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THEORY OF NOISE IN A MULTIDIMENSIONAL SEMICONDUCTOR WITH A P — N JUNCTION

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ABSTRACT: This thesis discusses the fluctuations of noise in a two and three dimensional semiconductor containing a p-n junction. We consider a rectangular parallelepiped single crystal. It is bisected in the longest dimension by a p-n junction. Since this dimension is several diffusion lengths it can be considered infinite. In the transverse plane we investigate the case where both dimensions are finite, and then the case where one is finite and the other infinite. In the p-n junction the noise is the result of fluctuations in the minority carrier density. In a p-n junction there are two classes of minority carriers: 1. holes in the n-type material, 2. electrons in the p-type material. Since both hole and electron density fluctuations are similar, we discuss only the former in detail. We investigate the differential equations for a two and three dimensional semiconductor with a p-n junction and find the inhomogeneous form of these equations. These equations are solved with the help of the scalar and tensor Green's function. The noise problem is solved by using these equations as Langevin equations and interpreting the distributed sources as random forces. Then the noise current spectrum is determined with stochastic process theory after deriving the sources from basic physical models and the theory of stationary, ergodic, Markovian processes. We

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consider two cases of surface recombination velocity on the transverse surfaces: infinite s and finite s. For the infinite case, we get the exact solution which provides an upper bound for the noise spectrum for large s. For an arbitrary s we get a solution but have confidence in the solution for only small s.

Therefore we have obtained a complete solution for the two cases of practical interest: large and small surface recombination velocity. These cases should prove of interest in the analysis of noise phenomena in semiconductors.

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CHAPTER I

INTRODUCTION

Semiconductor noise studies give useful information about the basic physical microscopic processes in semiconductors and in the solid state. Furthermore, noise becomes very important when a semiconductor device such as a transistor is used with signal levels comparable to the noise.

A semiconductor crystal which is p-type at one end and n-type at the other has a transition zone which is called a p-n junction (Shockley[1], Kittel[2]). Current is carried across the junction by minority carriers; that is, electrons in the p-type region and holes in the n-type region.

Petritz[3,4] has shown that noise in a p-n junction arises from fluctuations in the concentration of minority carriers. Considering a p-n junction as an ideal one dimensional structure, he has derived expressions for this noise. Van der Ziel[5] has extended the solution to the one-dimensional p-n-p transistor structure. In both studies the effects of surfaces were considered in an approximate manner.

However, surface conditions have been found to influence markedly the performance of p-n junction diodes and transistors (Kingston[6]). Considerable theoretical work has been done to understand the signal properties (voltage, current, frequency relations) of p-n junction devices, considered as three-dimensional structures (Shockley[7], Van Roosbroeck[8]). It is the

purpose of this thesis to develop a theory of noise which considers the p-n junction as a three dimensional system, and which treats the effects of surfaces in an exact manner.

A second objective of the thesis is to test and extend a powerful method developed by Petritz[3,4] for studying complicated random processes. This aspect of the work is of interest in the general theory of random processes.

CHAPTER II

PROBLEM AND METHOD OF SOLUTION

2.1 Introduction

We assume that semiconductor noise is a stationary, ergodic and Markovian random process.[3] Considering the local hole density, $p_t(x,y,z,t)$, as a random variable, this is a three-fold infinite random process. In order to solve such a complicated problem, we have generalized a method used originally by Petritz.[3] This method employs the Kolmogorov--Fokker-Planck (KFP) and the Langevin techniques to describe the noise (Feller[9], Chandrasekhar[10], Uhlenbeck and Ornstein[11], Wang and Uhlenbeck[12]).

2.2 The Kolmogorov-Fokker-Planck Equation Approach

The KFP equations[12] for the three dimensional semiconductor are given by

$$\frac{\partial P(m_o(r)|m(r),t)}{\partial t} = -P(m_o(r)|m(r),t)\sum_{k;k\neq m} Q(m(r)|k(r)) + \sum_{k;k\neq m} P(m_o(r)|k(r),t)\, Q(k(r)|m(r)) \quad (1)$$

$P(m_o/m,t)$ is the conditional probability of finding the random variable with a value m after the time t, if at zero time the random variable had a value m_o. The random variable is the hole density in the n-type semiconductor. The symbol r represents r(x,y,z), a function of the three rectangular coordinates. Q is a transition probability and is defined[3] by the equation,

$$P(k|m,\Delta t) = Q(k|m)\Delta t + \text{order}(\Delta t)^2. \quad (2)$$

Q describes how the system changes in an infinitesimal interval of time, Δt, and characterizes the stochastic process. For the semiconductor problem, Q is independent of time and the process is stationary. Q is non-zero and less than unity and the process is ergodic.

The interpretation of equation (1) is that the rate at which the conditional probability $P(m_o/m,t)$ changes with time results from transitions away from and to the desired state. Equation (1) is subject to the boundary condition

$$P(m_*|m,0) = \delta_{m_*,m} \quad , \tag{3}$$

where $\delta_{m_*,m}$ is the Kronecker delta.

Since a random process is characterized by transition probabilities, we list them for the p-n junction:

$Q_1(m(r)|m(r)-1) = m(r)/\tau$, bulk recombination; (4)

$Q_2(m(r)|m(r)+1) = \langle m(r)\rangle/\tau$, bulk emission; (5)

$Q_3(m(r)|m(r)-1) = m(r)/\tau_D$, bulk diffusion decrease; (6)

$Q_3(m(r)|m(r)+1) = \langle m(r)\rangle/\tau_D$, bulk diffusion increase. (7)

At the transverse surfaces, the surface transition probabilities are

$Q_4(m(r_s)|m(r_s)-1) = m(r_s)/\tau_s$, surface recombination; (8)

$Q_4(m(r_s)|m(r_s)+1) = \langle m(r_s)\rangle/\tau_s$, surface emission. (9)

r_s denotes that the random variable is evaluated only at the surface; τ is the bulk recombination lifetime of a hole in an excited state; τ_D and τ_S designate the lifetime of bulk diffusion and surface recombination respectively;

and $\langle\rangle$ designates ensemble average. For a stationary random process, time and ensemble averages are equal.

The KFP equations with the set of transition probabilities given in equations (1) and (4) to (9) comprise a three fold infinity of differential equations, since $p(r,t)$ depends continuously on x, y and z. We have not solved this complete set of equations, but we later use some KFP equations to solve the noise in an infinitesimal region of the semiconductor.

2.3 The Langevin Equation Approach

The Langevin equation is a deterministic equation of a system excited by random noise sources.[12] For a particle in a viscous medium it is

$$\frac{du}{dt} + \beta u = A(t) , \tag{10}$$

where u is the velocity of the particle, β is the viscosity, and A(t) is the random force. The two assumptions made are that A(t) is independent of u and that A(t) varies extremely rapidly compared to the variation of u.

By generalizing the above concept we have a suitable method for solving the p-n junction noise problem. The deterministic equations for minority carrier flow are:[1,3]

$$\frac{\partial p_t(r,t)}{\partial t} + \frac{(p_t(r,t)-p_n)}{\tau} + \frac{1}{q}\vec{\nabla}\cdot\vec{j}(r,t) = 0 \quad , \tag{11}$$

$$\vec{j}(r,t) = q\mu p_t(r,t)\vec{E}(r,t) - qD\vec{\nabla}p_t(r,t) \tag{12}$$

The subscript t denotes the total hole density, $\vec{j}$ is the current density, $\vec{E}$ is the electric field intensity, $p_t(r,t)$ is the hole concentration at $r(x,y,z)$ at time t, μ is the hole mobility, p_n is the hole concentration at thermal equilibrium, τ is the mean lifetime of a hole in bulk n-type

semiconductors, D is the diffusion constant, and q is the electronic charge. We assume that the diffusion current is much greater than the conduction current. By introducing appropriate noise sources into the above equations we have the three dimensional generalization of the simple Langevin equation (10):

$$\frac{\partial p(r,t)}{\partial t} + \frac{p(r,t)}{\tau} + \frac{1}{q}\vec{\nabla}\cdot\vec{j}(r,t) = S_r(r,t) \quad , \tag{13}$$

$$\vec{j}(r,t) + qD\vec{\nabla}p = \vec{S}_p(r,t) . \tag{14}$$

S_r and $\vec{S}_p$ are noise sources. The variable p is the deviation of the hole density from its equilibrium value, like Eq. (B1). (A letter preceding an equation number indicates the appendix in which the equation is found.)

It is important to note that there exists no a priori knowledge of the noise sources; their solution is a key part of this work. After finding expressions for these noise sources, we solve the deterministic Eqs. (13) and (14) and find the noise spectrum of the p-n junction. The latter step involves the use of scalar and tensor Green's functions.

CHAPTER III

THREE-DIMENSIONAL NOISE SOURCES FOR A p-n JUNCTION

3.1 Introduction

It is necessary to derive explicit expressions for the noise sources appearing in equations (13) and (14). We consider the analogous but considerably simpler problem of a one-dimensional transmission line.

3.2 The Inhomogeneous Transmission Line Equations[13]

The homogeneous differential equations for a one-dimensional transmission line with no series inductance are

$$\frac{\partial I(x,t)}{\partial x} = -C\frac{\partial V(x,t)}{\partial t} - GV(x,t) \quad , \tag{15}$$

$$\frac{\partial V(x,t)}{\partial x} = -RI(x,t) . \tag{16}$$

I is the current flowing in the line; V is the voltage across the line; C, G and R are the capacitance, shunt conductance and resistance per unit length of line respectively. An infinitesimal section of line is shown in Figure 1, page 8. Taking the Fourier transform[14] of the voltage and current (either represented by F) we have

$$F(t) = \int_{-\infty}^{\infty} F(f)\exp(i\omega t)\,df . \tag{17}$$

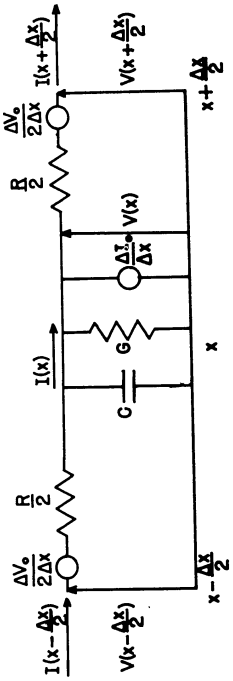

Fig. 1. Infinitesimal section of a transmission line showing the circuit elements, the distributed sources, and the voltage and current distributions. The series inductance is zero.

$$\frac{\partial I}{\partial x} = -(i\omega C + G)V = -YV \quad , \tag{18}$$

$$\partial V/\partial x = -RI \quad , \tag{19}$$

$$Y = i\omega C + G \quad , \tag{20}$$

$$\omega = 2\pi f \quad . \tag{21}$$

I and V are functions of frequency, f, and position, x.

We now consider voltage and current sources (generators) in the line; the total source in the series arm is $\Delta V_o(x,f)/\Delta x$, while in parallel with the admittance Y is a current source $\Delta I_o(x,f)/\Delta x$. In the limit as Δx approaches zero, we have

$$\lim_{\Delta x \to 0} \frac{\Delta F_o(x,f)}{\Delta x} = \frac{\partial F_o(x,f)}{\partial x} \quad , \tag{22}$$

where $F_o(x,f)$ represents either $V_o(x,f)$ or $I_o(x,f)$. The resulting inhomogeneous transmission line equations are

$$\frac{\partial I}{\partial x} + YV = -\frac{\partial I_o}{\partial x} \quad , \tag{23}$$

$$\frac{\partial V}{\partial x} + RI = -\frac{\partial V_o}{\partial x} \quad . \tag{24}$$

3.3 Analogy between the Transmission Line and the One Dimensional p-n Junction

The three dimensional p-n junction, Eqs. (13) and (14), reduce to one dimension when the functions considered are constant in the y and z directions. Using Fourier transforms, Eqs. (13) and (14) become

$$\frac{\partial j}{\partial x} + qDK^2 p = -\frac{\partial j_o}{\partial x} \quad , \tag{25}$$

$$\frac{\partial p}{\partial x} + \frac{j}{Dq} = -\frac{\partial p_o}{\partial x} \quad , \tag{26}$$

$$K^2 \doteq (1 + i\omega\tau)/D\tau \quad . \tag{27}$$

Here j and p are functions of frequency and x. The quantities in Eqs. (25) and (26) analogous to those in Eqs. (23) and (24) are

$$I \sim j \quad ; \quad V \sim p \quad ; \tag{28}$$

$$Y \sim q(i\omega + \tau^{-1}); \quad R \sim 1/qD \; ; \tag{29}$$

$$\partial I_o/\partial x \sim \partial j_o/\partial x \; ; \tag{30}$$

$$\partial V_o/\partial x \sim \partial p_o/\partial x \quad . \tag{31}$$

3.4 The Three Dimensional Inhomogeneous Differential Equations for a Semiconductor with a p-n Junction

Since the current density is a vector and the excess hole density is a scalar, there are four more equations like (25) and (26) for the y and z directions. These six equations can be written as

$$\vec{\nabla}\cdot\vec{j} + qDK^2 p = -\vec{\nabla}\cdot\vec{j}_o \quad , \tag{32}$$

$$\vec{\nabla} p + \vec{j}/qD = -\vec{\nabla} p_o \quad . \tag{33}$$

When the variables are separated, we get

$$\nabla^2 p - K^2 p = -S \quad , \tag{34}$$

$$S = S_b' + S_r = \nabla^2 p_o - \vec{\nabla}\cdot\vec{j}_o/qD \, , \tag{35}$$

$$\vec{\nabla}\vec{\nabla}\cdot\vec{j} - K^2\vec{j} = -\vec{S} \, , \tag{36}$$

$$\vec{S} = \vec{S}_r' + \vec{S}_b = \vec{\nabla}\vec{\nabla}\cdot\vec{j}_o - Dq K^2 \vec{\nabla} p_o \, . \tag{37}$$

3.5 Discussion of the Random Noise Sources

In the first order differential equations (32) and (33) there are two sources of noise: $\vec{\nabla}\cdot\vec{j}_o$ and $\vec{\nabla} p_o$. The first is the divergence of the hole current and is a scalar source. This is due to hole recombination with electrons. The second noise source is the gradient of the hole density, $\vec{\nabla} p_o$, and is a vector source. The diffusion current density is proportional to $\vec{\nabla} p_o$, and is in the direction from greater to lesser hole density.

In addition to the above noise sources there are sources which result from the recombination or emission of holes in surface states. This noise is related to the flow of holes into the surface and is directed normal to the surface.

The modified noise sources, S_b' and S_r', Eqs. (35) and (37), are not new sources but result from mathematical operations on the physical sources.

3.6 Method of Deriving the Noise Sources

We derive explicit expressions for the noise sources following the method first used by Petritz.[3,4] This method uses the KFP equation to solve for the spectrum of the noise in an

infinitesimal region of the semiconductor. Then an appropriate deterministic equation with unknown noise sources is set up for the same infinitesimal region (local Langevin equation). Knowing the spectrum from the KFP solution, one is able to derive expressions for the noise sources appearing in the local Langevin equations. These sources turn out to be appropriate for use in the Langevin equations of the whole semiconductor, Eqs. (34) to (37).

3.7 Models Used for Determination of Noise Sources

The determination of the noise sources is simplified because of the assumption of statistical independence of the various elementary processes. To set up a model which isolates each source, we cut the three dimensional semiconductor into infinitesimal cubes without changing their hole density, and apply the appropriate boundary conditions. For recombination the cube is an interior one with perfectly reflecting boundaries. The charge density remains uniform throughout; the only decay is due to the bulk recombination time constant since diffusion currents require gradients.

For the bulk diffusion sources the cube is an interior one with perfectly absorbing boundaries on two opposite faces and perfectly reflecting boundaries on the other faces. The volume is so small that the concentration gradients cause large diffusion currents while relatively few holes are lost by recombination.

For the surface recombination noise source, the cube is at the surface of the semiconductor. Its dimension perpendicular to the surface is very small compared to the others and the boundaries are perfectly reflecting except for the original semiconductor boundary. In this surface element the dominant process is recombination and emission from the surface states.

Because of the thinness of the element, bulk recombination and diffusion are relatively unimportant.

3.8 The Bulk Recombination Noise Source

The excess hole density Langevin equation for bulk recombination and emission is obtained from the model in Section 3.7 and from Eqs. (10) and (13),

$$\frac{\partial p}{\partial t} + \frac{p}{\tau} = \frac{\vec{\nabla}\cdot\vec{j}_e}{q} . \tag{38}$$

The random force is $\vec{\nabla}\cdot\vec{j}/q$. The time constant for bulk recombination τ determines the transition probabilities,

$$Q_r(N|N-1) = N/\tau , \qquad \text{bulk hole-electron recombination;} \tag{39}$$

$$Q_r(N|N+1) = \langle N\rangle/\tau , \qquad \text{excitation of a hole.} \tag{40}$$

Here N and $\langle N\rangle$ are the total and the average number of holes in the cube respectively. The minority charge density is assumed uniform over the volume v of the cube, thus

$$N = v\, p_t . \tag{41}$$

Substituting these transition probabilities into the KFP equation (1), we get for the small volume under consideration,

$$\frac{\partial P(N_o|N,t)}{\partial t} = -P(N_o|N,t)\ \frac{[N+\langle N\rangle]}{\tau} + P(N_o|N-1,t)\ \frac{\langle N\rangle}{\tau} + \tag{42}$$
$$+ P(N_o|N+1,t)\ \frac{[N+1]}{\tau} .$$

Multiplying equation (42) by N, the number of holes at time t if N_o existed at the initial time, and summing over the ensemble, there results

$$\partial \langle N_1 \rangle_o / \partial t = - \langle N_1 \rangle_o / \tau \quad , \tag{43}$$

$$N_1 = N - \langle N \rangle \quad , \tag{44}$$

and where the conditional average for N is defined as

$$\langle N \rangle_o = \sum_N P(N_o | N,t) \; N \; . \tag{45}$$

The solution of equation (43) is

$$\langle N_1 \rangle_o = N_{1o} \exp(-t/\tau) \quad , \tag{46}$$

where N_{10} is defined with the aid of Eq. (3) as

$$\langle N_1 \rangle_o = N_{1o} \; , \; t=0 \; . \tag{47}$$

This conditional average is transformed into the correlation function using[3]

$$\rho(t) = \langle N_1(t=0) \; N_1(t=t) \rangle \; ; \tag{48}$$

with equation (45) this expression becomes

$$\rho(t) = \sum_{N_{1o}} N_{1o} \; W(N_{1o}) \; \langle N_1 \rangle_o \; . \tag{49}$$

$W(N_{10})$ is the probability of N_{10}. When Eq. (46) is substituted into (49), the correlation function becomes

$$\rho(t) = \langle N_{1o}^2 \rangle \; \exp(-t/\tau) \; . \tag{50}$$

The Wiener-Khintchine Theorem[3] transforms the correlation function into the spectrum

$$w(f) = 4 \int_o^\infty \rho(t) \; \cos \omega t \; dt \; . \tag{51}$$

When equation (50) is substituted into equation (51) and the result integrated, there results

$$w(|N_1|^2) = 4 \langle N_{1o}^2 \rangle / \tau |K^4| D^2 \; . \tag{52}$$

K^2 is defined by Eq. (27) and the symbol $||$ means absolute value.

