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Meson Effects on Deuterium

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The various meson effects on deuterium are of great theoretical and experimental interest. Meson absorption by deuterium, as in the case of the photoeffect on deuterium, must be accompanied by specific decay of deuterium. This effect turns out to be very sensitive to the type of meson field (scalar, pseudoscalar, etc.). See E. L. Feynberg, Journal of Physics Vol 5, p177 (1941). During absorption by deuterium of a meson with integral spin, there appear two nucleons each with kinetic energy equal to half the meson's rest mass. If the spin of the absorbed meson is $\frac{1}{2}$, then the absorption of the meson must be accompanied by the emission of another particle with spin $\frac{1}{2}$. This fact must have as its consequence non-monochromaticity of the energy spectrum of the meson-nucleons (namely, nucleons of the disintegrated deuterium).

It is very essential that the effects, which are opposite to the preceding ones, possess unexpectedly large cross sections, as indicated by calculations. The capture of fast neutrons by protons, that is, deuterium formation, with emission, for example, of a neutral meson, considerably exceeds the radiation loss of mesons, at the threshold of the effect.

For neutral mesons the above-mentioned opposite effect can be one of the most interesting meson effects on deuterium. Let us therefore consider as an example the formation of neutral, scalar, and pseudoscalar mesons by such means.

I. Width of Potential Well Equals Zero.

(a) Scalar meson. Let us limit ourselves to an approximation of two-dimensional waves in the ground state of the nucleons.

Let p_n, r_n, and p_p, r_p be respectively the momentum and coordinate of the neutron and proton in the laboratory system of coordinates; $\chi_1^{m_s}$ is the spin function; E_μ and k_μ are the energy and wave vector of the meson; and V is the normed volume.

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Further let R be the coordinate of the nucleons' center of gravity and let r be the distance between the nucleons.

In this system of coordinates the wave function of the ground state and the function of interaction are respectively rewritten as follows:

$$\psi_0(\vec{R}\vec{r}) = \frac{1}{V} e^{i(\vec{p}_n-\vec{p}_p)\frac{\vec{R}}{\hbar}} \cdot e^{i(\vec{p}_n-\vec{p}_p)\frac{\vec{r}}{\hbar}} \cdot \chi_1^{m_s} = \frac{e^{i\vec{Q}\vec{R}}}{V} \cdot e^{i\vec{q}\vec{r}} \chi_1^{m_s}, \quad (1)$$

$$H'(\vec{R}\vec{r}) = g\hbar c\sqrt{\frac{2\pi}{E_\mu V}} \cdot a^+ (e^{-i\vec{k}_\mu\vec{r}/2} + e^{-i\vec{k}_\mu\vec{r}/2}) \cdot e^{-i\vec{k}_\mu\vec{R}}. \quad (2)$$

The wave function of the final state of the system is taken in the following form:

$$\psi_F = V^{\frac{1}{2}} \cdot e^{i\vec{Q}'\vec{R}} \cdot \phi(r) \chi_1^{m_s'}, \quad (3)$$

where phi $\phi(r)$ is the "internal" wave function of the deuterium, which in the case of a potential well of zero width is written in the well-known form:

$$\phi(r) = \sqrt{\frac{\alpha}{2\pi}} \cdot \frac{e^{-\alpha r}}{r} \quad (4)$$

here alpha $\alpha = \frac{\sqrt{M\varepsilon}}{\hbar}$, M is the nucleon's mass, and epsilon ε is the deuterium's bond energy.

Corresponding matrical elements are written in the following manner:

$$|H'|_\pm = \frac{g\hbar c}{V}\sqrt{\frac{\alpha}{E_\mu}} \int \frac{1}{r} e^{-\alpha r} \cdot e^{i(\vec{q} \pm \vec{k}_\mu/2)\vec{r}} \cdot dv_r. \quad (5)$$

Keeping in mind that deuterium is formed only for parallel spins of nucleons and that interaction (2) does not vary the orientation of the spins we select the ground states of nucleons with parallel spins. Then the differential cross section is obtained in the following form:

$$d\sigma = 12\pi\frac{g^2}{\hbar c} c^3 \frac{1}{\hbar^2}\sqrt{\frac{\varepsilon}{2E}} \cdot M^2 \left\{\frac{1}{\alpha^2+l^2} + \frac{1}{\alpha^2+l'^2}\right\}^2 \frac{p_\mu \cdot \sin\theta \cdot d\theta}{E_\mu + 2Mc^2}, \quad (6)$$

where $l^2 = (\vec{q} - \vec{k}_\mu/2)^2, \quad l'^2 = (\vec{q} - \vec{k}_\mu/2)^2$ [sic] (7)

$\hbar\vec{q} = \vec{p}$ is the momentum of the nucleon, E is the nucleon's energy in the centroid system; $\hbar\vec{k}_\mu = \vec{p}_\mu$ and E_μ are the momentum and energy of the meson.

