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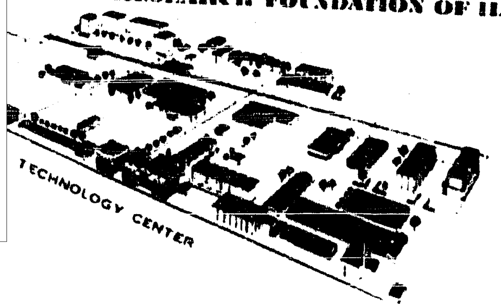

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RESEARCH AND DEVELOPMENT OF NEW DESIGN METHOD FOR POWER TRANSFORMERS

FINAL REPORT

Period covered: May 1, 1953 to Aug. 30, 1955

for

Electronic Parts and Materials Branch
Signal Corps Engineering Laboratories
Fort Monmouth, New Jersey

and

Electronic Components Laboratory
Wright Air Development Cent[redacted]

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ARMOUR RESEARCH FOUNDATION
of
Illinois Institute of Technology
Technology Center
Chicago 16, Illinois

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RESEARCH AND DEVELOPMENT OF NEW DESIGN METHOD FOR POWER TRANSFORMERS

Final Report

Period Covered: May 1, 1953 to Aug. 30, 1955

Object: To conduct research and development leading to a design method for power transformers, and to fabricate models for component testing in accordance with Squier Signal Laboratory Technical [redacted] of October 6, 1952.

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ARMOUR RESEARCH FOUNDATION
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RESEARCH AND DEVELOPMENT OF NEW DESIGN METHOD FOR POWER TRANSFORMERS

PURPOSE

The purpose of this investigation is the development of a new and improved method for the design of certain types of electronic power transformers. The method should yield an optimum design without the need for repetitive trial procedures, and should be readily understandable to an engineer not normally associated with the transformer industry.

ABSTRACT

The new design method for electric power transformers which was developed under Contract No. DA-36-039 SC-5519 has been extended and modified to make it suitable for the more special types of power transformers. It has been intended that the design methods for these transformers would be used by electrical engineers who are not normally associated with the transformer industry. Satisfactory designs can be obtained with little or no repetitive trial procedures. The following types of transformers have been investigated during the current contract:

1. Transformers with unbalanced magnetization.
2. Current-limiting or high-reactance transformers.
3. ~~Current-limiting transformers with unbalanced magnetization.~~
4. Vibrator-supply transformers.
5. Low-capacitance transformers.
6. Instrument transformers.

It is assumed that the transformer designer is given information on power rating, voltages, currents, frequencies, ambient temperature,

maximum temperature rise, and other requirements and limiting factors pertaining to the circuit in which the transformer must operate. The design procedures account for operating temperatures to 200°C, ambient temperatures to 200°C, absolute pressures between 30 and 1.32 inches of mercury, power ratings to 5 kilovolt amperes, RMS voltages up to 50 kilovolts, and frequencies from 25 to 2500 cycles per second.

Design methods have been developed for each of the above types by study of the theoretical principles of operation and by compilation of empirical data from developmental models. This approach has yielded empirical parameters which have been incorporated into design equations. Elimination of trial procedures has required that ultimate limitations of a given design be used in the initial design equations. The most universal design limitation is the operating temperature. Therefore parameters which are functions of losses and size are particularly important.

In order to provide supplements to well-known transformer theory and data, a study has been made to determine the approximate distribution of current densities which minimize losses and temperature rise. Another study has yielded optimum stacking ratios for given types of laminations. It was found necessary to obtain and compile new data on magnetic materials as a basis for the design of transformers with unbalanced magnitization. The development of design methods for current-limiting and low-capacitance transformers has required an investigation of transformer leakage flux and leakage reactance in order to determine how these quantities could be accounted for in the design. In addition to the development of theoretical relationships, there are presented detailed design procedures and examples.

ACKNOWLEDGMENTS

During the investigation valuable suggestions and guidance have been received from M. Irving Remis, project engineer for the Electronic Parts and Materials Branch, Signal Corps Engineering Laboratories, and from Mr. Gene Tarrants and Lt. Carl K. Greene, project engineers for the Electronic Components Laboratory, Wright Air Development Center. In addition, acknowledgment is due to the government representatives and industrial consultants of the Interservice Program and Guidance Group on Audio, Power, and Pulse Transformers who have offered many helpful suggestions.

The participation of the Gramer-Halldorson Transformer Corporation, Chicago, Illinois, as subcontractor has been very valuable, especially in the constructing and testing of experimental transformer models and in aid with transformer design problems. Contributions by Mr. Forrest E. Zimmerman, Design Engineer, and Mr. Fred R. Cooper, Vice President for Engineering. are gratefully acknowledged.

RESEARCH AND DEVELOPMENT OF NEW DESIGN METHOD FOR POWER TRANSFORMERS

I. INTRODUCTION

This is the final report on a research program conducted during the period May 1, 1953 to August 31, 1955. This study has been a continuation of the investigation carried out under Contract No. DA-36-039 SC-5519. The major objective of the study has been to develop design procedures for certain types of transformers which have special requirements. In addition to being characterized by special requirements, these special transformer types are used in relatively small quantities and comprise a small percentage of the total electronic power transformer production. Limited utilization and special design problems have resulted in the widespread use of repetitive trial design, model construction, and model test procedures for obtaining satisfactory designs. To eliminate or reduce repetition of design calculations, and to place the design problems on a more orderly basis, efforts have been directed toward the compilation of theoretical and experimental data, and application of the principles of the design method developed under Contract No. DA-36-039 SC-5519 to these transformer types. The special types of transformers studied may be grouped according to the design problems involved as follows:

(1) Transformers with unbalanced magnetization for use with rectifier supplies or combined rectifier and filament supplies,

(2) Current-limiting transformers for either rectifier or filament supplies,

(3) Current-limiting transformers with unbalanced magnetization for rectifier supplies,

(4) Vibrator-supply transformers,

(5) Low-capacitance filament transformers,

(6) Instrument transformers.

The ranges of electrical characteristics and operating conditions which have been given major consideration are:

1). Power output up to 5 kilovolt-amperes,

2) Operating voltages to 50 kilovolts,

3) Frequencies from 25 to 2500 cycles per second,

4) Pressures as low as 1.32 inches of mercury, corresponding to an altitude of about 70,000 feet.

5) Operating temperatures to 200 degrees C, and ambient temperatures from -55 to 200 degrees C.

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However, the design methods presented here may be found to be applicable outside of these ranges, such as for temperatures above 200°C. For related information dealing with the materials for and construction of miniaturized power transformers and inductors capable of satisfactory operation at ambient temperatures of 200°C and operating temperatures in the order of 325 to 350°C, attention is directed to the reports from Contract No. AF-33(600)-24120, "Miniature Power Transformers Having A Wide Temperature Range", Bell Telephone Laboratories.

This report has been divided into two parts. The material in the first part, which comprises the first ten chapters, gives the essentials of the basic design procedure as developed on Contract No. DA-36-039 SC-5519, and presents the theoretical considerations, experimental work, and derivation of the design procedure for each of the special transformer types which have been studied during this contract. Also included in this first part is a continuation of the study of optimum transformer proportions as they are affected by changes in the stack ratio of cores assembled from scrapless laminations. In addition, one chapter has been devoted to a consideration of winding current densities and how they influence transformer losses and heating.

In the second part are a step-by-step design procedure and an example design for each of the special transformer types considered. A summary of the basic design procedure and method for calculating temperature rise is also presented, together with the design procedures which were derived during the previous contract for ordinary filament transformers, autotransformers, and rectifier-supply transformers. The derivations of the design procedures and additional information about the last three transformers is contained in the final report of the previous contract. However, the design procedures for them have been repeated in this report in order to make it as comprehensive as possible. To provide correlation between the two final reports, the corresponding equation and figure numbers used in the final report of the previous contract and in this report are given in Appendix F.

II. ESSENTIALS OF DESIGN PROCEDURE WITH REVISIONS

The design procedure developed under Contract No. DA-36-039 SC-5519 is briefly discussed in this chapter. This material is also given here so that subsequent changes which have been made may be introduced. A step-by-step summary of the design method, as applied to filament transformers, is given in Chapter XI.

Basis of Design Procedure

For a sinusoidal variation in flux, the RMS voltage of any winding is

$$V = \sqrt{2}\,\pi\, f\, F_1\, B\, A_1\, N \times 10^{-5} \text{ volts,} \qquad (2\text{-}1)$$

where B = maximum flux density in kilolines per square inch,

f = frequency in cycles per second,

A_1 = gross cross-sectional area in square inches,

F_1 = core space factor, the fraction of core cross section occupied by magnetic material,

N = turns comprising winding.

The RMS current is

$$I = \frac{NI}{N} \text{ amperes,} \qquad (2\text{-}2)$$

and assuming that the winding being considered occupies half of the available window area and that current density is uniform in all windings, a substitution may be made for RMS ampere turns, NI, to give

$$I = \frac{F_c\, \Delta\, A_c}{2\, N}\, 10^3 \text{ amperes,} \qquad (2\text{-}3)$$

where F_c = winding space factor, the fraction of total core window area occupied by conductor cross section,

Δ = current density in the conductors in kiloamperes per square inch,

A_c = area of the core window in square inches.

Multiplying (2-1) and (2-3) gives an expression for rating

$$W_r = VI = \frac{\pi}{\sqrt{2}\,100}\, f\, F_c\, F_1\, B\, \Delta\, A_c\, A_1 \text{ volt amperes.} \qquad (2\text{-}4)$$

Combination of the constants gives

$$W_r = \frac{1}{45.0}\, f\, F_c\, F_1\, B\, \Delta\, A_c\, A_1 \text{ volt-amperes,} \qquad (2\text{-}5)$$

which is the general transformer equation. It may be of some aid in recognizing symbols used if it is noted that quantities with the subscript "i" refer to the iron core and those with subscript "c" refer to the winding, which is usually made of copper conductor.

Equation (2-5) may be transformed into another, which relates rating to temperature rise. This measure of temperature rise is taken as winding power dissipation per unit area of exposed winding surface, in watts per square inch. This quantity has been chosen because the transformer winding is the part most vulnerable to excessive temperature. The procedure requires the prediction of transformer temperature when a design is being begun, and the use of a simple, reasonable relation between temperature, transformer geometry and losses is the key to reduction of cut and try procedures in design. A study of the design algebra has shown that an error in the choice of allowable watts per square inch has a reduced effect on errors in core and wire sizes, a result supporting the validity of this approach.

In the design equations, transformer proportions are represented by dimensionless constants which are independent of size. Important dimensions, lengths, areas, and volumes are found by multiplying these dimensionless constants, or ratios by a function of the characteristic linear dimension, which is a measure of size.

Characteristic linear dimension:

$$\ell = \sqrt[4]{A_c A_i} \,. \qquad (2\text{-}6)$$

Mean length of magnetic circuit:

$$m_i = a\,\ell \text{ inches.} \qquad (2\text{-}7)$$

Mean length of turn of the winding:

$$m_c = b\,\ell \text{ inches} \qquad (2\text{-}8)$$

Gross cross-sectional area of core:

$$A_i = c\,\ell^2 \text{ square inches.} \qquad (2\text{-}9)$$

Area of window:

$$A_c = d\,\ell^2 \text{ square inches.} \qquad (2\text{-}10)$$

Exposed surface area of the winding (sum of all winding surfaces except those in contact with the core):

$$S_c = e\,\ell^2 \text{ square inches.} \qquad (2\text{-}11)$$

Exposed core surface area:

$$S_i = g\, \ell^2 \text{ square inches.} \qquad (2\text{-}12)$$

A general design equation is then developed as follows:

Winding losses, assuming that current density is about the same in all windings (the validity of this assumption is discussed in Chapter III):

$$W_c = \Delta^2 \rho \text{ times (conductor volume) watts,} \qquad (2\text{-}13)$$

where Δ = current density in kiloamperes per square inch,

ρ = material resistivity in microhm-inches.

Conductor volume = $m_c\, A_c\, F_c$ cubic inches,

where F_c is winding space factor.

Winding losses then become:

$$W_c = \Delta^2 \rho\, m_c\, A_c\, F_c \text{ watts.} \qquad (2\text{-}14)$$

The dissipation per unit exposed winding surface is

$$\frac{W_c}{S_c} = \frac{\Delta^2 \rho\, m_c\, A_c\, F_c}{S_c} \text{ watts per square inch.} \qquad (2\text{-}15)$$

Solving for current density:

$$\Delta = \sqrt{\frac{S_c}{\rho\, m_c\, A_c\, F_c} \cdot \frac{W_c}{S_c}} \text{ kiloamperes per square inch.} \qquad (2\text{-}16)$$

Substituting for Δ from (2-16) in equation (2-5) yields

$$W_r = \frac{1}{45.0} f\, F_c\, F_i\, B\, A_c\, A_i \sqrt{\frac{S_c}{\rho\, m_c\, A_c\, F_c} \cdot \frac{W_c}{S_c}} \text{ volt-amperes} \qquad (2\text{-}17)$$

Finally, substituting for A_c A_i from (2-6), m_c from (2-8), A_c from (2-10), and for S_c in the numerator from (2-11),

$$W_r = \frac{1}{45.0} f\, F_i\, B\, \ell^4 \sqrt{\frac{e\, \ell^2\, F_c}{\rho\, b\, \ell\, d\, \ell^2} \cdot \frac{W_c}{S_c}}$$

$$W_r = \frac{1}{45.0} f\, F_i\, B\, \ell^{7/2} \sqrt{\frac{e}{bd} \cdot \frac{F_c\, W_c}{\rho\, S_c}} \text{ volt-amperes,} \qquad (2\text{-}18)$$

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which is the design equation sought. The term $\sqrt{bd/e}$ is a combination of dimensionless ratios, and is designed K_0. Equation (2-18) is the basis for a design nomograph to be used for any type of core and proportions. This nomograph is given in Fig. 11-7. Its function is to aid in solving for the characteristic linear dimension, ℓ, which is a measure of physical size required.

The desirable proportions and type of core can be roughly estimated near the start of the design. Values of K_0 and other geometric parameters to be used have been tabulated for several types of cores in Figs. 11-3, 11-4, and 11-5. The constants a, b, c, d, e and g are calculated from proportions, while the constants K_0 through K_6 are functions of a, b, c, d, e and g, combined for ease in calculating design values. The proportions of the core illustrated are only a few of the many that can be used. For other proportions than those given by the figures, the designer can use constants for the core type most similar, or estimate the constants by interpolation.

For proportions which are greatly different from those given, a new set may be calculated. To do this, a core of the desired proportions, but of any size may be taken. First the product of window area and gross core cross section A_c A_i is calculated. Then the characteristic linear dimension, from (2-6),

$$\ell = \sqrt[4]{A_c A_i},$$

is found. Since the areas of the window and core cross section have been used, the parameters c and d can be found immediately from (2-9), $A_i = c\,\ell^2$, and (2-10), $A_c = d\,\ell^2$, respectively.

Mean length of magnetic circuit, m_i, is simply the average length of the core flux path. For this study the length is calculated assuming that the flux makes a right angle turn in the corners of stacked cores having square corners, and that the flux follows a circular path at the corners of wound-type cores. Then the parameter, a, is found from (2-7), $m_i = a\,\ell$.

Mean length of turn is used to find the parameter, b. Since coil corners are always rounded, the mean turn for the simple or shell types of core is the perimeter around the core leg plus pi times the window width; and for the core type, mean turn is the perimeter plus pi times half of the window width. Then b is found from (2-8), $m_c = b\,\ell$.

Exposed surface area of the winding is the sum of all outside surface areas except those facing core surfaces, assuming that the ends are smooth and that the coil exactly fills the window. Side and end areas are included. Then the constant e is found from (2-11), $S_c = e\,\ell^2$. Exposed surface area of the core is found in a manner similar to that for the winding, and the constant, g, is calculated from (2-12), $S_i = g\,\ell^2$.

Design Procedure

The first steps in the design procedure are the study of the specifications and the selection of a core type, core material, grade and thickness of lamination, and type of enclosure. A stack height to lamination width ratio, s, of 1.5 is recommended, since this value gives a core of reasonable proportions for most designs. A discussion of the influence of stack ratio is presented in Chapter IV.

The allowable winding dissipation W_c/S_c, which must be determined at the start of the design in order to apply the nomograph, is calculated from

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per square inch,} \qquad (2\text{-}19)$$

where ΔT = winding temperature rise, °C,

K = parameter from Table 11-1.

The parameter K, is used to relate the factors which have the most important effects on temperature rise. The values of K given in Table 11-1 and equation (2-19) were arrived at after evaluating considerable test data, various designs, and theories of heat transfer. The variables considered are: transformer type, that is, open, compound-filled, and oil-filled; type of core, simple, shell, or core; frequency; and ambient temperature.

Another important quantity which must be estimated at the start of a design is the winding space factor F_c. The factors which have the principal influence on space factor are: physical size of the transformer, number of windings, and operating voltage. These are related by the expression

$$F_c = .08 \log_{10} W_r' + F \qquad (2\text{-}20)$$

where W_r' = equivalent rating based on 60 cycles and 40°C rise,

F = factor from Fig. 11-2.

Figure 11-2, which is a revision of Fig. 40 of Contract DA-36-039 sc-5519, along with a discussion of high voltage designs, is given at the end of the chapter. Since the physical size of a transformer is affected by both frequency and temperature rise, the expression used to relate the equivalent rating W_r', to the actual rating W_r, is

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76}\left(\frac{\Delta T}{40}\right)^{.63}} \text{ volt-amperes} \qquad (2\text{-}21)$$

When estimates for the allowable winding dissipation and winding space factor have been made, nomograph scale values to be calculated are

$$\frac{K_0 W_r}{F_i f} \text{ and } \frac{F_c W_c}{\rho S_c},$$

where K_0 is found from Figs. 11-3, 11-4, or 11-5 for the appropriate core and stacking ratio,

W_r is output volt-amperes,

F_i is core space factor, which is generally specified by the manufacturer,

f is frequency, or if frequency is variable, the low end of the range,

ρ is winding resistivity, usually copper, the value of Fig. 11-6 increased by two per cent or more.

To find the characteristic dimension ℓ from the nomograph, the proper values are calculated for Scales A and F. The line between these points determines a point on Scale C. Next a flux density in kilolines per square inch is chosen using Table 11-2, to determine a point on Scale B. The line between the points on Scales C and B locates a point on Scale D, which determines ℓ in inches.

The next step is finding the core weight, and from this the core losses and excitation. Unless limited otherwise by specifications, it will usually be desirable to keep core losses below 20 per cent of output volt-amperes, and to keep excitation below 80 per cent of output volt-amperes. Core loss, excitation and regulation ranges of typical transformers which have been manufactured might serve as an additional guide in setting limits. These ranges are given in Table 11-4, for two frequencies. Core weight may either be calculated using the material density, or may be given by a lamination or core catalogue.

Core weight equals core volume times material density.

$$M_i = m_i A_i F_i \delta_i \text{ pounds,} \qquad (2\text{-}22)$$

where M_i = core weight in pounds,

m_i = mean length of magnetic circuit in inches,

A_i = cross sectional area of the core in square inches,

F_i = core space factor,

δ_i = density of core material in pounds per cubic inch.

Then for m_i and A_i may be substituted the quantities of (2-7) and (2-9), so that core weight becomes

$$M_i = a\, c\, F_i\, \delta_i\, \ell^3 = K_1\, F_i\, \delta_i\, \ell^3 \text{ pounds.} \qquad (2\text{-}23)$$

Since (a c) depends only on core proportions, this product is tabulated as K_1 in Figs. 11-3 or 11-5.

Now that the weight is known, total core loss, W_i, in watts, and excitation, W_{ex}, in volt-amperes can be calculated using curves giving the characterisitcs of the magnetic material corrected by the appropriate factors from Table 11-3. The curves give core loss and excitation in watts and volt-amperes per pound respectively for ideal conditions of utilization, and the correction factors account for additional loss and excitation due to joint effects, corners, stresses and other factors. Data for a frequency other than the desired value may be used to estimate the core performance, because core loss varies roughly as $f^{1.4}$, and excitation in volt-amperes varies as f, at a fixed density B. Core loss and excitation may be checked to find if one or both exceeds the specified values. If one value is excessive it will be necessary to choose a new flux density, find a new ℓ from the nomograph, and calculate a new core weight from (2-23). If both values are considerably below those specified it is desirable to raise flux density in order to reduce core size. When a value of ℓ appears to be satisfactory, all core dimensions may be found.

Figures 11-3 and 11-5 give the ratio of the width L to linear dimension. Then,

$$L = \left(\frac{L}{\ell}\right) \ell \text{ inches.} \qquad (2\text{-}24)$$

A lamination size is chosen, such that equation (2-6), $A_c A_i = \ell^4$, is satisfied. For stacked cores the stack height is calculated such that this is so. For choosing from completed cores, such as a wound type, a core should be selected such that the area product is approximately equal to the fourth power of the characteristic dimension. It may also be desirable to round off dimensions, and if ℓ is changed somewhat because of this, the new value is to be used in subsequent calculations.

A design value of use in checking core dissipation is the core surface area. In terms of ℓ this is

$$S_i = g\, \ell^2 \text{ or } K_2\, \ell^2 \text{ square inches,} \qquad (2\text{-}25)$$

where S_i is exposed core surface in square inches.

Core dissipation, in loss per unit exposed surface area, can be found as W_i/S_i in watts per square inch. To avoid excessive core temperatures which may be damaging, it seems advisable to keep this value from exceeding the dissipation per unit area of the winding, W_c/S_c.

Coil exposed surface area is

$$S_c = e\, \ell^2 = K_3\, \ell^2. \qquad (2\text{-}26)$$

Copper loss can now be estimated from

$$W_c = \frac{W_c}{S_c}\, S_c \text{ watts.} \qquad (2\text{-}27)$$

Per cent winding loss is then approximately

$$\frac{W_c}{W_r}\, 100 \text{ per cent.} \qquad (2\text{-}28)$$

The weight of conductor required for the design is volume of conductor times density.

$$M_c = m_c\, A_c\, F_c\, \delta_c \text{ pounds} \qquad (2\text{-}29)$$

where m_c is mean length of turn in inches,

δ_c is density of conductor material in pounds per cubic inch, for copper equal .321.

If substitutions are made for m_c and A_c from (2-8) and (2-10),

$$M_c = b\, d\, F_c\, \delta_c\, \ell^3 = K_4\, F_c\, \delta_c\, \ell^3 \text{ pounds.} \qquad (2\text{-}30)$$

The product (b d) has been tabulated as K_4 in Figs. 11-3 and 11-5.

The next design calculations are the wire sizes. An equation for circular mils per ampere is used. From (2-16) the reciprocal of current density in square inches per kiloampere is

$$\frac{1}{\Delta} = \sqrt{\frac{\rho\, F_c\, \ell}{\frac{W_c}{S_c}} \cdot \frac{b\, d}{e}} = F_c \cdot \sqrt{\frac{\ell}{\frac{F_c}{\rho}\frac{W_c}{S_c}} \cdot \frac{b\, d}{e}}$$

Then circular mils per ampere $= \dfrac{4000}{\pi\, \Delta}$.

$$\frac{CM}{amp} = \sqrt{\frac{\frac{\ell}{F_c W_c}}{\rho S_c}} \cdot (K_5 F_c), \qquad (2\text{-}31)$$

where $K_5 = \frac{4000}{\pi} \cdot \sqrt{\frac{b\,d}{a}}$.

The circular mils required for each winding is the result from (2-31) times rated current for the winding. While secondary current is specified, primary current is calculated from

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_c + W_i)^2 + W_{ex}^2 - W_i^2} \text{ amperes} \qquad (2\text{-}32)$$

Wire sizes are to be chosen from Table 11-5, using the closest values available.

From equation (2-1), the turns per volt of a winding is

$$\frac{N}{V} = \frac{10^5}{\sqrt{2}\,\pi f F_i B A_i} = \frac{10^5}{4.44 f F_i B A_i} . \qquad (2\text{-}33)$$

Substituting for A_i from (2-9),

$$\frac{N}{V} = \frac{K_6}{f F_i B \ell^2} \text{ turns per volt,} \qquad (2\text{-}34)$$

where $K_6 = \frac{10^5}{\sqrt{2}\,\pi c}$.

Equation (2-33) is preferable, but equation (2-34) is useful for estimating turns early in the design procedure. The turns for any winding may be found by multiplying (2-33) by the rated voltage of the winding. A correction should be made for regulation so that rated voltage is obtained from the transformer at full load. This could be done by decreasing primary turns, or increasing secondary turns, or both. The amount of the total change in per cent should be approximately equal to the per cent winding loss as determined earlier.

There have been applications of transformers in which part of a secondary winding supplies a separate load. In such cases, W_r should be calculated from the total load in the usual way. Then wire sizes for each part of each winding should be selected according to the RMS current in that part or winding.

The next step is the layout of the coil. Most commonly a layer-insulated winding will be used. Then wire with one coat of enamel or other film insulation will be used for sizes smaller than AWG No. 14, and either one or two coats of insulation above that size, depending on care taken in construction. Tables 11-6 and 11-7 are included to help in laying out the winding. The important check to be made on the coil layout is that the coil adequately fills the window space but is not too tight. Typical per cent build measured as a proportion of the window width is 80 to 90 per cent.

When a design is completed to this point, other checks may be made, particularly on those quantities close to their limiting values, such as losses and regulation. In particular, a check of the voltage ratio should be made. First, winding resistances are calculated by multiplying the resistance per unit length (corrected to operating temperature from Fig. 11-6) by the number of turns, and then by the mean length of turn. The mean length of turn equals the length of the inside turn, plus pi times the build-up of the winding. The primary voltage is then calculated from

$$V_p = n \left[V_s + I_s (R_s + R_p/n^2) \right] \text{ volts,} \qquad (2\text{-}35)$$

where R_s and R_p are the secondary and primary resistances respectively,

n is the ratio of primary to secondary turns.

The turns ratio should be adjusted if the calculated primary voltage differs appreciably from the specified voltage. Another calculation to be made is temperature rise. A method, which was developed on Contract No. DA-36-039 SC-5470, is summarized in Chapter XI.

Design of High-Voltage Transformers

The formula for calculation of winding space factors, as derived in the final report for Contract No. DA-36-039 SC-5519, is

$$F_c = .08 \log_{10} W_r' + F, \qquad (2\text{-}20)$$

where W_r' = equivalent rating based on 40°C rise and 60 cycles,

F is a term to account for number of windings and working voltage.

Shortly after that report was issued, it was found that the resultant space factors were too high for the higher-voltage designs, in that there was inadequate window space. The reason for the difficulty has been discovered and a revision of Figure 40 of the previous report has been made to obtain better values for the term "F". The revised values are given in Figure 11-2 of this report.

The difficulty results from the design practices used for the units upon which the original figure was based. These units had at least one high-voltage winding designed for a very low current density. Small wire sizes are so difficult to handle that it is common to select some minimum size, such as No. 40 AWG, for windings where heating would ordinarily permit a smaller cross section. For application of the design method in such cases, the remedy is to use a space factor, for purposes of calculation, which is smaller than the value obtained in the completed design. This is accomplished by use of the revised figure. Even though a wire size might be chosen which is much larger than would be required on the basis of heating limitations alone, there will usually be little effect on overall transformer size. This results from the fact that space occupied by a high voltage winding may be small compared to surrounding insulation, so that a change in wire size has little effect on insulation clearances.

The allowable winding dissipation W_c/S_c, for use in the nomograph, should be calculated in the usual way, but final values will often be less. It is not practical to attempt to design all high voltage units to operate near maximum permissible temperature rise, and even if this were done by using extremely small wire sizes, there would be little saving in space and weight.

However, the revised figure for "F" does not solve the problem in designs for which the calculated space factor F_c is negative, or for which the two right-side terms have opposite signs and about the same magnitude. This situation is likely to occur with high-voltage units of small equivalent ratings. When this is the case, it is recommended that the space factor F_c and the characteristic linear dimension ℓ be calculated by assuming that the working voltage is 5000. Then the calculated ℓ is increased by 0.3 inch for each additional 5000 working volts of the required design.

In either of two cases: (1) when a wire size larger than the calculated value is selected, or (2) when ℓ is increased by 0.3 inch per 5000 volts, modifications are necessary in the design procedure. In case (1) it may not be known that a wire size must be enlarged until it is selected, in which case the normal method for calculating winding losses, per cent regulation and conductor weight (which would have been calculated in previous-steps) are invalid. In case (2), calculation of these quantities should be deferred. Circular mils per ampere is calculated from the standard equation. (For case (2), use the value for ℓ corresponding to 5000 working volts). Wire sizes are next computed and increased to the minimum practical size where necessary, in either case.

Turns per volt should be calculated, and preliminary turns for each winding can then be determined. Next, resistances of each winding should be calculated. An approximate correction for the regulation for each winding can be made using the product of rated current times calculated resistance. The ratio of this value to the nominal voltage of each winding is the fraction by which secondary preliminary turns must be increased and primary turns decreased, respectively.

For use in temperature-rise calculations, total losses may be found as the sum of current squared times resistance for all windings. Winding exposed surface area is

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$$S_c = K_3 \ell^2 \qquad (2\text{-}26)$$

where K_3 is a geometric parameter given by Fig. 11-3, 11-4, or 11-5,

ℓ is characteristic linear dimension, inches.

(for case (2), use the final value after increasing ℓ by 0.3 inch per 5000 working volts.)

Finally, the weight of conductor for each winding is length times pounds per unit length.

III. WINDING CURRENT DENSITIES, LOSSES AND HEATING

In the calculation of wire sizes for the different windings of a transformer, the design procedure developed under Contract No. DA-36-039 SC-5519 yields equal current densities in all windings, at least to the extent that equal values can be achieved with a finite number of available wire sizes. The possibility that advantages might be derived by unequal current densities has been studied with respect to:

1) Conditions for minimum total winding losses,
2) Conditions for minimum hottest spot temperature rise,
3) Insulation selection - possible use of different materials for different parts of the transformer.

A straightforward solution may be obtained for the first problem, and it is shown in Appendix A that minimum total losses are obtained when the available window space is so apportioned that current densities are inversely proportional to square root of the product, mean length of turn times space factor of the windings. This holds for any number of windings, and for the different layers of any one winding.

A solution to the second problem is much too difficult to obtain except for simple geometric configurations. However, simple configurations are available as a qualitative guide to transformer characteristics. For such purposes, the transformer as a heat source may be likened to a sphere or to a section of a cylinder. Another possibility is that a side of the winding might be likened to part of a plane.

Several observations regarding temperature rise are applicable to a heat source in the form of a sphere, cylinder or plane. Hottest-spot temperature rise is the sum of the rise from the surface of a body to the ambient plus the rise from the hottest spot to the surface. In the steady-state, the surface rise of a simple body depends only upon total losses and not upon loss distribution, so long as the distribution is symmetrical. Therefore a transformer can be expected to have approximately minimum surface rise for a given rating when current densities satisfy the condition of Appendix A. For a simple body and a fixed amount of generated heat, the entire body would have the same temperature if the heat sources were distributed uniformly on the surface. In this case, hottest-spot temperature is the same as surface temperature. Although this condition is not desirable for application to transformer design because of the high resulting losses, it does indicate that there may be some advantage in generating more heat per unit volume near the surface than far from the surface. Similarly maximum coil rise is obtained if the same heat sources were placed at the point (sphere) or points (cylinder or plane) farthest away from the surface.

Therefore it is seen that the practice of choosing the same current density for all windings is a compromise which gives higher total losses and surface rise than the possible minimums, but which give a lower hottest spot-to-surface rise. It is difficult to say what distribution of current density is best, but factors of importance are the coil thickness, coil thermal conductivity, surface heat-transfer coefficient and the difference between inside and outside turns. An attempt has been made to analyze a cylindrical heat source, but the algebra is very involved. However, a heat source consisting of an infinite plane of thickness $2x_o$ has been studied. Conditions for minimum total rise of this source are given in Appendix B, with the restriction, for simplicity, that current density be linearly distributed from the center to the surface of the plane. For the plane source, it is found that if the coil thermal conductivity is very high, the current distribution should be about constant across the plane; and that if the surface rise is very small, the current density should be low at the center and high at the surface. These characteristics of the plane, modified by the condition for minimum total losses, justify the use of a uniform current density in transformers until more precise information is available.

The study of conditions for minimum loss and minimum transformer temperature rise indicate that it is not feasible to attempt to operate the transformer winding at an almost uniform temperature throughout. In units designed for a high temperature rise, the differences of temperature within the coil may be so high that different materials can be used for different portions. Thus it may be possible to use materials less resistant to highest temperatures for the coolest parts of the winding, thereby reducing expense.

IV. OPTIMUM CORE STACK RATIOS

The purpose of the study of optimum core stack ratios has been to carry further the investigation of optimum transformer proportions which was reported under Contract No. DA-36-039 SC-5519. This previous work considered general transformer proportions where all geometric ratios are assumed to be variable. However, certain widely-used EI and UI laminations have fixed proportions, so that the proportions of a core can only be varied by changing the stacking ratio. This is defined as the ratio of core stack height to lamination leg width (center leg of a shell-type core). Naturally it cannot generally be expected that a quantity to be minimized can be made as small by using laminations of fixed proportions as it would be for laminations having all proportions flexible.

Since the previous analysis was made, computing equipment has become available which has greatly simplified the work of calculation. Four types of transformers using scrapless laminations have been analyzed. Three of these use common laminations. The fourth is a transformer using the new EI lamination conceived during the previous contract. For a discussion of this lamination, see Appendix C. This is included for comparison, and as a guide in case this lamination is manufactured at some future time. The four transformers considered are:

1. Shell type using the scrapless EI lamination of Figure 4-1a.
2. Simple type using the scrapless UI lamination of Figure 4-1b. (The winding encircles one long leg.)
3. Core type using the UI lamination of Figure 4-1b. (The winding consists of two coils, one on each long leg.)
4. Shell type using the new EI scrapless lamination of Figure 4-1c.

The general equations for analyzing a transformer for total weight, volume, losses or cost are applicable to any of the types.

$$C = n_c\, m_c\, A_c = n_c\, V_c$$
$$I = n_i\, m_i\, A_i = n_i\, V_i \qquad (4\text{-}1)$$

where C = total weight, volume, losses or cost of the winding,

I = total weight, volume, losses or cost of the core,

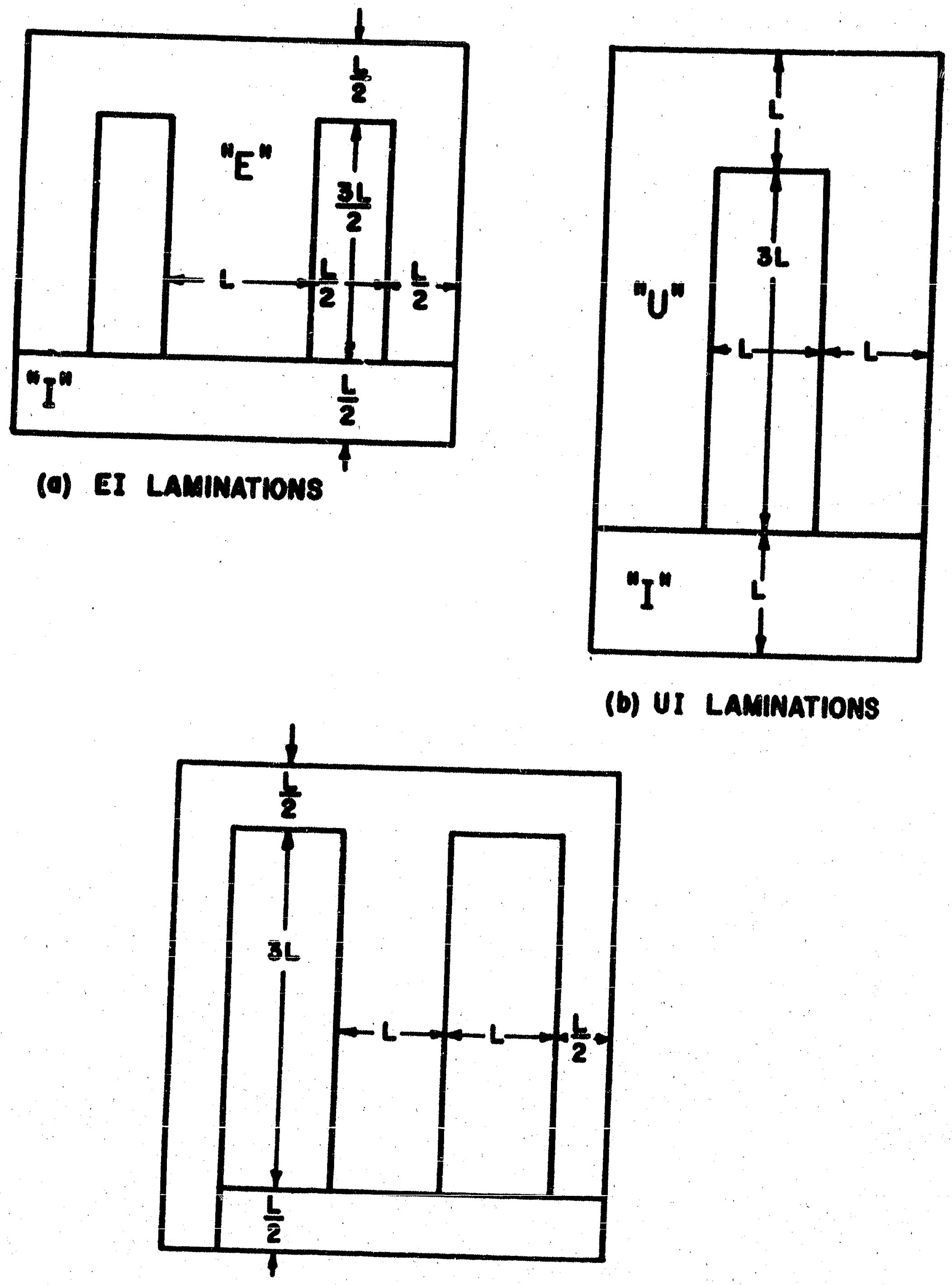
n_c = weight, volume, losses or cost per unit volume of the winding,

n_i = weight, volume, losses or cost per unit volume of the core,

m_c = mean length of winding turns,

m_i = mean length of core magnetic circuit,

A_c = area of core window,



(a) EI LAMINATIONS

(b) UI LAMINATIONS

(c) NEW EI LAMINATIONS

FIG. 4-1 — SCRAPLESS LAMINATIONS

A_i = area of core cross section,
V_c = total volume of the winding,
V_i = total volume of the core.

Another necessary quantity is a weighting factor for winding and core. This is

$$K = \frac{n_c}{n_i} . \tag{4-2}$$

1) For the shell-type transformer using the common EI laminations of Figure 4-1a, equations (4-1), with substituted values for mean lengths and areas, become

$$C = n_c \left(\frac{3}{4} L^2\right) \left(2 + 2s + \frac{\pi}{2}\right) L$$
$$I = n_i \left(sL^2\right) (6L), \tag{4-3}$$

where s is the core stacking ratio

Adding, dividing by n_1, and replacing the ratio n_c/n_i by K, gives

$$\frac{C + I}{n_1} = \left[K \left(\frac{3}{4}\right) \left(\frac{4 + \pi}{2} + 2s\right) + 6s\right] L^3 \tag{4-4}$$

The right side of (4-4) is a function of the three variables L, s and K. It is desirable to minimize this quantity, keeping the product of window and core cross sectional areas $A_c A_i$ constant. This product is

$$A_c A_i = k^4 = \left(\frac{3}{4} L^2\right) \left(sL^2\right). \tag{4-5}$$

where k is used in this chapter in place of ℓ for the characteristic linear dimension so as to avoid a script symbol.

Solving for L gives

$$L = \frac{k}{\left(\frac{3s}{4}\right)^{.25}} . \tag{4-6}$$

Substituting for L in (4-4) gives a function of stacking factor s and weighting factor K.

$$\frac{C + I}{6n_i k^3} = \left[K \left(\frac{1}{4}\right)\left(\frac{4 + \pi}{4} + s\right) + s\right] \left(\frac{4}{3s}\right)^{.75} \tag{4-7}$$

This function is plotted in Figure 4-2 with s as abscissa and K as parameter. Another function which has been calculated is the ratio of total winding weight, volume, losses or cost to total core weight, volume, losses or cost.

$$\frac{C}{I} = \frac{K(\frac{1}{4})(\frac{4 + \pi}{4} + s)}{s} \qquad (4\text{-}8)$$

2) Similar equations can be derived for the simple-type transformer using the UI laminations of Figure 4-1b. With values substituted in (4-1) for mean lengths and areas,

$$C = n_c\ (3L^2)(2 + 2s + \pi)L$$
$$I = n_i\ (sL^2)(12L) \qquad (4\text{-}9)$$

Also, $$\frac{C + I}{n_i} = \left[K\ (3)(\pi + 2 + 2s) + 12s\right] L^3. \qquad (4\text{-}10)$$

Since $$A_c\ A_i = k^4 = (3L^2)(sL^2), \qquad (4\text{-}11)$$

Then $$L = \frac{k}{(3s)^{.25}} \qquad (4\text{-}12)$$

Equation (4-10) becomes

$$\frac{C + I}{6n_i k^3} = \left[K\ (\frac{\pi + 2}{2} + s) + 2s\right] \frac{1}{(3s)^{.75}}\ . \qquad (4\text{-}13)$$

Similar to (4-8) is

$$\frac{C}{I} = \frac{K\ (\frac{\pi + 2}{2} + s)}{2s}\ . \qquad (4\text{-}14)$$

Equation (4-13) is plotted in Figure (4-3).

3) The equations for the core-type transformer using the UI laminations of Figure 4-1b are

$$C = n_c\ (3L^2)(2 + 2s + \frac{\pi}{2})\ L$$
$$I = n_i\ (5L^2)(12L) \qquad (4\text{-}15)$$

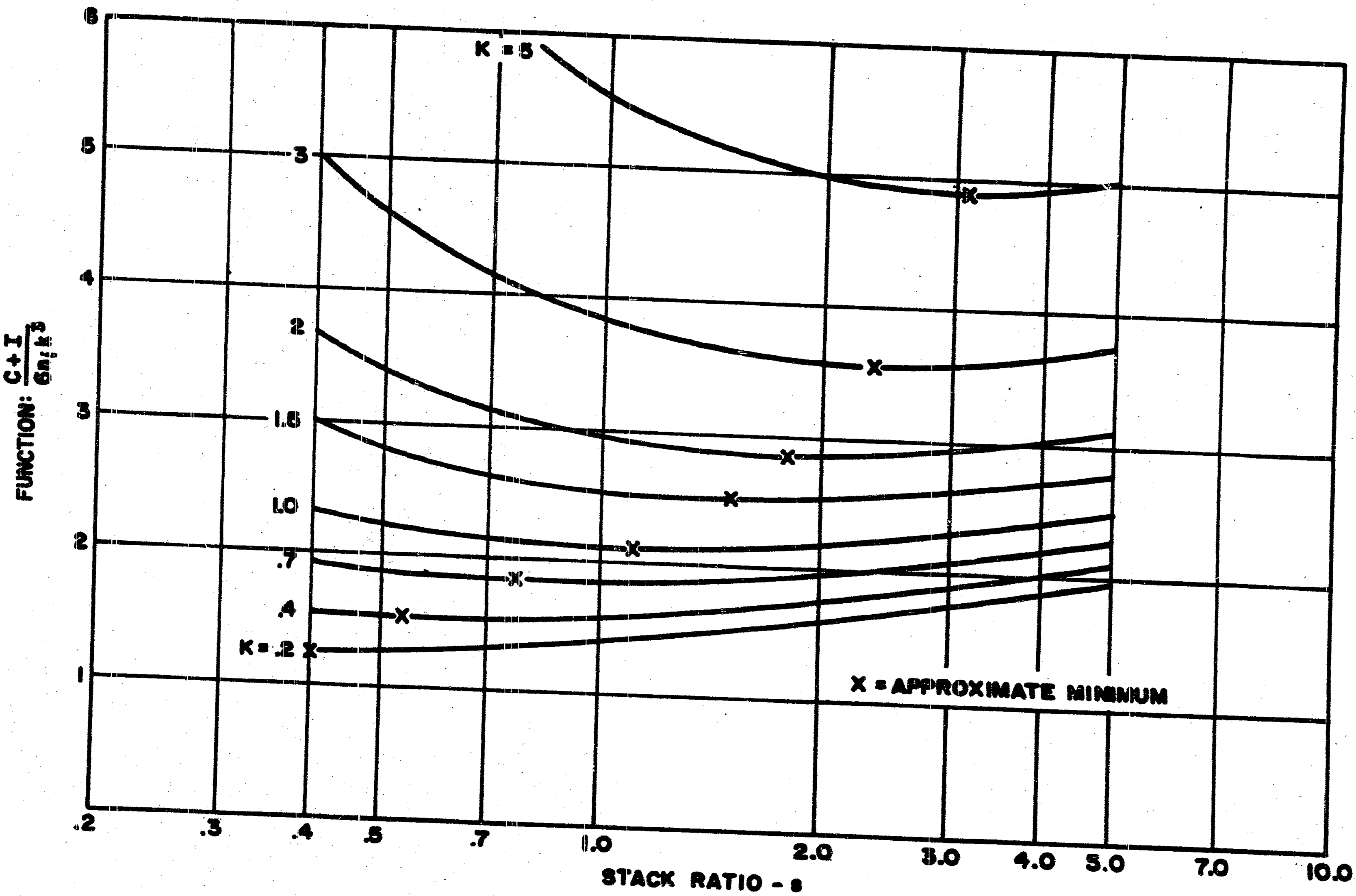


FIG. 4-2 -STACK RATIOS FOR SHELL TYPE WITH E I LAMINATIONS

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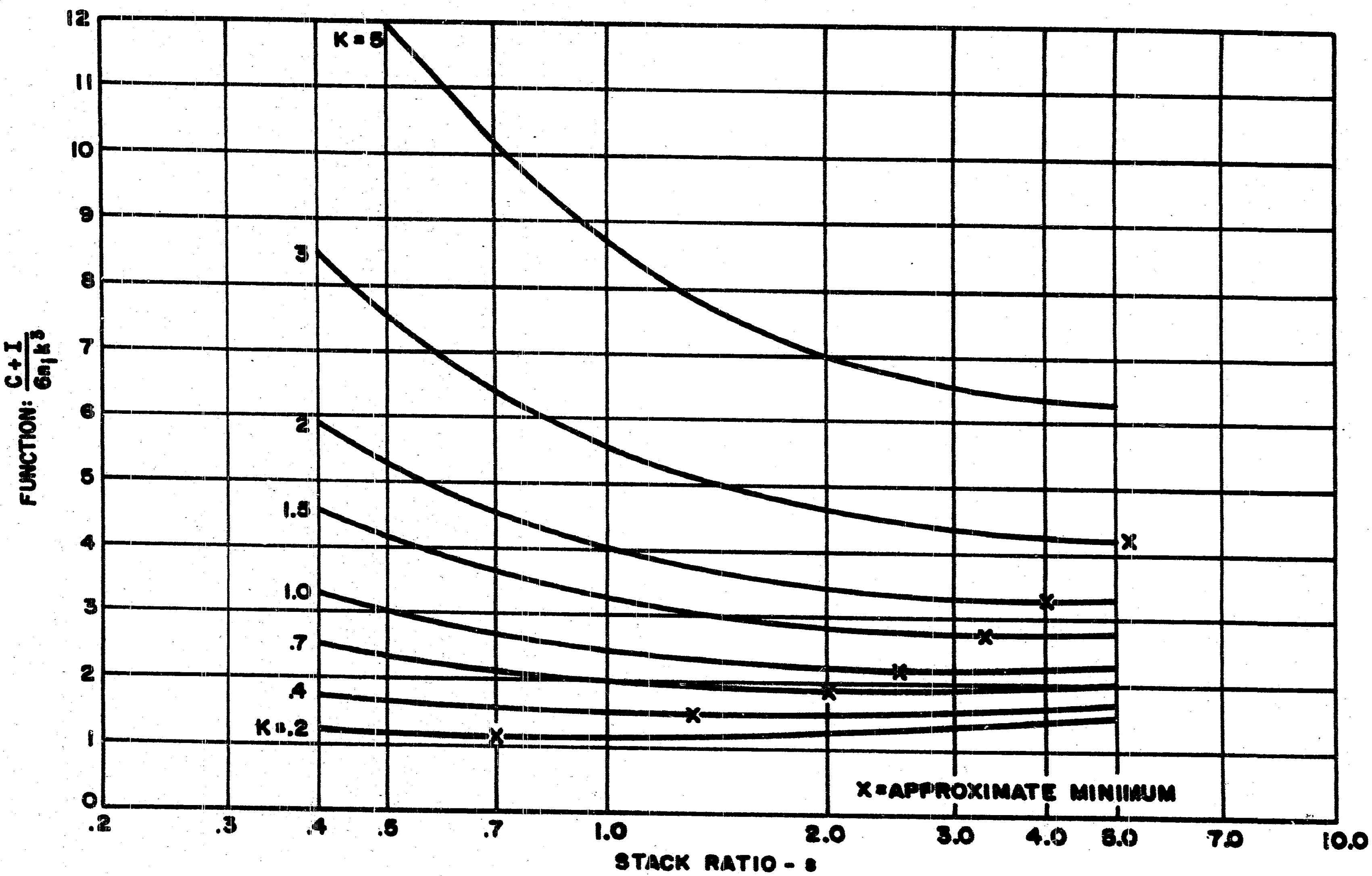

FIG. 4-3 - STACK RATIOS FOR SIMPLE TYPE WITH UI LAMINATIONS

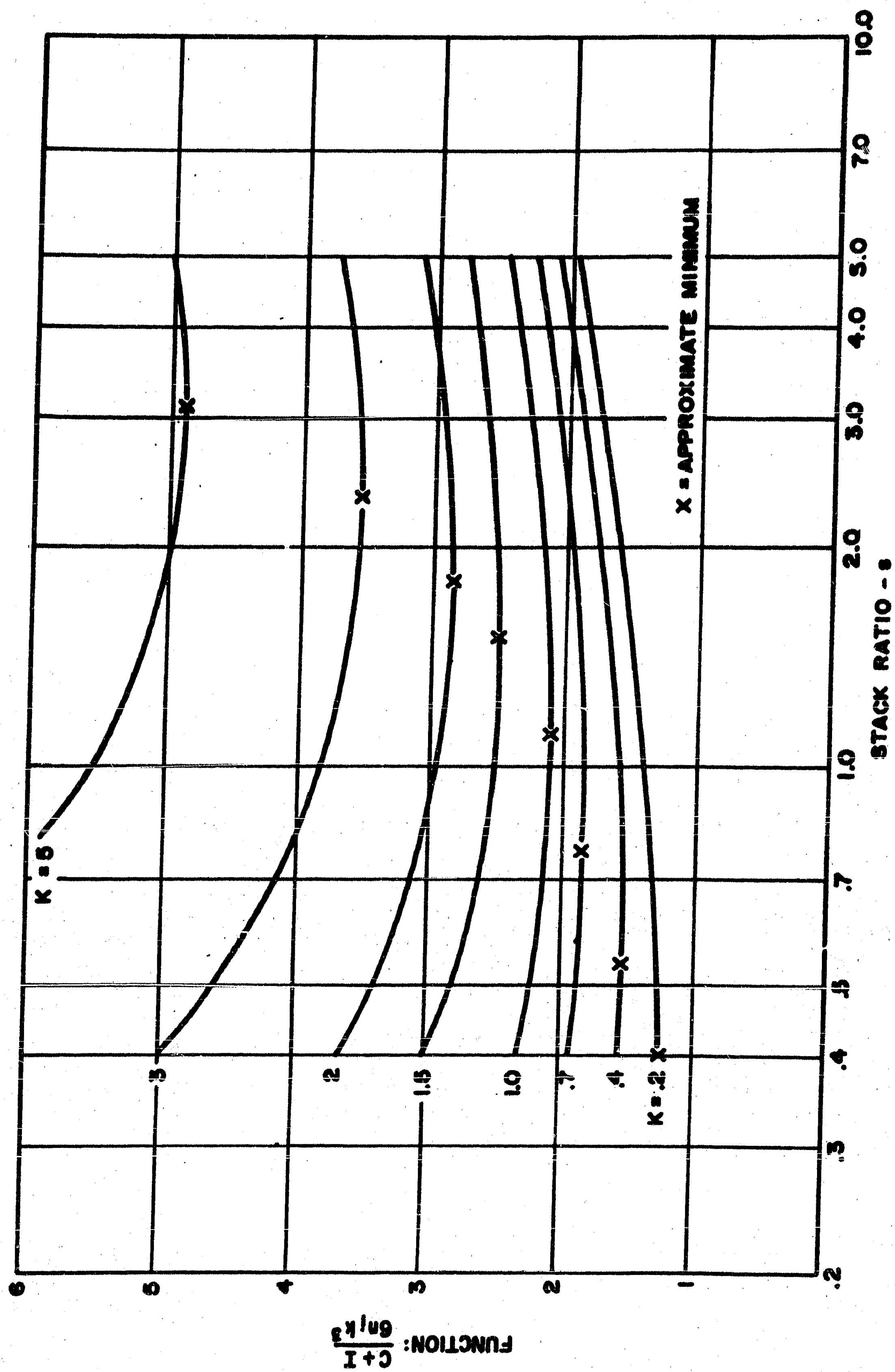

FIG. 4-2 -STACK RATIOS FOR SHELL TYPE WITH E I LAMINATIONS

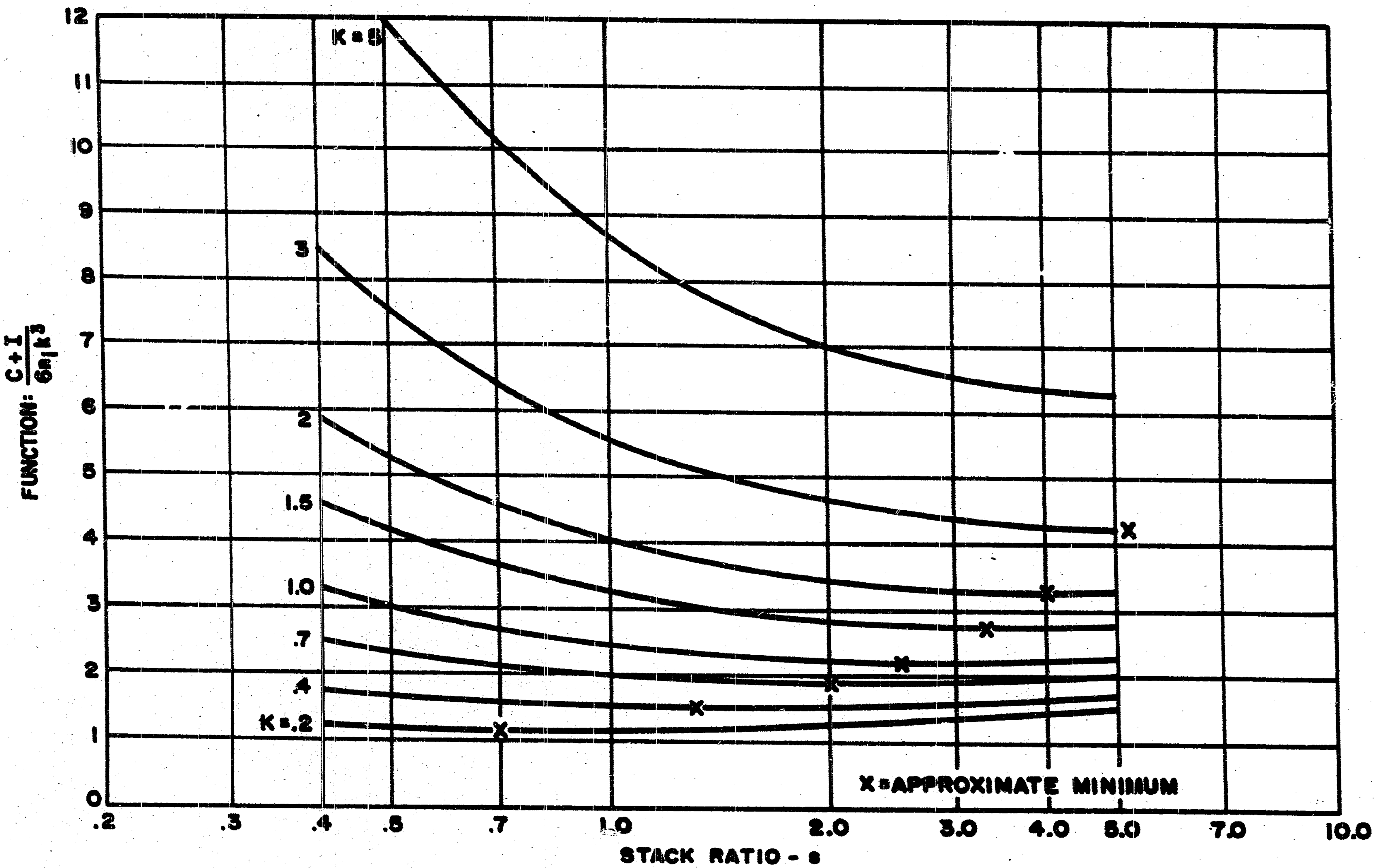

FIG. 4-3 - STACK RATIOS FOR SIMPLE TYPE WITH UI LAMINATIONS

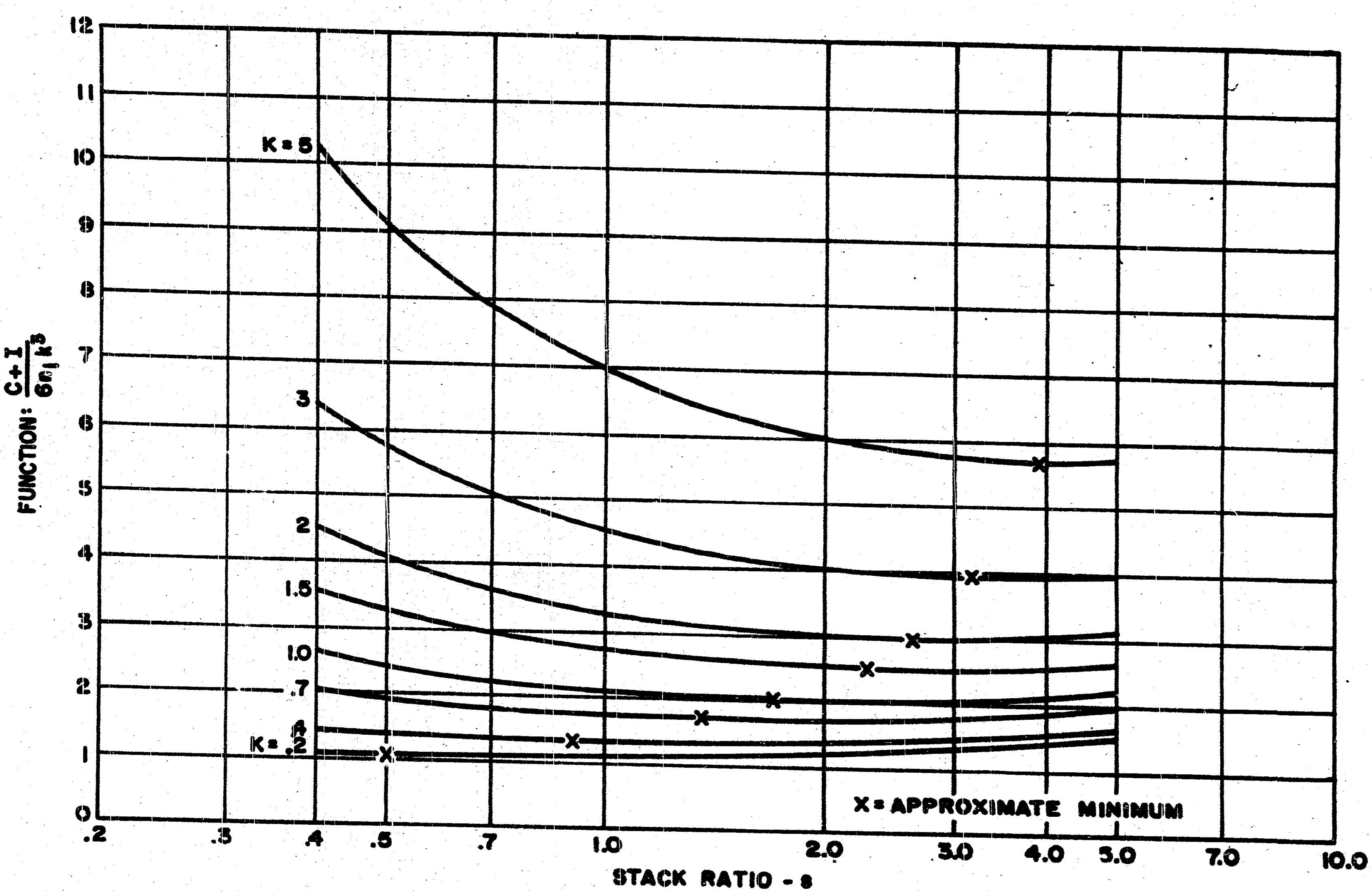

FIG. 4-4 - STACK RATIOS FOR CORE TYPE WITH UI LAMINATIONS

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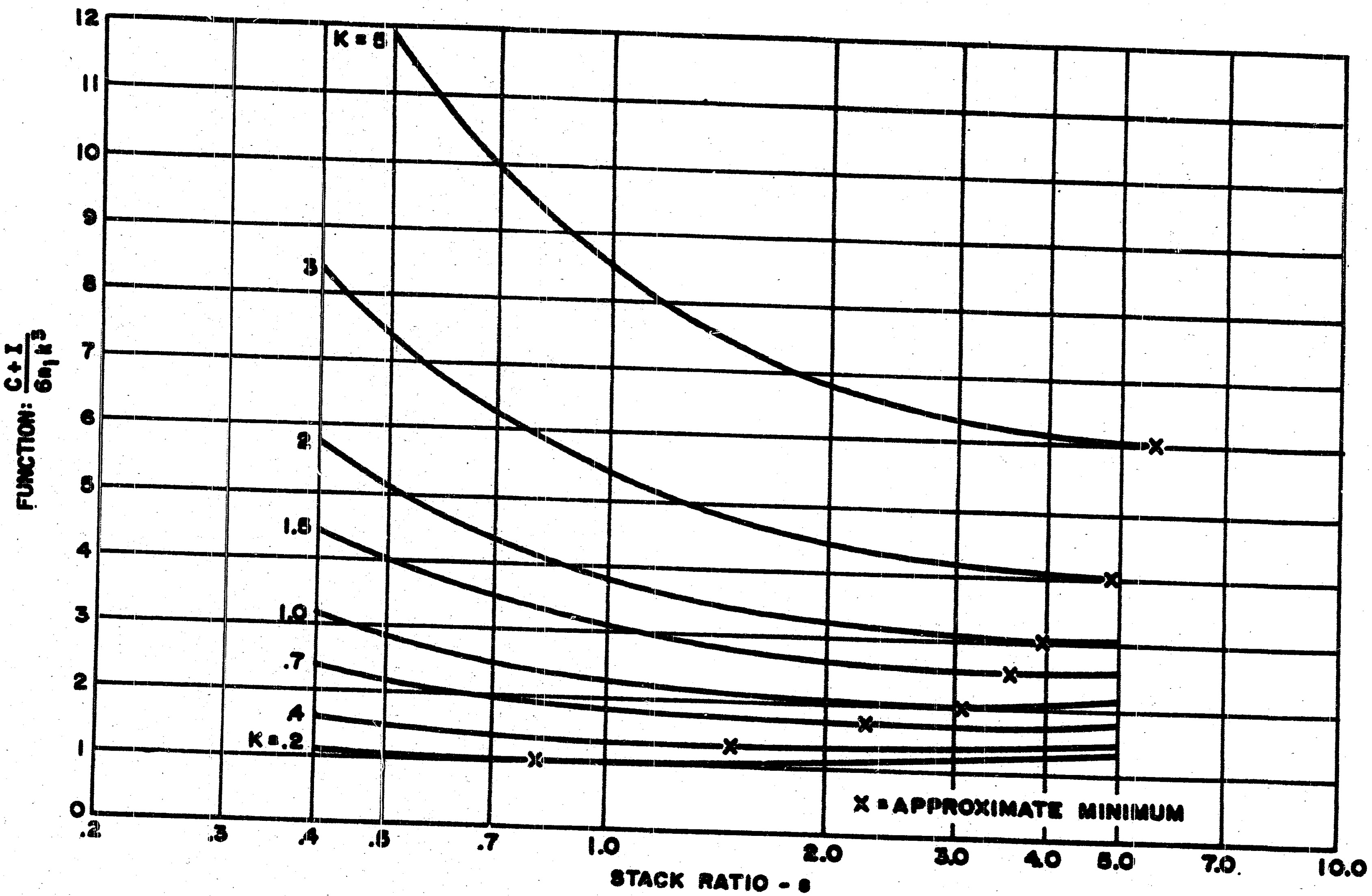

FIG. 4-5 -STACK RATIOS FOR SHELL TYPE WITH NEW EI LAMINATIONS

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$$\frac{C + I}{n_i} = \left[K\,(3)\left(\frac{4 + \pi}{2} + 2s\right) + 12s \right] L^3 \quad (4\text{-}16)$$

$$A_c\, A_i = k^4 = (3L^2)\,(sL^2) \quad (4\text{-}17)$$

$$L = \frac{k}{(3s)^{.25}} \quad (4\text{-}18)$$

$$\frac{C + I}{6n_i k^3} = \left[K\left(\frac{4 + \pi}{4} + s\right) + 2s \right] \frac{1}{(3s)^{.75}} \quad (4\text{-}19)$$

$$\frac{C}{I} = \frac{K\left(\frac{4 + \pi}{4} + s\right)}{2s} \quad (4\text{-}20)$$

Equation (4-19) is plotted in Figure 4-4.

4) The equations for the <u>shell-type</u> transformer using the <u>new EI scrapless lamination</u> of Figure 4-1c are

$$C = n_c\,(3L^2)(2 + 2s + \pi)\,L$$

$$I = n_i\,(sL^2)(10L) \quad (4\text{-}21)$$

$$\frac{C + I}{n_i} = \left[K\,(3)(2 + \pi + 2s) + 10s \right] L^3 \quad (4\text{-}22)$$

$$A_c\, A_i = k^4 = (3L^2)(sL^2) \quad (4\text{-}23)$$

$$L = \frac{k}{(3s)^{.25}} \quad (4\text{-}24)$$

$$\frac{C + I}{6n_i k^3} = \left[K\left(\frac{2 + \pi}{2} + s\right) + \frac{5s}{3} \right] \frac{1}{(3s)^{.75}} \quad (4\text{-}25)$$

$$\frac{C}{I} = \frac{K\left(\frac{2 + \pi}{2} + s\right)}{\frac{5s}{3}} \quad (4\text{-}26)$$

Figure 4-5 gives the results of equation (4-25).

The stack ratio used in Figures (4-2) to (4-5) effectively establishes the ratio of winding size to core size. Other ratios could also be used: the ratio of window area to core cross-sectional area A_c/A_i, the ratio C/I, or the ratio of winding volume to core volume V_c/V_i. Table 4-1 gives the optimum values of these quantities for several values of the weighting factor K. One interesting result is the fact that C/I has a very large range for weighting factors from .2 to 5. The range is much larger for scrapless laminations than for generally variable core proportions. It is concluded from this result that the practice of making the total value C associated with the winding equal to the total value I associated with the core, is not a very good approximate rule for scrapless laminations.

It is of interest to compare the different types of scrapless laminations with each other, and with the general types studied previously. This is readily done by calculating the function $(C + I)/n_i$ for the optimum stacking ratio in each case. The previous value for area product $A_c A_i = 6.25$ has been used, and thus the quantity k is determined. The results are given in Table 4-2. The results for transformers No. 5 to 8, which have generally variable proportions, are taken from the previous work. Although Nos. 5, 7, and 8 were calculated with rounded core corners, a comparison of Nos. 6 and 7 gives a rough gauge of the difference between rounded and square corners.

Several other comparisons can be made from Table 4-2. One noticeable feature is that Nos. 5, 7, and 8 are each more economical than Nos. 2 (simple), 1 or 4 (shell), and 3 (core) respectively. The given differences indicate the loss incurred by using scrapless laminations. Comparing the common scrapless EI lamination No. 1, with the new scrapless EI lamination No. 4, shows that No. 1 is more economical for large K (typified by total losses), whereas No. 4 is more economical for small K (typified by total weight).