Equation (52) is the spectrum of the total hole variation. However, we desire an expression for the noise source which appears in the Langevin frequency equation (34). We can solve for the spectrum of the source now that we have determined the spectrum of the total hole variation. This is the inverse of what is normally done with the Langevin equation. The normal procedure is to postulate a white noise source and to solve for the fluctuations in the total hole density. The magnitude of the white noise source is determined by considerations of statistical mechanics and thermal equilibrium. Since we are interested in non-equilibrium as well as thermal equilibrium noise sources, we cannot use this normal procedure. Instead the local KFP equation is used to determine the thermal equilibrium and nonequilibrium spectrum of the total hole variation, and the noise sources derived have general validity. This combined use of the KFP and the Langevin methods locally was first done by Petritz[2] and appears to be a powerful technique for solving complicated random processes.

We use this technique to transform the spectrum of the total hole variation, Eq. (52), into the noise source associated with the recombination dissipative process. This is done by integrating equation (38) over the volume of the cube; inserting

$$N_1 = \int p_1(r)\,dv \,, \tag{53}$$

where $p_1(r)$ is like $p_1(0)$, Eq. (B3); using

$$I_r = \int (\vec{\nabla}\cdot\vec{j}_o)\,dv \;; \tag{54}$$

and using Eq. (17). This yields

$$I_r = qDK^2 N_1 \,. \tag{55}$$

The spectrum of I_r is

$$w(|I_r|^2) = q^2 D^2 |K^4|\; w(|N_1|^2). \tag{56}$$

Equation (52) substituted into Eq. (56) obtains

$$w(|I_r|^2) = 4q^2\langle N_1^2\rangle/\tau \,. \tag{57}$$

This is the noise source associated with the recombination dissipative process. It is independent of frequency and therefore is a white noise source.

Equation (57) is not the recombination noise source in equation (35). This source is

$$w(|S_r|^2) = w_r\left(\left|\frac{\vec{\nabla}\cdot\vec{j}_o}{qD}\right|^2\right). \tag{58}$$

A relation between Eqs. (57) and (58) is determined with the equation

$$\lim_{\Delta v \to 0} \frac{I_r}{\Delta v} = \vec{\nabla}\cdot\vec{j}_o \tag{59}$$

and the generalized impedance theorem.

The impedance theorem (Lawson and Uhlenbeck[15]) states that by multiplying the current noise spectrum w_i in a linear system by the absolute value squared of its impedance function, $Z(f)Z^*(f)$, w_i is transformed into the voltage spectrum w_o. In symbols

$$w_o = w_i\, ZZ^*. \tag{60}$$

We generalize this impedance theorem by letting w_i and w_o be any two spectra in a linear system which are related by a factor of proportionality $Z(f)Z^*(f)$.

The explicit expression for the recombination source spectrum is

$$w(|S_r|^2) = 4\langle N_1^2\rangle/\tau D^2\, dv^2. \tag{61}$$

To express the recombination source, Eq. (61), in known parameters, $\langle N_1^2\rangle$ is evaluated by multiplying Eq. (42) by N^2 and summing; the result is

$$\frac{\partial\langle N^2\rangle_o}{\partial t} = \left(2\langle N\rangle\langle N\rangle_o + \langle N\rangle - 2\langle N^2\rangle_o + \langle N\rangle_o\right)/\tau \,. \tag{62}$$

When the time of observation of N goes to infinity,

$$\langle N_1^2\rangle = \langle N^2\rangle - \langle N\rangle^2 = \langle N\rangle \tag{63}$$

since

$$\langle N\rangle_{o,\,t\to\infty} = \langle N\rangle. \tag{64}$$

With Eqs. (63) and (41) Eq. (61) becomes

$$w(|S_r|^2) = 4\langle p_t(m)\rangle/\left(\tau D^2\, dv\right), \tag{65}$$

where $\langle p_t(r) \rangle$ is given by Eq. (B11). This is the desired noise source for bulk recombination.

3.9 The Diffusion Noise Source

Solving Eqs. (13) and (14) for p, letting S_r be zero and using the model for the diffusion noise source, Section (3.7), we get

$$\frac{\partial p}{\partial t} - D \sum_{i=1}^{3} \frac{\partial^2 p}{\partial x_i^2} = S_D \ , \qquad (66)$$

where x_i is x, y, or z. We assume that diffusion in the three directions is statistically independent. Equation (66) becomes a set of three:

$$\frac{\partial p}{\partial t} - D \frac{\partial^2 p}{\partial u^2} = S_D \ , \qquad (67)$$

where u stands for x, y or z.

To write equation (66) in the Langevin form, the spacial term is transformed to contain a time constant. We write the second derivative for the finite but small cube and use the densities

$$p(0) = 0 \ , \ p(\Delta u) = 0 \ , \ p\left(\frac{\Delta u}{2}\right) = p. \qquad (68)$$

In this differentiation the diffusing direction is u and the length of the cube in this direction is Δu. The second derivative is

$$\frac{\partial^2 p}{\partial u^2} = -\frac{2p}{\Delta u^2} \qquad (69)$$

and equation (67) becomes

$$\frac{\partial p}{\partial t} + \frac{p}{\tau_{Du}} = S_D \ , \qquad (70)$$

where $\tau_{Du} = \Delta u^2 / 2D$. (71)

With these time constants the transition probabilities for the u direction are

$$Q_{Du}(N_D | N_D - 1) = N_D / \tau_{Du} \ , \quad \text{loss by diffusion;} \qquad (72)$$

$$Q_{Du}(N_D | N_D + 1) = \langle N_D \rangle / \tau_{Du} \ , \quad \text{gain by diffusion.} \qquad (73)$$

Here N_D is the total number of holes in the cube with dimensions Δx, Δy, Δz and with the diffusion boundary conditions:

$$N_D = \int p \, dv = \Delta x \, \Delta y \, \Delta z \, p/2 \ . \qquad (74)$$

Using the techniques of the previous section, the noise source for diffusion in the u direction is

$$W(|S_{Du}|^2) = |K^4| 4 q^2 D \langle p_t(r) \rangle / dx \, dy \, dz \ . \qquad (75)$$

3.10 The Surface Recombination Noise Source

With the model for the surface recombination noise source, Section 3.7, the Langevin equation (10) becomes

$$\frac{\partial p_{is}}{\partial t} + \frac{p_{is}}{\tau_s} = \frac{\vec{\nabla} \cdot \vec{j}_{is}}{q} \ . \qquad (76)$$

The p_{is} is evaluated at the semiconductor boundary and the $\vec{j}_{is}$ is the current flowing in

the direction normal to the surface i. These surface recombination time constants define the transition probabilities:

$$Q_s(N_s|N_s-1) = N_s/\tau_s \qquad (77)$$

by recombination at the surface;

$$Q_s(N_s|N_s+1) = \langle N_s\rangle/\tau_s \qquad (78)$$

by emission at the surface.

N_s is the total number of minority carriers evaluated at the surface if the surface layer were Δx_i thick:

$$N_s = \int p_{is}\, dv_s = \Delta x_i\, \Delta x_j\, \Delta x_k\, p_{is} . \qquad (79)$$

Using the techniques demonstrated in Section 3.7, the surface recombination noise source is

$$w_s\left(\left|\frac{\partial p}{\partial x_i}\right|^2\right) = 4\langle p_{ts}\rangle s/D^2\, dx\, dx_k , \qquad (80)$$

where i and k are either the pair y, z or z, y; s is the surface recombination velocity and is assumed to be the same on the y and z surfaces. The relation for the surface recombination velocity is given by Rittner,[16]

$$s_i = \tau_s/\Delta x_i . \qquad (81)$$

The rate of surface recombination acts as if a current of holes were drifting into the surface with an average velocity s and being removed.[1]

3.11 Noise Source and p-n Junction Equations

The two bulk sources in equations (65) and

(75) are expressed per unit volume. Since the differential equations are for a small volume of the three dimensional semiconductor, the noise sources are in the correct form. That these sources correspond to physical processes was also shown by analogy with the transmission line, Section 3.3.

When the variables are separated in the two first order semiconductor equations, spacial differential operations are performed on some of the noise sources. This changes their nature from those calculated in Sections 3.8 and 3.9. In the equations for excess hole density (34) and (35) the nature of the recombination source S_r is not changed and these equations can be solved for the recombination noise spectrum. However, the diffusion source S_D has been differentiated spacially and becomes S_D'. By using the vector analogue of integration by parts, the diffusion noise source transforms to the correct form. This technique is used in Chapter VII.

In the diffusion current density equations (36) and (37) the diffusion source is still S_D. These equations can be solved for the diffusion noise spectrum with the tensor Green's function. This approach is followed in Chapter VI.

CHAPTER IV

SCALAR INHOMOGENEOUS SEMICONDUCTOR EQUATION AND GREEN'S FUNCTION

4.1 Formal Solution of the Scalar Inhomogeneous Equation

Having derived explicit expressions for the noise sources, we consider the inhomogeneous differential equations for the semiconductor with a p-n junction, Eqs. (34) to (37). Green's functions are useful for solving inhomogeneous partial differential equations. In our problem we use both scalar and tensor Green's functions, the latter because of the vector nature of the sources in Eq. (36). A summary of the properties of scalar and tensor Green's functions is given in Appendix A.

The formal solution of Eq. (34) in terms of a scalar Green's function is,[14]

$$p(r) = \int [G \vec{\nabla}_0 p(r_0) - p(r_0) \nabla_0 G] \cdot d\vec{A}_0 + \int S(r_0)\, G\, dv_0 \,. \quad (82)$$

Throughout the paper the zero subscript denotes the source coordinates, while the coordinates without subscripts are the observation ones. The surface integral gives the contribution for noise sources at the surfaces, while the volume integral is for the volume sources.

4.2 General Discussion of Scalar Eigenfunctions

We construct an explicit expression for the scalar Green's function in terms of a series of scalar eigenfunctions. An eigenfunction is the solution of an ordinary homogeneous differential equation containing a separation constant which satisfies simple boundary conditions. The values of the separation constants which allow the eigenfunction to fit the conditions are called eigenvalues. In physics it is assumed that the Dirac-delta function related to our Green's function can be expanded in terms of a complete, orthogonal set of eigenfunctions.

The orthogonality condition for a set of eigenfunctions is

$$\int F_{lmn} F_{l'm'n'}\, dv = \delta_{lmn,l'm'n'} L_{lmn} \,. \quad (83)$$

Here $\delta_{lmn,l'm'n'}$ is the Kronecker delta and L_{lmn} is the normalization constant. The differential equation the eigenfunctions of this problem must satisfy is

$$\nabla^2 F_{lmn} + K^2_{lmn} F_{lmn} = 0 \,. \quad (84)$$

K^2_{lmn} is the separation constant and specific values of the separation constant for which the above equation can be solved are the eigenvalues.

4.3 The Eigenfunction Expansion of the Scalar Green's Function for Arbitrary Surface Recombination Velocity

The scalar Green's function is now expanded in a series of scalar eigenfunctions:

$$G = \sum_{l,m,n} a_{lmn} F_{lmn}(r) \,. \quad (85)$$

This series is substituted into equation (A13), noting that the coefficients are not functions of r. Multiplying by $F_{l'm'n'}(r)$, integrating over the volume, and using Eqs. (83) and (84), we find

$$a_{lmn} = \cdot F_{lmn}(r_s) / L_{lmn} K^2_{Klmn} \qquad (86)$$

and $$G = \sum_{lmn} F_{lmn}(r_s) F_{lmn}(r) / L_{lmn} K^2_{Klmn} , \qquad (87)$$

where $$K^2_{Klmn} = K^2 + K^2_{lmn} = \frac{1}{D\tau} + K^2_{lmn} + \frac{i\omega}{D} . \qquad (88)$$

This Green's function satisfies the reciprocity condition since it is symmetrical in the source and observation coordinates.

The semiconductor geometry is shown in Figure 2. The p-n junction is located at the x=0 plane and the origin of coordinates lies at the center of this face. The rectangular parallelepiped is bounded by the planes x=0, x=a, y=b, y=-b, z=c, z=-c.

The three dimensional rectangular coordinate system is a separable system and the eigenfunction can be written as the product of three factors:

$$F_{lmn} = F_l F_m F_n . \qquad (89)$$

Each factor is the eigenfunction which satisfies the boundary condition in one coordinate. Furthermore Eq. (84) separates into three equations:

$$\frac{\partial^2 F_r}{\partial u^2} + K_r^2 F_r = 0 , \qquad (90)$$

where r is any index, l, m or n.

The boundary conditions in the x-direction are that p=o at x=o and x=a. This implies an ohmic contact at x=a and short-circuited conditions at the x=0 and x=a planes. The eigenfunction

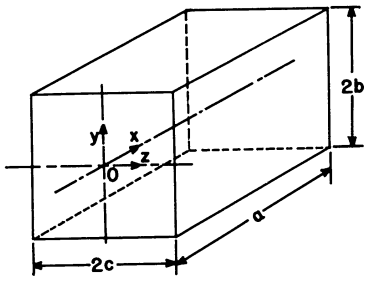

Fig. 2. The geometry and coordinate system for the three dimensional semiconductor with finite surface recombination velocity on the transverse surfaces y=b, y=-b, z=c, z=-c. The p-n junction is at the x=0 boundary and an ohmic contact is at the x=a boundary.

which satisfies these conditions is

$$F_x = \sin \pi l x/a \quad . \tag{91}$$

On the y and z boundaries current moving into the surface is proportional to the excess hole density.[1] The constant of proportionality is the surface recombination velocity s:

$$j_u/q = \pm sp \,, \qquad u = \pm \alpha \quad . \tag{92}$$

The coordinate u stands for either y or z and α stands for the y or z boundary surfaces. Using the homogeneous form of Eq. (14), Eq. (92) becomes

$$\partial p/\partial u = \mp sp/D \,, \qquad u = \pm \alpha \,. \tag{93}$$

Equations (90) and (93) have the following solutions: In the y-direction the cosine and sine eigenfunctions are

$$F_m = \cos \beta_m y \,; \; F_\mu = \sin \beta_\mu y. \tag{94}$$

In the z-direction the cosine and sine eigenfunctions are

$$F_n = \cos \beta_n z \,; \; F_\nu = \sin \beta_\nu z \,. \tag{95}$$

The boundary condition, Eq. (93), becomes for the cosine eigenfunctions:

$$\beta_r \alpha \tan \beta_r \alpha = s\alpha/D \,, \tag{96}$$

and for sine eigenfunctions:

$$\beta_\rho \alpha \cot \beta_\rho \alpha = -s\alpha/D \,. \tag{97}$$

The subscript r stands for either m or n, the subscript ρ stands for either μ or ν, and α stands for either $\pm b$ or $\pm c$, respectively. (The word "respectively" in this expression means that only the values $(m, \mu, \pm b)$ or $(n, \nu, \pm c)$ can occur together in the above equations.) We assume that the surface recombination velocity is the same for an opposite pair of surfaces.

Combining Eqs. (91), (94), and (95) the eigenfunctions for the rectangular parallelepiped semiconductor are

$$F_{lMN} = \sin \frac{\pi l x}{a} \begin{matrix} \cos \\ \sin \end{matrix} (\beta_m y) \begin{matrix} \cos \\ \sin \end{matrix} (\beta_n z) \,, \tag{98}$$

where all possible sine and cosine combinations are taken. The symbols M and N stand for a y or z index respectively. These eigenfunctions are complete since they satisfy Eq. (84) and the homogeneous boundary conditions.[14] The scalar Green's function Eq. (87) satisfies the same boundary conditions as the eigenfunctions. The eigenfunctions are orthogonal; from Eqs. (83) and (98) we find

$$\int F_{lmn} F_{l'm'n'} \, dv = L_{lmn} \, \delta_{lmn, l'm'n'} \tag{99}$$

and

$$\int F_{l\mu\nu} F_{l'\mu'\nu'} \, dv = L_{l\mu\nu} \, \delta_{l\mu\nu, l'\mu'\nu'}, \tag{100}$$

while any mixing of the m,μ and n,ν give a null integral. Since the eigenfunctions are separable we can integrate the various coordinate integrals separately:

$$L_{lMN} = L_l L_M L_N \tag{101}$$

$$\int_0^a \sin\frac{\pi l x}{a}\ \sin\frac{\pi l' x}{a}\ dx = \frac{a}{2}\delta_{l,l'} = L_l \delta_{l,l'} \ . \quad (102)$$

$$\int_{-\infty}^{\infty} \cos\beta_r u\ \cos\beta_{r'} u\ du = \pi\left(1+\frac{\sin 2\beta_r \pi}{2\beta_r \pi}\right)\delta_{r,r'} = L_r^+ \delta_{r,r'} . \quad (103)$$

$$\int_{-\infty}^{\infty} \sin\beta_\rho u\ \sin\beta_{\rho'} u\ du = \pi\left(1-\frac{\sin 2\beta_\rho \pi}{2\beta_\rho \pi}\right)\delta_{\rho,\rho'} = L_\rho^- \delta_{\rho,\rho'} . \quad (104)$$

$$\int_{-\infty}^{\infty} \cos\beta_r u\ \sin\beta_\rho u\ du = 0 \ . \quad (105)$$

The r stands for m or n while ρ stands for μ or ν.

