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PRESENT STATUS AND DEVELOPMENT OF COMPUTING TECHNIQUES

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PRESENT STATUS AND DEVELOPMENT OF COMPUTING TECHNIQUES

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1. The Role of Computing Techniques in the Development of Present-Day Science

Precise computing techniques are essential in solving problems of: structural mechanics, geodesy, geophysics, theory of linear and nonlinear oscillations, land, sea, and air artillery, bomb-sighting, anti-aircraft [redacted]rtillery, long-range rockets which are directed by special control instruments for firing and sighting. The examples given demonst[redacted]te the importance and diversity of the problems which must be solved by computing techniques rapidly and accurately.

The scale and nature of present-day calculating operations have determined their orientation. [redacted] "[redacted]roic" individual calculations (of Euler, Gauss, and others) have been replaced by "computing collectives" equipped with special computing techniques. Therefore, present-day computing techniques are characterized by various instruments, devices, and machines, commonly known as "computers". In a number of cases, these new techniques have made possible calculations so large-scale and rapid that they would have been impossible with previous methods. The great effectiveness of present-day military operations is due in a large measure to the perfection of computers.

In this report, we briefly review existing methods of speeding up computations, principles of computers, and ~~the~~ development of computing techniques.

2. Priority Problems in Applied Mathematics and Present-Day Methods for Solving Them

One trends in the development of present-day applied mathematics is a) the study and proof of existence theorems governing definite types of solutions of mathematical-physical and engineering problems and b) [redacted] uniqueness of solution, convergence of approximation [redacted] processes, asymptotic nature of solutions, etc. The theoretical and methodological value of these works is un[redacted]questioned, since these studies make possible qualitative pictures of

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various physical processes which could not be obtained in any other way. However, qualitative solutions alone are insufficient for the practical solution of most problems of physics and engineering.

Another trend equally important stems from the desire of physicists and engineers to reduce mathematical studies to numbers, tables, graphs, or nomograms; that is, to the stage closest to the consumer who requires practical answers to his problems from the mathematician.

Our report deals with the second trend; that is, with numerical methods for solving mathematical problems, the most important of which are:

1) ordinary differential equations;
2) boundary-value problems of mathematical physics and integral equations;
3) systems of algebraic linear equations with many unknowns;
4) high-order algebraic equations and transcendental equations;
5) harmonic analysis.

Numerical solution of any mathematical problem consists of its theoretical solution (in principle) that is, its reduction to existing "computing schemes", and calculations by means of these schemes. The theoretical solution, at the same time, depends upon the "scheme" to which it is directed. Each scheme has its area of applicability, i.e., a set of problems reducing to it.

Before we consider present calculating methods, we should like to point out existing methods of numerical solution of some important mathematical problems.

One of the most generally used devices for the numerical solution of problems of analysis is the method of series expansion and the "method of networks" ('metod setok': nets, meshes, grids, etc).

In series expansion, the functions both known and unknown, figuring in a problem are expanded in functional series and the relationships between them are transformed into ratios between their expansion coefficients. By resolving all series into a finite number of terms (determined by the accuracy desired), we get a finite system of equations and coefficients. We can approximately represent all functions f_i entering into a problem by the finite sums

$\boxed{f_i = \sum a_{ij} u_j}$

[redacted] (1), where u_j are functions used in the expansion found by iterative operations. For example, in expansion in orthogonal systems (trigonometric and Bessel functions, etc.), that is, so-called harmonic analysis, we have $a_{ij} = \int f_i u_j dx$ (2).

$\boxed{a_{ij}}$

The number of coefficients which must be calculated is sometimes very great. For example, in the solution of one integro-differential equation in the Division of Approximate Calculations, Institute of Mathematics, it was necessary to calculate 25 coefficients determined by the integrals (2) with infinite limits; and since this equation was solved for 30 different values of the parameters, a total of 750 such integrals had to be calculated.

