$$K_W(u_x, u_y, u_z, \tau; x, y, z, t) = B_W(x+u_x, y+u_y, z+u_z, t+\tau, x, y, z, t) \tag{15}$$

$$Q_W(\chi_x, \chi_y, \Omega; \kappa_x, \kappa_y, \omega) = \Psi_W(\kappa_x, \kappa_y, \omega; \kappa_x+\chi_x, \kappa_y+\chi_y, \omega+\Omega) \tag{16}$$

is the complex conjugation symbol.

We will express the autocorrelation function

$$B_0(t_1, t_2) = \langle u(t_1) u(t_2) \rangle, \tag{17}$$

the Rayevskiy spectrum

$$\varphi_u(\omega_1, t_2) = \frac{1}{2\pi} \langle u(t_2) S_u(\omega_1) \rangle e^{j\omega_1 t_2} \tag{18}$$

the frequency correlation function

$$\Psi_u(\omega_1, \omega_2) = \frac{1}{(2\pi)^2} \langle S_u(\omega_1) \bar{S}_u(\omega_2) \rangle, \tag{19}$$

the function of the difference arguments for time and frequency

$$(\tau, \Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K_u(\tau, t) e^{-j\Omega t}\, dt = \int_{-\infty}^{\infty} Q_u(\Omega, \omega) e^{-j\omega\tau}\, d\omega, \tag{20}$$

$$[K_u(\tau, t) = B_u(t + \tau, t), \tag{21}$$

$$Q_u(\Omega, \omega) = \Psi_u(\omega, \omega + \Omega)] \tag{22}$$

the antenna response u(t) through the spectral-correlation characteristics of the field $\varphi(x, y, z, t)$ in which it is situated.

As the initial expressions it is convenient to use:

$$(t) = \frac{1}{n_{11}}\iint_{-\infty}^{\infty} \varphi(x, y, z, t - v) W(x, y, z, v, t)\, dx, dv \tag{23}$$

$$S_u(\omega) = \frac{1}{(2\pi)^3 n_{11}} \iiint_{-\infty}^{\infty} S_\varphi(\omega'\alpha_x, \omega'\alpha_y, \omega') \times$$

$$(\omega'\alpha_x, \omega'\alpha_y, -\omega', -\omega)\omega'^2\, d\alpha_x\, d\alpha_y\, d\omega' \tag{24}$$

$$U(t) = \frac{1}{(2\pi)^4 n_{11}} \iiiint_{-\infty}^{\infty} S_\varphi(\omega'\alpha_x, \omega'\alpha_y, \omega') \times$$

$$(\omega'\alpha_x, \omega'\alpha_y, -\omega', -\omega)\exp(j\omega t)\omega'^2\, d\alpha_x, d\alpha_y\, d\omega'\, d\omega \tag{25}$$

We obtain analytical expressions relating the spectral-correlation characteristics of antenna response and the field affecting it by combining (23), (24) and (25).

For example, from (23), with (10) taken into account, it follows that:

$$B_u(t_1, t_2) = \frac{1}{n_{11}^2} \iiiint_{-\infty}^{\infty} B_\varphi(x_1, y_1, z_1, t_1 - v_1; x_2, y_2, z_2, t_2 - v_2) \times$$

$$\times B_W(x_1, y_1, z_1, v_1, t_1; x_2, y_2, z_2, v_2, t_2)\, dx_1\, dv_1\, dx_2\, dv_2 \tag{26}$$

On the basis of (25) and (12) we obtain

$$B_u(t_1, t_2) = \frac{1}{(2\pi)^6 n_{11}^2} \int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty} \Psi_\varphi(\omega'\alpha_x', \omega'\alpha_y', \omega'; \omega''\alpha_x'', \omega''\alpha_y'', \omega'') \times$$

$$\times \bar{\Psi}_W[\omega'\alpha_x', \omega'\alpha_y', \omega''\alpha_x'', \omega''\alpha_y'', -\omega', -\omega'', -(\omega_1 - \omega'), -(\omega_2 - \omega'')] \times$$

$$\times \exp[j(\omega_1 t_1 - \omega_2 t_2)]\omega'^2 \omega''^2\, d\alpha_x'\, d\alpha_y'\, d\omega'\, d\omega_1\, d\alpha_x''\, d\alpha_y''\, d\omega''\, d\omega_2 \tag{27}$$

Multiplying (23) and (25), with (11) taken into account, we obtain

$$B_u(t_1,t_2)=\frac{(2\pi)^4}{n_{II}^2}\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}\varphi_v(\omega'\alpha_x,\omega'\alpha_y,\omega';x,y,z,t_1-v)\cdot$$
$$\cdot\,\bar{\varphi}_w[\omega'\alpha_x,\omega'\alpha_y,-\omega',-(\omega-\omega');x,y,z,v,t_1]\exp[-j\omega(t_1-t_2)]\omega'^4\cdot \qquad (28)$$
$$\cdot\, dx\,dv\,d\alpha_x\,d\alpha_y\,d\omega'\,d\omega$$

It follows from (23), (24) and (11) that

$$\varphi_u(\omega_1,t_2)=\frac{(2\pi)^4}{n_{II}^2}\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}\varphi_v(\omega'\alpha_x,\omega'\alpha_y,\omega';x,y,z,t_2-v)\cdot \qquad (29)$$
$$\cdot\,\bar{\varphi}_w[\omega'\alpha_x,\omega'\alpha_y,-\omega',-(\omega_1-\omega');x,y,z,v,t_2]\omega'^4\,dx\,dv\,d\alpha_x\,d\alpha_y\,d\omega' \quad (29)$$

Multiplying (24) and (25), with (12) taken into account, we obtain

$$\varphi_0(\omega_1,t_2)=\frac{(2\pi)^8}{n_{II}^2}\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}\psi_v(\omega'\alpha_x',\omega'\alpha_y',\omega';\omega''\alpha_x'',\omega''\alpha_y'',\omega'')\cdot$$
$$\cdot\,\psi_w[\omega'\alpha_x',\omega'\alpha_y',\omega''\alpha_x'',\omega''\alpha_y'',-\omega',-\omega'',-(\omega_2-\omega'),-(\omega_1-\omega'')]\cdot \qquad (30)$$
$$\cdot\,\exp[j(\omega_1-\omega_2)t_2]\omega'^4\omega''^4\,d\alpha_x'\,d\alpha_y'\,d\omega'\,d\alpha_x''\,d\alpha_y''\,d\omega''\,d\omega_2$$

Expressions (24) and (12) are reduced to the expression

$$\psi_u(\omega_1,\omega_2)=\frac{1}{(2\pi)^6 n_{II}^2}\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}\psi_v(\omega'\alpha_x',\omega'\alpha_y',\omega';\omega''\alpha_x'',\omega''\alpha_y'',\omega'')\cdot$$
$$\cdot\,\bar{\psi}_w(\omega'\alpha_x',\omega'\alpha_y',\omega''\alpha_x'',\omega''\alpha_y'',-\omega',-\omega'',-(\omega_1-\omega'),-(\omega_2-\omega'')]\omega'^4\omega''^4\cdot \qquad (31)$$
$$d\alpha_x'\,d\alpha_y'\,d\omega'\,d\alpha_x''\,d\alpha_y''\,d\omega''$$

In order to obtain the expression $\varkappa(\Omega, \tau)$ we employ the expression

$$K_u(\tau,t)=\frac{1}{n_{II}^2}\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}K_v(u_x,u_y,u_z,\tau-\tau';x,y,z,t-t')\cdot$$
$$\cdot\,K_w(u_x,u_y,u_z,\tau',\tau;x,y,z,t',t)\,dx\,du_x\,d\tau'\,dt' \qquad (32)$$

With (13) and (18) or (23) taken into account, on the basis of (20) we obtain:

$$\varkappa(\tau,\Omega)=\frac{1}{n_{II}^2}\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int_{-\infty}^{\infty}\varkappa_v(\chi_x',\chi_y',\Omega';u_x,u_y,u_z,\tau-\tau')\cdot$$
$$\cdot\,\varkappa_w(\chi_x'',\chi_y'',\Omega-\Omega',t';u_x,u_y,u_z,\tau,\tau')\exp\{j[(\chi_x'+\chi_x'')x+ \qquad (33)$$
$$+(\chi_y'+\chi_y'')y-(\kappa_x'-\bar{\kappa}_x'+\kappa_x''-\bar{\kappa}_x'')z+(\chi_z'+\chi_z'')z]\}\cdot$$
$$\cdot\,\exp(-j\Omega't')\,d\chi_x'\,d\chi_y'\,d\Omega'\,d\chi_x''\,d\chi_y''\,du_x\,d\tau'\,dt'\,dx$$

Using formula (27) we obtain the known expression for the dispersion of response of a nonparametric inertialess antenna for a homogeneous stationary field through the field indicatrix and the antenna directional diagram

$$G_u^{\circ} = \frac{1}{\pi^2}\int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{\pi/2} g_\varphi(\vartheta,\theta,\omega)\left|R\left(\frac{\omega}{c}\sin\vartheta\cos\theta,\ \frac{\omega}{c}\sin\vartheta\sin\theta\right)\right|^2 \cdot \sin\vartheta\, d\vartheta\, d\theta\, d\omega \qquad (34)$$

Antenna array. We will examine a one-dimensional array with the interval Δ oriented along the x-axis. The weighting function of an element W[(n + ε) Δ, t − τ, t] is the antenna response to an effect in the form

$$\delta^{\circ}[x-(n+\varepsilon)\Delta]\delta(t-\tau) \qquad (35)$$

The complex frequency characteristic of the antenna

$$H^*(\omega\alpha_x,\varepsilon;\omega,t) = \Delta\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} W[(n+\varepsilon)\Delta,\vartheta,t]\exp[-j(\omega\vartheta-\omega\alpha_x n\Delta)]\,d\vartheta \qquad (36)$$

is a two-dimensional Fourier transform, discrete in space and continuous with respect to the time variable v.

If the antenna is oriented along an arbitrary straight line, its complex frequency characteristic is

$$H_0^*(\omega\alpha_x,\varepsilon_x;\omega\alpha_y,\varepsilon_y;\varepsilon_z;\omega,t) = \Delta\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} W[(n+\varepsilon_x)\Delta_x,(m+\varepsilon_y)\Delta_y,(z+\varepsilon_z)\Delta_z,\vartheta,t]\exp\{-j\omega[\vartheta-(\alpha_x n\Delta_x+\alpha_y m\Delta_y+\alpha_z z\Delta_z)]\}\,d\vartheta \qquad (37)$$

and we have

$$(m+\varepsilon_y)\Delta_y = f_1[(n+\varepsilon_x)\Delta_x], \qquad (38)$$

$$(z+\varepsilon_z)\Delta_z = f_2[(n+\varepsilon_x)\Delta_x], \qquad (39)$$

where the types of functional dependences are determined by equation (6). [The complex frequency characteristic of a discrete antenna was used in the studies of Yu. Z. Shlipchenko.]

The analytical expressions for the autocorrelation function, spectrum, and frequency correlation function for an antenna array are discrete equivalents of (10), (11) and (12).

We will match the x-axis of the rectangular coordinate system with (an arbitrarily oriented) antenna in such a way that the origin of the reading coincides with one of the receivers. Then

$$U(t) = \Delta_x\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\Psi'[n\Delta_x,0,0,t-\vartheta]\,W[n\Delta_x,\vartheta,t]\,d\vartheta \qquad (40)$$

or

$$U(t) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\pi/\omega'\Delta_x}^{\pi/\omega'\Delta_x} S_{\Psi'}^{*}(\omega,\alpha_x,\omega')\,\Gamma^*(\omega,\alpha_x,-\omega',-\omega)\exp\{j\omega t\}\,\omega'^2\,d\alpha_x'\,d\omega'\,d\omega \qquad (41)$$

and

$$S_u(\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\pi/\omega'\Delta_x}^{\pi/\omega'\Delta_x} S_{\Psi'}^{*}(\omega'\alpha_x,\omega')\,\Gamma^*(\omega'\alpha_x,-\omega',-\omega)\,\omega'^2\,d\alpha_x\,d\omega', \qquad (42)$$

where $\varphi'(x, y, z, t)$ is the $\varphi(x, y, z, t)$ field in the coordinate system related to the antenna

$$S'_{\varphi'}(\omega\alpha_x, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\varphi'}(\omega\alpha_x, \omega\alpha_y)\, \omega\, d\alpha_y, \tag{43}$$

$$S_{\varphi'}(\omega\alpha_x, \omega\alpha_y) = S_\varphi[\omega(n_{11}\alpha_x + n_{12}\alpha_y + n_{13}\alpha_z), \omega(n_{21}\alpha_x + n_{22}\alpha_y + n_{23}\alpha_z)]\exp\{j\omega(\alpha_x x_0 + \alpha_y y_0 + \alpha_z z_0)\} \tag{44}$$

Here (n_{11}, n_{12}, n_{13}) and (n_{21}, n_{22}, n_{23}) are the cosines of the angles formed by the old axes x and y with the axes of the new coordinate system; x_0, y_0, z_0 are the coordinates of the old origin of coordinates in the new coordinate system.

The response correlation function can be computed using expression (40) or expression (41) or the expressions (40) and (41).

Accordingly, we obtain:

$$(t_1, t_2) = \Delta_x^2 \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} \iint_{-\infty}^{\infty} B_{\varphi'}[n_1\Delta_x, 0, 0, t_1 - \vartheta_1; n_2\Delta_x, 0, 0, t_2 - \vartheta_2] \times B_w[n_1\Delta x, \vartheta_1, t_1; n_2\Delta x, \vartheta_2, t_2]\, d\vartheta_1 d\vartheta_2. \qquad (45) \tag{45}$$

$$t_2) = (2\pi)^3 \iiint_{-\infty}^{\infty} \int_{-\omega'\Delta_x}^{\omega'\Delta_x} \int_{-\omega''\Delta_x}^{\omega''\Delta_x} \Psi_{\varphi'}^{*}(\omega'\alpha_x', \omega''\alpha_x'', \omega', \omega'') \times [\omega'\alpha_x', \omega''\alpha_x'', -\omega_1', -\omega_1'', (\omega_1 - \omega'), -(\omega_2 - \omega'')]\exp\{j(\omega_1 t_1 - \omega_2 t_2)\}\omega'^2\omega''^2 \, d\alpha_x' d\omega' d\omega_1\, d\omega'' d\alpha_x'' d\omega_2, \qquad (46) \tag{46}$$

$$(t_1, t_2) = (2\pi)^2 \Delta_x \sum_{n=-\infty}^{\infty} \iiint_{-\infty}^{\infty} \int_{-\omega'\Delta_x}^{\omega'\Delta_x} \Phi_{\varphi'}^{*}[\omega'\alpha_x', \omega', \Delta_x n, 0, 0, t_1 - \vartheta] \times \Phi_w^{*}[\omega'\alpha_x, -\omega', -(\omega - \omega'), n\Delta x, \vartheta, t_1]\cdot\exp\{-j\omega(t_1 - t_2)\} \times \omega'^2 \, d\alpha_x\, d\vartheta\, d\omega' d\omega \qquad (47) \tag{47}$$

We obtain the antenna response spectrum on the basis of (40) and (42)

$$(\omega_1, t_2) = (2\pi)^2 \Delta_x \sum_{n=-\infty}^{\infty} \iint_{-\infty}^{\infty} \int_{-\omega'\Delta_x}^{\omega'\Delta_x} \Phi_{\varphi'}^{*}(\omega'\alpha_x, \omega', n\Delta, 0, 0, t_2\, \vartheta) \times \Phi^{*}[\omega'\alpha_x, -\omega', -(\omega_1 - \omega'), n\Delta_x, \vartheta, t_2]\omega'^2 \, d\alpha_x\, d\vartheta\, d\omega' \tag{48}$$

Employing (41) and (42), we obtain the antenna response spectrum in the form:

$$\Phi_U(\omega_1, t_1) = (2\pi)^3 \iiint_{-\infty}^{\infty} \int_{-\omega'\Delta_x}^{\omega'\Delta_x} \int_{-\omega''\Delta_x}^{\omega''\Delta_x} \Psi_{\varphi'}^{*}(\omega'\alpha_x', \omega', \omega''\alpha_x'', \omega'') \times \Psi_w^{*}[\omega'\alpha_x', \omega''\alpha_x'', -\omega', -\omega'', -(\omega_2 - \omega'), -(\omega_1 - \omega'')] \times \exp\{j(\omega_1 - \omega_2)t_2\}\omega'^2\omega''^2 \, d\alpha_x'\, d\alpha_x''\, d\omega' d\omega'' d\omega \tag{49}$$

The antenna response frequency correlation function looks as follows:

$$\Psi_v(\omega_1,\omega_2)=(2\pi)^3\iint_{-\infty}^{\infty}\int_{-\vec{\omega}'\Delta_1}^{\vec{\omega}'\Delta_1}\int_{-\vec{\omega}''\Delta_1}^{\vec{\omega}''\Delta_1}\Psi_{\psi'}^{\prime *}(\omega'\alpha_x',\omega;\omega''\alpha_x'',\cdot\omega'')\times \tag{50}$$

$$\times\bar{\Psi}_v^{*}[\omega'\alpha_x',\omega''\alpha_x'',-\omega',-\omega'',-(\omega_1-\omega'),-(\omega_2-\omega'')]\times$$

$$\times\,\omega'^2\,\omega''^2\,d\alpha_x'\,d\alpha_x''\,d\omega'\,d\omega''$$

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DETECTION OF SIGNALS AND SPATIAL LOCALIZATION OF THEIR SOURCES ON THE BASIS OF SPECTRAL ANALYSIS

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[Article by V. I. Chaykovskiy]

[Text] In connection with the improvement of algorithms, structure and increase in the speed of specialized Fourier processors (spectrum analyzers) of digital and analog types it is important to broaden the possibilities of their use in the information field. Examples of such an application are the spectral procedures for detection of a useful signal and the formation of the directional properties of a linear antenna considered below.

As is well known [1], for a model of a precisely known signal $S_i(t)$ and noise with a normal distribution law n(t), stipulated by the equation

$$y(t) = S_i(t) + n(t); \qquad 0 \le t \le T \tag{1}$$

the optimum detector is a set of matched filters or correlators performing weighted integration of the type

$$\alpha_i = \int_0^T y(t) z_i(t) dt \tag{2}$$

Integration is carried out for the entire set of possible signals (i = 1, 2, 3,...), so that on the basis of a comparison of the results the most probable of them is determined. The weighting characteristic for integration is unambiguously related by the expression [2]

$$z_i(t) = \frac{1}{2\pi} \int_{-\Delta\omega}^{\Delta\omega} \frac{S_i(\omega)}{P(\omega)} e^{j\omega t} d\omega \tag{3}$$

to the form of the useful signal spectrum $S_i(\omega)$ and the spectral density of noise $P(\omega)$, stipulated in the band $\Delta\omega$, so that for white noise

It follows from what has been cited above that the synthesis of an optimum detector is reduced to choice of an algorithm for determining the mean α_i.

Two methods for optimum detection on the basis of modeling of the weighted mean have been investigated in sufficient detail [3]:

-- the method for direct modeling of algorithm (2) using factoring devices, delay lines and integrators;

-- the frequency filtering method, using a matched filter, whose frequency characteristic is complexly conjugate with the weighting function $\eta_i(t)$.

Recently in the practice of signal detection use is being made of still another optimization method based on the carrying out of spectral analysis of the mixture y(t) and subsequent processing of the analytical results [4, 5]. This method is close to the frequency filtering method [6], but for selection of a signal hypothesis no reverse shifting in the time region is required. Using this method (spectral density method) all the initial data for the adoption of a decision are formed in terms of the spectral characteristics of a signal and noise.

In actuality, applying the Parseval theorem to determination of the weighted mean α_i, we obtain

$$\alpha_i = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega) R_i^*(\omega)\, d\omega \tag{5}$$

where $Y(\omega)$ is the Fourier transform of the record y(t), $R_i^*(\omega)$ is the complexly conjugate spectrum of the weighting function $\eta_i(t)$. The finiteness of the integration interval in (2) makes it possible to represent $Y(\omega)$ and $R_i^*(\omega)$ in the form of Kotel'nikov sums

$$Y(\omega) = \sum_k Y\left(k\frac{2\pi}{T}\right) \operatorname{sinc}\frac{T}{2}\left(\omega - k\frac{2\pi}{T}\right)$$

and

$$R^*(\omega) = \sum_k R_i^*\left(k\frac{2\pi}{T}\right) \operatorname{sinc}\frac{T}{2}\left(\omega - k\frac{2\pi}{T}\right)$$

whose substitution into (5), taking into account the property of orthogonality of kernels of the type sinc x, determines the result of weighted integration in the time region in the form of the weighted sum of the spectral components of the record of the investigated mixture

$$\alpha_i = \frac{1}{T}\sum_k Y\left(k\frac{2\pi}{T}\right) R_i^*\left(k\frac{2\pi}{T}\right) \tag{6}$$

A natural limitation on the effective zone of existence of the weighting function spectrum (useful signal spectrum) simplifies the derived expression, reducing it to the final sum of the weighted spectral components

$$\alpha_i = \frac{1}{T}\sum_{k=-Q}^{k=Q} Y\left(k\frac{2\pi}{T}\right) R_i^*\left(k\frac{2\pi}{T}\right) \tag{7}$$

[The spectral processing method has been described in detail only for the special case of reception of telemetric information in the case of multiple frequency manipulation [4] and the case of detection of complex narrow-band signals with a random initial phase [5]. Its effectiveness in the detection of signals of an arbitrary shape has not been evaluated.]

As noted above, the set α_i is a set of informative parameters from which is accomplished the optimum procedure of adoption of a decision. As a result, the derived expression can be regarded as the algorithm for the optimum processing of a mixture of a useful signal and noise, completely equivalent to the optimum correlation algorithm (2). The distinguishing characteristic of the formulated algorithm is that it is based on the weighted summation of the spectral components of the investigated mixture, whereas the correlation algorithm (2) assumes a weighting of the initial record y(t). In a number of cases this peculiarity ensures a determination of the advantage of systems for optimum detection based on spectral processing. The advantages are determined by the possibility of use of extremely effective modern means and methods of spectral analysis in the detection system.

For real records of a mixture of a useful signal and noise expression (7) and the optimum detection procedure corresponding to it can be simplified, taking into account the properties of the even and odd symmetry of the real and fictitious components of the signal and noise spectrum. In this case

$$\alpha_i \cong \frac{1}{T} Re\, Y_0\, Re\, R_{0i} + \frac{2}{T} \sum_{k=1}^{k=Q} \left\{ Re\, Y_k\, Re\, R_{ki} + Im\, Y_k\, Im\, R_{ki} \right\} \qquad (8)$$

where Y_k and R_{ki} are the sample values of the corresponding spectra at frequencies k $2\pi/T$.

The refinement of the spectral processing algorithm presented above makes it possible to represent the optimum detector of the considered type, or, which is the same, the Fourier optimum detector, in the form of a multichannel analyzer of the complex spectrum, in the analysis interval accomplishing a determination in real time Q + 1 of complex (or 2Q + 1 orthogonal) Fourier coefficients Y_k of a mixture of useful signal and noise, which by means of a set of matched (with the spectral characteristics ReR_{ki} and ImR_{ki}) pairs of multichannel weighting summators are added in accordance with the expressions

$$\sum Re\, Y_k\, Re\, R_{ki} \quad \text{and} \quad \sum Im\, Y_k\, Im\, R_{ki}$$

[The matching assumes a coincidence of the transfer coefficients in the channels of the summators and the corresponding readings of the spectral characteristics ReR_{ki} or ImR_{ki} of the weighting functions at the frequencies k $2\pi/T$.]

The results of the weighted summation in each summator of the i-th pair are combined, in accordance with expression (8) forming the informative parameter α_i. Since the quantitative values of the informative parameters α, determined in accordance with the spectral processing algorithm (8), all other conditions being equal, coincide with the quantitative values of the informative parameters α, determined in correlation processing, the conclusion must be drawn that there is a coincidence of the working characteristics and other properties of the optimum cross-correlation detector or a recursive matched filter and an optimum Fourier detector. The difference between the compared optimum detection methods is manifested only in apparatus for their realization.

As we have noted, the principal link in the optimum Fourier detector is a multichannel analyzer of the complex spectrum operating at a real time scale, that is, accomplishing a determination of all Q complex Fourier coefficients with a delay relative to the interval of existence of the useful signal, not exceeding the inverse value of the doubled width of the investigated spectral band. This makes it possible to ensure processing of information in adjacent intervals T of signal existence without losses. Since the number of coefficients to be determined is equal to the coefficient of signal complexity $T\Delta f$ and in practical cases can attain many hundreds, the used spectrum analyzer must have an extremely high speed. Quantitatively the value for this speed is evaluated on the basis of the mean time for determining one coefficient

$$\tau = \frac{T}{2(T\Delta f)^{2}} \qquad (9)$$

Guided by models of digital spectrum analyzers using fast Fourier transform algorithms [7], the τ parameter can be evaluated at tens of microseconds. In those cases when this speed is inadequate, it is possible to recommend use of digital synthesizers of the Fourier coefficients [8], making it possible to increase the speed τ to a few microseconds. Finally, in order to have analyzers with the speed τ of the order of fractions of a microsecond it is necessary to use a discrete-analog spectral analysis method [9]. A peculiarity of the discrete-analog spectral analysis method is a discrete representation of the investigated signals and an analog processing of the principal functional transformations constituting the content of the Fourier transform algorithms. Such a combination makes it possible to organize a simultaneous determination of the required number of complex Fourier coefficients at the rate of information receipt.

The possibility of synthesis of an optimum detector on the basis of spectral processing indicated above is based on the existence of a spectral (rigorously equivalent to the temporal) description of operation of a linear optimum filter or a correlation detector. It is natural to assume that the spectral models exist not only for linear systems for the processing of signals (time functions), but also for systems for the processing of distributions (space coordinate functions). An example of such a system is a linear (one-dimensional) antenna system or its discrete equivalent -- an equidistant antenna array. An analysis of the spectral model of a linear antenna makes it possible to clarify a number of useful peculiarities, whose use makes it convenient to synthesize a diagram-forming unit or a unit for controlling the directional properties, whose functioning is based on the spectral processing of information. The use of the spectral processing method ensures, as in the case of synthesis of an optimum detector, definite technical advantages of the considered systems.

The functioning of a band antenna with independent elements is formally reduced to the integration of the instantaneous values along its apertures, taking into account the distribution of the sensitivity of antenna elements G(x). In this case, using the generalized coordinates $u = \pi \sin \varphi$; $p = 2x/\lambda$; $N = 2L/\lambda$, where λ is the wavelength of the excited oscillations; x is the space coordinate; γ is distance; φ is the angle of incidence of the wave front; 2L is antenna length, the output signal is represented in the form

$$f(\tau; u; t) = \frac{\lambda}{2} \int_{-N}^{N} A_p(\tau; u; t)\, G(p)\, dp \tag{10}$$

As follows from (10), for a point source of harmonic oscillations situated in the distant zone, the dependence of the complex amplitude of the output signal F(u) on the angle of incidence, or what is the same, the complex directional diagram, with an accuracy to a constant factor is determined as a complexly conjugate Fourier transform of the distribution of sensitivity of elements in the system G(p) [10]

$$F(u) = \int_{-N}^{N} G(p)\, e^{jup}\, dp \tag{11}$$

Expression (11) is a symbolic description of the spectral model of an antenna which can serve as the basis for developing a method for the electric control of the directional diagram or organizing a multiray antenna on the basis of a fixed linear base. In actuality, it follows from (11) that for moving a directional diagram formed by an antenna with the spatial sensitivity G(p) by the value Δu it is necessary to use the sensitivity (response) distribution $G_{\Delta u}(p)$

$$G_{\Delta u}(p) = G(p)\, e^{-j\Delta u p} \tag{12}$$

since

$$F(u - \Delta u) = \int_{-N}^{N} G(p)\, e^{-j\Delta u p}\, e^{jup}\, dp \tag{13}$$

This situation is a direct consequence of the theorem known as the "movement theorem" in Fourier transform theory. Thus, in accordance with the modeling of functioning of a linear band antenna (10), for movement of the diagram it is necessary to carry out integration of the excitations in the aperture with a new weight. The output signal of the antenna is represented by the integral

$$f(u - \Delta u; \tau; t) = \int_{-N}^{N} A_p(\tau; u; t)\, G(p)\, e^{-j\Delta u p}\, dp \tag{14}$$

It follows from expression (14) that for movement of the directional diagram by the angle Δu it is necessary to use a diagram-forming unit which at the "frequency" Δu accomplishes a Fourier transform of the spatial continuum of the elementary excitations in the antenna aperture. The procedure for moving the directional diagram of an antenna with a stipulated distribution of response of its elements G(p) is thus reduced to the procedure of carrying out a spatial Fourier transform at a stipulated angular frequency and the diagram-forming unit is represented in the form of an analyzer of the spectrum of space harmonics.

Applicable to a band linear system not having branches along its aperture, it is virtually impossible to carry out the spatial Fourier transform procedure. However, it is quite easily carried out for a discrete equivalent of such a system -- an equidistant antenna array.

For a discrete equivalent of a linear antenna with a distance between the elementary detectors d equal to half the wavelength λ, the procedure of an integral Fourier transform in the space coordinate is replaced by the procedure of discrete inertialess weighted summation of the sample values of the continuum of perturbations

$$f(u-\Delta u;\tau;t) = \sum_{k=-N}^{k=N} A_k(\tau;u;t)G(k)e^{-j\Delta uk} \qquad (15)$$

where A_k is the instantaneous value of excitation of the k-th antenna array element, G(k) is the response of the k-th element. Then the procedure of directional diagram movement can be considered a discrete Fourier transform procedure weighted in accordance with the distribution law for excitation in the space of antenna array elements

$$f(u-\Delta u;\tau;t) = \sum_{k=-N}^{k=N} R_k(\tau;u,t)e^{-j\Delta uk} \qquad (16)$$

where R_k is the excitation of the k-th element of the antenna array, weighted with the coefficient G(k)

$$R_k(\tau;u;t) = A_k(\tau;u;t)\,G(k) \qquad (17)$$

Expression (16) is the algorithm for functioning of the diagram-forming unit accomplishing inertialess summation of the instantaneous values of the elementary excitations $R_k(\tau;\ u;\ t)$, weighted by the complex coefficients exp $[-j\Delta uk]$. Such a procedure, as noted, coincides with the procedure for forming of the complex Fourier coefficient for the discrete distribution of real readings R_k at the frequency Δu and can be reduced to two independent weighted summation procedures

$$Re[f(u-\Delta u;\tau;t)] = \sum_{k=-N}^{k=N} R_k \cos k\Delta u \qquad (18)$$

$$Im[f(u-\Delta u;\tau,t)] = \sum_{k=-N}^{k=N} R_k \sin k\Delta u \qquad (19)$$

Each of the weighted summation procedures (18) and (19) is easily realized using 2N + 1 channel analog summators in a bipolar operational amplifier with a set of weighting resistors stipulated for the required Δu. Variations of these resistors, matched with the required law of change Δu, make it possible to carry out plane or discrete measurement of the angle of movement of the directional diagram.

In order to organize a multiray system with 2Q +1 independent directional diagrams it is convenient to use as the multipole diagram-forming scheme a matrix weighting unit formed as a result of the parallel cutting-in of 2Q + 1 pairs of 2N + 1 channel analog summators. In this case each pair of summators contains a definite set of weighting resistors ensuring formation of the n-th partial directional diagram with a stipulated movement Δu_n. If the partial diagrams conform to the condition of uniform (with the interval Δu) movement of the generalized angle coordinate

$$\Delta u_n = n\Delta u \qquad (20)$$

the weight coefficients are determined by the values

$$\left.\begin{array}{l} C^{nk} = \cos nk\Delta u \\ S^{nk} = \sin nk\Delta u \end{array}\right\} |n| = 0, 1, \ldots, Q; \; |k| = 0, 1, \ldots, N \qquad (21)$$

with formation of the matrices $\| C^{nk} \|$ and $\| S^{nk} \|$, each with the dimensionality (2Q + 1) x (2N + 1). In this case the algorithm for functioning of the multipole diagram-forming scheme, realizing the processing of 2N+1 signals from the outputs of a linear antenna array for the formation of 2Q+1 partial directional diagrams, can be written in the form of a matrix product, separately for the real and fictitious componen's of the complex directional diagrams:

$$\left.\begin{array}{l} \| Re f_n \| = \| C^{nk} \| \times \| R_k \| \\ \| Im f_n \| = \| S^{nk} \| \times \| R_k \| \end{array}\right\} \qquad (22)$$

where $\| R_k \|$ is the columnar matrix of perturbations at the output of the antenna array channels.

The procedures for the formation of partial directional diagrams for a multiray antenna array, represented by expressions (22), are conveniently carried out using two scalar matrix inertialess weighting schemes with the dimensionality (2Q+1) x (2N+1), in essence representing a complex discrete-analog space harmonics spectrum analyzer [11].

