STAT

Page Denied

Ministry of Higher Education of the USSR

Moscow Aviation Technological Institute

MODERN PROBLEMS IN THE TECHNOLOGY OF ASSEMBLY IN INSTRUMENT MAKING

Editor

V. P. Chumakov, Cand. Tech. Sci., **Lecturer**

Transactions of the Institute

No. 27

State Publishing House for Defense Industries
Moscow 1956

Ministry of Higher Education of the USSR
Moscow Aviation Technological Institute

MODERN PROBLEMS IN THE TECHNOLOGY OF ASSEMBLY IN INSTRUMENT MAKING

Editor
V. P. Chumakov, Cand. Tech. Sci., **Lecturer**

Transactions of the Institute
No. 27

State Publishing House for Defense Industries
Moscow 1956

PREFACE

The fulfillment of the goals set by the Communist Party of the Soviet Union for the Soviet Instrument Builders requires further development of the technology of instrument building and specifically of the technology of assembly, regulation and inspection of instruments.

The most laborious and complicated processes in the production of modern instruments are assembly and regulation or control. Assembly and testing-control operations, according to the data of a number of instrument-building plants, represent more than 50 to 60% of the total labor in the production of the instruments. There are many different assembly operations in instrument building, which is explained by the variety of operating principles employed in modern instruments. Often one instrument incorporates mechanical, electrical, pneumatic, and other elements. Many subassemblies of the instruments must be selected so as to satisfy the requirements of interchangeability not only with respect to geometric parameters, but also with respect to a number of physical characteristics. The requirement of interchangeability of subassemblies with respect to elastic, magnetic, electrical, and other characteristics could be mentioned as an example.

At present, the demands on the technology of production of instruments are considerably increased. The technological processes of assembly, regulation, and control of the instruments should yield a high degree of accuracy and reliability of operation under difficult operating conditions; at the same time, these processes should in many cases be planned and designed for large series production, assembly and in-

[redacted]

spection of instruments for continuous flow, **on assembly lines.**

One of the main directions of work of the scientific-research work of the chair of "Technology of Aircraft Instrument Making" at the Moscow Aviation Technology Institute (MATI) is the study of questions of accuracy and production capacity of assembly and control processes in instrument making.

A number of investigations performed by members of the scientific staff of the institute in the foremost instrument plants was reported during the conference on the technology of instrument assembly, organized by the chair of Technology of Aircraft Instrument Making of MATI in 1954, in collaboration with representatives of industry and of Moscow Institutes.

During the conference it was resolved to publish the most pertinent reports and investigations about the assembly of instruments. That resolution is in some measure being fulfilled by the release of this collection.

Included in the articles of the collection are the results of investigations, theoretical relationships, announcements, classifications, descriptions of original designs, and other material which could be useful to engineering-technical and scientific employees of plants and Institutes.

The Chair will be obliged to all readers who will submit to the Institute their remarks and wishes regarding the articles in this collection.

V. P. CHUMAKOV

candidate in technical science,

Lecturer

Head, Chair of Technology of Aircraft

Instrument Making

[redacted]

STAT

PROBLEMS OF ACCURACY OF TECHNOLOGICAL PROCESSES IN THE WINDING OF ELECTRIC SUBASSEMBLIES

by

V. P. Chumakov, Cand. Tech. Sci., Docent

Many modern instruments and automatic devices are based on electric or electromechanical principles of operation. Windings of various types find numerous applications in instrument making.

Subassemblies of current-carrying coils of wires are called windings. Windings are used in coils, potentiometers, rotors, stators, and other electric subassemblies.

Almost all instrument-making plants concern themselves in one way or another with the production of windings. The technology of production of windings has its own peculiarities and is only little explored. The winding process itself is the most important and most difficult one in the production of windings. The quality of the windings produced, and consequently the quality of the instruments, depends on the proper processing practices in winding operations.

The quality of the windings is characterized by the ohmic resistance, the correctness of the geometric shape and dimensions, high insulation strength and resistance, and the number and arrangement of the coils. For certain windings, the following may be required: a predetermined magnetic flux, a certain space factor in the slots, rate of change of ohmic resistance, and other parameters.

The requirements for accuracy of the ohmic resistance of the windings vary; for

[redacted]

example, the deviation of the ohmic resistance may in general be 5 - 10%, and in specific cases not more than 3 - 1%; there are potentiometer applications with non-linearities of not more than 1 - 0.5% and even of 0.1 - 0.01%.

The technological process of winding should be so planned and carried out in actual production as to guarantee that the necessary quality of the entire series of windings is obtained with minimum losses in time and resources.

With any technological process of winding it is practically impossible to obtain truly accurate parameters of the windings. Because of detectable errors or deviations of dimensions and of electrical properties of the wire and tension forces during winding, the parameter obtained is in all cases larger or smaller than that given. Even under uniform conditions during the winding of two or more subassemblies deviations of dimensions and of electric characteristics of the windings are unavoidable. To obtain completely uniform characteristics of windings consistently is practically impossible.

Deviations in the technological process used from ideal conditions of manufacture of windings, under which the windings in the best case correspond to their nominal values, cause errors in the windings.

The direct basic reasons causing the appearance of errors in winding are considered in this article. The possibility of establishing a quantitative relationship between the production error and its cause is shown for several cases.

As a preliminary step we will briefly review the characteristics of systematic and random errors occurring in the manufacture of windings.

An error which, within the limits of the problem under consideration, remains constant or changes according to a law is called systematic. If, for example, the counter for the turns being wound, which transmits a stop signal to the machine after the winding of a given number of turns, has an instrument error of one revolution in 200, and if this error is not taken into account in setting up the machine, then all windings with a number of turns larger than 200 will be wound on

[redacted]

STAT

the machine with not less than one excessive coil.

An error which, within the limits of the problem under consideration, has different values (Bibl. 1) so that the time of its appearance and its exact value for each subassembly in the lot cannot previously be determined, is called a random error. Random errors are caused by the influence of factors whose regularity cannot be previously determined or by the influence of a large number of errors, which, even though subject to laws, enter into and disappear from the process at random.

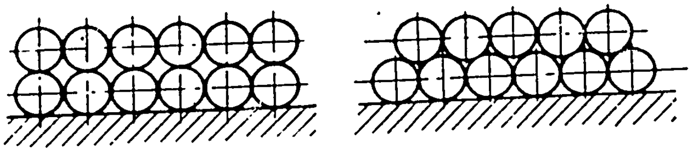


Fig.1 - Two Schematic Representations of Probable Correct Arrangement of Turns in a Winding

It would appear that the turns of wire in the winding are distributed correctly and evenly (Fig.1). Actually the turns arrange themselves with various, hardly detectable, deviations from the correct arrangement.

As shown in Fig. 2 the turns (1) and (2) of the second layer are located between the turns of the first layer, while the turns (3) and (4) are lying on top of the turns of the first layer. This could occur as the result of random factors: nonuniformity of the wire diameter, deformation of the wire, varying wire tension during the winding process, and other factors. The turns (3) and (4) are located higher than the turns (1) and (2) by the value Δr, and their perimeters are different. The turn (5) extends slightly above the plane of the second layer of coils; this may have been due to an elastic deflection of the wire, to insufficient tension in winding, to adherence of dust particles to the lacquer insulation of the wire, and to other reasons, For the same reasons, and possibly due to a deviation of the diameter and of the cross section of the wire or nonuniformity of the layer of the insulation, the turn (7) protrudes from the first layer of turns, entraining the turns (6) and (8) with which it is in contact. The coil (9) is raised since the gap between the preceding turns and the end of the body is too narrow.

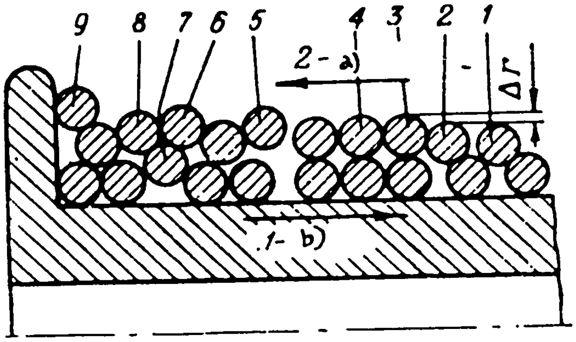


Fig.2 - Schematic Presentation of Possible Arrangement of Turns in a Winding

a) First layer; b) Second layer

There can be an infinite number of combinations of various arrangements and deviations of turns. Deformations of the cable and insulation are of great influence; of importance are also the angle of incidence of the lay of the turns, frictional forces between the turns and also frictional forces between the turns and the body, and other factors.

The transition to the second layer of turns during winding causes a change in the direction of lay of the turns and an increase in the length of the turn (Fig.3).

A change in the arrangement and in the density of lay of the turns is reflected in the overall size of the winding, the length of the wire, the ohmic resistance, and in other electric characteristics of the windings. Below, a typical example will be given for demonstrating the influence of nonuniformity of the lay on single-layer windings with a definite pitch.

The wiper of the potentiometer (Fig.4), after traveling past the first turn

STAT

immediately contacts turn (4), bypassing turns (2) and (3). This causes an error in the change of the ohmic resistance.

The position of the turns and their density of lay depends mainly on the method of winding. It was found that the density of lay of the turns in the slots of rotor windings of small electric machines is slightly higher if wound on winding machines with wire stays than on winding machines without wire stay although the difference is insignificant. Rotors wound by hand usually result in the highest density of lay of the turns.

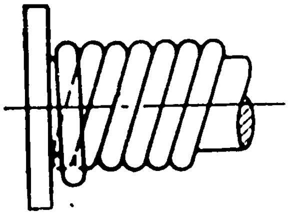


Fig.3 - Position of the Extreme Turn at the Point of Transition to the Next Layer of Turns

Therefore, due to various reasons influencing the quality of the windings during the process of winding, the windings obtained are not exactly uniform in terms of resistance, geometric dimensions, nonlinearity, and other characteristics.

Random errors occurring in different windings of a series being produced depend on the winding process and the combination of structural features of the windings; the method selected and the operating conditions during winding; nonuniformity of the diameter and of the mechanical properties of the wire; the quality of the insulation; and many other factors.

Direct basic reasons causing errors in the manufacture of windings are subdivided into two groups:

Group I - Errors caused by the winding materials;

Group II - Errors inherent in the winding process.

Of the causes of Group I, the following have a great influence on the accuracy of manufacture of windings:

1. Nonuniformity of diameter and of physical properties of the wire used in windings;
2. Low quality of the wire insulation;

STAT

3. Inaccuracies due to manufacture of the bodies.

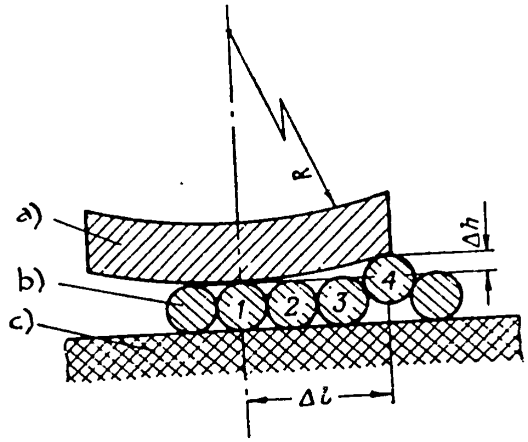
Reasons for errors caused by the winding process are:

1. Inaccuracies in the kinematic scheme adopted for obtaining the windings.
2. Geometric inaccuracy of equipment and apparatus.
3. Deformations of the wire, body, and machine parts.
4. Measurement inaccuracies during the winding process.
5. Inaccuracies in the construction of the winding machine.

ERRORS CAUSED BY THE WINDING MATERIALS

1. Nonuniformity of Diameter and Physical Properties of the Wire.

The experience of the winding departments of instrument-making plants shows that one of the basic conditions for obtaining windings which fully satisfy technical requirements and drawings in series production, is accuracy of dimensions and high quality of the wire. This is confirmed on the example for determining the influence of the variation in the parameters of the wire on its ohmic resistance



[illegible] - Touching of Turns of the Winding on Displacement of the Potentiometer Wiper

a) Wiper; b) Wire; c) Body

$$R=\frac{4\rho l}{\pi d^2}, \qquad (1)$$

where R is the ohmic resistance;

d is the wire diameter;

ρ is the resistivity;

l is the length of the wire.

For purposes of calculating the absolute error ΔR of the ohmic resistance we disregard small quantities of higher orders; setting the increment ΔR equal to a differential function of R, we obtain

$$\Delta R = \frac{4l}{\pi d^2}\Delta\rho + \frac{4\rho}{\pi d^2}\Delta l - \frac{8\rho l}{\pi d^3}\Delta d. \tag{2}$$

The value of the relative error for the ohmic resistance of the wire is equal to

$$\left|\frac{\Delta R}{R}\right| \leqslant \left|\frac{\Delta\rho}{\rho}\right| + \left|\frac{\Delta l}{l}\right| + 2\left|\frac{\Delta d}{d}\right|. \tag{3}$$

a) A considerable influence on the relative error of ohmic resistance of the wire is exerted by nonuniformity of its diameter. This is of especially great importance in the case of thin and very thin wire*.

The relative error of the ohmic resistance due only to nonuniformity of the wire diameter is equal to

$$\frac{\Delta_1 R}{R} = \frac{2\Delta d}{d}. \tag{4}$$

Equation (4) indicates that the relative error of the ohmic resistance for each rated wire dimension is different for the same absolute deviation of the diameter.

For copper wire of d = 0.1 ± 0.005 mm (according to GOST - Soviet State Standard - 2112-46)

$$\frac{\Delta_1 R}{R} = \pm\frac{2\cdot 0{,}005}{0{,}1} = +0{,}1.$$

Diameter variations of the wire within the limits of tolerance permitted by GOST may cause a change in the ohmic resistance of ± 10%, i. e., significantly more than the tolerance permitted for the overall ohmic resistance of many windings.

Equation (4) is based on a substitution of the differential dR for the increment ΔR, and the error caused by this is of practical significance in calculating $\frac{\Delta R}{R}$ for thin and very thin wire. More accurate results are given by a formula

---

*Wire of a diameter up to 0.02 mm is referred to as very thin or microwire.

whose use requires in every case that the direction of the diameter deviation from the rated value is taken into account; in the case of a positive deviation, the quantity Δd has to be taken with a plus sign and, in case of a negative deviation, with a minus sign.

$$\frac{\Delta_1 R}{R} = -\frac{2\frac{\Delta d}{d}}{1+2\frac{\Delta d}{d}}. \qquad (5)$$

In some cases, it is more convenient to use equation (5) in the arrangement

$$\frac{\Delta_1 R}{R} = -\frac{2\Delta d}{d+2\Delta d}. \qquad (5a)$$

For wires of a diameter of 0.05 ± 0.003 mm, the relative error of the ohmic resistance computed according to equation (4) equals ± 12%, and according to equation (5) is - 10.7 to ±13.6%.

Accurate calculations yield, correspondingly, values of -11 and ± 13.7%.

For accurate calculations, equation (4) should not be used. In several literature sources (Bibl.2), this is not taken into account.

According to GOST 2112-46 the permissible deviation of the diameter of copper wire is:

for diameters	up to	0,09 mm	. . . . . .	± 0,003 mm;
"	"	0,1—0,25 mm	. . . .	± 0,005 mm;
"	"	0,26—0,69 mm	. . .	± 0,01 mm.

The relative error of the ohmic resistance, caused by these deviations is shown on the graph (Fig.5).

The considerable magnitude of error caused by deviations of the wire diameter indicates that it is not possible (without making a sample winding and subsequent measurement) to make any automatic assumptions as to the ohmic resistance of precision windings (with small tolerances).

Measurements indicate that the nonuniformity of the diameter of samples taken from different spools of wire, received by the plant in one lot, is in many cases

STAT

[redacted]

less than that of the limiting values permitted by GOST. Consequently, the relative error of the ohmic resistance of the wire is also smaller. Nevertheless, the deviations of the ohmic resistance often exceed the permissible figures.

Investigations of a large number of samples of thin wire indicate that the variations of the wire diameter among spools of the same nominal diameter are subject to the laws of normal distribution. The absolute value of deviation limits of the wire diameter depends on the type and dimensions of the wire.

In the winding of multilayer (dense turns, one next to the other) slotted, and some other types of windings, uniformity of the layer of insulation and dimensional stability of the insulation along the length of the wire are of great importance. Usually, the overall size of the winding, nonlinear characteristics of a potentiometer, coefficient of space utilization in the slots, and other parameters, are dependent on the nonuniformity of the insulation layer.

For wire samples of the same diameter, taken from different spools, the mean-square deviation of the insulation diameter is usually somewhat larger than the mean-square deviation of the metal diameter.

Even within the limits of a single spool, the wire diameter is not exactly constant. According to our investigation, the wire diameter changes along its length irregularly within the limits of one coil.

b) The resistivity ρ of wire samples taken from different coils of the same type and diameter is not uniform. Slight variations in resistivity have even been observed within the limits of a single coil.

This must be taken into account in the design of windings and in planning the technological processes of production.

On the other hand, each type of wire has a probable mean value of resistivity which has to be assumed in calculations and design. Investigations performed by Graduate Student V. S. Loktayev showed that the resistivity value most often encountered in several types of annealed copper wire of a diameter of 0.15 mm, equals

[redacted]

STAT

0.017 ohm mm 2/m.

It will, therefore, be expedient to use this resistivity value instead of the maximum value of 0.01754 ohm mm ^{2}/m shown in GOST 2112-46, in the calculation and design of windings made of such wire.

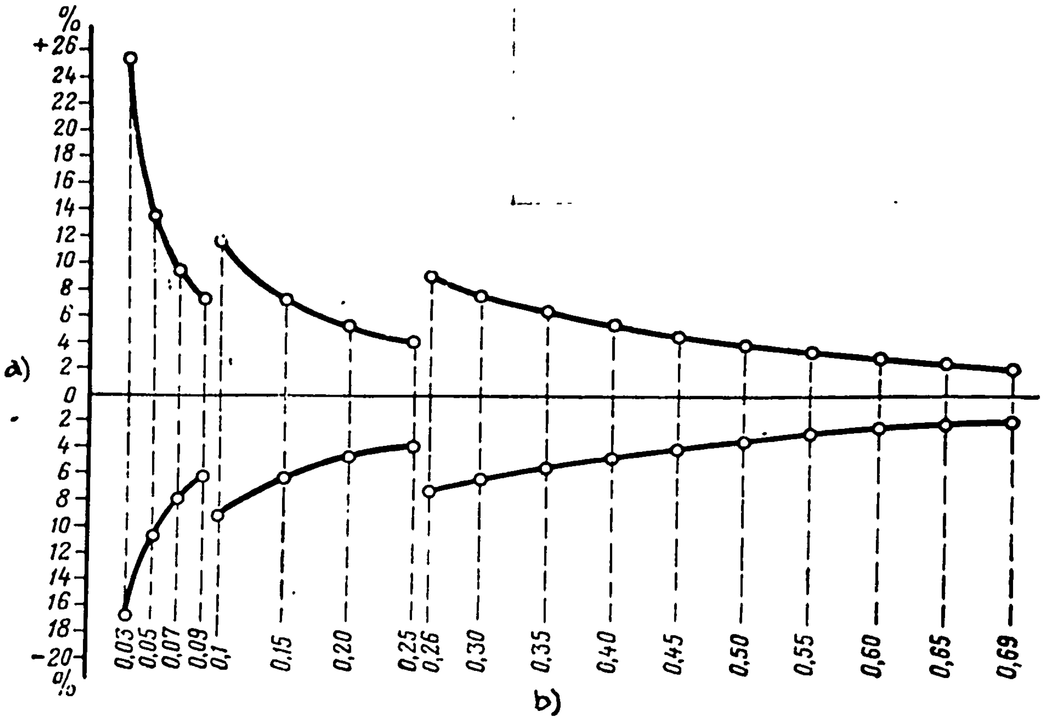


Fig.5 - Relative Error of Ohmic Resistance of Copper Wire as a Function of Diameter Tolerance Limits

a) Relative error of ohmic resistance; b) Wire diameter, in mm

Otherwise, the actual ohmic resistance of windings made of this given wire will on the average be 2 to 3% less than that calculated.

Figure 6 shows the curve of resistance deviations of samples of annealed copper wire PEL of a diameter of 0.07 mm and a length of 1 m, taken from different coils of a lot. The resistance variations reached ± 4.2% of the nominal value.

Investigations of wires of other diameters (from 0.05 to 0.5 mm) indicated that

[redacted]

the variations in resistance of annealed copper wire of a length of 1 m, on different coils of the same diameter are 3 - 8%. The resistance variation of 1 m of copper wire, within the limits of a single coil, did not exceed 3.5%. The resistance change of Nichrome and Constantan alloy wire of diameters up to 0.1 mm.within the limits of a single spool did not exceed 0.3%.

The sorting of wire according to its resistance is an excellent way of preventing extra work in connection with a trial winding operation in the manufacture of certain precision windings with close overall ohmic resistance tolerances.

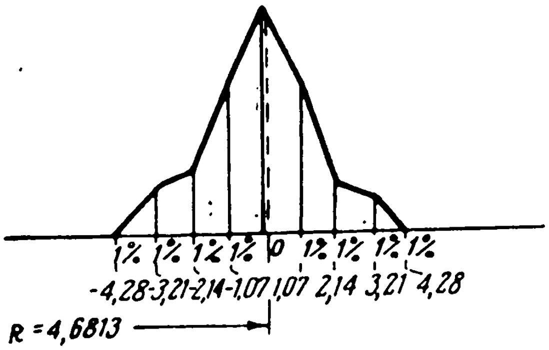


Fig.6 - Distribution Curve of Resistance Deviations of Samples of Copper Wire PEL of a Diameter of 0.07 mm and Length of 1 m, Taken from Different Coils of the Lot

c) The reasons for a change in the length of the wire in windings may be as follows:

Elongation due to pulling of the wire during the winding process; varying density and nonuniformity of lay of the turns; inaccurate number of turns; variations in shape and dimensions of bodies (also including thickness variations in insulating spacers); nonuniformity of wire diameter.

The last two reasons cause a change in the perimeter of the coils and the influence of these reasons on the ohmic resistance does not lend itself to calculations.

2. Low Quality of the Wire Insulation

During the production of the windings it is also important to obtain a high quality insulation of the wire for the coils. The quality of the insulation is checked for strength and insulation resistance according to GOST - Soviet Govern-

[redacted]

ment Standard - 2773-51 and checked also for short-circuiting of the coils.

The insulation can be damaged during the production of the wire, as well as during the process of winding.

During the production of wire the following defects in the insulation are possible: gaps, i.e., places not covered by enamel, damp enamel which becomes easily detached from the metal, uneven enamel with bubbles and various defects, cracks, and other flaws.

Enameled wire of diameters up to 0.35 mm, is inspected not only for breakdown strength, but also for detecting damage points. Damage point is the designation for places in the enamel cover with a lower dielectric strength of the insulation, i.e., spots which will not withstand a voltage of 60 v. Even with good eyesight, damage points cannot be seen, but they can be determined by running the wire through a mercury bath under direct current of a voltage of 60 v. For this purpose, one lead from the power source is connected to the mercury bath, and the other lead to the wire being inspected. In the presence of damages of the insulation, a closed electrical circuit is created and a signal lamp, connected in series in the circuit, lights up. The setup for inspection of the insulation is equipped with a special counter, which records the number of defects over a length of 15 m established by GOST - Soviet State Standards. The wire being passed through the mercury bath is wound from one spool onto another.

In practice, it is very difficult to produce wire without a single puncture in a length of 15 m, and therefore GOST and technical specifications for every type of wire establish a permissible number of defects. For example, for PEL wire of diameters of 0.05 - 0.35 mm, the permissible number of damage points for a length of 15 m should not exceed 10.

Cases of damage to the insulation during the process of winding are comparatively few. The insulation in most cases is damaged as a result of contact with fins or protruding parts, and also due to roughness of parts of the machine through

which the wire passes in winding.

Sharp parts of the body and burrs remaining on grooves of the stack may also tear or scratch the insulation. Aside from this, the insulation may be considerably deformed in these places, since, as a result of very small radii of curvature, it is subject to large specific stresses, as indicated by the equation

$$q = \frac{P_n}{R}, \tag{6}$$

where: q is the specific pressure of the coil on a unit length of the perimeter of the cross section of the frame,

P_n is the tensile force established during winding,

R is the radius of curvature of the frame.

Damage to the insulation due to pulling of the wire in the process of winding is fairly rare. Experiments have shown that in the process of pulling the wire to its proportional limit no damage to the insulation occurred. The PEL insulation was sufficiently elastic and stretched together with the metal without damage, up to tensions exceeding the yield point.*

Viniflex insulation of the PEV wires was stronger and more elastic and failed far beyond the yield point, much closer to the point of failure of the wire on the stress-strain diagram. The tensile strength of copper wire in the winding process is limited only by its mechanical properties.

3. Inaccuracy in the Production of Frames

All windings are obtained by means of winding wire on frames or forms (in the case of coreless windings). The shape, dimensions, and material, of the frames are determined by the structural requirements of the windings.

a) Deviations in shape and dimensions of the cross section of the frames from their calculated values have an influence on the dimensions and electrical charact-

---

* See transactions of MATI, No. 22.

eristics of the windings.

Deviations of the shape and dimensions of the frames can be of various kinds, including deviations of the geometric configurations of the second order (eccentricity, non-parallelism of the sides, etc.).

The category of errors under consideration includes changes in shape and dimensions of the frames caused by slack contact and variations in the thickness of the layers, which insulate the frame or its individual parts (slots). Variations in the cross-sectional areas of the frame are also taken into account below (see infra) in the example where errors of ohmic resistance during winding onto a frame of rectangular cross section are calculated.

Deviations of dimensions and of the shape of the frame, due to mechanical processes, are in most cases subject to the law of normal distribution. Inaccuracies in stamping the frames cause a systematic error. For precision windings manufactured in large quantities, the normal change of dimensions caused by the wear of the punch and die has to be taken into account. In the case of wear, the systematic error will be variable.

Manufacturing errors of parts molded from plastics depend on: the molding material used; tolerances in the preparation of the parts of the mold; wear of the surfaces of the mold; thickness variations of the lining; deposits of loose particles in the mold, and other reasons.

The largest influence on the manufacturing accuracy of frames is the variation in the setting time of the molding compound, which depends on its moisture content, the amount of volatile matter, the quantity of filler, and the conditions in molding.

Variations in the setting time of the molding material are subject to the law of normal distribution; variations in the dimensions and shapes of frames molded from plastics vary similarly.

b) The influence of residual internal stresses. The cold-working in the frames due to stamping and other operations should be eliminated. If the cold-working is

[redacted]

not completely removed, the frame will deform in time, as a result of equalization of internal stresses, leading to a change in the accuracy of the finished windings. The material of the frame manufactured from thermosetting plastics by means of a molding process should undergo complete polymerization.

The process of transition of the plastic into stage C - into a state of complete hardness - may cause some warping and change in dimensions and shape of the frames. If this process is not fully completed prior to winding, the dimensions of the frame may change with time, disturbing the required accuracy of the windings. For complete polymerization, the frames are sometimes heated in oil at a temperature of 120 to 140°C for 2 - 3 hrs.

c) Various defects of frames made of plastics. The most characteristic defects of frames are: porosity, bubbles, depressions, cracks, discoloration of various sections, low electric quality, etc. In most cases, these defects are due to lack of observation of proper molding practices, inaccuracies in the proportioning of the molding compounds, their unsuitability, and disregard of proper techniques.

All plastics and molding powders destined for frames should satisfy specific technological requirements, the most important of which are: flowability, rate of hardening, moisture content, set, grain size, and absence of extraneous admixtures. Deviation of the properties of plastics and molding powders beyond the limits established by the technical requirements causes a change in dimensions and physical properties of the core material (dielectric losses, thermal stability, thermal conductivity, and others).

The flowability of molding powders, i.e., the ability to fill all cavities in the mold under the action of temperature and pressure, is of greatest importance. Molding powders of low flowability inadequately fill the working space of the mold so that the molded product does not completely correspond to the required shape. On the other hand too much flow of the molding powder is the cause of the appearance of flash and sharp protrusions, as a result of the flow of the plastic into

[redacted]

gaps between the parts of the mold.

d) Frames for magnetic circuits. Magnetic circuits should have minimum losses from hysteresis and eddy currents, high magnetic permeability, considerable coercive force, maximum stacking factor, and the required shape and dimensions. Of great influence on the quality of magnetic circuits are the materials and their heat treatment. The production of magnetic circuits, starting with the stamping of laminations and ending with the assembly, requires high accuracy and conscientious execution.

Laminations are produced to accuracies of the second and third class. The accuracy of location of the slots around the circumference of the laminations is usually not above a tolerance of ± 10 to 15'. In the stamping of laminations, the magnetic properties of electrotechnical steel and iron-nickel alloys deteriorate due to internal stresses. The deformed zone with sharply decreased properties extends around the perimeter of the stamping to a distance of 0.5 to 4 mm, depending on the magnitude of the clearance between punch and die, and also on the thickness and the brand of steel. In assembly, the laminations are distributed in a specific fashion relative to one another, since in many steels used for magnetic circuits an inductive anisotropy may be present.

