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THE THEORY OF HELIUM-III

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Abstract: In Section 1 we determine the temperature dependence of thermal capacity, viscosity, and thermal conductivity in helium-3. In Section 2 we clarify the influence of exchange effects due to the nuclear spins of helium-3 atoms upon the phase transition of liquid helium-3 to the solid state. The heat of fusion of helium-3 at low temperatures must be negative and equal to $-R \cdot \log 2$. Theoretical possibilities of obtaining temperatures of the order of 10^{-6} to 10^{-7} degree during adiabatic freezing of liquid helium-3, are indicated. Certain peculiarities of the nuclear magnetic susceptibility are clarified.

1. Thermal Capacity, Viscosity, and Thermal Conductivity of Liquid Helium-3

Recently it has been established that liquid helium-3 in contrast to liquid helium-4 does not possess superfluidity down to the temperature 1.05°K (1). This result makes probable the absence of superfluidity in helium-3 for any temperature. In the case of helium-4 the Bose-Einstein statistics favors superfluidity (2). If a sufficiently rarefied gas is considered which obeys the Fermi-Dirac statistics, then the influence of the interaction of the atoms upon the properties of this gas can be explained by the methods of the theory of perturbations. The theory of perturbations is applicable for sufficiently small gas densities and in the case of forces acting in short ranges. Considerations and actual calculations such as mentioned do not indicate any superfluidity in helium-3. Under similar conditions a Bose gas reveals superfluidity in certain cases (2). These are the bases for not considering helium-3 superfluid. The energy of the excited states of helium-3 for small excitation energies is the sum of the elementary excitation energies of the "quasiparticles" (3). It is essential to consider that these excitations obey the Fermi-Dirac statistics, since in *this* case such excitations of the entire system pass continuously to the excited states in the sense of the forward motion of the helium-3 individual atoms, when the transition from the liquid to the gas occurs. The properties of the excitations

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of conducting electrons in metals lead one to similar considerations of excitations in liquid helium-3. As is well known, the electron thermal capacity of metals and a number of other properties of metals correspond with the Fermi-Dirac statistics for excitations. (The existence of superconductivity is connected with some small effects not taken into consideration in the rough Fermi-Dirac model. The smallness of these effects appear in the smallness of the temperatures at which superconductivity occurs, in comparison with the "natural" electron temperatures which possess the order of magnitude of the temperature governing the degeneracy of electrons.)

Well known is the case where the excitations of the system of interacting electrons obey the Bose statistics (4). This case corresponds to the states that differ slightly from the "null" states in which the electron spins are oriented to one side. A "null" state is a "singular point" and therefore excitations close to such a state also possess unusual properties. In the case of liquid helium-3 just as in the case of nonferromagnetic metals in nonsuperconducting states the lowest "null" state is characterized only by the lowest energy and nothing more. (In the ferromagnetic case it is further characterized by a large magnetic moment.) Since the wavelength of helium-3 atoms is of the order of the distance between them, the interaction between the helium-3 atoms depends upon the mutual orientation of the nuclear spins (in helium-3 the nuclear spin equals $\frac{1}{2}$) (5). Thus in liquid helium-3 the essential role is played by the exchange effects which are connected with the exchange of two helium-3 atoms. It is easy to show that these exchange effects apparently lead to the principal antiparallel orientation of the neighboring nuclear spins, so that liquid helium-3 is not a nuclear ferromagnetic but is an example of an exchange nuclear paramagnetic similar to an electron exchange paramagnetic of the solid oxygen type. In the case of two helium-3 atoms, in fact, which possess parallel spins, the coordinate part of the wave function must be antisymmetrical relative to the interchange between two atoms. Therefore the coordinate part can include the S function. Under these conditions the probability of small distances between helium-3 atoms is very slight in comparison with the

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case where the coordinate part of the wave function of two atoms can be included with the S function; that is, when the spins are antiparallel. If the rapid decrease in the negative energy of attraction of two helium-3 atoms with increasing distances between them is taken into consideration, then one can conclude that the energy of the two atoms of helium-3 will be less in the case where they can be closer to each other; that is, in the case of antiparallel orientation of the neighboring spins. Therefore one takes into consideration even small distances between the atoms, but all greater than r_0 , which is the distance at which intense repulsion sets in. This limitation is carried out well in liquid helium-3, since the average distance between the atoms is several times greater than r_0 . Namely there exists a liquid helium-3 in which the atoms attract each other on the average. Thus there is no reason to consider that liquid helium-3 is a nuclear ferromagnetic. Therefore one can also consider that close to the null state of helium-3 the excitation statistics coincide with atom statistics.