A diagram-forming unit of the type considered above for each space frequency (movement angle) Δu_k has two real outputs, representing the real $Re f_n (u - \Delta u; \tau; t)$ and fictitious $Im f_n(u - \Delta u; \tau_i t)$ components of the harmonic to be analyzed. This circumstance in a number of cases makes it possible to extract additional information on the parameters of the received signal. However, in the case of formation of the directional diagram of interest to us the presence of two separate reception channels is an undesirable factor, since for each of the channels the directional diagram is represented in the form of identical lobes symmetrically crossed at the angle $\pm\Delta u_k$. As a result, the amplitude directional diagrams for each output have an uncertainty with respect to the sign or direction of movement of the main maximum. In order to eliminate this uncertainty it is necessary to proceed from a complex diagram to its modulus, which is easily achieved with the quadrature summation of the output signals of the channels for each space harmonic. The formed amplitude directional diagram, all other conditions being equal (number of used detectors, distribution of response, angle of movement and frequency of the received oscillations), is completely equivalent to the directional diagram of an antenna array with signal phasing. A distinguishing characteristic and advantage of the spectral method for the forming of directivity is the use of resistor groups as the regulating elements instead of the phase inverters used in classical antenna arrays, which is manifested particularly significantly in the planning and use of antennas with a directional diagram controllable by program, since it ensures improvement of the weight and size characteristics of the diagram-forming units and the convenience of their tie-in to digital control units.

We note in conclusion that the effectiveness of the spectral models and data processing procedures indicated in the examples of determination of the cross-correlation coefficients for signals and the space harmonics spectrum is retained in the solution of a number of other problems of independent interest and falling outside the framework of this study.

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ONE METHOD FOR DETERMINING THE COORDINATES OF A LOCAL NOISE FIELD SOURCE

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 251-254

[Article by A. M. Derzhavin, L. A. Bespalov, O. L. Sokolov, O. Yu. Borenshteyn and A. G. Strochilo]

[Text] A determination of the coordinates of local sources on the basis of the noise field created by them is an extremely timely problem in statistical acoustics.

The considered method is among the rangefinder-difference methods for the determination of coordinates. The basis for the method is the use of a sufficiently great number of nondirectional detectors situated on a straight line parallel to the selected axis of Cartesian coordinates.

We will use the notation y_1, $y_2, \ldots, y_n$ to designate the vertical coordinates of these detectors and y to denote the coordinate of the noise source. For implementing the method it is necessary to satisfy the condition

$$y_1 \leqslant y \leqslant y_n$$

It is obvious that for estimating the coordinates of the source y it is sufficient only to establish the number of the detector closest to the source.

This can be done using the maximum difference of the distance from the noise source to the first detector and accordingly to each of the remaining detectors.

From the set of measured distance differences $\Delta R_i / i = 2, 3, \ldots, n /$ we select

$$\Delta R_j = \{(R_1 - R_i)_{i=j}\}_{max} > 0 , \qquad (1)$$

where R_1, R_i are the distances from the source to the first and i-th detectors.

In order to estimate the distance differences in the case of noise signals it is most common to measure the time lag of the cross-correlation function

The conditions for the applicability of correlation analysis for acoustic problems are analyzed in detail in [1]. It is shown, for example, that the use of correlation analysis is preferable only in those cases when the width of the signal spectrum is adequate for the forming of a pointed correlation function.

This condition applicable to the described method is expressed in the following way

$$\Delta f \tau_{i,i+1} \geqslant m \frac{R_{1,i}(\tau)}{R_{1,i+1}(\tau)} \tag{2}$$

where Δf is the width of the energy spectrum of the noise signal, $P_{1,i}(\tau)$, $R_{1,i+t}(\tau)$ are the maxima of the cross-correlation functions of the signals from the first and accordingly from the i-th and from the (i+1)-st detectors, $\tau_{i,\ i+1}$ is the time lag between the extrema of the cross-correlation functions $R_{1,i}(\tau)$ and $R_{1,i+1}(\tau)$, m is the error coefficient for evaluation of the correlation coefficient.

A preliminary evaluation using formula (2) shows that with lag durations $\tau_{i,\ i+1}$ of the order of msec, being usual for the correlation analysis of acoustic signals, the width of the spectrum must exceed hundreds of cps.

Expression (2) does not take into account the nonstationary nature of real noise signals. We will estimate the error in measuring the correlation function caused by the nonstationary state. Since the use of discrete methods is characteristic for developing the correlation analysis approach, we made an analysis of the error in the example of measurement of the autocorrelation function of a signal quantized by level and time.

For generality we will assume that the quantization levels can be variable but limited in value, that is

$$\Delta = |U_{j+1} - U_j| \neq const < +\infty, \qquad j = 1, 2, \ldots$$

Similarly, for time intervals the following is correct

$$\delta = |t_{i+1} - t_i| \neq const < +\infty, \qquad i = 1, 2, \ldots$$

We will also assume that the noise process X(t) has a zero mathematical expectation and the covariation function $B_X(t_1, t_2)$.

As an evaluation of the covariation function of the noise process X(t) we will take a random value determined by the expression

$$\hat{B}_x(t, \tau, \Delta t, N) = \frac{1}{N} \sum_{n=1}^{N} x(t + n\Delta t) \cdot x(t + \tau + n\Delta t), \tag{3}$$

where N is the number of observation points in the process x(t), obtained as a result of quantization of the X(t) process by levels and time; Δt is the time interval between adjacent observations.

For each t we use the notation $[t] = \max, \{t_1 \leqslant t\}$

Due to the fact that we have the inequality

$$\frac{\sup\{U_{i+1} - U_i\}}{2} > \min_{i > 1}\{|U_i - x([t])\},$$

we obtain the expression

$$x(t) - x([t]) = \delta([t]) < \frac{\Delta}{2} \qquad (4)$$

Then it is possible to write

$$x([t]) - x(t) = ([t] - t)x_t'(t) + 0(t - [t]) \qquad (5)$$

From the expressions (4) and (5) we obtain an expression for the quantized process

$$x(t) = x(t) + \delta([t]) - (t - [t])x_t'(t) + \delta(t - [t]) \qquad (6)$$

Using (6), it is possible to write an expression for the product of values of the quantized process at the moments in time t_1 and t_2, separated by the interval τ, and then, proceeding to the mathematical expectation we finally obtain

$$B_x(t_1, t_2) < B_x(t_1, t_2) - (t_1 - [t_1])\frac{\partial B_x(t_1, t_2)}{\partial t_1} - \\ - (t_2 - [t_2])\frac{\partial B_x(t_1, t_2)}{\partial t_2} - (\sigma_x(t_1) + \sigma_x(t_2)\frac{\Delta}{2} \qquad (7)$$

Using (7) it is possible to find the mathematical expectation of the evaluation $\hat{B}_z$ $(t, \tau, \Delta t, N)$, and then replacing the sums by the corresponding integrals and assuming $T = N\Delta t$, we arrive at the expression

$$M\hat{B}_x(t, \tau, \Delta t, N) < \frac{1}{T}\int_0^T B_x(t+z, t+\tau+z)dz + \\ + \frac{\delta}{T}\int_0^T\left(\left|\frac{\partial B_x(t+z, t+\tau+z)}{\partial t_1}\right| + \left|\frac{\partial B_x(t+z, t+\tau+z)}{\partial t_2}\right|\right)dz + \\ + \frac{\Delta}{2T}\int_0^T[\sigma_x(t+z) + \sigma_x(t+\tau+z)]dz + \frac{\Delta t}{2T}[B_x(t+T, t+\tau+T) - \\ - B_x(t, t+\tau)] \qquad (8)$$

Now the sought-for expression for the modulus of the bias of evaluations can be written at once:

$$|\Delta\hat{B}_x(t, \tau, \Delta t, N)| < |M\hat{B}_x(t, \tau, \Delta t, N) - B_x(t, t+\tau)| \leqslant \\ \leqslant \left|\frac{1}{T}\int_0^T B_x(t+z, t+\tau+z)dz - B_x(t, t+\tau)\right| + \frac{\delta}{T}\int_0^T\left(\left|\frac{\partial B_x(t+z, t+\tau+z)}{\partial t_1}\right| + \\ + \left|\frac{\partial B_x(t+z, t+\tau+z)}{\partial t_2}\right|\right)dz + \frac{\Delta}{2T}\int_0^T(\sigma_x(t+z) + \sigma_x(t+\tau+z))dz + \\ + \frac{\Delta t}{2T}|B_x(t+T, t+\tau+T) - B_x(t, t+\tau)|. \qquad (9)$$

Expression (9) makes it possible to compute the error in bias of the evaluation of the correlation function of the nonstationary noise signals, quantized by level and time. In a special case, assuming $\Delta \to 0$ and $\delta \to 0$, it is possible to compute the bias of the evaluation of the correlation function of a nonquantized noise signal.

As an example, we can consider a process of the type $x(t) = \varphi(t) \cdot Z(t)$, where $\varphi(t) = e^{\beta t}$, Z(t) is a Gaussian process with a zero mean and the correlation function

$$B_z(\tau) = \sigma_z^2 e^{-\alpha|\tau|}$$

In this case from formula (9) we obtain

$$|\Delta \hat{B}_x(t,\tau,\Delta t,N)| \leq \left|1-\frac{e^{2\beta T}}{2\beta T}\right| \sigma_z^2 \cdot e^{2\beta t+(\beta-\alpha)\tau} + \frac{\delta}{8\beta T}\sigma_z^2 (|\beta+\alpha| + |\beta-\alpha|) e^{2\beta t+(\beta-\alpha)\tau} \cdot (e^{2\beta T}-1) + \frac{\Delta}{4\beta T}\sigma_z^2 e^{2\beta t}(1+e^{2\beta T}) \cdot (e^{2\beta T}-1) + \frac{\Delta T}{2T}\sigma_z^2 e^{2\beta t+(\beta-\alpha)\tau}|e^{2\beta\tau}-1|.$$

The evaluation of the bias error makes it possible to clarify the influence of nonstationarity of the noise signals on the accuracy in measuring the coordinates of local sources of the noise field. It should be noted that this systematic error has a tendency to increase with an increase in the number of readings or an increase in integration time.

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INVESTIGATION OF THE NOISE IMMUNITY OF A STANDARD DETECTION CHANNEL IN THE RECEPTION OF A TWO-COMPONENT SIGNAL WITH A NARROW-BAND NOISE COMPONENT

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 255-262

[Article by V. M. P'yanov]

[Text] In some practical applications of hydroacoustics one must contend with the detection of a so-called two-component signal s_2 (t), one of whose components s(t) is a determined (or quasidetermined) component, the second $s_p(t)$ is the noise component (for example, see [1]). In this connection it can be of interest to investigate the noise immunity of a standard detection channel (SDC) with the reception of such a signal. In this study it is proposed that s(t) is a narrow-band radio pulse with a fixed amplitude A and a random initial phase β, existing also in $s_p(t)$, in the interval $[t_0, t_0 + T_s]$. The noise component is a random, normal quasistationary narrow-band process with a zero mathematical expectation and the correlation function

$$B_p(t,u) = \sigma_p^2 z_p(t,u) \cos \omega_p (t-u), \tag{1}$$

$$z_p(t,u) = \begin{cases} 1 - \frac{|t-u|}{T_s}, & |t-u| \leq T_s \\ 0 & |t-u| > T_s \end{cases} \tag{2}$$

$$\omega_p = \omega_0 + \Delta\omega_g$$

The power spectra $s_p(t)$ and s(t) differ only by a shift along the frequency axis by $\Delta\omega_g$.

As additive noise N(t) we used white noise with the correlation function

$$B_N(t,u) = \frac{g_N}{2} \delta(t-u) \tag{3}$$

We will examine a SDC consisting of a preselector, quadratic detector (QD) and low-frequency filter (LFF).

We will determine the signal, noise and signal-to-noise ratio at the SDC output with the arrival of a mixture of the above-mentioned signal $s_2(t)$ and noise at the channel input.

It is assumed that the effective transparency band of the preselector is

$$\Delta f_{\ni} \gg \Delta f_{\ni s} = \Delta f_{\ni p}$$

[$\ni$ = eff(ective)] where $\Delta f_{eff\ s}$ and $\Delta f_{eff\ p}$ is the effective width of the spectral band s(t) and $s_p(t)$. The response of the preselector to the noise and determined components of the signal $s_p(t)$ is

$$y_p(t) \approx s_p(t); \qquad y_s(t) \approx s(t)$$

In the presence of a signal at the preselector output we obtain

$$y_\Sigma(t) = s(t) + s_p(t) + y_N(t), \tag{4}$$

where $y_N(t)$ is the response of the preselector to noise.

As the signal C at the channel output we use [2] the increment of the mathematical expectation of voltage at the output, caused by the presence of a signal at the time of the reading $t_0 + T$. Although usually we select $T = T_s$, we cite the signal and noise values for a more general case when for one reason or another $T \leq T_s$.

In this case

$$C = a_2\sigma_N^2(q^2 + q_p^2)\int_0^T h_\delta(\tau)d\tau = \frac{a_2\sigma_N^2}{2M\rho}(\mu + 2\gamma)\int_0^T h_\delta(\tau)d\tau, \tag{5}$$

where a_2 is the detection constant, $h_\delta(\tau)$ is the "impulse" transfer characteristic of the LFF

$$\left.\begin{aligned} &q^2 = \frac{A^2}{2\sigma_N^2};\ q_p^2 = \frac{\sigma_p^2}{\sigma_N^2};\ \sigma_N^2 = \overline{y_N^2(t)};\ \sigma_p^2 = \overline{s_p^2(t)}; \\ &\mu = \frac{A^2 T_s}{g_N};\ \gamma = \frac{\sigma_p^2 T_s}{g_N};\ M = \Delta f_\ni T;\ \rho = \frac{T_s}{T} \geq 1 \end{aligned}\right\} \tag{6}$$

As the noise Π we use [2] the mean square value of channel response at the reading time.

Accordingly,

$$\Pi = \left[\iint_{t_0}^{t_0+T} h_\delta(t_0 + T - t_1)h_\delta(t_0 + T - t_2)B_{z\Sigma 0}(t_1; t_2)dt_1dt_2\right]^{1/2}, \tag{7}$$

where $B_{2\Sigma 0}(t_1;\ t_2)$ is the autocorrelation function of the low-frequency component of the process $z_\Sigma(t)$ at the detector output.

Taking into account the nondependence and normality of the processes $s_p(t)$ and N(t) and representing the the correlation functions of the noise component and the noise at the preselector output in the form

$$\left.\begin{aligned} &B_{yp}(t,u) = \sigma_p^2 r_{yp}(t,u)\cos[(\omega_0 + \Delta\omega_p)(t-u)] \\ &B_{yN}(t,u) = \sigma_N^2 r_{yN}(t,u)\cos\omega_0(t-u), \end{aligned}\right\} \tag{8}$$

after a number of transformations we obtain

$$\Pi = a_2 \sigma_N^2 \left[2\int_0^T F(\tau) J(\tau)\, d\tau\right]^{1/2}, \tag{9}$$

where

$$J(\tau) = \int_0^{T-\tau} h_g(t) h_g(t+\tau)\, dt, \tag{10}$$

$$F(\tau) = \frac{1}{\sigma_N^4}\{\sigma_p^4 r_p^2(\tau) + \sigma_N^4 r_{yN}^2(\tau) + 2\sigma_p^2 \sigma_N^2 r_p(\tau) r_{yN}(\tau) \cos \Delta\omega_g \tau +$$

$$+ H^2[\sigma_p^2 r_p(\tau) \cos \Delta\omega_g \tau + \sigma_N^2 r_{yN}(\tau)]\} \tag{11}$$

The results of computation of the signal-to-noise ratio for a case when the LFF is an ideal integrator (II) and the preselector is an individual resonance circuit (IRC) under the condition that $\Delta\omega_g = 0$ are given in Fig. 1.

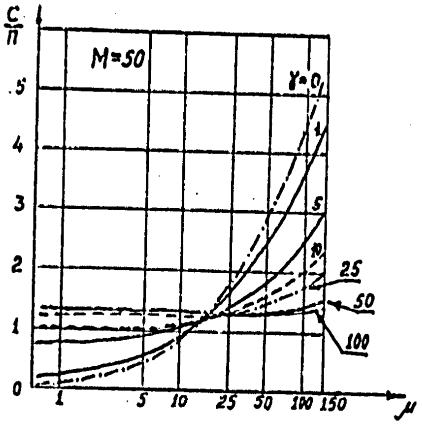

Fig. 1.

It follows from Fig. 1 that the signal-to-noise ratio increases with an increase in μ, but the rate of this increase is dependent on γ. In the case of large γ the signal-to-noise ratio is virtually not dependent on μ and on γ and approaches $\sqrt{2}$. It also follows from Fig. 1 that when $\mu < \mu_1$ an increase in γ causes an increase in the signal-to-noise ratio, whereas when $\mu > \mu_1$ there is a decrease in the signal-to-noise ratio. With M = 50 the μ_1 value is of the order of 15-20; a decrease (or increase) in M causes some decrease (or increase) in μ_1.

Thus, cases are possible when an increase in s_p (t) or s(t) at the channel output leads not to an increase, but a decrease in the signal-to-noise ratio at the channel output; an increase in γ or μ causes a greater increase in noise than the signal at the output.

In this connection the question arises of how an increase in γ or μ exerts an influence on the reliability of detection of a two-component signal. In order to obtain a rigorous answer to this question it is necessary to determine the distribution law for the process at the output and then on the basis of this law construct the detector detection characteristic curve (DCC). However, unfortunately, it was not possible to determine the precise distribution law for the process at the channel output. It is necessary to be satisfied with different approximate representations.

If the process at the SDC output is assumed to be normal, the DCC is determined by the parameters [2]

$$\sqrt{d_1} = \frac{c}{n}\ ;\ \sqrt{d_2} = \sqrt{\frac{\sigma^2_{uN0}}{\sigma^2_{u\Sigma 0}}}\ , \tag{12}$$

where σ^2_{uN0} and $\sigma^2_{u\Sigma 0}$ is the dispersion of the process at the channel output in the absence and in the presence of a signal at the input.

An increase in the dispersion at the output leads to a decrease in d_2 and accordingly to some increase (with a given d_1) in the probability of correct detection D. Thus, the influence of a decrease in d_1 with an increase in γ or μ, which was mentioned above, on the reliability of detection can to one degree or another be compensated by a decrease in d_2.

The question arises of the possibility of approximation, by a normal law, of the process at the LFF for a two-component signal.

It is known that the normalizing effect of LFF is manifested only in the case of a large M value. Therefore, for a two-component signal, to the usual condition of normalizability $M \gg 1$ is added the additional condition $\Delta f_{eff\ p} T \gg 1$, where $\Delta f_{eff\ p}$ is the effective width of the band $s_p(t)$ at the preselector output.

However, for $s_p(t)$ there is satisfaction only of the first condition and therefore the DCC, constructed on the assumption of normality of the process at the output, may be inadequately precise. Assume for the IRC-QD-II channel there is satisfaction of the condition $M \gg 1$ and the condition $\Delta f_{eff\ p} T < 1$ or $\Delta f_{eff\ p} < 2 \Delta F_{eff\ T}$, where $\Delta F_{eff\ T}$ is the II effective transmission band [2].

With satisfaction of these conditions the process at the LFF input (see expression (11)) can be divided into two groups: one group (consisting of several components), which with passage through the LFF is virtually normalized, and the second group (consisting of one component $s_p^2(t)$), for which passage through the LFF is virtually not related to the change in the distribution. In this case the distribution of the process at the LFF output is determined by the composition of the normal and exponential distributions; on the assumption of a nondependence of these distributions the resultant one-dimensional distribution density of the process has the form [3]

$$W_1(x) = \frac{1}{2\varepsilon}\exp\left\{\frac{1}{8\varepsilon^2} - \frac{x}{2\varepsilon}\right\}\varphi\left(\frac{\varepsilon x - 0{,}5}{\varepsilon}\right), \tag{13}$$

where φ (x) is the Laplace integral; $\varepsilon = \sigma_1^2/\sigma_2$; σ_1, σ_2 are the parameters of the exponential and normal initial distributions.

We will call (13) an exponential-normal distribution.

An equation for the detector working characteristic (DWC) follows from (13)

$$D = \varphi\left\{\frac{a-\beta}{\sigma_2} - \frac{\sigma_3}{\sigma_2}\varphi^{-1}(1-F)\right\} + 2\varepsilon W_1\left\{\frac{\beta-a}{\sigma_2} + \frac{\sigma_3}{\sigma_2}\varphi^{-1}(1-F)\right\}, \tag{14}$$

where φ^{-1} is a function inverse of φ ; σ_3, β are the dispersion and the mathematical expectation of the process at the channel output in the absence of a signal at the input; a is the mathematical expectation of a group of components having a normal distribution; F is the probability of a false alarm.

With a normal distribution the DWC equation has the form [2]

$$D = \varphi\left\{\sqrt{d_1} - \sqrt{d_2}\,\varphi^{-1}(1-F)\right\} \tag{15}$$

Thus, an examination of the SDC noise immunity with the reception of a two-component signal with a narrow-band noise component is carried out by successive approximations:

a) zero approximation -- evaluation of noise immunity from the signal-to-noise ratio;

b) first approximation -- determination of the DCC on the assumption of a normal distribution at the output;

c) second approximation -- determination of the DCC on the assumption that this distribution is an exponential-normal distribution.

Some results of computations of the DCC in the case of normal and exponential-normal distributions of the process are represented in Figures 2, 3 and 4, 5 respectively. A comparison of these curves indicates an insignificant difference between the DCCs in the first and second approximations. The difference increases with an increase in γ and when $\gamma = 0$ disappears; for $\gamma \leqslant 100$, $\mu \leqslant 150$ with a normal distribution the value is somewhat greater than in the case of an exponential-normal distribution. The DCC distribution in the first and second approximation with respect to the threshold values μ_{thr} and γ_{thr} is also insignificant and does not exceed (for the considered μ and γ values) 2-3 db. A common characteristic for DCC of both approximations is that with small μ values ($\mu \leqslant \mu_1$, where μ_1 is some "boundary" value) an increase in γ causes a substantial increase in D, whereas in the case of large μ values ($\mu > \mu_1$) an increase in γ causes some increase in D. This μ_1 value increases with an increase in P and M and a decrease in F. The μ_1 value with given M, P and F falls in some band dependent on γ ; with an increase in γ μ_1 increases somewhat. An increase in M causes a decrease in D, that is, an increase in μ_{thr}; with an increase in γ this change in D or μ_{thr} is less significant.

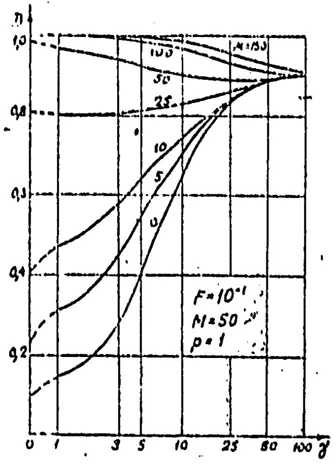

Fig. 2

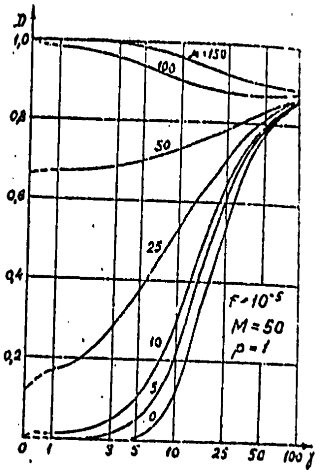

Fig. 3

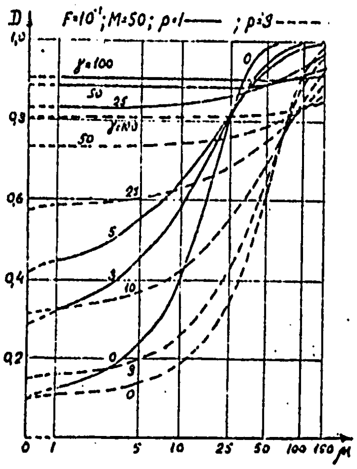

Fig. 4

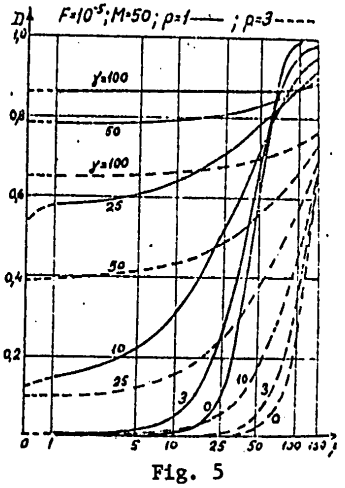

Fig. 5

The sole qualitative difference in DCC for both distributions is their behavior when $\gamma \to \infty$. For a normal distribution this leads to $D \to 0.92$, but for an exponential-normal distribution this leads to $D \to 1$. Although this difference still does not appear for the considered values $\gamma \leqslant 100$, it is evidence that the second approximation more precisely describes the process at the channel output.

These experimental investigations of SDC noise immunity with reception of the mentioned signal also confirmed the proposal of an exponential-normal distribution of the process.

Another question of interest is the matching of the SDC with the two-component signal in the preselector band. Below this problem is considered under the condition that $t_0 = 0$; $T_s = T$; $\Delta\omega_g = 0$.

Under the term "optimum transmission band of the preselector" we will understand the band $\Delta\omega$, which maximizes a ratio in the form [4]

$$\rho = \sqrt{\frac{P_{s\Sigma}(t_m;\Delta\omega)}{\sigma_N^2}}, \qquad (16)$$

where

$$P_{s\Sigma}(t_m;\Delta\omega) = s^2(t_m;\Delta\omega) + \sigma_p^2(t_m;\Delta\omega) \qquad (17)$$

is signal power at the preselector output, t_m is the reading time at which $P_{3\Sigma}$ attains a maximum.

It can be shown that $t_m = T$. In this case

$$P_{s\Sigma}(T,M) = \frac{A^2}{8}(1-e^{-2M})^2 + \frac{\sigma_p^2}{4}\left[e^{-4M}\left(\frac{1}{2M}+1\right) - \left(\frac{1}{2M}-1\right)\right] \qquad (18)$$

Taking (16) into account, we obtain

$$\rho(M) = \left\{\frac{\sigma_p^2 T}{2g_N}\left[\frac{A^2}{4\sigma_p^2 M}(1-e^{-2M})^2 + \frac{e^{-4M}}{2M}\left(\frac{1}{2M}+1\right) - \frac{1}{2M}\left(\frac{1}{2M}-1\right)\right]\right\}^{1/2} \qquad (19)$$

The dependence

$$\rho^2(M)\frac{2g_N}{\sigma_p^2 T} = \frac{A^2}{4\sigma_p^2}\frac{(1-e^{-2M})^2}{M} + \frac{e^{-4M}}{2M}\left(\frac{1}{2M}+1\right) - \frac{1}{2M}\left(\frac{1}{2M}-1\right) \qquad (20)$$

on M for values of the parameter $A^2/4\sigma_p^2 = 0$, 0.1, 1, 10 and ∞ ($\sigma_p^2 = 0$) is shown in Fig. 6.

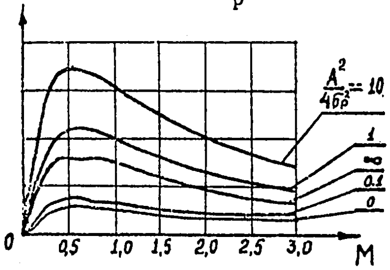

Fig. 6

It can be seen from Fig. 6 that the optimum value $M_{opt} = \Delta f_{eff\ opt} \cdot T =$ 0.5-0.7 with virtually any relationships between the determined and noise components of the signal.

Thus, the presence of the considered noise component exerts virtually no influence on the choice of the optimum transmission band of the filter, which is determined only by signal duration.

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DETECTION OF SIGNALS WITH UNKNOWN PARAMETERS ON A REVERBERATION BACKGROUND

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 263-269

[Article by G. S. Nakhmanson and V. V. Pavlov]

[Text] In a study of the problems involved in the detection [1-3] of signals against a noise background it is usually assumed that there is a coincidence of the parameters of the received and reference signals, which does not seem possible in a real situation. A mismatch of parameters can lead to an appreciable deterioration of the detection characteristics; therefore, simultaneously with detection it is necessary to evaluate the signal parameters [4]. In this connection it is of interest to examine the detection characteristics of a maximum probability detector, comparing the output signal maximum with the threshold.

Characteristics of Detection of Signal With Random Amplitude and Initial Phase

Assume that an additive mixture arrives at the detector input

$$x(t) = S(t, \ell_0, a_0, \varphi_0) + n_n(t) + n_r(t) \tag{1}$$

where $S(t,\ell_0,a_0,\varphi_0)=a_0 F(t,\ell_0)\cos[\omega t+\varphi(t,\ell_0)-\varphi_0]$

is a useful signal with a random initial phase, uniformly distributed in the interval $[0, 2\pi]$, and an amplitude conforming to a Rayleigh distribution

$W(a) = a\,exp\{-a^2/2\sigma_a^2\}/\sigma_a^2,\ a \geq 0.$ $F(t,\ell_0)$ и $\Phi(t,\ell_0)$ are

amplitude and phase modulations of the signal, dependent on the evaluated parameter ℓ_0, $n_{\pi}(t)$ is normal noise with a zero mean value and the correlation function $K_n(t_1, t_2)$, and

$$n_r(t) = \int_0^T A(t,\tau)f(t-\tau)\cos[(\omega+\Omega_p)(t-\tau)+\varphi_1(t-\tau)+\Psi(t,\tau)]\,dt, \tag{2}$$

is the reverberation noise signal, whose correlation function has the form [5]

$$K_z(\tau) = K_p(\tau)\, r(\tau) \cos(\omega + \Omega_p)\tau, \quad r(\tau) = [r_c^2(\tau) + r_s^2(\tau)]^{\frac{1}{2}}$$

$$\left.\begin{matrix} r_c(\tau) \\ r_s(\tau) \end{matrix}\right\} = \int_0^T f(t) f(t+\tau) \left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\} [\varphi_1(t+\tau) - \varphi_1(t)]\, dt \tag{3}$$

Ω_p is the frequency shift caused by the movement of scatterers, $K_\rho(\tau)$ is the Fourier transform of the channel scattering function [5]. f(t) and $\Phi_1(t)$ are the amplitude and phase modulation of the sounding signal.

In this case the logarithm of the functional of the probability ratio is determined by the expression

$$M(\ell) = \frac{1}{2} R^2(\ell) / [1 + Q(\ell)] - \ln[1 + Q(\ell)] \tag{4}$$

where

$$R(\ell) = [X^2(\ell) + Y^2(\ell)]^{\frac{1}{2}},$$

$$\left.\begin{matrix} X(\ell) \\ Y(\ell) \end{matrix}\right\} = \sigma_a \int_0^T x(t) V(t,\ell) \left\{\begin{matrix}\cos\\ \sin\end{matrix}\right\} [\omega t + \varphi_1(t,\ell)]\, dt$$

$$Q(\ell) = \frac{\sigma_a^2}{2} \int_0^T F(t,\ell) V(t,\ell) \cos[\varphi(t,\ell) - \varphi_1(t,\ell)]\, dt$$

V(t, ℓ) and $\Phi_1(t, \ell)$ are the laws of amplitude and phase modulation of the heterodyne reference signal, dependent on the evaluated parameter ℓ,

$$v(t,\ell,\varphi) = \sigma_a V(t,\ell) \cos[\omega t + \varphi_1(t,\ell) - \varphi]$$

in a general case not coinciding with the reference signal of the optimum detector, that is, not satisfying the equation [6]

$$\int_0^T K(t_1,t_2)\, v(t_2,\ell,\varphi)\, dt_2 = S(t_1,\ell,\varphi)$$

$$K(t_1, t_2) = K_n(t_1,t_2) + K_z(t_1,t_2) \tag{5}$$

The detector (4) with reception of the mixture (1) forms M(ℓ), determines M(ℓ_m), where ℓ_m is the evaluation of the maximum probability of the ℓ parameter, and compares it with the threshold M_0.

