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EVAPORATION OF VERY SMALL WATER DROPLETS

M. P. Timofeyev and M. Ye. Shvets

Although the evaporation of drops has been studied both experimentally and theoretically, the problem is still far from solved.

Besides the well-known classic works of Maxwell and Stefan experimental studies on the evaporation of drops have been contributed by Langmuir (1), Vyrubov (2), Frossling (3), Findeisen (4), Kiryukhin (5), Zak (6), Starokadomskaya (7), Baranayev (8), and others; Fuks (9), Leybenson (10), and Frankel' (11) written theoretical works on the subject. The results of these theoretical studies could not be used, however, in the study of the evaporation of small drops (radius less than 10 μ). One of the main defects of the theoretical works was the assumption that saturation vapor concentration existed on the surface of the evaporating drop. As follows from the kinetic theory of gases, this assumption placed an inaccurate condition upon the studies.

The present work takes into account the fact that the vapor concentration on the surface of an evaporating drop is different from saturation and leads to conclusions which differ notably from those obtained by the theory which did not make provision for this more accurate condition.

1. Vapor Concentration on the Drop's Surface

Evaporation under natural conditions consists of the following processes: transition of molecules from the liquid to the gaseous medium (actual evaporation), transition of the vapor molecules into the liquid state ("adhesion" of the vapor molecules to the liquid surface - that is, condensation), and diffusion of the vapor molecules from the liquid surface into the surrounding medium.

The first process is determined by the physical characteristics of the drop (temperature and molecular weight). The number of molecules cable of escaping from the liquid into the surrounding atmosphere is generally given by the well known formula.

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$$E = \alpha p_m \sqrt{\frac{M}{2\pi RT}} \qquad (1)$$

where p_m is maximum vapor pressure, M is molecular weight, T is the temperature of the liquid, R is the gas constant; and alpha α is the coefficient of accomodation (this coefficient has long been taken as unity for all liquids including water). This coefficient has been recently determined quite reliably for several liquids. According to Alti (12) the coefficient of accomodation for water is 0.034-0.36 for temperatures from 0 to 15°. This value agrees with the value of alpha α which Baranayev obtained from theoretical considerations.

The number of vapor molecules condensing on the liquid surface is determined by a similar formula. Diffusion of vapor molecules, which determines the speed of evaporation is obviously equal to the difference between the first (evaporation) and second (condensation) processes.

Designating the diffusion coefficient as D and the drop radius as r_0 and substituting in the formula the concentration c in the place of the vapor pressure p, we obtain the following condition on the drop's surface:

$$-D\frac{\partial c}{\partial r} = A(c_m - c) \quad \text{for } r = r_0 \qquad (2)$$

where A is defined by the formula $A = \alpha\sqrt{RT/2\pi M}$

In his work (3) Frossling gave as the formula for the speed of evaporation:

$$-D\frac{\partial c}{\partial r} = \frac{\alpha}{1-\frac{1}{2}\alpha}\sqrt{\frac{M}{2\pi RT}} \cdot (p_m - p) \qquad (2')$$

As is easily seen, this expression is practically identical with (2) for water (because of the small value of alpha α).

Formula (2) or (2') determines the concentration on the drop's surface.

II. Evaporation of an Immovable Drop. The Stationary Process.

The small drops ($r \leqslant 10^{-2}$ cm) which compose fogs and clouds have low falling velocities, and therefore under ordinary conditions they may be considered to be immovable with respect to air. This section therefore will apply not only to evaporation of immovable drops but also to evaporation of small drops under atmospheric conditions.

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The basic equation of diffusion in spherical coordinates for this case will be of the form

$$\frac{\partial}{\partial r} r^2 \frac{\partial c}{\partial r} = 0 \qquad (3)$$

The boundary conditions are:

$$\left.\begin{array}{l} \text{for } r = r_0, \quad -D\frac{\partial c}{\partial r} = A(c_m - c) \\ \text{and for } r = \infty, \quad c = 0. \end{array}\right\} \qquad (4)$$

The solution of equation (3) which satisfies the conditions (4) is:

$$c = \frac{Ac_m r_0}{(A + \frac{D}{r_0})r}. \qquad (5)$$

Hence, the vapor concentration on the drop surface is:

$$c(r_0) = \frac{Ac_m}{A + \frac{D}{r_0}} \qquad (6)$$

Formula (6) shows that the vapor concentration on the surface of small drops is notably different from the saturation c_m.

