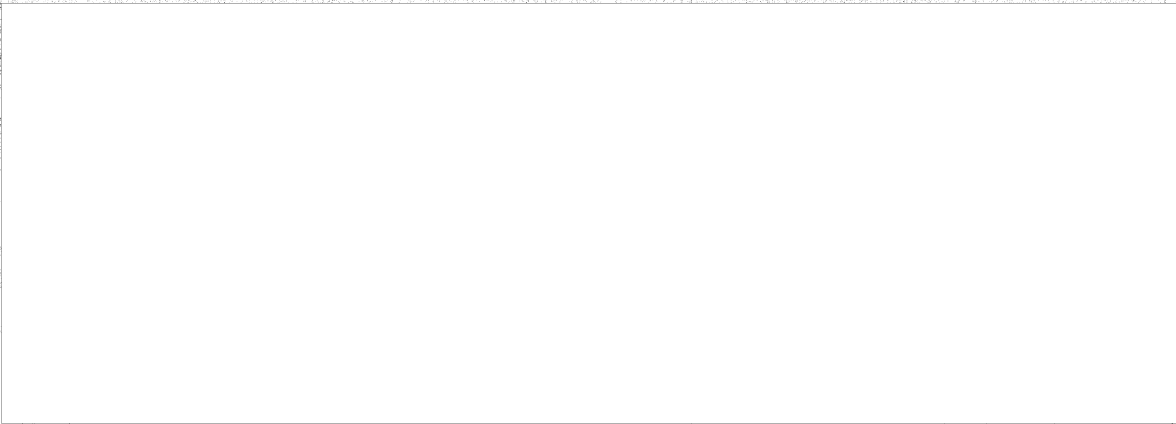


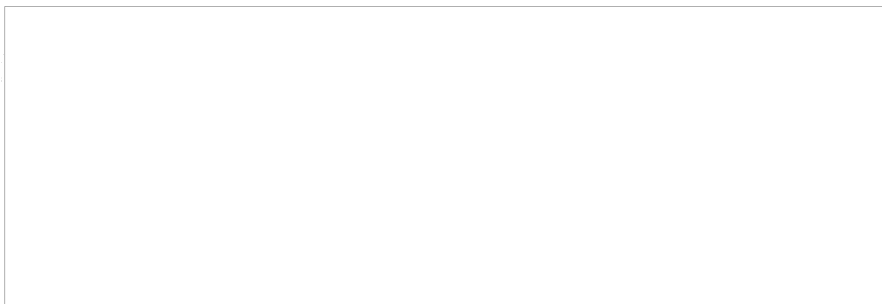
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REPENT SHOCKS IN SEISMIC OBSERVATIONS

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REPEAT SHOCKS IN SEISMIC OBSERVATIONS *

in
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Given here is a description of experiments on the determination of the function of the time of appearance of repeat shocks and their amplitude with respect to the magnitude of the charge Q and the depth h of an explosion. The graphs of the function of time ΔT between two successive shocks conform to the theory of vibrations of a gas globe in a liquid, taking into consideration the action of the boundaries of the section: the surface of the water and the bed of the water reservoir. The ratio (A_2 / A_1) of the amplitudes of the waves which correspond to the second shock (A_2) and the explosion (A_1) is diminished as Q is increased. With small values of Q repeat shocks were observed which were of greater intensity than the explosion. Theoretical research has been done on the problem of the intensity of repeat shocks. It has been shown that the cause of the observed ratios A_2 / A_1 may be the difference in the frequency spectrums of the explosion and the pulsation.

The presence of repeat shocks in explosions in water-reservoirs has been noted in a whole series of works [1-5, and others]. A. G. Ivanov [1] adduces some data on the difference in time ΔT between the explosion and the shock, and on the ratio of amplitude of the second shock to that of the first with different charges in the course of explosions in a river; in the work of A. A. Tsvetayev and N. I. Shapirovskiy [2] notice is taken of the fact of the presence of repeat shocks in explosions in the sea during

*[Note: See the original document for the figures 1-16; namely, *Izvestiya Akademii Nauk SSSR, Seriya Geofizicheskaya*, No 4 (July-August 1951), pages 43-60. See appendix for tracings of some figures.]

seismic prospecting using the reflected-wave method. The results of the majority of works by foreign authors, in which data are adduced on repeat shocks in explosions in water-reservoirs, are set forth in Koul's summarizing work [4].

In certain experimental projects notice has been taken of repeat shocks in explosions in drill-holes filled with water. But this phenomenon is encountered with considerably less frequency in seismic research, and we shall not go into it in this work.

The cause of repeat shocks is the vibrations of the gas bubble which is formed in explosions in water [3-5]. Photographs which illustrate the pulsation of the gas bubble are given in work [5].

Theoretical consideration of the problem of vibrations of a gas globe in water is given for the special case where the center of the bubble is motionless and the bubble is located in an unbounded liquid, and for the more general case where the influence of the boundaries of the section (the surface of the water and the floor or bed) and the migration of the bubble due to the action of gravitational force are taken into consideration [4].

In seismic investigations by the reflected-wave method and the correlation method of refracted waves, where a successive portion of the seismogram is used, the phenomenon of repeat shocks in explosions in water is a matter of great danger. Due to the presence of repeat shocks the wave picture may in some case be so strongly distorted that utilization of the successive portion of the seismogram, beginning with the time when the second shock is

registered, becomes impossible. If the presence of repeat shocks cannot be established by records, work with the successive portion of the seismogram may then lead to erroneous conclusions relative to the number of seismic boundaries of the section, their shape, and the angle of inclination. The well-known method of dealing with repeat shocks by decreasing the depth of the explosion or increasing the charge [4,5] is far from being always applicable. All this makes it essential to conduct special study of the phenomenon of repeat shocks in seismic investigations. Study of this phenomenon should, in particular, make it possible to determine the conditions under which repeat shocks do not occur or are of such small intensity that their presence cannot cause any serious distortion of the record.

In the seismic experiments of the Geophysical Institute in 1947-1949 we performed experiments to determine the dependence of the character of repeat shocks (amplitude, period, etc.) on the magnitude of the charge, the depth of the explosion, etc. We determined a number of dynamic peculiarities of the record of repeat shocks which had not been noted before. Below is given a description of the results obtained, with an attempt to interpret them.

1. THE RESULTS OF THE EXPERIMENTS

Conditions of the experiments.

The explosions were produced in water-reservoirs on the bottom or at some depth in a layer of water. In most cases the reservoirs were closed (quarries, ponds, etc.); the depth of the

reservoirs was from 1.5 to 15 meters, and they measured from 20 to 200 meters across. The explosions were usually produced in the approximate middle of the reservoir. To set up vibrations charges from 1 electro-detonator to 100 kilograms of dynamite were used. Registration of the vibrations was accomplished principally by means of a multiple-channel seismic station. The maximum of the frequency characteristic of the amplifiers was at a frequency of 60 cycles, and the width of the frequency-pass band, using a level of 0.7 of the maximum value, was equal to 100 cycles. A small scope of observations was achieved by means of apparatus tuned to register frequencies of the order of 80 cycles.

In the observations vertical electrodynamic seismographs with a period of natural oscillations of about 0.04 seconds were used. The distance from the seismographs to the point of explosion varied from 10 to 1000 meters. Repeat shocks were recorded for explosions in different reservoirs in the layer of water and on the bottom of the reservoirs.

Number of repeat shocks.

The maximum number of repeat shocks noted on the seismogram was four (Figure 1). In a large number of instances two shocks were noted on the records.

Magnitude of the charges and depth of the explosion.