Substituting Eq. (98) into (84), the eigenvalues are

$$K_{lMN}^2 = K_l^2 + K_m^2 + K_n^2 = \left(\frac{\pi l}{a}\right)^2 + \beta_M^2 + \beta_N^2 \ . \quad (106)$$

If we let a, the x-dimension of the parallelepiped, go to infinity, the discrete sum goes over to an integral and can be integrated. To carry out this limit operation, Green's function is written as

$$G = \sum_{MN} \frac{F_{MN}(r_0)\, F_{MN}(r)}{L_{MN}} \sum_l \frac{2}{a}\ \frac{\sin\frac{\pi l x}{a}\ \sin\frac{\pi l x_0}{a}}{K_l^2 + K_{KMN}^2} \ . \quad (107)$$

The variables γ and $\Delta\gamma$ take on the values $\pi l/a$ and π/a, respectively, and are put into Eq. (107).[14] When a goes to infinity, Eq. (107) becomes

$$G = \frac{2}{\pi} \sum_{MN} \frac{F_{MN}(r_0)\, F_{MN}(r)}{L_{MN}} \int_0^\infty \frac{\sin\gamma x\ \sin\gamma x_0\ d\gamma}{\gamma^2 + K_{KMN}^2} \ . \quad (108)$$

When we integrate equation (108), Green's function becomes

$$G = \frac{1}{2} \sum_{MN} \left[\frac{F_{MN}(r)\, F_{MN}(r_0)}{L_{MN}\, K_{KMN}}\right] \left[\exp\left(-K_{KMN}|x-x_0|\right) - \exp\left(-K_{KMN}|x+x_0|\right)\right], \quad (109)$$

$$F_{MN}(r) = F_M F_N = \begin{pmatrix}\cos\\ \sin\end{pmatrix}\beta_M y \begin{pmatrix}\cos\\ \sin\end{pmatrix}\beta_N z \Big), \quad (110)$$

$$L_{MN} = L_M L_N = \left\{b\left[1 \pm \frac{\sin(2\beta_M b)}{2\beta_M b}\right]\right\}\left\{c\left[1 \pm \frac{\sin(2\beta_N c)}{2\beta_N c}\right]\right\}, \quad (111)$$

$$K_{KMN}^2 = \frac{1}{D\tau} + \beta_M^2 + \beta_N^2 + \frac{j\omega}{D} \ . \quad (112)$$

L_{MN} is defined by Eqs. (101), (103) and (104) and K_{KMN} by Eqs. (106) and (88). The plus signs belong to the m,n indices, while the minus signs to the μ, ν indices. This is the desired form for the scalar Green's function for a rectangular parallelepiped semiconductor when the x-dimension goes to infinity. The assumption that a goes to infinity means physically that the diffusion length of the minority carriers is much less than a. Therefore minority carriers injected at x=o will recombine before they reach the contact at x=a. This assumption is valid for p-n junction diodes. In the case of transistors, a second p-n junction is at the distance a, and in this case, the diffusion length is much greater than a.

Our solution is therefore directly applicable to p-n junction diodes; transistors can be

handled in the same general way, but a is kept finite and the boundary condition at a is changed. For simplicity we confine this work to the p-n junction diode and assume a is infinite.

4.4 The Eigenfunction Expansion for the Scalar Green's Function for Infinite Surface Recombination Velocity

Green's function for the case of infinite surface recombination velocity on the transverse surfaces and zero hole density on the longitudinal surfaces is called the infinite s case. It is convenient to change the coordinate system from that of Figure 2 to that of Figure 3, in which the origin is at one corner of the p-n junction face rather than at its center. To designate that a symbol pertains to the case of infinite s, we affix the superscript ∞ to the symbol.

On the longitudinal surfaces the boundary conditions are not changed. On the transverse surfaces s is infinite. From Eq. (93), we get

$$\frac{1}{s} = \mp \frac{p}{D \frac{\partial p}{\partial u}} \quad , \qquad u = \pm \sigma \quad . \tag{113}$$

Here σ is the u-boundary surface: 0, B for y or 0, C for z. With $s = \infty$, either p is zero or $\partial p/\partial u$ is infinite. The last relation requires an infinite surface current and is physically impossible. Therefore p is zero on the surfaces which have infinite surface recombination velocity. Thus the excess hole density is zero on all surfaces.

Following the same procedure as for the case of arbitrary s, we find

$$G^{\infty} = \frac{1}{2} \sum_{mn} \frac{F^{\infty}_{mn}(r_1) F^{\infty}_{mn}(r)}{L^{\infty}_{mn} K^{\infty}_{kmn}} \left[\exp\left(-K^{\infty}_{kmn} |x - x_1|\right) - \exp\left(-K^{\infty}_{kmn} |x + x_1|\right) \right] , \tag{114}$$

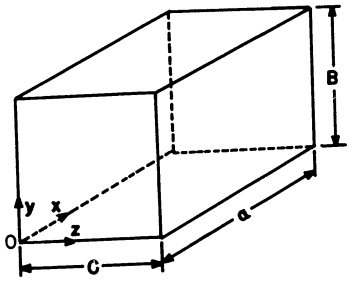

Fig. 3. The geometry and coordinate system for the three dimensional semiconductor with infinite surface recombination velocity on the transverse surfaces y=0, y=B, z=0, z=C. The p-n junction is on the x=0 boundary and an ohmic contact is on the x=a boundary.

$$F^{\infty}_{mn} = \sin \frac{\pi m y}{B} \quad \sin \frac{\pi n z}{C} \quad , \tag{115}$$

$$L^{\infty}_{mn} = \frac{B}{2} \quad \frac{C}{2} \quad , \tag{116}$$

$$K^{\infty 2}_{mn} = K^{\infty 2}_{m} + K^{\infty 2}_{n} + K^{2} = \left(\frac{\pi m}{B}\right)^2 + \left(\frac{\pi n}{C}\right)^2 + \left(\frac{1 + i\omega\tau}{D\tau}\right) . \tag{117}$$

Equation (114) is the scalar Green's function for infinite surface recombination velocity on the transverse surfaces.

CHAPTER V

TENSOR GREEN'S FUNCTION FOR THE SEMICONDUCTOR WITH A p-n JUNCTION

5.1 Formal Solution of the Diffusion Current Density with the Tensor Green's Function

In deriving the noise from a p-n junction the vector inhomogeneous differential equation, Eq. (36), must be solved; one method of solution is with a tensor Green's function. The general properties of tensor Green's functions are reviewed in Appendix A.

We solve the inhomogeneous current density equations (36) and (37) formally with the tensor Green's function, Γ, defined by Eq. (A14). Equation (36) is postmultiplied with Γ while equation (A14) is premultiplied with $\vec{j}$ and the two resulting expressions are subtracted. The following tensor identities are used:

$$\vec{\nabla}\cdot\left[(\vec{\nabla}\cdot\vec{j})\,\Gamma\right] = (\vec{\nabla}\cdot\vec{j})(\vec{\nabla}\cdot\Gamma) + (\vec{\nabla}\vec{\nabla}\cdot\vec{j})\cdot\Gamma \quad , \tag{118}$$

and

$$\vec{\nabla}\cdot\left[\vec{j}\,(\vec{\nabla}\cdot\Gamma)\right] = (\vec{\nabla}\cdot\vec{j})(\vec{\nabla}\cdot\Gamma) + \vec{j}\cdot(\vec{\nabla}\vec{\nabla}\cdot\Gamma). \tag{119}$$

The resulting equation is integrated over the volume using

$$\mathcal{L}\cdot\vec{F} = \vec{F}\cdot\mathcal{L} = \vec{F} \tag{120}$$

for any vector function $\vec{F}$ and for the idem-factor $\mathcal{I}$, Eq. (A10). The symbols r and r_o are interchanged and the reciprocity condition, Eq. (A22) is used. The current density is

$$\vec{j}(r) = \int\{[\vec{\nabla}_o \cdot \vec{j}(r_o)]\tilde{\Gamma} - \vec{j}(r_o)[\vec{\nabla}_o \cdot \tilde{\Gamma}]\} \cdot d\vec{A}_o + \int[\vec{S}(r_o)\cdot\tilde{\Gamma}]\,dv_o \,. \quad (121)$$

5.2 Formal Vector Eigenfunction Expansion of the Tensor Green's Function

In order to use Eq. (121) we must find an explicit expression for the tensor Green's function. This is accomplished formally with a complete, orthogonal series of vector eigenfunctions, $\vec{j}_{\ell mn}$. These vector eigenfunctions must satisfy the equation

$$\vec{\nabla}\vec{\nabla}\cdot\vec{j}_{\ell mn} + K^2_{\ell mn}\vec{j}_{\ell mn} = 0 \quad , \quad (122)$$

and the orthogonality condition

$$\int \vec{j}_{\ell mn}\cdot\vec{j}_{\ell' m' n'}\,dv = \Lambda_{\ell mn}\,\delta_{\ell mn,\ell' m' n'} \,. \quad (123)$$

We expand the tensor Green's function in a series of these vector eigenfunctions,

$$\Gamma = \sum_{\ell mn} \vec{A}_{\ell mn}\vec{j}_{\ell mn}(r_o) \,. \quad (124)$$

This series is put into Eq. (A14) and both sides of the resulting expression are multiplied by $\vec{j}_{\ell' m' n'}$ and integrated over the volume. Using Eqs. (123), (120), and (88) the vector coefficients are

$$\vec{A}_{\ell mn} = \vec{j}_{\ell mn}(r)/\Lambda_{\ell mn}K^2_{\ell mn} \,. \quad (125)$$

Substituting Eq. (125) into (124) the tensor Green's function becomes

$$\Gamma(r|r_o) = \sum_{\ell mn} \vec{j}_{\ell mn}(r)\,\vec{j}_{\ell mn}(r_o)\,/\Lambda_{\ell mn}K^2_{\ell mn} \,. \quad (126)$$

This is the formal expression for Green's tensor in terms of a complete orthogonal set of vector eigenfunctions.

5.3 Vector Eigenfunctions[14]

The vector eigenfunction solutions of the vector equation (36) are obtained from scalar eigenfunction solutions of the corresponding scalar eigenfunction equation (84). These vector eigenfunctions are written as

$$\vec{j}_{\ell mn} = \vec{l}_x X + \vec{l}_y Y + \vec{l}_z Z = \vec{L} + \vec{M} + \vec{N} \quad , \quad (127)$$

where $\vec{L} = \vec{\nabla}\varphi$; $\vec{M} = \vec{\nabla}\times(\vec{a}, w\psi)$; $\vec{N} = \vec{\nabla}\times\vec{\nabla}\times(\vec{a}, w\chi)/K_{\ell mn}$ (128)

and X, Y, Z or φ, ψ, χ are eigenfunction solutions of equation (84); $\vec{a}$, is the vector normal to the surface and w is a function of the coordinate in the direction $\vec{a}$, . The eigenvalues, K_{lmn}, and the relative magnitudes of X, Y, Z or φ, ψ, χ are adjusted to satisfy the boundary conditions.

In the p-n junction the diffusion current is proportional to the gradient of the hole density. Therefore a vector eigenfunction is the gradient of a scalar eigenfunction. The curl of these vector eigenfunctions is zero, and M and N, Eqs. (127) and (128), are zero.

For an arbitrary surface recombination velocity the vector eigenfunctions are

$$\vec{J}_{\ell mn} = -D\,q\,\vec{\nabla} F_{\ell mn} \quad , \tag{129}$$

where F_{lmn} are defined by Eq. (98), K^2_{lmn} by (106) and the boundary conditions by (96) and (97). These eigenfunctions satisfy the differential equation (84).

5.4 Non-orthogonality of the Vector Eigenfunctions for Finite s

To investigate the orthogonality of the vector eigenfunctions we write one of them in component form:

$$\vec{J}_{\ell MN} = \vec{I}_x j^x_{\ell MN} + \vec{I}_y j^y_{\ell MN} + \vec{I}_z j^z_{\ell MN} = \tag{130}$$

$$= -Dq \begin{bmatrix} \vec{I}_x \frac{\pi\ell}{a} & \cos\frac{\pi\ell x}{a} & \cos\beta_m y & \cos\beta_n z \\ & & \sin\beta_\mu y & \sin\beta_\rho z \\ +\vec{I}_y \beta_M & \sin\frac{\pi\ell x}{a} & (-)\sin\beta_m y & \cos\beta_n z \\ & & \cos\beta_\mu y & \sin\beta_\rho z \\ +\vec{I}_z \beta_N & \sin\frac{\pi\ell x}{a} & \cos\beta_m y & (-)\sin\beta_n z \\ & & \sin\beta_\mu y & \cos\beta_\rho z \end{bmatrix}$$

where M refers to m or μ and N to n or ν. All possible combinations of the cosine and sine factors are implied in each term. A component of $\vec{J}_{lmn}$ is denoted by the coordinate written as a superscript.

The orthogonality of the set of vector eigenfunctions is determined from Eq. (123):

$$\int \vec{J}_{\ell mn} \cdot \vec{J}_{\ell' m'n'}\, dv = \int J^x_{\ell mn} J^x_{\ell' m'n'}\, dv + \int J^y_{\ell mn} J^y_{\ell' m'n'}\, dv + \int J^z_{\ell mn} J^z_{\ell' m'n'}\, dv. \tag{131}$$

To evaluate these integrals, Eqs. (102) to (105) and the following expressions are used:

$$\int_{-\infty}^{\infty} \sin\beta_r u \sin\beta_{r'} u\, du = \frac{2s}{D(\beta_r^2 - \beta_{r'}^2)} \frac{\sin[(\beta_r - \beta_{r'})\alpha]}{\sin\beta_r\alpha} \frac{\sin[(\beta_r + \beta_{r'})\alpha]}{\sin\beta_{r'}\alpha} \tag{132}$$

$$= L^{+rr'} \quad ; \; r \neq r'$$

$$\int_{-\infty}^{\infty} \sin\beta_r u \sin\beta_{r'} u\, du = \alpha\left[1 - \left(\frac{\sin 2\beta_r\alpha}{2\beta_r\alpha}\right)\right] = L^-_r \quad ; \; r = r' \tag{133}$$

$$\int_{-\infty}^{\infty} \cos\beta_\rho u \cos\beta_{\rho'} u\, du = \frac{2s}{D(\beta_\rho^2 - \beta_{\rho'}^2)} \frac{\sin[(\beta_\rho - \beta_{\rho'})\alpha]}{\cos\beta_\rho\alpha} \frac{\sin[(\beta_\rho + \beta_{\rho'})\alpha]}{\cos\beta_{\rho'}\alpha} \tag{134}$$

$$= L^{-\rho\rho'} \quad ; \; \rho \neq \rho'$$

$$\int_{-\infty}^{\infty} \cos\beta_\rho u \cos\beta_{\rho'} u\, du = \alpha\left(1 + \frac{\sin 2\beta_\rho\alpha}{2\beta_\rho\alpha}\right) = L^+_\rho \quad ; \; \rho = \rho' \tag{135}$$

$$\int_{-\infty}^{\infty} \sin\beta_r u \cos\beta_\rho u\, du \equiv 0 \tag{136}$$

The superscript plus is not written unless there is a chance of ambiguity. Here r stands for m or n, ρ stands for μ or ν, and u for y or z, respectively.

The components of Eq. (131) are now

$$\int J^x_{\ell mn} J^x_{\ell' m'n'}\, dv = \left(\frac{Dq\pi\ell}{a}\right)^2 \frac{a}{2} L^\pm_m L^\pm_n \delta_{\ell mn, \ell' m'n'} \,. \tag{137}$$

The y component has two cases:

Case 1: $\ell=\ell'$, $M\neq M'$, $N=N'$

$$\int j^{y}_{\ell mn}\, j^{y}_{\ell' m'n'}\, dv = (Dq\beta_m)^2 \frac{a}{2} L^{\pm mn'} L^{\pm}_{n} \delta_{\ell n,\ell' n'} . \quad (138)$$

Case 2: $\ell=\ell'$, $M=M'$, $N=N'$

$$\int j^{y}_{\ell mn}\, j^{y}_{\ell' m'n'}\, dv = (Dq\beta_m)^2 \frac{a}{2} L^{\mp}_{m} L^{\pm}_{n} \delta_{\ell mn,\ell' m'n'} . \quad (139)$$

The z-component also has two cases:

Case 1: $\ell=\ell'$, $M=M'$, $N\neq N'$

$$\int j^{z}_{\ell mn}\, j^{z}_{\ell' m'n'}\, dv = (Dq\beta_m)^2 \frac{a}{2} L^{\pm}_{m} L^{\pm mn'} \delta_{\ell m,\ell' m'} . \quad (140)$$

Case 2: $\ell=\ell'$, $M=M'$, $N=N'$

$$\int j^{z}_{\ell mn}\, j^{z}_{\ell' m'n'}\, dv = (Dq\beta_n)^2 \frac{a}{2} L^{\pm}_{m} L^{\mp}_{n} \delta_{\ell mn,\ell' m'n'} . \quad (141)$$

M refers to either m or μ and N refers to n or ν. The $\delta i,j$ are Kronecker deltas. When two symbols are written in a column, the upper symbol refers to either m or n and the lower one to μ or ν. The integrals in Eqs. (138) and (140) do not vanish even when $M\neq M'$ or $N\neq N'$; the set of vector eigenfunctions is not orthogonal. Therefore we are not able to construct a tensor

Green's function from these vector eigenfunctions.

5.5 Vector Eigenfunctions for Infinite Surface Recombination Velocity

In the special case of infinite surface recombination velocity a complete set of orthogonal vector eigenfunctions can be derived for the three dimensional p-n junction. These eigenfunctions are found from the products of Eqs. (115) and (91) and the gradient operation of Eq. (129):

$$\vec{j}^{\infty}_{\ell mn} = -Dq F^{\infty}_{\ell mn} \left(\vec{i}_x \frac{\pi\ell}{a} \cot\frac{\pi\ell x}{a} + \vec{i}_y \frac{\pi m}{B} \cot\frac{\pi m y}{B} + \vec{i}_z \frac{\pi n}{C} \cot\frac{\pi n z}{C} \right) \quad (142)$$

The orthogonality relation is proved from Eqs. (142) and (123):

$$\int \vec{j}^{\infty}_{\ell mn} \cdot \vec{j}^{\infty}_{\ell' m'n'}\, dv = (Dq K^{\infty}_{\ell mn})^2 \frac{aBC}{8} \delta_{\ell mn,\ell' m'n'} = F^{\infty}_{\ell mn} \delta_{\ell mn,\ell' m'n'} . \quad (143)$$

Here a, B, C are shown in Figure 3 and $K^{\infty\,2}_{\ell mn}$ is given by

$$K^{\infty\,2}_{\ell mn} = K^{2}_{\ell} + K^{2}_{m} + K^{2}_{n} = \left(\frac{\pi\ell}{a}\right)^2 + \left(\frac{\pi m}{B}\right)^2 + \left(\frac{\pi n}{C}\right)^2 . \quad (144)$$

5.6 Eigenfunction Expansion of the Tensor Green's Function for Infinite Surface Recombination Velocity

Having found a complete set of orthogonal vector eigenfunctions for the infinite surface

recombination velocity case, we construct a tensor Green's function. Affixing the superscript ∞ onto the parameters in Eq. (126), Green's tensor is

$$\Gamma(\vec{r}|\vec{r}_0) = \sum_{lmn} \vec{J}_{lmn}(r)\vec{J}_{lmn}(r_0)/\Lambda_{lmn}\, K^{\infty\,2}_{Klmn}. \quad (145)$$

$K^{\infty\,2}_{Klmn}$ is derived from Eqs. (144), (88), and (27):

$$K^{\infty\,2}_{Klmn} = (D\tau)^{-1} + (\pi l/a)^2 + (\pi m/B)^2 + (\pi n/C)^2 + i\omega/D. \quad (146)$$

The tensor product of a vector eigenfunction pair is

$$\vec{J}(r)\vec{J}(r_0) = \begin{bmatrix} J_x J_{xo} & J_x J_{yo} & J_x J_{zo} \\ J_y J_{xo} & J_y J_{yo} & J_y J_{zo} \\ J_z J_{xo} & J_z J_{yo} & J_z J_{zo} \end{bmatrix}, \quad (147)$$

where the subscripts in the vector eigenfunctions are not shown. The denominator of Eq. (145) can be written as

$$(\Lambda_{lmn} K^{\infty\,2}_{Klmn})^{-1} = (8/\alpha A)\left(\tfrac{1}{2}K^2\right)\left[(K^{\infty}_{lmn})^{-2} - (K^{\infty}_{Klmn})^{-2}\right], \quad (148)$$

where Λ_{lmn}, K_{Klmn}, K^2, K_{lmn}, and $\mathbf{A}$ are given by Eqs. (143), (146), (27), (144), and (B34) respectively.