By employing the Fermi-Dirac distribution for the excited states one can easily derive a number of conclusions concerning certain properties of liquid helium-3. The Fermi surface in the considered case represents a sphere of radius p_0 which in order of magnitude will equal $(3 \cdot \pi^2)^{1/3} \cdot \hbar N^{1/3}$, where N is the number of atoms per 1 cm^3 (here it was learned that nuclear spin of helium-3 equals $\frac{1}{2}$). For temperatures small in comparison with temperatures of degeneracy the excitation will have momenta (impulses) close in magnitude to p_0 . Expanding the excitation energy ϵ in a power series in $(p - p_0)$ we get

$$\epsilon = v \cdot (p - p_0) \quad (1)$$

Since the main role at temperature T is played by excitations for which we have $\epsilon \sim T$ and the number of such excitations (per 1 cm^3) equals the quantity $p_0^2 \Delta p / \pi^2 \hbar^3 \sim p_0^2 \epsilon / \hbar^3 v \sim p_0^2 T / \hbar^3 v$, then the energy per 1 cm^3 of helium-3 depends upon temperature in the following manner

$$u = u_0 + \frac{1}{2} a T^2 \quad (2)$$

$$a = \gamma p_0^2 / \hbar^3 v, \text{ where } \gamma \sim \pi^{-2}. \quad (3)$$

Equations are true for temperatures sufficiently small in comparison with the

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temperature of degeneracy of the excitations T_0 . In order of magnitude T_0 is

$$T_0 \sim p_0^2/2m \sim (3\pi^2)^{2/3} \hbar^2 N^{2/3}/2m \sim 5^\circ .$$

According to equation (2) the thermal capacity of helium-3 turns out to be proportional to the temperature as follows

$$c = aT \quad (T \ll T_0) . \quad (4)$$

This conclusion gives the possibility of immediate experimental verification of the Fermi character of the energy spectrum. It is easy to establish the temperature dependence of certain kinetic characteristics of helium-3. To do this let us determine the path of free flight (mean free path) of excitations. The finiteness of the excitation free path is due to collisions. The amount of excitations per 1 cm³ is equal in the order of magnitude to

$$n \sim p_0^2 \Delta p / \hbar^3 \sim p_0^2 T / \hbar^3 v \sim NT / T_0 \ll N . \quad (5)$$

Since n is much less than N , $n \ll N$, the most probable are pair collisions. In each collision of two excitations both states that are obtained after collision must have momenta close to p_0 . This decreases the effective cross sections of such a collision since the probability of collision is proportional to the number of possible finite states. The number of states after collision is equal to

$$d k / (2\pi\hbar)^3 = k^2 d k \cdot d\Omega / (2\pi\hbar)^3 = p_0^2 d\varepsilon \cdot d\Omega / (2\pi\hbar)^3 v . \quad (6)$$

Here k is the momentum of one of the particles after collision; $d\Omega$ is an element of the solid angle characterizing the direction of propagation of a particle. In the integration over $d\varepsilon \cdot d\Omega$ a coefficient proportional to T arises. Thus the effective cross section σ for the collision of two excitations turns out to equal

$$\sigma = \sigma_0 \cdot (T/T_0) , \quad (7)$$

where σ_0 has the order of magnitude of gas-kinetic cross section (10^{-15} cm²). Combining equations (5) and (7) we find the free path of excitation

$$l \sim (1/N\sigma_0) \cdot (T_0/T)^2 . \quad (8)$$

This is the result obtained earlier in considerations of the free path of electrons in metals, due to the interaction of electrons with each other (6,7) (see also (8)). The free path found permits one to establish the dependence of viscosity in liquid helium-3 upon temperature, since all the kinetic properties of helium-3 can be found in a consideration of an excitation gas with the properties described by the equations (1) to (8). Since eta is $\eta = Nm l v$, we

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find after substituting from equation (8)

$$\eta = Nm \cdot (1/N\sigma_0) \cdot (T_0/T)^2 \nu = \Lambda/T^2 \quad (9)$$

The viscosity of liquid helium-3 must be inversely proportional to the temperature squared when T is much smaller than T_0 , $T \ll T_0$.

With the aid of equations (8) and (4) we obtain the thermal conductivity of liquid helium-3

$$\kappa = \sigma \lambda \nu = B/T \quad (10)$$

The thermal conductivity of liquid helium-3 must be inversely proportional to the absolute temperature for $T \ll T_0$.