Another basic method of algebraisation is the "method of networks". Values of an unknown function are sought at a certain system of points, i.e., nodes of the network. The integrals are replaced by sums of the values at these points, and the derivatives are replaced by ratio of differences. A system of equations is obtained which connects the values of the unknown functions at adjacent nodes of the network. The method of networks with consecutive transitions from node to node is a method for the numerical integration of ordinary differential equations. In boundary-value problems, a great number of algebraic equations connecting the unknown values at the inner nodes of the network with each other and with the known values on the edges must be solved simultaneously. The main difficulty here lies in the solution of this system, since the number of equations is frequently greater than in the method of series expansion; there are hundreds of nodes in plane problems and thousands in spatial problems. The numerical solution of systems of linear algebraic equations with many unknowns has therefore become one of the most fundamental problems in applied analysis.

The method of consecutive approximations is also frequently used in calculating techniques. The essential feature of all the methods cited is that they all require large-scale application of identical operations, e.g., identical operations at each node of the network in the method of networks, finding identical integrals in harmonic analysis (the method of series expansion, etc.).

Numerical solution of systems of algebraic equations also involves many monotypical (iterative) operations; many substitutions in the method of elimination of unknowns are of the same type; many calculations of monotypical linear forms are encountered in the method of successive approximations, etc.

Thus, the use of these methods for the numerical solution of problems in applied mathematics is connected with a single difficulty, i.e., the performance of large-scale iterative calculations.

3. Methods for Speeding Up and Automatizing the Computing Techniques

The importance of tables and graphical aids has not decreased with the appearance of special computers. In the first place, tables and nomograms are much easier to produce than special mathematical machines, and therefore they have and will continue to have wide application in individual calculations and in the work of small "computing bureaus". In addition, many computing operations performed on machines require the use of tables and many machines are equipped with special devices for automatic selection of tabular data.

No matter how important the role of tables, nomograms, and other aids in computing operations, their use alone cannot provide high effectiveness for computing techniques, since the performance of a great many computing operations even with special aids requires prolonged work to obtain decisive results.

Computers and special mathematical machines for mechanization and automatization of computing processes are the most effective and up-to-date devices for accelerating computing processes in the solution of present-day problems of applied mathematics. These constructions are being used extensively in present-day computing techniques and are becoming increasingly important in the solution of physics and engineering problems.

The remainder of our report is concerned with a discussion of the main types of computers used in present-day computing techniques.

4. Devices for Performing Mathematical Operations

Computing operations dealing with the numerical solution of problems of applied mathematics possess several main stages.

When numerical results cannot be obtained directly, we resort to approximate numerical methods. This means that complex operations are replaced by simpler but more numerous operations. The process of resolving complex calculations into more elementary ones may be continued further. Thus, any complex algorithm can be resolved into a series of elementary operations (as the arithmetic operations are usually considered), i.e., into "logical" operations (selection and combination of numerical data for operation on it, classification of data according to certain characteristics, selection from tables, etc.) and into determination of intermediate and end results.

Consequently, in the numerical solution of applied-mathematics problems, operations on continuous mathematical functions are for the most part replaced by operations on discrete mathematical quantities.

Present-day computers can be used for mechanizing computing processes at various stages in the numerical solution of a mathematical problem and are based upon the modeling of various mathematical functional relations or quantities.

All present-day computers can be divided into two main groups. The first group includes non-digital computers, which model continuous mathematical relations. They are based upon the fact that one and the same mathematical relation may describe various physical processes. The Laplace equation, for example, describes voltage in an electrostatic field, stationary distribution of heat, and the movement of an ideal non-eddied fluid. There is a correspondence between different physical phenomena in that they are described by the same mathematical equations. Among these physical systems, some are more fitted to the assignment of definite values of quantities and measurements. These can be used to model a given mathematical relation, and thus one can obtain the solution directly from the given equation without resorting to approximate numerical solution.

Instruments to model intermediate stages of the algorithm can be devised from the same principle. For example, the solution of systems of linear equations can be obtained by means of a model performing the algorithm by elimination of variables or by the method of iteration. In the first case the elimination of one unknown in each pair of equations is modeled, while in the second, the main operation of the individual iteration is modeled with the result transmitted for further iterative processing.