Table 4-1 REQUIRED VALUES FOR MINIMUM C + I

Transformer Type	Shell, EI Lamin.				Simple UI Lamin.				Core UI Lamin.				Shell New EI Lamin.			
K	s	A_c/A	C/I	V_c/V_i	s	A_c/A	C/I	V_c/V_i	s	A_c/A_i	C/I	V_c/V_i	s	A_c/A_i	C/I	V_c/V_i
.2	.40	1.9	.27	1.4	.7	4.3	.467	2.3	.5	6.0	.46	2.3	.8	3.8	.51	2.5
.4	.54	1.4	.43	1.1	1.3	2.3	.596	1.5	.9	3.3	.59	1.5	1.5	2.0	.65	1.6
1.0	1.1	.68	.66	.66	2.5	1.2	1.01	1.01	1.7	1.8	1.03	1.03	3.1	.97	1.10	1.10
2.0	1.8	.42	.98	.49	4.0	.75	1.64	.82	2.6	1.2	1.71	.85	3.9	.77	2.06	1.03
5.0	3.1	.24	1.9	.38	5.5	.55	3.7	.74	3.9	.77	3.6	.72	5.7	.53	4.1	.81

Table 4-2 COMPARISON OF OPTIMUM TRANSFORMERS OF VARIOUS TYPES ($A_c A_i = 6.25$)

Transformer Type	Function $\frac{C + I}{n_i}$			% $\frac{C + I}{n_i}$ based on No. 5		
	K = .2	1.0	5.0	K = .2	1.0	5.0
1. Shell, Scrapless EI	29.8	50.0	116.4	126	98.4	95.0
2. Simple, Scrapless UI	27.9	52.8	147	114	104	120
3. Core, Scrapless UI	25.6	48.3	136.3	104.4	95.2	111
4. Shell, New Scrapless EI	24.8	48.3	143	101	95.2	116
5. Optimum simple, round corners	24.5	50.8	122.6	100	100	100
6. Optimum shell, square corners	23.7	47.9	113.5	96.7	94.3	92.7
7. Optimum shell, round corners	23.2	46.5	108.9	94.8	91.6	88.9
8. Optimum core, round corners	21.6	46.2	115.3	88.2	91.1	94.2

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V. TRANSFORMERS WITH UNBALANCED MAGNETIZATION

A transformer with unbalanced magnetization presents a complex problem in finding relations among the electrical quantities. Difficulties arise as a result of the nonlinear relationship between the core induction or flux density and magnetic field strength. This factor is of less importance when either the unbalanced magnetization or the superimposed alternating induction is small, because existing design methods for such cases are reliable and widely used. In a transformer, however, alternating induction is always much larger, and the nonlinear characteristics of the material make it impractical to apply algebraic analysis for obtaining relations among the magnetic quantities. The principal variables are the instantaneous values of induction or flux density and the magnetizing force. Other variables are the configuration of the magnetic circuit, the type of joint used and the grade and thickness of the material.

The superposition of D-C and alternating components of magnetizing force on magnetic materials gives characteristics which are qualitatively similar in some respects to the case of an alternating magnetization alone. In both instances the magnetic quantities may be related by a hysteresis curve, which is usually given with induction or B as ordinate and magnetizing force or H as abscissa. With an unbalanced magnetization, the curve is dissymmetrical and has time-average values of B and H which are unequal zero. In addition, the average values of B and H are not simply related to each other by the D-C magnetization characteristic of the material. In power apparatus, the hysteresis loop is narrow, with or without an unbalanced magnetization, so that the relation between B and H is practically single-valued. Therefore the maximum and minimum values of induction and magnetizing force respectively are directly related by the D-C magnetization characteristic.

In general the parameters which must be determined in the design of magnetic circuits are: a suitable material, a satisfactory value of flux density, proper proportions for the magnetic circuit, and the non-magnetic gap in the core. It is desirable that the resulting device be as small as possible for the required power rating. This means that operating densities; flux density in the core, and current densities in the windings should be made as high as design limitations will permit. Such limitations may be core or winding losses, voltage regulation, efficiency of the unit or permissible heating. The most universal limitation in power and communication equipment is permissible temperature rise. Regardless of other important specifications to be met, excessive temperative must be avoided. A characteristic of almost all power equipment is that temperature rise increases as size decreases, for a given output. To obtain maximum rating with least material, the designer should approach the permissible operating temperatures of both core and winding. The quantities which determine whether the magnetic circuit has been properly designed are the core loss and the magnetizing reactive power required by the core. Either of these quantities may limit flux density in the core. In physically-small apparatus, the contribution of the magnetizing power to the heating of the windings is usually the important design limitation, while in large equipment, core loss usually fixes the limiting flux density.

References

Published works which have been found to be of the greatest value will be briefly reviewed. Two excellent and extensive bibliographies have been published by Rex[1] and by Miles[2], which include almost all of published references and patents which have been found on the subject of unbalanced magnetization. These lists were primarily intended to present a background on magnetic amplifiers.

Some of the earliest important contributions were made by Niwa and others[3,4] who found that for a certain D-C magnetization, the superposition of an alternating field may either raise or lower the average value of induction. Another study of the effects of a superposed alternating field on permeability and losses is that of Spooner[5]. Recent tests have been made by Battelle Memorial Institute[6] under contract No. W 36-039 SC-38255, but the results given apply only to relatively low values of D-C magnetization.

While some of the references give techniques and circuits for the measurement of magnetic properties with D-C magnetization, others emphasize and compare different circuits for obtaining losses and effective inductance. These circuits are in two general classes: null-balance or A-C bridge types, and direct measurement types. In the bridge circuits, a coil on the magnetic sample constitutes one arm of the bridge. The D-C magnetizing force can be supplied through the same winding or by a second winding on the sample. Harris[7] deals with different types of bridge circuits. Charlton and Jackson[8] have presented a circuit for direct measurements, using two similar cores with two windings on each. Windings of each coil are connected in series to the A-C supply and to the D-C supply, an arrangement which yields non-sinusoidal flux wave shapes in the cores, and which gives results difficult to interpret.

Many references deal with the characteristics and the design of transformers where one winding carries direct current. In 1927, Hanna[9] published a classic article on design of reactances and transformers, which relates inductance, direct current, magnetic field, core geometry and air gap in the core. Curves presented by Hanna make it possible to select an optimum air gap. However one sheet of design curves is valid for only one A-C flux density. Most of the data and design methods published subsequently are applicable only to much smaller A-C densities than would be used in transformers, and it is usually assumed that the incremental permeability of the magnetic material is independent of A-C density. Following Hanna, data and analyses have been given by many others such as Lee[10]. The book of the MIT Staff[11] gives typical data and points out that a core air gap may be used to obtain more nearly constant inductance over a cycle of operation. Legg[12] has extended the method of Hanna by devising an orderly design method for transformers and reactors carrying unbalanced direct current. Further analysis of this problem is given by Carter and Richards[13], who also show that average induction and average magnetic field strength are not simply related unless A-C induction of density is small.

An important factor in the design of transformers supplying a half-wave rectifier is the type of output filter circuit. Schade[14] has given widely used curves which relate transformer secondary and load quantities

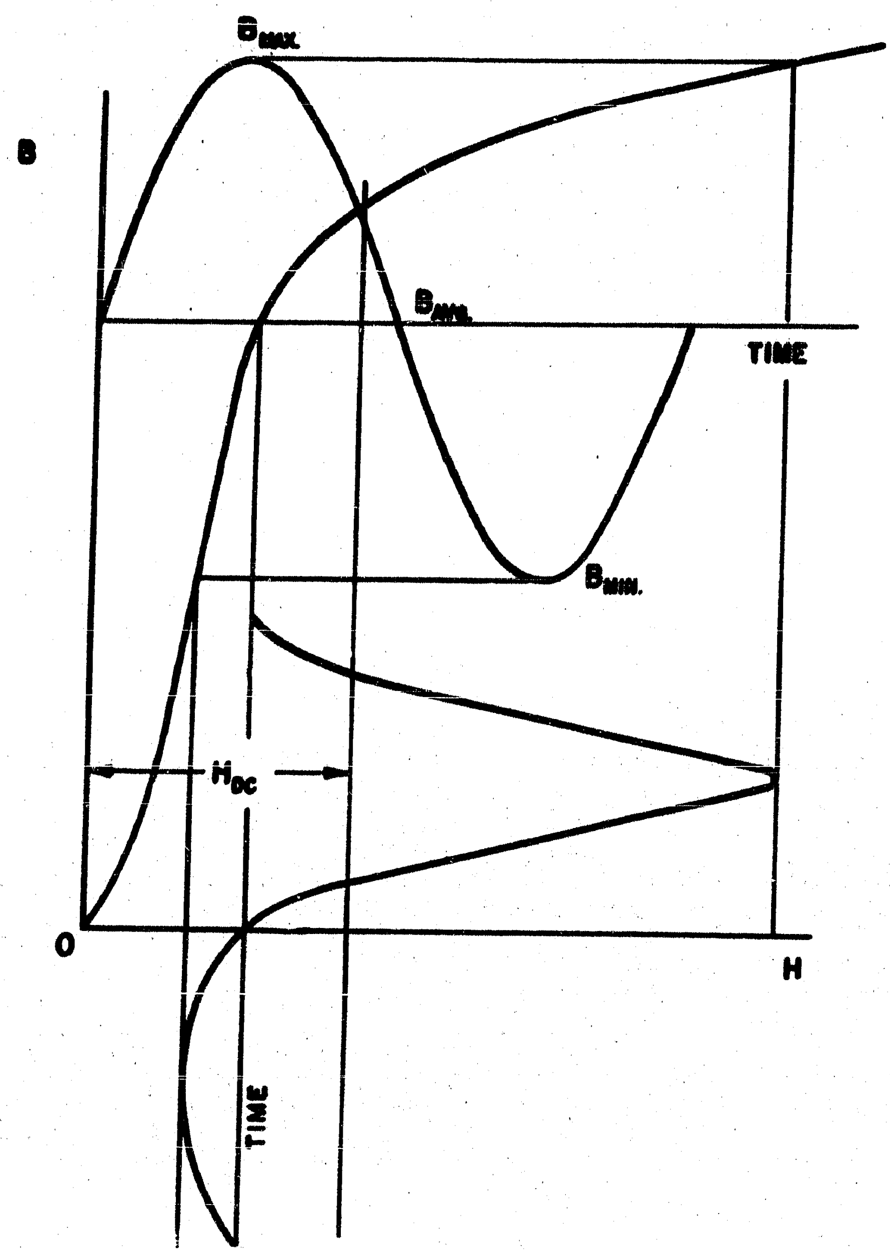

FIG. 5-1 —RELATIONS BETWEEN INDUCTION AND FIELD STRENGTH.

according to frequency, load resistance, filter capacitance and circuit resistance. Seely[15] has also studied the interesting half-wave rectifier output circuit consisting of an inductance and resistance in series.

General Properties of Magnetic Circuits with Unbalance

The non-linear characteristics of ferromagnetic materials cause considerable complication in an analysis of magnetic circuits with unbalanced magnetization. The problem consists essentially of relating induction or flux density B and magnetic field strength H. A typical D-C magnetization curve is given in Fig. 5-1. When the material is subjected to alternating values of B and H about some average values, the D-C curve describes the approximate performance. Qualitatively, there are two rather different conditions: comparatively small variations and comparatively large variations. As noted earlier, the references which give extensive algebraic relations among the circuit quantities cover the case for small variations in B and H. To solve circuit problems, it is desirable to use additional data in the form of incremental permeability, or the ratio of change in B to the change in H. When small B and H variations are occurring in the steep region of the magnetization curve, the value of incremental permeability is much less than the slope of the D-C magnetization curve. Also, the incremental permeability is not greatly affected by the magnitude of the B variation.

In the second case, where the variation in induction B is large, the D-C curve describes fairly well the relation between B and H, a fact which is used for the following qualitative analysis. During operation under unbalanced conditions, the exact function consists of a displaced (non-symmetrical) hysteresis loop, enclosing an area proportional to losses per cycle, as for magnetic materials operated without an unbalance. With a large variation in B, the quantity corresponding to incremental permeability is some average slope of the D-C characteristic, and this magnitude is greatly affected by the magnitude of the variation in density B. This added dependence makes the second case more complicated than the first, inasmuch as incremental permeability is no longer approximately constant as it is for low values of alternating density. Therefore the term incremental permeability has little significance for the second case. It is found that the most useful means for understanding the problem are: a graphical analysis of the magnetic quantities and a study of frequency components of the electrical quantities.

The important variables which determine the characteristics of a magnetic circuit are the average and variable components of induction B and magnetic field strength H, the geometry and material of the magnetic circuit and the frequency of variation. Average flux density is defined as B_o, and the average field strength is defined as H_{DC}. A sinusoidal component of flux density is assumed, which is defined as B. This is one half of the maximum variation about the average B_o. Important geometric parameters of the magnetic circuit are the dimensions and the type of joint used. It is assumed that the flux density is uniform in the core at every instant of time. In general, the presence of a joint in the magnetic circuit makes the field strength non-uniform around the magnetic circuit. Therefore the given average field H_{DC} is an average around the core circuit as well as being an average in time. This quantity can be related directly to the direct currents in the windings surrounding the core. If there is a direct current I_{DC} in one winding of N turns then the average field strength may be defined as

$$H_{DC} = \frac{.4 \pi NI_{DC}}{m_i} \text{ oersteds,} \qquad (5\text{-}1)$$

where m_i is the mean length of magnetic circuit in centimeters.

Average flux density is not simply related to the average field strength H_{DC}. The relation between these quantities is demonstrated by Fig. 5-1. If an average value of flux density B_o is assumed, with a superimposed variation B, then the resulting function of field strength H is uniquely determined by the D-C magnetization curve, insofar as the hysteresis effect can be neglected. The average of the H function must be H_{DC}. This establishes a relation between B_o and H_{DC}. It can be seen that average flux density cannot be readily determined from the magnetic characteristic by graphical means, since repeated trials would be required, assuming each time a certain value of average density. In a closed magnetic circuit it is also difficult to determine average density by experimental methods. The only possible way is to trace the magnetic history of an initially demagnetized specimen.

From the foregoing discussion it is found that the magnetic variables of a given magnetic circuit are uniquely determined by the average and varying components of flux density. Since both of these together determine the unbalanced magnetization H_{DC}, it can be reasoned that the performance could also be described by the values of B and H_{DC}, which are readily determined. Unbalanced magnetization is defined by equation (5-1). If a sinusoidal voltage or component of voltage is applied to a winding of the core, the voltages are

$$v = iR + N\frac{d\phi}{dt}, \qquad (5\text{-}2)$$

where i is the current function of time,
R is the resistance of the winding,
ϕ is the flux in the magnetic circuit,
N is number of turns,
t is time.

If it is assumed that the iR voltage term is negligible in comparison with the induced voltage term, and if the applied voltage is sinusoidal,

$$v = V_m \cos\omega t, \qquad (5\text{-}3)$$

then equation (5-2) can be solved for flux to obtain

$$\phi = \frac{V_m}{N}\int \cos\omega t\,dt = \frac{V_m}{N\omega}\sin\omega t + \phi_o = \phi_o + \phi_m \sin\omega t, \qquad (5\text{-}4)$$

where ϕ_o is the average component of flux,

ϕ_m is the peak of the alternating component.

This development shows that the existence of an average component of flux is compatible with the fundamental relations between applied potential and flux. Therefore the operation of a magnetic circuit can be described entirely in terms of B (which is ϕ divided by net cross-sectional area of the core) and H_{DC}.

Effect of a Non-Magnetic Gap

In addition to the other physical properties of a closed magnetic circuit, core geometry and material, an important variable is the non-magnetic gap which may be introduced in some cases to obtain improved characteristics. The presence of a core joint of any type has an effect on performance which is similar to that of the non-magnetic gap. In most applications of magnetic cores to power apparatus it is desired that the self inductance of a given winding on the core have the highest value possible. When there is no unbalanced magnetization present, the highest self inductance is obtained when a core joint of minimum reluctance is used. If the D-C magnetization curve of Fig. 5-1 is considered to be a plot of flux density or induction B against average magnetic field strength H around the entire magnetic path, then the introduction of a non-magnetic gap increases the value of H for each value of B, or bends the curve to the right. This effect is undesirable when there is no unbalance and maximum inductance is required.

A gap may yield an increase in self inductance when a core has an unbalanced magnetizing current in one winding, which can be demonstrated qualitatively. The change in the magnetization characteristic of the core then tends to reduce the average flux density B_o of the core. The quantities B and H_{DC} are considered constant and independent. Therefore the maximum flux density $(B + B_o)$ is decreased and so is the maximum field strength corresponding to $(B + B_o)$. The cost of obtaining this advantage is a decrease of slope of the B-H curve in the regions of low H. In another sense, instantaneous permeability (actually a variable during a cycle) is increased at high values of H and decreased at low values of H. Since it is apparent that a sufficiently large gap would decrease self inductance in any case, there may be some optimum value of non-magnetic gap, which depends on B, H_{DC}, core material and geometry. In a transformer with unbalanced magnetization, the requirement of maximum self inductance is equivalent to a requirement for minimum magnetizing current in the winding which provides A-C excitation to the core.

The quantities B and H_{DC} are used to predict magnetic characteristics of various cores of various shapes and sizes. Similarly, it is desirable to express non-magnetic gap in a manner such that the results for one size and shape can be used for others. It can be shown that per cent non-magnetic gap should be used, or the ratio of air gap length to magnetic circuit length. Consider a magnetic circuit having a uniform cross section and air gap carrying a flux which does not vary with time. The magnitude of flux is

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$$\phi_{DC} = \frac{\text{magnetomotive force}}{\text{reluctance}}$$

$$= \frac{.4 \pi N I_{DC}}{\frac{m_1}{\mu A_1} + \frac{m_g}{A_g}} \text{ maxwells or lines} \qquad (5\text{-}5)$$

where N = number of winding turns,

I_{DC} = winding current, amperes,

m_1 = length of magnetic iron circuit, centimeters,

μ = relative permeability of the magnetic material,

A_1 = net cross-sectional of the magnetic material, sq. cm.,

m_g = effective length of the non-magnetic gap, cm,

A_g = effective cross-sectional area of the non-magnetic gap, sq. cm.

The two terms in the denominator of (5-5) are the reluctances of iron and gap respectively. Then since $\phi_{DC} = B_{DC} A_1$, (5-5) gives for flux density

$$B_{DC} = \frac{.4 \pi N I_{DC}}{m_1 \left(\frac{1}{\mu} + \frac{m_g}{m_1} \cdot \frac{A_1}{A_g}\right)} \text{ lines per sq. cm.} \qquad (5\text{-}6)$$

H_{DC} has been defined in (5-1) as the average magnetomotive force in oersteds around the magnetic circuit, neglecting the length of the gap in comparison with the length of the iron, so that (5-6) becomes

$$B_{DC} = \frac{H_{DC}}{\left(\frac{1}{\mu} + \frac{m_g}{m_1} \frac{A_1}{A_g}\right)} \text{ lines per sq. cm.} \qquad (5\text{-}7)$$

Equations (5-6) and (5-7) show that only the ratio of lengths m_g/m_1, and not each independently, affects the relation between B_{DC} and H_{DC}. The ratio A_1/A_g is a correction factor for flux fringing at the gap, and has a value slightly less than one for relatively short gap lengths. Equations (5-6) and (5-7) do not hold in general when there is superimposed alternating flux in the magnetic circuit. They are given here only to show that the ratio of lengths is important.

Although it is evident that the ratio of gap length to magnetic circuit length is most important, another consideration is the minimum effective gap length obtainable in particular cores. This effective gap length is present because of the necessity for joints, and depends very little on the size of the core. Thus, the minimum effective gap ratio of a small core is higher than that of a large core, when both are constructed with the best possible joint. Table 5-1 gives suggested effective gap lengths for different cases.

Table 5-1

EFFECTIVE GAPS DUE TO JOINTS

Core	Gap-Inches
Stacked laminations, interleaved 1x1 or 2x2	.001
Stacked laminations, butt joint	.005
Wound core, two good-quality butt joints	.001

The values of Table 5-1 can be considered as a part of the term m_g. Actual total spacing to be placed in the core joints is then

$$m_g = \left(\frac{m_g}{m_i}\right) m_i \text{ inches,} \qquad (5\text{-}8)$$

where the ratio m_g/m_i is to be given from magnetic material curves, and m_i is found during the design. If the calculated m_g is equal to, or less than the value of Table 5-1, no additional spacing in the joint is necessary. If calculated m_g for a wound core were, for example, 10 mils, then 9 mils (total) of spacers should be placed in the joints.

Test Circuits for Obtaining Experimental Data

As discussed earlier, two major methods have been used to obtain data on magnetic materials subjected to superimposed A-C and D-C magnetization: bridge or null-balance circuits, and direct measurement. A basic objection to bridge circuits is that a non-linear impednace is being tested, which makes it impossible to obtain a true balance. For determining incremental permeabilities and core losses at small A-C flux densities, the American Society for Testing Materials (ASTM A34-48) specifies an Owen bridge and a magnetic sample with two windings, one to serve as an arm of the bridge, and the other to carry direct current. In addition to a distinction according to the various types of bridge circuits, the magnetic sample may have one or two windings for excitation. When only one winding is used, a combined A-C and D-C excitation source is used as the bridge input. This leads to some difficulty in control of the source. On the other hand, when two windings are used, precautions must be taken to prevent transfer of power from the A-C circuit to the D-C circuit. This is accomplished by using a large inductance in the D-C circuit.

Because of the difficulties associated with bridge circuits, a method of direct measurement is preferred. Direct methods, in general, do not usually yield the accuracy obtainable from a null-balance circuit. A suitable degree of accuracy in measurements on magnetic materials is achieved when the measurement error is a small fraction of the variability of the material among identical samples. From this standpoint a direct method should give reasonable absolute and relative accuracies for typical data. As for the bridge circuits, a direct measurement can be made on a sample which has either one or two windings. When one winding is used, A-C and D-C excitation sources are interconnected. Adjustment of the input and maintenance of sinusoidal A-C wave shape can be obtained only with testing apparatus of unusual versatility. Therefore a sample with two windings should be considered. The simplest example of a circuit with two windings is shown in Fig. 5-2. Ideally, the inductance in the D-C circuit should be infinite, such that no A-C current flows in the D-C winding. When this situation is obtained, the total core excitation at any time is the sum of the instantaneous primary current and the D-C secondary current. The power measured by the wattmeter is composed of the primary winding losses and the core losses, while the battery in the secondary circuit supplies the resistive losses of the secondary winding, the choke, and the control resistance. Practically, it is desirable to use sufficient impedance such that power transferred from the A-C winding is negligible. This could be accomplished with a choke many times the size of the sample under test, and suitable for carrying the direct current. An alternative is to measure the real and reactive power components in the D-C circuit.

A solution to the porblems presented by the circuit of Fig. 5-2 is provided by the circuit shown by Fig. 5-3. An A-C voltage can be introduced in the D-C circuit in such a manner as to oppose the voltage induced in the D-C winding around the test sample. This is accomplished with an auxiliary transformer which need have only the same turns ratio as that of the sample, and be capable of carrying the direct current. With this circuit there will still be a relatively small net A-C voltage in the D-C circuit which is due to distortion of the induced voltages from a sinusoidal wave shape. This in turn is caused by resistances in the primary circuits of both test sample and auxiliary transformer. The net induced voltage will be of harmonic frequencies. However, since the net voltage is of relatively low magnitude and of higher frequencies, a very small inductance in the D-C circuit is adequate. The presence of an A-C current can be detected with an oscilloscope or with the RMS-reading ammeter as shown. If the reading of the meter is the same with and without the primary or A-C winding being energized, then the A-C component is negligible.

The circuit of Fig. 5-3 has apparently not been used to obtain data on magnetic materials. The circuit is essentially the same as that of a simple, parallel-connected magnetic amplifier in which load resistance is very small and control-circuit (comparable to the D-C circuit of Fig. 5-3) impedance is high. The major difference between the magnetic amplifier and the test circuit is that the flux density variation in the magnetic amplifier is not sinusoidal, and the maximum value of B is limited to the saturation value. Also, the maximum current corresponding to maximum H in the magnetic amplifier is determined approximately by instantaneous applied voltage and load resistance, rather than by maximum B, as in the test circuit of Fig. 5-3. Because of these where I_{pL} = RMS load component of primary current,

I_s = RMS secondary current,

I_{DC} = average secondary current.

For the case of an unfiltered output resistance (the first load circuit of Fig. 5-4, the secondary RMS current is 1.57 I_{DC}, and the primary component of load current, from 5-10, is 1.21 I_{DC}. Therefore primary current may be less than secondary current, but the addition of excitation in practical designs will make it greater.

Total primary input can be calculated by summing in-phase and quadrature volt-ampere components. The input component of output volt-amperes is secondary voltage times the load component of primary current.

$$W_{pL} = V_s I_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} \qquad (5\text{-}11)$$

Other real power components are the winding losses W_c and core losses W_i. The magnetizing component of excitation power is

$\sqrt{W_{ex}^2 - W_i^2}$, where W_{ex} is the excitation volt-amperes given by

the curves. Leakage reactance is neglected, so that the difference between primary and secondary terminal voltages (with unity turns ratio) is due to resistance drops in the windings. Approximate primary current is approximate input volt amperes divided by primary voltage V_p.

$$I_p = \frac{W_{rp}}{V_p} = \frac{1}{V_p} \sqrt{(W_{pL} + W_c + W_i)^2 + W_{ex}^2 - W_i^2}, \qquad (5\text{-}12)$$

where W_{rp} = approximate total primary volt amperes,

V_p = primary voltage.

A more refined method than equation (5-12) is not justifiable in view of the variable nature of excitation among similar cores. However, the results obtained with the equation tend to be a few per cent low. A reason for this is that both W_{pL} and W_{ex} have higher harmonic components of the same frequency which are effectively added in quadrature regardless of actual phase. An inspection of Fig. 5-5c shows that the second harmonics of the two components are actually in phase (of the phase $+I_2 \cos 2\omega t$), so that the approximate calculation might be expected to give a low result.

Load Tests

During the magnetic tests which have been described, only direct current flowed in the secondary winding of the test transformers. In order to check these results, several tests were made on Models No. 1 and 3 loaded with a half-wave rectifier. The optimum values for the non-magnetic gap were found to check; that is, the same gap gives minimum primary current during load tests and minimum excitation during the no-load or core tests for corresponding A-C flux density and unbalanced magnetization.

B-H curves for the load tests were found to be similar to those for the core tests. The density B was applied to the vertical plates of an oscilloscope using induced voltage from a core winding fed through an integrating circuit. The magnetizing force H is proportional to the difference of primary and secondary currents. One primary and one secondary transformer lead were connected together and to one end of a small resistance. The primary and secondary circuits were then completed to the other end of the resistance. The voltage across this resistance measures the required current difference if winding polarities are correct.

Comparisons have also been made of calculated primary current using equation (5-12) and measured primary current. It is found that calculated current is lower than measured current by from four to 15 per cent for typical operating conditions. The principal reason for this discrepancy is that the method of combining primary current components is only approximate.

Another aspect of unbalanced operation is the effect of primary resistance, which tends to distort the flux wave shape. During one core test, external primary resistance of the order of the magnetizing impedance was added, but optimum core gaps are practically the same as with winding resistance alone, for the same RMS winding voltages and unbalanced magnetization.

Optimum Excitation and Flux Density

Part of a recent paper[16] is devoted to the development of criteria for selecting an optimum flux density for transformers without unbalanced direct current. The analytic approach used was to find the flux density which would yield maximum volt-ampere output from a given transformer, as primary voltage was varied After the optimum density, currents and voltages are found for a given transformer, the wire sizes and turns can then be adjusted so that the required voltages are obtained. However, this procedure is assumed in order to obtain analytic expressions for finding the optimum density. These equations can be used to evaluate a design. An attempt has been made to apply the same methods to transformers with unbalanced direct current. While it is probable that qualitatively-similar criteria exist as for the balanced types, preliminary work indicates that further efforts in this direction are not warranted. Certain interesting relations are briefly outlined.

Assuming that there is some optimum flux density, the RMS volt-ampere product of the secondary, $W_r = V_s I_s$, will vary little over a range of A-C densities B near the optimum density. It can also be assumed that the direct component of secondary current I_{DC} (and therefore the average unbalanced magnetomotive force H_{DC}) is proportional to RMS secondary current I_s. Since the number of turns is considered constant in an example, A-C

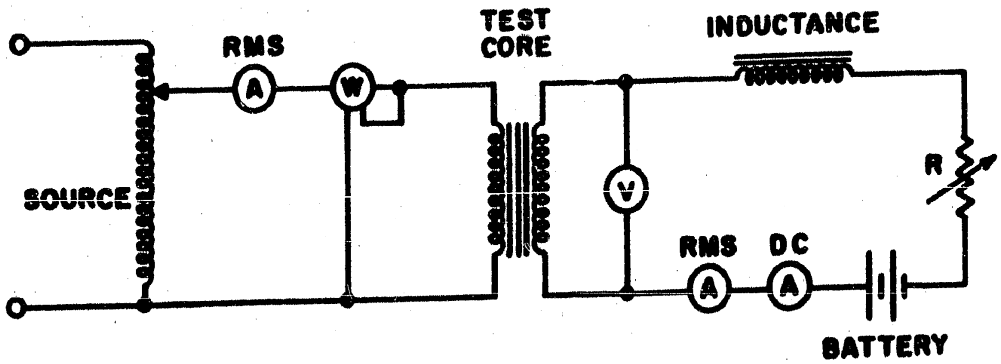


FIG. 5-2 CIRCUIT FOR MAGNETIC TESTS, REQUIRING A LARGE INDUCTANCE

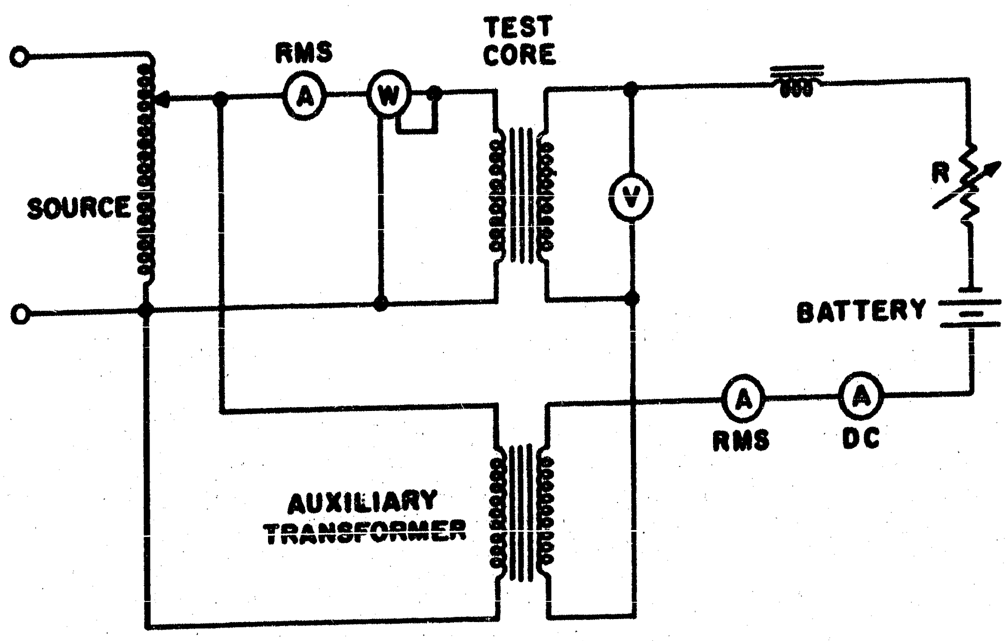


FIG. 5-3 CIRCUIT FOR MAGNETIC TESTS, USING AN AUXILIARY TRANSFORMER

factors, the data given for magnetic materials in following sections cannot be applied to magnetic amplifier problems. Magnetic amplifier data are often given in terms of voltages, current, impedances and winding parameters. It is believed that such data would be more general and readily applicable if quantities such as flux density, magnetic field strength and excitation in volt-amperes were used instead.

Test Results and Design Curves

The variables which have been studied experimentally to compile data for design use are A-C incremental flux density B, average magnetic field strength H_{DC}, length of non-magnetic gap, grade of core material and thickness, core geometry and frequency of the power supply. Considerable data have recently been made available by Battelle Memorial Institute[6]. However, one important variable not considered was the non-magnetic gap, and the values of unbalanced magnetic field strength are limited to four oersteds.

Four cores were selected as representative of geometry, size, lamination thickness and grade of material which are predominantly used in small electronic power transformers. In using the circuit of Fig. 5-3, it would be desirable to measure core loss directly with the wattmeter. However, the window areas of typical small cores is not sufficient to accommodate the large wire sizes in the A-C winding that would make winding losses negligible. This is a disadvantage which could be overcome by using a much smaller ratio of core cross section to window area. It is less likely that this difficulty would be encountered on much larger cores, because flux density is then limited by core loss rather than by exciting current. With the typical small cores selected, it is easy to measure winding resistance and to calculate winding loss. This is subtracted from total input losses to obtain core loss. It is desirable to use a low power factor type wattmeter for magnetic measurements. The one used for these tests was a Weston Model 310, Form 2.

Descriptions of the four cores and graphical results are given in Appendix E. These data are considered to be typical, but since only one sample of each was tested, the variations which would exist among cores of the same type and construction are not known. However, the relative characteristics should be the same. Figures E-1 to E-10 give excitation (volt amperes per pound) and core loss (watts per pound) for the four cores at typical A-C flux densities. The abscissa H_{DC} is defined by equation (5-1). The parameter is the effective per cent non-magnetic gap, or the ratio m_g/m_i times 100. The term m_g includes the appropriate weighting factor from Table 5-1, and is therefore equal to the given factor plus the sum of the actual gap lengths. The correction factor is particularly important for relatively small cores. Typical excitation data at one density, such as Fig. E-1, show the effect of the gap. For a particular value of average magnetic field strength H_{DC}, several values of excitation can be obtained with various gaps. Similarly, typical core loss data at one density are given, such as in Fig. E-3, and the magnitude of the gap also has an effect upon core loss.

Results of the tests have been studied to find if the gap which gives minimum excitation also yields minimum core loss. If the values of non-magnetic gap for minimum core loss and minimum excitation are appreciably different, then both should be considered in selecting a joint. In general, there is a fair correlation of conditions for minimum core loss and minimum excitation current. That is, minimum core loss and excitation, at the various values of density B and magnetization H_{DC}, are usually obtained with the same, or almost the same effective gap. There is better correlation between the conditions for minimum total loss and minimum excitation current. Total input loss in these tests is the sum of core loss and primary winding loss. Since core proportions and winding sizes in the experimental transformers are typical of those that might be used in production units, it is indicated that designing for minimum excitation will tend to yield minimum total losses, even though core losses are not quite at the minimum in all cases.

In accordance with the foregoing discussion, it is desirable to select a non-magnetic gap which yields minimum excitation. Therefore, the design conditions are determined by the envelopes of the curves for excitation at each flux density. From the data for a large range of flux densities, two design curves have been derived for each of the four cores as given in Fig. 13-1 to 13-8. Each design curve for excitation permits the determination of excitation (the ordinate) and non-magnetic gap from two independant quantities, flux density B (the abscissa) and magnetic field strength H_{DC} (the parameter). The appropriate values of effective gap are marked off on the curves. The design curves for finding core loss also yield this quantity as a function of density and magnetic field strength. The core loss values given are those for the per cent gap which yields minimum excitation. The fact that this does not always correspond to the condition for minimum core loss makes some of the core loss design curves appear to be somewhat erratic.

Qualitatively, it can be seen from the design curves that non-magnetic gap is likely to be indicated when flux density is intermediate or low, and field strength is high. Approximate empirical equations have been obtained to relate the three variables: per cent effective gap, flux density and magnetic field strength. For each of the four cores tested there is an equation of the form

$$\%\ \text{gap} = P\,H_{DC} - Q\,B + R, \qquad (5\text{-}9)$$

where P, Q, and R are parameters, which have the values given in Appendix E.

Observation of the wave shapes of field strength as a function of time and of B-H loops on an oscilloscope shows that the core performance is in agreement with the analysis presented earlier. An increase of the non-magnetic gap makes the wave shape of the excitation current more nearly sinusoidal, and decreases the peak-to-peak value of H or current. A comparison of B-H curves shows that an increase in gap decreases the slope of the loop for small H, and also decreases the maximum H in the saturation region.

Comparisons of Data

Test results for cores No. 1, 3, and 4 have been compared with the results recently obtained by Battelle Memorial Institute[6]. These comparisons are restricted to the conditions of minimum gap in the magnetic circuit, and to low values of magnetizing force H_{dc}, below four and two oersteds for the

60-cycle and 400-cycle tests, respectively. In view of material variability, the comparison of results can be considered good.

Battelle gives test results for three wound cores of 14 mil oriented material tested at 60 cycles, comparable to No. 1. The cores represented relatively good, average and poor characteristics of a number of samples. Excitation and core loss results obtained for core No. 1 were within or close to the range of values from the Battelle tests. Core No. 3 was compared with a wound core of five mil, oriented material tested at 400 cycles by Battelle. Losses of core No. 3 were found to be somewhat lower, but values for excitation are close.

Data are given by Battelle for a core of four mil laminations. Comparison of excitation values with the results for core No. 4 shows excellent agreement. Maximum differences are only ten per cent. Core loss values check well up to incremental flux densities of 70 kilolines, but from 80 to 100 kilolines, the losses of No. 4 average about 20 per cent lower, which is not an abnormal variation.

It may sometimes be desirable to use data for non-gapped cores to estimate the performance of cores with gaps. Since core loss is affected very little by small gaps, core loss data for the proper magnetization and density should give a reasonable value. However, the excitation values of non-gapped cores may be up to 40 per cent higher than those of a gapped core at the same magnetization and A-C or incremental flux density.

Properties of a Core Joint

From the tests, it has been found that core loss generally increases as unbalanced magnetization H_{DC} increases, for a certain incremental flux density B. With fixed B and H_{DC} and an increasing gap, the core loss variation is at first unpredictable. Some tests show an initial decrease in loss while others show an increase in loss. As the gap becomes fairly large, that is, considerably greater than the values for minimum excitation, the core loss tends to increase with gap length.

Increase in core loss for these conditions seems to be attributable only to flux-fringing losses at the non-magnetic gap. For a non-gapped core, the losses increase with increasing H_{DC}. Since H_{DC} is an average of the magnetomotive force around the core, an increase in gap length means that actual magnetomotive force in the steel must decrease, if H_{DC} is held constant. Therefore, it might be expected that core loss would decrease with an increase in gap, for constant B and H_{DC}. Since this is not what is found by test for large gaps, fringing of the flux at the gap is believed to cause the increase. Flux entering a lamination perpendicular to the plane of the lamination can induce much higher eddy currents than when entering parallel to the plane of the lamination.

Several 60-cycle tests have been made on core No. 1 (wound-type) to investigate joint losses. It is well known that fringing is greater from a joint not surrounded by a winding than from a joint beneath a winding. To find the effect of joint location on losses and excitation, one joint of core

No. 1 was ground down so that the gap could be located entirely in one core leg. Tests were than made with the winding structure located on the same leg as the gap, and on the leg opposite from the gap. Gap sizes of 0.0054, 0.0108 and 0.0216 inch were tried. Excitation and losses were measured for each of these gaps, and for the winding located on the same and opposite legs from the gap, at several values of incremental density and unbalanced magnetization.

It is found that excitation is lower by 15-20 per cent for the 0.0216 inch gap, when the gap is on the leg opposite the winding. This indicates greater fringing when the gap is outside of the winding, or a greater effective flux cross-sectional area, and therefore a higher magnetising inductance. Losses are found to be 10-20 per cent lower when the gap is on the leg opposite from the winding. This is evidently caused by a reduction of density in a large part of the core path outside of the winding. The core gap increases the reluctance of the outer part of the magnetic circuit so much that considerable flux may pass through long non-magnetic paths. The increase in total losses which might be expected with higher fringing losses when the gap is on the outside, is therefore overbalanced by the reduction in density outside of the winding.

To find if core loss could be decreased by reducing the fringing flux, a copper shield was placed around the core gap outside of the winding. This shield consisted to two turns of a thin copper sheet 1-1/2 inches wide, insulated between turns and from the core. Such a shield tends to confine the flux, because of the magnetomotive forces arising from eddy-currents in the plane of the conducting sheet. It was found that the shield has no measurable effect on losses or excitation. This result might be attributed to long leakage-flux paths completely outside the core and shield, as well as to fringing beneath the shield, since some separation between shield and core is unavoidable. In addition there are some eddy-current losses in such a shield.

Even though losses and excitation of a core are less when the gap is outside of the winding, it is normally better practice to place the gap inside when large gaps are used. With the gap outside, the higher stray fields can cause local heating in structural parts such as the case, or can cause noise in signal circuits. However, the effects of joints on core loss are small for the relatively small sizes of non-magnetic gap which are found to give minimum excitation. Therefore, the gap can best be provided in the easiest manner for a particular core. A wound core with two butt joints should have spacers in each joint to give the suitable total gap; grinding one joint is not justifiable. Similarly, laminations may be stacked with spacers placed where the butt joints normally occur.

Half-Wave Rectifier Supply Transformer

The application of the data and analysis to the half-wave rectifier supply transformer will be considered. In a full-wave rectifier circuit, the current in each half of the secondary has a direct-current component, but the total D-C ampere turns of the two halves is zero in the ideal case.

There may be some unbalance due to dissimilar rectifier characteristics, and there may be an unbalance due to dissymmetrical location of the two halves of the secondary windings, but these effects are usually negligible, particularly in small transformers. In large transformers, elimination of any dissymmetry is very desirable. In the event that a transformer has two different secondaries which are each to supply a half-wave rectifier, then terminal connections should be made so that the net unbalance affecting the core is the difference, rather than the sum of the two unbalancing magnetomotive forces.

The half-wave rectifier supply transformer connection, shown in Fig. 5-4, is almost always mentioned briefly in discussions of rectifier circuits, but its analysis and design problems have received very little attention. The half-wave rectifier supply is used in relatively few, but nevertheless important applications. Among these are electronic power supplies -- particularly for bias power, battery-charging circuits and high-potential sources such as those in electrostatic dust precipitators.

The design or evaluation of transformers used in half-wave power is similar to balanced types in that size depends upon rating, density in the core and current densities in the windings. In order to obtain the smallest size, it is necessary to use the highest flux and current densities that heating or other limitations will permit. Winding wire sizes should be chosen according to the root-mean-square (RMS) currents which determine losses and therefore heating. Secondary RMS current is a function of the secondary voltage, transformer impedances and the output circuit. Primary RMS current is principally a function of secondary current and excitation current. With unbalanced magnetization, the excitation current is not the same as no-load current, as it is, approximately, in balanced transformers. The problem in core design is to select the highest flux density in order to approach one of the two possible limits, either core loss or excitation volt amperes. Both of these quantities are a function of load current, flux density and geometry.

In order to design a transformer with a core of suitable cross-sectional area and window size and with windings of proper cross section and turns, it is necessary to have information on core characteristics under unbalanced conditions and to calculate winding currents and voltages for a given load circuit. The necessary data on magnetic materials have been presented in following chapters or in Appendix E, and following sections show how this information can be applied to the transformer. Secondary current may be calculated by well-known methods, but primary current presents more of a problem. The correct turns ratio must then be used in order to obtain specified winding voltages.

Two typical load circuits have been shown in Fig. 5-4, the resistance load, and a resistance load with capacitance filter. The latter circuit is also equivalent to the battery-charging circuit. It will be noted that an inductance-input filter is absent. Such a filter is used in other types of rectifiers to obtain an almost steady output current and voltage. This effect cannot be obtained with the half-wave circuit. With a sinusoidal

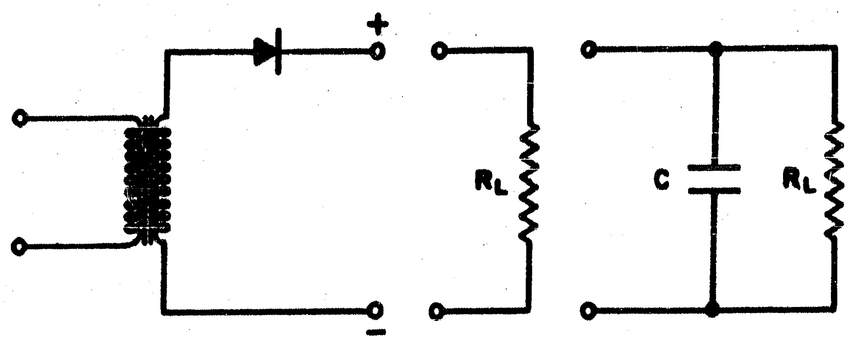


FIG. 5-4 BASIC HALF-WAVE RECTIFIER CIRCUITS

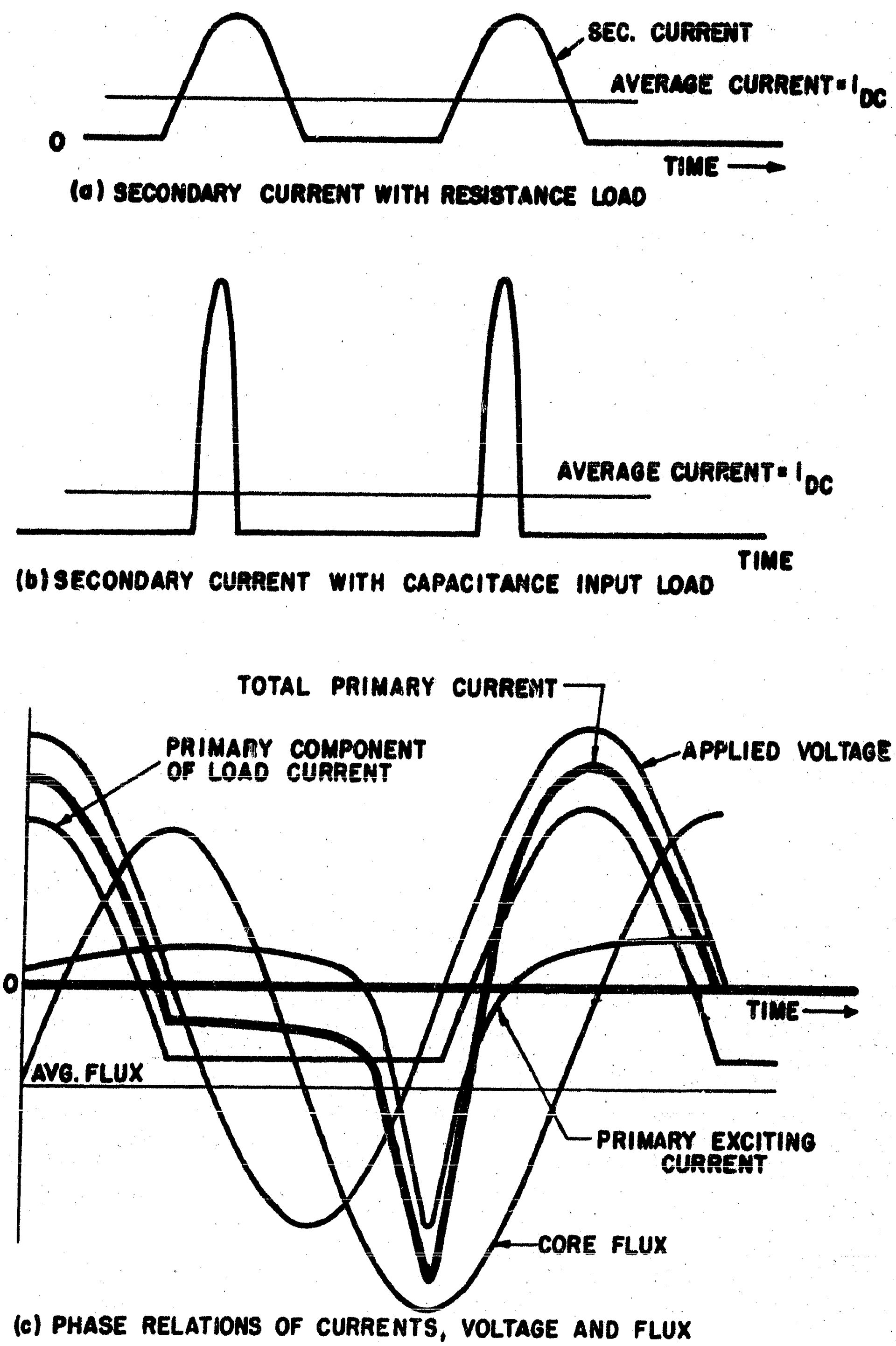


FIG. 5-5 WAVE SHAPES IN HALF-WAVE RECTIFIER SUPPLY TRANSFORMER

voltage input to the primary, the secondary current for the resistance load will be half-wave pulses of undirectional current. as shown in Fig. 5-5a. With the capacitance-input filter, the current consists of sharply-peaked pulses every cycle, as in Fig. 5-5b. For this filter, as for a battery charger, current only flows during the interval when the output voltage exceeds the voltage of the capacitance, With an inductance-input filter, Seely[15] shows that the peak magnitude of current is reduced over that of the resistance load, and that time of conductance becomes greater than one-half cycle. The average output current is reduced, depending on the size of the inductance. If it were possible to draw a steady, ripple-free current with a large inductance, then the rectifier could be removed. Obviously, this would not work. When suitable parameters are used, the two output circuits of Fig. 5-4 are equivalent to more complex filter circuits, insofar as the transformer is affected.

For the resistance load the wave-shape of secondary current is fixed, while for the capacitance-input load, wave shape depends upon the product of load resistance, capacitance and frequency, and upon the series resistance up to the load, including the winding, rectifier and leads. One problem is to find the relation of average load current to RMS currents in the windings. When the output of the rectifier is simply a resistance as in Fig. 5-5a, an ideal rectifier permits conduction for exactly half of each cycle, and the current is in phase with the secondary voltage. The average D-C current is found to be 0.318 times the peak current. Since the RMS value of this wave shape is 0.500 times peak current, the ratio of secondary RMS current to direct current is 0.500/0.318 = 1.57. Therefore the secondary winding will heat as though it carried 1.57 times the value of average load current. The ratio of voltages must also be determined. Since load voltage wave shape is similar to load current, average load voltage is 0.318 times the peak value. The secondary RMS voltage is 0.707 times its peak, so that the ratio of secondary RMS to average load voltage is 0.707/0.318 = 2.22.

With the capacitance input circuit, the peak of the current occurs in time slightly before the peak of the voltage because the difference between secondary and capacitor voltages is greater when secondary voltage is increasing toward, rather than decreasing from the peak value. The ratio of secondary RMS current to load direct current may be found from rather involved analysis, or use may be made of curves given by Schade[14]. The ratios are given directly as a function of circuit resistance and of load resistance divided by filter reactance = $2\pi fCR_L$ or 2π times frequency times capacitance times load resistance. For reference, some values from Schade's curves have been adapted and listed in Table 13-1 for the ratio of secondary RMS current to load direct current.

Table 13-2 also gives the ratio of peak secondary current to load direct current, a quantity useful for selecting a rectifier of proper peak current rating. Table 13-3 gives the ratio of RMS secondary voltage to average load voltage.

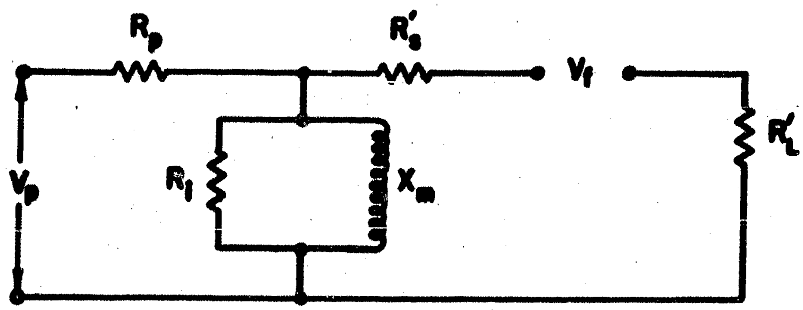
Transformer Circuit Frequency Components

Consideration of the different frequency components of currents and voltages leads to a relatively simple method for finding the primary current and the magnetic characteristics of the transformer with unbalanced magnetization. Although there is a direct-current component in the secondary caused by the rectifier, there can be no steady-state, D-C voltage in the primary, because the supply is assumed to be sinusoidal, and because an induced D-C voltage would require that the core flux increase continuously in one direction. Since there will be some resistance in the primary winding and connected supply, any unidirectional transient current will decay to zero. If a zero-resistance supply and winding were possible, then a primary direct current would also be possible. However, further consideration is limited to the practical steady-state condition.

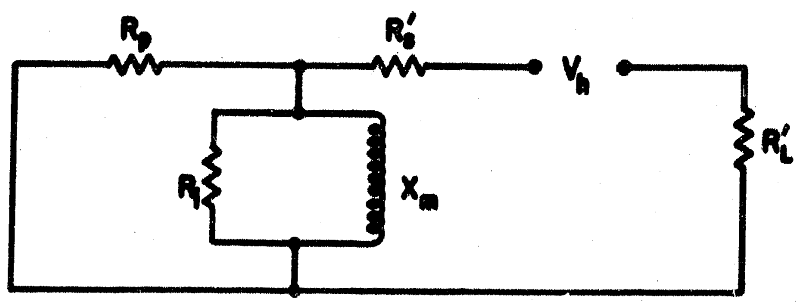
The principle of super position is the basis for a very useful method for analyzing linear circuits. If all impedances are linear, then the currents and voltages may be found by solving for the contributions resulting from each voltage or current source, and then adding these. Similarly, super position may be used for analyzing separately the voltages and currents due to each harmonic frequency of any one source, and then adding these components for each instant of time.

In the circuit consisting of a transformer supplying a half-wave rectifier, there are in general two nonlinear elements, the rectifier in the secondary circuit and the impedance corresponding to the excitation required to establish a varying flux in the core. An ideal rectifier may be considered as a voltage source instead of an impedance. As such, the rectifier has zero impedance during forward conduction, and is equivalent to a voltage equal and opposite to the impressed voltage during non-conduction. Thus when considered as a voltage source, the rectifier supplies a half-sine wave with magnitude corresponding to the secondary voltage during the non-conducting half cycles. This voltage function consists of an average or D-C component, a fundamental-frequency component which is approximately half of the secondary voltage, and of higher even-harmonic voltages. If, then, the magnetizing impedance of the transformer were linear, the equivalent circuit of the transformer consists of linear impedances and two voltage sources, the voltage applied to the primary and the rectifier voltage. The solution for currents and voltages can then be obtained by solving the individual equivalent circuits shown in Fig. 5-6. Fig. 5-6a is a circuit for fundamental-frequency currents; 5-6b is a circuit for harmonic frequencies; and 5-6c is a circuit for the D-C component.

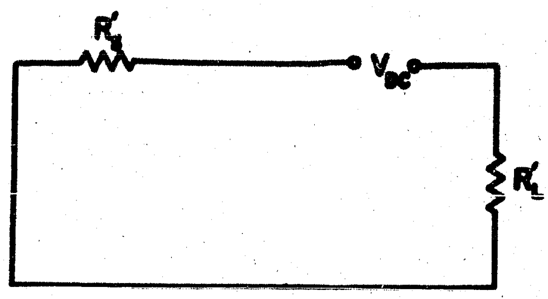
The component circuits Fig. 5-6 would be applicable in case the magnetic core material were operated in regions of induction B and magnetizing force H where the relation between these two is practically linear. The analysis would also apply for an air-cored transformer. Primary and secondary resistances are assumed to be relatively small, so that the harmonic voltage in Fig. 5-6b will be impressed almost entirely across the load resistance R_L. Similarly, the D-C voltage in Fig. 5-6c will almost all appear across the load resistance. However, the direct current which flows in this circuit provides an unbalanced magnetizing force on the core which



(a) FUNDAMENTAL-FREQUENCY CIRCUIT



(b) HARMONIC-FREQUENCIES CIRCUIT



(c) D-C COMPONENT CIRCUIT

FIG. 5-6 EQUIVALENT CIRCUITS OF COMPONENT VOLTAGES WITH LINEAR IMPEDANCES.

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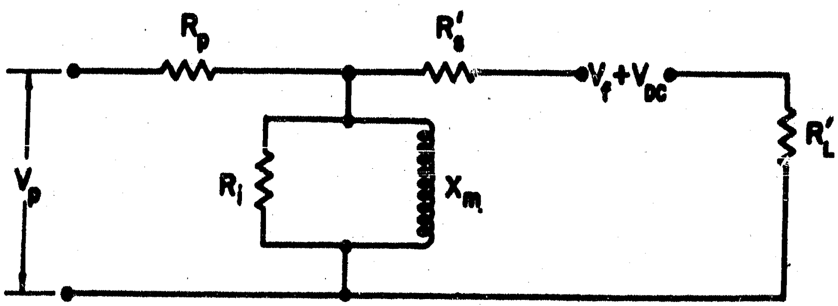
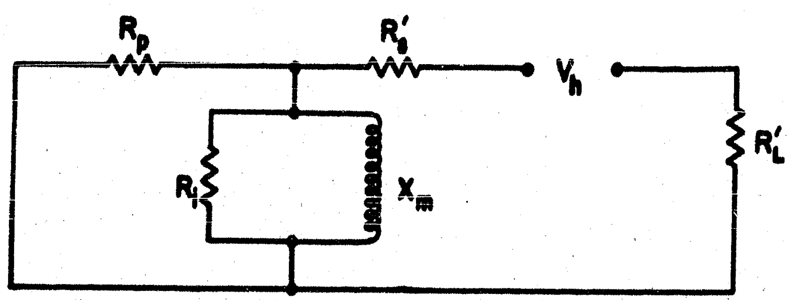


(a) FUNDAMENTAL-FREQUENCY AND DC COMPONENTS CIRCUIT



(b) HARMONIC-FREQUENCIES CIRCUIT

FIG. 5-7 EQUIVALENT CIRCUITS OF COMPONENT VOLTAGES WITH NON-LINEAR MAGNETIZING INDUCTANCE.

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is not represented by a voltage drop in the circuit.

Next, the effect of a nonlinear magnetizing impedance can be considered. The principle of super position prohibits separation of the current and voltage components associated with a nonlinear impedance. However, currents and voltages of harmonic frequencies across the magnetizing impedance X_m are very small, as can be deduced from Fig. 5-6b. Therefore it is possible, when winding resistances are relatively small, to analyze the circuit as in Fig. 5-7. Here the fundamental frequency and D-C components are together in one circuit. The equivalent rectifier voltage in Fig. 5-7a consists of fundamental-frequency and D-C components only.

These equivalent circuits are an aid in showing how the electrical quantities affect the operation of the core. If the applied voltage is nearly sinusoidal then the core flux will also be nearly sinusoidal. The other effect upon the core is that of the direct-current component in the secondary which is determined by the magnitude of the D-C voltage component and the secondary circuit resistance. The magnitude of the D-C magnetizing force resulting from this current depends upon the number of secondary turns and upon the core geometry, according to equation (5-1).

Primary Current

The significance of the equivalent-circuit analysis is that all alternating components of secondary ampere turns are equal to the load component of primary ampere turns. The sum of this load component and the excitation component is the primary current. The A-C excitation current function of time plus the secondary direct-current component determines the total magnetomotive force which is related to the core flux function by the magnetization curve of the material. The primary component of load current for a half-wave rectifier with a resistance load is shown in Fig. 5-5c as a half sine-wave function without an average component. It is next necessary to establish the excitation component in terms of phase and qualitative form. Since the peaks of the flux function lag the peaks of the applied voltage by one quarter cycle by definition, the peaks of the excitation component must also lag the voltage peaks by one quarter cycle or 90 electrical degrees.

It must be found which peak of the excitation current is sharper because of the saturation effect. To do this, the primary load component of Fig. 5-5c can be considered for the moment as two currents, one exactly like the secondary current, including D-C component, plus a constant negative part equal to the average of the other. If this second negative part were absent, then average core flux would be zero, because the sum of the first part and secondary current would provide no unbalanced magnetomotive force. The fact that the hypothetical second part is negative indicates that saturation tends to occur during the negative half cycles of the flux and excitation current, as shown in Fig. 5-5c. This reasoning establishes the qualitative form of excitation current and the displacement of the average flux as given in the figure. Next, the sum of the load and excitation components gives the time function of total primary current. For very small flux densities the primary current becomes very similar to the given load component.

It should be appreciated that an ideal core material having definite permeability cannot be assumed in an analysis such as this, because the presence of the unbalance in the secondary would yield an infinite flux. Therefore, an excitation component cannot be eliminated in the consideration of an ideal material. However excitation current could be made small compared to the primary load component of current if the A-C density were so small that the cyclic variation in magnetic field strength (and corresponding ampere turns) were relatively small compared to load-current ampere turns. These conditions require a magnetic material with less than infinite permeability.

The qualitative shape of primary current has now been established, and next it is necessary to find methods for calculating the RMS magnitude which determines the size of wire needed for the winding. One method for finding primary current is by graphical addition of the components as in Fig. 5-5c, followed by computation of the RMS value from the resultant time function, that is, by squaring the function, taking the time average and then the square root of the average. However, this procedure would be very time consuming for a design computation. A better method is based upon elementary A-C circuit algebra. In general, both excitation and load components of primary current are non-sinusoidal. If the magnitude and phase of all frequency parts of each component were known, then the value of primary current could be readily determined. The resultant of each frequency might be found by combining the parts of the same frequency as phasors. Finally, the total current is determined by adding in quadrature the contributions at all different frequencies. These calculations can be made using and obtaining RMS values of current.

The excitation and load components of primary current have fundamental-frequency components which are almost in quadrature, or one quarter cycle out of phase. If in addition the two components were sinusoidal, or if only one were sinusoidal and the other non-sinusoidal, or if the two components contained different higher harmonics, or if the corresponding harmonics in the components were in time quadrature, then the RMS values of the two components could be combined in quadrature to obtain accurately the total RMS primary current. Although none of these conditions is satisfied in general, the principles suggest combination of the two components in quadrature to obtain an approximation of primary current, and in practice, this is a suitable procedure.

For a two-winding transformer having a turns ratio of unity, the RMS load component of primary current is defined as the alternating parts of the secondary current function, which is simply the secondary current with the D-C component removed. Since the RMS value of secondary current is precisely the quadrature sum of D-C and all A-C components, it follows that

$$I_{pL} = \sqrt{I_s^2 - I_{DC}^2} \qquad (5\text{-}10)$$

where I_{pL} = RMS load component of primary current,

I_s = RMS secondary current,

I_{DC} = average secondary current.

For the case of an unfiltered output resistance (the first load circuit of Fig. 5-4, the secondary RMS current is 1.57 I_{DC}, and the primary component of load current, from 5-10, is 1.21 I_{DC}. Therefore primary current may be less than secondary current, but the addition of excitation in practical designs will make it greater.

Total primary input can be calculated by summing in-phase and quadrature volt-ampere components. The input component of output volt-amperes is secondary voltage times the load component of primary current.

$$W_{pL} = V_s I_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} \qquad (5\text{-}11)$$

Other real power components are the winding losses W_c and core losses W_i. The magnetizing component of excitation power is

$\sqrt{W_{ex}^2 - W_i^2}$, where W_{ex} is the excitation volt-amperes given by

the curves. Leakage reactance is neglected, so that the difference between primary and secondary terminal voltages (with unity turns ratio) is due to resistance drops in the windings. Approximate primary current is approximate input volt amperes divided by primary voltage V_p.

$$I_p = \frac{W_{rp}}{V_p} = \frac{1}{V_p} \sqrt{(W_{pL} + W_c + W_i)^2 + W_{ex}^2 - W_i^2}, \qquad (5\text{-}12)$$

where W_{rp} = approximate total primary volt amperes,

V_p = primary voltage.

A more refined method than equation (5-1` is not justifiable in view of the variable nature of excitation among similar cores. However, the results obtained with the equation tend to be a few per cent low. A reason for this is that both W_{pL} and W_{ex} have higher harmonic components of the same frequency which are effectively added in quadrature regardless of actual phase. An inspection of Fig. 5-5c shows that the second harmonics of the two components are actually in phase (of the phase $+I_2 \cos 2\omega t$), so that the approximate calculation might be expected to give a low result.

Load Tests

During the magnetic tests which have been described, only direct current flowed in the secondary winding of the test transformers. In order to check these results, several tests were made on Models No. 1 and 3 loaded with a half-wave rectifier. The optimum values for the non-magnetic gap were found to check; that is, the same gap gives minimum primary current during load tests and minimum excitation during the no-load or core tests for corresponding A-C flux density and unbalanced magnetization.

B-H curves for the load tests were found to be similar to those for the core tests. The density B was applied to the vertical plates of an oscilloscope using induced voltage from a core winding fed through an integrating circuit. The magnetizing force H is proportional to the difference of primary and secondary currents. One primary and one secondary transformer lead were connected together and to one end of a small resistance. The primary and secondary circuits were then completed to the other end of the resistance. The voltage across this resistance measures the required current difference if winding polarities are correct.

Comparisons have also been made of calculated primary current using equation (5-12) and measured primary current. It is found that calculated current is lower than measured current by from four to 15 per cent for typical operating conditions. The principal reason for this discrepancy is that the method of combining primary current components is only approximate.

Another aspect of unbalanced operation is the effect of primary resistance, which tends to distort the flux wave shape. During one core test, external primary resistance of the order of the magnetizing impedance was added, but optimum core gaps are practically the same as with winding resistance alone, for the same RMS winding voltages and unbalanced magnetization.

Optimum Excitation and Flux Density

Part of a recent paper[16] is devoted to the development of criteria for selecting an optimum flux density for transformers without unbalanced direct current. The analytic approach used was to find the flux density which would yield maximum volt-ampere output from a given transformer, as primary voltage was varied After the optimum density, currents and voltages are found for a given transformer, the wire sizes and turns can then be adjusted so that the required voltages are obtained. However, this procedure is assumed in order to obtain analytic expressions for finding the optimum density. These equations can be used to evaluate a design. An attempt has been made to apply the same methods to transformers with unbalanced direct current. While it is probable that qualitatively-similar criteria exist as for the balanced types, preliminary work indicates that further efforts in this direction are not warranted. Certain interesting relations are briefly outlined.

Assuming that there is some optimum flux density, the RMS volt-ampere product of the secondary, $W_r = V_s I_s$, will vary little over a range of A-C densities B near the optimum density. It can also be assumed that the direct component of secondary current I_{DC} (and therefore the average unbalanced magnetomotive force H_{DC}) is proportional to RMS secondary current I_s. Since the number of turns is considered constant in an example, A-C

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flux density is proportional to RMS voltage V_s. Therefore, if $W_r = V_s I_s$ is constant over a range of A-C densities, I_s, I_{DC} and H_{DC} are all inversely proportional to A-C density B. The design curves for primary excitation volt-amperes W_{ex} versus density B with the parameter H_{DC} have been studied to find how W_{ex} varies with B, subject to the condition that B is inversely proportional to H_{DC}. Such functions are readily derived point by point and are found to be much less steep than the given curves for W_{ex} versus B at a constant H_{DC}. Although not a precise linear relation, excitation volt-amperes W_{ex} are roughly proportional to A-C density B.

It is possible to derive equations for the unbalanced transformer which are similar to equations (7) and (10) of the reference[18]. However the above discussion indicates that the exponent n, defined by $W_{ex} = K_1 B^n$ is roughly equal unity. In view of this, the reference equations (7) and (10) show that the optimum is roughly independent of the selected A-C density B. Therefore, in the absence of a more concrete guide, it is recommended that designs be made so that excitation volt-amperes are in the range of 40 to 80 per cent of the secondary RMS volt-ampere product.

Example designs indicate that such a balance between excitation and load volt-amperes for a transformer with unbalanced direct current is obtained when the selected A-C flux density is about ten per cent lower than the density which would be used for a unit without the unbalance. Since desirable values for A-C density and per cent excitation are functions of unbalanced magnetization H_{DC} as defined by equation (5-1), some study has been given to the effects of size and proportions on H_{DC}. Total winding ampere turns are proportional to window area times current density times winding space factor. This indicates that, for constant current density and space factor, H_{DC} is proportional to window area divided by length of the magnetic circuit. In a given size and rating, H_{DC} can be reduced somewhat by increasing core cross section and reducing window area, a change which results in a higher proportion of core volume to winding volume.

Next, consider the effect of increasing size and rating, holding all geometric proportions fixed. The fact the H_{DC} is proportional to an area (window area) divided by a length means that H_{DC} would increase linearly with size if current density and space factor are constant. However it can be shown that to hold winding losses per unit of winding exposed surface area constant because of temperature-rise limitations, current density should be reduced, and changed inversely as the square root of linear size. The combined effect including geometric factors and current density indicates that H_{DC} is directly proportional to square root of linear size. Therefore unbalanced magnetomotive force will tend to be higher in larger transformers.

Regulation and Turns Ratio

In transformers carrying sinusoidal currents, corrections for voltage drops can be made using the RMS current and resistance for each winding (and the leakage-reactance, if appreciable). When secondary current or voltage wave shapes differ appreciably from sinusoidal forms, the usual methods will yield voltage ratios which are excessively in error. This problem has been met in tests on model transformers with unbalanced direct current, which usually have complex current wave shapes.

To find a sufficiently accurate relation between primary and secondary voltages, an analysis has been made which is based upon an assumed sinusoidal applied primary voltage. If the secondary current is not sinusoidal, secondary voltage must likewise be non-sinusoidal unless the transformer has zero equivalent series impedance. Secondary RMS voltage may be defined as an exact function of its several components.

$$V_s = \sqrt{V_{sf}^2 + V_{sh}^2 + V_{sDC}^2} \text{ RMS volts} \qquad (5\text{-}13)$$

where V_{sf} = RMS fundamental volts,

V_{sh} = RMS harmonic frequency volts,

V_{sDC} = voltage due to direct current.

The term V_{sh} is composed of the second and higher harmonics, all of which add in quadrature to yield V_{sh}.

Next, equations may be written for the secondary voltage components in terms of the respective current components and impedances. These are obtained by treating each of the three components of V_s independently.

$$V_{sf} = \frac{V_p}{n} - I_{sf} R', \qquad (5\text{-}14)$$

$$V_{sh} = I_{sh} R', \qquad (5\text{-}15)$$

$$V_{sDC} = I_{DC} R_s, \qquad (5\text{-}16)$$

where V_p = primary voltage

n = turns ratio

R' = transformer equivalent series resistance referred to the secondary, ohms

R_s = secondary resistance, ohms.

