On Long-Term Forecasting of the Critical Frequencies of the Ionosphere and of Cases of Disturbances in it

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ON LONG-TERM FORECASTING OF THE CRITICAL FREQUENCIES OF THE IONOSPHERE AND OF CASES OF DISTURBANCES IN IT

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Presented by Academician B. A. Vvedenskiy

The generally known circumstance that the attenuation of radio waves on passage through the ionosphere is less, the shorter the wave, compels selection, for uninterrupted radio communication, of the shortest wavelength which, while close to the critical wavelength, is still capable of being reflected from the ionized layers of the atmosphere. This wavelength, termed the optimum, is also the most advantageous with respect to obtaining the best possible transit of the waves.

The correlations that have been established between phenomena on the sun, on the one hand, and the variations that take place in the ionosphere and the earth's magnetic field, on the otherhand, make it possible to predict geomagnetic and ionospheric disturbances, critical frequencies and working wavelengths for radio communication, to the extent that we are able to foretell the development of solar activity.

The condition of solar activity is characterized by the following different indices, which correlate non-uniformly with the phenomena transpiring on the earth: the Wolf numbers, the areas occupied by sunspots, prominences, and faculae, the impulses of solar activity and the relative intensity in the sunspots and on the solar surface, of the Balmer series of hydrogen lines.

The non-uniform degree of correlation is due to the character of the radiation (ultraviolet or corpuscular radiation), and likewise to the direction of motion exhibited by the active regions: whether towards the poles or towards the equator of the sun.

The number and area of the active regions formed on the solar surface are not constant and fluctuate considerably from year to year, resulting in the fundamental "11-year" cycle of solar activity.

The motion of the active regions from higher to lower latitudes on the solar surface, which is due to the development of the fundamental cycle, their displacement in longitude caused by the rotation of the heliosphere, and also the inclination of the sun's axis to the plane of the ecliptic, together with other well-known causes, give rise, in the main, to five cycles of variation in the condition of the ionosphere: 11-year, annual, semiannual (spring-autumn maximum), seasonal and 27-day cycles.

In the present paper, we shall consider the correlation between the 11-year cycle of critical frequencies and of ionospheric and magnetic disturbances, on the one hand, and the Wolf numbers, on the other hand.

VARIATION IN THE VALUES OF THE CRITICAL FREQUENCIES, MAGNETIC AND IONOSPHERIC DISTURBANCES IN RELATION TO THE 11-YEAR CYCLE OF WOLF NUMBERS

Berkner and Wells [1], using the data of critical-frequency measurements by the ionosphere stations at Washington and Watheroo from 1934 to 1937, established a linear relation between the number of sunspots and the square of the mean annual value of the critical frequency for the F_2 layer.

Judson, in his paper "Comparison of data on ionosphere, sunspots and terrestrial magnetism" [2], and also Smith, Gilliland and Kirby [3], using the critical-frequency measurements of the Washington ionosphere station from 1933 to 1937, established a good correlation -- practically a strict one -- between the Wolf numbers and the critical frequencies for the E, F_1 and F_2 layers for noon, and for their diurnal minima.

By now the ionosphere stations of the world have accumulated enough experimental material to draw conclusions on the variation of the critical frequencies for the ionosphere during the entire 11-year period. The Washington ionosphere station has the fullest experimental data (Table 1), and regularly publishes the results of its measurements in the journal "Terrestrial Magnetism and Atmospheric Electricity".

Table 1

Year	$\bar{f}^{\alpha}_{F_2}$	$\bar{f}_E$	$\bar{f}^H$	$\bar{f}_{F_2}$	$\bar{N}$	$\bar{W}$
1933	5.15	2.6	1.98	3	13	5.9
1934	6.25	3.15	3.75	3.08	11	9.4
1935	6.8	3.35	4.23	3.62	18	36.5
1936	10	3.7	5.96	4.8	20	79.6
1937	11.45	3.8	6.2	5.68	35	113.2
1938	11.2	3.75	6.68	5.82	40	109
1939	10.2	3.66	5.94	4.8	38	92
1940	8.6	3.5	5.22	4.25	32	68
1941	7.5	3.37	4.5	3.43	24	48
1942	7	3	4.4	3.4	19	30
1943	6.7	2.94	4.9	3.2	18	15
1944	7	2.86	4.8	3.29	16	10

Figure 1 shows the curves characterizing the semiannual variation in the critical frequencies observed at Washington and in the number N of magnetic storms according to the Sverdlovsk catalog. The maximum values for these have been taken at 100 percent. The curves of distribution of the mean values for the critical frequencies $\bar{f}$ and the number of magnetic storms $\bar{N}$ are similar to the curve of solar activity, expressed as mean values of the Wolf numbers $\bar{W}$, but this resemblance of the curves in the pre-maximum from 1933 to 1937 is not fully maintained after the maximum. This results from the fact that besides the primary cause, being the process of solar outburst, there is also another cause that influences the character of the atmospheric ionization curves. This second cause consists in the fact that as the processes develop, the active zones of the sun move from the high solar latitudes towards the equator, in connection with which the radiation from these active areas towards the plane of the ecliptic (and consequently also towards the earth) is more considerable during the epoch of declining activity than during the epoch of its rise.