Some physical quantity, e.g., length, deflection angle, current, voltage, power, etc., serves as a model of a continuous mathematical quantity in non-digital computers modeling continuous mathematical relations. The dependence between these quantities in the instrument corresponds to the given mathematical functional relation.

The second group includes digital computers, which model discrete quantities and operations on them (calculating, calculating-analytical, calculating-impulse machines).

In these devices, the discrete mathematical quantity corresponds to a discrete value of some mechanical quantity enumerated in a decimal or other similar system. For example, in writing a given value of this quantity in a decimal system, the number of units of it corresponds to the same number of angular or linear displacements of the linkage which registers units; the number of tens corresponds to the same number of angular or linear displacements of the linkage which registers tens, etc.

Computers of the second group are equipped only to perform the four basic arithmetical operations on the given and newly obtained numbers. Their region of application, however, is almost unrestricted, since the solution of any mathematical problem can be reduced to an algorithm which can be performed by a series of arithmetical operations by means of successive resolving of complex calculating operations into elementary operations.

Computers, (i.e., non-digital calculators) are divided into mechanical, electrical, electromechanical, optical, etc. depending upon the method of assigning and measuring quantities and upon the method of implementing the mathematical dependency or upon the nature of the coupling or connection between the assigned and obtained quantities.

In mechanical computers, the assigned values are introduced mechanically, the quantities sought are obtained mechanically, and there is mechanical coupling between the assigned and obtained quantities. The machine for integrating differential equations constructed under the direction of I. S. Bruk, Corresponding Member of the Academy of Sciences USSR, in the Power Engineering Institute, Academy of Sciences USSR, and the calculating-analytical machine of the Powers firm are examples of mechanical computers, or calculators.

In electrical computers, quantities are assigned and obtained by the electrical method; the mathematical relation between assigned and desired quantities is also implemented electrically. The electro-integrator designed and constructed under the direction of Professor L. I. Gutenmakher in the Power Engineering Institute, Academy of Sciences USSR, and Mallock's machine for solving systems of linear equations are examples of electrical computers.

In electromechanical computers, part of the processes are carried out mechanically and part, electrically. For example, the mathematical dependency is effected electrically, while the quantities are obtained and introduced mechanically. One of the latest German bombsights for bombing when pulling out of a dive is an example of this type of electromechanical computer.

We now consider the general problem of accuracy of computers of both groups.

5. The Accuracy of Computers

One of the most important characteristics of computers is the accuracy with which a numerical solution of a given mathematical relation can be obtained. For a number of reasons, the mathematical functional relation effected by a computer is different from that given. In addition, there are errors in introducing the given quantities, and in measurements and observations of the desired quantities. Therefore, the numerical solution of a mathematical problem obtained by a computer is different from the true solution.

There are three groups of reasons for errors in solving problems by means of computers.

The first group involves errors caused by replacing the given mathematical relation with some approximate relation that is more convenient for numerical solution. For example, in the numerical integration of a differential equation, one of the methods of algebraization of this equation is usually used and thus slight inaccuracies are obtained in the solution of this equation (L. I. Gutenmakher's electro-integrator).

In solving mathematical problems connected with anti-aircraft artillery, bomb-sighting, etc., the form of the functions is often simplified considerably and thus the design of the computer can be simplified considerably. This also reduces errors due to inaccuracies of the instrument itself in the solution of problems. Errors caused by replacing the precise mathematical relation with some approximate relation, i.e., caused by simplification of the design of the computer, belong to the so-called systematic errors. These errors can be taken into consideration when the solution of the problem is obtained.

The second group includes errors caused by inaccuracy in introducing the assigned quantities into the computer and errors in measurements or observations of the results obtained. For example, in bomb-sighting computers for automatically calculating and constructing the sighting angle from the height and air speed introduced into the mechanism, an error in the sighting angle arises due to the inaccuracy in the introduction of these parameters. Errors caused by inaccuracy in the introduction and measurement of quantities depend substantially upon the scale in which these quantities are modeled in the computer.