The stacking factor, i.e., the relationship of the actual volume of the steel or alloy of the part to the total volume of the part, varies in practice from 0.80 to 0.97 and depends on the surface finish of the laminations or strip, the amount of their bending, the absence of burrs on the internal and external perimeters, uniform thickness of the material, and the degree of compression of the part by clamps. A decrease in the stacking factor to 0.8 - 0.85, at constant volume of the part, causes a decrease in the number of laminations, and as a consequence a considerable increase in the number of ampere-turns required for magnetization, while at the same time increasing the losses in the part.

The assembly of frames from laminations of iron-nickel alloys requires special precautions since not only impact, but also compression, elongation, and bending of

STAT

the laminations cause the appearance of residual stresses which lower the magnetic properties of the parts. In the case of exertion of pull on the laminations, the tractive force should not exceed the compressive limit at which the magnetic qualities of the stacks start to decrease.

A number of special investigations to determine the influence of errors in the frames on the accuracy of production of windings is required.

ERRORS DUE TO THE PROCESS OF WINDING

1. Inaccuracies in the Kinematic Arrangement for Forming the Windings

Errors of this type result from conscious deviations from a theoretically correct process arrangement for forming windings and substitution of an approximately equal arrangement.

Example I. In order to obtain the required pitch of the winding, a specific gear ratio in the kinematic linkage between the spindle of the machine and the spreader attachment is required. Frequently, it is difficult to obtain exactly this gear ratio (for example, this may require the making of special gears); in such a case, a gear ratio close to the calculated one is used, which may easily be obtained on a given machine by means of an approximate selection of intermediate gears between the spindle and the spreader attachment.

The difference between an approximate gear ratio and its theoretical value causes an error in the pitch of the assembly being wound.

Example II. In manual assembly, all sections of many two-layer slotted windings are, as a rule, first inserted into the slots of one side of the section, where each of these sides is placed in the lower part of the slot and then is placed in the other side of the section, where it occupies the upper part of the slot. Due to this, all sections of the winding are obtained in a uniform and symmetric manner. Using the described method for mechanical assembly is very difficult. In producing

[redacted]

such windings on machines, both sides of the sections are put into the slots simultaneously, so that the resultant sections are not uniform in length and are not symmetrical. Some sections have both sides located in the lower parts of the slots, other sections have only one side in the lower part of the slot and the other side in the upper part and, finally, still other sections have both sides in the upper parts of the slots.

Substitution of an approximate arrangement for the theoretical arrangement in the forming of windings should be resorted to when the exact arrangement, under the given conditions, can only be obtained with difficulty and is not economically justified. A similar substitution is permissible if the sum of errors of the approximate arrangement in winding does not exceed the tolerances shown in the drawings of the windings to be produced.

Inaccuracies of the basic kinematic diagram for forming windings will cause systematic errors, which can be calculated.

2. Geometric Inaccuracy of Equipment and Attachments

The precision of winding equipment and attachments, in contrast to metal working machines, has never been thoroughly investigated. There are no precision standards for winding machines and their assemblies. Data on the precision of machines in use is very scarce.

However, in many cases the accuracy of the winding processes depends largely on the winding equipment and attachments.

The following are primarily of influence on the accuracy of the winding operations:

a) Inaccuracy in Operation of Spreader Attachments. Spreader attachments determine the correctness of lay of the turns along the core, i.e., the accuracy of the winding pitch.

Reasons for inaccuracies in operation of the spreader attachment may be imper-

[redacted]

STAT

fections in the construction of the attachments and errors in the production of parts of the kinematic linkage of the machine, connecting the spindle and the spreader attachment.

Errors in the kinematic linkage may be due to the following:

Inaccuracy of pitch of the feed screw or of the cam profile driving the spreader. If the inaccuracies of the lead screw are fully transferred during threading to the pitch of the thread (i.e., in the case of tight fits), these inaccuracies are of considerably less definite influence during the winding process.

Inaccuracies in the production and mounting of gears of the linkage. These inaccuracies cause errors in the pitch of the part being wound, as a function of the period determined by the change in the gear ratio. The magnitude of error in the winding pitch is less than the corresponding inaccuracy in the cut of the gear teeth.

b) <u>Inaccuracy in the Performance of Devises Regulating the Tension.</u> Tension of the wire during the process of winding is the basic technical factor determining the resistance and overall shape of the windings and also the accuracy of lay of the pitch. Tension of the wire during winding is produced by a tension-regulating attachment.

The purpose of the tension-regulating attachment is not only the creation of the required tension on the wire, corresponding to its mechnical properties and the technical requirements of the winding, but also the maintenance of this tension within predetermined limits during the entire process of winding of the first as well as of the last windings obtained from the same spool of wire.

The tension force during winding should not exceed the force at which permanent elongation of the wire causes considerable changes of its electric properties and dimensions, exceeding those permissible for the given wire and the part being wound*. The tension force should guarantee the required shape and density of the winding, and also high-quality performance of the part. In factories, the amount of

---

* See transactions MATI, No. 22.

STAT

[redacted]

tension on the wire is established by the setup man, on the basis of experiments. Usually a completely even winding, without torn wires or excessive elongation, serves as a criterion.

In rare cases, the specifications for obtaining the necessary electric characteristics in all windings of a given series are used as a guide in establishing the tension on the wire.

At the plant of one instrument manufacturer, it was found that the magnitude of tension on the wire during winding of the same parts on a machine SNYa-4 varied within considerable limits, deviating from the mean value by 20 - 25%. This was reflected in the uniformity of the electric characteristics and overall contours of the windings produced.

Some of the tension-regulating attachments used in instrument-making plants do not completely satisfy the specifications and can only be used for a very narrow range of sizes and makes of wire.

In one plant, it was impossible for a long time to produce windings of armatures with PEV-2 wire of a diameter of 0.31 mm on the semiautomatic machine SNYa-1. After the tension-regulating attachment on the machine SNYa-1 was equipped with an additional braking attachment, the winding of the armatures became successful; at present, such windings are produced mechanically at a high output rate.

The braking attachments of many winding machines have no regulators. Such attachments produce large variations in the tension of the wire during the process of winding, which is nonuniform (especially during the winding of noncylindrical cores) and is accompanied by large inertia loads and elongations of the wire. Furthermore, the tension force also changes as a function of the amount of wire unwound from the spool.

c) <u>Inaccuracies in the Control Mechanisms of the Machine and of the Braking Attachments.</u> The main purpose of these mechanisms is shutting off the electric drive of the machine and simultaneously actuating the braking mechanism, upon completion

[redacted]

STAT

of the winding of the required number of turns. The mechanisms of modern winding machines receive the actuation signal from a counter which counts the number of turns wound and whose pointer or lever, at the proper instant, actuates a microswitch included in the electric circuit of the machine. Delay or premature actuation of the mechanism controlling the machine causes inaccuracy in the number of turns wound.

Inaccuracy in the operation of the mechanism may be due to inaccuracies of the counter. Certain braking attachments do not guarantee a sufficiently accurate stopping of the machine: depending on the inertia, the spindle undergoes a long deceleration or stops without reaching the position required for stopping.

3. Deformation of the Wire, Body, and Machine Parts

Deformation of the wires is of prime importance in the accuracy of the windings produced. In any type of winding operation, the wire is deformed. Deformation of the cores is observed in the case of low rigidity in the direction of tension of the wire or in the case of seizing during the clamping process. Deformation of machine parts occur relatively seldom: only during the winding of thick wire of high rigidity.

Deformations during the process of winding occur under the influence of wire tension required for bending it and for close contact with the frame, and also under the influence of inertia loads. In practice, it is difficult to obtain a winding process without accelerating the movement of the wire. Therefore, inertia loads appear to some degree at all times*.

a) Deformation of the Wire. During winding, the ohmic resistance of the wire increases due to the fact that the outer fibers of the wire, when bent around the core, undergo plastic deformation and also due to some work-hardening of the wire

---

* Reasons for acceleration during the process of winding and formulas for calculating accelerations during winding on cores of various cross sections are included in the Transactions MATI, No.22.

STAT

which, after being unwound from the spool, is straightened and bent several times on straightening rolls and the spreader mechanism of the machine. It was established that, during the winding of annealed copper wire of a diameter of 0.05 to 0.5 mm at the minimum tension required to bend it around a cylindrical core and at very low winding speeds, the ohmic resistance increases by 1 - 2% of the mean value of the resistance of the wire.

Inertia overloads, in turn, may cause elongation of the wire and as a consequence increase its ohmic resistance which is nonuniform in different windings of the series. The scatter of the ohmic resistance of the windings may be determined by experimental means.

The influence of elastic deflection of the wire (turn 4) on the nonlinearity of the potentiometer is illustrated in Fig.4.

b) Deformation of the Cores. Thin plates, shafts, and other bodies of low rigidity bend during winding under the action of the tension force of the wire. Deformation is possible also due to the forces in tightening when, for example, the cross-sectional area of the core has the shape of a thin wall bushing or of a box, assembled from thin sheets. If the bending of the cores takes place within the limits of elastic deformation, they resume their original shape upon completion of the process of winding or upon release of the cores from pressure, which may influence the distribution of the turns of the winding.

The deformation of the cores in many cases may be determined by calculation.

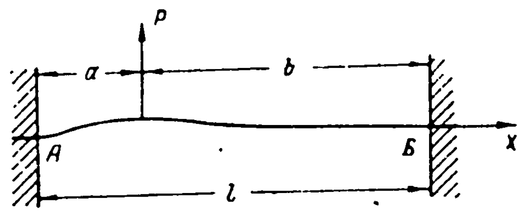


Fig.7 - Schematic Representation of the Action of Forces during Winding upon a Shaft with Fixed Ends

[redacted]

During the winding upon a shaft with fixed ends (Fig.7), the bending of any given point of the core may be determined by the following equations for an elastic line:

Along the length from A to P

$$y=\frac{Pb^2x^2}{6EIl^3}(3al-3ax-bx); \qquad (7)$$

Along the length from P to B

$$y=\frac{Pa^2(l-x)^2}{6EIl^3}[3bl-(3b+a)(l-x)], \qquad (8)$$

where P is the tension force of the wire during winding, in kg;

E is the modulus of elasticity of the material of the core in kg/cm^2;

I is the moment of inertia of the cross section of the core in cm^4.

The maximum deflection due to bending of the core

at $x=\frac{2al}{3a+b}$ (for $a>b$)

$$f_{\max}=\frac{2}{3}\frac{P}{EI}\frac{a^3b^2}{(3a+b)^2}, \qquad (9)$$

at $x=l-\frac{2bl}{3b+a}$ (for $a<b$)

$$f_{\max}=\frac{2}{3}\frac{P}{EI}\frac{a^2b^3}{(3b+a)^2}. \qquad (10)$$

If the body has the shape of a plate (Fig.8) and the ratio of its sides is $\frac{t}{h}=n>4$, then in determining the displacement of the various points of the core it is necessary to take torsional deformation into account in addition to bending.

The angle of twist (in radians) of the cores (Fig.9) in any given cross section perpendicular to the axis may be determined from the formula

$$\varphi_B = \frac{Mba}{lGI_{\kappa}}, \qquad (11)$$

where M is the torque in kg-cm. $M = P\frac{t}{2}$;

G is the modulus of shear in kg/cm^2;

I_K is the geometric modulus of rigidity in torsion, in cm^4.

$$I_{\kappa} = \frac{(m - 0{,}63)\,h^4}{3}, \qquad (12)$$

where

$$m = \frac{t}{h}.$$

The angle of twist of any given cross section of the core to the left of the section CC' may be determined on the basis of similiarity of the triangles ACC' and ADD' and, correspondingly for cross sections to the right of the section, from the triangles BCC' and BEE'.

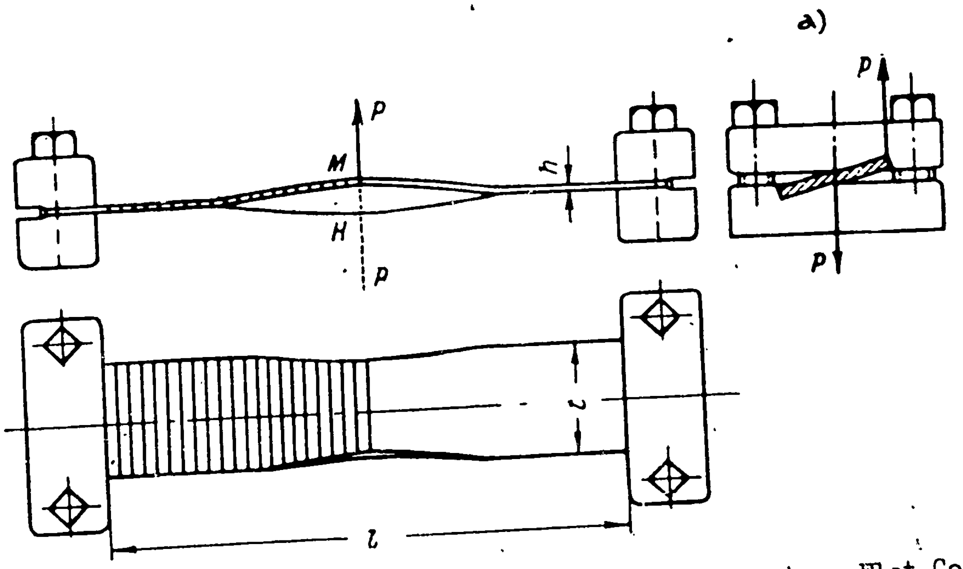


Fig.8 - Diagram of Action of Forces during Winding onto a Flat Core with Reinforced Ends. a) Section through HM

During the action of a torque applied at the center of the body (see Fig.8) the twist of the center section is determined from the formula

$$\varphi_{\max}=\frac{Ml}{4GI_{\kappa}}. \qquad (13)$$

For determining the linear displacement of points located at a distance z from the axis of the core we use eq.(11), obtaining

$$c=\varphi z=\frac{Mbaz}{lGI_{\kappa}}. \qquad (14)$$

The linear displacement of the points on the fins of the core is determined after making the substitution $z = \frac{t}{2}$

$$c_{p}=\frac{Mbat}{2lGI_{\kappa}}. \qquad (15)$$

For determining the total displacement (Fig.10) of the points of a given cross section of the core, the bending determined according to eq.(7) or eq.(8), has to be added to the linear displacement computed according to eqs.(14) or (15)*.

The maximum total displacement depends on the ratios of the bending and twisting moments acting on the body. In order to determine the cross section at which the total deflection is at a maximum, it is in every case necessary to perform calculations and to construct deformation diagrams for bending and twisting.

It should be mentioned that the deformation of the cores obtained by means of eqs.(7) - (15) are approximate, since the dynamic effects during winding and certain (always possible) deviations of actual conditions in fastening of the parts from theoretical ones are not considered in the calculation.

c) <u>Deformation of Machine Parts.</u> Under the action of forces during winding, such deformations may occur as a consequence of insufficient rigidity of the parts

---

* For simplicity in the given case, we do not consider bending of the core along its plane cross section.

STAT

and assemblies of the machine, and also due to deformation of abutting surfaces between the parts (contact deformations). Calculation of deformations of parts or assemblies of the machines may be performed according to formulas of the strength of materials and elastic theory.

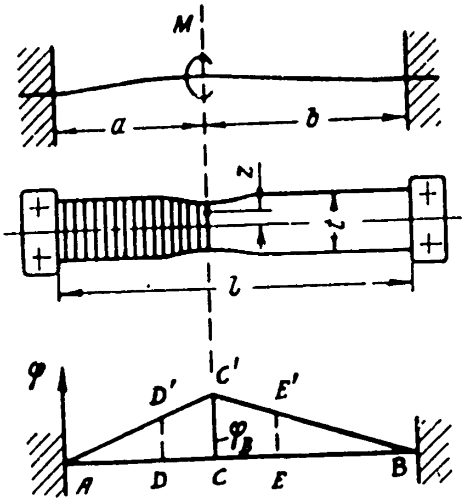


Fig.9 - Diagram of the Angles of Twist of Cross Sections of Plane Cores in Winding

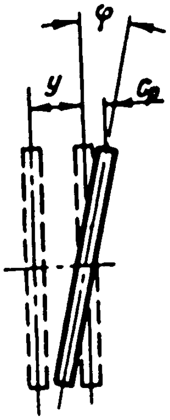


Fig.10 - Total Displacement of the Cross Section of a Plane Core due to Bending and Twisting

In a winding machine, deformation of sleeves and of face plates used for strengthening its frame, and bending of the spreader attachments was observed.

4. Inaccuracy of Measurements in the Process of Winding

Inaccuracies in measurements of the parameters of the windings, creating the wrong impression as to the quality of their production, may appear as a consequence of the fact that the testing instruments used during the production of the windings have inherent errors and that during the use of these instruments certain errors caused by subjective evaluation of the readings may appear.

The number of turns wound is read on a counter in which instrument errors are inherent. The number of divisions in counters with circular scales is insufficient.

[redacted]

Many winding operations are not performed with methods which automatically result in the required parameter of the windings. For example, during the production of certain types of windings, the worker stops the process of winding only after a trial measurement of the wound assembly has indicated that the winding has the necessary ohmic resistance. The accuracy of measurement of the ohmic resistance of the winding depends on the class of accuracy of the control instruments and the method of measurement.

The density and overall shape of the windings, and sometimes their ohmic resistance, are largely determined by the tension of the wire during winding. As a rule, the tension of the wire is determined "by eye", by experience, and by trial measurement of the first windings in the series produced first piece inspection.

On certain winding machines, special gram-meters for control of the tension of the wire are attached. They have considerable instrument errors, and their readings are being made difficult by strong fluctuations of the pointer, caused by inertia loads and vibrations.

5. Inaccuracies in Design of the Machine

Work on tooled-up machines is the most popular way of obtaining certain parameters. The method of trial winding is only used in cases where the method of work of the setup is, for some reason, not usable, for example, where the zone of scatter of the ohmic resistance of the wire within the limits of the spool exceeds the permissible region of ohmic resistance of the winding.

Winding machines are set up simultaneously for several parameters: for pitch or density of lay of the winding, for the number of turns, etc.

During setting up of winding machines it is necessary:

To regulate the tension mechanism of the machine for a given wire;

To correctly select the speed of winding and set the gears of the machine for that speed. Winding speeds which are too high cause rupture of the wire or unaccep-

STAT

tably excessive elongations of the wire. Low speeds affect the productivity of the machine. The speed depends upon the type and diameter of the wire and upon the shape and dimensions of the core;

To regulate the feed of the wire along the axis of the core for each turn of the spindle of the machine;

To regulate the starting and braking mechanisms of the machine;

To regulate the interaction of auxiliary mechanisms of the machine, which turn the part at the pitch for winding the following sections, which eject the loop for soldering of the wire to the collector and which serve other functions.

Despite preparatory setup of the machine the operator is required to regulate the work of the machine in a number of cases. For example, if the starting mechanism does not give a sufficiently smooth start, the operator is required to brake the flywheel of the spindle by hand during starting of the machine. With existing, not completely satisfactory, tension attachments the operator is often required to change the setting of this attachment during the process of winding, i.e., to change the magnitude of the braking moment according to the amount of wire unwound from the spool.

Inaccuracies in setups of the winding machines (similarly as in mechanical work) influence the quality of the produced assemblies. For example, our investigations of winding processes for PEL copper wire of a diameter of 0.05 mm on a core of square cross section indicated that, in cases of unsatisfactory regulation of the tension attachment (which could be seen on an oscillogram), the elongation of the wire and its ohmic resistance increased sharply.

The magnitude of tension on the wire is in most cases established on the basis of experience and "feel" of the foreman and operator. To date, there are no accurate and objective methods and instruments used in the factories to determine the magnitude of tension on the wire during the process of its winding. No methodology of setting up winding machines is used.

[redacted]

The development of such materials should be started.

Very important in setting up winding machines is the proper selection and establishment of the tension of the wire and of the speed of winding, and also that the center of the field of distribution of each parameter should be so regulated for the entire lot of windings produced, as to coincide with the center of the field of tolerances of the corresponding parameter.

Examples of determining relative errors of ohmic resistance of a flat potentiometer, caused by inaccuracy of preparation of the core and wire and by deviations in the density of the lay of the turns, are cited below. The resistance of the potentiometer (Fig.11) is equal to

$$R=\rho\frac{l}{S}, \qquad (16)$$

where ρ is the specific resistivity;

l is the length of wire;

S is the cross section of the wire $(S = \frac{\pi d^2}{4})$;

d is the diameter of the metal in the wire.

Substituting l = nw, we obtain

$$R=\rho\frac{nw}{S}=\rho\frac{2n(b+y+2d_1)k_1}{S}, \qquad (17)$$

where n is the number of turns,

w is the length of the turn*

$$w=2(b+y+2d_1)k_1; \qquad (18)$$

---

* The increase in length of the turn, due to the inclination of the turns is omitted, since in our case (dense winding of thin wire) this increase represents hundredths or thousandths of a percent.

STAT

d_1 is the diameter of the wire across the insulation

$$d_1=k_0d; \qquad (19)$$

k_0 is a coefficient allowing for insulation thickness;

k_1 is a coefficient allowing for loose fit between the turns and the cores (Fig.12) and dependent on the diameter and the elastic properties of the wire and its insulation, the tension force during winding, the dimensions and relationship between the sides of the cross section of the core, the radii of curvature of its fins and other factors.

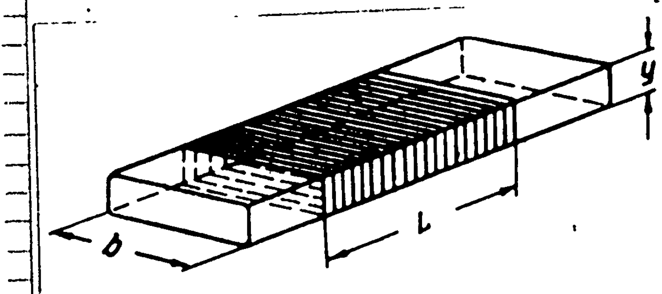


Fig.11 - Diagram of the Lay of Turns on a Flat Potentiometer

$$n=Ln_0, \qquad (20)$$

where n_0 is the number of turns per unit length of the winding.

$$n_0=\frac{1}{d_1k_2}, \qquad (21)$$

where k_2 is the coefficient of density of lay of the turns.

For our case we assume: k_2 = 1.03 to 1.08. Performing the substitution of n, S, and d_1 according to eqs.(20), (21), and (19), we obtain the following relation from eq.(17):

$$R=\rho\frac{8(b+y+2d_1)Lk_1}{\pi d^3k_0k_2}. \qquad (22)$$

For calculating the absolute error of the ohmic resistance Δ R, we disregard the magnitude of small quantities of a higher order; equating the increment ΔR to

[redacted]

the differential function R, we obtain

$$\frac{\Delta R}{R}=\frac{\Delta\rho}{\rho}+\frac{\Delta b+\Delta y+2\Delta d_1}{b+y+2d_1}+\frac{\Delta L}{L}+\frac{\Delta k_1}{k_1}-\frac{3\Delta d}{d}-\frac{\Delta k_0}{k_0}-\frac{\Delta k_2}{k_2}. \qquad (23)$$

The difference between 2 Δd_1 and 2 Δ d is small in comparison with $\Delta b + \Delta y$, and the difference between $2d_1$ and 2d is small in comparison with b + y; therefore, in the second term, d may be substituted for d_1. For more accurate determination of the influence of variations in the diameter of thin wire on the magnitude of the ohmic resistance, the following has to be used in place of the term $-\frac{3\Delta d}{d}$:

$$-\frac{3\,\frac{\Delta d}{d}}{1+3\,\frac{\Delta d}{d}}=-\frac{3\Delta d}{d+3\Delta d}. \qquad (24)$$

In eq.(23), this term is obtained with a negative sign if the diameter of the wire is less than nominal. We then obtain

$$\frac{\Delta R}{R}=\frac{\Delta\rho}{\rho}+\frac{\Delta b+\Delta y+2\Delta d}{b+y+2d}+\frac{\Delta L}{L}+\frac{\Delta k_1}{k_1}-\frac{3\Delta d}{d+3\Delta d}-\frac{\Delta k_0}{k_0}-\frac{\Delta k_2}{k_2}. \qquad (25)$$

The maximum relative error of ohmic resistance of the potentiometer is determined by

$$\frac{\Delta R}{R}=\frac{|\Delta\rho|}{\rho}+\frac{|\Delta b|+|\Delta y|+|2\Delta d|}{b+y+2d}+\frac{|\Delta L|}{L}+\frac{|\Delta k_1|}{k_1}+ \\ +\frac{|3\Delta d|}{d+3\Delta d}+\frac{|\Delta k_0|}{k_0}+\frac{|\Delta k_2|}{k_2}. \qquad (26)$$

Here, Δ d in the denominator of the term $\frac{3\Delta d}{d+3\Delta d}$ has a negative sign if the diameter of the wire is less than nominal.

As was shown in the beginning of the article, many primary errors of winding processes are random and depend upon the variation of one or several factors. These errors can be characterized by means of corresponding distribution curves.

For determining the resultant error of the controlling characteristic for wire of a certain type and dimensions, the character and magnitude of the constant errors has to be known and, for random errors, also the law of their distribution.

Compared to the mechanical working of metals, the field of production of windings still has insufficient empirical data for the establishing of guide lines on the calculation of winding processes for accuracy.

For practical calculations of the accuracy of technological processes in the production of windings, it is necessary to observe a large number of typical winding operations and to evaluate the laws of distribution of individual type of errors under various conditions of winding, on the basis of observation data.

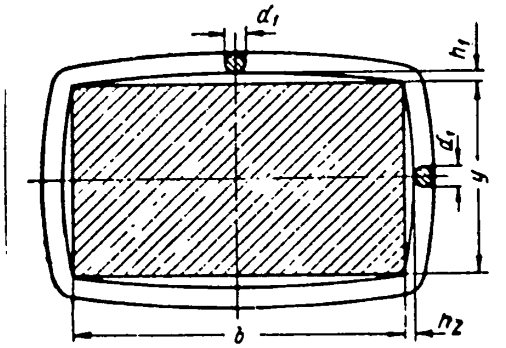


Fig.12 - Section of the Core of a Potentiometer with Applied Turns

This, in turn, permits a determination of individual deviation limits of given errors, at a certain selected level reliability.

In addition, it is necessary to determine the character of the relationship of the centers of the groups of these errors of their systematic elements with the factors to be controlled.

The assembled statistical material permits the establishment of norms of accuracy for typical winding operations.

We believe that statistical and mathematical-analytical methods for the determination of resultant errors in the production of windings will soon find wide application.

[redacted]

BIBLIOGRAPHY

1. Sokolovskiy,A.P. - Calculations of Accuracy of Work in Metal-Cutting Machines. Mashgiz (1952)

2. Bulovskiy,P.I. and Povalyayev,A.V. - Technology of Production of Parts and Assemblies of Electric Measuring Instruments. Mashgiz (1952)

3. Laktayev,V.S. - Investigation of the Technological Process of Production of Windings for Small Motors. Candidate's dissertation, MATI (1954)

4. Chumakov,V.P. - Two Basic Conditions of the Winding Operation. Volume on "Present-Day Problems of the Technology of Instruments". Transactions MATI, No. 22, Oborongiz (1954)

5. Yevteyev,F.Ye. and Zhukov,V.A. - Technology of Radio Equipment. Gosenergoizdat (1952)

6. Chumakov,V.P. and Nikolayev,N.Ye. - Scientific Investigation of the Chair of "Technology of Aircraft Instrument Manufacture". MATI, 1954 (unpublished).

[redacted]

STAT

ACCURACY AND PHYSICAL INTERCHANGEABILITY OF PICKUP ELEMENTS

by

Engineer D. A. Braslavskiy

Modern remote-indicating instruments and automatic devices contain a large number of pickup elements, utilized in the conversion of a measured or regulated (input) physical quantity into another (output), which is more convenient for transmission over a distance, for amplification, or for starting motion of a recording device.

The most common elements are represented in Table 1, where the horizontal columns show the input parameters, and the vertical ones the output parameters*.

One of the most effective means of decreasing the cost of mass production and of simplification in the use of complex instruments and automatic device is maximum interchangeability of their parts, subassemblies, and pickup elements.

While a sufficient specification as to interchangeability of parts and mechanical subassemblies is the retention of their geometric shape and dimensions within the limits of permissible tolerances, the interchangeability of pickup elements also requires maintenance of a given accuracy in its characteristics, expressing the functional dependence between input and output in physical quantities (Fig.1).