Using the abbreviations

$$\cos(\pi r u/\theta) = c_r\,; \quad \sin(\pi r u/\theta) = s_r\,; \quad (149)$$

$$\cos(\pi r u_o/\theta) = c_{ro}; \quad \sin(\pi r u_o/\theta) = s_{ro},$$

where the subscript r stands for l, m, or n while u stands for x, y, or z respectively. Green's tensor function for the infinite surface recombination velocity case is given as Equation (150) on page 42.

When the x dimension of the rectangular parallelepiped goes to infinity, a technique similar to that used in Section 4.3 transforms $\Gamma^{\infty}(r/r_0)$ into Eq. (151) which is given on page 43.

In Eq. (151) there are two classes of components: one contains K^{∞}_{mn}, obtained from Eq. (144) with ℓ equal to zero, while the other contains K^{∞}_{Kmn}, Eq. (117). To understand the meaning of the two classes of terms, let us review the physical significance of the Green's tensor function. Each tensor is composed of nine components. When the three diagonal terms are excited with Dirac delta functions, the Green's tensor describes the state of the system. This state depends on the properties and the geometry of the medium and the sources. For the semiconductor the properties appear as the diffusion constant D and the time constant τ. One of the properties of the source is its frequency.

The components containing K^{∞}_{Kmn} depend on the properties of the medium and the frequency ω of the sources as well as on the geometry. However, the terms containing K^{∞}_{mn} depend only on the geometry of the medium and therefore can contribute only an additive constant to our final result. We have examined this constant and found it without physical meaning.

$$\left[\sigma_{xy}(r|r_o)=\frac{8}{a^2B^2K^2}\left(\frac{1}{K^{m}_{1mn}}-\frac{1}{K^{m}_{2mn}}\right)\right]_{1mn}$$

$$\left[\begin{array}{lll}
\left(\frac{\pi z}{a}\right)^2 C_{1z}C_{1y}S_{1m}S_{1n}S_{1o} & \frac{\pi z}{a}\frac{\pi y}{B}C_{1z}S_{1m}S_{1n}C_{1m}S_{1o} & \frac{\pi z}{a}\frac{\pi}{C}C_{1z}S_{1m}S_{1n}S_{1o}C_{1o} \\
\frac{\pi m}{B}\frac{\pi z}{a}S_{1y}C_{1z}S_{1m}S_{1n}S_{1o} & \left(\frac{\pi m}{B}\right)^2 S_{1y}S_{1o}C_{1m}S_{1n}S_{1o} & \frac{\pi m}{B}\frac{\pi}{C}S_{1y}S_{1o}C_{1m}S_{1n}S_{1o}C_{1o} \\
\frac{\pi n}{C}\frac{\pi z}{a}S_{1y}S_{1o}S_{1m}S_{1n}S_{1o} & \frac{\pi n}{C}\frac{\pi m}{B}S_{1y}S_{1o}S_{1m}C_{1m}S_{1n}S_{1o} & \left(\frac{\pi n}{C}\right)^2 S_{1y}S_{1o}S_{1m}S_{1n}C_{1o}C_{1o}
\end{array}\right]$$

Equation (150)

$$\left[\sigma_{xy}(r|r_o)=\frac{2}{A^2B^2K^2}\left(\begin{array}{ll}
-S_{2m}S_{2o}S_{2n}S_{2o}K^{m}_{1mn} & \frac{\pi m}{B}C_{2m}S_{2n}S_{2o} \\
\left[\exp(-K^{m}_{1mn}|x_o+x|)+\exp(-K^{m}_{1mn}|x_o-x|)\right] & \left[\exp(-K^{m}_{1mn}|x_o+x|)-\exp(-K^{m}_{1mn}|x_o-x|)\right] \\
\frac{\pi n}{C}S_{2m}C_{2n}S_{2o} & \\
\left[\exp(-K^{m}_{1mn}|x_o+x|)-\exp(-K^{m}_{1mn}|x_o-x|)\right] &
\end{array}\right]_{mn}$$

$$\left[\begin{array}{ll}
\frac{\pi m}{B}S_{2m}C_{2n}S_{2o} & \frac{\pi n}{C}S_{2m}S_{2n}C_{2o} \\
\left[\exp(-K^{m}_{1mn}|x_o+x|)\pm\exp(-K^{m}_{1mn}|x_o-x|)\right] & \left[\exp(-K^{m}_{1mn}|x_o+x|)\pm\exp(-K^{m}_{1mn}|x_o-x|)\right] \\
-\left(\frac{\pi m}{B}\right)^2 C_{2m}S_{2n}S_{2o}K^{m-1}_{1mn} & -\frac{\pi n}{C}\frac{\pi m}{B}C_{2m}S_{2n}S_{2o}K^{m-1}_{1mn} \\
\left[\exp(-K^{m}_{1mn}|x_o+x|)\pm\exp(-K^{m}_{1mn}|x_o-x|)\right] & \left[\exp(-K^{m}_{1mn}|x_o+x|)\pm\exp(-K^{m}_{1mn}|x_o-x|)\right] \\
-\frac{\pi n}{C}\frac{\pi m}{B}S_{2m}C_{2n}S_{2o}K^{m-1}_{1mn} & \left(\frac{\pi n}{C}\right)^2 S_{2m}S_{2n}C_{2o}K^{m-1}_{1mn} \\
\left[\exp(-K^{m}_{1mn}|x_o+x|)-\exp(-K^{m}_{1mn}|x_o-x|)\right] & \left[\exp(-K^{m}_{1mn}|x_o+x|)-\exp(-K^{m}_{1mn}|x_o-x|)\right]
\end{array}\right]$$

$$+\frac{2}{A^2B^2K^2}\left(\begin{array}{ll}
S_{2m}S_{2n}S_{2o}K^{m}_{2mn} & -\frac{\pi m}{B}C_{2m}S_{2n}S_{2o} \\
\left[\exp(-K^{m}_{2mn}|x_o+x|)+\exp(-K^{m}_{2mn}|x_o-x|)\right] & \left[\exp(-K^{m}_{2mn}|x_o+x|)-\exp(-K^{m}_{2mn}|x_o-x|)\right] \\
-\frac{\pi n}{C}S_{2m}S_{2n}C_{2o} & \\
\left[\exp(-K^{m}_{2mn}|x_o+x|)+\exp(-K^{m}_{2mn}|x_o-x|)\right] &
\end{array}\right]_{mn}$$

Equation (151)
Note: When two signs are given the upper one is used when $x_o > x$ and the lower one when $x_o < x$.

Therefore to save space we neglect the terms depending on K_{mn}^{∞} in our further analysis.

In the subsequent work $x_o > x$ and the upper signs are used in Eq. (151). The tensor Green's function for $s = \infty$ and $a = \infty$ is given by Eq. (152) on page 45. Equation (152) is the required expression for the tensor Green's function for the three dimensional p-n junction with infinite s on the transverse faces.

$$\left[\hat{P}(r|r_o)\right] = \frac{2}{A^{\infty}K^2}$$

$$\begin{bmatrix} \frac{S_m S_{mo} S_n S_{no} K_{mnn}^{\infty}}{\left[\exp(-K_{mnn}^{\infty}|x_o+x|)+\exp(-K_{mnn}^{\infty}|x_o-x|)\right]} & -\frac{\pi m}{B} C_m S_{mo} S_n S_{no} K_{mnn}^{\infty-1} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] & -\frac{\pi n}{C} S_m S_{mo} S_n C_{no} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)+\exp(-K_{mnn}^{\infty}|x_o-x|)\right] \\ -\frac{\pi m}{B} C_m S_{mo} S_n S_{no} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] & \left(\frac{\pi m}{B}\right)^2 C_m S_{mo} S_n S_{no} K_{mnn}^{\infty-1} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] & \frac{\pi n}{C}\frac{\pi m}{B} S_{mo} C_n S_{no} K_{mnn}^{\infty-1} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] \\ -\frac{\pi n}{C} S_m S_{mo} C_n S_{no} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] & \frac{\pi n}{C}\frac{\pi m}{B} S_{mo} C_n S_{no} K_{mnn}^{\infty-1} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] & \left(\frac{\pi n}{C}\right)^2 S_m S_{mo} C_n S_{no} K_{mnn}^{\infty-1} \left[\exp(-K_{mnn}^{\infty}|x_o+x|)-\exp(-K_{mnn}^{\infty}|x_o-x|)\right] \end{bmatrix}_{mn}$$

Equation (152)
Note: A^{∞}, K^2, K_{mnn}^{∞} are given by Eqs. (B34), (27), and (117), while c_r, s_r, c_{ro}, and s_{ro} are given by Eq. (149).

CHAPTER VI

NOISE CURRENT SPECTRUM IN THE p-n JUNCTION WITH INFINITE SURFACE RECOMBINATION VELOCITY

6.1 Introduction

We have derived explicit expressions for the noise generators appearing in the Langevin equations of the semiconductor, and for the scalar and tensor Green's functions. We now use Green's functions to sum the contributions from the infinitesimal noise sources and derive expressions for the total noise spectrum of the p-n junction. The recombination noise spectrum is derived from the scalar hole density, Eq. (82), and the scalar Green's function, Eq. (114). The diffusion noise spectrum is found from the vector diffusion current density, Eq. (121), and the tensor Green's function, Eq. (152).

Since the recombination and diffusion noise spectra are the result of independent elementary processes, they are derived separately. These noise spectra are added together to obtain the total noise spectrum.

We consider first the case of infinite surface recombination velocity because we have been able to derive both the scalar and tensor Green's functions for this case.

6.2 Recombination Noise Current Spectrum

Let us examine the surface integrals in the expression for the hole density, Eq. (82):

$$p^{\infty}(r)\big|_s = -\int [G^{\infty} \vec{\nabla}_o p(r_o) - p(r_o) \vec{\nabla}_o G^{\infty}] \cdot d\vec{A}_o . \quad (153)$$

The subscript s denotes the contribution from the surface integrals, and the superscript ∞ indicates that we are considering the case of infinite surface recombination velocity. Since the excess hole density $p^{\infty}(r_o)$ on all the surfaces equals zero for the boundary conditions discussed in Section 4.4, the second term on the right hand side of Eq. (153) is zero. Furthermore an examination of Eqs. (114) and (115) shows that G^{∞} vanishes at the boundaries since

$$\left\{ \exp(-K^{\infty}_{Kmn}|x-x_o|) - \exp(-K^{\infty}_{Kmn}|x+x_o|) \right\}\Big|_{\substack{x_o=0 \\ x_o=\infty}} \equiv 0 \quad (154)$$

and

$$F^{\infty}_{mn}(r_o)\Big|_{\substack{y_o=0 \\ y_o=B}} \equiv 0 ; \quad F^{\infty}_{mn}(r_o)\Big|_{\substack{z_o=0 \\ z_o=C}} \equiv 0. \quad (155)$$

In each of these expressions there are two evaluation surfaces; each expression is to be evaluated separately at each surface. Therefore, the first term on the right hand side of Eq. (153) is zero and there is no contribution from the surfaces. Thus the excess hole density equation (82) becomes

$$p^{\infty}(r) = \int G^{\infty} S \, dv_o . \quad (156)$$

The excess hole density is transformed to current density at the x=0 plane with the homogeneous form of Eq. (14), and the current density is integrated over the x=0 plane. The

result is

$$I_x\Big|_{x=0} = \int_0^\infty \int_0^B \int_0^C S(r_o)\,dx_o\,dy_o\,dz_o \int_0^B \int_0^C \partial G^\infty/\partial x\Big|_{x=0} dy\,dz\,Dq. \quad (157)$$

Since $S(r_Q)$ is not a function of the observation coordinates, we get

$$\int_0^B \int_0^C \partial G^\infty/\partial x\Big|_{x=0} dy\,dz = (16/\pi^2)\sum_{\substack{m\,n\\ odd}} \left[\exp(-x_o K^\infty_{Kmn})\right] \frac{F_{mn}(r_o)}{mn}, \quad (158)$$

where G^∞ is given by Eqs. (114), (115), and (116). Substituting Eq. (158) into (157) the current at the x=0 plane is

$$I_r(f) = \quad (159)$$

$$-D_q(16/\pi^2)\sum_{\substack{m\,n\\ odd}} \frac{F_{mn}(r_o)}{mn} \int_0^\infty \int_0^B \int_0^C S_r(r_o) \exp(-x_o K^\infty_{Kmn})\,dv_o.$$

From now on the fact that the current is in the x-direction and is evaluated at the x=0 plane will not be shown. The subscript r designates the recombination process.

A noise process is characterized by its spectrum. To derive the recombination current spectrum, $w(|I_r(f)|^2)$, we use the relation between the spectrum and the Fourier frequency components given by Rice[17]:

$$w(|I_r|^2) = \lim_{T\to\infty} I_r(f)\, I_r^*(f)/T, \quad (160)$$

where T is the time interval. Substituting Eq. (159) into (160), using the relation for the recombination source spectrum,

$$\lim_{T\to\infty} S(r_o,f)\, S^*(r_o,f)/T = w(|S_r(r_o)|^2)\,\delta(r_o - r_o')\,dv_{o'} \quad (161)$$

and employing $\int f(x)\,\delta(x-x_o)\,dx = f(x_o)$, (162)

the recombination spectrum becomes

$$w(|I_r|^2) = \quad (163)$$

$$\left[\frac{16\,Dq}{\pi^2}\right]^2 \sum_{\substack{m\,n\\ m'n'\\ odd}} \int_0^\infty \int_0^B \int_0^C \frac{w(|S_r|^2)}{mm'nn'} S_{mo} S_{m'o} S_{no} S_{n'o} \exp\left[-x_o\left(K^\infty_{Kmn} + K^{\infty *}_{Km'n'}\right)\right] dv_o^2.$$

The symbols $w(|S_r|^2)$, s_{mo}, and K^∞_{Kmn}, are given by Eqs. (65), (149), and (117). The Dirac delta function in Eq. (161) expresses the fact that at each point of the semiconductor it is assumed that the noise source is uncorrelated with the noise sources at all other points.

The recombination source function, Eq. (65), contains $\langle p_t(r_o)\rangle$. This quantity is defined by Eqs. (B11) to (B13) and is made up of two parts, the thermal equilibrium hole density $\langle p_t(r_o)\rangle_{N_1}$ and the average excess hole density $\langle p_t(r_o)\rangle_E$. Integrating the part of Eq. (163) which contains the thermal equilibrium hole density, the Nyquist current spectrum is obtained:

$$w(|I_r^2|)_N = (16\,q/\pi^2) p_n A^\infty \tau^{-1} \sum_{\substack{m\,n\\ odd}} \left[m^2 n^2 \left(K^\infty_{Kmn} + K^{\infty *}_{Kmn}\right)\right]^{-1}, \quad (164)$$

where A^∞ and K^∞_{Kmn} are given by Eqs. (B34) and (117) respectively.

The excess recombination noise spectrum is derived by integrating the part of Eq. (163) which contains the average excess hole density,

Eq. (B21). Before integration this equation is

$$w(|I_r^2|)_E = \left[\frac{2^7 q}{\pi^3 \beta}\right]^2 P_b^{(0)} \qquad (165)$$

$$\sum_{\substack{m,m',m'' \\ n,n',n'' \\ \text{odd}}} \int_0^\infty \int_0^B \int_0^C S_{m_x} S_{m'_x} S_{m''_x} S_{n_y} S_{n'_y} S_{n''_y} S_{r'_0} \exp\left[-\chi_0 (K^{\infty}_{Kmn} + K^{\infty}_{Km'n'} + K^{\infty}_{Km''n''})\right] d v_0 .$$

Integrating Eq. (165) yields

$$w(|I_r^2|)_E = \left[\frac{2^9 q}{\pi^3 \beta}\right]^2 \frac{P_b A^{\infty}}{C} \qquad (166)$$

$$\sum_{\substack{m,m',m'' \\ n,n',n'' \\ \text{odd}}} \frac{\{[(m^2-m'^2-m''^2)^2-(2m'm'')^2][(n^2-n'^2-n''^2)^2-(2n'n'')^2]\}^{-1}}{K^{\infty}_{Kmn} + K^{\infty}_{Km'n'} + K^{\infty}_{Km''n''}},$$

where the symbols A^{∞}, $p_b(0)$, s_{mo}, K^{∞}_{Kmn}, and $K^{\infty}_{K'mn}$ are given by Eqs. (B34), (B5), (149), (117) and (B22) respectively. In deriving Eq. (166) the following integral has been used:

$$\int_0^{\theta} S_{r_0} S_{r'_0} S_{r''_0} d u_0 = -4\theta r r' r'' \left\{\pi\left[(r^2-r'^2-r''^2)^2-(2r'r'')^2\right]\right\}^{-1} \qquad (167)$$

where only those values of r, r', r'' appear for which the value of $|r \pm r' \pm r''|$ is odd. The symbol r stands for m or n; θ for B or C; and u for y or z, respectively. Therefore in Eq. (166) there appear only those values of m, m', m'' and n, n', n'' for which the expressions $|m \pm m' \pm m''|$ and $|n \pm n' \pm n''|$ are odd.

6.3 Diffusion Noise Current Spectrum for Infinite Surface Recombination Velocity

The expression for the diffusion current density is given by Eq. (121) with ∞ superscripts. Since $\vec{\nabla}\cdot\vec{j}^{\infty}$ and $\vec{\nabla}\cdot\vec{\Gamma}^{\infty}$ go to zero at the boundary, there is no contribution from the boundaries. For the x component of the current density, Eq. (121) becomes

$$j_x^{\infty}(r) = \int \left(S_{Dx_0}\Gamma^{\infty}_{x_0,x} + S_{Dy_0}\Gamma^{\infty}_{y_0,x} + S_{Dz_0}\Gamma^{\infty}_{z_0,x}\right) d v_0 . \qquad (168)$$

To get the total diffusion current $I_D(f)$ in the x direction at the x=0 plane, Eq. (168) is integrated over this plane. The result is

$$I_D(f) = \int_0^B \int_0^C \int_0^{\infty} \int_0^B \int_0^C \left(S_{Dx_0}\Gamma^{\infty}_{x_0,x}\Big|_{x=0} + S_{Dy_0}\Gamma^{\infty}_{y_0,x}\Big|_{x=0} + S_{Dz_0}\Gamma^{\infty}_{z_0,x}\Big|_{x=0}\right) dy_0 dz_0 dv_0 \qquad (169)$$

Performing the integration at the x=0 plane, using Eq. (152), the diffusion current becomes

$$I_D(f) = \frac{16}{K^2\pi^2} \sum_{\substack{m,n \\ \text{odd}}} \int_0^{\infty} \int_0^B \int_0^C \left(\frac{S_{Dx_0} S_{m_0} S_{n_0} K^{\infty}_{Kmn}}{m\,n} - \frac{\pi S_{Dy_0} C_{m_0} S_{n_0}}{n\,B} - \frac{\pi S_{Dz_0} S_{m_0} C_{n_0}}{m\,C}\right) e^{-K^{\infty}_{K_{mn}} x_0} dv_0 \qquad (170)$$

where S_{m_0} and C_{m_0} are given by Eq. (149) and S_{Du} are the u components of the diffusion sources.

