The Radiational Properties of St. Clouds

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Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya, No 4, pp 92-117, Russian (article) bi-mo per, Jul/Aug 1951 - 187 T 30

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An approximate solution was obtained for the equation of the transfer of radiation energy in a cloud for a nonspherical indicatrix of diffusion. The magnitude of luminosity at the boundaries of the cloud in relation to the rays was calculated. The formulas are given for the calculation of albedo and the coefficients of permeability of direct and diffused solar radiation. The distribution of the height of flow of the long wave radiation in a cloud was computed. Formulas were set up for the calculation of albedo and the coefficient of a naturally radiating cloud in the long wave part of the spectrum.

In sending radiation energy through a cloud, part of the former is absorbed and diffused by water vapor and droplets of water. The study of these processes is related to the solution of the equation for the transfer of radiation energy, which presents the condition of horizontal uniformity in the atmosphere.

$$\cos\theta \frac{\partial I_\lambda(z;r)}{\partial z} = \alpha_\lambda(z) B_\lambda(T) + \frac{\sigma_\lambda(z)}{4\pi}\int I_\lambda(z;r')\gamma_\lambda(z;r';r)\,d\omega - (\alpha_\lambda+\sigma_\lambda) I_\lambda(z;r) \qquad (1)$$

Here $I_\lambda(z;r)$ is the intensity of radiation in the direction r, $B_\lambda(T)$ is the radiation of a black body with temperature T $\gamma_\lambda(z;r';r)$ is the indicatrix of diffusion, dw' is the elementary solid angle (integration is performed over the surface of a sphere of unit radius), $\alpha_2(z)$ and $\sigma_2(z)$ are coefficients of absorption and diffusion, referring to the unit volume; each of them contributes to the sum of the corresponding coefficients for water vapor (α_λ and σ_λ) and for water ($\alpha_{\lambda 2}$ and $\sigma_{\lambda 2}$). Together with the volumes let us also present the masses of the coefficients of radiation and diffusion $\bar{\alpha}_{i\lambda}$ and $\bar{\sigma}_{i\lambda}$.

It is clear that: $$\alpha_{i\lambda}(z) = \bar{\alpha}_{i\lambda}\rho_i(z)$$

and

$$\sigma_{i\lambda}(z) = \bar{\sigma}_{i\lambda}\rho_i(z)$$

when $i = 1, 2$ ($\rho_i(z)$ is the density of water vapor and water, respectively).

In setting up values of $T(z)$, $\alpha_{i\lambda}$, $\sigma_{i\lambda}$, $\rho_i(z)$ and $\gamma_\lambda(r; r'; z)$ it is possible to determine from equation (1) the intensity of radiation $I_\lambda(z; r)$.

1. The coefficients of absorption of water vapor and water, used in the present work, are quoted in Tables 1 and 2. The tables are compiled on the basis of experimental data [1-3], assembled from the works of Albrecht (the values of α_1 , when $0.59 \leq \lambda \leq 0.98\mu$) of K. Ya Kondrat'yev (α when $\lambda \geq 5\mu$) and Dorsey (α_1 when $0.9 \leq \lambda \leq 5\mu$ and α_2 for $0.4 \leq \lambda \leq 18\mu$.

It is impossible, however, to consider the values quoted in Tables 1 and 2 as definitely determined. The measurements were as a rule derived under laboratory conditions, at temperatures and pressures that were different from atmospheric conditions. Various researchers give values of coefficients differing considerably among each other.

TABLE 1

Coefficients of the Absorption of Water Vapor in sq cm gram

λ_μ	α cm^2/g	λ	α	λ	α
0.59 -0.615	0.008	1.935	31.5	7 -7.5	50
0.635-0.665	0.008	1.97	20.1	7.5-8	16
0.69 -0.74	0.054	2.0	6.0	8 -9	3.1
0.80 -0.84	0.40	2.48	43.9	9-12	0.1
0.90 -0.98	1.00	2.55	43.3	12-13	0.47
0.95	1.4	2.585	108.0	13-14	0.84
1.12	2.5	2.618	151.0	14-15	1.84
1.35	7.7	2.65	99.0	15-16	1.72
1.37	14.6	2.82	39.8	16-17	10.1
1.40	32.4	3.19	8.5	17-18	1.61
1.45	19.1	3.26	5.2	18-19	1.38
1.50	7.1	5 -5.5	38.0	19-20	4.60
1.80	6.2	5.5 -6	138.0	20-21	25.0
1.885	43.3	6 -6.5	220.8	21-23	58.0
1.90	22.3	6.5 -7	157	22-22	64.0

TABLE 2

The Coefficients of Absorption of Water

λ	α	λ	α	λ	α	λ	α	λ	α
0.400	0.005	1.30	1.48	2.10	31.6	5.47	335	7.88	775
0.745	0.044	1.35	9.14	2.147	27.8	5.80	928	7.94	690
0.845	0.044	1.40	3.05	2.15	24.7	6.00	2120	8.0	766
0.850	0.069	1.44	2.94	2.237	19.6	6.09	2530	8.065	785
0.900	0.0161	1.45	20.1	2.30	25.9	6.5	1040	8.13	765
0.950	0.0311	1.47	29.9	2.35	33.0	6.73	870	8.16	785
0.970	0.448	1.50	26.4	2.40	40.3	6.765	880	8.22	715
0.980	0.142	1.56	15.0	2.6	530	6.92	820	8.28	765
0.995	0.415	1.60	9.0	2.8	2240	6.955	830	8.38	695
1.05	0.368	1.677	5.2	3.0	7330	7.00	820	8.43	755
1.085	0.333	1.708	11.4	3.02	2730	7.11	820	8.49	725
1.095	0.188	1.75	16.0	3.2	6640*	7.275	845	9	700
1.13	0.29	1.85	12.7	3.4	1440	7.41	790	10	705
1.17	1.12	1.90	31.5	3.6	490	7.44	810	11	1200
1.20	1.22	1.95	86.0	3.93	203	7.49	800	12	2590
1.21	1.30	1.96	125.0	4.5	450	7.545	810	13	2890
1.24	1.22	1.97	111.0	4.7	545	7.65	765	15	3570
1.25	1.24	2.00	70.0	5.27	308	7.7	785	18	2990
1.25	1.17	2.08	36.0	5.42	342	7.83	765		

* Is also found to equal 2590

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2. Information about the values $\rho_i(z)$ and T (z) of the stratus clouds, type St, investigated by us was taken from various sources since no one published work known to us was based on simultaneous measurements of temperature, density of water and water vapor.

In Ye. G. Zak's article [4], the mean curves of temperature according to height distribution and the relative humidity in a St cloud are cited (thickness of cloud 600 meters, temperature diminished linearly with height from 0^{o} on the lower boundary, the temperature gradient being nearly moist-adiabatic). Computed from the data of Ye G. Zak, the density of water vapor proves to be equal to $4.7 \cdot 10^{-6}$ grams per cubic centimeter at the lower boundary and $4 \cdot 10^{-6}$ grams per cubic centimeter on the upper boundary. The decrease in density also occurs approximately in a linear fashion.

The data of other authors confirms the indicated order of magnitudes and the character of the change with height of temperature and density of water vapor in St clouds.

The mean density of water in a cloud can be computed from well-known formulas for a given range of visibility. For St. clouds, this density amounts to 0.20-0.3 grams per cubic meter [5]. Of late an increasing amount of information about direct measurements of the density of water in clouds is being presented [6].

The data on the magnitude and the change with height of the density of water in St Clouds was presented in an article by Neyburger [7]. According to the measurements of this author, density increases linearly with height almost up to the top of the cloud. The density gradient equalled 0.13 grams per cubic meter for 100 meters.

3. It is known that the diffusion of light in a moist atmosphere is approximately proportional to $\lambda^{-1.75}$. It may be assumed that the main centers of diffusion here are droplets of water, and not molecules of water vapor, and that therefore Reley's Law ceases to apply.

The diffusion of water droplets in a cloud was examined by us separately so far as the diffusion in water vapor was concerned, we presupposed that this was purely molecular diffusion which obeyed Reley's law:

$$\sigma_\lambda = \frac{32\pi^3}{3N\lambda^4}(m_0 - 1)^2 \tag{2}$$

Here N is the number of particles in a unit of volume, m_0 is the coefficient of refraction for water vapor which is equal to 1.000253 when $\lambda = 0.54\mu$ and e = 760 mm.

If, for water vapor tension e = 760 mm, and temperature $t=0^\circ$, assuming density to equal $8 \cdot 10^{-4}$ grams per cubic centimeter, then for λ, expressed in microns, we obtain from equation 2

$$\sigma_\lambda = \frac{10^{-5}}{\lambda^4} \text{ sq cm/g}$$

When the density of water vapor in a cloud equals $5 \cdot 10^{-6}$ grams per cubic centimeter,

$$\sigma_\lambda = \frac{5}{\lambda^4} 10^{-6} \text{ km}^{-1}$$

Hence, when $\lambda = 0.1\mu$, $\sigma = 0.005\ km^{-1}$,
when $\lambda = 1\mu$, $\sigma = 5 \cdot 10^{-6}\ km^{-1}$

4. The coefficient of diffusion for water droplets was determined by us on the basis of the calculations of Stratten and Houghton [8].

Assuming that the conductivity drop equals zero, these authors computed from Mie's formula some non-uniform characteristics of diffusion $K(\lambda, \rho)$ for the radii, commensurate with the length of the waves. The so-called "effective cross section" $\varepsilon(\lambda; \rho)$ is linked to the magnitude of K by the expression:

$$\varepsilon(\lambda; \rho) = 2\pi\rho^2 K(\lambda, \rho).$$

When $\frac{\lambda}{\rho} \to 0$ such that, beginning with $\lambda = \rho/6$, one may take approximately

$$\varepsilon(\lambda; \rho) = 2\pi\rho^2,$$

independently from the length of the wave.

The volumteric coefficient of diffusion for particles of radius ρ is expressed by $\varepsilon(\lambda; \rho)$ in the following form:

$$\sigma_{2\lambda}(\rho; z) = \varepsilon(\lambda; \rho) N(\rho; z),$$

where N $(\rho; z)$ is the number of particles with radius ρ, contained in a unit of volume of height z. The mean coefficient of diffusion $\sigma_{2\lambda}(z)$ of all particles contained in a unit of volume may be naturally defined in the following manner:

$$\sigma_{2\lambda}(z) = \bar{\varepsilon}(\lambda; z)\, n(z)$$

Here $\bar{\varepsilon}(\lambda; z)$ is the mean effective section which has been defined by the formula:

$$\bar{\varepsilon}(\lambda; z) = \frac{\int_0^\infty \varepsilon(\lambda; \rho) N(\rho; z)\, d\rho}{\int_0^\infty N(\rho; z)\, d\rho}$$

n(z) is the overall quantity of drops in unit volume by this relationship

$$n(z) = \int_0^\infty N(\rho; z)\, d\rho;$$

and by this relationship

$$\sigma_{2\lambda}(z) = \int_0^\infty \varepsilon(\lambda; \rho)\, N(\rho; z)\, d\rho. \qquad (3)$$

The magnitude $N(\rho; z)$ is probably derived from observation.