The speed of evaporation E is obtained from (5) by simple differentiation:

$$E = -D\frac{\partial c}{\partial r}\Big|_{r=r_0} = \frac{ADc_m}{Ar_0 + D}. \qquad (7)$$

This equation shows that very small drops will have high speeds of evaporation in dry air.

The change in drop mass is equal to

$$-\frac{dm}{dt} = \frac{4\pi r_0 ADc_m}{A + D/r_0} \qquad (8)$$

The change in the drop's surface is equal to

$$-\frac{ds}{dt} = 8\pi r_0 \frac{ADc_m}{(D + Ar_0)\rho}, \qquad (9)$$

where ρ is the density of the liquid.

Formulas 6-9 change into Maxwell's classic formulas under the condition that, $D \ll Ar_0$.

The change of drop radius during evaporation is defined by the formula:

$$-\frac{dr_0}{dt} = \frac{ADc_m}{D + Ar_0}\frac{1}{\rho} \qquad (10)$$

From this formula, the lifetime of a drop of initial radius r_0 is given by the following equation:

$$t = \frac{D + \frac{1}{2}r_0 A}{ADc_m} r_0 \rho. \qquad (11)$$

The formulas obtained hold only for dry air, but they are easily generalized for moist air. To do this, $c - c_\infty$ and $c_m - c_\infty$, respectively, should be substituted in all the formulas for c and c_m, where c_∞ is the concentration at a great distance from the drop.

III. Evaporation of an Immovable Drop. The Non-Stationary Process

The effect of the non-stationary condition in the evaporation of an immovable drop was investigated by Fuks (2), but again under the erroneous condition that saturation vapor concentration prevailed on the drop's surface. It would therefore be interesting to reopen this question for the more accurate boundary conditions indicated above.

The basic equation for the non-stationary process can be written in the form:

$$\frac{\partial (cr)}{\partial t} = D\frac{\partial^2 (cr)}{\partial r^2} \tag{12}$$

The boundary conditions are:

$$c = 0 \text{ for } t = 0, \text{ and}$$
$$-D\frac{\partial c}{\partial r} + Ac = Ac_m \text{ for } r = r_0 \ (t > 0). \tag{13}$$

We set

$$\left.\frac{\partial (cr)}{\partial r}\right|_{r=r_0} = F(t)$$

then, by using boundary condition (13), we find:

$$F(t) = c_m + D\left(\frac{1}{A} + \frac{r_0}{D}\right)\left.\frac{\partial c}{\partial r}\right|_{r=r_0};$$

whence the flow is

$$q = -D\left.\frac{\partial c}{\partial r}\right|_{r=r_0} = \frac{AD}{D+Ar_0}\left[c_m - F\right].$$

The solution of equation (12) is given by the expression

$$cr = -\sqrt{\frac{D}{\pi}}\int_0^t \frac{d\tau}{\sqrt{t-\tau}}\cdot \exp\left[-\frac{(r-r_0)^2}{4D(t-\tau)}\right]F(\tau). \tag{14}$$

By satisfying the boundary condition on the surface, we obtain the following integral equation for determination of $F(t)$:

$$F(t) + \frac{\rho}{\sqrt{\pi}}\int_0^t \frac{F(\tau)\,d\tau}{\sqrt{t-\tau}} = -\frac{c_m r_0 A}{D}, \tag{15}$$

where

$$\rho = \frac{1}{r_0}\cdot\frac{D+Ar_0}{\sqrt{D}}.$$

The solution of this equation can be represented in the following form:

$$F(t) = -\frac{c_m r_0 A}{D}\, e^{\xi}\left[1 - K(\sqrt{\xi})\right],$$

where $\xi = \rho^2 t$, and K is the ~~Kamp~~ Kramp function. It is assumed for the meantime that c_m does not ~~change~~ vary with time. Since the parameter ρ is large, we use the asymptotic expansion of the ~~Kamp~~ Kramp function.

$$1 - K(\sqrt{\xi}) = \frac{e^{-\xi}}{\sqrt{\pi\xi}} \cdot \left(1 - \frac{1}{2\xi} + \cdots\right)$$

and hence obtain

$$F(t) = -c_m \frac{r_0 A}{D} \frac{1}{\sqrt{\pi\xi}} \left(1 - \frac{1}{2\xi} + \cdots\right).$$

Now we obtain for the flow the formula which holds for the non-stationary process:

$$q = \frac{A c_m}{1 + \frac{r_0 A}{D}} \left[1 + \frac{A r_0^2}{\sqrt{D}(A r_0 + D)} \cdot \frac{1}{\sqrt{\pi t}} + \cdots\right]$$

This formula enables us to evaluate the influence of ~~the~~ the non-stationary process upon flow (that is, upon evaporation). The first ~~member~~ term of the series corresponds to the stationary process while the following ~~members~~ terms introduce the correction for the non-stationary process (actually, the order of the correction is determined by the order of A).