Equations (4), (9), and (10) give the possibility of complete experimental verification of the theory. It is necessary therefore to note once more that we did not utilize in the conclusion of equations (4), (9), and (10) the model of an ideal gas in application to helium-3 atoms.

2. The Influence of Exchange Effects in Liquid Helium-3 upon the Phase Transition of Liquid Helium-3 to Solid Helium-3

Although solid helium-3 has not yet been obtained, still it can scarcely be doubted that under pressures of the same order of magnitude as in the case of helium-4 liquid helium-3 will be solidified. The reason for solidification under high pressures is that as the volume decreases (under the action of the applied pressures) the ratio of the amplitude of zero oscillations to the distances between the atoms decreases. If for small pressures (large volumes) this ratio in the case of liquid helium-4 and helium-3 turns out to be of the order of unity (which precludes solidification), then with decreasing volume this ratio becomes less than unity and thus the formation of crystals becomes possible. For demonstration of the statements just made we equate the amplitude of zero oscillations $\sqrt{\hbar/m\omega}$ and the interatomic distance b . By ω we must understand a certain mean frequency connected with the potential energy of interaction of two atoms in the following manner

$$m \omega^2 \approx (d^2u/dr^2)_{r=b}$$

Setting $u = \alpha/r^n$ where $n \sim 6$ to 8 (9), we obtain

$$\sqrt{\frac{\hbar}{m\omega}} \cdot \frac{1}{b} \sim \left(\frac{\hbar}{m\sqrt{\alpha}}\right)^{1/2} \cdot b^{(n-2)/4} \rightarrow 0$$

in view of the fact that $n \gg 2$.

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It is easy to show that the phase transition: liquid helium-3 — solid helium-3 must reveal certain unusual peculiarities due to the exchange effects of helium-3 atoms in the liquid state. To clarify these peculiarities let us compare the entropy S of the liquid and solid phases of helium-3. In the liquid state the entropy S tends to zero as the temperature falls to 0. If the Fermi-Dirac distribution for the excitations is correct, then S must be proportional to T (10). It is necessary to note, however, that it is not essential to know further what kind of function S is of T in the liquid phase. Solely important is the fall in S with decreasing T . It is important to emphasize that here what falls is the part of S that is due to the nuclear spins. In the absence of exchange effects due to nuclear spin the spins of the nuclei could be freely oriented in space down to temperatures so low that the magnetic interaction of the nuclear spins with each other would become significant. These temperatures in the order of magnitude equal

$$T_M \sim \mu^2 / kb^3 \sim 10^{-7} \text{ degree, where } \mu \sim 10^{-23}. \quad (11)$$

Thus a part equal to $R \cdot \log 2$ (per one gram-atom) would enter the expression for S in the absence of exchange effects for $T \gg T_M$. The exchange effects lead to the appearance of correlation between the orientations of the spins of neighboring nuclei of helium-3 even for temperatures of the order of degrees; that is, of the order of "natural" helium temperatures. This conclusion follows from the fact that the difference in the energy of interaction of two helium-3 atoms in the case of parallel and antiparallel orientation of spins possesses the same order as the energy of interaction itself (see Section 1). It is not essential here whether helium-3 is a liquid exchange nuclear paramagnetic or a ferromagnetic. The nature of the correlation in these two cases differs, but the existence itself of correlation must be noted in each of these cases for $T \leq 1^\circ \text{K}$. Hence follows the fall in that part of S due to nuclear spins to values less than $R \cdot \log 2$, even for $T \leq 1^\circ \text{K}$. Since the entropy which is not connected with the nuclear spin also falls, then even for $T \leq 1^\circ \text{K}$ the entire entropy of liquid helium-3 must fall to values less than $R \cdot \log 2$. Let us now compare this entropy with the entropy of solid helium-3. The existence of a crystal lattice in which the amplitude of the zero oscillations are con-