Then the density distributions M(ℓ_m) in the presence and absence of a signal assume the form

$$\begin{matrix} W_{s+n}(M) = \sigma_1^{-1} \exp\{-(M-C)/\sigma_1\}, \\ W_n(M) = \sigma_2^{-1} \exp\{-(M-C)/\sigma_2\}, \end{matrix} \quad M > 0 \tag{6}$$

where

$$\sigma_1 = [G_1^2(\ell_m,\ell_m) + G^2(\ell_0,\ell_m)] / [1 + Q(\ell_m)], \quad \sigma_2 = G_1^2(\ell_m,\ell_m) / [1 + Q(\ell_m)],$$

$$C = -\ln[1 + Q(\ell_m)], \qquad G(\ell_1,\ell_2) = [G_c^2(\ell_1,\ell_2) + G_s^2(\ell_1,\ell_2)]^{\frac{1}{2}},$$

$$\left.\begin{matrix} G_c(\ell_1, \ell_2) \\ G_s(\ell_1, \ell_2) \end{matrix}\right\} = \frac{\sigma_u^2}{2}\int_0^T F(t, \ell_1) V(t, \ell_2)\left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\}[\varphi(t, \ell_1) - \varphi_1(t, \ell_2)]\,dt$$

$$G_1(\ell_1, \ell_2) = [G_{1c}^2(\ell_1, \ell_2) + G_{1s}^2(\ell_1, \ell_2)]^{1/2}$$

$$\left.\begin{matrix} G_{1c}(\ell_1, \ell_2) \\ G_{1s}(\ell_1, \ell_2) \end{matrix}\right\} = \frac{\sigma_u^2}{2}\iint_0^T K(t_1, t_2) V(t_1, \ell_1) V(t_2, \ell_2)\left\{\begin{matrix}\cos \\ \sin\end{matrix}\right\}[\omega(t_1 - t_2) + \varphi_1(t_1, \ell_1) - \varphi_1(t_2, \ell_2)]\,dt_1 dt_2$$

and the expression for the probability of correct detection by our detector can be represented as

$$D(\ell_m) = \exp\{\ln F[1 + G^2(\ell_0, \ell_m) G_1^{-1}(\ell_m, \ell_m)]\} \tag{7}$$

With satisfaction of the reliable evaluation conditions it is possible to seek, similarly [6]

$$\ell_m = \tilde{\ell}_0 + \varepsilon \ell_{10} + \delta \ell_{01} + \ldots + \varepsilon^i \delta^j \ell_{ij} + \ldots \tag{8}$$

Here

$$\varepsilon = [G^2(\ell_0, \tilde{\ell}_0) / G(\tilde{\ell}_0, \tilde{\ell}_0)]^{-1/2}, \quad \delta = [G(\ell_0, \tilde{\ell}_0)]^{-1/2}$$

are values inverse to the square roots of the signal-to-noise ratio and the signal maximum at the detector output, and $\tilde{\ell}_0$ is a solution of the equation

$$\left\{\frac{d}{d\ell}[G^2(\ell_0, \ell) / G_c(\ell, \ell)]\right\}_{\tilde{\ell}_0} = 0 \tag{9}$$

Substituting (8) into (7) and expanding $D(\ell_m)$ into a series in powers of ε and δ we have

$$D(\ell_m) = D_0 + \varepsilon D_{10} + \delta D_{01} + \ldots + \varepsilon^i \delta^j D_{ij} + \ldots \tag{10}$$

where

$$D_0 = D(\tilde{\ell}_0), \quad D_{10} = e^{\beta_0}\beta_1 \ell_{10}, \quad D_{20} = \frac{e^{\beta_0}}{2}[2\beta_1 \ell_{20} + (\beta_1^2 + \beta_2)\ell_{10}^2]$$

$$D_{02} = e^{\beta_0}\beta_1 \ell_{02}, \quad D_{12} = e^{\beta_0}[\beta_1 \ell_{12} + (\beta_1^2 + \beta_2)\ell_{10}\ell_{02}], \quad D_{01} = D_{21} = D_{03} = 0,$$

$$D_{30} = e^{\beta_0}[\beta_1 \ell_{30} + (\beta_1^2 + \beta_2)\ell_{10}\ell_{20} + \frac{1}{6}(\beta_1^3 + 3\beta_1\beta_2 + \beta_3)\ell_{10}^3], \ell_{01} = \ell_{21} = \ell_{03} = 0 \tag{11}$$

$$\beta_i = \left\{\ln F \frac{d^i}{d\ell^i}[G^2(\ell_0, \ell) / G_1(\ell, \ell) + 1]^{-1}\right\}_{\tilde{\ell}_0}$$

Then in the second approximation the mean value and the dispersion will have the form

$$\begin{aligned} \langle D\rangle &\approx D_0 + e^{\beta_0}\left[\beta_1(\langle l_{20}\rangle + \langle l_{02}\rangle) + \frac{1}{2}(\beta_1^2+\beta_2)\langle l_{10}^2\rangle\right] \\ \sigma^2(D) &= e^{2\beta_0}\Big\{\varepsilon^2\beta_1^2\langle l_{10}^2\rangle + \varepsilon^4\Big[\langle l_{10}^2\rangle\left(\frac{3}{2}\beta_1^4 + 4\beta_1^2\beta_2 + \beta_1\beta_3 + \frac{\beta_2^2}{2}\right) + \\ &+ \beta_1(\beta_1^2+\beta_2)(3\langle l_{10}^2 l_{20}\rangle - \langle l_{10}^2\rangle\langle l_{20}\rangle + \beta_1^2(2\langle l_{10} l_{30}\rangle + \langle l_{20}^2\rangle - \langle l_{20}\rangle^2)\Big] + \\ &+ \varepsilon^2\delta^2\left[\langle l_{10}^2\rangle\langle l_{02}\rangle\beta_1(\beta_1^2+\beta_2) + \beta_1^2\langle l_{10} l_{12}\rangle\right]\Big\} \end{aligned} \qquad (12)$$

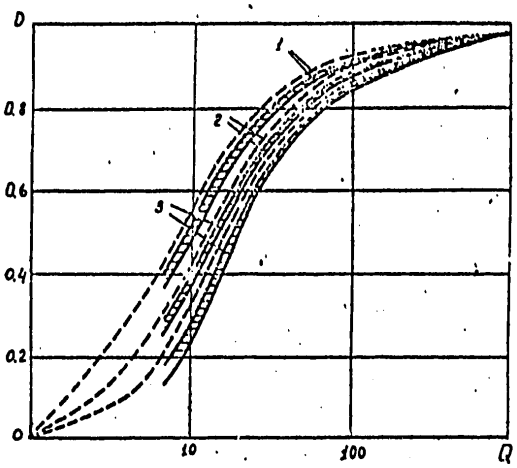

Fig. 1.

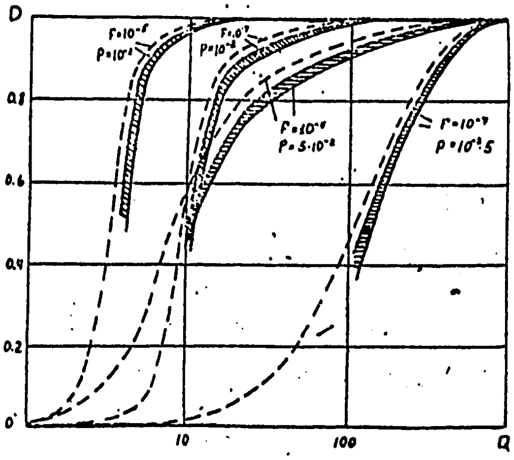

Fig. 2.

It is easy to confirm that in the case of optimum reception (that is $V(t, \ell, \varphi)$) $\beta_1 = 0$ satisfies (5) and in the first approximation $\langle D\rangle = D_0$, $\sigma^2(0) = 0$ and the detection characteristic (7) coincides with those known from the literature [1,2].

Figure 1 shows the detection characteristics of a bell-shaped pulse

$$s(t,\theta_0,\varphi_0)=a_0\exp\{-(t-\theta_0)^2\tau_0^{-2}\}\cos[\omega_0 t-\varphi_0]$$

with random initial phase and amplitude against a reverberation background with the correlation function

$$K_z(\tau)=\sigma_z^2 e^{-\Delta^2\tau^2}\cos(\omega+\Omega_p)\tau(1-\tfrac{|\tau|}{\tau_0})$$

and white noise with the spectral density N_0, with a detector optimum for reception against a background of white noise, as functions of

$$Q_0 = Q_0^2\tau_0\sqrt{\pi}/2\sqrt{2}\,N_0$$

(signal-to-noise ratio). The family of curves 1, 2, 3 corresponding to probabilities of a false alarm 10^{-3}, 10^{-4}, 10^{-5} were obtained on the assumption that the reverberation noise ratio is equal to unity. The dashed curve shows the dependences corresponding to D_0 and the shaded regions correspond to the values of probabilities of correct detection falling between the curves $\langle D\rangle$ and $\langle D\rangle - \sigma(D)$. It follows from the dependences shown in the figure that with signal-to-noise ratios of about 10-80 allowance for a second approximation introduces a correction of about 18±3%. With an increase in Q the value of the correction tends to zero.

Characteristics of Detection of Noiselike Echo Signal

Assume that a mixture

$$x(t)=S(t)+n_n(t)+n_z(t)$$

arrives at the detector input during the time T.

It is assumed that $S(t)$, $n_n(t)$ and $n_z(t)$ are normal, random, independent processes having zero mean values and the correlation functions

$$\langle S(t)S(t+\tau)\rangle = K_S(\tau,\ell_0),\quad \langle n(t)n(t+\tau)\rangle = K_n(\tau)$$
$$\langle n_z(t)n_z(t+\tau)\rangle = K_z(\tau) \tag{13}$$

Then we will assume that T is much greater than the correlation time x(t).

In this case the maximum probability detector forms an output effect [7]

$$M(\ell)=2T\int_0^T K(\tau,\ell)K_1(\tau)d\tau-A(\ell) \tag{14}$$

determines $M(\ell_m)$ and compares it with the threshold in (14)

$$K'(\tau,\ell)=\frac{1}{2\pi}\int_{-\infty}^{\infty}K_s(\omega,\ell)K^{-2}(\omega)[1+K_s(\omega,\ell)K^{-1}(\omega)]^{-1}e^{i\omega\tau}d\omega$$

$$K_1(\tau)=\frac{1}{T}\int_0^T x(t)x(t+\tau)dt,\quad A(\ell)=\frac{T}{2\pi}\int_{-\infty}^{\infty}\ln[1+K_s(\omega,\ell)K^{-1}(\omega)]d\omega. \tag{15}$$

$K_S(\omega, \ell)$ and $K(\omega)$ are the Fourier transforms of $K_S(\tau, \ell)$ and $K(\tau) = K_{11}(\tau) + K_r(\tau)$.

By virtue of the restrictions presented above the distribution laws in (14) in the presence and absence of a signal can be considered normal [8].

$$W_{s+n}(M)=(2\pi D_s)^{-\frac{1}{2}}\exp\{-(M-m_s)^2D_s^{-1}\},$$

$$W_n(M)=(2\pi D_n)^{-\frac{1}{2}}\exp\{-(M-m_n)^2D_n^{-1}\}. \tag{16}$$

Then the expression for the characteristics of detection of echo signals can be written similar to (7)

$$D(\ell_m)=1-\Phi(\gamma(\ell_m)),\qquad \Phi(x)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{x}\exp(-t^2/2)dt$$

$$\gamma(\ell)=\text{æ}\sqrt{D_n(\ell)D_s^{-1}(\ell)}-\Delta(\ell)\sqrt{D_s^{-1}(\ell)}. \tag{17}$$

$$D_s(\ell)=\frac{T}{\pi}\int_{-\infty}^{\infty}\frac{K_s^2(\omega,\ell)}{K^2(\omega)}\frac{[K_s(\omega,\ell_0)+K(\omega)]^2}{[K_s(\omega,\ell)+K(\omega)]^2}d\omega,$$

$$D_n(\ell)=\frac{T}{\pi}\int_{-\infty}^{\infty}K_s^2(\omega,\ell)/[K(\omega)+K_s(\omega,\ell)]^2d\omega.$$

$$\Delta(\ell)=m_s-m_n=\frac{T}{2\pi}\int_{-\infty}^{\infty}\frac{K_s(\omega,\ell)K_s(\omega,\ell_0)d\omega}{K(\omega)[K_s(\omega,\ell)+K(\omega)]},\quad \text{æ}=\Phi^{-1}(1-F),$$

where F is the probability of a false alarm. With satisfaction of the conditions for a reliable evaluation [6] ℓ_m can be sought in the form [6, 7]

$$\ell_m=\ell_0+\varepsilon\ell_1+\varepsilon^2\ell_2+\varepsilon^3\ell_3 \tag{18}$$

where $\varepsilon^{-2}=\langle M(\ell_0)\rangle^2/\langle[M(\ell_0)-\langle M(\ell_0)\rangle]^2\rangle$

is the signal-to-noise ratio at the detector output (14).

Expanding (17) into a series of ε similar to (10), we determine the coefficients D_i, knowing which it is easy to obtain expressions for the statistical characteristics $D(\ell_m)$. In the second approximation the mean value and the dispersion can be represented in the following way

$$\langle D\rangle=D_0-\varepsilon^2\frac{e^{-\frac{1}{2}\gamma_0^2}}{\sqrt{2\pi}}[\gamma_1\langle\ell_2\rangle+\frac{1}{2}(\gamma_2-\gamma_1^2\gamma_0)\langle\ell_1^2\rangle]$$

$$\sigma^2(D)=\varepsilon^2(2\pi)^{-1}e^{-\gamma_0^2}\{\gamma_1^2\langle\ell_1^2\rangle+\varepsilon^2[\gamma_1^2(\langle\ell_2^2\rangle-\langle\ell_2\rangle^2)+\gamma_1(\gamma_2-\gamma_1^2\gamma_0)(3\langle\ell_1^2\ell_2\rangle- \tag{19}$$

$$-\langle\ell_2\rangle\langle\ell_1^2\rangle+2\gamma_1^2\langle\ell_1\ell_3\rangle+\langle\ell_1^4\rangle(\frac{\gamma_2^2}{2}-4\gamma_0\gamma_1^2\gamma_2+\frac{3}{2}\gamma_1^4\gamma_0^2+\gamma_1\gamma_3-\gamma_1^4)]\}$$

Figure 2 shows the characteristics of signal detection with the correlation functions $K_S(\tau, \theta_0) = \sigma_S^2 \theta^{-\alpha|\tau|} \cos \theta_0 \tau$ against a background of reverberation noise with the correlation function $K_l(\tau) = \sigma^2 e^{-\alpha|\tau|} \cos \omega_l \tau$ and white noise with the spectral density N_0 as a function of $Q = 2\sigma_\tau^2/N_0$ -- the signal-to-noise ratio for different values of the parameter ρ (α)* -- the ratio of the correlation time of the process S(t) to the observation time in the first (dashed curve) and in the second (shaded region) approximations. It was assumed in this case that the reverberation-noise ratio in this case is equal to unity. [*illegible]

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STABILIZATION OF A FALSE ALARM IN HYDROACOUSTIC DETECTION CHANNELS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 277-287

[Article by S. N. Gerasimenko, V. P. Ovsyanik and S. V. Pasechnyy]

[Text] In hydroacoustics extensive use is made of standard detection channels (SDC) designed on the "filter-detector-filter" principle. The noise immunity of such channels has been investigated in a whole series of studies under conditions of both stationary [1] and nonstationary noise [2]. The results of these investigations are essentially as follows: in the detection of a signal against a background of nonstationary noise in the form

$$N(t) = m(t)n(t), \tag{1}$$

(where $m(t) = 1 + \sum_{i=1}^{n} m_i \cos 2\pi f_{mi}$ is a modulating function, m is modulating intensity, f_m is modulation frequency, n(t) is a stationary Gaussian random process) the noise immunity of the SDC decreases in comparison with cases of signal detection against a background of equivalent (with respect to mean intensity) stationary noise $n_{equiv}(t)$.

The following methods for stabilizing a false alarm are known:
- -- making the noise stationary by means of automatic volume control (AVC) [3],
- -- rigorous limitation [4],
- -- trimming of the threshold level at the SDC output [5],
- -- use of a phase autocorrelator circuit [5], and so forth.

In [2] it was demonstrated that the ideal reduction of noise to a stationary state in the case of a precisely known form of the modulating function considerably improves the noise immunity of the standard detection channel. Thus, for a probability of correct detection $D = 0.9$, the probability of a false alarm $F = 10^{-3}$ and a modulating function $m(t) = 1 + m \cos 2\pi f_m t$ with $m = 0.9$ and $f_m T = 1$ the gain in the threshold signal-to-noise ratio attains 13.5 db.

An investigation of the noise immunity of channels with rigorous restriction on the detection of weak signals is equivalent to the ideal reduction of noise to a stationary state; in the detection of strong signals the noise immunity of a channel with a limitation at 3 db is greater than for a channel without reduction of noise to a stationary state.

Channels with trimming of the threshold level in the case of nonstationary interference are inferior in noise immunity to channels with stationary noise.

Source [3] gives schemes of stabilization of a false alarm using AVC, controllable by a low-frequency filter for filtering the envelope of the stabilized process. This method has the important shortcoming that for the effective stabilization of a false alarm all the components of the modulating function m(t) must fall into the band of a LF filter from whose output AVC is accomplished. However, with the reception of pulsed signals of a great duration most of the energy spectrum of the signal envelope falls in the band of the controlling LF filter. In this case there is a substantial distortion of the received signals and the noise immunity of the detection channel is not improved and even deteriorates.

Thus, each of the known methods for stabilizing the false alarm has adequate effectiveness only for restricted detection conditions (in particular, for a limited change in the signal-to-noise ratio, signal duration, frequency and intensity of nonstationary noise, etc.). Therefore, it is of great interest to seek and investigate the effectiveness of detection algorithms invariant to the variability of the conditions for detection of signals against a background of nonstationary noise of the type (1).

We will examine also methods for determining the noise immunity of an adaptive receiving channel based on automatic volume control in conformity to a law formed by a device for measuring a priori information on the nature and parameters of the noise.

Figure 1 is a structural diagram of a device for realizing the false alarm method at the SDC output using AVC, controllable by the discriminated modulating function of the stabilized process, where 1, 2 are band filters with the transparency band Δf_{Q7} at the level 0,7, 3 -- device for AVC. 4,8 -- linear detectors, 5 - 5; 7 - 7 -- narrow-band filters with the band $\Delta f'_{Q7}$ at the level 0.7 covering the frequency range of the modulating function, 6_1 - 6_n -- comparison circuits, 9_1 - 9_n -- key devices, 10 -- summator, 11 -- SDC.

The process to be stabilized is fed to the input 1 of the circuit, whereas the equivalent (with respect to mean intensity) stationary Gaussian random process is fed to input 2.

If the process fed to circuit input 1 is an amplitude-modulated Gaussian random process, in the LF part of the spectrum at the output of the linear detector there will be harmonic components with amplitudes proportional to the

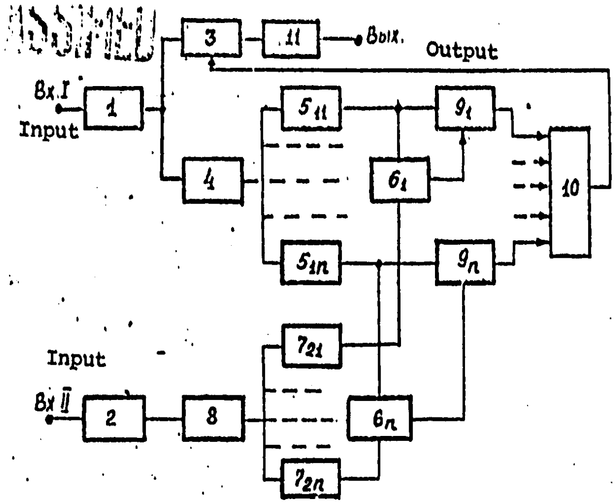

3
11
Вых.
Output
Вх.I
Input
1
5₁₁
9₁
4
6₁
10
5₁n
9n
Input
Вх II
2
8
7₂₁
6n
7₂n

Fig. 1

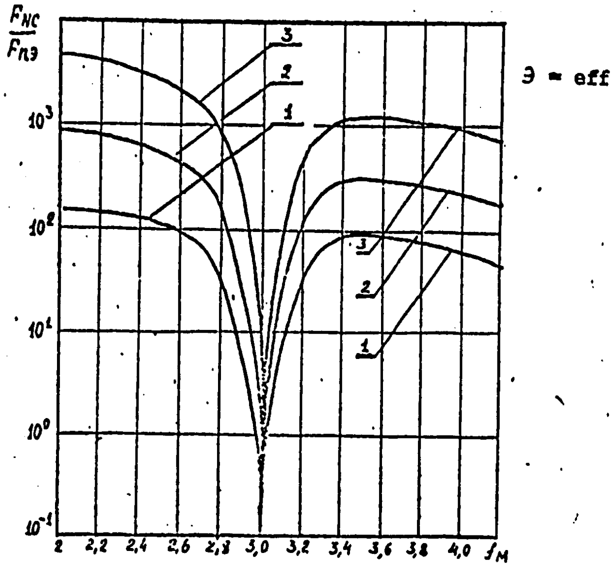
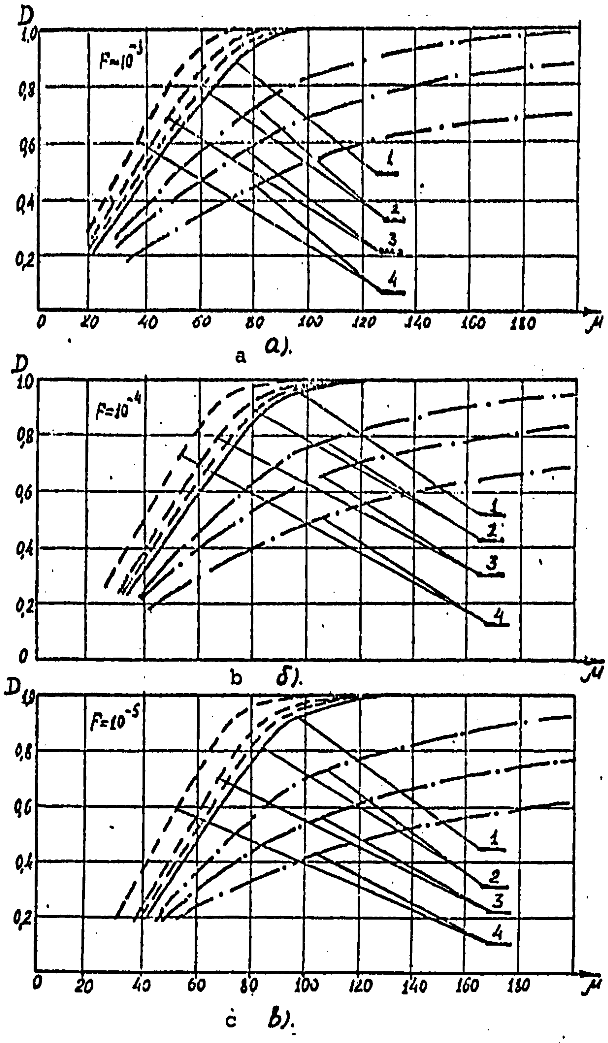

$\frac{F_{HC}}{F_{пэ}}$
Э = eff
10^3
10^2
10^1
10^0
10^{-1}
3
2
1
3
2
1
2
2,2
2,4
2,6
2,8
3,0
3,2
3,4
3,6
3,8
4,0
f_M

Fig. 2

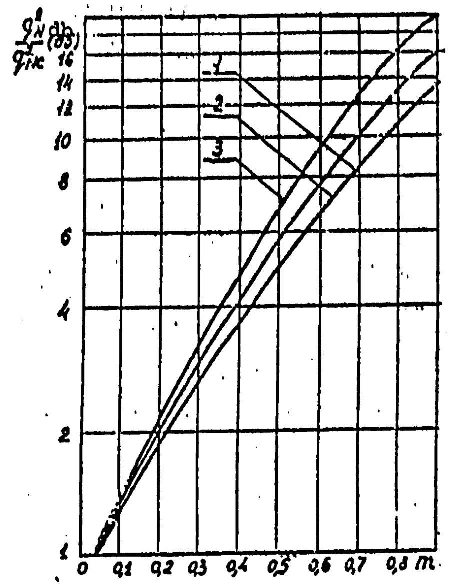

D
1,0
0,8
0,6
0,4
0,2
0
F=10^-3
F=10^-4
F=10^-5
1
2
3
4
20
40
60
80
100
120
140
160
180
μ
a).
б).
в).

a

b

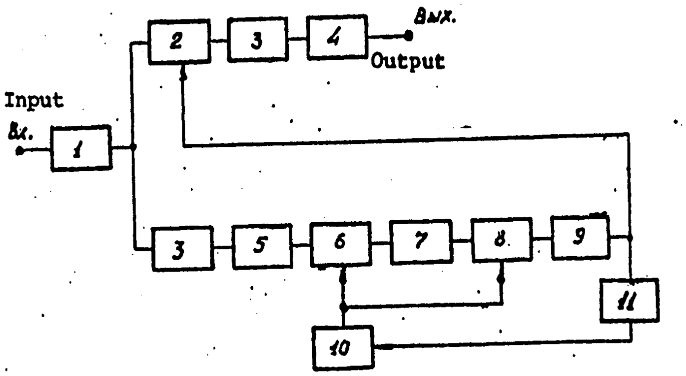
c

Fig. 3



$\frac{q_N^1}{q_{1K}}$ (дб)
16
14
12
10
8
6
4
2
1
1
2
3
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
m

Fig. 4



Input
Вх.
1
2
3
4
Вых.
Output
3
5
6
7
8
9
10
11

Fig. 5

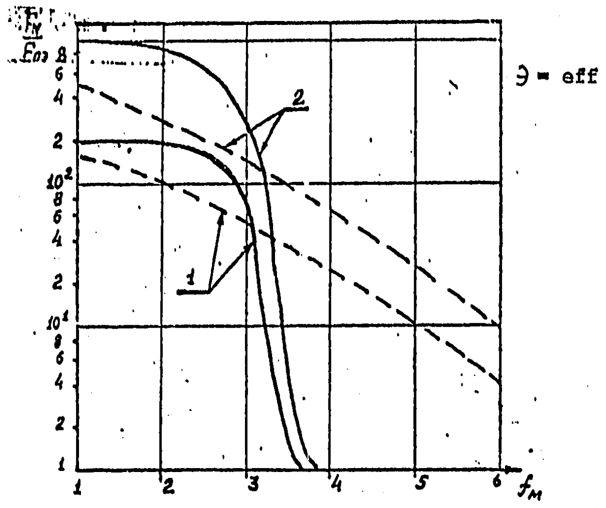


Fig. 6

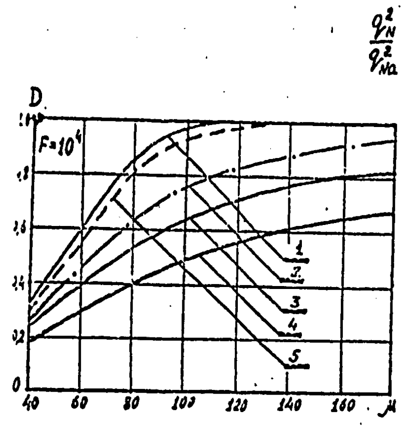


Fig. 7

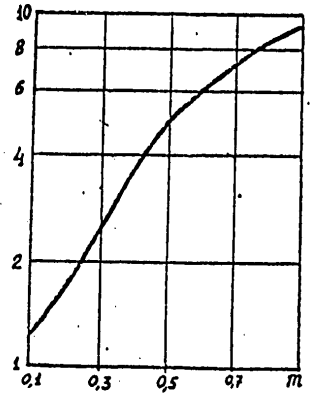


Fig. 8

corresponding intensities of modulation and frequencies equal to the frequencies of the modulating function. Therefore, at the outputs of the comparison circuits, to one of whose inputs is fed a signal from band filters with central frequencies equal to the frequencies of the modulating function, the comparison signals will be greater than unity. These signals close the switching circuits and the processes from the outputs of the corresponding filters are summed and fed as controlling signals to the AVC circuit.

Experimental investigations were carried out with $\Delta f_{0.7} = 200$ cps, $\Delta f'_{0.7} = f_{cen}/30$ (where f_{cen} is the central frequency of the narrow-band filter). As the narrow-band filters use is made of active RC filters for the upper and lower frequencies. A "Razdan" computer is used for determining the probabilities of a false alarm F and correct detection D.

Figure 2 shows typical graphs of the dependence of the ratio of probabilities of a false alarm $F_{NC}/F_{n\ eff}$ at the SDC (with AVC) output, organized in accordance with the scheme in Fig. 1, F_{NC} is the probability of a false alarm (determined from the relative time of presence of the process over the threshold level) with an influence on the channel input from the process $N(t) \simeq (i + m \cos 2\pi f_M t)n(t)$ with a modulation intensity $m = 0.5$, $F_{n\ eff}$ is the same under the influence of the process $n_{eff}(t)$. For the sake of clarity in evaluating the effectiveness of false alarm stabilization use was made only of band filters with $f_{cen} = 3$ cps. The curves 1, 2, 3 correspond to the probability of a false alarm $F_{n\ eff} = 10^{-3}, 10^{-4}, 10^{-5}$.

As can be seen from the graphs, the false alarm at the SDC output under the influence at its input of nonstationary noise with $f_M = f_{cen}$ is even less than in the case of equivalent (in mean intensity) stationary noise.

Figures 3,a,b,c show typical detection characteristic curves for the considered channel, where $\mu = 2Mq^2$, $M = \Delta f_{0.7} \cdot T$, $q^3 = A^2/2\sigma^2$ is the signal-to-noise ratio at the channel input, A is signal amplitude, σ is the mean square noise value. Curve 1 corresponds to a case when the noise is a stationary Gaussian random process $n_{eff}(t)$, 2, 3, 4 are curves corresponding to the case when the noise is an amplitude-modulated Gaussian random process with a modulation frequency $f_M = 3$ cps and modulation intensities $m = 0.3, 0.5, 0.8$ respectively. The dot-dash lines are curves obtained with stabilization of the false alarm. The curves were constructed for a probability of a false alarm $F = 10^{-3}$ (Fig. 2a), $F = 10^{-4}$ (Fig. 3b) and $F = 10^{-5}$ (Fig. 3c).

An analysis of the curves shows a quite high effectiveness of the proposed method for stabilizing a false alarm. The noise immunity of the channel when there is nonstationary noise at the input and when employing the proposed scheme (Fig. 1) is even somewhat higher than in the presence of stationary noise. This is attributable to the fact that the dispersion of a nonstationary process in the case of complete stationarity is less than the dispersion of the equivalent (with respect to mean intensity) stationary process, and accordingly, the signal-to-noise ratio increases.

Figure 4 shows graphs of the dependence of the gain in the threshold signal-to-noise ratio $g^2{}_N/g^2{}_{Nt}$ on the intensity of modulation of noise in the form (1) for a nonstationarity frequency $f_M = 3$ cps, $m = 0.9$ and $F = 10^{-3}, 10^{-4}, 10^{-5}$ (curves 1, 2, 3 respectively), $g^2{}_N$, $g^2{}_{Nt}$ are the signal-to-noise ratios at the output of a band filter without use of a false alarm stabilization scheme and with its use respectively.

As indicated by the graphs of experimental investigations, use of the false alarm stabilization circuit proposed here gives a substantial gain in the threshold signal-to-noise ratio. Thus, when $D = 0.9$ and $F = 10^{-3}$ this gain attains 13 db.

In a case when the process to be stabilized can with an adequate accuracy be represented by a model in the form

$$N(t) = (1 + m \cos 2\pi f_M t)\, n(t) \qquad (2)$$

it is desirable to use a false alarm stabilization scheme represented in Fig. 5, where 1 is the band filter, 2 is the AVC unit, 3 is the linear rectifier, 4 is the LF filter, 5 is a LF filter discriminating the envelope of the process to be stabilized, 6 is the multiplier, 7 is a narrow-band filter, 8 is a multiplier, 9 is a LF filter, 10 is a tunable heterodyne, 11 is a frequency trimming unit. The use of such a scheme substantially simplifies and reduces the cost of the false alarm stabilization unit, since only one band filter is used.

The operating principle for the system is as follows. If the process to be stabilized is (in the form (2)) fed to the system input, the frequency trimming device sets such a heterodyne frequency that the harmonic component in the spectrum of an envelope with the frequency f_M and an amplitude proportional to the intensity of modulation m is heterodyned into the frequency region f_{cen} of the band filter. From the output of the band filter a signal with the frequency f_{cen} and an amplitude proportional to m is fed to one input of the multiplier, to whose second input is fed a signal from the heterodyne. From the output of the LF filter 9 the discriminated signal with the difference frequency $f_{cen} - f_{um} = f_M$ with an amplitude proportional to the intensity of modulation of the process to be stabilized is fed to the AVC circuit 2. The effectiveness of such a false alarm stabilization scheme, under the influence of processes of the type (2), is equivalent to the effectiveness of the stabilization scheme shown in Fig. 1.

In those cases when the detection is accomplished under conditions when the upper frequency of nonstationarity of noise is $f_{M\ 1} > 1/T$, the use of relatively complex stabilization methods as described above is undesirable. In this case an effective false alarm stabilization can be obtained by a rational choice of the parameters of the post-rectifier LF filters. For example, with a modulation frequency equal to 3-4 cps there will be effective stabilization when $\tau_{RC} = 3\text{-}4$ sec.

The use of pulses of a lesser duration with these same nonstationarity frequencies can be effective with an increase in the steepness of the frequency characteristic of the post-rectifier LF filter.

Figure 6 shows typical graphs of the dependence of the ratio $F_N/F_{n\ eff}$ on modulation frequency f_M of a process of the type (2) for a modulation intensity m = 0.5.

The lines represent curves obtained when using active RC LF filters as post-rectifier filters. The dashed lines represent the same passive RC filters. Both filters are matched with a signal with the duration T = 300 msec.

Curves 1, 2 were obtained for threshold levels for which $F = 10^{-3}, 10^{-4}$ respectively. F_N is the probability of a false alarm at the SDC output when noise in the form (2) arrives at its input, $F_{n\ eff}$ is the probability of a false alarm under the influence of equivalent (in mean intensity) stationary Gaussian noise.

The graphs show that the use of active RC filters virtually completely stabilizes the false alarm at the SDC output with arrival of a process of the type (2) at its input in a case when the modulation frequency $f_M > 1/T$.

Figure 7 shows the detection characteristics for values of probability of a false alarm $F = 10^{-4}$, where $\mu = 2Mq^2$.

Curve 1 corresponds to the case of stationary noise $n_{eff}(t)$; 2, 3, 4 --- these curves correspond to noise representing a nonstationary amplitude-modulated Gaussian random process with a modulation frequency $f_M = 3.5$ cps and a modulation intensity m = 0.3, 0.5, 0.8 respectively. Curve 5 is the same with a modulation intensity m = 0.5, but the SDC makes use of a post-rectifier active RC filter.

Experimental investigations were made for $\Delta f_{0.7} = 200$ cps, T = 300 msec.

Figure 8 shows a graph of the dependence of the gain in the threshold signal-to-noise ratio q_N^2/q_{Na}^2 on modulation intensity m for $f_M = 3.5$ cps, D = 0.9 and $F = 10^{-4}$, q_N^2 and q_{Na}^2 are the signal-to-noise ratios at the output of a band SDC with a post-rectifier passive RC filter and an active RC filter respectively.