This correction is negligible for water drops, because evaporation of an immovable drop can be considered for the fairly accurately as a non-stationary process.

It is more difficult to evaluate the effect of the decrease in drop size during its evaporation

The rough evaluation of this effect which [redacted] Fuks made (8) shows that it cannot be disregarded. There are serious mathematical obstacles preventing the accurate evaluation of this effect.

It is sometimes necessary to consider that the maximum concentration c_m varies with time in order to calculate the speed of evaporation of the drop.

If c_m is assumed to vary with time, we have instead of (15)

$$F(t) + \frac{\rho}{\sqrt{\pi}} \int_0^t \frac{F(\tau)\,d\tau}{\sqrt{t-\tau}} = \varphi(t)$$

where

$$\varphi(t) = -\frac{A r_0}{D} c_m(t).$$

The solution of the last equation is easily found and is of the form

$$F(t) = \frac{d}{dt} \int_0^t e^{-\rho^2(\sigma - t)} \varphi(\sigma)\, \mathrm{erfc}\left[\sqrt{\rho^2(t-\sigma)}\right] d\sigma, \quad (16)$$

where the error function is erfc $y = 2 - K(y)$. If the function $f(t)$ is known all the necessary calculations can quite easily be carried through completely.

§ IV. Decrease in the Drop's Temperature Due to Evaporation

In order to evaluate the cooling of the drop due to evaporation, we find the influx of heat to the drop, disregarding the effect of [illegible] and convection -- that is, considering only the influx due to conduction. [illegible] the temperature decrease of the drop due to [illegible]

We consider the [redacted] heat diffusion equation

$$\frac{\partial (rT)}{\partial t} = D_T \frac{d^2(rt)}{dr^2}$$

and the boundary conditions

$$\text{at } t = 0 \quad T = 0 \quad \text{for } r \ldots$$

$$\text{at } r = \infty \quad T = 0 \quad \text{for } t \ldots$$

We introduce the designations

$$\left.\frac{\partial (rT)}{\partial r}\right|_{r=r_0} = \Phi(t) \text{ and } T(r_0 t) = T_0 .$$

The heat flow will be equal to

$$q_T = -D_T \frac{\partial T}{\partial r} = \frac{D_T}{r_0} \left[T_0 - \Phi(t) \right]$$

according to (14)

$$T_0 = -\frac{1}{r_0}\sqrt{\frac{D_T}{\pi}} \int_0^t \frac{\Phi(\tau)\, d\tau}{\sqrt{t-\tau}}$$

and hence

$$\Phi(t) = -\frac{r_0}{\sqrt{\pi D_T}} \frac{d}{dt} \int_0^t \frac{T_0\, d\tau}{\sqrt{t-\tau}} . \qquad 18$$

The heat balance may be written with sufficient accuracy in the following form:

$$\ell D \frac{\partial c}{\partial r} = D_T \frac{\partial T}{\partial r} \text{ for } r = r_0$$

(here ℓ is the latent heat of evaporation), or

$$\frac{D_T}{r_0}\left[T_0 - \Phi\right] = \frac{AD\ell}{D + Ar_0}(c_m - F) .$$

Substituting the values of Φ and F from (18) and (16), we obtain the following integral equation for determination of the temperature decrease:

$$\frac{D_T}{r_0} T_0 + \sqrt{\frac{D_T}{\pi}} \frac{d}{dt} \int_0^t \frac{T_0(\alpha)\, d\alpha}{\sqrt{t-\alpha}} = \frac{AD\ell}{D + Ar_0}\left[c_m - \frac{d}{dt}\int_0^t c_m P(\sigma - t, \rho^2)\, d\sigma\right]$$

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where $$P(\sigma-t,\rho^2)=e^{-\rho^2(\sigma-t)}\,\mathrm{erfc}\left(\sqrt{\rho^2(t-\sigma)}\right)$$; here

c_m is a known function of T_0.