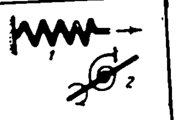
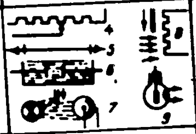
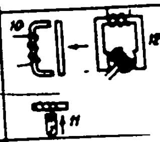
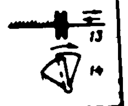
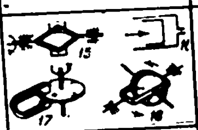
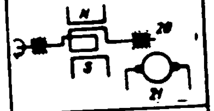
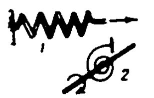
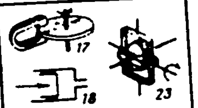
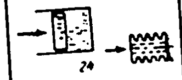
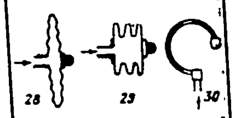
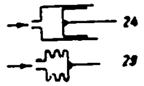
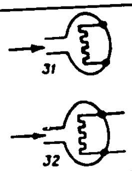
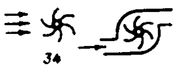
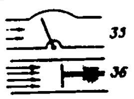
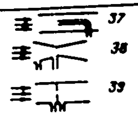
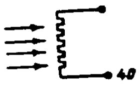
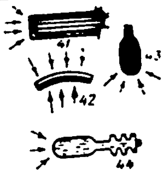
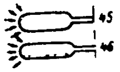
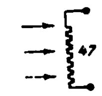
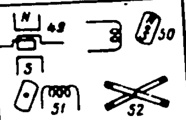
The term "physical interchangeability" originates from this, and is understood to represent an interchangeability of elements according to their physical charact-

---

* For more detailed information on the function of pickup elements shown in Table 1, see the book by D. A. Braslavskiy, S. S. Logunov and D. S. Pelpor (Bibl.3).

STAT

Table 1
Pickup Elements of Measuring Instruments

Input parameters	Output parameters							
	Mechanical parameters				Electrical parameters			
	Displacement	Velocity	Force Moment of force	Liqu. or gas pressure	Electric Resistance	Inductance, Mutual ind.	Electr. capacit.	Electromotive force
Displacement								
Velocity								
Acceleration								
Force, moment of force								
Liquid or gas pressure								
Velocity, consumpt. of liquid, gas								
Temperature								
Electr. Current								

eristics.

In an analysis of the accuracy and of the physical interchangeability of a pickup element, not only its shape and dimensions but also the physical parameters of the materials from which this element is produced must be taken into consideration: specific gravity, modulus of elasticity, specific electric conductivity, temperature coefficients, etc., which have an influence upon its physical characteristics.

In the following article, general methods of calculating the errors of a separate pickup element (see Chapter 1) are considered, and a relationship between the error of the measuring instruments and its elements (see Chapter II) is established.

Only statistical errors of the elements are discussed.

As a basis for the calculation of errors a method developed by N. Bruyevitch* is used; in this method a small movement of a system, caused by a deflection of the actual system from the ideal one, is linearized and studied separately from the main movement.

I. CALCULATION OF ERRORS OF A PICKUP ELEMENT

1. Determination and Classification of Errors

We will consider the pickup element as the converter of the general input coordinate X into the output coordinate Y (Fig.2).

The absolute error of the pickup elements will be used for denoting the difference between the actual and ideal (required) values of the output coordinates, at a certain constant value of the input coordinate.

The absolute error may be found analytically or graphically.

If the absolute expression of the characteristic of the ideal element is of the type

$$Y_0=f_0(X), \tag{1}$$

---

* For data, see Bruyevitch (Bibl.4)

where Y_0 is the ideal value of the output coordinate, and the characteristic of the actual element is

$$Y=f(X), \tag{2}$$

then the absolute error of the element is equal to

$$\Delta Y=Y-Y_0=f(X)-f_0(X)=\varphi(X). \tag{3}$$

If the ideal and actual characteristics are graphically represented (curves 1

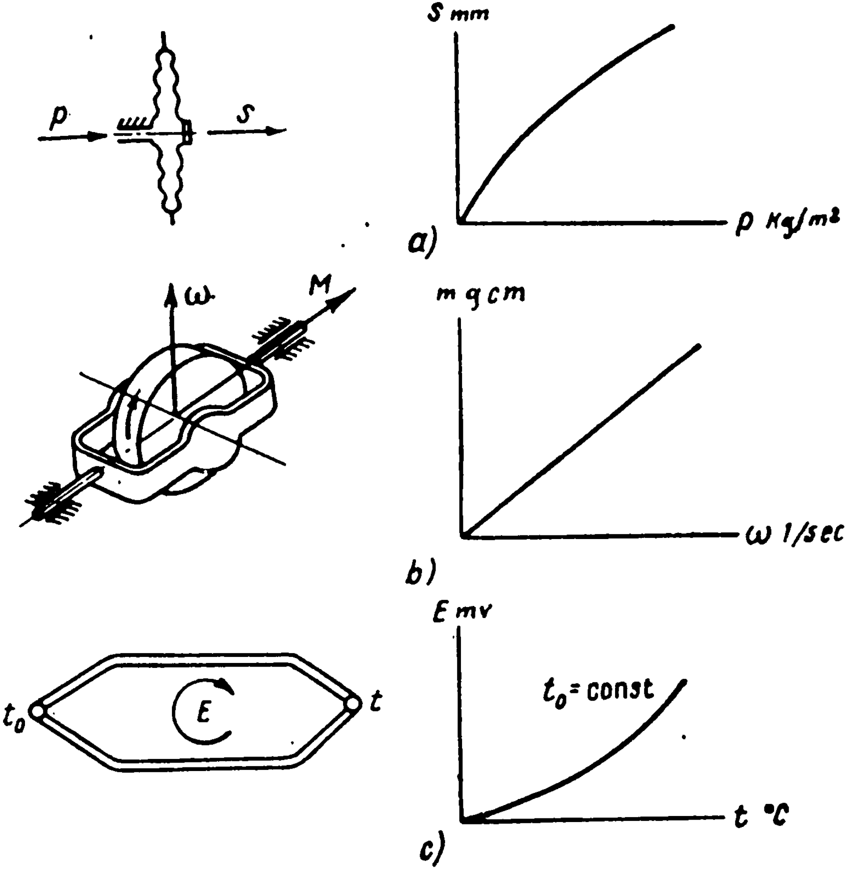


Fig.1 - Characteristics of Pickup Elements a - Elastic pickup element; δ - Gyroscope with two degrees of freedom; b- Thermocouple. p - Pressure; s - Displacement; w - Angular velocity; M - Moment of forces; t - Temperature; E - Electromotive force.

and 2, Fig.3), then the error ΔY is determined as the difference of the ordinates of curves 2 and 1.

The absolute error of the element may be also expressed in dimensions of the input coordinate ΔX if, on the graph in Fig.3, the difference of the abscissas of curves 2 and 1 is used.

In view of the relatively small magnitude of the errors, their transformation from one dimension to another may be performed by utilizing a transformation coefficient*.

$$\Delta Y = K \Delta X \quad or \quad \Delta X = \frac{\Delta Y}{K},$$

where

$$K = \left(\frac{\partial Y}{\partial X}\right)_0 = \frac{m_Y}{m_X} \operatorname{tg} \Theta;$$

m_x and m_y are the scales of the graph on the x and y axes;

θ is the angle between the tangent and the ideal curve 1.

The subscript zero of the partial derivative indicates that the ideal characteristic is being differentiated.

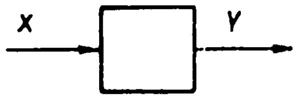


Fig.2 - Generalized Pickup Element

x - Generalized input coordinate; Y - Generalized output coordinate.

We further decide to express the absolute error of the element in dimensions of the output coordinate.

The relative error of the element will be referred to as the relation of its absolute error to the ideal value of the output coordinate at a given point:

$$\frac{\Delta Y}{Y_0} = \frac{\varphi(X)}{f_0(X)} = \varphi_1(X). \qquad (4)$$

---

* Together with the term "transformation coefficient" the literature also uses the following terms: transformation ratio, sensitivity, transconductance, steepness of characteristics, amplification factor.

[redacted]

The total relative error of the element will be referred to as the relationship of its absolute error to the largest ideal value of the output coordinate

$$\frac{\Delta Y}{Y_{0\,\max}} = \frac{\varphi(X)}{f_0(X_{\max})} = \varphi_2(X). \qquad (4')$$

From eqs.(3), (4), and (4') it follows that the errors of the elements are functions of the input coordinate.

The general error of the element consists of individual (partial) errors which, depending upon the reasons causing them, may be divided into the following basic types:

a) Methodical errors, appearing in the design of the element in cases where, for simplification of construction of the element, the exact (required) function of transformation is substituted by an approximate one;

b) Manufacturing errors, produced in the manufacture of the element, due to inaccurate execution of the given geometric and physical parameters which have an influence upon the characteristics of the elements (for the sake of brevity we will call these the main parameters);

c) Temperatures errors, caused by a change in the main parameters of the element during a change in its temperature;

d) Errors caused by friction, in the case where the element does work of which a part is expended in overcoming frictional forces;

e) Hysteresis errors, caused by internal friction in the material of the element;

f) Errors due to clearance, caused by additional (parasitic) displacements of the moving system of the element within the limits of the clearance in supports or guides.

The manner of summarizing partial errors of the element depends on whether they are systematic or random errors.

Systematic errors are added algebraically. Random errors are added with consideration of the probability of compounding these random errors.

STAT

When the overall error of one complete sample element is determined, then the partial errors (see paragraphs a, b, and c) should be related to the systematic ones and the remainder to random ones*. If, however, the overall error of a large lot (series) of elements of the same type is to be found, then the manufacturing errors (paragraph b) also pass into the category of random errors since the distribution of the main parameters of the elements during their serial production is mainly due to random factors.

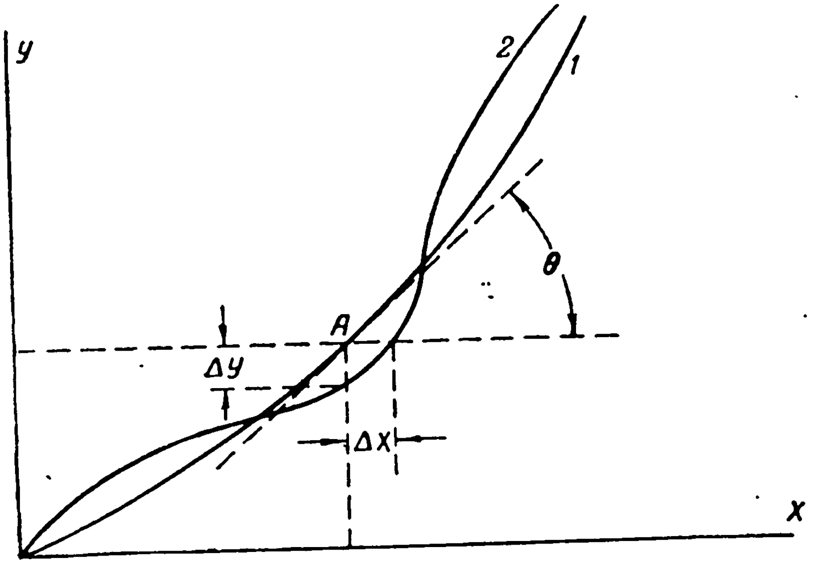


Fig.3 - Graphic Determination of Error of the Element

1 - Ideal characteristic; 2 - Actual characteristic.

We will consider methods of determining partial errors of the element.

2. Methodic errors

During the design of the element given by a function (1), some approximate function may be substituted for the ideal function $Y = f_0(X)$ in order to simplify the construction:

$$Y=f(X,\ q_1,\ q_2, \ldots q_n), \quad (5)$$

---

* In certain cases, methodical errors should also be related to random ones, as shown below in example 1.

STAT

where q_i is the main parameter of the element (geometric dimensions and physical coefficients, having an influence on the characteristics of the element).

The methodic error is equal to the difference of eqs.(5) and (1), where the parameters of q_i in eq.(5) are assumed to be equal to their ideal (calculated) values:

$$\Delta Y_{met} = Y - Y_0 = f(X, q_1, q_2, \cdots q_n) - f_0(X). \qquad (6)$$

Example 1. Let us find the methodical error of the linear rheostat* (Fig.4).

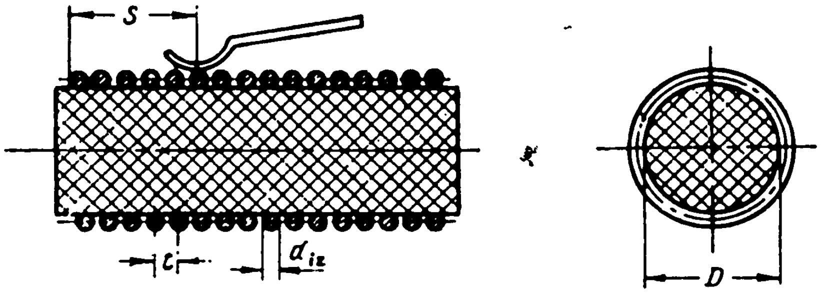


Fig.4 - Linear Rheostat

The ideal characteristic of the linear rheostat represents a proportional ratio of electric resistance to the displacement s of the glider, represented in Fig.5a by the broken line OA:

$$r_0 = Ks = \frac{R}{wt}s, \qquad (7)$$

where K is the transfer coefficient, $K = \frac{R}{wt}$ in ohm/mm;

R is the overall resistance of the rheostat in ohms;

w is the overall number of turns;

t is the lay of the winding, in mm.

In an actual rheostat, even produced with ideal accuracy, the output coordinate does not change proportionally to s, but in steps. In Fig.5a the characteristic of

---

* This error is known as error of turn or of resolving power of the rheostat.

the actual rheostat is represented by the stepped line, with the number of steps equal to the number of turns. The width of each step is equal to t, the height is equal to $\frac{R}{w}$.

The stepped characteristic is described by the equation of horizontal sections

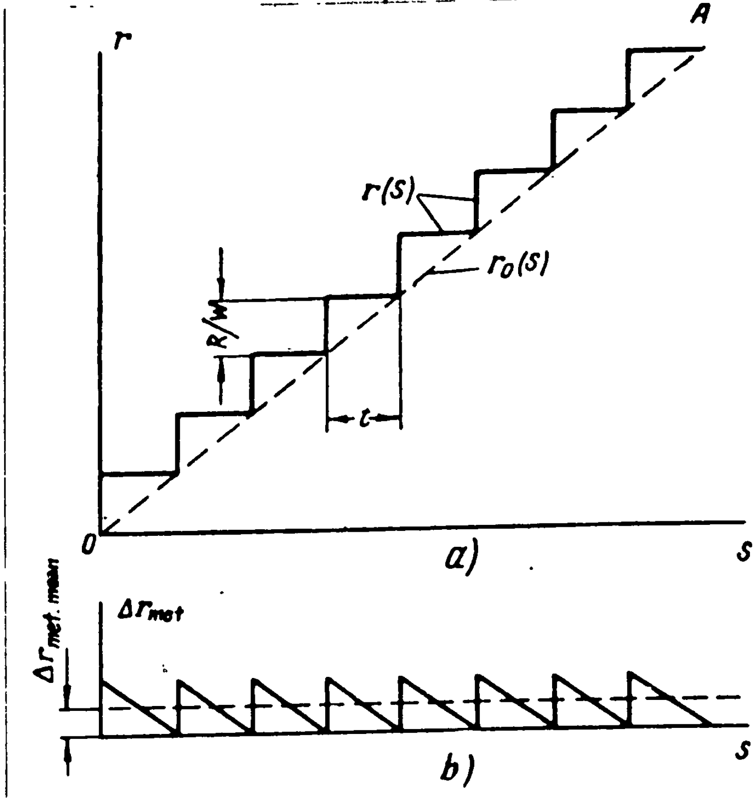


Fig.5 - Methodical Errors of a Linear Rheostat

a - Characteristic of the rheostat; δ - Change of methodical error.

on sectors of the width t.

In the first section $(0 < s < t)$: $r = \frac{R}{w}$

In the second section $(t < s < 2t)$: $r = 2\frac{R}{w}$

. . . .

In the n^{th} section $[(n-1)t < s < nt]$: $r = n\frac{R}{w}$

The methodical error in any given section is equal to

$$\Delta r_{met} = r - r_0 = n\frac{R}{w} - \frac{R}{wt}s = \frac{R}{w}\left(n - \frac{s}{t}\right),$$

where $(n - 1)t < s < nt$; $n = 1, 2, \ldots w$.

At $s = (n - 1)t$, the maximum error is equal to $\frac{R}{w}$; at $s = nt$, the error is equal to zero.

The graph for the change in the methodical error is shown in Fig.5b.

In view of the small magnitude of the step t, in comparison with the overall length of the rheostat, the methodical error may be considered as random, the appearance of which within the limits $\Delta r_{met} = 0$ to $\Delta r_{met} = \frac{R}{w}$ is equally probable.

In this case the mean value (mathematical probability) of the methodic error is equal to

$$\Delta r_{met\,mean} = \frac{R}{2w},$$

and the mean square deviation

$$\sigma_{met} = \frac{1}{2\sqrt{3}} \cdot \frac{R}{w}.$$

The mean value of the reduced methodic error, related to the overall resistance of the rheostat, is equal to

$$\left(\frac{\Delta r_{met}}{R}\right)_{cp} = \frac{1}{2w}.$$

The mean square deviation is

$$\frac{\sigma_{met}}{R} = \frac{1}{2\sqrt{3}w}.$$

For example, if $w = 100$, the mean value of the total methodical error is equal to 0.005 or 0.5%, and the mean square deviation is equal to 0.003 or 0.3%.

3. Manufacturing Errors

In listing manufacturing errors, the methodical errors are considered to be equal to zero and therefore the approximate function (5) is assumed to be ideal.

[redacted] STAT

(The parameters q_i are considered equal to their ideal values):

$$Y_0 = f(X,\ q_1,\ q_2 \dots q_n). \tag{8}$$

The characteristic of the actual element is of the type

$$Y = f(X,\ q_1 + \Delta q_1,\ q_2 + \Delta q_2 \dots q_n + \Delta q_n), \tag{9}$$

where Δq_i are primary errors (deviations of the main parameters from ideal values), appearing during manufacture of the element.

The absolute production error is equal to the difference of eqs.(9) and (8):

$$\Delta Y_{pr} = Y - Y_0 = f(X,\ q_1 + \Delta q_1,\ q_2 + \Delta q_2 \dots q_n + \Delta q_n) - \\ - f(X,\ q_1,\ q_2 \dots q_n). \tag{10}$$

In view of the relatively small magnitude of the increments Δq_i, the absolute production error may be represented as a linear function of primary errors, if eq.(10) is expanded into a Taylor series around the ideal values of the main parameters and is limited to the zero and first members of the series:

$$\Delta Y_{pr} = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial q_i}\right)_0 \Delta q_i. \tag{11}$$

The relative manufacturing errors may be found on substituting eq.(11) into eqs.(4) and (4').

In concrete calculations, the expression for relative errors is often considerable simplified.

We will consider a specific case which is fairly popular in practice, where the characteristic of the ideal element is expressed as a function of the type

$$Y = f(X) q_1^{a_1},\ q_2^{a_2} \ .\ .\ .\ q_n^{a_n} = f(X) \prod_{i=1}^{n} q_i^{a_i}, \tag{12}$$

[redacted]

STAT

where the parameters q_i and exponents a_i are independent of X.

In this case a simple expression is obtained for the relative manufacturing error, which may be found if eq.(12) is expressed logarithmically and the full differential of the logarithm is taken, assuming X = const:

$$\left(\frac{\Delta Y}{Y}\right)_{pr} = a_1 \frac{\Delta q_1}{q_1} + a_2 \frac{\Delta q_2}{q_2} + \ldots + a_n \frac{\Delta q_n}{q_n} = \sum_{i=1}^{n} a_i \frac{\Delta q_i}{q_i}. \tag{13}$$

Consequently, the manufacturing error of the element, having characteristics of the type (12), is equal to the sum of the products of relative primary errors in the exponents of the steps of the corresponding main parameters.

Example 2. Let us find the manufacturing error of a linear rheostat, shown in Fig.4.

In determining the manufacturing error of the rheostat, we will consider its methodical error equal to zero and, furthermore, for simplifying the calculation, assume the primary errors of the rheostat to be constant, i.e., independent of the displacement s.

We will express the overall resistance of the rheostat by its parameters:

$$R = \frac{\rho L}{q} 10^{-3} = \frac{4 \cdot 10^{-3} \rho (D + d_{iz}) w}{d^2}, \tag{14}$$

where ρ is the specific resistivity of the wire in $\frac{\text{ohm} - \text{mm}^2}{\text{m}}$;

L is the developed length of the wire, in mm;

q is the cross section of the wire without insulation, in mm^2;

d is the diameter of the wire without insulation, in mm;

d_{iz} is the diameter of the wire with insulation, in mm;

D is the diameter of the body, in mm (we will assume a round cross section of the body).

Substituting eq.(14) into eq.(7), we obtain the ideal characteristic of the

[redacted]

rheostat, expressed by its main parameters:

$$r_0 = \frac{4 \cdot 10^{-3} \rho (D + d_{iz})}{d^2 t} s.$$

Analogous to eq.(13), we obtain the relative manufacturing error of the rheostat, expressed by its relative primary errors.

$$\left(\frac{\Delta r}{r}\right)_{pr} = \frac{\Delta \rho}{\rho} + \frac{\Delta D + \Delta d_{iz}}{D + d_{iz}} - 2\frac{\Delta d}{d} - \frac{\Delta t}{t}.$$

For example, if in a specific rheostat the relative primary errors have the value $\frac{\Delta\rho}{\rho} = -0.05$; $\frac{\Delta D + d_{iz}}{D + d_{iz}} = +0.02$; $\frac{\Delta d}{d} = -0.05$; $\frac{\Delta t}{t} = +0.03$, then the overall conductivity error will be equal to

$$\left(\frac{\Delta r}{r}\right)_{pr} = -0,05 + 0,02 - 2(-0,05) - 0,03 = 0,04 \quad \text{or} \quad 4\%.$$

If the primary errors are not constant along the length of the rheostat, then the calculation of the manufacturing error becomes much more complicated, since it is necessary to consider the law of change of the primary errors as a function of the displacement of the glider.

In this case, the manufacturing errors of the rheostat are determined by experiment.

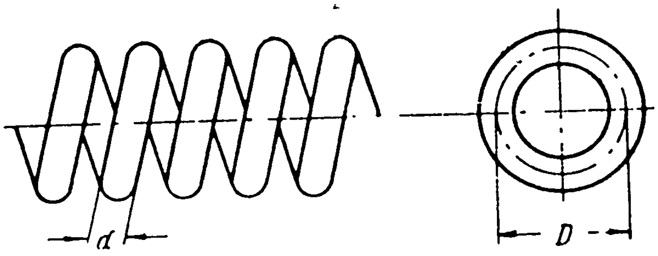


Fig.6 - Cylindrical Spring

Example 3. Let us find the manufacturing error of the cylindrical spring (Fig.6).

The characteristics of such a spring are determined by the formula

$$s = \frac{8D^3 n}{Gd^4} P, \qquad (15)$$

where s is the displacement, in mm;

P is the force, in kg;

D is the mean diameter of the turn, in mm;

n is the number of turns;

G is the modulus of shear, in kg/mm^2;

d is the diameter of the wire, in mm.

The manufacturing error is equal to

$$\frac{\Delta s}{s} = 3\frac{\Delta D}{D} + \frac{\Delta n}{n} - \frac{\Delta G}{G} - 4\frac{\Delta d}{d}.$$

Limiting Values of Manufacturing Errors

To find the limiting values of the absolute manufacturing errors of a group of elements of the same type, by the "maximum-minimum" method, we will use eq.(11), taking into account that the partial derivatives $\left(\frac{\partial y}{\partial q_i}\right)$ may be positive as well as negative.

We will assume that, of the overall number n of partial derivatives, there are m positive and n - m negative.

The upper limiting value of the manufacturing error is determined from the expression

$$\Delta Y_{pr}^{upper} = \sum_{i=1}^{m} \left(\frac{\partial Y}{\partial q_i}\right)_0 \Delta q_i^{upper} + \sum_{i=m+1}^{n} \left(\frac{\partial Y}{\partial q_i}\right)_0 \Delta q_i^{lower}.$$

The lower limiting value, from

$$\Delta Y_{pr}^{lower} = \sum_{i=1}^{m} \left(\frac{\partial Y}{\partial q_i}\right)_0 \Delta q_i^{lower} + \sum_{i=m+1}^{n} \left(\frac{\partial Y}{\partial q_i}\right)_0 \Delta q_i^{upper}. \qquad (16)$$

[redacted]

For elements with a characteristic of the type (12) we obtain from eq.(13) the following expressions for the limiting values of relative manufacturing errors

$$\left(\frac{\partial Y}{Y}\right)_{\text{pr}}^{\text{upper}}=\sum_{i=1}^{m} a_i\left(\frac{\Delta q_i}{q_i}\right)^{\text{upper}}+\sum_{i=m+1}^{n} a_i\left(\frac{\Delta q_i}{q_i}\right)^{\text{lower}}$$

$$\left(\frac{\Delta Y}{Y}\right)_{\text{pr}}^{\text{lower}}=\sum_{i=1}^{m} a_i\left(\frac{\Delta q_i}{q_i}\right)^{\text{lower}}+\sum_{i=m+1}^{n} a_i\left(\frac{\Delta q_i}{q_i}\right)^{\text{upper}}. \qquad (16')$$

In eqs.(16) and (16'), the terms Δq_i upper and Δq_i lower are the upper and lower deviations of the main parameters from the ideal values shown in drawings and technical specifications for materials.

It must be mentioned that the calculation of the limiting manufacturing errors of elements by the "maximum-minimum" method cannot be recommended for accurate calculations, since the laws of the theory of probability are not taken into account, which leads to an uncorrected tightening of tolerances of prime errors. The applicability of calculations, taking into account the laws of the theory of probability, is outlined at the end of Chapter II.

4. Temperature Errors

Temperature errors appear as a result of additional increases of the main parameters q_i with changes of temperature.

For calculating temperature errors we will use eq.(10) but will assume that the element is produced to ideal accuracy (manufacturing errors equal to zero) and that increments Δq_i are caused by the influence of temperature.

With an accuracy sufficient in practice it can be assumed that the increments Δq_i are proportional to the change in temperature

$$\Delta q_i = q_i \alpha_i \Delta t, \qquad (17)$$

where q_i is the value of the main parameter at normal temperature;

a_i is the temperature coefficient of the parameter;

Δt is the change in temperature as compared to normal temperature.

Substituting eq.(17) into eq.(10), we obtain an expression for the absolute temperature error:

$$\Delta Y_{\text{temp}} = f[X, q_1(1+\alpha_1\Delta t), q_2(1+\alpha_2\Delta t), \dots \dots q_n(1+\alpha_n\Delta t)] - f(X, q_1, q_2 \dots q_n). \tag{18}$$

Equation (18) should be used when the product $a_i \Delta t$ is relatively large (for example, if it exceeds 0.1).

For the majority of elements, the temperature coefficients are so small that justifiably the inequality is $a_i \Delta t \ll 1$ and the temperature error can be linearized by means of eq.(11), where the value of Δq_i is substituted from eq.(17):

$$\Delta Y_{\text{temp}} = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial q_i}\right)_0 q_i \alpha_i \Delta t. \tag{19}$$

In the specific case when an element has the characteristic of eq.(12), its relative temperature error is determined by a formula obtained by means of substituting eq.(17) into eq.(13):

$$\left(\frac{\Delta Y}{Y}\right)_{\text{temp}} = \sum_{i=1}^{n} a_i \alpha_i \Delta t. \tag{20}$$

The overall temperature coefficient* of such a pickup element is obtained from eq.(20), assuming that $\Delta t = 1^\circ$C; then

$$\alpha = \sum_{i=1}^{n} a_i \alpha_i. \tag{21}$$

Example 4. Let us find the overall temperature coefficient of a cylindrical spring, whose characteristic is expressed by eq.(15).

On the basis of eq.(21) and (15), the overall temperature coefficient of the spring may be represented in the form

$$\alpha = 3\alpha_D - \alpha_G - 4\alpha_d,$$

---

* The overall temperature coefficient of a pickup element we denote as the relative change of its output coordinate during a temperature variation of 1°C.

STAT

[redacted]

where a_D, a_G and a_d are temperature coefficients of the parameters D, G, and d.

If it is considered that a_D and a_d are equal to the relatively small coefficient of linear expansion of the spring material and, consequently, one is equal to the other, then the overall temperature coefficient of the spring will be equal to the temperature coefficient of the modulus of shear, taken with the reverse sign:

$$\alpha \approx -\alpha_G.$$

For example, for phosphorous bronze we have $\alpha_G \approx -5\times10^{-4}$, from which $\alpha \approx 5\times10^{-4}$.

Consequently, during a temperature variation of $\Delta t = 100^{\circ}C$, the temperature error is equal to

$$\alpha\Delta t = 5\cdot10^{-4}\cdot100 = 0{,}05 \quad \text{or} \quad 5\%.$$

5. Errors Caused by Friction

We will separate the elements with an output parameter P (see Table 1).

These elements in which the output parameter is the force P or the moment of forces M we will, in view of these conditions, call "forces"elements.

In the construction of instruments and automatic equipment, the force element is usually mechanically connected with one or more mechanisms or other elements. Consequently, the force developed by the force element is able to overcome friction forces acting not only within the given element but also in the mechanisms and other elements to which it is connected.

For mechanical systems with one degree of freedom, all forces of friction, overcome by the force element, may be substituted by an assumed (equivalent) friction force, applied at some point A of the element, or by an applied friction moment, applied at some axis zero.