An analysis of the graphs makes it possible to evaluate the effectiveness of use of active RC filters for increasing the SDC noise immunity in the case of nonstationary noise in the form (2) at its input.

Thus, for $F = 10^{-4}$ and D = 0.9 the gain in the threshold signal-to-noise ratio with $f_M = 3.5$ cps and m = 0.9 is 8.9 db.

In summary, the following conclusions can be drawn.

1. The cited analysis of schemes for the stabilization of a false alarm F indicated that with a change in the parameters of nonstationarity of noise and the durations of signals in a sufficiently broad range the most effective variant is an adaptive variant of construction of the receiving channel (Figure 1, 5). For example, in the case of nonstationary noise (2) with a modulation intensity m = 0.5 the probability of a false alarm F_{NC} at the output of such a channel is approximately 10 times less than the probability of a false alarm $F_{n\ eff}$ (for the case $F_{n\ eff} = 10^{-5}$) with an equivalent (in mean intensity) stationary Gaussian noise.

The gain in the threshold signal-to-noise ratio for D = 0.9 and $F = 10^{-3}$ with $f_M = 3$ cps and m = 0.9 is approximately 13 db, which approaches the noise immunity of an optimum detector with the reduction of noise to a stationary state with a known modulating function.

2. Comparison of the effectiveness of the proposed scheme for a detector with a "wide band filter - limiter - narrow band filter" (WBF-L-NBF) channel shows that in the reception of weak signals the noise immunity of these channels is approximately identical, whereas in the detection of strong signals the WBF-L-NBF channel is inferior in noise immunity to the proposed channel with AVC by approximately 10 db.

It can be noted that the WBF-L-NBF channel, in contrast to the proposed channel, does not stabilize a false alarm with additive nonstationary noise.

3. The use of post-detector LF filters with a frequency characteristic close to ideal considerably broadens the possibilities of use of SDC with nonstationary noise. For example, with $f_M = 3.5$ cps, m = 0.5, $F_{n\ eff} = 10^{-4}$ and LF filters matched with the duration of the pulsed signal T = 300 msec the probability of a false alarm at the output of a channel with a passive RC filter is 100 times greater than for a channel with an active filter.

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For D = 0.9 and $F = 10^{-4}$ with $f_M = 3.5$ cps and m = 0.9 the gain in the threshold signal-to-noise ratio in the case of use of active RC filters attains 8.9 db.

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SUBOPTIMUM DETECTION OF HYDROACOUSTIC ECHO SIGNALS ON AN ELECTRONIC COMPUTER

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 288-294

[Article by B. P. Brezhnev, V. D. Dubovets and V. V. Balagin]

[Text] For the simultaneous primary and secondary processing of sonar information for the purpose of detection and evaluation of the parameters of motion of hydroacoustic objects in some cases it is desirable to use electronic computers. The algorithms for such processing have a considerable flexibility and make it possible to introduce adaptation. This is important because under real conditions the hydroacoustic complexes must function under conditions of "a priori uncertainty" caused by temporal variability of the characteristics of the medium. However, the direct realization of optimum analog methods for primary processing on electronic computers frequently encounters "the curse of dimensionality."

The report discusses algorithms for the detection and evaluation of the parameters of motion of the ranged object (range -- D and radial velocity - V_p), which insignificantly yielding in the characteristics to the optimum case, substantially decrease the memory volume and time necessary for their realization.

The optimum linear methods for detection, jointly with evaluation of D and V_p of the ranged object, as is well known, represent a multichannel generalized matched filtering of the received signal or its generalized correlation reception. The synthesis of structure of an optimum detector involves solution of integral equations. However, their solution under conditions of hydroacoustics is an extremely difficult problem, and in addition, optimum in a statistical sense (Karunen-Loew expansion) spectral representation is ill-suited for computations on a digital computer. Therefore, it is desirable to use suboptimum representations (Fourier, Walsh), having "fast" algorithms. For realization of such processing it is necessary to have corresponding densities of the useful and interfering signals and their intensity in the range strobes.

We will discuss the possibility of using an electronic computer for realizing generalized suboptimum matched filtering when using rectangular tonal sounding signals.

The interfering signals are: reverberation $x(t)$, sea noise $\varepsilon(t)$, inherent noise of the sonar system carrier $\xi(t)$ if the carrier moves. We note that the processes $\varepsilon(t)$ and $\xi(t)$ have identical stochastic characteristics in any range strobe.

With movement of the object in a regime of well-developed hydrodynamic cavitation in the immediate neighborhood of the object a two-phase medium is formed: liquid-gas bubbles. This medium "effectively absorbs and scatters sound" [1]. Accordingly, in a cavitation regime with an increase in the velocity of movement of the object the reflection of sound from the "body" decreases, whereas scattering on cavitation bubbles increases. Under these conditions with stern aspects of irradiation the echo signal from the "body" can be completely absent. Therefore, for the purpose of obtaining more information the useful part of the echo signal must include not only the component $z(t)$, caused by reflection from the body of the object, but also reflection from the inhomogeneous medium near the body $u(t)$ [2].

On the basis of [3 and others] it is easy to derive expressions for the spectral densities of the useful and interfering signals [2]. However, it is difficult to check the characteristics of the medium and therefore empirical evaluations must be used as the statistical characteristics of the interfering signals under conditions which cannot be checked. First with use of the FFT and well-known averaging and smoothing methods it is possible to obtain evaluations of spectral density $\hat{S}_e(\omega_k) + \hat{S}_\xi(\omega)$ of samples of signals $\varepsilon(t) + \xi(t)$ corresponding in volume to the range strobe.

The $x(t)$ process in the near zone consists of surface, volume and bottom reverberation. The appearance of surface and bottom reverberation against a background of volume reverberation is accompanied by a relatively brief increase in the intensity of the processes $x(t)$, since surface and bottom reverberation attenuate more rapidly than volume reverberation. The times of appearance of surface and bottom reverberations can be determined easily. Taking into account the noted peculiarities of the $x(t)$ process, it seems reasonable to evaluate the normalized (with respect to dispersion) spectral density $S_x^n(\omega_k)$ and the law of decrease in mean intensity only of volume reverberation. This leads, as will be clear from the text which follows, to a situation in which the proposed algorithm will reveal bottom reflection and surface reverberation, but the latter only in a case when its spectral density differs substantially from the spectral density of volume reverberation.

The procedure of evaluations of parameters of the law of decrease in mean intensity of volume reverberation and obtaining $S_x^n(\omega_k)$ is as follows. A test sounding is made and the received signal $y(t)$ after heterodyning is registered in the computer memory. Assuming a law of decrease in the mean level of volume reverberation of the form $A\, e^{-\alpha t}$, the least squares method with correction is used in finding the parameters A and α. Then the adopted record is broken down into blocks, in time equal to the duration of the sounding signal, and these blocks are subjected to a fast Fourier transform. The unsmoothed evaluation $\tilde{S}_x^n(\omega_x)$ is obtained using the expression:

$$\tilde{S}_x^n(\omega_k) = \frac{I_i(\omega_k) - \hat{S}_\varepsilon(\omega_k) - \hat{S}_\xi(\omega_k)}{\sum_{i=0}^{n-1}[I_i(\omega_k) - \hat{S}_\varepsilon(\omega_k) - \hat{S}_\xi(\omega_k)]}; \tag{1}$$

where n is the number of strobes, N is the number of readings in the strobe $I_i(\omega_k)$ (k = 0.1,..., [N - 1/2]) are the ordinates of the periodogram of the i-th block.

The evaluation $\hat{S}_x^n(\omega_k)$ is obtained from $S_x^n(\omega_k)$ by smoothing with use of one of the spectral weighting functions [7].

The parameters of the spectral densities of the useful signals are dependent on the velocity and course angle of motion of the sounded object. A priori information on the parameters of motion is unavailable. Therefore, for carrying out suboptimum generalized matched filtering it is necessary to store in the computer memory a two-dimensional matrix of standard spectral densities for a definite set of velocities and course angles. This very greatly overloads the memory and the carrying out of filtering requires great expenditures of computer time ("the curse of dimensionality").

In order to overcome the "curse of dimensionality" it is proposed that the solution of the problem be divided into detection and evaluation, that is, that a two-stage procedure be used. In the first stage, employing definite algorithms, we solve the problem of detection of the useful signal in the range strobes jointly with the problem of preliminary evaluation of the radial velocity of the object. This evaluation makes it possible to form a relatively small set of standards for the second stage of the procedure, in which there is a refinement, if there is a need for this in secondary processing, of the evaluation made in the first stage.

At the basis of the detection procedure is the "whitening" principle in the spectral region of interfering signals. We use the notation $\hat{S}_{mc_2}(\omega_k) = \sigma_x^2\hat{S}_x^n(\omega_k)+\hat{S}_n(\omega_k)+\hat{S}_\xi(\omega_k)$. The dispersion of reverberation σ_x^2 is assumed to be constant in the range strobe and variable from strobe to strobe. We compute the evaluation $\hat{\sigma}_x^2$ from the determined parameters A and α and the time corresponding to the middle of the range strobe.

If the interfering signals [denoted "ms"] pass through a filter with the frequency characteristic

$$|W(j\omega)|^2 = \frac{1}{\hat{S}_{mc}(\omega)}, \tag{2}$$

then at the output its signal on the average will have a uniform spectrum. In a case when the input of such a filter receives a mixture of interfering and useful signals the spectral density at the output as an average for the records could not be identical for all frequencies. It is proposed that this fact also be used for detection of the sounded object. The zero hypothesis of absence of a useful signal in the range strobe is reduced to the hypothesis of uniformity of spectral density at the output of the whitening filter. The competing hypotheses to the effect that the strobe contains a mixture of interfering and any type of useful signals lead to the hypothesis that the spectral density at the filter output is not constant. Thus, the values of the periodogram $I_i(\omega_k)$ of the range strobe signal, obtained using the fast Fourier transform, are divided by $\hat{S}_{mc}(\omega_k)$ of this strobe and by means of a suitable criterion a check is made to ascertain whether the series $\{ I_i(\omega_k)/\hat{S}_{mc}(\omega_k)$ is homogeneous. The deviation of the $I_i(\omega_k)/\hat{S}_{mc}(\omega_k)$ estimates for competing hypotheses from the corresponding evaluations for the zero hypothesis is due more to the change in their mean values than the form of their distributions. Otherwise the deviation from the zero hypothesis in essence has the form of a nonmonotonic trend.

It is known that the random values of the periodogram of a normal correlation random process have a distribution proportional to x^2 -- a distribution with two degrees of freedom, and asymptotically nondependent relative to N. For N = 1024 there is full basis for assuming that the series $\{I_i(\omega_k)/\hat{S}_{mc}(\omega_k)$ consists of independent random values.

Due to the type of competing hypotheses which we have considered, for checking the zero hypothesis a suitable checking criterion is the homogeneity of the Bartlett dispersions, being a modification of the similarity ratio criterion for checking the homogeneity of the dispersions. For using the Bartlett test we break down ℓ values of the evaluations $I(\omega_k)/\hat{S}_{mc}(\omega_k)$ into r adjacent groups with ν elements in each, so that $\ell = r\nu$ and we use the notation

$$S_i^2 = \sum_{k=(i-1)\nu+1}^{i\nu} \frac{I(\omega_k)}{\hat{S}_{mc}(\omega_k)}, \quad (i=1,2,\ldots,r) \tag{3}$$

The statistics for this test then has the form

$$H = \frac{\left\{2\ell \ln\left(\frac{\sum_{i=1}^{r} S_i^2}{2\ell}\right) - \sum_{i=1}^{r} 2\nu \ln\left(\frac{S_i^2}{2\nu}\right)\right\}}{(6\nu-\ell)/(6\nu-3)} \tag{4}$$

It is known that if 2ν is greater than 5, the statistics H has an approximately x^2 distribution with $r - 1$ degrees of freedom and the zero hypothesis is refuted for large values of the critical function H.

The grouping of $I(\omega_k)/\hat{S}_{mc}(\omega_k)$ values is carried out not only so that it is possible to use the approximate H distribution for the zero hypothesis, but also in order to increase the effectiveness of the test. The $S_i^2/2\nu$ value is proportional to the smoothed evaluation of the spectral density; the smoothing is carried out using a uniform weighting scheme. It is therefore clear that it is desirable to select r sufficiently large that the peculiarities of the trend of the rejected spectral density of the mixture of useful and interfering signals are not smoothed. However, in this case the ν value must be sufficiently large that the dispersions of the $I(\omega_k)/\hat{S}_{mc}(\omega_k)$ values are relatively small. The value $\ell<[N - 1/2]$ is selected in such a way that the H statistics contains those $I(\omega_k)/\hat{S}_{mc}(\omega_k)$ which differ considerably in the presence and absence of useful signals. The reasoning with respect to the choice of ℓ is the same as when selecting the filter band for increasing the signal-to-noise ratio. The ν value must be selected on the basis of the duration of the trend caused by the presence in the strobe of a signal reflected from the body of the object. Such a conclusion can be drawn from the following considerations. With movement of the sounded object in a precavitation regime such a trend always occurs, whereas in a cavitation regime, although z(t) is small, the spectral density of the signal differs from the spectral density of the interfering signals in an interval of frequencies occupied by the signal z(t). Therefore, the choice of ν must be such as not to smooth the essentially "Doppler" trend. As indicated by experiments, ν must be selected equal to 8-12.

The threshold with which the statistics H is compared is selected on the basis of the probability of false detection in one range strobe.

This criterion must be supplemented by another test, making possible a clearer detection of the local trend in the sequence $\{I(\omega_k)/S_{mc}(\omega_k)\}$ during the movement of the object in a precavitation regime and with forward angles of irradiation. Such a supplementation makes possible not only an increase in the probability of proper detection of an object but also an evaluation of its radial velocity.

The proposed test uses the moving sum

$$B_{jL} = \sum_{k=j-L}^{j} \frac{I(\omega_k)}{S_{mc}(\omega_k)}, \quad j = 1, 2, \ldots, \ell - L \tag{5}$$

Here L is the number of terms in the moving sum.

The B_{jL} value is compared with a constant (for different j) threshold which when exceeded gives a solution with respect to the presence of a signal z(t) in the range strobe. The threshold c is selected on the basis of the probability of a false alarm in one strobe, with the same value as in the first criterion. The probability of a false alarm is equal to the probability of intersection by a discrete random process B_{jL} with a normalized correlation function

$$R(j_1, j_2) = \begin{cases} 1 - \frac{|j_1 - j_2|}{L} & \text{when } |j_1 - j_2| \leq L \\ 0 & \text{when } |j_1 - j_2| > L \end{cases} \tag{6}$$

of the threshold c in the absence of a signal z(t). In this case the values B_{jL} are distributed in conformity to the law x^2 with 2L degrees of freedom. The threshold c is found from the following considerations. The terms B_{jL} are at the distance L from one another; in accordance with (6) they are uncorrelated and accordingly in the sequence B_{jL}, j = 1, 2,

..., ℓ - L there are ℓ- L/L independent terms with identical probabilities of exceeding the threshold c. If the probability of false detection of a local trend should be equal to $P_\wedge$, and c is selected in such a way that each term of the sequence $\{B\}$ can exceed it with the probability $P_\wedge L/\ell - L$, the probability of a false alarm will be somewhat less than $P_\wedge$.

The number L is selected in dependence on the duration of the sounding signal.

An evaluation of the Doppler frequency shift is made using the maximum B_{jL} value, lying above the c threshold.

We note that the use only of the additional test is equivalent to simple carrying out of suboptimum generalized multichannel matched filtering.

The described algorithm with poorly defined competing hypotheses in the distant zone with the appearance of reverberation already on the basis of the first test (criterion) will give a false solution concerning the presence of a signal from an object moving in a cavitation regime. In order to preclude such spurious detections at great distances the following checking procedure is proposed. After making a decision about the presence of a target in the strobe on the basis of the difference $\sum_k[I(\omega_k) -\hat{S}_\varepsilon(\omega_k) - S_\xi(\omega_k)] = \sigma^2$ the intensity of the signal in the strobe is evaluated without ε (t) and ξ (t). Then the following sequence is computed

$$I_1^0(\omega_k) = \frac{I(\omega_k)}{\sigma^2 S^{H}_{\text{дал}}(\omega_k) - \hat{S}_\varepsilon(\omega_k) - \hat{S}_\xi(\omega_k)}, \quad k=1,2,\ldots,\left[\frac{N-1}{2}\right] \qquad (7)$$

where $S^{H}_{\text{dis reverb}}$ is the spectral density of distant reverberation, and the considered detection algorithm is again used.

If with such checking a second decision is not made that an object is present the conclusion is drawn that distant reverberation entered the strobe. In a case when useful signals actually are present in the strobe, the redistribution of their estimated energy by frequencies in accordance with $S^{H}_{\text{dis reverb}}(\omega_k)$ does not lead to a uniformity of the sequence $I_1(\omega_k)$ and with a definite probability a decision will again be made that an object is present.

In conclusion we will discuss the possibility of using the fast Walsh transform in the proposed processing procedure. It has an advantage over the fast Fourier transform with respect to required computation time. In applying algorithms with specialized apparatus this leads to a superiority of the fast Walsh transform over the fast Fourier transform in the sense of complexity of the equipment and speed.

From the point of view of adaptation to changes in characteristics of the medium it is also desirable that the Walsh spectral intensities be replaced by their empirical evaluations. The random values of the sequential representation of the correlated random process, with their natural ordering, in contrast to a harmonic representation, no longer are asymptotically independent relative to N and therefore a theoretical analysis of the algorithm is difficult, as a result of which it is still not completed. The preliminary results of an experimental investigation, with simple replacement of the fast Fourier transform by the fast Walsh transform in the algorithm, indicated that detection on the basis of H statistics when using the fast Walsh transform is somewhat poorer than when using the fast Fourier transform. The detection of the signal z(t) is dependent both on its intensity and on the Doppler frequency shift.

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REVIEW OF ADAPTATION METHODS IN STATISTICAL HYDROACOUSTIC PROBLEMS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 298-319

[Article by G. N. Belonozhko and V. V. Ol'shevskiy]

[Text] Part I. Antenna Arrays. #1. Principal Reasons for Use of Adaptive Procedures

Adaptation theory is a branch of mathematical statistics and the theory of checking of statistical hypotheses. The idea of an adaptive approach to the synthesis of optimum systems was initially developed in automatic control theory, and even then was used in solution of other problems related to data processing. The development of adaptation theory within the framework of statistical hydroacoustics is in the initial stage and is a promising direction in increasing the effectiveness of hydroacoustic systems of different kinds.

In the opinion of specialists there are several factors which lead to the need for using adaptive (or adapting) systems.

The first, and indeed, the principal factor is related to the need for operation of the system under conditions of uncertainty of external effects on the system and inadequacy of a priori information. In such cases we cannot employ the well-developed classical approach of mathematical statistics because this requires a complete a priori knowledge of the statistical characteristics of input situations. The adaptive approach is used for overcoming this a priori difficulty. Such an opinion is shared by specialists on theory and adaptation both in the field of control systems [1, 2] and in the field of radar and the theory of communications [3-5]. In particular, it is noted in [2]: "There is a need for using a teaching system in those cases when the system must operate under uncertainty conditions and the available a priori information is so limited that there is no possibility of advanced planning of a system with fixed properties which would operate quite well."

The second factor determining the necessity for using adaptive systems is situations when the statistical characteristics of input processes change with the course of time or in space. The law of their change is random.

This factor, in particular, is noted in [6]. Examining an adaptive antenna array, which is recommended for use in hydroacoustics, the author of this article writes that "...the advantage of the described adaptive method is manifested in tuning out from slowly changing noise fields."

The third factor is the natural simplicity of solution of problems in selecting variants in the use of adaptive procedures. A number of authors note that even in those cases when a priori difficulty can be overcome by means of preliminary additional measurements it is useful to employ adaptive methods for the processing of information for simplification of computations. For example, in [6-8] use is made of synthesis of optimum filters for the processing of signals of antenna arrays. It has been demonstrated that for multielement antenna arrays the computation of the weighting coefficients of optimum filters is impossible using modern computer systems on a real time scale since this involves too great a number of operations and memory units. The use of an iterative adaptive procedure considerably reduces the quantity of computations and the necessary memory volume due to an increase in computation time, which in the opinion of the author of [6] does not introduce serious difficulties.

The fourth reason for the use of adaptive procedures must be considered the need for creating universal systems which would operate quite well with different models of input situations. We find a mention of this factor in [3]: "...an important problem in adaptation theory...is the reliability problem, the 'sensitivity' of the results, that is, the problem of nondependence...on the change in a priori distributions, in other words, on the change in operating conditions of an optimum synthesizable system." For solution of the problem of "insensitivity" of the system to a definite class of models of input situations use is also made of nonparametric systems, but the problem of optimization of the system characteristics for each class of models is not raised.

Thus, we have the following basic reasons for use of the adaptive approach:
-- inadequacy of a priori information;
-- change in the statistical characteristics of input situations in time and in space;
-- simplification of mathematical computations in the synthesis of complex systems;
-- striving to create systems with universal characteristics relative to different models of situations.

#2. Adaptation and Variability of Models of Hydroacoustic Conditions

The adaptation concept was taken from biology and medicine, where it characterizes the capacity of a biological object to adapt to environmental changes.

In the theory of technical systems "adaptation" means a change in the structure and characteristics of the system in the process of functioning for the purpose of improving the quality index with an initial uncertainty or changing operating conditions.

In adaptation, for the purpose of overcoming a priori uncertainty, use is made of "teaching." Therefore, in an adaptive system there is always a measuring channel whose purpose is to obtain current information which is then used for "teaching" the system.

Now we will examine the points of view of different authors on adaptation and teaching.

The author of [3] notes that adaptation theory is characterized by the following specific aspects: allowance for characteristics associated with a priori uncertainty; use of stochastic iteration methods; replacement of a priori characteristics with empirical evaluations.

Source [4] gives a definition of the following three criteria which are characteristic of an adaptive communication system: the system must control the quality of characteristic functioning; it must study the operating conditions; the system must change its structure or characteristic if changing conditions lead to a decrease in the quality index. Now we will discuss some peculiarities of the adaptation problem in statistical hydroacoustics. As mentioned above, the principal studies of adaptation theory belong to the field of automatic control systems.

However, whereas in this field the principal adaptation problem is the overcoming of a priori uncertainty [1, 2], in statistical hydroacoustics it is statistical models of input situations which change in time and space which stand at the forefront. In this sense the adaptation problem in statistical hydroacoustics is close to adaptation problems in the theory of communications and radar [3, 4].

However, the formulation of the adaptation problem in hydroacoustics has its peculiarities in comparison with the radar case.

The principal difference is that the time for solution of the sonar problem is considerably greater than the time for solving the radar problem due to the substantially different velocities of signal propagation. If the time of computer operation is not taken into account, it can be assumed that the difference in the time for solving the corresponding problems attains four orders of magnitude. In addition, the change in the characteristics of the medium in both cases occurs at an identical rate. In actuality, atmospheric cyclones and underwater currents, winds and waves at the sea surface, incoming and outgoing tides, movement of atmospheric layers and water masses have one and the same mean period of variability, since in the last analysis they are dependent on common physical factors -- rotation of the earth, motions of the sun and moon. Thus, with respect to the time of solution of the problem the statistical characteristics of the input processes in sonar change considerably more rapidly than in radar. These differences with respect to time are three-four orders of magnitude.

As a result of the mentioned specific factors, there can be a sound formulation of the problem of adaptive optimization of systems specially for the case of hydroacoustics.

When carrying out measurements at sea it must be taken into account that models of hydroacoustic conditions have a significant spatial-temporal variability. As a result of this variability, the statistical characteristics of signals, noise and conditions for underwater observation cannot be predicted in advance.

Now we will examine the principal factors determining the noted variability of hydroacoustic conditions.

Sea noise is the result of superposing of noise of different genesis: noise of a wave-covered water surface, acoustic signals of biological origin, seismic noise, noise from shipping, etc. Each of these types of noise is described by statistical characteristics specific for it, which at the same time change with time. In addition, under different conditions the individual components in the resultant noise are represented differently. As a result the spatial-temporal variability of sea noise attains an energy of tens of decibels and its statistical characteristics also change greatly.

Sea reverberation is caused by the scattering of acoustic signals on inhomogeneities in the water medium and on its boundaries. Depending on the state of the water surface, bottom material, propagation conditions and a number of other factors the reverberation levels change by several tens of decibels. In addition, a whole series of its statistical characteristics are dependent on the movement of scatterers and also acoustic antennas and are also subject to considerable variability.

The radiated and reflected signals, the latter from underwater objects, are described by characteristics highly dependent both on the types of these objects and their movement and on the conditions for the propagation of acoustic waves. It is found that the signal levels can vary by tens of decibels and the statistical characteristics considerably change their form in dependence on different conditions.

Thus, in statistical hydroacoustics we deal with signals, noise and conditions of underwater observation which are described by dynamic stochastic models. These models are considerably diversified.

Precisely this determines the need for using adaptive methods for optimizing hydroacoustic systems.

Now we will give a definition of an adaptive hydroacoustic system.

An adaptive hydroacoustic system is one which, first, has a channel for measuring the changing characteristics of signals, noise and conditions of underwater observation; second, it monitors the index of quality of its functioning; third, it changes the characteristics or structure for the purpose of optimization when using information arriving in the measurement channel.

The following principal directions can be defined in investigation of adaptive methods in hydroacoustic systems:
-- synthesis of adaptive hydroacoustic antennas;
-- adaptive optimization of the emitted signals (active sonar, communication);
-- synthesis of adaptive devices for the secondary processing of hydroacoustic information;
-- adaptive optimization of characteristics of the measurement channel.

Among the presently known publications related to investigation of the adaptation problem in hydroacoustics, most have been devoted to the synthesis of adaptive antennas [6-15]. In a whole series of articles [16-20] the authors examine problems related to the adaptive optimization of measurements. Recently studies have appeared which are devoted to the adaptive processing of hydroacoustic information [21, 22].

In the first part of the review we will examine studies devoted to investigation of the adaptive optimization of hydroacoustic receiving antennas.

#3. Algorithms for Adaptation of Hydroacoustic Antenna Receiving Arrays

Now we will examine studies which describe different adaptive algorithms for optimizing the spatial-temporal processing of hydroacoustic information using antenna arrays. In most of these studies the authors examine the problem of detection and discrimination of a signal with a known bearing in the noise field, with unknown spatial-temporal statistical characteristics.

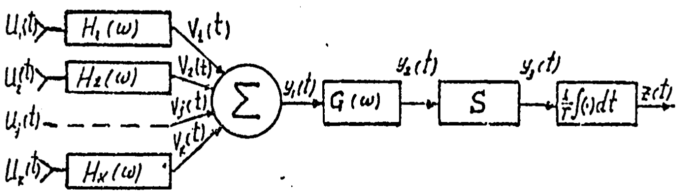

Fig. 1. Structural diagram of antenna filter.

The antenna is represented in the form of a spatial-temporal filter whose structural diagram is shown in Fig. 1.

This system contains k acoustic detectors, frequency filters $\{H_j(\omega)\}$ at the output of each detector, a summator Σ, a post-summation filter, a squarer S and an averaging filter $\frac{1}{T}\int_T(\cdot)dt$ at the output.

Each filter $H_j(\omega)$, in accordance with [7, 10], is designed in the form of a delay line with lead-offs and a device with weighted summation (see Fig. 2).

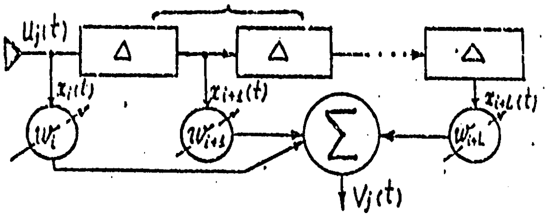

Fig. 2. Filter in the form of a delay line with lead-offs and weighted summation

KEY:

1. delay elements

The delay time between the lead-offs, as well as the weighted (amplitude) coefficients $\{w_2\}$ are selected in such a way that as a result there is modeling of a frequency filter $H_j(\omega)$ with a stipulated amplitude-phase characteristic. In the case of wide-band input processes $u_j(t)$ the number of delay elements n is selected sufficiently large that there is coverage of all the components in the stipulated frequency range with the corresponding phase shifts. In the case of narrow-band processes $u_j(t)$ it is sufficient to have only one delay line. The Δ value is selected in such a way that there will be a phase shift between the input process and the process at the delay line output.

Naturally, such a design of the filter makes it possible to change its characteristic $H_j(\omega)$ in a broad range by means of choice (regulation) of the weighting coefficients $\{w_\ell\}$.

Next we will discuss the operation of a spatial-temporal filter with respect to its adaptive optimization.

An important factor is the determination of the stochastic characteristics of models of the signals and noise acting on the antenna array.

The process arriving at the filter input constitutes an additive mixture of one or more spatially localized sources, isotropic noise and one or more sources of local noise. Both the signals and noise are described by normal random processes with zero mean values.

The direction of arrival of a signal, if it is present, is assumed to be a priori precisely known. In a number of studies [6, 7, 9, 10] the statistical characteristics of the signal are assumed to be fully known (the correlation function or the energy spectrum are stipulated), and in a number of studies [8, 11, 12, 13] they are known partially (only the frequency band is stipulated). The signal level is assumed to be unknown.

The statistical characteristics of the noise field are considered to be a priori unknown. Isotropic noise can be present or may be absent. It is unknown whether there are sources of local noise and how many there are; the direction to these sources is also unknown. The useful signal and the total noise are assumed to be statistically independent. The wave fronts of the signal and local noise are considered plane in the length of the receiving array.

Under conditions of such great a priori uncertainty with respect to models of signal and noise, the authors of [6-13] propose the use of an adaptive method for the spatial-temporal processing of hydroacoustic information.

In constructing the system use is made of measurements or evaluations of the characteristics of the input and output signals. The system is optimized by means of an iterative procedure of "trimming" the weighting coefficients $\{w_\xi\}$ in the filters $\{H_j(\omega)\}$; the scheme for one of these is shown in Fig. 2, on the basis of the Robins-Monroe stochastic approximation method. In the course of the iterative procedure for "trimming" the coefficients the system improves its characteristics and tends to optimum from the point of view of the selected criterion.

Now we will examine a spatial-temporal filter formed by acoustic detectors. To each of these detectors is joined a branching delay line containing L lead-off lines, that is, there are L - 1 ideal delay links for the time Δ each. The signal in the lead-off is multiplied by the variable weighting coefficient w_i, which can assume positive or negative values. The signal across the processing system output is formed by the summation of all the delayed and weighted signals.

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Such a system contains KL variable weighting coefficients (w_1, w_2,..., w_{KL}), the set of which can be represented in the form of a KL-dimensional vector,

$$\vec{W} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_i \\ \vdots \\ w_{KL} \end{bmatrix} \tag{1}$$

The signal in the i-th "branch" at the moment in time corresponding to the k-th adaptation cycle is equal to $x_i(k)$ and represents the sum of signal and noise

$$x_i(k) = s_i(k) + n_i(k),$$

where $n_i(k)$ is a result of the effect of all types of interference, and $s_i(k)$ is the result of passage of the useful signal through the array

$${}_i(k) = F_i[g(k)],$$

where $F_i[\cdot]$ is some linear function and g(k) is the useful signal received at the origin of space coordinates by non-noise nondirectional lattice elements.

The signal $y_1(k)$ at the output of the processing unit at the time corresponding to the k-th adaptation cycle (see Figures 1 and 2) is equal to

$$y_i(k) = \sum_{i=1}^{KL} x_i(k) w_i(k). \tag{2}$$

For the purposes of compactness of the recording of formulas we will turn to vector notations. We will denote the vectors, representing the total process, the useful signal and interference in all the "branches" of the antenna array, by the symbols $\vec{X}$, $\vec{S}$ and $\vec{N}$ respectively.

$$\vec{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_{KL} \end{bmatrix}; \quad \vec{S} = \begin{bmatrix} s_1 \\ \vdots \\ s_i \\ \vdots \\ s_{KL} \end{bmatrix}; \quad \vec{N} = \begin{bmatrix} n_1 \\ \vdots \\ n_i \\ \vdots \\ n_{KL} \end{bmatrix}$$

We will use the symbol $\vec{X}(k)$ to denote the set of signals in all "branches" of the array at the time corresponding to the k-th adaptation cycle,

$$\vec{X}(k) = \vec{S}(k) + \vec{N}(k).$$

The expression for the signal $y_1(k)$ at the output of the processing unit in vector form assumes the form

$$y_1(k) = \vec{X}^T(k) \cdot \vec{W}(k) = \vec{W}^T(k) \cdot \vec{X}(k), \qquad (3)$$

where the superscript T placed upward to the right of the symbol denoting the matrix indicates that the matrix is transposed.

In actuality, $\vec{X}(k)$ is a vector-column, that is, a matrix of the order of (KL x 1); $\vec{X}^T(k)$ represents a matrix of the order of (1 x KL), that is, a vector-row; $\vec{W}$ is a vector column of the order (KL x 1).