Repeat shocks were observed with charges Q from 1 electro-detonator up to the maximum useable charge, equal to 100 kilograms. The depths h of the explosion varied from 1 to 15 meters. In Figures 1, 2(a,b), and 3 are given the seismograms obtained with

various charges; the repeat shocks are registered on them.

For a fixed depth h and an increase in the charge Q , the interval of time ΔT between two successive shocks (Figure 2, a, b) increases, and beginning with some limiting value of the charge Q_{lim} the repeat shocks disappear (Figure 2, c). The value Q_{lim} increases with increased depth, and is different for different reservoirs. For example, for one of the reservoirs, about 3 meters in depth with an explosion depth (from the surface of the water) of $h = 1.3$ meters, Q_{lim} was equal to 300 grams, and with $h = 2.5$ meters Q_{lim} was equal to 5 kilograms. For another reservoir, 15 feet deep, at $h = 1$ meter Q_{lim} was equal to 600 grams, and at depth $h = 2.5$ meters and with Q_{lim} equal to 5 kilograms a second shock was still observed. This difference in the values of Q_{lim} is associated, in all probability, with the different depths and cross-measurements of the reservoirs, i.e. with the different action of the boundaries of the section on the vibrations of the gas bubble.

The observed values of Q_{lim} at which the second shock disappears, are close to values determined from the following equation:

$$Q = \frac{(h + 10) h^3}{38}, \quad (1)$$

where Q is expressed in kilograms, and h in meters.

Q = 150 grams

h = 1.8 meters

Figure 1. Seismogram on which 4 repeat shocks are indicated.

Q = 1 el. det.

a.
h = 1.3 meters

Q = 75 grams

b.
h = 1.3 meters

Q = 300 grams

c.
h = 1.3 meters

Figure 2. Seismograms obtained with different explosions.

Q = 50 kilograms

h = 15 meters

Figure 3. Seismogram obtained with Q = 50 kilograms and h = 15 meters.

Q = 30 kilograms

h = 15 meters

Figure 4. Seismogram and photograph of water upheaval corresponding to it.

Equation (1) is obtained from the formula which related the maximum radius A_m of the bubble during the first pulsation with the quantities Q and h [4]:

$$A_m = 3.37 \left(\frac{Q}{h + 10} \right)^{1/3} \quad (2)$$

Assuming in formula (2) that $A_m = h$, which corresponds to the boundary case when during the first pulsation the radius of the bubble becomes equal to the depth of the explosion and the bubble bursts, we obtain equation (1).

In Table 1 are given values of Q calculated according to formula (1).

Table 1

h, m	1	2	3	5	8	10	15	20	30
Q, kilo-									
grams	0.3	2.5	9.2	49	240	525	2200	6300	27400

Repeat shocks and upheaval of water.

In some articles and reports the opinion has been expressed that when there is an upheaval of water with an escape of the gas repeat shocks should not be observed. In the work of Koul [4] a description is furnished of cases where repeat shocks were observed when there was an enormous upheaval of water, and an explanation of this phenomenon is given whose essence is as follows. A bubble which has formed during an explosion at a certain depth is raised upward due to the action of gravitational forces, and in so doing ^{executes} accomplishes pulsating vibrations. With a sufficient depth of explosion the bubble succeeds in ^{executing} accomplishing one or more vibratory movements between reaching the surface and bursting. To these vibratory movements correspond the repeat shocks on seismograms, and the upheaval of water corresponds to the bursting of the bubble.

In our experiments repeat shocks were also observed when there was an upheaval of water, which conforms to the results of work [4]. In some cases the upheaval was in the form of a large vertical column of water and was accomplished by the escape of

gases; in other cases a small upheaval of water was observed in the form of a column of spray rising with a rather large area. The cause of the upheaval in the first case was probably the rupturing of the gas bubble, which occurred at the moment when the pulsating bubble reached the surface of the water. The cause of the upheaval in the second case may have been the action on the surface of the water of the shock wave which was formed during the explosion and propagated in the water.

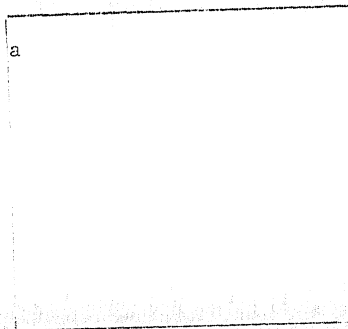
In Figure 4 is given a seismogram on which are noted repeat shocks and a photograph of the surface effect in the zone of the explosion on which corresponds to this record.

The interval of time between two successive shocks.

We shall denote by $\Delta T_{i,i+1}$ the interval of time between i and $(i+1)$ shocks, holding it to be the case that the first shock is caused by the explosion, and that successive shocks are caused by the vibrations of the gas bubble. The results of certain investigations performed for the purpose of determining the dependency of $\Delta T_{i,i+1}$ on the magnitude of the charge Q and the depth of the explosion h are given in Figure 5, a and b. In Figure 5a are represented graphs of $\Delta T_{12} = f(Q)$ for $h = 3, 5,$ and 8 meters, which have been drawn according to the data of observations of explosions in a body of water 15 meters deep. In Figure 6 are represented graphs of $\Delta T_{12} = f(Q)$ and $\Delta T_{23} = f_2(Q)$ when $h = 5$ meters, which have been drawn according to the data of observation of explosion in another body of water about 18 meters deep.

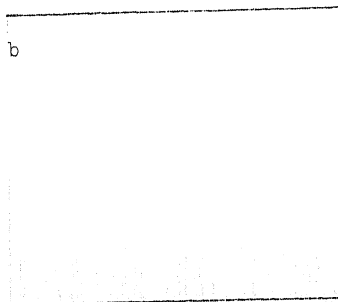
Consideration of the graphs indicates that the value $\Delta T_{i,i+1}$ increases with an increase in Q and a decrease in h , the quantity $\partial(\Delta T)/\partial Q$ decreasing with an increase in Q . The time

ΔT_{12} , seconds



Q , kilograms

ΔT_{12} , seconds



Q , kilograms

Figure 5. Graphs of the dependence of ΔT_{12} on Q : a gives the observed curves (solid lines) and the theoretical curves (dotted lines) according to formula (3); b gives the same curves according to formula (4). The individual dots on both graphs correspond to the observed data.

$\Delta T_{i,i+1}$ decreases with an increase in the number of the shock. The ratio $\Delta T_{23}/\Delta T_{12}$ changes little with a change in Q and h and is approximately equal to 0.7. The ratio $\Delta T_{34}/\Delta T_{23}$ was determined less reliably on the basis of a considerably smaller number of observations, and is approximately equal to 0.8-0.85.

The experimental data obtained have been compared with the theory of vibrations of a gas globe in water expounded in work [4], for two cases: (a) that of an unbounded medium and (b) that of a medium bounded by the surface of the water and the floor of the body of water.

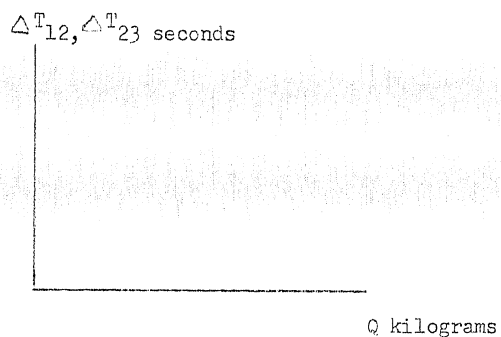


Figure 6. Graphs of the dependence of ΔT_{12} and ΔT_{23} on Q : (1) -- $\Delta T_{12} = f_1(Q)$ and $\Delta T_{23} = f_2(Q)$ are constructed according to the data of observations for $h = 5$ meters, $H = 18$ meters; (2) -- $\Delta T_{12} = f(Q)$ is calculated according to formula (4) for $h = 5$ meters, $H = 18$ meters.