The selection of the point A or of the axis zero is generally arbitrary, but it is more convenient to select a point of the force element where it is connected with other elements (in the case of translatory motion of the element), or an axis along which it is connected with other elements (in the case of rotational motion of the element).

The frictional force P'_{tp} applied at the point A is determined by conditions of equality of the elementary work of this force through a small displacement ds and on the basis of the sum of the elementary work of the frictional forces on the actual parts of the moving system, during corresponding displacement*

$$P'_{tp} = \pm\left(\sum_{i} P_i \frac{dy_i}{ds} + \sum_{k} M_k \frac{d\alpha_k}{ds}\right), \qquad (22)$$

where P_i is the friction force of the parts in translatory motion;

$\frac{dy_i}{ds}$ is the translatory relationship of point A to the i^{th} part;

M_k are the friction moments of the parts in rotating motion;

$\frac{d\alpha_k}{ds}$ is the translatory ratio of point A to the k^{th} part.

In an analogous manner, the friction moment applied at the axis zero is determined

$$M'_{tp} = \pm\left(\sum_{i} P_i \frac{dy_i}{d\varphi} + \sum_{k} M_k \frac{d\alpha_k}{d\varphi}\right), \qquad (23)$$

where $\frac{dy_i}{d\varphi}$ - is the translatory ratio of the axis zero to the i^{th} part;

$\frac{d\alpha_k}{d\varphi}$ is the translatory ratio of the axis zero to the k^{th} part.

The sign ± in eqs.(22) and (23) means that the error due to friction is not single-valued since the frictional forces change their direction during a change of direction of motion of the moving system.

Equations (22) and (23) give the limiting values of the absolute error of the force element due to friction, expressed in units of the output coordinate (force P or moment of forces M).

If eqs.(22) and (23) are divided by a translatory coefficient of a force element, then we obtain limiting values of the absolute error, caused by friction, ex-

---

* For data, see Tikhmenev (Bibl.2)

[redacted]

STAT

pressed in units of the input coordinate and called the threshold of sensitivity or the zone of rest of the system.

$$\Delta X_{tp}=\frac{P'_{tp}}{K_P}=\pm\frac{\sum_i P_i\frac{dy_i}{ds}+\sum_k M_k\frac{d\alpha_k}{ds}}{\left(\frac{\partial P}{\partial X}\right)_0} \qquad (24)$$

or

$$\Delta X_{tp}=\frac{M'_{tp}}{K_M}=\pm\frac{\sum_i P_i\frac{dy_i}{d\varphi}+\sum_k M_k\frac{d\alpha_k}{d\varphi}}{\left(\frac{\partial M}{\partial X}\right)_0} \qquad (25)$$

A change of the input coordinate within the zone of rest does not cause movement of the system, since the increase of the resultant input coordinate by the force element is insufficient for overcoming the applied force (moments) of friction.

Example 5. We will find the error due to friction of the electromechanical pressure indicator, consisting of a membrane (force element), kinematically connected across a translatory mechanism with the rheostat slide (Fig.7).

For simplicity, we will only consider the frictional force of the slide on the rheostat and will assume the angles of rotation of the lever to be small, so that

$$\frac{dy}{ds}\approx\frac{b}{a},$$

where s is the displacement of the membrane;

y is the displacement of the slide.

According to eq.(22), the friction force applied at point A, is equal to

$$P'_{tp}=\pm\frac{b}{a}P_{sl},$$

where P_{sl} is the friction force of the slide on the rheostat.

According to eq.(24), we determine the error of the indicator due to friction, expressed in units of the pressure being measured:

$$p_{tp} = \frac{P'_{tp}}{\frac{dP}{dp}} = \pm \frac{bP_{sl}}{aF},$$

where $F = \frac{dP}{dp}$ is the effective area of the membrane. If, for example, $a = 3$mm, $b = 15$mm, $P_{sl} = 0.5$ gm, $F = 5\ c^2$, then the error due to friction is equal to

$$p_{tp} = \pm \frac{15\cdot 0,5}{3\cdot 5} = \pm 0,5\ \text{g/cm}^2 = \pm 5\ \text{mm water col.}$$

6. Errors Due to Hysteresis

Errors due to hysteresis, similarly to errors caused by friction, do not definitely depend upon the input coordinate, i.e., one and the same value of the input coordinate corresponds to various values of the output coordinate, depending upon the character of the change of the input coordinate prior to the time it assumed the established value.

Errors due to hysteresis are determined by means of measuring the output coordinate first during the increase in input coordinate and later during its decrease.

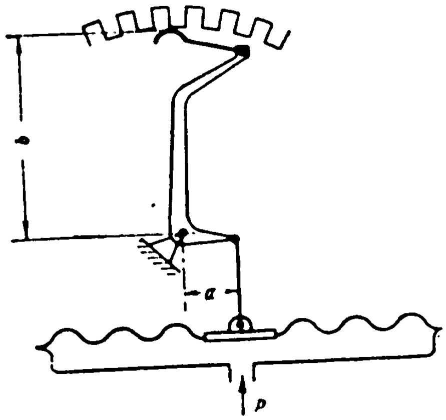


Fig.7 - Electromechanical Pressure Indicator.

The graph of the characteristic of an element obtained in a similar manner has the appearance of a closed loop (Fig.8), whose axis (broken line OB) corresponds to the characteristic of the ideal element, in the case where the element does not have any errors other than those due to hysteresis. The difference of the ordinates of the upper part of the loop OBC and of the axial line OB gives the upper limiting value of the absolute error due to hysteresis, while the difference of the ordinates of the lower part of the loop OAB and of the axial line OB gives the lower limiting value for the absolute error due to hysteresis.

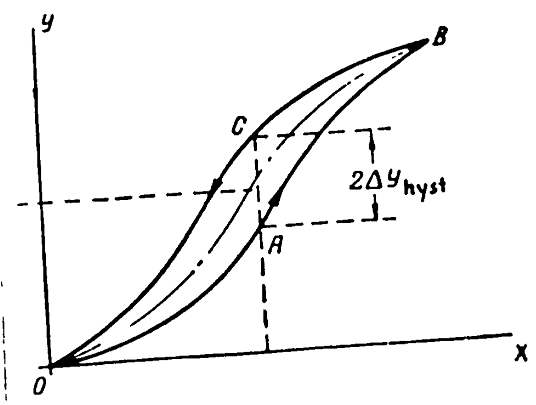


Fig.8 - Hysteresis Loop

AOB - Curve during rise; BCO - Curve during decrease

If the pickup element also has errors due to friction, then the experimentally obtained graph of Fig.8 will express the sum of the errors due to friction and hysteresis. In order to eliminate errors due to friction, the hysteresis loops are recorded in practice after lightly tapping the element or after a short-time buzzing (light vibration).

7. Errors Due to Clearance

We will consider errors due to clearance in the pickup elements, containing a moving system with one degree of freedom, which is secured by application of suitable kinematic couples, i.e., guides permitting freedom of translatory motion, or supports, giving freedom of rotational motion.

In the presence of clearances in kinematic couples, the moving system may have, in addition to freedom of motion in the main direction, also additional small (within the limits of the clearances) displacements in other directions, resulting in an error of the element.

The error is equal to the increase of the output coordinate, caused by additional small displacements of the moving system at a certain constant value of the input coordinate.

In the presence of clearances, the moving system in a general case may obtain five additional degrees of freedom, above the one degree of freedom in the main direction.

For calculating the errors due to clearances, we will construct around the element a coordinate system $\xi\eta\zeta$ oriented with respect to the element in such a way, that the main direction of motion takes place downward (or about) the axis ξ (Fig.9). The two other axes are selected in an arbitrary direction.

If the main motion of the element is translatory, then the absolute error of the output coordinate due to clearances will be equal to

$$\Delta Y_{clea}=\left(\frac{\partial Y}{\partial \eta}\right)_0 \Delta\eta+\left(\frac{\partial Y}{\partial \zeta}\right)_0 \Delta\zeta+\left(\frac{\partial Y}{\partial \varphi_\xi}\right)_0 \Delta\varphi_\xi+\left(\frac{\partial Y}{\partial \varphi_\eta}\right)_0 \Delta\varphi_\eta+\left(\frac{\partial Y}{\partial \varphi_\zeta}\right)_0 \Delta\varphi_\zeta, \qquad (26)$$

where $\Delta\eta$ and $\Delta\zeta$ are small translatory motions of the system along the axes η and ζ; $\Delta\varphi_\xi$, $\Delta\varphi_\eta$, $\Delta\varphi_\zeta$ are small rotational displacements of the system about the axes ξ, η and ζ.

If the main motion of the element is rotational, then the error of the output coordinate is equal to

$$\Delta Y_{clea}=\left(\frac{\partial Y}{\partial \xi}\right)_0 \Delta\xi+\left(\frac{\partial Y}{\partial \eta}\right)_0 \Delta\eta+\left(\frac{\partial Y}{\partial \zeta}\right)_0 \Delta\zeta+\left(\frac{\partial Y}{\partial \varphi_\eta}\right)_0 \Delta\varphi_\eta+\left(\frac{\partial Y}{\partial \varphi_\zeta}\right)_0 \Delta\varphi_\zeta, \qquad (27)$$

where $\Delta\xi$ is a small forward motion of the system along the axis ξ.

The partial derivatives in eqs.(26) and (27) and the magnitudes of possible displacements are determined from the analysis of the specific construction arrangement of the element.

For determining the mean and limiting values of errors due to clearances, calculation methods worked out in the field of theory on the accuracy of mechanisms are applied.

Example 6. Let us find the error due to clearance, for a linear rheostat whose slide undergoes rotational motion (Fig.9). We will select the axis of coordinates $\xi\eta\zeta$ in such a manner that the axis ξ coincides with the axis of rotation of the five additional degrees of freedom, above the one degree of freedom in the main direction.

For calculating the errors due to clearances, we will construct around the element a coordinate system $\xi\eta\zeta$ oriented with respect to the element in such a way, that the main direction of motion takes place downward (or about) the axis ξ (Fig.9). The two other axes are selected in an arbitrary direction.

If the main motion of the element is translatory, then the absolute error of the output coordinate due to clearances will be equal to

$$\Delta Y_{clea}=\left(\frac{\partial Y}{\partial \eta}\right)_0 \Delta\eta+\left(\frac{\partial Y}{\partial \zeta}\right)_0 \Delta\zeta+\left(\frac{\partial Y}{\partial \varphi_\xi}\right)_0 \Delta\varphi_\xi+\left(\frac{\partial Y}{\partial \varphi_\eta}\right)_0 \Delta\varphi_\eta+\left(\frac{\partial Y}{\partial \varphi_\zeta}\right)_0 \Delta\varphi_\zeta, \qquad (26)$$

where $\Delta\eta$ and $\Delta\zeta$ are small translatory motions of the system along the axes η and ζ; $\Delta\varphi_\xi$, $\Delta\varphi_\eta$, $\Delta\varphi_\zeta$ are small rotational displacements of the system about the axes ξ, η and ζ.

If the main motion of the element is rotational, then the error of the output coordinate is equal to

$$\Delta Y_{clea}=\left(\frac{\partial Y}{\partial \xi}\right)_0 \Delta\xi+\left(\frac{\partial Y}{\partial \eta}\right)_0 \Delta\eta+\left(\frac{\partial Y}{\partial \zeta}\right)_0 \Delta\zeta+\left(\frac{\partial Y}{\partial \varphi_\eta}\right)_0 \Delta\varphi_\eta+\left(\frac{\partial Y}{\partial \varphi_\zeta}\right)_0 \Delta\varphi_\zeta, \qquad (27)$$

where $\Delta\xi$ is a small forward motion of the system along the axis ξ.

The partial derivatives in eqs.(26) and (27) and the magnitudes of possible displacements are determined from the analysis of the specific construction arrangement of the element.

For determining the mean and limiting values of errors due to clearances, calculation methods worked out in the field of theory on the accuracy of mechanisms are applied.

<u>Example 6.</u> Let us find the error due to clearance, for a linear rheostat whose slide undergoes rotational motion (Fig.9). We will select the axis of coordinates $\xi\eta\zeta$ in such a manner that the axis ξ coincides with the axis of rotation of the

slide, while the axis ζ passes through the center of rotation O and the contact point of the slide C as well as the axis η are perpendicular to the axes ξ and ζ.

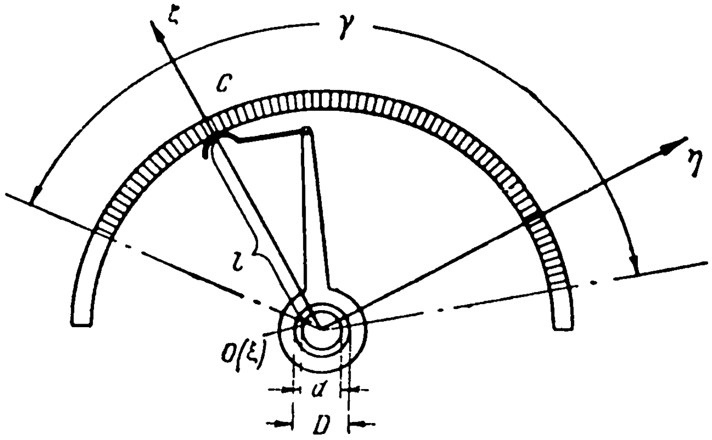


Fig.9 - For Calculating the Error due to Clearance

The main motion is a rotation of the slide about the axis ξ. In the presence of clearance between the axis of the slide and the bushing, the moving system (slide) attains an additional five degrees of freedom and the error due to clearance in the general case is determined according to eq.(27), as follows:

$$\Delta r_{clea} = \left(\frac{\partial r}{\partial \xi}\right)_0 \Delta\xi + \left(\frac{\partial r}{\partial \eta}\right)_0 \Delta\eta + \left(\frac{\partial r}{\partial \zeta}\right)_0 \Delta\zeta + \left(\frac{\partial r}{\partial \varphi_\eta}\right)_0 \Delta\varphi_\eta + \left(\frac{\partial r}{\partial \varphi_\zeta}\right)_0 \Delta\varphi_\zeta,$$

where Δr_{clea} is the change in the output resistance. However, according to Fig.9 it is obvious that, within the selected system of coordinates, we have

$$\left(\frac{\partial r}{\partial \xi}\right)_0 = \left(\frac{\partial r}{\partial \zeta}\right)_0 = \left(\frac{\partial r}{\partial \varphi_\eta}\right)_0 = \left(\frac{\partial r}{\partial \varphi_\zeta}\right)_0 = 0.$$

The partial derivative $\left(\frac{\partial r}{\partial \eta}\right)_0$ is equal to

$$\left(\frac{\partial r}{\partial \eta}\right)_0 = \frac{R}{\gamma l},$$

STAT

[redacted]

where R is the overall resistance of the rheostat;

γ is the angle of the rheostat winding (in radians);

l is the lever arm OC.

Consequently, the error due to clearance is equal to

$$\Delta r_{clea} = \left(\frac{\partial r}{\partial \eta}\right)_0 \Delta\eta = \frac{R}{\gamma l}\Delta\eta.$$

The largest possible displacement $\Delta\eta$ is equal to

$$\Delta\eta = \frac{1}{2}(D-d),$$

where D is the diameter of the bushing;

d is the diameter of the axis.

The maximum value of the error is equal to

$$\Delta r_{clea\,\max} = \frac{R(D-d)}{2\gamma l}.$$

The total value of the maximum error is equal to

$$\left(\frac{\Delta r_{clea}}{R}\right)_{\max} = \frac{D-d}{2\gamma l}.$$

If, for example, $D - d = 0.1$ mm; $\gamma = 120^\circ = \frac{2\pi}{3}$; $l = 20$ mm,

$$\text{то}\left(\frac{\Delta r_{clea}}{R}\right)_{\max} = \frac{0{,}1\cdot 3}{2\cdot 2\pi\cdot 20} = 0{,}0012 \text{ or } 0{,}12\%.$$

STAT

[redacted]

II. RELATIONSHIP OF THE OVERALL ERROR OF THE MEASURING INSTRUMENT TO THE ERRORS OF THE ELEMENTS

1. Structural Arrangement of the Measuring Instrument

Since the action of the measuring instrument, in the final analysis, is due to the displacement of the counting or recording mechanism, corresponding in a predetermined scale to the quantity to be measured, then any given measuring instrument may be considered as a functional transformer of the physical quantity to be measured into mechanical displacement.

The order of this transformation, determined by the number and type of the elements of which the instrument consists, is necessarily performed by making use of a structural layout which divides the instrument into simpler transformer links.

Figure 10 shows the principal and structural arrangement of a thermoelectric aircraft thermometer. The structural arrangement (Fig.10b) of the instrument consists of four links:

1 - Thermocouple transforming the temperature T to be measured into the electromotive force E;

2 - Electric circuit transforming the electromotive force E into the electric current I;

3 - Electro magnetic system (frame in the magnetic field), transforming the electric current I into the mechanical moment M;

4 - Spring element (two helical springs), transforming the moment M into displacement of the pointer*.

In the given instrument, the links (1), (3), and (4) represent pickup elements,

STAT

---

* Links (3) and (4) represent together the electromagnetic galvanometer which may be considered as a complex (2-link) element, transforming the current I into the displacement α.

[redacted]

whereas the link (1) (thermocouple) represents the primary pickup element, directly reacting to the magnitude of T to be measured.

In instruments, based on indirect methods of measurement, the primary pickup element does not always react to the magnitude to be measured by the instrument, but has a certain other physical magnitude which is functionally connected with the one to be measured.

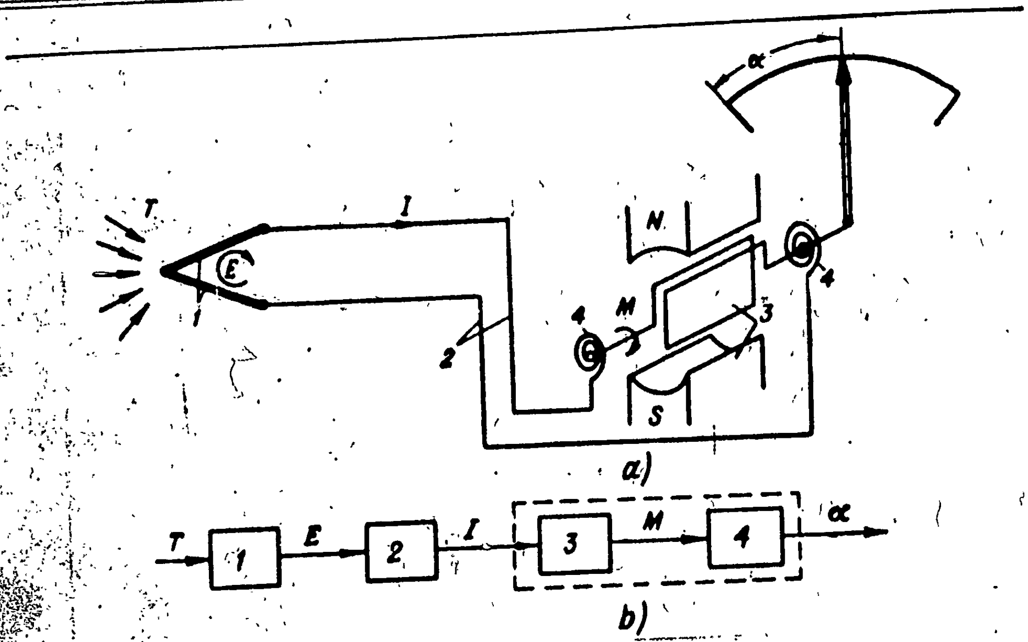


Fig.10 - Thermoelectric Thermometer

a - Principal view; b - Structural arrangement.

For example, the primary pickup element of the barometric aircraft altimeter does not record the altitude of flight but the atmospheric pressure (Fig.11).

During an analysis of errors of similar instruments it is necessary to supplement the structural arrangement by a provisional link, establishing the functional relationship between the quantity to be measured and the quantity recorded by the instrument.

For example, the provisional link of the altimeter shown by dots in Fig.11

[redacted]

gives the relationship of the atmospheric pressure to the altitude.

An introduction of the provisional link into the structural arrangement permits consideration not only of the error of the elements but also of the error of the method of measurement.

The structural arrangements shown in Figs.10 and 11 are called "open undeveloped" arrangements. All links in these arrangements are connected in series, and the output parameter of each link forms the input parameter of the following link.

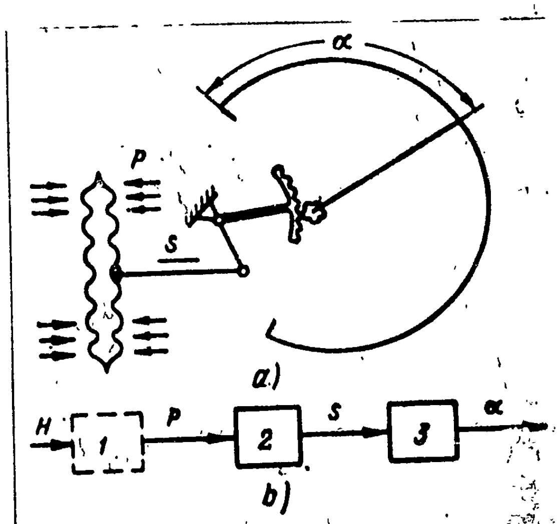


Fig.11 - Barometric Altimeter

a - Principal view; δ - Structural arrangement; H - Altitude to be measured; p - Atmospheric pressure; s - Motion of the aneroid box; α - Angle of deflection of the pointer

The structural arrangements of certain instruments contain parallel links. The presence of parallel links is characteristic of computing instruments, e.g., true air speed indicators, speed, autopilots, and others.

Finally, checking systems and automatic regulators, as well as instruments based upon compensating methods of measurement, which are widely used in various fields of technology, also have feedback loops, whose presence makes the structural scheme a closed one.

In this article, only open structural arrangements without parallel links, characteristic of most measuring instruments, are considered.

Figure 12 represents a general open structural arrangement of a measuring instrument, consisting of n links in series.

The general input coordinate X represents the physical quantity to be measured. The intermediate coordinates X_1, X_2, etc. are physical quantities whose increase

[redacted]

depends upon the operating principle of the elements of the instrument. The output coordinate $X_n = \alpha$ represents the deflection of the reading or recording device.

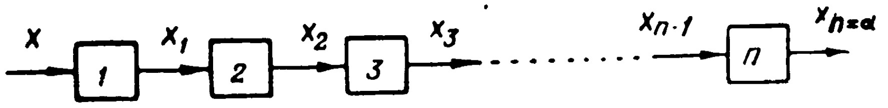


Fig.12 - Generalized Open Structural Arrangement

X - Input coordinate (magnitude to be measured); X_1 X_2... X_{n-1} - Intermediate coordinates, $X_n = \alpha$ - Output coordinate (deflection of the reading device).

2. Equation of the Scale of the Instrument. Absolute and Relative Errors

The static characteristics of the measuring instrument, known as the equation of the scale and indicating the ratio of the deflection of the reading device to the value of the quantity to be measured, may be expressed by the formula

$$\alpha = f(X). \qquad (28)$$

This ratio can be found by solving a system of equations, expressing the characteristics of the links in the structural scheme.

For example if in the arrangement shown in Fig.12 the number of links is n = 4, then the equations for the links will be

$$\begin{aligned} X_1 &= f_1(X); \\ X_2 &= f_2(X_1); \\ X_3 &= f_3(X_2); \\ \alpha &= f_4(X_3). \end{aligned} \qquad (29)$$

As a result of the solution of the system of equations (29) we obtain the equation for the scale of the instrument

$$\alpha = f_4\{f_3[f_2|f_1(X)|]\} = f(X). \qquad (30)$$

Example 7. Let us find the equation of the scale of a thermoelectric thermometer, shown in Fig.10.

The equation for the first link (thermocouple) is,

$$E=aT+bT^2,$$

where E is the electromotive force;

a and b are coefficients dependent on the materials of the thermoelectrodes;

T is the temperature.

The equation of the second link (electric link) is

$$I=\frac{E}{\sum R},$$

where Σ R is the sum of the electric resistances connected in series within the circuit of the thermocouple.

The equation for the third link (electromagnetic element) is

$$M=\frac{BSw}{9810}I,$$

where B is the magnetic induction in the air gap;

S is the area of the frame;

w is the number of turns.

The equation for the fourth link (two helical springs) is

$$\alpha=\frac{1}{2}\,\frac{12l}{bh^3\varepsilon}M,$$

where l is the free length of the spring;

b and h are the width and thickness of the cross section of the spring;

ε is the modulus of elasticity.

Solving this system of equations for the links according to eq.(30), we obtain the equation of the scale of the instrument:

$$\alpha = \frac{6l}{bh^{3\varepsilon}} \frac{BSw(aT + bT^2)}{9810 \sum R}.$$

The characteristic of the scale of the instrument may be constructed by graphical methods, if the characteristics of the links are given as graphs or in Tables.

Equation (30) gives the characteristic of the ideal instrument, if the output characteristics, given in eq.(29), are taken as ideal elements

$$\alpha_0 = f_0(X). \tag{31}$$

If, however, the characteristics of actual elements are to be considered, then the deflection of the reading device will be different from ideal and will be expressed by the formula

$$\alpha = f(X). \tag{32}$$

The absolute error of deflection of the reading device is equal to the difference between its actual (32) and ideal (31) values, being

$$\Delta\alpha = \alpha - \alpha_0 = f(X) - f_0(X). \tag{33}$$

The sensitivity of the instrument is the partial derivative of deflection of the specific device with respect to the quantity to be measured:

$$K = \left(\frac{\partial \alpha}{\partial X}\right)_0. \tag{34}$$

The subscript o indicates that the ideal equation (31) of the scale is being differentiated.

The sensivity of the instrument with an open structural arrangement is equal to the product of the transfer coefficients of all of its links:

$$K = \left(\frac{\partial \alpha}{\partial X}\right)_0 = \left(\frac{\partial X_1}{\partial X}\right)_0 \left(\frac{\partial X_2}{\partial X_1}\right)_0 \cdots \left(\frac{\partial X_{n-1}}{\partial X_{n-2}}\right)_0 \left(\frac{\partial X_n}{\partial X_{n-1}}\right) = \tag{35}$$

$$= K_1 K_2 \ldots K_{n-1} K_n = \prod_{i=1}^{n} K_i.$$

[redacted]

The absolute errors of the instrument are usually expressed in dimensions of the quantity measured, reading

$$\Delta X = X_{pr} - X,$$

where X_{pr} is the reading of the instrument in units of the quantity being measured, corresponding to the deflection α of the reading device;

X is the actual value of the quantity measured, causing a deflection α of the reading device.

In order to transform the absolute error of the instrument from the dimension α into the dimension X, it suffices to divide the error $\Delta\alpha$ (33) by the sensivity of the instrument (35):

$$\Delta X = \frac{\Delta\alpha}{K} = \frac{\Delta\alpha}{\prod\limits_{i=1}^{n} K_i}. \qquad (37)$$

The relative error of the instrument is equal to

$$\frac{\Delta X}{X}. \qquad (38)$$

The total relative error is equal to

$$\frac{\Delta X}{X_{max}}. \qquad (39)$$

Finding the Instrument Error from the Errors of its Elements

In general we consider the absolute instrument error to be that caused by a deviation of the characteristics of its elements from ideal values.

Let us assume that, as a result of calculation or of experimental investigation, the errors of all links in the structural arrangement are found, expressed in dimensions of the output coordinate of the corresponding link

(40)

Error of the 1st link | $\Delta X_1 = \varphi_1(X).$

$$\left.\begin{array}{ll}
\text{Error of the 2}^{\text{nd}}\text{ link} & \Delta X_2 = \varphi_2(X_1). \\
\dots\dots\dots\dots\dots\dots\dots\dots & \\
\text{Error of the i}^{\text{th}}\text{ link} & \Delta X_i = \varphi_i(X_{i-1}). \\
\dots\dots\dots\dots\dots\dots\dots\dots & \\
\text{Error of the n}^{\text{th}}\text{ link} & \Delta X_n = \varphi_n(X_{n-1}).
\end{array}\right\} \qquad (40)$$

In finding the overall error of the instrument we will consider errors of ΔX_i as primary.

In the beginning, we assume that all elements of the instrument, with exception of the i^th^ element, are ideal.

The partial error of deflection of the reading device, resulting from the error of the i^th^ element may, with an accuracy sufficient for practical purposes, be considered as a linear function of the primary error ΔX_i:

$$\Delta\alpha^{(i)} = \left(\frac{\partial\alpha}{\partial X_i}\right)_0 \Delta X_i. \qquad (41)$$

The index i of the error $\Delta\alpha$ indicates, that this error is partial, caused by the influence by the i^th^ element.

The linear approximation is completely proper in the given case, since all partial errors of the instrument lie within the limits of the field of tolerances, whose dimensions are very small relative to the length of the scale of the instrument.

