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Title: THE STRENGTH OF FROZEN SOILS UNDER BUILDING FOUNDATIONS

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THE STRENGTH OF FROZEN SOILS UNDER BUILDING FOUNDATIONS

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Studies of the resistance of various soils to external loads show that soils can be divided into three basic groups according to internal resistance and deformation: 1) soils in which resistance is provided by internal friction; 2) soils in which resistance is created mainly by cohesion; and lastly 3) soils in which resistance is provided by both internal friction and cohesion.

We shall not enter into a detailed classification of these three groups of soils because this complex problem should be the subject of a special study; we will merely show the place occupied by frozen soils in this classification.

We shall consider some research data on the behavior of frozen soils under load. Figure 1 shows typical compression (stress-strain) diagrams obtained by N. A. Tsytovich, Corresponding Member, Academy of Sciences USSR, for cubes of frozen clay at -10.4°C (Figure 1, right) and frozen sandy loam at -3.1°C (Figure 1, left).

In the diagrams, the proportional limit and the section showing rectilinear dependence between stress and strain are both clearly apparent. With a further increase in applied load (beyond the proportional limit), the diagrams become quite different. At low temperatures (-10.4°), strain increases only as the applied load is increased almost until the latter reaches the rupture value, i.e., there is no definite yield point. This type of rupture is similar to that observed in the cleavage of cubes of brittle solids.

In the second case, for a relatively high temperature (-3.1°), the yield point is clearly seen after a certain period of increasing strain with increasing stress, i.e., strain continues to increase markedly with no increase in the applied load and the deformation speed remains almost constant.

We also observed this yield effect (experiments in 1946 by the Institute of Permafrost Studies, Academy of Sciences USSR, in the Leningrad Construction Engineering Institute) in pressing a 3 x 20 cm die into frozen clay at -3°, humidity 28% and cohesion 2.28 kg/cm^2; the yield point appeared at a unit stress of 12.2 kg/cm^2.

Consequently, the type of deformation of frozen soils under load indicates that soils at low temperatures behave similar to brittle solids, while at relatively high temperatures, they acquire properties similar to those of plastic solids.

In both cases, resistance is created by cohesion. Thus, experimental data forces the conclusion that the strength of frozen soils with the exception of dry sandy soils (the so-called "dry frost") is provided mainly by cohesion. A theoretical study leads to the same conclusion. In order to show this, we briefly consider the basic principles underlying any theoretical determinations of the rupture load.

For the simplest case of a uniformly-distributed bar load (the two-dimensional problem), it is frequently recommended that soil's bearing capacity limit be theoretically determined by plasticity theory Prandtl's (Sokolovskiy, V. V., The Statics of a Friable Medium, Izd. AN SSSR, 1942).

However, as studies have shown, Prandtl's system of sliding surfaces must be changed slightly in order to bring the theoretical picture of soil rupture into agreement with the effects actually observed in nature. This change relates to the soil region situated under a foundation, in which no sliding surfaces actually occur (contradicting Prandtl's ideas).

A bearing bulb forms beneath foundations, which together with the foundation deforms the surrounding ground. With accuracy sufficient for practical purposes, we can assume that this bearing bulb has the form of a rectangular prism (i.e. has a right-triangle cross-section). Then the formation of sliding surfaces will be as in Figure 2. This system has already brought us considerably closer to the picture observed in nature.

The limit of soil's bearing capacity is expressed by Prandtl's formula, which we have changed slightly:

$$P_{rup} = k_1 c + k_2 q; \tag{1}$$

Here c is cohesion; q is the intensity of a uniformly distributed load on the soil's surface, whose action is equivalent to the action of the soil's

$$K_1 = (1 + \cot \Phi)\left[(1 + \sin \Phi) \cdot \exp\{(\pi + \Phi) \tan \Phi\} - (1 - \sin \Phi)\right]$$
$$K_2 = (1 + \tan \Phi) \cdot (1 + \sin \Phi) \cdot \exp\{(\pi + \psi) \tan \psi\} \tag{2}$$

Where Φ is the soil's angle of internal friction.