At present there is being published a sufficiency of material which concerns observations of the microstructure of clouds, and which also contains distribution curves of particles according to size. These curves generally deviate not from the unit of volume, but from the overall number of drops which have collected, on the instrument.

Assuming that the same distribution according to size is maintained in each cubic centimeter of volume from which drops fall onto the instrument, we determined $N(\rho; z)$ from

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the relation

$$N(\rho; z) = n(z)\, p(\rho; z),$$

where $p(\rho; z)$ is the per cent of particles of radius ρ removed with distribution curve.

If, instead of the usual number of particles, the density of water $\rho_2(z)$ is known, then

$$n(z) = \frac{\rho_2(z)}{\frac{4}{3}\pi \int_0^\infty \rho^3 p(\rho; n)\, d\rho} \qquad (4)$$

whence

$$\sigma_{2\lambda}(z) = \rho_2(z) \frac{\int_0^\infty \varepsilon(\rho) p(\rho; z)\, d\rho}{\frac{4}{3}\pi \int_0^\infty \rho^3 p(\rho; z)\, d\rho} \qquad (5)$$

After dividing the last equation by $\rho_2(z)$ we obtain an expression for the mass coefficient of diffusion which, as we will see, depends on the height, if the form of the distribution curve changes with height.

We computed the coefficient of diffusion $\sigma_{2\lambda}(z)$ from formula (5), given $\rho_2(z)$ and $p(\rho; z)$ from the data of Neyburger [6]:

$$\rho_2(z) = 0.13 \text{ g/cu M.} \qquad (6)$$

where z is expressed in hundreds of meters. The function is derived from the data of Table 3.

TABLE 3

$\rho\mu$	2-3	3-5	5-6	6-10	10-11	11-15	15-17	17-19
$p(\rho)$ %	2.7	3.4	10.4	15.4	7.4	2.7	0.7	1.3

The percentages computed for the interval of the length are equal to one micron. Such a distribution curve, according to Neyburger, corresponds to a height z=850 feet. Comparing the distribution curves pertaining to the different heights, this author arrived at the conclusion that with a change in height, the form of the curve is not changed. Other researchers indicated a change in the magnitude of $p(\rho)$ with height. However, for simplicity, we see that the actual work follows the hypothesis of Neyburger.

Computations from (4), (5) and (6) give

$$n(z) = 35 z \frac{1}{CM},$$

where z is measured in hundreds of meters

$$\sigma_{2\lambda}(z) = 35 z f(\lambda) \frac{1}{CM},$$

$$\bar{\sigma}_{2\lambda} = 2.7 \cdot 10^{8} f(\lambda) \frac{CM^2}{g},$$

where

$$f(\lambda) = \int_0^\infty \varepsilon(\lambda;\rho)\, p(\rho)\, d\rho.$$

In Table 4, the reductions are derived values of the coefficients

TABLE 4

λ_μ	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5	2	3	4	5	6	7	8	9
$\frac{1}{Z}\sigma_{2\lambda}(Z)$	21	21	21	21	21	21	21	21	22	22	22	22	24	27	28	29
$\bar{\sigma}_{2\lambda}$	1640	1640	1640	1640	1640	1640	1640	1640	1690	1690	1690	1690	1850	2070	2160	2200

Continuation of Table 4

λ_μ	10	11	12	13	14	15	16	17	18	19	20
$\frac{1}{Z}\sigma_{2\lambda}(Z)$	29	28	27	26	25	24	23	22	20	19	17
$\bar{\sigma}_{2\lambda}$	2220	2160	2100	2030	1930	1850	1740	1650	1540	1450	1330

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Usually in investigations of the microstructure of clouds, drop distribution curves are used which are similar dimensionally to curve 1, as represented in Figure 1. The data of Findeisen is an exception [9]. Curve 2, also in Figure 1, represents a typical distribution curve. The first type of curve is derived from the path of microphotographs of drops in a cloud; this being the case, as is known, the larger part of the small particles is lost. It is extremely probable that the Findeisen curves, derived by a method free from the above errors, is dimensionally nearer the true distribution of drops.

It is evident that if one can correct curve 1 in the area of the small radii so that it would become similar to curve 2, then the coefficient of diffusion can also be increased.

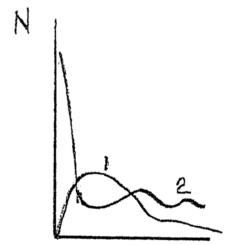

Figure 1

In order to ascertain how great this increase would be, we set up the coefficients side by side in Table 4, calculated the coefficients of diffusion for distribution curves of the second type, which coincided in the regions of large radii ($\rho \geq 4\text{-}5\mu$) with the given data presented in Table 3.

It has been shown that an increase of 700 particles of radius from 0.4 to 1 μ increases the coefficient of diffusion only in the light part of the spectrum and at the same time increases only 5 per cent of its previous size.

The variation of the distribution curve in the interval of radius 1-6 μ, increased the coefficient of diffusion 6-8 per cent for $0.4\mu \leq \lambda \leq 6\mu$. For very long waves, the value of σ is increased all the more significantly, reaching 0.5 per cent. In this way, with the addition of a specified number of small particles, it is not possible to provoke a noticeable increase in the value of the diffusion coefficients even in the light part of the spectrum. The particles with radii of 1-5 μ play a considerably greater role, and finally, exerting an influence on the size of the coefficient of diffusion the whole spectrum of long waves, it shows the presence of coarse particles $(\rho > 5\mu)$. The last deduction is linked to the extremely rapid growth of $\varepsilon(\lambda; \rho)$ for increases in radius.

In I. A. Khvostikov's article [10] it is shown that if the gravimetric concentrations of particles with a radius of 0.3 μ is only 5 per cent of the weight of the concentration of the particles with radius 6 μ, the first produces a diffusion of light similar to the second. Having continued these calculations, we deduced that for the accumulation of 5 per cent of the weight, it is necessary for each particle with radius 6 μ to have 400 particles with radius 0.3 μ. One particle of 20 μ diffuses similarly to 3500 particles of radius 0.3 μ. In a cloud with a volume of 1 cubic centimeter, there must always be more than ten coarse particles $(\rho > 5\text{-}6\mu)$. The reduced number shows that in 1 cubic centimeter there are present only ten thousand so-called "submicroscopic" particles; the coefficient of diffusion can, at their expense, be substantially increased in comparison with what was computed for curves of type 1.

From all that has been said, one may, it appears, to us, draw the following conclusion. For accurate determinations of the coefficient of diffusion in a cloud in the light and infra-red parts of the spectrum, it is extremely essential to know how to estimate the degree of loss of the small microscopic particles $(1\mu \leq \rho \leq 5\mu)$ in the process of measuring. Also very essential it is to clarify in the determination of the coefficient of diffusion in clouds of light waves, whether 1 cubic centimeter of volume can, in the active regions, contain ten thousand submicroscopic particles.

It is known that the experimental determination of the coefficient of diffusion in a cloud is derived from light wave lengths [11]. This being the case, these values were obtained: σ =38 km^{-1} for Scu clouds; 26 km^{-1} for St and 19.5 for Ast (12). In the latest work it was shown that these are probable and larger values of σ.

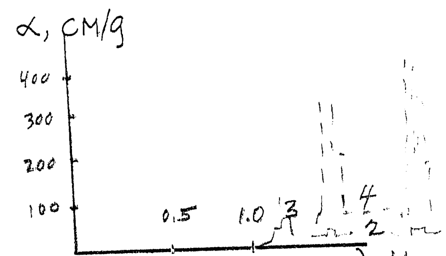

Figure 2.

Consequently, the experimental data confirms the value of the coefficient σ for light wave lengths computed by us.

In the long wave part of the spectrum, in view of the larger values of the coefficient of absorption, one can expect marked corrections in the calculations of the values of σ computed by us. Unfortunately, there is no data by which it would be possible to guess at the size of these corrections.

5. For the shortwave part of the spectrum, equation (1) can be written in the form:

$$\cos\theta \frac{\partial I_\lambda}{\partial z} = \frac{\sigma_\lambda}{4\pi}\int I_\lambda(z;r')\gamma_\lambda(z;r';r)\,d\omega' + \frac{\sigma_\lambda S_\lambda}{4} e^{-\sec\zeta\int_z^\infty(\sigma_\lambda+\alpha_\lambda)\rho(z)\,dz}\,\gamma(z;r_0;r)-(\alpha_\lambda+\sigma_\lambda)\,I_\lambda(z;r). \tag{7}$$

Here $I_\lambda(z;r)$ is the intensity of radiation diffusion, πS_λ is the intensity of direct solar radiation at the upper boundary of the cloud r_0 is the direction of decrease of the solar ray, ζ is the zenith distance of the sun. One may disregard the internal radiation of particles $\alpha_\lambda B_\lambda(T)$ under investigation in the interval of long waves.

In Figure 2, the mass coefficients of absorption and diffusion are represented: $\bar{\sigma}_{2\lambda}$ (curve 1), $\bar{\alpha}_{2\lambda}$ (curve 2), $\bar{\alpha}_{1\lambda}$ (curve 3), $10\bar{\alpha}_{1\lambda}$ [redacted] (curve 4).

The coefficient of diffusion of water vapor calculated in paragraph 3, is so small that it is immediately possible to disregard it.

The curves show that absorption is small in comparison to diffusion.

Since the density of water vapor is on an average ten times greater than the density of water in a cloud, the role of $\bar{\alpha}_{1\lambda}$ increases. However, a comparison of curve 1 with curve 4, which represents the coefficient of absorption as increased ten times, we see that even the latter is small everywhere in comparison to the coefficient of diffusion except in two parts of the spectrum, corresponding to the zones of absorption Ψ and Ω.

The comparison of coefficients shows that diffusion in droplets of water is a basic factor influencing short wave radiation passing through a cloud.

We shall solve equation (7) with the calculation of only one diffusion. The question of the role of absorption will be investigated separately.