In the case of an unbounded medium the dependence of the value ΔT_{12} on Q and h is expressed by the following formula:

$$\Delta T = KQ^{1/3} z_0^{-5/6} \quad (3)$$

where $z_0 = (h + 10)$ meters, and K is a constant coefficient which depends on the type of explosive substance.

In Figure 5a are given curves which have been calculated according to formula (3) for $K = 2.4$. The value K was determined on the basis of the data of the observations when $h = 8$ meters.

A comparison of the data of the observations and the calculation according to formula (3) shows that the observed values of ΔT_{12} depend considerably less on the depth h than follows from formula (3). By changing the value of K it is possible to achieve a satisfactory coincidence between the data of the theoretical calculations and one of the experimental curves ($h = \text{const}$) in a certain region of values of Q , but at the same time considerable departures take place for all other curves which correspond to other values of h and for the given curve with other values of Q . This is evidence of the inapplicability of formula (3) to the conditions of the experiment being described.

In the case of a medium bounded by a free surface and a bottom, the dependence of ΔT_{12} on Q and z_0 has the following form:

$$\Delta T_{12} = KQ^{1/3} z_0^{-5/6} \left[1 + \frac{0.905 Q^{1/3}}{b z_0^{1/3}} F(x) \right] \quad (4)$$

where $x = \frac{h - b}{h + b}$, and b and h are the corresponding distances from the site of the explosion to the bottom of the body of water and to the surface of the water; $F(x)$ is some function of x . Calculated values of $F(x)$ are given in work [4].

In Figure 5b are given curves calculated according to formula (4) when $K = 2.55$, for $h = 3, 5, \text{ and } 8$ meters with an overall depth of the body of water of $H = h + b$ equal to 15 meters. In Figure 6 are given curves calculated for $h = 5$ meters and $H = 18$ meters.

From consideration of Figures 5b and 6 it follows that the experiment is in close conformity to theory. This indicates that for the conditions of our experiment (relatively shallow bodies of water) in calculating the time ΔT_{12} it is necessary to take into consideration the action of the free surface of the water and the floor of the body of water. The presence of side boundaries of the body of water sufficiently removed from the point of the explosion, and likewise migration of the bubble due to the action of the force of gravity apparently have significant effect on the period of pulsation ΔT_{12} under the conditions of the experiment which is being described.

Shape of the record.

The shape of the record and the vibration frequencies predominating on the records remain in most cases approximately the same for successive shocks as for the first shock, which was caused by the explosion (Figures 1a; 2b; 7a); the direction of motion also does not change. On individual records a certain attenuation of the

high-frequency components of the vibrations caused by the shock is noticeable by comparison with the vibrations caused by the explosion (Figure 1b). On other seismograms (Figures 3,4) a weakening in the intensity of the high-frequency initial phases of the vibrations is noticeable, while the ensuing lower-frequency phases have an intensity close to the intensity of the corresponding phases of the explosion. A possible cause of these phenomena will be discussed below.

Amplitude of waves caused by repeat shocks.

Let us denote by A_1 , A_2 , and A_3 the amplitudes of the seismic waves caused respectively by the first, second, and third shocks (the first shock is caused by the explosion). Figure 8 gives graphs of the dependence of A_2/A_1 and A_3/A_2 on Q for small values of Q (of the order of hundreds of grams), constructed on the basis of the data of observations of explosions at depths of 1.3 and 1.75 meters in a body of water with overall depth of 1.75 meters. Figure 9 gives graphs of the dependence of A_2/A_1 on Q for large values of Q (of the order of tens of kilograms), constructed on the basis of data of observations of explosions at a depth of 5 meters in a body of water with overall depth of 15 meters. From consideration of the graphs of Figures 8 and 9 and the seismogram of Figures 1, 2a, and 2b it is evident that with small charges, Q , the amplitude of the second, and in certain cases also of the

third, wave exceeds the amplitude of the first shock; the ratio A_2/A_1 reaches 3.3, and the ratio A_3/A_1 reaches 1.5. The ratio A_2/A_3 is usually less than unity. With an increase in Q the ratios A_2/A_1 and A_3/A_2 decrease, with the decrease taking place more rapidly with small values of Q and considerably more slowly with large values of Q .

Thus, from consideration of the experimental data it is evident that with values of the charge Q from 1 electro-detonator up to as much as 100 kilograms the relative amplitude of the repeat shocks may be great, and their presence may cause a fundamental distortion of the record. With small values of Q the amplitude of the second shock considerably exceeds the amplitude of the first shock, caused by the explosion, in a number of cases. The amplitude of the repeat shocks caused by the pulsations decreases with an increase in the number of the shock in the overwhelming majority of cases.

$Q = 300$ grams
 $h = 2.5$ meters

Figure 7. Seismogram obtained for $Q = 300$ grams and $h = 2.5$ meters.

Figure 8. Graphs of the dependence of A_2/A_1 and A_3/A_2 on Q ; $h = 1.3$ meters and $h = 1.75$ meters.

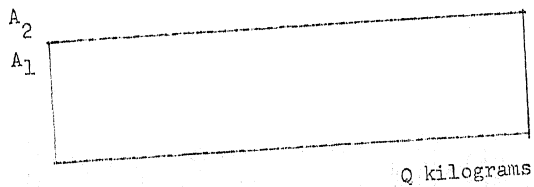


Figure 9. Graphs of the dependence of A_2/A_1 on Q ; $h=5$ meters.

$Q = 400$ grams
 $h = 1.3$ meters

Figure 10. Seismogram on which have been recorded two waves with different predominant frequencies.

Amplitude of sonic waves.

On some records sound waves (velocity 330 meters/second), apart from the seismic waves, are registered, corresponding to the first, second, and sometimes to the third shocks (Figures 1, 2a, 2b). The predominant frequency of the sound waves on the records is about 80 cycles, and that of the seismic waves is 50 to 60 cycles. The magnitude of ΔT_{ij+1} is identical for the sonic and the seismic waves. The amplitude ratios A_2/A_1 and A_3/A_2 for the seismic waves and B_2/B_1 and B_3/B_2 for the sound waves are different in the predominant number of cases.

Table 2 gives the observed values of B_2/B_1 and B_3/B_2 and the values of A_2/A_1 and A_3/A_2 which correspond to them, which were obtained in a body of water 1.75 meters deep (measurements of the amplitudes A and B were made on the same seismograms).

Table 2

h meters	Q grams	B_2/B_1	B_3/B_2	A_2/A_1 [sic]	A_3/A_2
1.3	1 electric detonater	1.98		3.28	
	20	0.81	0.5	1.3	1.12
	50	0.5		1.33	
	75	0.71	0.41	1.29	0.84
	100	0.55	0.44	1.12	1.0
	150	0.38		0.40	
1.75	50	0.70		0.89	
	75	0.33		0.65	
	100	0.51		0.76	0.66
	150	0.33		0.71	
	200	0.43		0.59	

As may be seen from the data given, the ratio B_2/B_1 is less than A_2/A_1 and B_3/B_2 is less than A_3/A_2 , which is evidence of the fact that with repeat shocks the conditions for the formation of high-frequency sonic waves are worse than for the lower-frequency seismic waves.

This phenomenon is not specific for sound waves. Qualitatively analogous relationships were obtained for two seismic waves corresponding to different boundaries of the section and characterized by a different predominant frequency. Figure 10 gives a seismogram on which have been recorded a wave t_1 with frequency of about 100 cycles and amplitude A, and a wave t_2 with frequency of about 200 cycles and amplitude B. The ratio of amplitudes $A_2/A_1 \approx 0.6$, and $B_2/B_1 \approx 0.3$; i.e. just as in the case of sound waves, with repeat shocks high-frequency waves are recorded with less intensity than low-frequency waves.