The last integral equation can be changed to the form

$$\int_0^t T_0(\alpha)\left[\frac{D_T}{r_0}+\sqrt{\frac{D_T}{\pi}}\frac{1}{\sqrt{t-\alpha}}\right]d\alpha = \frac{ADl}{D+Ar_0}\int_0^t c_m(1-P)\,d\sigma.$$

This equation can be solved by the method of consecutive approximations, not without considerable difficulty, however.

The temperature decrease of the drop can be found very simply if we consider the quasi-stationary process. The heat flow in this case is expressed by the same formula as the speed of evaporation:

$$Q = 4\pi r_0 D_T (T_1 - T_0),$$

where T_1 is the temperature at a sufficiently great distance from the drop and T_0 is the drop's temperature.

The loss of heat due to evaporation is equal to

$$Q' = 4\pi r_0 l \frac{ADc_m}{A+\frac{D}{r_0}}$$

But in the stationary state $Q' = Q$, and consequently

$$T_1 - T_0 = \frac{lAc_m}{A+\frac{D}{r_0}}\cdot\frac{D}{D_T}.$$

Despite its simplicity this formula [illegible] for evaluating the possible decrease in temperature [illegible] due to evaporation. In particular, this formula shows that the temperature decrease of a water drop is negligible.

V. Evaporation of a Drop in Flow for Small Reynolds Numbers

In order to determine the speed of evaporation of a moving drop, we must use, in addition to the diffusion equation, the equations of hydromechanics to determine the speed of flow around the drop.

These equations in spherical coordinates are quite complex in form, but Boltze (13) showed that they can be greatly simplified if one considers the flow in the boundary lamina of a body of rotation.

If we assume that the y-coordinate is directed along the normal -- that is, along the drop's radius -- and the x-coordinate is along the meridian from the point O (Figure 1), and designate the velocity projections on these axes as u and v, and the velocity potential on the boundary of the laminar [illegible] as $\bar{u}$, then, according to Boltze, the equations of motion and continuity will have the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\bar{u}\frac{\partial \bar{u}}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (19)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{u}{r_0}\cot\frac{x}{r_0}=0,$$

where ν is the kinematic coefficient of viscosity, and r_0 is the drop's radius.

The diffusion equation can be shown in the form

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \qquad (20)$$

On the surface of the streamlined drop (y = 0), the following boundary conditions hold

$$\left.\begin{aligned} u=v&=0\\ -D\frac{\partial c}{\partial y}&=A(c_m-c)\end{aligned}\right\} \qquad (21)$$

[illegible] from the drop [illegible] we have

$$\left.\begin{aligned} u&=\bar{u}\\ c&=0\end{aligned}\right\} \qquad (22)$$

[illegible] concentration [illegible] the drop [illegible] flow [illegible], we [illegible] the diffusion [illegible] equations

[illegible] to the dimensionless variables [illegible]

$$x=x_1 d,\ u=u_1 U,\ c=c_1 c_m$$

$$y=y_1\frac{d}{\sqrt{Re}},\ v=v_1\frac{1}{\sqrt{Re}},\ \bar{u}=\bar{u}_1 U,$$

where d is the drop's diameter, U is the characteristic velocity, and $Re=\frac{Ud}{\nu}$ is the Reynolds number.

[illegible] new variables, the equations and boundary conditions have the form

$$\left.\begin{aligned} u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y_1}&=\bar{u}_1\frac{\partial \bar{u}_1}{\partial x_1}+\frac{\partial^2 y}{\partial y_1^2}\\ \frac{\partial u_1}{\partial x_1}+\frac{\partial v_1}{\partial y_1}+2u_1\cot 2x_1&=0\end{aligned}\right\} \qquad (19')$$

$$u_1\frac{\partial c}{\partial x_1}+v_1\frac{\partial c_1}{\partial y_1}=\sigma\frac{\partial^2 c_1}{\partial y_1^2} \qquad (20')$$

$$\text{for } y_1=0:\ u_1=v_1=0,\ -\frac{\partial c_1}{\partial y_1}=B(1-c_1),\ B=\frac{Ad}{D\sqrt{Re}} \qquad (21')$$

$$\text{for } y_1=\infty:\ u_1=\bar{u}_1,\ c_1=0. \qquad (22')$$

Since this system of equations does not contain the Reynolds number, the velocities u_1 and v_1 are functions of x_1 and y_1, σ and consequently, c_1 will be a function of x_1, y_1, σ, and B.