Since in regions of few long waves, $\bar{\sigma}_{2\lambda}$, roughly speaking, does not depend on a long wave, the equation of transfer can be written for the integral of radiation intensity I(z;r), where

$$I(z;r)=\int_0^{\bar{\lambda}} I_\lambda(z;r)\,d\lambda$$

when $\bar{\lambda}=2\mu$ (it is supposed that the indicatrix likewise does not depend on long waves). Let us introduce visible thickness of the atmosphere:

$$\tau = \int_0^z \bar{\sigma}_2(z)\,dz,$$

where $\bar{\sigma}_2(z)=210z\ km^{-1}$, if z is expressed in kilometers. Hence

$$\tau=105\,z^2;\ \tau^*=105\,H^2$$

where H is the thickness of the cloud.

In this manner we obtain:

H_{km}	1	0.8	0.6	0.5	0.4	0.3	0.2	0.1
τ^*	105	67	38	26	17	9	4	1

Equation (7) now appears as:

$$\cos\theta \frac{\partial I}{\partial \tau} = \frac{1}{4}\int I(\tau; r')\,\gamma(\tau; r'; r)\,d\omega' + \frac{S}{4} e^{-\sec\zeta(\tau^* - \tau)}\,\gamma(z; r_0; r) - I(\tau; r). \quad (8)$$

The intensity of solar energy at the upper boundary of a cloud is defined from the relationship:

$$S = \bar{S} e^{-\alpha \sec\zeta \int_{z^*}^{\infty} \rho(z)\,dz},$$

$\pi\bar{S}$ is the solar constant, α is the mean coefficient of absorption of water vapor in short wave parts of the spectrum, z^* is the height of the upper boundary of the cloud.

From calculations it was derived that

$$\alpha = \frac{\int_0^{\bar{\lambda}} \alpha_\lambda d\lambda}{\Delta\lambda} \approx 0.3\ \mathrm{cm^2/g},$$

$$\rho(z) = \rho_0 e^{-\frac{(z - z^*)}{l}},$$

where $\rho_0 = 4.10^{-6}$ grams per cubic centimeter; $l = 2.6$ kilometers.

Because of the nature of border conditions, let us set the radiation intensity on the lower boundary of the cloud at $\theta < \frac{\pi}{2}$ (θ is the angle of ray r with the z axis) and on the upper boundary at $\theta > \frac{\pi}{2}$. Let us suppose, moreover, that the intensity of the radiation diffusion which comes from beyond the boundary of the cloud does not depend upon direction and is linked to the corresponding currents by these relationships:

$$I_1(0; \theta) = \frac{1}{\pi} F(0), \quad \theta \leqslant \frac{\pi}{2}$$

$$I_2(\tau^*; \theta) = \frac{1}{\pi} F_2(\tau^*), \quad \theta \geqslant \frac{\pi}{2} \quad (9)$$

Until now the indicatrix of diffusion γ (z;r';r) remained indeterminate. To begin with, let us suppose that γ depends only on cos (r';r). Let us present this dependence in the form of a series according to the Lagrange polynomial, whereupon we confine ourselves to breaking down the third term:

$$(r';r) = 1 + C_1P_1[\cos (r';r)] + C_2P_2[\cos (r';r)] \qquad (10)$$

When $C_1 = 1.425$ and $C_2 = 0.74$ we obtain the following values of (r';r):

(r';r)	0	15	30	45	60	75	90	105
(r';r)	3.165	3.063	2.716	2.222	1.620	1.069	0.630	0.339
(r';r)	120	135	150	175	180			
(r';r)	0.196	0.148	0.208	0.271	0.316			

We will solve approximately, equation (8), for the boundary conditions (9) and indicatrix (10), employing the method developed by Ye. S. Kuznechov [13].

If the indicatrix of diffusion was presented in series [10], one may point out that the derivation will have the form (Generally, the derivation will still contain the series with sin Ψ and sin 2Ψ. The coefficients for this series converge to zero, if, from the beginning of azimuth taking, the vertical plane of the sun is fixed, and if, furthermore, the boundary conditions do not depend on the azimuth):

$$I(\tau;r) = \frac{1}{2}A_0(\tau;\theta) + A_1(\tau;\theta)\cos\Psi + A_2(\tau;\theta)\cos 2\Psi, \qquad (11)$$

Where ψ is the azimuth of ray r.

The undetermined functions $A_0(\tau;\theta)$; $A_1(\tau;\theta)$; and $A_2(\tau;\theta)$ are determined from the system of equations:

$$\cos\frac{\partial A_\nu}{\partial \tau} = \frac{1}{2}\int_0^\pi A_\nu(\tau;\theta)\left\{\sum_{k=\nu}^{2} C_k \frac{(k-\nu)!}{(k+\nu)!} P_k^\nu(\cos\theta) P_k^\nu(\cos\theta')\right\}\times$$
$$\times\sin\theta' d\theta' + \frac{S}{2} e^{-\sec\zeta(\tau^*-\tau)} \sum_{k=\nu}^{2} (-1)^k C_k \frac{(k-\nu)!}{(k+\nu)!}\times \tag{12}$$
$$\times P_k^\nu(\cos\zeta) P_k^\nu(\cos\theta) - A_\nu(\tau;\theta), \quad \nu = 0, 1, 2.$$

The boundary conditions in relation to equation (9) must take the form:

$$\left.\begin{aligned} A_0(0;\theta) &= \frac{2}{\pi} F_1(0) \\ A_1(0;\theta) &= 0 \\ A_2(0;\theta) &= 0 \end{aligned}\right\} \text{for } \theta < \frac{\pi}{2}$$

$$\left.\begin{aligned} A_0(\tau^*;\theta) &= \frac{2}{\pi} F_2(\tau^*) \\ A_1(\tau^*;\theta) &= 0 \\ A_2(\tau^*;\theta) &= 0 \end{aligned}\right\} \text{for } \theta > \frac{\pi}{2} \tag{13}$$

Let us assume that the values of the function $A_\nu(\tau;\theta)$ are known to us at some n points $\mu_1, \ldots, \mu_n$, where $\mu = \cos\theta$. Let us compute $A_\nu(\tau;\mu)$ by its values at these points by means of interpolation of Lagrange's formulas:

$$A_\nu(\tau;\mu) = \sum_{j=0}^{n} A_\nu(\tau;\mu_j)\frac{(\mu-\mu_0)\cdots(\mu-\mu_{j-1})(\mu-\mu_{j+1})\cdots(\mu-\mu_n)}{(\mu_j-\mu_0)\cdots(\mu_j-\mu_{j-1})(\mu_j-\mu_{j+1})\cdots(\mu_j-\mu_n)} + R_n$$

Substituting the last expression in (12), assuming, moreover, that $\mu = \mu_1, \mu_2 \cdots \mu_n$, successively, and disregarding the remaining number R_n, we obtain n equations for the determination of the approximate value of the function $A_\nu(\tau; \mu)$ in n units of interpolation. Let us define the approximate expression $A_\nu(\tau; \mu)$ at the point $\mu_j - A_\nu^j(\tau)$. The equations which determine $A_\nu^j(\tau)$ will have the form:

[redacted]

$$\mu_j \frac{dA_\nu^j}{d\tau} = \frac{1}{2} \sum_{j=0}^{n} \frac{A_\nu^j(\tau)}{(\mu_j - \mu_0) \cdots (\mu_j - \mu_{j-1})(\mu_j - \mu_{j+1}) \cdots (\mu_j - \mu_n)} \times$$

$$\times \int_{-1}^{+1} (\mu' - \mu_0) \cdots (\mu' - \mu_{j-1})(\mu' - \mu_{j+1}) \cdots (\mu' - \mu_n) K_\nu(\mu'; \mu_j)\, d\mu' +$$

$$+ \frac{S}{2} e^{-\sec \zeta (\tau^* - \tau)} \sum_{k=\nu}^{2} (-1)^k C_k \frac{(k-\nu)!}{(k+\nu)!} P_k^\nu(\cos\theta) P_k^\nu(\cos\zeta) - A_\nu^j(\tau; \theta)$$

$$\nu = 0, 1, 2$$

Where

$$K(\mu'; \mu_j) = \sum_{k=\nu}^{2} C_k \frac{(k-\nu)!}{(k+\nu)!} P_k^\nu(\mu_j) P_k^\nu(\mu').$$

To each value of ν there corresponds n equations, where n is the number of units interpolated in Lagrange's formula.

Altogether in our case there will be 3n equations, which are separated into three systems which are uncombined among themselves, according to the n equations in each one. We confine ourselves to two groups of interpolation:

$$\mu_1 = -\frac{1}{2};\ \mu_2 = \frac{1}{2}$$

Then the equations will become:

$$-\frac{1}{2}\frac{dA_0^{(1)}}{d\tau} + A_0^{(1)}(\tau) = \frac{A_0^{(1)}}{2}\left(1+\frac{C_1}{3}\right) + \frac{A_0^{(2)}}{2}\left(1-\frac{C_1}{3}\right) +$$

$$+\frac{S}{2}e^{-(\tau^*-\tau)\sec\zeta}\left\{1+\frac{C_1}{2}\cos\zeta - \frac{C_2}{8}\left(\frac{3}{2}\cos^2\zeta - \frac{1}{2}\right)\right\};$$

$$\frac{1}{2}\frac{dA_0^{(2)}}{d\tau} + A_0^{(2)}(\tau) = \frac{A_0^{(1)}}{2}\left(1-\frac{C_1}{3}\right) + \frac{A_0^{(2)}}{2}\left(1+\frac{C_1}{3}\right) + \qquad (14)$$

$$+\frac{S}{e}e^{-(\tau^*-\tau)\sec\zeta}\left\{1-\frac{C_1}{2}\cos\zeta - \frac{C_2}{8}\left(\frac{3}{2}\cos^2\zeta - \frac{1}{2}\right)\right\};$$

$$-\frac{1}{2}\frac{dA_1^{(1)}}{d\tau} + A_1^{(1)}(\tau) = \frac{\pi\sqrt{3}}{32}\left(C_1+\frac{3}{4}C_2\right)A_1^{(1)} + \frac{\pi\sqrt{3}}{32}A_1^{(2)}\left(C_1-\frac{3}{4}C_2\right) -$$

$$-\frac{\sqrt{3}}{8}Se^{-(\tau^*-\tau)\sec\zeta}\sin\zeta\left\{C_1+\frac{3}{2}C_2\cos\zeta\right\}.$$

$$\frac{1}{2}\frac{dA_1^{(2)}}{d\tau} + A_1^{(2)}(\tau) = \frac{\pi\sqrt{3}}{32}\left(C_1-\frac{3}{4}C_2\right)A_1^{(1)} + \frac{\pi\sqrt{3}}{32}A_1^{(2)}\left(C_1+\frac{3}{4}C_2\right) -$$

$$-\frac{\sqrt{3}}{8}Se^{-(\tau^*-\tau)\sec\zeta}\sin\zeta\left(C_1-\frac{3}{2}C_2\cos\zeta\right).$$

$$-\frac{1}{2}\frac{dA_2^{(1)}}{d\tau} + A_2^{(1)} = \frac{3}{32}C_2A_2^{(1)} + \frac{3}{32}C_2A_2^{(2)} + \frac{9SC_2}{64}\sin^2\zeta e^{-(\tau^*-\tau)\sec\zeta},$$

$$\frac{1}{2}\frac{dA_2^{(2)}}{d\tau} + A_2^{(2)} = \frac{3}{32}C_2A_2^{(1)} + \frac{3}{32}C_2A_2^{(2)} + \frac{9SC_2}{64}\sin^2\zeta e^{-(\tau^*-\tau)\sec\zeta}$$