The remarkable fact should be noted that not all of the maxima of the 11-year curve of critical frequencies coincide with the maxima of the Wolf numbers. The maximum number of magnetic storms also lags by about a year. The curves of critical frequencies $\bar{f}^{d}_{F_2}$ of the noon values for the F_2 layer and of the frequencies $\bar{f}_E$ for the E layer coincide with the maximum of solar activity, but the minimum values of early morning critical frequencies $\bar{f}^{m}_{F_2}$ and also those at midnight $\bar{f}^{n}_{F_2}$, lag by a year in their maxima behind the "solar curve" $\bar{W}$, and coincide instead with the curve of the number of magnetic storms. These facts, remarked by us as early as

1941 for the critical frequencies and the magnetic storms, and later again, in 1943, for the number of magnetic storms, were confirmed by S. K. Vsekhsvyatskiy [5] on the basis of his study of 1103 magnetic storms in the Slutsk catalog [6] from 1878 to 1940.

The emphatic discrepancy between the maxima of the daily values for the critical frequencies $\bar{f}^{d}_{F_2}$ and $\bar{f}_E$, on the one hand, and the nocturnal $\bar{f}^{n}_{F_2}$ and minima $\bar{f}^{M}_{F_2}$ (early morning) values of the critical frequencies, on the other, may be explained by the existence of two basic components in the solar radiation, both playing a substantial part in the ionization of the atmosphere, namely the ultraviolet component and that of corpuscular radiation.

During the daytime, when the principal ionizer (supplementing the corpuscular and other radiation) is the ultraviolet radiation, ~~coincidence occurs between~~ the maxima of the 11-year march of the critical frequencies of the E layer and of the noon values of the critical frequencies for the F layer, agree with the maximum of the curve of the Wolf numbers (Fig. 1).

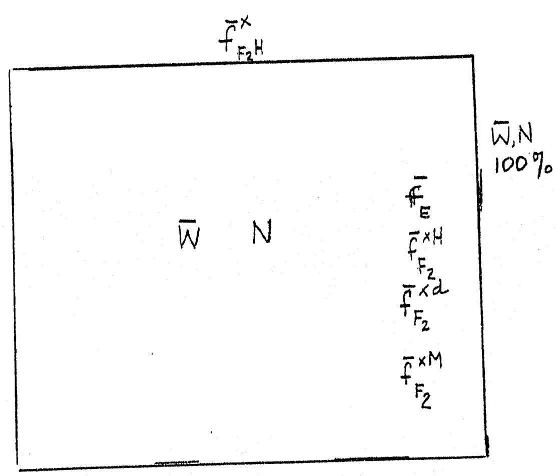

Figure 1

At night and during the pre-dawn hours, when the dark side of the terrestrial sphere is inaccessible to the ultraviolet radiation of the sun, the corpuscular radiation plays a substantial role in the ionization of the F_2 layer, which is excellently confirmed by the 11-year march of the critical frequencies $\bar{f}^{n}_{F_2}$ and $\bar{f}^{m}_{F_2}$ (Fig. 1).

The lag in the maximum of these curves and of the curve of magnetic storms is due to the same reason: motion towards the solar equator by the sources of corpuscular radiation having low solid angles (8 - 9 degrees, as was established by M. Gnevichev and A. Ol' [7].

The remarkable effect of a separation between the curves of critical frequencies during the day and those in the night, as well as the coincidence between the march of the curve of the number of magnetic storms with the curves of the nocturnal critical frequencies for the F_2 layer, confirm the corpuscular origin of magnetic storms. This constitutes an additional argument in favor of the Chapman-Ferraro theory of magnetic storms [8], according to which they are caused by a neutral corpuscular stream ejected by the sun, and against the theory of ~~Khel'bert~~ Hulburt ascribing them to ultraviolet acting as a disturbing influence on corpuscles of terrestrial origin.

Thus, the physical dependence of the 11-year march of the critical frequencies for the ionosphere on the solar activity of the same period becomes obvious. But to forecast the 11-year march of the critical frequencies we must have a quantitative estimate of the correlation between the values that characterize the 11-year process on the sun and those that characterize the ~~march~~ behavior of ionization and

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the ~~march~~ (behavior) of magnetic storms.

The significant statistical constants in the study of this quantitative relation are the correlation coefficient, in the case of linear correlation, and the correlation ratio, in the case of non-linear correlation.

After determination of the correlation coefficient, a correlation equation is set up, by means of which the critical frequencies or the number of magnetic storms are computed.

Before setting up the correlation equation, we must first establish a criterion of linearity together with its probable error. The criterion of linearity makes it possible for us to judge whether it is sufficient to confine ourselves to a linear correlation equation to express the relation between the statistical values of the Wolf numbers and those of the critical frequencies, or whether it is necessary for us to pass to equations of higher degree.

The correlation coefficient r and the correlation ratio η may differ only slightly from each other for a distribution of W, f and N. In such a case the question arises as to whether or not such divergence is significant.