To include the effects of leakage reactance, total impedance Z' could be substituted in equations (5-14) and (5-15) for R', but it would be necessary to consider the terms of the equations as phasors rather than as magnitudes. The calculation of secondary RMS load voltage V_s from equations (5-13) to (5-16) can be considered as successive operations on the no-load secondary voltage V_p/n. First a term $I_{sf} R'$ is subtracted algebraically, then two other terms are added successively in quadrature. However, in transformers the voltage drop terms are always small compared to V_p/n. Only the term

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subtracted directly, according to (5-14) makes an appreciable change in V_p/n; the addition of the quadrature terms can be neglected. It follows that V_s is closely equal to V_{sf}, and that the difference between V_p/n and V_s depends only upon the voltage drop of the fundamental-frequency component of secondary current through the transformer equivalent resistance R .

The next problem is finding the fundamental-frequency current magnitude from the RMS current value. This depends upon the type of load circuit. For the case of a half-wave rectifier and resistance load without a capacitance filter, the current is a half-sine wave, and the RMS value of the fundamental component is 70.7 per cent of the total RMS secondary current. Another important ratio for this case is that of the secondary RMS current to its D-C component, which is 1.57. Regulation is

$$\% \text{ Reg} = \frac{V_p/n - V_s}{V_s}\,100 = \frac{I_{sf} R'}{V_s}\,100 = \frac{0.707\, I_s R'}{V_s}\,100$$

$$= \frac{0.707\, I_s^2 R'}{V_s \cdot I_s}\,100 = \frac{0.707\, W_c}{W_r}\,100 \qquad (5\text{-}17)$$

where W_c = winding losses, watts,

W_r = secondary RMS volts times RMS amperes.

A capacitance filter is commonly used across the resistance load of a half-wave rectifier. This tends to increase the ratio of secondary RMS current to D-C component above 1.57: a typical value is 2.0. The current wave shape with a filter becomes more peaked and current flows less than half of the entire period. Therefore the ratio of secondary RMS fundamental current to total RMS current will be less than 0.707, about 0.5 for the typical case, a value checked by tests of a model transformer.

The foregoing discussion shows that the previous equation for per cent regulation should be multiplied by some factor less than one. The two examples given, without and with a filter, show how an estimate can be made. It is observed that RMS fundamental current is more dependent on the D-C component than on the total secondary current. Therefore it is suggested that the following factor be used in (5-17) in place of the quantity 0.707.

$$\text{Correction Factor} = \frac{I_{sf}}{I_s} = \frac{1.1\ I_{DC}}{I_s} = \frac{1.1}{I_s/I_{DC}} \qquad (5\text{-}18)$$

The ratio I_s/I_{DC} is determined from Table 5-2 as a function of the circuit parameters.

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Design Procedure for a Transformer with Unbalanced Magnetization

The first step in the design of a transformer with unbalanced direct current is determining the equivalent secondary volt-ampere rating. Secondary RMS current is the D-C load current required, times the proper ratio of Table 13-1. If series resistance, load resistance, and filter capacitance are not known, a ratio of 2.0 can be considered typical. Similarly secondary RMS voltage is found as the D-C load voltage times the appropriate ratio of Table 13-3, or it may be specified for the transformer designer. If circuit conditions are unknown, a value of 1.1 is typical. Then the product of RMS current and RMS voltage is defined as equivalent secondary rating. If secondaries with balanced loads are present, the ratings of these are to be added.

The design procedure is then carried out similarly to that for the balanced transformer except that the flux density from Table 11-2 should be decreased up to about 10 or 15 per cent depending upon whether there are additional secondaries supplying balanced loads. The maximum reduction is used when only one secondary with unbalanced direct current is present, but higher values of flux density are permissible when secondaries with balanced loads are present. The core loss and excitation are found in order to ascertain whether the flux density is reasonable. This requires the use of the design curves, Fig. 13-1 through 13-8. The appropriate unbalanced magnetizing force H_{DC}, for use with the design curves is calculated in the following manner. The mean length of the magnetic circuit m_i is

$$m_i = a\,\ell \text{ inches,} \tag{2-7}$$

where a = constant from Fig. 11-3 or 11-5,

ℓ = characteristic linear dimension from nomograph, Fig. 11-7

The approximate secondary turns N_s, is calculated from

$$N_s = \frac{K_6 V_s}{f F_1 B \ell^2} \text{ turns,} \tag{2-34}$$

where K_6 = constant from Fig. 11-3, 11-4, or 11-5,

V_s = secondary RMS voltage in volts,

f = frequency in cycles per second,

F_1 = core space factor,

B = flux density in kilolines per square inch.

The approximate unbalanced magnetizing force H_{DC} is then

$$H_{DC} = \frac{.495\ N_s\ I_{DC}}{m_1} \text{ average oersteds,} \qquad (5\text{-}19)$$

where m_1 = mean length of magnetic circuit in inches,

I_{DC} = average load current in amperes.

(Note: the constant, $.495 = .4\pi/2.54$, so that m_1 can be inches).

Then the core loss, excitation, and nonmagnetic gap are obtained from the design curves (Fig. 13-1 to 13-8).

The next modification of the design procedure is in the calculation of the primary current. The primary component of load volt-amperes W_{pL}, is calculated from

$$W_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} \text{ volt amperes,} \qquad (5\text{-}11)$$

where I_s = secondary RMS current in amperes,

I_{DC} = average load current in amperes.

The primary current I_p, is then obtained from (5-12), neglecting W_i^2 in comparison with W_{ex}^2, and including the ratings of additional secondaries W_{r2}, where applicable,

$$I_p = \frac{1}{V_p} \sqrt{(W_{pL} + W_{r2} + \ldots + W_c + W_i)^2 + W_{ex}^2} \text{ amperes} \qquad (5\text{-}20)$$

The value obtained for I_p should then be increased 10 per cent for a transformer with one secondary which supplies unbalanced direct current. If secondaries with balanced loads are present, a smaller per cent increase should be used, proportional to the fraction of the unbalanced winding rating to the total rating of all secondaries.

The correction of the turns to account for regulation follows the normal procedure with the exception that the correction for regulation is made in accordance with equation (5-17). Therefore the expressions for winding turns for the primary and secondary are:

$$N_p = V_p \frac{N}{V} \left(1 - \frac{.707\ W_c}{2\ W_r}\right) \text{ turns,}$$

$$N_s = V_s \frac{N}{V} \left(1 + \frac{.707\ W_c}{2\ W_r}\right) \text{ turns,} \qquad (5\text{-}21)$$

where N/V = calculated turns per volt.

Where a capacitance filter is used, the factor 0.707 is replaced by the correction factor of (5-18). The design is then completed in the usual manner, including design checks. When the voltage ratio is checked, the following expression should be used,

$$V_p = n \left[V_s + 1.1\, I_{DC} \left(R_s + R_p/n^2 \right) \right] \text{ volts.} \qquad (5\text{-}22)$$

The design summary and calculation of temperature rise are the same as for the transformer with balanced loads.

VI. CURRENT-LIMITING TRANSFORMERS

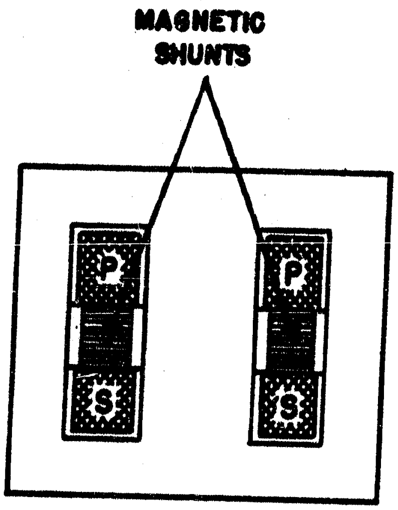
Some transformer loads have a very low initial resistance when power is applied, but a higher resistance after the load has heated. For such loads it is often necessary to limit initial current in some manner. One way of accomplishing this is to apply a reduced initial voltage, but the most common method is to design the transformer such that it has a high equivalent series or leakage reactance. Such transformers are principally used for tube filaments, although there are other possible applications. Among these might be a current-limiting rectifier supply to limit current when charging batteries or a current limiting supply to restrict damage in a circuit in the event of a short circuit. Since the main application of current-limiting transformers is for a filament supply, these will be discussed specifically, with the understanding that the same principles would apply to a current-limiting rectifier supply.

Requirements and Construction

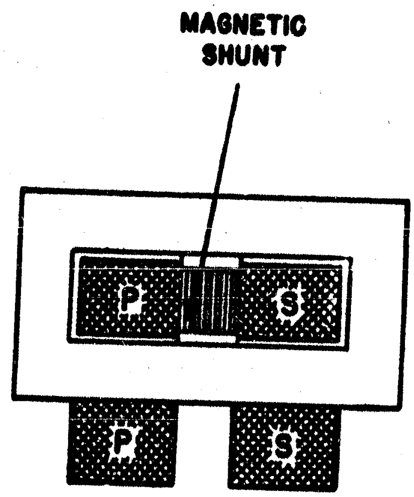
Materials used for electronic tube filaments have a high positive-resistance temperature coefficient. The cold-filament resistance may be low enough to result in an initial current which is many times rated current. In tubes with small filaments having short thermal time constants and relatively low rated currents, the initial current has little if any detrimental effect. However, the current for large filaments must be limited to prevent damage to the filament or cathode resulting from thermal changes and magnetic forces. The resistance of a cold tungsten filament is approximately one-tenth of its resistance when hot. Usually the electronic-equipment engineer will specify complete requirements for the transformer engineer. However the latter should be aware of the possible current-limiting requirement whenever the rated filament current is greater than about twenty amperes. A typical requirement is that the cold-filament starting current be limited to 150 or 200 per cent of the rated current.

A current limiting transformer provides current-limiting action as the result of high-leakage reactance. It is feasible to make a design using air-spaced coils to provide the leakage path, but this is likely to present problems to the electronic equipment manufacturer. An air-spaced, high leakage-reactance design may be unsatisfactory if the leakage flux path is greatly affected by the location of other components of ferromagnetic materials in the vicinity of the transformer. Not only will the stray field of magnetic flux be affected by components located near the transformer, but this stray field may interfere with the operation of other equipment as well.

A much more satisfactory design can be made by providing a low-reluctance, leakage flux path in the magnetic circuit which will confine the flux to a rather definite path. The use of a magnetic shunt as shown in Fig. 6-1 is the usual manner of providing the leakage-flux path. With no secondary load, the magnetic flux path of least reluctance is through the secondary winding, so that very little of the flux passes through the shunts, thus providing nearly the same open circuit secondary voltage as



a) HIGH-LEAKAGE REACTANCE SHELL-TYPE TRANSFORMER



b) HIGH-LEAKAGE REACTANCE SIMPLE-TYPE TRANSFORMER

FIG. 6-1 EXAMPLES OF HIGH-LEAKAGE REACTANCE TRANSFORMERS.

without the shunts. As load on the secondary winding is increased, the secondary ampere turns oppose the flux induced by the primary winding, and part of the flux induced by the primary follows the magnetic shunt path. Consequently, the secondary voltage will drop since secondary voltage is proportional to flux linking the secondary winding.

The actual physical design is little different from conventional transformers except for the shunt structure. The secondary coil for a low voltage output is frequently wound with strip copper. The strip copper, when used, may be the full width of the secondary coil less necessary margins. Strip copper is usually used in thicknesses of from .010 to .032 inches. The total height of the copper in each turn is built up to the desired thickness with enough strips to carry the rated current. Usually no allowance is made in the current rating of either primary or secondary conductor cross section for the current at cold-filament starting, because this is only an infrequent transient condition, and the thermal capacity of the transformer will take care of this current for the short-time starting period.

Tests and inspection for a high-reactance filament transformer include measurement of winding resistances, high potential test, open-circuit ratio test, measurement of insulation resistance, and inspection of mechanical details. In addition, two other tests must be made. A load test must be made to assure that the transformer will supply rated voltage at rated current. One commonly-used specification requires that the transformer supply rated current at rated voltage with a tolerance on the rated voltage of plus or minus three per cent. The other is a load test using a load equal the cold filament starting resistance to check the cold-filament initial current. A short-circuit test is frequently substituted since the cold-filament initial current and the short-circuit current are nearly the same.

To design a current-limiting transformer, the desired leakage reactance must be determined from the given conditions. Once the leakage reactance is known, the leakage flux may be deduced. Hence, for any given flux density in the primary portion of the core, the flux density in the secondary portion of the core may be calculated. By using this secondary flux density together with the rated secondary output, the design procedure becomes similar to that for an ordinary filament transformer. First, expressions will be derived for the leakage reactance and no-load voltage. With the aid of these expressions, the ratio of flux density in the secondary portion to that in the primary portion can be readily obtained.

Leakage Reactance and No-load Voltage

The quantities that are usually specified for the current-limiting transformer are:

V_p = primary voltage,

V_s = rated secondary voltage,

I_s = rated secondary current,

p = ratio of short-circuit current to rated secondary current.

When the transformer is delivering rated current at rated secondary voltage to a resistive load, the relationship among quantities shown in the equivalent circuit of Fig. 6-2 is:

$$\frac{V_p}{n} = \sqrt{(V_s + I_s R)^2 + (I_s X)^2} \quad \text{volts,} \qquad (6\text{-}1)$$

where $n = N_p/N_s$ = ratio of primary turns to secondary turns,

$R = R_p/n^2 + R_s$ = equivalent winding resistance referred to the secondary,

X = leakage reactance referred to the secondary.

When the load resistance is very small, such as that presented by the cold resistance of vacuum-tube filaments, the secondary current is very nearly the short-circuit current of the transformer. For this condition, the flux density in the shunt is much higher than during the rated load condition. The reluctance of the shunt path is increased at the higher flux density, and the leakage reactance is reduced. If reluctance of the leakage flux path were due entirely to the non-magnetic portions, or if steel permeability were constant, there would be no such change in reactance. In order to account for this variation in leakage reactance, the following ratio is introduced.

$$q = \frac{\text{reactance at short circuit}}{\text{reactance at rated current}} \qquad (6\text{-}2)$$

A typical value of q is .8 or .9. Higher values are representative when shunt flux density during short circuit is not very high compared to saturation density for the material.

The relationship of voltages during short circuit, similar to equation (6-1), is

$$V_p/n = pI_s \sqrt{R^2 + (q\,X)^2} \quad \text{volts.} \qquad (6\text{-}3)$$

Eliminating leakage reactance X, or primary voltage V_p from (6-1) and (6-3) gives the following equations.

$$V_p/n = pq \sqrt{\frac{V_s(V_s + 2I_s R) + I_s{}^2 R^2 (1-1/q^2)}{p^2 q^2 - 1}} \quad \text{volts,} \qquad (6\text{-}4)$$

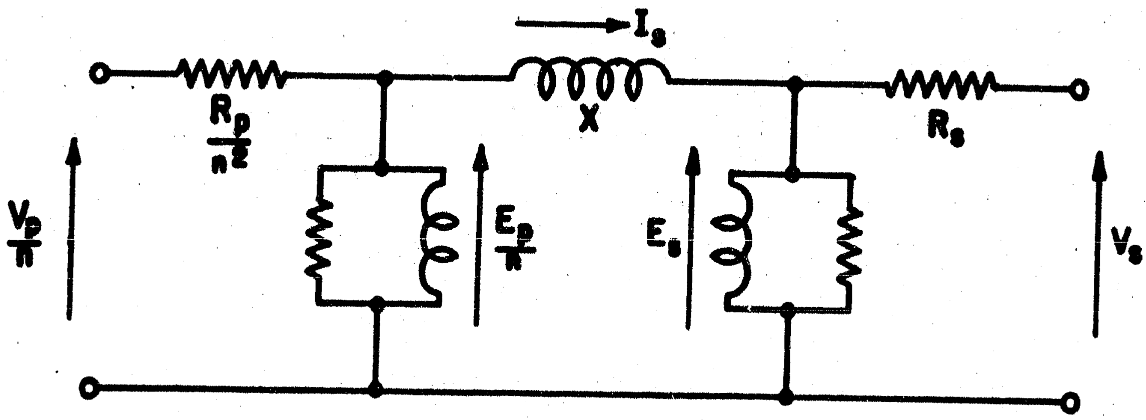


FIG. 6-2 EQUIVALENT CIRCUIT OF A TRANSFORMER WITH QUANTITIES REFERRED TO THE SECONDARY SIDE.

$$X = \sqrt{\frac{V_s(V_s + 2I_sR) - I_s{}^2R^2(p^2 - 1)}{I_s^2\,(p^2\,q^2 - 1)}} \quad \text{ohms.} \qquad (6\text{-}5)$$

Neglecting R, approximate expressions for the no-load voltage and leakage reactance are:

$$V_p/n = \frac{p\,q\,V_s}{\sqrt{p^2\,q^2 - 1}} \quad \text{volts,} \qquad (6\text{-}4)$$

$$X = \frac{V_s}{I_s\sqrt{p^2\,q^2 - 1}} \quad \text{ohms.} \qquad (6\text{-}7)$$

The relationship between the flux density in the portion of the core which is surrounded by the primary winding and the primary voltage is

$$E_p = 4.44\,f\,B_p\,A_i\,F_i\,N_p\,10^{-5} \text{ volts,} \qquad (6\text{-}8)$$

where N_p = number of primary turns,

B_p = flux density in portion of core surrounded by the primary winding in kilolines per square inch,

E_p = induced primary voltage,

A_i = core cross-sectional area in square inches,

F_i = core space factor,

f = frequency in cycles per second.

Similarly, the relationship for the secondary is

$$E_s = 4.44\,f\,B_s\,A_i\,F_i\,N_s\,10^{-5} \text{ volts,} \qquad (6\text{-}9)$$

where N_s = number of secondary turns,

B_s = flux density in portion of core surrounded by the secondary winding in kilolines per square inch,

E_s = secondary voltage induced as a result of flux B_s.

The ratio of equation (6-8) to (6-9) is

$$\frac{B_s}{B_p} = \frac{E_s N_p}{E_p N_s} . \qquad (6\text{-}10)$$

Since $n = N_p/N_s$, equation (6-10) becomes

$$\frac{B_s}{B_p} = \frac{E_s}{E_p/n} . \qquad (6\text{-}11)$$

Referring to Fig. 6-2, it is apparent that E_s and E_p may be approximated by V_s and V_p if the winding resistances are neglected. Then

$$B_s = B_p \frac{V_s}{V_p/n} \text{ kilolines per square inch,} \qquad (6\text{-}12)$$

where V_p/n is given by equation (6-6). Equation (6-12) indicates that for short-circuit conditions $B_s = 0$, and for open-circuit conditions $B_s = B_p$, approximations for quantities in the actual transformer.

Non-Magnetic Gap

Another design problem is the calculation of the proper gap in the magnetic shunt path. The actual gap as determined from trial and test is usually different than that calculated. This is partly attributed to fringing effects of the flux path around the gap. An examination of some core and shunt structures reveals that much of the apparent error in gap length is the result of imperfections in lamination dimensions and stacking workmanship. The normal dimensional tolerances and stacking irregularities add up to more than the allowable gap tolerance. Success in efforts to improve the accuracy of gap calculations depends upon the precision maintained in the manufacturing. The use of trials for determining the final gap dimension is predominant in manufacturing. The improvement of manufacturing methods necessary to eliminate trials may be impracticable.

The actual shunt air gap is fixed, but an effective air gap can be defined as a length depending upon the total reluctance of the shunt flux path. Leakage reactance is inversely proportional to such an effective gap. Since the ratio of leakage reactance at short circuit to leakage reactance during load is defined as q,

$$\frac{m'_{g\,sc}}{m'_g} = \frac{1}{q} \qquad (6\text{-}13)$$

where m'_g = effective gap at rated load, inches,

$m'_{g\,sc}$ = effective gap at short circuit, inches.

The effective gap length may be calculated using the principle that the total magnetomotive force around a closed path is zero. During short circuit, it is assumed that the total magnetomotive force around the secondary appears across the shunt flux path, and therefore across the effective gap.

$$.4 \pi N_s (p \sqrt{2}\ I_s) = H_{g\,sc} (2.54\, m'_{g\,sc}) \qquad (6\text{-}14)$$

where $H_{g\,sc}$ = magnetic field intensity in the gap during short circuit, in oersteds.

Since for a non-magnetic material, magnetic field intensity equals flux density, $B_{g\,sc}$ may be substituted for $H_{g\,sc}$. Changing units for density, and solving for effective short-circuit gap, gives

$$m'_{g\,sc} = \frac{4.52\, N_s\, p\, I_s}{10^3\, B_{g\,sc}} \text{ inches} \qquad (6\text{-}15)$$

where $B_{g\,sc}$ is the flux density in the gap during short circuit in kilolines per square inch. The effective gap during rated load should be closer than the above to the actual gap length. Combining equations (6-13) and (6-14),

$$m'_g = \frac{4.52\, p\, q\, N_s\, I_s}{10^3\, B_{g\,sc}} \qquad (6\text{-}16)$$

For a shell-type core, where two shunts are required, each shunt should have the gap length given by Eq. (6-16). The flux density in the gap $B_{g\,sc}$, corresponds to the short-circuit condition and may be calculated by assuming that all the primary flux is carried by the shunt during this condition. A correction factor to account for fringing of the flux does not appear to be warranted, to judge from test results and calculations. A reactance to give the correct load and short-circuit condition can usually be achieved by altering the thickness of the shunt through adding or subtracting a few of the shunt laminations, or by changing the length of the gap.

Design Procedure

The design of a current-limiting transformer follows the design procedure for an ordinary filament transformer after the addition of several important modifications. These modifications have been introduced into the design procedure by making use of the results which were derived in the previous two sections.

The transformer rating should be based on the secondary full load output, using the secondary voltage and current during full load.

$$W_r = V_s I_s$$

The winding dissipation is found in the normal manner, although the exposed winding surface area, S_c, is increased slightly (over the one winding which would fill the same window without the shunt) because of separation of primary and secondary. Nevertheless the surface area of a winding which fills the window can be used consistently in the calculations, and this practice is justifiable from thermal considerations.

Due to the presence of the shunt and the need for additional winding margins adjacent to the shunt, the window area available for windings is greatly reduced. The space factor for a transformer using a shunt, F_c', may be estimated from the space factor for a unit without a shunt. The winding space factor for a current-limiting transformer is

$$F_c' = .6\, F_c\,, \qquad (6\text{-}17)$$

where F_c is the copper space factor from Eq. (2-20).

The factor .6 is used for a scrapless lamination, and the most suitable value is nearer to .5 for units less than 50 volt-amperes. For a lamination with larger windows than the scrapless type, the factor is usually somewhat greater than .6.

The flux density in the portion of the core surrounded by the primary winding is selected using Table 11-2 as a guide. The secondary flux density when the transformer is carrying full load is then approximately

$$B_s = B_p \frac{V_s}{V_p/n} \text{ kilolines per square inch,} \qquad (6\text{-}12)$$

$$\text{where } V_p/n = \frac{pqV_s}{\sqrt{p^2 q^2 - 1}} \text{ volts.} \qquad (6\text{-}6)$$

The scale values should be calculated using F_c' in place of F_c,

$$\frac{K_0 W_r}{F_i f} \quad \text{and} \quad \frac{F_c' W_c}{\rho S_c}$$

The characteristic linear dimension is then found using the scale values and the secondary flux density B_s, since the transformer rating is based

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on the secondary full-load output. In checking the primary and secondary flux densities to determine that allowable core loss and excitation are not exceeded, half of the core weight should be used with each flux density.

The calculations involving the core dimensions follow the normal procedure. However, some changes are required in the winding calculations. Calculation of conductor weight should be omitted because the shunts occupy only a part of the window area and equation (2-30) is no longer valid. The primary current should not be based on Eq. (2-32), since the leakage reactance cannot be neglected. Instead the expression for calculating the primary current is:

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_c + W_i)^2 + (W_{ex} + I_s^2 X)^2} \text{ amperes} \quad (6\text{-}18)$$

where W_r = rated secondary output in volt-amperes,

W_c = winding losses in watts,

W_i = core losses in watts (neglecting shunt losses),

W_{ex} = excitation volt-amperes.

The leakage reactance volt-amperes, from Eq. (6-7) are:

$$I_s^2 X = \frac{V_s I_s}{\sqrt{p^2 q^2 - 1}} \quad (6\text{-}19)$$

The primary and secondary wire sizes may then be readily found.

The determination of winding turns involves a correction for transformer impedance in two steps. Nominal values for turns per volt are obtained for both primary and secondary. The fact that these are different indicates the correction for leakage reactance. Then a correction for winding resistance drops are made in the usual way, that is, by adding turns to the secondary and subtracting turns from the primary. The nominal primary and secondary turns per volt are given by the following:

$$\frac{N_p}{V} = \frac{10^5}{4.44 \, f \, A_i \, F_i \, B_p} \quad (6\text{-}20)$$

and

$$\frac{N_s}{V} = \frac{10^5}{4.44 \, f \, A_i \, F_i \, B_s} \quad (6\text{-}21)$$

where B_p is the chosen primary flux density, and B_s is the calculated secondary flux density according to Eq. (6-12).

In determining the winding layout, space must be provided for the magnetic shunt or shunts, depending upon the core type. Because the flux in the shunt is at a maximum only during short circuit, the shunt cross-sectional area need not be as high as that of the remainder of the core. It is suggested that shunt area be at least two-thirds that of the core, and more, if necessary, such that shunt flux density during short circuit is never higher than 130 kilolines per square inch. High shunt densities are to be avoided in cases where the ratio p is below about 1.3, so that changes in leakage reactance can be kept small.

The proper cross-sectional area of shunt may be obtained by varying shunt thickness in the direction parallel to the coil axes, or to a lesser extent, by varying length in the direction through the window. Normally the latter dimension will be the same as the core stack. For a simple-type core, shunt thickness is about (2/3)L or more.

Subtracting the thickness of the shunt from the window length gives the space remaining for windings and margins. Usually the shunt will not be exactly in the center of the window. By moving it off center, the winding space may be used more efficiently. With these considerations taken into account, the winding layout follows the pattern of the general design method.

After the winding layout is completed, the actual winding resistances should be calculated. First calculate the mean length of turn of each winding (length of the inside turn of the winding + π times the build-up of the winding). Resistance equals resistance per unit length (corrected to operating temperature from Fig. 11-6) times mean length of turn times number of turns. Once the winding resistances are determined, the design should be checked by calculating the primary voltage in order to ascertain that the turns ratio is correct. Rewriting Eq. (6-1) in a slightly different manner gives an expression for the primary voltage,

$$V_p = n\sqrt{\left[V_s + I_s\,(R_s + R_p/n^2)\right]^2 + (I_s X)^2} \quad \text{volts.} \qquad (6\text{-}22)$$

The turns ratio should be adjusted if the calculated primary voltage differs appreciably from the specified voltage. The winding resistances need not be re-calculated if the turns are altered, since the change in resistance will be small.

As shown in Fig. 6-1, the gap is ordinarily divided into two parts, one on each side of the shunt, so that fringing is minimized and it is easier to force the shunt into position. The total non-magnetic gap associated with a shunt used in a simple-type transformer, or with each shunt used in a shell-type transformer, is

$$m_g' = \frac{4.52\ p\ q\ N_s\ I_s}{B_{g\,sc}\ 10^3} \quad \text{inches} \qquad (6\text{-}16)$$

where $B_{g\,sc}$ is the flux density in the gap during short circuit—in kilolines per square inch.

VII. CURRENT-LIMITING TRANSFORMERS WITH UNBALANCED MAGNETIZATION

This type of transformer has: (1) a high-leakage reactance which is usually obtained by using a magnetic shunt, and (2) an unbalanced magnetization component of the core caused by direct current flowing in the secondary winding. Both of these characteristics have been studied separately. The design method for a current-limiting transformer with unbalanced magnetization is developed by combining the two procedures and by accounting for certain new problems.

Design Procedure

The design procedure for a current-limiting transformer with unbalanced magnetization follows the basic procedure as presented for a filament transformer, with few modifications. Only the deviations required to adapt the procedure to a current-limiting transformer with unbalanced magnetization are presented here.

An equivalent rating for the transformer is based on RMS secondary voltage and current. These are related to load voltage and current, filter capacitance, circuit resistance and frequency in the same manner as for transformers with only unbalanced magnetization. For a resistive load, the secondary RMS current is obtained by multiplying the average load current by 1.57; whereas for a capacitance-filtered load, a ratio from Table 13-1 should be used. An inductance-input filter is seldom used with a half-wave rectifier. When no filter is used, the secondary RMS voltage equals 2.22 times the sum of the average load voltage plus rectifier forward voltage drop and any other circuit resistance voltage drops. When a capacitance-input filter is used, the secondary RMS voltage is obtained by multiplying the average load voltage by a ratio from Table 13-3.

Since window space must be provided for the magnetic shunt, the winding space factor is reduced in the same manner as for the current-limiting transformer supplying a balanced load. The ratio of the leakage reactance at short circuit to that at rated current is expressed by the factor q, which has a typical value of 0.8.

The flux density in the secondary portion of the core is calculated from the primary flux density in the same manner as for the current-limiting transformer with a balanced load. The selected primary flux density should be about 10 to 15 per cent lower than that which would be used for transformers without unbalance in order to obtain designs with reasonable values of excitation in comparison with rating.

The design curves presented in Fig. 13-1 through 13-8 are used to determine core loss and excitation as functions of flux density and unbalanced magnetization. Unbalanced magnetizing force is different in the primary and secondary portions of the core. If the value given by equation (5-19) is defined as H_{DC}, then measurements and analysis indicate that the unbalanced magnetizing force in the primary portion of the core is about

$$H_{DCp} = .7\ H_{DC} \text{ average oersteds,} \tag{7-1}$$

The unbalanced magnetizing force in the secondary portion of the core is about

$$H_{DCs} = 1.3\ H_{DC} \text{ average oersteds.} \tag{7-2}$$

These equations are readily justified qualitatively by comparing a core before and after the insertion of the shunt, for a given value of unbalancing (D-C) ampere turns. Before the shunt is inserted, the direct component of field strength will be about the same all around the core path. With the shunt in place, the reluctance of the magnetic circuit as seen from the secondary winding will be decreased, so that average (in time) secondary flux and H_{DCs} will be increased. Also the shunt will certainly reduce the average magnetomotive force across the primary part of the core, so that average primary flux and H_{DCp} will be decreased. However these statements cannot be used as a simple basis for quantitative analysis because nonlinear relations of the magnetic quantities prohibit separate consideration of A-C and D-C components.

A gap in the secondary portion of the core will often be indicated by the design curves because of the relatively low flux density and high unbalanced magnetizing force in the secondary portion. Sometimes a gap will also be indicated for the primary portion. The type of core construction will decide whether it is possible to employ the exact gaps. The designer should attempt to use the value of gap indicated by the design curves.

To calculate the primary current, the leakage reactance volt-amperes must be determined. The leakage reactance is

$$X = \frac{V_{sf}}{I_{sf}\sqrt{p^2 q^2 - 1}} \text{ ohms,} \tag{7-3}$$

where V_{sf} = RMS fundamental component of secondary voltage,

I_{sf} = RMS fundamental component of secondary current,

p = ratio of short-circuit current to rated current,

q = ratio of leakage reactance at short circuit to leakage reactance at rated current.

Equation (7-3) is similar to (6-7), which is the expression for the leakage reactance of an ordinary current-limiting transformer. It is obtained in an analogous manner by considering only the fundamental components in the voltage equations. The RMS fundamental secondary voltage, V_{sf}, is the major component in the transformer RMS secondary voltage, and therefore V_s may be used with little error. The RMS fundamental secondary current, I_{sf}, may be assumed equal to 1.1 times I_{DC} according to the discussion in Chapter V. The expression for leakage reactance becomes

$$X = \frac{V_s}{1.1\, I_{DC} \sqrt{p^2 q^2 - 1}} \text{ ohms.} \qquad (7\text{-}4)$$

The reactive power absorbed by the transformer leakage reactance is defined as leakage-reactance volt-amperes, and is approximately equal to

$$X\,(I_s{}^2 - I_{DC}{}^2) \qquad (7\text{-}5)$$

Only the alternating-current components of the secondary current contribute to the leakage reactance volt-amperes.

The primary current may be calculated from

$$I_p = \frac{1.1}{V_p} \sqrt{(W_{pL} + W_c + W_i)^2 + \left[W_{ex} + X(I_s{}^2 - I_{DC}{}^2)\right]^2} \text{ amperes,} \qquad (7\text{-}6)$$

where W_{pL} = primary component of load voltage-amperes according to equation (5-11),

W_c = winding loss in watts,

W_i = core loss in watts (neglecting shunt losses),

W_{ex} = excitation in volt-amperes.

X = leakage reactance in ohms referred to secondary winding.

The calculation of turns per volt is made in the same manner as for an ordinary current-limiting transformer. The difference between primary and secondary turns per volt accounts for the leakage-reactance voltage drop. The corrections for winding-resistance voltage drops are made by adjusting the turns in the same way as for a transformer with unbalanced magnetization only. When the transformer design is completed, the voltage ratio should be checked. In accordance with the reasoning given in Chapter V, only the fundamental-frequency components should be considered. From the equivalent circuit for the fundamental-frequency components, the primary voltage is

$$V_p = n \sqrt{\left[V_{sf} + I_{sf}\,(R_s + R_p/n^2)\right]^2 + (I_{sf}\,X)^2} \text{ volts,} \qquad (7\text{-}7)$$

where V_{sf} = RMS fundamental secondary voltage,

I_{sf} = RMS fundamental secondary current,

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R_s = secondary resistance,

R_p/n^2 = primary resistance referred to secondary,

X = leakage reactance referred to secondary winding.

When approximate relationships are substituted for fundamental components, equation (7-7) becomes

$$V_p = n\sqrt{\left[V_s + 1.1\ I_{DC}\ (R_s + R_p/n^2)\right]^2 + (1.1\ I_{DC}\ X)^2} \text{ volts} \quad (7\text{-}8)$$

If the calculated primary voltage differs appreciably from the specified voltage, the turns are altered, since the change in resistance will be small.

VIII. VIBRATOR-SUPPLY TRANSFORMER

The design of a transformer for a vibrator supply is only a part of the more general problem presented by the entire supply circuit. The transformer, vibrator, and timing capacitor must be integrated so as to achieve a satisfactory power supply. Emphasis is placed here on the requirements and design of the transformer, since a study of the complete supply circuit is beyond the scope of this project. Recourse to the references,[17-32] in particular the Vibrator Data Book[17] of P. R. Mallory and Company, will provide the designer with additional information. Material of a general nature is given by Connelly[18] and Distin[19], whereas Evans[20] presents a detailed and mathematical treatment of vibrator-supply circuits. Vibrator power supplies normally do not exceed a rating of 50 to 60 watts at 300 to 400 volts DC output. Dixey and Wilman[21] discuss ratings greater than 50 watts, and general trends in vibrator power supply developments are reported by Mitchell[22].

A typical vibrator supply circuit with contact-driving coil omitted is given in Fig. 8-1. The capacitor shown, called the timing capacitor or buffer capacitor, is used across either one or both the windings of the transformer. Capacitance is necessary to prolong contact life and to decrease stress on transformer insulation. Numerous circuits are used, but as far as transformer operation is concerned, most circuits perform similarly to Fig. 8-1. The middle contact is vibrated by a relay coil which can be incorporated into the circuit in many ways. As the middle contact meets one of the others, battery voltage minus circuit drop is applied across half of the primary winding. When the contact reverses, voltage is applied across the other half of winding in an opposite direction. The result is an alternating voltage induced across the entire primary, and therefore across the secondary.

A second set of vibrator contacts, operated by the same relay, is sometimes placed in the secondary to yield a rectified output, the so-called self-rectifying type of circuit. When only one pair of stationary contacts is used as in Fig. 8-1, the vibrator is termed an interrupter type. In this case, a rectified output may be obtained by placing either a metallic or tube rectifier across the transformer secondary. For a properly adjusted vibrator, the self-rectifying type is the more efficient. The contact travel time, usually referred to as the "off-contact" time, ranges from 15 to 30 per cent of a complete cycle. Expressed another way, the "time efficiency" which is the ratio of vibrator contacting time to half a period is normally between 0.7 and 0.85.

Frequency

The frequency of operation is generally 115 cycles per second, although some vibrators have been made for 250 and 400 cycles. The obvious advantage of a reduction in transformer size as a result of using a frequency greater than 115 cycles does not always result in an overall improvement because of the influence of other factors. The driving power for the vibrator coil increases with the third power of the frequency. This effect is compensated for by using a permanent magnet which, however, increases the

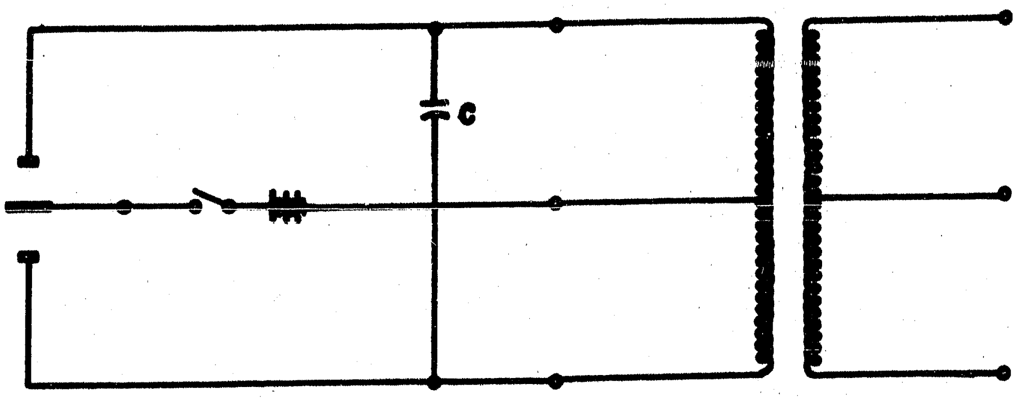

FIG. 8-1 VIBRATOR, TIMING CAPACITOR, AND TRANSFORMER

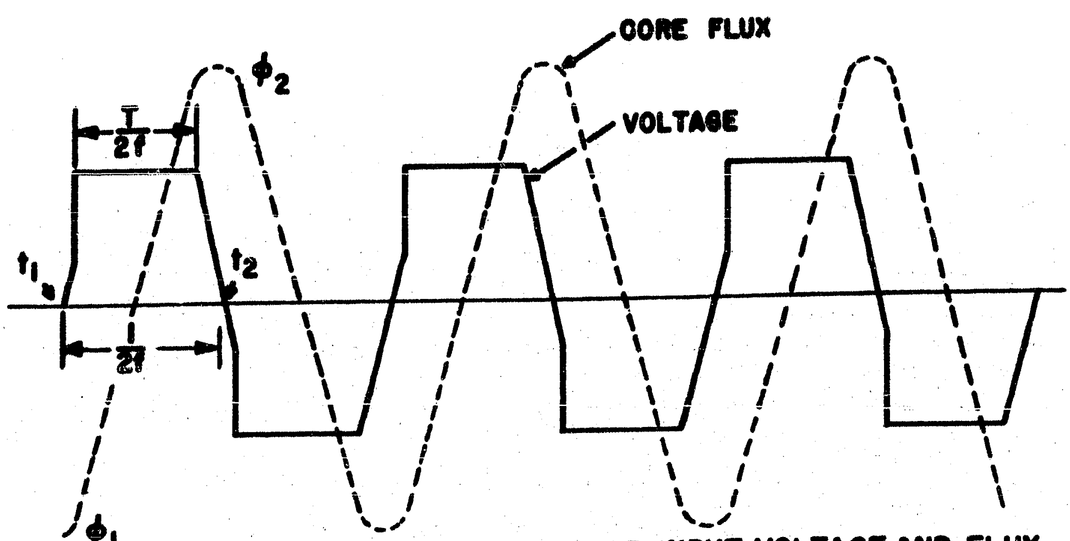

FIG. 8-2 —WAVE SHAPE OF TRANSFORMER INPUT VOLTAGE AND FLUX SHOWING EFFECT OF TIMING CAPACITANCE vibrator cost. Manufacturing costs are further increased since adjustment of a vibrator which operates at a higher frequency is more critical. The time efficiency decreases with an increase in frequency since a finite amount of "off-contact" time is necessary to avoid destructive arcing. Larger and more expensive filters are required since the vibrator is operating in a frequency range which is more likely to cause interference with communication equipment. Furthermore, contact life would be shortened at a higher frequency as a result of the greater number of operations per unit time.

Flux Density

The vibrator contacts are usually made from tungsten. As the contacts wear, the time efficiency decreases, which greatly affects the waveform and hence the transformer characteristics. To avoid contact deterioration, the normal practice is to design the transformer with a relatively low flux density so as to reduce the exciting current. Where additional protection of the contacts is required, Dixey and Wilman[5] suggest inserting a choke in the transformer secondary and Kiltie[23] describes a vibrator circuit for reducing contact current to zero before opening occurs. Others, such as Allen[24] and Hunt[25], also have considered the problem of reducing contact deterioration.

Starting of a vibrator is especially critical, since it is during this time that the contacts may be completely destroyed. When the battery is connected, the flux density after the first contact closure may exceed the normal maximum flux density if the residual magnetization of the transformer core is of an adverse polarity. Even more significant is the fact that while the vibrator is coming up to normal operating speed, the exciting current on successive half cycles may be different due to unequal contact closures. As a result, during the initial seven or eight cycles, the transformer may be subjected to a uni-directional component of magnetization. All these factors may produce an extremely high flux density and a high exciting current.

Table 19-1 gives suggested flux densities which should result in satisfactory contact life. The flux densities given are for the maximum anticipated voltage. The supply voltage is usually a battery which may be in various states of charge. Table 19-2 gives typical operating voltage ranges corresponding to the nominal voltages. For economic reasons, vibrator-supply transformers are most often designed with 24 gage (.025 inch thick) non-oriented, silicon steel of approximately AISI M-22 grade. Better grades of steel are sometimes used in order to operate the transformer at higher flux densities and at the same time avoid an excessive exciting current. The same advantage is obtained by using wound cores of oriented steel.[26]

In order to reduce the saturating effects and limit the exciting current during starting, a gap in the transformer core is sometimes used. However, this practice is ordinarily not recommended since it results in a large steady-state exciting current. Another way by which the exciting current may be limited is by inserting resistance in series with the battery or primary winding. This has a self-regulating effect, since the voltage across the primary is reduced when the transformer draws a large exciting current. For this reason some manufacturers prefer to place the primary winding over the secondary winding to increase the mean length of turn. Methods have been devised whereby a relay automatically inserts resistance only during the starting period. A variable resistance which is reduced to zero after starting often is necessary, especially when the supply voltage exceeds 12 volts.

The principal reason that starting is more difficult with higher supply voltages is that arcing at the vibrator contacts is much more severe. For tungsten contacts an arcing voltage in excess of 14 or 16 volts makes it difficult to interrupt the current. For this reason the currents which the vibrator contacts can adequately handle at the higher supply voltages must be reduced so as to insure that exciting current, especially during starting, does not become excessive. When series resistance is not used with 24 and 32 volt vibrator supplies, the design flux densities should be somewhat less than those given by Table 19-1. Capacitance across each of the contacts is effective in extinguishing the arc, but it is not recommended since it greatly reduces contact life. However, a modification of this idea is used, since for 24 and 32 volt vibrators, it is customary to place some capacitance across the entire primary winding. A portion of the timing capacitance is usually used for this purpose.

Vibrator Voltage Relationship

When a transformer is supplied by a battery and vibrator without using a timing capacitor, the idealized wave shape for primary voltage is an alternating series of rectangular pulses. When a timing capacitor of proper value is used, the wave shape is in most cases similar to that shown in Fig. 8-2. The main function of the timing capacitance is to supply the transformer exciting current during the contact-off periods. Otherwise there would be a rapid collapse of the flux as the contacts opened, resulting in high induced voltages and destructive arcing at the contacts. The wave shape of the core flux also shown in Fig. 8-2 is easily determined from the applied voltage wave shape by use of the basic equation of induction,

$$v = N \frac{d\phi}{dt} 10^{-5} \quad \text{volts,} \tag{8-1}$$

where v = instantaneous voltage,

N = number of turns across which voltage is applied,

ϕ = core flux in kilolines.

Since previously derived equations relating core flux and voltage are intended for a sinusoidal input, they are not applicable for the vibrator transformer. There is a more general equation found by integrating the basic equation of induction. This gives,

$$\int_{\phi_1}^{\phi_2} d\phi = \frac{10^5}{N} \cdot \int_{t_1}^{t_2} v\,dt. \tag{8-2}$$

If the times t_1 and t_2 are chosen such that t_1 is the time at which voltage is zero and increasing, and t_2 is time at which voltage is zero and decreasing, and there is no voltage zero between, then the right side of (8-2) has a maximum value. The value of the right side is proportional to the average voltage during the period t_1 to t_2, times the time, (t_2-t_1). The left side of the equation can be readily integrated, and the result is

$$\phi_2 - \phi_1 = \frac{10^5}{N} (t_2 - t_1) V_{avg} \qquad (8\text{-}3)$$

Since times are chosen for which the right-side term is maximum, then the left term must also be a maximum, such that ϕ_2 is a positive peak and ϕ_1 is a negative peak. If there is no bias flux, and applied voltage has the property of half-wave symmetry about zero (successive half waves are mirrored images about the time axis), then $(\phi_2 - \phi_1) = 2\,\phi_m$, where peak flux ϕ_m is measured from zero. The conditions for the times are such that $(t_2 - t_1)$ is one-half cycle equal $\frac{1}{2f}$ where f is frequency. Substituting in (8-3) gives

$$2\,\phi_m = \frac{10^5}{N} \frac{V_{avg}}{2f} \qquad (8\text{-}4)$$

$$V_{avg} = 4\,f\,N\,\phi_m\,10^{-5} \text{ volts.} \qquad (8\text{-}5)$$

This is a basic equation independent of voltage wave shape, except for the restriction to half-wave symmetry. Effective or RMS voltage is average voltage times the form factor. For flux can be substituted net area times density, and (8-5) becomes

$$V = 4\,f_x\,f\,N\,F_i\,B\,A_i\,10^{-5} \text{ RMS volts,} \qquad (8\text{-}6)$$

where f_x = voltage-wave form factor,

F_i = core space factor,

B = flux density in kilolines per sq. in.,

A_i = core cross section in square inches.

For a sinusoidal wave shape, form factor is 1.11, and (8-6) is the familiar relation for that case. For vibrator-supply transformers, the form factor varies because of changes in wave shape. The principal reason for wave shape change is contact wear which increases the "contact-off" time. Different types of loads also greatly influence wave shape. To obtain some idea of the form factor associated with a vibrator, a simplified wave shape consisting of an alternating series of rectangular pulses separated by the time required for contact travel will be considered. The RMS voltage V of such a rectangular wave is related to its maximum value V_M by

$$V = V_M \sqrt{T} \text{ RMS volts,} \qquad (8\text{-}7)$$

where T = the ratio of vibrator contacting time to half a period, between 0.7 and 0.85.

Similarly, the average value V_{avg} is

$$V_{avg} = V_M T \text{ volts.} \tag{8-8}$$

Hence, voltage form factor is

$$f_x = \frac{V}{V_{avg}} = \frac{1}{\sqrt{T}} \tag{8-9}$$

For the normal range of T, f_x is between 1.19 and 1.08. It should be noticed that these values bracket 1.11, which is the form factor for a sinusoidal wave shape.

Core Loss and Exciting Volt-Amperes

The core loss of a vibrator-supply transformer at a fixed flux density will depend upon the form factor of the applied voltage wave. Using the somewhat artificial approach of dividing core loss into "hysteresis" and "eddy-current" losses, the following expression for core loss may be written for any wave shape of flux,

$$W_i = P B^n + Q V^2, \tag{8-10}$$

where P, n, and Q are approximately constants,

B is maximum flux density,

V is RMS volts of a winding.

Then since $V = f_x V_{avg}$ and since V_{avg} is proportional to flux density B, (8-10) may be rewritten

$$W_i = P B^n + Q' (f_x B)^2. \tag{8-11}$$

Equation (8-11) shows that for a fixed flux density, the core loss of a vibrator-supply transformer will be the same as that for a transformer supplied by a sinusoidal voltage, provided f_x is 1.11. Since it has been previously shown that the voltage form factor is usually close to 1.11, the core loss characteristics taken for an applied sinusoidal voltage may be used with a reasonable degree of accuracy.

Excitation power is unlike that of other transformers, because here it is real power dissipated. Opening of vibrator contacts prevents excitation power from returning to the source. The value of this power is the average voltage-current product. Excitation volt-amperes may be roughly estimated

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as average battery voltage times the current corresponding to the maximum flux density, or may be taken directly from material characteristics in volt-amperes per pound. It might seem that average volt-amperes should be half of this, since current is initially zero at the beginning of each half cycle. However, during a half-cycle of voltage, flux changes from negative maximum to positive maximum and there must be a corresponding change in exciting current. Since initial current during each voltage half cycle is zero, the peak exciting current should correspond to twice the usual value for maximum flux density. For the circuit of Fig. 8-1 the heating effect of excitation current in each half of the primary is the same as though each half carried .707 of the total. Therefore the resulting excitation input to both windings is 1.414 W_{ex}, where W_{ex} is the excitation required on the basis of core weight and maximum flux density. Total primary input, including load component, excitation and winding losses, is

$$W_{rp} = W_r + 1.414\, W_{ex} + W_c \text{ volt-amperes,} \qquad (8\text{-}12)$$

where W_r = load component,

W_{ex} = excitation as found from core weight and flux density,

W_c = winding losses.

Timing Capacitance

If the timing capacitor were not present, the core flux would drop to its residual value during the interval that is required for the moving vibrator contact to travel from one stationary contact to the other. By placing a timing capacitor across either the primary or the secondary of the transformer, the energy stored in the capacitor during one contact closure controls the flux in the core until subsequent closure with the other stationary contact.

Consider the transformer, capacitor, and vibrator contact arrangement shown in Fig. 8-1. During closure of the moving contact with the upper contact, half the primary is energized. As a result of the induced voltage in the other half of the primary, the capacitor is charged to a voltage equal to almost twice the battery voltage. When the contacts open, the timing capacitor and the magnetizing inductance of the transformer comprise a free oscillatory circuit. During each "contact-off" time, it would be desirable to have the voltage across the transformer primary change from twice battery voltage of one polarity to twice battery voltage of the opposite polarity. In this way, the voltage across the primary would equal the voltage which would be applied when the contacts meet. In other words, a total change of approximately four times battery voltage is required across the primary, and hence also across the timing capacitance, during each contact-travel time. The timing capacitance must supply the transformer exciting current during this period, so the charge required is approximately equal to the peak value of the exciting current multiplied by the contact-travel time. If T is defined as the ratio of vibrator contacting time to half a period, then from Fig. 8-2 it is seen that each contact-travel time is (1-T)/2f second, where f is the vibrator frequency in cycles per second.

From the foregoing, an expression for the timing capacitance is

$$C = \frac{i_m \, (1-T)/2f}{4 \, E_b} \, 10^6 \text{ microfarads.} \qquad (8\text{-}13)$$

where i_m = peak value of exciting current,

T = ratio of vibrator contacting time to half a period,

f = vibrator frequency,

E_b = battery voltage.

To obtain an expression for the peak value of exciting current, assume that the excitation is the product of battery voltage times average no-load current. Since peak exciting current may be approximated as equal twice the average no-load current, equation (8-13) becomes,

$$C = \frac{(1-T) \, W_{ex} \, 10^6}{4 \, f \, E_b^{\,2}} \text{ microfarads,} \qquad (8\text{-}14)$$

where W_{ex} = excitation as found from core weight and flux density.

The value of capacitance calculated by equation (8-14) should be divided by 0.6 in accordance with recommended practice. This is done mainly to give satisfactory operation when the time efficiency, T, decreases due to contact wear. Furthermore, a larger capacitance helps starting and it is less damaging to the contacts than not enough capacitance. Equation (8-14) shows that the required timing capacitance varies with changes in supply voltage. Also, a higher supply voltage means a slight increase in vibrator frequency and duration of contact-closure time. If the vibrator transformer is operated with a flux density which does not produce appreciable saturation under the highest input voltage, a timing capacitance can be selected which provides satisfactory vibrator operation over the expected range of input voltages. From the foregoing it is apparent that many factors influence and alter the optimum value of timing capacitance. It is suggested that the calculated value of timing capacitance be used only as a rough approximation, and that the most satisfactory value be determined by viewing the primary voltage wave shape on a cathode ray oscilloscope. When the correct timing capacitance is used, the wave shape at no load should be similar to Fig. 8-2.

The timing capacitor is usually placed on the secondary side since a smaller size is required, capacitance being inversely proportional to the turns squared. In applications where the input voltage exceeds 12 volts, part of the timing capacitance should be placed on the primary side to alleviate arcing. With the timing capacitor on the secondary side, the capacitor must have a much higher voltage rating so all of the gain due to reflecting the capacitance is not realized. A general rule is to use a capacitor having a voltage rating about four times the secondary no-load voltage corresponding to the highest anticipated input voltage. In general, a resistor should be placed in series with the capacitor when it is placed on the secondary side. The purpose of the resistance is to damp oscillations arising from the presence of the leakage reactance, and to limit, during contact make, the capacitor charging current which results from using too large a timing capacitance. A resistor should never be used in series with the portion of the timing capacitance on the primary side, since the function of this capacitance is to alleviate arcing.

Vibrator Transformer Operation With Unbalanced Magnetization

The undesirable effects of a high exciting current as the result of unbalanced magnetization during vibrator starting have already been mentioned. Because of these effects, half-wave rectification is normally not used on the output of the customary circuit of Fig. 8-1. However, a satisfactory design can be achieved when using a half-wave rectifier by designing the transformer to operate with a somewhat lower flux density than would normally be used. This problem is similar to the unbalanced magnetization of a transformer supplied from a sinusoidal voltage. Particular attention must be given to the magnitude of the exciting current to avoid damaging the contacts. A gap in the core structure may in some cases be desirable in order to limit the exciting current during starting. Also a gap may help to reduce the excitation volt-amperes, as shown by the curves given in Chapter V.

Another circuit arrangement which results in unbalanced magnetization is a non-center-tapped transformer together with a single-contact vibrator. A single-contact vibrator may be obtained from a conventional vibrator by connecting the two stationary contacts together. If the secondary load is not rectified, the direct component of the primary current will produce an unbalanced magnetization of the core. To obtain satisfactory operation with this condition, a low flux density should be chosen, and an attempt should be made to obtain a low residual magnetism, possibly by the use of a gap in the core structure. However, if the secondary load is rectified, it may be possible to balance the secondary and primary ampere turns so that a net unbalance magnetization of the core does not occur.

A small unbalanced magnetization of the core can occur in still another way even though the circuit of Fig. 8-1 is used with a full wave rectifier. This results from the fact that with a shell-type construction the two halves of the primary (also applies to the secondary) will have a different winding drop due to different mean lengths of turn when one half is wound over the other half. This can be avoided by using a bifilar winding, or by winding each half of the primary and secondary side by side rather than by winding one on top of the other.

Leakage Reactance and Winding Layout

The last mentioned scheme has the disadvantage that leakage reactance will very probably be increased. High leakage reactance is undesirable in a vibrator transformer since it causes high induced voltages upon contact opening, and causes undesired oscillations with the timing capacitor. To obtain a low leakage reactance with the mentioned scheme, the primary and secondary windings should be subdivided into sufficiently many parts so as to distribute each winding more effectively. With a scheme such as this, the various coils must be externally connected so that those coils which are carrying current at any particular time are adjacent to one another. With a normal layer winding, the usual manner for reducing leakage reactance is by inter-leaving portions of the primary and secondary. This type of winding is the most effective for obtaining a low leakage reactance. Each time the windings are subdivided, the length of the leakage flux path is greatly increased with only a slight increase in the cross sectional area of the leakage flux path. However, as additional subdivisions are made the advantage dimishes and the cost of the winding increases.

The manner in which the windings are placed on the core is even more important from the leakage reactance standpoint for other arrangements. For example, if a full-wave-rectifier load is supplied by a core-type transformer with both primary and secondary center tapped, half of the primary and half of the secondary should be placed on one leg with the other two halves on the other leg. Care must be taken in making the external connections to be sure that both the windings on any one leg are conducting at the same time. If a non-rectified load is supplied by a core-type transformer with primary winding center tapped, the secondary should be split with half on each leg, and the primary should be divided into four parts with two parts to each leg. In this way, when either half of the primary is energized, an adjacent section of the secondary is also conducting, providing the external connections are made properly. The general rule is to equalize primary and secondary load ampere turns at each instant, for each leg of the core structure.

In addition to the special techniques already mentioned, the windings of the vibrator transformer differ from those of conventional transformers in a number of other ways. Additional insulation is often required in order to avoid breakdown from the high induced voltages developed during contact opening. Heavier insulation is also required to help support the primary winding on low voltage designs, since only a few layers of rather heavy wire are usually required. When heavy wire is used, the primary winding is often placed over the secondary to facilitate winding and to take advantage of the increase in resistance resulting from the greater mean length of turn.

Because of the induced voltages developed during contact opening, a great deal of high-frequency interference is presented by a vibrator-supply circuit. To eliminate some of the interference, a copper shield is sometimes placed between the primary and secondary windings and then grounded to the core. When the use of a copper shield is not justified, some degree of shielding can be obtained by "inverting" the secondary winding. This is accomplished by bringing out all the leads for both halves of the secondary. and then externally joining and grounding the further outside and inside leads of the secondary winding. This requires that additional insulation be used between the middle two layers of the secondary, since the entire secondary voltage appears here. When neither of these two special techniques appear to be justified for eliminating high frequency interference, satisfactory operation can often be achieved by simply encasing the entire transformer and properly placing it and the vibrator with respect to the frequency sensitive equipment. Other techniques employing additional circuit components are usually used, but these will not be discussed since these are not directly related to the transformer design.

Design Procedure

The transformer rating will in most cases be based on RMS voltages and currents. However, for some low-voltage vibrator supplies, the primary resistance drop may be great enough to necessitate a larger primary wire size together with a larger transformer. A check should be made at the completion of the design to determine whether a revision is necessary.

The equivalent RMS secondary current is determined from the DC load current and the filter requirements. If a vibrator transformer is supplying a full-wave rectified load through an infinite inductance-input filter, the RMS current in each half of the secondary would be .707 times the DC load current, since each half of the secondary supplies half of the load current. If a filter were not used, then the current in each half of the secondary would flow for less than the entire half cycle as determined by the time efficiency. For a rectangular wave, the DC load current should be multiplied by $.707/\sqrt{T}$. For $T = .81$, the multiplying factor becomes .786 which is the same as for a sine-wave. For a capacitance-filtered load, the ratio depends on the amount of filtering. Since both sine and vibrator wave shapes give the same ratio for an infinite inductance-input filter, and for no filter with a reasonable value of T, it is reasonable that the ratios of RMS secondary current to DC load current given in Table 12-2, which are for sine waves, also be used for the vibrator transformer.

The equivalent RMS voltage for the secondary winding is usually specified; otherwise it may be estimated from the required DC load voltage plus estimated rectifier forward drop and other series resistance voltage drops multiplied by 1.11 for an infinite inductance-input filter, or multiplied by $1/\sqrt{T}$ if a filter is not used; or DC load voltage multiplied by the ratio from Table 12-4 for a capacitance-filtered load.

The total secondary rating is twice the rating of each half of the winding, so the equivalent rating becomes

$$W_r = 2\, I_s\, V_s/2 \text{ volt-amperes,} \qquad (8\text{-}15)$$

where I_s = secondary RMS current,

$V_s/2$ = half the secondary RMS voltage.

For exactness in using the nomograph, this rating should be multiplied by a correction factor which depends on the form factor of the voltage wave. Referring to equation (8-6), it is seen how the form factor enters in the general relationship between RMS voltage and flux density. It should be recalled that the nomograph has been constructed for a sinusoidal wave shape. However, since the form factor for a vibrator wave shape is near to that for a sine wave, in accordance with the discussion following equation (8-5), the rating given by (8-15) can be used for the nomograph with negligible error. The design method then follows the general design procedure for a filament transformer with only slight changes.

The winding space factor F_c should be obtained from Fig. 11-2 in the usual way, with each half of a center-tapped winding counted as a separate winding. Construction of models indicates that this procedure allows for the reduction in winding space factor which results from bringing out center taps and the use of additional insulation to protect against induced voltages.

In the selection of flux density, a low value must be chosen for the reasons listed previously. Table 19-1 gives suggested flux densities corresponding to maximum anticipated voltage, and Table 19-2 shows typical voltage variations for the nominal voltage systems. The flux density to be used in the design procedure is selected from Table 19-1, and then decreased by the ratio of most probable operating voltage to maximum operating voltage. Next, the characteristic linear dimension may be determined from the nomograph. Core loss and excitation are then found. In general the core loss and exciting volt-amperes which result from selecting the flux density in the foregoing manner will be acceptable unless some special requirements must be met. In determining the lamination size and stack from the characteristic linear dimension, an attempt should be made to minimize the exciting current by obtaining a low core weight. This means that a lamination with a large window area per unit core cross-sectional area should be chosen, if available. Also a low stack will reduce core weight for some types of laminations.

The next modification in the design procedure occurs in the calculation of the RMS value of primary current. For a vibrator transformer total primary input power is,

$$W_{rp} = W_r + W_c + 1.414\, W_{ex} \text{ volt-amperes,} \qquad (8\text{-}16)$$

where W_r is given by Eq. (8-15),

W_c and W_{ex} are calculated in the design procedure.

The excitation volt-amperes are multiplied by the factor 1.414 to account for the heating effect of the exciting current which flows only during half a period in each half of the primary. The primary RMS current for use in calculating wire sizes, is half of the total primary power input divided by the RMS voltage across half of the winding $V_p/2$,

$$I_p = \frac{W_{rp}}{2(V_p/2)} \text{ amperes.} \tag{8-17}$$

An expression for the RMS voltage across half the primary winding, allowing one volt for contact drop, follows from equation (8-7):

$$V_p/2 = (V_b - 1)\sqrt{T} \text{ RMS volts,} \tag{8-18}$$

where V_b = battery voltage.

After circular mils per ampere are found in the normal manner, the wire sizes are calculated from the RMS primary current given by (8-17) and from the RMS secondary current which was determined at the start of the design.

The exact equation giving turns per volt is

$$\frac{N}{V} = \frac{10^5}{4\, f_x\, f\, F_1\, B\, A_1} \text{ turns per volt,} \tag{8-19}$$

where V = RMS voltage,

f_x = form factor, ratio of RMS voltage to average voltage.

Since the form factor for a vibrator voltage wave approximates that for a sine wave, the standard expression may be used with reasonable accuracy.

The primary RMS voltage of half the winding is given by equation (8-18), whereas that for the secondary was determined at the start of the design. To calculate the total turns in each winding, the result must be multiplied by two and then corrected for regulation in the normal manner.

Finally the windings should be laid out to see if the window is properly filled, and other checks on the completed design made to find if any critical limitation has been exceeded. Adequate insulation should be used to insure against breakdown from high induced voltages.

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IX. LOW-CAPACITANCE FILAMENT TRANSFORMER

Construction

This type of transformer is used to supply filament heater power in a circuit where it is necessary to have a low-capacitance from the filament circuit to ground. Frequently the transformer is operated such that the secondary has a high voltage with respect to ground and to the primary, although the voltage difference across the secondary is small. A low value of capacitance can only be achieved by providing a large physical separation between the secondary and other parts. Secondary supports should have a low dielectric constant and should occupy as little space as practical. An open transformer, with air comprising most of the space around the secondary, has a lower capacitance than the same unit immersed in a compound or oil. The presence of adjacent equipment raises the effective secondary capacitance.

Special construction or provision to meet low-capacitance requirements is normally necessary for values up to 50 micro-microfarads, and perhaps even higher. Since capacitance is a function of size as well as of proportions, construction will depend on rating, frequency, temperature rise and perhaps test voltage, since all of the quantities affect size to some degree. It has been found that conventional 60-cycle filament transformers with ratings from about 28 to 250 volt-amperes have capacitance values ranging upward from 100 micro-microfarads. This gives a rough guide to the values for which special construction is required.

To obtain very low values of capacitance, in the order of 5 to 30 micro-microfarads, an arrangement of core and coils as shown in Fig 9-1 can be used. Although no secondary support is shown, some scheme is necessary, and the design of supports depends principally on shock and vibration which the unit must withstand. Primary and secondary windings may be placed on the same leg (as in Fig 9-1) or on opposite legs of the core, but the former is usually preferred because of the lower leakage reactance. In some cases where the low-capacitance winding is operated at a high voltage with respect to ground, the spacing necessary to obtain low capacitance is adequate to withstand the voltage stress. A check should be made in every case. Since insulation strength usually depends on the length of the creepage path, the secondary mechanical supports should be designed for adequate creepage length.

To obtain intermediate values of capacitance, in the order of 50 to 100 micro-microfarads, a construction very similar to that of conventional filament transformers can be used, provided that margins and inter-coil insulation are increased over normal values.

Calculation of Capacitance

The capacitance between electrodes of any shape is based upon the simple capacitance relation for parallel plates. Extension of the basic parallel-plate formula to complex shapes can be accomplished by dividing up all of the space between the two electrodes into infinitely small sections to which the parallel-plate formula can be applied with negligible error. Then the proper combination of the infinitely small sections, in series and

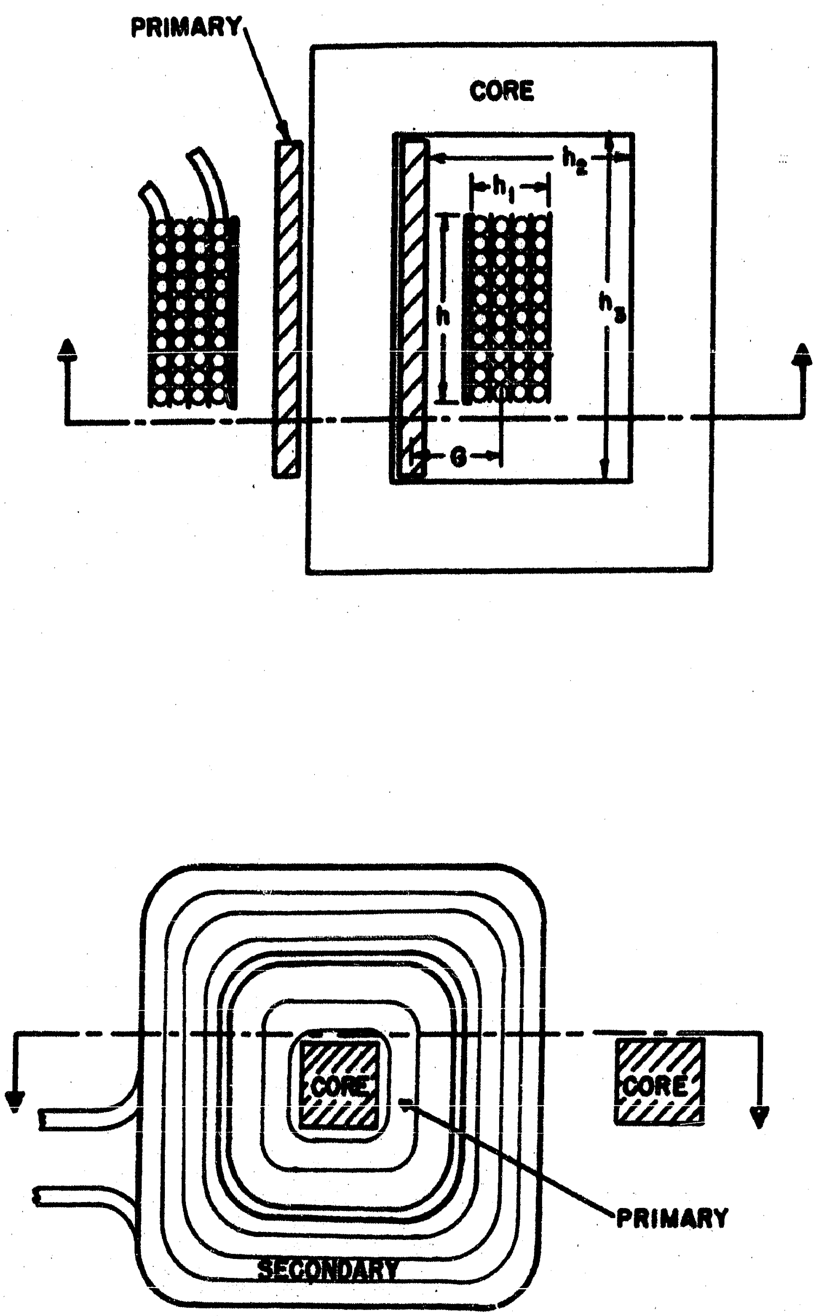

FIG.9-1 CORE AND WINDINGS OF LOW-CAPACITANCE TRANSFORMER

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parallel, yields the capacitance of a complex shape. In the report on Contract No. DA-36-039 sc-5519 the capacitance between parallel plates with negligible fringing has been shown to be

$$C = \frac{.225\ KA}{t} \text{ micro-microfarads,} \qquad (9\text{-}1)$$

where K = dielectric constant,

A = area of the plates, sq. in.,

t = separation of the plates, inches.