As is well known, the product of the two matrices $\vec{A}$ and $\vec{B}$ of the orders (n x p) and (p x m) respectively could form a matrix $\vec{C}$ of the order of (n x m). If the $\vec{A}$ matrix consists of the elements α_{ij}

$$\vec{A} = \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1p} \\ \vdots & & \vdots \\ \alpha_{n1} & \cdots & \alpha_{np} \end{pmatrix} = (\alpha_{ij});$$

and the matrix $\vec{B}$ consists of the elements β_{ij}

$$\vec{B} = \begin{pmatrix} \beta_{11} & \cdots & \beta_{1m} \\ \vdots & & \vdots \\ \beta_{p1} & \cdots & \beta_{pm} \end{pmatrix} = (\beta_{ij});$$

then the $\vec{C}$ matrix consists of the elements γ_{ij}

$$\vec{C} = \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1m} \\ \vdots & & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nm} \end{pmatrix} = (\gamma_{ij}),$$

such that

$$\gamma_{ij} = \sum_{l=1}^{p} \alpha_{il} \beta_{lj},$$

In our case

$$\vec{A} = \vec{X}^T(k), \quad \vec{B} = \vec{W}(k), \; n=1, \; p=KL$$

and m = 1. Accordingly, the matrix is $\vec{C} = \vec{X}^T(k)\vec{W}(k) = \gamma_{11}$

that is, consists of one element. In this case

$$\gamma_{11} = \sum_{l=1}^{KL} \alpha_{1l} \beta_{l1},$$

that is

$$y_1(k) = \vec{X}^T(k)\,\vec{W}(k) = \sum_{i=1}^{KL} x_i(k)\, w_i(k).$$

Thus, the writing in the form of the sum (2) and the vector writing (3) of the process at the output of the processing unit are identical. Henceforth everywhere we will use vector writing as the most compact and it is possible to describe all the processes transpiring in all "branches" of the space-frequency filter.

It follows from formulation of the problem that

$$\langle \vec{S}(k)\rangle = \langle \vec{N}(k)\rangle = 0,$$
$$\langle \vec{S}(k)\vec{S}^T(k)\rangle = \vec{R}_{ss},$$
$$\langle \vec{N}(k)\vec{N}^T(k)\rangle = \vec{R}_{NN},$$
$$\langle \vec{S}(k)\vec{N}^T(k)\rangle = 0,$$

where the symbol $\langle\cdot\rangle$ denotes the mathematical expectation, $\vec{R}_{SS}$ is the correlation matrix of the signal, $\vec{R}_{NN}$ is the correlation matrix of noise.

It is easy to see that in the considered case the correlation matrix of the input processes is equal to the sum of the correlation matrices of the signals and noise.

$$\vec{R}_{xx} = \langle \vec{X}(k)\vec{X}^T(k)\rangle = \vec{R}_{ss} + \vec{R}_{NN}.$$

For evaluating the working characteristics of the described processing unit we will use the least mean square error LMSE criterion (test). The error is

$$\varepsilon(k) = g(k) - y_1(k), \tag{4}$$

and the quality measure is determined as

$$\sigma^2 = \langle \varepsilon(k)\rangle \tag{5}$$

The min σ^2 value is attained by the weighting vector W corresponding to the Wiener filter. As demonstrated in [7],

$$opt\,\vec{W} = \vec{R}_{xx}^{-1}\,\vec{P}_g,$$

where $\vec{P}_g$ is the vector of cross-correlation between the vector of the observed signal and $\vec{X}(k)$ and the useful signal g(k).

$$\vec{P}_g = \langle g(k)\vec{X}(k)\rangle = \langle g(k)\vec{S}(k)\rangle.$$

For computing the minimum value of the mean square error min σ^2 we substitute (3) and (4) into (5). We obtain

$$\min\sigma^2 = \langle [g(k) - opt\,y_1(k)]^2\rangle = \langle g^2(k)\rangle - \vec{P}_g^T\,\vec{R}_{xx}^{-1}\,\vec{P}_g.$$

For computing the optimum vector $\vec{W}$ it is necessary to know in advance the correlation matrix $\vec{R}_{xx}$ and the cross-correlation vector $\vec{P}_g$, that is, it is necessary to know the correlation properties of both the signals and noise.

In the studies considered here, as mentioned above, the direction of signal arrival is known, and in a number of studies [6, 7, 9, 10] the power spectrum of the signal from the target is also known, that is, the vector $\vec{P}_g$. However, the correlation matrix of noise $\vec{R}_{NN}$ remains unknown. In order to overcome this difficulty we use different adaptive algorithms, not leading to optimum solutions, but convergent on an optimum solution.

We will write an expression for the mean square error σ^2

$$\sigma^2 = \langle [g(k) - y(k)]^2 \rangle = \langle g^2(k) \rangle - 2\vec{P}_g^T \vec{W} + \vec{W}^T \vec{R}_{xx} \vec{W}. \qquad (6)$$

Equation (6) corresponds to a sole minimum. It can be sought using gradient descent methods.

In particular, the "most rapid descent" method (iteration of the weighting vector is accomplished in the direction of the negative gradient of the surface of errors) gives the algorithm [6]

$$\vec{W}(k+1) = \vec{W}(k) + \mu[\vec{P}_g - \vec{R}_{xx} \vec{W}(k)], \qquad (7)$$

where μ is the interval of the iteration procedure during adaptation.

For computations in accordance with the most rapid descent algorithm it is necessary to know both the correlation matrix $\vec{R}_{XX}$ and the cross-correlation vector $\vec{P}_g$. Due to a lack of knowledge of $\vec{R}_{NN}$ the most rapid descent algorithm cannot be applied to the considered problems.

The second gradient descent method, in which the $\vec{R}_{XX}$ matrix is not required, was proposed by Widrow and Hoff [6]. According to this method, the increments of the weighting vector occur in the direction of the negative gradient of the instantaneous value of the square error $\varepsilon^2(k)$.

$$\vec{W}(k+1) = \vec{W}(k) + \mu[g(k)\vec{X}(k) - \vec{X}(k)\vec{X}^T(k)\vec{W}(k)] = \vec{W}(k) + \mu[g(k) - y(k)]\vec{X}(k). \qquad (8)$$

This algorithm found use in solving problems related to recognition of images, identification of systems and modeling.

Comparison of (7) and (8) shows that the Widrow-Hoff method is obtained from the most rapid descent method by a replacement of the mean values $\vec{R}_{XX}$ and $\vec{P}_g$ by the corresponding instantaneous values.

When making computations by the Widrow-Hoff algorithm the $\vec{R}_{XX}$ matrix is not used. But a new difficulty arises. With each adaptation cycle it is necessary to have a signal from the target g(k). In the problem considered here we do not know this signal and the filter is intended precisely for its evaluation.

Figure 3 illustrates a basic adaptive element.

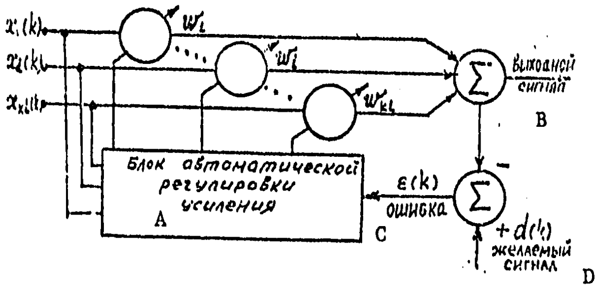

Fig. 3. Adaptive processing unit.

KEY:
- A. AVC unit
- B. Output signal
- C. Error
- D. Desired signal

If this element is connected to an output signal quantizer, we obtain an adaptive linear threshold device ("Adalina").

As has already been demonstrated, for the best reproduction of a useful signal and the suppression of noise in the sense of a minimum of the mean square error it is necessary that the signal d(k) be a real signal

$$d(k) = g(k).$$

However, the signal is unknown. If it was known, a receiving antenna would not be required. The

The authors of [7] gave a method for overcoming this difficulty. A control signal generator is used for this purpose. For the first time the idea of a control signal generator was advanced in [9]. In this method the output signals of the array elements are summed with the signals produced by the generator of control signals. These signals are shaped in such a way that their characteristics are identical to the characteristics which the signal from the target has according to the estimate.

The authors of [7] present two different methods for regulating the variable weighting coefficients of the system processor.

First method. Two-regime adaptation of the weighting coefficients in which the following occur alternately:
-- adaptation only using the signals produced by the control signal generator;
-- adaptation only using signals received by the array elements (real signal).

Second method. Single-regime adaptation, accomplished using the sum of signals of the control signal generator and the real received signal.

The author of [6] proposed an algorithm representing a combination of the most rapid descent and Widrow-Hoff algorithms

$$\vec{W}(k+1) = \vec{W}(k) + \mu[\vec{P}_g - \vec{X}(k)\vec{X}'(k)\vec{W}(k)] = \tag{9}$$
$$= \vec{W}(k) + \mu[\vec{P}_g - y(k)\vec{X}(k)].$$

In this algorithm use is made of a combination of mean and instantaneous values. Its merit is that all its components are either known in advance ($\vec{P}_g$) or are obtained from data by means of direct measurements (y(k) and $\vec{X}(k)$).

In sources [8, 11, 13] the correlation matrix of the signal $\vec{R}_{33}$ and the vector $\vec{P}_g$ are also unknown. The authors of these papers propose a minimum mean square error algorithm with limitations (MMSEA). For using this algorithm it is necessary to know a priori only the direction of arrival of the signal and the frequency band. The author of [8] obtained the following MMSE algorithm

$$\begin{cases} \vec{W}(0) = \vec{F} \\ \vec{W}(k+1) = \vec{P}[\vec{W}(k) - \mu y(k)\vec{X}(k)] + \vec{F}, \end{cases} \tag{10}$$

where $\vec{P}$ is a matrix with the dimensionality (KL x KL), determined from the limitations.

The use of the MMSE algorithm for processing signals from the arrays is limited by the requirement that signals with a stipulated direction and noise from other directions are uncorrelated.

Since limitations are imposed on the weighting coefficients of the array, for application of the algorithm it is not necessary to know a priori the statistical characteristics of the signal and noise. The MMSE algorithm ensures satisfaction of the limitations and prevents the accumulation of quantization errors in a discrete realization.

In the papers considered above the direction of arrival of the useful signal was assumed to be known. In [14] a study was made of the influence of errors in stipulating direction on the characteristics of an adaptive array. In particular, it has been demonstrated that if the signal-to-noise ratio at the output of the antenna array is 100, an error in stipulating direction by 0.04° reduces this ratio by half. But, on the other hand, due to this property the adaptive array can ensure a high resolution and a precise localization of the source in the investigation of surrounding space.

Source [15] describes the results of experiments with the practical realization of adaptive antennas. The authors examine a two-element array with a unit for regulating the weighting coefficients, at whose input a useful signal and spatially localized noise is received. The adaptive antenna forms a diagram zero in the direction of the noise. The "depth" of the zero is dependent on signal and noise intensity and the difference in the angles of their arrival. The authors of the article introduce the concept of system adaptivity, by which they mean improvement in the signal-to-noise ratio (SNR) in the adaptive system in comparison with a rigid system "tuned" to a stipulated direction of signal arrival.

Adaptivity = SNR after adaptation/SNR before adaptation.

The experiment demonstrated that even in the case of a two-element antenna, an improvement in the SNR for the greater part of the angles of arrival of noise is 10 db, whereas for some angles it attains 30 db.

In addition to the algorithms described above, some authors propose other simplified algorithms for the purpose of reducing the complexity of the computations and the electronic apparatus.

For example, the authors of [7] propose a so-called relaxation adaptation algorithm. This algorithm is based on relaxation or approximate computation methods which were developed by Sautell and recommended by him for different technical computations [23]. [Translator's note: The published bibliography does not include this item.] In the relaxation algorithm use is made of the same error signal as in the LMSE method. For determining the necessary values of the weighting coefficients we use the evaluation of the mean square error found by squaring and averaging of this error signal in a finite time interval. With the relaxation algorithm at each particular moment in time there is a change only of one weighting coefficient; the regulation is accomplished in a cyclic sequence. However, in the LMSE algorithm there is simultaneous regulation of all the coefficients. The relaxation procedure gives a greater "mismatch" that the LMSE method, but with a fixed mismatch the duration of the adaptation process in the relaxation method is greater than in the LSME method. However, the relaxation method can be easily realized using electronic computers and can considerably reduce the complexity of the apparatus in a number of variants of adaptation schemes.

#4. Convergence of Adaptation Algorithms. Convergence Variants

Now we will examine the problem of convergence of the vector of weighting coefficients to the optimum vector opt $\vec{W}$. For describing the properties of this convergence we will determine the mean weighting vector $\vec{M}_W(k)$, the weighted autocorrelation matrix $\vec{R}_{WW}(k)$ and the weighted covariation matrix $\vec{G}_{WW}(k)$.

It is obvious that all these values are a function of the number of iterations k.

$$\vec{M}_w(k) = \langle \vec{W}(k) \rangle,$$

$$\vec{R}_{ww}(k) = \langle \vec{W}(k)\, \vec{W}^T(k) \rangle$$

$$\vec{C}_{ww}(k) = \langle [\vec{W}(k) - \vec{M}_w(k)][\vec{W}(k) - \vec{M}_w(k)]^T \rangle =$$

$$= \vec{R}_{ww}(k) - \vec{M}_w(k)\, \vec{M}_w^T(k).$$

In source [6] there is a discussion of the problem of convergence of the algorithm (9). It was demonstrated in the study that for any $\varepsilon > 0$ there is such a μ_ε which for any μ satisfying the inequality $0<\mu<\mu_\varepsilon$,

$$\lim_{k\to\infty} \sup(\langle\|\vec{W}_k - \text{opt}\,\vec{W}\|\rangle) < \varepsilon$$

Here the symbol $\|\ldots\|$ denotes the norm or length of the vector. Thus, with a suitable choice μ the weighting vector can be made arbitrarily close to the vector opt $\vec{W}$. However, if it is assumed that the vectors $\vec{X}(k)$, $X(k+1)$, ... obtained by successive measurements are statistically uncorrelated and Gaussian (and this should be correct in practical cases when the time interval between successive applications of the algorithm is great in comparison with the intervals of correlation of the results of measurements), convergence is ensured with less rigorous conditions. Specifically, if the scalar component μ satisfied the inequality

$$0 < \mu < \frac{2}{\lambda_{max}}$$

where λ_{max} is the maximum eigenvalue of the correlation matrix $\vec{R}_{xx}$, then when the number of adaptations tends to infinity the mean weighting vector converges as a sequence to opt $\vec{W}$.

Source [6] describes experiments with modeling on a computer carried out as an illustration of the properties of convergence of the new adaptive algorithm (9). The results of modeling were used in constructing graphs for the mean square error corresponding to the adaptive algorithm (9) and the mean square error corresponding to the mean weighting vector. It can be concluded from the graphs that the characteristic curve for the adaptive weighting vector is very close to the characteristic curve for the mean value of the weighting vector, especially during the time of the initial iterations. When the number of iterations becomes greater, the curve for the mean weighting vector converges to the optimum solution, whereas the curve for the adaptive weighting vector fluctuates relative to the optimum straight line. These fluctuations occur as a result of the nonzero value of the coefficient μ. However, attempts at a decrease in μ were accompanied by a corresponding decrease of convergence.

In contrast to the new adaptive algorithm (9), proposed by Griffiths, the Widrow-Hoff algorithm (8) gives an unbiased value of the vector of the weighting coefficients.

It was demonstrated in [6] that in the case of a two-regime adaptation procedure by means of a generator of control signals

$$\lim_{k\to\infty} \vec{M}_w(k) \approx \left[\frac{\mu_I\, m_I}{\beta^2 \mu_I\, m_I}\, \vec{R}_u + \hat{\vec{R}}_{ss}\right] \hat{\vec{P}}_g ,$$

where $k = m_I + m_{II}$, M_I is the number of adaptations in regime I, m_{II} is the number of adaptations in regime II, μ_I and μ_{II} are constants determining the magnitude of the interval in regimes I and II respectively, β is the amplitude of the oscillations at the output of the generator of control signals, $\hat{\vec{R}}_{ss}$ is the correlation matrix of the control signal, $\hat{\vec{P}}_g$ is the vector of cross correlation between the vector of the control signal $\hat{\vec{S}}(k)$ and the control signal $\hat{g}(k)$ received at the origin of coordinates of a non-noise array element invariant relative to direction

$$\hat{\vec{R}}_{ss} = \langle \hat{\vec{S}}(k)\, \hat{\vec{S}}^{T}(k) \rangle, \quad \hat{\vec{P}}_{g} = \langle \hat{g}(k)\, \hat{\vec{S}}(k) \rangle .$$

In the case of a single-regime adaptation procedure

$$\lim_{k \to \infty} \vec{M}_{w}(k) = \left[\frac{\vec{R}_{uu}}{\rho^{2}} + \hat{\vec{R}}_{ss} \right]^{-1} \hat{\vec{P}}_{g} .$$

The mean vector $\vec{M}_W$, obtained as a result of use of both single- and two-regime adaptation procedures with a control signal, in a general case will not be equal to the weighted vector, corresponding to the criterion of a minimum of the mean square error opt $\vec{W}$. This is correct even in those cases when the control signal ideally reproduces the real signal from the target $\hat{g}(k) = g(k)$. A bias of the solution arises because the input signals of the processor usually contain not only a control signal, but also the real signal from the target. The bias can be eliminated either by carrying out the adaptation process in the course of intervals when there is no real signal from the target or by means of use of a control signal whose amplitude is small in comparison with the amplitude of the signals actually received by the array.

The author of [8] examined the conditions for convergence of the algorithm (10) for the vector of the coefficients in the case of an unknown correlation structure of the signal. As demonstrated in the study, the evaluation (10) is unbiased. The rate of convergence and the dispersion of the evaluation, as before, is determined by the choice of the coefficient μ.

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Summary

This paper constitutes a review of methods for the adaptation of hydroacoustic systems, for the most part, hydroacoustic receiving antenna arrays.

The article gives an analysis of algorithms obtained by different authors for the adaptive optimization of antenna systems intended for the detection and discrimination of a useful signal and the suppression of noise signals under conditions of a priori ambiguity or changing input effects.

The considered systems consist of linear, plane or spatial antenna arrays and a processing unit designed in the form of a set of delay lines and lead-offs, with weighted summation and subsequent averaging. The adaptive algorithms are used for iterative "adjustment" of the weighting coefficients in accordance with the criterion of minimum of the mean square error. Such an antenna system can be used in frequency and direction filtering. In the course of the work it changes the directional diagram proper, the frequency characteristics or other parameters by means of an internal feedback and thereby increases the response to the useful signal and reduces response to noise signals, asymptotically tending to an optimum variant.

In most of the analyzed studies it is postulated that the sources of the useful signal are localized in space and can be regarded as point sources. The noise is caused either by point sources in the medium surrounding the array elements or by the thermal noise of the amplifiers. The useful signals and the signals from the local noise sources are propagated in space as uniform plane waves. The space in which the antenna system functions is linear; the influence of space on the signals is reduced only to their time delay.

Most of the cited adaptive algorithms were developed for a case when the direction to the source of the signal and its spectral characteristics or frequency bands are assumed to be known a priori. However, the direction to the noise sources, the amount of noise, its spectral and correlation characteristics may not be known, the noise can change its position and characteristics, can appear and disappear in the course of reception. Such a formulation is characteristic for problems in underwater communications. Available a priori information on the useful signal makes it possible for the antenna system to form its diagram in such a way that its main lobe is directed toward the signal source and makes it possible to determine the width of the main lobe and the frequency characteristic of the filter. With arrival of signals from directions other than the stipulated direction, the system classifies these signals as noise signals and forms diagram zeroes in the direction of their arrival.

However, in the most extensive and important class of cases of practical interest the direction to the source of the useful signal not only is not known, but frequently must be determined. In this case the problem

arises of using adaptation for seeking a signal and determining its bearing.

The problem is to detect a signal in the field of distributed and local noise. Neither the direction of arrival of the signals nor the direction of arrival of the noise are known. It is of interest to examine a class of noise broader in comparison with point sources, with more complex spatial-temporal characteristics, for example, sea noise, reverberation noise and other types of noise characteristic for hydroacoustic applications. It is necessary to synthesize such a spatial-frequency filter which on the basis of use of characteristic criteria of a useful signal (for example, spectral, correlation and other properties) will ensure reception of this signal and simultaneous suppression of all noise. This problem in part is dealt with in [14, 15]and elsewhere .

Notations

$u_j(t)$ is the signal at the output of the j-th detector in the antenna array, $j = 1, 2, 3, \ldots;$
$H_j(\omega)$ is the filter at the output of the j-th antenna array detector, the array being designed in the form of a delay line with L lead-offs;
$V_j(t)$ is the signal at the filter output $H_j(\omega)$;
Σ is the summator;
$Y_1(t)$ is the signal at the output of the summator Σ;
$G(\omega)$ is the post-summator filter;
$Y_2(t)$ is the signal at the filter output $G(\omega)$;
S is the squarer;
$Y_3(t)$ is the signal at the output of the squarer S;
$\frac{1}{T}\int(\cdot)dt$ is the averager;
$Z(t)$ is the signal at the output of the antenna filter;
Δ is the delay time between filter lead-offs $H_j(\omega)$;
$x_i(t)$ is the signal in the i-th lead-off of the processing unit, $i = 1, 2 \ldots, KL$;
$x_i(k)$ is the signal in the i-th lead-off corresponding to the k-th adaptation cycle;
w_i is a weighting coefficient by which the signal $x_i(t)$ is multiplied;
$n_i(k)$ is the total noise in the i-th lead-off in the k-th adaptation cycle;
$s_i(k)$ is the useful signal in the i-th lead-off in the k-th adaptation cycle;
$\vec{W}$ is the vector of the weighting coefficients w_i;
$\vec{X}(k)$ is the vector of the signals $x_i(k)$;
$\vec{S}(k)$ is the vector of useful signals $s_i(k)$;
$\vec{N}(k)$ is the vector of noise $n_i(k)$;
$g(k)$ is the signal from the target perceived at the origin of space coordinates by a non-noise array element invariant relative to direction;
R_{SS} is the correlation matrix of the useful signal;
R_{NN} is the correlation matrix of noise;
R_{XX} is the correlation matrix of the process at the input of the processing unit;

$\varepsilon(k)$ is the error in evaluating the signal $g(k)$;
σ^2 is the mean square error in evaluating the signal $g(k)$;
opt $\vec{W}$ is the optimum weighting vector, corresponding to the Wiener filter;
$\vec{P}_g$ is the vector of cross-correlation between the vector of the observed signal $\vec{X}(k)$ and the signal from the target $g(k)$;
μ is the interval of the iteration procedure during adaptation;
$d(k)$ is the "desired" signal or the signal which must be obtained at the output of the processing unit;
$M_W(k)$ is the mean weighting vector;
$R_{WW}(k)$ is the weighted autocorrelation matrix;
λ_{max} is the maximum eigenvalue of the correlation matrix R_{XX};
m_I, m_{II} is the number of adaptation cycles in regime I and in regime II respectively in the case of a two-regime adaptation procedure;
μ_I, μ_{II} are constants determining the magnitude of the interval in regimes I and II respectively;
ρ is the amplitude of the oscillations at the output of the generator of control signals;
R_{SS} is the correlation matrix of the control signal;
P_g is the vector of cross correlation between the vector of the control signal and the signal from the target.

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ALGORITHMS FOR PROCESSING SONAR INFORMATION UNDER A PRIORI UNCERTAINTY CONDITIONS

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 320-327

[Article by Yu. Ye. Sidorov]

[Text] Under the real conditions of sonar observation the statistical characteristics of the registered processes vary in a broad range and cannot be a priori completely known to the observer. This circumstance makes it necessary to solve the problems involved in the statistical synthesis of algorithms for the processing of sonar information under a priori uncertainty conditions.

The detection algorithms, the algorithms for determining coordinates (primary processing) and trajectory of movement of objects (secondary processing) are based on use of the principles of nonbias [1, 2], invariance [1] and the theory of rank criteria [1, 3] when checking complex statistical hypotheses, which makes it possible to synthesize methods with a structure which is extremely stable relative to real observation conditions.

I. Rule for Detection of Echo Signal

The detection of a signal reflected from an object in the form of a packet of pulses (such a type of signal is very common and is created, for example, with the use of a periodic source of the explosive type) must be accomplished in two stages and in each of these stages there is an independent optimization of the processing rules. The first stage is the stage of binary quantization of the received oscillation, whereas in the second stage there is an accumulation of quantized signals for the purpose of detecting the packet. The rules for each stage are formulated as the rules for checking the hypothesis H_0 of absence of a signal relative to the alternative H_1 of its presence.

The binary quantization rule is based on a comparison of oscillations ("contrast" method) received from n ($n \geqslant 2$) elementary radar range resolution sectors. An "inspection" of the distance is made by the sequential method on the assumption that the target can be situated only in the last, n-th, of the compared range sectors. The rule does not require a priori information on the distribution law for the noise background and uses ranks of initial observations. [By "targets" is meant single or group reflectors, such as sea animals, surface layer scatterers, bottom, etc.] It can be demonstrated that this rule is the uniformly most clearly expressed invariant rank rule in the form:

$$\Phi(R_0) = \begin{cases} 1 \text{ when } R_0 > C, \\ \gamma \text{ when } R_0 = C, \\ 0 \text{ when } R_0 < C, \end{cases} \qquad (1)$$

where R_0 is the rank of the value V, being a reading of the envelope of the oscillations in the n-th range sector in the j-th period of repetition of the pulses, $j = 1, \ldots, h_j$; $\Phi(R_0)$ is the probability of a decision in favor of the H_1 hypothesis. The threshold number C and the probability γ are determined unambiguously with respect to the stipulated probability of a false alarm α_1 from the condition:

$$\alpha_1 = E_0[\Phi(R_0)] \qquad (2)$$

Here the averaging of E_0 is carried out using the distribution of the rank R_0 with H_0, which is equal to 1/n. This indicates that rule (1), (2) has a constant probability of false alarm with any noise intensity and any law of distribution of the noise background.

Computation of the effectiveness of rules (1), (2) with the approximation of noise by a normal law, being typical in the reception of an echo signal against a background of reverberation noise [5, 6], and with "harmonious" fluctuations of pulses in conformity to the Rayleigh and Rice laws, show that the losses in the signal-to-noise ratio caused by a lack of knowledge of the distribution law for the noise background and its intensity, and the use of rank values instead of observed values, with an increase in n decrease and when $n > 10$ for all practical purposes tend to zero.

The rule for the second stage is a randomized invariant rule (RIR) in the form:

$$\Phi(x) = \begin{cases} 1 \text{ when } X > M, \\ \delta \text{ when } X = M, \\ 0 \text{ when } X = M, \end{cases} \qquad (3)$$

where $X = \sum_{j=1}^{h} x_j$,

and x_j assume the values "1" or "0" with the unknown probabilities p_j and $1-p_j$ and are solutions indicating the presence or absence of a signal respectively, used in the first stage. The threshold M and the probability

δ are determined on the basis of the given resultant probability α_2 of a false alarm from the condition

$$\alpha_2 = 1 - \sum_{k=0}^{M} C_h^k \alpha_1^k (1-\alpha_1)^{h-k} + \delta C_h^M \alpha_1^M (1-\alpha_1)^{h-M}, \tag{4}$$

where c_h^k is the number of combinations of h relative to k.

It follows from condition (4) that the numbers M and δ are determined only by the probability α_2 and are not dependent on the unknown probabilities p_j, which in the presence of a target do not remain constant with time as a result of nonstationarity (in a general case) of the noise background and the reflected signal. Accordingly, the rule (3), (4) is not dependent on the a priori unknown noise intensity, on the distribution law for the mixture of signal and noise and operability in the case of a nonstationary state of the observed process.

The effectiveness of the rule (3), (4) is determined by the intensity function, which with identical $p_j = p$ has the form:

$$\beta_2(p) = 1 - \sum_{k=0}^{M} C_h^k p^k (1-p)^{h-k} + \delta C_h^M p^M (1-p)^{h-M}$$

and increases with an increase in the number h of pulses in the packet. Due to the fact that the algorithm for detection of the packet is "two-peaked" and the dependence $\alpha_2 = f(\alpha_1)$ is known, it is possible to construct a family of curves for detection of the packet for different values α_1, α_2, n, h, by using which it is possible to select the becessary work "regime" for a two-stage detector in general. Comparison of such a detector with the Manna-Whitney detector, in which in the second stage use is made of the sum of the weighted rank statistics (which leads to a result close to optimum), and not the adopted decisions themselves, shows that the losses in the signal-to-noise ratio of the synthesized detector in comparison with the Manna-Whitney detector with an increase in the number of accumulation cycles decreases monotonically and when $h \geqslant 30$ do not exceed 1.5 db (in the comparison the noise was assumed to be Gaussian and the signal was assumed to fluctuate in conformity to the Rayleigh law, $\beta_2 = 0.9$, $\alpha_2 = 10^{-2}$). It should be noted that the Manna-Whitney detector is considerably more complex in its circuitry than the synthesized two-stage detector, which moreover can be constructed completely using components from computer technology.

II. Rule for Determining Coordinates of Object [This rule was formulated jointly with L. A. Zhivotovskiy.]

The rule for determining coordinates is based on use of the information spatial-temporal relationships between the target and a group of sensors at the time of adoption of a decision. The essence of the rule is as follows. Assume that the processing of information is accomplished simultaneously from z sensors (in z channels), $z = 1, 2, \ldots$, each of which has its effective range. Decisions concerning the presence of a target are made using data from the output of the two-stage detector of each channel for each range sector. Assume that in each of any two range distances there is one such decision or solution. On the basis of these decisions, knowing the coordinates of the sensors and the source, it is possible to construct two position lines for the target (in this case -- ellipses), at whose intersection several points are formed; a target can be situated at each of these. For each of these "suspicious" points at the next distances it is possible to determine in advance the range sectors in one of which the signal should fall if it was actually reflected from the target. Geometrically this means that all the position lines in the presence of a target should intersect at one of the "suspicious" points, which also will determine its coordinates. If the "suspect" sectors at each of the distances are connected by lines, we obtain "paths" characterizing the coordinates of possible target position. Thus, the problem of determining coordinates is reduced to the choice of a target "path" among a set of "paths." For this purpose we successively carry out a paired comparison of any two "paths" for the purpose of selecting the true path, which in turn is compared with the new path, etc., until all the paths have been "sorted." As a result, only a single path should remain, and this will be considered the true one. Geometrically this means that we have determined the point of intersection of the greatest number of position lines.

The rule for determination of coordinates is the invariant uniformly strongest and has the form:

$$\varphi(X) = \begin{cases} 1 \text{ when } \sum_{i=1}^{z} X_i > c, \\ \mu \text{ when } \sum_{i=1}^{z} X_i = c, \\ 0 \text{ when } \sum_{i=1}^{z} X_i < c, \end{cases} \tag{5}$$

$$\alpha_3 = 1 - \sum_{k=0}^{c} C_z^k (1/2)^z + \mu C_z^c (1/2)^z, \tag{6}$$

where X_i = 1 when $x_{1i} > x_{2i}$; $X_i = 0$ with $x_{1i} < x_{2i}$ and x_{1i} and x_{2i}, assuming the values "1" and "0," are decisions concerning the presence ("1") or absence ("0") of a target in the i-th compared pair of suspicious points on the first and second "paths" respectively, $i = 1,\ldots,z$; α_3 is the probability of error of the first kind. It can be seen from formulas (5), (6) that the rule is not dependent on the unknown probabilities $p_i = p\{x_i - 1\}$ and has a constant probability α_3 for any p_i. The probability of a correct determination of coordinates when $p_i = p$ is

$$\beta_3(p) = 1 - \sum_{k=0}^{c} C_z^k p^k (1-p)^{z-k} + \mu C_z^c p^c (1-p)^{z-c}$$

This probability increases with an increase in the number of sensors z. For determining the dependence of p on the signal-to-noise ratio q at the input of a two-stage detector it is possible to use the formula:

$$p(q) = 0.5[1 - \alpha_2 + \beta_2(q)]$$

Having the p(q) and $\beta_3(p)$ dependences it is easy to form a dependence which is the "through" and final stochastic characteristic for the primary processing of sonar information.

III. Rules for Detection of Trajectory of Object

The first stage in the secondary processing of information is the autointerception of the trajectory of the moving target, in the course of which there is a tie-in of the sounding signal for the readings made in each period to the trajectory to be detected. As a result, true and spurious trajectories are formed, whose selection is accomplished in accordance with the adopted autointerception criterion, at the basis of which we should have the statistical differences between the trajectories of the targets and the "trajectories" of the noise.

The algorithms for detection of the trajectory obtained below are based on use of absence of a correlation between spurious readings obtained in adjacent periods of the sounding signal train [7]. The following premises were adopted in formulating the algorithms.

1. Two sets of random values are observed $\{x_1i;\ y_1i\}$ and $\{x_{2i};\ y_{2i}\}$, $i = 1, \ldots, d = \nu/3$, which are the coordinates of the instantaneous target in two adjacent periods of the sounding signal train, in each of which the measurements of the target position are made i times using i "trios" of sensors (observation model with time quantization).

2. The functional of the continuous distribution law of each of the sets $\{x_{1i};\ y_{1i}\}$ and $\{x_{2i}y_{2i}\}$ is considered unknown.

3. An informative criterion indicating that the readings obtained in two adjacent periods of the train of signals belong to the true trajectory is the presence of correlation of the sets $x_1 = \{x_{1i}\}$ and $x_2 = \{x_{2i}\}$, $y_1 = \{y_{1i}\}$ and $y_2 = \{y_{2i}\}$. Otherwise we use the solution of a false trajectory detection. By virtue of the nondependence of the sets $\{x_{1i};\ x_{2i}\}$ and $\{y_{1i};\ y_{2i}\}$ it is sufficient to obtain a detection algorithm for one of them, for example, for $\{x_{1i};\ x_{2i}\}$, since for the second it will be similar.