As a criterion of linearity, Blackman [9] has proposed that the correlation be considered linear if the ratio between the measure of linearity ζ and the probable error of the measure of linearity is less than 3, i.e. if

$$\frac{\zeta}{\sigma_\zeta} < 3 \qquad (1)$$

The measure of linearity is represented by

$$\zeta = \eta^2 - r^2 \tag{2}$$

The probable error of the measure of linearity is approximately equal to

$$\sigma_\zeta = \frac{2}{\sqrt{n}} \sqrt{\zeta \left[(1-\eta^2)^2 - (1-r^2)^2 + 1\right]}, \tag{3}$$

where n is the full number of observations.

Thus, to accomplish the task set, that is, to forecast the critical frequencies f, we must find the correlation coefficient r between the curve of sunspot distribution W and the curve of critical-frequency distribution f, and then, if we are not convinced that the regression (The term "regression" currently adopted in the statistical literature was introduced by Galton. It is not entirely apt. It would be clearer if we were to write, instead of the words" "curve of regression" or "equation of regression", the words: "correlation curve" or "correlation equation") of the statistical set S (W_h , f_i , n_{hi} , n) is linear, we must determine the correlation ratio <u>eta</u> (η). For this we construct correlation curves of the paired distribution of the Wolf numbers $\bar{W}$ and the critical frequencies of the F_2 layer: the noon value f (Fig. 2), the midnight value $\bar{f}^{n}_{F_2}$ (Fig. 3), the diurnal minimum value $\bar{f}^{m}_{F_2}$ (Fig. 4), the noon value for the E layer $\bar{f}_E$ (Fig. 5), and the number of magnetic storms $\bar{N}$ (Fig. 6). The absolute values have been plotted on these curves, taking the maximum as unity.

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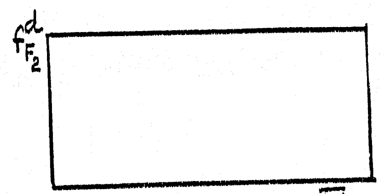


Fig. 2

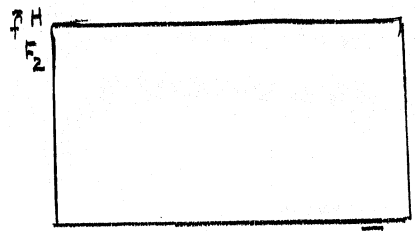


Fig 3

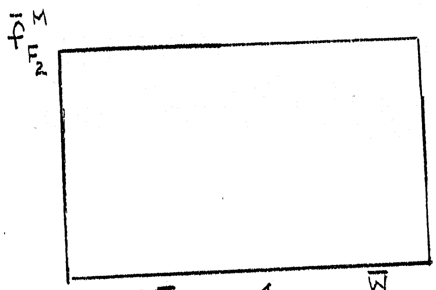


Fig. 4

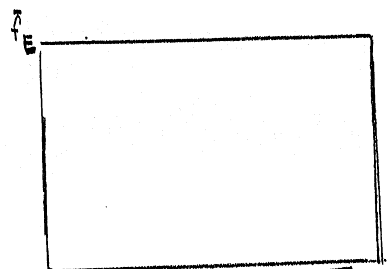


Fig. 5

In contradistinction to the law of full distribution of statistical elements (cf. Fig. 1), the concept of paired distribution is used in this case; that is, in studying the distribution of one statistical element -- in this case $\bar{f}^{d}_{F_2}$ etc. -- the distribution of another statistical element is considered, i.e. $\bar{W}$ (Figs. 2-6).

In making a combined study of two or more statistical elements, when only a small number of observations is involved, the series of distributions is investigated, but with a fairly large number of observations, distribution tables of the distribution of those values is studied instead, which makes it possible to make a complete study of their distribution and of the relation between the paired means $\bar{W}_i$ and $\bar{f}^{d}_{F_2 h}$, $\bar{f}^{H}_{F_2 h}$, $\bar{f}_{Eh}$, N_h.

As is commonly known [10, 11, 12], for a small number of observations the correlation coefficient r

$$r = \frac{\mu_{11}}{\sigma_W \sigma_f} = \frac{\frac{1}{n}\sum (W_i-\alpha)(f_i-\beta) - (\bar{W}-\alpha)(\bar{f}-\beta)}{\sqrt{\frac{1}{n}\sum (W_i-\alpha)^2 - (\bar{W}-\alpha)^2}\sqrt{\frac{1}{n}\sum (f_i-\beta) - (\bar{f}-\beta)^2}} \qquad (4)$$

For a sufficiently large number of observations

$$r = \frac{\mu_{11}}{\sigma_W \sigma_f} = \frac{\frac{1}{n}\sum_{h,i} n_{h_i}(W_h-\alpha)(f_i-\beta) - (\bar{W}-\alpha)(\bar{f}-\beta)}{\sqrt{\frac{1}{n}\sum_h n_h (W_h-\alpha)^2 - (\bar{W}-\alpha)^2}\sqrt{\frac{1}{n}\sum_i n_i (f_i-\beta)^2 - (\bar{f}-\beta)^2}} \qquad (5)$$

The coefficients of correlation between the Wolf numbers and the values of the critical frequencies for the E and F_2 layers, as observed by the Washington ionosphere station, and the number of magnetic storms, were computed by formula (4), and the standard deviations σ_W and σ_f were also determined.