Equation (9-1) is given here to show that capacitance has the dimension of length, in that dielectric constant K is dimensionless, area A is a length squared and separation t is a length. Extension of this principle to any pair of complex electrodes means that capacitance varies linearly with size for constant proportions.

Exact values of capacitance can be readily calculated for a few simple geometric electrode shapes, such as parallel planes, parallel cylinders and spheres. Many fairly intricate two-dimensional electrode shapes can be handled by the method of complex variables to obtain exact capacitance values. However the low-capacitance transformer presents a three-dimensional configuration which is much too intricate to obtain exact values by analytical means, and even approximate calculations would be fairly involved and subject to error. Therefore data have been compiled for the purpose of developing an empirical formula for use in calculating capacitance.

Capacitance measurements have been made on four 60 cycle transformer development models. Each of these had the secondary mounted on the same leg as the primary, and supported by four wooden blocks which were about the same length as the secondary winding axial length. The primary was connected to the core, and capacitance was measured (using a General Radio 716 C Capacitance Bridge) between secondary and core for three conditions: with wooden blocks in place, with the secondary suspended by strings and blocks removed, and with the secondary suspended around the core leg opposite from the primary. Capacitance was measured at 1000 cycles. Capacitance measurements and other data for these models, identified as K2, K3, K4, and K5, are given in Table 9-1.

Extensive tests were made on model K2 to determine the effect of moving the secondary winding around in the core window, but about the same leg. It was found that capacitance is not sensitive to a change in the position until clearance in any one direction is reduced to a small value. The winding supplied with K2 was removed, and measurements were made using single turns of wire and copper strips of different sizes. In addition a core of the same cross section but with a smaller window area was constructed, and measurements of capacitance between the core and single turns of wire were made.

The data obtained from all tests were compiled and studied to find an empirical formula for capacitance. The form of the equation given in the report supplement of Contract DA 36-039 sc-5519 was tried, and found to be very satisfactory.* The resulting equation is

$$C = \frac{1.35\, k_c\, m_{cs}}{\ln \frac{P_1}{P_2}} \text{ micro-microfarads} \qquad (9\text{-}2)$$

where m_{cs} = mean length of secondary turn, inches,

P_1 = perimeter of open core window space with primary in place, equals $2(h_2 + h_3)$ of Fig 9-1

P_2 = perimeter of secondary cross section neglecting outside insulation, equal $2(h + h_1)$ of Fig 9-1,

k_c = correction factor for secondary supports and dielectric between secondary and core,

ln = logarithm to Napierian base, e.

* This form was originally suggested by W. C. Bloch of the General Electric Company

TABLE 9-1 Data for Low-Capacitance Models

Quantity	K2	K3	K4	K5
Rating at approx. 35° C rise, volt amperes	38.5	4.2	148	36
Core type	"L" laminations - - - - - - -			"C"
Core cross section, inches	.719x1.125	.5x.5	1x1.187	.75x1
Window, inches	2.5x2.5	1x1.5	4x4	1.25x2
Primary build, inches	.13	.13	.25	.38
Mean length of magnetic circuit, m_1, inches	12.9	7.0	20.0	9.5
Primary turns	690	2070	433	552
Mean length of secondary turn, m_{cs}, inches	12.0	6.5	15.7	8.75
Winding space factor, F_c	.0385	0658	.0512	.0563
Effective gap, pri to sec.	.66	.35	1.5	.5
Axial length of sec. layer, h, inches	.375	.221	.815	.282
Capacitance with blocks, C, micro-microfarads	10.0	7.2	14.8	8.9
Capacitance without blocks, mmf	9.0	5.6	12.3	7.6
Capacitance without blocks, sec. on opposite leg, mmf	8.0	4.3	11.0	6.2
Ratio of C, Sec. on same leg to sec. on opposite leg (without blocks)	1.13	1.30	1.12	1.22
Meas. Reactance at 60 cps, same leg; ohms	32.9	176	14.2	13.0
Meas. Reactance, opp. leg, ohms	73.8	384	35.9	34.6
Ratio of X, opp. leg to same leg	2.24	2.18	2.53	2.66
Calculated reactance, same leg-ohms	30.2	133	16.4	14.2

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The factor k_e is a ratio greater than one which accounts for the fact that the secondary is not suspended in air. The factor k_e can be estimated from the type of construction, proximity of equipment to be mounted adjacent to the transformer, and the dielectric constant of materials used for supporting the secondary, as given in Table 9-2.

Table 9-2

DIELECTRIC CONSTANTS

Material	Dielectric Constant
Air	1.0
Asbestos pressed fibers	40 - 250
Bakelite	4.5-5.5
Glass	5.4-9.9
Oil	2.2-4.7
Paper (dry)	2.0-2.6
Polystyrene	2.4-2.9
Porcelain	5.7-6.8
Wood	2.5-7.7

The factor k_e is lower than the dielectric constant of the surrounding material or supports unless the entire space between the secondary and the core is occupied by a solid or liquid dielectric. Tests made on models of low-capacitance transformers provide a guide for selecting k_e. When wood blocks are used to support the secondary, capacitance is increased about 20 per cent, so k_e is 1.2. Tests made with porcelain supports also increased capacitance about 20 per cent. Table 9-2 lists the dielectric constant of asbestos treated fibers, a material which is applicable for 200°C operation. The fibers are pressed into a board-like material from which blocks may be cut for secondary supports. Because of its high dielectric constant, measurements on model transformers using this material showed an increase in capacitance of about 50 per cent over values obtained when the secondary was supported in air with strings. Naturally, materials with lowest dielectric constants are preferred for this type of transformer.

Table 9-1 shows that capacitance values obtained from the transformer models with secondary suspended around the core leg opposite from the primary are 12 to 30 per cent lower than the values for the secondary suspended around the same leg encircled by the primary. From these data, an average decrease of about 20 per cent can be expected for the same length of secondary mean turn. However, if designs are made so that the secondary is equidistant from primary and core, a secondary opposite from the primary will have a smaller mean turn than one on the same leg, and consequently, an even lower capacitance, according to equation (9-2).

To confirm the applicability of equation (9-2), a plot of the quantities m_{oo}/C and P_1/P_2 was made on the semilogarithm paper, with P_1/P_2 on the log scale. It can be noted that both of these ratios are dimensionless, the first being so because C is a length. The points gave a straight line relationship, which is in accordance with the equation. The transformer data obtained without solid supports were used.

Since equation (9-2) checked measured capacitance when using small sizes of wire, one limiting configuration is accounted for. Another limiting case is obtained when the secondary almost fills the window. As P_1 approaches P_2, the denominator of (9-2) approaches zero, and capacitance C tends to a very high value for very small spacings to primary and core. Values obtained from (9-2) for small secondary spacing have been compared with results of the parallel-plate formula, and are found to agree well.

Leakage Reactance

One basic formula is used for practically all calculations of inductance or reactance of coils and transformers. For a flux path in a non-conducting material having relative permeability of unity, reactance or leakage reactance is

$$X = \frac{20.1}{10^8} fN^2 \frac{A}{h} \text{ ohms,} \qquad (9\text{-}3)$$

where f = frequency, cycles per second,

N = turns of winding to which reactance is referred

A = cross-sectional area of flux path, square inches,

h = length of flux path, inches.

A derivation and discussion of (9-3) has been given in Chapter II of the final report for Contract DA 36-039 sc-5519. One important principle is that reactance (or inductance) has the dimension of length, since an area appears in the numerator and a length appears in the denominator. This is also a feature of the basic formula for capacitance. The significance is that reactance per turn squared (for constant frequency) is directly proportional to linear size, if proportions of a transformer are fixed. It has also been shown that for a non-uniform magnetic field, the geometric terms A and h depend principally on those parts of the field where the flux is most dense. In any transformer, including the low-capacitance type, the greatest leakage flux density occurs between primary and secondary windings. However a low-capacitance transformer has a very complex magnetic field distribution which makes impossible a precise calculation of leakage reactance.

To obtain an empirical formula, data obtained from the models of Table 9-1 have been studied to find how equation (9-3) could be applied. The equation recommended for reactance of low-capacitance units with concentric windings is

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$$X = \frac{5}{10^8} fN^2 \frac{m_{cs} G}{h} \text{ ohms} \qquad (9\text{-}4)$$

where m_{cs} = mean length of secondary turn, inches,

G = effective magnetic separation of primary and secondary in inches, equal to actual separation plus one third the sum of primary and secondary radial builds,

h = axial length of secondary in inches, equal turns per layer times wire diameter

In table 9-1 are given reactances referred to the primary as calculated from (9-3), measured reactances for primary and secondary windings around the same core leg and for windings around opposite core legs. The agreement between measured and calculated values is quite good except for model K3. Values for reactance with windings on opposite legs are found to be 2.18 to 2.66 times the values for concentric windings; the average is 2.4.

The so-called measured values of reactance given in Table 9-1 were actually calculated from short-circuit tests. The secondary winding is short-circuited through an ammeter, and reduced voltage is applied to the primary. Readings are taken of primary voltage and secondary current. It is also necessary to measure resistances of both windings and of the ammeter. Leakage reactance referred to the primary can then be calculated from

$$V_p = \frac{I_s}{n} \sqrt{\left[R_p + n^2 (R_s + R_m)\right]^2 + X^2} \text{ volts} \qquad (9\text{-}5)$$

where V_p = applied primary volts,

I_s = secondary short-circuit current, amperes,

R_p = primary resistance, ohms,

R_s = secondary resistance, ohms,

R_m = ammeter resistance, ohms,

X = leakage reactance referred to the primary, ohms,

n = turns ratio, primary to secondary.

Equation (9-5) is a quadrature sum of equivalent real and reactive voltage drops which yield the primary voltage. Although the model transformers were designed for 60 cycles, the reactance of one unit was measured at 400 cycles, and a close check (3 per cent difference) of the 60-cycle value was obtained.

The measured values of leakage reactance for all models was checked by calculating primary voltage for load conditions, and comparing with measured values. Such calculations can be made with the formula,

$$V_p = \sqrt{(nV_s + \frac{I_s}{n} R_p + nI_sR_s)^2 + (\frac{I_s}{n}X)^2}, \qquad (9\text{-}6)$$

where V_s is secondary terminal voltage.

Calculated primary voltages from (9-6) were within five per cent of the measured values. An interesting observation is that the (I_sX/n) term in (9-6) contributes at most two per cent to the value of primary voltage V_p for these particular models, when the windings are concentric. The significance of this is that the regulation of low-frequency, low-temperature-rise units depends almost entirely upon resistance drop in the windings, particularly that of the secondary. The secondary resistance voltage drop is typically four times that of the primary, due to the large mean length of secondary turn. The leakage-reactance drop term is more appreciable when the windings are on opposite core legs. It is also higher at higher frequencies, and should not be neglected. The effect of leakage reactance voltage drop tends to be greater for higher temperature-rise units because all impedance voltage drops in (9-6) increase in comparison with nV_s. If a low-temperature transformer were re-rated for a high rise by a change of materials, the currents would be increased and turns ratio n would be decreased (by increasing secondary turns) to maintain proper secondary voltage.

It should be noted that the empirical leakage reactance formula (9-4) gives a value which is about one-fourth to one-third of that which would be obtained from the basic formula (9-3), for which the term A is usually set equal to $m_c G$ (mean length of all turns times effective separation). The result of (9-4) is not as low as one-fourth because m_{os} is greater than m_c. It is apparent that (9-4) should not be applied to transformers where the windings occupy most of the window space. To obtain one formula valid for both regular high-capacitance construction and low-capacitance units, it would be necessary to multiply (9-4) by a dimensionless correction factor based upon proportions of the configuration. This is a common practice in the calculation of coil inductances which makes the reactance formula (9-3) applicable to any size or shape.

Capacitance and Leakage Reactance Checks

Measurements were made on additional low-capacitance models to compare square and circular configurations of the secondary winding, and to verify the empirical equations for capacitance and leakage reactance.

One model identified as D5, consisted of model K3 with a new secondary substituted for the previous one. The new secondary contains twice as many layers of wire and twice as many turns per layer, giving a total of four times the turns of the previous secondary. The same wire size was used. Secondary capacitance was measured under various conditions and was also calculated by equation (9-2). Results are as follows:

	Micro-Microfarads
Capacitance - calculated, without blocks	8.5
Capacitance - measured, without blocks, concentric windings	9.8
Capacitance - measured, with blocks, concentric windings	12.8
Capacitance - measured, without blocks windings on opposite legs	7.4

The agreement between the calculated value of 8.5 and the measured value of 9.8 is considered to be satisfactory, and within the accuracy limitations of the empirical formula.

Another model identified as D3, consisted of a new secondary on Model K4. The new secondary was made in a square rather than in a circular shape. It has the same number of turns, number of layers, turns per layer and wire size. The over-all size of the new secondary is such that the minimum inside and outside dimensions are equal respectively to the inside and outside diameters of the previous circular winding. Capacitance values were measured and calculated for the square secondary, and are compared with the circular secondary as follows:

	Square	Circular
Capacitance - calculated without blocks	16.7	13.5
Capacitance - measured, without blocks, concentric windings	13.5	12.3
Capacitance - measured, with blocks concentric windings	16.2	14.8
Capacitance - measured, without blocks, windings on opposite legs	12.1	11.0

From these values it can be seen that capacitance of the circular winding is less than that of the square by about 10 per cent for all measurements. It is also interesting to note that the formula, although empirical, accounts for the direction of change in capacitance from circular to square shape. It might perhaps have been expected that the square secondary would have a lower capacitance than the round because it has a somewhat greater average separation from the core. The fact that this is not the case can be explained in a qualitative manner by noting that the surface area of the circular secondary considered as an electrode, is less than that of the square secondary.

It should also be noted that the circular secondary has a mean turn of about 25 per cent less than the square, and would therefore have correspondingly less secondary winding losses, an additional advantage.

Values of leakage reactance were measured and calculated for the square secondary in the same manner as for the round secondary. Results at 60 cycles are as follows:

	Square	Circular
Calculated reactance, windings concentric, ohms	20.5	16.4
Measured reactance, windings concentric, ohms	17.5	14.2
Measured reactance, windings on opposite legs, ohms	37.2	35.9
Ratio of measured reactances, opposite leg to concentric	2.13	2.53

These results show that the circular winding is preferable to the square because of the lowered reactance obtained in every case. Therefore it appears that the circular winding is preferable in all respects considered here, and in addition, is easier to wind than the square shape. It is concluded that the round secondary can be used to advantage except in designs where the primary is almost square and spacing between secondary and primary is comparatively small.

Regulation and Size

In the design of low-capacitance transformers, it is desirable to achieve minimum size and weight in meeting the circuit requirements. The relations between regulation and transformer size have been studied to find how a designer should be guided in obtaining these minimums. Another purpose has been to find whether it is feasible to operate such a transformer at high values of temperature rise. The low-capacitance unit is characterized by higher equivalent series impedance than conventional filament transformers. The secondary resistance is higher because of the long secondary turns, and leakage reactance is higher because of the low reluctance of the leakage-flux path.

It is a well-known principle that maximum power output from a source is obtained when load resistance equals source impedance. Applied to the low capacitance transformer, maximum power output occurs when

$$n^2R_L = \sqrt{(R_p + n^2R_s)^2 + X^2}, \qquad (9\text{-}7)$$

where R_L = load resistance

Although secondary voltage of a given transformer would vary as the load resistance changes, the secondary wire size and turns could be varied to keep secondary voltage and conductor volume constant for different values of power. For most transformers limiting temperature rise is reached long before power output is increased to the theoretical maximum determined by (9-7). However, it may be found that operating some low-capacitance transformers at temperatures near maximum permissible values for the insulation materials would result in having transformer impedance higher than load impedance. If this occurs it is possible to deliver the same load with lower transformer losses, using the same or less weight of secondary conductor. This is accomplished by changing secondary wire size and turns. This is a condition that should be checked in low-capacitance designs.

The foregoing discussion has covered the effect of varying the power output from a transformer of fixed weight. It remains to be shown how weight varies with regulation for a fixed power output. Among the several designs that might be made to meet certain requirements, one is a unit with low-temperature rise and low regulation, and another is a unit with high-temperature rise and high regulation. Regulation, which expresses the impedance voltage drop from no load to full load is defined as

$$\text{Regulation} = \frac{V_p - nV_s}{nV_s} \; 100 \text{ per cent,} \qquad (9\text{-}8)$$

where V_p = applied primary voltage,

V_s = secondary terminal voltage at full load.

Of the two examples given, the transformer with high temperature rise and high regulation has a smaller wire size than the other, but more secondary turns must be supplied to maintain specified secondary voltage. For identical load requirements, the two examples have different values of turns ratio n, which enters into equation (9-6). It is desirable to find if the greater number of turns of the first example yields a smaller or greater secondary winding weight than the second example, in spite of the saving in wire size.

To answer this question partially, consider that the primary turns of a particular low capacitance unit with concentric windings are constant, such that flux density is essentially constant. Actually leakage flux tends to reduce flux density in the portions of the core outside of the primary. Also primary resistance voltage drop decreases flux density in all parts of the core, but this impedance component is small compared to others present, so that the effect on induced voltage is not appreciable.

Equation (9-6) is a general relation between transformer voltages. An attempt has been made to find secondary weight as a function of regulation accounting for all terms of (9-6), but the algebra is too formidable. Therefore a simplified case may be considered as a qualitative guide. Primary resistance and leakage reactance are neglected, since the term nI_sR_s, is by far the most important in low-frequency units. Then induced secondary voltage is proportional to secondary turns. Secondary resistance is proportional to turns and inversely proportional to wire cross-sectional area. For constant terminal voltage and current, a voltage equation for the secondary is

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$$k_1 N_s = V_s + k_2 \frac{N_s}{A_w} \qquad (9\text{-}9)$$

where k_1 and k_2 are constants

N_s = secondary turns, a variable

A_w = secondary wire cross-sectional area, a variable.

Solving for N_s gives

$$N_s = \frac{V_s}{k_1 - \frac{k_2}{A_w}}. \qquad (9\text{-}10)$$

Now secondary conductor weight, for constant mean turn, is proportional to number of turns times wire area. Secondary weight, using (9-10), is

$$W_{cs} = k_3 N_s A_w = \frac{k_3 V_s A_w^2}{k_1 A_w - k_2}. \qquad (9\text{-}11)$$

Since weight becomes high for large wire size, A_w, a minimum may exist for some smaller wire size. Differentiating (9-11) with respect to A_w,

$$\frac{d\,W_{cs}}{d\,A_w} = k_3 V_s \frac{2 A_w (k_1 A_w - k_2) - k_1 A_w^2}{(k_1 A_w - k_2)^2}$$

Equating this to zero gives a condition for minimum weight:

$$k_2 = \frac{k_1 A_w}{2} \qquad (9\text{-}12)$$

Substituting for k_2 in (9-12) gives

$$k_1 N_s = V_s + \frac{k_1 N_s}{2} \qquad (9\text{-}13)$$

This shows that minimum weight is obtained when regulation is 100 per cent, or when the secondary terminal voltage is equal to half of the secondary induced voltage.

The solution of the weight and regulation problem for the special case gives the same result as the condition for maximum power from a transformer, equation (9-7). It is suggested that this similarity might exist for the general case in which primary resistance and leakage reactance were accounted for.

The conclusion is that a transformer should not be designed for maximum permissible temperature rise if the load resistance is less than the transformer equivalent series impedance referred to the secondary. Violation of this principle results in higher transformer losses than necessary, and in larger weight than necessary.

Modification of Basic Design Procedure

The design procedure for low-capacitance transformers is a modification of the basic method developed under Contract No. DA 36-039 SC-5519. The basis for this special design method is an analysis of geometry and electrical relations, with reliance upon dimensional principles and upon tests of experimental models.

The design nomograph may be used for design of low-capacitance units provided the various parameters appearing on the nomograph scales are properly selected. One of these is W_c/S_c, defined as the ratio of winding losses to exposed winding surface area. In a properly designed low-capacitance transformer, primary and secondary operate at roughly the same temperature, and most of the winding losses occur in the secondary. Therefore, it is most important to establish a suitable value for secondary losses in relation to size and proportions. This is done by using the secondary dissipation per unit secondary surface area in one nomograph scale factor. This ratio is calculated as:

$$\frac{W_{cs}}{S_{cs}} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per sq. in.} \qquad (-14)$$

where W_{cs} = secondary winding losses, watts

S_{cs} = secondary exposed surface area, sq. in.

ΔT = maximum permissible temperature rise, °C

K = a parameter depending on ambient temperature.

Recommended values of the parameter K, as determined from tests of models, are given in Table 21-1 for open core and winding construction. With the maximum permissible rise and the given K, the value of W_{cs}/S_{cs} for use with the nomograph is determined. Inspection of equation (9-14) shows that too large values of K are conservative, in that they tend to give an actual temperature which is low. It is desirable that K be somewhat conservative so that actual rise of a series of designs will average less than maximum permissible values, simply to avoid too many rejects. Unpredictable variations in temperature rise will always be present due to slight differences in design (which cannot reasonably be considered during design), manufacturing tolerances and errors in testing.

Another parameter which must be considered is the function of dimensionless geometric ratios, K_o. The design nomograph is simply a means for solving a general design equation. The design equation, as it applies to low-capacitance units, has been compared with the equation for common transformer types. A comparison of terms and selection of constant based on model data give

$$K_o = \frac{.22}{F_c^{.6}}, \qquad (9\text{-}15)$$

where F_c = winding space factor, ratio of total wire cross-sectional area to window area.

Winding Space Factor

Space factor is a very fundamental quantity in the design of all types of transformers, but in the low-capacitance type it has a new significance, because it helps determine secondary capacitance. In units where the winding nearly fills the window, secondary capacitance can be reduced by increasing secondary spacing, which in effect requires a lower winding space factor. However, there is no simple relation between space factor and capacitance because capacitance is also a function of size. That is, for fixed proportions, capacitance is directly proportional to linear size, which in turn is a function of rating, temperature and frequency. Therefore, two transformers having different ratings but the same capacitance have different sizes and different shapes (determined by space factor).

In order to deal with transformer size, it is necessary to obtain a function of rating from which the effects of temperature rise and frequency have been eliminated. This is done by calculating an equivalent volt-ampere rating which is a measure of physical size. This equivalent, based on 60 cycles frequency and 40°C rise has been shown (Contract DA 36-039 SC-5519) to be

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76}\left(\frac{\Delta T}{40}\right)^{.63}} \qquad (2\text{-}21)$$

where f = frequency, cycles per sec.,

ΔT = maximum temperature rise, °C.

The problem is to obtain space factor F_c as a function of equivalent rating W_r' and of secondary capacitance C. The starting place for such a function is the capacitance formula:

$$C = \frac{1.35\, k_c\, m_{cs}}{\ln \frac{P_1}{P_2}} \text{ micro-microfarads,} \qquad (9\text{-}2)$$

where m_{cs} = mean length of secondary turn, inches,

k_c = correction factor for secondary supports and any dielectric between secondary and core,

P_1 = perimeter of remaining window space with primary in place, inches,

P_2 = perimeter of secondary cross section, around wire only, inches.

In equation (9-2) it is necessary to replace m_{cs} and P_1/P_2 by functions of W_r' and F_c. The derivation of these substitutions is briefly outlined.

To obtain a function for m_{cs}, one relation needed is equivalent rating in terms of geometry and space factor. This has been deduced from the general transformer equation as it applies to low-capacitance transformers.

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$$W_r' = kA_1 (F_c A_c)^{3/4} \qquad (9\text{-}16)$$

where k = some constant,

A_1 = core cross-sectional area,

A_c = window area.

Another relation is needed to incorporate preferred transformer proportions. A way of doing this is to require that core cross-sectional area be proportional to total conductor cross-sectional area in the window. A suitable relation is

$$A_1 = 2.5\, F_c A_c \qquad (9\text{-}17)$$

Next, substituting for A_c in (9-16) according to (9-17) gives

$$W_r' = k' A_1^{7/4}, \qquad (9\text{-}18)$$

where k' is another constant.

From a study of transformer geometry only, it has been found that mean secondary turn is related to the areas and to space factor only.

$$m_{cs} = \sqrt{\frac{A_1}{F_c}} \left(2 + 4\sqrt{F_c} + 2.8\, F_c \right) \qquad (9\text{-}19)$$

Finally, an expression for m_{cs} is obtained by solving for A_1 in (9-18) and substituting in (9-19).

$$m_{cs} = k'' \frac{W_r'^{\,2/7}}{F_c^{\,1/2}} \left(2 + 4\sqrt{F_c} + 2.8\, F_c \right), \qquad (9\text{-}20)$$

where k'' is another constant.

The ratio of P_1/P_2 in equation (9-2) can be expressed in terms of winding space factor as

$$\frac{P_1}{P_2} = \frac{1.66 - 1.16\, F_c}{\sqrt{F_c}\,(F_c + 1.63)}. \qquad (9\text{-}21)$$

In an effort to simplify the function, it was found that the ratio is very closely equivalent to

$$\frac{P_1}{P_2} = 2.6 \sinh \left(\ln \frac{1}{\sqrt{F_c}} \right), \qquad (9\text{-}22)$$

where sinh means hyperbolic sine.

This was obtained by expanding the denominator of (9-21) according to the binomial theorem.

Therefore, from equations (9-2), (9-20), and (9-22), and by evaluation of the constant using empirical data, one can obtain a complicated but fairly accurate relation among W_r', C and F_c.

$$\frac{k_c W_r'^{\,2/7}}{C} = \frac{1.28\sqrt{F_c}\ \ln 2.6 \sinh \left(\ln \frac{1}{\sqrt{F_c}}\right)}{1 + 2\sqrt{F_c} + 1.4 F_c} \qquad (9\text{-}23)$$

Winding space factor F_c is an unknown quantity which cannot be explicitly found from (9-23), but Fig 21-1 gives a plot of F_c versus the left side term of (9-23). The fact that there are two possible values of F_c for one value of the ratio, $k_c W_r'^{\,2/7}/C$, indicates that two designs would give the required rating and capacitance. Of these, the higher F_c will yield the smaller transformer. However, this unit may not have sufficient window area for insulation of the secondary, a factor to be checked in the design procedure.

Figure 21-1 shows a maximum value for the ordinate of .32. This means that there is a minimum capacitance which can be obtained for any particular rating. If the calculated ordinate from specified rating and capacitance exceeds this, then the conditions cannot be satisfied. This maximum occurs at about F_c = .05, corresponding to a ratio of perimeters, from equation (9-21), of P_1/P_2 = 5.5. For a certain rating, secondary capacitance will be higher for either higher or lower space factors and ratios of perimeters. The almost flat part of the curve where F_c varies from about .03 to .07 is a region where choice of F_c has little effect on capacitance for a given equivalent rating.

Transformer Layout

Because desirable proportions of low-capacitance transformers vary greatly with requirements, the geometric factors used in the basic design method are not applicable. However, the definition of characteristic linear dimension is unchanged.

$$\ell = \sqrt[4]{A_c A_i} \qquad (2\text{-}6)$$

Eliminating first A_c, then A_i between equations (2-6) and (9-17) gives the formulas:

$$A_i = 1.6\,\ell^2 \sqrt{F_c}\,, \qquad (9\text{-}24)$$

$$A_c = \frac{.63\,\ell^2}{\sqrt{F_c}}\,. \qquad (9\text{-}25)$$

To obtain an estimate of total winding losses at an early point in the design of the windings, the following empirical equation can be used.

$$W_c = 22\,\ell^2 \frac{W_{cs}}{S_{cs}} \text{ watts.} \qquad (9\text{-}26)$$

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The formula for circular mils per ampere which has been used in the basic procedure is satisfactory if the quantity, 1000 K_o, (K_o from (9-15) is used instead of K_r. Although the current density is based on secondary winding temperature rise, tests of models show that reasonable values of primary rise are obtained by using approximately the same current density for the primary.

Nominal turns per volts should be calculated as in the basic procedure. To correct for regulation, it is recommended that nominal primary turns be unchanged, but that nominal secondary turns be increased by the ratio W_o/W. This is a correction for resistance drop, made entirely in secondary turns because most of the transformer equivalent series resistance is due to the secondary, a direct result of the longer mean length of turn. No correction in turns should be made at this point for voltage regulation due to leakage-reactance voltage drop, because this factor is usually negligible.

Design Checks

After wire size, number of turns and physical layout of the winding have been determined, the design should be checked. Checks are of particular importance where values are critical or where quantities depend on rough approximations. Insulation of the windings, especially from secondary to primary and core, should be checked. For working (peak) voltage over 700 volts or .7KV, the winding must be able to withstand a test voltage of

$$KV_T = \sqrt{2}\, KV_W + 1 \text{ kilovolts, RMS,} \qquad (9\text{-}27)$$

where KV_T = test kilovolts, RMS,

KV_W = working (peak) kilovolts.

Another important check is made to see that equivalent transformer series impedance is less than load resistance. This requires calculation of winding resistances and of leakage reactance.

A proper voltage ratio at the specified load is usually very important. This can be checked by calculating the primary voltage which would yield the required load conditions. Resistance and leakage reactance drops are added to secondary voltage.

$$V_p = \sqrt{\left(nV_s + \frac{I_s}{n} R_p + n I_s R_s\right)^2 + \left(\frac{I_s X}{n}\right)^2} \text{ volts} \qquad (9\text{-}6)$$

where n = turns ratio, N_p/N_s,

V_s = secondary volts,

I_s = secondary current, amperes,

R_p = primary resistance, ohms,

R_s = secondary resistance, ohms,

X = leakage reactance referred to the primary, ohms.

A check of approximate secondary temperature rise may be made using equation (9-14) and the values of K from Table 21-1. Calculated secondary losses are used for the term W_{cs}. Approximate exposed secondary surface can be found from

$$S_{cs} = 1.5\, m_{cs} P_2 \quad \text{sq. in.,} \qquad (9\text{-}29)$$

where m_{cs} = mean length of secondary turn, inches,

P_2 = perimeter of secondary cross section, around wire only, inches.

The factor 1.5 is introduced because P_2 as defined is not as large as the effective heat-dissipating perimeter of the winding.

X. INSTRUMENT TRANSFORMERS

Instrument transformers provide low voltages or low currents which are proportional respectively to higher voltages or currents. The two types are potential transformers which provide a low-voltage output, and current transformers which provide a low-current output. These transformers are used for measurement, protection and/or control of quantities in the higher-voltage or higher-current circuit. The use of instrument transformers also avoids a direct connection with high-voltage circuits and heavy, current-carrying conductors. Therefore it is possible to measure electrical quantities with safety and to connect to transformer secondaries meters or devices which have low-voltage or current ratings.

Potential Transformers

In the design of a potential transformer, it is necessary to keep voltage-ratio and phase-angle errors within prescribed limits over the operating range. These errors result from resistance and reactance voltage drops which are functions of the load and exciting currents. At one particular load (load is usually referred to as a burden of instrument transformers) the error in voltage ratio may be compensated for by slightly increasing the secondary turns or decreasing the primary turns. However errors will be obtained for primary voltage or burdens different from the design values. Phase-angle errors cannot be corrected by adjusting the turns ratio, but can only be minimized by special design of the windings and core. Resistances are kept down by using sufficiently large wire sizes and by making the mean length of winding as short as possible. The exciting current is minimized by using high-grade materials for the core together with low flux densities, and by keeping the magnetic path as short as possible. These requirements for reduction of errors may appear to be conflicting. However, the errors in a given design can be reduced by an increase in overall transformer size.

Potential transformers usually have a secondary rating of 115 volts. In most European countries, the standard is 110 volts. Since the primary voltage is usually quite high, adequate insulation must be used between the primary and secondary windings. Furthermore, since potential transformers are frequently used with electric power distribution systems, overvoltages resulting from faults, lightning discharges and switching may occur. Insulation should be adequate to protect against unusual conditions. The secondary is usually wound next to the core. Core type or simple type construction with the windings arranged concentrically, is usually employed.

The design procedure for a potential transformer is the same as that for a filament transformer. However the designer should be aware of the special considerations which have been mentioned. Values of maximum primary voltages should be specified. These include the voltage across the winding and the maximum voltages to ground if one end of the primary is not at ground potential. Then the highest working voltage is used to calculate winding space factor. Transformer power rating is the product of secondary voltage and secondary current, which is the burden. The design can be calculated in the usual way (as for filament transformers), and the final design should be checked to see whether or not ratio and phase-angle errors are within limits. If not, then a higher value of ℓ should be selected, and the design repeated. In order to keep the exciting current down, a high grade steel should be used for the core, and a conservative or low flux density be selected. Omit calculation of core dissipation per unit area and winding losses. Particular attention should be given to checks at the completion of the design to insure that adequate insulation has been used and that internal impedance drops are not great enough to cause excessive voltage ratio and phase-angle errors over the specified ranges of burden and primary voltage.

Current Transformers

Internal losses and impedances also cause ratio and phase-angle errors in current transformers. It is particularly important to keep flux density and therefore excitation low in current transformers because exciting current causes a deviation from the ideal ratio of primary and secondary currents, and in addition, exciting current is a nonlinear function of the primary load-current component. Current transformers must often operate within limited errors over a large range of currents, such as to several times the rated value of a circuit. One limitation on upper current values of the transformer is set by saturation effects of the core, which cause a radical departure from the nominal ratio of currents. Therefore, current transformers are designed with low flux densities, which may be necessary because of normal-load error requirements or because of overcurrent ratio limits. For example, it may be specified that a transformer have a certain maximum error at rated current, and that it will not saturate at 10 times rated current.

Magnetizing and loss components of the exciting current may be limited by using high quality materials, thin laminations, and high quality joints in the core structure. Special core materials are sometimes used which have values of core loss much less than for silicon steel. These special core materials are alloys of 50 per cent nickel and 50 per cent iron, and nickel alloys having small percentages of copper and molybdenum or chromium. If a high degree of accuracy is not required, or if the current transformer is not to be used to obtain the difference between several large quantities, a silicon steel core may be used with a flux density that enables operation to be well below a point where excitation becomes appreciable. From the foregoing, it is apparent that the application of the current transformer will determine accuracy requirements.

The physical appearance and construction of current transformers is quite different from most transformers since extremely large currents are usually to be measured. Often only one primary turn is required. When this is the case, the core may be toroidally shaped with a bar in the center or simply a hole through which a conductor may be inserted. Since it is necessary to open the heavy current carrying conductor in order to insert the transformer with conventional current transformers, another type of construction whereby the core may be separated is sometimes used. An example of this type is the typical clamp-on ammeter. Since butt gaps are present in the core, the exciting current is increased. With this type of core or with a stacked core it is not as easy to achieve the same degree of accuracy as with a one-piece wound core upon which the secondary windings are wound. A secondary current of five amperes is standard for current transformers. With a one-turn primary, taps are placed on the secondary winding. For current transformers where the maximum current is less than several hundred amperes, a number of primaries may be wound on one core together with a single secondary winding. The voltage of the conductor of which the current is to be measured also influences construction. For example, insulating a high-voltage bar-type transformer is readily accomplished by surrounding the bar with an insulating tube. For the type which has a hole for the primary conductor, no additional insulation is required provided the high-current conductor is adequately insulated.

The design procedure for a current transformer is similar to that for a filament transformer. The transformer rating should be based on the rated secondary current (usually five amperes) and the highest secondary voltage which will be required for the impedance placed across the secondary. Winding space factor may be reduced somewhat because of the insulation between the primary and secondary and primary and core. If the core is of the wound type, the geometric constants of Fig. 11-3 and 11-5 may be used. New geometric constants may be estimated or calculated in the manner indicated in Chapter II for other types. A low flux density should be selected. The value depends upon the type of core material and accuracy required by the transformer. Calculations of core exposed surface area, core dissipation, winding exposed surface area, winding losses, and conductor weight can be omitted.

XI. SUMMARY OF DESIGN PROCEDURE AND TEMPERATURE RISE CALCULATION

This chapter includes the step-by-step design procedure as developed on Contract No. DA-36-039 SC-5519, and in addition a method for calculating transformer temperature rise as developed on Contract No. DA-36-039 SC-5470. The design procedure is applicable to designs for frequencies 25 to 2500 cycles per second, for ambient temperatures to 200°C, and for operating temperatures to 200°C. Thus the method can be used in producing designs for high ambient temperatures and low temperature rise, or designs for low ambient temperatures and high temperature rise. In each case the method should give a compact design having the minimum size and weight possible for the type of core chosen.

Step-by-Step Design Procedure

1) **Specifications:**

Frequency, voltages, secondary currents, rectifier filter circuit (where applicable), temperatures (ambient and rise), regulation, grade of protection.

2) **Chosen Quantities:**

Type of core, grade and thickness of lamination, limits for core loss and excitation, core stacking ratio, type of construction (open, compound-filled, or oil-filled).

3) **Nomograph Values:**

a) Secondary rating W_r

$W_r = V_s I_s$ volt-amperes,

where V_s = secondary RMS voltage, volts,
I_s = secondary RMS current, amperes.

b) Allowable winding dissipation W_c/S_c

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per sq. in.,} \qquad \text{(2-19) or Fig. 11-1}$$

where W_c = winding losses, watts,
S_c = winding exposed surface area, square inches,
K from Table 11-1,
ΔT = maximum permissible winding temperature rise, °C.

Note: See Table 11-1A for most commonly-used conditions.

c) Copper space factor F_c

$$F_c = .08 \log_{10} (W_r') + F \qquad (2\text{-}20)$$

$$W_r' = \frac{W_r}{(\frac{f}{60})^{.76} (\frac{\Delta T}{40})^{.63}} \qquad (2\text{-}21)$$

where W_r' = equivalent rating based on 60 cycles and 40°C rise,
F is factor from Fig. 11-2
f = frequency, cps.
ΔT = maximum permissible winding temperature rise, °C.

d) Nomograph scale A factor $\frac{K_o W_r}{F_i f}$

where K_o from Fig. 11-3, 11-4, or 11-5,
f = given frequency,
F_i = core space factor, as given by manufacturer,
W_r = secondary rating, from 3a.

e) Scale F factor $\frac{F_c W_c}{\rho S_c}$

where F_c from 3c,
W_c/S_c from 3b,
ρ = resistivity of conductor at design temperature, the value from Fig. 11-6, increased by 2 per cent.

f) Select flux density B in kilolines per sq. in., from Table 11-2

g) Find characteristic linear dimension ℓ from nomograph, Fig. 11-7.

Note: The following steps h) and i), can be omitted at this point unless it is desired to obtain approximate core loss and excitation. Then, following step 4f, weight, core loss and excitation would be obtained from manufacturer's data.

h) Core weight M_1

$$M_1 = (K_1 F_i \delta_i) \ell^3 \text{ pounds,} \qquad (2\text{-}23)$$

where K_1 is from Fig. 11-3, 11-4, or 11-5,
F_i = core space factor, as given by manufacturer,
δ_i = core steel density in lbs. per cu. in., .276 for cold-rolled, oriented silicon steel, and .272 for hot-rolled, non-oriented silicon steel.
ℓ is from 3g.

(Note: For standard laminations, weight is given by manufacturer's catalogue)

i) Core loss W_i and excitation W_{ex} (to check flux density, B)

Use core weight, material curves, and correction factors from Table 11-3.

Use Table 11-4 as a guide for typical values of core loss and excitation.

4) Core Dimensions:

a) Core exposed surface area S_i

$$S_i = K_2 \ell^2 \text{ sq. in.,} \tag{2-25}$$

where K_2 is from Fig. 11-3, 11-4, or 11-5,

ℓ^2 is from 3g.

b) Core dissipation per unit area W_i/S_i

Use W_i from 3i, and S_i from 4a.

c) Core width (width x stack = cross-sectional area)

$$L = \ell \frac{L}{\ell} \text{ inches,} \tag{2-24}$$

where ℓ is from 3g,
L/ℓ is from Fig. 11-3, 11-4, or 11-5.

d) Select a lamination having a width close to the calculated value. If a wound core is to be used, skip this step and select a core with an area product close to that obtained in 4e.

e) Calculate area product $A_c\ A_i$

$$A_c\ A_i = \ell^4 \tag{2-6}$$

where A_c = window area, sq. in.,
A_i = gross core cross-sectional area, sq. in.

f) Calculate stack height, sL, for stacked cores

$$sL = \frac{A_c\ A_i}{A_c\ L} \text{ inches,}$$

where $A_c\ A_i$ is found from 4e,
A_c = window area of the chosen lamination,
L = lamination width selected in 4d.

5) **Winding Calculations:**

a) Winding exposed surface S_c

$$S_c = K_3\, \ell^2 \text{ sq. in.,} \tag{2-26}$$

where K_3 is from Fig. 11-3, 11-4, or 11-5,
ℓ^2 is from 3g.

b) Approximate winding losses W_c

$$W_c = \frac{W_c}{S_c} S_c \text{ watts} \tag{2-27}$$

where $\frac{W_c}{S_c}$ is found from 3b,
S_c is found from 5a.

c) Approximate per cent regulation $= \frac{W_c}{W_r} 100,$ (2-28)

where W_c is from 5b,
W_r is from 3a.

d) Conductor weight Mc

$$M_c = (K_4\, F_c\, \delta_c)\ell^3 \tag{2-30}$$

where K_4 is from Fig. 11-3, 11-4, or 11-5,
F_c is winding space factor from 3c,
δ_c is conductor material density equal .321 lbs. per cu. in. for copper.

e) Circular mils per ampere

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c}{\rho}\frac{W_c}{S_c}}}\,(K_5 \cdot F_c), \tag{2-31}$$

where ℓ is from 3g,

$\frac{F_c}{\rho}\frac{W_c}{S_c}$ is from 3e,

K_5 is from Fig. 11-3, 11-4, or 11-5,
F_c is from 3c.

f) Primary current I_p

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_i + W_c)^2 + W_{ex}^2 - W_i^2} \text{ amperes,} \qquad (2\text{-}32)$$

where V_p = given primary voltage

W_r = secondary volt-amperes, from 3a,

W_i = core loss, watts, from 3i,

W_c = winding losses, watts, from 5b,

W_{ex} = excitation volt-amperes, from 3i.

g) Calculate wire sizes in circular mils

$= (\frac{\text{Circular mils}}{\text{ampere}})$ amperes,

where circular mils per ampere is from 5e.

Then select a wire for each winding from Table 11-5.

h) Turns per volt N/V

$$\frac{N}{V} = \frac{10^5}{4.44 \, f \, F_i \, B \, A_i} \qquad (2\text{-}33)$$

where

f = frequency, cycles per second,

F_i = core space factor, as given by manufacturer,

B = flux density in kilolines per square inch, from 3f,

A_i = gross core cross-sectional area, sq. in.

i) Calculate turns of each winding, correcting for regulation

Nominal turns = $\frac{N}{V}$ times voltage of the winding.

where

$\frac{N}{V}$ is from 5h.

Correct for regulation by adding turns to secondaries and subtracting turns from primary, using per cent regulation from 5c. In most cases secondary turns are increased by a fraction equal one-half of the regulation, and primary turns are decreased by the same fraction. However, exceptions may occur, such as when there is difficulty in providing an integral number of turns for several windings. Calculate any winding taps.

6) Winding Layout

a) Find winding width, equal window length minus margins from Table 11-6.

b) Find turns per layer from Table 11-5 and calculate number of layers.

c) Choose a tube thickness from Table 11-7 and layer insulation from Table 11-6. Check voltage stresses if above 250 volts.

7) Check the Coil Build

Add tube thickness, wire, layer insulation, wrappers, shield (if any). The sum should be about 80 to 90 per cent of window width.

8) Summarise the Design

List core material and dimensions, tube, winding wire sizes, total turns, turns per layer, number of layers, taps, layer insulation, wrappers, and shield data.

9) Check of Winding Resistances

a) Calculate resistance of each winding, equal to resistance per unit length (corrected to operating temperature from Fig. 11-6), times mean length of turn, times number of turns. Resistance also equals resistivity, times mean length of turn, times number of turns, divided by wire cross-sectional area.

b) Calculate mean length of turn of each winding, which is equal to the length of the inside turn of the winding, plus pi times the build-up of that winding.

10) Check of Voltage Ratio

Calculate primary voltage

$$V_p = n \left[V_s + I_s \left(R_s + R_p/n^2\right)\right] \text{ volts} \qquad (2\text{-}35)$$

where R_s and R_p are obtained from 9.

Adjust the turns ratio if the calculated primary voltage differs appreciably from the specified voltage.

11) Special Calculations and Design Checks (when necessary or when a value is close to limit)

a) Winding losses W_c

W_c = sum of current squared times resistance for each winding.

b) Conductor weight M_c

M_c equals length of conductor times pounds per unit length; or, is length in inches times cross-sectional area in square inches times density (.321 lbs. per cu. in.).

c) Exposed winding surface area S_c
Add all outside coil side and end surfaces except those facing the core.

d) Calculate winding loss per unit exposed surface area, $\frac{W_c}{S_c}$, using values from 11a and 11c.

e) Find core weight M_i from lamination handbook for appropriate stack height. This is also

$$M_i = m_i A_i F_i \delta_i, \tag{2-22}$$

where m_i = mean length of magnetic circuit,

A_i = gross core cross-sectional area,

F_i = core space factor,

δ_i = core material density.

Use dimensions for actual core.

f) Check flux density B in kilolines per square inch, from (2-33)

$$B = \frac{10^5}{4.44 \, f \, F_i \, A_i \, (N/V)}$$

g) Check core loss and excitation, as in 3i.

h) Exposed core surface area S_i
Add all outside edge and face surfaces of the core except those in contact with the winding.

i) Core loss per unit exposed core surface $\frac{W_i}{S_i}$

Divide W_i from 11g, by S_i from 11h.

12) Comparison of Design Procedure and Calculated Values

Compare values from the design method with the detailed checks in step 11, when these are made.

13) Calculation of Temperature Rise

See method in section following.

TABLE 11-1

VALUES OF K FOR APPROXIMATE TEMPERATURE-RISE EQUATION

Transformer Type		Open				Compound-Filled			Oil-Filled		
Type of Core		Shell	Shell (End Cases)	Simple	Core	Shell	Simple	Core	Shell	Simple	Core
Ambient Temp.-°C	Freq. C.P.S.										
25	25	76	90	92	107	70	84	98	58	69	80
25	60	79	94	95	110	74	88	103	62	74	86
25	200	89	107	107	123	85	101	117	73	87	101
25	400	99	122	119	137	98	117	136	85	101	117
25	800	108	132	129	149	107	128	149	95	113	131
25	2500	112	136	131	151	110	131	152	98	116	134
50	25	72	87	86	100	68	81	94	56	67	78
50	60	75	91	90	104	72	86	100	60	71	83
50	200	84	104	101	117	83	99	115	71	84	97
50	400	94	119	112	129	95	113	131	83	98	114
50	800	102	129	122	139	104	124	144	92	110	128
50	2500	106	132	125	143	107	128	149	95	113	131
65	25	69	85	83	96	67	80	93	55	65	76
65	60	72	90	87	100	71	84	98	59	70	81
65	200	81	102	97	113	82	98	114	69	83	95
65	400	91	117	108	125	93	111	129	81	96	112
65	800	98	126	118	134	102	121	141	90	107	124
65	2500	102	129	121	138	105	125	146	93	110	128
75	25	68	84	81	94	66	79	93	54	64	74
75	60	71	89	85	98	70	83	97	58	69	80
75	200	80	100	95	100	81	97	113	68	82	94
75	400	89	115	105	122	92	110	128	80	95	110
75	800	96	125	115	131	100	120	139	88	105	122
75	2500	100	128	118	134	103	124	144	91	108	126
85	25	66	83	79	92	65	78	92	53	63	73
85	60	69	88	83	96	69	82	96	57	68	79
85	200	78	99	93	107	80	96	112	67	80	93
85	400	87	114	103	119	91	109	127	79	94	109
85	800	94	123	112	128	99	118	137	87	103	120
85	2500	97	126	115	131	102	122	142	90	107	124
125	25	62	79	74	86	63	76	88	50	60	70
125	60	65	84	78	90	66	79	92	54	64	74
125	200	72	96	86	99	76	91	106	64	76	88
125	400	80	108	96	111	87	104	121	75	89	103
125	800	87	118	103	119	95	113	131	83	99	105
125	2500	90	120	107	123	98	117	136	85	101	117
200	25	52	74	63	73	58	69	80	46	55	64
200	60	54	77	66	75	61	73	85	49	58	67
200	200	62	88	73	85	70	84	98	58	69	80
200	400	68	100	81	94	80	96	112	68	81	94
200	800	74	109	88	101	87	104	121	75	89	103
200	2500	76	111	91	104	90	107	124	77	92	106

Table 11-1A-Values of $(\Delta T/K)^{1.25}$ for Standard Conditions

Temp. Rise Deg C	Ambient Temp. Deg C	Frequency cps	Open core and coil			Compound			Oil		
			Shell	Simple	Core	Shell	Simple	Core	Shell	Simple	Core
40	65	60	0.476	0.384	0.322	0.50	0.438	0.333	0.625	0.50	0.416
40	65	400	0.358	0.294	0.247	0.352	0.278	0.232	0.417	0.333	0.278
115	85	60	1.95	1.52	1.26	1.95	1.55	1.26	2.45	1.95	1.62
115	85	400	1.46	1.19	0.975	1.36	1.10	0.895	1.12	1.31	1.10

Table 11-2

SUGGESTED FLUX DENSITIES FOR SILICON STEELS AT VARIOUS FREQUENCIES

(Kilolines per square inch)

Material and Core	Frequency - cycles per second 25	60	400	800	1600	2500
Non-oriented Steel, Stacked Core	85-100	80-98	60-95	45-65	25-45	18-35
Oriented Steel, Stacked Core	95-103	90-100	70-97	50-70	30-50	22-40
Oriented Steel, Wound Core	98-108	95-105	80-100	55-80	35-55	25-45

Table 11-3

TYPICAL CORE LOSS AND EXCITATION OF TRANSFORMER CORES AS PERCENT OF EPSTEIN VALUES FOR SILICON STEEL

Material and Core	Core Loss Percent	Excitation Percent
Non-oriented steel, stacked core	120-140	150-300
Oriented steel, stacked core	130-160	300-1000
Oriented steel, wound core with two butt joints	105-135	200-500

Table 11-4

TYPICAL VALUES FOR CORE LOSS, EXCITATION, AND REGULATION

Rating, volt-amperes	Frequency	Core loss, %	Excitation, %	Regulation,* %
10	60	5-15	40-60	10-30
10	400	10-20	20-40	6-12
100	60	3-6	24-45	4-10
100	400	4-8	5-15	2-5
1000-5000	60	1-5	10-50	1-4
1000-5000	400	1-5	10 or less	1-3

* The data for regulation are confined to low temperature (Class A), low reactance designs with unity power factor loads. In this report, regulation is calculated using no-load and full-load voltages obtained with the windings at temperatures corresponding to full-load.

Table 11-5 - COPPER WIRE DATA

Size AWG	Area in Circular Mils	Diameter - Inches Bare Wire	1 Layer Enamel	2 Layers Enamel	Turns/Inch (1 Layer Enamel)	Ohms/1000 ft. at 20°C, 100% Cond.	Lbs. per 1000 ft., bare
4	41,740	.2043				.248	126.4
5	33,100	.1819				.313	100.2
6	26,250	.1620				.395	79.5
7	20,820	.1443				.498	63.0
8	16,510	.1285	.1305	.1324	7	.628	50.0
9	13,090	.1144	.1164	.1182	8	.792	39.6
10	10,380	.1019	.1039	.1056	9	.999	31.4
11	8,234	.0907	.0927	.0943	10	1.260	24.9
12	6,530	.0808	.0827	.0842	11	1.588	19.77
13	5,178	.0720	.0738	.0753	12	2.003	15.68
14	4,107	.0641	.0659	.0673	13.5	2.525	12.43
15	3,257	.0571	.0588	.0602	15	3.184	9.86
16	2,583	.0508	.0525	.0539	17	4.016	7.82
17	2,048	.0453	.0469	.0482	19	5.064	6.20
18	1,624	.0403	.0418	.0432	21	6.385	4.92
19	1,288	.0359	.0374	.0387	24	8.051	3.90
20	1,022	.0320	.0334	.0346	27	10.15	3.09
21	810	.0285	.0300	.0310	30	12.80	2.45
22	624.4	.0253	.0267	.0278	34	16.14	1.94
23	509.5	.0226	.0238	.0249	39	20.36	1.54
24	404.0	.0201	.0213	.0224	43	25.67	1.22
25	320.4	.0179	.0191	.0201	48	32.37	.970
26	254.1	.0159	.0170	.0180	54	40.81	.769
27	201.5	.0142	.0153	.0161	60	51.47	.610
28	159.8	.0126	.0136	.0145	67	64.90	.484
29	126.7	.0113	.0122	.0130	75	81.83	.384
30	100.5	.0100	.0109	.0116	84	103.2	.304
31	79.50	.0089	.0100	.0105	94	130.1	.241
32	63.21	.0080	.0088	.0095	104	164.1	.1913
33	50.13	.0071	.0078	.0085	117	206.9	.1517
34	39.75	.0063	.0070	.0075	131	260.9	.1203
35	31.52	.0058	.0062	.0067	146	329.0	.0954
36	25.00	.0050	.0056	.0060	162	414.8	.0757
37	19.83	.0045	.0050	.0055	183	523.1	.0600
38	15.72	.0040	.0045	.0050	204	649.6	.0476
39	12.47	.0035	.0040	.0043	227	831.8	.0377
40	9.89	.0031	.0035	.0038	261	1,049	.0299
41	7.84	.0028	.0031	.0036	296	1,323	.02374
42	6.22	.0025	.0028	.0032	326	1,668	.01882
43	4.93	.00222	.00237	----	384	2,103	.01493
44	3.91	.00198	.00213	----	426	2,652	.01184
45	3.10	.00176	.00188	----	483	3,344	.00939
46	2.46	.00157	.00169	----	538	4,217	.00745
47	1.95	.00140	.00151	----	603	5,320	.00590
48	1.55	.00124	.00135	----	674	6,710	.00468
49	1.227	.001107	.00121	----	742	8,460	.00371
50	.973	.00986	.00108	----	830	10,670	.00294

Table 11-6

LAYER INSULATION AND MARGINS FOR MECHANICAL STRENGTH

Wire Size AWG	Layer Insulation, mils	Margins at each end, inches
10-16	10.0	5/32
17-19	7.0	"
20-21	5.0	1/8
22-24	3.5	"
25-27	2.2	"
28-31	1.5	"
32-33	1.3	3/32
34-38	1.0	"
39-41	.7	1/16
42-44	.5	"

Table 11-7

TUBE THICKNESS FOR MECHANICAL STRENGTH

Smallest Core Dimension, inches	Tube Thickness, mils of paper
to 1/2	10-20
1/2 to 5/8	15-30
5/8 to 3/4	17-35
3/4 to 7/8	20-40
7/8 to 1	25-45
1 - up	30-50

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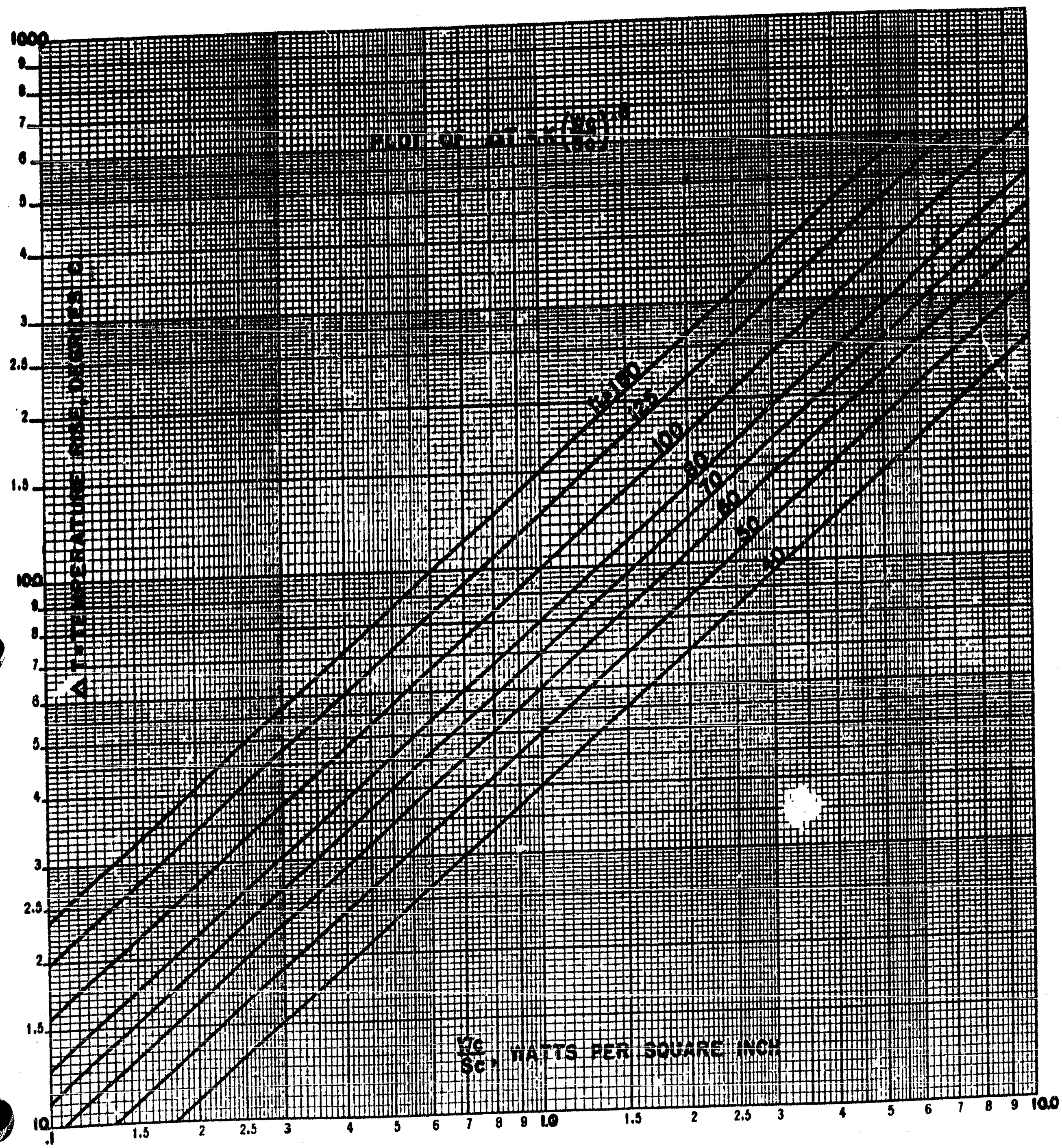

FIG. 11-1—TEMPERATURE RISE AND WINDING DISSIPATION

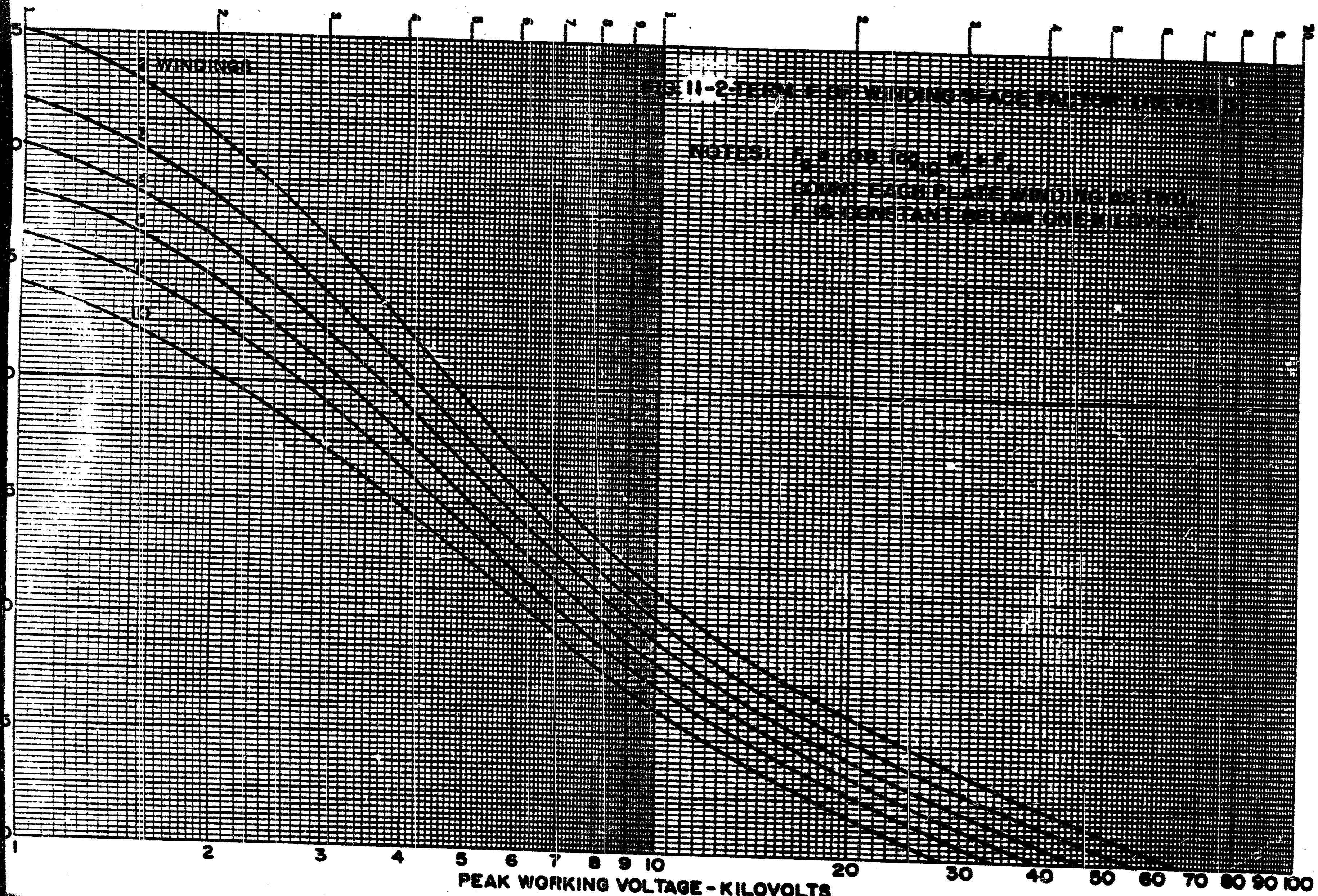

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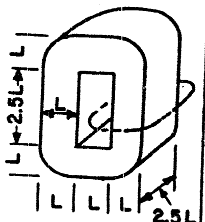
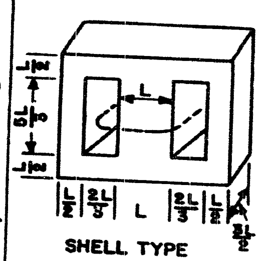
FIG. 11-3 CONSTANTS FOR DESIGN EQUATIONS

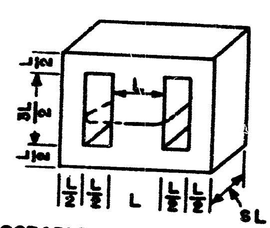
CORE SKETCH AND DESCRIPTION



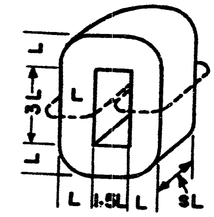
SIMPLE TYPE (TYPICAL PROPORTIONS)



SHELL TYPE (AVERAGE PROPORTIONS)



SCRAPLESS E-I SHELL TYPE



CORE TYPE (AVERAGE PROPORTIONS)

	Simple Type (Typical Proportions)	Shell Type (Average Proportions)	Scrapless E-I Shell Type					Core Type (Average Proportions)		
s			1	1½	2	2½	3	1	1.5	2.25
AlAc/L⁴	6.25	1.667	0.750	1.125	1.50	1.875	2.25	4.5	6.75	10.12
L/2	0.633	0.880	1.077	0.970	0.902	0.854	0.817	.687	.621	.561
a	6.42	5.87	6.45	5.82	5.42	5.13	4.90	8.35	7.55	6.82
b	6.42	6.24	6.00	6.36	6.84	7.32	7.82	4.37	4.57	4.97
c	1.00	1.162	1.155	1.411	1.632	1.826	2.00	.472	.578	.707
d	1.00	0.860	0.886	0.706	0.612	0.548	0.500	2.12	1.732	1.414
e	16.9	12.22	13.02	10.61	9.20	8.23	7.51	29.5	25.8	23.1
g	16.9	21.7	23.1	24.0	25.3	26.6	28.0	10.17	10.08	10.44
K_0	0.616	0.661	0.630	0.649	0.675	0.696	0.720	.560	.554	.552
K_1	6.42	6.84	7.45	8.23	8.86	9.35	9.80	3.94	4.37	4.82
K_2	16.9	21.7	23.1	24.0	25.3	26.6	28.0	10.17	10.08	10.44
K_3	16.9	12.22	13.02	10.61	9.20	8.23	7.51	29.5	25.8	23.1
K_4	6.42	5.37	5.20	4.49	4.19	4.01	3.91	9.27	7.92	7.03
K_5	785	842	803	826	860	886	917	712	705	703
K_6	22,500	19,370	19,480	15,920	13,780	12,310	11,250	47,700	38,900	31,800

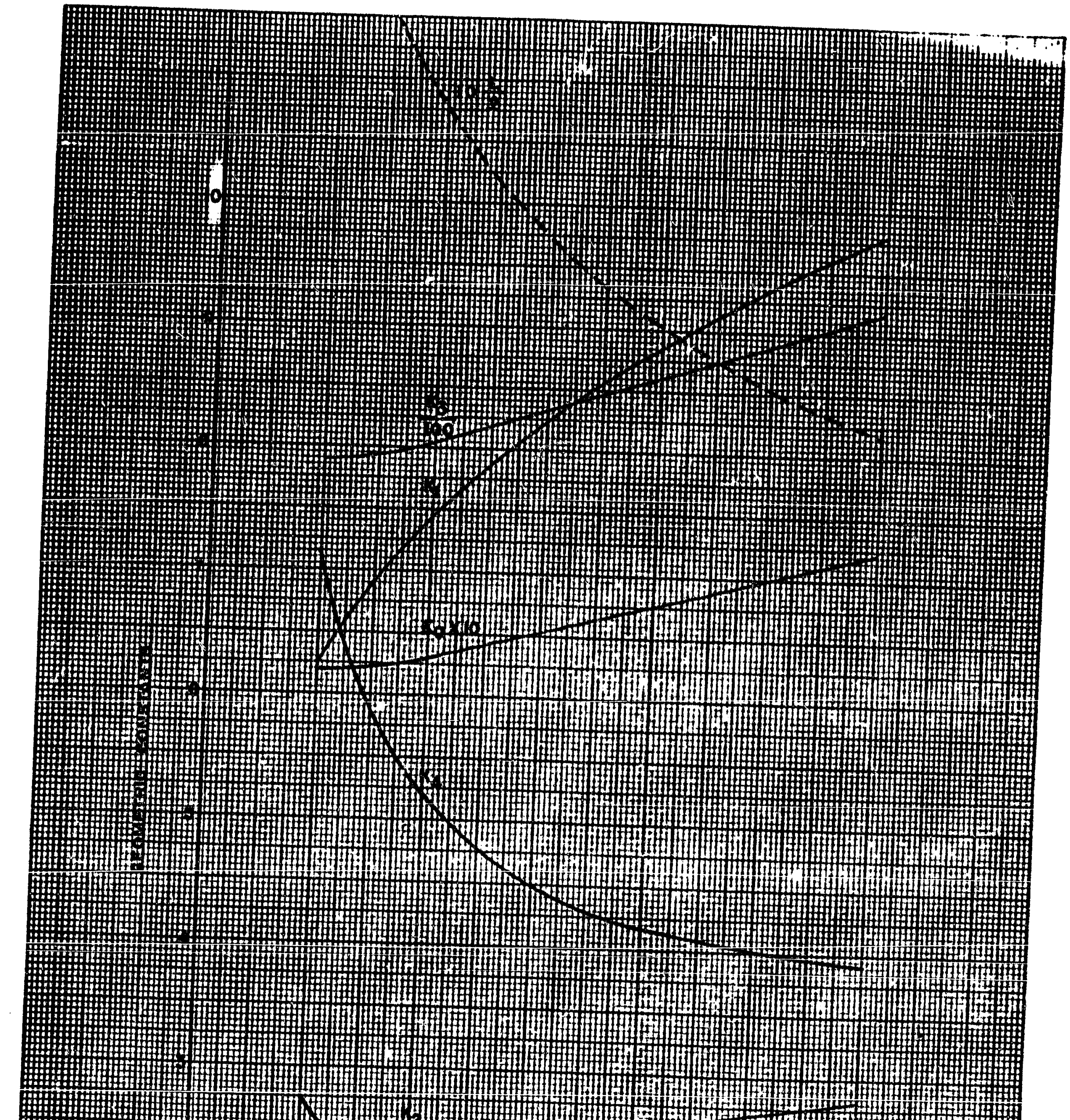


FIG. II-4 GEOMETRIC CONSTANTS FOR

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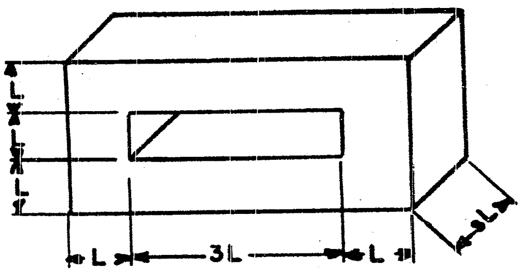
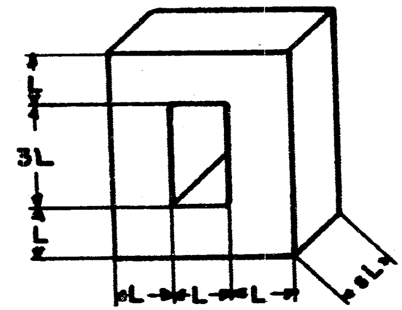
FIG. II-5 CONSTANTS FOR CORES WITH SCRAPLESS UI LAMINATIONS



SIMPLE TYPE WITH SCRAPLESS UI LAMINATIONS

S	1.0	1.5	2.25
$A_1 A_c/L^4$	3.0	4.5	6.75
L/ℓ	.760	.689	.620
a	9.12	8.26	7.44
b	5.43	5.62	5.98
c	.579	.713	.866
d	1.74	1.43	1.16
e	23.2	20.3	17.9
g	17.9	17.8	18.2
K_0	.638	.629	.623
K_1	5.28	5.89	6.44
K_2	17.9	17.8	18.2
K_3	23.2	20.3	17.9
K_4	9.46	8.04	6.94
K_5	814	802	795
K_6	38,900	31,600	26,000



CORE TYPE

S	1.0	1.5	2.25
$A_1 A_c/L^4$	3.0	4.5	6.75
L/ℓ	.760	.689	.620
a	9.12	8.26	7.44
b	4.23	4.53	5.01
c	.579	.713	.866
d	1.74	1.43	1.16
e	26.6	23.8	21.6
g	12.7	12.8	13.3
K_0	.526	.521	.518
K_1	5.28	5.89	6.44
K_2	12.7	12.8	13.3
K_3	26.6	23.8	21.6
K_4	7.36	6.48	5.81
K_5	659	663	660
K_6	38900	31600	26000

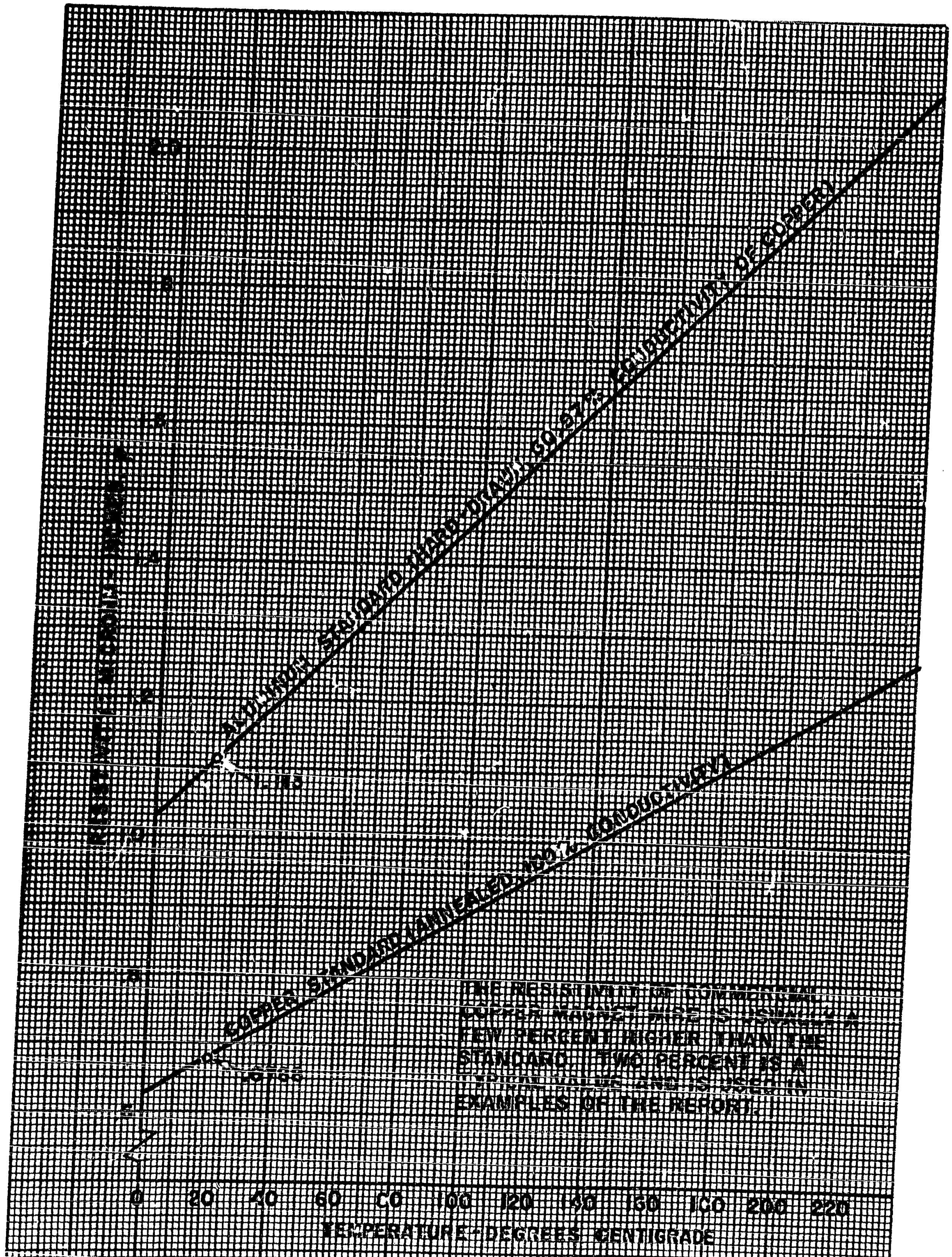
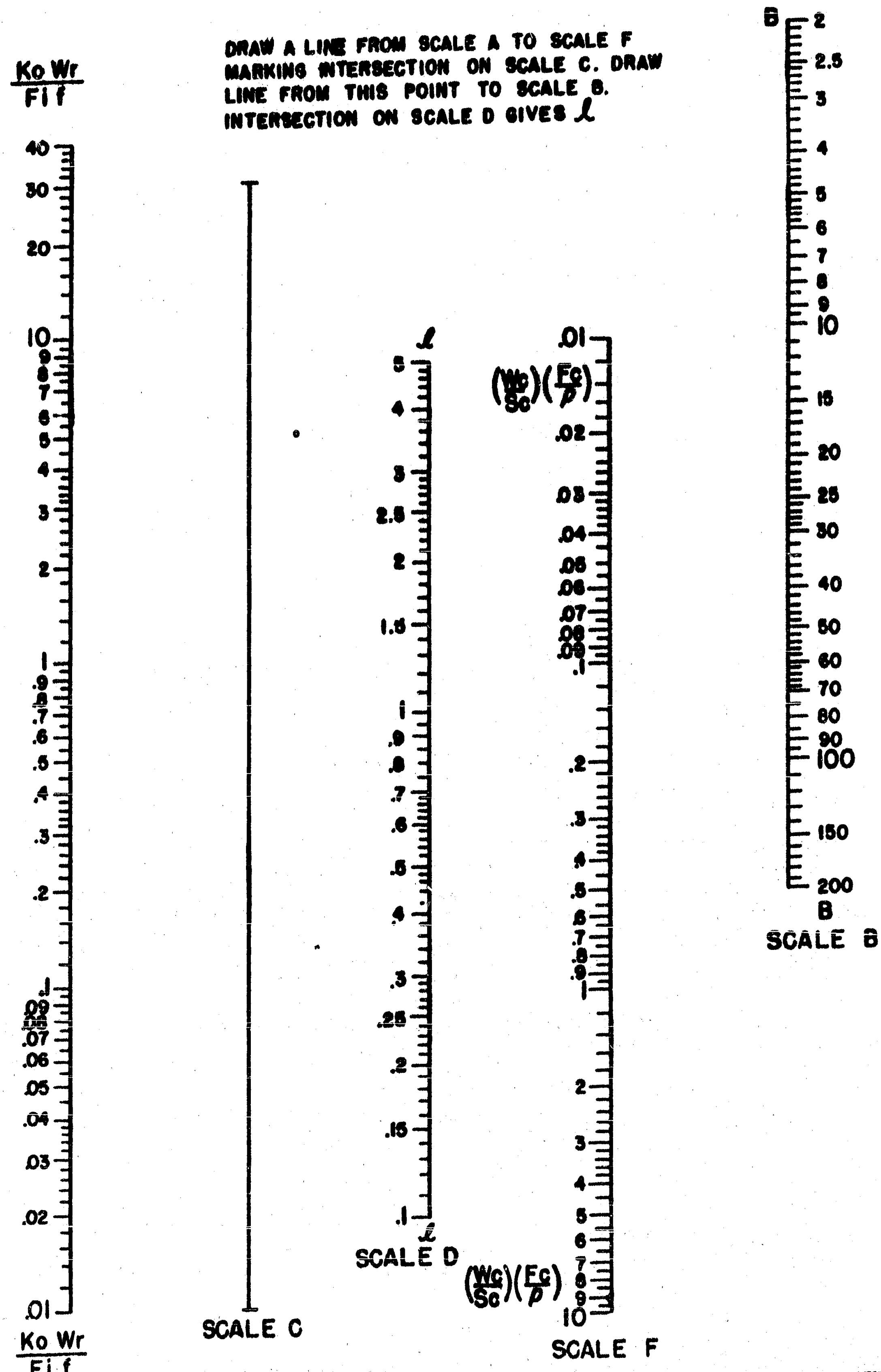


FIG. II-6 RESISTIVITIES OF COPPER AND ALUMINUM

FIG. 11-7 POWER TRANSFORMER NOMOGRAPH



Calculation of Temperature Rise

No design of a power transformer is complete without either a calculation of its winding temperature rise, or a comparison with previously manufactured transformers to make sure that temperature rise will not be excessive. The design method includes maximum temperature rise as a required specification. Equation 2-19 introduces temperature rise into the design calculations as one of the main factors determining size. The parameter, K, from Table 11-1, is an average based on past experience with standard types of construction.