As seen from the form of (1), its first term gives the resistance due to cohesion while the second term gives the resistance due to the loading action of the ground's own weight and to the internal friction caused by this action.

$$\xcancel{k_1 = (1 + \cot \Phi)\left[(1 + \sin \Phi) \cdot \exp\{(\pi + \Phi) \tan \Phi\} - (1 - \sin \Phi)\right]}$$
$$\xcancel{k_2 = (1 + \tan \Phi) \cdot (1 + \sin \Phi) \cdot \exp\{(\pi + \psi) \tan \psi\}}$$

We now investigate for different angles of internal friction, φ the relation between these two terms for foundations of ordinary sizes frequently encountered in practice. Figure 3 shows a graph describing the variation of each term in formula (1) as a function of internal friction angle for the case of a foundation 3.0 meters wide sunk 2.0 meters deep into the ground and having a density of 1.8 tons cubic meter and cohesion of 2.0 kg/cm^2. The graph shows that the "specific weight" of the second term in formula (1) is small for low values of ϕ. Thus, at $\phi = 30°$ the term k_2q is 18% of the total resistance; at $\phi = 20°$ it is 13%; and at $\phi = 15°$ it is only 11%. For foundations wider than 3 meters, the value of the second term of (1) will be still less. Thus, analysis of the formula for the bearing capacity limit shows that in soils having cohesion 2.0 kg/cm^2 and up the resistance of internal friction can in most cases be disregarded as of negligible influence and that soil's strength under load can be considered as provided solely by cohesion for internal-friction angles 20° and lower. Most frozen soils have very low internal friction and considerable cohesion.

The sharp reduction of internal friction in frozen soils in comparison with soils having temperatures above 0°C is explained by the considerable dissociation of soil particles due to water freezing in the pores. In the deformation of soils under load, internal friction decreases because of the presence of hydrodynamic processes, even though the particles come closer to each other.

Taking those effects into consideration, we should think that in both clayey and sandy frozen soils the angle of internal friction is usually less than 20°.

For the characteristic of the amount of cohesion, we employed N. A. Tsytovich's data on natural frozen soils' average temporary resistance to compression (temporary resistance of a cube) when completely saturated with water.

If we assume that the cubes could be normally deformed and that the cohesion was numerically equal to the tangential breaking point under experimental conditions, then according to the above data the cohesion of frozen soils varies from 2.5 to 18 kg/cm^2.

The temporary resistance to compression is given by the following figures:

Temperature:	Up to -0.5°	From -0.5° to -1.5°	From -1.5° to -2.0°
Sands	22	27	36
Sandy Loams	11	22	--
Clayey Soils	--	20	26
Clays	6	17	--
Dusty Silt Soils	5	15	23

Consequently, theoretical studies of resistance forces accounting for strength under load fully confirm the conclusion that for most frozen soils these resistance forces are created mainly by cohesion alone. Starting from this basic principle, we consider it correct to adopt the Saint-Venant theory of strength [the so-called maximum strain theory] to determine frozen soils' resistance to load.

We first consider the case, very important in engineering, of the effect of a uniformly-distributed bar load (the two-dimensional problem). We must first clarify how valid is the assumption of uniform transmission of pressures to the soil by rigid foundations (with which we must deal most frequently) for loads near critical values.

Rigid foundations within elasticity limits transmit pressure to the soil quite nonuniformly; moreover if the soil has considerable cohesion, the pressure distribution diagram along the foundation's bottom has a saddle-like form (see Figure 4).

The boundary ordinates of this diagram are much greater than the ordinate for the average uniformly-distributed pressure P_a; therefore for a value of P_a far from that causing plastic deformation of the soil, local plastic strains will begin to appear at the foundation's edges. Since plastic strains emerge in extremely small zones relative to the foundation's total width, the zones do not appear on load-strain diagrams, which (i.e. diagrams) therefore cannot indicate when the proportional limit has been reached.

The emergence of these boundary zones of plastic strains immediately causes the stresses to be redistributed along the foundation's base -- the stress distribution becoming nearly uniform.

This effect gives us sufficient basis to make the following conclusions: 1) zones of plastic strains at the foundation's edges are localized, and plastic strains cease in these zones after a certain time; and 2) for further increase of load, the pressure distribution along the foundation's base can be considered approximately uniform.