The partial derivative $\left(\frac{\partial\alpha}{\partial X_i}\right)_0$ may be represented as the product of transfer coefficients of links from (i + 1) to n:

$$\left(\frac{\partial\alpha}{\partial X_i}\right)_0 = \left(\frac{\partial X_{i+1}}{\partial X_i}\right)_0 \left(\frac{\partial X_{i+2}}{\partial X_{i+1}}\right)_0 \cdots \left(\frac{\partial X_{n-1}}{\partial X_{n-2}}\right)_0 \left(\frac{\partial X_n}{\partial X_{n-1}}\right)_0 = \qquad (42)$$

$$= K_{i+1}K_{i+2} \cdots K_{n-1}K_n = \prod_{s=i+1}^{n} K_s.$$

The overall error of deflection of the reading device, during simultaneous action of errors of all links, is equal to the sum of the partial errors

$$\Delta\alpha=\sum_{i=1}^{n}\Delta\alpha^{(i)}=\sum_{i=1}^{n}\left(\Delta X_i\prod_{s=i+1}^{n}K_s\right). \qquad (43)$$

In order to find the absolute error of the instrument, expressed in units of the quantity measured, the error of deflection (43) should be divided by the sensitivity of the instrument; according to eq.(37) we obtain

$$\Delta X=\frac{\Delta\alpha}{K}=\frac{\sum\limits_{i=1}^{n}\left(\Delta X_i\prod\limits_{s=i+1}^{n}K_s\right)}{\prod\limits_{i=1}^{n}K_i}=\sum_{i=1}^{n}\frac{\Delta X_i}{\prod\limits_{s=1}^{i}K_s}. \qquad (44)$$

The relative error of the instrument is computed by substituting eq.(44) into eq.(38), and the total relative error by substituting eq.(44) into eq.(39).

For specific calculations, the expressions for the relative error of the instrument are often considerably simplified.

We will consider the simplest specific case, where the instrument consists of linear links, whose characteristics are expressed by the equation

$$\left.\begin{aligned}X_1&=K_1X;\\X_2&=K_2X_1;\\X_3&=K_3X_2;\\&\;\cdots\cdots\\X_i&=K_iX_{i-1};\\&\;\cdots\cdots\\X_n&=K_nX_{n-1}.\end{aligned}\right\} \qquad (45)$$

Using eqs.(45), any intermediate coordinate may be expressed in terms of X:

$$\begin{aligned}X_1&=K_1X;\\X_2&=K_2X_1=K_1K_2X;\\X_3&=K_3X_2=K_1K_2K_3X;\\&\cdots\cdots\cdots\cdots\cdots\cdots\end{aligned}$$

$$X_l = K_1 K_2 K_3 \ldots K_l X = X \prod_{s=1}^{l} K_s,$$

from which

$$\prod_{s=1}^{l} K_s = \frac{X_l}{X}. \qquad (46)$$

Substituting eq.(46) into eq.(44) and dividing both parts of the equation by X we obtain:

$$\frac{\Delta X}{X} = \sum \frac{\Delta X_l}{X_l}. \qquad (47)$$

From eq.(47) it follows that the relative error of the instrument, consisting of linear links, is equal to the sum of the relative errors of all links.

Analogously, it may be shown that the total relative error of the instrument, consisting of linear links, is equal to the sum of the total relative errors of all links:

$$\frac{\Delta X}{X_{\max}} = \sum_{l=n}^{n} \frac{\Delta X_l}{(X_l)_{\max}}. \qquad (48)$$

Example 8. Let us find the overall relative error of a thermoelectric (Fig.10), where for simplification we will assume the characteristic of the thermocouple to be linear, i.e.,

$$E = aT.$$

In this case, the overall relative error of the instrument may be expressed by the relative errors of its links by means of eq.(47), as follows:

$$\frac{\Delta T}{T} = \frac{\Delta E}{E} + \frac{\Delta I}{I} + \frac{\Delta M}{M} + \frac{\Delta a}{a},$$

STAT

136

where $\frac{\Delta E}{E}$ is the relative error of the thermocouple;

$\frac{\Delta I}{I}$ is the relative error of the electric circuit;

$\frac{\Delta M}{M}$ is the relative error of the electromagnetic element;

$\frac{\Delta \alpha}{\alpha}$ is the relative error of the springs.

3. Conditions of Physical Interchangeability of Elements of a Measuring Instrument

The physical interchangeability of pickup elements, as well as the dimensional interchangeability of parts and subassemblies, may be obtained by one of five methods, accepted in the construction of machinery (Bibl.4):

Method of absolute interchangeability;

Method of incomplete (partial) interchangeability;

Method of selection or of selective assembly;

Method of fitting;

Method of regulation.

The conditions of absolute physical interchangeability are determined by means of eq.(44) if, in the latter, concrete values of errors of the elements are substituted by their limiting values (calculations of "maximum-minimum"), taking into account the signs of the transfer coefficient.

Let us assume that, of the overall number n of links, there are m links for which the product $\prod_{s=1}^{i} K_s$ is positive and (n-m) links, for which the product $\prod_{s=1}^{i} K$ is negative.

Then the upper limiting error of the instrument is equal to

$$\Delta X^{\text{upper}} = \sum_{i=1}^{m} \frac{\Delta X_i^{\text{upper}}}{\prod\limits_{s=1}^{i} K_s} + \sum_{i=m+1}^{n} \frac{\Delta X_i^{\text{lower}}}{\prod\limits_{s=1}^{i} K_s}; \qquad (49)$$

where $\frac{\Delta E}{E}$ is the relative error of the thermocouple;

$\frac{\Delta I}{I}$ is the relative error of the electric circuit;

$\frac{\Delta M}{M}$ is the relative error of the electromagnetic element;

$\frac{\Delta \alpha}{\alpha}$ is the relative error of the springs.

3. Conditions of Physical Interchangeability of Elements of a Measuring Instrument

The physical interchangeability of pickup elements, as well as the dimensional interchangeability of parts and subassemblies, may be obtained by one of five methods, accepted in the construction of machinery (Bibl.4):

Method of absolute interchangeability;
Method of incomplete (partial) interchangeability;
Method of selection or of selective assembly;
Method of fitting;
Method of regulation.

The conditions of absolute physical interchangeability are determined by means of eq.(44) if, in the latter, concrete values of errors of the elements are substituted by their limiting values (calculations of "maximum-minimum"), taking into account the signs of the transfer coefficient.

Let us assume that, of the overall number n of links, there are m links for which the product $\prod_{s=1}^{i} K_s$ is positive and (n-m) links, for which the product $\prod_{s=1}^{i} K$ is negative.

Then the upper limiting error of the instrument is equal to

$$\Delta X^{\text{upper}} = \sum_{i=1}^{m} \frac{\Delta X_i^{\text{upper}}}{\prod_{s=1}^{i} K_s} + \sum_{i=m+1}^{n} \frac{\Delta X_i^{\text{lower}}}{\prod_{s=1}^{i} K_s}; \qquad (49)$$

and the lower limiting error of the instrument is equal to

$$\Delta X^{\text{lower}} = \sum_{i=1}^{m} \frac{\Delta X_i^{\text{lower}}}{\prod_{s=1}^{i} K_s} + \sum_{i=m+1}^{n} \frac{\Delta X_i^{\text{upper}}}{\prod_{s=1}^{i} K_s}, \qquad (50)$$

where $\Delta X_i^{\text{upper}}$ and $\Delta X_i^{\text{lower}}$ are the upper and lower values of the errors of the elements, permitted by the technical specifications for a given element.

If the instrument consists of linear links, eq.(47) will yield the following expression for the limiting relative errors of the instrument:

$$\left(\frac{\Delta X}{X}\right)^{\text{upper}} = \sum_{i=1}^{m} \left(\frac{\Delta X_i}{X_i}\right)^{\text{upper}} + \sum_{i=m+1}^{n} \left(\frac{\Delta X_i}{X_i}\right)^{\text{lower}}, \qquad (51)$$

$$\left(\frac{\Delta X}{X}\right)^{\text{lower}} = \sum_{i=1}^{m} \left(\frac{\Delta X_i}{X_i}\right)^{\text{lower}} + \sum_{i=m+1}^{n} \left(\frac{\Delta X_i}{X_i}\right)^{\text{upper}}. \qquad (52)$$

Analagous expressions are obtained for the limiting values of total relative errors, if eq.(48) is used.

The method of absolute interchangeability, the basis for which is the "maximum-minimum" calculation, is principally and practically without justification in most cases, if it is taken into account that primary errors are mainly random quantities and the probability of addition of their extreme values, especially in multilink chains is quite negligible.

N. A. Borodachev cites, for example, the following data for chains with ten primary errors, uniform in magnitude. The probability of obtaining a resultant error within the limits of 0.9 to 1.0 of its value, computed as a maximum, is as follows:

a) With distribution of primary errors according to the law of equal probability (which is rarely encountered in practice) it is less than one case out of a million.

b) With distribution of primary errors according to the normal law (close to

STAT

[redacted]

practical cases) it is less than one case out of 10^{15} cases.

The method of partial interchangeability, where the overall error of the instrument is computed in accordance with the rules of the theory of probability, seems to be more justified, as accepted for measuring and kinematic linkages.

The applicability of this method of computing is primarily derived from an explanation of the laws of distribution of primary errors by means of statistical treatment of the results of control of a large number of elements and, secondly, from the application of the following rules for the addition of errors (Bibl.2):

a) Systematic errors and quantities, characterizing centers of grouping of deviations of random errors, are added algebraically;

b) Quantities, characterizing the scatter of deviations of independent random errors, are added quadratically.

The possibility of appearance of any possible value of a partial error of the instrument, independently of the value of the other errors, is referred to as the independence of errors.

The formula for computing the tolerances of the overall error of the instrument, derived on the basis of the above rules, is of the type

$$\Delta X_{\Sigma} = \Delta X_0 + \delta X, \tag{53}$$

where ΔX_0 is the systematic part of the overall instrument error, equal to

$$\Delta X_0 = \sum_{i=1}^{n} \left(\frac{\partial X}{\partial X_i}\right)_0 (\Delta_{0i} + \alpha_i \delta_i); \tag{54}$$

Here, δX is the practical limiting random part of the overall error, equal to

$$\delta X = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial X}{\partial X_i}\right)_0^2 \left(\frac{\lambda_i}{\lambda}\right)^2 \delta_i^2} = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial X}{\partial X_i}\right)^2 k_i^2 \delta_i^2}. \tag{55}$$

In eqs.(54) and (55)

$\left(\frac{dX}{dX_i}\right)_o$ is the transfer coefficient from the input coordinate of the instrument X to the output coordinate of the i^{th} link;

Δ_{oi} is the coordinate of the center of the field of tolerances of the i^{th} primary error (if this error is random) or its algebraic value (if the i^{th} error is systematic),

α_i is the coefficient of relative asymmetry of the i^{th} error;

δ_i is one-half of the absolute magnitude of the field of tolerances of the i^{th} random error (if the i^{th} error is systematic, then $\delta_i = 0$);

λ_i is the relative mean quadratic deviation of the law of distribution of the i^{th} error $\left(\lambda_i = \frac{\sigma_i}{\delta_i}\right.$, where σ_i is the mean quadratic deviation of the i^{th} error);

λ is the relative mean quadratic deviation of the overall instrument error;

$$\lambda = \frac{\sigma}{\delta X},$$

where σ is the mean quadratic deviation of the instrument error;

$k_i = \frac{\lambda_i}{\lambda}$ is the coefficient of relative scatter of the primary error.

If the fields of tolerances of all primary errors are given and the law of their distribution is known, then all coefficients of the right side of eq.(55) are determined, with the exception of the coefficient λ which may be made variable.

An increase in λ permits a decrease of the tolerance of the instrument error, but in this case the so-called percentage of risk, i.e., the probable percentage of instruments with errors exceeding the limits of established tolerances, is increased.

In determining the percentage of risk, it has been accepted to use the quantity

$$t = \frac{1}{\lambda}.$$

Since, in most cases of practical problems, the distribution of the overall error may be considered subject to the normal law, the percentage of risk is determined according to the formula

$$P\% = 100\,[1-\Phi(t)] = 100\left[1-\frac{2}{\sqrt{2\pi}}\int_0^t e^{-\frac{t^2}{2}}\,dt\right], \qquad (56)$$

where Φ (t) is a Laplace function.

The computed values of the integral Φ(t) are given in Tables used in courses on the theory of probability and mathematical statistics.

Table 2 gives certain values of the integral Φ (t) and the percentage of risk, as a function of the parameter t.

Table 2

t	$\Phi(t)$	P %	t	$\Phi(t)$	P %
1,00	0,6827	32	3,00	0,9973	0,27
1,65	0,9011	10	3,29	0,9988	0,12
2,00	0,9545	4,5	3,89	0,999	0,01
2,57	0,9901	1,0			

In practice the triple mean-square deviation is considered to be the limit of the normal curve of distribution, i.e., $\delta X = 3\sigma$, which corresponds to the percentage of risk P = 0.27% (for t = 3).

Applying this condition, Tables yielding the values for the coefficients a_i and k_i for the most frequently encountered laws of distribution of primary errors in manufacturing, have been prepared.

After determining the coefficient k_i and computing the error δX, the upper and lower limits of the overall instrument error can be found from eqs.(53), (54), and (55).

$$\Delta X_{\Sigma}^{\text{upper}} = \Delta X_0 + \delta X = \sum_{i=1}^{n}\left(\frac{\partial X}{\partial X_i}\right)_0 (\Delta_{0i} + \alpha_i\delta_i) + \qquad (57)$$

$$\left.\begin{aligned}
&+\sqrt{\sum_{i=1}^{n}\left(\frac{\partial X}{\partial X_i}\right)_0^2 k_i^2\delta_i^2},\\
\Delta X_{\Sigma}^{\text{upper}}=\Delta X_0-\delta X=&\sum_{i=1}^{n}\left(\frac{\partial X}{\partial X_i}\right)_0(\Delta_{0i}+\alpha_i\delta_i)-\\
&-\sqrt{\sum_{i=1}^{n}\left(\frac{\partial X}{\partial X_i}\right)_0^2 k_i^2\delta_i^2}.
\end{aligned}\right\} \qquad (57)$$

For an open undeveloped structural arrangement, the transfer coefficient $\left(\frac{\partial X}{\partial X_i}\right)_0$ is inversely proportional to the product of the transfer coefficients of the links from 1 to i, i.e.,

$$\left(\frac{\partial X}{\partial X_i}\right)_0=\frac{1}{\frac{\partial X_1}{\partial X}\cdot\frac{\partial X_2}{\partial X_1}\cdots\frac{\partial X_i}{\partial X_{i-1}}}=\frac{1}{K_1K_2\ldots K_i}=\frac{1}{\prod\limits_{s=1}^{i}K_s}. \qquad (58)$$

Substituting eq.(58) into eq.(57), we obtain the final expression for the tolerance of the overall instrument error, constructed according to an open structural arrangement,

$$\left.\begin{aligned}
\Delta X_{\Sigma}^{\text{upper}}&=\sum_{i=1}^{n}\frac{\Delta_{0i}+\alpha_i\delta_i}{\prod\limits_{s=1}^{i}K_s}+\sqrt{\sum_{i=1}^{n}\frac{k_i^2\delta_i^2}{\prod\limits_{s=1}^{i}K_s}},\\
\Delta X_{\Sigma}^{\text{lower}}&=\sum_{i=1}^{n}\frac{\Delta_{0i}+\alpha_i\delta_i}{\prod\limits_{s=1}^{i}K_s}-\sqrt{\sum_{i=1}^{n}\frac{k_i^2\delta_i^2}{\prod\limits_{s=1}^{i}K_s}}.
\end{aligned}\right\} \qquad (59)$$

Equations (59) are also conditions of partial interchangeability of the elements of the measuring instrument.

The method of selective assembly, widely used in instrument making, and especially the method of regulation require special consideration. It can only be indicated that these are also based on assumptions which have been accepted for analogous methods in the theory of measuring and kinematic linkages.

STAT

BIBLIOGRAPHY

1. Balashkin,B.S. - Technology of Machine Building. Mashgiz (1949)
2. Borodachev,N.A. - Analysis of the Quality and Accuracy of Manufacturing. Mashgiz (1946)
3. Braslavskiy,D.A., Logunov,S.S., and Pelpor,D.S. - Calculation and Construction of Aircraft Instruments. Oborongiz (1954)
4. Bruyevich,N.G. - Accuracy of Mechanisms. Tekhteoretizdat (1946)
5. Bykhovskiy,M.L. - Accuracy of Electrical Computing Linkages. Izvestiya AN SSSR, OTN, No.8 (1948)
6. Gavrilov,A.N. - Technology of Aircraft Instrument manufacture. Oborongiz (1951)
7. Kutay,A.K. - Interchangeability, Tolerances, and Adjustments in Instrument Making. Mashgiz (1947)
8. Malikov,M.F. - Fundamentals of Meteorology , Committee on Measures and Measuring Instruments (1949)
9. Tikhmenev,S.S. - Theory of Aircraft Instruments. VVIA imeni Zhukovskiy (1940)
10. Yakhin,A.B. - Technology of Precision Instrument Manufacture. Oborongiz (1940)

STAT

ABBREVIATION OF THE EQUATION OF MOTION OF A RAPIDLY ROTATING GYROSCOPE ON GIMBALS AND THE INFLUENCE OF STATIC IMBALANCE OF THE GYROMOTOR ON THE BEHAVIOR OF THE GYROSCOPE

by

Cand. Tech. Sci., Docent G. A. Slomyanskiy

We will consider two of the most popular cases of arrangement of the symmetrical gyroscope in a gimbal suspension (Figs.1a and 1b).

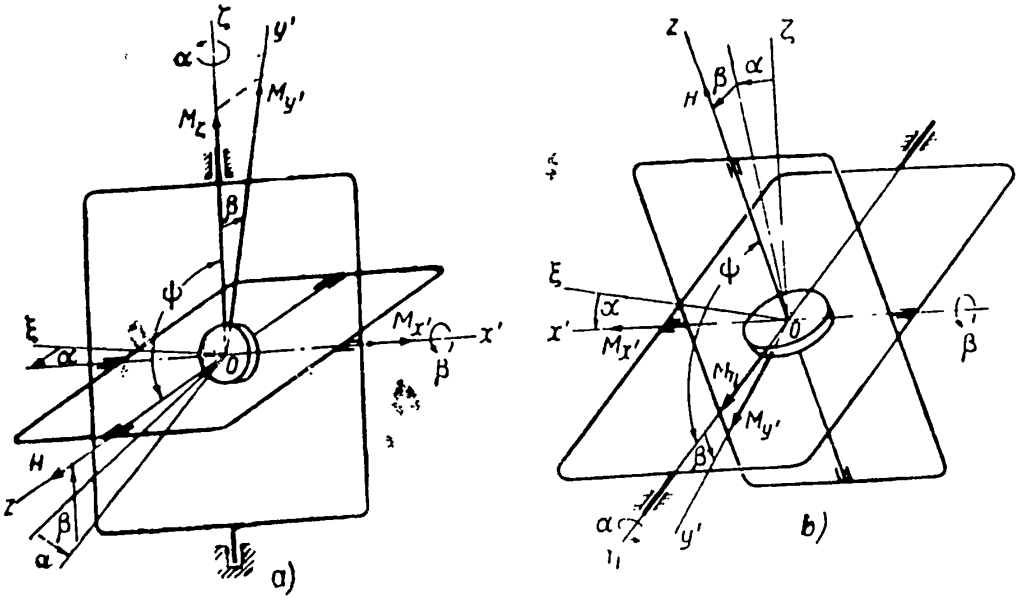


Fig.1 - Arrangement of a Gyroscope on Gimbals

a - Vertical axis of the outer frame of the gyroscope, δ - Horizontal axis of the outer frame of the gyroscope.

The system $\xi\eta\zeta$ is stationary, the axis x'y'z rigidly connected with the inner frame, the angle α is the angle of rotation of the outer frame, the angle β is the angle of rotation of the inner frame, the angles α and β are indicated in a positive direction.

If the inertia of the frames of the suspension is not taken into account, then

[redacted]

moment of the gyroscope and causing nutation,

the equations of motion of the axis of the gyroscope for both cases of arrangement of the gyroscope in a universal suspension are of the following type (Bibl.1):

$$\left.\begin{aligned} A(\ddot{\alpha}\cos\beta - 2\dot{\alpha}\dot{\beta}\sin\beta) + H\dot{\beta} &= M_{y'}, \\ A(\ddot{\beta} + \dot{\alpha}^2\sin\beta\cos\beta) - H\dot{\alpha}\cos\beta &= -M_{x'}, \\ \dot{H} &= M_z, \end{aligned}\right\} \qquad (1)$$

Where $M_{x'}$, $M_{y'}$, and M_z are the projections, upon the axes x', y', and z, of the moment M of the internal forces applied to the gyroscope. Then, equating $M_z = 0$, we will have H = const according to the third equation of the system (1). In this case, the motion of the axis of the gyroscope is fully characterized by the two first equations of the system (1). Only moments (or their projections), directed along the axes of rotation of the frames of the suspension, i.e., along the axes x' and ζ for the gyroscope shown in Fig.1a and along the axes x' and η for the gyroscope shown in Fig.1b, can show any influence upon the behavior of the axis of the gyroscope in universal suspension. We will transform the first two equations of the system (1) in such a fashion that they will include only moments directed along the axes of rotation of the inner and outer frames of the universal suspension. These moments we correspondingly designate by M_{in} and M_{out}. Since $M_{in} = M_{x'}$ then it is only necessary to transform the first of the equations in the system (1). Multiplying this equation by cos β and taking into account that $M_{y'} \cos\beta = M_{out}$ (in the case of Fig.1a, the moment $M_{out} = M_\zeta$ and in the case of Fig.1b the moment $M_{out} = M_\eta$) we will rewrite the first two equations of the system (1) as follows:

$$\left.\begin{aligned} A(\ddot{\alpha}\cos\beta - 2\dot{\alpha}\dot{\beta}\sin\beta)\cos\beta + H\dot{\beta}\cos\beta &= M_{out} \\ A(\ddot{\beta} + \dot{\alpha}^2\sin\beta\cos\beta) - H\dot{\alpha}\cos\beta &= -M_{in} \end{aligned}\right\} \qquad (2)$$

Abreviation of the Equation of Motion of a Rapidly Rotating Gyroscope in Gimbals

We will assume that the gyroscope rotates rapidly. In order to obtain abbreviated equations, characterizing the basic precessional motion of the gyroscope in gimbals, the terms in the equations of the system (2) containing A, depending upon the equatorial component of the kinetic moment of the gyroscope and causing nutation, will have to be discarded. Then we obtain

$$\left.\begin{aligned} H\dot{\beta}\cos\beta &= M_{out}, \\ H\dot{\alpha}\cos\beta &= M_{in}. \end{aligned}\right\} \qquad (3)$$

Equations (3) are applicable to any given values of the angle β, satisfying the inequality $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$. They differ in this respect from the usual abbreviated equations

$$H\dot{\beta} = M_{y'}, \quad H\dot{\alpha} = M_{x'},$$

applicable only for small values of the angle β.

Law of Precession of the Rapidly Rotating Gyroscope in Gimbals

We will designate the angle between the frames of the universal suspension by ψ. Figure 1 indicates that $\psi = \frac{\pi}{2} - \beta$. Solving eqs.(3) with respect to $\dot{\alpha}$ and $\dot{\beta}$ and introducing the angle ψ we obtain

$$\dot{\alpha} = \frac{M_{in}}{H\sin\psi}; \quad \dot{\beta} = \frac{M_{out}}{H\sin\psi}.$$

These expressions determine the rate of precession $\dot{\alpha}$ and $\dot{\beta}$ of the rapidly rotating gyroscope about the axes of rotation of the outer and inner frames of the suspension, and are applicable to any given values of the angle ψ, except for $\psi = 0$, permitting the formulation of the law of precession of a rapidly rotating gyroscope in gimbals in the following manner: The internal moment M, acting upon the gyroscope about the axis of rotation of any of the frames of the universal suspension, causes the axis of the gyroscope to rotate (precess) about the axis of rotation of the other frame with a speed

$$\Omega = \frac{M}{H\sin\psi}. \qquad (4)$$

During this rotation, the vector H tends to coincide with the vector M. In this fashion, the same moments acting about the axes of rotation of the frames of the universal suspension increase the rate of precession as the angle between the frames deviates from a right angle. The smallest values of the rates of precession $\dot{\alpha}$ and $\dot{\beta}$ are obtained when the frames of the universal suspension are mutually perpendicular ($\beta = 0, \psi = \frac{\pi}{2}$). From this follows a conclusion important for the technical utilization of the gyroscope, namely that at mutually perpendicular frames of the universal suspension, a rapidly rotating gyroscope possesses maximum stability with respect to upsetting moments applied to it. The more the angle between the frames of the universal suspension differs from a right angle, the less stable will the gyroscope be. Therefore, in cases when it is absolutely necessary that moments of friction on the axis of the universal suspension, moments caused by insufficient balances of the frames of the suspension, and also moments of transfer forces of inertia, have as little an influence as possible on the location of the axis of the gyroscope, then special devices for automatic suspension of the frames of the universal suspension in a mutually perpendicular position have to be considered.

Equation (4) is also applicable in cases, when the freedom of rotation of the gyroscope about its dead point is secured not by means of a universal suspension, but in some other fashion; as an example, the following may be mentioned: gyrostat, a gyroscope suspended by one end from a string, and others. In that case, the angle ψ is the angle between H and Ω, if $\widehat{H,M} = \frac{\pi}{2} = \text{const}$; if, under the action of the moment M, the angle between H and M changes, then $\psi = H, M$.

As an example of application of the laws of precession formulated by us, we will consider how a displacement of the center of gravity of the gyromotor (the internal frame with the rotor of the gyroscope) in the direction of the axes y' and z, at various values of the angle ψ between the frames of the universal suspension, will influence the gyroscope shown in Fig.1a. We will assume that o, y', and z are the coordinates of the center of gravity of the gyomotor and that the weight of the gyromotor is equal to P. Then, at an angle between the frames equal to $\psi < \frac{\pi}{2}$, the moment

$$M_{in} = P(z \sin\psi - y' \cos\psi),$$

will act on the gyroscope, which in accordance with the law of precession (4) will cause a rotation of the gyroscope about the axis of the outer frame with a speed of

$$\Omega = \dot{\alpha} = \frac{M_{in}}{H \sin\psi} = \frac{P}{H}(z - y' \operatorname{ctg}\psi).$$

This expression indicates that a displacement of the center of gravity of the gyromotor along the axis z causes a precession whose speed does not depend on the angle ψ between the frames. The rate of precession, caused by a displacement of the center of gravity along the axis y', is proportional to cot ψ.

Integration of Abbreviated Equations of Motion of the Gyroscope

The possibility of integration of eqs.(3) depends on the character of the moments M_{in} and M_{out}. We will consider certain important cases where integration of these equations is possible. We will assume that the moment M_{out} is a function only of the angle β (or is constant), i.e., that $M_{out} = M_{out}(\beta)$. Then, assuming, that, during the instant of initial motion $\beta = \beta_0$ at $t = 0$, we obtain the general solution of the first equations of the system (3) of the following type:

$$t = H \int_{\beta_0}^{\beta} \frac{\cos\beta \, d\beta}{M_{out}(\beta)}. \qquad (5)$$

It is obvious that the possibility of expressing the given integral in terms of elementary functions is determined only by the type of the function $M_{out}(\beta)$. The relation (5) permits a determination of the time of transition of the gyroscope from one position to another. For integration of the second equation in the system (3), we will initially assume that the moment M_{in} depends only on the angle β

(or is constant), i.e., $M_{in} = M_{in}(\beta)$ and, therefore, that this moment is a function of the angle α only (or is constant), i.e., $M_{in} = M_{in}(\alpha)$. In each of these cases, the general solution of the equation under consideration can be expressed in quadratics and, under the conditions that $\alpha = \alpha_0$ at $t = 0$, is of the type

$$\alpha = \alpha_0 + \frac{1}{H}\int_0^t \frac{M_{in}(\beta)}{\cos\beta}\,dt \quad (\text{for} \quad M_{in} = M_{in}(\beta)); \tag{6}$$

and

$$H\int_{\alpha_0}^{\alpha} \frac{d\alpha}{M_{in}(\alpha)} = \int_0^t \frac{dt}{\cos\beta} \quad (\text{for} \quad M_{in} = M_{in}(\alpha)). \tag{7}$$

In a number of practically important cases, the integrals appearing in eqs.(5), (6), and (7) may be expressed in terms of elementary functions.

Dividing the equations of the system (3), we obtain the differential equations of the trajectory of the top of the gyroscope along the surface of a unit sphere:

$$\frac{d\beta}{d\alpha} = \frac{M_{out}}{M_{in}}. \tag{8}$$

From eq.(8) it follows that the type of trajectory, described by the top of the gyroscope, depends only on the character of the moments acting on the gyroscope about the axis of rotation of the inner and outer frames of the universal suspension. The entire fixed value of the ratio $\frac{M_{out}}{M_{in}}$ corresponds to the fixed value of the ratio $\frac{d\beta}{d\alpha}$, independently of the angle between the frames of the universal suspension.