Finally, the third group includes errors caused by variations in the parameters of the computer itself. If only kinematic parameters of a mechanism are used in modeling a mathematical operation (this corresponds to the use of steady-state conditions in electrical devices), the only errors which will appear will be those caused by deviations in the actual values of the parameters of the device from those planned (variations in sizes of the linkages, gaps in kinematic couples, variations in electrical resistances, etc). These errors depend upon the design of the instrument and quality of its manufacture. For example, in summation summation with differential gears in [illegible] of inaccuracies in production and position of the differential gear wheels. In an electrical multiplying device of the bridge type (with active resistances), errors arise in the product because of inaccuracies of the red resistances.

If a mathematical problem solved by a computer depends upon the dynamic parameters of the mechanism (this corresponds to utilization of non-steady-state or transient conditions in electrical devices), we have in addition so-called inertial errors, caused by the mechanical or electrical inertia of the system. For example, if a centrifugal tachometer is used as a differential device, an error may arise in the derivative because of free oscillations in the tachometer coupling. In an electrical device containing inductance and capacitance, errors arise because of natural oscillations in the circuit. Under certain conditions, inertial errors may become extremely important. They can be eliminated by correct selection of the dynamic parameters of the device.

Errors of the last two groups, i.e., errors caused by inaccuracies in introduction and measurement and inaccuracies of the computer itself are of a random nature. The influence of these errors upon accuracy of solution can be determined by the theory of probabilities.

From the standpoint of accuracy, non-digital calculators, which model continuous mathematical dependencies, and digital calculators, which model discrete quantities, are quite different.

Non-digital computers always possess errors due to inaccuracies in introduction and measurement of the numerical data and inaccuracies of the machine itself; and, therefore, the accuracy of these machines is limited. These errors can be modified by efficient design and high quality in manufacturing and adjusting.

As was pointed out above, non-digital computers can model a given operation directly without preliminary simplification, and in this case, the numerical solution obtained will not contain systematic errors.

Digital computers, which model discrete quantities, are free from random errors caused by inaccuracies in introduction and measurement of numerical data. As for errors of the machine itself, the operations are carried out on discrete quantities and thus errors of the mechanism either do not influence the result (when they are small) or else cause a complete breakdown of the mechanism (when its operation is grossly inaccurate due to errors). Thus, computing operations performed by digital computers are free from random errors if the machine is correctly regulated. The accuracy of these devices is almost unlimited.

Systematic errors in the solution of a problem are caused by simplifications (by a transformation of a given relation to one between discrete values, for example) and by a deviation between the actual number of significant figures in the discrete quantity and the number of signs introduced into the machine.

At the present stage in the theory of accuracy of mechanisms, it is possible to clarify the reasons for the inaccuracies of each mechanism, to evaluate the accuracy of various types of mechanisms, to establish the effect of various primary errors upon the accuracy of the machine, and thus to secure a high degree of accuracy. Academician N. G. Bruyevich studied general methods for calculating the accuracy of mechanisms and made important contributions to the theory of accuracy of mechanisms.

6. Principles Governing Mechanical Devices for Modeling Continuous Mathematical Relations

Present mechanical computers (i.e., non-digital calculators), which model continuous functional relations, are in most cases based upon the kinematic principle of modeling. Kinematic circuits are used in which the dependency between the movements of the individual linkages correspond to the given mathematical dependency. The driving linkages are the ones whose laws of motion correspond to the laws governing the independent variables. Their number is always equal to the number of independent variables. The motion of the driven linkage corresponds to the law of variation of the dependent variable.

Several mechanisms, differing from each other in design and construction, can be selected for even the simplest functions. For example, a bar mechanism with knee couples (Fig. 1) or a cam mechanism (Fig. 2) can be used to obtain the sine function. A differential gear (Fig. 3) or a bar addition mechanism (Fig. 4) can be used to add two continuous quantities.

A mechanical computer for modeling a complex functional relation is usually a set of separate mechanisms, each of which effects a simple functional relation.