FREQUENCY SPECTRA OF THE EXPLOSION AND PULSATION

In works with which we are familiar, which consider the problem of repeat shocks, no theoretical explanation has been given of the dynamic peculiarities of the seismic records of repeat shocks and, in particular, of the amplitude ratio of the waves caused by the various shocks and the explosion.

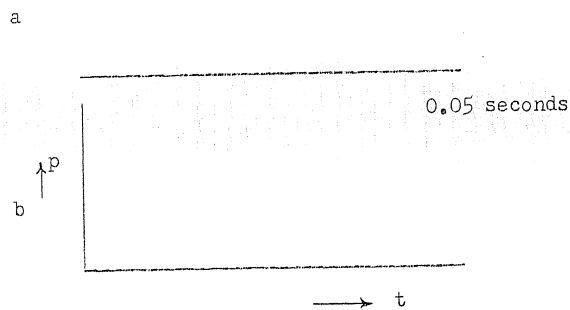


Figure 11. Experimental curves of pressure versus time for $p = p(t)$

We have made an attempt at qualitative explanation of the observable dynamic peculiarities of records of repeat shocks, using as a basis consideration of the frequency spectra of the explosion and the pulsations.

Measurements of the pressure in the water near the explosion point showed [3,4] that the variation in pressure in the course of the explosion and pulsation takes place according to various principles, and is distinguished by the velocity of pressure change and maximum values of pressure. When a high explosive is detonated the pressure increases instantaneously to the maximum value P_1 and then decreases rather rapidly. In the course of the pulsation the pressure changes takes place comparatively slowly -- the pressure increases first to the value P_i ($i = 2, 3, \dots, n$), and then falls off smoothly. The ratio P_1/P_2 , as follows from work [4], remains within the limits 5-10 in most cases. With an increase in the distance from the point of explosion the maximum values of pressure decrease, and the duration of the pressure impulse increases.

Figure 11 gives two experimental curves of $p=p(t)$; curve a for a charge of trotyl of $Q=200$ grams at a depth of 170 meters, recorded at a distance of 70 meters from the source of excitation of the vibrations (the curve is taken from work [3]; curve b for a charge of trotyl of $Q=137$ ^{kilo}grams at a depth of 15 meters, recorded at a distance of about 18 meters from the source (the curve is taken from work [4]).

As may be seen from Figure 11, the nature of the pressure

change is different in time with the explosion and with the pulsation, which should cause a difference in the frequency spectra of the vibrations produced.

It is possible to approximate the observed curve of pressure versus time $p = p(t)$ for the explosion with an exponential curve of the following type:

$$p = P_1 e^{-\beta_1 t} \quad (5)$$

We shall introduce the value Δt_1 , which defines the length of the impulse. We shall consider, according to Itskhoki [6], that the value Δt_1 is equal to the interval of time between the points at which the function $p(t)$ is equal to 0.1 of the maximum value. Under these conditions β_1 , and Δt_1 , will be related by the relationship:

$$\beta_1 \approx \frac{2.3}{\Delta t_1} \quad (6)$$

Substituting β_1 for Δt_1 in formula (5), we obtain

$$p = P_1 e^{-2.3 t / \Delta t_1} \quad (7)$$

The spectral function of the impulse of exponential form defined by formula (5), in which β_1 is expressed by Δt_1 , has the following form [6]:

$$s_1 = \frac{P_1 \Delta t_1}{14.4 \sqrt{1 + 7.44 f^2 \Delta t_1^2}} \quad (8)$$

where f is the frequency in hertzes*. The quantity $S_1(f)$ characterizes the amplitude of a harmonic component with frequency f . Drawing 12 represents the functions $p_1(t)$ and S_1/S_0 , where S_0 is the value of S when $f = 0$.

The graph in Figure 11 shows that the impulse of pressure, in the course of the pulsation, has a shape close to that of the bell-shaped function:

$$p = P_0 e^{-\beta t^2}, \quad (9)$$

except for the region of maximum values of pressure, where the pressure-time curve is closer in shape to a triangular impulse.

Figure 12 shows the bell-shaped and triangular impulses and the spectral functions which correspond to them. As may be seen from Figure 12, the spectra of the impulses of bell and triangular form are very close to each other at frequencies $f \leq \frac{2}{\Delta t}$, where Δt is the duration of the impulse. From subsequent exposition it will be clear that it is just the region of these frequencies that has the fundamental magnitude in seismic investigations.

The approximately uniform character of the spectral functions for the bell and triangle impulses, each of which approximates the observed curve $p = p(t)$ with varying degree of precision, makes it possible later on to consider a single one of these impulses. We shall approximate the observed pressure-time curve for the pulsation with a bell function of the form of (9). In so doing

* [Note: Hertz \equiv 1 cycle/second.]

the expression for the pressure which is developed by the i th pulsation is written in the form

$$p_i = P_i e^{-\beta_i t^2}. \quad (9')$$

The quantities β_i and Δt_i are related by the relationship:

$$\beta_i \approx \frac{9.2}{\Delta t_i^2}; \quad (10)$$

the value Δt_i is determined in a manner analogous to the foregoing (Figure 12).

Substituting Δt_i for β_i in formula (9'), we obtain

$$p_i = P_i e^{-9.2(t/\Delta t_i)^2}. \quad (11)$$

The spectral function of an impulse of type (11) has the following form [6]:

$$S_i = \frac{P_i \Delta t_i}{10.8} e^{-1.07 f^2 \Delta t_i^2}. \quad (12)$$

From consideration of formulas (8) and (12) and from the curves constructed according to them it follows that the spectral composition $S_1(f)$ and $S_i(f)$ of the time functions $p_1(t)$ and $p_i(t)$ is distinguished mainly by the fact that in the spectrum $S_1(f)$ the high frequencies have a larger relative weight than in the spectrum $S_i(f)$. In the spectrum of the bell impulse $S_i(f)$ frequencies $f > \frac{2}{\Delta t}$ are not present, and in the spectrum of the exponential impulse

the high frequencies have a greater intensity.

Figure 12. The pressure-time curves of $p = p(t)$ and their spectral functions S/S_0 : 1 -- the spectrum of the triangular impulse; 2 -- the spectrum of the bell impulse; 3 -- the spectrum of the exponential impulse.

For comparison of the theoretical data with the data of observations it is necessary to proceed from pressure to the velocity of displacement of the particles of the medium, since the electrodynamic seismographs used in the research register a value close to the velocity of displacement (the frequency of inherent oscillations of the seismograph was about 25 cycles and the predominant frequency of the recorded vibrations was to 60-100 cycles.)

In the case of flat waves being propagated in a liquid, and also in the case of spherical waves with sufficiently large distances r from the source of the vibrations, the incremental pressure and velocity of displacement are expressed by functions of identical type [7].

For an approximate estimation of the nature of the phenomenon which is being described, we shall assume that in the case under investigation the velocity of displacement is expressed by a function of the same type as the pressure, differing only in the constant multiplier m . The expression for the velocity of displacement v is written respectively for the explosion and the pulsation in the form:

$$v_i = A e^{-2.3t/\Delta t_1}, \quad (7')$$

$$v_i = B_i e^{-9.2(t/\Delta t_i)^2}, \quad (11')$$

where $A = mP_1$; $B_i = mP_i$; m is the coefficient.