In a number of technical gyroscopes the moments M_{out} and M_{in} are functions of the angle β only (or are constant), i.e., $M_{out} = M_{out}(\beta)$ and $M_{in} = M_{in}(\beta)$. In this case, the general solution of eq.(8), under the initial conditions $\alpha = \alpha_0$ and $\beta = \beta_0$ at $t = 0$, is of the type

$$\alpha = \alpha_0 + \int_{\beta_0}^{\beta} \frac{M_{in}(\beta)}{M_{out}(\beta)}\,d\beta. \tag{9}$$

STAT

In cases when the moment M_{out} depends only on the angle β (or is constant), i.e., when $M_{out} = M_{out}(\beta)$, and the moment M_{in} is a function of the angle α only (or is constant), i.e., when $M_{in} = M_{in}(\alpha)$, the general solution of eq.(8), under the same initial conditions, is of the type:

$$\int_{\alpha_0}^{\alpha} \frac{d\alpha}{M_{in}(\alpha)} = \int_{\beta_0}^{\beta} \frac{d\beta}{M_{out}(\beta)} . \qquad (10)$$

In a number of practically important cases, the integrals in eqs.(9) and (10) may be expressed in terms of elementary functions.

We will note that, at $\frac{M_{in}}{M_{out}} = k = \text{const}$, the trajectory of the top of the gyroscope along the surface of a unit sphere is determined, as follows from eq.(9), by the equation

$$\alpha = \alpha_0 + k(\beta - \beta_0).$$

If $\frac{M_{out}}{M_{in}} = q \cos\beta$ where (q = const), the top of the gyroscope will describe a loxodrome on the surface of a unit sphere.

The relationships considered permit an investigation of the behavior of technical gyroscopes for any values of the angles α and β, except where $\beta = \pm\frac{\pi}{2}$.

Influence of Static Imbalance of the Gyromotor on the Behavior of the Gyroscope

We will consider a gyroscope, shown in Fig.1a, and assume that the axis of rotation of the supporting frame is vertical.

We will assume that the center of gravity of the gyromotor does not coincide with the center of rotation of the inner frame (axis x') and is located at the point C with the polar coordinates l and ψ (Fig.2). Here, l is the distance from the axis x' to the point C, ψ is the angle between the plane $x'z$ and the plane passing through the axis x' and the point C; the angle ψ is considered positive in the direction of positive y'. The weight of the gyromotor will be denoted by P. The

STAT

following moment with respect to the axis of rotation of the inner frame will then act on the gyroscope:

$$M_{P\,in} = lP\cos(\beta + \varphi).$$

The moment will be considered positive, if it tends to rotate the gyromotor counterclockwise, looking at it from the positive end of the axis x' (Figs.1a and 2).

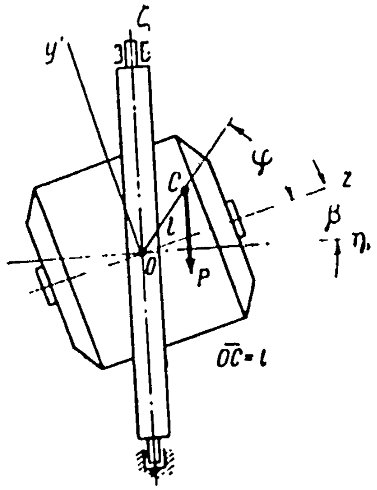


Fig.2 - Schematic of the arrangement and polar coordinates l, ψ of the center of gravity C of the gyromotor.

Aside from this, friction moments will act on the gyroscope in the bearings of the frames of the universal suspension and will be constant in magnitude and opposite in direction to the angular velocities $\dot{\alpha}$, $\dot{\beta}$. Since, at positive $\dot{\alpha}$ and $\dot{\beta}$, the vectors of these angular velocities are erected along the axes ($+\zeta$) and ($-x'$), the friction moments along the axes x' and ζ are equal to

$$M_{T.in}\ \text{sign}\,\dot{\beta} \text{ and } -M_{T.out}\ \text{sign}\,\dot{\alpha},$$

where $M_{T.in}$ and $M_{T.out}$ are the absolute values of the friction moments about the axes x' and ζ.

Thus, in the case under consideration, the moments applied to the gyroscope with respect to the axis of rotation of the inner and outer frames of the universal suspension, are equal to

$$\left.\begin{aligned} M_{in} &= lP\cos(\beta + \varphi) + M_{T.in}\ \text{sign}\,\dot{\beta}, \\ M_{out} &= -M_{T.out}\ \text{sign}\,\dot{\alpha}. \end{aligned}\right\} \qquad (11)$$

In order to set the gyroscope in motion (on a rigid support) it is necessary

STAT

following moment with respect to the axis of rotation of the inner frame will then act on the gyroscope:

$$M_{P\,in} = lP\cos(\beta + \varphi).$$

The moment will be considered positive, if it tends to rotate the gyromotor counterclockwise, looking at it from the positive end of the axis x' (Figs.1a and 2).

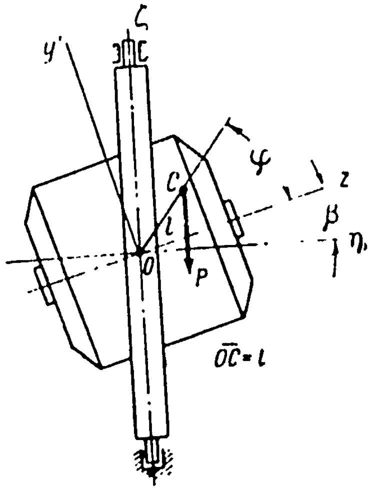


Fig.2 - Schematic of the arrangement and polar coordinates l, ψ of the center of gravity C of the gyromotor.

Aside from this, friction moments will act on the gyroscope in the bearings of the frames of the universal suspension and will be constant in magnitude and opposite in direction to the angular velocities $\dot{\alpha}$, $\dot{\beta}$. Since, at positive $\dot{\alpha}$ and $\dot{\beta}$, the vectors of these angular velocities are erected along the axes ($+\zeta$) and ($-x'$), the friction moments along the axes x' and ζ are equal to

$$M_{T.in}\,\mathrm{sign}\,\dot{\beta} \text{ and } -M_{T.out}\,\mathrm{sign}\,\dot{\alpha},$$

where $M_{T.in}$ and $M_{T.out}$ are the absolute values of the friction moments about the axes x' and ζ.

Thus, in the case under consideration, the moments applied to the gyroscope with respect to the axis of rotation of the inner and outer frames of the universal suspension, are equal to

$$\left.\begin{aligned} M_{in} &= lP\cos(\beta + \varphi) + M_{T.in}\,\mathrm{sign}\,\dot{\beta}, \\ M_{out} &= -M_{T.out}\,\mathrm{sign}\,\dot{\alpha}. \end{aligned}\right\} \qquad (11)$$

In order to set the gyroscope in motion (on a rigid support) it is necessary to have the absolute value of the moment $M_{P\,in}$ larger than the friction moment $M_{T.in}$, i.e., it should satisfy the inequation

$$lP\,|\cos(\beta_0+\varphi)| > M_{T.in}, \tag{12}$$

where β_o is the initial value of the angle β.

If the unbalance lP is equal to or less than the friction moment $M_{T.in}$ motion of the gyroscope is impossible for any values of the angle β_o. If, however, $lP > M_{T.in}$ then the gyroscope will only start moving at angles β_o satisfying the inequation (12).

Substituting in expression (12) an equal sign for the sign $>$ we obtain an equation for the limits of the region of rest. The graph in Fig.3 has been constructed according to this equation. If $\frac{P}{M_{T.in}}$ and $(\beta_o + \varphi)$ are such that the point corresponding to them in Fig.3 lies in the hatched region, which is the region of rest, then the motion of the gyroscope cannot start since the inequation (12) is not satisfied. On the other hand, the graph in Fig.3 indicates that, at $lP > M_{T.in}$, there always exist such values of β_o at which the inequation (12) is satisfied, and also values at which it is not satisfied. Therefore, the balancing of the gyromotor and the checking of its quality must be done at several values of the angle β_o.

Furthermore, the graph in Fig.3, shows that it is impermissible to limit the balancing of the gyromotor to only two values of the angle β_o, 90° apart, since such a method of balancing may prevent an unbalance from becoming apparent. This is possible, if $\frac{lP}{M_{T.in}}$ and $(\beta_o + \varphi)$ are such that the point corresponding to them in the graph of Fig.3 falls into the crosshatched region. For example, if $\frac{lP}{M_{T.in}} = 1.2$ and the balancing is in one case done at $\beta_o + \varphi = 40^\circ$ and in the other case at $\beta_o + \varphi + 90^\circ = 130^\circ$ or at $\beta_o + \varphi - 90^\circ = -50^\circ$, then an unbalance will not become apparent, since the corresponding points A_1, A_2, and $A_{2'}$ (Fig.3) are located in the region of rest, whereas the point A_1 is in the crosshatched region.

Considering further, that the inequation (12) has been satisfied, we obtain,

[redacted]

on the basis of eqs.(3) and (11), the following expressions for the rates of precession $\dot\beta$ and $\dot\alpha$:

$$\left.\begin{aligned}\dot\beta &= -\operatorname{sign}\dot\alpha\,\frac{M_{T.out}}{H\cos\beta},\\ \dot\alpha &= \frac{lP}{H\cos\beta}\cos(\beta+\varphi)+\operatorname{sign}\dot\beta\,\frac{M_{T.in}}{H\cos\beta}.\end{aligned}\right\} \qquad (13)$$

If in the expression for $\dot\alpha$, the member containing $M_{T.in}$ is omitted and if it is assumed that $\varphi = 0$ or π , i.e., if it is assumed that the center of gravity of the gyromotor lies in the plane x'z, then we obtain the result that the velocity of precession $\dot\alpha$, caused by the unbalance of lP, is equal to $\pm \frac{lP}{H}$ = const and, consequently, does not depend on the angle β or on the angle between the frames of the

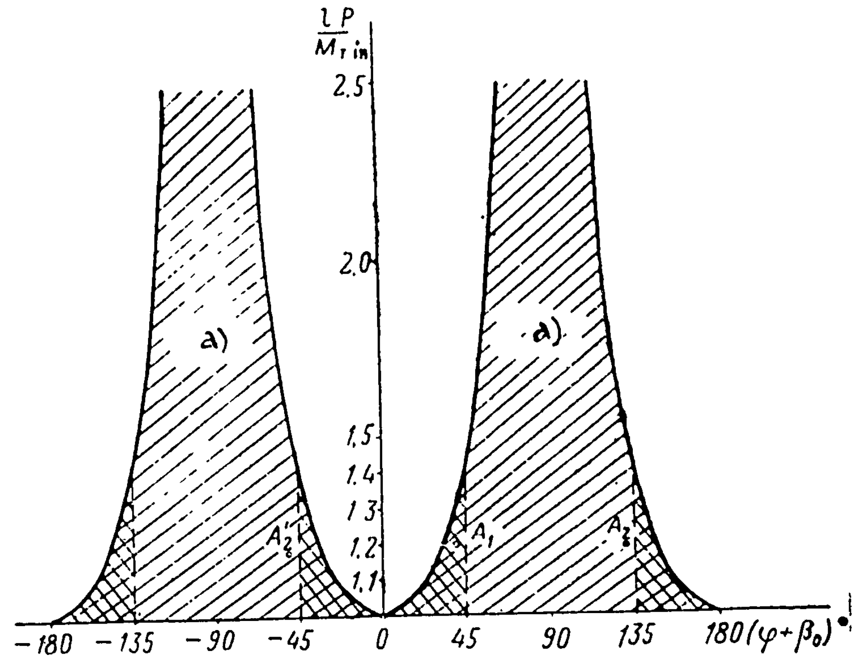


Fig.3 - Regions of Rest of the Gyromotor

a) Region of rest

universal suspension. In all other cases of location of the center of gravity C (see Fig.2), and also at $M_{T.in} \neq 0$, this velocity is a function of the angle $\dot\beta$. The velocity $\dot\beta$, as may be seen from eq.(13), does not depend on the magnitude of

[redacted]

the unbalance lP. The absolute value of the velocity β increases as a function of the increase of $|\beta|$ from 0 to $\frac{\pi}{2}$. We should mention that $\dot{\beta}$ is independent of lP only in the case where the axis of rotation of the inner frame is vertical, which was also assumed in the beginning of this Section.

Substituting eq.(11) for M_{out} in eq.(5) and integrating, we obtain

$$t = -\operatorname{sign}\dot{\alpha}\,\frac{H}{M_{\text{T. out}}}\,(\sin\beta - \sin\beta_0). \qquad (14)$$

From this we obtain the law of change of the angle β with time

$$\beta = \arcsin\left(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\,\frac{M_{\text{T. out}}}{H}\,t\right). \qquad (15)$$

Substituting in eq.(6) the moment M_{in} by its value from eq.(11), we obtain

$$\alpha = \alpha_0 + \frac{1}{H}\int_0^t \frac{M_{\text{T. in}}\operatorname{sign}\dot{\beta} + lP\cos(\beta+\varphi)}{\cos\beta}\,dt.$$

Eliminating, by means of eq.(15), the angle β and applying the substitution:

$$\sin\beta_0 - \operatorname{sign}\dot{\alpha}\,\frac{M_{\text{T. out}}}{H}\,t = X;$$

$$\sin\beta_0 = X_0; \quad -\operatorname{sign}\dot{\alpha}\,\frac{M_{\text{T. out}}}{H}\,dt = dX,$$

we obtain

$$\alpha = \alpha_0 - \frac{\operatorname{sign}\dot{\alpha}}{M_{\text{T. out}}}\left\{M_{\text{T. in}}\operatorname{sign}\dot{\beta}\int_{X_0}^{X}\frac{dX}{\sqrt{1-X^2}} + lP\cos\varphi\int_{X_0}^{X}dX - \right.$$

$$\left. - lP\sin\varphi\int_{X_0}^{X}\frac{X\,dX}{\sqrt{1-X^2}}\right\}.$$

Using the integrals which are part of the last expression, we have

$$\alpha = \alpha_0 - \frac{\operatorname{sign}\dot{\alpha}}{M_{\text{T. out}}}\,|M_{\text{T. in}}\operatorname{sign}\dot{\beta}\arcsin X +$$

$$+ lP\cos\varphi X + lP\sin\varphi\sqrt{1-X^2}\Big|_{X_0}^{X}.$$

[redacted]

The first of the equations in the system (13) indicates that, at $H > 0$ and $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$, the sign of the velocity $\dot{\beta}$ is always opposite to that of the velocity $\dot{\alpha}$ and that, at $\frac{\pi}{2} < \beta < \frac{3}{2}\pi$, the velocities $\dot{\alpha}$ and $\dot{\beta}$ are always single-valued.

We will further consider motions of the gyroscope at which $\frac{\pi}{2} < \beta < \frac{3}{2}\pi$.

In this case,

$$\operatorname{sign}\dot{\alpha}\,\operatorname{sign}\dot{\beta} = -1. \qquad (16)$$

Substituting the limits in the expression for α and taking into account the equality (16), we obtain the following final expression for the law of change of the angle α with time:

$$\alpha = \alpha_0 + \left(\frac{lP}{H}\cos\varphi\right)t + \operatorname{sign}\dot{\alpha}\,\frac{lP}{M_{T.\,out}}\sin\varphi\Big(\cos\beta_0 - \sqrt{1-\left(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\,\frac{M_{T.\,out}}{H}t\right)^2} - \frac{M_{T.\,in}}{M_{T.\,out}}\Big[\beta_0 - \arcsin\left(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\,\frac{M_{T.\,out}}{H}t\right)\Big]. \qquad (17)$$

In order to utilize the derived equations, it is necessary to know the sign of the velocity $\dot{\alpha}$. For this purpose, we will use the second of the expressions in (13). Above, we limited ourselves to a consideration of the cases where $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$. Under this condition, we find that $\dot{\alpha} > 0$, if $-\frac{\pi}{2} < \beta + \varphi < \frac{\pi}{2}$ and $\dot{\alpha} < 0$ at $\frac{\pi}{2} < \beta + \varphi < \frac{3}{2}\pi$.

We will prove that, in the process of the motion of the gyroscope being considered, the signs of the velocities $\dot{\alpha}, \dot{\beta}$ do not change, remaining the same as they were, at the instant of incipient motion. The sign of the velocity $\dot{\alpha}$ can only change in the case where the sign of the moment M_{in} changes. The latter, as may be seen from eq.(11), consists of the friction moment $M_{T.in}$ and the moment due to unbalance M_{Pin}. During the process of motion, the signs of $M_{T.in}$ and M_{Pin} are always opposite to each other. It was mentioned above that, during motion, the absolute value of M_{pin} must be larger than that of M_{Tin}. During the process of motion in connection with a change in the angle β, the value of $|M_{Pin}|$ may increase as well as decrease

[redacted]

in comparison with its value at the instant of incipient motion. It is apparent that an increase in $|M_{Pin}|$ cannot change the initial sign of M_{in}. A decrease in $|M_{Pin}|$ can only take place up to such a time when $|M_{Pin}|$ does not equal the friction moment $M_{T.in}$ since, at the instant of reaching this equality, the motion of the gyroscope stops. Consequently, even a decrease of $|M_{Pin}|$ cannot cause a change in the sign of M_{in}. Thus, at all times during the motion of the gyroscope according to the laws being considered, the sign of M_{in} and consequently, also the sign of $\dot{\alpha}$ remain the same as they were at the instant of incipient motion. From the fact that the sign of $\dot{\alpha}$ cannot change, it follows that the sign of $\dot{\beta}$ cannot change, which may be seen from the first expression in the system (13).

This will give

$$\left.\begin{array}{lll} \dot{\alpha} > 0; \quad \dot{\beta} < 0 & \text{for} & -\frac{\pi}{2} < \beta_0 + \varphi < \frac{\pi}{2}, \\ \dot{\alpha} < 0; \quad \dot{\beta} > 0 & \text{for} & \frac{\pi}{2} < \beta_0 + \varphi < \frac{3}{2}\pi. \end{array}\right\} \tag{18}$$

As a result of an analysis of the signs of the velocities $\dot{\alpha}$ $\dot{\beta}$ the following rule can be formulated: A change in angle β takes place in the direction of action of the moment of unbalance. This simple rule should be utilized in balancing of an operating gyromotor.

Eliminating from eq.(17), by means of expression (14), the time t, we obtain the equation of the trajectory of the top of the gyroscope along the surface of a unit sphere of the type

$$\begin{aligned} \alpha = \alpha_0 + \operatorname{sign}\dot{\alpha}\,\frac{IP}{M_{T.\,out}}\,[\sin(\beta_0+\varphi) - \sin(\beta+\varphi)] - \\ - \frac{M_{T.\,in}}{M_{T.\,out}}(\beta_0 - \beta). \end{aligned} \tag{19}$$

If from the instant of incipient motion and up to the point of its stopping, the angle β remains small, then it can be assumed that $\beta \approx \beta$ and cos $\beta \approx 1$, so that the equation of the trajectory (19) may be approximately represented in the form

$$\alpha=\alpha_0+\frac{1}{M_{\text{T. out}}}(\operatorname{sign}\dot{\alpha} lP\cos\varphi-M_{\text{T. in}})(\beta_0-\beta). \qquad (19')$$

The motion of the gyroscope according to the laws under consideration will continue, depending on the magnitude of lP up to the time when the velocity $\dot{\alpha}$ is not equal to zero, which will take place as soon as the moment M_{in} becomes zero or up to the instant the angle β reaches a value of $\pm\frac{\pi}{2}$ which corresponds to the loss of one degree of freedom by the gyroscope.

We will find the value of the angle $\beta = \beta^*$ up to which the moment M_{in} and, consequently, the velocity $\dot{\alpha}$ become equal to zero. For this purpose, we equate the first of the expressions in the system (11) to zero; solving the resultant equation for β, we find

$$\beta^*=\begin{cases} -\arccos\frac{M_{\text{T. in}}}{lP}-\varphi & \text{for } -\frac{\pi}{2}<\beta_0+\varphi<\frac{\pi}{2}, \\ \pi+\arccos\frac{M_{\text{T. in}}}{lP}-\varphi & \text{for } \frac{\pi}{2}<\beta_0+\varphi<\frac{3}{2}\pi. \end{cases} \qquad (20)$$

From this it can be seen that, if $lP\sin\varphi = M_{T.in}$, then $\beta^* = \pm\frac{\pi}{2}$. In this fashion, at $lP\sin\varphi \geq M_{T.in}$ (which in turn is possible at $0<\varphi<\pi$), the frames of the universal suspension will, in time, coincide and the gyroscope will lose one degree of freedom.

At $lP\sin\varphi < M_{T.in}$ (if $\pi<\varphi<2\pi$, then this will hold for any lP), the frames will not align. In this case, the motion of the gyroscope will continue for the duration of the time t^*, after which the gyroscope will attain an equilibrium position, determined by the coordinates α^* and β^*, where $-\frac{\pi}{2}<\beta^*<\frac{\pi}{2}$. Using $\beta=\beta^*$ in eqs.(14) and (19), we obtain for t^* and α^* the following expressions:

$$t^*=-\operatorname{sign}\dot{\alpha}\frac{H}{M_{\text{T. out}}}(\sin\beta^*-\sin\beta_0), \qquad (21)$$

[redacted]

$$\alpha^* = \alpha_0 + \operatorname{sign}\dot{\alpha}\,\frac{lP}{M_{T.\,out}}\left[\sin(\beta_0+\varphi) - \sin(\beta^*+\varphi)\right] - \\ - \frac{M_{T.\,in}}{M_{T.\,out}}(\beta_0 - \beta^*). \qquad (22)$$

From a consideration of eqs.(20), (21), and (22), the following conclusions may be drawn. The angle β^* is a function only of the angle φ and of the ratio $\frac{M_{T.in}}{l\,P}$. An increase in the ratio $\frac{l\,P}{M_{T.in}}$ causes an increase in the time t^*, as well as in the angles α^*, β^*. The time t^* and the angle α^* become smaller with increasing friction moment in the bearings of the supporting frame $M_{T.out}$. This is explained in the following manner: In the case under consideration, the moment $M_{T.out}$ causes a precession of the gyroscope with a velocity $\dot{\beta}$, directed in such a way that it changes the moment M_{in} to zero. The larger $M_{T.out}$ the larger will be $\dot{\beta}$. Therefore, the larger $M_{T.out}$ the more rapidly will M_{in} become zero; consequently, other conditions being equal, the smaller will be the deviation of the gyroscope by an angle α (angle α^*), caused by the moment M_{in}. However, with an increase in $M_{T.out}$ the value of the transmitted velocity, at which the gyroscope is driven by translatory rotation, will also increase. In cases when $\pi < \varphi < 2\pi$, i.e., when the center of gravity of the gyromotor is located below the plane x'z, the gyroscope will more rapidly reach an equilibrium position (α^*, β^*), and the angles α^*, β^* will be smaller than at $0 < \varphi < \pi$ i.e., when the center of gravity of the gyromotor is located above the plane x'z. This is illustrated in Fig.4, which gives the graphs of the ratios t^*, α^*, β^*, as a function of φ for values of the ratio $\frac{l\,P}{M_{T.in}}$ equal to 1.1, 1.2, and 1.4 at $\alpha_0 = \beta_0 = 0$. The graphs in Fig.4 further indicate that β^* is considerably larger than α^*. For a given value of $\frac{l\,P}{M_{T.in}}$, the angle α^* decreases as the ratio $\frac{M_{T.in}}{M_{T.out}}$ decreases, that is the larger $M_{T.out}$.

[redacted]

We will consider the character of change in the angles α and β with time. In order not to restrict the calculation to specific values of H and $M_{T.out}$, we will

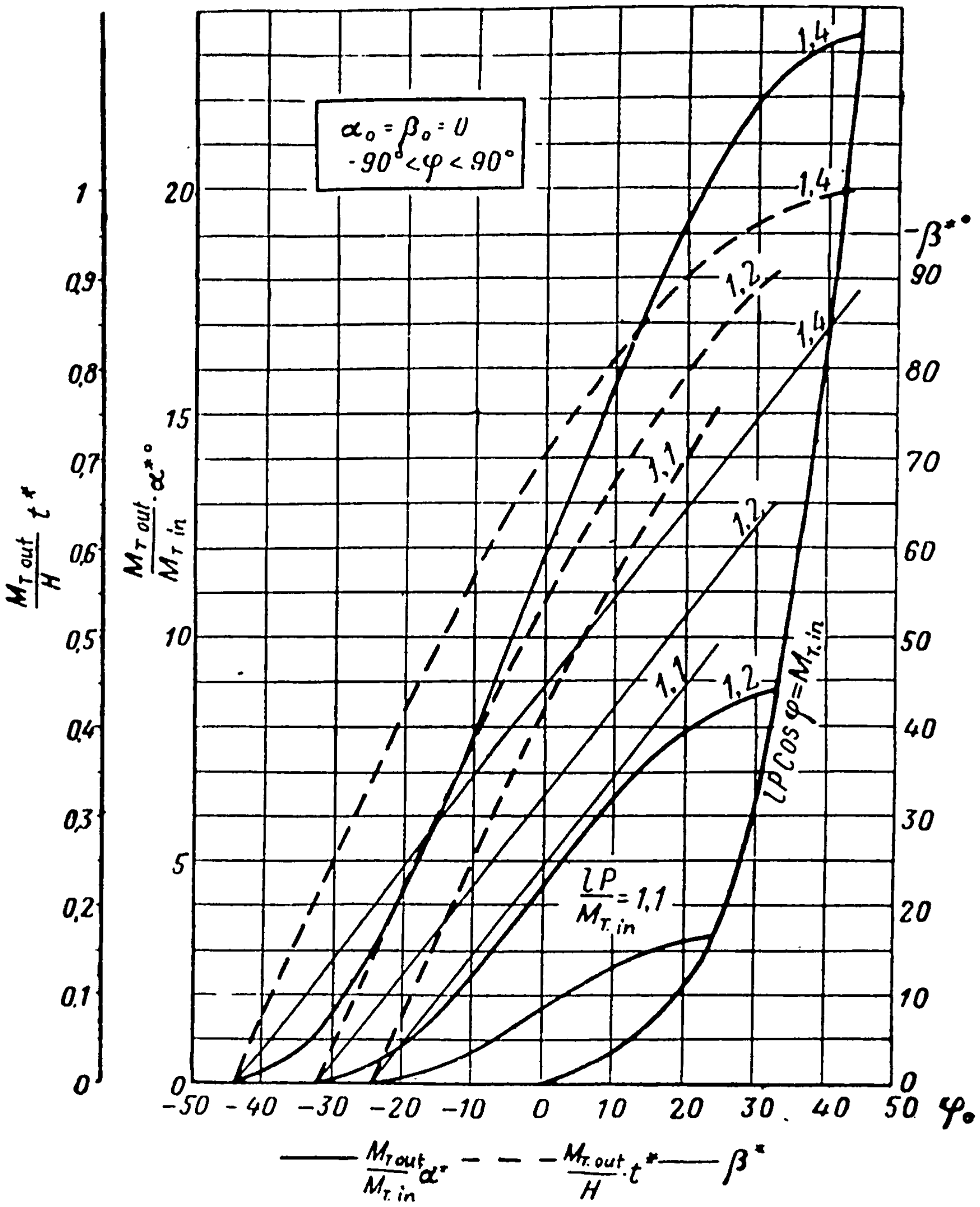


Fig.4 - Graphs of the Ratio of the Coordinates of Location of the Equilibrium of the Gyromotor α^*, β^* and of the Time t^* of Motion to this Location to the Angle φ, Unbalance lP, and Moments of Friction $M_{T.in}$ and $M_{T.out}$ under the Condition that, at the Instant of Incipient Motion, i.e., at $t = 0$, we have $\alpha = \beta = 0$.

substitute the time t in eqs.(15) and (17) for α and β by the dimensionless time

$$\tau = \frac{M_{T.\,out}}{H} t; \tag{23}$$

and thus obtain

$$\beta = \arcsin(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\tau), \tag{24}$$

$$\alpha = \alpha_0 + \frac{M_{T.\,in}}{M_{T.\,out}} \left\{ \frac{lP}{M_{T.\,in}} \left[\tau\cos\varphi + \operatorname{sign}\dot{\alpha}\sin\varphi\,(\cos\beta_0 - \right.\right.$$

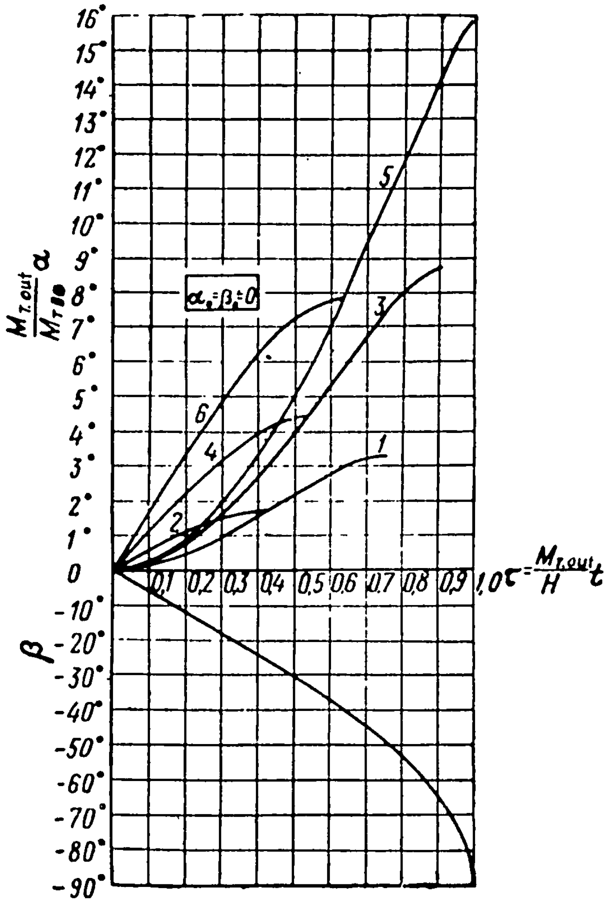
$$\left.\left. - \sqrt{1-(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\tau)^2}\,\right] - \beta_0 + \arcsin(\sin\beta_0 - \operatorname{sign}\dot{\alpha}\tau) \right\}. \tag{25}$$

The graphs in Fig.5 are constructed according to these formulas.