The boundary conditions for approximated equations we likewise retain, which are also for the points:

$$A_0^{(2)}(0) = A_0(0;\mu_2) = \frac{2}{\pi}F_1(0)$$

$$A_0^{(1)}(\tau^*) = A_0(\tau^*;\mu_1) = \frac{2}{\pi}F_2(\tau^*)$$

$$A_1^{(1)}(\tau^*) = A_2^{(1)}(\tau^*) = 0 \qquad (15)$$

$$A_1^{(2)}(0) = A_2^{(2)}(0) = 0$$

Determining $A_\nu^j(\tau)$, we compute $A_\nu(\tau;\mu)$:

$$A_\nu(\tau;\mu) = A_\nu^{(1)}(\tau)\frac{\mu-\mu_2}{\mu_1-\mu_2} + A_\nu^{(2)}\frac{\mu-\mu_1}{\mu_2-\mu_1},$$

but then for $I(\tau;\mu;\psi)$

$$I(\tau;\mu;\psi) = \left[\tfrac{1}{2}A_0^{(1)}(\tau) + A_1^{(1)}(\tau)\cos\psi + A_2^{(1)}\cos 2\psi\right]\frac{\mu-\mu_2}{\mu_1-\mu_2} +$$

$$+\left[\tfrac{1}{2}A_0^{(2)}(\tau) + A_1^{(2)}(\tau)\cos\psi + A_2^{(2)}(\tau)\cos 2\psi\right]\frac{\mu-\mu_1}{\mu_2-\mu_1}$$

$$-1 \leq \mu \leq 1.$$

Let us separate the rising intensity of radiation ($\theta \leq \frac{\pi}{2}$; $0 \leq \psi \leq 2\pi$) from the descending intensity of radiation ($\theta' = \pi - \theta$; $\psi' = \psi + \pi$). First is stated the formula for $\mu_1 = -\frac{1}{2}$; $\mu_2 = \frac{1}{2}$:

$$I_1(\tau;\mu;\psi) = \left[\tfrac{1}{2}A_0^{(1)}(\tau) + A_1^{(1)}(\tau)\cos\psi + A_2^{(1)}(\tau)\cos 2\psi\right]\left(\tfrac{1}{2}-\mu\right) +$$

$$+\left[\tfrac{1}{2}A_0^{(2)}(\tau) + A_1^{(2)}(\tau)\cos\psi + A_2^{(2)}(\tau)\cos 2\psi\right]\left(\tfrac{1}{2}+\mu\right). \quad (16)$$

The formula for the second is in the form:

$$I_2(\tau;\mu;\psi) = \left[\tfrac{1}{2}A_0^{(1)}(\tau) - A_1^{(1)}(\tau)\cos\psi + A_2^{(1)}(\tau)\cos 2\psi\right]\left(\tfrac{1}{2}+\mu\right) +$$

$$+\left[\tfrac{1}{2}A_0^{(2)}(\tau) - A_1^{(2)}(\tau)\cos\psi + A_2^{(2)}(\tau)\cos 2\psi\right]\left(\tfrac{1}{2}-\mu\right), \quad (17)$$

$$0 \leq \mu \leq 1;\ 0 \leq \psi \leq 2\pi$$

Relations (16) and (17) in the sense of clarification of the dependence of the unknown function $I(\tau; r)$, still gives to μ a very small but completely linear interpolation. In order to obtain the closest approximation to the exact solution, let us place sub-relations (16) and (17) in the main part of the accurate equations of transfer which are easy to determine from equation (8):

$$I(\tau;r) = I_1(0;r)e^{-\tau/\mu} + \frac{1}{\mu}\int_0^{\tau} K_1(t;r)e^{-\frac{\tau-t}{\mu}}\,dt + \frac{Se^{-\sec\zeta(\tau^*-\tau)}}{4\mu(\sec\zeta+\frac{1}{2})}\gamma_{21}(r_0;r) \tag{18}$$

$$I_2(\tau;r) = I_2(\tau^*;r)e^{-\frac{\tau^*-\tau}{\mu}} + \frac{1}{\mu}\int_{\tau}^{\tau^*} K_2(t;r)e^{-\frac{(t-\tau)}{\mu}}\,dt + \frac{S\left[e^{-\frac{\tau^*-\tau}{\mu}} - e^{-\sec\zeta(\tau^*-\tau)}\right]}{4\mu(\sec\zeta-\frac{1}{\mu})}\gamma_{22}(r_0;r)$$

Here

$$K_1(\tau;r) = \frac{1}{4\pi}\int I_1(\tau;r')\gamma_{11}(r';r)\,d\omega' + \frac{1}{4\pi}\int I_2(\tau;r')\gamma_{21}(r';r)\,d\omega'$$

$$K_2(\tau;r) = \frac{1}{4\pi}\int I_1(\tau;r')\gamma_{12}(r';r)\,d\omega' + \frac{1}{4\pi}\int I_2(\tau;r')\gamma_{22}(r';r)\,d\omega' \tag{19}$$

Integration is carried out for a hemisphere of unit radius.

$$\gamma_{11}(r';r) = \gamma_{22}(r';r) = 1 + C_1\mu\mu' + \frac{C^2}{4}(3\mu^2-1)(3\mu'^2-1) + \cos(\psi-\psi')[C_1 + 3C_2\mu\mu']\sqrt{1-\mu^2}\sqrt{1-\mu'^2} + \frac{3C^2}{4}(1-\mu^2)(1-\mu'^2)\cos(\psi-\psi'),$$

$$\gamma_{12}(r';r) = \gamma_{21}(r';r) = 1 - C_1\mu\mu' + \frac{C^2}{4}(3\mu^2-1)(3\mu'^2-1) - \cos(\psi-\psi')[C_1 - 3C_2\mu\mu'\sqrt{1-\mu^2}\sqrt{1-\mu'^2} + \frac{3}{4}C^2(1-\mu^2)(1-\mu'^2)\cos 2(\psi-\psi'). \tag{20}$$

According to the last formula the intensity of radiation arising from a cloud through its boundary was computed:

$$I_1(\tau^*;\mu;\psi) = \frac{1}{\pi}F_1(0)e^{-\frac{\tau^*}{\mu}} + \frac{S\gamma_{21}(r_0;r)}{4\mu(\sec\zeta_0+\frac{1}{\mu})} +$$
$$+\frac{1}{4}(1-\frac{2}{3}C_1\mu)\int_0^{\tau^*}A_0^{(1)}(t)e^{-\frac{\tau^*-\tau}{\mu}}dt + \frac{1}{4\mu}(1+\frac{2}{3}C_1\mu)\int_0^{\tau^*}A_0^{(2)}(t)e^{-\frac{\tau^*-\tau}{\mu}}dt +$$
$$+\frac{\pi\sqrt{1-\mu^2}}{16\mu}\cos\psi\Big\{\Big[C_1-\frac{3}{2}\mu C_2\Big]\int_0^{\tau^*}A_1^{(1)}(t)e^{-\frac{\tau^*-\tau}{\mu}}dt +$$
$$+\Big[C_1+\frac{3}{2}C_2\mu\Big]\int_0^{\tau^*}A_1^{(2)}(t)e^{-\frac{\tau^*-t}{\mu}}dt\Big\} +$$
$$+\frac{C_2}{8\mu}\cos 2\psi(1-\mu^2)\int_0^{\tau^*}\Big[A_2^{(1)}(t)+A_2^{(2)}(t)\Big]e^{-\frac{\tau^*-\tau}{\mu}}dt. \tag{21}$$

$$I_2(0;\mu;\psi) = \frac{1}{\pi}F_2(\tau^*)e^{-\tau^*/\mu} + \frac{S\left[e^{-\sec\zeta_0\tau^*}-e^{-\tau^*/\mu}\right]}{4\mu(\frac{1}{\mu}-\sec\zeta_0)}\gamma_{22}(r_0;r) +$$
$$+\frac{1}{4}\mu(1+\frac{2}{3}C_1\mu)\int_0^{\tau^*}A_0^{(1)}(t)e^{-t/\mu}dt + \frac{1}{4\mu}(1-2/3\,C_1\mu)\int_0^{\tau^*}A_0^{(2)}(t)e^{-t/\mu}dt +$$
$$+\frac{\pi\sqrt{1-\mu^2}}{16\mu}\cos\psi\Big\{\Big[C_1+\frac{3}{2}\mu C_2\Big]\int_0^{\tau^*}A_1^{(1)}(t)e^{-\frac{t}{\mu}}dt + \Big[C_1-\frac{3}{2}C_2\mu\Big]\times$$
$$\times\int_0^{\tau^*}A_1^{(2)}(t)\Big\} + \frac{C_2}{8\mu}\cos 2\psi(1-\mu^2)\int_0^{\tau^*}\Big[A_2^{(1)}(t)+A_2^{(2)}(t)\Big]e^{-t/\mu}dt \tag{22}$$

Relation (22) is significantly reduced, if the thickness of the cloud exceeds 100 meters. For such clouds, the value of τ^* is so great that the series of formula (22), containing $e^{-\tau^*/\mu}$ and $e^{-\sec\zeta_0\tau^*}$, seems small in comparison with other series.