The results of the computations are presented in Table 2.

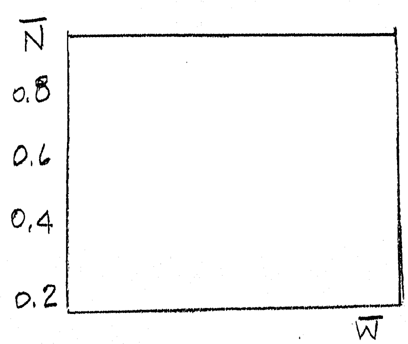


Table 2

	r	σ_W	σ_f
W and $f^d_{F_2}$	0.974	38.4	1.98
W and $f^n_{F_2}$	0.881	38.4	1.032
W and $f^m_{F_2}$	0.945	38.4	0.975
W and f_E	0.925	38.4	0.394
W and N	0.931	38.4	10.2

On thus determining the correlation coefficients and likewise the correlation ratios according to formulae which (cf. [10], [11] and [12]) are of the form

$$\eta_{Wf} = \frac{\sigma(\bar{W}_f)}{\sigma_W} \frac{\sqrt{\frac{1}{n}\sum_i n_i(\bar{W}_i-\gamma)^2-(\bar{W}-\gamma)^2}}{\sqrt{\frac{1}{n}\sum_h n_h(\bar{W}_h-\alpha)^2-(\bar{W}-\alpha)^2}} \qquad (6)$$

for the correlation between W and f, and of the form

$$\eta_{fW} = \frac{\sigma(\bar{f}_W)}{\sigma_f} \frac{\sqrt{\frac{1}{n}\sum_h n_h(\bar{f}_h-\gamma)^2-(\bar{f}-\gamma)^2}}{\sqrt{\frac{1}{n}\sum_i n_i(\bar{f}_i-\beta)^2-(\bar{f}-\beta)^2}} \qquad (7)$$

for the correlation between f and W, we then proceed to find the measure of linearity zeta (ζ) and its standard error sigma sub-zeta (σ_ζ), according to the expressions (2) and (3), which enables us to set up a criterion of linearity under the condition of (1).

LINEAR CORRELATION

Having thus established measures of the correlative relation between the above statistical elements, we now proceed to establish the correlation equations necessary to discover the dependence between the separate pairs of series: W and $f^d_{F_2}$, W and $f^n_{F_2}$, W and $f^m_{F_2}$, W and $f_{E/\Lambda}$, and W and N.

Our specific task now reduces down to the following. Given the tables:

$$\left.\begin{array}{l} W_1,\ W_2,\ W_3 \cdots\cdots\cdots\cdots W_s, \\[2ex] \bar{f}^{d}_{F_2 1},\ \bar{f}^{d}_{F_2 2},\ \bar{f}^{d}_{F_2 3} \cdots\cdots\cdots\cdots \bar{f}^{d}_{F_2 s}, \\[2ex] n_1,\ n_2,\ n_3 \cdots\cdots\cdots\cdots n_s, \end{array}\right\}\ \text{Table 3}$$

$$\left.\begin{array}{l} W_1,\ W_2,\ W_3 \cdots\cdots\cdots\cdots W_s, \\[2ex] \bar{f}^{d}_{F_2 1},\ \bar{f}^{d}_{F_2 d},\ \bar{f}^{d}_{F_2 3} \cdots\cdots\cdots\cdots \bar{f}^{d}_{F_2 s}, \\[2ex] n_1,\ n_2,\ n_3 \cdots\cdots\cdots\cdots n_s, \end{array}\right\}\ \text{Table 4}$$

$$\left.\begin{array}{l} W_1,\ W_2, W_3 \cdots\cdots\cdots\cdots W_s, \\[2ex] \bar{f}_{E_1},\ \bar{f}_{E_2},\ \bar{f}_{E_3}, \cdots\cdots\cdots\cdots \bar{f}_{E_s}, \\[2ex] n_1,\ n_2, n_3 \cdots\cdots\cdots\cdots n_s, \end{array}\right\}\ \text{Table 5}$$

$$\left.\begin{array}{l} W_1,\ W_2, W_3 \cdots\cdots\cdots\cdots W_s, \\ \bar{f}_{E_1},\ \bar{f}_{E_2},\ \bar{f}_{E_3}, \cdots\cdots\cdots\cdots \bar{f}_{E_s}, \\ n_1\ \ n_2\ \ \ n_3 \cdots\cdots\cdots\cdots n_s, \end{array}\right\}\ \text{Table 6}$$

$$\left.\begin{array}{l} W_1,\ \ W_2,\ \ W_3 \cdots\cdots\cdots\cdots W_s, \\[2ex] \bar{N}_1,\ \ \bar{N}_2,\ \ \bar{N}_3 \cdots\cdots\cdots\cdots \bar{N}_s \\[2ex] n_1,\ \ n_2,\ \ n_3 \cdots\cdots\cdots\cdots n_s \end{array}\right\}\ \text{Table 7}$$