The method of calculating temperature rise presented here is based on an analysis of the heat sources within a transformer and the paths by which this heat flows to the ambient (Contract No. DA-36-039 SC-5470). By making some simplifying assumptions, the heat flow diagram of Fig. 11-8 can be applied to any standard type of power transformer. The heat generated by core and coil losses flows to the surface of the core and coil respectively, and for open types of construction, it is then transmitted directly to the surrounding medium. For encased types of construction, it flows from the coil and core surfaces, across the impregnant, oil or compound, to the case. From the case surface, the heat is transmitted by convection and radiation to the surrounding medium. Conduction of heat from the case can also occur through the transformer mounting. However, conduction losses are usually small, and in any event, the transformer designer seldom has control over mounting conditions.

There are three transformer temperature gradients that are important. As shown on Fig. 11-8, these are the surface gradient, θ_{surf}, the impregnant gradient, θ_{imp}, and the coil gradient, θ_h. Each is independent of the other, but is dependent on the type of construction, and on the transformer losses. For an open type transformer, θ_{imp} is zero.

1) Surface Temperature Rise

The following equations are used to calculate the surface rise over the ambient:

$$\theta_{surf} = F_{surf} \frac{(W_c + W_i)}{(S_c + S_i)(h_c + h_r)} \text{ degrees C} \qquad (11\text{-}1)$$

$$h_c = 3.75 \times 10^{-3} \frac{(\theta_{surf})^{0.22}}{(S_c + S_i)^{0.17}} p^{.44} \frac{\text{watts}}{\text{in}^2 \, ^\circ\text{C}} \qquad (11\text{-}2)$$

$$h_r = e \frac{3.70 \times 10^{-3}}{\theta_{surf}} \left[\left(\frac{T_{surf}}{100}\right)^4 - \left(\frac{T_{amb}}{100}\right)^4 \right] \frac{\text{watts}}{\text{in}^2 \, ^\circ\text{C}} \qquad (11\text{-}3) \text{ or Fig. 11-9}$$

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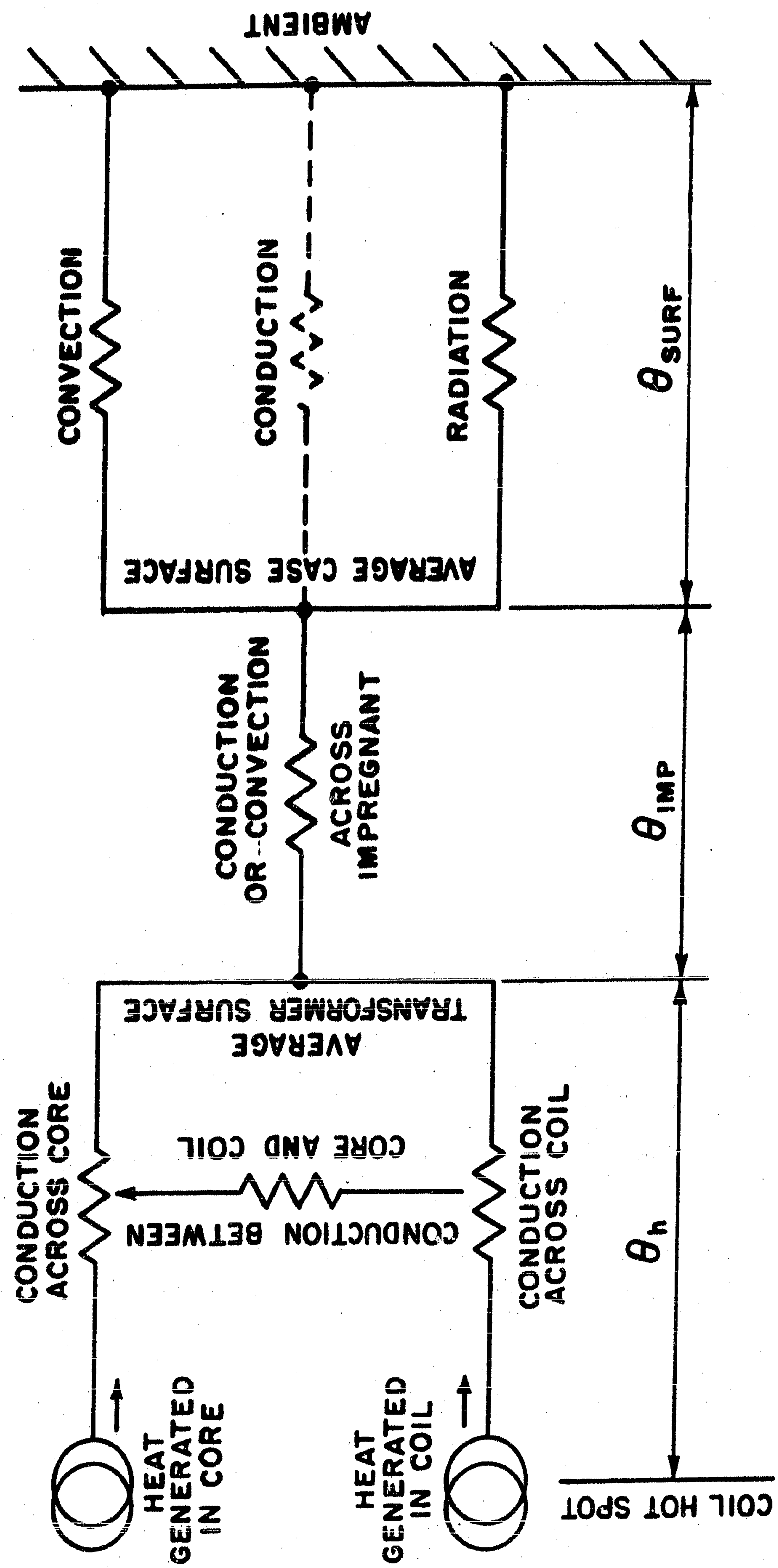

FIG. 11-8 HEAT FLOW ANALOGUE OF A TRANSFORMER

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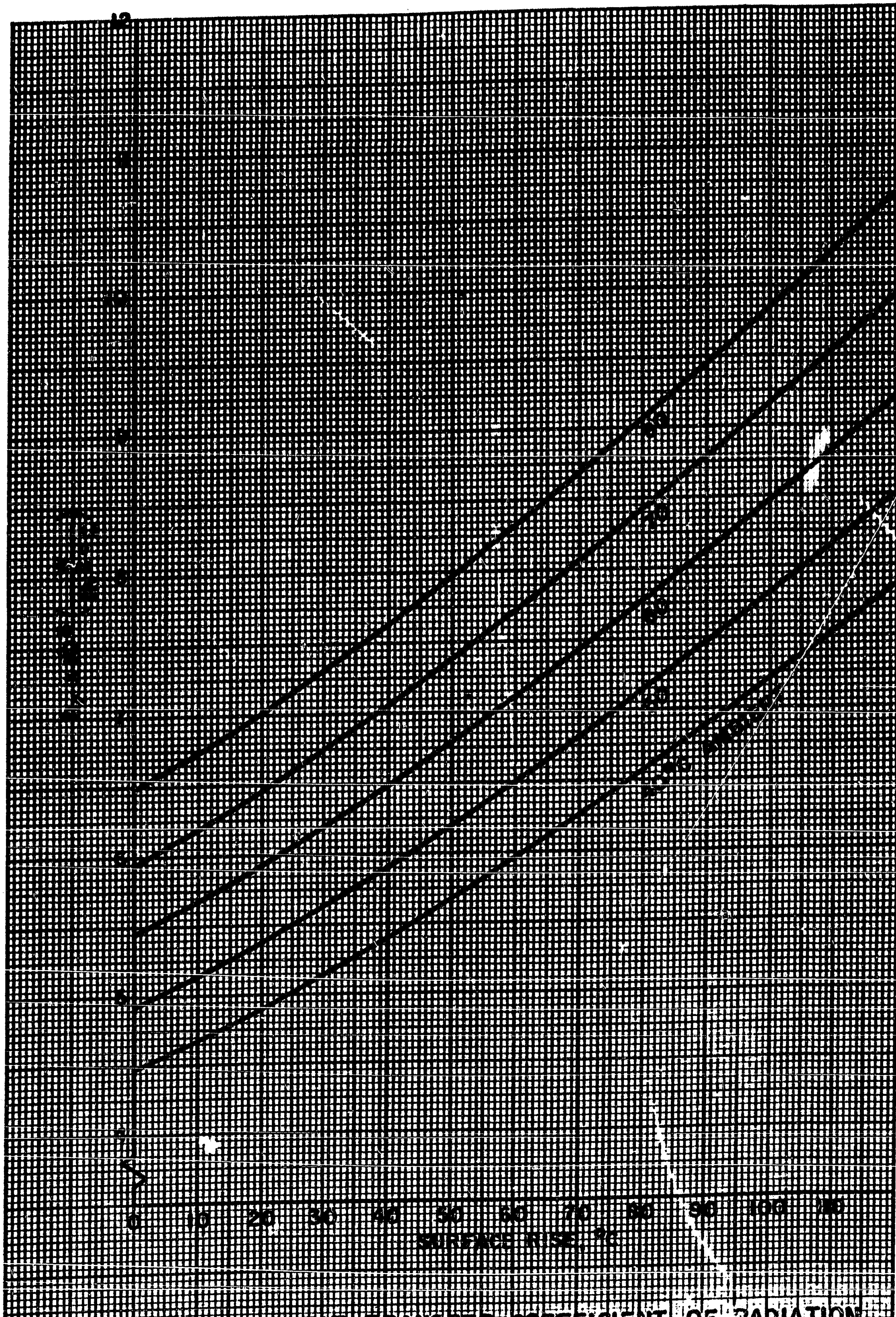

FIG. II-9 HEAT TRANSFER COEFFICIENT OF RADIATION
EMISSIVITY = 1

where W_c = winding loss, watts,
W_i = core loss, watts,
S_c = exposed coil surface, square inches.
S_i = exposed core surface, square inches,
p = air pressure in atmospheres,
h_c = coefficient of free convection, $\frac{\text{watts}}{\text{sq. in. °C}}$,
h_r = coefficient of radiation, $\frac{\text{watts}}{\text{sq. in. °C}}$,
T_{surf} = absolute temperature of surface, °Kelvin (°C+273),
T_{amb} = absolute temperature of ambient, °Kelvin,
e = emissivity of surface,
F_{surf} = form factor of surface.

It will be noted that h_c and h_r used in equation (11-1) to calculate θ_{surf}, are themselves functions of θ_{surf}. This necessitates a trial procedure, whereby an assumed θ_{surf} is used to find h_c, h_r, which in turn are used to calculate θ_{surf}. If the calculated value is not close to the assumed value, then the calculation should be repeated.

In an attempt to eliminate all further trials beyond the second, a guide has been devised* for selecting the second assumed value. This guide, believed to be sufficiently accurate if the first calculated value is less than the first assumed value, is

$$\theta_{surf} = .1\theta_{assumed} + .9\theta_{calc}. \tag{11-4}$$

where θ_{surf} is value to be used for the second trial,
$\theta_{assumed}$ is initially assumed value,
θ_{calc} is the result of the first calculation.

Figure 11-10 gives the ratio of θ_{surf} to θ_{calc}, the correction factor to be applied to θ_{calc}, as a function of $\theta_{assumed}$ to θ_{calc}. Equation (11-4) defines the curve only above (1.1). The accuracy of the function of Fig. 11-10 below (1.1) has not been well confirmed. The first assumption for θ_{surf} (which is $\theta_{assumed}$) should be somewhat over half, such as 60 per cent, of the maximum permissible winding temperature rise, so that the abscissa of Fig. 11-10 will be greater than 1.0.

* Mr. I. Remis of the Signal Corps suggested this guide.

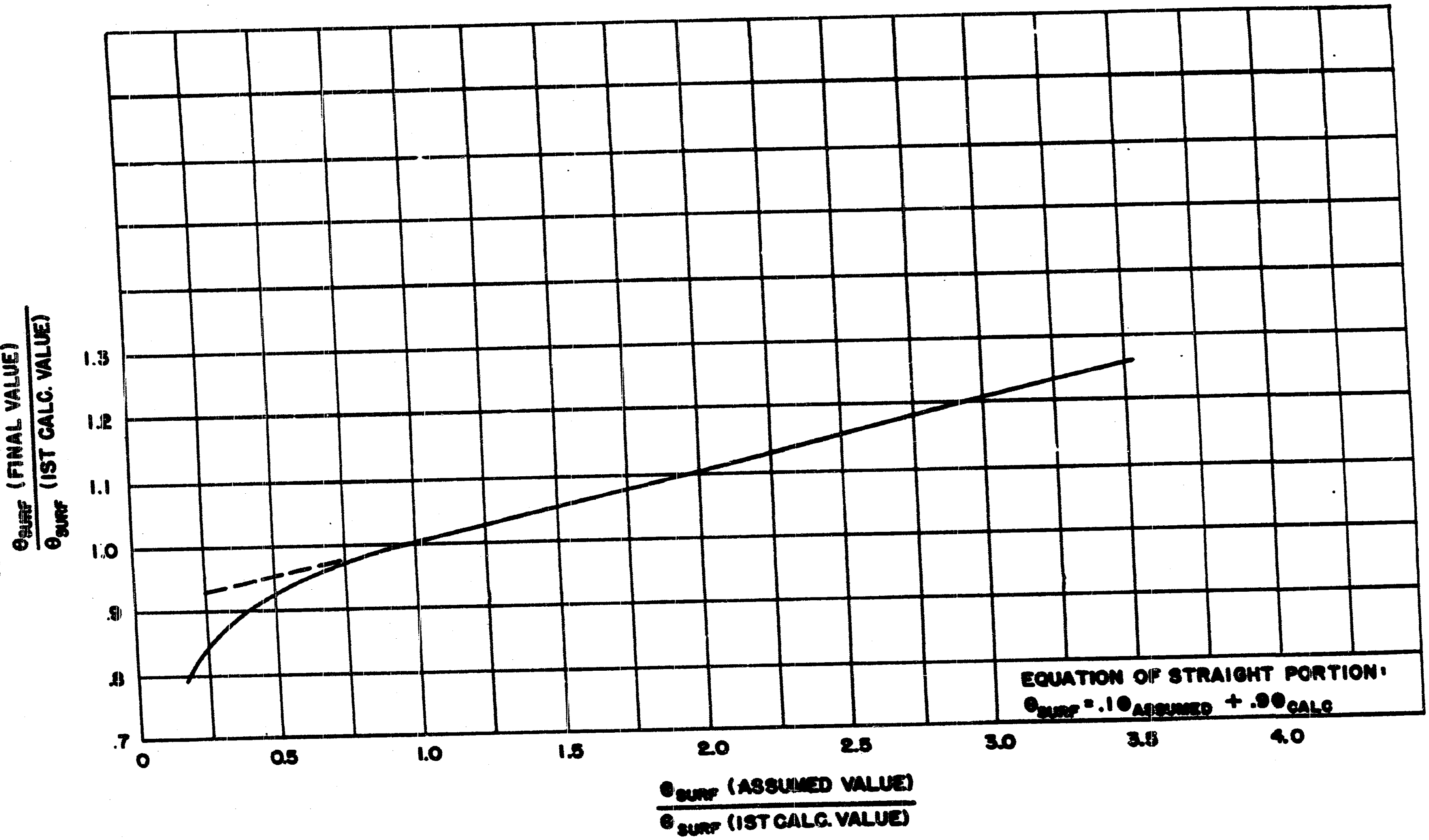

FIG. II-10 GUIDE TO SURFACE TEMPERATURE RISE

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The following tables give values for e and F_{surf}.

Table 11-8 - EMISSIVITY OF SURFACES

Surface	Emissivity
oil paint (any color)	0.92 - 0.96
enamel (any color)	0.88 - 0.91
varnish	0.88 - 0.91
black lacquer	0.80 - 0.95
aluminum paint	0.27 - 0.67
dull sheet steel	0.80

Table 11-9 - SURFACE FORM FACTORS

Type of Transformer	F_{surf}
Open (shell or core)	0.9
Potted	1.1
Oil-filled	1.2

In using Table 11-8, the value of emissivity to choose within the range for any particular surface, depends upon the glossiness of the surface. Dull surfaces have a higher emmisivity.

2) Gradient Across Impregnant

a. Compound-filled transformers:

The average temperature drop, Θ_{imp}, across the compound of a potted transformer can be expressed as:

$$\Theta_{imp} = F_{imp} \frac{(W_c + W_i)m}{S \quad k} \text{ degrees C,} \tag{11-5}$$

where

m = average thickness of compound, inches,

S = average area of compound, square inches,

k = thermal conductivity of compound, $\frac{\text{watts}}{\text{in. °C}}$,

F_{imp} = correlation factor.

The average compound area is defined as the mathematical average of the case surface area and the transformer area.

$$S = \frac{1}{2} (S_{case} + S_c + S_i) \text{ square inches.} \tag{11-6}$$

The average compound thickness m, is defined as the difference between the radii of two spheres whose areas are equal to the surface areas of the case and of the transformer.

$$m = \sqrt{\frac{S_{case}}{4\pi}} - \sqrt{\frac{S_c + S_i}{4\pi}} \text{ inches.} \qquad (11\text{-}7)$$

An empirical value for F_{imp} is 1.75. This applies to transformers potted in rectangular cases such as the MIL T-27 series. Typical values for the conductivity of potting compounds are given in Table 11-10.

Table 11-10

THERMAL CONDUCTIVITY OF POTTING COMPOUNDS

Type of Compound	K - watts/in.°C
100% bitumin	.008
bitumin - 45% silica	.015
bitumin - 55% silica	.016

b. Encapsulated Units:

Transformers sealed in plastic compounds fall into the same class as compound-filled units. If the final shape is rectangular, an F_{imp} = 1.75 would apply. If the transformer is coated with a uniform thickness of plastic, F_{imp} = 1.0, and m equals the actual thickness.

c. Oil Filled Transformers:

Accurately predicting the temperature gradient across the oil in a small oil-filled transformer is very difficult. In any specific transformer it is difficult to determine the percentage of heat being transferred through the oil by convection and that transferred by conduction. However, there are equations giving approximate results. If previous data are available for a particular transformer the oil gradient may be found from the following equation:

$$\Theta = C\,(W_c + W_i)^{1.25} \text{ degrees C,} \qquad (11\text{-}8)$$

where

W_c = copper loss, watts,

W_i = core loss, watts,

C = constant.

The constant, C, is found by substituting previous data into equation 11-8.

The following empirical equation may be used for a transformer filled with an organic insulating oil such as "Wemco C" (Westinghouse):

$$\Theta = 130 \frac{m}{S} (W_c + W_i)^{.78} \text{ degrees C,} \qquad (11\text{-}9)$$

where

S = oil area from equation (11-6)

m = oil thickness from equation (11-7)

3) Gradients Across the Coil

a. Coil hot spot:

For the usual case, the hot spot of a coil is located in the center of the coil cross section. The gradient from average transformer surface temperature to hot-spot temperature may be found from the following:

$$\Theta_h = F W_c^x \left(\frac{m}{k S_c}\right)^y \text{ degrees C,} \qquad (11\text{-}10)$$

where

Θ = coil hot spot to transformer surface gradient, degrees C

W_c = copper loss, watts,

m = distance from coil surface to hot spot (assumed to be 1/2 coil build), inches,

k = coil conductivity, watts/in.°C,

S_c = exposed coil surface area, square inches,

F, x, y are parameters dependent on construction.

Typical experimental values for F, x and y are to be found in Table 11-11.

Table 11-11

COIL GRADIENT PARAMETERS

Type	F	x	Y
open	1.2	.85	1.4
potted	.32	1.0	2.0
oil	.32	1.0	2.0

The thermal conductivity of layer wound coil is given by

$$k = k_i \left(\frac{R + 1}{.11R + 1} \right) \qquad (11\text{-}11)$$

k = coil conductivity, $\frac{\text{watts}}{\text{in. °C}}$,

R = ratio of bare wire diameter to total insulation thickness per layer,

k_i = thermal conductivity of composite insulation, $\frac{\text{watts}}{\text{in. °C}}$.

The total insulation thickness used for R means the interlayer insulation thickness plus twice the wire radial insulation thickness. The thermal conductivity of the composite insulation refers to an insulation which is equivalent to the interlayer insulation, plus the impregnant, plus any voids. A typical value of k_i for a varnish impregnated coil with kraft paper insulation is .003 $\frac{\text{watts}}{\text{in. °C}}$.

b. Average Winding Rise

Temperature distribution throughout a winding is such that the average winding rise over the transformer surface temperature is directly related to the hot spot rise, depending on the location within the coil of the particular winding.

$$\Theta_w = C\,\Theta_h \text{ degrees C,} \qquad (11\text{-}12)$$

Θ_w = average winding rise over the trnasformer surface temperature, degrees C,

C = constant.

If m is the distance measured radially from the first layer of a coil to the surface, Table 11-12 gives values for the constant, C, depending on winding position within the coil.

Table 11-12

AVERAGE WINDING RISE PARAMETER

Winding Space % of m	Constant, C Open	Potted	Oil-Filled
0 - 50	.90	.90	.80
50 - 100	.80	.65	.60
0 - 25	.80	.80	.67
25 - 50	.97	.97	.93
50 - 75	.99	.92	.86
75 - 100	.62	.42	.35

4) Summary

To find the temperature rise of any particular part of a transformer, it is necessary to add up the gradients from the ambient to the part for which the rise is required. Adding the ambient temperature to the rise gives actual operating temperature.

For example, the hot spot rise over ambient equals:

$$\Theta = \Theta_{surf} + \Theta_{imp} + \Theta_{h}. \qquad (11\text{-}13)$$

The hot spot temperature equals:

$$T_{h} = T_{amb} + \Theta_{surf} + \Theta_{imp} + \Theta_{h}. \qquad (11\text{-}14)$$

If the primary of a particular transformer occupies the first half of the coil, its average temperature is

$$T_{pri} = T_{amb} + \Theta_{surf} + \Theta_{imp} + C\,\Theta_{h}. \qquad (11\text{-}15)$$

Table 11-13

DESIGN EQUATIONS

$W_r = V_s I_s$	$S_1 = K_2 \ell^2 =$
$\frac{W_c}{S_c} = (\frac{\Delta T}{K})^{1.25}$	$\frac{W_1}{S_1}$
$F_c = .08 \log_{10} W_r{}' + F$	$L = \ell(L/\ell)$
$W_r{}' = \frac{W_r}{(\frac{f}{60})^{.76} (\frac{\Delta T}{100})^{.63}}$	$A_c A_1 = \ell^4$
$\frac{K_0 W_r}{F_1 f}$	A_c
$\frac{F_c W_c}{\rho S_c}$	$A_1 = \frac{\ell^4}{A_c}$
B	$sL = \frac{A_1}{L}$
ℓ	$s = \frac{sL}{L}$
$M_1 = K_1 F_1 \delta_1 \ell^3$	$S_c = K_3 \ell^2$
Core loss/pound	$W_c = \frac{W_c}{S_c} S_c$
W_1	$M_c = K_4 F_c \delta_c \ell^3$
~~Volt-amperes/pound~~	$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c W_c}{\rho S_c}}} K_5 F_c$
W_{ex}	

Table 11-13 (Cont'd)

DESIGN EQUATIONS

$I_p = \frac{1}{V_p}\sqrt{(W_r + W_i + W_c)^2 + W_{ex}^2 - W_i^2}$	Layers in primary
	Layers in secondary
CM_p	Coil Build: Tube ___ layers of ___ wire ( ) ___ layer of paper ( ) Wrapper ___ layers of ___ wire ( ) ___ layers of paper ( ) Wrapper Build =
CM_s	
$\frac{N}{V} = \frac{10^5}{4.44\ f\ F_i\ A_i\ B}$	
$\frac{W_c}{W_r}$	
$N_p = \frac{N}{V} V_p (1 - \frac{W_c}{2 W_r})$	$m_{cp} =$
	$R_p =$
$N_s = \frac{N}{V} V_s (1 + \frac{W_c}{2 W_r})$	$m_{cs} =$
Winding length	$R_s =$
Primary turns per layer	$V_p = \frac{N_p}{N_s}\left[V_s + I_s\ (R_s + R_p/n^2)\right]$
Secondary turns per layer	

TEMPERATURE CALCULATIONS

Assumed θ_{surf}	$m = \sqrt{\frac{S_{case}}{4\pi}} - \sqrt{\frac{S_c + S_i}{4\pi}}$
$h_c = 3.75 \times 10^{-3} \frac{\theta_{surf}^{.22}\ p^{.44}}{(S_c + S_i)^{.17}}$	$\theta_{imp(oil)} = C\,(W_c + W_i)^{1.25}$
$h_r = e\,\frac{3.7 \times 10^{-3}}{\theta_{surf}} \left[\left(\frac{T_{surf}}{100}\right)^4 - \left(\frac{T_{amb}}{100}\right)^4 \right]$	$\theta_h = F\ W_c^{\,x} \left(\frac{m}{k\,S_c} \right)^y$
$\theta_{surf} = F_{surf}\ \frac{W_c + W_i}{(S_c + S_i)(h_c + h_r)}$	$k = k_i \left(\frac{1 + R}{1 + .11R} \right)$
$\theta_{surf} = .1\,\theta_{assumed} + .9\,\theta_{calculated}$	R
$\theta_{imp} = F_{imp}\ \frac{(W_c + W_i)\,m}{S\,k}$	$T_{avg(pri)} = T_{amb} + \theta_{surf} + \theta_{imp} + C_p\theta_h$
$S = 1/2(S_{case} + S_c + S_i)$	$T_{avg(sec)} = T_{amb} + \theta_{surf} + \theta_{imp} + C_s\,\theta_h$

Table 11-14

XII. DESIGN PROCEDURES PREVIOUSLY PRESENTED

The three types of transformers considered in this chapter were analyzed in the final report for the previous Contract, No. DA-36-039 SC-5519. In order to make this report as inclusive as possible, abstracts of the design procedures for filament transformers, autotransformers, and rectifier-supply transformers are presented in this chapter.

Filament Transformers

The design of filament transformers is carried out following the procedure given in Chapter XI. Temperature rise may also be calculated according to the method given. Normal-type filament transformers, isolation transformers, or any single-phase type where each output winding carries sinusoidal current, and where sinusoidal voltage is supplied across the primary may be designed. Current-limiting filament transformers are considered in another chapter.

Autotransformers

To apply the procedure in the design of an autotransformer, an equivalent two-winding transformer rating must be determined from the autotransformer rating. This equivalent transformer becomes the autotransformer merely by proper inter-connection of windings. The rating of the equivalent two-winding transformer is

$$W_r = \left[1 - \frac{V_2}{(V_1 + V_2)}\right] W_{ra} \text{ volt-amperes,} \tag{12-1}$$

where W_{ra} = rating of autotransformer,

V_2 = smaller autotransformer voltage,

$(V_1 + V_2)$ = larger autotransformer voltage.

The voltages of the equivalent two-winding transformer are V_1 and V_2. When the autotransformer increases the input voltage, the primary voltage of the equivalent two winding transformer is $V_p = V_2$, and the secondary voltage of the equivalent two-winding transformer is $V_s = V_1 = (V_1 + V_2) - V_2$. When the autotransformer decreases the input voltage, then $V_p = V_1$ and $V_s = V_2$. Similarly I_1 and I_2 (in the windings with voltages V_1 and V_2 respectively) are the load current components in the equivalent two winding transformer. However load current to the high voltage winding of the autotransformer is I_1, and load current to the low voltage winding is $I_1 + I_2$. Thus, neglecting losses and excitation, the volt-amperes of the two-winding equivalent transformer are

$$W_r = V_1 I_1 = V_2 I_2, \tag{12-2}$$

and for the autotransformer as connected, are

$$W_{ra} = (V_1 + V_2)\, I_1 = V_2 (I_1 + I_2) \tag{12-3}$$

From equation (12-1), it is seen that if the voltage ratio of an autotransformer were two, the equivalent two-winding transformer has a nominal rating equal half that of the autotransformer. The advantage in reduced physical size and reduced equivalent rating for a certain output rating, as obtained with an autotransformer, decreases as the voltage transformation ratio increases. This can readily be seen from equation (12-1), in that the ratio W_r/W_{ra} approaches unity for large ratios of $(V_1 + V_2)/V_2$. On the other hand, when this voltage ratio is very close to unity the ratio W_r/W_{ra} becomes very small, and the common turns would consist of comparatively small wire relative to the V_1 winding. The rating W_r to be used in the design should be increased to accommodate no-load current if the voltage ratio of the autotransformer is very close to unity, such as within 25 per cent. When the voltage transformation ratio is close to two, the current is almost the same in all turns of the transformer, and one winding, tapped near the center turn, satisfies the requirements.

Secondary current is:

$$I_s = \frac{W_r}{V_s} \text{ amperes.} \qquad (12\text{-}4)$$

If a tap is to be made on an autotransformer winding, the tap wire size must be large enough to carry the rated output current (rather than a winding current) of the autotransformer.

Primary current is:

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_i + W_c)^2 + W_{ex}^2 - W_i^2} \text{ amperes.} \qquad (2\text{-}32)$$

When exciting current flows through the entire winding, extra conductor area may be required in both primary and secondary of the equivalent two-winding transformer. If the low-voltage winding were the primary, then only the lower half of the autotransformer winding carries the exciting current.

With the exception of exciting current considerations, the design method for an autotransformer (by way of the equivalent two-winding transformer) is the same as that for a filament transformer. Choice of winding space factor should be made using the equivalent rating W_r, and the highest working voltage of the actual autotransformer.

Rectifier-Supply Transformers

Only rectifier transformers for balanced operation will be considered. The treatment of unbalanced operation, such as with a half-wave rectifier, is presented in another chapter. The design precedure for rectifier-supply transformers is precisely that given in Chapter XI with the following supplements.

1) Specifications should include type of rectifier and filter.

2) $W_r = V_s I_s$, secondary RMS volt-amperes.

V_s is RMS volts across the entire secondary. For the full-wave rectifier, V_s is two times the factor of Table 12-1 (or Table 12-4) times D-C load voltage. For the bridge type, V_s is one times the factor of Table 12-1 (or 12-4) times load voltage. Add secondary circuit voltage drop to D-C load voltage when using Table 12-1.

I_s is RMS secondary current. For the full-wave rectifier, I_s is the factor of Table 12-1 (or Table 12-2) times D-C load current. For the bridge type I_s is the factor of Table 12-1 times load current, but for capacitance-input filter, I_s is 1.414 times the factor of Table 12-2 times D-C load current.

It is common practice for a designer to be given specified values of RMS secondary voltage and D-C load current. His work is easier if he is given all transformer RMS quantities.

3) Calculate primary current from equation (2-32) for the bridge-type rectifier.

For the full-wave rectifier, calculate primary current from

$$I_p = \frac{1}{V_p} \sqrt{(.707\, W_r + W_i + W_c)^2 + W_{ex}^2 - W_i^2} \text{ amperes.} \quad (12\text{-}5)$$

Table 12-1

CONSTANTS FOR RECTIFIER TRANSFORMERS AND CIRCUITS

Type of Rectifier	Full-Wave		Bridge	
	Filter*	No Filter	Filter	No Filter
Average D.C. load volts	1	1	1	1
Average D.C. load amperes	1	1	1	1
Average D.C. volt-amperes	1	1	1	1
Sec. RMS volts (per leg if center-tapped)	1.11	1.11	1.11	1.11
Pri. RMS volts	1.11	1.11	1.11	1.11
Sec. RMS amperes	.707	.786	1.00	1.11
Pri. RMS amperes	1.00	1.11	1.00	1.11
Sec. volt-amperes, RMS (both legs where center tapped)	1.57	1.74	1.11	1.23
Pri. volt-amperes, RMS	1.11	1.23	1.11	1.23
Average volt-amperes, RMS	1.34	1.49	1.11	1.23
Ratio, Sec. VA to Pri. VA (RMS)	1.414	1.414	1.0	1.0
Sec. Utility Factor	.637	.574	.90	.812
Pri. Utility Factor	.900	.812	.90	.812
Transformer Utility Factor	.746	.673	.90	.812

* Infinite inductance-input filter

Table 12-2

RATIO OF RMS SECONDARY CURRENT TO AVERAGE LOAD CURRENT FOR FULL-WAVE RECTIFIER

$\frac{R}{R_L} = \frac{\text{Series Res.}}{\text{Load Res.}}$	$2\pi fCR_L$				
	.05	.5	5	50	500
.0002	.790	.90	1.40	2.30	2.80
.001	.790	.90	1.37	2.00	2.25
.005	.790	.88	1.35	1.68	1.74
.05	.790	.85	1.20	1.21	1.23
.25	.790	.83	.95	.98	1.00

Table 12-3

RATIO OF PEAK SECONDARY CURRENT TO AVERAGE LOAD CURRENT FOR FULL-WAVE RECTIFIER

$\frac{R}{R_L} = \frac{\text{Series Res.}}{\text{Load Res.}}$	$2\pi fCR_L$				
	.05	.5	5	50	500
.0002	1.57	1.90	5.75	13.5	17.0
.001	1.57	1.85	5.50	9.5	10.5
.005	1.57	1.80	4.75	6.5	6.5
.05	1.57	1.80	3.05	3.2	3.2
.25	1.57	1.75	2.20	2.3	2.3

Table 12-4

RATIO OF RMS SECONDARY VOLTAGE OF HALF THE WINDING TO AVERAGE LOAD VOLTAGE FOR FULL-WAVE RECTIFIER

$\frac{R}{R_L} = \frac{\text{Series Res.}}{\text{Load Res.}}$	$2\pi fCR_L$				
	.1	1.0	10	100	1000
.005	1.11	1.06	.79	.73	.72
.02	1.1	1.07	.80	.78	.78
.10	1.22	1.15	.95	.93	.93
.25	1.65	1.62	1.48	1.47	1.47
.50	1.65	1.62	1.48	1.47	1.47

R is total series resistance in one branch of rectifier circuit excluding load resistance. This includes secondary resistance from center tap to one end, resistance from end of winding to one side of the load, and resistance from center tap to the other side of the load.

C is shunt capacitance across load.

f is supply frequency.

XIII. DESIGN PROCEDURE: TRANSFORMER WITH UNBALANCED MAGNETIZATION

1) Specifications

Frequency, voltages, load and filter requirements, temperatures (ambient and maximum rise), grade of protection.

2) Chosen Quantities

Type of core, grade and thickness of lamination, core space factor, preferred stack ratio, type of construction. Factors to be considered in the choice of the type of core are extensively discussed in the final report for Contract DA 36-039 SC-5519, Chapter VII.

3) Nomograph Values

Secondary RMS current:

I_s is average load current I_{DC}, multiplied by 1.57 if no filter is used, or multiplied by a ratio from Table 13-1 for a capacitance-filtered load.

Secondary RMS voltage (winding supplying rectifier):

V_s is average load voltage V_{DC} plus rectifier average forward drop and other circuit voltage drops multiplied by 2.22 if no filter is used, or V_{DC} multiplied by the ratio from Table 13-3 for a capacitance-filtered load.

Equivalent secondary rating:

$$W_r = V_s I_s + W_{r2} + \ldots \text{ volt amperes,}$$

where W_{r2}, etc. are the ratings of any additional secondary windings supplying balanced loads.

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per sq. in.,} \qquad \text{(2-19) or Fig. 11-1}$$

Winding space factor:

$$F_c = .08 \log_{10} W_r' + F \qquad \text{(2-20)}$$

Nomograph scale factors:

$$\frac{K_0 W_r}{F_i f} \text{ and } \frac{F_c W_c}{\rho S_c}$$

Flux density:

Select flux density using a value from Table 11-2 decreased about 10 to 15 per cent according to the percentage of the total secondary rating which is supplying an unbalanced load.

Characteristic linear dimension:

Use nomograph, Fig. 11-7 to obtain ℓ.

Approximate core weight:

$$M_i = K_1 F_i \delta_i \ell^3 \qquad (2\text{-}23)$$

Mean length of magnetic circuit:

$$m_i = a\ell \text{ inches.} \qquad (2\text{-}7)$$

Approximate secondary turns:

$$N_s = \frac{K_6 V_s}{f F_i B \ell^2} \text{ turns} \qquad (2\text{-}34)$$

Approximate unbalanced magnetizing force:

$$H_{DC} = \frac{.495 N_s I_{DC}}{m_i} \text{ average oersteds.} \qquad (5\text{-}19)$$

Core loss, excitation, and gap:

Use design curves, Fig. 13-1 through 13-8, to calculate W_i, W_{ex}, and non-magnetic gap.

4) <u>Core Dimensions</u>

Area product:

$$A_c A_i = \ell^4 \qquad (2\text{-}6)$$

Lamination leg width:

$$L = \frac{L}{\ell}\ell \text{ inches} \qquad (2\text{-}24)$$

Window area:

Calculate A_c from lamination dimensions.

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Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} \text{ sq. in.}$$

Stack height:

$$sL = \frac{A_i}{L} \text{ inches.}$$

Stack ratio:

$$s = \frac{sL}{L}$$

Core exposed surface area:

$$S_i = K_2 \ell^2 \text{ sq. in.} \qquad (2\text{-}25)$$

Core dissipation per unit area:

$$W_i / S_i \text{ watts per sq. in.}$$

5) Winding Calculations

Winding exposed surface:

$$S_c = K_3 \ell^2 \text{ sq. in.} \qquad (2\text{-}26)$$

Approximate winding loss:

$$W_c = \frac{W_c}{S_c} S_c \text{ watts.} \qquad (2\text{-}27)$$

Conductor weight:

$$M_c = K_4 F_c \delta_c \ell^3 \qquad (2\text{-}30)$$

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c W_c}{\rho S_c}}} \cdot (K_5 F_c) \qquad (2\text{-}31)$$

Primary component of load volt-amperes:

$$W_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} \text{ volt-amperes.} \qquad (5\text{-}11)$$

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Primary current:

$$I_p = \frac{1}{V_p} \sqrt{(W_{pL} + W_{r2} + \dots + W_c + W_i) + W_{ex}^2} \text{ amperes} \qquad (5\text{-}20)$$

Increase I_p up to 10 per cent according to the number of secondaries which are supplying unbalanced loads.

Wire sizes:

Calculate from circular mils per ampere using primary and secondary RMS currents.

Turns per volt:

$$\frac{N}{V} = \frac{10^5}{4.44\ f\ A_i\ F_i\ B} \text{ turns per volt.} \qquad (2\text{-}33)$$

Turns:

$$N_p = V_p \frac{N}{V} \left(1 - \frac{.707 W_c}{2 W_r}\right) \text{ turns.} \qquad (5\text{-}21)$$

$$N_s = V_s \frac{N}{V} \left(1 + \frac{.707 W_c}{2 W_r}\right) \text{ turns.}$$

For capacitance filter, replace .707 by $\frac{1.1}{I_s/I_{DC}}$ in equations (5-21).

6) Winding Layout

Winding length:

Window length minus two margins.

Turns per layer:

Appropriate turns per inch times winding length.

Layers:

Appropriate turns divided by turns per layer.

7) Check of Coil Build

Choose tube, layer insulation, and wrappers, and check build to insure that it is between 80 and 90% of window width.

8) Summary of Design

List core material, dimensions, weight, tube, winding wire sizes, total turns, taps, turns per layer, number of layers, layer insulation, wrappers, and shield data.

9) **Check of Winding Resistances**

Resistance equals resistance per unit length (corrected to operating temperature from Fig. 11-6), times mean length of turn, times number of turns.

Mean length of turn equals length of inside turn, plus pi times build-up of winding.

10) **Check of Voltage Ratio**

Calculate primary voltage:

$$V_p = n \left[V_s + 1.1 \; I_{DC} \left(R_s + R_p/n^2 \right) \right] \quad \text{volts} \qquad (5\text{-}22)$$

where R_s and R_p are obtained from step 9.

Adjust the turns ratio if the calculated primary voltage differs appreciably from the specified voltage.

11) **Special Calculations and Design Checks**

Apply when necessary

12) **Calculation of Temperature Rise**

Follow basic method.

Table 13-1

RATIO OF RMS SECONDARY CURRENT TO AVERAGE CURRENT FOR HALF-WAVE RECTIFIER

$\frac{R}{R_L}$ = Series Res. / Load Res.	$2\pi fCR_L$				
	.1	1.0	10	100	1000
.0002	1.58	1.80	2.80	4.60	5.60
.001	1.58	1.80	2.75	4.00	4.50
.005	1.58	1.75	2.70	3.37	3.48
.05	1.58	1.70	2.40	2.42	2.45
.25	1.58	1.65	1.90	1.97	2.00

Table 13-2

RATIO OF PEAK SECONDARY CURRENT TO AVERAGE CURRENT FOR HALF-WAVE RECTIFIER

$\frac{R}{R_L}$ = Series Res / Load Res.	$2\pi fCR_L$				
	.1	1.0	10	100	1000
.0002	3.14	3.8	11.5	27	34
.001	3.14	3.7	11.0	19	21
.005	3.14	3.6	9.5	13	13
.05	3.14	3.6	6.1	6.3	6.3
.25	3.14	3.5	4.4	4.5	4.5

Table 13-3

RATIO OF RMS SECONDARY VOLTAGE TO AVERAGE LOAD VOLTAGE FOR HALF-WAVE RECTIFIER

$\frac{R}{R_L}$ = Series Res. / Load Res.	$2\pi fCR_L$				
	.1	1.0	10	100	[illegible]
.005	2.17	1.87	.89	.75	
.02	2.18	1.91	.94	.83	
.10	2.45	2.02	1.14	1.09	
.25	3.25	2.36	1.54	1.49	
.50	3.3	2.95	2.08	2.02	

Notes: R is total series resistance in rectifier circuit excluding load resistance. This includes transformer secondary resistance and resistance from ends of the winding to the filter capacitor.

C is shunt capacitance across load.

F is supply frequency

If insufficient load circuit data are available, typical values may be used: V_s/V_{DC} = 1.4; I_s/I_{DC} = 2.

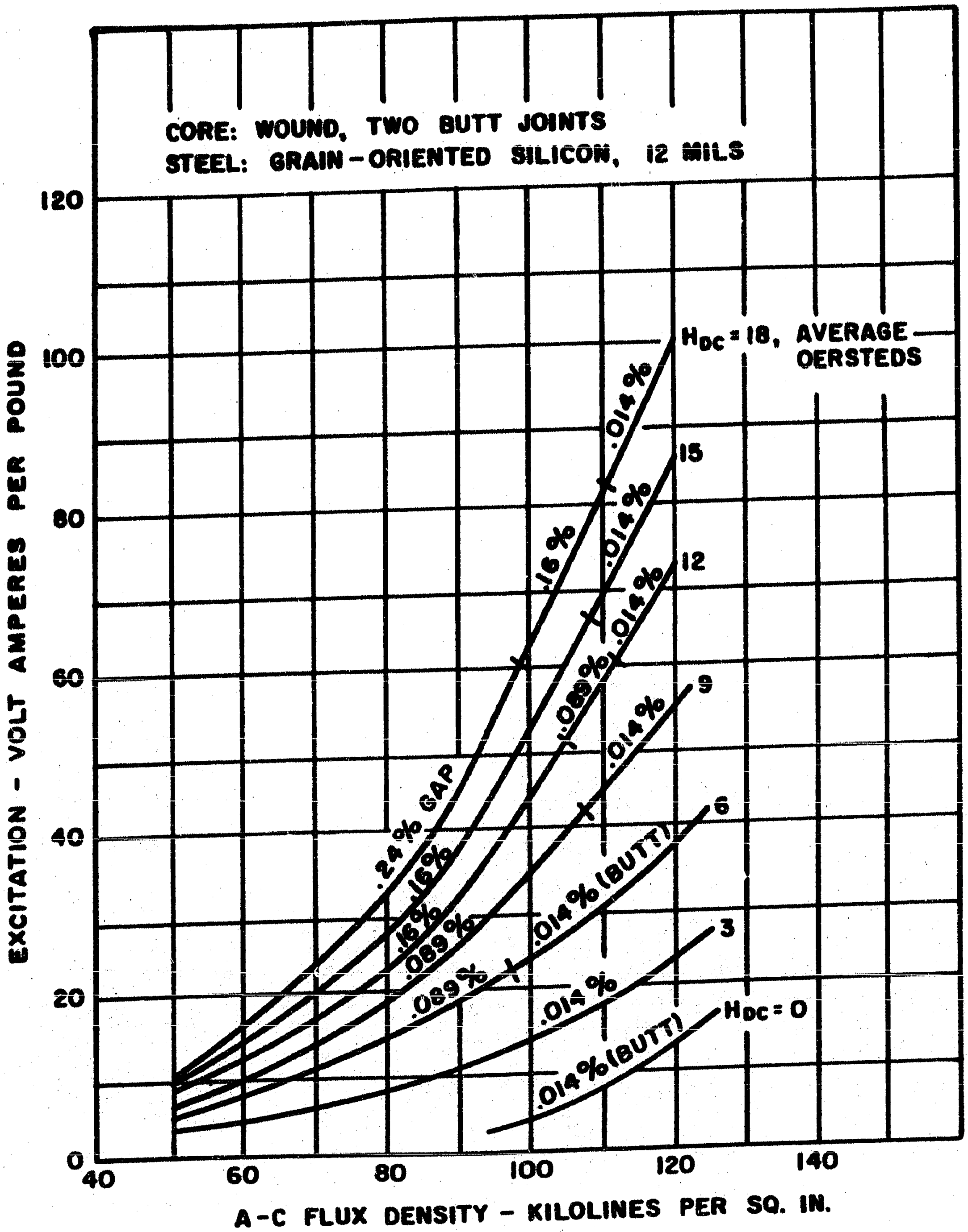

FIG. 13-1 EXCITATION AND GAP FOR WOUND CORE — DESIGN CURVES (60 CPS)

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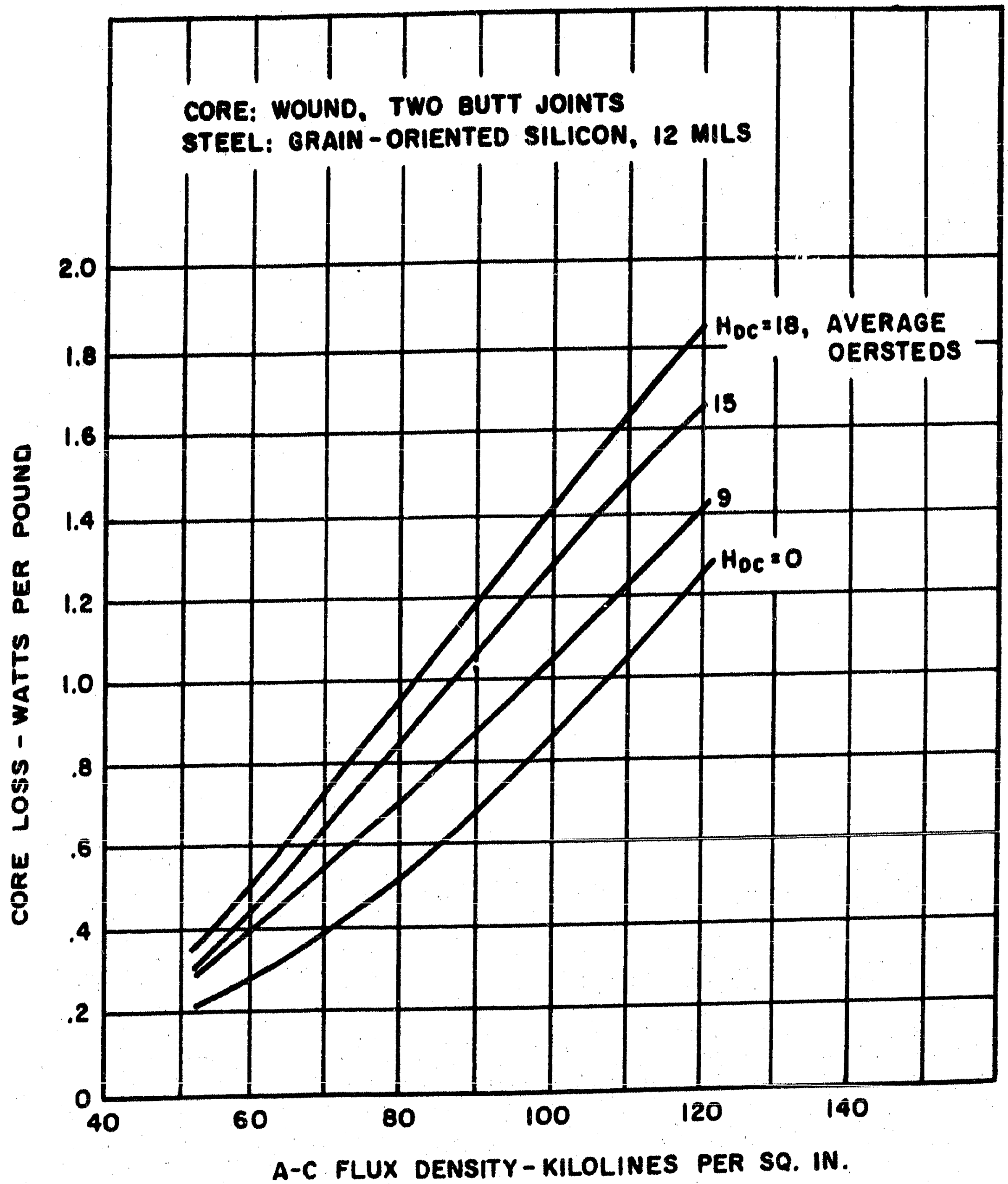

FIG. 13-2 CORE LOSS OF WOUND CORE — DESIGN CURVES (60 CPS)

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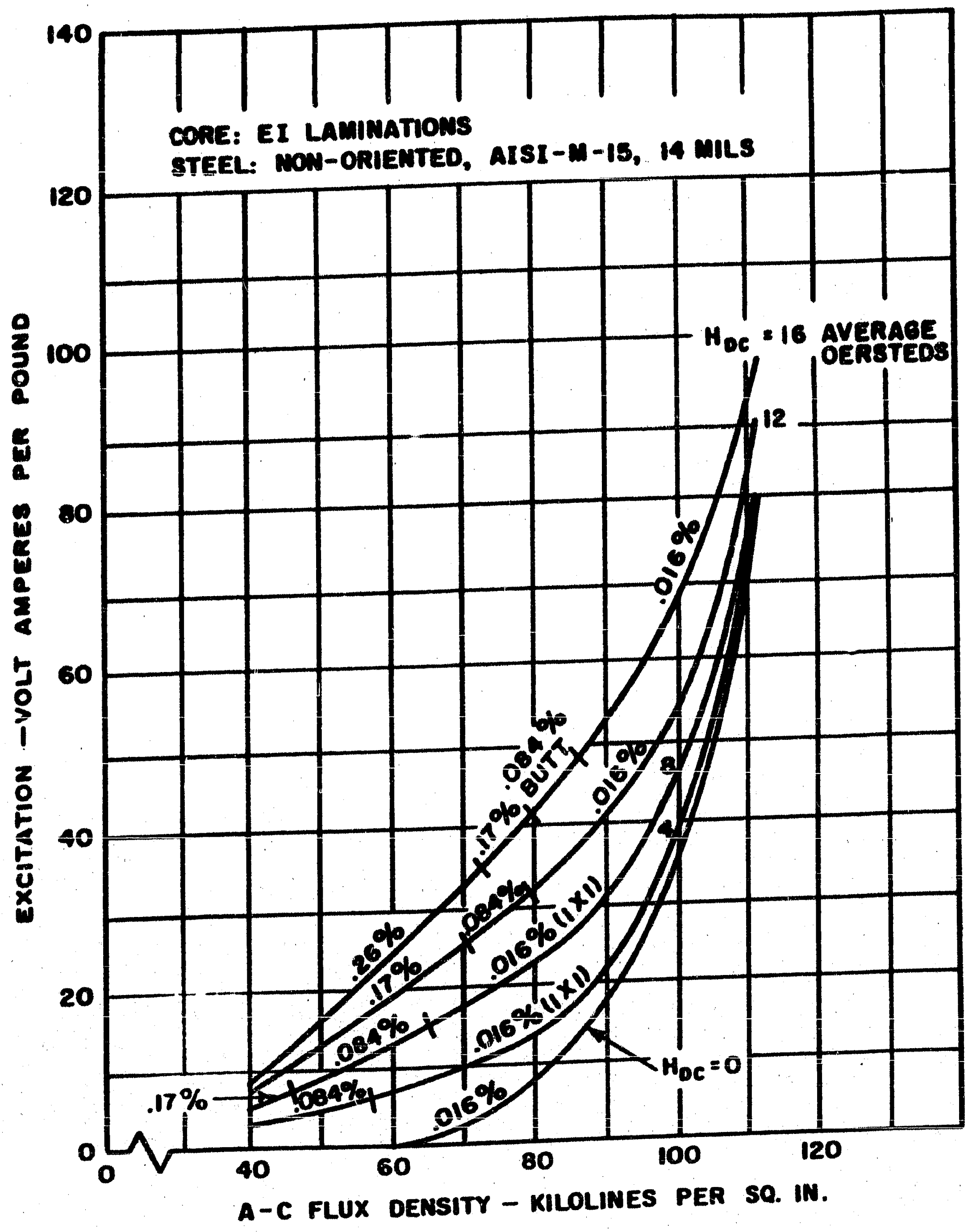

FIG. 13-3 EXCITATION AND GAP FOR STACKED CORE-DESIGN CURVES (60 CPS)

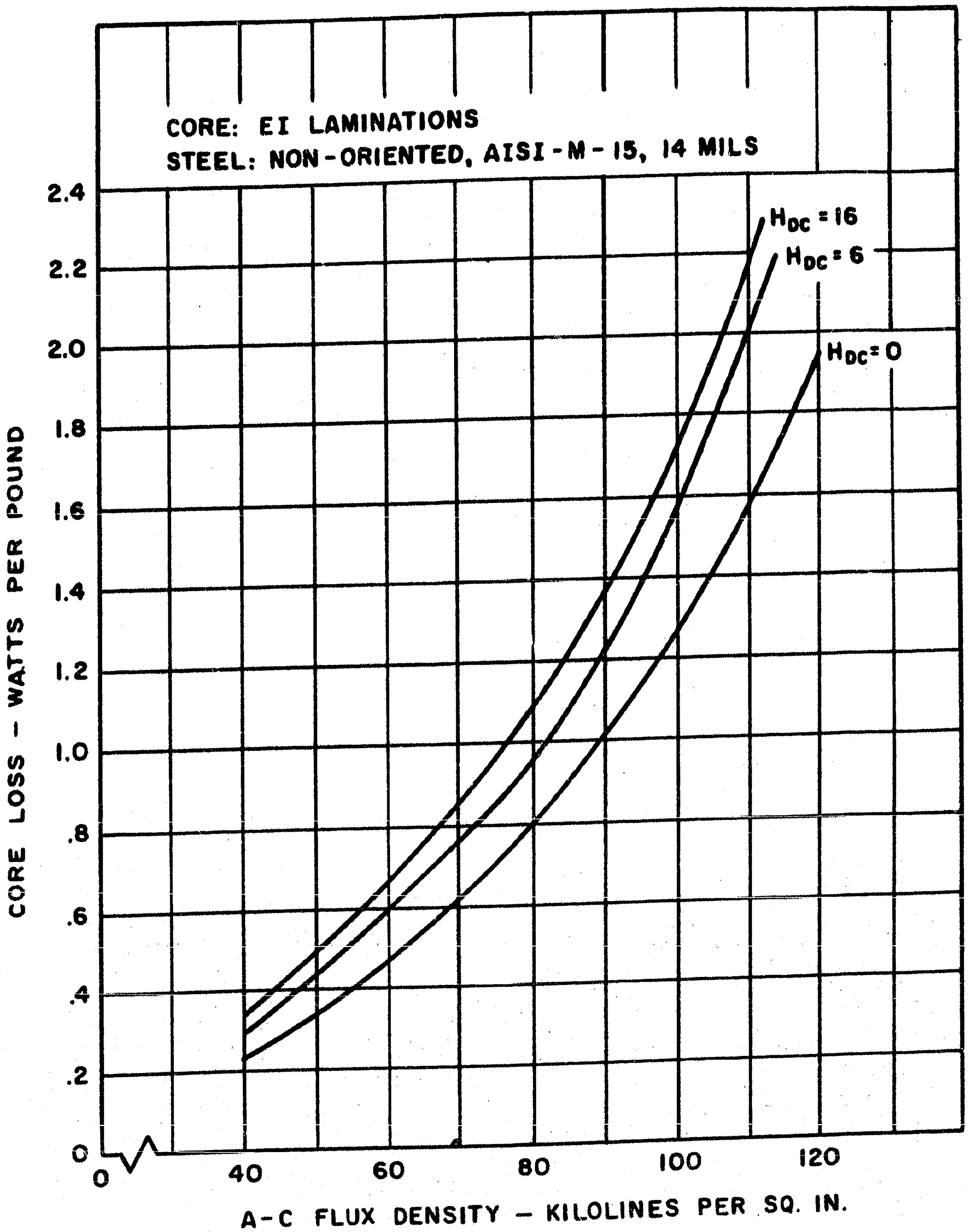

FIG. 13-4 CORE LOSS OF STACKED CORE — DESIGN CURVES (60 CPS)

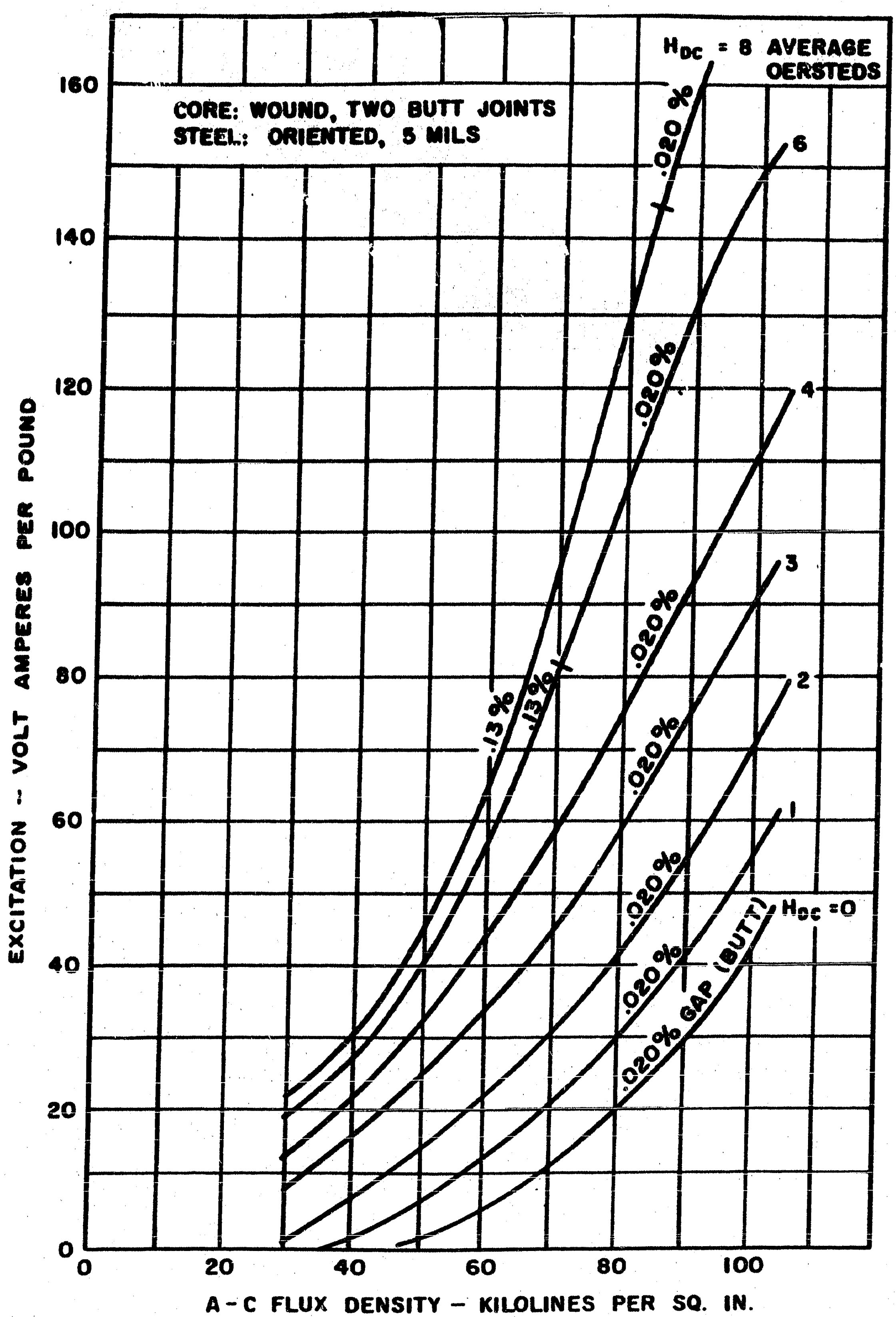

FIG. 13-5 EXCITATION AND GAP FOR WOUND CORE — DESIGN CURVES (400 CPS)