Equation (24) and Fig.5 show that the angle β is a function only of the dimensionless time τ; the rate of increase in the angle β increases with an increase in its absolute value. The character of the change in the angle α with time, as may be seen from Fig.5, depends on the magnitude of the angle $\beta_0 + \varphi$. At $\beta_0 + \varphi > 0$, the rate of increase of the angle α increases in the beginning, reaches a certain maximum, and then decreases to zero. If $\beta_0 + \varphi \leq 0$, then the velocity $\dot{\alpha}$ has its largest value at the instant t = 0, after which it will gradually decrease to zero. The curves 1, 3, and 5 are constructed for values of φ corresponding to the limits of the region of rest.

The equilibrium values of the angles α and β, obtained in this case equal α* and β* are the maximum possible ones at given values of $\frac{lP}{M_{T.in}}$. The curves 2, 4, and 6 are constructed for φ = 0 and are characteristic of the maximum possible initial velocity α.

It may be seen from Fig.5 that the deviation of the gyroscope by an angle takes place considerably faster than by an angle α. The graphs in Fig.5 permit an evaluation of the behavior of gyroscopes, characterized by any given values of the quantities H, $M_{T.out}$, $\frac{M_{T.out}}{M_{T}}$ at values of $\frac{lP}{M_{T.in}}$ equal to 1.1, 1.2, 1.3. As an example, we will determine the deviation of a gyroscope by the angles α and β at t = 15 min, if H = 1200 gm-cm-sec, $\frac{lP}{M_{T.in}} = 1.1$, $\varphi = 0$, $M_{T.out} = 0.3$ gm-cm, and $\frac{M_{T.out}}{M_{T.in}} = 1.2$. According to eq.(23), we find: $\tau = 0.225$. From the graph in Fig.5, along curve 2,



№ of curve	$\frac{lP}{M_{T.in}}$	φ
1	1,1	24°30′
2		0
3	1,2	32°30′
4		0
5	1,3	39°30′
6		0

Fig.5 - Graphs for the Change of the Angles α and β, as a Function of the Dimensionless Time τ at Various Values of the Angle φ and the Ratio $\frac{lP}{M_{T.in}}$.

we have: $\frac{M_{T.out}}{M_{T.in}}\alpha = 1.2^\circ$.

Considering that $\frac{M_{T.out}}{M_{T.in}} = 1.2$, we obtain $\alpha = 1^\circ$. For the angle β we immediately obtain the value $\beta = 13^\circ$. Equations (23), (24), (25) and the graphs in Fig.5

permit the following conclusions on the influence of the quantities lP, $M_{T.in}$, and $M_{T.out}$ exerted on the magnitude of the angles by which the gyroscope declines from initial position $\alpha_0 = \beta_0 = 0$ at $t = 0$ through the time t $(t < t^*)$. An influence on the magnitude of the angle β is exerted only by $M_{T.out}$. The larger $M_{T.out}$, the larger will be the angle β. The angle α increases with an increase of lP and decreases with an increase in $M_{T.in}$ or the larger the ratio $\frac{lP}{M_{T.in}}$, the larger will be the angle α.

If $\pi \leq \varphi \leq 2\pi$, the increase in $M_{T.out}$ leads to a decrease in the angle α. If, however, $0 < \varphi < \pi$ then for all values of $M_{T.out}$ at which the bisectrix of the angle passing through the arc OC (Fig.2) during the time t lies above the plane of the horizon, the angle α will be smaller in proportion to $M_{T.out}$. In cases when, at $0 < \varphi < \pi$, the mentioned bisectrix lies below the plane of the horizon, a decrease in $M_{T.out}$ will cause an increase in the angle α. Thus, if the gyromotor has a low tendency to oscillate, the increase in $M_{T.out}$ will lead to a decrease in the deviation of the gyroscope by an angle α during the time t. At a maximum tendency to oscillate, an increase in $M_{T.out}$ leads, in some cases, to an increase in the angle α and, in other cases, its decrease. Thus, the decrease in the deviation of the gyroscope by an angle α during the time t may be obtained by increasing the accuracy of static balance, i.e., by decreasing the unbalance lP, as well as by increasing the fixed moments $M_{T.in}$ and $M_{T.out}$ (in specific cases, a decrease of $M_{T.out}$ may be required). However, in no case should $M_{T.in}$ and $M_{T.out}$ be increased, since the increase in the friction moments decreases the sensivity of the gyroscope to translatory rotation and adversely affects the performance of the instrument on a moving and oscillating support. Therefore, it follows that $M_{T.in}$ and $M_{T.out}$ should be decreased if possible. For a decrease in the deviation of the gyroscope by an angle α, the quality of the static balancing of the gyromotor should be increased, which will cause a decrease in the value of the ratio $\frac{lP}{M_{T.in}}$ due to a decrease in the value of l. The balancing should be done with the gyromotor operating, i.e., with $M_{T.in}$ in the oper-

STAT

ating condition of the instrument. If it is practically impossible to obtain the desired accuracy of static balancing of the gyromotor and to decrease the quantities $M_{T.in}$ and $M_{T.out}$, it becomes necessary to increase H. It must be mentioned that an increase in H should not be done by increasing the moment of inertia of the gyroscope, but by increasing the velocity of its specific rotation, while simultaneously decreasing its weight.

BIBLIOGRAPHY

1. Bulgakov, B.V. - Theory of Application of the Gyroscope. GONTI (1939)

GENERAL CLASSIFICATION OF WINDING MACHINES IN INSTRUMENT

by

Cand. Tech. Sci., Docent V.P.Chumakov, Senior Lecturer Ye.M.Nikolayev*

A large number of winding machines of various types are used in instrument making. The total number of types of winding machines exceeds 150. Many models of winding machines, used for the same types of windings and differing only in accuracy and productivity, could be mentioned. Such a diversity of winding equipment is explained not only by the considerable nomenclature of the windings used in modern instruments and automatic devices, but also by the fact that, up to this time, little attention was paid to the calculation and construction of winding machines and that these machines were developed by many organizations at times without a sufficient theoretical foundation or investigation of winding processes. Technologists do not always rationally select winding equipment, in planning winding processes.

One of the more important and timely problems in the field of the production of windings is the typification of technological processes of winding, to assist in the creation of high productivity and accurate processes. Work on the standardization of technological processes of winding requires the classification of winding machines. The absence of such a classification causes considerable difficulties at present in the planning of technological processes and the selection of winding machines. The classification of winding machines should serve as basic principles in the unification of winding equipment and the development of types of machines. At the same time, a classification will be of value in the creation of uniform terminology in the field of winding equipment.

It can be stated that the planning and utilization of winding machines at the

---

* The first two Chapters were written by V. P. Chumakov; the third Chapter was written by V. P. Chumakov in cooperation with Ye. M. Nikolayev.

[redacted]

present time is at approximately the same level as it was in our country twenty to twenty-five years ago with respect to metal working machines, at which time a large variety of models and types of metal working machines, and specifically many lathe-type machines, were in use. ENIMS* and other organizations, within the last years have done a large amount of work on the classification and unification of metal-cutting machines and the development of various types of machines. The work performed played an important role in the development of machine-building and metal-working. Similar work should be performed on winding equipment.

The starting materials in the development of classifications of winding machines were: 1) a classification of windings by technological criterions developed by us, 2) a classified compilation of types of windings, 3) investigations of winding equipment. Data on more thoroughly perfected domestic equipment were mainly utilized in the compilation of the classification.

As a basis for classification, the principle of formation and lay of the windings in the process of windings was used. From this point of view, it will serve the purpose to divide all winding machines into three classes:

I. Machines for coil (open) windings;

II. Machines for closed windings;

III. Machines for slotted windings.

As coil windings we consider windings, which are obtained by means of windings wire onto a core rotating about its own axis. During this process, the wire or the core, in addition to the basic motion, frequently also has an additional regular motion along the axis of the coil, required for a given lay of the turns.

Circular (toroidal) windings are formed by winding wire around the axis of the cross section of a core which has the shape of a ring or semicircle, and which rotates slowly relative to its own axis, thus attaining the required lay of the turns. In order to run around the cross section of the body, the wire must be fed through

---

* Experimental Scientific Research Institute for Metal-Cutting Machines

STAT

the inside of the contour; this is obtained by special circular shuttles.

For slotted windings, the wire is layed in the slots of the rotor or stator stack.

Each of the mentioned classes of machines are divided into separate types, groups, and kinds. For convenient illustration, three separate Tables of classifications by winding classes have been prepared.

In the classifications of winding machines presented, the structural and production characteristics of the machines are not mentioned, and models or overall dimensions are not indicated. Only a brief technological characteristic of the winding equipment is given, and general criteria of separate kinds, groups, and types of machines are differentiated. It is apparent that more detailed classification schemes for various kinds of machines along construction and usage characteristics could be proposed in the future.

Class I: Machines for Coil Windings

Entirely different coil windings have different accuracy, distribution of turns, diameters of wire, cross-sectional shapes of the cores, and other data.

In instrument-making plants winding machines differing in construction, productivity, and overall dimensions are used for core windings.

According to technological criteria, we divide the machines for coil windings into:

I. Machines for general or wide application;

II. Machines for special application;

III. Machines of high productivity (for simultaneous winding of several assemblies).

I. Machines for general application are useful for various types of spool windings: layer windings, single-layer and multilayer windings, crossed-over windings, bifilar windings, and other types.

STAT

These are obtained not only by means of structural possibilities of the machines, but also as a result of utilization of various types of interchangeable fixtures and attachments, mounted to the machines. For example, attachments for cross-over and functional windings, two spool-holders and chucks on the spreader, and others. The machines have large speed and feed ranges and may be utilized for the winding of cores of various cross sections and dimensions.

Modern machines for general application are equipped with braking mechanisms, actuated by a counter after the winding of a given number of turns, mechanisms for automatic spreading of the turns, infinitely variable speed and feed control, and other mechanisms increasing the quality and automation of the winding processes. Such machines find widest application in experimental and small-series production.

II. Under machines for special applications we include machines, utilized for special types of coil windings, cross sections and dimensions of bodies, and diameters of wires. These machines will, in their construction, closely resemble machines for general application, but the design of each will be adapted for a special type of winding. Newer machines for special applications also have a number of improved attachments and mechanisms, used in a manner similar to that in general-purpose machines. However, older models of machines are still being utilized in plants (mainly those of foreign make) for example, machines without automatic spreading of turns (make "Scincilla") or without speed control (make "Boston" and "Mikafil").

Brief technological characteristics of the various types of winding machines for special applications are given in Table 1.

1. Machines for layer windings are divided into:

   a) Machines without spreader mechanisms, utilized for free windings ("bulk" windings);

   b) Machines for single-layer windings with attachments for automatic spreading of the turns; the majority of such machines have no automatic shifting of the spreader for the return sequence.

STAT

c) Machines for multilayer windings. The most popular machines of this type are equipped with attachments for automatic spreading of the turns and shifting for the return sequence.

2. Machines for polar coils and coils of heavy wire (of diameters of 0.6 - 3 mm). These machines are of one and the same type as the machines of the preceding group but have stronger braking attachments, spreaders operated by a cam, and other attachments.

The winding of polar coils and coils of heavy wire on other machines for layer windings is usually made difficult because of the large diameters of the bodies.

3. Machines for windings of thin wire of diameters of 0.008 - 0.07 mm, used for accurate resistances, frames of measuring instruments, and other assemblies. Wire of diameters up to 0.02 mm is often called very fine or microwire. Machines of this type are characterized by high accuracy of production, low moment of inertia of rotating and moving parts, and work at low speeds; thus permit return winding of the wire from the core onto the spool.

4. Machines for winding of flat cores (rectangular, trapezoidal, or curvilinear). Characteristic of the construction of machines of this type is the transmission of rotation to the headstock as well as tailstock. Rotation of the tailstock eliminates twisting of the bodies, held in the spindles of the headstock and tailstock during the winding process. A special differential transmission to the spreader is mounted on these machines in order to increase the accuracy of spreading of the turns and adjustment of the lay, which changes as a function of the profile of the body.

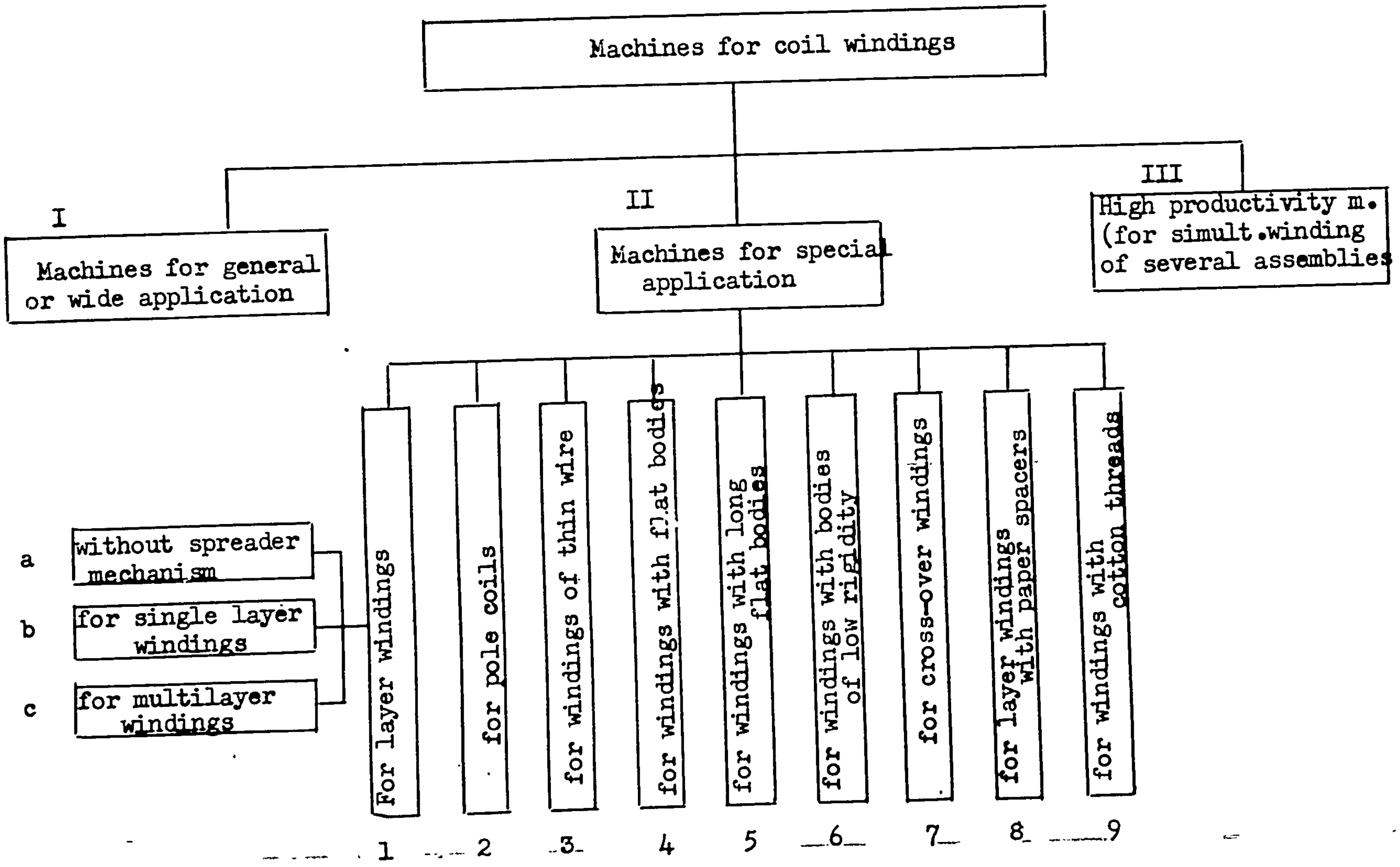
5. Machines for winding of flat cores of great lengths (up to 0.5 m) were developed due to the requirement for spreads of high accuracy (tolerance of lay of the spread, ± 0.01), complicated profiles and, mainly, large dimensions of the bodies.

To avoid considerable differences in tension of the wire during winding and in order to increase the accuracy of spreading of the turns, the machines are equipped with special differential mechanisms and other special devices.

STAT

Table 1

Class I



[redacted]

STAT

6. Machines for winding of bodies of low rigidity, i.e., of thin plates made of plastics and other materials; wire is also used for the bodies. The bodies are stretched by clamping between the headstock and the tailstock. The latter has a drive shaft and turns with the same velocity as the headstock. A moving support of the machine supports the body, protecting it from bending during the winding process. The moving support is displaced together with the carriage, on which the spreader mechanism for the wire is mounted, and supports the body in the place where the laying of the turns takes place.

7. Machines for cross-over windings. The main feature of machines for cross-over windings is the relatively large speed of the return motion of the dog with the wire. During the cycle of this motion, the spindle, with the body being wound, makes one revolution (for universal windings) or two and more revolutions (for cross-over multiple turn windings). Machines of this kind come in two types: a) with slide-block mechanisms (Fig.1), which permits the winding of spools of various widths by means of changing the ratio of the levers of the slide; b) with shifting of the dog from the cam; for each width of winding in most cases a separate cam is required.

8. Machines for layer windings with paper spacers. These machines have a special attachment for cutting off and positioning the insulating paper. The paper is cut off for the required length which increases with each new layer of turns, and is automatically placed on the spool after the end of winding of each layer of turns of wire. The transport mechanism of the paper as well as the mechanism for positioning the wires, is usually put into motion by means of a friction clutch.

9. Machines for layer windings with cotton threads. Simultaneously with the wire, cotton thread is wound between the turns and between the layers of wire. For winding the thread, a special mechanism is provided. Cotton thread is used for increasing the electric and mechanical strength of the windings. In coreless coils, the thread serves as a connecting element.

III. Machines of high productivity. This group includes machines on which sev-

STAT

eral assemblies (more than four) are wound simultaneously at high speeds, with the use of attachments. The machines are intended for large series production and are

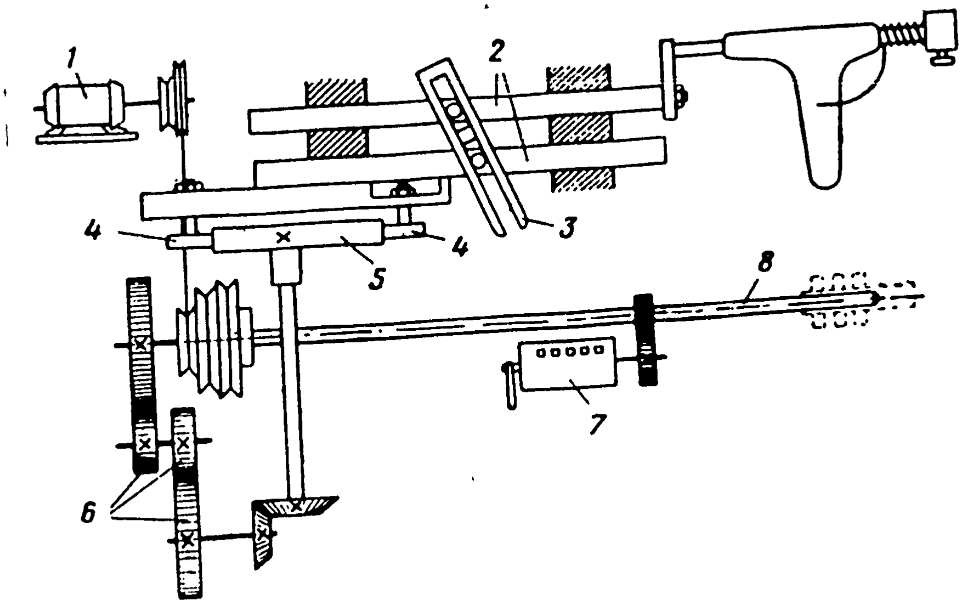


Fig.1 - Machine for Cross-Over Windings

1 - Electric motor; 2 - Moving racks; 3 - Slide block;
4 - Roller; 5 - Eccentric cam; 6 - Changeable gears;
7 - Counter of number of turns; 8 - Spindle of machine.

poorly adaptable for quick changes from one winding to another.

High production machines are divided into: 1) single-spindle machines, on which all assemblies are located on a single spindle and 2) multispindle machines, in which each assembly is located on its own spindle, being driven from the main shaft.

Class II: Machines for Ring Windings

Machines for ring windings (Table 2) are subdivided into:

I. Machines for general or wide application;

II. Machines for special applications;

III. High-productivity machines (multispindle).

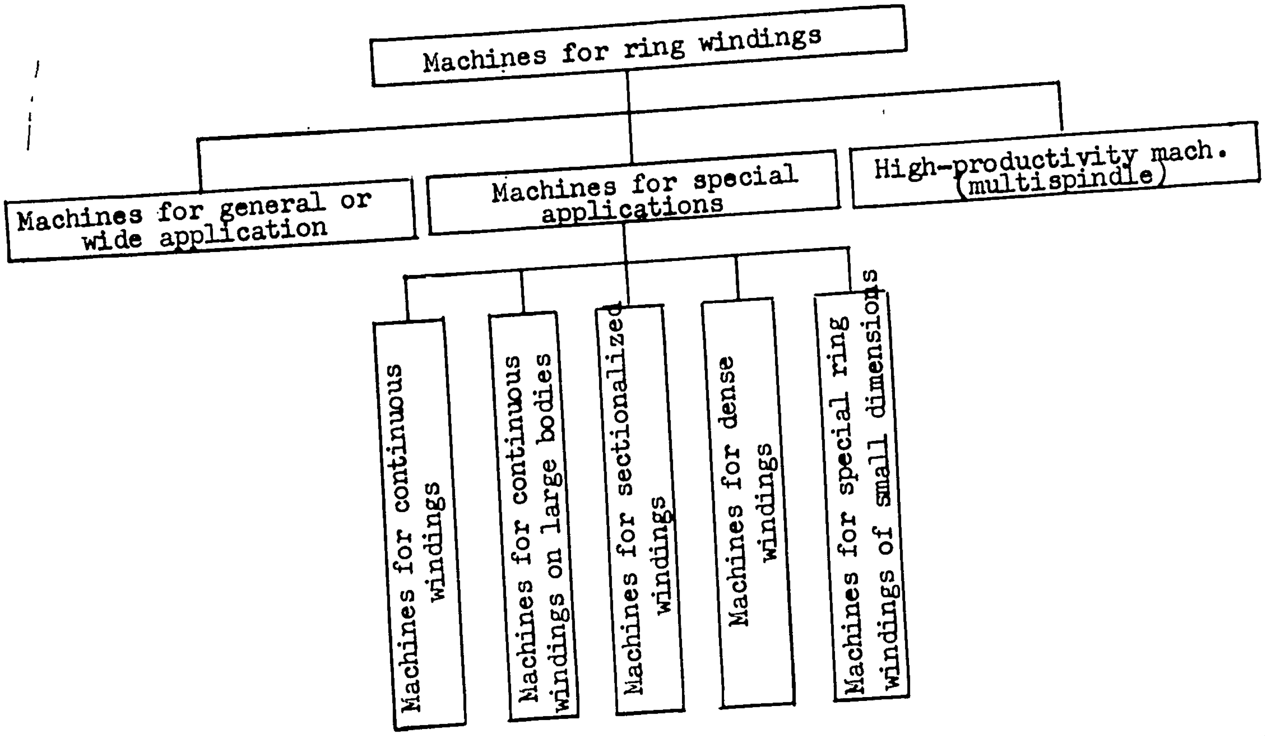
I. On machines for general use, the following can be produced:

a) Single-layer windings, dense or with a pitch, which may be uniform or change according to a predetermined functional relationship;

b) Multilayer windings and other types of windings.

The design of these machines includes a number of attachments to produce: windings with a functional relationship, sectionalized windings, and others. Furthermore, the machines are equipped with various interchangeable attachments, which increases the accuracy of the winding and extends the fields of application. Sometimes magnifying glasses for observation of the process of winding the turns onto the body are attached.

Table 2



II. Machines for special applications include machines used for specific ring windings, body dimensions, and diameters of wire. The design of the machines is adapted for one or the other type of winding.

Machines for special applications are subdivided into the following types:

1. Machines for continuous windings (Fig.2) are usually used for single-layer

STAT

[redacted]

windings with a constant pitch or for dense windings. The range of diameters of the cores which can be wound on such machines is rather large and runs from 20 to 120 mm.

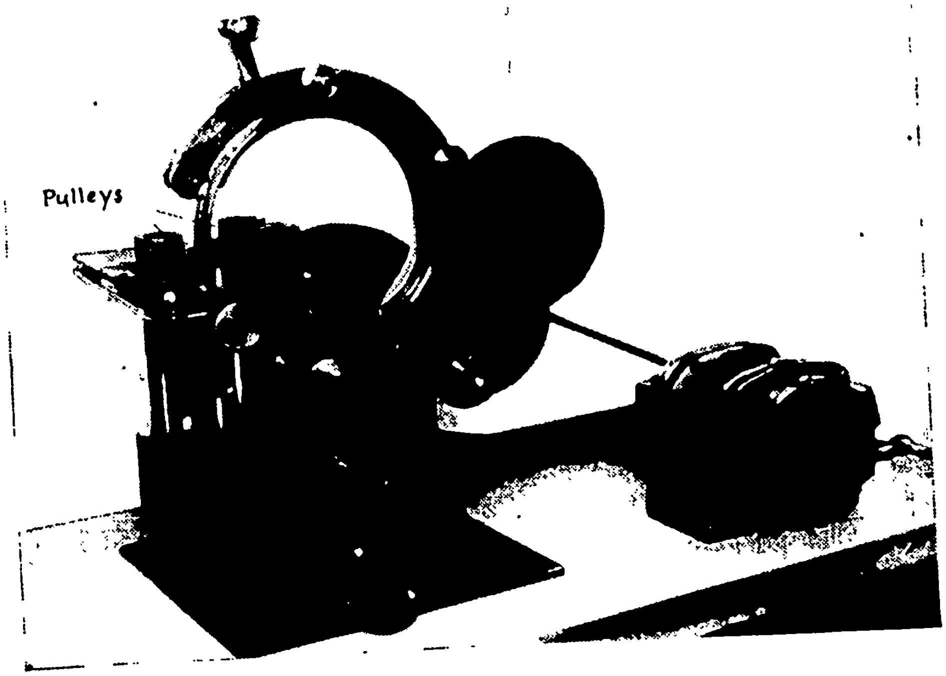


Fig.2 - Machines for Continuous Ring Windings

The transfer of motion to the rolls which turn the core to be wound, is obtained by means of connecting rods and gear trains. The core or frame can be set in motion by means of a flexible metal strip, which gives a smoother transmission.

2. Machines for continuous windings on large cores of diameters up to 400 mm. Machines for such windings should contain suitable attachments for fastening and rotating the bodies.

3. Machines for sectionalized windings, which consist of several sections separated by spacers insulating plates secured to the core. The sections consist of several layers of closely spaced wire and are symmetrically arranged.

A peculiarity of the machines for sectionalized windings is the successive forward and reverse motion of the table, on which the body to be wound is mounted. The angle of rotation of the table corresponds to the arc of the wound section. The

STAT

table is made to rotate by means of the connecting rod (1) and ratchet (2) mechanisms (Fig.3). The rotational speed of the table depends on the diameter of the wire and on the rotational speed of the shuttle (3). After the winding of a section, the machine is stopped and the core turned by hand for winding of the next section.

4. Machines for dense windings (abutting turns). The density of winding on the machines is attained by a pair of special levers (1) (Fig.4) which, during the winding process, force the turn against the row of previously wound turns. Such a system of dense winding of turns is used only for thin wire of a diameter of 0.2 - 0.04 mm and for small cores with an outside diameter of 10 - 30 mm of a height of 15 mm.