Performing the integration and disregarding the small values, we determine for $I_2(0;\mu;\psi)$ the following simple expression)

$$I_2(0;\mu;\psi)=\frac{1}{\pi}F_1(0)\left[1-\frac{\mu+1/2}{(1-C_1/3)\tau^*}\right]+\frac{1}{\pi}$$

$$+\frac{1}{\pi}F_2(\tau^*)\frac{\mu+1/2}{1+(1-C_1/3)\tau^*}+S\cos\zeta\left(\mu+\frac{1}{2}\right)\left\{\frac{2\cos\zeta+\frac{1}{2}}{2[1+(1-C_1/3)\tau^*]}-\right. \tag{23}$$

$$\left.-\frac{\frac{C_1}{3}\cos\zeta+\frac{C_2}{8}(3\cos^2\zeta-1)\left[1-2\cos\zeta\left(\frac{C_1}{3}-1\right)\right]}{4\left[1+\left(1-\frac{C_1}{3}\right)\tau^*\right]}\right\}$$

Formula (23) illustrates that if the thickness of a cloud is not exceedingly small, the intensity of radiation, arising externally through its lower boundary, does not depend on the azimuth. The dependence on , just prior to the substitution in the accurate equations, remains linear, while this dependence diminishes with the amount of thickness of the cloud. Similar transformations can be carried out also in formula (21), in addition to that, however, it does not result in significant simplification of its form. In figures 5 and 6, the results of the values computed for the sizes of $I_2(0;r)$ and $I_1(\tau^*;r)$ for the following values of numerical parameters are height at the lower boundary of the cloud, 600 meters, thickness of it, 300 meters, $\tau^* = 9$; $S = 0.360$ gram-calories per square centimeter per minute, $\zeta = 60°$; $F_1(0) = 0.0367$ gram-calories per square centimeter per minute; $F_2(\tau^*) = 0.138$ gram-calories per square centimeter per minute.

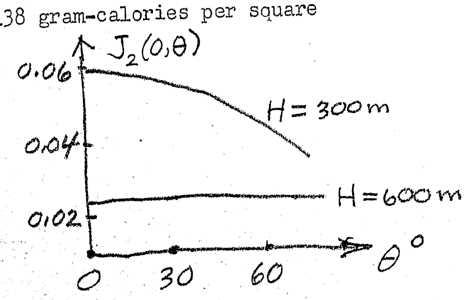

Figure 3.

These values of the current of radiation are derived from theoretical considerations. The calculations of $I_2(0;\theta$ are also carried out, and for H 600 meters, $\tau^* = 38$, with the retention of previous values of all remaining parameters.

TABLE 5

Values of I_2 $(0;\theta)$

$\theta°$	0	30	45	60	75
$I_2(0;\theta)$ for H = 300 M.	0.0608	0.0564	0.0513	0.0445	0.0365
$I_2(0;\theta)$ for H = 600 M.	0.0242	0.0241	0.0240	0.0239	0.0238

TABLE 6

Values of I_1 (   )

$\theta° \backslash \psi°$	0	45	90	135	180
0	0.172	0.172	0.172	0.172	0.172
15	0.171	0.171	0.175	0.181	0.182
30	0.176	0.177	0.182	0.195	0.203
45	0.187	0.187	0.196	0.222	0.227
60	0.205	0.204	0.020	0.267	0.295
75	0.234	0.232	0.261	0.339	0.384
90	0.297	0.298	0.343	0.471	0.543

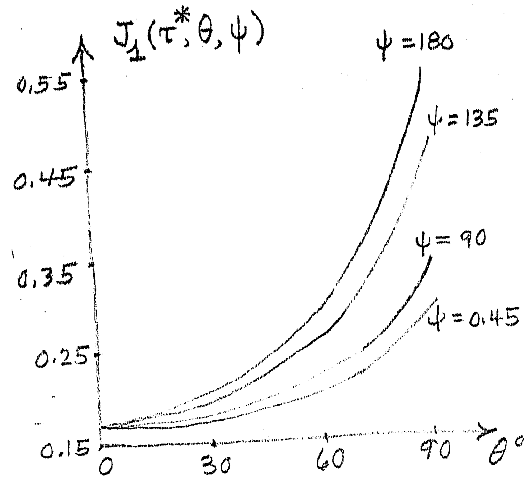

Figure 4.

The graph illustrates that $I_2(0,\theta)$ decreases with the magnitude of θ, i. e., according to the direction of the horizon. For the thickness of a cloud of 600 meters, the dependence of $I_2(0,\theta)$ upon direction becomes nearly imperceptible. $I_1(\tau^*; \theta)$ increases with the magnitude of azimuth and becomes maximal for $\psi = 180°$, i.e., for observations against the sun. On the plane of each azimuth, $I_1(\tau^*; r)$ increases with respect to direction, with the horizon.

Let us examine the current of radiation which arises from the boundary of a cloud. For a sufficient cloud thickness, disregarding small values, we obtain

$$F_1(\tau^*) = \int I_1(\tau^*, r)\cos\theta \, d\omega = F_2(\tau^*)\left[1 - \frac{7}{6[1+(1-C_1/3)\tau^*]}\right] + \frac{7F_1(0)}{6[1+(1-C_1/3)\tau^*]} + \pi S\cos\zeta \Bigg\{1 - \frac{7}{6}\frac{2\cos\zeta+1}{[1+(1-C_1/3)\tau^*]} + \frac{7}{6}\,\frac{\cos\zeta\frac{C_1}{3} + \frac{C_2}{8}(3\cos^2\zeta-1)\left[1-2\cos\zeta\left(\frac{C_1}{3}-1\right)\right]}{4\left[1+\left(1-\frac{C_1}{3}\right)\tau^*\right]} \qquad (24)$$

$$F_2(0) = \int I_2(\tau^*; r) \cos\theta \, d\omega = F_1(0)\left[1 - \frac{7}{6[1+(1-c_1/3)\tau^*]}\right] + \frac{7F_2(\tau^*)}{6\left[1+(1-\frac{c_1}{3})\tau^*\right]} + \pi S \cos\zeta \left\{ \frac{7}{6} \, \frac{2\cos\zeta + 1}{2\left[1+(1-\frac{c_1}{3})\tau^*\right]} - \frac{7}{6} \, \frac{\frac{c_1}{3}\cos\zeta + \frac{c_2}{8}(3\cos^2\zeta - 1)\left[1 - 2\cos\zeta\left(\frac{c_1}{3} - 1\right)\right]}{4\left[1+(1-\frac{c_1}{3})\tau^*\right]} \right. \tag{25}$$

Each number of the last formula has a simple physical meaning. The first factors in both expressions return to the atmosphere as a part of the incident diffusion on the boundary of a cloud of radiation. The albedo of a cloud of diffusion radiation equals

$$A_d = 1 - \frac{7}{6\left[1+(1-\frac{c_1}{3})\tau^*\right]} \tag{26}$$

on the lower, as well as the upper boundary. The equation of albedo on the boundaries is a consequence of the hypothesis of independence of mass diffusion coefficients according to height.

The second set of components in formulas (24) and (25) determine the permeability of a region of diffusion radiation.

The coefficient of permeability is

$$P_d = \frac{7}{6\left[1 + \left(1 - \frac{\tau^*}{3}\right)\tau^*\right]} \tag{27}$$

Formulas (26) and (27) show that, with the magnitude of cloud thickness, A_d increases as P_d decreases. For $\tau^* \to \infty$, $A_d \to 1$, $P_d \to 0$. In the case of nonspherical diffusion, permeability is great, but the albedo is smaller than in the spherical case.

In Figure 5 and Table 7, A_d and P_d are presented as functions of cloud thickness for the spherical case and selected by us as non-spherical diffusion indicatrices.

With an increase in cloud thickness, A_d and P_d at first change rapidly, and then the speed of change suddenly decreases.

The third number in the first part of expression (24) gives the reflection incidence on the upper boundary of rays of solar radiation. The corresponding series in (25) is a part of the solar energy of radiation, coming from the diffusion through the lower boundary of the cloud. A part of the rays of solar radiation must go through the lower boundary as parallel beams, the intensity of which is equal to

$$I_0 e^{-\sigma \sec \zeta \int_0^H \rho(z)\, dz}.$$

Since σ is large, this part of radiation converges to zero for essentially all thick clouds. Albedo for rays of solar radiation may be shown to equal

$$A_s = 1 - \frac{7}{12}\,\frac{2\cos\zeta_\odot}{1 + (1 - C_1/3)\tau^*} +$$

$$+ \frac{7}{24}\,\frac{\frac{C_1}{3}\cos\zeta + \frac{C_2}{8}(3\cos^2\zeta_\odot - 1)\left[1 - 2\cos\zeta_\odot\left(\frac{C_1}{3} - 1\right)\right]}{1 + \left(1 - \frac{C_1}{3}\right)\tau^*} \qquad (28)$$

In the case of spherical indicatrices,

$$A_s = 1 - \frac{7}{12}\,\frac{2\cos\zeta_\odot + 1}{(1 + \tau^*)}$$

The change of A_s with the change in the zenith distance of the sun is depicted in Figure 6 and Table 8.

TABLE 7

H, in km	A_d,% non-spher.	A_d,% spher.	P_d,% non-spher.	P_d, % spher.
0.1	23.	41	77	59
0.2	62	77	38	23
0.3	80	88	20	12
0.4	88	94	12	6
0.5	92	96	8	4
0.6	94	97	6	3
0.8	97	98	3	2
1.0	98	99	2	1

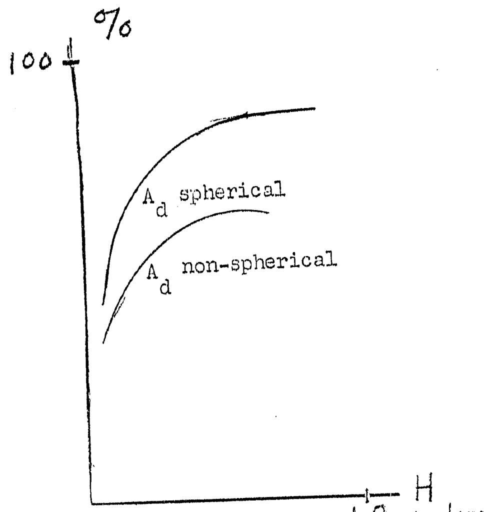

Figure 5.

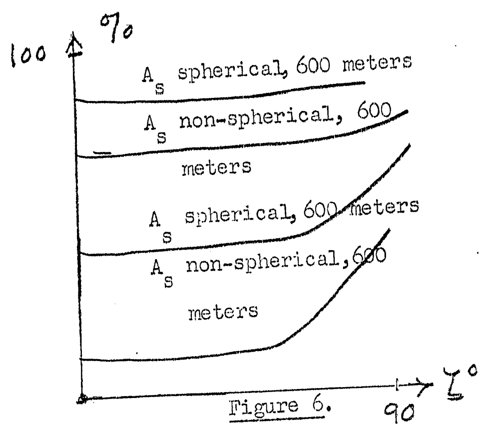

Figure 6.