Integrating (6) with respect to the angle and regarding α^2 as small in comparison with l^2 and l'^2, we obtain the total cross section of the effect:

$$\sigma_s = 48\pi\frac{g^2}{\hbar c}\left(\frac{\hbar}{\mu c}\right)^2 \sqrt{\frac{\mu c^2}{2E}} \left(\frac{Mc}{\mu c}\right)^2 (\mu c)^4 \left\{\frac{1}{(p^2 - \frac{p_\mu^2}{2})^2} + \frac{1}{(p^2+p_\mu^2/2)p p_\mu} \cdot \ln\frac{p+p_\mu/2}{p-p_\mu/2}\right\} \cdot (p_\mu c/(E_\mu + 2Mc^2)) \quad (8)$$

where mu μ is the meson's mass.

The magnitude of the meson's momentum p can be obtained from the law of preservation of energy:

$$2E + \varepsilon = E_D + E_\mu = p_\mu^2/4M + \sqrt{p_\mu^2 c^2 + \mu^2 c^4}. \quad (9)$$

Here $2E$ is the kinetic energy of the nucleons in the centroid system and E_D is the kinetic energy of the deuterium. From (9) it follows

$$p_\mu^2 = 4M\cdot\left[2E+\varepsilon+2Mc^2-\sqrt{(2E+\varepsilon+2Mc^2)^2-(2E+\varepsilon)^2+\mu^2c^4}\right] \quad (10)$$

Setting $g^2/\hbar c = 1/6$, $M/\mu = 6.4$, $\varepsilon/\mu c^2 = 0.015$,

$$(\hbar/\mu c)^2 = 1.6\cdot 10^{-26}\,cm^2,\ p^2 = 2ME,\ x = 2E/\mu c^2;\ y = p_\mu/\mu c, \quad (11)$$

we obtain (8) and (10) in the following expression:

$$\sigma \approx \frac{10^{-24}}{\sqrt{x}}\left\{\frac{1}{(6.4x-\frac{1}{4}y^2)^2}+\frac{\ln\frac{\sqrt{6.4x}+\frac{1}{2}y}{\sqrt{6.4x}-\frac{1}{2}y}}{(6.4x+\frac{1}{4}y^2)\sqrt{6.4x}\,y}\right\}\cdot\frac{y}{\sqrt{1+y^2}+12.8} \quad (12)$$

for

$$y^2 = 25.6\left\{x+12.8-\sqrt{(x+12.8)^2-x^2+1}\right\}.$$

Figure 1 shows a curve describing the dependence of cross section upon energy (Note: Miss Chekhovich did the computation necessary for the drawing of these curves). The maximum cross section reaches the value of approximately 10^{-27} cm^2 when the nucleons's energy equal 1.3 ($2E = 1.3$) in units of meson mass.

(b) Pseudoscalar meson. The interaction of nucleons with pseudoscalar mesonic field is described in the centroid system in the following form:

$$H'_{ps} = \frac{\hbar}{imc}\,g\hbar c\sqrt{\frac{2\pi}{E_\mu V}}\left[\sigma_p\vec{k}_\mu\, e^{i\vec{k}_\mu\vec{r}/2}+\sigma_n\vec{k}_\mu\, e^{-i\vec{k}_\mu\vec{r}/2}\right]\cdot e^{i\vec{k}_\mu\vec{R}}. \quad (15)$$

By ordinary methods we obtain the integral cross section corresponding to this case: $\sigma_{ps} =$

$$\frac{1.28\cdot 10^{-24}}{\sqrt{x}}\left\{\frac{1}{(6.4x-y^2/4)^2}+\frac{1}{2}\,\frac{\ln\frac{\sqrt{6.4x}+\frac{1}{2}y}{\sqrt{6.4x}-\frac{1}{2}y}}{(6.4x-y^2/4)\sqrt{6.4x}\,y}\right\}\frac{y^3}{\sqrt{1+y^2}+12.8} \quad (13)$$

Comparing the pseudoscalar case (13) with the scalar case (12), we see that the spin operators entering the interaction (15) changed somewhat the numerical coefficients of the terms in the expression of the integral cross section (12), but the derivative in the interaction of the pseudoscalar case led to the appearance in (13) of an extra y^2 (that is, the square of the meson's momentum). This fact causes a very different behavior in the phenomenon near the threshold of the same effect (small momenta of the generated mesons). Curve 1 in figure 1 gives the variation, with energy of the neutrons, of the cross section of capture of neutrons by protons for the case where pseudoscalar mesons are created.

The absence of a maximum in the second curve -- in every case close to the threshold of the effect -- and the small absolute values in this region in comparison with the scalar case give one the theoretical possibility of experimentally differentiating these cases. More accurately, the behavior of the effect near the threshold is distinguishable depending upon whether the the nuclear forces contain a derivative or this derivative is absent.

We've considered the case where the potential well is of zero width. Investigations show that the effect is very strongly dependent upon the width of the potential well.