Required, to find the equations:

$$\bar{f}_{F_2W} = \varphi(W),$$

$$\bar{f}_{F_2W} = \varphi(W),$$

$$\bar{f}_{F_2W} = \varphi(W),$$

$$\bar{f}_{EW} = \varphi(W),$$

$$\bar{N}_W = \varphi(W),$$

which, as is commonly known [10, 11, 12], may be expressed in the following way:

$$\bar{f}^d_{F_2W} = \bar{f}^d_{F_2} + r\frac{\sigma_f}{\sigma_W}(W - \bar{W}),$$

$$\bar{f}^d_{F_2W} = \bar{f}_{F_2} + r\frac{\sigma_f}{\sigma_W}(W - \bar{W}),$$

$$\bar{f}^M_{F_2W} = \bar{f}^M_{F_2} + r\frac{\sigma_f}{\sigma_W}(W - \bar{W}),$$

$$\bar{f}_{EW} = \bar{f}_E + r\frac{\sigma_f}{\sigma_W}(W - \bar{W}),$$

$$\bar{N}_W = \bar{N} + r\frac{\sigma_N}{\sigma_W}(W - \bar{W}),$$

We assume that the initial values of the full means f_{F_2}, f_E, $\bar{N}$ and $\bar{W}$ are equal to their maxima (cf. Romanovskiy [10], p. 350 and Mitropol'skiy [12], p. 213, and also Table 1), and substitute these initial values, together with the values for r, σ_f, and σ_W (cf. Table 2) in equations (8) to (12).

We thus obtain:

(a) the noon values of the critical frequencies for the F_2 layer:

$$\bar{f}^{d}_{F_2W} = 5,77 + 0,05\ W,$$

(b) the midnight values of the critical frequencies for the F_2 layer:

$$\bar{f}^{n}_{F_2W} = 3,52 + 0,0237\ W,$$

(c) the diurnal minimum values of the critical frequencies for the F_2 layer:

$$\bar{f}^{m}_{F_2W} = 3,1 + 0,024\ W,$$

(d) the noon values for the critical frequencies for the E layer:

$$\bar{f}_E = 2,725 + 0,01\ W,$$

(e) the number of magnetic storms:

$$\bar{N} = 12 + 0,247\ W.$$

Check computations were made by these formulae, and on this basis theoretical forecasting graphs of the 11-year course of the critical frequencies and the number of magnetic storms were prepared (Figs. 7-11). On the same graphs the mean values of the experimentally determined critical frequencies were also plotted.

As may be seen from these forecasting graphs, there is satisfactory coincidence between the experimentally determined points and the theoretical curves, which enables us to make reliable forecasts of the critical frequencies and of the number of magnetic storms for any year of the 11-year epoch, if the Wolf number W is known.

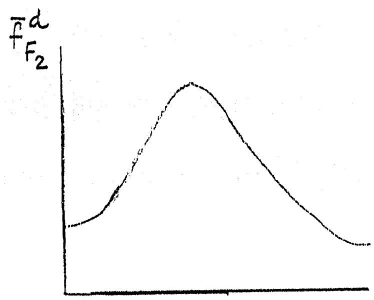


Figure 7

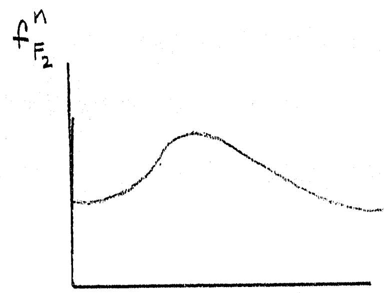


Figure 8

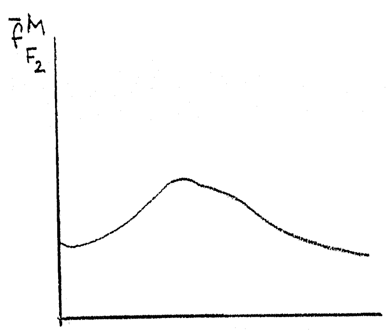


Figure 9

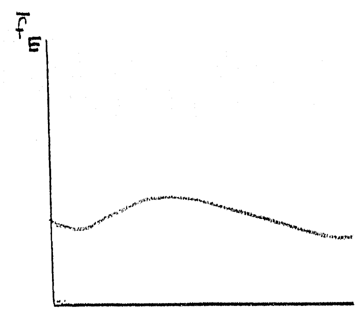


Figure 10

Such prognoses may be made for any point on earth having an ionosphere station that has assembled enough experimental material to construct the correlation equations.

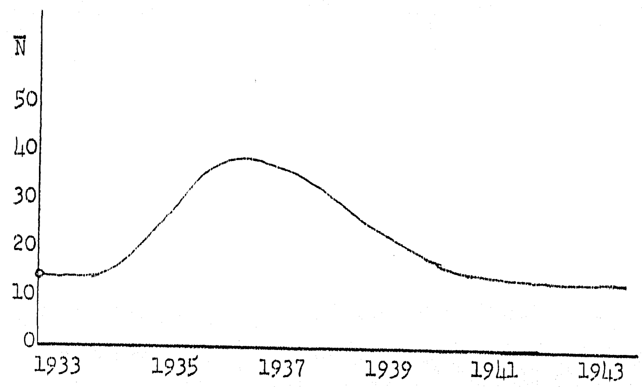


Figure 11.

