STAT

SUMMARY OF THE WORK 'REDUCIBLE SYSTEMS'

N. P. Yerugin

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Included below are the Foreword and Table of Contents, given first.7

Foreword

The investigation here is conducted on the basis of the theory of functions of matrices. All the knowledge necessary for this investigation, both relative to the theory of functions of matrices and relative to the analytical theory of differential equations, can be found in Volume III of V. I. Smirnov's Course. The starting point is the definition of reducible systems, which was given by Lyapunov /Liapounoff7 in his book General Problem of the Stability of Motion (ONTI, 1935; page 44).

In the completion of this work I was given substantial assistance by academician V. I. Smirnov, to whom I express my profound gratitude.

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SUMMARY /English-Language in the Original/

REDUCIBLE SYSTEMS

By N. Erougin /N. P. Yerugin/

Consider a system of homogeneous linear differential equations in the matric form

$$\frac{dX}{dt} = XP \tag{1}$$

where the elements of the matrix P are bounded continuous functions on the interval (0, ∞). Introduce the new unknown matrix Y by means of the equality

$$Y = XZ \tag{2}$$

where the elements of the matriz Z are differentiable functions on (0, ∞). We shall have an equation of the form

$$\frac{dY}{dt} = YB \tag{3}$$

where the matrix B depends on P, Z, and $\frac{dZ}{dt}$. The system (1) is said to be reducible if there exists a matrix Z bounded, together with $\frac{dZ}{dt}$ and the inverse of the determinant $D(Z^{-1})$, on $(0, \infty)$ such that all the elements of B in (3) are constant. A. Liapounoff in his famous paper "Le problème général de stabilité du mouvement" (Annales de Toulouse, 1907) gave the definition of reducible systems and revealed their role for the problem of stability of solutions of non-linear systems of differential equations. In the present paper the general theory of reducible systems is built, on the base of which the reducibility of some systems of differential equations of considerably general form is investigated. We give two necessary and sufficient criteria of reducibility and make clear to what extent the choice of the transformation matrix Z is arbitrary.

The first criterion of reducibility characterizes completely the solutions of reducible systems. We also give a rather simple necessary condition of reducibility. We give a number of methods for investigation of the systems from the viewpoint of their reducibility, which can be divided into two groups: the methods of successive approximations and the methods of reduction of the given systems to the systems of special form, the reducibility of which can be easily established.

The reducibility of the systems with the matrices P having limit values at infinity is completely studied, namely, some simple necessary and sufficient conditions of reducibility of such systems are given. Solutions of such systems are represented as series uniformly converging on the whole range of the independent variable t. This takes place, for instance, in the case, where the elements of the matrix P are analytic functions and $t = \infty$ is an irregular singular point of the system.

Certain systems, whose coefficients have no limit values at infinity, are also investigated.

We have also made clear under what variations of the coefficients at infinity the reducibility of the system and the reduced system rest invariant.

We have succeeded in generalizing the notion of reducibility to the case, where the coefficients of the system are not bounded. This has enabled us to establish the existence of bounded solutions of such systems and to represent the solutions as series uniformly converging on the whole infinite range of the independent variable.

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