We calculate evaporation from the drop's surface assuming laminar flow. If we change to dimensionless quantities, then we have

$$\frac{c}{c_m} = \Phi\left(\frac{x}{d}, \frac{y\sqrt{Re}}{d}, \sigma, B\right).$$

Because of the boundary condition (2), the speed of evaporation is

$$-\frac{dm}{dt} = -D\int\left(\frac{\partial c}{\partial y}\right)_{y=0} dF = Ac_m\int\left(1-\frac{c}{c_m}\right)_{y=0} dF$$

or

$$-\frac{dm}{dt} = \frac{Ac_m\pi d^2}{4}\int_0^{\pi}\left[1-\Phi\left(\frac{x}{d},0,\sigma,B\right)\right]\sin\Theta\, d\Theta \quad \left(\frac{\Theta}{2} = \frac{x}{d}\right)$$

Since the integral is a function of only σ and B, the speed of evaporation is

$$-\frac{dm}{dt} = Ac_m d^2 f(\sigma, B)$$

The form of the function f [illegible]

The rate of laminar flow [illegible] sphere by a slow [redacted] [illegible] ment has been investigated by Stokes ([illegible]).

The following expression [illegible] velocity component at a distance r from the [redacted] center of [illegible]

$$u_\Theta = U\sin\Theta\left[1 - \frac{3}{4}\frac{r_0}{r} - \frac{1}{4}\frac{r_0^3}{r^3}\right]$$

Introducing the quantity y from the [illegible]

$$r = r_0 + y$$

we obtain for small y's ([redacted] that is, close to the drop surface)

$$u_\Theta = 3V\frac{y}{d}\sin\Theta$$

or for the dimensionless velocity

$$u_{1t} = 3\frac{y_1}{\sqrt{Re}}\sin\Theta.$$

Following Leybenzon (10), we represent the diffusion equation in the form

$$u_{1t}\frac{\partial c_1}{\partial x_1} = \sigma\frac{\partial^2 c_1}{\partial y_1^2}.$$

or, substituting for [redacted] u_{1t},

$$\frac{3\sin\Theta}{\sigma\sqrt{Re}}\, y_1\frac{\partial c_1}{\partial x_1} = \frac{\partial^2 c_1}{\partial y_1^2}. \qquad (2?)$$

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The boundary conditions are of the form

$$
\begin{aligned}
&\text{at } y_1 = 0 \text{ and } x_1 > 0 \; : \; -\frac{dc_1}{dy_1} = B(1-c_1),\\
&\text{at } x_1 = 0 \text{ and } y_1 > 0 \; : \; c_1 = 0,\\
&\text{at } y = \infty \; : \; c_1 = 0.
\end{aligned}
\qquad (24)
$$

We introduce the new independent variable τ according to the formula $y_1 = \frac{2}{B}\sqrt{\tau}$; hence the equation (23) and the boundary conditions (24) take on the form

$$\frac{\partial c_1}{\partial x_1} = \frac{B^3\sigma\sqrt{Re}}{3\sin\Theta}\frac{\partial}{\partial\tau}\sqrt{\tau}\frac{\partial c_1}{\partial\tau} \qquad (23')$$

$$
\begin{aligned}
&\text{at } \; = 0 \; : -\sqrt{\tau}\frac{\partial c_1}{\partial\tau} = 1 - c_1,\\
&\text{at } x = 0 \; : \; c_1 = 0,\\
&\text{at } \; = \infty \; : \; c_1 = 0.
\end{aligned}
\qquad (24')
$$

We let $\frac{B^3\sigma\sqrt{Re}}{6\sin\Theta} = \beta$ and assume that beta β is a constant to simplify integration. If we set $-\sqrt{\tau}\frac{\partial c_1}{\partial\tau}\Big|_{\tau=0} = q(x)$, the solution of equation (23) will be

$$c_1(x_1,\tau) = \frac{\beta^{1/3}}{(\frac{2}{3})^{1/3}\Gamma(\frac{2}{3})}\int_0^x \frac{q(\alpha)\cdot\exp\left[-\frac{4\tau^{3/2}}{9\beta(x_1-\alpha)}\right]d\alpha}{(x_1-\alpha)^{2/3}}$$