With these premises it is possible to synthesize two algorithms for autointerception of the trajectory.

1. The most effective unbiased algorithm, based on transpositions of the form:

$$\Psi(x_1; x_2) = \begin{cases} 1 \text{ with } h(x_1, x_2) > K[T(x_1, x_2)], \\ \varkappa \text{ with } h(x_1, x_2) = K[T(x_1, x_2)], \\ 0 \text{ with } h(x_1, x_2) < K[T(x_1, x_2)], \end{cases} \tag{7}$$

where the threshold is dependent on the rank statistics $T(x_1, x_2) = (x_1, \ldots x_1^{(d)}; x_2^{(d)}), x_1^{(1)}, <\ldots x_1^{(d)}; x_2^{(1)} < \ldots < x_2^{(d)}$, which is the complete and adequate statistics for the unknown distribution of the readings $h(x_1, x_2)$. The probability of error of the first kind α_4 of the criterion (7) is constant for any form of the densities $h(x_1, x_2)$, as can be seen from the condition: $\alpha_4 = [1/(d!)^2](d + x)$, where a is the number of points in the set A formed from the observed point $(x_1, x_2) = (x_{11}, \ldots, x_{1d}; x_{21}, \ldots, x_{2d})$ by all possible transpositions x_{1i} and x_{2i} separately and containing only $(d!)^2$ points at which the hypothesis of detection of a spurious trajectory is refuted. For computing the effectiveness of the criterion (7) the density $h(x_1, x_2)$ can be approximated by a two-dimensional normal distribution with unknown parameters. In this case the rule (7) is the uniformly most effective unbiased transposition rule with a critical region in the form:

$$\sum_{i=1}^{d} x_1^{(i)} x_{2j_i} > K[T(x_1, x_2)] \tag{8}$$

2. The unbiased rank algorithm has the form:

$$Q = \sum_{i=1}^{d} R_i S_i = \sum_{i=1}^{d} i T_i > C \tag{9}$$

where $R_1, \ldots, R_d$ are the ranks $x_{11}, \ldots x_{2d}$ (ordered in increasing values); $S_1, \ldots, S_d$ are the ranks $x_{21}, \ldots, x_{2d}$ (ordered in increasing values), and Ti is the rank of the x_2 value related to the i-th minimum x_1 value. The threshold constant C is determined using the stipulated significance level α_4 from the condition: $\alpha_4 = E[\varphi(T)]$. Here $\varphi(\cdot)$ is the probability of deviation of the hypothesis of detection of a spurious trajectory; the averaging E is carried out using the distribution of statistics Q with that hypothesis which is equal to $1/d!$. It can therefore be seen that the threshold C is not dependent on the distribution of the observed readings.

On the assumption of a normal distribution law for the observed readings (with a priori unknown parameters) on the basis of use of this same criterion of absence of correlation between spurious readings in adjacent periods of repetition it was possible to obtain two algorithms.

1. The uniformly most effective unbiased algorithm has the form:

$$V = \frac{|R|}{\sqrt{(1-R^2)/(d-2)}} > C, \quad \alpha_4 = 2\int_C^{\infty} t_{d-2}(w)\,dw; \tag{10}$$

where R is the sample correlation coefficient, $t_{d-2}(w)$ is the central t-distribution with (d - 2) degrees of freedom. In the case of large d this algorithm serves as a good approximation to the transposition algorithm (8), which is impractical in the case of a considerable volume of the sample.

2. The uniformly most effective invariant rule for trajectory detection is based on use of the correlations between the readings from a target, obtained in P ($P \geqslant 2$) adjacent periods of transmission of the sounding signal.

This rule is a generalization of the rule (10), obtained for the case P = 2, and has the form:

$$W^* = \frac{(d-p)R^2}{(p-1)(1-R^2)} > c, \quad \alpha_\psi = \int_c^\infty F_{p-1,\, d-p}(w^*)\,dw^*, \qquad (11)$$

where R^2 is the square of the sample multiple correlation coefficient; $F_{p-1,\ d-p}(w^*)$ is the central Fisher distribution F with (p-1, d-p) degrees of freedom.

The effectiveness of the resulting rules (8), (9), (10), (11) can be computed easily using, for example, the tables in [8].

Summary

1. In order to carry out statistical synthesis of algorithms for the primary and secondary processing of sonar information under real conditions of underwater observation, characterized by an absence of full a priori information on the statistical characteristics of the input data, it is extremely productive to employ the nonbias and invariance principles and the theory of rank criteria in the checking of complex statistical hypotheses. In contrast to classical synthesis methods, requiring full a priori information and as a rule leading to systems "sensitive" to change in the input conditions, the application of these principles made it possible to synthesize algorithms which are applicable under the unknown and changing observation conditions and having stable stochastic characteristics with any change in external conditions.

2. The extremely stable structure of synthesized algorithms under conditions of a priori uncertainty and their applicability using the element base of discrete microelectronics makes it possible to create on their basis automated digital systems for the processing of sonar information.

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NONPARAMETRIC METHODS FOR THE PROCESSING OF HYDROACOUSTIC INFORMATION

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[Article by N. G. Gatkin, A. Ya. Kalyuzhnyy and L. G. Krasnyy]

[Text] I. Introduction. By the term "nonparametric methods" is meant those which ensure an invariance of some characteristics of the processing system (for example, the probabilities of a false alarm in the detection problem) to the properties of noise. A distinguishing characteristic of these methods is the reduction of the initial data to symbolic or rank information. In this paper we examine methods based on ranks.

We will mention some definitions. Assume that there is a sample $\{x_i\}$ ($i = 1,\ldots,n$). Arranging its elements in an increasing order, we obtain the ordered sample $x^{(1)} < x^{(2)} < \ldots < x^{(n)}$, called a variational series. The sequence number of the element x_i in this series is called its rank R_i. The procedure for computing R_i can be represented in the form

$$R_i = \sum_{j=1}^{n} u(x_i - x_j) + 1, \qquad (1)$$

where

$$u(z) = \begin{cases} 1, & z > 0 \\ 0, & z \leq 0 \end{cases} \qquad (2)$$

It is obvious that $1 \leq R_i \leq n$. The totality of the ranks of all elements in the sample, that is, some transposing of the whole numbers from 1 to n, is called the rank vector $\{R_i\}$. If the elements of the initial sample are independent and identically distributed, it is easy to show that all the records of the rank vector are equiprobable, regardless of the specific form of the sample distribution. Therefore, any statistics based on ranks will ensure a constancy of the false alarm level, independently of the distribution of noise, provided that the above-mentioned conditions are satisfied for the noise readings. However, the detection of a signal is possible because its presence leads to a disruption of these conditions, which is disclosed by rank statistics.

Thus, the nonparametric properties of the rank algorithms are ensured by transformation from the space of observations with an arbitrary distribution function F(x) to the space of the ranks having a known distribution in the absence of a signal. This transformation can be interpreted in the following way [6]. We will write the empirical distribution function of the initial sample

$$F^*(x) = \frac{1}{n}\sum_{j=1}^{n} u(x - x_j) \tag{3}$$

It follows from a comparison of (3) and (1) that $R_i = nF^*(x_i) - 1$. In accordance with the Glivenko theorem, $F^*(x) \xrightarrow{P} F(x)$, and so

$$\frac{1}{n}R_i \xrightarrow{P} F(x_i) \tag{4}$$

It therefore follows that the ranks can be considered as a sound evaluation of a one-dimensional distribution law. The transformation on the right-hand side of (4) for continuous distributions is reversible, and accordingly, does not lead to an information loss. It therefore follows that for each specific distribution it is possible, in principle, to form asymptotically optimum rank statistics. To be sure, if the noise distribution is known, there is no need to shift from observations to ranks and it is better to use the results of classical theory. However, a priori information on the properties of noise as a rule is not entirely reliable. Therefore there is a risk that a parametric device optimum for a definite type of noise can be far from optimum under real conditions and does not ensure a stipulated false alarm level. However, rank devices optimized for any most probable distribution ensure asymptotically optimum results for it and at the same time have an insensitivity (in the sense of the false alarm level) to possible deviations from the adopted model. However, if there is absolutely no a priori information concerning the properties of the noise, ordinary devices cannot be used, at the same time that rank devices fully retain their operability and ensure, as will be demonstrated below, an adequately high noise immunity.

In general, the determination of ranks is dependent on the specific nature of the particular detection problem. In some cases for rank detection it is insufficient to have only a working sample $\{x_i\}$. It is also necessary to have a teaching or reference sample of noise $\{Y_j\}$. In this case the ranking, that is, the evaluation of the one-dimensional distribution law, can be accomplished both solely on the basis of a reference sample and on the basis of a composite sample formed from the reference and working samples. The need for a reference sample arises in a case when the presence of a signal does not lead to a disruption of the uniformity of the distribution of elements of the working sample and the appearance of a dependence between them (for example, the readings of a signal of an identical strength). Then the detection is accomplished by means of a comparison of the properties of the working and reference samples. However, if the readings are for signals not of equal level, a reference sample is not required because the presence of a signal leads to the appearance of a definite rank order in the working sample, dependent on signal shape. However, if the signal shape is unknown, a reference sample is also necessary.

It should be noted that the rank algorithms known in mathematical statistics have nonparametric properties only under the condition that the noise readings are independent and identically distributed in the observation interval. However, if these conditions are disrupted, systems based on ordinary ranks lose their nonparametric properties. In order to overcome these shortcomings it is possible to suggest two approaches. The first is such a change in the ordering procedure as will ensure the necessary invariant properties even with disruption of these conditions. As was indicated above, the ranking in the case of independent samples in essence represented an evaluation of the one-dimensional distribution law. Similarly, ranking for dependent samples should represent, in one form or another, an evaluation of a multidimensional distribution law. For example, it is possible to obtain evaluations on the basis of the following transformation

$$Q_i = F_i [x_i / x_{i-1}, \ldots, x_1] \; (i = 1, \ldots, n) \qquad (5)$$

where $F_i [\cdot]$ is the i-dimensional conditional distribution of the sample. As a result of such a transformation, from a sample $\{x_i\}$ with an arbitrary multidimensional distribution it is possible to obtain a sample $\{Q_i\}$of independent uniformly distributed values. Their sound and unbiased evaluations $\{Q_i^*\}$ should have (at least asymptotically) these same properties. However, such a procedure is exceedingly unwieldy and is scarcely applicable in the case of a large sample.

The second approach is more productive and involves a reasonable combination of ordinary and nonparametric processing methods. On its basis the literature (including in this study) proposes a number of algorithms characterized by simplicity and strong nonparametric properties for a broad class of noise. The essence of these methods is set forth below.

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2. Nonparametric Processing Algorithms

It is possible to distinguish the following nonparametric processing methods.
1. Traditional methods;
-- successive use of procedures known in mathematical statistics in the problem of signal detection.
2. Hybrid or "mixed" methods;
-- the essence of these methods is a preliminary transformation of the input information by usual methods (for example, filtering, detection, etc.) and then the use of nonparametric processing. In addition, nonparametric processing may not be applied to the entire mass of data, but to its individual parts with subsequent accumulation of the results. This makes it possible to reduce the volume of observations.
3. The method of nonparameterization of decision rules proposed by the authors. The processing is carried out primarily by usual methods and nonparametric methods (algorithms) are used in the final stage of adoption of the decision. Using this method it is possible to make any known detector nonparametric.

Now we will examine the application of these methods in specific signal detection problems.

Detectors of Determined and Quasidetermined Signals

Assume that the received signal contains P components $\{S_1(t),\ldots,S_P(t)\}$, which can differ, for example, either with respect to the reception points or the time shift or the central frequencies. The voltage across the input of the detector $\{x_1(t),\ldots, x_P(t)\}$, in addition to the signal, contains the additive noise $\{N_1(t),\ldots, N_P(t)\}$. After evening-out the time delays between the components, the voltage is time-quantized. Thus, the initial data for the processing represent the totality P of the samples $\{x_j(t_i)\}$ $(j = 1,\ldots, P;\ i = 1,\ldots, n)$. In addition to the working samples it is assumed, in a general case, that it is possible to obtain q reference samples of noise $\{Y_k(t_i)\}$ $(k = 1,\ldots,q)$, whose statistical characteristics coincide with the characteristics of noise at the input.

Traditional methods. The general form of the processing algorithm for determined signals is as follows:

$$T_1 = \sum_{j=1}^{P} \sum_{i=1}^{n} s_j(t_i) h\{R[x_j(t_i)]\}, \qquad (6)$$

where $R[x_j(t_i)]$ is the rank of the value $x_j(t_i)$, $h\{\cdot\}$ is some transformation of ranks. With a corresponding choice of this transformation [2] the statistics (6) can have an asymptotic optimality for some specific distribution. The ranks R can be computed by different methods. Multichannel (in particular, spatial-temporal) processing affords additional possibilities for ranking:
Method I. The ranking is accomplished in each channel separately. In this case, in general, to the working sample in each channel it is possible to add the corresponding reference sample. In this method only the

stationarity of the noise is important. The properties of noise in the different channels can be different. For example, this means that with spatial-temporal processing this method ensures invariance to the form of nonuniformity of the noise field, but is sensitive to nonstationarity. This method corresponds to evaluation of the distribution function with time.

Method II. The outputs of the channels are ranked at a fixed moment in time. The reference sample can also participate in the ranking. This method ensures invariance relative to the nonstationary state of noise, but the properties of noise in different channels must be identical. With spatial-temporal processing this method corresponds to evaluation of the distribution function in space.

If the signal contains one or more unknown parameters, it is possible to use all the procedures known in classical detection theory. In this case for each set of parameters it is necessary to carry out processing in accordance with (6) and select channels with the greatest effects. In the case of a narrow-band signal with a random initial phase it is possible to use the well-known idea of quadrature processing or proceed to the signal envelopes.

A merit of traditional processing methods is the possibility of attaining an asymptotically optimum quality of detection. The shortcomings include a substantial complexity of the record and rather limited nonparametric properties, since for stability of the false alarm level there must be a nondependence of the noise readings both in time and between channels.

Hybrid or "mixed" methods. Ranking is a rather time-consuming operation. Thus, in the ranking of n samples the number of required operations is of the order of n^2. The authors of 8 proposed a method for reducing the number of operations. For this purpose the ranked sample is broken down into k groups of equal size and the ranking is carried out within each group. The results of nonparametric processing for each group are then summed. This makes it possible to reduce the number of operations by a factor of k. The breakdown can be accomplished both respect to time and channels. For example, with a breakdown into groups on the basis of time the processing algorithm assumes the form:

$$T_\ell = \sum_{j=1}^{n}\sum_{\ell=1}^{k}\sum_{i=1}^{N/k} s_j[t_{(\ell-1)N/k+i}]\cdot h\{R_\ell[x_j(t_{(\ell-1)N/k+i})]\}, \qquad (7)$$

where R_ℓ is the rank, computed for the ℓ-th group. The loss of algorithm (7) with respect to noise immunity in comparison with the traditional method (6) is already insignificant with a volume of the groups of about 10 samples.

The number of operations can be reduced still further if there is a preliminary reduction of the initial data by usual methods. For example, if the signal components differ only with respect to the time delays, after evening out of the delays the channel voltages can be summed and then single-channel nonparametric processing can be carried out. This not only reduces the number of operations, but also strengthens the nonparametric properties of the algorithm. For example, in the case of spatial separation of channels this ensures invariance to the spatial properties of noise.

Other methods for the preliminary transformation of input information (filtering, rectification, etc.) are also possible. The merit of this group of methods in comparison with the preceding methods is the simplicity of application and stronger invariant properties. But here, as a rule, it is already impossible to attain asymptotically optimum results, since a preliminary reduction of the volume of data leads to information losses.

Methods for nonparameterization of decision rules. Almost all the processing is accomplished by usual methods and its results are subjected to nonparametric processing. Two such processing methods are possible: parallel and sequential.

Method I. In each channel there is processing of the signal using any known algorithm (for example, by matched filtering or in a standard receiving channel). By the end of the observation interval random values, such as $\lambda_1, \ldots, \lambda_p$ are formed at the channel outputs. The output effects of the noise reference channels $\eta_1, \ldots, \eta_q$ obtained during this same observation interval must be added to them. The composite sample formed in this way is ranked and the following statistics are formed

$$T_j = \sum_{j=1}^{p} a_j g\{R[\lambda_j]\}, \qquad (8)$$

which is then compared with the threshold and a decision is made concerning the presence or absence of a signal. Here $R[\lambda_j]$ is the rank of the random value λ_j in the composite sample, $g\{\cdot\}$ is some transformation of the ranks, a_j are regressive constants, determined by the expected level of the useful effect in the corresponding channel. For registry of the false alarm level it is only important to have the random values $\{\lambda_j\}$ and $\{\eta_k\}$ nondependent and identically distributed. This means, for example, that when using spatial channels it is important to have uniformity of the noise field in space and a nondependence between the channels; the temporal properties of the noise do not play a role. However, if time channels are used (that is, pulse packets), the stationarity of noise and the nondependence of its values after an interval equal to the pulse repetition rate are known; the spatial properties of the noise are of no importance. In addition, if the packet modulation law is known and is not rectangular, a reference sample is not mandatory.

Method II. The decision interval is broken down into two segments. First there are several detection teaching cycles without a signal and at the output a reference sample $\{\eta_k\}$ is formed, which is stored. Then several working cycles take place and a working sample $\{\lambda_j\}$ is formed at the output.

Then there is nonparametric processing similar to (8). Since the output effects of different detection cycles are usually independent, for practical purposes for stabilizing the false alarm there is a need only for the stationarity of the noise in the decision interval. The remaining properties of the noise can be completely variable. This method is extremely similar to an adaptive detector. Its advantages are: 1) there is assurance of strong invariant properties, and 2) any known detector can be made nonparametric by connecting to its output a device computing the statistics (8). Naturally, for this it is necessary to sacrifice the quality of detection, but as indicated by the results cited below the losses are small.

We note in conclusion that this same group of methods is invariant to signal shape; the specifics of the signal must be taken into account by the preceding processing.

Detectors of Random Signals

It is postulated that the reception is accomplished at P points in space. After evening-out of the time delays between the spatial channels there is a discretization of the voltage with time. Thus, the initial data are represented, as above, by the totality P of samples $\{x_j(t_i)\}$ $(j = 1,\ldots, P;\ i = 1_1,\ldots,n)$. In the detection of determined signals the information criteria for the presence of a signal was either the difference between the working and reference samples or the appearance of a definite rank order in the sample. In this case the information criterion of signal presence can also be the difference between the working and the reference samples, and in addition, the appearance of a statistical correlation between the channels caused by the presence of one and the same random signal in them. In the latter case detection can also be carried out without reference samples.

The algorithm for spatial-temporal processing in a general case has the form:

$$T_4 = \sum_{i=1}^{n} \{ \sum_{j=1}^{P} f\{R[x_j(t_i)]\} \}^2 \qquad (9)$$

The ranks, the same as above, can be determined by two methods, that is, ranking either in time or in space, both with and without a reference sample. If a reference sample is used, we find both the difference between the working and reference samples and the appearance of the statistical correlation between the working samples. In the absence of a reference sample only the appearance of a statistical correlation between working samples is detected.

Hybrid methods and nonparameterization methods in this case are similar to the idea considered above.

3. Stability of Nonparametric Processing Methods

As indicated by the results in the preceding section, absolute invariance to the properties of interference cannot be achieved in all cases. It was postulated above that the elements of a ranked sample must be independent and identically distributed. However, if these conditions are not satisfied the rank detectors lose their nonparametric properties.

The influence of the dependence of sample elements was investigated, for example, in [4, 9-11]. However, the influence of nonuniformity of the distribution of sample elements to all intents and purposes has not been investigated. In most studies the stability of the detectors to deviation from the adopted interference model has been investigated on the basis of change in noise immunity. We will study stability from the point of view of stability of the false alarm level.

An exhaustive characteristic of the stabilizing properties of the detector is the dependence of the actual false alarm level F on some set of destabilizing parameters $\vec{\alpha}$ with a stipulated false alarm level F_0. This characteristic is an analog of the detection characteristic and it can be called the nonparametric characteristic. In particular, for normal statistics

$$F = \varphi\left\{\frac{\mu(\vec{\alpha})-\mu_0}{\sigma(\vec{\alpha})} - \frac{\sigma_0}{\sigma(\vec{\alpha})}\varphi^{-1}(1-F_0)\right\} \qquad (10)$$

where $\varphi\{\cdot\}$ is the Laplace integral, μ_0 and σ_0^2 are the mathematical expectation and the dispersion of statistics corresponding to the proposed interference model, $\mu(\vec{\alpha})$ and $\sigma^2(\vec{\alpha})$ are the same parameters corresponding to its real properties.

However, computation of this characteristic in many cases meets with substantial mathematical difficulties, and in addition, such a complete characteristic is not always necessary. Sometimes it is sufficient to characterize the relative improvement or worsening of the stabilizing properties of one detector relative to the other. As such an index it is possible to use the value

$$\varepsilon = \lim \frac{F_1(\vec{\alpha},F_0)-F_0}{F_2(\vec{\alpha},F_0)-F_0}, \qquad (11)$$

where θ is a zero vector which we call the relative nonparametric effectiveness of the detector 2 in comparison with the detector 1. If $\varepsilon>1$, then the detector 2 ensures a better stabilization of F than the detector I. II has an especially simple form for normal statistics under the condition that $\vec{\alpha} = \alpha$ and $\mu(\alpha) = \mu_0$. Substituting (10) into (11) and expanding the uncertainty, we obtain

$$\varepsilon = \left.\frac{\frac{1}{\sigma_{01}}\cdot\frac{d^\nu\sigma_1(\alpha)}{d\alpha^\nu}}{\frac{1}{\sigma_{02}}\cdot\frac{d^\nu\sigma_2(\alpha)}{d\alpha^\nu}}\right|_{\alpha=0}. \qquad (12)$$

where ν is the least order of magnitude of the derivative, different from zero. Below we investigate the stability of statistics similar to the widely known Manna-Whitney statistics, but modified by us somewhat so that it will be suitable for the detection of a sign-variable signal as well:

$$W = \sum_{i=1}^{n} S_i \left(\frac{1}{m} \sum_{k=1}^{m} u(x_i - y_k) - \frac{1}{2} \right) \tag{13}$$

The stabilizing properties of these statistics will be compared with the properties of the correlation detector

$$\lambda = \sum_{i=1}^{n} S_i \cdot x_i \tag{14}$$

The statistics (13) and (14) are asymptotically normal under sufficiently broad conditions. Therefore ε can be computed using formula (12).

Influence of Dependence of Samples

It is not possible to compute the dispersion directly for the statistics (13) with dependent samples. Therefore, we will consider its asymptotically ($m \to \infty$) equivalent statistics

$$W_\infty = \sum_{i=1}^{n} S_i F_0(x_i), \tag{15}$$

where $F_0(\cdot)$ is real interference. In a Gaussian case we obtain

$$\sigma^2_{W_\infty} = \frac{1}{12} \sum_{i=1}^{n} S_i^2 + \frac{1}{2\pi} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \sum S_i S_j \arcsin \frac{\alpha|\rho_{ij}|}{2} \tag{16}$$

where ρ_{ij} are the elements of the sample correlation matrix. The first term in (16) is the dispersion of statistics in the case of independent samples, the second is the "increment" to dispersion, caused by the dependence. As we see, in this case the dispersion is dependent on the type of correlation function of interference, and accordingly, the statistics (15) lose their nonparametric properties. We will compute ε for these statistics relative to the statistics (14). For the latter under these same conditions we have

$$\sigma_\lambda^2 = \sum_{i=1}^{n} S_i^2 + \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \sum \alpha \rho_{ij} S_i S_j \tag{17}$$

Substituting (16) and (17) into (12) we find that $\varepsilon = \pi/3$ regardless of the type of correlation function.

Thus, the instability of a false alarm in rank statistics (15) was approximately the same (with a "weak" dependence) as for an ordinary parametric situation. This indicates that the dependence of the samples can represent a serious problem in creating nonparametric detectors.

Influence of Nonuniformity of Distribution of Sample Elements

Assume that $F_i(x)$, $w_i(x)$ are the distribution function and probability density of the i-th element of the working sample, $F_{0k}(x)$, $w_{0k}(x)$ are these same characteristics for the k-th element of the reference sample. If the distribution of interference is symmetric relative to the mathematical expectation, for the statistics (13) we have

$$\mu_w = 0$$

$$\sigma_w^2 = \sum_{i=1}^{n} S_i^2 \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{m} \sum_{k=1}^{m} F_{0k}(x) \right]^2 w_i(x)\,dx - \frac{1}{4} \right\} +$$
$$+ \frac{1}{4m} \sum_{i=1}^{n} S_i^2 - \frac{1}{4m} \left(\sum_{i=1}^{n} S_i \right)^2 - \frac{1}{m^2} \sum_{i=1}^{n} \sum_{k=1}^{m} S_i^2 \times$$
$$\times \int_{-\infty}^{\infty} [F_{0k}^2(x) w_i(x) + F_i^2(x) w_{0k}(x)]\,dx + \tag{18}$$
$$+ \frac{1}{m^2} \int_{-\infty}^{\infty} \left[\sum_{i=1}^{n} S_i F_i(x) \right]^2 \sum_{k=1}^{m} w_{0k}(x)\,dx$$

As we see, the dispersion of statistics, and accordingly, the false alarm level, are dependent on the functional form of noise distribution, that is, the invariance is impaired.

For the statistics (14) under these same conditions

$$\mu_\lambda = 0$$
$$\sigma_\lambda^2 = \sum_{i=1}^{n} S_i^2 \int_{-\infty}^{\infty} x^2 w_i(x)\,dx \tag{19}$$

If the nonhomogeneity is manifested only in the scale parameter, that is

$$F_i(x) = F\left(\frac{x}{1 + \alpha \Delta_i} \right), \tag{20}$$

it can be shown that the nonparametric effectiveness of the statistics (13) relative to the statistics (14), regardless of the form of the nonhomogeneity, is equal to (in the case of large n and m):

$$\varepsilon_{w/\lambda} = \left\{ 24 \int_{-\infty}^{\infty} x F_0(x) w_0^2(x)\,dx \right\}^{-1} \tag{21}$$

For a Gaussian distribution this value is equal to 0.907, that is, also, as in the case of a dependence of the samples, the instability of a false alarm of a rank detector is of the same order of magnitude as for an ordinary parametric detector. However, in this case the situation can improve.

Assume that $m/n = \delta$ is a whole number and the reference sample repeats by δ times the working sample in its statistical properties, that is

$$F_{0i}(x) = F_{0[i+(l-1)n]}(x) = F_i(x) \quad (i=1,\dots,n;\ l=1,\dots,\delta) \tag{22}$$

In (13) we assume that $S_i = 1$ (this is possible in the case of a unipolar signal). Then (18) assumes the form

$$\sigma_W^2 = \frac{n(\delta+1)+1}{12\delta} - \frac{1}{\delta n^2}\sum_{i=1}^{n}\sum_{k=1}^{n}\left\{\int_{-\infty}^{\infty}[F_k^2(x)w_i(x)+F_i^2(x)w_k(x)]dx - \frac{2}{3}\right\} \qquad (23)$$

The first term is the dispersion of Manna-Whitney statistics in the case of a uniform sample, the second is the increment to dispersion caused by the nonhomogeneity. When $n \to \infty$ this increment decreases in comparison with the first term. Thus, Manna-Whitney statistics has an asymptotic nonparametric character even in the case of a nonuniform sample. It is also easy to apply this result to other linear rank statistics.

However, in the case of a sign-variable signal $S_i \neq 1$ the rank statistics do not have this property. However, in this case it is possible to propose a somewhat modified ordering procedure (which we will call the weighted ranking procedure), due to which in this case an asymptotic nonparametricity is attained. Applicable to the statistics (13) it is necessary to introduce the following changes:

$$W_s = \sum_{i=1}^{n} S_i\left\{\frac{\sum_{k=1}^{m} S_k^2\, u(x_i - y_k)}{\sum_{k=1}^{m} S_i^2} - \frac{1}{2}\right\} \qquad (24)$$

(For the constants S_k there should be satisfaction of an expression similar to (22)). In this case with the adopted assumptions we have:

$$\mu_{W_s} = 0$$

$$\begin{aligned}\sigma_{W_s}^2 = {} & \frac{E_s}{12} + \frac{3}{4\delta E_s}\sum_{k=1}^{n} S_k^4 - \frac{1}{4\delta E_s^2}\Big(\sum_{i=1}^{n} S_i\Big)^2\Big(\sum_{k=1}^{n} S_k^4\Big) - \\ & - \frac{1}{\delta E_s^2}\sum_{i=1}^{n}\sum_{k=1}^{n} S_i^2 S_k^2\int_{-\infty}^{\infty}[F_k^2(x)w_i(x) + F_i^2(x)w_k(x)]dx + \\ & + \frac{1}{\delta E_s^2}\sum_{k=1}^{n} S_k^4\int_{-\infty}^{\infty}\Big[\sum_{i=1}^{n} S_i F_i(x)\Big]^2 w_k(x)dx,\end{aligned} \qquad (25)$$

where $E_s = \sum_{i=1}^{n} S_i^2$.

It follows from (25) that when $\delta \to \infty$ the dispersion is not dependent on the distribution of interference and tends to the value $E_s/12$. If the mean signal value is equal to zero and the number of readings in the period is great, in (25) it is possible to discard the third and last terms. Then asymptotic nonparametricity is achieved also with finite δ and $n \to \infty$.

We will apply this result to arbitrary linear rank statistics

$$T^* = \sum_{i=1}^{n} S_i h\left\{U_{Ni} - \frac{1}{2}\right\}, \qquad (26)$$

where V_{Ni} is the "weighted" rank of an i-th element of the working sample, equal to

$$V_{Ni} = \frac{\sum_{k=1}^{N} S_k^2 u(x_i - x_k)}{\sum_{k=1}^{N} S_k^2}, \tag{27}$$

$\{x_i\}$ is an aggregate sample (i = 1,...,N), the first n elements of which are working elements and the next N - n = m elements are reference elements. First of all, we note that the statistics (26) are asymptotically equivalent to the statistics

$$T = \sum_{i=1}^{n} S_i h\left\{\frac{\sum_{k=1}^{N} S_k^2 F_k(x)}{\sum_{k=1}^{N} S_k^2} - \frac{1}{2}\right\}, \tag{28}$$

and therefore it is sufficient to investigate the properties of the statistics, since they asymptotically coincide with the properties of T^*. For the statistics T it is possible to demonstrate the following result. If 1) m/n = δ is a whole number and the condition (22) is correct; 2) the distribution of interference is symmetric relative to the mathematical expectation, which is identical for all samples; 3) the function h $\{\cdot\}$ is odd; 4)

$$\frac{\sum_{k=1}^{n} S_k^2}{\max\limits_{1 \le k \le n} S_k^2} \xrightarrow[n \to \infty]{} \infty,$$

then the statistics (and accordingly also the statistics T^*) are asymptotically normal with parameters not dependent on the type of distribution of interference.

In actuality, it is easy to confirm that by virtue of assumptions 2 and 3

$$\mu_T = 0 \qquad \sigma_T^2 = \sum_{i=1}^{n} S_i^2 \int_{-\infty}^{\infty} h^2\left\{\frac{\sum_{k=1}^{N} S_k^2 F_k(x)}{\sum_{k=1}^{N} S_k^2} - \frac{1}{2}\right\} W_i(x)\,dx \tag{25}$$

[Translator's note: There is a duplication of numbers of formulas.]

In accordance with assumption 1 we can write

$$\sigma_T^2 = \sum_{i=1}^{n} S_i^2 \int_{-\infty}^{\infty} h^2\left\{\frac{\sum_{k=1}^{n} S_k^2 F_k(x)}{\sum_{k=1}^{n} S_k^2} - \frac{1}{2}\right\} W_i(x)\,dx$$

Putting the sum sign beneath the integral sign and making the replacement

$$\left(\sum_{k=1}^{n} S_k^2\right)^{-1}\left\{\sum_{k=1}^{n} S_k^2 F_k(x)\right\} = y + \frac{1}{2}$$

we obtain

$$\sigma_T^2 = 2E_1 \int_0^a h^2(y)\,dy \qquad (26)$$

Thus, the dispersion and mathematical expectation T are not dependent on the distribution of interference even in the case of a nonuniform sample. The proof of asymptotic normality of T involves checking of the Lindberg condition and does not cause substantial difficulties. We will not cite it due to limitations on space.

Thus, as a result of our investigation of the influence of nonhomogeneity the following conclusions can be drawn. In the case of a unipolar signal (that is, with post-detector processing) the usual linear rank statistics have asymptotic nonparametricity; this, in particular, is correct for all systems constructed by the method of nonparameterization of the decision rules. In the case of a sign-variable signal (that is, with predetector processing) the stabilizing properties of ordinary rank detectors deteriorate substantially and become comparable with the properties of ordinary detectors. In order to improve the stabilizing properties in this case it is possible to recommend use of the procedure of "weighted" ranking.

4. Noise Immunity of Nonparametric Processing Methods

Usually it is not possible to compute the detection characteristics for rank detectors. Therefore, in the statistical literature a generally accepted quality criterion [1] for nonparametric algorithms is the asymptotic relative effectiveness (ARE) of the investigated detector relative to some standard detector. For asymptotically normal statistics

$$\text{ARE} = \lim_{\substack{q \to 0 \\ n \to \infty}} \frac{\Delta\mu^2(q)/\sigma^2}{\Delta\mu_0^2(q)/\sigma_0^2} \qquad (27)$$

where $\Delta\mu(q)$, $\Delta\mu_0(q)$ is the increment of the mathematical expectation of the investigated and standard statistics, σ^2 and σ_0^2 are the dispersions of statistics, q is the signal-to-noise ratio at the detector input, a is the volume of the sample.