The diffusion current spectrum is

$$w(|I_D^2|) = \left(\frac{4}{|K|\pi}\right)^4 \sum_{\substack{m,n \\ m',n' \\ \text{odd}}} \int_0^\infty \int_0^B \int_0^C \left\{\exp\left[-x_o\left(K^\infty_{Kmn} + K^{\infty *}_{Km'n'}\right)\right] dv_o^2\right\} \left\{\begin{array}{l} w(|S_{Dx_o}|^2)\, \frac{S_{mo}S_{m'o}S_{no}S_{n'o}}{m m' n n'} K^\infty_{Kmn} K^{\infty *}_{Km'n'} \\ + \left[\pi^2 w(|S_{Dy_o}|^2)\, C_{mo}C_{m'o}S_{no}S_{n'o}\right]/mn'B^2 \\ + \left[\pi^2 w(|S_{Dz_o}|^2)\, S_{mo}S_{m'o}C_{no}C_{n'o}\right]/mm'C^2 \end{array}\right\} . \quad (171)$$

Here we have used an equation similar to (160) and the diffusion source spectrum given by

$$\lim_{T\to\infty} \frac{S_{Du_o} S^*_{Du_o}}{T} = w(|S_{Du_o}|^2)\,\delta(r_o - r_o')\, dv_o . \quad (172)$$

At an arbitrary point the diffusion sources in the three orthogonal directions are independent. Furthermore, the diffusion sources at an arbitrary point are assumed to be independent of all other points.

In Eq. (171) the noise sources, Eq. (75), contain the average hole density $\langle p_c^\infty(r)\rangle$. As in Section 6.2 the part of Eq. (171) which contains the thermal equilibrium hole density gives the Nyquist noise while the part which contains the excess hole density gives the excess noise.

Integrating the Nyquist part of Eq. (171), the Nyquist diffusion noise is

$$w(|I_D^2|)_N = \frac{A^\infty 2^8 q^2 p_b}{\pi^4} \sum_{\substack{m,n \\ \text{odd}}} \frac{\left(|K^\infty_{Kmn}|^2 + K^{\infty 2}_{mn}\right)}{m^2 n^2 \left(K^\infty_{Kmn} + K^{\infty *}_{Kmn}\right)} , \quad (173)$$

where A^∞ and K^∞_{Kmn} are from Eqs. (B34) and (117)

respectively, and K^∞_{mn} is from Eq. (144) with ℓ equal to zero.

The excess noise part of Eq. (171) is

$$w(|I_D|^2)_E = \left(A^\infty 2^{14} q^2 D\, p_b(0)/\pi^6\right) \sum_{\substack{m,n \\ m',n' \\ \text{odd}}} \int_0^\infty \int_0^B \int_0^C \left\{\exp\left[-x_o\left(K^\infty_{Kmn} + K^{\infty *}_{Km'n'} + K^\infty_{K'm''n''}\right)\right] dv_o\right\} \left(\begin{array}{l} \frac{S_{mo}S_{m'o}S_{m''o}S_{no}S_{n'o}S_{n''o}}{m m' m'' n n' n''} K^\infty_{Kmn} K^{\infty *}_{Km'n'} \\ + \frac{\pi^2 C_{mo}C_{m'o}S_{m''o}S_{no}S_{n'o}S_{n''o}}{m'' n n' n'' B^2} \\ + \frac{\pi^2 S_{mo}S_{m'o}S_{m''o}C_{no}C_{n'o}S_{n''o}}{m m' n'' C^2} \end{array}\right) . \quad (174)$$

Here Eq. (B21) is substituted for $\langle p_c(x_o)\rangle$ in (75), which in turn is put into (171).

Integrating Eq. (174) shows the excess diffusion current spectrum to be

$$w(|I_D|^2)_E = \left(A^\infty 2^{18} q^2 D\, p_b(0)/\pi^8\right) \sum_{\substack{m,m',m'' \\ n,n',n'' \\ \text{odd}}} \frac{K^\infty_{Kmn} K^{\infty *}_{Km'n'} + \frac{\pi^2}{2B^2}(m^2+m'^2-m''^2) + \frac{\pi^2}{2C^2}(n^2+n'^2-n''^2)}{\left(K^\infty_{Kmn} + K^{\infty *}_{Km'n'} + K^\infty_{Km''n''}\right)\left[(m^2-m'^2-m''^2)-(2m'm'')^2\right]\left[(n^2-n'^2-n''^2)-(2n'n'')^2\right]}$$

where A^∞, $p_b(0)$, K^∞_{Kmn}, and $K^\infty_{K'mn}$ are given by Eqs. (B34), (B5), (117), and (B22). Here we used Eq. (167) and the integral

$$\int_0^{\infty} c_m c_{m'} \sin\theta_m \sin\theta_{m'} \, d\omega_{r''} = (2\pi''\theta/\pi)\left(r^2+r'^2-r''^2\right)\left[(r^2-r'^2-r''^2)^2-(2r'r'')^2\right]^{-1}, \quad (176)$$

where r is either m or n. In Eqs. (167) and (176) only those values of r, r', r'' appear for which the expression $|r \pm r' \pm r''|$ is odd.

6.4 Total Nyquist Noise: Stochastic Theory and Nyquist Law

The total noise spectrum calculated from stochastic theory is composed of two parts: the recombination current spectrum and the diffusion current spectrum. Since the recombination and diffusion processes are independent statistically, we add the spectral densities. For the total Nyquist noise current spectrum Eqs. (164) and (173) are summed to get

$$w(|I^2|)_N = \frac{A^{\infty 2} 2^8 q^2 p_n D}{\pi^4} \sum_{\substack{m,n \\ odd}} \frac{1}{m^2 n^2} \left[\frac{|K^{\infty}_{K'mn}|^2 + K^{\infty 2}_{K'mn}}{2\mathcal{R}(K^{\infty}_{K'mn})} \right] \quad (177)$$

The symbol $\mathcal{R}(K^{\infty}_{K'mn})$ means the real part of $K^{\infty}_{K'mn}$. From equations (B30), (B22) and (117) we get

$$\mathcal{R}(K^{\infty}_{K'mn}) = \frac{1}{\sqrt{2}} \left[\left(K^{\infty 4}_{K'mn} + \frac{\omega^2}{D^2} \right)^{\frac{1}{2}} + K^{\infty 2}_{K'mn} \right]^{\frac{1}{2}} . \quad (178)$$

With Eq. (178), Eq. (177) becomes

$$w(|I^2|)_N = \frac{2^6 A^{\infty 2} q^2 p_n D}{\pi^4} \sum_{\substack{m,n \\ odd}} \frac{\mathcal{R}(K^{\infty}_{K'mn})}{m^2 n^2} . \quad (179)$$

As a check on the above method, the Nyquist noise current spectrum is calculated from the conductance at the x=0 plane. From the Nyquist law[15] the noise spectrum is

$$w(|I^2|)_N = 4KTG , \quad (180)$$

where G is the real part of the thermal equilibrium admittance of the p-n junction. At thermal equilibrium V_0 is zero in the expression for the admittance, Eq. (B33), and Eq. (180) is identical with Eq. (179). Thus the result of the stochastic analysis is correct, giving us confidence in the method of tensor Green's functions. We shall see in Chapter VII, on the other hand, that the stochastic result obtained by the use of scalar Green's functions does not check the result obtained from the Nyquist law.

6.5 Total Excess Current Spectrum with Infinite Surface Recombination Velocity

Adding the excess recombination and the excess diffusion current spectra, Eqs. (165) and (175), the total noise spectrum is found to be

$$w(|I^2|)_E = \left[2^{18} q^2 D p_0 (0) A^{\infty 2} / \pi^8 \right] \sum_{\substack{m,n \\ odd}} \sum_{\substack{m',n' \\ odd}} \left(K^{\infty}_{K'mn} + K^{\infty *}_{K'm'n'} + K^{\infty}_{K'm'n'} \right)^{-1}$$
$$\times \left[\frac{1}{D\tau} + K^{\infty}_{K'mn} K^{\infty *}_{K'm'n'} + \frac{\pi^2}{2^6} (m^2+m'^2-m''^2) + \frac{\pi^2}{2^6} (n^2+n'^2-n''^2) \right]$$
$$\times \left[(m^2-m'^2-m''^2)^2 - (2m'm'')^2 \right]^{-1} \left[(n^2-n'^2-n''^2)^2 - (2n'n'')^2 \right]^{-1} \Big\} , \quad (181)$$

where A^{∞}, p_0, $K^{\infty}_{K'mn}$, and $K^{\infty}_{K'mn}$ are given by Eqs. (B34), (B5), (117), and (B22). Here r represents either m or n and only those values of r, r', r'' appear for which $|r \pm r' \pm r''|$ is odd.

It is interesting to see if the excess spectrum is proportional to the steady current flowing at x=0. The dc current density is

derived from Eqs. (12) and (B21). Integrating over the x=0 plane gives for total current

$$I^{\infty} = [2^6 q D \rho_b(0) A^{\infty}/\pi^4] \sum_{m,n} K_{\infty mn}/m^2 n^2 . \quad (182)$$

Comparing the dc current with the excess current spectrum, Eq. (181), the two relations can be related. However, the factor of proportionality is a complicated function of frequency, semiconductor parameters, and geometry.

6.6 Divergence of the Expression for the Nyquist Noise

Let us examine the noise spectra for convergence at each frequency. The Nyquist current spectrum, Eq. (179), is converted into a double integral and integrated. All terms in Eq. (179) are positive, and we investigate the range for large m and n. Making the transformations

$$\frac{\pi m}{B} = x \ , \ \Delta x = \frac{\pi}{B} \ , \ y = \frac{\pi n}{C} \ , \ \Delta y = \frac{\pi}{C} , \quad (183)$$

and neglecting $1/D\tau$ and ω/D with respect to m and n, the series in Eq. (179) becomes when Δx and Δy go to zero

$$\sum_{m,n} \frac{\mathcal{R}(K^{\infty}_{\infty mn})}{m^2 n^2} > \frac{\pi^2}{BC} \int_{m>>1}^{\infty} \int_{n>>1}^{\infty} \frac{(x^2+y^2)^{1/2}\, dx\, dy}{x^2 y^2} . \quad (184)$$

With the transformations

$$x = \rho \cos\theta \ , \ y = \rho \sin\theta , \ dx\,dy = \rho\, d\rho\, d\theta , \quad (185)$$

Eq. (184) becomes

$$\sum_{m,n} \frac{\mathcal{R}(K^{\infty}_{\infty mn})}{m^2 n^2} > \frac{4\pi^2}{BC} \int_{\rho_0}^{\infty} \frac{d\rho}{\rho^2} \int_{\frac{\pi}{C\rho}}^{\frac{\pi}{2} - \frac{\pi}{C\rho}} \frac{d\theta}{\sin^2 2\theta} = \infty . \quad (186)$$

Therefore $w(|I^2|)_N$ becomes infinite.

To determine the physical significance of this divergence we investigate the admittance with $V_0=0$ at the x=0 plane, since the Nyquist current spectrum is proportional to its real part. For arbitrary surface recombination velocity the admittance at the x=0 plane is given by Eq. (B28). The factors in the denominator of the infinite series are each bounded and are neglected for discussions of convergence. Furthermore, from the boundary conditions Eqs. (96) and (97)

$$\beta_r^{\alpha} = r\pi + \Delta\theta_r \ , \quad (187)$$

where r stands for m, n; α for b, c respectively. Only for infinite surface recombination velocity does $\Delta\theta_r$ always remain $\pi/2$ independent of r. Otherwise $\Delta\theta_r$ approaches zero as r goes to infinity.

If we take r finite but so large that $\Delta\theta_r$ is small, $[\beta_m^2+\beta_n^2] >> [(D\tau)^{-1} + i\,\omega/D]$, and the identity for large positive integers m and n

$$(m^2+n^2)^{1/2} \leq mn \ , \quad (188)$$

the series in Eq. (B28) becomes

$$\sum_{m,n} \left(\frac{\sin\beta_m b}{\beta_m b}\right)^2 \left(\frac{\sin\beta_n c}{\beta_n c}\right)^2 K_{\infty mn} \leq \sum_{m,n} \frac{(\Delta\theta_m)^2 (\Delta\theta_n)^2}{\pi^3 mn} . \quad (189)$$

As m and n individually go to infinity the terms go to zero because $(\Delta\theta_m)^2$ or $(\Delta\theta_n)^2$ goes to zero. The value of the input admittance is finite except when $\Delta\theta_r$ is $\pi/2$, when it is infinite. When $s=\infty$ the surface of the semiconductor seems to be covered with a perfectly conducting layer which short-circuits the semiconductor. Therefore the Nyquist noise current spectrum becomes infinite when the surface recombination velocity is infinite.

A convergent expression for the case of arbitrary s can be written down directly from the Nyquist Law, Eq. (180) and the real part of the admittance, Eq. (B31). This is discussed further in Chapter VII for the case of arbitrary s. Now we examine this expression for the case of large but finite s.

From the boundary conditions, Eqs. (96) and (97),

$$\psi = s\alpha/D = \beta_r\alpha \tan \beta_r\alpha. \tag{190}$$

When ψ is large we can write $\beta_r\alpha$ as [7]

$$\beta_r\alpha = \frac{r\pi}{2} - \frac{r\pi}{2\psi} = \frac{r\pi}{2} - \epsilon_r , \tag{191}$$

where r is odd. Substituting Eq. (B31) into (180), letting ψ be large and the dc voltage at x=0 be zero, we get for the range of small ϵ_r,

$$\begin{aligned} w(|I^2|)_N &= 8\sqrt{2}\, A D q^2 p_n \\ &\times \sum_{m,n\ \text{odd}} \left[\frac{\cos(m\pi D/2sb)\cos(n\pi D/2sc)}{(m\pi/2)(1-D/sb)(n\pi/2)(1-D/sc)}\right]^2 \\ &\times \left[1 + \frac{\sin(m\pi D/sb)}{m\pi(1+D/sb)}\right]^{-1}\left[1 + \frac{\sin(n\pi D/sc)}{n\pi(1+D/sc)}\right]^{-1} \\ &\times \left\{\left\{\left[(D\tau)^{-1} + \left[(m\pi/2b)(1-D/sb)\right]^2 + \left[(n\pi/2c)(1-D/sc)\right]^2\right]^2 + \omega^2/D^2\right\}^{1/2}\right. \\ &\left. + (D\tau)^{-1} + \left[(m\pi/2b)(1-D/sb)\right]^2 + \left[(n\pi/2c)(1-D/sc)\right]^2\right\}. \end{aligned} \tag{192}$$

If we let sb/D and sc/D become very large while ϵ_m and ϵ_n remain small, Eq. (192) becomes identical with the Nyquist current spectrum with $s=\infty$, Eq. (179); to make the two equations agree the conversion factors between Figures 2 and 3 are used:

$$B = 2b; \quad C = 2c. \tag{193}$$

6.7 Convergence of the Series for the Excess Noise Spectrum

To discuss the convergence of the excess noise spectrum, we investigate Eq. (181). In the denominator of this equation there are two factors which contain minus signs, one in m and one in n. Taking the factor in m (identical results are obtained with n) the indices m, m' and m" are related so that $|m \pm m' \pm m''|$ must be odd. For the whole factor to be zero

$$(m^2 - m'^2 - m''^2)^2 = (2m'm'')^2. \tag{194}$$

Solving we get

$$m \pm m' \pm m'' = 0. \tag{195}$$

Since zero is an even number, the factors cannot vanish. Furthermore the factor in m is negative whenever $m < m' + m''$.

The question of convergence of the excess noise spectrum is a very difficult one to answer. Converting the sum to an integral is not permissible since some of the values of r, r' and r" are not present. Furthermore only conditional convergence and not absolute convergence is required. Putting test values into the series the denominator increases very much faster than the numerator. It appears that the sum converges rapidly, but a rigorous proof would require numerical evaluation of the excess noise spectrum.

Equation (181) can be used to calculate the excess current spectrum for infinite surface recombination velocity. From the above discussion of the Nyquist noise, we can expect that Eq. (181) provides an upper bound for the excess noise spectrum when the surface recombination velocity is large but not infinite.

6.8 Contribution of Electron Density Fluctuation in the p-Type Material

Our analysis has only considered hole conduction in the n-region. Shockley[7] has shown that electron conduction in the p-region is simply an additive effect to the hole current. The noise resulting from the concentration fluctuations of electrons is statistically independent of the hole fluctuations and the analysis is similar to that made above. When the p and n symbols in the expressions for the hole fluctuations are interchanged and when the values of D and τ for electrons in p-type material are used, the derived spectra pertain to electron fluctuations in the p-type material.

The total current spectrum is the sum of the hole current spectrum and the electron current spectrum.

6.9 Summary of Chapter VI

Using tensor and scalar Green's functions, we have derived explicit expressions for the thermal equilibrium noise spectrum, Eq. (179), and the excess noise spectrum, Eq. (181), of the p-n junction when the transverse surface recombination velocity approaches infinity. In actual practice the surface recombination velocity is never infinite, but may be very large. Since Eq. (180) is convergent, it should be used to calculate the thermal noise when the surface recombination velocity is large,

but finite. The excess noise spectrum can be evaluated from Eq. (181) and should give a good upper bound for the noise when the surface recombination velocity is large, but finite.

CHAPTER VII

NOISE IN A p-n JUNCTION WITH ARBITRARY SURFACE RECOMBINATION VELOCITY

7.1 Introduction

In the discussion of the noise current spectrum with an infinite surface recombination velocity, we had a very important check on our work. We derived the Nyquist noise current spectrum by two independent methods: the Nyquist law using the input conductance and the stochastic process theory using basic physical principles. These spectra are identical. In this chapter we get the current spectrum for the case of finite surface recombination velocity. When we compare the current spectrum of the Nyquist noise derived from the Nyquist law and from stochastic theory, we find that only for small values of surface recombination velocity do the two methods agree. Since the stochastic process method uses basic physical principles and rigorous methods, we present it, but we do not resolve the question of which result is correct.

For the case of arbitrary surface recombination velocity we use the rectangular coordinate system shown in Figure 2 with the p-n junction at the x=0 plane. The origin is at the center of the p-n junction.