Mechanical computers permit operations on complex numbers (plane vectors) as well as on real numbers. For this purpose, the so-called pantographs are used, with the help of which a given curve can be transformed into a curve similar to it, thus corresponding to multiplication of a variable complex number by a constant real number. The curve can also be turned through a certain angle, which, together with a change of dimensions, gives multiplication by a complex number. By generalizing this latter fact, S. A. Gershgorin proved the important formula: a bar mechanism can be designed to effect any algebraic integral (entire) function of a complex variable. Thus, it is possible to design mechanisms which effect conformal transformations corresponding to these functions.

In principle, almost any mathematical relation can be realized by a combination of simple mechanisms. In the practical solution of complex problems, however, many difficulties may arise in connection with providing normal operating conditions for the computer (providing the proper accuracy, efficient size, etc). The accuracy of computers is a particularly important problem, since the accuracy of a machine may drop considerably with an increase in the number of mechanisms entering into its construction.

Mechanical computers have been used extensively in military devices and as machines for solving systems of algebraic equations and ordinary differential equations. At the present time, they compete successfully in these fields with electrical computers.

We do not propose to give in this report a detailed survey of various military devices or of those used for the solution of systems of linear equations, since these instruments are not essentially different from other computers. Military computers are subjected to narrowly specialized aims and are usually synthesized from functional mechanisms. The development of machines for the solution of systems of linear algebraic equations follows the same general pattern. Inasmuch as the number of mechanisms entering a machine increase substantially with the increase in the number of equations, machines for systems having more than 15-20 equations are obviously inefficient to design. The machines built so far have been limited to this number of equations.

Mechanical computers have been used extensively for integration of systems of ordinary differential equations. The main element of a machine for integration of differential equations is the integrator, proposed in 1876 by Lord Kelvin. The following paragraph is given to the operating principle of Kelvin's integrator.

A machine for integration of differential equations consists of integrators (their number corresponding to the order of the equation) and a series of functional mechanisms or patterns. The drive is affected through shafts of the independent and dependent variables.

Fig. 5 shows the plan of a device for integrating the differential equations $dy/dx = x^2 \sin x$.

The device consists of one integrator (J), one sine mechanism (S), one power (exponential) mechanism (m), and one multiplying mechanism (M). The independent variable x is transmitted to the integrator disc, to the driving linkage of the sine mechanism, and to the driving linkage of the power mechanism (drawing A) from the shaft (x). The sin x and x^2, respectively, are taken off the driven linkages of the last two mechanisms. They are multiplied by means of the multiplying mechanism, and the product is transmitted to the integrator carriage. The value of the function sought is taken from the integrator carriage.

Drawing B of Fig. 5 shows a device for the same equation using a pattern. The pattern is traced according to the right part of the equation and displaced from the shaft (x). The ordinates of the pattern are shown by the integrator carriage.

One of the first machines for integration of systems of four linear differential equations, constructed in 1912 by Academician A. N. Krylov, was never used in practice because of its low accuracy. The main reason for the high errors introduced by the machine was slippage in the friction drive of the integrator. At that time, there were no special devices to counteract this slippage. A little later, a special regulator was invented to relieve the integrator disc of external loads in the form of the driven linkages of the mechanism. This regulator considerably furthered the development of mechanical devices for the integration of differential equations. [The next four paragraphs deal with Bush's differential analyser and its modifications. An improved (1931) model of the differential analyzer was constructed in the Power Engineering Institute, Academy of Sciences USSR.]

At the present time, the development of computers for integration of partial differential equations is still in its initial stages. Gershgorin proposed a special mechanism for integrating the Laplace equation in finite differences, i.e., by the "method of networks". This proposal, however, did not obtain practical application. A machine for the solution of partial differential equations, based upon a hydraulic principle, was constructed by Luk'yanov.

7. Modeling of Continuous Mathematical Relations By Means of Electrical Devices

Modeling of continuous mathematical relations by means of electrical devices is based upon the linear dependency, expressed by Ohm's law and Kirchhoff's laws, that exists between the main parameters of an electrical circuit, i.e., voltage, current, and conductance.