In the case of independent readings of stationary and uniform noise the asymptotically optimum statistics for detection of a determined signal has the form

$$\lambda_0 = \sum_{j=1}^{\ell} \sum_{i=1}^{n} s(t_i)\varphi[x_j(t_i)], \qquad (28)$$

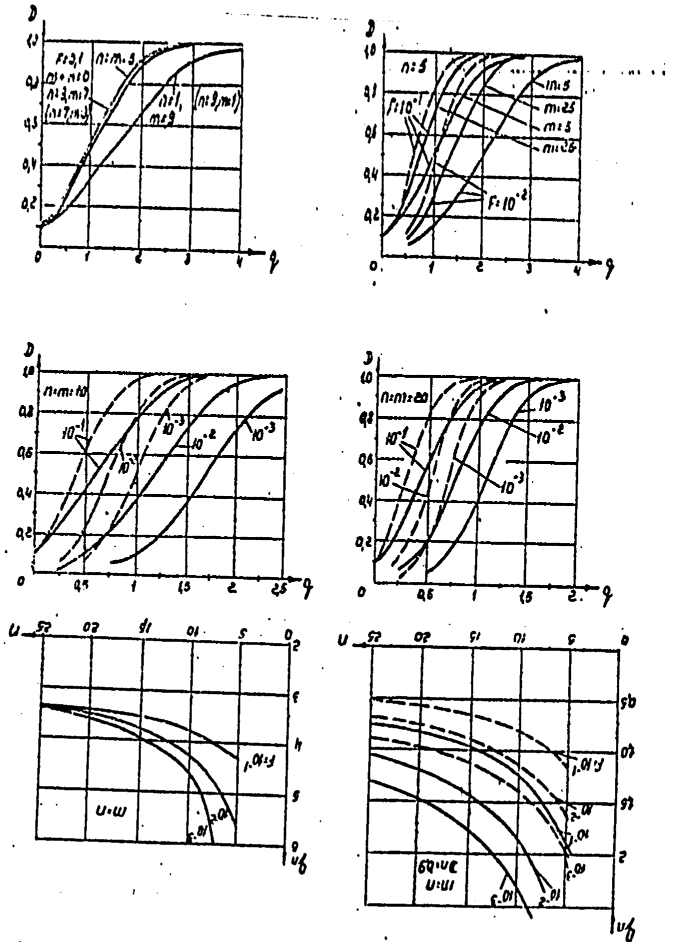
where

$$\varphi(x) = -\frac{\left[\frac{\partial}{\partial s} w(x/s)\right]_{s=0}}{w(x/0)}, \qquad (29)$$

w(x/s), w(x/0) is the probability density of the observed data in the presence and absence of a signal.

The ARE of rank statistics (6) relative to (28) is equal to

$$\mathrm{ARE}_{T/R_1}=\frac{\left[\int_0^1 h(u)\varphi[F^{-1}(u)]du\right]^2}{\left\{\int_0^1 h^2(u)du-\left(\int_0^1 h(u)du\right)^2\right\}\left\{\int_{-\infty}^{\infty}\varphi^2(x)w(x/0)dx\right\}} \qquad (30)$$



If h(u) = [FF^{-1}(u)], then the statistics (6) becomes asymptotically optimum and ARE = 1. But even with a nonoptimum sample h the rank statistics have a high effectiveness. For example, if h(u) = u (Wilcoxson statistics), ARE = 0.95 in a Gaussian case. Moreover [12], it has been demonstrated that in this case for the most favorable distribution ARE> 0.864. The ARE of rank detectors can also be considerably greater than 1 if the distribution of interference differs from that adopted in optimization. For example, the so-called "normal marks criterion" or the Van der Varden method, equivalent to it, corresponding to $h(u) = \varphi^{-1}(u)$, where $\varphi^{-1}(u)$ is a function inverse of the Laplace integral, have ARE $\geqslant 1$ for any symmetric distribution and the minimum ARE value, equal to unity, is attained for a normal density [12].

However, the results cited above characterize the properties of rank statistics only in the case of large sizes of samples and weak signals. In many cases, for example, when using mixed methods or parameterization methods, the sizes of the sample can be small. In these cases the only acceptable method for investigating noise immunity is the Monte Carlo method. Using this method, in [5] the author computed the characteristics of detection and the threshold signal-to-noise ratios for Kendall and Spearman statistics with Gaussian interference.

We carried out similar computations for Wilcoxson two-sample statistics. The results of the computations are given in Figures 1-6. For computing one point of the detection characteristic curve an average of 500 repetitions were carried out. The threshold values were taken from the tables [7]. Using the results of these computations, it is possible to obtain the detection characteristics for any algorithm obtained by the nonparameterization method if we know the relationship between the signal-to-noise ratio at the processing input and output preceding the nonparametric determination. The root of the signal-to-noise ratio has been plotted along the x-axis on the cited detection characteristic curves (Figures 1-4). On these same graphs the dashed curve represents the detection characteristics for optimum processing, which in this case involves a simple averaging of the initial data.

The results of the computations have been summarized in Fig. 5, where we have shown the dependence of the threshold signal-to-noise ratio corresponding to a probability of correct detection 0.9 on the sample volume with probabilities of a false alarm 10^{-1}, 10^{-2} and 10^{-3}. On the next graph (Fig. 6) we have shown the losses in the threshold signal-to-noise ratio for investigated processing in comparison with the optimum situation. As we see, the losses decrease rapidly with an increase in n and already with $n = 20$ are close to asymptotic, which constitute a little more than 3 regardless of the false alarm level.

In some cases the total volume of the working and reference sample can be limited. For example, when using nonparameterization algorithms the observation interval is fixed. Part of this interval can be used for obtaining reference samples; the remainder -- for obtaining working samples. Figure 1 illustrates the influence of the relationship between the magnitude of the working and reference samples. The highest noise immunity is attained under the condition that the volumes of the working and reference samples are identical.

5. Summary

Our investigations show that nonparametric processing methods are a promising means for overcoming a priori uncertainty. They make it possible to achieve an insensitivity (in the sense of the false alarm level stability) to a change in the properties of noise for most cases of practical importance. In some cases, however, when this cannot be achieved (due to the influence, for example, of nonhomogeneity), the use of these methods is nevertheless more preferable than ordinary parametric methods due to their lesser sensitivity to noise properties. It is characteristic that in a number of situations such invariant properties of these detectors, valuable for practical purposes, are achieved at the price of a small loss in noise immunity. In addition, one of the merits of these methods is the possibility of their relatively simple realization by use of modern digital computers.

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POSSIBILITIES OF APPLICATION OF METHODS OF NONPARAMETRIC STATISTICS IN SONAR

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 346-347

[Article by F. P. Tarasenko]

[Text] One of the principal problems in sonar is the synthesis of instrument complexes for the processing of the received signal for the detection of reflecting objects, discrimination (classification) of targets, determination of the necessary parameters of their motion (coordinates, direction and velocity, etc.). Since modern technology of production of instrumentation and computers makes it possible to apply virtually any algorithm for the processing of signals, the central problem in statistical sonar is becoming the synthesis of processing procedures satisfying definite requirements on the probabilities of errors and the degree of accuracy of the adopted statistical solutions. The abundant experience gained in statistical radar demonstrates the great value of the methods for statistical synthesis of receivers and gives rise to hopes for similar success in sonar. However, the specific results of statistical radar are actually useless for sonar. The reason for this is the exceedingly important difference in the physical conditions under which radar and sonar are used. The most important of these are: the difference between the speeds of light and sound; the presence of reverberation; degree of saturation of the medium by natural reflecting and scattering objects; complex acoustic inhomogeneity of the medium; rather rapid change in the parameters of the medium; the higher level of noise and interference and their nonstationarity. As a result, a model of a received signal used successfully in radar is inadequate for sonar.

The development of theoretical models of sonar signals (V. V. Ol'shevskiy, D. Middleton, and others) made it possible to reduce the complex structure of signals to a specific form, but precisely this complexity does not make it possible (in any case -- for many situations of practical importance) to obtain analytical expressions for the distributions of probabilities necessary for proceeding to a statistical synthesis of procedures. On the other hand, experimental investigations of real signals also reveal an exceedingly complex picture: even a simple change in the aspect of the target leads to unrecognizable changes in signal statistics.

The direct approach to statistical synthesis is a continuation of attempts to obtain, theoretically or experimentally, the necessary distributions. In this respect it is extremely promising to use the modeling programs proposed by V. V. Ol'shevskiy and D. Middleton [1, 2].

However, there is also a circuitous approach, involving departure from a too detailed study of the statistics of sonar signals and then direct advance to the synthesis of procedures not requiring a knowledge of the functional type of distributions. This approach is assumed to be nonparametric statistical. It is characteristic for this branch of statistics that it requires an extremely small amount of a priori information: it is sufficient to know only some general differences between noise and a mixture of signal and noise of the type "a distribution shift appears," "its scale parameter changes," "the distribution form changes," "the symmetry of the distribution is impaired," "a dependence between the signal readings appears," etc. This information already makes it possible to formulate a successfully operating procedure for the detection of the signal and evaluation of some parameters. Despite the paucity of a priori information, nonparametric procedures are insignificantly less effective than parametric methods in the case of correctness of the parametric information and considerably surpass the latter in those cases when the true distribution differs from the postulated distribution. However, it should not be surmised that parametric statistics already has ready answers to all problems in hydroacoustics. Even in the simplest problem of signal detection in noise much will be dependent on what is actually known about the differences between the distributions of noise and its mixture with the useful signal. The creation of adaptive or self-teaching classification algorithms is also not a simple problem. Finally, the specific nature of sonar undoubtedly requires the formulation of procedures taking into account the nonstationary nature of the sample and this branch of statistics in general and nonparametric statistics in particular for the time being is still extremely poorly developed.

Despite the mentioned difficulties, hydroacoustics is an exceedingly very promising field of application of nonparametric statistics.

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ALGORITHMS FOR SIGNAL RECOGNITION WITH INCOMPLETE A PRIORI INFORMATION

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 348-350

[Article by V. P. Vagin and V. D. Petukhov]

[Text] In a real situation the developer of an information system frequently does not have all the necessary a priori information in order to make use of the results of the classical theory of statistical derivations. It is therefore not surprising that the creation of methods for removal of a priori uncertainty became the most timely direction in development of the theory of statistical synthesis of information systems. One of the possibilities of overcoming these difficulties is the use of nonparametric methods, that is, methods invariant to the functional form of the noise distribution law and the method of its interaction with the signal.

The fields of applicability of the above-mentioned methods are the problems: dichotomy -- division of the entire alphabet of classes of objects into two contradictory types of concept, of which one negates the other; taxonomy, when a set of space vectors must be divided in a definite way using decision rules, etc. [1].

This sort of problem arises quite frequently in statistical measurements in hydroacoustics [2, 3, 4].

The problem of signal recognition is represented in the form of the problem of checking the statistical hypothesis $[F_m(x), F_n(x)] \in F_0$ against the general alternatives $[F_m(x), F_n(x)] \in F \setminus F_0$. We will determine the "input effect" as the random process $\{X(t); 0 \leqslant t \leqslant T\}$ or as a finite sequence of random values $\{x_i; i = 1, 2, \ldots, n\}$. Two assumptions are possible: 1) X represents "pure noise" or 2) X is a "mixture of signal and noise"; it is necessary to obtain a solution on the basis of a finite sequence $(x_1, x_2, \ldots, x_n)$ of independent observations of the input process. For solution of the above-mentioned problems it is possible to use nonlinear rank statistics. In this respect the Kolmogorov-Smirnov criterion is of the greatest interest, being well-founded in comparison with the general alternatives [5, 6].

$$D_{mn} = \sup |F_m(x) - F_n(x)|$$

where $F_m(x)$ and $F_n(x)$ are empirical coefficients of samples of the dimensionality m and n respectively (without loss of generality $n \geqslant m$). The Hodges method (1957) is used for determining $P(D_{mn} < d)$. This involves computations using recurrent formulas derived by Massey in [7] with more general boundary conditions, that is, $m \neq n$ [8]. The peculiarities of use of the algorithm were presented in points A, B, C.

A. The algorithm gives a precise computation of $P(D_{mn} < d)$ with satisfaction of any of the three restrictions on the sample:

1) $n \leqslant 50$, $m \leqslant [0,1n]$ for $m/n \leqslant [0,1n]$;
2) $n \leqslant 100$, $m \geqslant [0,1n]$ for $n = km$, $k = 1, 2, 3, \ldots$;
3) $n \leqslant 50$, $m \geqslant [0.1n]$ for $n \neq km$, $k = 1, 2, 3, \ldots$;

B. With the restrictions:

1) $100 > n \geqslant 50$, $m \leqslant [0.1n]$ when $m/n \leqslant [0.1n]$;
2) $n \geqslant 100$, $m < 80$ the D_{mn} distribution uses the Kolmogorov approximation with the Miller correction (1956) (on the D_{mn} scale)

$$S = \sqrt{m} \left(D_{mn} - \frac{1}{2n} \right)$$

C. With the restrictions:
1) $n \geqslant 100$, $m \geqslant 80$;
2) $100 \leqslant n \leqslant 50$, $m \geqslant [0.1n]$ with $n \neq km$, $k = 1, 2, 3 \ldots$ the D_{mn} distribution uses the Smirnov approximation. The observed S value is corrected by the correction $(1/2\sqrt{n})$ (on the S scale).

$$S = \sqrt{\frac{mn}{m+n}} D_{mn} + \frac{1}{2\sqrt{n}}$$

The algorithm was programmed in FORTRAN language for a BESM-6 computer.

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OPTIMUM DETECTION AND DIFFERENTIATION OF SIGNALS UNDER CONDITIONS OF RESTRICTED A PRIORI INFORMATION

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 351-360

[Article by L. G. Krasnyy]

[Text] Introduction. In classical detection theory the optimization of processing algorithms is possible only with the availability of full a priori information on signals and interference. As is well known, the optimum (in the sense of mean risk) processing algorithm is a functional of the probability ratio

$$\lambda_i(u) = \frac{\mu(du/H_i)}{\mu(du/H_0)}, \qquad (1)$$

where $\mu(\cdot/H_i)$ and $\mu(\cdot/H_0)$ are measures corresponding to the hypothesis H_i (presence of the i-th signal) and the zero hypothesis H_0 (absence of signals), and the solution γ_K in favor of the K-th hypothesis is obtained in accordance with the rule:

$$\gamma_k : \{\lambda_k(u) > \lambda_j(u),\ k \neq j\} \qquad (2)$$

However, in most cases information on signals and interference has a limited character. In this case computation of the probability ratio is impossible and accordingly algorithm (1) cannot be realized.

In this connection it is of interest to formulate the problem of optimization in a form differing from the classical formulation: it is necessary to synthesize a processing algorithm $F_0(u)$ which is optimum (using the selected criterion) with a stipulated level of a priori information on the signal and interference and asymptotically convergent on $\lambda(u)$ with an increase in a priori data.

The pioneering work in this direction is the work of N. G. Gatkin [1]. There, applicable to the signal detection problem, he proposed the Δ-criterion ("signal-to-noise" maximum criterion), making it possible to synthesize detection algorithms without recourse to computation of $\lambda(u)$. Our paper develops the ideas proposed in [1]. Its purpose is the selection of a quality criterion satisfying the enumerated conditions and the optimizing of processing functionals on its basis.

ε-Criterion for Optimizing Processing Algorithms

We will examine the problem of differentiating "n" signals $s_1(t,\vec{x}),\ldots,s_n(t,\vec{x})$ in the noise field with an arbitrary distribution law. Assume that in the presence of an i-th signal the adopted realization $u(t,\vec{x})$ has the form:

$$u(t,\vec{x})=f_i[s_i(t,\vec{x});N(t,\vec{x})]+n(t,\vec{x}), \tag{3}$$

where $f_i(\cdot)$ is a function determining the method of interaction of the i-th signal and noise $N(t,\vec{x})$, $n(i,\vec{x})$ -- spatial-temporal white noise.

We introduce the random vector $\vec{\theta}$, assuming the value $\vec{\theta}_i$ in the case of the hypothesis H_i. If it is assumed that the i-th coordinate $\vec{\theta}_i$ is equal to unity, and the others -- to zero, then in place of (3) we have

$$u(t,\vec{x})=(\vec{\theta},\vec{f}(t,\vec{x}))+n(t,\vec{x}), \tag{4}$$

where $(\cdot,\cdot)$ is the symbol for the scalar product,

$$\vec{f}(t,\vec{x}):\{f_1(\cdot),\ldots,f_n(\cdot)\}.$$

In such a formulation, the problem of differentiating signals can be reformulated into the problem of measuring the random vector $\vec{\theta}$. As an evaluation $\vec{\theta}^*$ of the $\vec{\theta}$ vector we will use a Bayes evaluation with a quadratic loss function. Then the optimum evaluation is the vector-functional $\vec{F}_0(u)$, minimizing the measurement error

$$\varepsilon(\vec{F})=E\{\|\vec{F}(u)-\vec{\theta}\|^2\}, \tag{5}$$

where $E\{\cdot\}$ is the mathematical expectation symbol, $\|\cdot\|$ is the norm in the Hilbert space of the functionals $\vec{F}(u)$.

We will show that with complete a prior information on signals and interference, optimization using the criterion (5) (we call it the ε-criterion) leads to the same results as the traditional mean risk criterion.

Assume that the appearance of each of the signals is equiprobable. Then (5) can be represented in the form:

$$\varepsilon(\vec{F})=\frac{1}{n}\sum_{i,j=1}^{n}\int[F_j(u)-\theta_{ji}]^2\lambda_i(u)\mu(du/H_0), \tag{6}$$

where $F_j(u)$ and θ_{ji} respectively are the j-th components of $\vec{F}(u)$ and $\vec{\theta}_i$.

Varying this expression for $F_K(u)$, we obtain

$$F_{0K}(u)=\frac{\sum_{i=1}^{n}\theta_{Ki}\lambda_i(u)}{\sum_{i=1}^{n}\lambda_i(u)}$$

or since $\theta_{KI} = \delta_{Ki}$ (δ_{Ki} is the Kronecker symbol)

$$F_{o\kappa}(u) = \frac{\lambda_\kappa(u)}{\sum_{i=1}^{m}\lambda_i(u)} \tag{7}$$

The solution γ_K of the presence of the K-th signal is found using the rule:

$$\gamma_\kappa : \{F_{o\kappa}(u) > F_{oj}(u),\ \kappa \neq j\},$$

which, with (7) taken into account, coincides with (2).

In a two-alternative case, for example, with the detection of the signal $s(t,\vec{x})$, it follows from (7) that

$$F_o(u) = \frac{\lambda(u)}{1+\lambda(u)}, \tag{8}$$

that is, the optimum processing functional is a monotonic function of the probability ratio

$$\lambda(u) = \frac{\mu(du/s \neq 0)}{\mu(du/s = 0)} \tag{9}$$

Thus, with full a priori information on signals and interference, the $\mathcal{E}$-criterion is equivalent to the mean risk minimum criterion.

$\mathcal{E}$ - Optimum Processing Functionals

Now we will examine the synthesis of $\mathcal{E}$-optimum processing functionals under conditions of restricted a priori information.

Assume that we know only the first 2p moments of the adopted realization $u(t, \vec{x})$ in the presence and absence of signals. We will seek the $\mathcal{E}$-optimum processing algorithm in the class $\mathcal{F}_p$ of p-linear functionals in the form:

$$F_j(u) = \sum_{\kappa=1}^{p} \int_{\Omega,T}\cdots\int u(t_1,\vec{x}_1)\cdots u(t_\kappa,\vec{x}_\kappa)\varphi_{j\kappa}(t_1,\ldots,t_\kappa;\vec{x}_1,\ldots,\vec{x}_\kappa)\,dt_1\cdots d\vec{x}_\kappa \tag{10}$$

Substituting (10) into (5), varying (5) with respect to $\varphi_{jK}(\cdot)$ and solving the resulting equation, we obtain the following system of integral equations for determining the kernel $\varphi^0_{jK}(\cdot)$, $\mathcal{E}$ is the optimum functional of (10):

$$\sum_{\ell=1}^{p}\int_{\Omega,T}\cdots\int\int\Big\{\sum_{i=1}^{m} M_{\kappa\ell}^{(i)}(\bar{t}_\kappa,\bar{\tau}_\ell;\bar{x}_\kappa,\bar{y}_\ell)\Big\}\varphi_{j\ell}^{0}(\bar{\tau}_\ell;\bar{y}_\ell)\,d\bar{\tau}_\ell\,d\bar{y}_\ell = M_\kappa^{(j)}(\bar{t}_\kappa;\bar{x}_\kappa), \tag{11}$$

where $\bar{t}_\kappa:(t_1,\ldots,t_\kappa)$, $\bar{x}_\kappa:(\bar{x}_1,\ldots,\bar{x}_\kappa)$; $M_{\kappa\ell}^{(i)}(\cdot)$

are the moments of the $(K + \ell)$-th and K-th orders $u(t, \vec{x})$ in the presence of the i-th signal.

In the special case of detection of a signal the system (11) assumes the form:

$$\sum_{\ell=1}^{p}\int_{\Omega,T}\cdots\int\int\{M_{\kappa\ell}^{sN}(\bar{t}_\kappa,\bar{\tau}_\ell;\bar{x}_\kappa,\bar{y}_\ell)+M_{\kappa\ell}^{N}(\bar{t}_\kappa,\bar{\tau}_\ell;\bar{x}_\kappa,\bar{y}_\ell)\}\varphi_\ell^{0}(\bar{\tau}_\ell;\bar{y}_\ell)\,d\tau_\ell\,dy_\ell = M_\kappa^{sN}(\bar{t}_\kappa;\bar{x}_\kappa), \tag{12}$$

where $M_{K\ell}^{SN}(\cdot)$ and $M_K^{SN}(\cdot)$ are the moments $u(t, \vec{x})$ in the presence of a signal, $M_{K\ell}^{N}(\cdot)$ are the moments $u(t, \vec{x})$ in the absence of a signal.

It follows from expressions (11) and (12) that the use of the $\mathcal{E}$-criterion for synthesis of the processing functionals does not require other information on the investigated processes other than the one at hand.

However, if information on the moments figuring in equations (11)-(12) is a priori absent, there can be a changeover to adaptive processing algorithms in which, in accordance with [2], in place of $M_{K\ell}^{(i)}(\cdot)$ and $M_K^{(i)}$ we should have their $\mathcal{E}$-optimum evaluations obtained in the teaching stage.

$\mathcal{E}$-Optimum Recurrent Processing Functionals

In those cases when the solution of the systems of equations (11)-(12) is difficult, it is possible to formulate an iterative procedure for the synthesis of $\mathcal{E}$-optimum processing functionals. For this we will minimize (5) under the condition that in algorithm (10) only the functions $\varphi_{jp}(\cdot)$ are unknown, whereas $\varphi_{j1}(\cdot),\ldots,\ \varphi_{jp-1}(\cdot)$ are stipulated.

Solving the corresponding variational problem, we obtain

$$\sum_{i=1}^{n}\int\cdots\int_{\Omega,T} M_{pp}^{SN}(\bar{t}_p,\bar{\tau}_p;\bar{x}_p,\bar{y}_p)\varphi_{jp}^{\circ}(\bar{\tau}_p;\bar{y}_p)d\bar{\tau}_p\,d\bar{y}_p = M_p^{(i)}(\bar{t}_p;\bar{x}_p) - \sum_{\kappa=1}^{p-1}\int\cdots\int_{\Omega,T}\Big\{\sum_{i=1}^{n} M_{p\kappa}^{(i)}(\bar{t}_p,\bar{\tau}_\kappa;\bar{x}_p,\bar{y}_\kappa)\Big\}\varphi_{j\kappa}(\bar{\tau}_\kappa;\bar{y}_\kappa)d\bar{\tau}_\kappa\,d\bar{y}_\kappa; \quad (13)$$

Applicable to the detection problem

$$\int\cdots\int_{\Omega,T}\{M_{pp}^{SN}(\bar{t}_p,\bar{\tau}_p;\bar{x}_p,\bar{y}_p)+M_{pp}^{N}(\bar{t}_p,\bar{\tau}_p;\bar{x}_p,\bar{y}_p)\}\varphi_p^{\circ}(\bar{\tau}_p;\bar{y}_p)d\bar{\tau}_p\,d\bar{y}_p = M_p^{SN}(\bar{t}_p;\bar{x}_p) - \sum_{\kappa=1}^{p-1}\int\cdots\int_{\Omega,T}\{M_{p\kappa}^{SN}(\bar{t}_p,\bar{\tau}_\kappa;\bar{x}_p,\bar{y}_\kappa)+M_{p\kappa}^{N}(\bar{t}_p,\bar{\tau}_\kappa;\bar{x}_p,\bar{y}_\kappa)\}\varphi_\kappa(\bar{\tau}_\kappa;\bar{y}_\kappa)d\bar{\tau}_\kappa\,d\bar{y}_\kappa. \quad (14)$$

Accordingly, in the case of an iterative optimization procedure instead of a system of integral equations it is necessary to have a successive solution for only one integral equation. The corresponding $\mathcal{E}$-optimum processing functional is described by the following recurrent algorithm:

$$F_{on}^{(p)}(u) = F_{on}^{(p-1)}(u) + \Delta F_{on}^{(p)}(u), \quad (15)$$

where $$\Delta F_{on}^{(p)}(u) = \int\cdots\int_{\Omega,T} u(t_1,\vec{x}_1)\cdots u(t_p,\vec{x}_p)\varphi_p^{\circ}(t_1,\ldots,t_p;\vec{x}_1,\ldots,\vec{x}_p)dt_1\cdots d\vec{x}_p$$

$\mathcal{E}$-Optimum Detection and Differentiating of Signals as a Problem in Nonlinear Filtering

In the preceding sections it was demonstrated that the use of the $\mathcal{E}$-criterion makes it possible to reformulate the problem of optimum differentiation (detection) of signals in the optimum evaluation (filtering) problem for the random vector $\vec{\theta}$. Such a point of view makes it possible to use a different approach to the synthesis of optimum processing algorithms based on the theory of nonlinear filtering of random processes [3-5] and fields [6]. According to this theory, an optimum evaluation is the conditional mathematical expectation, conforming to some stochastic differential equation. However, this equation is not closed, which considerably reduces its practical significance. The use of the $\mathcal{E}$-criterion eliminates this difficulty. The specific feature of $\mathcal{E}$-optimization is that there is filtering of the components of the θ_K vector, which with equiprobability assume only two values: "1" or "0." Accordingly, all the higher moments θ_K are expressed through the first moment, which makes it possible to close the filtering equation. We will demonstrate this.

We will introduce the field $\tilde{u}(t,\vec{x})=\int_0^t u(\tau,\vec{x})d\tau$,

conforming, in accordance with (4), to a stochastic differential equation:

$$d\tilde{u}(t,\vec{x})=(\vec{\theta},\vec{f}(t,\vec{x}))dt+dw(t,\vec{x}), \tag{16}$$

where

$$w(t,\vec{x})=\int_0^t n(\tau,\vec{x})d\tau$$

is the Wiener field introduced into [7].

Then the $\mathcal{E}$-optimum evaluation θ_K is the conditional mathematical expectation θ_K in the case of observation of $\tilde{u}(\tau,\vec{x})$ in the interval [0,t], that is

$$F_{0K}^t(u)=E\{\theta_K/\tilde{u}(\tau,\vec{x}),\ \tau\in[0,t]\} \tag{17}$$

As demonstrated in [6], $F_{0K}^t(u)$ is a quadratically-integrable martingale for which the following representation is correct

$$F_{0K}^t(u)=F_{0K}^{t=0}(u)+\int_0^t\int_X\psi(\tau,\vec{x})d\nu(\tau,\vec{x})d\vec{x}, \tag{18}$$

where

$$d\nu(t,\vec{x})=d\tilde{u}(t,\vec{x})-h^*(t,\vec{x})dt,$$

$$h^*(t,\vec{x})=E\{h(t,\vec{x})/\tilde{u}(\tau,\vec{x}),\ \tau\in[0,t]\},\quad h(t,\vec{x})=(\vec{\theta},\vec{f}(t,\vec{x})). \tag{19}$$

We will find the kernel $\psi(t,\vec{x})$ in the representation (18). For this we introduce the process $\eta(t)$, allowing, as also in (18), the representation

$$\eta(t)=\int_0^t\int_X\varphi(\tau,\vec{x})d\nu(\tau,\vec{x})d\vec{x} \tag{20}$$

and we will compute the mathematical expectation $E\{\eta(t)\overset{\circ}{\theta}_K\}$ where $\overset{\circ}{\theta}_K = \theta_K - E(\theta_K)$. Since from (16) and (19)

then $d\nu(t,\vec{x})=[h(t,\vec{x})-h^*(t,\vec{x})]dt+dw(t,\vec{x})$.

$$E\{\eta(t)\overset{\circ}{\theta}_K\}=E\{\int_0^t\int_X\theta_K\varphi(\tau,\vec{x})[h(\tau,\vec{x})-h^*(\tau,\vec{x})]d\tau d\vec{x}= \tag{21}$$

$$=E\{\int_0^t\int_X\varphi(\tau,\vec{x})E[\theta_K[h(\tau,\vec{x})-h^*(\tau,\vec{x})]/\tilde{u}(s,\vec{x}),\ s\in[0,\tau]\}d\tau d\vec{x}.$$

Taking into account the equality $E\{d\nu(t,\vec{x})d\nu(\tau,\vec{x})\} = \delta(t-\tau)dt\,d\tau$ expression (21) is easily transformed in the following way:

$$E\{\eta(t)\dot{\theta}_k\} = E\{\eta(t)\int_0^t\int_X [E\{\theta_k[h(\tau,\vec{x}) - h^*(\tau,\vec{x})]/\vec{u}(s,\vec{x}), s\in[0,\tau]\} d\nu(\tau,\vec{x})d\vec{x}\} \tag{22}$$

On the other hand

$$E\{\eta(t)\dot{\theta}_k\} = E\{\eta(t)\cdot E\{\dot{\theta}_k/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\}\} = E\{\eta(t)[F_\omega^t(u) - F_\omega^{t,0}(u)]\}. \tag{23}$$

It follows from a comparison of (22) and (23) that:

$$\Psi(t,\vec{x}) = E\{\theta_k h(t,\vec{x})/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\} - E\{\theta_k/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\}\cdot E\{h(t,\vec{x})/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\}. \tag{24}$$

For computing the conditional mathematical expectations entering into (24) we will use the above-mentioned property of the moments of the random values θ_K:

$$E\{\theta_k^2/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\} = E\{\theta_k/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\}.$$

Hence

$$E\{\theta_k h(t,\vec{x})/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\} = f_k^*(t,\vec{x})F_\omega^t(u) \tag{25}$$

and

$$E\{h(t,\vec{x})/\vec{u}(\tau,\vec{x}), \tau\in[0,t]\} = \sum_{k=1}^n f_k^*(t,\vec{x})F_\omega^t(u) \tag{26}$$

where $f_K^*(t, \vec{x})$ is the mean square evaluation $f_k\{s_k(t,\vec{x}); N(t,\vec{x})\}$, computed with the hypothesis H_K, that is

$$f_k^*(t,\vec{x}) = \int f_k\{s_k(t,\vec{x}); N(t,\vec{x})\}\mu(ds_k, dN/H_k; u(\tau,\vec{x}), \tau\in[0,t]).$$

Substituting (24)-(26) into (18) and taking into account the expression $d\vec{u}(t, \vec{x}) = u(t, \vec{x})dt$, we obtain a closed stochastic differential equation for the ε-optimum processing functional:

$$dF_\omega^t(u) = F_\omega^t(u)\int_X\{f_k^*(t,\vec{x}) - \sum_{k=1}^n f_k^*(t,\vec{x})F_\omega^t(u)\}\cdot\{u(t,\vec{x}) - \sum_{k=1}^n f_k^*(t,\vec{x})F_\omega^t(u)\}d\vec{x}\,dt, \tag{27}$$

with the initial condition

$$F_\omega^{t,0}(u) = \frac{1}{n}.$$

In the case of signal detection expression (27) assumes the form:

$$dF_1^t(u) = F_1^t(u)[1 - F_1^t(u)]\int_X\{[f_1^*(t,\vec{x}) - f_0^*(t,\vec{x})]\}\{u(t,\vec{x}) - f_0^*(t,\vec{x}) - F_1^t(u)[f_1^*(t,\vec{x}) - f_0^*(t,\vec{x})]\}d\vec{x}\,dt;\ F_1^{t,0}(u) = \frac{1}{2}. \tag{28}$$

where $f_1^*(t, \vec{x})$ is the evaluation of $f\{s(t, \vec{x}); N(t, \vec{x})\}$, $f^*_2(t, \vec{x})$ is the evaluation of $f(0; N(t, \vec{x}))$.