To find the current spectrum for the three dimensional p-n junction, the excess hole density p is derived with the aid of the

scalar Green's function. This density p is obtained from Eq. (82). Since the Green's function satisfies the same homogeneous boundary conditions as the unknown function, Eqs. (96) and (97), the surface integrals in Eq. (82) are zero. For the surfaces $x_o=0$ and $x_o=\infty$ this is proved with a technique similar to that used in Section 6.2.

The vanishing of the remaining surface integrals is proved in the following manner. They can be written as

$$p(r)\big|_s = \int [G \nabla_o p(r_o) - p(r_o) \nabla_o G] \cdot d\vec{A}_o \,, \tag{196}$$

and the mixed boundary conditions, Eq. (93), can be written as

$$\nabla_o p = \mp (\vec{i}_y + \vec{i}_z) s\, p/D, \tag{197}$$

where the symbol ∇_o is defined by

$$\nabla_o = \vec{i}_y \frac{\partial}{\partial y_o} + \vec{i}_z \frac{\partial}{\partial z_o} \,. \tag{198}$$

Green's function satisfies Eq. (197). To prove this, Green's function, Eq. (109), is written as

$$G = \sum_{M,N} \begin{bmatrix} \cos\beta_r u_o \\ \sin\beta_r u_o \end{bmatrix} R_{MN} \,, \tag{199}$$

and is differentiated to get

$$\frac{\partial G}{\partial u_o} = \sum_{M,N} \begin{bmatrix} -\beta_r \sin\beta_r u_o \\ \beta_r \cos\beta_r u_o \end{bmatrix} R_{MN} \,, \tag{200}$$

where r denotes m or n, ρ denotes μ or ν, and

u_o denotes y_o or z_o respectively. Eqs. (199) and (200) are evaluated at the boundaries with the aid of the eigenfunction form of the mixed boundary conditions, Eqs. (96) and (97). When we compare the resulting equations, we see that Green's function satisfies Eq. (197).

Substituting Eq. (197) for $\nabla_o^2 p$ and a similar expression for $\nabla_o^2 G$ into Eq. (196), $p(r)\big|_S$ is found to be identically zero and Eq. (82) becomes

$$p(r) = \int S(r_o)\, G(r|r_o)\, dv_o \,. \tag{201}$$

Here $S(r_o)$ and G are given by Eqs. (35) and (109) respectively.

7.2 Bulk Recombination Current Spectrum for Finite Surface Recombination Velocity

The recombination current spectrum derivation is exactly the same as that for the infinite surface recombination velocity. The Nyquist current spectrum is

$$\omega(|I_r|^2)_N = 4q^2 A p_n \tau^{-1} \sum_{m,n} \gamma_{mn}^2 L_{mn} \left(K_{Kmn} + K^*_{Kmn}\right) , \tag{202}$$

where A, γ_{mn}, L_{mn}, and K_{Kmn} are given by Eqs. (B20), (B19), (111) and (112); and the symbols q and τ are defined after equation (12).

The excess current spectrum is

$$\omega(|I_r|^2)_\epsilon = 4q^2 A^3 p_o(0)\, \tau^{-1} \sum_{\substack{m,m',m'' \\ n,n',n''}} \gamma_{mn}\, \gamma_{m'n'}\, \gamma_{m''n''}\, E^{(m)}_{ccc} E^{(n)}_{ccc} \left(K_{Kmn} + K^*_{K'm'n'} + K_{K'm''n''}\right), \tag{203}$$

where

$$E^{(r)}_{ccc} = \int_\alpha c^f_{ro}\, c^f_{r'o}\, c^f_{r''o}\, du_o = \left[\frac{2}{(\beta_r^2+\beta_{r'}^2-\beta_{r''}^2)^2 - 4\beta_{r'}^2\beta_{r''}^2}\right] \tag{204}$$

$$\times\Big[(\beta_r^2+\beta_{r'}^2-\beta_{r''}^2)(\beta_r s^f_{r\alpha} c^f_{r'\alpha} c^f_{r''\alpha} + \beta_{r'} c^f_{r\alpha} s^f_{r'\alpha} c^f_{r''\alpha} - \beta_{r''} c^f_{r\alpha} c^f_{r'\alpha} s^f_{r''\alpha})$$

$$-2\beta_{r'}\beta_{r''}(\beta_r c^f_{r\alpha} s^f_{r'\alpha} c^f_{r''\alpha} + \beta_{r'} s^f_{r\alpha} c^f_{r'\alpha} c^f_{r''\alpha} + \beta_{r''} s^f_{r\alpha} c^f_{r'\alpha} s^f_{r''\alpha})\Big] .$$

c^f_{ro}, s^f_{ro}, $c^f_{r\alpha}$, and $s^f_{r\alpha}$ are defined by

$$\begin{aligned} c^f_{ro} &\equiv \cos\beta_r u_o , \qquad s^f_{ro} \equiv \sin\beta_r u_o , \\ c^f_{r\alpha} &\equiv \cos\beta_r \alpha , \qquad s^f_{r\alpha} \equiv \sin\beta_r \alpha , \end{aligned} \tag{205}$$

where r stands for m or n; α for b or c; and u_o for y_o or z_o respectively. $p_o(0)$ is given by Eq. (B5). The symbol $K_{K'mn}$ is given by Eq. (B18) and the other symbols are identified after Eq. (202).

7.3 The Diffusion Current Spectrum for Finite Surface Recombination Velocity

The diffusion current spectrum for the finite transverse surface recombination velocity requires a new approach. We reexamine the solution for the excess hole density, Eq. (201). By transforming this equation the diffusion noise source takes on the same form as that derived in Chapter III. Gradients of the scalar Green's function result from this transformation.

We start the derivation of the diffusion current spectrum with the solution for excess hole density. The source function S is now S_D, Eq. (35), and Eq. (201) becomes

$$p_o(r) = \int [\nabla_o^2 p(r_o)]\, G\, dv_o \,. \tag{206}$$

The appearance of the Laplacian in the source function makes it impossible to use the physical source function, Eq. (75). We transform Eq. (206) into

$$p(r) = \int [G \nabla_o p_o] \cdot dA_o - \int [\nabla_o p_o] \cdot [\nabla_o G]\, dv_o \qquad (207)$$

with the vector identity

$$G[\nabla_o^2 p] = \nabla_o \cdot [G \nabla_o p] - [\nabla_o p] \cdot [\nabla_o G] \qquad (208)$$

and Gauss' theorem.[14]

The surface integrals in Eq. (207) result from the diffusion current to the transverse surfaces of the semiconductor. This current is caused by the surface recombination and is related to the surface recombination velocity. It is important to note that the surface sources did not enter prior to the transformation of Eq. (207). The three volume integrals are due to the volume diffusion.

In order to derive the diffusion current spectrum, we solve for the total diffusion current, $I_D(f)$, in the x direction at the x=0 plane. When Eq. (207) is written in component form and integrated over the x=0 plane, the diffusion current becomes

$$I_D(f) = AD_q \sum_{m,n} \left[\gamma_{mn} \int_0^\infty \exp(-K_{Kmn} Y_o)\, dY_o \right] \qquad (209)$$

$$\times \left\{ -\int_{-c}^{c} c_{mb}^f c_{no}^f \left(\frac{\partial p_o}{\partial y_o}\Big|_b\right) dz_o + \int_{-c}^{c} c_{mb}^f c_{no}^f \left(\frac{\partial p_o}{\partial y_o}\Big|_{-b}\right) dz_o - \int_{-b}^{b} c_{mo}^f c_{nc}^f \left(\frac{\partial p_o}{\partial z_o}\Big|_c\right) dy_o \right.$$

$$\left. + \int_{-b}^{b} c_{mo}^f c_{nc}^f \left(\frac{\partial p_o}{\partial z_o}\Big|_{-c}\right) dy_o - \iint_{-b-c}^{b\,c} \left[\left(\frac{\partial p_o}{\partial Y_o}\right) K_{Kmn} c_{mo}^f c_{no}^f + \left(\frac{\partial p_o}{\partial y_o}\right) \beta_m s_{mo}^f c_{no}^f + \left(\frac{\partial p_o}{\partial z_o}\right) \beta_n c_{mo}^f s_{no}^f \right] dy_o\, dz_o \right\}.$$

Here p_o is substituted for $p(r_o)$. The symbols A, γ_{mn}, K_{Kmn}, and β_m are given by Eqs. (B20), (B19), (112), and (96) respectively. The symbols D and q are defined after Eq. (12). In deriving Eq. (209) the following facts and equations are used: the scalar Green's function is zero at the planes x=0 and x=∞; Eq. (14); in Eq. (207) only Green's function depends on the observation coordinates.

The diffusion current spectrum is derived by substituting the diffusion current, Eq. (209), into an equation similar to (160) and is

$$\omega(|I_p|^2) = 4 D_q^2 A^2 \sum_{m,m',n,n'} \left[\gamma_{mn} \gamma_{m'n'} \int_0^\infty \exp\left[-Y_o (K_{Kmn} + K^*_{Km'n'})\right] dY_o \right. \qquad (210)$$

$$\times \left\{ \frac{2s}{D} \left[\int_{-c}^{c} \langle p_t(b) \rangle c_{mb}^f c_{m'b}^f c_{no}^f c_{n'o}^f\, dz_o + \int_{-b}^{b} \langle p_t(c) \rangle c_{mo}^f c_{m'o}^f c_{nc}^f c_{nc}^f\, dy_o \right] \right.$$

$$+ \int_{-b}^{b} \int_{-c}^{c} \langle p_t \rangle \left[K_{Kmn} K^*_{Km'n'} c_{mo}^f c_{m'o}^f c_{no}^f c_{n'o}^f + \beta_m \beta_{m'} s_{mo}^f s_{m'o}^f c_{no}^f c_{n'o}^f \right.$$

$$\left. \left. \left. + \beta_n \beta_{n'} c_{mo}^f c_{m'o}^f s_{no}^f s_{n'o}^f \right] dy_o\, dz_o \right\} \right],$$

where the symbols $c_{r\alpha}^f$, $s_{r\alpha}^f$, $c_{r\alpha}^{\dagger}$, and $s_{r\alpha}^{\dagger}$ are defined by Eq. (205), while the other symbols are identified after Eq. (209). The factor two in the surface integrals comes from the algebraic addition of the two equal uncorrelated noise spectra produced by currents to opposite transverse surfaces. In the derivation of Eq. (210) the relation characterizing the surface sources is

$$\lim_{T \to \infty} \frac{1}{T} \left[\left(\frac{\partial p_o}{\partial u_o}\right) \left(\frac{\partial p_o}{\partial u_o}\right)^* \right]_\alpha = \omega_s (|\tfrac{\partial p_o}{\partial u_o}|^2)_\alpha\, \delta(Y_o - Y_o')\, \delta(y_o - y_o')\, dY_o\, dy_o\,, \qquad (211)$$

while the relation characterizing the bulk diffusion sources is

$$\lim_{T\to\infty}\frac{1}{T}\left[\left(\frac{\partial p_r}{\partial u_o}\right)\left(\frac{\partial p_{r'}}{\partial u_o}\right)^*\right] = \omega_p\left(\left|\frac{\partial p_r}{\partial u_o}\right|^2\right)\delta(r_o - r_o')\,dv_o . \qquad (212)$$

In Eq. (211) u_o stands for y_o or z_o while in Eq. (212) u_o stands for x_o, y_o, or z_o; v_o denotes any coordinate except u_o. In Eq. (211) the symbol $w_s\left(\left|\frac{\partial p_r}{\partial u_o}\right|^2\right)$ is given by Eq. (80) while in Eq. (212) the symbol $w_p\left(\left|\frac{\partial p_r}{\partial u_o}\right|^2\right)$ is derived from Eqs. (75), (60), and the homogeneous form of (14).

To evaluate the diffusion current spectrum it is divided into Nyquist noise and excess noise. For the Nyquist noise p_n is substituted for $\langle p_t(r)\rangle$ in Eq. (210) and the equation integrated,

$$w(|I_{Dl}|^2)_N = 4q^2A^2D\beta_r\left\{\sum_{m,m',n}\frac{\gamma_{mn}\gamma_{m'n}}{K_{Kmn}+K^*_{Kmn}}\left[2s(\cos\beta_m b)(\cos\beta_{m'}b)L_n + \beta_m\beta_{m'}L^{mm'}L_n\right]\right.$$

$$\left.+\sum_{m,n,n'}\frac{\gamma_{mn}\gamma_{mn'}}{K_{Kmn}+K^*_{Kmn'}}\left[2s(\cos\beta_n b)(\cos\beta_{n'}b)L_m + \beta_n\beta_{n'}L^{nn'}_{m}L^{mn'}\right] + \sum_{m,n}\frac{\gamma_{mn}^2 K^*_{Kmn}L_mL_n}{K_{Kmn}+K^*_{Kmn}}\right\} \qquad (213)$$

where the integrals $L^{rr'}$, L_r, and L_r^- are given by Eqs. (132), (103), and (133) respectively; when $r=r'$, $L^{rr'}$ is replaced by L_r; s is the surface recombination velocity. The other symbols are identified after Eq. (209).

For the excess noise spectrum, $P_b(r)$, Eq. (B17), is substituted for $\langle p_t(r)\rangle$ in Eq. (210). Integrating the resulting equation, the excess diffusion current spectrum is

$$w(|I_{Dl}|^2)_E = 4q^2A^2p_b^{(0)}D \qquad (214)$$

$$\times\sum_{\substack{m\,m'\,m''\\ n\,n'\,n''}}\left\{\left[\frac{\gamma_{mn}\gamma_{m'n'}\gamma_{m''n''}E^{(m)}_{ccc}E^{(n)}_{ccc}}{K_{Kmn}+K^*_{Km'n'}+K_{K'm''n''}}\right]\right.$$

$$\times\left[K_{Kmn}K^*_{Km'n'} + \beta_m\beta_{m'}\frac{E^{(m)}_{ssc}}{E^{(m)}_{ccc}} + \beta_n\beta_{n'}\frac{E^{(n)}_{ssc}}{E^{(n)}_{ccc}}\right.$$

$$\left.\left.+\frac{2s}{D}\left[\frac{\cos\beta_m\ell\cos\beta_{m'}\ell\cos\beta_{m''}\ell}{E^{(m)}_{ccc}} + \frac{\cos\beta_n c\cos\beta_{n'}c\cos\beta_{n''}c}{E^{(n)}_{ccc}}\right]\right]\right\}.$$

The symbols $E^{(r)}_{ccc}$ and $p_b^{(0)}$ are given by Eqs. (204) and (B5); symbol $E^{(r)}_{ssc}$ is given by

$$E^{(r)}_{ssc} = \int_{-a}^{a}\sin\beta_r u_o\,\sin\beta_{r'}u_o\,\cos\beta_{r''}u_o\,du_o = \qquad (215)$$

$$= 2\left[\beta_r\beta_{r'}(\beta_r s^f_{ra}c^f_{r'a}c^f_{r''a} + \beta_{r'}c^f_{ra}s^f_{r'a}c^f_{r''a} - \beta_{r''}c^f_{ra}c^f_{r'a}s^f_{r''a})\right.$$

$$\left.-(\beta_r^2+\beta_{r'}^2-\beta_{r''}^2)(\beta_r c^f_{ra}s^f_{r'a}c^f_{r''a} + \beta_{r'}s^f_{ra}c^f_{r'a}c^f_{r''a} + \beta_{r''}s^f_{ra}s^f_{r'a}s^f_{r''a})\right]$$

$$\times\left[(\beta_r^2+\beta_{r'}^2-\beta_{r''}^2)^2 - 4\beta_r^2\beta_{r'}^2\right]^{-1}$$

where c^f_{ra} and s^f_{ra} are defined by equation (205); the other symbols are identified after equation (209).

7.4 Total Nyquist Noise for Finite Surface Recombination Velocity: Stochastic Process Method and Nyquist Law Method

The total Nyquist current spectrum determined by the stochastic process method equals the algebraic sum of recombination current spectrum, Eq. (202), and the diffusion noise spectrum, Eq. (213), and is

$$W(|I^2|)_N^S = 4q^2A^2\beta_n D\left\{\sum_{m\,n}\left[\frac{\gamma_{mn}^2}{K_{Kmn}+K_{Kmn}^*}\right]\right. \qquad (216)$$

$$\times\left[L_{mn}\left((D\tau)^{-1}+|K_{Kmn}|^2\right)+2sD^{-1}\left(L_n\cos^2\beta_m b+L_m\cos^2\beta_n c\right)+\beta_m^2 L_m^- L_n\right.$$

$$\left.+\beta_n^2 L_m L_n^-\right]+\sum_{\substack{m\,m'\\ n}}\left[\frac{\gamma_{mn}\gamma_{m'n}L_n}{K_{Kmn}+K_{Km'n}^*}\right]\left[2sD^{-1}\left(\cos\beta_m b\right)\left(\cos\beta_{m'}b\right)+\beta_m\beta_{m'}L^{mm'}\right]$$

$$\left.+\sum_{\substack{m\,n\,n'\\ m}}\left[\frac{\gamma_{mn}\gamma_{mn'}L_m}{K_{Kmn}+K_{Kmn'}^*}\right]\left[2sD^{-1}\left(\cos\beta_n c\right)\left(\cos\beta_{n'}c\right)+\beta_n\beta_{n'}L^{nn'}\right]\right\}.$$

The Nyquist current spectrum for the p-n junction is now derived from the Nyquist law, Eq. (180), and the input conductance, Eq. (B31); it is

$$W(|I^2|)_N^C = 2\sqrt{2}A^2Dq^2\beta_n\sum_{m\,n}\gamma_{mn}^2 L_{mn}$$

$$\times\left\{\left[K_{K'mn}^4+(\omega/D)^2\right]^{1/2}+K_{K'mn}^2\right\}^{1/2}. \qquad (217)$$

Comparing Eqs. (216) and (217) we see that the two current spectra are not the same. The former has a triple infinite series while the latter has a double infinite series. When the parameters sb/D and sc/D become very small, agreement is attained between the two expressions for the Nyquist current spectrum. In this case the γ factors converge very rapidly, as shown by Shockley,[7] and the first terms of the series represent the Nyquist spectrum adequately. For the usual p-n material $1/D\tau$ is large with respect to the β_o values in the K_{Koo} factor, Eq. (112).

For small $s\alpha/D$ values the current spectrum by both the stochastic process and the Nyquist law methods is

$$W(|I^2|)_N^S = W(|I^2|)_N^C = 4q^2A\beta_n D\,\mathcal{R}[K_{Koo}] \qquad (218)$$

where

$$\mathcal{R}[K_{oo}] = 2^{-1/2}\left\{\left[\left[(1/D\tau)+sD^{-1}(b^{-1}+c^{-1})\right]^2\right.\right. \qquad (219)$$

$$\left.\left.+(\omega D^{-1})^2\right]^{1/2}+\left[\left[(D\tau)^{-1}+sD^{-1}(b^{-1}+c^{-1})\right]\right]\right\}^{1/2}.$$

Concerning the difference between the stochastic process spectrum and the Nyquist law spectrum for large values of $s\alpha/D$, we note that the transformation giving Eq. (207), which was made in order to use the physical diffusion noise source, results in the gradient operation on the scalar Green's function. While this seems correct formally, it may be that it throws the problem into the domain of the tensor Green's function. The original reason for investigating the tensor Green's function was to use the diffusion noise sources without an additional transformation. Unfortunately an explicit expression for the tensor Green's function for the case of arbitrary surface recombination velocity has not been found. Therefore, we are not able to determine whether the scalar method for the diffusion spectrum is in error or whether the Nyquist theorem is not general enough to handle noise in a p-n junction with mixed boundary conditions.