The spectral composition of the functions $v_1(t)$ and $v_i(t)$ is defined by the following formulas:

$$\frac{S_1}{A} = \frac{\Delta t_1}{14.4 \sqrt{1+7.44 f^2 \Delta t_1^2}}, \quad (8')$$

$$\frac{S_i}{B_i} = \frac{\Delta t_i}{10.8} e^{-1.07 f^2 \Delta t_i^2}. \quad (12')$$

In order to calculate the spectral functions according to formulas (8') and (12') it is necessary to estimate the order of the possible values of Δt_1 and Δt_i , which define the length of the impulse. It is obvious that the values of Δt will be a function of the magnitude of the charge Q . For a given distance from the source small values of Δt_1 and Δt_i should be characteristic for small charges Q , since with small charges the processes of explosion and pulsation run their course more rapidly than with large charges. Large values of Δt_1 and Δt_i should occur with large charges. The ratio $\Delta t_2/\Delta t_1$ is always greater than two, since the fall in pressure occurs more rapidly for the explosion than for the pulsation. The ratio $\Delta t_{i+1}/\Delta t_i$ is always greater than unity, since each successive pulsation is more prolonged than the preceding one [4].

We have at our disposal a limited number of data as to the absolute values of the quantities Δt_1 and Δt_2 ; the order of the maximum possible values of Δt_1 and Δt_2 may be determined on the basis of the seismic records. On the seismograms the duration of the record of the first wave, caused by the repeat shock, usually does not exceed 0.04 seconds with a charge $Q \ll 100$ kilograms. Taking into consideration that the duration τ of the residual processes is of the order of 0.015 seconds for the apparatus which was used, we determine that the maximum duration of the impulse must be less than 0.025 seconds. The corresponding maximum value Δt_1 must be less than 0.012 seconds (from the condition that $\Delta t_2/\Delta t_1 < 2$).

The following values of Δt_1 and Δt_2 for a charge $Q = 200$ grams may be determined on the basis of the curves in Figure 11: $\Delta t_1 \approx 0.0005$ seconds; $\Delta t_2 \approx 0.0015$ seconds; $\Delta t_3 \approx 0.003$ seconds.

On the basis of the data given we have adopted the following limits for the values Δt_1 and Δt_2 : $0.0001 \leq \Delta t_1 \leq 0.001$ seconds; $0.001 \leq \Delta t_2 \leq 0.03$ seconds.

As noted above, the ratio $\Delta t_2/\Delta t_1$ should be greater than two. For the curves of Figure 11a this ratio is approximately equal to three, and for the curves of Figure 11b it is equal approximately to five. We do not know what may be the maximum value of this ratio. Nor is it clear how the magnitude of the ratio $\Delta t_2/\Delta t_1$, varies with a change in the charge Q .

Figure 13 shows, on a semilogarithmic scale, curves of the dependence of S_1/A and S_2/B_2 on the frequency, for values of Δt_1 equal to 0.001 , 0.003 , 0.001 , 0.003 , and 0.01 seconds; and for values of Δt_2 equal to 0.001 , 0.003 , 0.006 , 0.01 , 0.015 , 0.02 , and 0.03 seconds.

A consideration of the curves of Figure 13 shows that the spectral composition of the explosion and the pulsation is different. In the spectrum of the explosion (the curves of S_1/A) the amplitude of the vibrations composing the spectrum remains almost unchanged with frequencies from 0 to 400 cycles; only with relatively large values of $\Delta t_1 > 0.003$ seconds (a prolonged duration of the impulse may correspond to large charges) does the decrease in the magnitude of S_1 become more perceptible in the region of low frequencies. In the region of high frequencies, at the same time, the change in S_1/A takes place comparatively slowly.

Figure 13. Graphs of S_1/A and S_2/B with respect to the frequency f .

In the spectrum of the pulsation (the curves of S_2/B_2) the energy of the impulse is concentrated in a more narrow band of frequencies than in the spectrum of the explosion. The oscillations with frequency $f = 0$ possess the greatest amplitude. With an increase in frequency the amplitude decreases rapidly, this decrease occurring more abruptly as the length of the impulse Δt_2 becomes greater.

THE AMPLITUDES OF THE WAVES

Using the information obtained above with respect to the frequency spectra of the explosion and the pulsations, let us attempt to explain the data observed with respect to the relationships of the ^{the} amplitudes of the waves caused by the explosion and the various pulsations. In particular, let us attempt to explain the cause of the large amplitude of the oscillations caused by the pulsation, by comparison with the amplitude of the oscillations caused by the explosion.

The relationship of the amplitudes of the waves caused by the first pulsation and the explosion.

Let us find the relationship of the amplitudes of the identical harmonics composing the spectrum of the oscillations caused by the first pulsation and the explosion. According to formulas (12') and (8'), the ratio S_2/S_1 depends on the frequency f in the following manner:

$$\frac{S_2}{S_1} = 1.34 \frac{P_2}{P_1} \sqrt{\frac{(\Delta t_2)^2}{(\Delta t_1)^2} + 7.44 f^2 \Delta t_2^2 e^{-1.07 f^2 \Delta t_2^2}} \quad (13)$$

Figure 14. Graphs of the dependence of S_2/S_1 on the frequency f ;

$$\frac{P_2}{P_1} = 0.2.$$

Figure 14 gives the curves of the dependence of S_2/S_1 on the frequency f for the same values of Δt_2 as were adopted in

calculating the curves of Figure 13 and for a ratio of $\Delta t_2 / \Delta t_1$ equal to 3, 5 and 10. The ratio P_2/P_1 , according to experimental data [3,4], was taken as equal to 0.2.

Analysis of formula (13) and consideration of the curves of Figure 14 show that the ratio S_2/S_1 decreases with an increase in the frequency f ; the decrease in S_2/S_1 occurs the more abruptly as Δt_2 and $\Delta t_2 / \Delta t_1$ become larger. When $f = \text{const}$ S_2/S_1 is greater as $\Delta t_2 / \Delta t_1$ becomes greater and Δt_2 becomes smaller. When $\frac{\Delta t_2}{\Delta t_1} > 3.7$ (determined on the basis of formula (13) for

$\frac{S_2}{S_1} = 1$) the ratio S_2/S_1 is greater than unity in a certain

region of frequencies near $f = 0$. This band of frequencies becomes broader as $\frac{\Delta t_2}{\Delta t_1}$ becomes greater and Δt_2 becomes smaller.

Thus, from the calculations which have been performed it follows that the ratio of the amplitudes of the oscillations which correspond to the repeat shock and the explosion should decrease with an increase in the duration of the impulses, i.e. with an increase in the charges. With a short duration of the impulses and a ratio $\frac{\Delta t_2}{\Delta t_1} > 3.7$ the amplitude of the second shock may be greater than the amplitude of the explosion in a certain band of frequencies.

Seismic apparatus is usually endowed with selectivity. In tuning the apparatus to various ranges, different relationships of

the amplitudes of the oscillations corresponding to the repeat shock and the explosion should be obtained. This ratio of amplitudes, with other conditions equal, should be greater as lower-frequency apparatus is used in the observations. When the apparatus is tuned to register frequencies of the order of 30 to 80 cycles, which usually takes place in seismic exploration, the amplitude of the second shock may be greater than the amplitude of the explosion when the impulse are of short duration and when

$$\frac{\Delta t_2}{\Delta t_1} > 4.$$

The conclusion reached as to the fact that at frequencies of 30 to 80 cycles the amplitude A_2 of the shock may be larger than the amplitude A_1 of the explosion when the impulse is of short duration, i.e. when the charges are small, is in good correlation with the data of the observations. The ratio $\frac{A_2}{A_1} > 1$ occurred with small

charges $< 100-300$ grams. Comparison of the results of the calculation and the observations makes it possible to assume that with small charges the ratio $\frac{\Delta t_2}{\Delta t_1}$ may be greater than four.

The relationship of the amplitudes of waves caused by two successive pulsations.