Albedo grows with the increase of ζ, which has nothing to do with cloud thickness, nevertheless this increase is significant. The path of the curves in Figures 5 and 6 is in agreement with observed data. The calculations as to the value of albedo appear as rather dependent in comparison to observed values. It is possible to point out three reasons for obtaining too large values of albedo:

TABLE 8

	H = 300 M		H = 600 M	
ζ°	A_s, % nonspherical	A_s, % spherical	A_s, % spher.	A_s,% nonspher.
0	74	83	92.8	95.5
30	75	84	93.3	96.0
45	77	86	93.8	96.4
60	81	88	94.7	97.0
75	85	91	95.8	97.7
85	88	93	96.7	98.2
90	-	94	98.0	98.5

1. The principle selected by us for the distribution of the density of water according to height

$$\rho(z) = 0.13z \ g/M^3$$

gives, apparently, too large values of density on the upper layers of the cloud.

2. The series of observations shows that the predominant radius of drops in St clouds equals 4-6μ. The displacement to the left of the maximal distribution curves, in the same way as the decreased density of water, leads to a decreased value of τ^*, but subsequently, as formulas (26) and (28) show, also to a diminished value of the albedo.

3. Figures 5 and 6 show that for nonspherical diffusion the albedo is less than for spherical diffusion. Simple physical considerations show that the more elongated the indicatrix, the smaller the albedo. The true indicatrix of diffusion is more elongated than indicatrix (10), which diffuses little at the back, but very much on the side. The procedure for calculating a more elongated indicatrix, clearly must be derived from the smaller values of the albedo.

The choice of a more elongated indicatrix is connected to the increase in the number of terms separated (10). This being the case, consequently there is an increase in the number of systems of equations from which the values $A_j^j(\tau)$ are determined. By this method, the calculations become complicated, however, not so much so that working with them proves to be difficult in practice.

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6. If a long wave is greater than 2μ, it is possible to disregard the internal radiation of the atmosphere. At the same time, solar radiation takes place to a small degree in this part of the spectrum. The equation of transfer of radiation energy takes the form:

$$\cos\theta = \frac{\partial I_\lambda}{\partial z} = \alpha_\lambda B_\lambda(z) + \frac{\sigma_\lambda}{4\pi}\int I_\lambda(z; r')\gamma(r'; r)\,d\omega' - (\alpha_\lambda + \sigma_\nu) I_\lambda(z; r'). \tag{29}$$

In Figure 7, the coefficients $\bar{\sigma}_{2\lambda}$; $\bar{\alpha}_{2\lambda}$; $\bar{\alpha}_{1\lambda}$ and $10\bar{\alpha}_{1\lambda}$ are given for $\lambda \geqslant 2\mu$ (curves 1, 2, 3, 4, respectively).

The curves illustrate that $\bar{\alpha}_{2\lambda}$ is of a form similar to $\bar{\sigma}_{2\lambda}$. The coefficient $10\bar{\alpha}_{1\lambda}$ is small everywhere in comparison to $\bar{\alpha}_{2\lambda}$ and $\bar{\sigma}_{2\lambda}$ with the exception of the two intervals of long waves. All three coefficients change significantly with a change in the long wave. Consequently, it is necessary to introduce visible thickness for each long wave.

Neglecting the absorption of water vapor, we assume

$$\tau_\lambda = (\bar{\alpha}_{2\lambda} + \bar{\sigma}_{2\lambda})\int_0^z \rho(z)\,dz;$$

Hence

$$\tau^* = 0.065\,H^2(\bar{\alpha}_{2\lambda} + \bar{\sigma}_{2\lambda})$$

where H is in kilometers, and $\bar{\sigma}_{2\lambda}$ and $\bar{\alpha}_{2\lambda}$ are in square centimeters per gram.

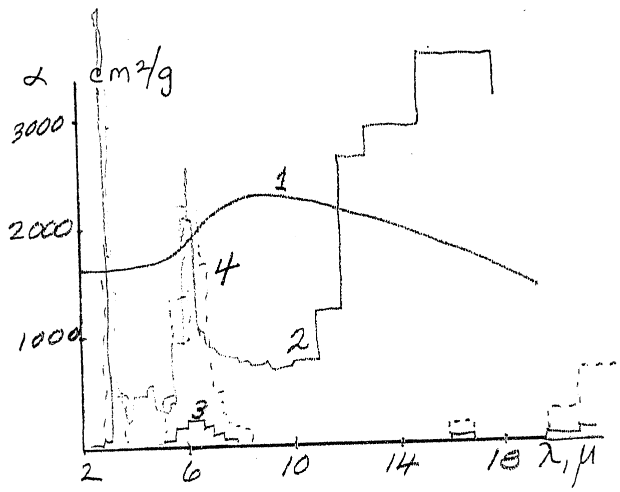

Figure 7.

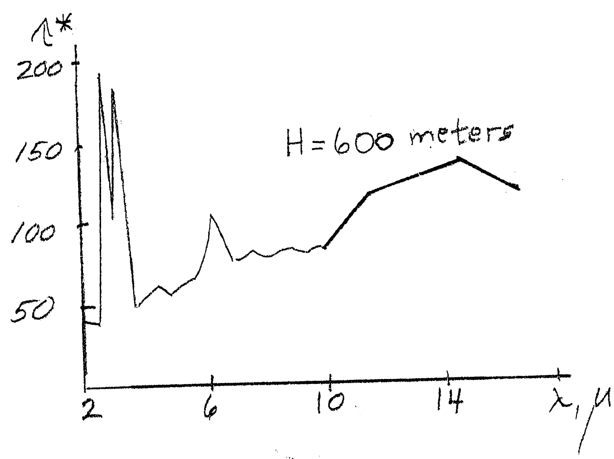

Figure 8.

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In Figure 8, the function $\tau^*(\lambda)$ is presented where H=600 meters. The minimum value for τ^*, equalling 39, correlates with $2\mu \leq \lambda \leq 2.6\mu$.

Equation (29), after substituting z for τ, appears as:

$$\cos\frac{\partial I_\lambda}{\partial \tau} = (1-q_\lambda) B_\lambda(\tau) + \frac{q_\lambda}{4\pi}\int I_\lambda(\tau; r')\,\gamma(r'; r)\,d\omega - - I_\lambda(\tau; r'), \qquad (30)$$

$$q_\lambda = \frac{\sigma_\lambda}{\alpha_\lambda + \sigma_\lambda}$$

For the spherical indicatrix of diffusion and for two sets of interpolation $\mu_1 = -\frac{1}{2}$ and $\mu_2 = \frac{1}{2}$, the approximated equations which correspond to equation (30) conform accurately to the equations of Schwartzhild. Solving the last equation for a given distribution of temperature according to height and a given current $F_{1\lambda}(0)$ and $F_{2\lambda}(\tau^*)$, we obtain the following expression for radiation current:

(31)

[See next page for formulas 31 and 32]

(32)

In Table 10 and Figure 9, calculations are presented according to these formulas of current distribution according to height. The computations resulted from six zones of long waves, which, as presented in Figure 9, it may be assumed with rough approximation, are constant.

In Table 9 the selected parts of long waves are quoted as

$$F_{1\lambda}(\tau) = \frac{1-\sqrt{1-g_\lambda}}{1+\sqrt{1-g_\lambda}}\left[F_{2\lambda}(\tau^*_\lambda) - \pi\left(1-\sqrt{1-g_\lambda}\right)\sqrt{1-g_{\lambda_0}} \times \right.$$

$$\left. \times \int_0^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}(\tau^*_\lambda - t)} dt\right] e^{-2\sqrt{1-g_\lambda}(\tau^*_\lambda - \tau_\lambda)} +$$

$$+\left[F_1(0) - \pi\left(1-\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda}\int_0^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}t} dt\right] e^{-}$$

$$^{-2\sqrt{1-g_\lambda}\tau_\lambda} + \pi\left(1+\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda}\int_0^{\tau_\lambda} B_\lambda(t) e^{-}$$

$$^{-2\sqrt{1-g_\lambda}} dt + \pi\left(1-\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda}\int_{\tau_\lambda}^{\tau^*_\lambda} B_\lambda(t) e^{-}$$

$$^{-2\sqrt{1-g_\lambda}(t-\tau_\lambda)} dt; \quad (31)$$

$$F_{2\lambda}(\tau) = \left[F_{2\lambda}(\tau^*_\lambda) - \pi\left(1-\sqrt{1-g_\lambda}\right)\right.$$

$$\left.\sqrt{1-g_\lambda} \times \int_0^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}(\tau^*_\lambda - t)} dt\right] e^{-2\sqrt{1-g_\lambda}(\tau^*_\lambda - \tau_\lambda)} +$$

$$+ \frac{1-\sqrt{1-g_\lambda}}{1+\sqrt{1-g_\lambda}}\left[F_1(0) - \pi\left(1-\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda} \times \int_0^{\tau^*_\lambda} B_\lambda(t) e^{-}\right.$$

$$\left.^{-2\sqrt{1-g_\lambda}t} dt\right] e^{-2\sqrt{1-g_\lambda}\tau_\lambda} + \pi\left(1-\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda}$$

$$\int_0^{\tau_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}(\tau_\lambda - t)} dt + \pi\left(1+\sqrt{1-g_\lambda}\right)\sqrt{1-g_\lambda})$$

$$\sqrt{1-g_\lambda}\int_{\tau_\lambda}^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}(t-\tau_\lambda)} dt. \quad (32)$$

well as values of the parameters (H = 600 meters) which correspond to them.