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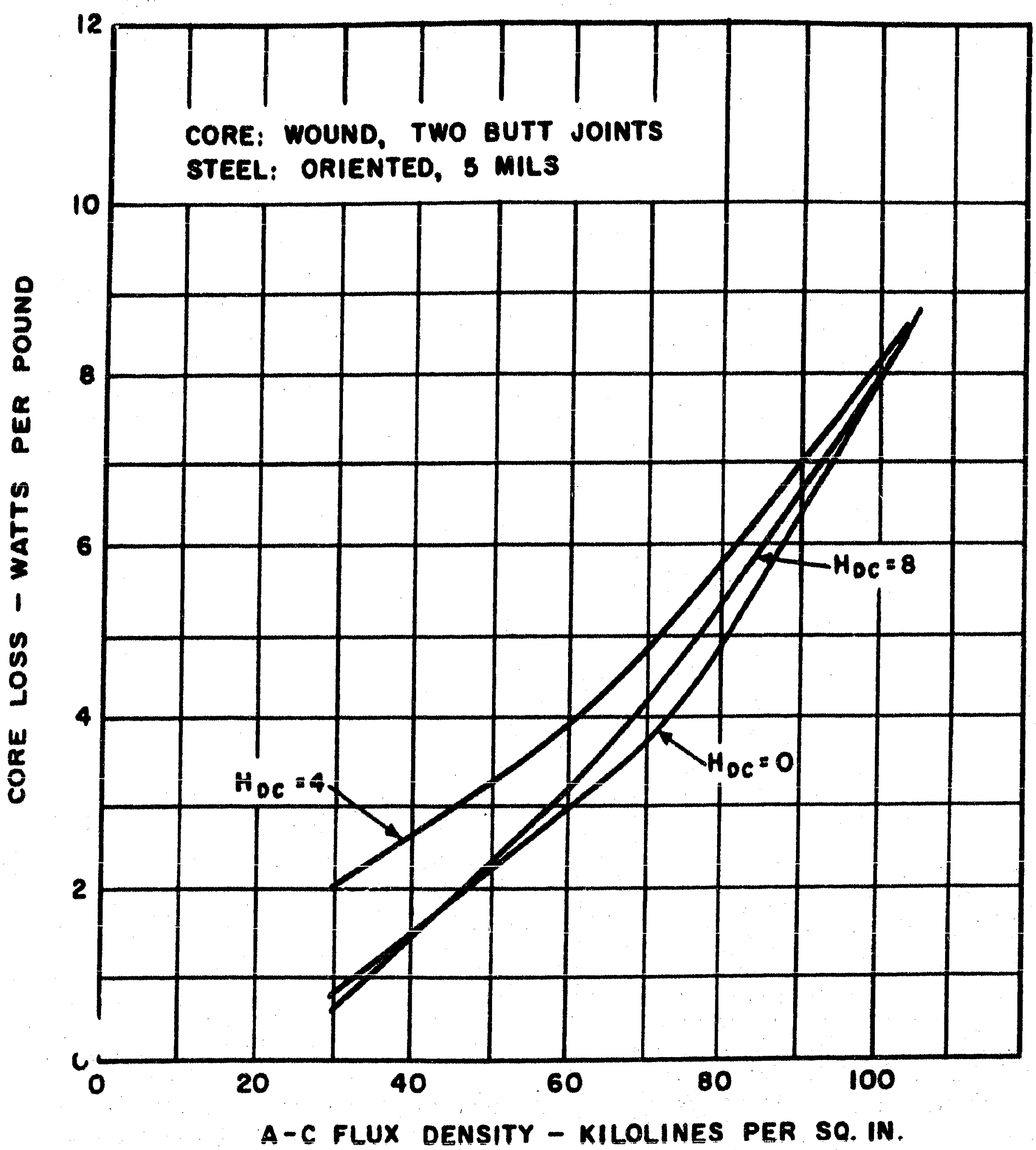

FIG. 13-6 CORE LOSS OF WOUND CORE — DESIGN CURVES (400 CPS)

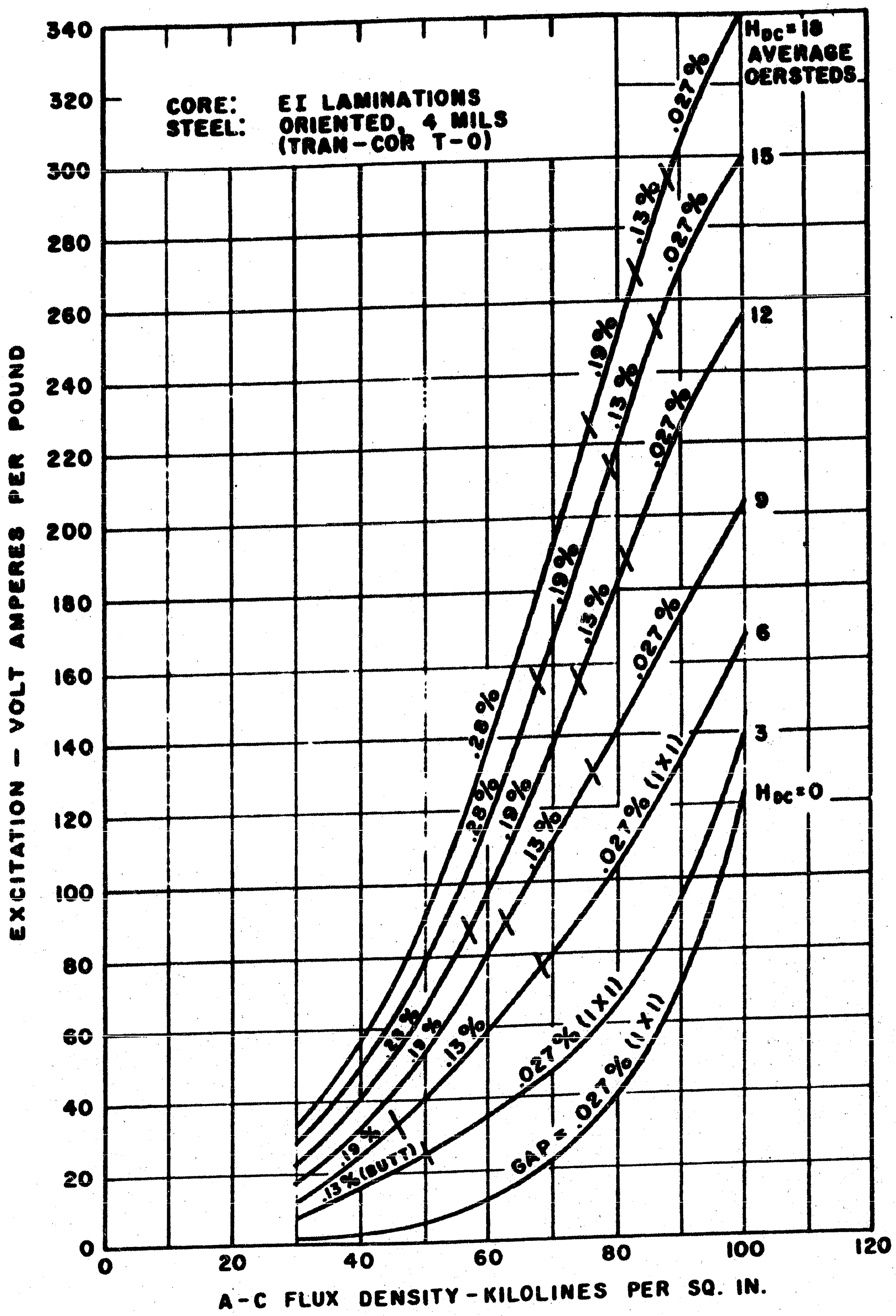

FIG. 13-7 EXCITATION AND GAP FOR STACKED CORE-DESIGN CURVES (400 CPS)

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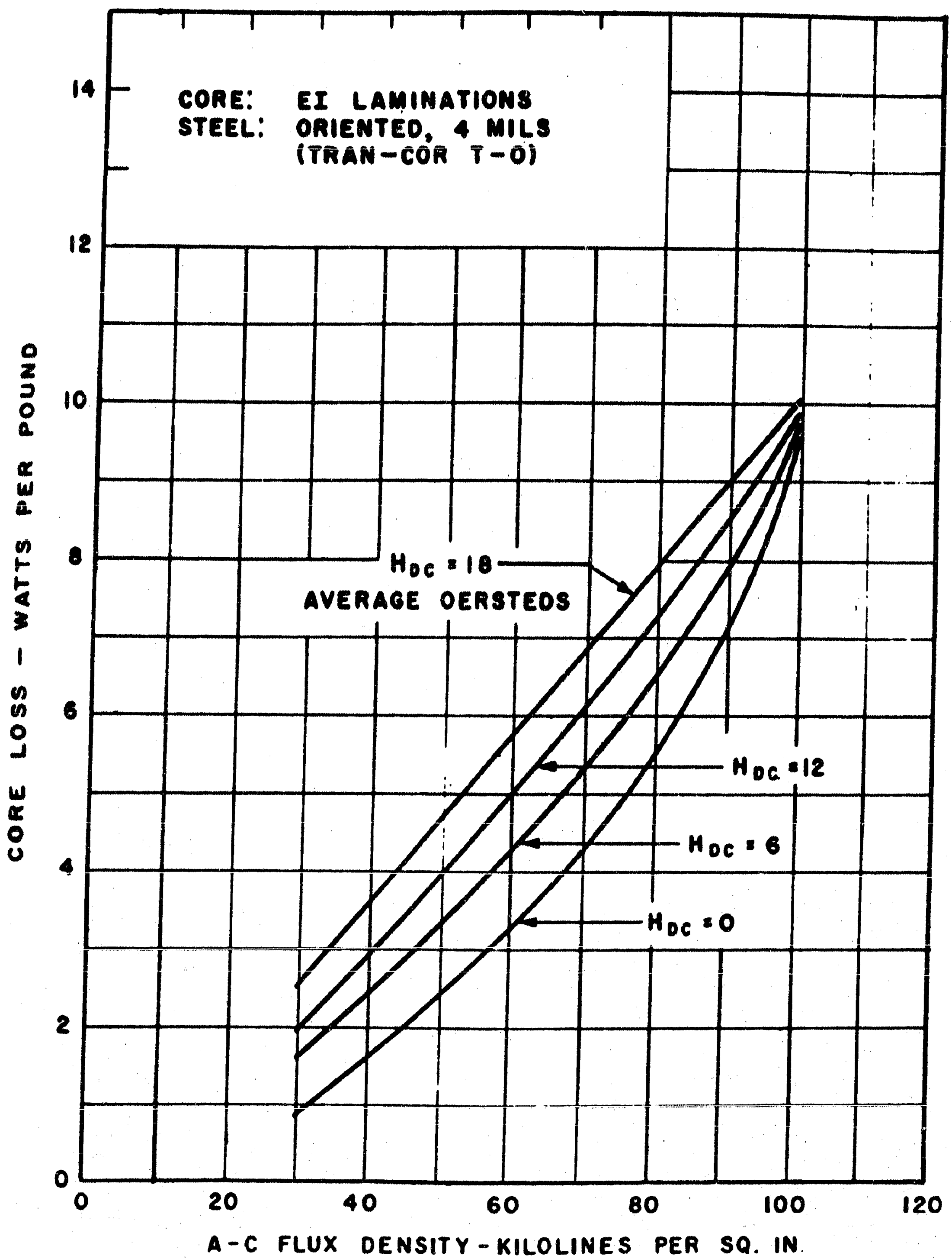

FIG. 13-8 CORE LOSS OF STACKED CORE - DESIGN CURVES (400 CPS)

XIV. EXAMPLE: TRANSFORMER WITH UNBALANCED MAGNETIZATION

1) Specifications

Frequency: 400 cycles per second.

Ambient temperature: 85°C.

Maximum temperature rise: 115°C.

Primary: 115 volts.

Secondary: 560 volts RMS, 1.0 ampere RMS, 0.50 ampere DC, half-wave rectifier with capacitance-input filter.

Protection: Grade 2 (less resistant to adverse environmental conditions).

2) Chosen Quantities

Core: Scrapless EI laminations.

Steel: Oriented silicon, 0.004 inch thick, (Armco Tran-Cor T-0 grade).

Construction: Open core and coils.

Core space factor: 0.9

Approximate stack ratio: 1.5

3) Nomograph Values

Secondary RMS voltage:

V_s = 560 volts RMS.

Secondary RMS current:

I_s = 1.0 amperes RMS.

Equivalent secondary rating:

$W_r = V_s I_s = (560)(1.0) = 560$ volt-amperes.

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} = \left(\frac{115}{87}\right)^{1.25} = 1.4 \text{ watts per sq. in.,}$$

$\Delta T = 115$ = maximum temperature rise in °C,

$K = 87$ = constant from Table 11-1.

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Winding space factor:

$$F_c = .08 \log_{10} W_r' + F = .08 \log_{10} 68 + .12 = .27$$

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76} \left(\frac{\Delta T}{40}\right)^{.63}} = \frac{560}{\left(\frac{400}{60}\right)^{.76} \left(\frac{115}{40}\right)^{.63}} = 68 \text{ volt-amperes,}$$

$F = .12$ = constant from Fig. 11-2,

$f = 400$ = frequency in cycles per second.

Nomograph scale factors:

$$\frac{K_0 W_r}{F_i f} = \frac{(.649)(560)}{(.9)(400)} = 1.01,$$

$$\frac{F_c W_c}{\rho S_c} = \frac{(.27)(1.4)}{1.15} = 0.33,$$

$K_0 = 0.649$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.5$,

$F_i = 0.9$ = core space factor,

$\rho = 1.15$ = resistivity, the value from Fig. 11-6 corresponding to 190°C, increased 2 per cent.

Flux density:

Select 78 kilolines per square inch with the aid of Table 11-2.

Characteristic linear dimension:

$\ell = 1.0$ from nomograph, Fig. 11-7.

Approximate core weight:

$$M_i = K_1 F_i \delta_i \ell^3 = (8.23)(.9)(.276)(1.0)^3 = 2.04 \text{ pounds,}$$

$K_1 = 8.23$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.5$,

$\delta_i = .276$ = core steel density in lb. per sq. in.

Approximate mean length of magnetic circuit:

$$m_1 = a\,\ell = (5.82)(1.0) = 5.82 \text{ inches,}$$

a = 5.82 = constant from Fig. 11-3 corresponding to s = 1.5.

Approximate secondary turns:

$$N_s = \frac{K_6 V_s}{f F_1 B \ell^2} = \frac{(15{,}920)(560)}{(400)(.9)(78)(1.0)^2} = 318 \text{ turns,}$$

K_6 = 15,920 = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5,

B = 78 = flux density in kl. per sq. in.

Approximate unbalanced magnetizing force:

$$H_{DC} = \frac{.495\, N_s\, I_{DC}}{m_1} = \frac{(.495)(318)(.5)}{5.82} = 13.5 \text{ average oersteds,}$$

I_{DC} = 0.5 = average load current in amperes.

Core loss, excitation, and gap:

For B = 78 kl per sq. in. and H_{DC} = 13.5,

Core loss = 7.2 watts per lb. from Fig. 13-8,

Excitation = 180 volt-amperes per lb. from Fig. 13-7,

Gap = .13% from Fig. 13-7.

W_i = (2.04)(7.2) = 15 watts (2.7% of rating).

W_{ex} = (2.04)(180) = 370 volt-amperes (66% of rating).

Effective gap = $\frac{.13}{100}$ (5.82) = .0075 inch.

4) Core Dimensions

Core exposed surface area:

$$S_i = K_2\,\ell^2 = (24.0)(1.0)^2 = 24 \text{ square inches,}$$

K_2 = 24.0 = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5.

Core dissipation per unit area:

W_i/S_i = 15/24 = .625 watts per square inch.

Lamination width:

$L = \ell(L/\ell) = (1.0)(.97) = .97$ inch, use L = 1.0 inch,

$L/\ell = .97$ = constant from Fig. 11-3 corresponding to s = 1.5.

Area product:

$A_c A_i = \ell^4 = 1.0$.

Window area:

$A_c = .75$ square inch.

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} = \frac{1.0}{.75} = 1.33 \text{ square inches.}$$

Stack height:

$$sL = \frac{A_i}{L} = \frac{1.33}{1.0} = 1.33 \text{ inches, approximately 1-5/16".}$$

Stack ratio:

$$s = \frac{sL}{L} = \frac{1.33}{1.0} = 1.33.$$

5) <u>Winding Calculations</u>

Winding exposed surface area:

$S_c = K_3\ell^2 = (10.61)(1.0)^2 = 10.61$ square inches,

$K_3 = 10.61$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5.

Approximate winding losses:

$W_c = S_c\,(W_c/S_c) = (10.61)(1.4) = 14.8$ watts.

Conductor weight:

$M_c = K_4 F_c \delta_c \ell^3 = (4.49)(.30)(.321)(1.0)^3 = 0.433$ lb.,

$K_4 = 4.49$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5,

$\delta_c = 0.321$ = copper density in lb. per sq. in.

Circular mils per amperes:

$$\frac{CM}{amp} = \sqrt{\frac{s}{\frac{F_c W_c}{\rho S_c}}}\, K_5 F_c = \sqrt{\frac{1.0}{.33}}\,(826)(.27) = 388,$$

K_5 = 826 = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5.

Primary component of load volt-amperes:

$$W_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} = 560\sqrt{(1.0)^2 - (.5)^2} = 485 \text{ volt-amperes.}$$

Primary current:

$$I_p = \frac{1.1}{V_p}\sqrt{(W_{pL} + W_c + W_i)^2 + (W_{ex})^2}$$

$$= \frac{1.1}{115}\sqrt{(485 + 14.8 + 15)^2 + (370)^2} = 6.0 \text{ amperes.}$$

Primary wire size:

CM = (388)(6.0) = 2330. Use No. 17 AWG (2048 CM)

Secondary wire size:

CM = (388)(1.0) = 388. Use No. 24 AWG (404 CM).

Turns per volt:

$$\frac{N}{V} = \frac{10^5}{4.44\, f\, B\, A_i\, F_i} = \frac{10^5}{(4.44)(400)(78)(1.33)(.9)} = 0.604$$

Correction for resistance drop:

$$Reg = \frac{W_c}{W_r} \cdot \frac{1.1}{I_s/I_{DC}} = \frac{(14.8)(1.1)}{(560)(2)} = .0146.$$

Turns:

$$N_p = V_p \frac{N}{V}(1 - 1/2\ Reg) = (115)(.604)(.993) = 69 \text{ turns,}$$

$$N_s = V_s \frac{N}{V}(1 + 1/2\ Reg) = (560)(.604)(1.007) = 340 \text{ turns.}$$

6) Winding Layout

Winding length equals window length minus two margins,

1.5 - 2(.1563) = 1.1874 inches.

Turns per layer:

Primary: (19)(1.1874) = 22.6, use 23 turns per layer,

Secondary: (42)(1.1874) = 49.8, use 49 turns per layer.

Layers:

Primary: 69/23 = 3 layers,

Secondary: 340/49 = 6.94, use 7 layers.

Tube:

.030 inch thick, 1-1/64 x 1-5/16 x 1-7/16 inches long.

Shield:

.002 inch thick copper sheet.

7) Check of Coil Build

		Thickness - Inches
Tube		.030
3 layers of No. 17	(3)(.0469) =	.141
2 layers of high temp. insulation	(2)(.009) =	.018
Wrapper of " " "		.009
Copper shield		.002
Wrapper of high temp. insulation		.009
7 layers of No. 24	(7)(.0213) =	.149
6 layers of high temp. insulation	(6)(.009) =	.054
Wrapper of " " "		.012
		.424

Build = .424/.500 = 85%

8) Summary of Design

Core:

Lamination: scrapless EI with center leg width of 1",
Steel: oriented silicon, (Armco Tran-Cor T-O grade), .004 inch thick,
Stack: 1-5/16 inches,
Construction: butt joint.

Tube:

Dimensions: .030 inch thick, 1-1/64 x 1-5/16 x 1-7/16 inches long,

Material: suitable high-temperature insulation.

Primary winding - 115 volts, (next to core):

Wire size: No. 17 AWG, high-temperature insulation

Turns per layer: 23,

Layers: 3,

Turns: 69,

Layer insulation: .009 inch, high temperature insulation,

Wrapper: .009 inch, high temperature insulation

Shield - ground to core:

Material: one layer of .002 inch thick copper sheet,

Wrapper: .009 inch, high temperature insulation

Secondary winding - 560 volts RMS:

Wire size: No. 24 AWG, high temperature insulation

Turns per layer: 49,

Layers: 7,

Turns: 340,

Layer insulation: .006 inch, high temperature insulation,

Wrapper: .012 inch, high temperature insulation

9) <u>Check of Winding Resistances</u>

Resistance equals resistance per unit length (correct to operating temperature from Fig. 11-6), times mean length of turn, times number or turns.

Mean length of turn equals length of inside turn, plus pi times build-up of winding.

Primary:

$$R_p = \frac{(5.064)(1.13)}{(12000)(.6788)} (5.40)(69) = .261 \text{ ohms},$$

$$m_{op} = 2(1.3125 + .06) + 2(1.0156 + .06) + \pi (.159) = 5.40 \text{ inches}.$$

Secondary:

$$R_s = \frac{(25.67)}{(12000)} \frac{(1.13)}{(.6788)} (6.85)(340) = 8.31 \text{ ohms},$$

$$m_{os} = 2(1.3125 + .06) + 2(1.0156 + .06) + 2\pi (.209 + .102) =$$

6.85 inches.

10) Check of Voltage Ratio

Primary voltage:

$$V_p = n \left[V_s + 1.1\, I_{DC} (R_s + R_p/n^2) \right]$$
$$= \frac{69}{340} \left[560 + (1.1)(.5)(14.65) \right] = 115 \text{ volts},$$

$$R_s + R_p/n^2 = 8.31 + .261 \cdot \frac{(340)^2}{(69)^2} = 14.65 \text{ ohms}.$$

11) Special Calculations and Design Checks (none required)

12) Calculation of Temperature Rise

Surface temperature rise:

$$\theta_{surf} = F_{surf} \frac{W_c + W_i}{(S_c + S_i)(h_c + h)} = \frac{(.9)(17.71 + 15)}{(10.61 + 24)(.0051 + .0079)} = 65.4°C,$$

$$W_c = I_s^2 R_s + I_p^2 R_p = (1.0)^2(8.31) + (6.0)^2(.261) = 17.71 \text{ watts},$$

$W_i = 15$ watts,

$S_c = 10.61$ square inches,

$S_i = 24$ square inches,

$F_{surf} = .9$ = form factor of surface from Table 11-9,

$$h_c = 3.75 \times 10^{-3} \frac{\Theta_{surf}^{.22}}{(S_c + S_1)^{.17}} P^{.44} = 3.75 \times 10^{-3} \frac{66^{.22}}{(10.61+24)^{.17}} = .0051,$$

$h_r = (.0088)(.9) = .0079,$

Θ_{surf} is assumed to be 66°C,

$\Theta_{surf} = .1\ \Theta_{assumed} + .9\ \Theta_{calc} = (.1)(66) + (.9)(65.4) = 65.5°C.$

Coil hot spot temperature:

$$\Theta_h = F W_c^{x} \left(\frac{m}{k S_c}\right)^{y} = (1.2)(17.71)^{.85} \left(\frac{.250}{.0127 \times 10.6}\right)^{1.4} = 32.7°C,$$

$m = 1/2\ (.5) = .250$ inch,

$$k = k_i \left(\frac{R+1}{.11 R + 1}\right) = .003 \left(\frac{7}{1.66}\right) = .0127,$$

$R = 6$ (estimate for high temperature insulated wire).

Average winding temperatures:

Primary:

$$T_{pri} = T_{amb} + \Theta_{surf} + C\ \Theta_h$$

$$= 85 + 65.5 + (.9)(32.7) = 180°C, \ (\Delta T = 95°C).$$

Secondary:

$$T_{sec} = T_{amb} + \Theta_{surf} + C\ \Theta_h$$

$$= 85 + 65.5 + (.8)(32.7) = 176.7°C, \ (\Delta T = 91.7°C).$$

XV. Design Procedure: Current-Limiting Transformers

1) Specifications

Frequency, voltages, secondary load and short-circuit currents, temperatures (ambient and maximum rise), grade of protection.

2) Chosen Quantities

Type of core, grade and thickness of lamination, maximum core loss and excitation, core space factor, preferred stack ratio, type of construction.

3) Nomograph Values

Rating based on secondary output:

$$W_r = V_s I_s \text{ volt amperes.}$$

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts/sq. in.} \qquad \text{(2-19) or Fig. 11-1}$$

Winding space factor:

$$F_c' = .6F_c, \qquad \text{(6-17)}$$

$$\text{where } F_c = .08 \log_{10} W_r' + F. \qquad \text{(2-20)}$$

The factor .6 is for a scrapless lamination, and is nearer to .5 for units less than 50 volt-amperes.

For a non-scrapless lamination, the factor is usually greater than .6.

Nomograph scale values:

$$\frac{K_0 W_r}{F_1 f} \text{ and } \frac{F_c' W_c}{\rho S_9}$$

Flux density:

Select B_p using Table 11-2 as a guide.

$$B_s = B_p \frac{V_s}{V_p/n} \text{ kilolines per sq. in} \qquad (6\text{-}12)$$

$$\text{where } \frac{V_p}{n} = \frac{pq\, V_s}{\sqrt{p^2 q^2 - 1}} \text{ volts,} \qquad (6\text{-}6)$$

n = ratio of primary turns to secondary turns,

$p = \dfrac{\text{short-circuit current}}{\text{rated current}}$,

$q = \dfrac{\text{leakage reactance at short circuit}}{\text{leakage reactance at rated current}}$

= .85 typically but is nearer to 1.0 if shunt flux density during short circuit is low.

Characteristic linear dimension:

Using B_s enter nomograph, Fig. 11-17, and obtain ℓ.

Approximate core weight:

$$M_1 = K_1 F_1 \delta_1 \ell^3 \text{ pounds.} \qquad (2\text{-}23)$$

Core loss and excitation:

Use material curves, correction factors (Table 11-3), and half of the core weight with each flux density to calculate W_i and W_{ex}.

4) Core Dimensions

Area product:

$$A_c A_i = \ell^4 \qquad (2\text{-}6)$$

Lamination leg width:

$$L = \ell \frac{L}{\ell} \qquad (2\text{-}24)$$

Window area:

Calculate A_c from lamination dimensions.

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} \text{ sq. in.}$$

Stack height:

$$sL = \frac{A_i}{L} \text{ inches}$$

Stack ratio:

$$s = \frac{sL}{L}.$$

Core exposed surface area:

$$S_i = K_2 \ell^2 \text{ sq. in.} \qquad (2\text{-}25)$$

Core dissipation per unit area:

W_i / S_i watts per sq. in.

5) <u>Calculations for Windings</u>

Equivalent winding exposed surface:

$$S_c = K_3 \ell^2 \text{ sq. in.} \qquad (2\text{-}26)$$

Approximate winding loss:

$$W_c = \frac{W_c}{S_c} S_c \text{ watts.} \qquad (2\text{-}27)$$

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c'}{\rho} \frac{W_c}{S_c}}} (K_5 F_c'). \qquad (2\text{-}31)$$

Primary current:

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_c + W_1)^2 + (W_{ex} + I_s^2 X)^2} \text{ amperes,} \qquad (6\text{-}18)$$

$$\text{where } I_s^2 X = \frac{V_s I_s}{\sqrt{p^2 q^2 - 1}} \text{ volt amperes.} \qquad (6\text{-}19)$$

Wire sizes:

Calculate from circular mils per ampere using calculated primary current and rated secondary load current.

Turns per volt:

$$\frac{N_p}{V} = \frac{10^5}{4.44 f A_i F_i B_p} \text{ turns per volt.} \qquad (6\text{-}20)$$

$$\frac{N_s}{V} = \frac{10^5}{4.44 f A_i F_i B_s} \qquad (6\text{-}21)$$

Primary turns:

$$N_p = \frac{N_p}{V} V_p \left(1 - \frac{W_c}{2 W_r}\right) \text{ turns.}$$

Secondary turns:

$$N_s = \frac{N_s}{V} V_s \left(1 + \frac{W_c}{2 W_r}\right) \text{ turns.}$$

6) Winding Layout

Total available winding length:

Window length minus (four margins plus magnetic shunt thickness) Assume magnetic shunt thickness (dimension parallel to coil axes) to be at least (2/3) L for a simple-type core, and (1/3) L for a shell-type core. In any case, the shunt flux density should not exceed 130 kilolines per square inch during short circuit. (Shunt flux density is equal to the difference of primary flux and secondary flux divided by the shunt net area.)

Turns per layer:

Primary turns per layer equals turns per inch times .6 of winding length.
Secondary turns per layer equal turns per inch times .4 of winding length.

Layers:

Appropriate turns divided by appropriate turns per layer.

7) Check of Coil Build

Choose tube, layer insulation, and wrappers, and check build of both primary and secondary windings to insure that each is 80 to 90 per cent of window width.
To equalize builds, re-apportion the available winding length between the primary and secondary when necessary.

8) Summary of Design

List core material, dimensions, weight, tube, winding wire sizes, total turns, taps, turns per layer, number of layers, layer insulation, wrappers, and shield data.

9) Check of Winding Resistance

Resistance equals resistance per unit length (corrected to operating temperature from Fig. 11-6, times mean length of turn, times number of turns.
Mean length of turn equals length of inside turn, plus pi times build-up of winding.

10) Check of Voltage Ratio

Calculate primary voltage:

$$V_p = n \sqrt{\left[V_s + I_s\,(R_s + R_p/n^2)\right]^2 + (I_s X)^2} \text{ volts,}$$

where R_p and R_s are obtained from step 9.

Adjust the turns ratio if the calculated primary voltage differs appreciably from the specified voltage.

11) **Special Calculations and Design Checks**

Magnetic shunt(s):

Thickness (in the direction of long window dimension) of magnetic shunt(s) has previously been determined in step 6).

Width of magnetic shunt(s) (in the direction perpendicular to plane of window) is approximately equal to stack height of the core, sL.

Length of magnetic shunt(s) is equal to window width minus air gap.

An expression for the air gap associated with the magnetic shunt, or with each shunt if there are two, is:

$$m_g{}' = \frac{4.52\ p\ q\ N_s\ I_s}{B_{g\ sc}\ 10^3}\ \text{inches.} \qquad (6\text{-}16)$$

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XVI. EXAMPLE: CURRENT-LIMITING TRANSFORMER

1) Specifications

Frequency: 60 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Primary: 125/115/105 volts.

Secondary: 5.5 volts, 10 amperes, 13.5 amperes short-circuit current.

Protection: Grade 1 (most resistant to adverse environmental conditions).

2) Chosen Quantities

Core: Scrapless EI laminations.

Steel: AISI M-10 grade, oriented silicon, .014 inch thick.

Construction: Encased, hermetically sealed, with sand-loaded asphalt filling compound.

Core space factor: 0.87

Approximate stack ratio: 1.0.

3) Nomograph Values

Rating based on secondary output:

$$W_r = V_s I_s = (5.5)(10) = 55 \text{ volt-amperes.}$$

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} = .49 \text{ watts per sq. in.}$$

$\Delta T = 40$ maximum temperature rise in °C,

$K = 71$ = constant from Table 11-1.

Winding space factor:

$$F_c' = .6\, F_c = (.6)(.289) = .173,$$

$$F_c = .08 \log_{10} W_r + F = (.08)(1.74) + .15 = .289,$$

$F = 0.15$ = constant from Fig. 11-2

Nomograph scale values:

$$\frac{K_0 W_r}{F_i r} = \frac{(.630)(55)}{(.87)(60)} = 0.66,$$

$$\frac{F_c' W_c}{\rho S_c} = \frac{(.173)(.49)}{(.930)} = 0.091,$$

$K_0 = 0.630$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.0,

$F_i = 0.87$ = core space factor,

$\rho = .930$ = resistivity, the value from Fig. 11-6 corresponding to 105°C, increased 2 per cent.

Flux density:

Select B_p = 100 kilolines per square inch,

$$B_s = B_p \frac{V_s}{V_p/n} = \frac{(100)(5.5)}{11.2} = 49 \text{ kilolines per square inch,}$$

$$V_p/n = \frac{p\,q\,V_s}{\sqrt{p^2\,q^2 - 1}} = \frac{(1.35)(.85)(55)}{\sqrt{(1.35)^2\,(.85)^2 - 1}} = 11.2 \text{ volts,}$$

$$p = \frac{13.5}{10} = 1.35,$$

q = .85 (estimated factor which accounts for change in leakage reactance).

Characteristic linear dimension:

ℓ = 1.2 inches from nomograph, Fig. 11-7.

Approximate core weight:

$$M_1 = K_1 F_i \delta_1 \ell^3 = (7.45)(.87)(.276)(1.2)^3 = 3.1 \text{ lb.,}$$

$K_1 = 7.45$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.0,

δ_i = .276 = core steel density in lb. per sq. in.

Core loss and excitation:

For B_p = 100 kl. per square inch:

core loss = 1.0 watts per lb.

excitation = 3 volt-amperes per lb.

For B_s = 49 kl. per square inch,

core loss = .27 watts per lb.,

excitation = .35 volt-amperes per lb.

W_i = (3.1/2)(1.0 + .27)(1.4) = 2.7 watts (4.9% of rating),

W_{ex} = (3.1/2)(3.0 + .35)(4) = 21 volt-amperes (38% of rating).

4) Core Dimensions

Core exposed surface area:

$S_i = K_2 \ell^2 = (23.1)(1.2)^2$ = 33 square inch,

K_2 = 23.1 = constant from Fig. 11-3 or 11-4 corresponding to s = 1.0.

Core dissipation per unit area:

W_i/S_i = 2.7/33 = .082 watts per square inch.

Lamination width:

$L = \ell\,(L/\ell)$ = (1.2)(1.07) = 1.28 inches, use L = 1.25 inches,

L/ℓ = 1.07 = constant from Fig. 11-3 or 11-4 corresponding to s = 1.0.

Area product:

$A_c A_i = \ell^4 = (1.2)^4$ = 2.07 inches^4.

Window area:

A_c = 1.172 square inches.

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} = \frac{2.07}{1.172} = 1.76 \text{ square inches.}$$

Stack height:

$$sL = \frac{A_i}{L} = \frac{1.76}{1.25} = 1.41 \text{ inches, approximately 1-3/8 inches.}$$

Stack ratio:

$$s = \frac{sL}{L} = \frac{1.41}{1.25} = 1.13$$

5) Winding Calculations

Equivalent winding exposed surface area:

$$S_c = K_3 \ell^2 = (13.02)(1.2)^2 = 18.8 \text{ square inches,}$$

$K_3 = 13.02$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Approximate winding losses:

$$W_c = S_c(W_c/S_c) = (18.8)(.49) = 9.2 \text{ watts.}$$

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c'}{\rho}\frac{W_c}{S_c}}}\, K_5 F_c' = \sqrt{\frac{1.2}{.091}}(803)(.173) = 500,$$

$K_5 = 803$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Primary current:

$$I_p = \frac{1}{V_p}\sqrt{(W_r + W_c + W_i)^2 + (W_{ex} + I_s^2 X)^2}$$

$$= \frac{1}{115}\sqrt{(55 + 9.2 + 2.7)^2 + (21 + 97)^2} = 1.17 \text{ amperes.}$$

$$I_s^2 X = \frac{I_s V_s}{\sqrt{p^2 q^2 - 1}} = \frac{(10)(5.5)}{\sqrt{(1.35)^2(.85)^2 - 1}} = 97 \text{ volt-amperes.}$$

where X is leakage reactance referred to secondary winding.

Primary wire size:

CM = (500)(1.17) = 585. Use No. 22 AWG (624.4 CM).

Secondary wire size:

CM = (500)(10) = 5000. Use No. 13 AWG (5178 CM).

Primary turns per volt:

$$\frac{N_p}{V} = \frac{10^5}{4.44 \, f \, B \, A_i \, F_i} = \frac{10^5}{(4.44)(60)(100)(1.76)(.87)} = 2.45.$$

Secondary turns per volt:

$$\frac{N_s}{V} = \frac{10^5}{4.44 \, f \, B \, A_i \, F_i} = \frac{10^5}{(4.44)(60)(49)(1.76)(.87)} = 5.0$$

Correction for resistance drop:

$$\frac{W_c}{W_r} = \frac{9.2}{55} = .1675$$

Turns:

$$N_p = V_p \frac{N_p}{V}\left(1 - \frac{W_c}{2 W_r}\right) = (125)(2.45)(.916) = 280 \text{ turns.}$$

Place taps at 258 and 235 turns.

$$N_s = V_s \frac{N_s}{V}\left(1 + \frac{W_c}{2 W_r}\right) = (5.5)(5)(1.084) = 30 \text{ turns.}$$

6) Winding Layout

Total available winding length equals window length minus margins and magnetic shunt thickness.

$$1\frac{7}{8} - \left[(2)(1/8) + 2(5/32) + (1/3)(1.25)\right] = .895 \text{ inches.}$$

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Turns per layer

Primary:

(34)(.6)(.895) = 18.2, use 18 turns per layer,

Secondary:

(12)(.4)(.895) = 4.3, use 4 turns per layer.

A check of the secondary build shows that it is excessive while the primary build is low. Both winding lengths are then changed so as to use the window area more effectively.

Revised turns per layer:

Primary: use 17 turns per layer,
winding length = 17/34 = .500 inch,
tube length is 11/16 inch.

Secondary: use 5 turns per layer,
winding length = 5/12 = .416 inch
tube length is 11/16 inch.

Layers:

Primary: 280/17 = 16.5, use 17 layers.

Secondary: 30/5 = 6 layers.

7) **Check of Coil Build**

Primary:

		Thickness (inches)
Tube		.040
17 layers of No. 22	(17)(.0276) =	.455
16 layers of paper	(16)(.033) =	.048
Wrapper		.010
		.553

Build = .553/.625 = 88.5%

Secondary:

Tube		.040

6 layers of No. 13	(6)(.0753) =	.452
5 layers of paper	(5)(.010) =	.050
Wrapper		.010
		.552

Build = .552/.625 = 88.5%

8) <u>Summary of Design</u>

Core:

Lamination: scrapless EI with center leg width of 1-1/4 inches,

Steel: oriented silicon, AISI M-10 grade, .014 inch thick,

Stack: 1-3/8 inches,

Tubes:

Both are .040 inch thick, 1-1/4 x 1-7/16 x 11/16 inch long.

Primary winding (125/115/105 volts):

Wire size: No. 22 AWG single-layer enamel, copper wire,

Turns per layer: 17,

Layer: 17,

Turns: 280, taps at 258 and 235 turns,

Layer insulation: .003 inch paper.

Wrapper: .010 inch paper

Secondary winding (5.5 volts):

Wire size: No. 13 AWG double-layer enamel, copper wire,

Turns per layer: 5,

Layers: 6,

Turns: 30,

Layer insulation: .010 inch paper.

Wrapper: .010 inch paper.

Magnetic shunts:

See part (11).

9) **Check of Winding Resistance**

Primary:

Resistance equals ohms per inch times mean length of turn times turns times correction to operating temperature.

$$R_p = \frac{(16.14)(.93)}{(12000)(.679)}(7.27)(280) = 3.75 \text{ ohms,}$$

Mean length of turn equals length of inside turn plus pi times build-up of winding.

$$m_{cp} = 2(1.25 + .080) + 2(1.4375 + .080) + \pi(.503) = 7.27 \text{ inches.}$$

Secondary:

$$R_s = \frac{(2.003)(.93)}{(12000)(.679)}(7.27)(30) = .0498 \text{ ohms,}$$

$$m_{cs} = 2(1.25 + .080) + 2(1.4375 + .080) + \pi(.502) = 7.27 \text{ inches.}$$

10) **Check of Voltage Ratio**

Primary voltage:

$$V_p = n\sqrt{\left[V_s + I_s(R_s + R_p/n^2)\right]^2 + (I_s X)^2}$$

$$= \frac{280}{30}\sqrt{\left[5.5 + 10(.0928)\right]^2 + (10)(.97)^2} = 109 \text{ volts}$$

$$R_s + R_p/n^2 = .0498 + 3.75\frac{(30)^2}{(280)^2} = .0928 \text{ ohms.}$$

This indicates that the secondary voltage will be somewhat high, but it is not necessary to change the turns ratio since a slight change in the leakage reactance by means of adjusting the magnetic shunts will give the correct voltage.

11) **Special Calculations and Design Checks**

Magnetic shunts:

Thickness of each shunt (along the long window dimension) = .50 inch (max.),
Width of each shunt: aL = 1-3/8 inches,

Length of each shunt (window width minus air gap) = .625 - .016 = .609 inch

Air gap:

$$m_g' = \frac{4.52\ p\ q\ N_s\ I_s}{10^3\ B_{g\ sc}} = \frac{(4.52)(1.35)(.85)(30)(10)}{100 \times 10^3} = .016 \text{ inch}$$

12) **Calculation of Temperature Rise**

Surface temperature rise:

$$\theta_{surf} = F_{surf} \frac{W_c + W_i}{(S_{case})(h_c + h_r)} = \frac{(1.1)(10.12 + 2.7)}{(87.4)(.00342 + .0056)} = 18.0°C$$

$$W_c = I_s^2 R_s + I_p^2 R_p = (10)^2(.0498) + (1.17)^2(3.75) = 10.12 \text{ watts},$$

W_i = 2.7 watts,

S_{case} = 87.4 = case area in square inches (3.875 x 3.300 x 4.313 inches),

F_{surf} = 1.1 = form factor of surface from Table 11-9,

$$h_c = 3.75 \times 10^{-3} \frac{\theta_{surf}}{S_{case}^{.17}} p^{.44} = 3.75 \times 10^{-3} \frac{20^{.22}}{87.4^{.17}} = .00342,$$

h_r = (.0062)(.9) = .0056,

θ_{surf} is assumed to be 20°C,

$$\theta_{surf} = .1\ \theta_{assumed} + .9\ \theta_{calculated} = (.1)(20) + (.9)(18.0)$$

$$= 18.2°C$$

Temperature difference across impregnant:

$$\theta_{imp} = F_{imp} \frac{(W_c + W_i)(m)}{Sk} = \frac{(1.75)(10.12 + 2.7)(.58)}{(69.6)(.015)} = 12.5°C$$

F_{imp} = 1.75 = correlation factor,

$S = 1/2\ (S_{case} + S_c + S_i) = 1/2\ (87.4 + 18.8 + 33)$

$= 69.6$ square inches,

$S_c = 18.8$ square inches,

$S_i = 33$ square inches,

$k = .015$ = thermal conductivity from Table 11-10,

$$m = \sqrt{\frac{S_{case}}{4\pi}} - \sqrt{\frac{S_c + S_i}{4\pi}} = \sqrt{\frac{87.4}{4\pi}} - \sqrt{\frac{18.8 + 33}{4\pi}} = .58 \text{ inches.}$$

Coil hot spot temperatures:

Primary:

$$\Theta_{hp} = F\, W_{cp}^{\ x} \left(\frac{m}{k\, S_{cp}}\right)^y = (.32)(5.14)^{1.0}\left(\frac{.3125}{.0127 \times 9.4}\right)^{2.0} = 11.5°C$$

$W_{cp} = I_p^{\ 2}\, R_p = (1.17)^2(3.75) = 5.14$ watts,

$m = 1/2\ (.625) = .3125$ inch,

$S_{cp} = 18.8/2 = 9.4$ square inches,

$$k = k_i \left(\frac{R + 1}{.11R + 1}\right) = .003\left(\frac{7.05}{1.66}\right) = .0127,$$

$$R = \frac{.0254}{.0042} = 6.05.$$

Secondary:

$$\Theta_{hs} = F\, W_{cs}^{\ x} \left(\frac{m}{k\, S_{cs}}\right)^y = (.32)(4.98)^{1.0}\left(\frac{.3125}{.0128 \times 9.4}\right)^{2.0}$$

$= 10.0°C$,

$W_{cs} = I_s^{\ 2}\, R_s = (10)^2\ (.0498) = 4.98$ watts,

S_{cs} = 18.8/2 = 9.4 square inches,

$$k = k_1 \left(\frac{R + 1}{.11 R + 1}\right) = .003 \left(\frac{7.15}{1.67}\right) = .0128,$$

$$R = \frac{.07196}{.0117} = 6.15$$

Average winding temperatures:

Primary:

$$T_{pri} = T_{amb} + \Theta_{surf} + \Theta_{imp} + .77\Theta_{hp}$$

$$= 65 + 18.2 + 12.5 + (.77)(11.5) = 104.6°C \; (\Delta T = 39.6°C).$$

Secondary:

$$T_{sec} = T_{amb} + \Theta_{surf} + \Theta_{imp} + .77\,\Theta_{hs}$$

$$= 65 + 18.2 + 12.5 + (.77)(10.0) = 103.4°C. \; (\Delta T = 38.4°C).$$

The factor .77 is the average of .90 and .65 from Table 11-12.

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XVII. DESIGN PROCEDURE: CURRENT-LIMITING TRANSFORMER WITH UNBALANCED MAGNETIZATION

1) Specifications

Frequency, voltages, secondary load and short-circuit currents, type of load filter, temperatures (ambient and maximum rise), grade of protection.

2) Chosen Quantities

Type of core, grade and thickness of lamination, maximum core loss and excitation, core space factor, approximate stack ratio, type of construction.

3) Nomograph Values

Secondary RMS voltage:

V_s is the sum of average load voltage V_{DC} plus rectifier forward voltage drop and circuit resistance voltage drops multiplied by 2.22 if no filter is used, or average load voltage multiplied by a ratio from Table 13-3 for a capacitance-filtered load.

Secondary RMS current:

I_s is average load current I_{DC}, multiplied by 1.57 if no filter is used, or by a ratio from Table 13-1 for a capacitance-filtered load.

Equivalent secondary rating:

$W_r = V_s I_s$ volt amperes,

Winding dissipation:

$\frac{W_c}{S_c} = (\frac{\Delta T}{K})^{1.25}$ watts per sq. in. (2-19) or Fig. 11-1

Winding space factor:

$F_c' = .6 F_c$ (6-17)

where $F_c = .08 \log_{10} W_r' + F$, (2-20)

The factor in (6-17) allows space for the shunt, and is taken as 0.6 for a scrapless lamination. However a value of 0.5 is to be used for units rated less than about 50 volt amperes. For a non-scrapless lamination, the factor is usually greater than 0.6

and can be estimated by allowing space for a shunt and extra margins. The total width (in the direction of the long window dimension) of the space allowed should be a little greater than the outside leg width of the core.

Nomograph scale values:

$$\frac{K_0 W_r}{F_1 f} \text{ and } \frac{F_c' M_c}{\rho S_c}$$

Flux density:

Select B_p, using a value from Table 11-2 decreased about 10 per cent. This reduction is made to obtain reasonable values of excitation volt-amperes, which would otherwise tend to be excessive because of the unbalanced magnetization.

$$B_s = B_p \frac{V_s}{V_p/n} \text{ kilolines per sq. in.} \qquad (6\text{-}12)$$

$$\text{where } V_p/n = \frac{p\, q\, V_s}{\sqrt{p^2 q^2 - 1}} \text{ volts,} \qquad (6\text{-}6)$$

$$p = \frac{\text{short-circuit current,}}{\text{rated current}}$$

$$q = \frac{\text{leakage reactance at short circuit}}{\text{leakage reactance at rated current}}$$

= .8 typically.

Characteristic linear dimension:

Obtain ℓ from nomograph, Fig. 11-7, using B_s.

Approximate core weight:

$$M_1 = K_1 F_1 \delta_1 \ell^3 \text{ pounds.} \qquad (2\text{-}23)$$

Mean length of magnetic circuit:

$$m_1 = a \ell \text{ inches.} \qquad (2\text{-}7)$$

Approximate secondary turns:

$$N_s = \frac{K_6 V_s}{f F_1 B_s \ell^2} \text{ turns.} \qquad (2\text{-}34)$$

Factor for calculating unbalanced magnetizing forces:

$$H_{DC} = \frac{.495 \, N_s \, I_{DC}}{m_1} \qquad (5\text{-}19)$$

Approximate unbalanced magnetizing force in primary portion of core:

$$H_{DC\,p} = .7 \, H_{DC} \text{ average oersteds} \qquad (7\text{-}1)$$

Approximate unbalanced magnetizing force in secondary portion of core:

$$H_{DC\,s} = 1.3 \, H_{DC} \text{ average oersteds} \qquad (7\text{-}2)$$

Core loss and excitation:
Use design curves (Fig. 13-1 through 13-8), and half of the core weight with each flux density and magnetizing force to obtain total W_1 and W_{ex}.

4) <u>Core dimensions</u>

Core exposed surface area:

$$S_1 = K_2 \ell^2 \text{ sq. in.} \qquad (2\text{-}25)$$

Core dissipation per unit area:

W_1/S_1 watts per sq. in.

Area Product:

$$A_c \, A_1 = \ell^4. \qquad (2\text{-}6)$$

Lamination leg width:

$$L = \ell \frac{L}{\ell} \text{ inches} \qquad (2\text{-}24)$$

Window area:

Calculate A_c from lamination dimensions.

Core cross-sectional area:

$$A_1 = \frac{\ell^4}{A_c} \text{ sq. in.}$$

Stack height:

$$SL = \frac{A_1}{L} \text{ inches}$$

Stack ratio:

$$s = \frac{aL}{L}$$

5) **Winding Calculations**

Equivalent winding exposed surface:

$$S_c = K_3 \ell^2 \text{ sq. in.} \qquad (2\text{-}26)$$

Approximate winding loss

$$W_c = \frac{W_c}{S_c} S_c \text{ watts.} \qquad (2\text{-}27)$$

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c' W_c}{\rho S_c}}} (K_5 F_c') \qquad (2\text{-}31)$$

Primary component of load volt-amperes:

$$W_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} \text{ volt-amperes} \qquad (5\text{-}11)$$

Primary current:

$$I_p = \frac{1.1}{V_p} \sqrt{(W_{pL} + W_c + W_i)^2 + \left[W_{ex} + X(I_s^2 - I_{DC}^2)\right]^2} \text{ amperes} \qquad (7\text{-}6)$$

$$\text{where } X = \frac{V_s}{1.1\, I_{DC} \sqrt{p^2 q^2 - 1}} \text{ ohms} \qquad (7\text{-}4)$$

Wire sizes:

Calculate from circular mils per ampere using RMS primary current and rated RMS secondary current.

Turns per volt:

$$\frac{N_p}{V} = \frac{10^5}{4.44\, f\, A_i\, F_i\, B_p} \text{ turns per volt,}$$

$$\frac{N_s}{V} = \frac{10^5}{4.44\, f\, A_i\, F_i\, B_s} \text{ turns per volt.}$$

Turns:

$$N_p = \frac{N_p}{V} V_p \left(1 - \frac{.707\, W_c}{2\, W_r}\right) \text{ turns,}$$

$$N_s = \frac{N_s}{V} V_s \left(1 + \frac{.707\, W_c}{2\, W_r}\right) \text{ turns}$$

For a capacitance filter, replace .707 by $\frac{1.1}{I_s/I_{DC}}$ in the two previous equations.

6) Winding Layout

Total available winding length:

Window length minus (four margins plus magnetic shunt thickness). Assume magnetic shunt thickness (dimension parallel to coil axes) to be at least (2/3) L for a simple-type core, and (1/3) L for a shell-type core. Increase shunt thickness when necessary so that shunt flux density does not exceed 130 kilolines per square inch during short circuit. (See Ch. XV, paragraph 6)

Turns per layer:

Primary turns per layer are approximately equal to turns per inch times .6 of total available of winding length.

Secondary turns per layer are approximately equal to turns per inch times .4 of total available winding length.

Layers:

Turns divided by turns per layer.

7) Check of Coil Build

Choose tube, layer insulation, and wrappers, and check build of both primary and secondary windings to insure that each is 80 to 90 per cent of window width.

To equalize builds, re-apportion the available winding length between the primary and secondary when necessary.

8) Summary of Design

List core material, dimensions, weight, tube, winding wire sizes, total turns, taps, turns per layer, number of layers, layer insulation, wrappers, and shield data.

9) Check of Winding Resistances

Resistance equals resistance per unit length (corrected to operating temperature from Fig. 11-6) times mean length of turn, times number of secondary turns.

Mean length of turn equals length of inside turn of the winding plus pi times the build-up of winding.

10) <u>Check of Voltage Ratio</u>

Calculate primary voltage:

$$V_p = h \sqrt{\left[V_s + 1.1\, I_{DC}\, (R_s + R_p/n^2)\right]^2 + (1.1\, I_{DC} X)^2}$$

where R_p and R_s are obtained from step 9.

Adjust the turns ratio if the calculated primary voltage differs appreciably from the specified voltage.

11) <u>Special Calculations and Design Checks</u>

Magnetic shunt:

Thickness of shunt(s) (along the long window dimension) has previously been determined in step 6.

Width of shunt(s) is approximately to core stack height, sL.

Length of shunt(s) is equal window width minus air gap.

An expression for the air gap associated with the shunt, or with each shunt if there are two is

$$m_g' = \frac{4.52\, p\, q\, N_s\, I_s}{B_{g\,sc}\, 10^3} \text{ inch.} \qquad (6\text{-}16)$$

12) <u>Calculation of Temperature Rise</u>

Follow basic method.

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XVIII. EXAMPLE: CURRENT-LIMITING TRANSFORMER WITH UNBALANCED MAGNETIZATION

1) Specifications

Frequency: 400 cycles per second.

Ambient temperature: 65°C

Maximum temperature rise: 40°C

Primary: 60/65/71 volts.

Secondary: 65 volts RMS, 1.2 amperes RMS, .75 ampere DC, 1.65 amperes RMS short-circuit current, half-wave rectifier with resistance load and no filter.

Protection: Grade 2 (less resistant to adverse environmental conditions).

2) Chosen Quantities

Core: Scrapless EI laminations.

Steel: AISI-10 grade, oriented silicon steel, .004 inch thick.

Construction: Open core and coils.

Core space factor: .85

Approximate stack ratio: 1.0

3) Nomograph Values

Secondary RMS voltage

$$V_s = 65 \text{ volts RMS.}$$

Secondary RMS current

$$I_s = 1.2 \text{ amperes RMS.}$$

Equivalent secondary rating

$$W_r = V_s I_s = (65)(1.2) = 78 \text{ volt-amperes.}$$

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} = \left(\frac{40}{91}\right)^{1.25} = 36 \text{ watts per square inch,}$$

$\Delta T = 40$ = maximum temperature rise in °C,
$K = 91$ = constant from Table 11-1

Winding space factor:

$$F_c' = .6\,F_c = (.6)(.25) = .15,$$

$$F_c = .08 \log_{10} W_r' + F = .08 \log_{10} 18.5 + .15 = .25,$$

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76}\left(\frac{\Delta T}{40}\right)^{.63}} = \frac{78}{\left(\frac{400}{60}\right)^{.76}} = 18.5 \text{ watts},$$

$f = 400$ = frequency in cycles per second,

$F = .15$ = constant from Fig. 11-2

Nomograph scale values:

$$\frac{K_o\,W_r}{F_i\,f} = \frac{(.630)(78)}{(.85)(400)} = .145,$$

$$\frac{F_c'\,W_c}{\rho\,S_c} = \frac{(.15)(.36)}{(.93)} = .058$$

$K_o = .630$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$

$F_i = .85$ = core space factor.

$\rho = .930$ = resistivity, the value from Fig. 11-6 corresponding to 105°C and then increased 2 per cent.

Flux density:

Select $B_p = 50$ kilolines per square inch to account for unbalanced magnetization

$$B_s = B_p \frac{V_s}{V_p/n} = 50\,\frac{65}{160}\;20.5 \text{ kilolines per square inch},$$

$$V_p/n = \frac{p\,q\,V_s}{\sqrt{p^2\,q^2 - 1}} = \frac{(1.37)(.8)(65)}{\sqrt{(1.37)^2\,(.8)^2 - 1}}$$

$$p = \frac{1.65}{1.2} = 1.37,$$

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$q = .8$ (estimated factor to account for change in leakage reactance).

Characteristic linear dimension:

$\ell = 1.05$ inches from nomograph, Fig. 11-7.

Approximate core weight:

$W_1 = K_1 F_1 \delta_1 \ell^3 = (7.45)(.85)(.276)(1.05)^3 = 2.02$ pounds,

$K_1 = 7.45$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

$\delta_1 = .276$ = core steel density in pounds per square inch.

Mean length of magnetic circuit:

$m_1 = a\ell = (6.45)(1.05) = 6.77$ inches,

$a = 6.45$ = constant from Fig. 11-3 corresponding to $s = 1.0$.

Approximate secondary turns:

$$N_s = \frac{K_6 V_s}{f F_1 B_s \ell^2} = \frac{(19,480)(65)}{(400)(.85)(20.5)(1.05)^2} = 165 \text{ turns,}$$

$K_6 = 19,480$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Factor for calculating unbalanced magnetizing forces:

$$H_{DC} = \frac{.495\, N_s I_{DC}}{m_1} = \frac{(.495)(165)(.75)}{6.77} = 9.05,$$

$I_{DC} = .75$ = average load current in amperes.

Approximate unbalanced magnetizing force in primary portion:

$H_{DC\,p} = .7\, H_{DC} = (.7)(9.05) = 6.3$ average oersteds.

Approximate unbalanced magnetizing force in secondary portion:

$H_{DC\,s} = 1.3\, H_{DC} = (1.3)(9.05) = 12$ average oersteds.

Core loss and excitation:

For $B_p = 50$ kl per sq. in. and $H_{DC\,p} = 6.3$,

core loss = 3.4 watts per lb. from Fig. 13-8,
excitation = 40 volt-amperes per lb. for Fig. 13-7,
gap = .13% from Fig. 13-7.

For B_s = 20.5 kl per sq. in. and $H_{DC\,s}$ = 12,

core loss = 1.5 watts per lb. from Fig. 13-8
excitation = 15 volt-amperes per lb. from Fig. 13-7.
gap = .28% from Fig. 13-7.

$W_i = (2.02/2)(3.4 + 1.5) = 5$ watts.

$W_{ex} = (2.02/2)(40 + 15) = 55$ volt-amperes.

Effective gap = (.0028)(6.77) = .019. Use butt joint with .014 inch of paper in secondary portion.

4) <u>Core Dimensions</u>

Core exposed surface area:

$S_i = K_2\ell^2 = (23.1)(1.05)^2 = 25.5$ square inches,

$K_2 = 23.1$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Core dissipation:

$W_i/S_i = 5/25.5 = .196$ watts per square inch.

Lamination width:

$L = \ell(L/\ell) = (1.05)(1.077) = 1.13$ inches, use $L = 1.125$ inches.

$L/\ell = 1.077$ from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Area product:

$A_c A_i = \ell^4 = (1.05)^4 = 1.215$ inches^4.

Window area:

$A_c = .95$ square inch.

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} = \frac{1.215}{.95} = 1.28 \text{ square inches.}$$

Stack height:

$$sL = \frac{A_i}{L} = \frac{1.28}{1.125} = 1.14 \text{ inches, approximately 1-3/16 inches.}$$

Stack:

$$s = \frac{sL}{L} = \frac{1.14}{1.125} = 1.01.$$

5) **Winding Calculations**

Equivalent winding exposed surface area:

$S_c = K_3 \ell^2 = (13.02)(1.05)^2 = 14.4$ square inches,

$K_3 = 13.02$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Approximate winding losses:

$W_c = (W_c/S_c)\, S_c = (.36)(14.4) = 5.2$ watts.

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c' W_c}{\rho S_c}}}\, K_5 F_c' = \sqrt{\frac{1.05}{.058}}\,(803)(.15) = 510,$$

$K_5 = 803$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.0$.

Primary component of load volt-amperes:

$$W_{pL} = V_s \sqrt{I_s^2 - I_{DC}^2} = 65\sqrt{(1.2)^2 - (.75)^2} = 61 \text{ volt-amperes.}$$

Primary current:

$$I_p = \frac{1.1}{V_p}\sqrt{(W_{pL} + W_c + W_i)^2 + \left[W_{ex} + X(I_s^2 - I_{DC}^2)\right]^2}$$

$$= \frac{1}{65}\sqrt{(61 + 5.2 + 5)^2 + (55 + 156)^2} = 3.78 \text{ amperes,}$$

$$X = \frac{V_s}{1.1\, I_{DC}\sqrt{p^2 q^2 - 1}} = \frac{65}{(1.1)(.75)\sqrt{(1.37)^2 (.8)^2 - 1}}$$

= 177 ohms referred to secondary winding,

$$X(I_s^2 - I_{DC}^2) = 177\left[(1.2)^2 - (.75)^2\right] = 156 \text{ volt-amperes.}$$

Primary wire size:

CM = (510)(3.78) = 1920. Use No. 17 AWG (2048 CM).

Secondary wire size:

CM = (510)(1.2) = 612. Use No. 22(624.4 CM).

Primary turns per volt:

$$\frac{N_p}{V} = \frac{10^5}{4.44\, f\, B_p\, A_i\, F_i} = \frac{10^5}{(4.44)(400)(50)(1.28)(.85)} = 1.03$$

Secondary turns per volt:

$$\frac{N_s}{V} = \frac{10^5}{4.44\, f\, B_s\, A_i\, F_i} = \frac{10^5}{(4.44)(400)(20.5)(1.28)(.85)} = 2.52$$

Correction for resistance drop:

$$\frac{W_c}{W_r} = \frac{5.2}{78} = .0668.$$

Turns:

$$N_p = V_p \frac{N_p}{V} \left(1 - \frac{.707\, W_c}{2\, W_r}\right) = (71)(1.03)(.976) = 71 \text{ turns.}$$

Place taps at 65 and 60 turns.

$$N_s = V_s \frac{N_s}{V} \left(1 + \frac{.707\, W_c}{2\, W_r}\right) = (65)(2.52)(1.024) = 168 \text{ turns.}$$

6) Winding Layout

Total available winding length equals window length minus the sum of shunt width plus margins. Shunt width is approximately 1/3 of lamination center leg width.

$$1.6875 - \left[(4)(1/8) + (1/3)(1.25)\right] = .8125 \text{ inch.}$$

Turns per layer:

Primary: (19)(.6)(.8125) = 9.26, use 9 turns per layer.
Secondary: (34)(.4)(.8125) = 11.04, use 11 turns per layer.

Layers:

Primary: 71/9 = 7.9, use 8 layers
Secondary: 168/11 = 15.3, use 16 layers

7) Check of Coil Build

Primary:	Thickness - inch
Tube	.030
8 layers of No. 17	(8)(.0469) = .375
7 layers of paper	(7)(.007) = .050
Wrapper	.010
	.465

Build = .465/.5625 = 83%

Secondary:	Thickness - inch
Tube	.030
16 layers of No. 22	(16)(.0267) = .427
15 layers of paper	(15)(.003) = .045
Wrapper	.010
	.512

Build = .512/.5625 = 91%

8) Summary of Design

Core:

Lamination: scrapless EI, with center leg width of 1-1/8 inches,
Steel: oriented silicon, AISI M-10 grade, .004 inch thick,
Stack: 1-3/16 inches.
Construction: Butt joint with .007 inch paper in secondary portion of core.

Tubes:

Primary: .030 inch thick, 1-1/8 x 1-3/8 x 11/16 inch long,
Secondary: .030 inch thick, 1-1/8 x 1-3/8 x 9/16 inch long.

Primary winding (60/65/71 volts):

Wire size: No. 17 AWG single-enamel copper wire,
Turns per layer: 9,
Layers: 8,
Turns: 71. taps at 65 and 60 turns,
Layer insulation: .007 inch paper
Wrapper: .010 inch paper

Secondary winding (65 volts RMS):

Wire size No. 22 single-enamel copper wire,
Turns per layer: 11,
Layers: 16,
Turns: 168,
Layer insulation: .003 inch paper,
Wrapper: .010 inch paper

9) Check of Winding Resistances

Resistance equals ohms per inch, times correction to operating temperature from Fig. 11-6, times mean length of turn, times turns. Mean length of turn equals length of inside turn, plus pi times build-up of winding.

$$R_p = \frac{(5.064)(.93)}{(12000)(.679)} (6.57)(71) = .269 \text{ ohms},$$

$$m_{cp} = 2(1.125 + 060) + 2(1.375 + .060) + \pi(.425) = 6.57 \text{ inches.}$$

$$R_s = \frac{(16.14)(.93)}{(12000)(.679)} (6.72)(168) = 2.08 \text{ ohms.}$$

$$m_{cs} = 2(1.125 + .060) + 2(1.375 + .060) + \pi(.472) = 6.72 \text{ inches.}$$

10) Check of Voltage Ratio

Primary voltage:

$$V_p = n \sqrt{\left[V_s + 1.1\, I_{DC} (R_s + R_p/n^2)\right]^2 + (1.1\, I_{DC}\, X)^2}$$

$$= \frac{71}{168} \sqrt{\left[65 + (1.1)(.75)(3.59)\right]^2 + \left[(1.1)(.75)(177)\right]^2}$$

$$= 68 \text{ volts.}$$

$$R_s + R_p/n^2 = 2.08 + .269 \frac{(168)^2}{(71)^2} = 3.59 \text{ ohms}$$

This indicates that the secondary voltage would be slightly low, but it may not be necessary to change the turns ratio since a slight change in the leakage reactance by means of adjusting the magnetic shunts would give the correct voltage.

11) Special Calculations and Design Checks

Magnetic shunts:

Thickness of each shunt (along the long window dimension) = 7/16 inch (max.),
Width of each shunt is equal core stack height: sL = 1-3/16 inches,
Length of each shunt equals window width minus gap = .5625 - .0199 = 5426 inch.

Gap:

$$m_g' = \frac{4.52\, p\, q\, N_s\, I_s}{B_{g\,sc}\, 10^3} = \frac{(4.52)(1.37)(.8)(168)(1.2)}{50 \times 10^3} = .0199 \text{ inch.}$$

12) Calculation of Temperature Rise

Surface temperature rise:

$$\theta_{surf} = F_{surf} \frac{W_c + W_i}{(S_c + S_i)(h_c + h_i)} = (.9) \frac{(6.84 + 5.0)}{(39.9)(.00994)} = 27.0^\circ \text{ C},$$

$$W_c = I_s^2 R_s + I_p^2 R_p = (1.2)^2(2.08) + (3.78)^2(.269) = 6.84 \text{ watts}$$

W_i = 5.0 watts,

S_c = 14.4 square inches,

S_i = 25.5 square inches,

F_{surf} = .9 = form factor of surface from Table 11-9,

$$h_c = 3.75 \times 10^{-3} \frac{\Theta_{surf}^{.22}}{(S_c + S_i)^{.17}} P^{.44} = 3.75 \times 10^{-3} \frac{28^{.22}}{(14.4 + 25.5)^{.17}}$$

$$= .00418,$$

$$h_r = (.0064)(.9) = .00576,$$

Θ_{surf} is assumed to be 28°C.

$$\Theta_{surf} = .1\Theta_{assumed} + .9\Theta_{calculated} = (.1)(28) + (.9)(27.0)$$

$$= 27.1°\text{C}.$$

Coil hot-spot temperature

Primary:

$$\Theta_{hp} = F\, W_{cp}^{\,x} \left(\frac{m}{k\, S_{cp}}\right)^y = (1.2)(3.84)^{.85} \left(\frac{.281}{.0119 \times 7.2}\right)^{1.4} = 19.9°\text{C},$$

$$W_{cp} = I_p^2 R_p = (3.78)^2\, .269 = 3.84$$

$$m = (1/2)(.5625) = .281 \text{ inch},$$

$$S_{cp} = 14.4/2 = 7.2 \text{ square inches},$$

$$k = k_1 \left(\frac{R + 1}{.11 R + 1}\right) = .003\ \frac{6.3}{1.58} = .0119,$$

$$R = \frac{.0453}{.0086} = 5.3$$

Secondary:

$$\Theta_{hs} = F\, W_{cs}^{\,x} \left(\frac{m}{k\, S_{cs}}\right)^y = (1.2)(3.0)^{.85} \left(\frac{.281}{.0124 \times 7.2}\right)^{1.4} = 15.2°\text{C},$$

$$W_{cs} = I_s^2 R_s = (1.2)^2\,(2.08) = 3.0 \text{ watts},$$

S_{cs} = 14.4/2 = 7.2 square inches,

$$k = k_i \left(\frac{R + 1}{1 + .11R} \right) = .003 \; \frac{6.8}{1.64} = .0124,$$

$$R = \frac{.0253}{.0044} = 5.8.$$

Average winding temperatures:

Primary:

$$T_{pri} = T_{amb} + \theta_{surf} + .85\, \theta_{hp}$$

$$= 65 + 23.9 + (.85)(19.9) = 105.9°C \;(\Delta T = 40.9°C)$$

Secondary:

$$T_{sec} = T_{amb} + \theta_{surf} + .85\, \theta_{hs}$$

$$= 65 + 23.9 + (.85)(15.2) = 101.8°C \;(\Delta T = 36.8°C).$$

The factor .85 is an average of .90 and .80 from Table 11-12.