7.5 Total Excess Current Spectrum for Finite Surface Recombination Velocity

The total excess current spectrum for arbitrary surface recombination velocity is obtained by adding the excess current spectrum for bulk recombination and for diffusion, Eqs. (203) and (214) respectively. These two statistically independent spectra add algebraically, and the total excess current spectrum is

$$w(|I^2|)_E = 4 q^2 A^3 p_b(0) D \qquad (220)$$

$$\times \sum_{\substack{m\,m'\,m'' \\ n\,n'\,n''}} \left[\gamma_{mn}\,\gamma_{m'n'}\,\gamma_{m''n''}\,E^{(m)}_{ccc}\,E^{(n)}_{ccc}\,(K_{Kmn} + K^{*}_{Km'n'} + K_{k'm''n''})^{-1}\right.$$

$$\times \left\{(D\tau)^{-1} + K_{Kmn}\,K^{*}_{Km'n'} + \beta_m\,\beta_{m'}\,E^{(m)}_{ssc}\,(E^{(m)}_{ccc})^{-1}\right.$$

$$+ \beta_n\,\beta_{n'}\,E^{(n)}_{ssc}\,(E^{(n)}_{ccc})^{-1} + 2\,s\,D^{-1}\left[(c\cos\beta_m t)(\cos\beta_{m'} t)(\cos\beta_{m''} t)(E^{(m)}_{ccc})^{-1}\right.$$

$$\left.\left.\left. + (\cos\beta_n c)(\cos\beta_{n'} c)(\cos\beta_{n''} c)(E^{(n)}_{ccc})^{-1}\right]\right\}\Big].$$

Let us relate the dc current in the x direction at the x=0 plane to the excess current spectrum. The diffusion current density in the x direction is derived from Eq. (B17) and the homogeneous Eq. (14). Integrating this current density over the x=0 plane, the total dc current, I_{dc}, is

$$I_{dc} = p_b(0) A^2 D q \sum_{mn} \gamma^2_{mn}\,K_{K'mn}\,L_{mn}; \qquad (221)$$

where the symbols A, γ_{mn}, $K_{K'mn}$, β_r, $p_b(0)$ and L_{mn} are given by Eqs. (B20), (B19), (B18), (96), (B5), and (111) respectively; the symbols D and q are defined after Eq. (12).

Comparing this dc current with the excess current spectrum, Eq. (220), we find a relation between the quantities, but the factor of proportionality is a complicated function of frequency, geometry, and semiconductor constants. When s is small the excess noise spectrum, Eq. (220), becomes

$$w(|I^2|)_E = 4 q^2 A\, p_b(0) D\left[K^2_{K'oo} + (K^4_{K'oo} + \omega^2/D^2)^{1/2}\right] \qquad (222)$$

$$\times \left\{\left[2^{1/2}(K^4_{K'oo} + \omega^2/D^2)^{1/2} + K^2_{K'oo}\right]^{1/2} + K_{K'oo}\right\}^{-1}$$

and the dc current, Eq. (221), becomes

$$I_{dc} = p_b(0) A^2 D q\, K_{K'oo}, \qquad (223)$$

where $$K^2_{K'oo} = (D\tau)^{-1} + s(\ell^{-1} + c^{-1})/D. \qquad (224)$$

When

$$\omega/D \ll K_{K'oo},$$

$$w(|I^2|)_E = (8/3)\, q\, I_{dc}. \qquad (225)$$

This has the familiar form of the shot-effect phenomenon in a temperature-limited diode,[15] except that the constant for the diode is two, while Eq. (225) gives 8/3.

7.6 Spectrum of Electron Density Fluctuations in p-Type Material

As in Section 6.8 the noise current spectrum of the electron density fluctuations in a p-type material is similar to the spectrum of the hole density fluctuations in n-type material. The derivation of the electron density fluctuation spectrum is the same as that described in Section 6.8. The total current spectrum is again equal to the sum of the hole and electron current spectra.

7.7 Summary of Chapter VII

Using scalar Green's functions we have derived explicit expressions for the Nyquist noise, Eq. (216), and the excess noise, Eq. (220), for the case of arbitrary surface recombination. This would appear to constitute a complete solution to the three-dimensional p-n junction noise problem. However, because of the lack of

agreement between our result for the thermal equilibrium noise, Eq. (216), and the result obtained from the Nyquist theorem, Eq. (217), it is recommended that further study be made before complete reliance is placed on the stochastic results.

CHAPTER VIII

CURRENT SPECTRUM IN A TWO-DIMENSIONAL SEMICONDUCTOR WITH A p-n JUNCTION

8.1 Introduction

In many noise experiments the semiconductor sample has a two-dimensional nature. The sample is several diffusion lengths in the longitudinal or x dimension and for all practical purposes may be considered infinite in this direction. In one of the transverse directions, say the y direction, the filament is narrow and the surface recombination velocity is important. In the other transverse direction, the filament is so wide that the effect of surface recombination velocity is negligible. A bias voltage across the p-n junction causes a current to flow in the x direction.

Since there is no variation in the z direction, the del-operator in Eqs. (34) to (37) becomes

$$\nabla = \bar{1}_x \frac{\partial}{\partial x} + \bar{1}_y \frac{\partial}{\partial y} \qquad (226)$$

Except for this change the current spectrum is derived with the technique used to determine the spectrum of the three dimensional semiconductor containing a p-n junction.

8.2 Two Dimensional Current Spectrum for Infinite Surface Recombination Velocity

The Nyquist noise current spectrum is

$$\omega(|I^2|)_N = A^\infty (4/\pi)^4 q^2 p_n D \sum_m R(K^\infty_{Km})/m^2, \tag{227}$$

where (228)

$$R(K^\infty_{Km}) = 2^{-\frac{1}{2}}\left\{\left[\left([\pi m/B]^2+1/D\tau\right)^2+[\omega/D]^2\right]^{\frac{1}{2}}+\left([\pi m/B]^2+1/D\tau\right)\right\}^{\frac{1}{2}}$$

The Nyquist noise spectrum derived with the Nyquist law and the conductance of the two dimensional p-n junction is the same as Eq. (227).

The excess noise spectrum is

$$\omega(|I|)^2_\epsilon = 2^{18}\pi^{-8}A^{\infty 2} q^2 p_b(0) D \tag{229}$$
$$\times \sum_{m,m',m''} \frac{(1/D\tau)+K^\infty_{Km}K^{\infty *}_{Km'}+\pi^2(m^2+m'^2-m''^2)/2B^2}{\left(K^\infty_{Km}+K^{\infty *}_{Km'}+K^\infty_{Km''}\right)\left([m^2-m'^2-m''^2]^2-[2m'm'']^2\right)}$$

where the only values of m, m', m" appearing are those for which $|m \pm m' \pm m''|$ is odd. The symbols $p_b(0)$ and A^∞ are given by Eqs. (B5) and (B34), respectively; the symbols D, q and τ are defined after Eq. (12). The symbols K_{Km} and $K_{K'm}$ are defined as

$$K^\infty_{Km} \equiv [(\pi m/B)^2+(i\omega/D)+1/D\tau]^{\frac{1}{2}} \tag{230}$$

$$K^\infty_{Km} \equiv [(\pi m/B)^2+1/D\tau]^{\frac{1}{2}} \tag{231}$$

In two and three dimensions the current spectra behave the same (see Section 6.6); the Nyquist current spectrum diverges, but the

excess current spectrum appears to converge rapidly. The excess current spectrum can be used as an upper bound for large values of surface recombination velocity. As in the three dimensional case, the proportionality between the excess noise spectrum, Eq. (229), and the dc current at the x=0 plane

$$I^\infty_{dc} = \frac{2^6 q D p_b(0) A^\infty}{\pi^4} \sum_m K^\infty_{K'm}/m^2, \tag{232}$$

is very complicated.

8.3 Two Dimensional Current Spectrum with Finite Surface Recombination Velocity

The Nyquist current spectrum obtained by the stochastic process method is

$$\omega(|I^2|)^S_N = 4q^2A^2p_n D \tag{233}$$
$$\times\left\{\sum_m \frac{\gamma^2_m}{(K_{Km}+K^2_{Km})}\left[L_m\left(|K_{Km}|^2+1/D\tau\right)+\beta^2_m L^-_m+2sD^{-1}(L_m\cos^2\beta_m b)\right]\right.$$
$$\left.+\sum_{\substack{m,m'\\ m\neq m'}}\gamma_m\gamma_{m'}\left(2sD^{-1}\cos\beta_m b\cos\beta_{m'}b+\beta_m\beta_{m'}L^{mm'}\right)/\left(K_{Km}+K^*_{Km'}\right)\right\}.$$

The Nyquist current spectrum obtained by the Nyquist law and the input conductance method is

$$\omega(|I^2|)^C_N = 2\sqrt{2}AD q^2 p_n \sum_m \gamma^2_m L^+_m R(K_{Km}). \tag{234}$$

The symbols A, L^+_m, L^-_m, $L^{mm'}$, and β_m are given by Eqs. (B20), (103), (133), (132), and (96) respectively; and the following expressions define the remaining parameters:

$$\gamma_m = (\sin\beta_m b)/(\beta_m b L^+_m) \tag{235}$$

$$K^2_{K'm} = \frac{1}{D\tau} + \beta^2_m \quad , \tag{236}$$

$$K^2_{K-m} = K^2_{K'm} + \frac{i\omega}{D} \quad , \tag{237}$$

$$\mathcal{R}(K_{Km}) = \left[\left(K^4_{K'm} + \frac{\omega^2}{D^2}\right)^{\frac{1}{2}} + K^2_{K'm}\right]^{\frac{1}{2}} . \tag{238}$$

When sb/D becomes small, the Nyquist spectrum derived by the Nyquist law and by the stochastic process method is

$$w(|I^2|)^C_N = w(|I^2|)^S_N = 4q^2 A p_n D \mathcal{R}(K_{Ko}) \quad , \tag{239}$$

where $$\mathcal{R}(K_{Ko}) = \frac{1}{\sqrt{2}}\left\{\left[\left(\frac{1}{D\tau} + \frac{s}{Db}\right)^2 + \frac{\omega^2}{D^2}\right]^{\frac{1}{2}} + \frac{1}{D\tau} + \frac{s}{Db}\right\}^{\frac{1}{2}} . \tag{240}$$

In two dimensions as in three dimensions the Nyquist current spectra obtained by the stochastic processes method and by the Nyquist law method become the same only when sb/D is small.

The excess current spectrum with finite surface recombination velocity is

$$w(|I^2|)_E = 4q^2A^2 p_{bo}^{(0)} D \sum_{m,m',m''} \{\gamma_m \gamma_{m'} \gamma_{m''} E^{(m)}_{ccc} (K_{Km} + K^*_{Km'} + K_{K'm''}) \cdot X [(D\tau)^{-1} + K_{Km} K^*_{Km'} + \beta_m \beta_{m'} E^{(m)}_{ssc} (E^{(m)}_{ccc})^{-1} + 2s (D E^{(m)}_{ccc})^{-1} \cos\beta_{m,b} \cos\beta_{m',b} \cos\beta_{m'',b}]\} . \tag{241}$$

The symbols $p^{(0)}_{bo}$, $E^{(m)}_{ccc}$ and $E^{(m)}_{ssc}$ are given by Eqs. (B5), (204), and (215) respectively; the symbols D, q, and τ are defined after equation (12); and the rest of the symbols are defined after Eq. (234).

The dc current is

$$I_{dc} = p^{(0)}_b A^2 D q \sum_m \gamma^2_m K_{K'm} L_m . \tag{242}$$

As in three dimensions, Section 7.5, a complicated proportionality factor relates the excess current spectrum and the dc current for arbitrary surface recombination velocity. For small sb/D the excess current spectrum is

$$w(|I^2|)_E = 2\sqrt{2}\, q^2 A p^{(0)}_b D \left\{(D\tau)^{-1} + s(Db)^{-1} + \left[\left[(D\tau)^{-1} + s(Db)^{-1}\right]^2 + (\omega D^{-1})^2\right]^{1/2}\right\} \times \left\{\left\{\left[\left[(D\tau)^{-1} + s(bD)^{-1}\right]^2 + (\omega D^{-1})^2\right]^{1/2} + (D\tau)^{-1} + s(bD)^{-1}\right\}^{1/2} + \left[(D\tau)^{-1} + s(bD)^{-1}\right]^{1/2}\right\}^{-1} . \tag{243}$$

When sb/D is small and when $(\omega/D) \ll K_{K'o}$, the excess noise spectrum becomes

$$w(|I^2|)_E = (8/3)\, q^2 A p^{(0)}_b D K_{K'o} . \tag{244}$$

For small sb/D the dc current, Eq. (242), becomes

$$I_{dc} = p^{(0)}_b A D q K_{K'o} . \tag{245}$$

Thus the relation between the excess current spectrum and the dc current is

$$w(|I^2|)_E = (8/3)\, q\, I_{dc} . \tag{246}$$

The contribution from the electron density fluctuations in p-type material is similar to the three dimensional case, Section 6.8.

8.4 Comparison with Experiment

Experiments designed to study surface phenomena are generally performed with thin slices of germanium or silicon and fit the two dimensional geometry quite accurately. Experimental methods are available to vary the surface recombination velocity. Certain gaseous ambients, such as water vapor, have been found to increase the surface recombination velocity greatly.[6] Our results can be used to predict the effect of such surface treatment on the minority carrier noise of a p-n junction. For the case of arbitrary surface recombination velocity, the Nyquist contribution to the noise can be calculated from equation (234). An accurate approximation to the excess noise contribution can be calculated from Eqs. (243) and (229) for the limiting cases of small and large values of s, respectively. For large surface recombination velocity a triple infinite sum is involved and the sum appears to converge rapidly. For the case of small surface recombination velocity the single term given in Eq. (243) should be a good approximation. The two dimensional solution should be useful for comparing our theoretical results with experiment.

CHAPTER IX

SUMMARY AND CONCLUSIONS

9.1 Summary of Procedure

The noise in a semiconductor having a p-n junction results from the fluctuations of the minority carrier density. In the p-n junction there are two different types of minority carriers: holes in n-type material and electrons in p-type material. The two types of minority carriers behave similarly and their current spectra are independent so we discuss only the spectrum of the hole density fluctuations.

The solution of the noise current spectrum in a p-n junction is complicated if only the Kolmogorov-Fokker-Planck (KFP) equations are considered. However, with the modified Langevin approach due to Petritz[3] the problem is tractable. This method uses the KFP equations locally to derive the noise sources and then uses the generalized Langevin equation to transfer this noise to the p-n junction terminal. In order to apply this method, the inhomogeneous semiconductor equations are determined using the transmission line-semiconductor analogy. From these inhomogeneous equations the scalar differential equation for the hole density and its associated scalar Green's function are found. The bulk recombination current spectrum can now be solved since the recombination process has only a scalar source.

The diffusion noise source has the characteristics of a vector source and the scalar Green's function is not directly applicable. In solving the two first order semiconductor equations for the hole density, the diffusion source is changed and cannot be determined from stochastic process theory. We use two methods to remove this difficulty. 1. The vector current density equation and its associated tensor Green's function are investigated. 2. The scalar hole density equation is transformed so that the diffusion sources can be calculated from stochastic process theory.

To carry out the first approach, an explicit expression for the tensor Green's function is required. A complete set of orthogonal vector eigenfunctions is necessary, but is not found for an arbitrary surface recombination velocity on the transverse surfaces. The desired set is found for the special case of infinite surface recombination velocity on the transverse surfaces. The tensor Green's function is determined explicitly and the diffusion current spectrum is derived. Adding the recombination and diffusion noise spectra, the total current spectrum for infinite surface recombination velocity is determined. This spectrum consists of two parts: the thermal equilibrium Nyquist noise and the excess noise. Comparing the Nyquist noise spectrum with that derived with the Nyquist law, we find that both spectra are the same and are infinite. The excess noise spectrum appears to be finite and is an upper bound for the excess noise spectrum for semiconductors with large surface recombination velocity on the transverse surfaces.

Then to investigate the second method of deriving the diffusion noise spectrum, the scalar hole density equation is transformed by means of the vector integration by parts and the diffusion source takes the proper form. A gradient of the scalar Green's function appears which gives the Green's function a tensor characteristic. The diffusion noise spectrum is

found for arbitrary surface recombination velocity on the transverse surfaces. Adding the recombination and diffusion noise spectra, the total noise spectrum is derived. The Nyquist part of this spectrum is compared with the Nyquist noise derived with the Nyquist law. Agreement is found only for small surface recombination velocity. The excess noise spectrum appears to be finite.

9.2 Discussion of the Noise Current Spectra

The noise current spectrum depends on the area transverse to the infinite direction; it depends on the characteristics of the semiconductor through the diffusion constants, the time constants and the surface recombination velocity; it depends on the geometric shape of the semiconductor through the spacial harmonics. The spectrum is flat at low frequencies. Only for small surface recombination velocity and low frequencies is there a simple proportionality between the excess spectrum and the dc longitudinal current at the p-n junction.

If the noise spectrum is integrated over the whole frequency range, an infinite noise current results. This is equivalent to the infinite energy from a black body which was eliminated by Planck's quantum hypothesis. This shows that our equations do not pertain to very high frequencies where quantum conditions become important.

It should be possible to compare our results with experiment for the limiting cases of large and small values of surface recombination velocity. For three dimensions Eq. (217) can be used to calculate the Nyquist noise for large s, and Eq. (181) the excess noise. For two dimensions the corresponding equations are (234) and (229). For small s Eqs. (218) and (222) and Eqs. (239) and (243) are the Nyquist and excess noise contributions for three and two dimensions respectively.

9.3 Conclusions

In this thesis we have contributed to the development of the solution of a class of noise problems. The method due to Petritz[3,4] has been generalized; using Langevin's deterministic equations, noise sources from stochastic process theory, and scalar and tensor Green's functions we have solved the p-n junction noise problem. This method can be applied to other three dimensional noise problems. Furthermore, the Green's functions derived can be used to solve deterministic semiconductor problems.

It would be desirable in the future to extend the solution obtained with the tensor Green's function to the case of arbitrary surface recombination velocity. A set of orthogonal vector eigenfunctions is required for this. Since we have a complete set of independent, non-degenerate eigenfunctions, it may be possible to construct a set of orthogonal vector eigenfunctions by means of Schmidt's orthogonalization process.[18] With a set of orthogonal vector eigenfunctions it may be possible to construct a tensor Green's function and carry out the solution of the problem.