The spectral functions of the two successive shocks caused by the i th and $(i+1)$ th pulsations are expressed by the following formulas:

$$S_i = \frac{B_i \Delta t_i}{10.8} e^{-1.07 f^2 \Delta t_i^2}, \quad (12'')$$

$$S_{i+1} = \frac{B_{i+1} \Delta t_{i+1}}{10.8} e^{-1.07 f^2 \Delta t_{i+1}^2} \quad (12''')$$

The ratio S_{i+1}/S_i will be expressed by the formula:

$$\frac{S_{i+1}}{S_i} = \frac{P_{i+1}}{P_i} \frac{\Delta t_{i+1}}{\Delta t_i} e^{-1.07 f^2 \Delta t_i^2 \left[\left(\frac{\Delta t_{i+1}}{\Delta t_i} \right)^2 - 1 \right]} \quad (14)$$

Each successive shock is characterized by the fact that $P_{i+1} < P_i$, $\Delta t_{i+1} > \Delta t_i$ ($i > 2$).

In the calculations the same values of Δt_i were adopted as were adopted above for Δt_2 . When $i > 2$, $\Delta t_i > \Delta t_2$. The ratio

$\Delta t_{i+1}/\Delta t_i$ must be greater than unity. On the basis of a small number of experimental pressure-time curves a ratio

$\Delta t_3/\Delta t_2$ was determined which did not exceed 2 to 3. In calculations the maximum value of $\Delta t_{i+1}/\Delta t_i$ was adopted as equal to 10.

Figure 15 gives graphs of the dependence of S_{i+1}/S_i on f when Δt_i is equal to 0.001, 0.006, 0.01, 0.02, and when $\Delta t_{i+1}/\Delta t_i$ is equal to 2 and 5. The ratio P_{i+1}/P_i was taken as equal to 0.2, according to the experimental data (Figure 11).

Analysis of formula (14) and consideration of the graphs of Figure 15 show that the ratio S_{i+1}/S_i decreases with an increase in frequency. The decrease in S_{i+1}/S_i with frequency occurs more abruptly as Δt_i and $\Delta t_{i+1}/\Delta t_i$ become larger. For the values of P_{i+1}/P_i and $\Delta t_{i+1}/\Delta t_i$ adopted in the calculations the ratio S_{i+1}/S_i does not exceed unity, i.e. the amplitude of the successive shock is less than the amplitude of the preceding one. This conforms well to the experimental data: in the observations amplitude of the succeeding shock was less than the amplitude of

the preceding one. This conforms well to the experimental data: in the observations the amplitude of the succeeding shock was less than the amplitude of the preceding one in the overwhelming majority of cases.

The amplitudes of waves with different predominant frequency.

The differences observed in the relationship of the amplitude of waves with different predominant frequencies -- of a sound and seismic wave or of two seismic waves -- may be associated with the difference in the frequency spectra of the explosion and the pulsation.

The oscillations caused by the explosion are more favorable to the creation of comparatively high-frequency waves than the oscillations caused by the pulsation, since in the spectrum of the oscillations caused by the explosion the high frequencies have a larger relative weight than in the spectrum of the oscillations caused by the pulsation. This may be the cause of the large relative attenuation during pulsation of the high-frequency sound and seismic waves by comparison with the lower-frequency seismic waves.

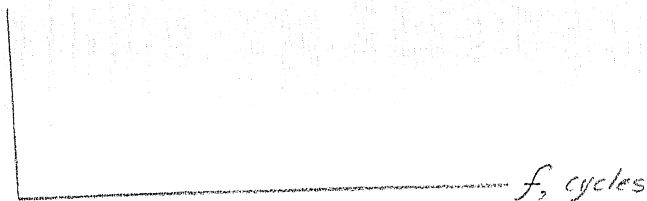


Figure 15. Graphs of the dependence of S_{i+1}/S_i on the frequency f ; $P_{i+1}/P_2 = 0.2$.

Thus, the difference in frequency spectra of the explosion and the pulsation may explain the observed relationships of the amplitudes of the records of the first repeat shock and the explosion, of two successive repeat shocks, and the character of the change in amplitudes in the course of the repeat shocks for waves with different predominant frequency. In particular, the larger amplitudes of the repeat shock by comparison with the explosion noted on the records obtained by means of comparatively narrow-pass apparatus may be explained.

THE SHAPE OF THE RECORD AND THE OSCILLATION FREQUENCIES
WHICH PREDOMINATE ON THE RECORDS

The frequency of the oscillations registered in the course of the observations depends mainly on the spectrum of the induced oscillations, on the absorbent properties of the medium and on the frequency characteristic of the apparatus. In the first approximation let us assume that absorption is not present in the medium.

When a given spectrum $S(f)$ of oscillations is fed to the input of a linear system with frequency characteristic $A(f)$ the spectrum of the oscillations obtained at the output of this system may be determined by means of multiplication of $S(f)$ by $A(f)$ for each given frequency. The frequency spectra S_1 and S_2 of oscillations created by the explosion and by the pulsation were determined above and set forth in figure 13. The frequency characteristic $A = A(f)$ which applies for the observational apparatus was also set forth in figure 13. Let us take note that the seismic apparatus is a linear system. Under these conditions the frequency spectrum of recorded oscillations $U = U(f)$ is determined by means of multiplication of the value of S_1 or S_2 by A for each given frequency.

Figure 16 gives curves of $U_1 = U_1(f)$ which were obtained in multiplying $A = A(f)$ by the corresponding curve of $S_1 = S_1(f)$, and curves of $U_2 = U_2(f)$ which were obtained in multiplying $A = A(f)$ by the corresponding curve of $S_2 = S_2(f)$.

As may be seen from Figure 16, with small values of Δt_1 and Δt_2 the position of the maxima of the functions U_1 and U_2 coincides with the position of the maximum of the frequency characteristic of the apparatus $A = A(f)$; with larger values of Δt_1 and Δt_2 the maximum of the curves of $U_1(f)$ and $U_2(f)$ shift in the direction of the lower frequencies, the shifts being larger for the curves of $U_2(f)$. The falling off of the curves of $U_1(f)$ takes place very slowly in the region of the frequencies $f > 60$ to 70 cycles. The falling off of the curves of $U_2(f)$, in the region of the same frequencies, takes place abruptly, the decrease in the magnitude of U_2 associated with an increase in f taking place the more rapidly as Δt_2 is larger. Only when $\Delta t_2 \leq 0.001$ are the curves of $U_2(f)$ characterized by a sloping right-hand slope when $f_{\max} < f < 400$ cycles.

Table 3 gives the values of the increment $\Delta f_{\max} = f_{\max 1} - f_{\max 2}$ and $f_{\max 1}$ and $f_{\max 2}$ are the respective frequencies of the maxima of the curves U_1 and U_2 taken from the curves of Figure 16 for various values of Δt_2 and $\Delta t_2 / \Delta t_1$.

A comparison of the curves of U_1 and U_2 and consideration of Table 3 makes it possible to draw the following conclusions as to the probable frequency peculiarities of the records of the explosion and the pulsation obtained with the apparatus whose frequency characteristic is given in Figure 13.

With small values of Δt_1 and Δt_2 the frequencies of the maxima of the function U_1 and U_2 are identical, and the falling off of the values of U_1 and U_2 in the region of the high frequencies

takes place slowly. This indicates that with small values of Δt_1 and Δt_2 (for small charges) the predominant frequencies on the records of the explosion and the pulsation should coincide, and the shape of the records should be identical.