TABLE 9

λ, μ	$\frac{(\alpha+\sigma)}{cm^2/g}$	q	τ^*	$F_1(0)$ $\frac{gr/cal}{cm^2/min}$	$F_2(\tau^*)$ $\frac{gr/cal}{cm^2/min}$
2.0-2.6	1700	0.98	40	$0.0^{4}166$	$0.0^{5}528$
2.6-3.4	8000	0.2	180	$0.0^{3}113$	$0.0^{4}785$
3.4-5.8	2000	0.75	47	0.01256	0.00800
5.8-6.4	4000	0.50	94	0.01004	0.00628
6.4-11.8	2900	0.70	68	0.1540	0.1193
11.8-18	5000	0.40	117	0.1507	0.1256

TABLE 10

	$2\mu \leq \lambda \leq 2.6\mu$			$2.6\mu \leq \lambda \leq 3.4\mu$		
z, KM	F_1 (z)	F_2 (z)	πB (z)	F_1 (z)	F_2 (z)	πB (z)
0	$0.0^{4}1660$	$0.0^{4}154$	$0.0^{4}120$	$0.0^{3}113$	$0.0^{3}983$	$0.0^{4}975$
0.05	$0.0^{4}160$	$0.0^{4}148$	$0.0^{4}114$	$0.0^{4}978$	0.03957	$0.0^{4}957$
0.10	$0.0^{4}149$	$0.0^{4}139$	$0.0^{4}1117$	$0.0^{4}950$	$0.0^{3}950$	$0.0^{4}950$
0.20	$0.0^{4}122$	$0.0^{4}118$	$0.0^{4}107$	$0.0^{4}937$	$0.0^{4}937$	$0.0^{4}937$
0.30	$0.0^{4}107$	$0.0^{4}106$	$0.0^{4}103$	$0.0^{4}927$	$0.0^{4}927$	$0.0^{4}927$
0.40	$0.0^{4}103$	$0.0^{5}997$	$0.0^{5}995$	$0.0^{4}915$	$0.0^{4}915$	$0.0^{4}916$
0.50	$0.0^{5}951$	$0.0^{5}954$	$0.0^{5}958$	$0.0^{4}906$	$0.0^{4}906$	$0.0^{4}906$
0.55	$0.0^{5}903$	$0.0^{5}889$	$0.0^{5}939$	$0.0^{4}899$	$0.0^{4}899$	$0.0^{4}899$
0.60	$0.0^{5}699$	$0.0^{5}628$	$0.0^{5}916$	$0.0^{4}886$	$0.0^{4}785$	$0.0^{4}893$

z, Km	$3.4\mu \le \lambda \le 5.8\mu$ F_1 (z)	F_2 (z)	πB (z)	$5.8\mu \le \lambda \le 6.4\mu$ F_1 (z)	F_2 (z)	πB (z)
0.0	0.0126	0.0112	0.0105	0.01004	0.00873	0.00846
0.05	0.0119	0.0107	0.0102	0.00894	0.00836	0.00824
0.10	0.0107	0.0102	0.0100	0.00821	0.00817	0.00816
0.20	0.00979	0.00978	0.00978	0.00802	0.00802	0.00802
0.30	0.00963	0.00963	0.00963	0.00790	0.00790	0.00790
0.40	0.00945	0.00945	0.00945	0.00775	0.00775	0.00775
0.50	0.00927	0.00927	0.00927	0.00763	0.00763	0.00763
0.55	0.00916	0.00916	0.00916	0.00755	0.00755	0.00755
0.60	0.00670	0.00800	0.00906	0.00726	0.00618	0.00748

z, Km	$6.4\mu \le \lambda \le 11.8\mu$ F_1(z)	F_2(z)	πB(z)	$11.8\mu \le \lambda \le 18\mu$ F_1(z)	F_2(z)	πB(z)	$2\mu \le \lambda \le 18\mu$ F_1(z)	F_2(z)	$\overline{F}_1$(z)-$\overline{F}_2$(z)
0	0.1570	0.1440	0.1387	0.151	0.141	0.140	0.330	0.305	0.0255
0.05	0.1485	0.140	0.1366	0.142	0.138	0.138	0.311	0.297	0.0136
0.10	0.138	0.136	0.135	0.137	0.137	0.137	0.294	0.292	0.0024
0.20	0.134	0.134	0.134	0.136	0.136	0.136	0.288	0.288	0
0.30	0.132	0.132	0.132	0.135	0.135	0.135	0.285	0.285	0
0.40	0.131	0.131	0.131	0.134	0.134	0.134	0.282	0.282	0
0.50	0.129	0.129	0.129	0.133	0.133	0.133	0.280	0.280	0
0.55	0.128	0.128	0.128	0.132	0.132	0.132	0.278	0.278	0
0.60	0.125	0.119	0.128	0.131	0.126	0.126	0.272	0.259	0.0131

B (τ) for a fixed temperature is determined as an integral of the planck function according to the corresponding interval of long waves. The boundary conditions were fixed so that F_1 (0) in each part was larger than $\pi B(0)$, and F_2 (τ^*) was smaller than $\pi B(\tau^*)$.

In Table 10 alongside of the values of F_1 (z) and F_2 (z) are given the values of $\pi B(z)$ in each of the investigated intervals of long waves. The last columns of the table present a summary of the current F_1 (z) and F_2 (z), and the total current of radiation energy in the interval $(2\mu; 18\mu)$ is equal to $\overline{F}_1$ (z)-$\overline{F}_2$(z). These values are also presented in Figure 9 (the rates for F_i(z) and $\overline{F}_1(z) - \overline{F}_2(z)$ are chosen to be different).

The computations and the graph illustrate that only on the boundary layers is (F_1(z) different from F_2(z) and from $\pi B(z)$ In the interior parts of a cloud

$$F_1(z) = F_2(z) = \pi B(z) \tag{33}$$

for each of the long wave intervals studied. The total current, moreover, is equal to zero in a layer 100 meters $\leq$ z $\leq$ [redacted] 550 meters. The computed result is easy to explain, if one recalls relationship (14):

$$\lim_{\alpha_\lambda \to \infty} \alpha_\lambda \left[I_\lambda(z;r) - B_\lambda(z)\right] = 0 \tag{34}$$

when $0 \leq \theta \leq \frac{\pi}{2}$; $z_0 \leq z \leq z_1$; $z_0 > 0$; $z_1 < H$

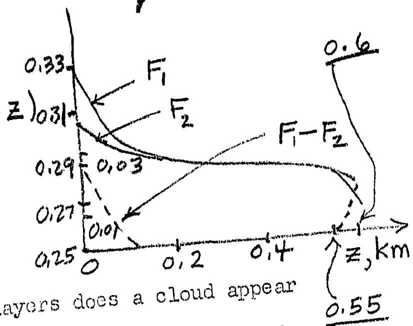
The distribution coefficients of a cloud are so large that for its interior layers that it accurately fulfills the equation

$$I_{\lambda i}(z;r) = B_\lambda(z)$$

Figure 9

whence

$$F_{\lambda i}(z) = \pi B_\lambda(z)$$



when i = 1.2.

In this way, only, on the boundary layers does a cloud appear active in relation to long wave radiation. It distributes energy in the lower layers, if $F_1(0) > \pi B(0)$, and it diffuses energy in the upper layers, if $F_2(\tau^*) < \pi B(\tau^*)$. According to the over-all thickness of a cloud, the total current of long wave radiation $F_1(z) - F_2(z)$ converges to zero and, moreover, in the heat process no influence is exerted.

For greater detail in the calculation of the spectrum, there are distributions, perhaps, in areas with sufficiently small distribution coefficients in which equation (34) will not be satisfied.

In the diffusion of the energy of appropriate long waves along with other aspects of heat inflow, heat exchange in a cloud will also be determined. We shall return to this question later in the article.

Let us examine the radiation current from a cloud through its boundary. For $\tau = \tau^*$ and $\tau = 0$, formulas (31) and (32) give, respectively

$$F_{1\lambda}(\tau^*_\lambda) = \frac{1-\sqrt{1-g_\lambda}}{1+\sqrt{1-g_\lambda}} F_{2\lambda}(\tau^*_\lambda) + 4\pi \frac{1-g_\lambda}{1+\sqrt{1-g_\lambda}} \int_0^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}(\tau^*_\lambda - t)} dt.$$

$$F_{2\lambda}(0) = \frac{1-\sqrt{1-g_\lambda}}{1+\sqrt{1-g_\lambda}} F_{1\lambda}(0) + 4\pi \frac{1-g_\lambda}{1+\sqrt{1-g_\lambda}} \int_0^{\tau^*_\lambda} B_\lambda(t) e^{-2\sqrt{1-g_\lambda}\, t} dt \qquad (35)$$

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The first terms of this formula are the reflected part of incident rays on the boundary of the radiation cloud.

The albedo of the cloud for a long wave λ is, disregarding cloud thickness, equal to

$$A_\lambda = \frac{1 - \sqrt{1 - q_\lambda}}{1 + \sqrt{1 - q_\lambda}} .$$

Strictly speaking, albedo, of course depends upon cloud thickness, but this dependence is expressed by exponential functions which converge to zero even for volumes of thin clouds.

In case of complete absorption $q_\lambda = 0, A_\lambda = 0$. For complete diffusion $q_\lambda = 1, A_\lambda =$ 100 percent. If the diffusion equals absorption $q_\lambda = 0.5$; $A =$ 18 percent.

In the second group of terms of formula (35) energies finally appear as arising from the boundary of the cloud at the expense of internal radiation.

We see that the current $F_{1\lambda}(\tau^*)$ and $F_{2\lambda}(0)$ in a general case cannot be expressed in the form

$$F_1(\tau^*) = AF_2(\tau) + (1+A)\pi B(\tau^*),$$

$$F_2(0) = AF_1(0) + (1-A)\pi B(0), \qquad (36)$$

and, consequently, radiation at the boundary of a cloud does not comply with Kirchhoff's Law (as was seen earlier, in the interior layer Kirchhoff's Law is satisfied).

If, in formula (35), applying the theory of the mean, the following is used:

$$F_1(\tau^*) = AF_2(\tau^*) + (1-A)\,\pi B(\xi_1),$$

$$F_2(\tau^*) = AF_1(0) + (1-A)\,\pi B(\xi_2). \tag{37}$$

Comparing expressions (36) and (37), we see that if the temperature of the cloud decreases with height, internal radiation on the upper boundary is greater, but on the lower boundary is less than the radiation according to Kirchhoff's Law. For an increase in temperature with height, all relationships are reversed, and only for constant temperature of the cloud is radiation from its boundary exactly conformant with Kirchhoff's Law.

For sufficiently thick clouds, slight deviations from Kirchhoff's Law are noted. In order to understand this, let us examine one of the integrated terms of formula (34), for example

$$4\pi \frac{1-g}{1+\sqrt{1-g}} \int_0^{\tau^*} B(t)\, e^{-2\sqrt{1-g}\,(\tau^*-t)}\, dt$$

For a change in t, B(t) changes much more slowly than $e^{-2\sqrt{1-g}\,(\tau^*-t)}$ as will be seen from the following relation.

Setting $\tau^* - t = a$, where a is chosen in order that $e^{-2\sqrt{1-g}\,a}$ is small in comparison to unity. The given difference $\tau^* - t$ corresponds to the change in z in the expression $H - z = \left(1 - \sqrt{1 - \frac{a}{\tau^*}}\right)$

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which will be much smaller than the larger value τ^* In the very unfavorable case $(2\mu \leqslant \lambda \leqslant 2.6\mu)$ when q = 0.98; $\tau^* = 40$, $e^{-2\sqrt{1-q}a}$ is small in comparison to unity for a=10. Then H-z=0.15 H=90 meters, if H=60, meters.

If height decreases from H to H-90 meters the temperature according to Ye. G. Zak rises from 1.9 to 1.4°, but the function $B_\lambda(\tau)$ always changes only about 5-6 per cent.

In the case of a larger q and a larger τ^*, i. e., for $\lambda \gg 3\mu$, $B(\tau)$ changes still less according to the exponential functions.