NON-LINEAR CORRELATION

We now proceed to determine the non-linear correlation equations. Except for the normal distribution of statistical elements, little work has been done to develop the theoretical basis for solving this problem. In this case, therefore, various empirical rules for finding curvilinear regressions are often used; and if these yield no results, recourse is then had to fitting a parabolic curve by the method of least squares, or to using P. L. Chebyshev's method.

Preliminary study of the curves shown in Figures 2 to 6 allows us to apply Chebyshev's method of parabolic fitting in a simplified form to some of them, without the use of continued fractions.

The Chebyshev method of determining the parabola [10, 13], under the condition that the variation in the value of the independent variable W be taken at equal intervals, consists in the successive computations of the terms of the following series:

$$\bar{f}_W = A_0 \varphi_0(W) + A_1 \varphi_1(W) + \dots + A_p \varphi_p(W), \qquad (18)$$

where $\varphi_i(W)$ are the polynomials of i-th degree in W that satisfy the conditions of orthogonality (cf. p. 41, R. Kurant Courant and D. ~~Gilbert~~ Hilbert /14/).

$$(\varphi_i \varphi_k) = 0 \qquad (i \neq k; \ i, k = 0, 1, 2, \dots, p),$$
$$(\varphi_i \varphi_i) \neq 0 \qquad (i = 0, 1, 2, \dots, p),$$

where the first of these conditions is equivalent to

$$(\varphi_i \varphi_k) = \sum_h n_h \varphi_i(W_h) \varphi_k(W_h);$$

and the second to

$$(\varphi_i \varphi_i) = \sum_h n_h \varphi_i(W_h) \varphi_i(W_h).$$

The determination [10] of the coefficients A_k and the polynomials φ_i (W) that enter into equation (18) enables us to obtain non-linear correlation equations for calculation, in those cases where, according to the criterion of linearity, non-linear equations should be employed.

We thus obtain for the ascending (A) branch of the cycle of ionization of the F_2 layer, at noon:

$$\bar{f}^{d}_{F_2W} = \frac{(\bar{f}^{d}_{F_2}W\varphi^{0})}{(\varphi_0\varphi_0)}\varphi_0 W + \frac{(\bar{f}_{F_2}W\,\varphi_1)}{(\varphi_1\,\varphi_1)}\left\{W - \frac{W\varphi_0}{(\varphi_0\varphi_0)}\varphi_0(W)\right\} +$$

$$+ \frac{\bar{f}^{d}_{F_2W}\varphi_2}{(\varphi_2\,\varphi_2)}\left\{W^2 - \frac{W^2\varphi_0}{(\varphi_0\varphi_0)}\varphi_0(W) - \frac{(W^2\varphi_1)}{(\varphi_1\varphi_1)}\varphi_1(W)\right\} + \dots \quad (20)$$

$$\dots + \frac{\bar{f}^{d}_{F_2W}\varphi_k}{(\varphi_k\,\varphi_k)}\left\{W^i - \frac{W^i\varphi_0}{(\varphi_0\varphi_0)}\varphi_0(W) - \dots - \frac{W^i\varphi_{i-1}}{(\varphi_{i-1},\varphi_{i-1})}\varphi_{i-1}(W)\right\}$$

$$(k = 0, 1, 2, \dots, p;\ i = 0, 1, 2, \dots, S-1).$$

For the descending (D) branch of the cycle of minimum values of ionization of the F_2 layer, we have:

$$\bar{f}^{M}_{F_2W} = \frac{(\bar{f}^{M}_{F_2W}\,\varphi_0)}{(\varphi_0\,\varphi_0)}\varphi_0(W) + \frac{(\bar{f}^{M}_{F_2}W\varphi_1)}{(\varphi_1\,\varphi_1)}\left\{W - \frac{W\varphi_0}{(\varphi_0\varphi_0)}\varphi_0(W)\right\} +$$

$$+ \frac{\bar{f}^{M}_{F_2W}\varphi_2}{(\varphi_2\varphi_2)}\left\{W^2 - \frac{(W^2\varphi_0)}{(\varphi_0\,\varphi_0)}\varphi_0(W) - \frac{W^2\varphi_1}{(\varphi_1\,\varphi_1)}\varphi_1(W)\right\} + \dots \quad (21)$$

$$\dots + \frac{\bar{f}^{M}_{F_2W}\varphi_k}{(\varphi_k\,\varphi_k)}\left\{W^i - \frac{W^i\varphi_0}{(\varphi_0\,\varphi_0)}\varphi_0(W) - \dots - \frac{W^i\varphi_{i-1}}{(\varphi_{i-1},\varphi_{i-1})}\varphi_{i-1}(W)\right\}$$

$$(k = 0, 1, 2, \dots, p;\ i = 0, 1, 2, \dots, S-1)$$

For the complete (A) and (D) 11-year march of nocturnal ionization of the F_2 layer we have:

$$\bar{f}^{n}_{F_2W} = \frac{(\bar{f}^{n}_{F_2W}\,\varphi_0)}{(\varphi_0\,\varphi_0)}\varphi_0(W) + \frac{(\bar{f}^{n}_{F_2W}\,\varphi_1)}{(\varphi_1\,\varphi_1)}\left\{W - \frac{W\varphi_0}{(\varphi_0\varphi_0)}\varphi_0(W)\right\} +$$

$$+ \frac{(\bar{f}^{n}_{F_2W}\,\varphi_2)}{(\varphi_2\,\varphi_2)}\left\{W^2 - \frac{W^2\varphi_0}{(\varphi_0\,\varphi_0)}\varphi_0(W) - \frac{(W^2\varphi_1)}{(\varphi_1\,\varphi_1)}\varphi_1(W)\right\} + \dots \quad (22)$$

$$\dots + \frac{(\bar{f}^{n}_{F_2W}\varphi_k)}{(\varphi_k\varphi_k)}\left\{W^i - \frac{(W^i\varphi_0)}{(\varphi_0\,\varphi_0)}\varphi_0(W) - \dots - \frac{W^i\varphi_{i-1}}{\varphi_{i-1}\,\varphi_{i-1}}\varphi_{i-1}(W)\right\}$$

$$(k = 0, 1, 2, 3, \dots, p)\ (i = 0, 1, 2, \dots, S-1)$$

For the ascending branch (A) of the 11-year march of the number of magnetic storms, we have:

$$\bar{N}_W = \frac{(\bar{N}_W \varphi_0)}{(\varphi_0 \varphi_0)} \varphi_0(W) + \frac{(\bar{N}_W \varphi_1)}{(\varphi_1 \varphi_1)} \left\{ W - \frac{W\varphi_0}{(\varphi_0 \varphi_0)} \varphi_0(W) \right\} +$$

$$+ \frac{(\bar{N}_W \varphi_2)}{(\varphi_2 \varphi_2)} \left\{ W^2 - \frac{(W^2 \varphi_0)}{(\varphi_0 \varphi_0)} \varphi_0(W) - \frac{(W^2 \varphi_1)}{(\varphi_1 \varphi_1)} \varphi_1(W) \right\} + \ldots \qquad (23)$$

$$\ldots + \frac{(\bar{N}_W \varphi_k)}{(\varphi_k \varphi_k)} \left\{ W^i - \frac{(W_i \varphi_0)}{(\varphi_0 \varphi_0)} \varphi_0(W) - \ldots - \frac{(W^i \varphi_{i-1})}{(\varphi_{i-1} \varphi_{i-1})} \varphi_{i-1}(W) \right\}$$

$$(k = 0, 1, 2, 3, \ldots, p;\ i = 0, 1, 2, \ldots, s-1).$$

For the numerical expression of the equations (20) to (23) so obtained, we calculate the coefficients A_k and the polynomials (W); and guiding ourselves by correlation table 1, we set up Table 8, below, which enables us to calculate the basic totals (cf. bottom line of Table 8).

(See following page for Table 8)

Such tables were set up for each of the curves on Figures 2, 3, 4 and 6, the branches of which, for the sake of precision in establishing the regressions, it is convenient to express as a non-linear function with the aid of the Chebyshev polynomial.

Expressing the totals through scalar ~~derivatives~~ products [14]:

$$\sum n_h = n = (\varphi_0 \varphi_0) \qquad \sum n_h W_h = (W) = (W\ \varphi_0),$$

$$\sum n_h W_h^2 = (W^2) = (W^2\ \varphi_0), \qquad \sum n_h W_h^3 = (W^3) = (W^3 \varphi_0),$$

$$\sum n_h W_h^4 = (W^4) = (W^4\ \varphi_0), \qquad \sum n_h \bar{f}_h = (\bar{f}_W) = (\bar{f}_W \varphi_0),$$

$$\sum n_h \bar{f}_h^2 = (\bar{f}_W^2) = (\bar{f}_W \bar{f}_W), \qquad \sum n_h \bar{f}_h W_h = (\bar{f}_W W),$$

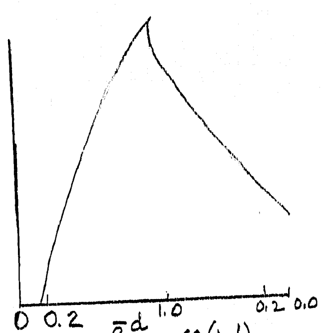
$$\sum n_h \bar{f}_h W_h = (\bar{f}_W^2 W),$$