The condition governing the emergence of plastic strains at a given point, according to Saint-Venant, is expressed by the equation:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = C \qquad (3)$$

where σ_1 and σ_2 are the principal stresses.

For a uniformly distributed bar load on a surface, the principal stresses at any point of an elastic or linearly-deformed half-space are determined by the Michell formulas:

$$\left.\begin{aligned} \sigma_1 &= \frac{P}{\pi}(2\beta + \sin 2\beta) \\ \sigma_2 &= \frac{P}{\pi}(2\beta - \sin 2\beta) \end{aligned}\right\} \qquad (4)$$

where 2β is the "angle of visibility" of a loading bar, i.e., the angle between the lines drawn from a given point to the boundary points of the loading bar.

Substituting the expressions for σ_1 and σ_2 from (4) into (3), we obtain the condition governing emergence of plastic strains in the following form:

$$\sin 2\beta = \frac{\pi c}{p} \quad \text{or} \quad p = \frac{\pi c}{\sin 2\beta} \tag{5}$$

These expressions show that plastic strains begin to appear at a load equal to [redacted]

$$p = \pi c \tag{6}$$

since this value corresponds to the minimum at which condition (5) is satisfied. From condition (5) it follows that $\sin 2\beta = 1$; i.e. $2\beta = \pi/2$ and consequently the plastic strains will initially emerge at those points of the half-space for which the angle $2\beta = \pi/2$. This condition is satisfied by points situated on the semicircumference passing through the boundary points of the loading bar (see Figure 5).

We believe it correct to consider the load determined by formula (6) as the proportional limit of the soil.

For a further increase of load, plastic regions will begin to develop in the depths and at the sides; as a result of these spreading plastic deformations i.e., a shift of some part of the soil along sliding surfaces, the soil is ruptured.

One cannot study the development of regions of plastic strains by using formula (5), because the law of propagation of stresses in a medium having both plastic and elastic regions will be different from the law in a linearly-deformed half-space and Michell's formula holds only for the latter. In order to study the stressed state beyond the proportional limit, we must solve the mixed problem involving elasticity theory and plasticity theory; unfortunately there is still no solution of this problem for the given case.

Experiments show that in the development of plastic regions in frozen soils as well as in soils above 0°C, a bearing bulb forms under the punch, which with the punch deforms the surrounding medium and presses the soil out towards the sides.

We observed this effect in pressing a punch (in the form of a bar) into frozen clay (t = 3°); the effect was very clearly detected in Nadai's experiments (see Figure 6).

By analogy with solids, we can consider for frozen soils that this bearing bulb is of the form of a prism whose cross-section is an isosceles right triangle.

To obtain the value of the breaking load (bearing capacity limit), we can solve the differential equations in Prandtl's plasticity theory of solids. We must keep in mind, however, that there are actually no plastic strains in the triangular region of soil directly under the punch, in which according to Prandtl the plastic state emerges. This region should be considered as elastic (see Figure 7).

Instead of Prandtl's three plastic regions, therefore, we will have two: one region in the form of a wedge with the vertex at edge of the load (in this region, the sliding lines are the radius-vectors emerging from the edge point and the circle having a center at the same point); and the second region in the form of a right triangle with the hypotenuse emerging on the surface (in this region, the sliding lines are straight lines at an angle of 45° with the horizontal).

An elastic body forms under the punch (namely, the foundation) for the following reasons: 1) Plastic regions do not occur immediately in the form corresponding to that at the time when the soil ruptures, but develop gradually; Prandtl's solution therefore does not conform with reality, since it ignores the time factor in studying the process in which a body changes from the elastic to the plastic state. 2) The rigidity and roughness of the punch are redistributed and cause additional stresses (because of soil friction at the base of the punch), and this is also ignored by the Prandtl solution.

This correction of the Prandtl notion is very important, but does not affect the end result.