In those cases when the problem of differentiating (detecting) signals is regular, from solutions of equations (27)-(28) it is possible to discriminate adequate statistics also being the $\mathcal{E}$-optimum processing algorithms (by analogy with [8] we will call them evaluation-correlation algorithms). For example, the sufficient statistics corresponding to (27) have the form:

$$F_\varepsilon(u) = \int_X\int_T [u(t,\vec{x}) - f_2^*(t,\vec{x})][f_1^*(t,\vec{x}) - f_2^*(t,\vec{x})]\,dt\,d\vec{x} - \frac{1}{2}\int_X\int_T [f_1^*(t,\vec{x}) - f_2^*(t,\vec{x})]^2\,dt\,d\vec{x}. \quad (29)$$

Expression (29) is a generalization of the results of studies [8] and [9], in the case of spatial-temporal processing and arbitrary (non-Markov) signals and interference. It follows from these expressions that the principal element of the $\mathcal{E}$-optimum evaluation-correlation processing functionals is the filtering block

$$f_k\{s_k(t,\vec{x}); N(t,\vec{x})\}.$$

The filtering block, by analogy with (10), can be described by integral polynomials [10]:

$$f_j^*(t,\vec{x}) = \sum_{k=0}^{\infty}\int_{X,T}\cdots\int u(t_1,\vec{x}_1)\cdots u(t_k,\vec{x}_k)\varphi_{jk}(t,t_1,\ldots,t_k;\vec{x},\vec{x}_1,\ldots,\vec{x}_k)\,dt_1\cdots d\vec{x}_k \quad (30)$$

whose optimization is reduced to solution of the following system of integral equations relative to the kernel $\varphi_{jK}(\cdot)$:

$$\sum_{l=0}^{\infty}\int_{X,T}\cdots\int M_{kl}^{(j)}(\bar{t}_k,\bar{\tau}_l;\bar{x}_k,\bar{y}_l)\varphi_{jl}(t,\bar{\tau}_l;\vec{x},\bar{y}_l)\,d\bar{\tau}_l\,d\bar{y}_l = m_k^{(j)}(t,\bar{t}_k;\vec{x},x_k) \quad (31)$$

where

$$m_k^{(j)}(t,\bar{t}_k;\vec{x},\bar{x}_k) = E\{f_j(t,\vec{x})u(t_1,\vec{x}_1)\cdots u(t_k,\vec{x}_k)/H_j\}.$$

$\mathcal{E}$- Optimum Functionals for the Processing of Gaussian Signals and Interference

As an illustration of the results we will examine the problem of detection of determined and random signals in the field of additive Gaussian noise.

Determined signal. In this case

$$f\{s(t,\vec{x}); N(t,\vec{x})\} = s(t,\vec{x}) + N(t,\vec{x})$$

and the $\mathcal{E}$-optimum processing functional (29) assumes the form:

$$F_\varepsilon(u) = \int_X\int_T [u(t,\vec{x}) - N_0^*(t,\vec{x})][s(t,\vec{x}) + N_1^*(t,\vec{x}) - N_0^*(t,\vec{x})]\,dt\,d\vec{x} - \frac{1}{2}\int_X\int_T [s(t,\vec{x}) + N_1^*(t,\vec{x}) - N_0^*(t,\vec{x})]^2\,dt\,d\vec{x}, \quad (32)$$

where $N_1^*(t, \vec{x})$ and $N_0^*(t, \vec{x})$ are the mean square evaluations of interference $N(\vec{x}, t)$, computed in the presence and absence of a signal.

Since $N(t, \vec{x})$ is a Gaussian field, the evaluations $N_0^*(t, \vec{x})$ are linear functionals of $u(t, \vec{x})$ and in accordance with (30)-(31) are equal to:

$$N_\theta^*(t,\vec{x}) = \int\limits_{\Omega}\int^{t} \varphi^*(t,\tau;\vec{x},\vec{y})[u(\tau,\vec{y}) - \theta s(\tau;\vec{y})]d\tau d\vec{y}, \quad \theta = 0,1, \tag{33}$$

where $\varphi^*(t,\tau;\vec{x}\,\vec{y})$ is solution of the integral equation

$$\frac{G_0}{2}\varphi^*(t,t_1;\vec{x},\vec{x}_1) + \int\limits_{\Omega}\int^{t}\varphi^*(t,\tau;\vec{x},\vec{y})K_N(\tau,t_1;\vec{y},\vec{x}_1)d\tau d\vec{y} = K_N(t,t_1;\vec{x},\vec{x}_1)$$

or in operator form we have

$$\Psi^*(G_0/2\, I + K_N) = K_N. \tag{34}$$

Here $K_N(t, t_1; \vec{x}, \vec{x}_1)$ is a spatial-temporal correlation function $N(t, \vec{x})$, G_0 is the spectral density of white noise $n(t, \vec{x})$, I is a unit operator.

From (33) $$N_1^* - N_0^* = -\Psi^* s$$

hence $$s + N_1^* - N_0^* = (I - \Psi^*)s.$$

Substituting this expression into (32), we have

$$F_0(u) = \langle (I-\Psi^*)u, (I-\Psi^*)s\rangle - \frac{1}{2}\langle (I-\Psi^*)s, (I-\Psi^*)s\rangle = \langle u, (I-\Psi^*)^2 s\rangle - \frac{1}{2}\langle s, (I-\Psi^*)^2 s\rangle, \tag{35}$$

where <., .> is the symbol for the scalar product in space $\vec{\Omega}$ x T. Using the "generating process method" [11], it can be demonstrated that

$$(I-\Psi^*)^2 = (G_0/2\, I + K_N)^{-1}. \tag{36}$$

The substitution of (36) into (35) gives

$$F_0(u) = \langle u, \beta\rangle - \frac{1}{2}\langle s, \beta\rangle = \int\limits_{\Omega}\int\limits_{T} u(t,\vec{x})\beta(t,\vec{x})dt\,d\vec{x} - \frac{1}{2}\int\limits_{\Omega}\int\limits_{T} s(t,\vec{x})\beta(t,\vec{x})dt\,d\vec{x}, \tag{37}$$

where $\beta(t, \vec{x})$ is solution of the equation

$$(G_0/2\, I + K_N)\beta = s.$$

The synthesized ε-optimum detection algorithm coincides with the known result, obtained earlier [12] from the probability ratio.

Random signal. In this case, in accordance with (30)-(31):

$$F_0(u) = \langle (u - N_0^*), (s^* + N_1^* - N_0^*)\rangle - \frac{1}{2}\langle (s^* + N_1^* - N_0^*), (s^* + N_1^* - N_0^*)\rangle, \tag{38}$$

where in Gaussian form $s(t, \vec{x})$ and $N(t, \vec{x})$:

$$s^0 = \Psi_S^0 u, \quad N_0^0 = \Psi_{N0}^0 u; \quad /\theta = q/. \tag{39}$$

the integral operators Ψ_S^0 and Ψ^0_N figuring in (39) are found from the equations:

$$\begin{aligned} &\Psi_S^0(\theta/2\, I + K_N + K_S) = K_S, \\ &\Psi_{N0}^0(\theta/2\, I + K_N + \theta K_S) = K_N, \end{aligned} \tag{40}$$

in which $K_S(t, t_1; \vec{x}, \vec{x}_1)$ is the correlation function of the signal.

With (39) and (40) taken into account:

$$s^0 + N_1^0 = \Psi_{SN}^0 u,$$

where

$$\Psi_{SN}^0(\theta/2\, I + K_N + K_S) = K_N + K_S.$$

Hence

$$F_0(u) = \langle (I - \Psi_{N0}^0)u, (\Psi_{SN}^0 - \Psi_{N0}^0)u \rangle - \frac{1}{2}\langle (\Psi_{SN}^0 - \Psi_{N0}^0)u,$$
$$, (\Psi_{SN}^0 - \Psi_{N0}^0)u \rangle = \frac{1}{2}\langle u, (I - \Psi_{N0}^0)^{-1} u \rangle - \frac{1}{2}\langle u, (I - \Psi_{SN}^0)^{-1} u \rangle.$$

By analogy with (36)

$$(I - \Psi_{N0}^0)^{-1} = (\theta/2\, I + K_N)^{-1}$$

and

$$(I - \Psi_{SN}^0)^{-1} = (\theta/2\, I + K_N + K_S)$$

hence

$$F_0(u) = \frac{1}{2}\langle u, Hu \rangle = \frac{1}{2}\int_{A,T}\cdots\int u(t_1, \vec{x}_1)u(t_2, \vec{x}_2)H(t_1, t_2; \vec{x}_1, \vec{x}_2)\,dt_1 \cdots d\vec{x}_2, \tag{41}$$

where

$$H = (\theta/2\, I + K_N)^{-1} - (\theta/2\, I + K_N + K_S)^{-1}.$$

Also, as in the case of a determined signal, the functional (41) coincides with the known result [13], obtained from the probability ratio.

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SEQUENTIAL PROCEDURE OF IDENTIFICATION IN HYDROACOUSTIC RESEARCH

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 361-365

[Article by G. S. Lbov and V. I. Kotyukov]

[Text] In connection with the recently increasing timeliness of automation of oceanographic research the use of recognition or identification systems and algorithms for the purposes of computer detection and classification of hydroacoustic signals is acquiring great importance.

The methods of the theory of automatic recognition of images are coming into wide use not only for the purposes of preliminary processing of the collected information, but also in the systematic study of many oceanographic objects: bottom structure, biosphere, distribution of different layers in the water thickness, and others. In all these tasks problems arise in the automation of the search for informative criteria from the signals from objects and the adoption of different rational decisions on their basis.

In a number of well-known studies the use of hierarchical recognition systems in hydroacoustics [3] is proposed for the case of complex classification problems. These make it possible to minimize the cost of measurement of criteria for the object to be identified with a stipulated reliability of its classification.

It is assumed that the "teaching" process on the basis of available statistical material is already completed and the identifying tool (algorithm) is used in a regime of classification of the newly arriving objects. Allowance for the relationship between losses from recognition errors and the cost of measuring the values of the criteria leads to a sequential process of their measurement.

Naturally, the criteria must be measured for an object subject to recognition in such a sequence and in such a quantity that the required reliability of its classification is ensured at a minimum measurement cost.

Usually in the literature [1, 2, 3] authors have proposed measurement procedures which do not take into account the statistical interrelationship of the criteria although it is evident that the creation of a method easily applied on an electronic computer using this additional information would lead only to a decrease in the cost of measurements in the recognition process.

In this study we have developed an algorithm for sequential measurement for the case of dependent criteria.

#1. Formulation of Problem

The problem of minimizing the cost of measurements of criteria is solved with the following restrictions:

a) it is assumed as a simplification that the cost of measuring each of N criteria $\{x_1,\dots,x_N\}$ is equal;
b) the repeated measurement of any of the criteria x_i for the object does not carry additional information;
c) for each i-th class (i = 1,...,m) we know its a priori probability P_i, and also as a result of the preliminary teaching process we determine the form and parameters of the probability density function $P_i(x_1,\dots,x_N)$.

Assume that in the n-th interval of the object recognition procedure we measure only some n criteria (n< N), whose values are denoted $(\tilde{x}^1,\dots,\tilde{x}^n)$. The unmeasured criteria, in turn, are denoted $\{x^1,\dots,x^\omega,\dots,x^{N-n}\}$; the a posteriori probabilities of the classes for a particular unknown object are equal to:

$$q_i(n) = \frac{P_i^0 \cdot P_i(\tilde{x}^1,\dots,\tilde{x}^n)}{\sum_{i=1} P_i^0 \cdot P_i(\tilde{x}^1,\dots,\tilde{x}^n)}, \qquad i = 1,\dots,m \tag{1}$$

We note that before the beginning of measurements we had

$$\{q_i(0) = P_i^0\}.$$

In this paper we selected the following rule as the criterion for stopping the measurement procedure. The successive measurement of the values of the criteria ends and a decision is made concerning whether an object belongs to the S-th class if

$$F(n) = \frac{q_s(n)}{q_2(n)} \geq B, \tag{2}$$

where

$$q_s(n) = \max_i \{q_i(n)\}; \quad q_2(n) = \max_{i,\, i \neq s} \{q_i(n)\};$$

is the value of some threshold related to the required probabilities of an erroneous classification of the first and second kinds [3].

We will denote by n_0 the number of measurements made, for which expression (2) is already satisfied.

Thus, in the considered formulation the problem involves attaining satisfaction of expression (2) with a minimum (average) number of measurements

$$M(n) \longrightarrow \min \tag{3}$$

M is the mathematical expectation symbol.

In the proposed strategy, if in the n-th interval we have $F(n) < \beta$, in the next (n+1)-st procedure interval for the object to be recognized we measure the value of that criterion for which with the available information $(\tilde{x}^1, \ldots, \tilde{x}^n)$ the anticipated value of the parameter F(n+1) is maximum.

We note that the algorithm considered below, with small changes, is suitable also for some other criteria [3] different from (2).

#2. Description of Algorithm

For each of the still unmeasured criteria x^{ω} the determination of the anticipated value F(n+1), denoted by $F_{ant}(n+1)$, is accomplished using the following scheme.

1. In the entire space of unmeasured criteria $\{x^1, \ldots, x^{\omega}, \ldots, x^{N-n}\}$ for each i-th class we find the vector of conditional mathematical expectations of the values of the corresponding criteria

$$x_{i\,\text{ож}} = (x^1_{i\,\text{ож}}, \ldots x^{\omega}_{i\,\text{ож}}, \ldots, x^{N-n}_{i\,\text{ож}})$$

[ож = ant(icipated)] with the already available measurements $(\tilde{x}^1, \ldots, \tilde{x}^n)$. A simple numerical method for finding $x_{i\ ant}$ (i = 1.....m) will be described below.

2. Assuming $\tilde{x}^{n+1}_{ant} = x^{\omega}_{i\,ant}$ we find the anticipated a posteriori probabilities of classes:

$$q_{j\,\text{ож}}(n+1)^{\omega}_{i} = \frac{P^0_j \cdot P_j(\tilde{x}^1, \ldots, \tilde{x}^n, x^{\omega}_{i\,\text{ож}})}{\sum\limits_{j=1}^{m} P^0_j \cdot P_j(\tilde{x}^1, \ldots, \tilde{x}^n, x^{\omega}_{i\,\text{ож}})}, \quad j = 1, \ldots, m.$$

[ож= ant(icipated)]

3. We compute

$$F^{\omega}_{i\,\text{ож}}(n+1) = \frac{q_{s\,\text{ож}}(n+1)^{\omega}_{i}}{q_{z\,\text{ож}}(n+1)^{\omega}_{i}}, \tag{5}$$

where

$$q_{s\,\text{ож}}(n+1)^{\omega}_{i} = \max_{j} \{q_{j\,\text{ож}}(n+1)^{\omega}_{i}\},$$

$$q_{z\,\text{ож}}(n+1)^{\omega}_{i} = \max_{j,\, j \neq s} \{q_{j\,\text{ож}}(n+1)^{\omega}_{i}\}.$$

In other words $F^{\omega}_{i\ ant}(n+1)$ is the value of the criterion F for the vector

$$(\tilde{x}^1, \ldots, \tilde{x}^n, x^{\omega}_{i\ ant}).$$

4. Similarly we will determine the F values for all vectors

$$(\tilde{x}^1, \ldots, \tilde{x}^n, x^{\omega}_{1\text{ож}}), \ldots, (\tilde{x}^1, \ldots, \tilde{x}^n, x^{\omega}_{m\text{ож}})$$

that is $$F^{\omega}_{1\text{ож}}(n+1), \ldots, F^{\omega}_{m\text{ож}}(n+1)$$

5. For the criterion x^{ω} we compute the sought-for value of the mathematical expectations of the F criterion

$$F'^{\omega}_{\text{ож}}(n+1) = \sum_{i=1}^{m} q_i(n) \cdot F^{\omega}_{i\text{ож}}(n+1). \tag{6}$$

In precisely the same way we determine the values F^{ω}_{ant} (n+1) for all the still unmeasured criteria -- $\omega = 1, \ldots, N - n$.

We find the value

$$F^{k}_{\text{ож}}(n+1) = \max_{\omega}\{F^{\omega}_{\text{ож}}(n+1)\}. \tag{7}$$

The k-th criterion is measured in the (n+1)-st interval.

#3. Determination of the Vector $x_{i\ ant}$ for the Case of a Normal Distribution

Assume for the i-th class that $P_i(x_1, \ldots, x_N) = N(\mu^i, A^i)$ where $\mu^i = (\mu^i_1, \ldots, \mu^i_N)$ is the vector of the mathematical expectations of the criteria and $A^i = \|\sigma^i_{jk}\|$ is the matrix of covariations.

Henceforth the superscript i will be omitted as a simplification. Since the criteria are statistically dependent, the mathematical expectation of the random value x_j is some function of the variables $\{x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N\}$:

$$\mu_j(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N) = \int x_j \cdot p(x_j / x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N)\, dx_j, \quad \text{где } x_j \in D_j \tag{8}$$

$$p(x_j / x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N) = \frac{p(x_1, \ldots, x_N)}{p(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N)}.$$

For a normal distribution of the function (8) has the form [4]:

$$\mu_j(x) = b_{j1}x_1 + \ldots + b_{jj-1}x_{j-1} + b_{jj+1}x_{j+1} + \ldots + b_{jN}x_N + b_{j0}, \tag{9}$$

where: $$b_{jk} = \beta_{jk} \cdot \frac{\sigma_{jj}}{\sigma_{kk}}; \quad b_{j0} = \mu_j - \sum_{k=1}^{N} b_{jk}\mu_k; \quad k = 1, \ldots, N; \quad k \neq j.$$

We note that if $\{x_1 = \mu_1, \ldots, x_{j-1} = \mu_{j-1}, x_{j+1} = \mu_{j+1}, \ldots, x_N = \mu_N\}$, then $\mu_j(x) = \mu_j$.

The β_{jk} coefficients are found by solving a system of linear algebraic equations [4]:

$$\left.\begin{aligned} r_{11}\beta_{j1}+\ldots+r_{1j-1}\beta_{jj-1}+r_{1j+1}\beta_{jj+1}+\ldots+r_{1N}\beta_{jN}&=r_{j1}\\ r_{N1}\beta_{j1}+\ldots+r_{Nj-1}\beta_{jj-1}+r_{Nj+1}\beta_{jj+1}+\ldots+r_{NN}\beta_{jN}&=r_{jN}\end{aligned}\right\} \qquad (10)$$

where

$$r_{ik}=\frac{\sigma_{jk}}{\sqrt{\sigma_{jj}}\sqrt{\sigma_{kk}}}$$

Thus, for the i-th class on the basis of the vector μ and the matrix A already in the teaching stage it is possible to determine (the coefficients $\{\beta_{jk}\}$ were found) the system of linear algebraic equations

$$\left.\begin{aligned} \mu_1(x)&=\sum_{k=2}^{N} b_{1k}x_k+b_{10}\\ &\cdots\cdots\cdots\\ \mu_j(x)&=\sum_{k=1,\,k\neq j}^{N} b_{jk}x_k+b_{j0}\\ &\cdots\cdots\cdots\\ \mu_N(x)&=\sum_{k=1}^{N-1} b_{Nk}x_k+b_{N0}\end{aligned}\right\} \qquad (11)$$

Since as the sought-for values x_j we will use their conditional mathematical expectations $\mu_j(x)$, in the system of equations (11) the left-hand sides $\{\mu_j(x)\}$ can be replaced by $\{x_j\}$ respectively.

In the n-th interval of the recognition procedure we will assume that we have the measured vector of values $(\tilde{x}',\ldots,\tilde{x}^n)$. In order to determine the vector of the conditional mathematical expectations $x_{i\exp}=(x'_{i\exp},\ldots,x^{\omega}_{i\exp},\ldots,x^{N-n}_{i\exp})$ it is necessary that the determined values $(\tilde{x}',\ldots,\tilde{x}^n)$ be substituted into the corresponding places in system (11). We note that the equations for which $\{\mu_j(x)\}$ correspond to the measured criteria $(\tilde{x}',\ldots,\tilde{x}^n)$ are not considered. After simple transformations we will have a system of (N - n) equations with (N - n) unknowns $\{x',\ldots,x^{\omega},\ldots,x^{N-n}\}$, by solving which we obtain the sought-for numerical vector $(x'_{i\exp},\ldots,x^{\omega}_{i\exp},\ldots,x^{N-n}_{i\exp})$.

The considered recognition algorithm was realized in the form of a program for the "Minsk-22" electronic computer.

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SUPPRESSION OF SIDE LOBES OF AN ANTENNA DIRECTIONAL DIAGRAM BY A METHOD BASED ON TEMPORAL CHANGE OF APERTURE SIZE

Novosibirsk TRUDY SHESTOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1975 pp 372-378

[Article by D. K. Solov'yev and P. Ya. Krasinskiy]

[Text] One of the methods for reducing the influence of noise arriving at a receiving device in the side lobes is the forming of the directional diagram for an antenna with the distribution of the noise field taken into account.

The construction of antennas with minimizing of the mean level of the side lobes of the directional diagram, and also elimination or limitation of the influence of strong point noise with known angular coordinates of their sources, was examined in the literature [1, 2, 3].

It is important to note the influence of some peculiarities on the applicability of different methods for forming the diagram in hydroacoustic devices. The methods for selecting excitation ensuring suppression of the side field in a stipulated direction meet with a number of difficulties. It appears that it is difficult to realize optimum excitation in a broad frequency range of complex configurations of antennas with allowance for the directivity of elements. In addition, it is necessary to ensure an adequately rapid scanning with the main ray of the directional diagram with a constant direction of the diagram minima.

The use of methods involving the forming of a directional diagram by means of averaging of a signal arriving from an antenna with a variable aperture affords great possibilities for control of the antenna side field.

For acoustic antennas the realization of a controllable change of aperture size with time cannot involve fundamental difficulties. As an example, Fig. 1 shows a block diagram for a variant of realization of controllable suppression of the side lobes with use of a linear array.

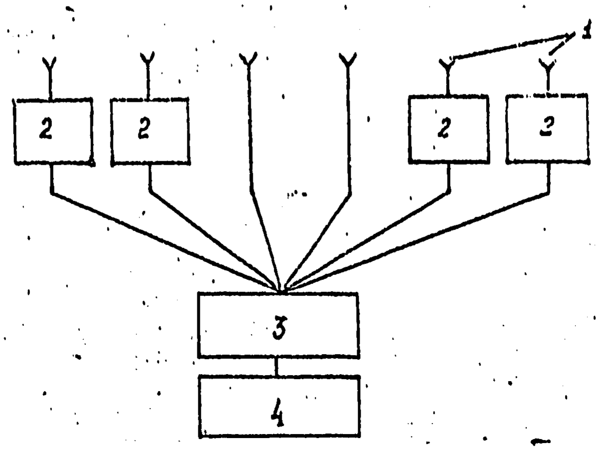

Fig. 1

In Figure 1: 1 -- hydrophones, 2 -- commutators, 3 -- summator, 4 -- filter. A change in aperture size with time is accomplished by means of periodic activation and deactivation of the commutators 2. The report given below is devoted to a study of such methods for control of the side field of a linear array.

We will examine the directional diagram of an antenna with a change in aperture size with time. The antenna directional diagram with a temporal change in the aperture can be written in the form:

$$F(Q,t)=\int_{-a(t)}^{a(t)} P(\xi)\exp[jk\xi(\sin Q-\sin Q_0)]d\xi \tag{1}$$

The mean current or voltage is equal to

$$F(Q)=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\int_{-a(t)}^{a(t)} P(\xi)\exp[jk\xi(\sin Q-\sin Q_0)]d\xi dt \tag{2}$$

This expression formally coincides with the directional diagram of a two-dimensional antenna. We will examine the case of a linear array uniformly excited in amplitude and phase. Assume that along the edges of an array consisting of M elements m elements with $m_0 = m/2$ are switched off on each side. The law of temporal change of the number of elements will be $M(t) = M + \beta m$

(3)

where

$$\beta = \begin{cases} 0 \text{ when } 0 \leqslant t \leqslant T/2 \\ -1 \text{ when } -T/2 \leqslant t \leqslant 0 \end{cases} \tag{4}$$

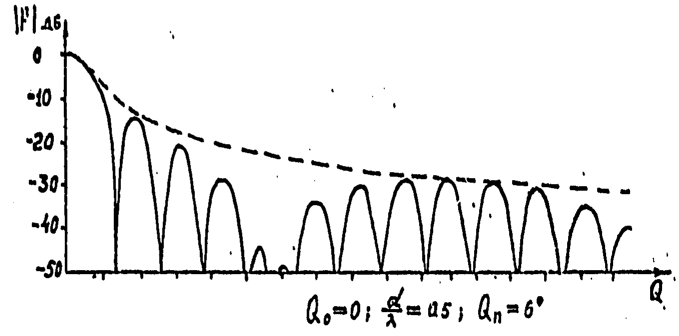

Fig. 2

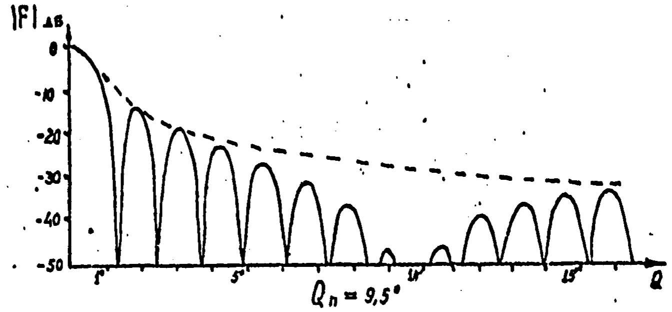

Fig. 3

In this case signal processing is reduced to use of a time-averaged component

$$F(Q)=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\sum_{n=-N(t)}^{N(t)} P_n \exp[j\kappa nd(\sin Q-\sin Q_0)]\,dt \tag{5}$$

where Q is the angle coordinate, Q_0 is the direction of compensation, K is the wave number, d is the array interval, 2N + 1 is the current number of elements in the array, T is the averaging period.

The mean value of voltage sums during the time T is given by the expression:

$$F(Q)=\sum_{n=-N}^{N} P_n \exp[j\kappa nd(\sin Q-\sin Q_0)]+ \tag{6}$$

$$+\frac{1}{2}\sum_{n=-J}^{J} P_n \exp[j\kappa nd(\sin Q-\sin Q_0)].$$

where $N-J=m_0$.

We will consider the amplitude distribution in the array to be uniform. By turning the geometrical progression in formula (6) we obtain:

$$F(Q)=\frac{\sin\left[\frac{(2M-m)}{2}\cdot\frac{\pi df}{c}(\sin Q-\sin Q_0)\right]\cos\left[\frac{m}{2}\cdot\frac{\pi df}{c}(\sin Q-\sin Q_0)\right]}{\frac{2M-m}{2}\cdot\sin\left[\frac{\pi df}{c}(\sin Q-\sin Q_0)\right]} \qquad (7)$$

Thus, we obtain a resultant directional diagram in the form of the product of the directional diagram of an array consisting of 2M ~ m/2 radiators and a minimizing factor in the form:

$$\cos\left[\frac{m}{2}\cdot\frac{\pi df}{c}(\sin Q-\sin Q_0)\right] \qquad (8)$$

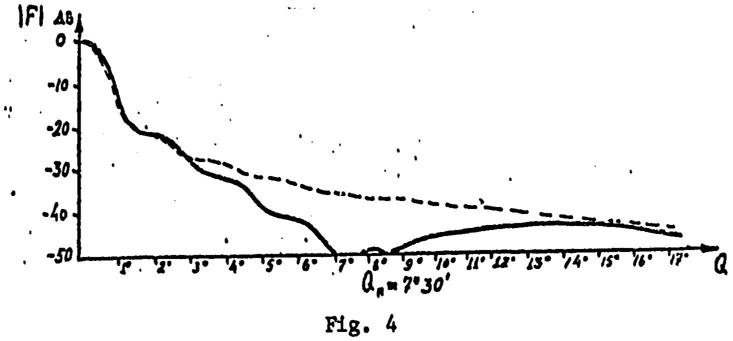

Fig. 4

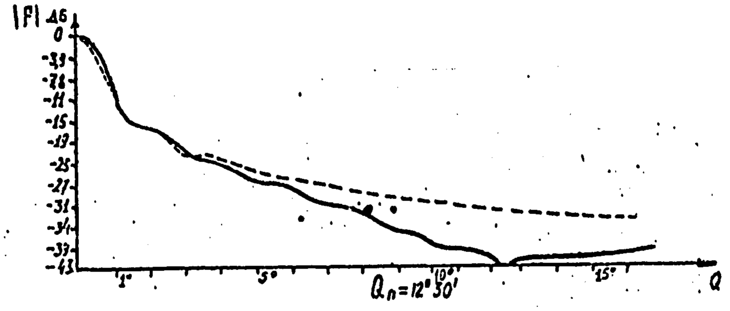

Fig. 5

It is evident that in the neighborhood of the point at which the cosine factor becomes equal to zero a suppression zone will appear. The direction in which the suppression zone is oriented is given by the formula:

$$Q_n = \arc\sin\left[\sin Q_0 + \frac{(1+2n)c}{mdf}\right] \qquad (9)$$

It is therefore clear that with one and the same number of cut-out elements there will be formation of several suppression zones whose coordinates are given by expression (9). With n = 0 the first suppression zone, closest to the main maximum, is formed in the diagram. The width of the suppression zone is dependent on its position relative to the main maximum. The suppression sector is narrowest near the main maximum of the directional diagram and broadens during movement of the suppression zone in the direction from the main lobe. Figures 2 and 3 show the directional diagrams of rays with a ray width 1°, M = 101, computed on a digital computer. The suppression zone is symmetric relative to the main maximum and with a change in its direction is displaced accordingly by this same angle.

Now we will consider how the cutting-off of some of the elements exerts an influence on the width of the main lobe. Whereas before cutting-off of some of the elements the width of the main lobe was

$$\Delta Q = 0{,}886 \frac{\lambda}{M\alpha} \operatorname{cosec}(Q_0) \tag{10}$$

after cutting-off of some of the elements the width of the main lobe becomes equal to

$$\Delta Q = 0{,}886 \frac{\lambda}{\frac{2M-m}{2} d} \operatorname{cosec}(Q_0) \tag{11}$$

Thus, with suppression of the side lobes the width of the directional diagram is increased

$$\frac{\Delta Q_n}{\Delta Q} = 1 + \frac{m}{2M-m} \tag{12}$$

Since usually m< M, the m/2M - m value is small. It can therefore be seen that controllable suppression of the side lobes exerts no appreciable influence on the width of the main maximum of the directional diagram, especially in the case of large suppression angles.

Now we will determine the antenna concentration coefficient with a change in aperture size with time and subsequent signal averaging. We use d max to denote the maximum aperture size. Then if

$$P_t(\xi,t) = \begin{cases} P(\xi) & \text{when } |\xi| \leq |a(t)| \\ 0 & \text{when } |\xi| > |a(t)| \end{cases} \tag{13}$$

it is possible to write $F(Q) = \int_{-a_{max}}^{a_{max}} \int_{-\frac{T}{\xi}}^{\frac{T}{\xi}} P_t(\xi,t) \exp[jk\xi(\sin Q - \sin Q_0)] d\xi\, dt$

here $\int_{t}^{-t/2} P_t(\xi, t)dt$ is an amplitude distribution giving a directional diagram equivalent to the directional diagram of an array with a change in size of the aperture and subsequent signal averaging in time. Hence, the concentration coefficient for an antenna with suppression of the side lobes can be computed using an equivalent amplitude distribution. For the case of formation of one suppression zone in the directional diagram of a linear equivalent array with a uniform amplitude distribution, when the distances between adjacent elements are multiples of 0.5 λ, the concentration coefficient will be equal to: $K = [2M - m]^2/4M - 3\ m$, where M is the number of elements in the array, m is the number of cut-off elements.

Even in a case when the suppression zone is close to the main maximum of the directional diagram, the concentration coefficient is insignificantly less than the concentration coefficient for an antenna without suppression of the side lobes.

Now we will examine operation of an antenna with suppression of the side lobes in the frequency range. The directional diagram of an antenna array in the frequency range can be written in the form [4]:

$$F(Q)=\left[\int_{f_1}^{f_2}\left[\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left[\sum_{n=-N}^{N} P_n \exp\left[\frac{j2\pi n d f}{c}(\sin Q - \sin Q_0)\right]dt\right]^2 df\right]^{1/2} \qquad (15)$$

We introduce weight filtering, varying with time, into the channels of a hydroacoustic antenna. In particular, this can be accomplished by periodic cutting-in of band filters with a distribution of limiting frequencies in the aperture. It can be seen from expression (8) that if the number of cut-off elements is dependent on frequency and the angle of compensation in accordance with the law:

$$m = \frac{c}{df(\sin Q_n - \sin Q_0)} \qquad (16)$$

we obtain a factor having a stable position of the minimum not dependent on frequency. The directional diagram of an antenna without stabilization of the width of the main maximum and with the suppression of the side lobes in the frequency band will have the form:

$$F(Q)=\left[\int_{f_1}^{f_2}\left[\frac{1}{T}\ \frac{\sin\left[\frac{2M-3}{2}\cdot\frac{\pi d f}{c}(\sin Q-\sin Q_0)\right]}{\sin\left[\frac{\pi d f}{c}(\sin Q-\sin Q_0)\right]}\right]df\right]^{1/2}\cos\left[\frac{\pi}{2}\,\frac{(\sin Q-\sin Q_0)}{(\sin Q_n-\sin Q_0)}\right] \qquad (17)$$

Thus, we obtain a controllable zone of a reduced level of the additional lobes in the frequency range caused by a minimum of the cosine factor.

Figures 4 and 5 show the directional diagrams of an array with degree noise in a frequency range equal to one octave.

The mean intensity of antenna reception in the suppression sector is more than 50 db lower than the intensity of reception from the main direction and is approximately 20-30 db lower than the intensity of reception in this sector of the array with a uniform distribution of amplitude. The described method for suppressing the side field makes it possible, relatively simply, to control the position of the suppression zone both in the harmonic signal and in the frequency range. In this connection there will be no restrictions on the scanning speed.

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