One of the parameters, usually conductance, varies according to a law of variation of the physical constants; voltage or current is used as the dependent variable. Electrical systems made up of active resistances and operating on direct current are usually used to model mathematical relations that describe "static" phenomena. Systems made up of inductances and capacitances and operating on direct current with transient conditions in the circuit are used for modeling dynamic process, i.e., for modeling processes that vary in time.

The linear relationship between the parameters of an electrical circuit accounts for the extreme simplicity in design of machines for modeling algebraic and transcendental functions. In particular, the operations of adding and subtracting voltages by means of a potentiometer and of multiplication and division using ordinary bridge circuits are very simple.

So-called "profile" rheostats or functional resistances are used to obtain trigonometric and other simple non-algebraic functions.

Thus, electrical functional devices can model simpler mathematical relations with the help of potentiometers, bridges, and "profile" rheostats. These devices replace the functional mechanisms used in mechanical computers. Electrical models for complex mathematical relations are made in the form of a set of separate functional devices, as was the case for mechanical models.

Electrical functional devices are used widely for the solution of mathematical problems in military instruments. The advantages of electrical devices in comparison with mechanical, i.e., low weight, small size, and, most important, simplicity in remote transmission and in obtaining data, are particularly important in military use. The role of electrical computers for military use has become particularly important in connection with radar methods of obtaining data, which is introduced into the device as independent variables. For example, in present anti-aircraft artillery, the coordinates of the airplane are determined by radar and introduced into the fire-control instrument as independent variables. From this data, the computer automatically solves the "encounter problem", i.e., determines the spatial position of the point of encounter of airplane and shell.

Electrical devices are also of great importance for modeling ...

value problems of mathematical physics. Electrical devices, or, as they are usually called, electrical models, are used almost exclusively at present for integrating the partial differential equations most commonly used in physics and engineering (Poisson, Laplace, Fourier equations, etc.).

The difficulties of problems of mathematical physics in many cases cannot be overcome by present-day computational mathematics. These difficulties are created by the complex conditions of reality in which the phenomena under study take place, i.e., the geometrical characteristics and physical properties of the system, the action of the surrounding medium upon the external points of the system, and its initial state.

A method of reducing partial differential equations to difference equations, i.e., to the use of the "method of networks", has been developed for the numerical integration of these equations.

Electrical devices for integration of partial differential equations are based upon modeling by the "method of networks" and are designed in the form of electrical networks made up of resistances with junctions which coincide with points at which values of the unknowns are sought. The resistance parameters are determined by the physical constants entering the equations.

S. A. Gershgorin is credited with the idea of using electrical networks for integrating partial differential equations (the Laplace equation), while L. I. Gutenmakher developed the idea for the main differential equations and devised a practical method. On the basis of this method, Gutenmakher constructed several units, known as electrointegrators.

The principle of modeling with the help of electrical networks can be clarified from the following elementary considerations.

Let us suppose that there is a long conductor through which a direct current i flows along the axis of the independent variable x.

We assume that the specific resistance of the conductor r is a continuous and differentiable function of the independent variable x:

$$r = r(x).$$

Further, we consider an element of the conductor dx, located at a distance x from the origin of the coordinates. The voltage difference at the ends of this element is:

$$du = ir(x)dx,$$

from which we obtain the voltage difference in two arbitrary points of the conductor:

$$u = i \int_{x_0}^{x_1} r(x)dx.$$

Instead of resistance, it is more convenient to use conductance = $1/r$: $A = 1/r$. Then

$$u = i \int_{x_0}^{x_1} \frac{dx}{A_x}.$$

If there are no branches in the conductor, there will be no change of current along the conductor, i.e.

$$\frac{di}{dx} = 0, \text{ or } \frac{d}{dx}\left(A_x \frac{du}{dx}\right) = A_x u''_x + A'_x u'_x = 0.$$

The last equation describes the voltage distribution along a conductor of variable conductance.