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siderably less than the distance between atoms leads either to the disappearance of the effects connected with the exchange of two atoms or to their considerable decrease. In solid helium-3, therefore, the nuclear spins must be freely oriented down to temperatures $T \sim T_M$. Consequently the entropy of solid helium-3 has a part equal to $R \cdot \log 2$, if $T \gg 10^{-7} \text{ }^\circ \text{K}$. Since the part of S due to oscillations is proportional to T^3 and in the order of magnitude equals $(12/5)\pi^4 R(T/\theta)^3$, where $\theta \geq 30^\circ$ (note: The theta θ of liquid helium-4 equals 30° (11), and that for solid helium-3 must be of the same order of magnitude.), then the entire entropy of solid helium-3 for $T \sim 1^\circ$ turns out to be a constant equal to $R \cdot \log 2$. Only for $T \leq T_M$ does the magnetic interaction of the nuclear spins in a crystal quickly decrease the entropy S down to zero for $T \ll T_M$. An example of dependence of S upon T in the two phases of helium-3 is shown in figure 1. As we note, T_1 is of the order of magnitude 1° . We see that for temperatures such that $10^{-7} \text{ }^\circ \sim T_M \ll T \ll T_1 \sim 1^\circ$, the entropy of solid helium-3 turns out to be greater than in liquid phase; that is, there exists a relation that is reverse to the ordinary one. Therefore for an isothermal transition of solid helium-3 to liquid helium-3 heat must not be absorbed, but released, and conversely heat must be absorbed during isothermal solidification. We have here negative heat of fusion Q which for $T_M \ll T \ll T_1$ equals

$$Q = T(S_{\text{liq}} - S_{\text{sol}}) = -RT \cdot \log 2. \quad (12)$$

At the same temperatures we obtain an unusual dependence for the pressure at which the phase transition of the temperature occurs. According to the Clausius-Clapeyron relation $dp/dT = (S_{\text{sol}} - S_{\text{liq}})/(\nu_{\text{sol}} - \nu_{\text{liq}})$

(sol refers to the solid phase, and liq refers to the liquid phase), we have

$$dp/dT = R \log 2 / (\nu_{\text{sol}} - \nu_{\text{liq}}) = -R \log 2 / (\nu_{\text{liq}} - \nu_{\text{sol}}) < 0. \quad (13)$$

It is taken into consideration here that the liquid possesses a volume that is greater than the crystal's volume.

Beginning with certain temperatures of the order of degrees the entropy S_{liq} will be greater than S_{sol} . Therefore p just as function T will have the form shown in figure 2. The existence of ~~#####~~ a rectilinear section with a negative slope for small temperatures qualitatively differentiates helium-3 from helium-4, in which the dependence of p upon T has a completely

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different character (12) (see figure 3).

We note that for $T \ll T_M$ the entropy S_{sol} tends towards zero in view of the onset of correlation between spins in the crystal. Here dn/dT tends toward zero. If one considers the adiabatic solidification of helium-3, then as is seen from figure 1 one discovers the possibility, in principle, of obtaining solid helium-3 for extremely low temperatures of the order 10^{-6} to 10^{-7} °K. To do this it is necessary to compress adiabatically liquid helium-3 having a temperature lower than T_1 down to pressures for which solidification sets in.

The condition governing the equality of entropy in the solid and liquid states results in the temperature of the crystal being obtained of the order of T_M (the transition is completed along the dotted line in figure 1).

The application of thermodynamic relations to the phase transition of liquid helium-3 to the solid state assumes that all corresponding times of the relaxation are small. Explanation of the feasibility of this assumption requires special considerations.

The exchange effects in liquid helium-3 must be due also to the nuclear magnetism of helium-3, since the free orientation of the spins of nuclei ceases not for $T \sim T_M$ as in almost all bodies (excluding compounds of the type ortho- or parahydrogen), but for $T \sim 1^\circ$. Beginning with these temperatures the nuclear magnetic moment of liquid helium-3 in an external magnetic field need not be proportional to the temperature (13), but must depend upon the temperature just as the susceptibility of exchange electron paramagnetics of the type solid hydrogen depends upon the temperature. In the model of liquid helium-3 discussed in Section 1 the nuclear magnetic susceptibility of helium-3 need not depend upon the temperature for $T \leq 1^\circ$.

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(3 figures follow)

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Figure 1

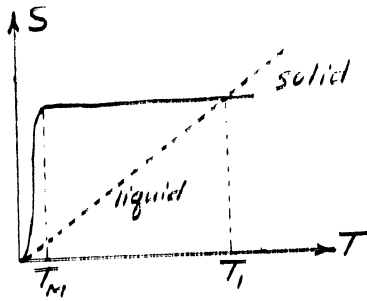


Figure 2

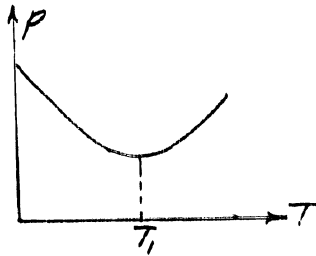
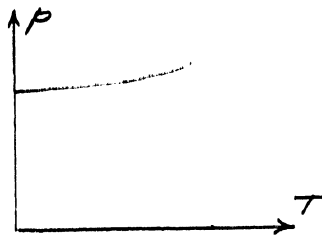


Figure 3



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