Such a solution, in addition to being of interest in semiconductor noise theory, would also shed some light on the disagreement between our stochastic result obtained by scalar Green's function and the result obtained by Nyquist's law.

APPENDIX A

DISCUSSION OF GREEN'S FUNCTIONS

A.1 Forward and Backward Equations

The semiconductor equations as functions of time are affected by the direction of time. When time increases, these equations are called the forward equations; Eqs. (13) and (14) with variables separated are

$$\nabla^2 p_t(r,t) - \frac{1}{D}\frac{\partial p_t(r,t)}{\partial t} - \frac{p_t(r,t)}{D\tau} = -\nabla^2 p_{to}(r,t) + \frac{1}{Dq}\vec{\nabla}\cdot\vec{j}_o(r,t) \quad , \qquad (A1)$$

$$\vec{\nabla}\vec{\nabla}\cdot\vec{j}(r,t) - \frac{1}{D}\frac{\partial \vec{j}(r,t)}{\partial t} - \frac{\vec{j}(r,t)}{D\tau} = -\vec{\nabla}\vec{\nabla}\cdot\vec{j}_o(r,t) + q\frac{\partial \vec{\nabla}}{\partial t}p_{to}(r,t) + \frac{q}{\tau}\vec{\nabla}p_{to}(r,t) . \qquad (A2)$$

When time reverses, the semiconductor equations are called the backward equations and are written as

$$\nabla^2 p_t(r,-t) + \frac{1}{D}\frac{\partial p_t(r,-t)}{\partial t} - \frac{p_t(r,-t)}{D\tau} = -\nabla^2 p_{to}(r,-t) + \frac{1}{Dq}\vec{\nabla}\cdot\vec{j}_o(r,-t) \quad , \qquad (A3)$$

$$\vec{\nabla}\vec{\nabla}\cdot\vec{j}(r,-t) + \frac{1}{D}\frac{\partial \vec{j}}{\partial t}(r,-t) - \frac{\vec{j}(r,-t)}{D\tau} = -\vec{\nabla}\vec{\nabla}\cdot j_o(r,-t) - q\frac{\partial \vec{\nabla}}{\partial t}p_{to}(r,-t) + \frac{q}{\tau}\vec{\nabla}p_{to}(r,-t). \qquad (A4)$$

A.2 Equations for the Green's Functions and Causality

Each inhomogeneous semiconductor equation, (A1) to (A4), defines an equation which must be satisfied by its associated Green's function. The scalar forward equation (A1) defines the equation for the scalar forward Green's function:

$$\nabla^2 G(r,t|r_o,t_o) - \frac{1}{D}\frac{\partial G}{\partial t}(r,t|r_o,t_o) - \frac{G(r,t|r_o,t_o)}{D\tau} = -\delta(r-r_o)\delta(t-t_o) \quad . \quad (A5)$$

The scalar backward equation (A3) defines the equation for the scalar backward Green's function:

$$\nabla^2 G(r,-t|r_o,-t_o) + \frac{1}{D}\frac{\partial G}{\partial t}(r,-t|r_o,-t_o) - \frac{G(r,-t|r_o,-t_o)}{D\tau} = -\delta(r-r_o)\,\delta(t-t_o). \quad (A6)$$

The vector forward equation (A2) defines the equation for the tensor forward Green's function:

$$\vec{\nabla}\vec{\nabla}\cdot\Gamma(r,t|r_o,t_o) - \frac{1}{D}\frac{\partial \Gamma}{\partial t}(r,t|r_o,t_o) - \frac{\Gamma(r,t|r_o,t_o)}{D\tau} = -\mathcal{I}\,\delta(r-r_o)\,\delta(t-t_o). \quad (A7)$$

The vector backward equation (A4) defines the equation for the vector backward Green's function:

$$\vec{\nabla}\vec{\nabla}\cdot\Gamma(r,-t|r_o,-t_o) + \frac{1}{D}\frac{\partial \Gamma}{\partial t}(r,-t|r_o,-t_o) - \frac{\Gamma(r,-t|r_o,-t_o)}{D\tau} = -\mathcal{I}\,\delta(r-r_o)\,\delta(t-t_o). \quad (A8)$$

Here

$$\delta(r-r_o) = \delta(x-x_o)\;\delta(y-y_o)\;\delta(z-z_o) \quad (A9)$$

and $\delta(t-t_o)$ are the Dirac delta functions.[14] The symbol $\mathcal{I}$ is the idemfactor[14] and in dyadic notation is

$$\mathcal{I} = \vec{i}_x\vec{i}_x + \vec{i}_y\vec{i}_y + \vec{i}_z\vec{i}_z \quad , \quad (A10)$$

where $\vec{i}_u$ is the unit vector in the u-direction.

A Green's function is interpreted as the response at the point r and time t to a unit impulse source placed at r_o and t_o. Therefore the forward Green's functions satisfy the causality condition:

$$\Psi(r,t|r_o,t_o) = 0 \quad , \quad \text{if} \quad t<t_o \, , \quad (A11)$$

while the backward or adjoint Green's functions satisfy the different causality condition:

$$\Psi(r,-t|r_o,-t_o) = 0 \quad , \quad \text{if} \quad t>t_o \, , \quad (A12)$$

Ψ denotes either the scalar or tensor Green's functions.

A.3 The Frequency Domain

When the semiconductor equations and their associated Green's functions are transformed to the frequency domain, the forward and backward equations become the same. Using the technique of Section 3.2, the scalar equations (A1) and (A3) become Eqs. (34) and (35), while the vector equations become Eqs. (36) and (37). Equations (A5) and (A6), which the scalar Green's functions satisfy, transform to

$$\nabla^2 G - K^2 G = -\delta(r-r_o) \quad , \quad (A13)$$

while Eqns. (A7) and (A8) which the tensor Green's functions satisfy transform to

$$\vec{\nabla}\vec{\nabla}\cdot\Gamma - K^2\Gamma = -\mathcal{I}\,\delta(r-r_0) \,. \qquad \text{(A14)}$$

The physical concept involved in the frequency domain is that the sources vary with a steady-state sinusoisal frequency. A steady-state source is isotropic in time and the forward and backward equations are the same. In the frequency domain for a linear medium the causality condition for the forward Green's functions is

$$-\infty \le \psi(r, t_0 | r_0, t_0) \le \infty \quad , \quad -\infty \le t \le \infty \,, \qquad \text{(A15)}$$

and likewise for the backward Green's functions

$$-\infty \le \psi(r, t_0 | r_0, t_0) \le \infty \quad , \quad -\infty \le t \le \infty \,. \qquad \text{(A16)}$$

A.4 Reciprocity Relations

Green's functions satisfy a reciprocal relation.[14] The scalar Green's function for a single frequency satisfies the relation

$$G(r_1|r_0) = G(r_0|r_1) \,. \qquad \text{(A17)}$$

This equation states that with a given harmonic excitation, interchanging the source and the observation point does not affect the behavior of the system.

The tensor Green's function satisfies a reciprocity relation which is now derived (see Morse and Feshbach,[14] pp. 877-883). Equation (A14) can be written as the operator equation

$$\mathcal{Q}_D\Gamma = -\mathcal{I}\,\delta(r-r_0) \qquad \text{(A18)}$$

where the differential operator $\mathcal{Q}_D$ is defined as

$$\mathcal{Q}_D \equiv \vec{\nabla}\vec{\nabla}\cdot - K^2 \,. \qquad \text{(A19)}$$

This differential operator can be represented as the continuous matrix $\mathcal{Q}(r|r_1)$. With this matrix, Eq. (A18) becomes

$$\mathcal{Q}(r|r_1)\cdot\Gamma(r_1|r_0) = -\mathcal{I}\,\delta(r-r_0) \,. \qquad \text{(A20)}$$

Interchanging rows and columns and reversing the positions of the factors of the product gives the adjoint of Eq. (A20):

$$\tilde{\Gamma}(r_1|r_0)\cdot\tilde{\mathcal{Q}}(r|r_1) = -\mathcal{I}\,\delta(r-r_0) \,. \qquad \text{(A21)}$$

The tensor Green's function for the adjoint operator $\tilde{\mathcal{Q}}$ is the adjoint of the tensor Green's function for $\mathcal{Q}$:

$$\tilde{\Gamma}(r|r_0) = \Gamma(r_0|r) \,. \qquad \text{(A22)}$$

This is the reciprocity condition which the tensor Green's function must obey.

APPENDIX B

EXCESS CHARGE DENSITY AND p-n JUNCTION INPUT ADMITTANCE

B.1 Definition of Terms

We use the scalar Green's function, Chapter IV, to solve two problems: the excess hole density resulting from a dc voltage applied to the plane x=0; and the input admittance of the p-n junction to an alternating voltage applied at x=0.

In the excess hole density problem we investigate the hole density at the x=0 plane. When no voltage is applied at the x=0 plane, the thermal equilibrium hole density p_n exists throughout the semiconductor. After applying the voltage V(0) at the x=0 plane, the hole density at x=0 increases to $p_t(0)$:

$$p_t(0) = p_n + p(0) = p_n + p_n[\exp(qV(0)/kT) - 1] \tag{B1}$$

where q is the electronic charge, k is Boltzmann's constant, and T is the absolute temperature. The symbol p denotes the excess hole density.

Putting V(0) equal to the sum of a large dc component V_o and a very small ac component v_1,

$$V(0) = V_o + v_1 , \tag{B2}$$

the exponential in equation (B1) is expanded in a power series in v_1. Keeping the first two terms, the hole density is

$$p_t(0) = p_n + p_b(0) + p_1(0)$$
$$= p_n + p_n[\exp(qV_o/kT) - 1] + p_n v_1 q(kT)^{-1}[\exp(qV_o/kT)]. \tag{B3}$$

The excess hole density is made up of two parts: $p_b(0)$, the excess hole density from the dc voltage, V_o; and $p_1(0)$, the excess hole density from the ac voltage v_1. Therefore,

$$p(0) = p_b(0) + p_1(0) , \tag{B4}$$

$$p_b(0) = p_{n'}[\exp(qV_o/kT) - 1] , \tag{B5}$$

$$p_1(0) = p_{n'} qv_1/kT , \tag{B6}$$

where $$p_{n'} = p_n[\exp(qV_o/kT)]. \tag{B7}$$

The average value of the total hole density at the x=0 plane, $\langle p_t(0)\rangle$, is found by taking the ensemble average of $p_t(0)$. Since the average value of the alternating component $p_1(0)$ is zero, we get

$$\langle p_t(0)\rangle = p_n + p_b(0) \tag{B8}$$

and define $$\langle p_t(0)\rangle_N = p_n \tag{B9}$$

and $$\langle p_t(0)\rangle_E = p_b(0) , \tag{B10}$$

where the subscripts N and E denote the thermal equilibrium and the excess part of the average hole density, respectively.

When a voltage is applied at the x=0 plane, an excess hole density also appears throughout the whole semiconductor and Eqs. (B8) to (B10) become

$$\langle p_t(r)\rangle = p_n + p_b(r) \tag{B11}$$

$$\langle p_t(r)\rangle_N = p_n \tag{B12}$$

$$\langle p_t(r)\rangle_E = p_b(r). \tag{B13}$$

These equations refer to finite surface recombination velocity s on the transverse surfaces. For the case of infinite s we put ∞ superscripts on the variables.

B.2 Excess Hole Density and Input Admittance

Let us evaluate the excess hole density in the semiconductor with the p-n junction. Assuming that all sources are zero except on the x_o=0 plane, Eq. (82) reduces to

$$p(r) = \int_{-b}^{b}\int_{-c}^{c} p(r_o)\,(\partial G/\partial x_o)\,dy_o\,dz_o \qquad \text{(B14)}$$

$$p^{\infty}(r) = \int_{o}^{B}\int_{o}^{C} p^{\infty}(r_o)\,(\partial G^{\infty}/\partial x_o)\,dy_o\,dz_o\,, \qquad \text{(B15)}$$

where no superscript and the ∞ superscript stand for finite and infinite s cases respectively.

For the finite s case $(\partial G/\partial x_o)$ is calculated from Eq. (109). When x_o goes to zero, we get

$$\left.\frac{\partial G}{\partial x_o}\right|_{x_o\to o} = \sum_{M,N} F_{MN}(r_o)\,F_{MN}(r)\,L_{MN}^{-1}\,\exp[-x\,K_{KMN}]. \qquad \text{(B16)}$$

Since $p(r_o)$ is constant over the x_o=0 plane, and Eq. (B5) defines $p_b(0)$, the excess dc hole density is

$$p_b(r) = A\,p_b(0)\sum_{m,n}\gamma_{mn}(\cos\beta_m y)(\cos\beta_n z)\exp(-x\,K_{K'mn}), \qquad \text{(B17)}$$

where
$$K_{K'mn}^2 = (1/D\tau) + \beta_m^2 + \beta_n^2 \qquad \text{(B18)}$$

$$\gamma_{mn} = (\sin\beta_m b)(\sin\beta_n c)/(\beta_m b\,\beta_n c\,L_{mn}) \qquad \text{(B19)}$$

$$A = 4bc\,, \qquad \text{(B20)}$$

and L_{mn} is given by Eq. (111). This value of the

excess hole density agrees with that given by Shockley.[1]

The infinite s case is similar to the finite s case, and we get

$$p_b^{\infty}(r) = 16\,\pi^{-2}\,p_b(0)\sum_{\substack{m,n\\ \text{odd}}}(mn)^{-1}(\sin\pi m B^{-1}y)(\sin\pi n C^{-1}z)\exp(-K_{K'mn}^{\infty}x) \qquad \text{(B21)}$$

$$K_{K'mn}^{\infty 2} = (D\tau)^{-1} + (\pi m B^{-1})^2 + (\pi n C^{-1})^2. \qquad \text{(B22)}$$

The input admittance to the p-n junction is calculated by finding the total current in the x direction at the x=0 plane and dividing by the ac voltage v_1 applied at this plane. The current density results from the homogeneous form of Eq. (14) and from Eqs. (B14) and (B15). To get the total current we integrate over the observation coordinates at the x=0 plane. With a constant source function $p_1(0)$, Eq. (B6), at the x=0 plane, the current in the x direction is

$$I(r)\Big|_{\substack{x_o=0\\ x=0}} = -q\,p_{b'}\,v_1\,\mu\sum_{MN}\int_{-b}^{b}\int_{-c}^{c}\int_{-b}^{b}\int_{-c}^{c}\left.(\partial^2 G/\partial x\,\partial x_o)\right|_{\substack{y_o=0\\ y=0}} dy\,dz\,dy_o\,dz_o \qquad \text{(B23)}$$

$$I^{\infty}(r)\Big|_{\substack{x_o=0\\ x=0}} = -q\,p_{b'}\,v_1\,\mu\sum_{mn}\int_{o}^{B}\int_{o}^{C}\int_{o}^{B}\int_{o}^{C}\left.(\partial^2 G^{\infty}/\partial x\,\partial x_o)\right|_{\substack{y_o=0\\ x=0}} dy\,dz\,dy_o\,dz_o \qquad \text{(B24)}$$

where μ is the mobility

$$\mu = D_q/kT\,, \qquad \text{(B25)}$$

and the symbols p_b', v_1, G, and G^{∞} are given by Eqs.

(B7), (B2), (109) and (114) respectively; the symbols q, k, and T are defined after Eq. (B1); and D is defined after Eq. (12).

For finite s the expression for $\partial^2 G/\partial x \partial x_0 \big|_{\substack{x_0=0 \\ x=0}}$ is

$$\frac{\partial^2 G}{\partial x \partial x_0}\bigg|_{\substack{x_0=0 \\ x=0}} = -\sum_{MN} F_{MN}(r_0) F_{MN}(r)\, K_{KMN}\, L^{-1}_{MN}. \tag{B26}$$

Putting it into Eq. (B23) and defining the input admittance at the x=0 plane as

$$Y\big|_{x=0} = I v_1^{-1}\big|_{x=0} \tag{B27}$$

we get

$$Y\big|_{x=0} = A^2 \mu \varepsilon \beta_b \sum_{m,n} \gamma_{mn}^2\, K_{Kmn}\, L_{mn}, \tag{B28}$$

where **A**, γ_{mn}, L_{mn}, K_{Kmn}, μ, and p_b are defined by Eqs. (B20), (B19), (111), (112), (B25) and (B7) respectively.

To separate Y into its real and imaginary parts, K_{Kmn} is so separated. Let

$$K_{Kmn} = (H_r + i H_i)^{1/2} \tag{B29}$$

where H is any complex quantity and the subscripts r and i stand for real and imaginary parts respectively; then

$$K_{Kmn} = 2^{-1/2}\left\{\left[\left(H_r^2 + H_i^2\right)^{1/2} + H_r\right]^{1/2} + i\left[\left(H_r^2 + H_i^2\right)^{1/2} - H_r\right]^{1/2}\right\}. \tag{B30}$$

Using Eq. (B30), Eq. (B28) becomes

$$Y\big|_{x=0} = 2^{-1/2} A^2 \mu \varepsilon \beta_b \sum_{m,n} \gamma_{mn}^2 L_{mn} \left\{\left[\left(K_{K'mn}^4 + \omega^2 D^{-2}\right)^{1/2} + K_{K'mn}^2\right]^{1/2} + i\left[\left(K_{K'mn}^4 + \omega^2 D^{-2}\right)^{1/2} - K_{K'mn}^2\right]^{1/2}\right\}, \tag{B31}$$

where $K_{K'mn}$ is defined by Eq. (B18) and the other symbols are defined after Eq. (B28). This is the expression for admittance which was derived by Shockley.

In the infinite s case the input admittance is

$$Y^{\infty}\big|_{x=0} = 64\pi^{-4} A^{\infty} \mu \varepsilon \beta_b \sum_{\substack{m,n \\ \text{odd}}} K^{\infty}_{Kmn} / m^2 n^2. \tag{B32}$$

With the real and imaginary parts shown explicitly, Eq. (B32) becomes

$$Y^{\infty}\big|_{x=0} = 64\pi^{-4} 2^{-1/2} \mu \varepsilon A^{\infty} \beta_b \sum_{\substack{m,n \\ \text{odd}}} (mn)^{-2} \left\{\left[\left(K^{\infty 4}_{K'mn} + \omega^2 D^{-2}\right)^{1/2} + K^{\infty 2}_{K'mn}\right]^{1/2} + i\left[\left(K^{\infty 4}_{K'mn} + \omega^2 D^{-2}\right)^{1/2} - K^{\infty 2}_{K'mn}\right]^{1/2}\right\}, \tag{B33}$$

where μ, K^{∞}_{Kmn}, and $K^{\infty}_{K'mn}$ are given by Eqs. (B25), (117) and (B22) respectively, and

$$A^{\infty} = 8c. \tag{B34}$$

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