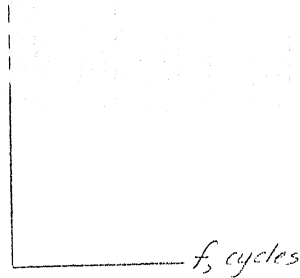


Figure 16. Graphs of $U_1 = U_1(f)$ and $U_2 = U_2(f)$

With an increase in Δt_1 and Δt_2 the frequency of the maximum of the function U_2 may be somewhat smaller than the frequency of the maximum of the function U_1 , and at the same time the spectrum of the function U_2 may be narrower than the spectrum of the function U_1 . This may lead to the situation where, with large values of Δt_1 and Δt_2 (large charges), the predominant frequencies on the records of pulsation will be lower than on the records of the explosions, and the records of the pulsation will be bereft of high-frequency components by comparison with the records of the explosions.

A comparison of the calculational data which have been obtained with the results of the observations shows that there is a close conformity between the observed data and the calculations obtained for $\Delta t_2 < 0.006$ second. According to the calculations, for these values of Δt_2 the predominant frequencies on the records of the shock and the explosion should be identical, and for $\Delta t_2 \approx 0.006$ seconds there may take place only a certain attenuation of

the high frequency components, by comparison with the records of the explosion, of the records of the repeat shocks.

Table 3

Δt_2 seconds	Δf_{\max} cycles		
	$\frac{\Delta t_2}{\Delta t_1} = 3$	$\frac{\Delta t_2}{\Delta t_1} = 5$	$\frac{\Delta t_2}{\Delta t_1} = 10$
0.001	0	0	0
0.003	3	3	3
0.006	4	8	8
0.01	7	11	15
0.015	10	14	21
0.02	12	16	22
0.03	15	19	24

Conclusions

The experimental study of the phenomenon of repeat shocks was performed with explosion in various bodies of water differing in transverse dimensions and depth. Up to four repeat shocks were noted on the seismograms. The repeat shocks were observed with explosions in the layer of water and on the bottom, with charges from 1 electro-detonator to 100 kilograms. The difference in time of registration of the oscillations caused by the explosion and the repeat shock, ΔT , increases with an increase in the charge Q and a decrease in the depth h . The observed data on the dependence of ΔT on Q and h are in good conformity with the theory of oscillations of a gas globe, which takes into consideration the effect of the boundaries

of the section -- the surface of the water and the bottom of the body of water.

The shapes of the records of the successive shocks and the first shock caused by the explosion are practically identical in the overwhelming majority of cases; the ratio of amplitudes A_2/A_1 of the successive and the first shocks decreases with an increase in charge. With small charges ($Q < 300$ grams) repeat shocks are observed which are more intense than the first shock; the ratio of amplitudes A_2/A_1 reaches 3.3. With large charges ($Q > 1$ kilogram) the successive shocks are usually weaker than the first shock. The amplitude of the successive shocks caused by the pulsation of the gas bubble decreases with an increase in the number of the shock.

It has been shown that the observed dynamic peculiarities of the records of the repeat shocks -- the relationship of the amplitudes, the predominating frequencies, etc. -- may be explained from the qualitative standpoint by the difference in frequency spectra of the explosion and the pulsation. In the frequency spectrum S_1 of an impulse of exponential shape, corresponding approximately to the explosion, there is little change in the amplitudes of the harmonic components when the frequency changes from 0 to 400 cycles with a comparatively small duration of the impulse. In the frequency spectrum S_2 of an impulse of bell shape, corresponding approximately to the pulsation, the maximum amplitudes are possessed by the low-frequency components; the decrease in amplitudes with frequency is the more abrupt as the duration of the impulse caused by the pulsation becomes

greater. In spite of the fact that the maximum pressure developed during the pulsation is considerably less than the pressure developed during the explosion, the ratio of the amplitudes of the waves corresponding to the pulsation and the explosion may be larger than unity due to the fact that the apparatus used registers the rather low frequencies created principally during the pulsation.

In further experiments on repeat shocks it appears expedient to perform a frequency analysis of the records of the repeat shocks and the explosion. Aside from this, light must be shed on problems of the directional characteristics of explosions and pulsations, on the relationship between repeat shocks and the character of the bottom of the water-body, etc.

The carrying out of further experiments on repeat shocks will make it possible to determine in more detail the physical nature of the phenomenon and will facilitate the quest for new means of attenuating the intensity of repeat shocks or of complete avoidance of them.

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[Appendix follows]

APPENDIX

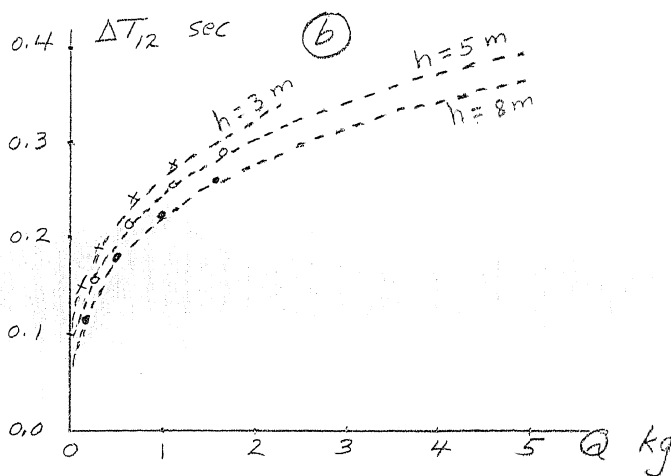
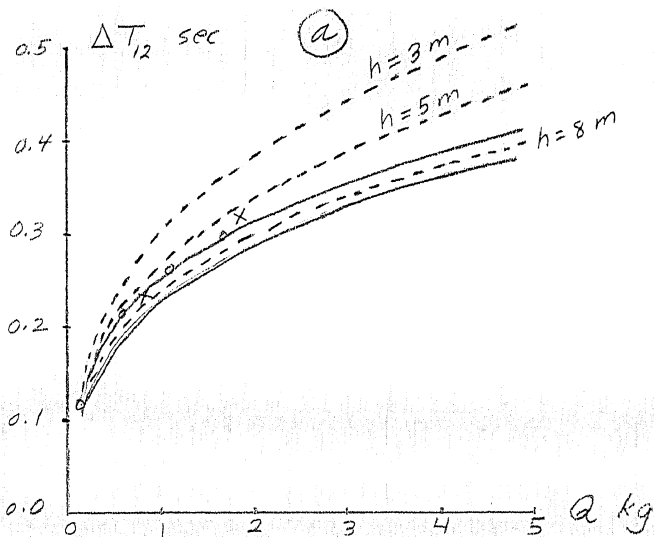


Figure 5. Graphs showing the dependence of ΔT_{12} upon Q :

- a - observational curve (continuous lines) and theoretical (dotted) curves according to formula (3)
- b - the same curves according to formula (1)

The separate points on both graphs correspond to observed data.

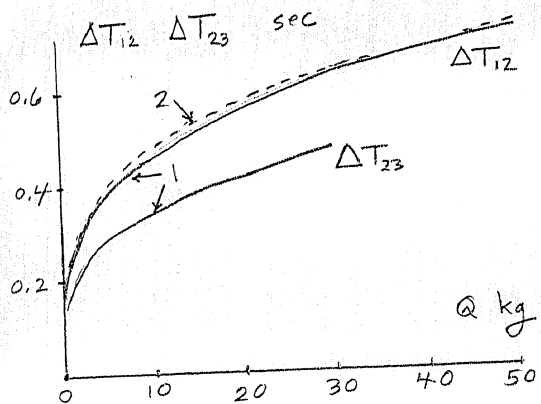


Figure 6. Graphs showing the dependence of ΔT_{12} and ΔT_{23} upon Q :

1 - $\Delta T_{12} = f_1(Q)$ and $\Delta T_{23} = f_2(Q)$ are constructed according to observational data for $h=5m, H=18m$.
 2 - $\Delta T_{12} = f(Q)$ is for $h=5m, H=18m$ by formula (4).