Thus, it is possible to integrate and model second-order differential equations without a right-hand member by means of a variable conductance.

The inconvenience lies in the necessity of creating special conductances corresponding to the form of the function for each separate function. To obtain flexibility and convenience, continuous distribution of conductance along the conductor is replaced by lumped conductances. For this purpose, the conductor is divided into sections of incremental length Δx and the conductance of each section is replaced by the lumped conductance equal to it. The chain shown in Fig. 6 is obtained. Moreover, a generalized coordinate $\underline{x}$, determining the position of each lumped conductance can be used instead of the conductor length. For this chain, the equation connecting voltage with the $\underline{x}$ coordinate will be the difference equation

$$[illegible] = 0.$$

$$u = i \sum [illegible]$$

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which may be considered a differential equation approximately.

Resistance lamps or rheostats can be used for the lumped conductances.

We now consider another chain, differing from the first in that additional current sources are connected to the ~~junction points~~ nodes and current is also drawn from these points. The currents supplied to the ~~junction points~~ nodes are independent of pressures in the chain and are functions of the x coordinate. Current is drawn from these points according to the same principle.

The equation of voltage distribution in this chain can be written by using Kirchhoff's law. It will correspond approximately to the following differential equation:

$$\frac{d}{dx}\left(A_x\frac{du}{dx}\right) + B(x)u = F(x),$$

where $B(x)$ and $F(x)$ are quantities depending upon the conductances of the circuits supplying current to and drawing ~~off~~ current from the ~~junction points~~ nodes.

Thus, the chain just described models a difference equation which corresponds approximately to a nonhomogeneous differential equation of the second order in one independent variable, x.

This method can be easily generalized for modeling partial differential equations in two independent variables. In this case, the circuit is no longer linear, but is ~~met~~ now planar, with conductances depending accordingly upon x and y. The currents supplied to and drawn from its ~~junctions~~ nodes depend upon both generalized coordinates of the given ~~junctions~~ nodes. This circuit models approximately (in differences) the following differential equation:

$$\frac{\partial}{\partial x}\left(A_x\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_y\frac{\partial u}{\partial y}\right) + B(x,y)u = F(x,y).$$

When no current is drawn from the ~~junctions~~ nodes, the conductances are constant and the coefficients A_x, A_y are constants, so that the circuit models the Poisson equation for a two-dimensional region:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,y).$$

If no additional current is supplied to the ~~junctions~~ nodes, the first part of the last equation becomes zero and the Laplace equation is obtained.

Models for three-dimensional and polydimensional problems can be obtained in the same way.

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The boundary conditions in these models are assigned by virtue of the fact that the currents supplied to (or drawn from) the [redacted], the coordinates of which lie on the boundaries of the region under study, are dependent upon the boundary conditions.

Similar circuits, made up of inductances and capacitances and operating on direct current with transient conditions in the circuit, are used to model processes that vary in time.

The field of application of electric models and methods of constructing them are not limited to the cases discussed in this report. At the present time, extensive work is being conducted on the application of various electronic devices to modeling of physical processes. These works should expand considerably the potentialities of electrical devices. One should remember, however, that modeling by means of electrical devices, for all its apparent simplicity, is often accompanied by a series of unexpected and unpleasant surprises with regard to providing the required accuracy and stability of operation.

8. Devices for Operation on Discrete Quantities

Computers which model continuous mathematical relations are used to solve certain types of problems. Naturally, these cannot be used for all the diverse computing operations which are encountered in the practical solution of various problems of contemporary science.

Moreover, the design and production of a device such as a machine for the integration of differential equations, for example, is very difficult, and their mass production is impossible, and therefore, unique. The total number of such machines operating in the whole world is very small and scarcely numbers 10 or 20. At the same time, the need for devices to carry out diverse large-scale computing processes is very great, and can be satisfied only by a great number of standard versatile devices. Such devices are calculating-analytical automatic punched card machines, which operate on discrete quantities. Up until recently, these machines were used chiefly for automatization of computing processes in statistics, accounting, and other computing operations. Later, they began to find use in astronomical observatories for large-scale astronomical calculations.