Table 8

W_h	$\bar{f}_h$	n_h	$n_h W_h$	$n_h W_h^2$	$n_h W_h$	$n_h W_h^4$	$n_h \bar{f}_W$	$n_h \bar{f}_h$	$n_h \bar{f}_h^2$	$n_h W_h^2 \bar{f}_h$
W_1	$\bar{f}_1$	n_1	$n_1 W_1$	$n_1 W_1^2$	$n_1 W_1^3$	$n_1 W_1^4$	$n_1 f W_1$	$n_1 \bar{f}_1$	$n_1 \bar{f}_1$	$n_1 W_1 \bar{f}_1$
W_2	$\bar{f}_2$	n_2	$n_2 W_2$	$n_2 W_2^2$	$n_2 W_2^3$	$n_2 W_2^4$	$n_2 \bar{f} W_2$	$n_2 \bar{f}_2$	$n_2 \bar{f}_2^2$	$n_2 W_2^2 \bar{f}_2$
W_3	$\bar{f}_3$	n_3	$n_3 W_3$	$n_3 W_3^2$	$n_3 W_3^3$	$n_3 W_3^4$	$n_3 \bar{f} W_3$	$n_3 \bar{f}_3$	$n_3 \bar{f}_3$	$n_3 W_3 \bar{f}_3$
-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-
W_s	$\bar{f}_s$	n_s	$n_s W_s$	$n_s W_s^2$	$n_s W_s^3$	$n_s W_s^4$	$n_s f W_s$	$n_s \bar{f}_s^2$	$n_s \bar{f}_s^2$	$n_s W_s^2 \bar{f}_s$
		$\sum n_h$	$\sum n_h W_h$	$\sum n_h W_h^2$	$\sum n_h W_h^3$	$\sum n_h W_h^4$	$\sum n_h \bar{f} W_h$	$\sum n_h \bar{f}_h$	$\sum n_h \bar{f}_h^2$	$\sum n_h W_h^2 \bar{f}_k$

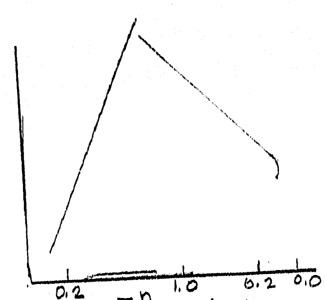
we calculate the coefficients of the polynomial (18) and of the polynomials $\varphi_i(W)$ for formulae (20) to (23).

Figure 12



$\bar{f}^{d}_{F_2} = \varphi(W)$

Figure 13



$\bar{f}^{n}_{F_2} = \varphi(W)$

As a result of this calculation we obtain, in place of formulae (20) to (23), the following computational equations for non-linear correlations:

$$\text{(B)} \quad \bar{f}^{d}_{F_2W} = 0{,}576 + 0{,}2\,W + 0{,}268\,W^2\,, \qquad (24)$$

$$\text{(H)} \quad f^{M}_{F_2W} = 0{,}46 + 0{,}384W + 0{,}108W^2\,, \qquad (25)$$

$$\text{(B)} \quad \bar{f}^{M}_{F_2W} = 0{,}484 + 0{,}68W - 0{,}094W^2\,, \qquad (26)$$

$$\text{(H)} \quad \bar{f}^{H}_{F_2W} = 0{,}647 + 0{,}3W - 0{,}018W^2\,, \qquad (27)$$

$$\text{(B)} \quad \bar{N}_{W} = 0{,}289 + 0{,}81W - 0{,}205W^2 \qquad (28)$$

Verifying calculations were made on the basis of these formulae, and from such calculations the theoretical graphs of the 11-year march of the critical frequencies and magnetic sttorms were then plotted (cf. Fig. 12 - 15). The values experimentally found were also plotted on these graphs. As may be seen from the sketches, there is satisfactory coincidence of the experimental curves with the theoretical ones, which enables us to make reliable forecasts of the critical frequencies and numbers of magnetic disturbances for any year of the 11-year period.

Figure 14

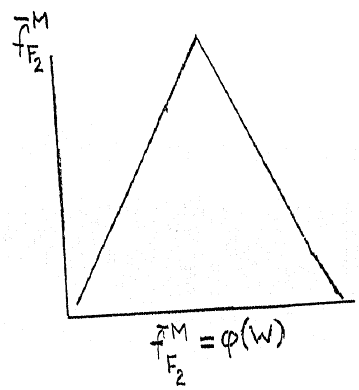


Figure 15,

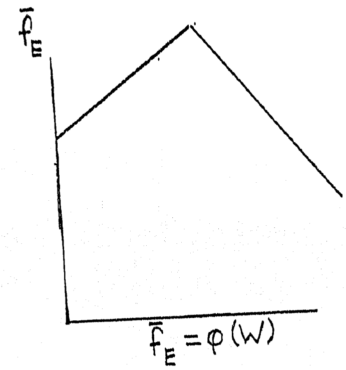
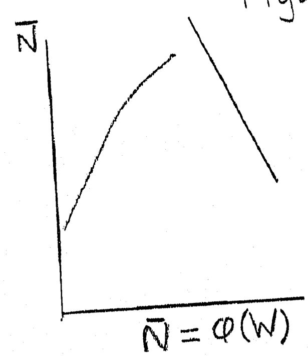


Figure 16.



Comparison of the results of the calculations obtained by using the linear correlation equations (cf. Figures 7-11) with the results using non-linear equations (cf. Figures 12-16) prove the superiority of the latter. But the accuracy is not so much increased in the latter case to make us abandon the use of the less cumbersome linear correlation equations in solving the problem of making forecasts in first approximation, which is adequate for practical purposes.

As experimental material accumulates at the Soviet ionosphere stations, the method of forecasting we have set forth may gain acceptance on the territory of the USSR; and it is proposed to devote one of the subsequent papers to its elucidation.

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