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"THE ANTICOMMUTATIVE MATRICES IN THE MESON THEORY"

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The first-order wave equations for the meson field can be written in the form[1]:

$$B_n \cdot \partial\psi/\partial x_n + k_0\psi = 0 \quad (k_0 = E_0/\hbar c) \tag{1}$$

where psi ψ is a uni-columnar matrix containing the components of the tensor quantities that characterize the field. The matrices B_i satisfy the rules of commutation:

$$B_i B_k B_l + B_l B_k B_i - B_i \delta_{kl} - B_l \delta_{ki} = 0 . \tag{2}$$

In the four-dimensional space there are three irreducible non-Euclidean representations of this algebra: 10-row, 5-row, and 1-row; the first representation is used to describe vectorial (V) or pseudovectorial (PV) field, the second is for the description of scalar (S) or pseudoscalar (PS) field, and the last representation $B_i = (0)$ is trivial.

Employment of the reducible 16-row representation of the beta-algebra (10-row plus 5-row plus 1-row) gives the wave equation for a mixture of meson fields (V + PS or PV + S mixture). It is possible to expand this scheme by including in the general wave equation a description of all four types of meson fields.

Let us use the ordinary form of the reducible representation[2] and let us designate the matrices $B_i^{(1)}$, so that the wave equation corresponds to the V + PS mixture:

$$B_n^{(1)} \cdot \partial\Psi/\partial x_n + \Psi = \Phi \tag{3}$$

where psi Ψ is a 16-component wave function composed of a scalar and components of the four-dimensional vector and completely antisymmetric tensors of first, third, and fourth rank; phi Φ is the function characterizing the source of the field.

From this equation one can obtain the wave equation for the PV + S mixture by way of a transformation of the functions Ψ and Φ by means of a 16-row Hermitian matrix Γ_0, which is connected with transformations of the mirror-reflection type. The sought-for equation will be

$$- B_n^{(1)} \partial \Psi'/\partial x_n + k_0 \Psi' = \Phi' , \tag{4a}$$

where $\Psi' = i\Gamma_0\Psi, \quad \Phi' = i\Gamma_0\Phi$
$(\Psi' = i\Psi'\Gamma_0, \quad \Phi = i\,\Phi'\Gamma_0).$ . . . . (5)

From these relations that are given above it follows that (4a) is equivalent to:

$$B_n^{(11)} \partial\Psi/\partial x_n + k_0\Psi = \Phi \; \ldots.(4b), \text{ where } B_i^{(11)} = -\Gamma_0 B_i^{(1)} \Gamma_0 . \tag{6}$$

Thus the equation of the form $\Gamma_n \cdot \partial\Psi/\partial x_n + k_0\Psi = \Phi$ (7), where $\Gamma_i = B_i^{(1)} + B_i^{(11)}$ (8), describes a mixture of the general type containing all the tensor fields. The 16-row Hermitian matrices Γ_i satisfy the simple commutative algebra. Actually, the matrix of transformations (5) and (6) of Γ_o possesses the following properties:

$$\Gamma_o^2 = I, \quad B_i\Gamma_oB_k + B_k\Gamma_oB_i = 0, \quad \Gamma_oB_iB_k\Gamma_o + B_kB_i - \delta_{ik}\cdot I = 0. \tag{9}$$

Using these relations one can easily show that Γ_i are anticommutative:

$$\tfrac{1}{2}(\Gamma_i\Gamma_k) - \delta_{ik}\cdot I = 0 . \tag{10}$$

Since, moreover, one can set $(\Gamma_i\Gamma_o) = 0$, it turns out that

$$\Gamma_o = \Gamma_1\Gamma_2\Gamma_3\Gamma_4 . \tag{11}$$

In the group of Γ-matrices there are 16 matrical operators which in combination with Ψ^* and Ψ give the tensor densities of the fundamental physical quantities of the field. The energy density, in the general case, is not positively definite, although the field even possesses integral spin. The further development of the theory is formally analogous to the theory of the electron. Use of this parallelism can facilitate the solution of a number of problems in the theory of meson fields. It is possible also by way of analogy to clarify the physical content of certain unclear relations in the electron theory.

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1. N. Kemmer, Proc Royal Society, A, 173, 91 (1939).
2. W. Pauli , Relativistic Theory of Elementary Particles, Part II, § 4, 1947 (book translated into Russian, at Moscow).

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