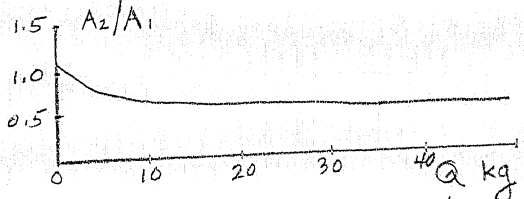


Figure 9. Graph of dependence A_2/A_1 on Q ; $h = 5$ meters.

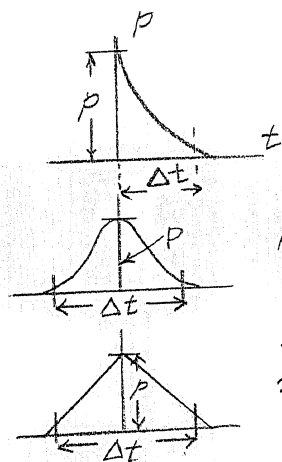


Figure 12. Curves of pressure-time $p=p(t)$ and their spectral functions S/S_0 .
 1 - spectrum of triangular impulse.
 2 - spectrum of bell-shape impulse.
 3 - spectrum of exponential impulse.

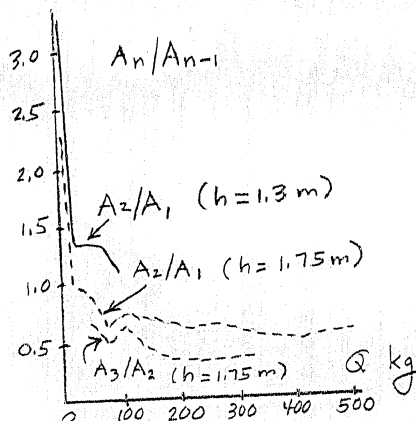


Figure 8. Graphs of dependence A_2/A_1 and A_3/A_2 on Q ; $h=1.3m$ and $h=1.75m$.

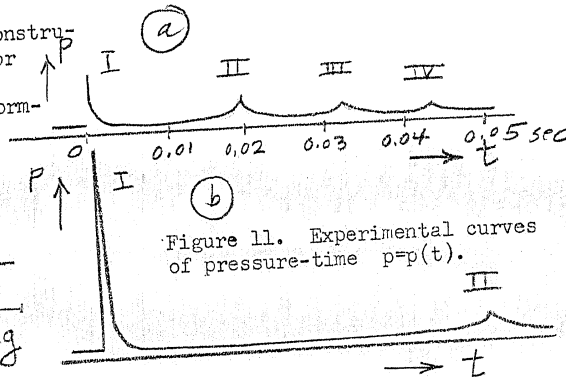


Figure 11. Experimental curves of pressure-time $p=p(t)$.

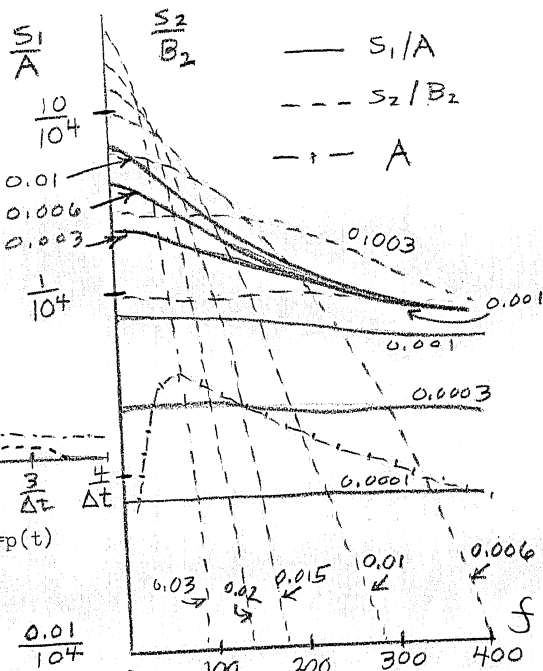
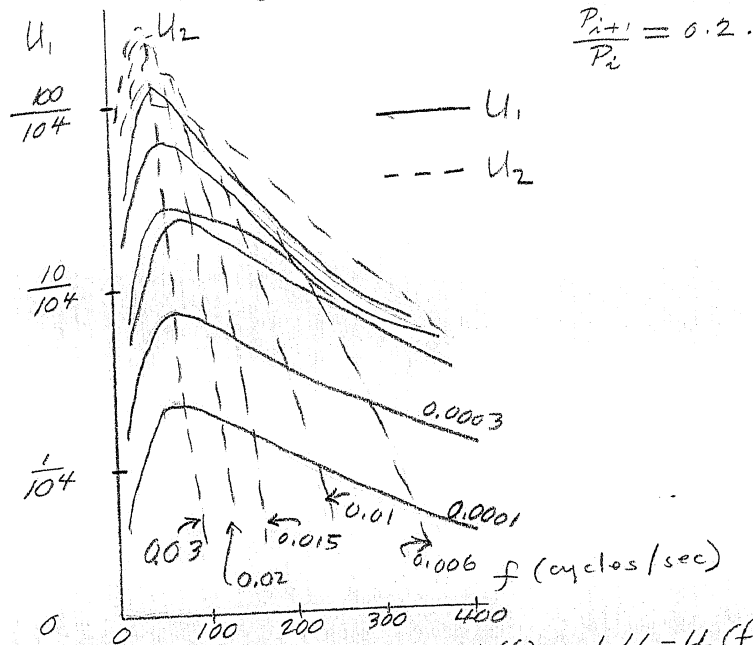
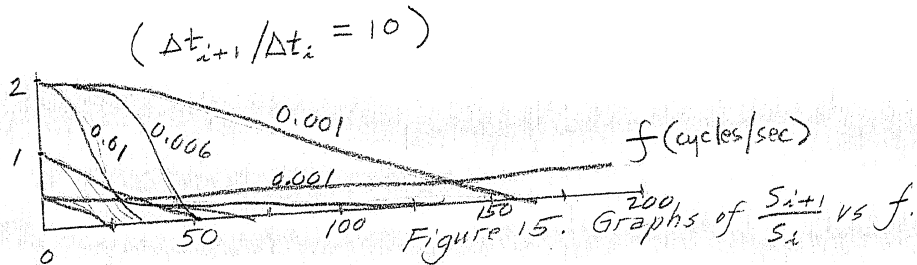
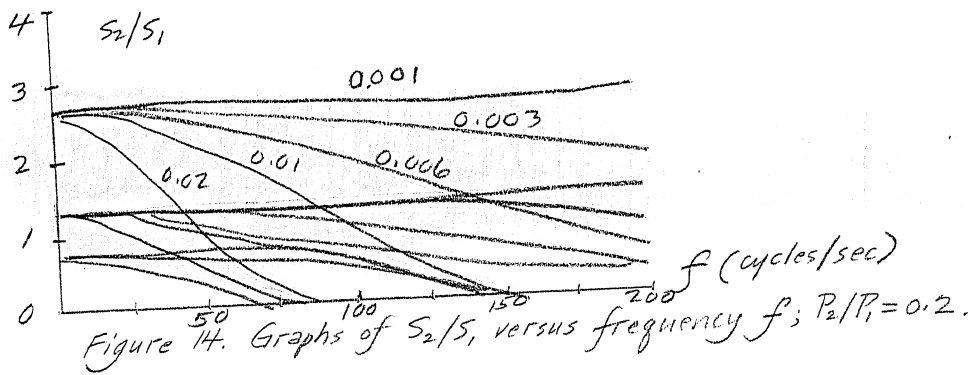


Figure 13. Graphs of S_1/A and S_2/B versus f (frequency).

B



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