(Γ is the gamma-function); hence

$$c_1(x_1,0) = \frac{\beta^{1/3}}{(\frac{2}{3})^{1/3}\Gamma(\frac{2}{3})}\int_0^x \frac{q(\alpha)d\alpha}{(x_1-\alpha)^{2/3}}.$$

Using the boundary condition (24'), we obtain for the determination of the function $q(x_1)$ the integral equation

$$q(x_1) + \frac{\rho}{\Gamma(\frac{1}{3})}\int_0^x \frac{q(\alpha)d\alpha}{(x_1-\alpha)^{2/3}} = 1$$

$$\rho = \frac{\Gamma(\frac{1}{3})\beta^{1/3}}{(\frac{2}{3})^{1/3}\Gamma(\frac{2}{3})} \approx 1.76\beta^{1/3}.$$

Solution of this integral equation gives the expression

$$q = e^{-\rho^3 x}\left[1 + \sum_{j=1}^{2}\frac{(-1)^j}{\Gamma(\frac{j}{3})}\int_0^{\rho^3 x_1} e^t t^{\frac{j}{3}-1}dt\right] \qquad (25)$$

The total amount of water vapor taken per unit time from the spherical drop by the flow is

$$-\frac{dm}{dt} = \frac{1}{2}Ac_m\pi d^2\int_0^\pi q\cdot\sin\Theta\, d\Theta$$

Further use of formula (25) for q is inconvenient and therefore we transform it, making use of the fact that $\rho^3 x_1$ is large even for very small drops.

It is easily seen that

$$\int_0^{\rho^3 x_1} e^t t^{\frac{1}{3}-1}\,dt = \int_0^{-\infty} \frac{e^t dt}{e^{1+\frac{1}{3}}} + \int_{-\infty}^{\rho^3 x_1} e^t t^{\frac{1}{3}-1}\,dt,$$

where $\int_0^{-\infty} \frac{e^t dt}{e^{1-\frac{1}{3}}} = -\Gamma(\frac{1}{3})$, and

$$\int_{-\infty}^{\rho^3 x_1} e^t t^{\frac{1}{3}-1}\,dt = \frac{e^{\rho^3 x_1}}{(\rho^3 x_1)^{1-\frac{1}{3}}}\left[1 - \frac{\frac{1}{3}-1}{\rho^3 x_1} + \frac{(\frac{1}{3}-1)(\frac{1}{3}-2)}{(\rho^3 x_1)^2} + \dots + \frac{(-1)^k(\frac{1}{3}-1)(\frac{1}{3}-2)\cdots(\frac{1}{3}-k)}{(\rho^3 x_1)^k} + R_k\right].$$

Where R_k is the remainder.

We can consider with sufficient accuracy that

$$q = \sum_{j=1}^{2} \frac{(-1)^j}{\Gamma(\frac{j}{3})(\rho^3 x)^{1-\frac{j}{3}}}.$$

Replacing q by its equivalent and substituting $x_1 = \frac{\pi}{2}$ we find after simple transformations

$$-\frac{dm}{dt} = \frac{1}{2} A C_m \pi d^2 \sum_{j=1}^{2} \frac{a_j}{(\sigma B' Re)^{1-\frac{j}{3}}}$$

where

$$a_j = \frac{(-1)^j}{\Gamma(\frac{j}{3})(3.46)^{1-\frac{j}{3}}} \int_0^{\pi} \left(\frac{\sin\theta}{\theta}\right)^{1-\frac{j}{3}} \sin\theta\, d\theta$$

Setting [illegible] we find from the last equation that

$$\frac{dm}{dt} \sim \frac{1}{2}\pi d D C_m \sqrt[3]{\frac{Re}{\sigma}}.$$

This formula gives the [illegible] of the speed of evaporation upon the parameters under the assumption [illegible] are right on the drop's surface.

~~[illegible]~~ The speed of evaporation corresponding to [illegible] according to (8) should be added to the speed of evaporation found from the above equation.

We should like to note that ~~[illegible]~~ very difficult to experiment with small drops, and therefore any theoretical study of this problem is of some interest. The very meager and not especially reliable experimental data available ~~[illegible]~~ prevented the authors from checking the theoretical conclusions.

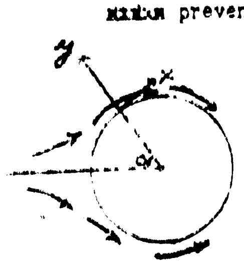

Figure 1

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