Consequently, we can roughly assume satisfaction of the following equations:

$$\int_0^{\tau^*} B(t)e^{-2\sqrt{1-q}(\tau^*-t)}dt = B(\tau^*)\int_0^{\tau^*} e^{-2\sqrt{1-q}(\tau^*-t)}dt,$$

$$\int_0^{\tau^*} B(t)e^{-2\sqrt{1-q}t}dt = B(0)\int_0^{\tau^*} e^{-2\sqrt{1-q}t}dt.$$

With such an assumption, the relations (35) develop into relations (36) and on the boundaries of the cloud they will satisfy Kirchhoff's Law. One can say that in case of larger τ^* on the boundaries of the cloud, the conditions of local thermodynamic balance are satisfied for the temperatures of these boundaries.

Let us compute by formula (35) the effective current on the cloud boundary.

For the lower boundary, we obtain

$$F_2(0) - F_1(0) = \frac{2\sqrt{1-q}}{1+\sqrt{1-q}}\left[\pi B(\tau^*) - F_2(\tau^*)\right].$$

For the upper boundary, we obtain

$$F_1(\tau^*) - F_2(\tau^*) = \frac{2\sqrt{1-g}}{1+\sqrt{1-g}}\left[\pi B(\tau^*) - F_2(\tau^*)\right].$$

If the temperature outside the cloud decreases with height, on the upper boundary effective radiation is always positive, and consequently, the cloud in its upper layers sends heat into the atmosphere and cools off.

In comparison to the lower boundary, it is not possible to arrive at such absolute conclusions. One can state that on the boundaries of transparent intervals in the spectrum of water vapor absorption, if diffusion of the earth is equal to black radiation, the effective radius will be negative, and the cloud will become warm. Generally, if assuming that $F_1(0) > B(0)$, and such an assumption is sufficiently evident, we obtain

$$F_2(0) - F_1(0) < 0.$$

The total amount of heat Q absorbable by a cloud is computed according to the formula:

$$Q = \frac{2\sqrt{1-g}}{1+\sqrt{1-g}}\left[F_2(\tau^*) + F_1(0) - \pi B(0) - \pi B(\tau^*)\right].$$

7. Let us examine once again Figure 2 and attempt to separate such part of the short wave spectrum in which it would pay to take into account absorption, aside from diffusion,

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The values $\overline{\alpha}_{1\lambda}$ and $\overline{\alpha}_{2\lambda}$ in some measure comparable with $\overline{\sigma}_{2\lambda}$, are all inclosed in the interval $1.3 \leqslant \lambda \leqslant 2\mu$. Particularly since here the coefficient of $10\alpha_{1\lambda}$ attains its greatest value.

At the first glance, it seems, however, that for the solution of our problems in regions of long wave lengths ($1.3;2\mu$), it is necessary to allow for diffusion in droplets of water and absorption of water vapor. But in these zones of the spectrum, where α is large, on the upper boundary of a cloud, solar energy is not emitted either partially or completely. The more it absorbs in the upper layers of the atmosphere, the more a small part of energy enters the spectrum of solar radiation in the long waves.

Thus, the effect of absorption of radiation energy in a cloud can be found only in these long waves for which $\overline{\alpha}_{1\lambda}$ is similarly small, and $\overline{\alpha}_{2\lambda}$ takes on its greatest values. One may point ot one such zone $\lambda \approx 1.8\mu$, in which $\overline{\alpha}_{1\lambda} \approx 0, \overline{\alpha}_{2\lambda} \approx 30\ \text{см}^2/\text{г}, \pi S = 0.08$ gram-calories per square centimeter per minute.

Let us assume σ = 100 sq centimeters per gram; then

$$q = \frac{\sigma}{\alpha + \sigma} = \frac{1000}{1030}.$$

The approximate equations of transfer in this case not being equal to zero, absorptions differ from equation (14) in that in the first part of these equations, for terms containing A_i^j and S, a factor q appears. In solving the equation, it is possible to take into account the total amount of heat which is absorbed in this cloud, according to the formula:

$$Q = \pi S \cos\zeta + F_1(0) + F_2(\tau^*) - F_1(\tau^*) - F_2(0).$$

For a given value of q, Q is approximately equal to zero. If by increasing α/σ, i. e., by changing q, then Q increases; for $\alpha = \sigma$, Q constitutes 72 per cent of all incident rays under study in the zone of long waves on the boundary of a cloud radiation. We see, consequently that the amount of absorption in cloud radiation energy depends upon the relation between the coefficients of absorption and diffusion. For these relations between the coefficients, which are located in the short wave part of the spectrum, absorption is so small that it is impossible even to locate it for an approximate solution to the equation of transfer. The small amount of absorption in a cloud of short wave radiation energy still does not denote that it does not influence the temperature of the cloud at all.

Let us write the equation of the balance of temperature

$$4\pi \int_{\bar{\lambda}}^{\infty} \alpha_\lambda B_\lambda d_\lambda = \int_0^{\bar{\lambda}} \alpha_\lambda \int I_\lambda \, d\omega \, d\lambda + \int \alpha_\lambda \int I_\lambda \, d\omega \, d\lambda + \varepsilon, \qquad (38)$$

where ε is the influx of temperature at the expense of all other forms of heat exchange.

Let us assume that in the long wave part of the spectrum, the coefficient of absorption does not depend on the wave length. Then equation (38) can be given in the following form.

$$4\pi B(\tau) = \int_0^{\bar{\lambda}} \frac{\alpha\lambda}{\alpha} \int I\lambda \, d\omega \, d\lambda + \int I \, d\omega + \varepsilon. \qquad (39)$$

The last expression shows that the role of short wave energy in the balance of temperature is ascertained by the ratio α_λ / α, i. e. as the relation between the coefficients of absorption in both short and long wave parts of the spectrum.

Let us assume

$$\alpha = 600 \ \text{см}^2/\text{г}$$

$$\alpha_\lambda = \bar{\alpha}_{1\lambda} + \bar{\alpha}_{2\lambda}$$

When $\lambda \leqslant 1.3\mu$ the last sum is small in relation to α.

In the region of greater wave lengths it is necessary to exclude the zones in which $\bar{\alpha}_{1\lambda}$ is great, since in these zones the intensity of radiation I_λ converges to zero.

It still remains to examine the zone, $\lambda \approx 1.8\mu$, in which $\alpha_{1\lambda} \approx 0$; $\alpha_{2\lambda}$ = 30 square centimeters per gram and $\frac{\alpha_\lambda}{\alpha}$ = 1/20.

While computing two integral terms in the right part of the balance, equation, we were convinced that the first of these consists of only 0.5 per cent of the second on the upper layer of atmosphere; at the lesser height, the role of the first term becomes even smaller. However, it would be premature on the basis of these computations to draw a conclusion concerning the small influence of short wave radiation in relation to long waves.

Let us give formula (38) the following form:

$$\int_0^\infty \alpha_\lambda \int I_\lambda \, d\omega \, d\lambda + \int_\lambda^\infty \alpha_\lambda \left[I_\lambda - B_\lambda \right] d\omega \, d\lambda + \varepsilon = 0. \qquad (40)$$

As was shown above, the coefficients of absorption in a cloud are so small that the subintegrated expression in the number of equation (40) converges to zero for all z in the interval $(z_0; z_1)$ and for all λ except, perhaps, in separate, narrow zones of the long wave spectrum.

Thus, in the second layer of a cloud the role of long wave radiation diminishes almost to zero and, correspondingly, the role of short wave radiation increases. The need of taking into account other forms of heat exchange increases also. It is precisely in a cloud, that the previous simplification of the heat exchange cycle for which the temperature is computed according to Stephan's Law, is especially unsatisfactory, but diffusion of short wave energy was simply disregarded. Only simultaneously, total and detailed calculation of all short wave radiation and of other aspects of temperature inflow will permit an accurate solution to the problem of heat exchange in a cloud.

THE END

BIBLIOGRAPHY

1. Albrecht, F. Eine einheitliche Darstellung des Absorptionsspektrums von wasserdampfhaltiger Luft and flussigem Wasser, Zeistschrift fur Meteorologie, No 7, 1947.

2. Kondrat'yev, K. Ya (On the Distribution of Long Wave Radiation in the Atmosphere), O pogloshchenii dlinnovolnov radiatsii v. atmosphere, Meteorology and Hydrology, No 6, 1947.

3. Dorsey, E, Property of Ordinary Water Substance, Reinhold Publishing Corporation, 1940.

4. Zak, Ye. G, Kharakteristika frontal'noy oblachnosti po dannym samoletnykh pod'yemov, (Characteristics of Front Cloudiness according to Data of Aircraft Elevation, Meteorology and Hydrology, No 8, 1937.

5. Diem, M., Messungen der Grosse fon Wolkenelementen. II. Met. Rundschau, H 9/10, 1948.

6. Zaytsev, V A, Vodnost' i raspredelenive kapel' v kuchevykh, oblakakh, Trudy GGO, (Water Content and Distribution of Drops in Cloud Masses, Proceedings of GGO), 19, 1950.

7. Neiburger, M, Reflection, Absorption and Transmission of Isolation by Stratus Clouds, Journal of Meteorology, volume 6, No 2, 1949.

8. Stratton, J A, and Houghton, H. G., A Theoretical Investigation of the Transmission of Light through Fog, Physical Rev., Volume 38, 1931.

9. Findeisen, W, Messungen der Grosse und Anzahl der Nebeltropfen zum Stadium der Koagulation inhomogenen Nebels, Gerlands Beitrage z. Geophysik, Volume 35, No 3/4, 1932.

10. Khvostikov, I A, Nekotorye voprocy fiziki tumanov, Izv, AN SSSR, Seriya geograf. i. geofiz., Some Problems of the Physics of Fog, No 3, 1942.

11. H. J. aufm Kampe, Visibility and Liquid Water Contents in Clouds in the Free Atmosphere, Journ. of Meteorology, Volume 7, No 1, 1950.

12. Bullrich, R, Lichtdurchlassigkeit in Wolken, Zeitschrift fur Meteorologie, No 11, 1948.

13. Kiznetsov, Ye S, Obshchiy Metod postroyeniva priblizhennykh uravneniy perenosa luchistoy energiv,(A General Method of Formation of Approximated Equations of Transfer of Radiation Energy), Izv. AN SSSR, Ser. No 4, 1951.

14. Feygel'son, E M, Raspredeleniye temperatury zemnoy atmosfery po vysote pri nalichii luchistogo i vertikal'nogo turbulentnogo teploobmena, Izv. AN SSSR, ser. geog. i geof., (The Distribution of Temperature in the Terrestrial Atmosphere according to Height of Effective Radiation and Vertical Turbulence of Heat Exchange, No 4, 1950.