The breaking load is given by the formula:

$$P = C(\pi + 2) = 5.14C \qquad (7)$$

The uses of this formula for the strength characteristic of frozen soils was proposed by Tsytovich in 1937 [N. A. Tsytovich and M. I. Sumgin, Principles Governing the Mechanics of Frozen Soils, (Osnovaniya mekhaniki merzlykh gruntov), Izd. An SSSR, 1937].

Experiment shows that formula (7) gives fairly good results. For example, for clays having a temperature -3°C, humidity 28% and cohesion 2.28 kg/cm^2, we have according to formula (7): $P_{rup} = 5.14 \times 2.28 = 11.7$ kg/cm^2; a value of 12.2 kg/cm^2 was found experimentally -- a discrepancy of 4.1%.

It is extremely difficult to obtain experimentally the value of the breaking load for soils having very low temperatures, because very high pressures are required. Therefore the use of formulas (6) and (7) is quite justifiable even in this case.

One should designate the permissible stress from the value of the proportional limit calculated by formula (6), after introducing a safety factor of at least 1.5 to the latter. At the same time, one should keep in mind that the use of the above formulas to determine the permissible stress presupposes constant soil temperatures. In order that the soil should maintain its proposed bearing capacity, all necessary measures must be taken to preserve its frozen state.

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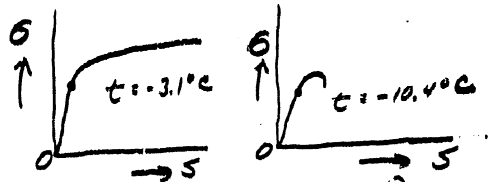

Fig 1 - Compression Diagrams For Frozen Soils; a - Sandy Loam for $t = -3.1°C$; b - Clays at $t = -10.4°C$ (p 49)

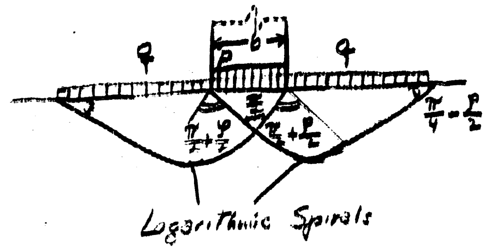

Fig. 2. Outline of Sliding Surfaces in Soils of the 3rd Group For a Bar Load (p 49)

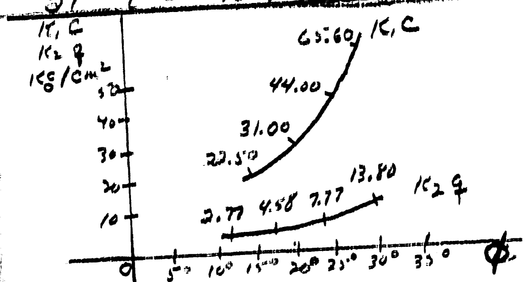

Fig. 3. Graph of the Change in the Value of the Members of Formula (1) as a Function of Φ. (p 50)

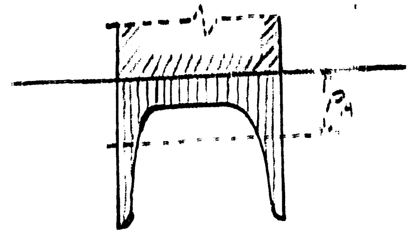

Fig 4. Diagram of Pressure Distribution Under the Bottom of a Rigid Foundation (p 51)

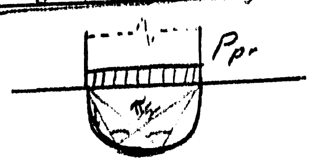

Fig 5. Diagram of Initial Region of Plastic Deformations in Frozen Soils (p 52)

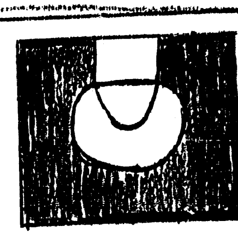

Fig 6 - Diagram of Bearing Bulb Under a Punch in Nadai Experiments (p 53)

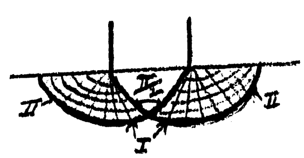

Fig 7 - Diagram of Sliding Surfaces in Frozen Soils For a Bar Load (p 53)

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