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XII. DESIGN PROCEDURE: VIBRATOR-SUPPLY TRANSFORMERS

1) **Specifications**

Frequency (usually 115 cps), supply voltage, load, type of filter, temperatures (ambient and maximum rise), grade of protection.

2) **Chosen Quantities**

Type of core, grade and thickness of lamination, core space factor, approximate stack ratio, type of construction.

3) **Nomograph Values**

RMS voltage of half the secondary:

$V_s/2$ is average load voltage V_{DC}, plus estimated rectifier average forward drop plus other series-resistance voltage drops multiplied by 1.1 for an infinite inductance-input filter, or multiplied by $1/\sqrt{T}$ if no filter is used (T is the ratio of vibrator contacting time to half a period, between .70 and .85), or V_{DC} multiplied by the ratio from Table 12-4 for a capacitance-filtered load.

RMS current in secondary:

I_s is average load current I_{DC}, multiplied by .707 for an infinite inductance-input filter, multiplied by $.707\sqrt{T}$ if no filter is used, or multiplied by the ratio from Table 12-2 for a capacitance-filtered load.

Equivalent secondary rating:

$$W_r = 2(V_s/2)I_s \text{ volt amperes.} \qquad (8\text{-}15)$$

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per square inch.} \qquad (2\text{-}19) \text{ or Fig. 11-1}$$

Winding space factors:

$$F_c = .08 \log_{10} W_r' + F \qquad (20\text{-}20)$$

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Nomograph scale values:

$$\frac{K_0 W_r}{F_i f} \text{ and } \frac{F_c W_c}{\rho S_c}$$

Flux density:

Select flux density from Table 19-1 and then decrease this value by the ratio of most probable operating voltage to the maximum operating voltage using Table 19-2.

Characteristic linear dimension:

Use nomograph, Fig. 11-7 to obtain ℓ.

Approximate core weight:

$$M_i = K_1 F_i \delta_i \ell^3 \text{ pounds.} \qquad (2\text{-}23)$$

Core loss and excitation:

Use material curves, correction factors (Table 11-3) and core weight to calculate W_i and W_{ex}.

4) Core Dimensions

Area product:

$$A_c A_i = \ell^4 \qquad (2\text{-}6)$$

Lamination leg width:

$$L = \ell \frac{L}{\ell} \text{ inches.} \qquad (2\text{-}24)$$

Window area:

Calculate A_c from lamination dimensions.

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} \text{ sq. in.}$$

Stack height:

$$sL = \frac{A_i}{L} \text{ inches.}$$

Stack ratio:

$$s = \frac{sL}{L}$$

Core exposed surface area:

$$S_1 = K_2\, \ell^2 \text{ sq. in.} \qquad (2\text{-}25)$$

Core dissipation per unit area:

$$W_1/S_1 \text{ watts per sq. in.}$$

5) **Winding Calculations**

Winding exposed surface area:

$$S_c = K_3\, \ell^2 \text{ sq. in.} \qquad (2\text{-}26)$$

Approximate winding loss:

$$W_c = \frac{W_c}{S_c}\, S_c \text{ watts.} \qquad (2\text{-}27)$$

Conductor weight:

$$M_c = K_4\, F_c\, \delta_c\, \ell^3 \text{ pounds.} \qquad (2\text{-}30)$$

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{F_c \dfrac{W_c}{\rho\, S_c}}}\,(K_5\, F_c). \qquad (2\text{-}31)$$

Primary input power:

$$W_{rp} = W_r + W_c + 1.414\ \cdot_{ex} \text{ watts.} \qquad (8\text{-}16)$$

RMS voltage of half the primary:

$$V_p/2 = (V_b - 1)\sqrt{T} \text{ RMS volts,} \qquad (8\text{-}18)$$

where V_b is the supply voltage, and one volt is assumed for contact drops.

RMS current in the primary:

$$I_p = \frac{W_{rp}}{2(V_p/2)} \text{ RMS amperes.} \qquad (8\text{-}17)$$

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Wire sizes:

Calculate from circular mils per ampere using primary and secondary RMS currents.

Turns per volt:

$$\frac{N}{V} = \frac{10^5}{4.44\ f\ A_i\ F_i\ B} \text{ turns per volt.} \qquad (2\text{-}33)$$

Primary turns:

$$N_p = 2(V_p/2)\ \frac{N}{V}\left(1 - \frac{W_c}{2\ W_r}\right) \text{ turns.}$$

Secondary turns:

$$N_s = 2(V_s/2)\ \frac{N}{V}\left(1 + \frac{W_c}{2\ W_r}\right) \text{ turns.}$$

6) Winding Layout

Winding length:

Window length minus two margins.

Turns per layer:

Appropriate turns per inch times winding length.

Layers:

Appropriate turns divided by turns per layer.

7) Check of Coil Build

Choose tube, layer insulation, and wrappers, and check build to make sure that it is between 80 and 90 per cent of window width.

8) Summary of Design

List core material, dimensions, weight, tube, winding wire sizes, total turns, taps, turns per layer, number of layers, layer insulation, wrappers, and shield data.

9) Check of Winding Resistances

Resistance equals resistance per unit length (corrected to operating temperature from Fig. 11-6) times mean length of turn, times number of secondary turns.

Mean length of turn equals length of inside turn, plus pi times build-up of winding.

10) <u>Special Calculations and Design Checks</u>

These are to be made when a quantity is near its maximum permissible limit, or to check operation in other ways.

11) <u>Calculation of Temperature Rise</u>

Follow basic method.

TABLE 19-1

SUGGESTED FLUX DENSITIES FOR SILICON-STEEL CORES IN VIBRATOR-SUPPLY TRANSFORMERS

(Kilolines per square inch at maximum voltage

	Frequency - cycles per second		
Material and core	115	250	400
Non-oriented steel, stacked core	60	55	40
Oriented steel, stacked core	70	65	50
Oriented steel, wound core	75	70	55

TABLE 19-2

TYPICAL OPERATING VOLTAGE RANGES FOR NOMINAL VOLTAGE SYSTEMS

Nominal Voltage	Operating Range
4	3-5
6	5-8
12	10-16
24	20-30
32	27-44

XI. EXAMPLE: VIBRATOR-SUPPLY TRANSFORMER

1) Specifications

Frequency: 115 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Supply: 24 volts DC.

Load: .050 amperes DC from full-wave rectifier with an inductance-input filter.

Secondary voltage: 572 volts RMS

Protection: Grade 1 (most resistant to adverse environmental conditions).

2) Chosen Quantities

Core: scrapless EI laminations.

Steel: AISI-M-22 grade, hot-rolled silicon steel, .025 inch thick.

Construction: encased, hermetically sealed, with sand-loaded asphalt filling compound.

Core space factor: .9.

Approximate stack ratio: 1.5.

3) Nomograph Values

RMS voltage of half the secondary:

$V_s/2 = 572/2 = 286$ volts RMS.

RMS current in half the secondary:

$I_s = .707\ I_{DC} = (.707)(50) = .0354$ amperes RMS

where the factor .707 is the suitable ratio of currents for an infinite inductance-input filter.

Equivalent secondary rating:

$W_r = 2(V_s/2)\ I_s = (2)(286)(.0354) = 20.3$ volt-amperes.

Winding dissipation:

$$\frac{W_c}{S_c} = \left(\frac{\Delta T}{K}\right)^{1.25} = \left(\frac{40}{75}\right)^{1.25} = .45 \text{ watts per square inch,}$$

$K = 75$ = constant from Table 11-1,

$\Delta T = 40$ = maximum temperature rise in °C.

Winding space factor:

$$F_c = .08 \log W'_r + F = .08 \log 12.4 + .10 = .19,$$

$$W'_r = \frac{W_r}{(\frac{f}{60})^{.76}(\frac{\Delta T}{40})^{.63}} = \frac{20.3}{(\frac{115}{60})^{.76}} = 12.4 \text{ volt-amperes},$$

F = .10 from Fig. 11-2 since both primary and secondary are center-tapped,

f = 115 = frequency in cycles per second

Scale values:

$$\frac{K_0 W_r}{F_i f} = \frac{(.649)(20.3)}{(.9)(115)} = .127,$$

$$\frac{F_c W_c}{\rho S_c} = \frac{(.19)(.45)}{(.930)} = .092,$$

K_0 = .649 from Fig. 11-3 or 11-4 corresponding to s = 1.5,

F_i = .9 = core space factor,

ρ = .930 = resistivity, the value from Fig. 11-6 corresponding to 105°C, increased 2 percent.

Flux density:

$B = 60 \frac{24}{30} = 48$ kilolines per square inch from Tables 19-1 and 19-2.

The supply voltage is nominally 24 volts, but may be assumed to be as high as 30 volts.

Characteristic linear dimensions:

ℓ = .76 inch from nomograph, Fig. 11-7.

Approximate core weight:

$$M_i = K_1 F_i \delta_i \ell^3 = (8.23)(.9)(.272)(.76)^3 = .89 \text{ pound},$$

K_1 = 8.23 from Fig. 11-3 or 11-4 corresponding to s = 1.5,

δ_i = .272 pound per cubic inch.

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Core loss and excitation:

From the curves for the material at B = 48 kilolines per square inch,

core loss = 1.15 watts per pound,

excitation = 2.2 volt-amperes per pound.

Applying correction factors from Table 11-3, and multiplying by core weight,

$$W_i = (.89)(1.15)(1.3) = 1.3 \text{ watts},$$

$$W_{ex} = (.89)(2.2)(2.5) = 4.9 \text{ volt-amperes}$$

4) Core Dimensions

Core exposed surface area:

$$S_i = K_2 \ell^2 = (24)(.76)^2 = 13.8 \text{ square inches},$$

$K_2 = 24$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5.

Core dissipation per unit area:

$$W_i/S_i = 1.3/13.8 = .094 \text{ watts per square inch.}$$

Lamination center leg width:

$$L = \ell(L/\ell) = (.76)(.97) = .737 \text{ inch, use } L = .75 \text{ inch},$$

$L/\ell = .97$ = constant from Fig. 11-3 or 11-4 corresponding to s = 1.5.

Area product:

$$A_c\, A_i = \ell^4 = (.76)^4 = .334 \text{ inch}^4.$$

Window area:

$$A_c = .422 \text{ square inch.}$$

Core cross-sectional area:

$$A_i = \frac{\ell^4}{A_c} = \frac{.334}{.422} = .79 \text{ square inch.}$$

Stack height:

$$sL = \frac{A_i}{L} = \frac{.79}{.75} = 1.055 \text{ inches, approximately 1-1/16 inches.}$$

Stack ratio:

$$s = \frac{sL}{L} = \frac{1.055}{.75} = 1.41$$

5) Winding Calculations

$$S_c = K_3 \ell^2 = (10.61)(.76)^2 = 6.12 \text{ square inches,}$$

$K_3 = 10.61$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.5$.

Approximate winding loss:

$$W_c = \frac{W_c}{S_c} S_c = (.45)(6.12) = 2.75 \text{ watts.}$$

Conductor weight:

$$M_c = K_4 F_c \delta_c \ell^3 = (4.49)(.19)(.321)(.76)^3 = .12 \text{ pound,}$$

$K_4 = 4.49$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.5$

$\delta_c = .321$ = conductor material density in pounds per cubic inch.

Circular mils per ampere:

$$\frac{CM}{amp} = \sqrt{\frac{\ell}{\frac{F_c W_c}{\rho S_c}}} (K_5 F_c) = \sqrt{\frac{.76}{.092}} (826)(.19) = 450,$$

$K_5 = 826$ = constant from Fig. 11-3 or 11-4 corresponding to $s = 1.5$

Primary input power:

$$W_{rp} = W_r + W_c + 1.414 W_{ex} = 20.3 + 2.75 + 1.414 (4.9)$$

$$= 30 \text{ volt amperes.}$$

RMS voltage of half the primary:

$V_p/2 = (V_b - 1)\sqrt{T} = (24-1)\sqrt{.85} = 21.2$ RMS volts,

$V_b = 24$ = supply voltage,

$T = .85$ = ratio of vibration contacting time to half a period.

Primary RMS current:

$$I_p = \frac{W_{rp}}{2V_p/2} = \frac{30}{(2)(21.2)} = .708 \text{ RMS amperes.}$$

Primary wire size:

CM = (450)(.708) = 318. Use number 25 AWG wire (320.4 CM).

Secondary wire size:

CM = (450)(.0354) = 15.9. Use number 38 AWG wire (15.72 CM).

Turns per volt:

$$\frac{N}{V} = \frac{10^5}{4.44\, f\, F_i\, A_i\, B} = \frac{10^5}{(4.44)(115)(.9)(.79)(48)} =$$

5.73 turns per volt.

Correction for winding resistance drop:

$$\frac{W_c}{W_r} = \frac{2.75}{20.3} = .135$$

Primary turns:

$$N_p = 2(V_p/2)\frac{N}{V}\left(1 - \frac{W_c}{2W_r}\right) = (5.73)(2)(21.2)(1-1.35/2)$$

= 222 turns.

Secondary turns:

$$N_s = 2(V_s/2)\frac{N}{V}\left(1 + \frac{W_c}{2W_r}\right) = (5.73)(286)(2)(1 + .135/2)$$

= 3500 turns.

6) <u>Winding Layout</u>

Winding length

$= 1.125 - 2 (\frac{1}{8}) = .875$ inch.

Turns per layer:

Primary: (48)(.875) = 42 turns per layer.

Secondary: (204)(.875) = 178 turns per layer.

Layers:

Primary: 222/42 = 5.3 use 6 layers

Secondary: 3500/178 = 19.7 use 20 layers.

Revised turns per layer:

Primary: 37 turns per layer.

Secondary: 175 turns per layer

7) Check of Coil Build:

		Thickness (inches)
Tube		.030
20 layers of No. 38 AWG wire	(20)(0045) =	.090
19 layers of paper	(19)(.002) =	.038
Wrapper		.020
6 layers of No. 25 AWG wire	(6)(.0191) =	.114
5 layers of paper	(5)(.002) =	.010
Wrapper		.020
		.322

$$\text{Build} = \frac{.322}{.375} 100 = 86\%$$

8) Design Summary

Core:

Lamination: scrapless EI with center leg width of 3/4 inch,

Steel: Hot-rolled silicon, AISI-M-22 grade, .025 inch thick,

Stack: 1-1/16 inches.

Tube: .03 inch thick, 3/4 x 1-3/32 x 1-1/16 inch long.

Secondary winding (next to core):

Wire size: No. 38 AWG single-enamel copper wire,

Turns per layer: 175,

Layers: 20,

Turns: 3500, tap at 1750 turns,

Layer insulation: .002 inch paper,

Wrapper: .02 inch paper.

Primary winding:

Wire size: No. 25 AWG single-enamel copper wire,

Turns per layer: 37,

Layers: 6,

Turns: 222, tap at 111 turns,

Layer insulation: .002 inch paper,

Wrapper: .02 inch paper.

9) Check of Winding Resistances

Mean length of turn equals length of inside turn, plus pi times build-up of winding.

Resistance equals ohms per inch, times mean length of turn, times turns, times correction to operating temperature.

Secondary:

$$m_{cs} = 2(.75 + .060) + 2(1.094 + .060) + \pi\,(.188) = 4.33 \text{ inches},$$

$$R_s = \frac{(649.6)(.93)}{(12000)(.679)}(4.33)(3500) = 1122 \text{ ohms}.$$

Primary:

$$m_{cp} = 2(.75 + .060) + 2(1.094 + .060 + 2\pi\,(.240) = 5.44 \text{ inches},$$

$$R_p = \frac{(32.37)(.93)}{(12000)(.679)}(5.44)(222) = 4.46 \text{ ohms}.$$

10) Calculation of Temperature Rise

Surface temperature rise:

$$\theta_{surf} = F_{surf}\frac{W_c + W_i}{(S_{case})(h_c + h_r)} = \frac{(1.1)(3.64 + 1.3)}{(42.6)(.0042 + .0062)} = 12.3°C,$$

$$W_c = I_s^2(R_s) + I_p^2(R_p) = (.0354)^2(1122) + (.708)^2(4.46) = 3.64 \text{ watts},$$

W_i = 1.3 watts,

S_{case} = 42.6 square inches (3.19 x 2.63 x 3.07 inches),

F_{surf} = 1.1 = form factor of surface from Table 11-9,

$$h_c = 3.75 \times 10^{-3} \frac{\Theta_{surf}^{.22}}{S_{case}^{.17}} p^{.44} = 3.75 \times 10^{-3} \frac{(30)^{.22}}{(42.6)^{.17}} = .0042,$$

h_r = (.0069)(.9) = .0062,

Θ_{surf} is assumed to be 30°C.

$$\Theta_{surf} = .1\ \Theta_{assumed} + .9\ \Theta_{calculated} = (.1)(30)+(.9)(12.3) = 14.1°C$$

Temperature difference across impregnant:

$$\Theta_{imp} = F_{imp} \frac{(W_c + W_i)(m)}{Sk} = \frac{(.175)(3.64 + 1.3)(.58)}{(31.3)(.015)} = 10.7°C,$$

F_{imp} = 1.75 = correlation factor,

$S = 1/2\ (S_{case} + S_c + S_i) = 1/2\ (42.6 + 6.12 + 13.8) = 31.3$ square inches,

S_c = 6.12 square inches,

S_i = 13.8 square inches,

k = .015 = thermal conductivity from Table 11-10,

$$m = \sqrt{\frac{S_{case}}{4\pi}} - \sqrt{\frac{S_c + S_i}{4\pi}} = \sqrt{\frac{42.6}{4\pi}} - \sqrt{\frac{6.12 + 13.8}{4\pi}} = .58 \text{ inches.}$$

Coil hot-spot temperature

$$\Theta_h = F\ W_c^{\ x} \left(\frac{m}{k\ S_c}\right)^y = (.32)(3.64)^{1.0}\left(\frac{.188}{.0122 \times 6.12}\right)^{2.0} = 7.4°C,$$

$$m = 1/2(.375) = .188 \text{ inch},$$

$$k = k_1\left(\frac{R + 1}{1 + .11 R}\right) = .033 \ \frac{6.6}{1.62} = .0122,$$

$$R = \frac{.0179}{.0032} = 5.6$$

Average winding temperatures:

Primary: $T_{pri} = T_{amb} + \Theta_{surf} + \Theta_{imp} + .65\ \Theta_h$

$$= 65 + 14.1 + 10.7 + (.65)(7.4) = 94.6°C (\Delta T = 29.6°C)$$

Secondary: $T_{sec} = T_{amb} + \Theta_{surf} + \Theta_{imp} + .9\ \Theta_h$

$$= 65 + 14.1 + 10.7 + (.9)(7.4) = 96.5°C (\Delta T = 31.5°C).$$

XXI. DESIGN PROCEDURE: LOW-CAPACITANCE TRANSFORMERS

1) List Specifications

Frequency, voltages, secondary currents, capacitance from secondary to primary and core, secondary working voltage, temperatures (ambient and maximum rise), grade of protection.

2) Chosen Quantities

Type of core, grade and thickness of lamination or strip steel, core space factor, type of construction.

3) Nomograph Values

Secondary Rating:

$W_r = V_s I_s$ volt-amperes,

where V_s = secondary RMS volts,

I_s = secondary RMS amperes.

Allowable secondary winding dissipation:

$$\frac{W_{cs}}{S_{cs}} = \left(\frac{\Delta T}{K}\right)^{1.25} \text{ watts per sq. in.,} \qquad (9\text{-}14)$$

where W_{cs} = secondary losses, watts,

S_{cs} = secondary exposed surface area, sq. in.,

ΔT = maximum permissible temperature rise, °C,

K = parameter from Table 21-1.

Equivalent rating (based on 60 cycles and 40°C rise):

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76}\left(\frac{\Delta T}{40}\right)^{.63}} \text{ volt amperes} \qquad (2\text{-}21)$$

where f = frequency, cps,

ΔT = maximum temperature rise, °C

Rating and capacitance function:

$$\frac{k_c W_r'^{\,2/7}}{C} \text{ or } \frac{k_c W_r'^{\,.286}}{C}$$

where W_r' = equivalent rating (60 cps, 40°C rise)

C = desired secondary capacitance

k_c = correction factor for secondary supports and any dielectric between secondary and core.

Winding space factor:

F_c is found from Fig. 21-1.

Highest of two values for F_c is preferable.

Geometric factor:

$$K_0 = \frac{.22}{F_c^{\,.6}} \qquad (9\text{-}15)$$

Nomograph scale factors:

Find $\frac{K_0 W_r}{F_i f}$ and $\frac{F_c W_{cs}}{\rho S_{cs}}$

where W_r = secondary rating, volt amperes,

F_i = core space factor (usually given by manufacturer),

f = frequency, cps,

ρ = resistivity of conductor material, microhm-inches (For copper wire, increase the standard for 100 per cent conductivity by about two per cent).

Flux density: Choose B in kilolines per sq. in.

Characteristic linear dimension: Find ℓ from nomograph.

Area product: Calculate $A_c A_i = \ell^4$, (2-6)

where A_c = area of core window,

A_i = gross cross-sectional area of core.

4) **Core Properties:**

Approximate core cross-sectional area:

$$A_i = 1.6\ell^2 \sqrt{F_c} \quad \text{sq. in.} \tag{9-24}$$

where ℓ = characteristic linear dimensions,

F_c = winding space factor.

Approximate core window area:

$$A_c = \frac{A_c A_i}{A_i} \quad \text{or}$$

$$A_c = \frac{.63\ell^2}{\sqrt{F_c}} \ . \tag{9-25}$$

Select a lamination and core dimensions:

Lamination thickness should be suitable for frequency. Assembled core should have approximately the calculated A_c and A_i. If it is suspected that working voltage is too high for the core size, a rough check may be made by considering that assembled primary will occupy about 140 times F_c per cent of the window, and that secondary will occupy about 100 times F_c per cent of the window. If clearances are inadequate, a smaller F_c should be chosen to find a larger ℓ.

Core weight:

This can be found using data of manufacturer, including space factor, or from

$$M_i = m_i A_i \delta_i F_i \text{ lbs.,} \tag{2-22}$$

where m_i = mean length of magnetic circuit, in.,

A_i = core cross-sectional area, sq. in.

δ_i = core material density, lbs. per cu. in.,

F_i = core space factor.

Excitation (W_{ex}) and core loss (W_i):

Volt amperes and watts respectively are each calculated as the Epstein values per pound (functions of density, B) times correction factors (to account for increases over Epstein due to joints and other factors) times core weight in pounds.

5) __Winding Calculations__

Estimate of Winding Losses:

$$W_c = 22\delta^2 \frac{W_{cs}}{S_{cs}} \text{ watts.} \qquad (9\text{-}26)$$

Circular Mils per Ampere:

$$\text{Find } \sqrt{\frac{\delta}{\frac{F_c W_{cs}}{\rho S_{cs}}}} (1000\ K_o\ F_c).$$

Primary Current (For resistance load):

$$I_p = \frac{1}{V_p} \sqrt{(W_r + W_c + W_i)^2 + (W_{ex} + I_s'^2 X)^2} \text{ amperes,}$$

where V_p = primary volts,

W_r = secondary rating, volt amperes,

W_c = estimate of winding losses, watts,

W_i = core losses, watts,

W_{ex} = excitation, volt-amperes,

$I_s'^2 X$ = leakage-reactance volt-amperes, estimated as 10% of W_r for concentric windings.

Wire Sizes:

Calculate circular mils cross section for each winding: equal circular mils per ampere times amperes. Then select the standard wire sizes having areas closest to the results.

Turns per Volt:

$$\frac{N}{V} = \frac{10^5}{4.44\ f\ F_i\ A_i\ B} \qquad (2\text{-}33)$$

where f = frequency, cps

F_i = core space factor

A_i = gross core cross-sectional area, sq. in.

B = flux density, kl. per sq. in.

Winding Turns:

Primary: $N_p = \frac{N}{V} \times V_p$ turns

Secondary: $N_s = \frac{N}{V} \times V_s \left(1 + \frac{W_c}{W_r}\right)$, turns,

where the term in parenthesis corrects for resistance drop.

6) Winding Layout:

Primary: Find winding axial length, equal window length minus margins. Find permissible turns per layer, from winding length and permissible turns per linear inch. Then calculate number of layers. Choose a tube, layer insulation, and wrapper, and calculate radial build.

Secondary: Choose number of layers, turns per layer, tube size and insulation such that secondary is centered in remaining space, preferably so that it is about equidistant from core and primary. The secondary tube may be made round if tube diameter must be large compared to primary size. Otherwise a square shape with rounded corners is necessary to obtain equidistant spacing.

7) Check of Secondary Insulation

Secondary test voltage may be a little over half of predicted breakdown voltage. For a creepage path, breakdown voltage may be estimated as:

$$KV = 18t^{.7} \text{ kilovolts RMS}$$

where t = length of path, inches.

For breakdown by strike through air, a relation for typical irregular electrode shapes is:

$$KV = 28t^{.7} \text{ kilovolts RMS.}$$

Permissible working (peak) voltage for a straight creepage path equal to secondary spacing is found by calculating breakdown and choosing a test voltage. Working voltage is then:

$$KV_W = \frac{KV_T - 1}{\sqrt{2}} \text{ kilovolts (peak),} \qquad (9\text{-}27)$$

where KV_T = test kilovolts, RMS.

(This relation is the same as the common: RMS test volts = 2 times RMS rated volts plus 1000.) If permissible working voltage is not high enough, a new design using a larger F_c may be made, or it may be possible to raise breakdown by increasing creepage paths in the same design. In the latter case the equation for breakdown by strike may be applicable.

8) Winding Resistances and Losses:

Find mean length of turn of each winding, equal to the length of the inside turn plus pi times the radial build of that winding. Resistance of each winding is length times resistance per unit length times a correction for operating temperature.

Losses of each winding are current squared times resistance.

The sum is W_c.

9) Capacitance (For concentric windings, from secondary to primary and core):

$$C = \frac{1.35\, k_c\, m_{cs}}{\ln \frac{P_1}{P_2}} \text{ micro-microfarads,} \qquad (9\text{-}2)$$

where m_{cs} = mean length of secondary turn, inches,

k_c = correction factor for secondary supports and any dielectric between secondary and core. (Typical values are 1.2 to 1.5.)

P_1 = perimeter of remaining window space with primary in place, inches,

P_2 = perimeter of secondary cross section, around wire only, inches.

10) Leakage Reactance (Concentric Windings):

$$X = \frac{5}{10^8} fN^2 \frac{m_{cs}\, G}{h} \text{ ohms,} \qquad (9\text{-}4)$$

f

where f = frequency, cycles per sec.,

N = turns of winding to which X is referred,

G = effective separation from secondary to primary, equal to actual separation of closest wires plus one-third sum of radial builds, inches,

h = axial length of secondary, equal turns per layer times wire diameter, inches.

11) <u>Transformer Impedance and Load Resistance:</u>

Turns Ratio:

$$n = \frac{N_p}{N_s}$$

Load resistance, referred to primary:

$$R_L' = n^2 \frac{V_s}{I_s} \text{ ohms,}$$

where V_s = load volts,

I_s = load current, amperes.

X referred to primary and R_L' may be compared to check the accuracy of the assumption that $I_s'^2 X$ is 10 per cent of $W_r = I_s'^2 R_L$. Equivalent transformer resistance, referred to primary, is

$$R = R_p + n^2 R_s \text{ ohms,}$$

where R_p = primary resistance, ohms,

R_s = secondary resistance, ohms.

Transformer impedance (referred to primary):

$$Z = \sqrt{R^2 + X^2} \text{ ohms.}$$

Z and R_L' should be compared to see that Z is less than R_L'. Otherwise power output could be increased and temperature rise decreased by increasing load resistance. This indicates that wire sizes should be increased and turns ratio (as defined) should be increased.

12) Check of Voltage Ratio:

Primary voltage (needed to give the specified output):

$$V_p = \sqrt{\left(nV_s + \frac{I_s}{n} R_p + n\, I_s R_s\right)^2 + \left(\frac{I_s X}{n}\right)^2} \quad \text{volts} \qquad (9\text{-}6)$$

where I_p = primary current, amperes,

I_s = secondary current, amperes

V_s = secondary or load volts,

X = leakage reactance referred to primary.

If calculated V_p is not sufficiently close to required V_p, secondary turns should be changed. This changes n, but has practically no effect on X. The term nI_sR_s is unchanged. Therefore, it is not necessary to recalculate impedances if change of n is small.

13) Check of Secondary Dissipation:

Approximate Secondary Surface Area:

$$S_{cs} = 1.5\, m_{cs}\, P_2 \text{ sq. in.,} \qquad (9\text{-}29)$$

where m_{cs} = mean length of secondary turn, inches,

P_2 = perimeter of secondary cross section, around wire only, inches.

Secondary Dissipation is $\frac{W_{cs}}{S_{cs}}$,

where W_{cs} is calculated secondary loss, watts.

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If this is too much higher than the preliminary value, wire sizes should be increased and number of secondary turns reduced. If it is too much lower, wire sizes can be decreased and secondary turns increased. In the latter case, the gain in weight may be too small to warrant redesign, particularly if transformer series impedance is an appreciable fraction of load resistance.

14) <u>Design Summary</u>:

List core dimensions, lamination thickness, steel grade, winding tubes, wire sizes, wire insulation, total turns, location of taps, layers, turns per layer, layer insulation, and wrappers.

TABLE 21-1 TEMPERATURE-RISE PARAMETER OF LOW-CAPACITANCE TRANSFORMER (OPEN-TYPE CONSTRUCTION)

Ambient Temperature, °C	K
25	130
50	124
65	120
75	117
85	114
115	108
125	106
200	90

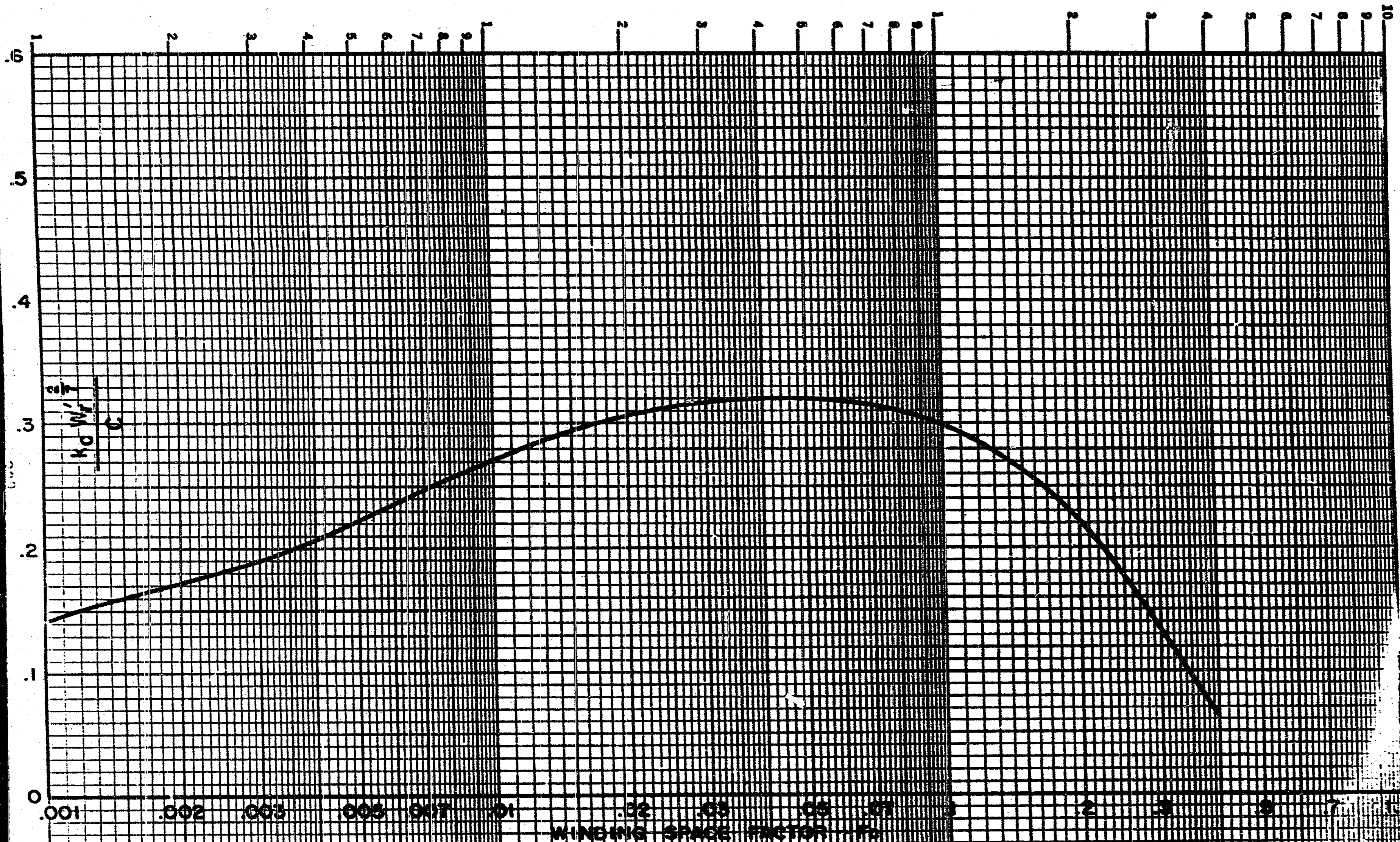

FIG. 21-1 CAPACITANCE, RATING, AND SPACE FACTOR FUNCTION.

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XXII. EXAMPLE: DESIGN OF LOW-CAPACITANCE TRANSFORMER

1) Specifications:

Frequency: 400 cps.

Primary: 115 volts.

Secondary: 10 volts, 5 amperes.

Capacitance from secondary to primary and core: 9 micro-microfarads.

Secondary working voltage: 2 kilovolts.

Ambient temperature: 85°C

Maximum rise: 115°C.

Grade of protection: Grade 2(less resistant to adverse environmental conditions).

2) Chosen Quantities:

Core: Laminations will be selected to yield an approximately square core cross-section and a square window.

Core steel: 14 mil non-oriented silicon steel AISI-M-19 grade. (This thickness and grade were chosen only because of availability.)

Core space factor: .88

Construction: Open core and coils, primary and secondary to be placed around the same core leg.

3) Nomograph Values:

Secondary rating:

$$W_r = V_s I_s = (10)(5) = \text{volt-amperes.}$$

Allowable secondary winding dissipation:

$$\frac{W_{cs}}{S_{cs}} = \left(\frac{\Delta T}{K}\right)^{1.25} = \left(\frac{115}{114}\right)^{1.25} = 1.01 \text{ watts per sq. in.}$$

ΔT = 115 = maximum temperature rise in °C,

K = 114 = constant from Table 21-1.

Equivalent rating (based on 60 cycles and 40°C rise):

$$W_r' = \frac{W_r}{\left(\frac{f}{60}\right)^{.76}\left(\frac{\Delta T}{40}\right)^{.63}} = \frac{50}{\left(\frac{400}{60}\right)^{.76}\left(\frac{115}{40}\right)^{.63}} = 6.05 \text{ volt-amperes,}$$

$f = 400$ = frequency in cycles per second.

Rating and capacitance function:

$$\frac{k_c W_r'^{2/7}}{C} = \frac{1.5\,(6.05)^{2/7}}{9} = .28$$

$k_c = 1.5$ = factor which accounts for high temperature material to be used for supports,

$C = 9$ = capacitance in micro-microfarads.

Winding space factor:

$F_c = .12$, from Fig. 21-1.

Geometric factor:

$$K_0 = \frac{.22}{(F_c)^{.6}} = \frac{.22}{(.12)^{.6}} = .787.$$

Nomograph scale factors:

$$\frac{K_0 W_r}{F_i f} = \frac{(.787)(50)}{(.88)(400)} = .112,$$

$$\frac{F_c W_{cs}}{\rho S_{cs}} = \frac{.12}{1.18}\, 1.01 = .103,$$

where ρ is taken as 1.16 from Fig. 11-6 increased 2 percent.

Flux density: Choose B = 65 kl. per sq. in.

Characteristic-linear dimension, from nomograph:

$\ell = .67$ inches.

Area product:

$$A_c\,A_i = \ell^4 = .67^4 = .201 \text{ in}^4.$$

4) <u>Core Properties:</u>

Cross-sectional area:

$$A_i = 1.6\ell^2 \sqrt{F_c} = 1.6 \times .67^2 \sqrt{.12}$$

$$= .248 \text{ sq. in.}$$

Window area:

$$A_c = \frac{A_c\,A_i}{A_i} = \frac{.201}{.248} = .81 \text{ sq. in.}$$

The nearest size in stock is the Allegheny-Ludlum L-4 lamination, which has width of 1/2", and yields a window of 3/4" x 1-1/2" = 1.125 sq. in. A square cross section may be used to make the stack 1/2". Although this gives an area product somewhat larger than calculated, the windings will be somewhat smaller.

Core weight: (Using manufacturer's data)

$$M_i = F_i \text{ times weight of solid square stack}$$

$$= .88\ (.442) = .388 \text{ lbs.}$$

Excitation in volt-amperes per pound is approximately proportional to frequency. The 60 cycle Epstein value at 65 kl. density is 2.4. Increasing by 2.0 because of joints and other such factors, applying a correction for frequency, and multiplying by weight gives:

$$W_{ex} = 2.4 \times 2.0 \times \frac{400}{60} \times .388 = 12.4 \text{ volt-amperes.}$$

Core loss in watts per lb. is approximately 13.0 (Epstein) at 400 cycles, and the correction is estimated at 1.2. Total core loss is then:

$$W_i = 13 \times 1.2 \times .388 = 6.1 \text{ watts.}$$

5) <u>Winding Calculations:</u>

Estimate of winding losses

$$W_c = 22\,\ell^2 \frac{W_{cs}}{S_{cs}} = 22\,(.67)^2\,1.01 = 10 \text{ watts.}$$

Circular mils per ampere =

$$\sqrt{\frac{\ell}{\frac{F_c W_{cs}}{\rho S_{cs}}}}(1000\ K_0\ F_c) = \sqrt{\frac{.67}{.103}}\ (1000 \times .787 \times .12) = 241.$$

Primary current:

$$I_p = \frac{1}{V_p}\sqrt{(W_r + W_c + W_i)^2 + (W_{ex} + I'^{\,2}_s X)^2}$$

Estimate $I'^{\,2}_s X$ as 10% of W_r.

Then

$$I_p = \frac{1}{115}\sqrt{(50 + 10 + 6.1)^2 + (12.4 + 5.0)^2} = .595 \text{ amperes.}$$

Wire sizes:

Primary: .595 x 241 = 144 CM. Use No. 28 wire.

Secondary: 5 x 241 = 1205 CM. Use No. 19 wire.

Turns per volt:

$$\frac{N}{V} = \frac{10^5}{4.44\ f\ F_i\ A_i\ B} = \frac{10^5}{(4.44)(400)(.88)(.248)(65)} = 3.95.$$

Primary turns:

$$N_p = \frac{N}{V} \times V_p = 3.95\ (115) = 455.$$

Secondary turns:

$$N_s = \frac{N}{V} \times V_s\ (1 + \frac{W_c}{W_r})$$

$$= 3.95 \times 10 \times (1 + .2) = 47 \text{ turns.}$$

6) <u>Winding Layout</u>:

Primary:

Winding length = 1-1/2" minus 2 x (1/8")

= 1.25"

Turns per layer = 1.25 x 67
= 84, use 76.

Layers = $\frac{455}{84}$ = 5.42, use 6 layers.

Primary build:

	inches
Tube	.025
Wire: 6 x .0136"	.082
Insulation: 5 x .0015	.007
Wrapper	.010
Total	.125 in.

Secondary: (To be concentric with primary)

The space remaining, after the primary is inserted, is .625" x 1.5". To obtain a secondary cross section of roughly similar proportions, use 3 layers and 16 turns per layer.

Winding length is 16 x .0374 = .60".

Use tube length = .73".

Secondary build:

	inches
Tube	.040
Wire 3 x .0374"	.112
Insulation 2 x .007"	.014
Wrapper	.010
Total	.176

The secondary tube may be made in the shape of a square with rounded corners. The flat portions should be at least .5" long in the direction of the wire. Inside clearance between flat sides, so that the secondary is properly centered in the window, should be

.5" + 2(.125") + 2(.350") = 1.45"

7) Check of Secondary Insulation:

Minimum separation from secondary to the primary or core is about .35 inches. If straight secondary supports are used, having a creepage path only this long, breakdown voltage would be approximately

$$KV = 18t^{.7} = 18\,(.35)^{.7}$$

$$= 8.65 \text{ kilovolts.}$$

For a safety factor of almost two, test voltage can be 4.4 kilovolts. Permissible working voltage is therefore about

$$KV_w = \frac{KV_T - 1}{\sqrt{2}} = \frac{4.4 - 1}{\sqrt{2}} = 2.4 \text{ kilovolts.}$$

Thus the straight supports are adequate for the required working voltage of 2KV.

8) Winding Resistances and Losses:

Primary: (No. 28, 455 turns)

Mean length:

$$m_{cp} = 4(.5) + \pi\,(.125) = 2.39"$$

Resistance at 200°C:

$$R_p = \text{length} \times (\text{ohms}/1000') \times \text{temp. correction}$$

$$= \frac{2.39 \times 455}{12{,}000} \times 64.9 \times \frac{1.16}{.679} = 9.83 \text{ ohms.}$$

Secondary: (No. 19, 47 turns)

Mean length:

$$m_{cs} = 4(.5) + \pi\,(1.45 - .5) + \pi\,(.176) = 5.54"$$

Resistance: (Add length of two turns for leads)

$$R_s = \frac{5.54 \times 49}{12{,}000} \times 8.05 \times \frac{1.16}{.679} = .311 \text{ ohms.}$$

Winding losses:

$$W_c = .595^2 \times 9.83 + 5^2 \times .311$$

$$= 3.49 + 7.77 = 11.26 \text{ watts.}$$

9) **Capacitance:**

$$C = \frac{1.35\ k_c\ m_{cs}}{\ln \frac{P_1}{P_2}}$$

$$= \frac{1.35 \times 1.5 \times 5.54}{\ln \frac{2(.625 + 1.5)}{2(.60 + .126)}}$$

(.126 is build of secondary wire and interlayer insulation)

C = 10.4 micro-microfarads.

10) **Leakage Reactance** (referred to primary, for concentric windings):

$$X = \frac{5}{10^8} f N^2 \frac{m_{cs}\ G}{h} \text{ ohms}$$

$$= \frac{5}{10^8} 400 (455)^2 \frac{5.54 \times \left(\frac{.125 + .126}{3} + .40 \right)}{.60}$$

(.40 = approx. separation between primary and secondary)

X = 18.4 ohms.

11) **Transformer Impedance and Load Resistance:**

Since the turns ratio is tentatively

$$n = \frac{N_p}{N_s} = \frac{455}{47} = 9.68,$$

the nominal load resistance referred to the primary is

$$R_L' = n^2 \frac{V_s}{I_s} = 9.68^2 \frac{10}{5} = 187 \text{ ohms.}$$

This shows that assuming $I_s'^2$ to be 10 per cent of W_r was good because X is about 10 per cent of R_L'.

At this point a check may also be made to see that transformer series impedance is less than load resistance. Equivalent transformer resistance referred to the primary is

$$R = R_p + n^2 R_s = 9.83 + 9.68^2 (.311)$$
$$= 38.9 \text{ ohms}$$

Transformer impedance referred to the primary is:

$$Z = \sqrt{R^2 + X^2} = \sqrt{38.9^2 + 18.4^2}$$

$$= 43.1 \text{ ohms.}$$

Since Z is less than R_L', the transformer is not operating in the undesirable region where watt output could be increased with a lower temperature rise.

12) Check of Voltage Ratio:

Calculate primary voltage:

$$V_p = \sqrt{(n\,V_s + I_p\,R_p + n\,I_s\,R_s)^2 + (\frac{I_s X}{n})^2}$$

$$= \sqrt{(9.68 \times 10 + .595 \times 9.83 + 9.68 \times 5 \times .311)^2 + (\frac{5 \times 18.4}{9.68})^2}$$

$$= \sqrt{117.8^2 + 9.5^2} = 118 \text{ volts}$$

This is the primary voltage needed to yield a secondary voltage of 10 under the specified conditions. However, since the temperature rise factor K is intentionally conservative, the rise will probably be somewhat less than maximum, and resistances will then be less than those above. This yields a primary voltage closer to the nominal 115.

13) Secondary Dissipation

Exposed secondary surface area is approximately

$$S_{cs} = 1.5\, m_{cs} P_2$$

$$= 1.5 \times 5.54\ (2 \times .726) = 12.0 \text{ sq. in.}$$

Therefore

$$\frac{W_{cs}}{S_{cs}} = \frac{7.77}{12.0} = .65 \text{ watts per sq. in.}$$

This is considerably lower than the preliminary value. One reason is that wire sizes are slightly larger than the calculated values. Another reason is that the core is larger than calculated, making the winding smaller. Since the calculation of current density did not account for this, secondary losses were reduced more than surface area. In some cases it would be desirable to modify the design to obtain a higher temperature rise.

14) **Design Summary:**

Note: Use materials suitable for 200°C operation

Core:

Laminations: Type L-4, 1/2" wide

Window: 3/4" x 1-1/2"

Stack: 1/2"

Space factor: .88

Steel: AISI-M-19 grade, .014" thick (29 gage)

Primary: (115 volt)

Wire size: No. 28 AWG, single enamel

Turns: 455

Layers: 6

Turns/layer: 76

Tube: .025" x .5" x .5" x 1.5" long

Layer insulation: .003"

Wrapper: .009"

Secondary: (10 volts, 5 amperes)

Mount concentric with primary

Wire size: No. 19 AWG, single enamel

Turns: 47

Layers: 3

Turns/layer: 16

Tube: .040" thick, .73" long; 1.45" square (inside dim.) with rounded corners, flat portions of sides are 1/2".

Layer insulation: .009"

Wrapper: .010"

XXIII. CONCLUSIONS

1. The basic design procedure which was developed under Contract No. DA-36-039 SC-5519 has been extended to special types of transformers, including transformers with unbalanced magnetization, current-limiting transformers, current-limiting transformers with unbalanced magnetization, vibrator-supply transformers, low-capacitance transformers, and instrument transformers. It is possible to design these types of transformers with very little trial procedure, and the methods given should be understandable to an engineer not normally associated with the transformer industry. The design method accounts for operating temperatures to 200°C, ambient temperatures to 200°C, pressures between 30 inches and 1.32 inches of mercury, power ratings to 5 kilovolt amperes, RMS voltages to 50 kilovolts, and frequencies between 25 and 2500 cycles per second.

2. The design of transformers with unbalanced magnetization requires suitable data on magnetic materials under unbalanced conditions and suitable relations among the electrical circuit quantities. Data have been compiled to give core loss, excitation and non-magnetic gap as functions of AC flux density and DC or average magnetization. It has been found that the desired non-magnetic gap (if any) should be based upon the conditions which give minimum excitation current. It is possible to compute secondary voltage and current from circuit constants and with the aid of published data. Primary current is computed by approximate equations which include load current, losses, and excitation as terms. The design of transformers with unbalanced magnetization may be accomplished using the previously-developed design procedure with a few modifications.

3. Current-limiting transformers require the calculation of proper turns, turns ratio, and magnetic shunt. Relations have been obtained among primary and secondary flux densities, voltages and currents such that a design may be made in a straightforward manner. These relations account for the required ratio of short-circuit to load current and the change in leakage reactance between normal load and short-circuit operating conditions. It is necessary that the turns ratio be correct; otherwise it is not possible to select a shunt which will yield proper circuit characteristics. Shunt gaps are so small and so critical that manufacturers will probably be unable to eliminate production tests in order to make sure that proper output is obtained. The design procedure for current-limiting transformers with unbalanced magnetization combines principles of the two individual types.

4. Vibrator-supply transformers are designed in a manner similar to that for the more common filament or plate transformers, but special consideration must be given to insulation problems, the timing capacitance, and to the effects of the vibrator on transformer operation. A proper timing capacitance is necessary in order to give a satisfactory voltage wave shape and to prevent extremely high induced voltages in the transformer windings. It is necessary to design these transformers using comparatively low flux densities because of the large supply-voltage variations which are frequently encountered and because of excessive currents which would occur if the vibrator causes a circuit unbalance during starting or normal operation. It is common practice to place the primary winding over the secondary in order to have the higher

primary resistance than would be obtained with the primary next to the core. This aids in keeping starting currents low. However, if primary resistance is increased by reduction in wire size over the values that would normally be used, the design would tend to become uneconomical because of unequal current densities in the different windings. Vibrator supplies operating from a source over about 15 volts must be designed with special care because contact arcing may be excessive. Series resistances are sometimes incorporated into the primary circuit to provide improved starting characteristics.

5. Low-capacitance transformers are required when it is necessary to supply a load which must have a low-capacitance path to the power supply. These transformers are often used to supply a low voltage difference to a filament circuit which has a high voltage to ground. Empirical equations have been derived from measurements on models and from theoretical studies to establish important relations among power rating, space-factor, and desired capacitance. It is found that for a given rating there is a minimum value of capacitance which can be obtained for any spacing of the secondary winding. In low-frequency units, leakage reactance has only a minor effect on the value of voltage regulation. It is important that voltage drops due to winding resistances be accounted for in order to obtain the required voltage ratio. Leakage reactance will become more important at frequencies over 60 cycles, but in all cases it should be computed and used in the design equations.

6. Instrument transformers for measurement of voltage or current may be designed using the basic design procedure. It is necessary that the burden be used as the power rating. Current transformers must be designed with a very low flux density in order that they may provide a reasonable ratio of load to instrument currents for load currents above the normal rating of the circuit. When the design of an instrument transformer is completed it may be checked to determine whether ratio and phase angle errors are within the limits required for the particular design.

7. An analysis has been made to determine how winding current densities should be selected, that is, whether different densities should be used for inner and outer windings. It has been found that minimum losses would be obtained if current densities were somewhat higher in the inside windings. However, if total densities were constant, then temperature rise would be minimized for higher densities in the outside windings. But the assumption of constant losses and higher densities in the outside windings would result in a lower power rating. Therefore it is recommended that current densities be uniform, as a compromise for reasonable losses and heating.

8. A study of optimum core portions which was carried out during the previous contract has been extended. The recent work gives optimum proportions for certain laminations. The proportions of these specific laminations represent restrictions which make it impossible to obtain the over-all optimum core proportions, but it is possible to obtain certain most favorable proportions for each lamination. Results are given in the form of optimum core-stack ratios. These ratios are functions of the relative costs of the core and winding per unit volume where "cost" may be taken as weight, volume, losses, or manufacturing expense.

9. Test results and calculated results have been compared for developmental models constructed during the course of the contract. Important comparisons are those between test and calculated operating temperatures. It is important that the maximum be approached as closely as practicable, but not exceeded. Actual temperatures of a given design can be expected to vary somewhat with manufacturing practice. It has been the intention to establish design parameters such that the maximum temperature rises are used in the calculations, and such that the resulting designs will have temperature rises ranging from 75 to 100 per cent of the maximum.

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XXIV. RECOMMENDATIONS

It is recommended that the design method be applied by manufacturing concerns and by government agencies to the types and ranges of transformers which have been analyzed. When a designer has gained experience in the use of these methods he should be able to devise short-cuts in the selection of design parameters and to omit some of the calculations. When a manufacturer has gained experience by production and evaluation of large quantities of transformers, it will probably be found that more accurate parameters can be specified in some cases. This is particularly true where the parameters depend on manufacturing practice and upon choice of materials, variations which it has been impossible to account for in a study of this kind.

It is believed that the basic philosophy used in the development of the design procedures could be applied to other electrical apparatus with advantage. Examples might be inductances, relay coils, and rotating machinery.

XXV. LOGBOOKS

The data obtained on this project are recorded in the following Armour Research Foundation logbooks: C-3280, C-3296, C-3598, C-3723, C-3858, and C-4296.

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APPENDIX A - CONDITIONS FOR MINIMUM TOTAL WINDING LOSSES

Consider first a two winding transformer; then the results can be extended to any number of windings. Winding losses are the sum of current density squared times resistivity times conductor volume for the windings.

$$W_c = \Delta_p^2 \rho \, m_{cp} A_{cp} F_{cp} + \Delta_s^2 \rho \, m_{cs} A_{cs} F_{cs} \text{ watts,} \qquad \text{(A-1)}$$

where Δ = current density of the winding denoted by subscript, kiloamperes per square inch,

ρ = conductor resistivity, microhm-inches,

m_c = mean length of winding denoted by second subscript, inches,

A_c = window space occupied by winding denoted by subscript, including its insulation and clearances, square inches,

F_c = space factor of winding denoted by subscript.

For each winding, ampere turns must be constant, and are equal to

$$N_p I_p = \Delta_p A_{cp} F_{cp} \text{ ampere turns,} \qquad \text{(A-2)}$$

$$N_s I_s = \Delta_s A_{cs} F_{cs} \text{ ampere turns.}$$

Substituting for Δ_p and Δ_s from (A-2) into (A-1) gives

$$W_c = \frac{(N_p I_p)^2}{A_{cp} F_{cp}} \rho \, m_{cp} + \frac{(N_s I_s)^2}{A_{cs} F_{cs}} \rho \, m_{cs} \text{ watts.} \qquad \text{(A-3)}$$

But since the total space available for windings is constant,

$$A_{cp} + A_{cs} = C, \text{ a constant.} \qquad \text{(A-4)}$$

Substituting for A_{cs} in (A-3) according to (A-4), differentiating with respect to A_{cp}, then removing the constant C gives

APPENDIX A - CONDITIONS FOR MINIMUM TOTAL WINDING LOSSES

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where Δ = current density of the winding denoted by subscript, kiloamperes per square inch,

ρ = conductor resistivity, microhm-inches,

m_c = mean length of winding denoted by second subscript, inches,

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F_c = space factor of winding denoted by subscript.

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Substituting for Δ_p and Δ_s from (A-2) into (A-1) gives

$$W_c = \frac{(N_p I_p)^2}{A_{cp} F_{cp}} \rho \, m_{cp} + \frac{(N_s I_s)^2}{A_{cs} F_{cs}} \rho \, m_{cs} \text{ watts.} \qquad \text{(A-3)}$$

But since the total space available for windings is constant,

$$A_{cp} + A_{cs} = C, \text{ a constant.} \qquad \text{(A-4)}$$

Substituting for A_{cs} in (A-3) according to (A-4), differentiating with respect to A_{cp}, then removing the constant C gives

$$\frac{1}{\rho} \frac{d W_c}{d A_{cp}} = - \frac{(N_p I_p)^2 m_{cp}}{F_{cp} A_{cp}^2} + \frac{(N_s I_s)^2 m_{cs}}{F_{cs} A_{cs}^2}. \qquad (A-5)$$

For minimum loss, this is zero. Equations (A-2) can be used again to give

$$-F_{cp} m_{cp} \Delta_p^2 + F_{cs} m_{cs} \Delta_s^2 = 0,$$

or

$$\frac{\Delta_p}{\Delta_s} = \sqrt{\frac{F_{cs} m_{cs}}{F_{cp} m_{cp}}} \qquad (A-6)$$

This is the relation for two windings. The equation can be extended immediately to any number of windings. The criterion must be satisfied by any pair of windings in the space occupied by the pair.

APPENDIX B-OPTIMUM CURRENT DENSITY DISTRIBUTION IN A PLANE

The distribution of heat sources is to be found which gives minimum total temperature rise from the center of a plane of thickness $2x_0$ to the outside ambient. The heat sources are assumed to be currents (flowing parallel to the surface), of density which varies linearly from the center to the surface. For a medium of constant resistivity, a linear distribution of current yields a parabolic distribution of heat sources. An exception is the case of a constant current density, which yields a constant distribution of heat sources.

To simulate a transformer coil of fixed load, total current through the plane must be kept constant, although distribution is changed. Therefore minimum total losses in the plane are obtained when density is constant, as can be seen from Appendix A, for the case of equal path lengths. A function of density which satisfies the requirements is

$$\Delta = K_0 + K_1 \left(x - \frac{x_0}{2}\right), \tag{B-1}$$

where Δ = density, assumed symmetrical about center of plane,

K_0 = a constant, equal to the average value of density,

K_1 = a parameter to change distribution of density,

x = distance from center of plane.

The basic equation of Poisson for one-dimensional heat flow in a medium containing heat sources is

$$\frac{d^2 T}{dx^2} = - \frac{W}{k}$$

where T = temperature, degrees C,

W = loss per unit volume, watts per cubic inch,

k = thermal conductivity, watts/inch -°C.

Since loss per unit volume is proportional to density squared, Poisson's equation becomes

$$\frac{d^2 T}{dx^2} = - \frac{C\Delta^2}{k} \tag{B-2}$$

where C is a constant of proportionality.

Substituting the value of density from (B-1) into (B-2), integrating twice and using the fact that $dT/dx = 0$ at the center ($x=0$), gives

$$T_1 - T_0 = \frac{C\,x_0^2}{2\,k}\left(K_0^2 - \frac{K_0\,K_1\,x_0}{3} + \frac{K_1^2\,x_0^2}{12}\right) \qquad \text{(B-3)}$$

where T_1 = temperature at the center of the plane,

T_0 = temperature at surface.

To obtain results without unreasonable complication requires a fairly simple relation between total losses and surface rise. The one chosen is

$$\Delta T = KW_t$$

where ΔT = surface rise, degrees C,

K = surface rise parameter,

W_t = watts per square inch transferred from the surface.

Total loss in a square inch cross section from the center of the plane to the surface is

$$W_t = \int_0^{x_0} W\,dx = C\int_0^{x_0} \Delta^2\,dx \qquad \text{(B-5)}$$

Substituting for density and integrating gives

$$W_t = C\,x_0\left(K_0^2 + \frac{K_1^2\,x_0^2}{12}\right) \qquad \text{(B-6)}$$

The sum of surface rise and coil rise, from (B-3), (B-4) and (B-6), is

$$T_t = \Delta T + (T_1 - T_0)$$

$$= K\,C\,x_0\left(K_0^2 + \frac{K_1^2\,x_0^2}{12}\right) + \frac{Cx_0^2}{2\,k}\left(K_0^2 - \frac{K_0K_1x_0}{3} + \frac{K_1^2\,x_0^2}{12}\right) \qquad \text{(B-7)}$$

Differentiating (B-7) with respect to the parameter K_1 (which varies density distribution), and setting the result equal to zero gives the condition for minimum total rise

$$K_1 = \frac{K_0}{K\,k + \frac{x_0}{2}} \qquad \text{(B-8)}$$

Since K_1 is the slope of the density function (B-1), a very small value for K_1 indicates almost constant density, while a large value indicates a low density near the center increasing to a large value at the surface. Typical values for the thermal parameters are K = 100 and k = .01.

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APPENDIX C - NEW SCRAPLESS LAMINATION

The size of a transformer is a function of power rating. However, several different types of construction and many different proportions can be used to fulfill a given set of design requirements. For a transformer or inductor constructed with conductor material and a magnetic core two extremes in proportions are possible: a relatively large quantity of conductor material and a small quantity of core material might be used, or a large proportion of magnetic to conductor materials might be used. The compromise between these possibilities depends upon the relative importance of weight, volume, losses and cost. The rating of a transformer is approximately proportional to the product of core window area and core cross-sectional area. A relatively large window area compared to core section indicates that the volume of the winding structure, including conductor and insulation, is generally larger than the volume of core material. The converse holds when the ratio of window area to core cross section is small.

Conventional Scrapless Lamination

For reasons of economy, the scrapless EI lamination has been used for most single-phase, shell-type transformers requiring laminations small enough to be punched and handled readily. Fig. C-1 shows how two E's and two I's are cut from a section of the material without any waste. Fig. C-2 shows how one E and one I are laid to form a layer of a transformer core. The other layers of a core can either be placed the same way to form a butt joint, or alternate layers can be reversed to form a butt-lapped joint, in which the abutting edges in one layer are bridged over by another layer.

Since the proportions of the scrapless lamination shown in Fig. C-1 and C-2 are fixed, the proportions of a transformer core can be varied only by changing the height of the stack of laminations. Most common ratios of stack height to center leg width fall within the range from 1:1 to 2:1. Reasons for this are: (1) a winding is more easily wound on a square form than on a rectangle with two very much longer sides, and (2) a ratio somewhat larger than 1:1 is usually the most economical shape within the limitations imposed by the use of this lamination.

A survey of currently-available laminations reveals an interesting situation in that the conventional scrapless lamination has extreme or unusual proportions compared to special or non-scrapless laminations. These special laminations are used to a much lesser extent because of the wasted material. They are characterized by their greater window area for a given center-leg size than has the conventional scrapless. This indicates very significantly that another scrapless lamination having a larger window would meet a need in the industry.

The New Lamination

A search for new scrapless laminations led to the scheme shown in Figs. C-3 and C-4. Fig. C-3 shows the cutting pattern which yields two sets of E's and I's from a rectangular piece of material, and Fig. C-4 shows how one layer is formed for a single-phase, shell-type transformer core. If and when this lamination is produced, it should be very valuable for many transformer designs and applications. However it is intended to supplement, and not to replace the conventional lamination.

The new lamination offers a saving in weight over the conventional lamination for almost any design. This reduction is achieved because designs with the new lamination tend to have a higher proportion of winding volume to the core volume than those made with the conventional lamination. Although the density of copper is about 15 per cent greater than that of steel, typical transformer windings are practically always less than 30 per cent copper by volume, the rest being paper and impregnant. In high voltage designs typical copper volume is only a few per cent of winding volume. It is estimated that transformer weights can be reduced at least 25 per cent. This would be of great importance in military requirements. Direct manufacturing costs would also be reduced if the expense of the additional copper wire and insulation were more than compensated for by the reduced magnetic material.

The new lamination should be particularly well suited for high-voltage designs, which need adequate winding clearances and space for solid insulation. Required clearances with a core using the new lamination could be obtained only with a heavier or poorly-proportioned core using the conventional lamination.

Another desirable feature of the new lamination is that the dissymmetry of the E part can be used to advantage for reducing the no-load or excitation current of a transformer. In the conventional transformer, layers of laminations can only be stacked two different ways, whereas they can be stacked four different ways with the new lamination. This makes it possible to distribute in twice as many places the abutting lamination edges at the corner joints. This yields a better core because crowding of flux in the butt-bridging laminations is somewhat alleviated. However the two joints at the ends of the center leg are unchanged in cores made with the new lamination.

In small transformers the effect of core joints is to increase no-load current from about two to four times the values which would be obtainable if there were no core joints, depending on the length of the flux path. It is estimated that improvement of the corner joints in a shell-type core will give reductions of from 15 to 30 per cent in no-load current over the conventional scrapless lamination. However this feature need not be utilized since it is possible to stack lamination layers only two ways as before.

A disadvantage of the new lamination is that it would probably involve an increase in punching expenses over the conventional lamination since the latter is very readily produced with a progressive die. However, if material costs greatly outweigh punching costs, as appears to be the case, then a more complex punching operation is not a formidable obstacle to the production of the new lamination.

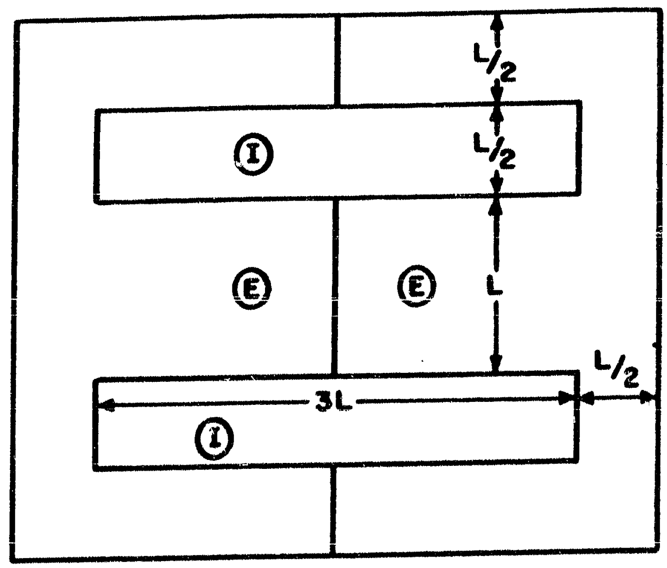

FIG. C-1 CONVENTIONAL SCRAPLESS EI LAMINATIONS AS CUT FROM SHEET MAGNETIC STEEL (TWO SETS)

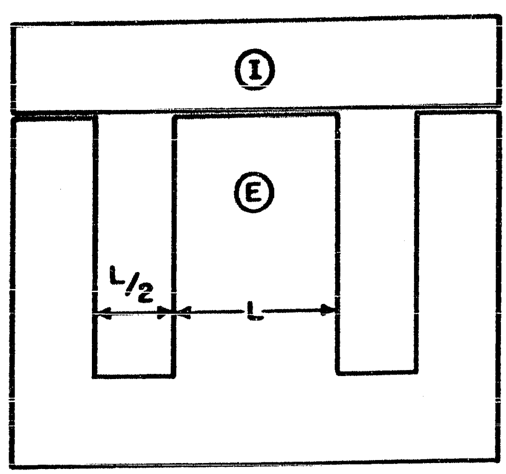

FIG. C-2 ASSEMBLY OF CONVENTIONAL SCRAPLESS LAMINATIONS (ONE LAYER)

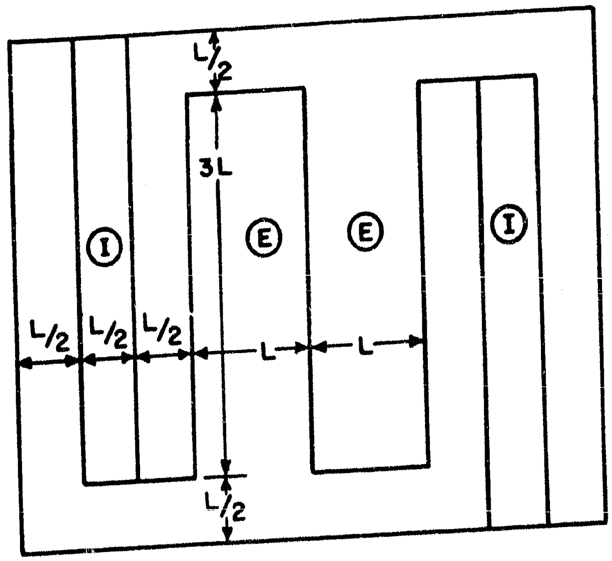

FIG. C-3 NEW SCRAPLESS EI LAMINATIONS AS CUT FROM SHEET MAGNETIC STEEL (TWO SETS)

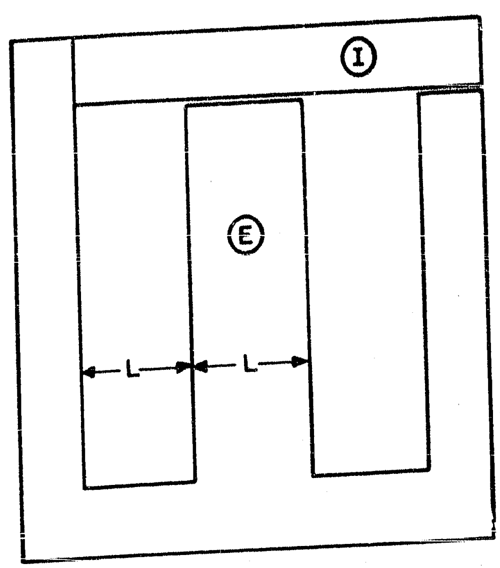

FIG. C-4 ASSEMBLY OF NEW SCRAPLESS LAMINATIONS (ONE LAYER)

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APPENDIX D: MODEL TRANSFORMER SPECIFICATIONS AND TEST RESULTS

Photographs of ten experimental transformers which were submitted as models are shown in Fig. D-1 and D-2. A circular secondary winding which may be substituted for the square secondary winding of one of the low-capacitance transformers is also shown. The purpose of constructing experimental transformers was to obtain empirical data and to verify the design procedures. Specifications and temperature data for ten experimental transformers which were selected to be submitted as models are presented on the following pages. In addition, temperature data on four of the examples in the final report of the previous contract (No. DA-36-039 SC-5519) are included.