II. Width of Potential Well Differs From Zero.

Let us employ for the deuterium function within and outside the well the following expressions respectively:

$$\psi_D = (2\pi R^3)^{-1/2}\left(\frac{a}{1+a}\right)^{1/2} \cdot \sin bz/z \quad , \text{ for } z<1$$

$$\psi_D = (2\pi R^3)^{-1/2}\left(\frac{a}{1+a}\right)^{1/2} \cdot \sin bz/z \cdot e^{-a(z-1)} \quad , \text{ for } z>1$$

where $z = r/R$, R is the width of the potential well, $a = R(M\varepsilon)^{1/2}/\hbar$, $b = \frac{R}{\hbar}\sqrt{M(V-\varepsilon)}$, epsilon ε is the bond energy in the deuterium, V is the depth of the potential well, and $b \cdot \cot b = -a$. (Rose & Goertzel, Phy Rev, 72, 749, 1947.)

The differential cross section for the scalar case has the following form:

$$d\sigma \approx 100\left(\frac{R\mu c}{\hbar}\right)R^2 \frac{y\frac{a}{1+a}(a^2+b^2)^2\sin^2 b}{\sqrt{x}\,(\sqrt{1+y^2}+12.8)} \cdot \left\{\frac{\cos\zeta + a\sin\zeta/\zeta}{(a^2+\zeta^2)(b^2-\zeta^2)} + \right.$$

$$\left. + \frac{\cos\zeta' + a\sin\zeta'/\zeta'}{(a^2+\zeta'^2)(b^2-\zeta'^2)}\right\} \cdot \sin\theta \cdot d\theta \, ;$$

here
$$\zeta = \left(\frac{R\mu c}{\hbar}\right)^2 (6.4x + \tfrac{1}{4}y^2 + \sqrt{6.4x}\; y\cos\theta$$
$$\zeta' = \left(\frac{R\mu c}{\hbar}\right)^2 (6.4x + \tfrac{1}{4}y^2 - \sqrt{6.4x}\; y\cos\theta \; .$$

The integral cross section obtained graphically is shown in figure 2 for various widths of the potential well. We see that the absolute values of the effect depend strongly upon dimensions of the potential well.

From the data on the scattering of slow neutrons by free protons and ortho and parahydrogens it follows that in the triplet state for R the approximate values are probably 1.5 to $1.8 \cdot 10^{-13}$ cm. We see that in comparison with the case $R = 0$ the absolute values of the effect at the most decreases 5 to 10 times and the maximum itself is displaced closer to the threshold of the effect (from $x = 1.3$ for $R = 0$ to $x = 1.1$ in the region $R = 1.6 \cdot 10^{-13}$).

The maximum is closest to the threshold for R in the interval 1.6 to $1.7 \cdot 10^{-13}$ and after that it again (with increasing R) shifts to large energies, as seen in figure 3.

Meson effects with the radiation of charged mesons are forbidden for the scalar meson of small energies by the Pauli exclusion principle. In such cases where the spin operators enter the law of interaction of nucleons with the meson field, these processes are permitted. The preceding considerations are required in a number of essential refinements and improvements of accuracy. We for example took for the ground states of the nucleons the two-dimensional waves, but the presence of nuclear forces must distort them. True, certain evaluations existing in the literature for the case of the photoeffect on deuterium show that in this region of energies the attainable absorption due to the non-orthogonality of the states is less than 30%. Possibly in our case this absorption alters the result in an essential manner. A decrease of the order of 2-3 times will leave the effect experimentally observable. (Note: In connection with the photoeffect on deuterium, see M. E. Rose and G. Goertzel, Phys Review, Vol 72, p 749, 1947.)

Submitted by Academician S. I. Vavilov
1 Nov' 1950.

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[Figures appended.]

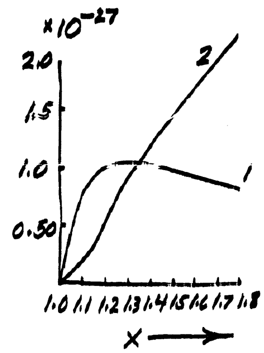

Figure 1.

1 : scalar meson.

2 : pseudoscalar meson.

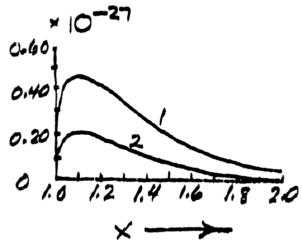

Figure 2.

1 : $R = 1.6 \cdot 10^{-13}$

2 : $R = 1.8 \cdot 10^{-13}$

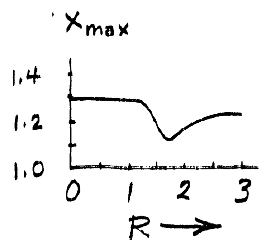

Figure 3.

Approximate position of x_{max}.

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