From 1942 to 1946, the Division of Approximate Calculations in the Institute of Mathematics, Academy of Sciences USSR, devised methods of solving mathematical problems through the use of these machines. The Division's successful solution of a number of complex problems of mathematical physics, harmonic analysis, etc., attests to the vast potentialities of punched card machines in applied mathematics.

The solution of mathematical problems on these machines is based upon the fact that the algorithm of any problem can be reduced by various methods to elementary operations (arithmetic operations, selection, and classification of data, and other logical operations). Therefore, any mathematical problem can in principle be solved by a specially selected set of punched-card machines capable of realizing the individual stages of the solution.

The operating principle of punch-card machines is based upon identification of a single-valued number with the turning of a certain wheel (the counter) through a definite angle. A multi-valued number can be identified with a set of wheels, the number of which is equal to the number of values of the given discrete quantity. Addition of two numbers corresponds to two consecutive turns of the wheels through angles determined by these numbers. This principle, which has proven highly useful, was proposed long ago by Pascal and Leibnitz. Its practical implementation has come a long way from the first mechanical counters to the present-day punched-card machines.

The main machine in the set of punched card machines is the tabulator, which automatically adds numbers sequentially introduced into it. The main part of the tabulator is the counting system. It consists of a certain number of counters, each of which are a group of number wheels. A number wheel is a disc whose circumference is divided into 10 parts; numbered 0, 1, 2, ...9. Each number wheel corresponds to a definite place (units, tens, hundreds) of a single-valued number.

Two numbers are added by two consecutive turns of the number wheels of the corresponding place. Since the sum of two numbers in any place can be greater than 9, the possibility of transferring each ten from a lower place to a higher place had to be provided for.

The operation of the counters is controlled by punch cards.

/Approximately four more paragraphs giving relatively elementary discussion of electrical and mechanical operation of punched card machines from punch cards/.

-END-

FIGURES

Figure 1.

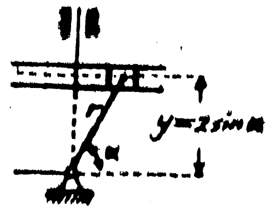
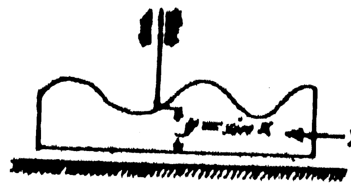


Figure 2.



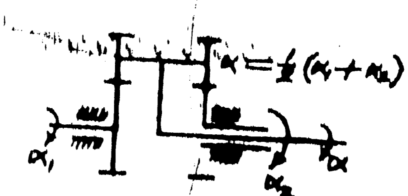


Figure 3.

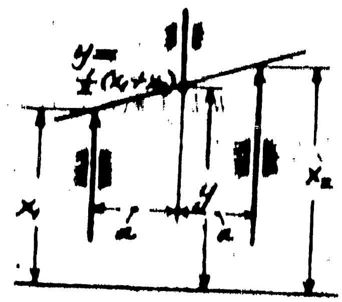


Figure 4.

Figure 5

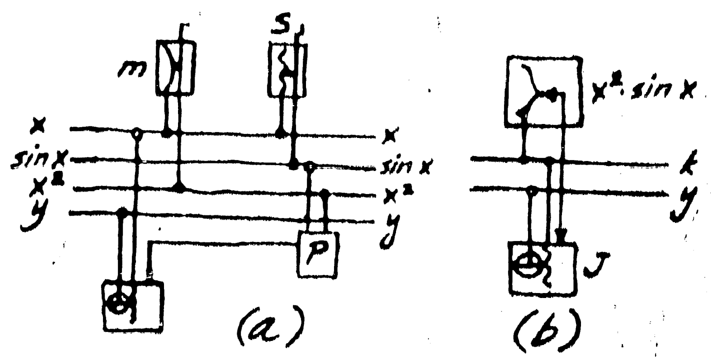


Figure 6.

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