FIG. D-1 PHOTOGRAPH OF CURRENT-LIMITING TRANSFORMERS AND TRANSFORMERS WITH UNBALANCED MAGNETIZATION

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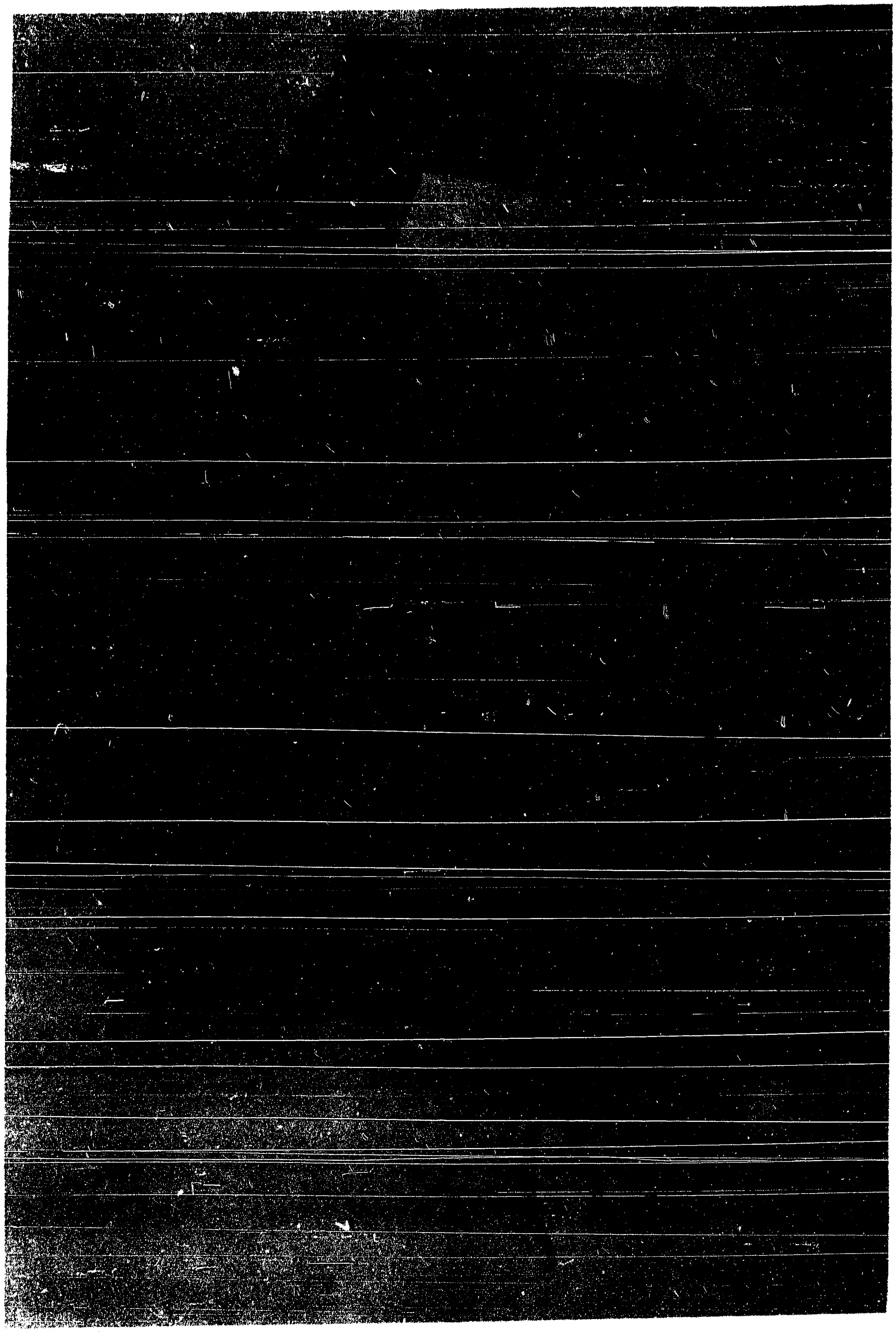

FIG. D-2 PHOTOGRAPH OF VIBRATOR-SUPPLY AND LOW-CAPACITANCE TRANSFORMERS

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K-12: Plate Transformer with Unbalanced Magnetization

Requirements:

Frequency: 400 cycles per second.

Ambient temperature: 85°C

Maximum temperature rise: 115°C

Primary: 115 volts.

Secondary: 560 volts RMS, 1.0 ampere RMS, 0.50 ampere DC, half-wave rectifier with capacitance-input filter.

Protection: Grade 2 (less resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with 1 inch center leg width.

Steel: 1-5/16 inches.

Construction: Butt joint.

Tube:

Dimensions: .030 inch thick, 1-1/64 x 1-5/16 x 1-7/16 inches long.

Material: Quinterra.

Primary winding (next to core, 115 volts):

Wire size: No. 17 AWG, teflon-coated wire,

Turns per layer: 23,

Layers: 3,

Turns: 69,

Layer insulation: .009 inch Quinterra,

Wrapper: .009 inch Quinterra.

Shield (connect to core):

Material: One layer of .002 inch thick copper sheet,

Wrapper: .009 inch Quinterra.

Secondary winding (560 volts RMS):

Wire size: No. 24 AWG, teflon-coated wire,

Turns per layer: 49,

Layers: 340,

Layer insulation: .006 inch Quinterra,

Wrapper: .012 inch Quinterra.

Measured Resistances (at 76°C)

Primary: 0.1975 ohms,

Secondary: 5.87 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 95°C,

Secondary: 92°C.

Measured average winding temperature rise:

Primary: 113°C,

Secondary: 96°C.

K-6: Plate Transformer with Unbalanced Magnetization

Requirements

Frequency: 60 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Primary (3 to 4, 5, or 6): 105/115/125 volts.

Secondary (1 to 2): 180 volts RMS, .11 amperes RMS, .055 amperes DC, half-wave rectifier with capacitance-input filter.

Protection: Grade I (most resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with 11/16 inch center leg width.

Steel: Non-oriented silicon, .014 inch thick, AISI M-15 grade.

Stack: 1-3/16 inches

Construction: Lap joint, laminate 2 x 2.

Tube:

Dimensions: .030 inch thick, 11/16 x 1-3/16 x 1-1/32 inch long,

Material: Paper

Secondary winding (next to core, 180 volts RMS):

Wire size: No. 33 AWG single enamel copper wire,

Turns per layer: 94,

Layers: 13,

Turns: 1214,

Layer insulation: .001 inch paper,

Wrapper: .010 inch paper.

Shield (over secondary, connect to core):

Material: One layer of .002 inch thick copper sheet,

Wrapper: .010 inch paper.

Primary winding (outside, 105/115/125 volts):

Wire size: No. 30 AWG plain enamel copper wire,

Turns per layer: 71,

Layers: 12.

Turns: 778, taps at 716 and 653,

Layer insulation: .0015 inch paper,

Wrapper: .010 inch paper.

Case:

Dimensions: 2.5625 x 2.125 x 3 inches.

Filling: Sand-loaded asphalt compound.

Measured Resistances (at 65°C)

Primary: 39.02 ohms (115 volt tap),

Secondary: 106.45 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 28°C,

Secondary: 29°C.

Measured average winding temperature rise:

Primary: 35°C,

Secondary: 39°C.

K-22: Current-Limiting-Filament Transformer

Requirements

Frequency: 60 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Primary (1 to 2, 3, or 4): 105/115/125 volts.

Secondary (5 to 6): 5.5 volts, 10 amperes, 13.5 amperes short-circuit current.

Protection: Grade I (most resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with center leg width of 1-1/4 inches.

Steel: Oriented silicon, .014 inch thick, AISI M-10 grade.

Stack: 1-3/8 inches.

Construction: Lap joint, laminate 2 x 2.

Tubes (primary and secondary):

Dimension: .040 inch thick, 1-1/4 x 1-7/16 x 11/16 inch long.

Material: Paper.

Primary winding (105/115/125 volts):

Wire size: No. 22 AWG single enamel copper wire.

Turns per layer: 17,

Layers: 17,

Turns: 280, taps at 258 and 235,

Layer insulation: .003 inch paper,

Wrapper: .010 inch paper.

Secondary winding (5.5 volts):

Wire size: No. 13 AWG double enamel copper wire,

Turns per layer: 5,

Layers: 6,

Turns: 30,

Layer insulation: .010 inch paper,

Wrapper: .010 inch paper.

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Magnetic shunts (two required):

Thickness of each shunt corresponds to 9 laminations, each .025 inch thick.

Length of each shunt (length of each lamination) = 1-3/8 inches.

Width of each shunt (width of each lamination) = .605 inch.

Case:

Dimensions: 3.875 x 3.300 x 4.313 inches,

Filling: Sand-loaded asphalt compound.

Measured Resistances (at 65°C)

Primary: 2.91 ohms (115 volt tap),

Secondary: 0.0454 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 40°C,

Secondary: 38°C

Measured average winding temperature rise:

Primary: 39°C,

Secondary: 34°C.

K-23: Current-Limiting-Filament Transformer

Requirements

Frequency: 400 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Primary: 115 volts.

Secondary: 6.3 volts, 5 amperes, 10 amperes short-circuit current.

Protection: Grade 2 (less resistant to adverse environmental conditions).

Design

Core:

Lamination: "L" type having a width of 1/2 inch, window dimensions are 1/2 x 1-1/2 inches.

Steel: Oriented silicon, .006 inch thick.

Stack: 1/2 inch.

Construction: Lap joint, laminate 2 x 2.

Tubes (primary and secondary):

Dimensions: .030 inch thick, 1/2 x 1/2 x 1/2 inch long.

Material: Paper.

Primary winding (115 volts):

Wire size: No. 27 single enamel copper wire,

Turns per layer: 19,

Layers: 20,

Turns: 378,

Layer insulation: .002 inch paper,

Wrapper: .010 inch paper.

Secondary winding (6.3 volts):

Wire size: No. 17 single enamel copper wire.

Turns per layer: 5,

Layers: 6,

Turns: 28,

Layer insulation: .007 inch paper,

Wrapper: .010 inch paper.

Magnetic shunt:

Thickness of shunt corresponds to 14 laminations, each .0185 inch thick.

Length of shunt (length of each lamination) = 1/2 inch.

Width of shunt (width of each lamination) = 0.490 inch.

Measured Resistances (at 65°C)

Primary: 6.45 ohms,

Secondary: 0.0515 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 36°C,

Secondary: 36°C.

Measured average winding temperature rise:

Primary: 34°C,

Secondary: 30°C.

K-26: Current-Limiting Transformer with Unbalanced Magnetization

Requirements

Frequency: 400 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C

Primary: 60/65/71 volts.

Secondary: 65 volts RMS, 1.2 amperes RMS, .75 amperes DC, 165 amperes RMS short-circuit current, half-wave rectifier with resistance load and no filter.

Protection: Grade 2(less resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with center leg width of 1-1/8 inches.

Steel: Oriented silicon, .004 inch thick.

Stack: 1-3/16 inches.

Construction: Butt joint with .007 inch paper in secondary portion of core.

Tubes:

Primary: .030 inch thick, 1-1/8 x 1-3/8 x 11/16 inch long,

Secondary: .030 inch thick, 1-1/8 x 1-3/8 x 9/16 inch long,

Material: Paper.

Primary winding (60/65/71 volts):

Wire size: No. 17 AWG single enamel copper wire,

Turns per layer: 9,

Layers: 8,

Turns: 71, taps at 65 and 60,

Layer insulation: .007 inch paper,

Wrapper: .010 inch paper.

Secondary winding (65 volts RMS):

Wire size: No. 22 AWG single enamel copper wire,

Turns per layer: 11,

Layers: 16,

Turns: 168,

Layer insulation: .003 inch paper,

Wrapper: .010 inch paper.

Magnetic shunts (two required):

Thickness of each shunt corresponds to 18 laminations, each .025 inch thick.

Length of each shunt (length of each lamination) = 1-3/16 inches,

Width of each shunt (width of each lamination) = .5425 inch.

Measured Resistances (at 61.5°C)

Primary: 0.2205 ohms (65 volt tap),

Secondary: 1.786 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 44°C,

Secondary: 40°C,

Measured average winding temperature rise:

Primary: 39°C,

Secondary: 36°C.

K-27: Vibrator-Supply Transformer

Requirements

Frequency: 115 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Supply: 24 volts DC.

Load: .050 amperes DC from full-wave rectifier with an inductance-input filter.

Secondary voltage: 572 volts RMS.

Protection: Grade 1 (most resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with center leg width of 3/4 inch.

Steel: Hot-rolled silicon, .025 inch thick, AISI M-22 grade.

Stack: 1-1/16 inches.

Construction: Lap joint, laminate 2 x 2.

Tube:

Dimensions: .030 inch thick, 3/4 x 1-3/32 x 1-1/16 inch long.

Material: Paper.

Secondary winding (1 to 3, 2 center-tap) - next to core:

Wire size: No. 38 AWG single enamel copper wire,

Turns per layer: 175,

Layers: 20,

Turns: 3500, tap at 1750 turns,

Layer insulation: .002 inch paper,

Wrapper: .020 inch paper.

Primary winding (4 to 6, 5 center-tap):

Wire size: No. 25 AWG single enamel copper wire,

Turns per layer: 37,

Layers: 6,

Turns: 22, tap at 111 turns,

Layer insulation: .002 inch paper,

Wrapper: .020 inch paper.

Measured Resistances (at 62°C)

Primary: 3.74 ohms,

Secondary: 957 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 30°C,

Secondary: 32°C.

Measured average winding temperature rise:

Primary: 30°C,

Secondary: 33°C.

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K-28: Vibrator - Supply Transformer

Requirements

Frequency: 115 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Supply: 12 volts DC.

Load: .11 amperes DC from a full-wave rectifier with capacitance filter.

Secondary: .094 amperes RMS, 544 volts RMS.

Protection: Grade 1 (most resistant to adverse environmental conditions).

Design

Core:

Lamination: Scrapless EI with center leg width of 1 inch.

Steel: Non-oriented silicon, .01875 inch thick, AISI M-15 grade.

Stack: 1-3/32 inches.

Construction: Lap joint, laminate 2 x 2.

Tube:

Dimensions: .040 inch thick, 1 x 1-3/32 x 1-7/16 inches long.

Material: Paper.

Secondary winding (1 to 3, 2 CT)- next to core:

Wire size: No. 34 AWG single enamel copper wire,

Turns per layer: 146,

Layers: 18

Turns: 2618, tap at 1309 turns,

Layer insulation: .002 inch paper.

Wrapper: .030 inch paper.

Primary winding (4 to 6, 5 center tap):

Wire size: No. 18 single enamel copper wire,

Turns per layer: 21,

Layers: 4,

Turns: 84, tap at 42,

Layer insulation: .005 inch paper,

Wrapper: .020 inch paper.

Measured Resistances (at 64.5°C)

Primary: .363 ohms,

Secondary: 343 ohms.

Temperature Data

Calculated average winding temperature rise:

Primary: 35°C,

Secondary: 37°C.

Measured average winding temperature rise:

Primary: 34°C,

Secondary: 39°C.

K-24: Low-Capacitance Transformer

Requirements

Frequency: 400 cycles per second.

Ambient temperature: 85°C.

Maximum temperature rise: 115°C.

Primary: 115 volts.

Secondary: 10 volts, 5 amperes.

Secondary working voltage: 2 kilovolts.

Capacitance from secondary to primary and core: 9 micro-microfarads.

Protection: Grade 2 (less resistant to adverse environmental conditions).

Design

Core:

Lamination: "L" type having a width of 1/2 inch.

Steel: Oriented silicon, .014 inch thick, AISI M-19 grade.

Stack: 1/2 inch.

Window: 3/4 x 1-1/2 inches.

Tubes:

Primary: .025 inch thick, 1/2 x 1/2 x 1-1/2 inch long.

Secondary: .040 inch thick, 1.45 x 1.45 x .73 inch long,

Material: Quinterra.

Primary winding (115 volts):

Wire size: No. 28 AWG teflon-coated wire,

Turns per layer: 76,

Layers: 6,

Turns: 455

Layer insulation: .003 inch Quinterra,

Wrapper: .009 inch Quinterra.

Secondary winding (10 volts):

Wire size: No. 19 AWG teflon-coated wire,

Turns per layer: 16,

Layers: 3,

Turns: 47,

Layer insulation: .009 inch Quinterra

Wrapper: .010 inch Quinterra.

Measured Resistances (at 84°C)

Primary: 7.9 ohms,

Secondary: 0.218 ohms.

Temperature Data

Calculated average winding temperature rise:

Secondary: 82°C.

Measured average winding temperature rise:

Primary: 62°C

Secondary: 87°C.

Measured Capacitance

Capacitance from secondary to primary and core equals 14 micro-microfarads when "Quinterrabord" supports are used and windings are concentric.

K-20: Low-Capacitance Transformer

Requirements

Frequency: 60 cycles per second.

Ambient temperature: 65°C.

Maximum temperature rise: 40°C.

Primary: 115 volts.

Secondary: 6.3 volts, 20 amperes.

Capacitance from secondary to primary and core: 18 micro-microfarads.

Protection: Grade 2 (less resistant to adverse environmental conditions).

Design

Core:

Lamination: Special type giving two core leg widths of 1-1/4 inches and the other two 1-5/32 inches.

Window: 4-3/4 x 6-9/16 inches.

Steel: Non-oriented silicon, .0185 inch thick, AISI M-19 grade.

Stack: 1-5/32 inches.

Tubes:

Primary: .065 inch thick, 1-5/32 x 1-5/32 x 6-7/16 inch long.

Secondary: 1/8 inch thick wall, with 5.0 inch outside diameter.

Material: Paper for the primary and phenolic resin for secondary.

Primary winding (115 volts):

Wire size: No. 19 AWG single-enamel copper wire,

Turns per layer: 112,

Layers: 4,

Turns: 448,

Layer insulation: .005 inch paper,

Wrapper: .010 inch paper.

Secondary winding (6.3 volts):

Wire size: No. 7 AWG double enamel copper wire,

Turns per layer: 6,

Layers: 5,

Turns: 28,

Layer insulation: .007 inch paper,

Wrapper: .010 inch paper.

Measured Resistances (at 23.5°C)

Primary: 1.755 ohms,

Secondary: 0.0222 ohms.

Temperature Data

Calculated average winding temperature rise:

Secondary: 21°C.

Measured average winding temperature rise:

Primary: 16°C,

Secondary: 23°C.

Measured Capacitance

Capacitance from secondary to primary and core equals 17 micro-microfarads when polystyrene supports are used and windings are concentric.

K-4: Low-Capacitance Transformer

Requirements:

Frequency: 60 cycles per second.

Ambient temperature: 65°C

Maximum temperature rise: 40°C

Primary: 115 volts.

Secondary: 8.0 volts, 15 amperes.

Capacitance from secondary to primary and core: 16 micro-microfarads.

Protection: Grade 2 (less resistant to adverse environmental conditions).

Design

Core:

Lamination: "L" type with leg width of 1.0 inch.

Steel: Non-oriented silicon, .0185 inch thick, AISI M-27 grade.

Stack: 1-1/4 inches.

Window: 4 x 4 inches.

Tubes:

Primary: .040 inch thick, 1 x 1-1/4 x 3-15/16 inches long.

Secondary: .125 inch thick, 4 x 4 x [illegible]-3/8 inches long.

Material: Paper.

Primary winding (115 volts):

Wire size: No. 19 AWG single-enamel copper wire,

Turns per layer: 87,

Layers: 5,

Turns: 433,

Layer insulation: .005 inch paper,

Wrapper: .010 inch paper.

Secondary winding (8.0 volts):

Wire size: No. 9 AWG single-enamel copper wire,

Turns per layer: 6,

Layers: 6,

Turns: 37,

Layer insulation: .010 inch paper,

Wrapper: .010 inch paper.

Measured Resistances (at 28°C)

Primary: 1.66 ohms,

Secondary: 0.0478 ohms.

Temperature Data

Calculated average winding temperature rise:

Secondary: 27°C.

Measured average winding temperature rise:

Primary: 33°C,

Secondary: 46°C.

Measured Capacitance

Capacitance from secondary to primary and core equals 16 micro-mircofarads when supports are wood blocks and windings are concentric.

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The design data for the following four examples are given in the final report on Contract No. DA-36-039 SC-5519.

Example	Maximum rise	Calculated average winding rise		Measured average winding rise	
		Pri	Sec.	Pri	Sec
Design A, Filament Transformer	40°C	37°C	36°C	31°C	28°C
Design B, Autotransformer	40°C			43°C	
Design C, Full-Wave Rectifier Transformer	40°C	34°C	32°C	48°C	45°C
Design E, High Temperature Rectifier and Filament Supply Transformer	85°C	77°C	71°C	64°C	66°C

APPENDIX E: TEST DATA FOR TRANSFORMERS WITH UNBALANCED MAGNETIZATION

Pertinent data on four typical transformer cores tested are:

1) Wound-type core, windings on one leg

Two butt joints

Mean length of magnetic circuit: 7.22 inches

Net cross-sectional area of core: 0.753 square inches

Core weight: 1.5 pounds

Steel: Oriented silicon

Thickness: 12 mils

Test frequency: 60 cycles per second

2) Shell-type core, EI laminations

Leg width: 1 inch

Mean length of magnetic circuit: 6 inches

Core weight: 2.13 pounds

Steel grade: Audio type equivalent to AISI-M-15, non-oriented silicon

Thickness: 14 mils

Test frequency: 60 cycles per second

3) Wound-type core, coils on one leg

Two butt joints

Mean length of magnetic circuit: 4.93 inches

Core weight: 0.5 pounds

Steel: Oriented silicon

Thickness: 5 mils

Test frequency: 400 cycles per second

4) Shell-type core, EI laminations

Leg width: 0.625

Mean length of magnetic circuit: 3.75 inches

Core weight: 0.3907 pounds

Steel: Grain-oriented silicon (Armco Tran-Cor T-0)

Thickness: 4 mils

Test frequency: 400 cycles per second

The following equations give the approximate per cent gap which results in minimum excitation. In each of the cores it has been found that the per cent gap is almost independent of flux density below some value of flux density. This behavior can be accounted for by using the indicated value in the equations.

1) % gap = $0.015\ H_{DC} - 0.003\ B + 0.24$
   (Use B = 90 if density is less than 90),
   where H_{DC} = magnetic field strength in oersteds, 0.495 times average ampere-turns per inch,
   B = flux density in kilolines per square inch

2) % gap = $0.021\ H_{DC} - 0.016\ B + 1.0$
   (Use B = 70 if density is less than 70)

3) % gap = $0.025\ H_{DC} - 0.0052\ B + 0.29$
   (Use B = 70 if density is less than 70)

4) % gap = $0.019\ H_{DC} - 0.008\ B + 0.5$
   (Use B = 60 if density is less than 60)

Application of these equations might yield a negative value for the per cent gap. In this case the core joint with minimum effective gap should be used, a butt joint for cut, wound cores, and a lapped-butt joint for stacked cores.

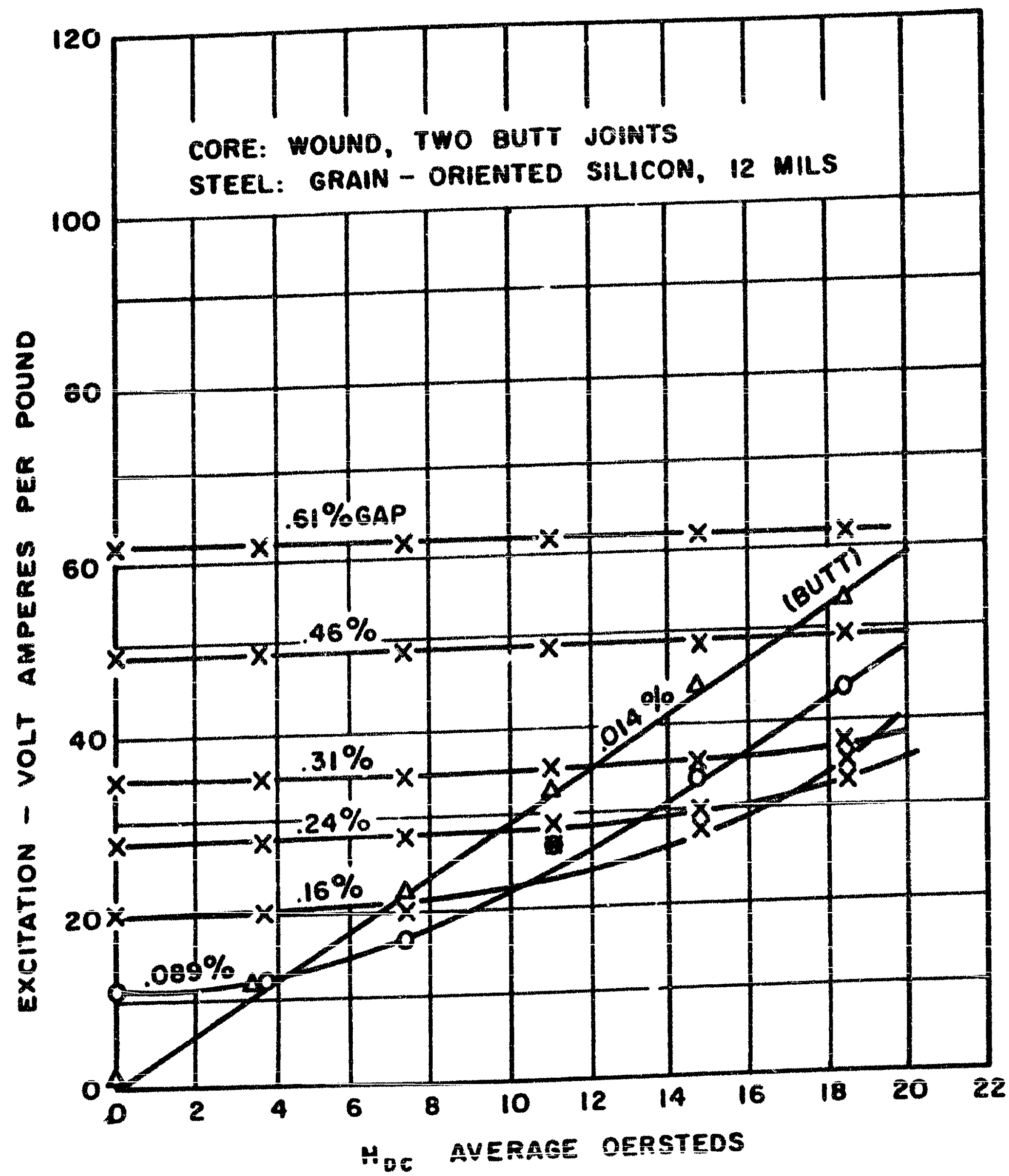

FIG. E-1 EXCITATION OF WOUND CORE AT 80 KILOLINES PER SQ. IN. (60 CPS)

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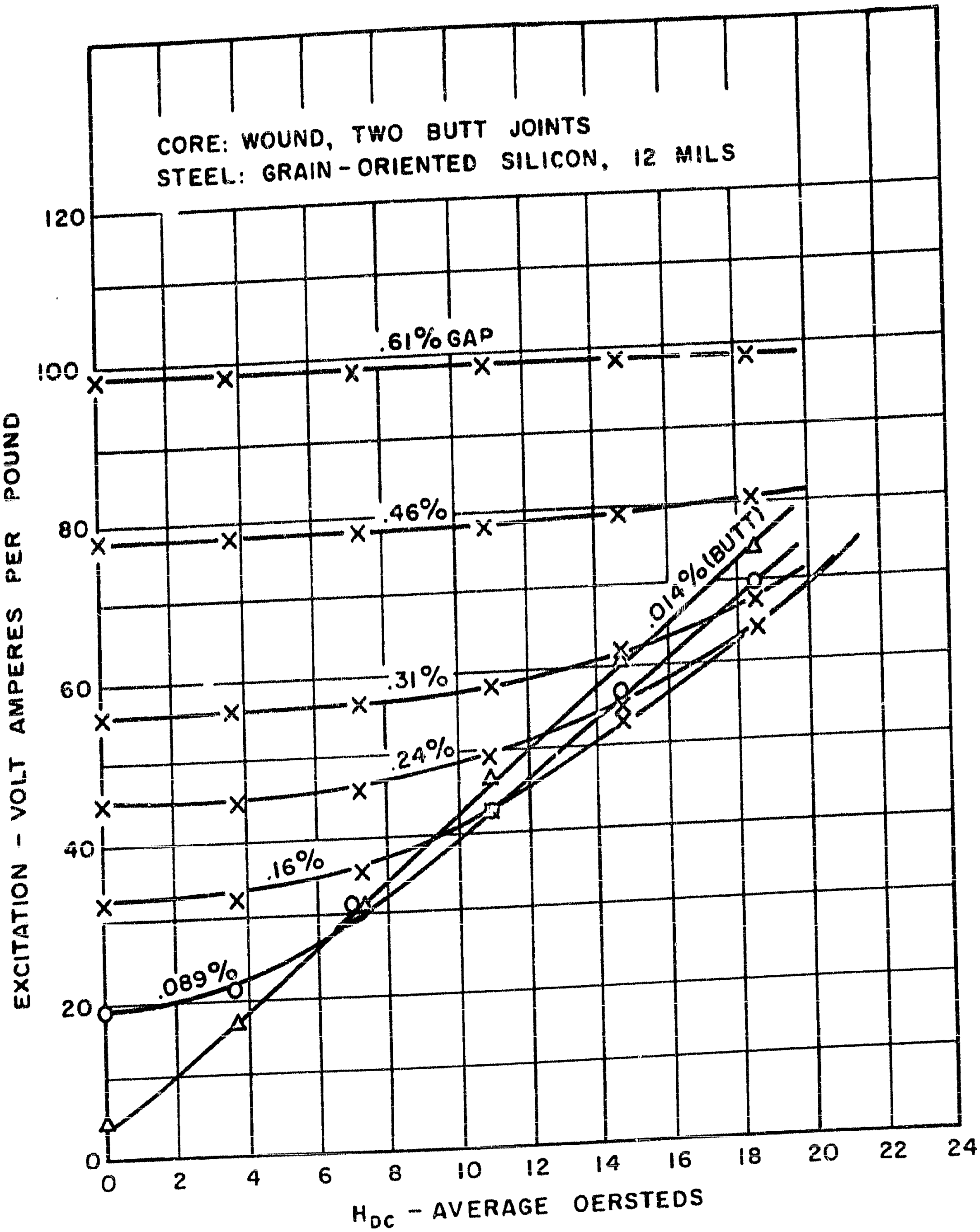

FIG. E-2 EXCITATION OF WOUND CORE AT 100 KILOLINES PER SQ. IN. (60 CPS)

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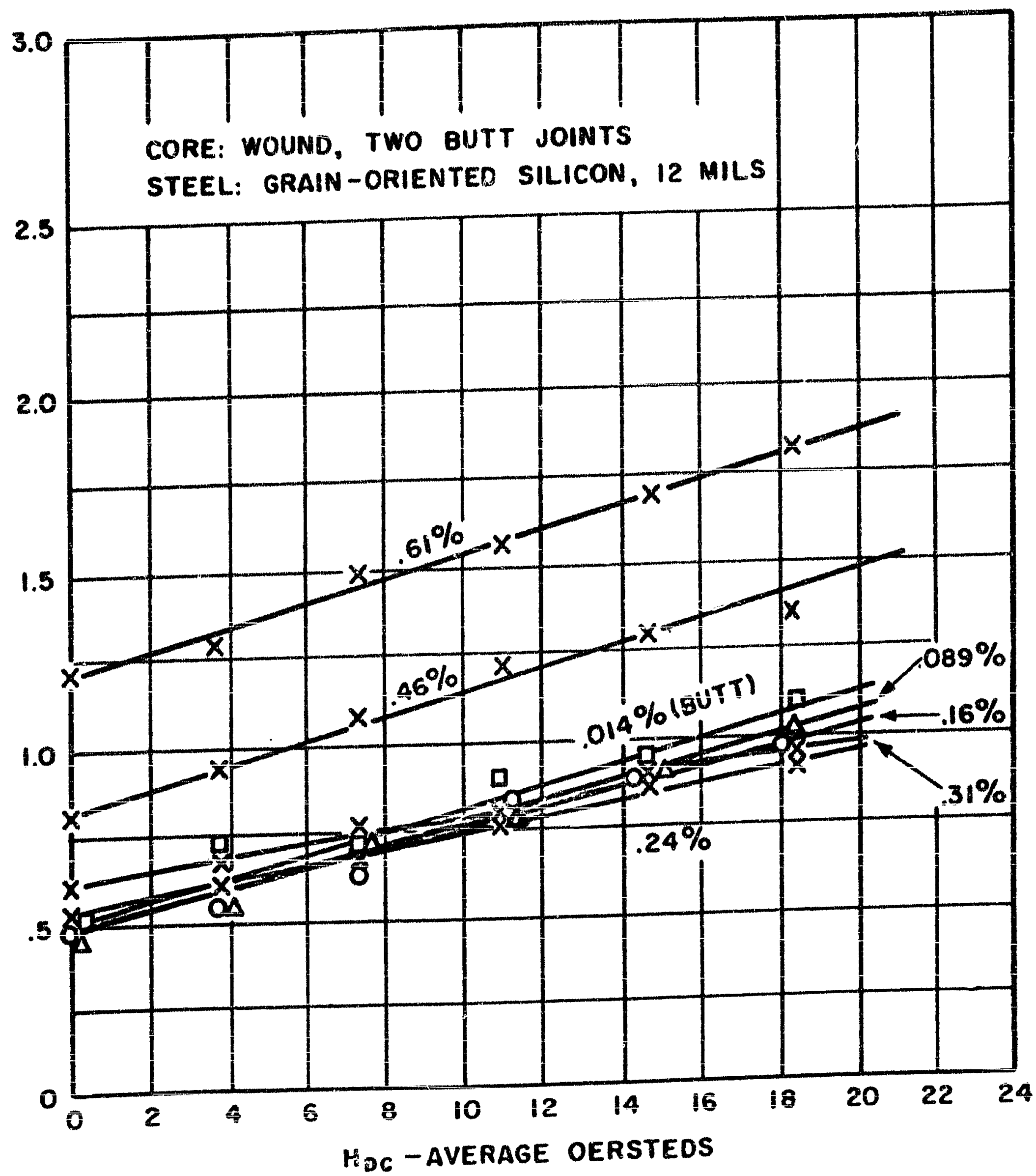

FIG. E-3 CORE LOSS OF WOUND CORE AT 80 KILOLINES PER SQ. IN. (60 CPS)

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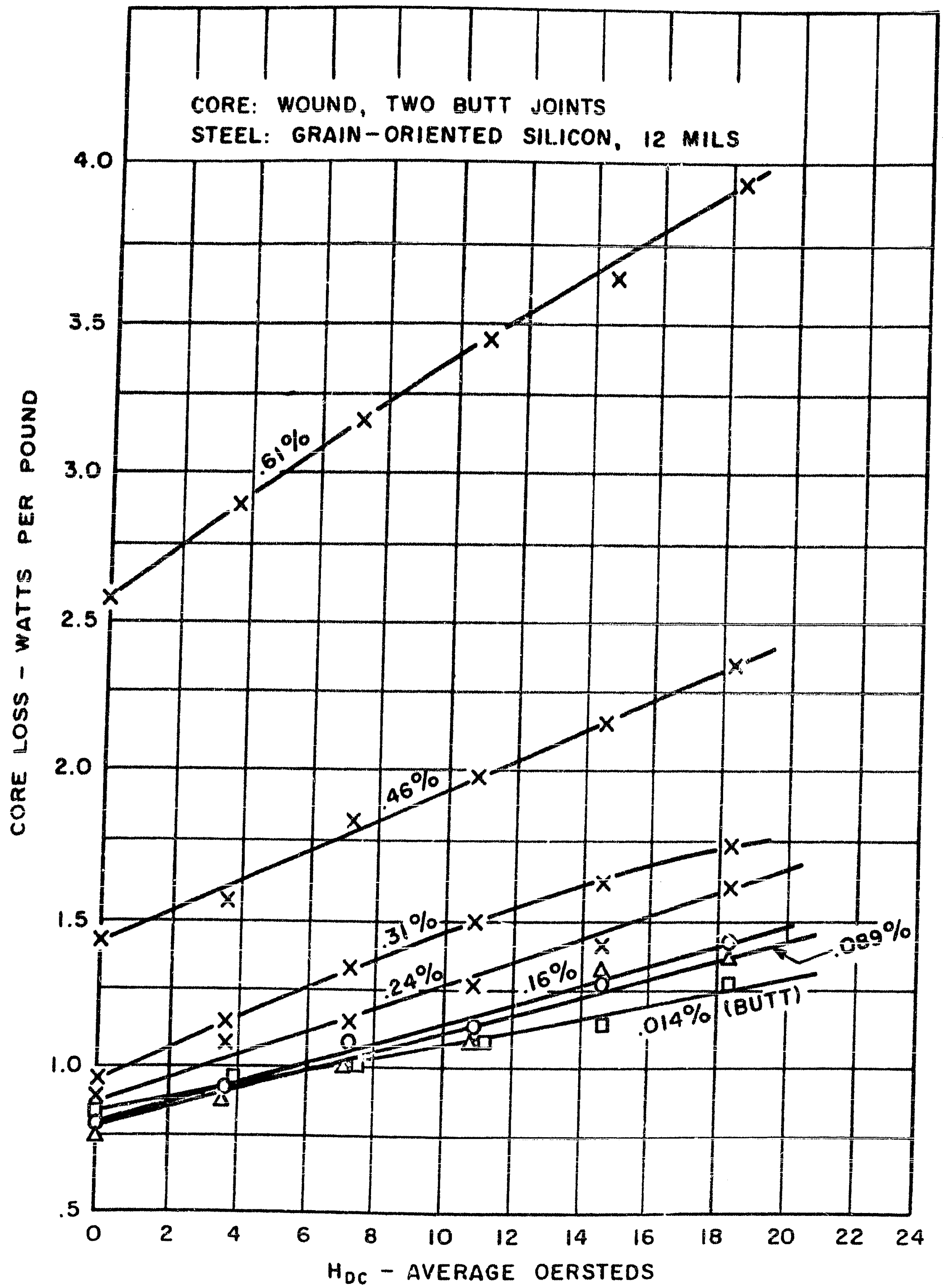

FIG. E-4 CORE LOSS OF WOUND CORE AT 100 KILOLINES PER SQ. IN. (60 CPS)

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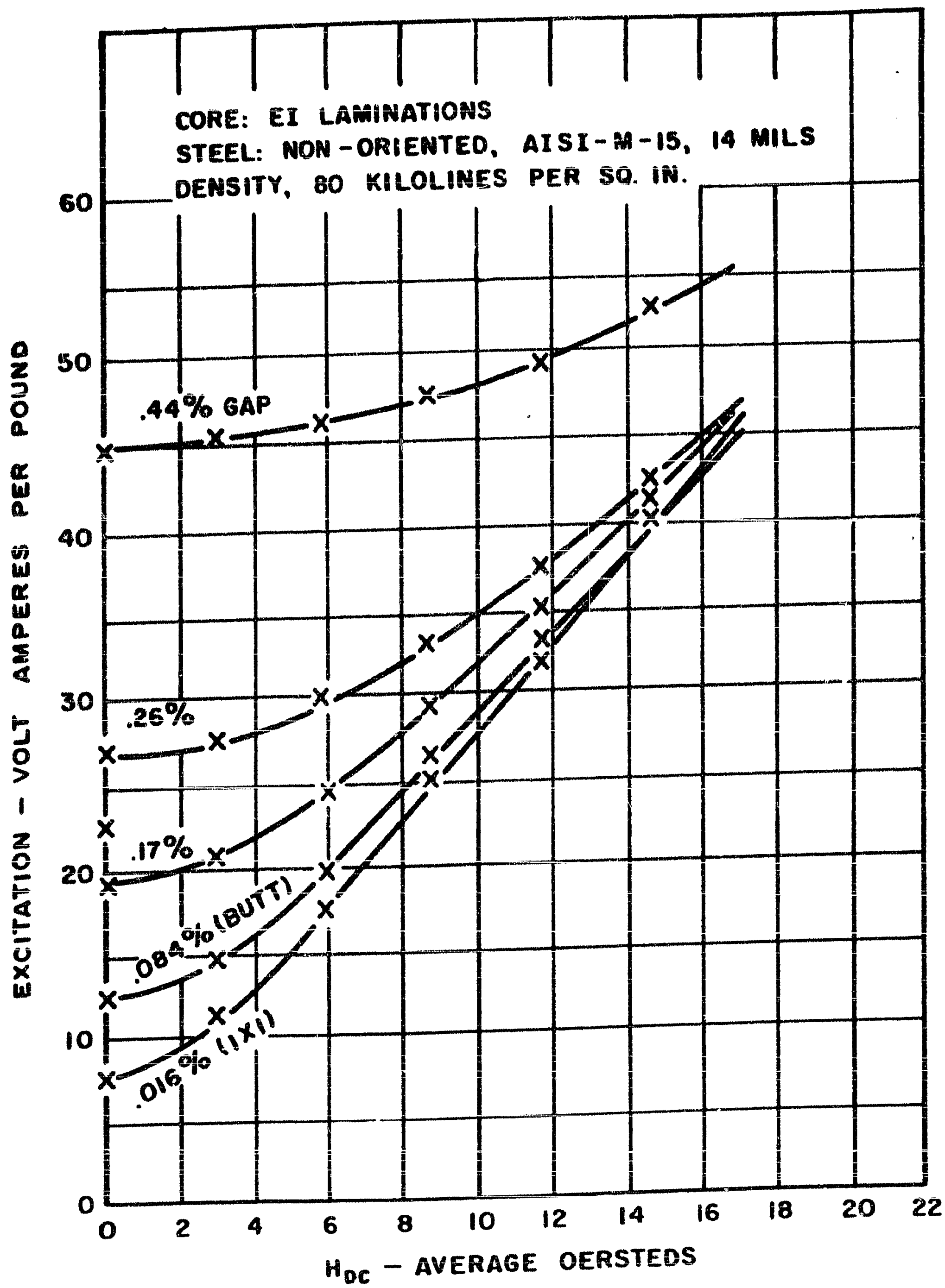

FIG. E-5 EXCITATION OF STACKED CORE (60 CPS)

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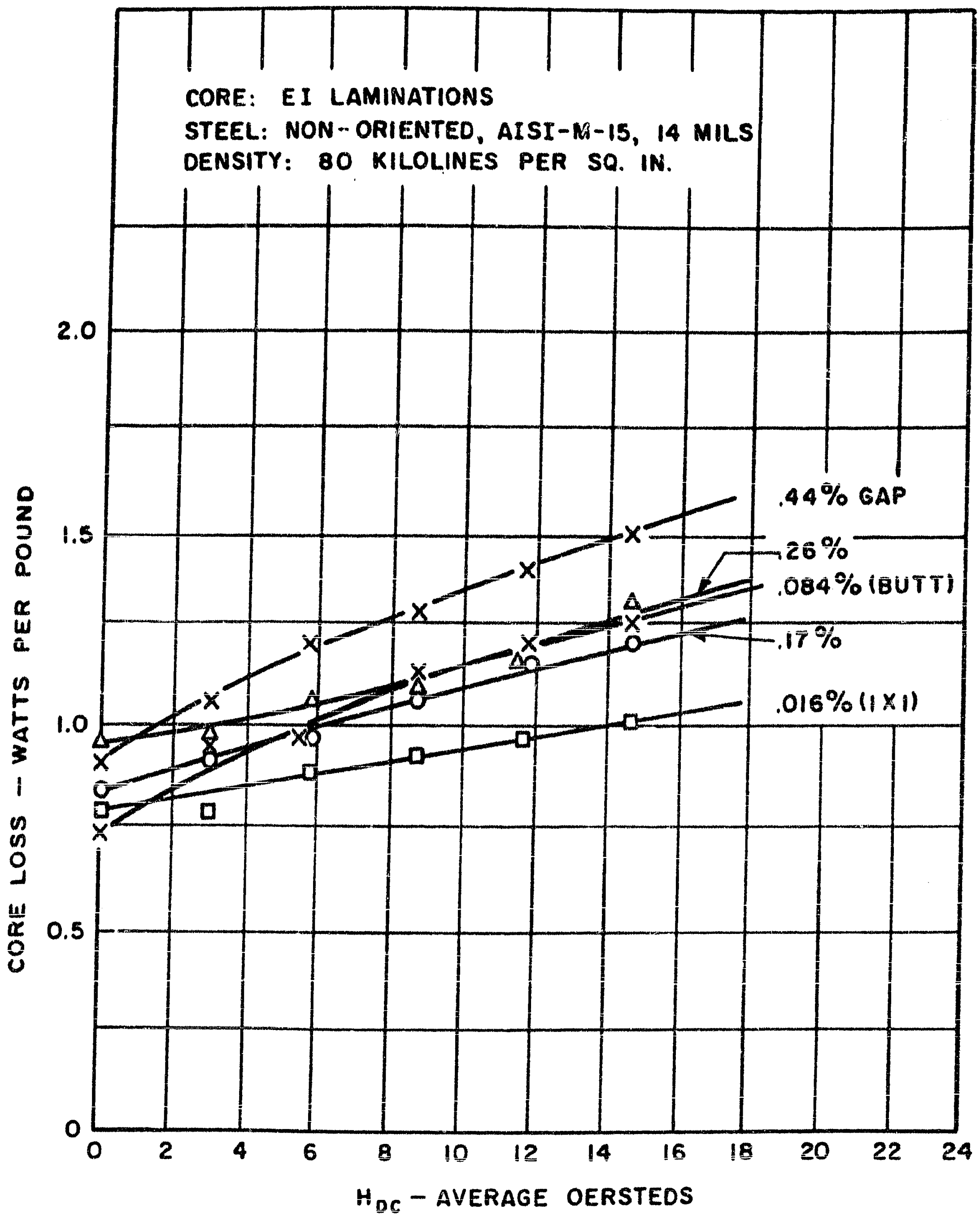

FIG. E-6 CORE LOSS OF STACKED CORE (60 CPS)

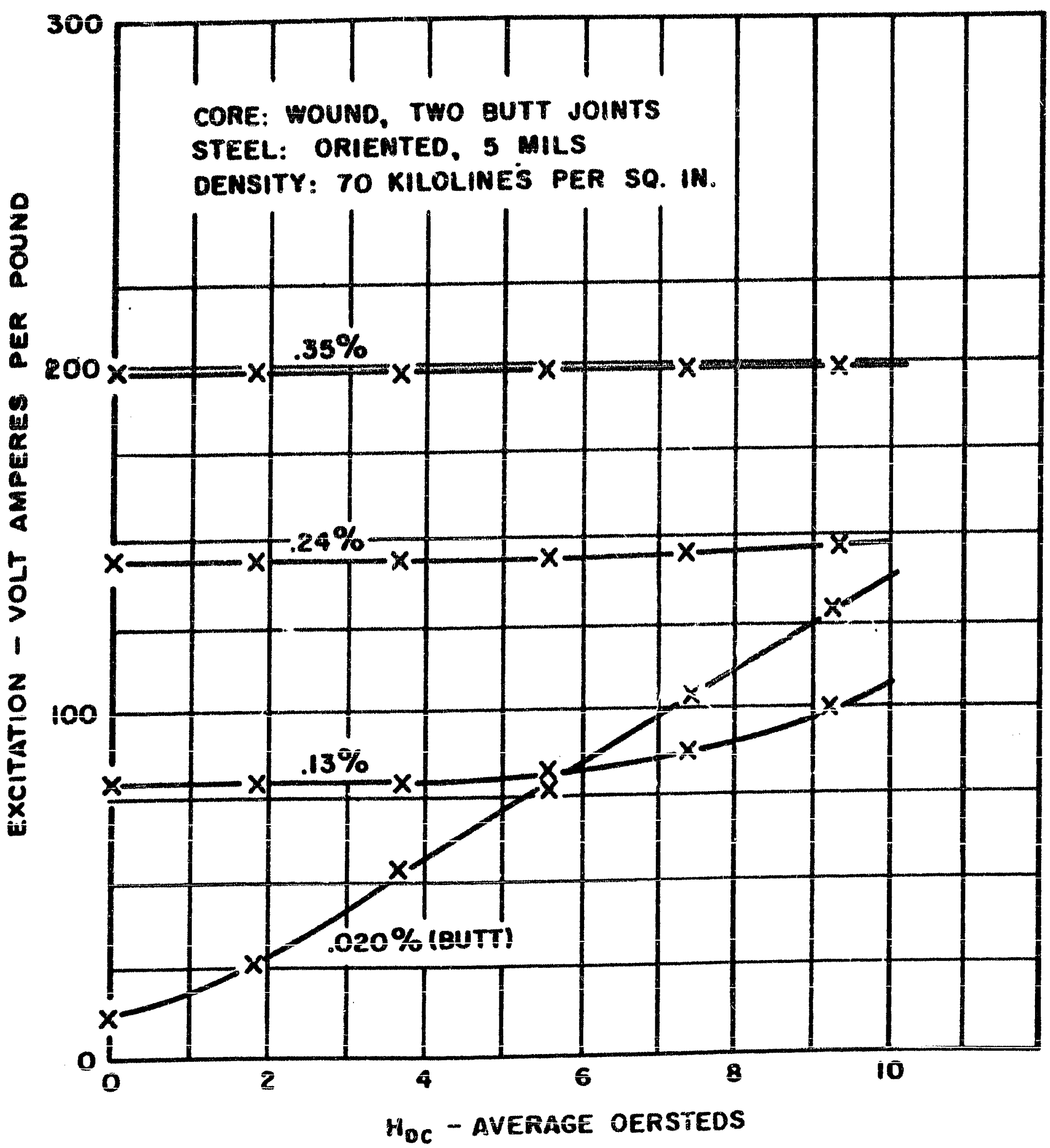

FIG. E-7 EXCITATION OF WOUND CORE (400 CPS)

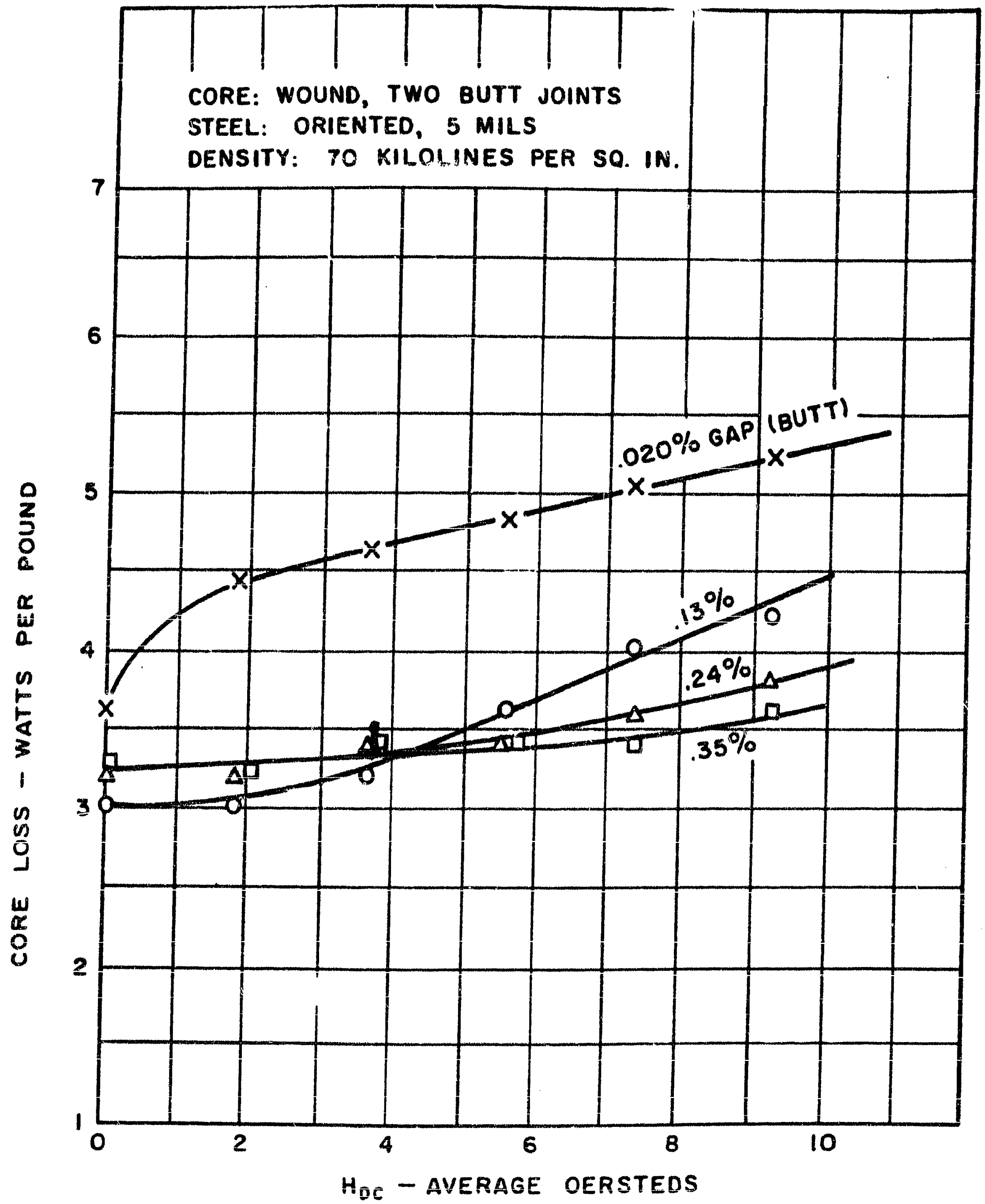

FIG. E-8 CORE LOSS OF WOUND CORE (400 CPS)

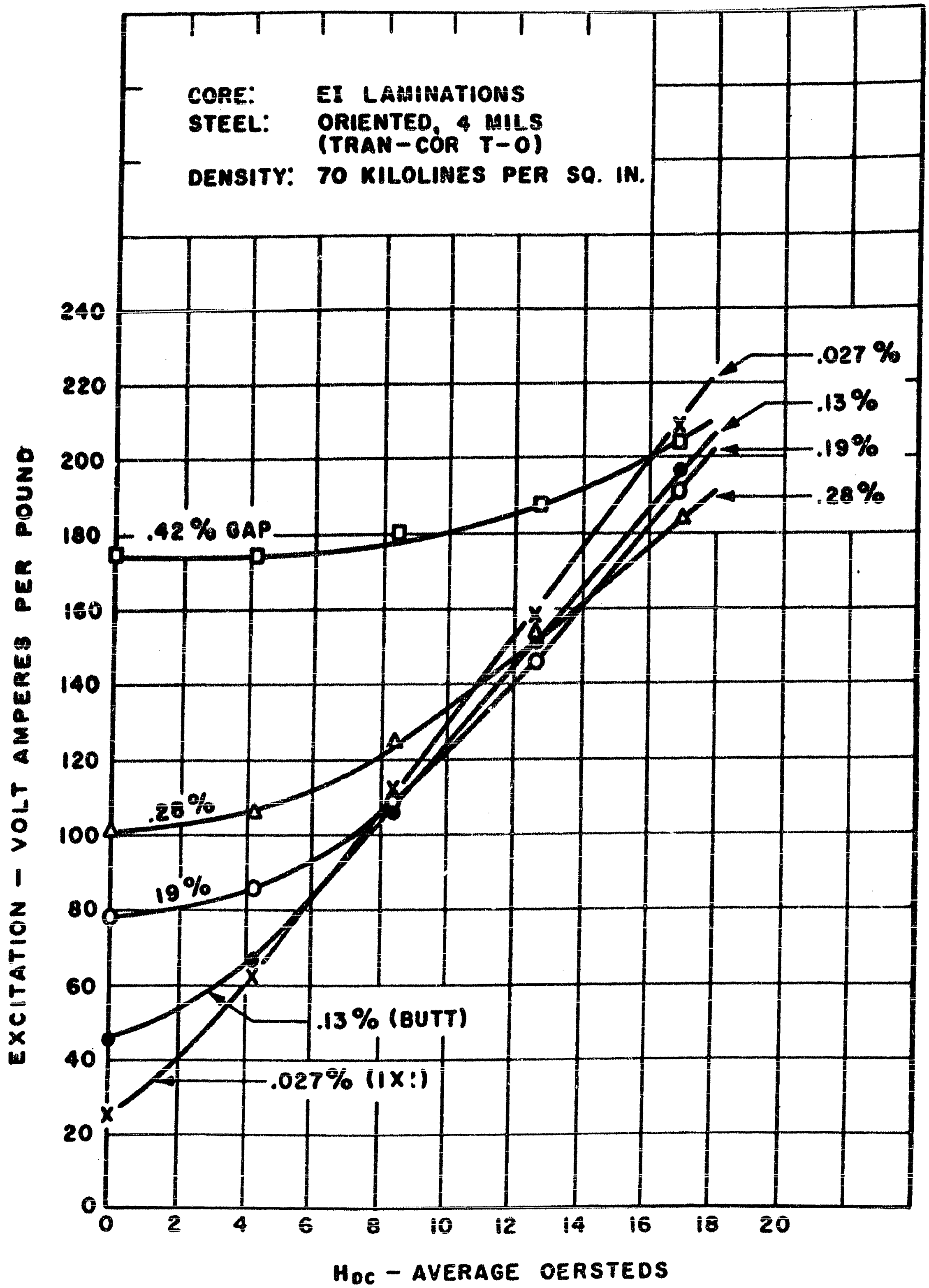

FIG. E— 9 EXCITATION OF STACKED CORE (400 CPS)

ARMOUR RESEARCH FOUNDATION OF ILLINOIS INSTITUTE OF TECHNOLOGY

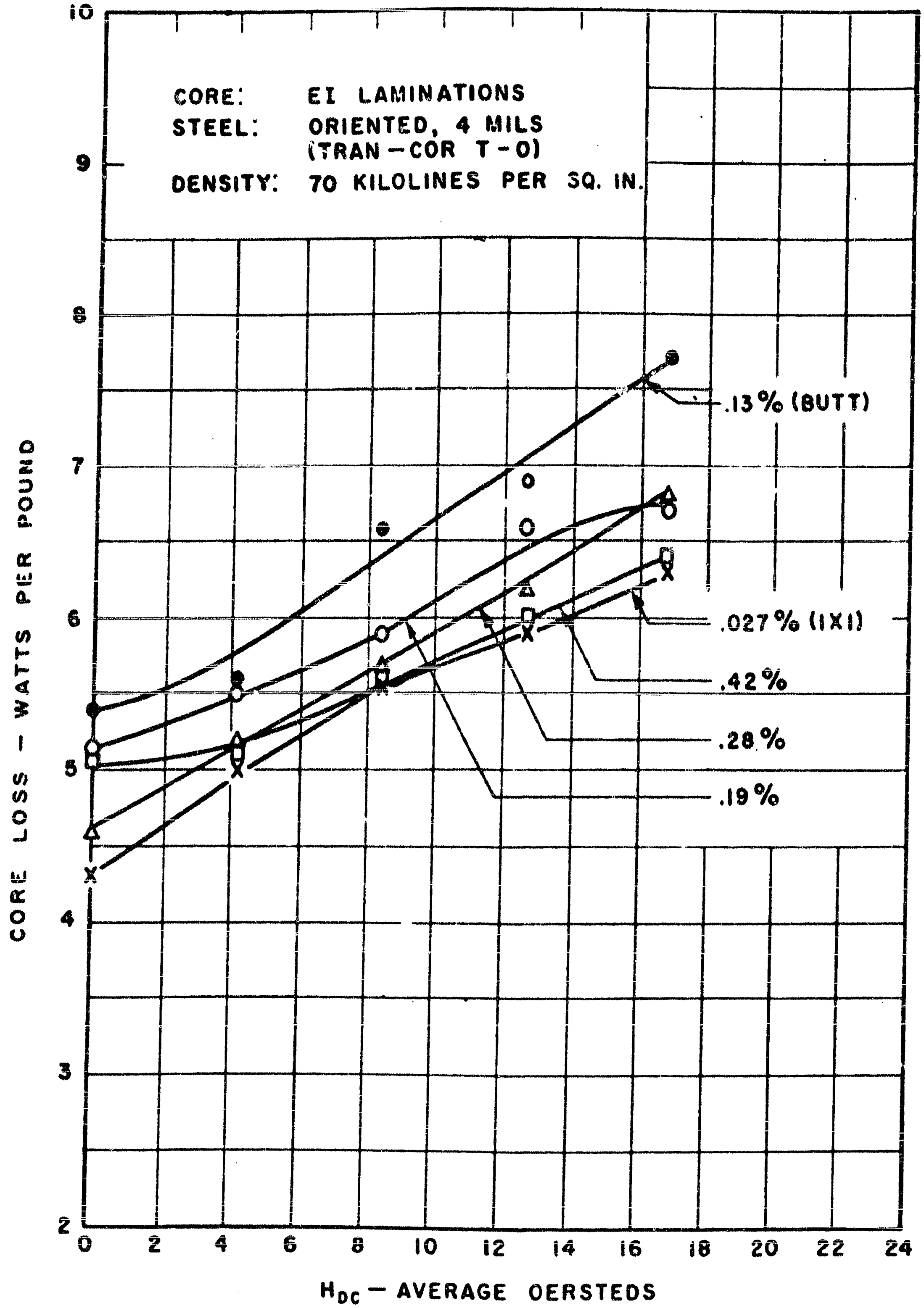

FIG. E- 10 CORE LOSS OF STACKED CORE (400 CPS)

APPENDIX F: CORRELATION OF EQUATION, TABLE, AND FIGURE NUMBERS

Correlation between the numbers for the equations, tables, and figures which are used in this final report of Contract No. DA-36-039 SC-52656, and which also were used in the final report of Contract No. DA-36-039 SC5519 follow.

Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-5519	Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-5519
Equation Numbers		**Equation Numbers (cont)**	
2-1	2-8	2-20	9-13
2-2	2-9	2-21	9-12
2-3	2-10	2-22	10-14
2-4	2-11	2-23	10-15
2-5	2-12	2-24	10-16
2-6	10-11	2-25	10-17
2-7	10-1	2-26	10-18
2-8	10-2	2-27	10-19
2-9	10-3	2-28	2-3
2-10	10-4	2-29	10-20
2-11	10-5	2-30	10-21
2-12	10-6	2-31	10-22
2-13	10-7	2-32	2-6
2-14	10-8	2-33	10-23
2-15	10-9	2-34	10-24
2-16	10-10	11-1	12-1
2-17	10-12	11-2	12-2
2-18	10-13	11-3	12-3
2-19	9-3	11-4	12-3A

Equation Numbers (Cont.)

Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-5519
11-5	12-4
11-6	12-5
11-7	12-6
11-8	12-7
11-9	12-8
11-10	12-9
11-11	12-10
11-12	12-11
11-13	12-12
11-14	12-13
11-15	12-14
12-1	14-2
12-4	14-5
12-5	15-1

Table Numbers

Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-5519
11-1	9-1
11-2	6-3
11-3	6-4
11-4	10-1
11-5	9-2
11-6	9-7
11-7	9-6
11-8	12-1
11-9	12-2
11-10	12-3

Table Numbers (Cont.)

Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-52656
11-11	12-4
11-12	12-5
12-1	15-1
12-2	15-2
12-3	15-3
12-4	15-4

Figure Numbers

Contract No. DA-36-039 SC-52656	Contract No. DA-36-039 SC-52656
11-1	37
11-2	40
11-3	44
11-5	45
11-6	42
11-7	43
11-8	47
11-9	47A

APPENDIX G: LIST OF PRINCIPAL SYMBOLS

W_r rating, volt-amperes
f frequency, cycles per second
F_c copper space factor
F_i core space factor
B flux density, kilolines per square inch
Δ current density, kiloamperes per square inch
A_c window area, square inches
A_i core cross-sectional area, square inches
V RMS potential, volts
I RMS current, amperes
m_c mean length of winding, inches
m_i mean length of core, inches
K temperature-rise parameter
ΔT temperature-rise, degrees centigrade
ℓ characteristic linear dimension, inches (equal $\sqrt[4]{A_c A_i}$)
ρ conductor resistivity, microhm-inches
W_c winding loss, watts
W_i core loss, watts
S_c exposed area of winding, square inches
S_i exposed area of core, square inches
N turns of winding
n turns ratio, primary to secondary
W_{rp} total primary volt-amperes
W_{ex} excitation, volt-amperes
a, b, c, d, e, g: dimensionless ratios
K_0 through K_6: combinations of a, b, c, d, e, g
M_i core weight, pounds
δ_i density of core material, pounds per cu. in.
M_c winding weight, neglecting insulation, pounds
δ_c density of conductor material, pounds per cu. in.
X leakage reactance, ohms
R equivalent series resistance, ohms
n_c weight, loss, volume or cost per unit volume of winding
n_i weight, loss, volume or cost per unit volume of core
V_c volume of winding, cubic inches
V_i volume of core, cubic inches
W_r' equivalent rating of a given transformer, which indicates approximate rating of that unit if operated at 60 cycles and 40°C rise, volt-amperes.
L lamination leg width, inches.
s ratio of stack height to leg width of a laminated core.
CM circular mils
F term in space factor equation 2-20 which is found in Figure 11-2
h_c heat transfer coefficient of free convection.
h_r heat transfer coefficient of radiation.
θ certain temperature differences.

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m a thickness used in temperature calculations, inches.

C a capacitance

R_L load resistance, ohms.

W_{pL} a component of primary volt-amperes for units with unbalanced magnetization, volt-amperes.

H_{DC} unbalanced magnetizing force, averaged in time and around the length of a core, oersteds.

p ratio of short-circuit current to rated current

q ratio of leakage reactance during short circuit to leakage reactance at rated load.

m'_g effective air gap, inches

F'_c space factor of current-limiting transformers.

k_c correction for supports in calculation of capacitance of low-capacitance units.

P a perimeter, inches.

Z impedance, ohms.

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FINAL REPORT

[redacted] STAT

Period 1 March 1953 to 30 August 1955

DESIGN METHOD FOR POWER TRANSFORMERS

Armour Research Foundation
Chicago 16, Illinois

[redacted] STAT

This contract is supervised by Electronic Parts & Materials Branch, Components Department, SCEL. For further technical information contact Components Department, Fort Monmouth, New Jersey. [redacted] STAT

[redacted] STAT

INDUCTIVE PARTS SECTION
ELECTRONIC PARTS AND ASSEMBLIES BRANCH
ELECTRONIC PARTS AND MATERIALS DIVISION
COMPONENTS DEPARTMENT

[redacted] **EVALUATION REPORT** 22 October 1956

[redacted]

1. **INTRODUCTION:**

This report covers the entire contract period 1 May 1953 to 30 August 1955. The report was due 31 January 1956, was received 16 April 1956 and was accepted 14 June 1956.

2. **CONFERENCES DURING LAST QUARTER OF REPORT PERIOD:**

[redacted]

It was agreed that the design procedures presented in previous quarterly reports would be presented in outline form in the seventh quarterly report. Each design presented in the final report would include component temperature calculations.

[redacted]

The contents and the format of a draft of the Final Report were discussed. Various additions, modifications, and corrections were made. It was decided that the basic design procedure, as well as the design methods developed for the various types of transformers [redacted] would be included in this Final Report. Therefore, this report will contain all of the design information necessary for the types of power transformers developed under the original contract and this extension of the work.

[redacted]

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3. DISCUSSION AND EVALUATION:

This contract was a twenty-four (24) month extension of work conducted on Contract DA36-039 SC-5519 which resulted in a simplified design method for filament transformers; plate transformers with no unbalanced dc; plate and filament transformers with no unbalanced dc; and autotransformers. The design method utilized a nomograph, charts and curves. Work on this contract has extended the design method to include low capacitance transformers; current limiting transformers with or without unbalanced dc; plate transformers with unbalanced dc; and vibrator supply transformers.

The combined work on both contracts has resulted in a simplified design method for all of the types of power transformers listed above. The work on this second contract resulted in the modifications needed for the design of the special types of transformers listed. Some of the major modifications or additions to the basic design procedure are as follows:

Transformers with Unbalanced DC

Data in the form of curves were compiled to give core loss, excitation, and non-magnetic gap as functions of ac flux density and dc magnetization. Formulae and charts were included for the computation of secondary voltage and current from circuit constants, and for primary current and regulation.

Current Limiting Transformers

Relations were obtained among primary and secondary flux densities, and voltages, and current. The equation for winding space factor was modified. Guides were given for the selection of the magnetic shunts.

Vibrator Supply Transformers

Guides were given for the selection of flux density. An equation for primary voltage was included to account for time constant. Design information was given for procedures to keep the starting current low.

Low Capacitance Transformers

Empirical equations were derived from measurements and theoretical studies to establish relations among power ratings, space factor and desired capacitance. Equations for computing capacitance and leakage resistance were included.

In addition to the above an analysis was made relating to the selection of current densities. A study of optimum core proportions were made. Results were given in the form of optimum core stack height to width ratios.

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In effect these ratios reflect the relative costs of the core and winding per unit volume, where cost may be function of weight, volume, losses, or manufacturing expenses.

The design procedures developed under this contract are simple to follow and may be used by engineers not normally associated with the transformer industry. This will be of great importance in times of emergency in the event of a shortage of experienced transformer design engineers. In fact the benefits can even now be realized in view of the present national concern with the lack of scientific personnel.

4. FUTURE PLANS:

None

5. RECOMMENDATIONS:

None

6. REMARKS

None

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