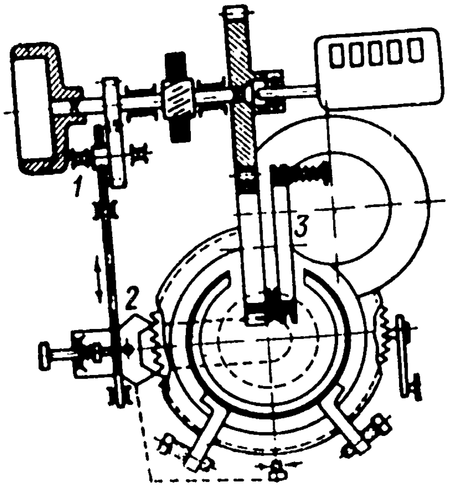


Fig.3 - Machine for Sectionalized Ring Windings

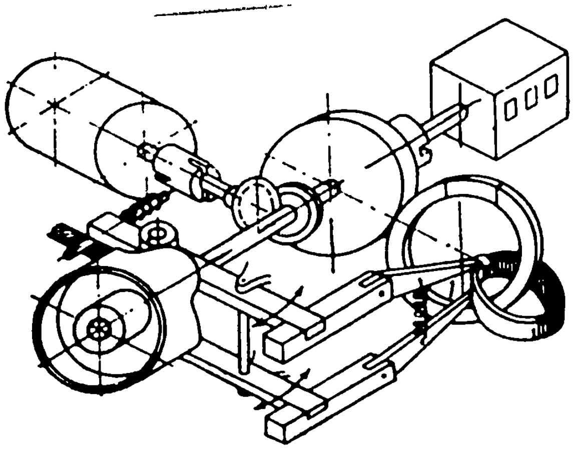


Fig.4 - Machine for Dense Ring Windings

5. Machines for special ring windings of small dimensions. In instrument making, ring windings with very small dimensions, such as windings with overall core diameters of 5 mm, wire diameters of 0.015 - 0.02 mm, and 10,000 turns are used. Such windings cannot be obtained in machines of the type described above.

At present, machines for such windings are still in the planning stage.

III. Machines for high productivity are multispindle machines of the caroussel type. The spindles (heads) of the machines are located on a rotating table. There are six-spindle machines of the type developed by Engineer V. I. Grishin. Winding on these machines is performed simultaneously by five heads, with the sixth head disconnected; the machines are used for set-up and supplemental work such as: replacement or turning of cores, threading of wire onto spools, etc.

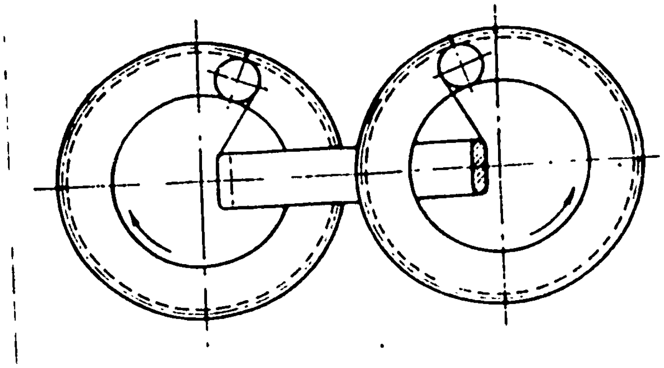


Fig.5 - Schematic Sketch of Winding Two Sections of One Core on the Machine Developed by V. I. Grishin.

The winding of each body is performed simultaneously by two shuttles, each of which winds its own section (Fig.5). The machines are mainly meant for sectionalized windings, but may easily be adapted for various other types of windings, specifically for the winding of rectilinear cores of short length.

Class III: Machines for Slotted Windings

This class of machines includes machines for producing slotted windings for small electric equipment, i.e., machines in which the outside diameter of the stacks with external slots and the inside diameter (diameter of the hole) of stacks with internal slots does not exceed 60 mm. This limit, it should be understood, is conditional. Most of the slotted windings in instrument making have smaller corresponding dimensions. However, there are winding assemblies with diameters that are somewhat larger than 60 mm. Packets with internal and external slots have a length of 3 - 100 mm.

For windings of small electric machines, thin copper wire (according to GOST -

[redacted]

Soviet State Standard - 6324-52) with diameters of 0.05 - 1.2 mm are used, usually with enameled (PEL, PEV, PEM) insulation, and less frequently with silk (PELShO) or cotton (PELKhO) insulation.

The basic classification of a given class of machines is the grouping of slotted windings according to technological application and dimensions developed by us (see Table 3).

Each of the machines, included by us in Class III, is characterized by its method of laying the turns into the stacks and is primarily intended for one type of windings or for several types which are similar in shape and spacing of turns.

Many slotted windings, up to this time, are being made by hand, since no machines, generally applicable to series production, are available.

Some types of slotted windings or groups of windings have characteristic features which require special machines for their production.

The classification of machines developed by us reflects, to a certain extent, the state of presently available winding equipment in instrument making. Machines for chord windings with internal slots have found the widest application.

Some of these machines are successfully used in series production. These include types of machines with simplest automation, belonging to the subgroups of machines with rotating packet and with rotating reel, and also semiautomatic machines of the first subgroup.

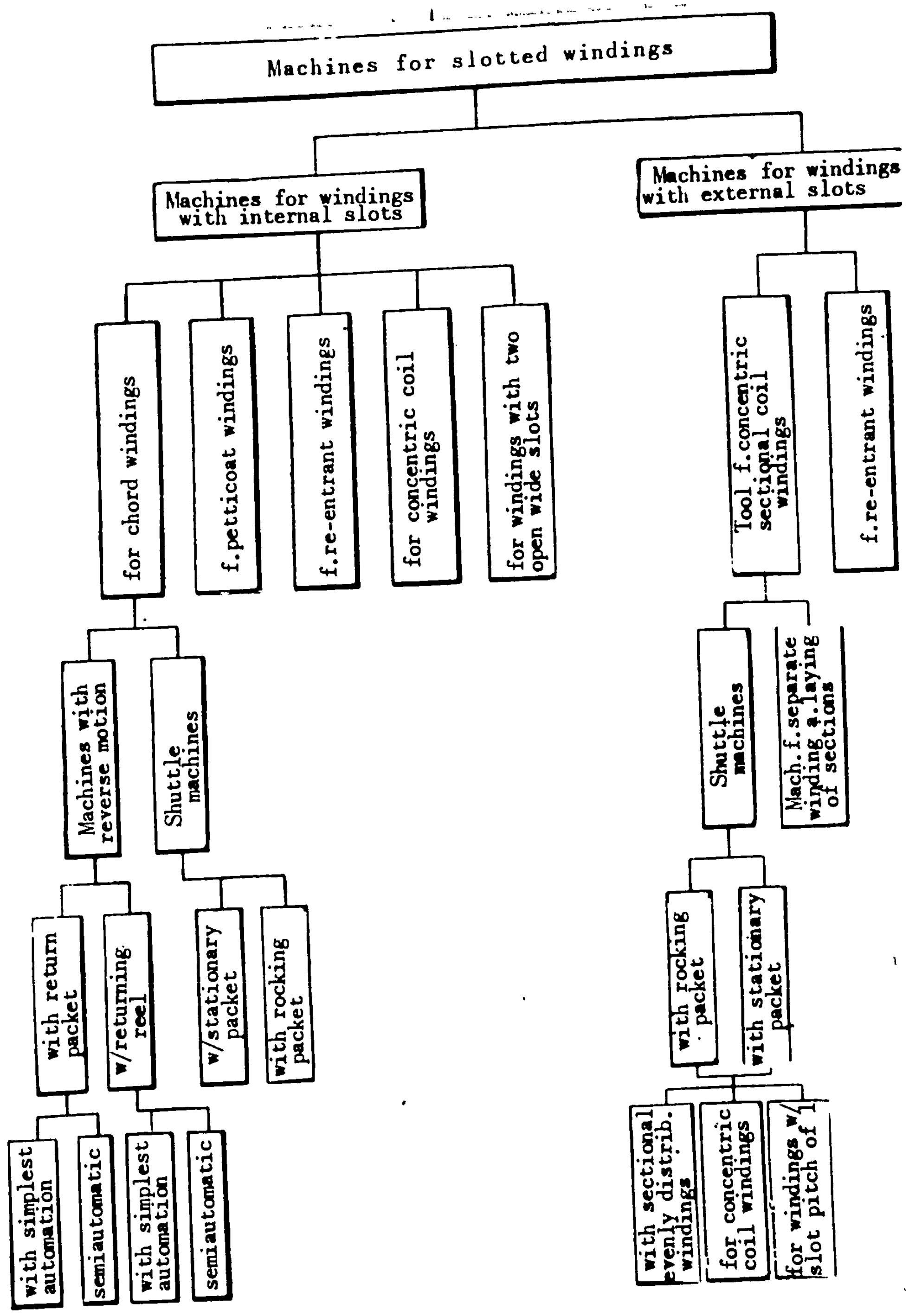
There are no machines for the winding of internal slots available, but investigative work is being done on the creation of machines included in Table 3 in the category of machines for windings with internal slots. Work is also being done on a machine for separate winding and laying of sections (machine developed by Ye. D. Klyushnev).

The slots in the packets may be located along the line of generation of the packet (straight slots) or along a winding line (slanted slots). In many cases one and the same machines may be used for packets with straight slots and for packets

STAT

[redacted]

Class III



[redacted]

STAT

with slanted internal slots. Sometimes various machines can be adapted for both straight and slanted slots.

All machines for slotted windings are subdivided into machines for windings with external slots on the packets and machines for windings with internal slots.

Machines for chord windings. Among windings with internal slots the most popular are windings which can be grouped under the term "chord windings"; these include the following types: single-chord, two-chord, christmas tree, single-chord with variable pitch of slots, three-phase of equal sections (evenly distributed), with a small number of slots per pole and phase, single-phase, two-phase, and sinusoidal.

With respect to the techniques characteristic of single-chord windings, there is the positioning of windings of all sections in one direction during the winding process. Figure 6 shows turns layed counterclockwise. The winding of the sections is done along one chord from slot to slot without change of direction.

The winding of two-chord windings is done along two chords, symmetrical with respect to the axis of the packet of the armature winding (Fig.7).

Looking at sections corresponding to chords during winding (in the direction of the arrows A and B), the direction of the sections will differ (Fig.8). This is characteristic of two-chord windings. From here on, all types of chord windings in which the direction of winding, when going from one section to another, has to be changed, will be referred to as two-chord windings.

Included in two-chord windings in this general sense are specifically two-chord (christmas tree), three-phase, equispaced, and other windings.

Since, in the production of two-chord windings, the direction of rotation of the spindle of the machine has to be changed, the machines for such windings should be reversible. An exception is the machine for the winding of small rotors and stators with internal slots (see below), in which the packet is clamped in two pairs of guide jaws. In this case, the direction of rotation of the spindle of the machine does not have to be changed. In passing to the winding of the following sec-

STAT

[redacted]

tion, the wire is transferred from one pair of jaws to the other.

In small electric machines the majority of windings are two-pole windings.

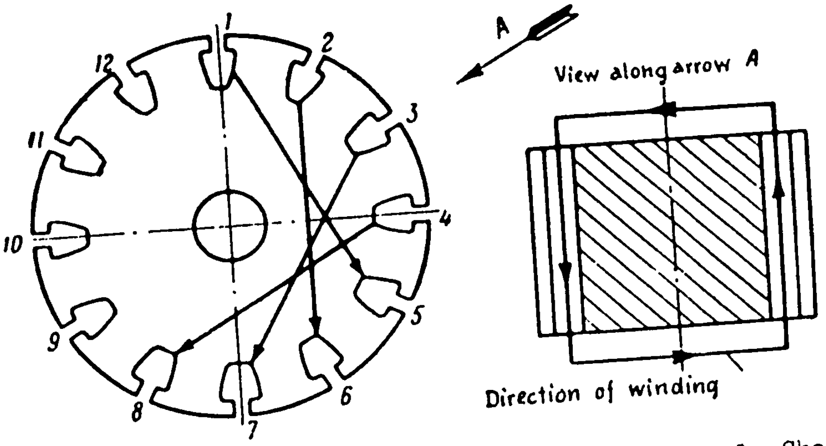


Fig.6 - Schematic Sketch of the Winding Process of a Single-Chord Winding.

In chord windings of two-pole electric machines the pitch of the slots (or the number of slots included in the section of turns wound), corresponds as a rule to a central angle of ~180°, but in some instances of two-phase windings, the pitch is equal to 90°.

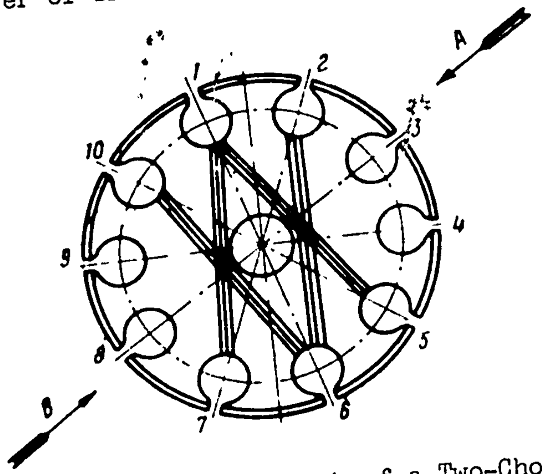


Fig.7 - Schematic Sketch of a Two-Chord Armature Winding

In chord windings for multipole electric machines, the pitch of the slots is equal to or less than 90° (with windings of four-pole electric machines, with an elongated pitch, the spacing of the slots may be slightly more than 90°); therefore the winding of armatures of multipole electric machines by simple rotational motion is fairly difficult.

Machines for chord windings can be subdivided into two basic groups:

The first group comprises machine whose main motion is rotation about a stationary axis. This group is divided into two subgroups.

The first subgroup comprises machines on which the packet of the rotor or stator to be wound rotates and winds the wire onto itself.

Machines of the first subgroup are used for winding of small packets of diameters not above 40 - 50 mm and a width of less than 40 mm.

There are two types of such machines: machines with "simplest automation" and semiautomatic machines.

Machines with "simplest automation" are used for winding armatures and rotors of two-pole electric machines, where the turns are laid into the slots of the packets of the type of single-chord or two-chord windings. These machines also include the machine for winding small rotors and stators with internal slots* and the SNYa-4 machine (Fig.9).

The SNYa-4 machine, apart from the slotted windings mentioned, can produce various types of coil windings: series windings, sectionalized windings, inductionless windings, etc., since this machine is equipped with special attachments for automatic positioning of the turns.

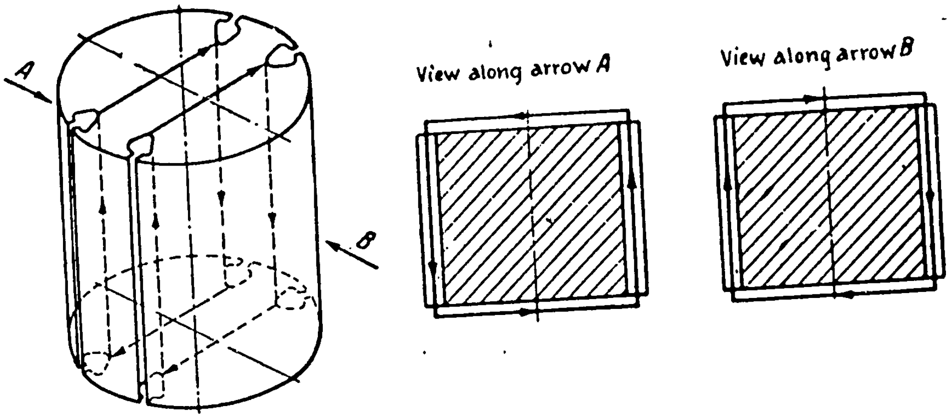


Fig.8 - Direction of Turns for a Two-Chord Winding.

The majority of supplementary operations during winding of packets on machines with simplest automation is performed manually. After winding a given number of

---

* See next Chapter of this book.

turns, the machine stops automatically.

Semiautomatic machines are used for many operations without attention by the operator, such as forming and threading of the wire loop onto the pin of the mandrel, release of the guide jaws, return of the packet for winding of the following section, placement of the guide jaws, starting of the machine, winding of sections, etc. On semiautomatic machines SNYa-1, ATE-1 (Fig.10), in use at the present time, only the production of single-chord windings for two-pole electric machines is possible since these machines are nonreversible. The semiautomatic machine ATE-1 is earmarked for

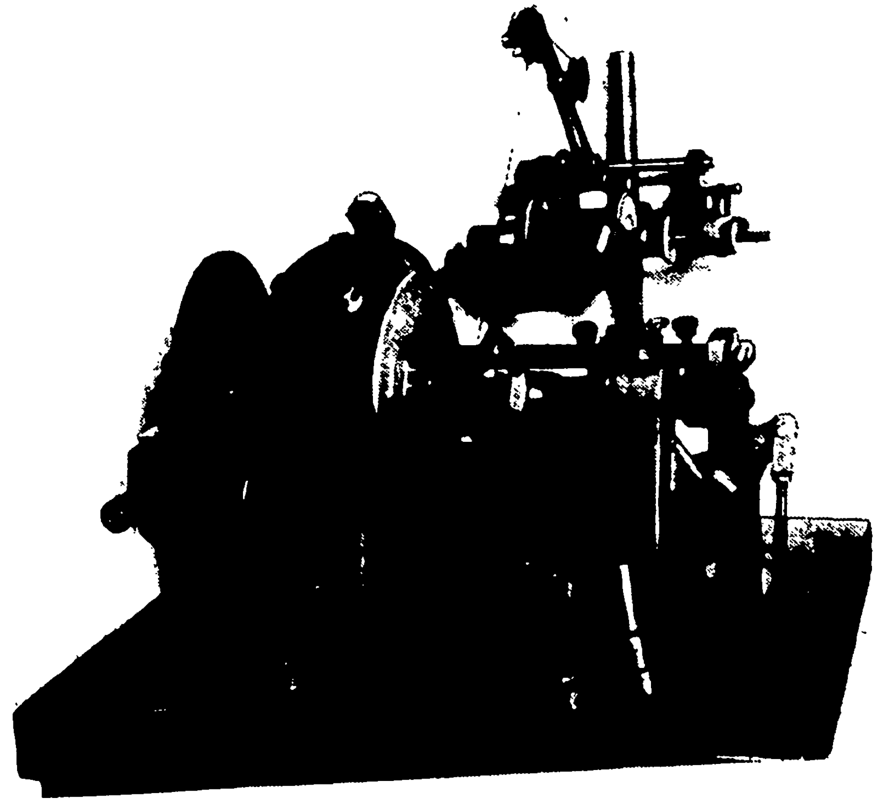


Fig.9 - Machine SNYa-4 for Slotted Windings.

single-chord windings of armatures with approximately eight turns in a section. The winding is performed simultaneously with two wires of two sections in the same pair of slots.

The productivity during winding of armatures on the machine ATE-1 is 50 to 60

STAT

[redacted]

armatures per hour (during manual winding, four to five armatures per hour).

The second subgroup of machines of the first group comprises machines in which the reel rotates, winding the wire onto a stationary stack of the rotor or stator. Representative of the second subgroup of machines is the machine produced by the factory "Schetmash"* (see below). Larger stacks, of diameters above 40 - 50 mm are wound on machines of the second subgroup.

Both subgroups are in the same (first) group since in all machines included in these subgroups the relative motion of the wire with respect to the stack to be wound is of the same type.

The stacks of rotors and stators, except for rare exceptions, have semi-closed slots. The wire falls into the slots through narrow slits in the slots. Therefore, a required attachment for all machines of this group are guide jaws along which the wire slides into the slots of the stack.

The manufacture of semiautomatic machines for the second subgroup with rotating reels is fully possible, since they are structurally simpler than semiautomatic machines with rotating stacks. Consequently, we should also specify types of semiautomatic machines in the second subgroup.

Second group - shuttle machines, in which the reel performs a complex motion, somewhat similar to the motion of a human hand during manual winding of a stack of the rotor with wire, and in which the stack performs a rocking motion about its geometric axis (for stacks with straight slots, the latter motion is absent).

Shuttle machines can be divided into two subgroups:

The first subgroup comprises machines in which, during the winding of sections, the stack remains stationary while the second subgroup is composed of machines with a rocking stack.

Shuttle machines, at present, are in the development stage and are not yet used in production. The proposed field of application of these machines is the winding

STAT

---

* Later in text.

of stacks with small slot pitch (in multipole rotors) and with slanted slots (at a large angle of inclination of the slots).

Existing machines are only used for chord windings of two-pole machines. Chord windings of multipole machines are made manually. However, not all types and dimensions of chord windings of two-pole machines can be made on the above machines. Difficulties are created by certain design feature of the assemblies:

a) Presence of collectors, plates and other parts, attached to the axis of the stack (armature) and interfering with the laying of the wire into the slots;

b) Necessity of tight laying of the wire on the face parts of the stacks; in manual work, the wires are usually clamped several times;

c) Large angle of inclination of the slots in their plane;

d) High space factor of the slots;

e) Requirement for identity of the sections; in mechanical winding, the difference in ohmic resistance of the section sometimes exceeds the established tolerance.

For the following types of slot winding, corresponding machines for special applications should be produced (see Table 3).

For "petticoat" windings, each turn coming from the slot is not bent around the end of the stack as in other windings, but forms loops on both sides of the cylindrical surface of the armature (Fig.11). The loops of each row of turns of all slots form something like a "petticoat". The front part may consist of several layers or separate "petticoats". The number of the "petticoats" is equal to the number of turns in the section. The individual loops and layers ("petticoats") are separated by insulation. In instrument making, armatures with several layers (up to four) are used.

For "petticoat" windings, no machines are in existence and no methods for mechanization of the winding have ever been reported.

For re-entrant windings, the stacks (cores) have open slots. The sections of the turns of the wire bent around the teeth of the stacks in a predetermined sequence, are distributed into the slots (Fig.12).

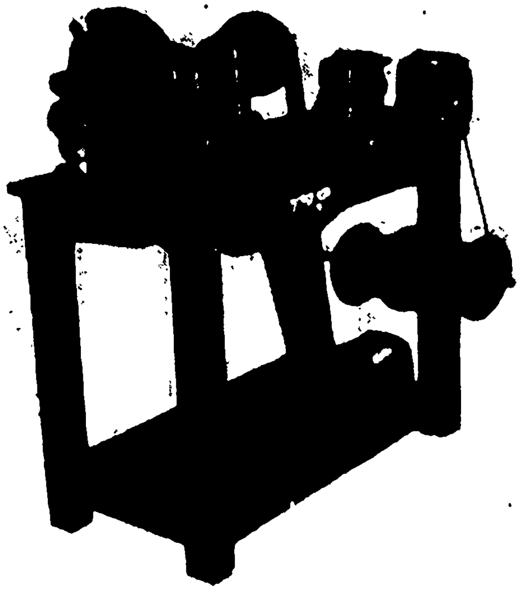


Fig.10 - Semiautomatic Machines for Slotted Windings ATЪ-1

Re-entrant windings with internal slots do not differ in principle from such windings with external slots, but the methods of their mechanical production may differ. No machines for re-entrant windings are in existence. At present in the production of windings, sections of wire are wound onto patterns which are then manually inserted tooth by tooth into the slots of the stacks and separated by insulating inserts.

A characteristic peculiarity of concentric coil windings is the concentric distribution of the sections or coils within the slots of the stack (Fig.13). In concentric coil windings, the spacing of the slots of the winding for internal (a) and external (b) turns located in different slots, will differ.

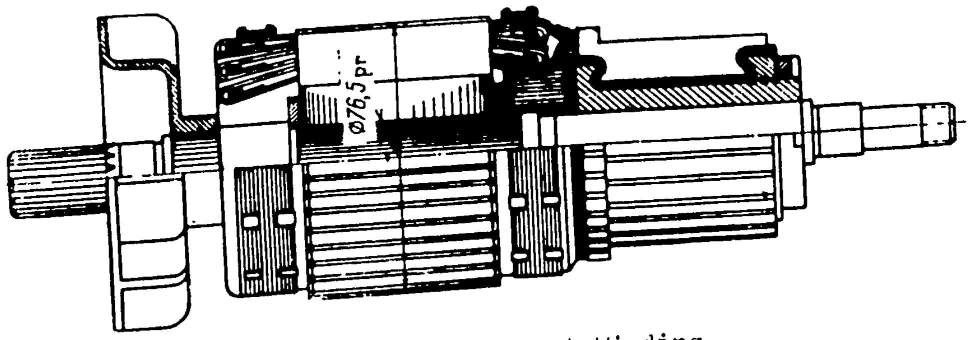


Fig.11 - Petticoat Winding

[redacted]

STAT

Concentric coil windings are used in stacks with internal and external slots. The methods for mechnaical production of such and other types of windings will differ. There are no machines for concentric coil windings.

A special group are windings with internal slots, with a slot spacing equal to unity, i.e., with the section wound around one tooth of the stack (Fig.14). In the production of these windings, nothing interferes with mechanization in forming the front parts of the windings. Therefore, a mechanization of the production of such concentric coil windings presumably will be easier than for other windings with internal slots.

Windings with two open wide slots (Fig.15) are produced by winding two sections, located on both sides of the axis of the assembly, in sequence. During the winding of each section, a spreader for the wire within the slot has to be used. The slots of the stacks of this type of windings may be straight or slanted. There are no special machines for windings with two open wide slots. It is specifically difficult to mechanize the production of windings with slanted slots of the stacks. Machines presently in operation, specifically the machine SNYa-4, will not give uniform laying of the turns in the slots of the stacks.

Under sectionalized windings with internal slots we include three-phase equisection (equally distributed), single-phase, two-phase (Fig.16), and sinusoidal types. The sections of the windings are located in the stack along the chords with constant slot pitch and in most cases with nonuniform direction of the turns.

Up to this time the problem of mechanization of the winding processes for stacks with internal slots has not been solved in instrument making plants. This can be explained by the considerable technical difficulties encountered in the solution of the given problem. The greatest difficulty is presented in forming the front part of the windings and in reaching a satisfactory space factor in the slots. At present, experimental machines for sectional and concentric coil windings with internal slots, which can be subdivided into two groups, are being made and tested.

[redacted]

The first group comprises shuttle machines, i.e., machines in which the windings are placed by a special shuttle, describing (relative to the stator) a trajec-

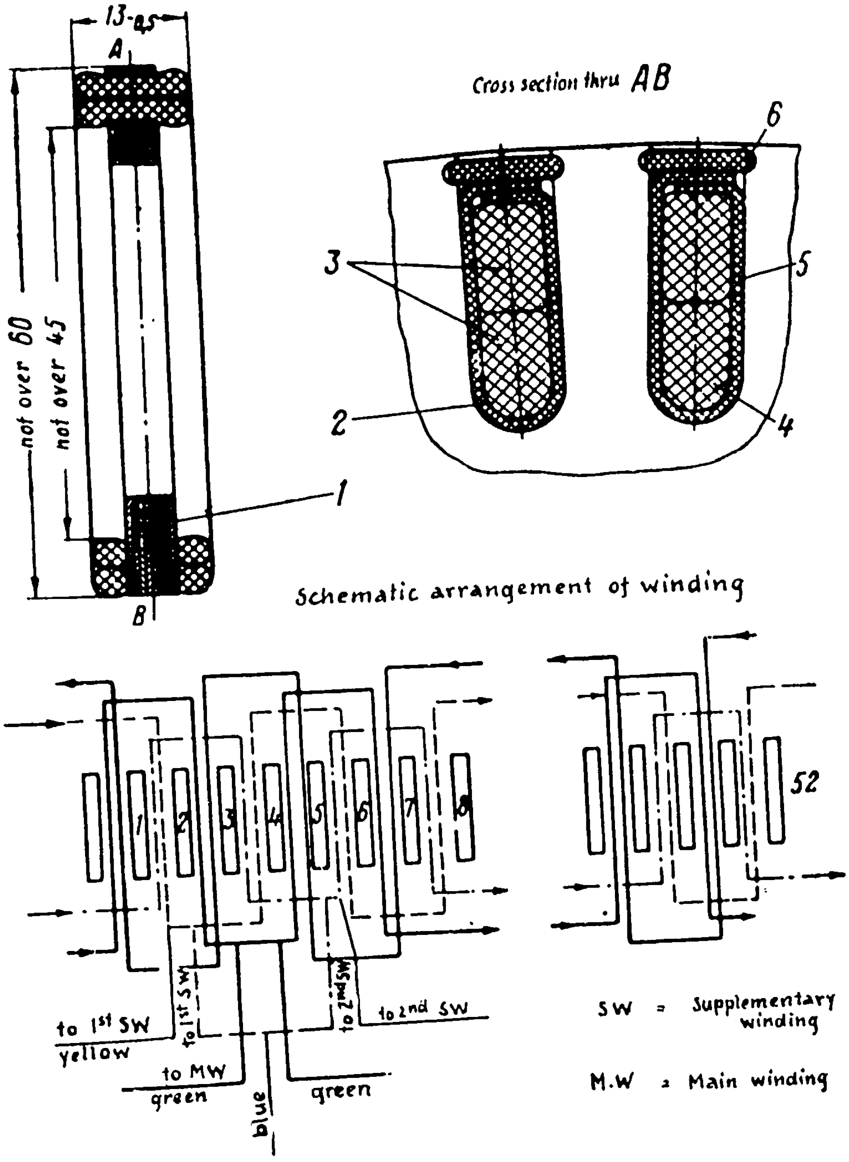


Fig.12 - Re-entrant Winding with Internal Slots.

—— main winding; ----- first supplementary excitation winding; -.-.-. second supplementary excitation winding. 1 - Rotor stack; 2 - Slot insulation; 3 - Main winding; 4 - Supplementary winding (first); 5 - Supplementary winding (second); 6 - Wedge.

tory which follows the outline of the coil, located in the slots of the stator stack.

The second group includes machines with separate winding and placement of the sections into the slots of the stator stack.

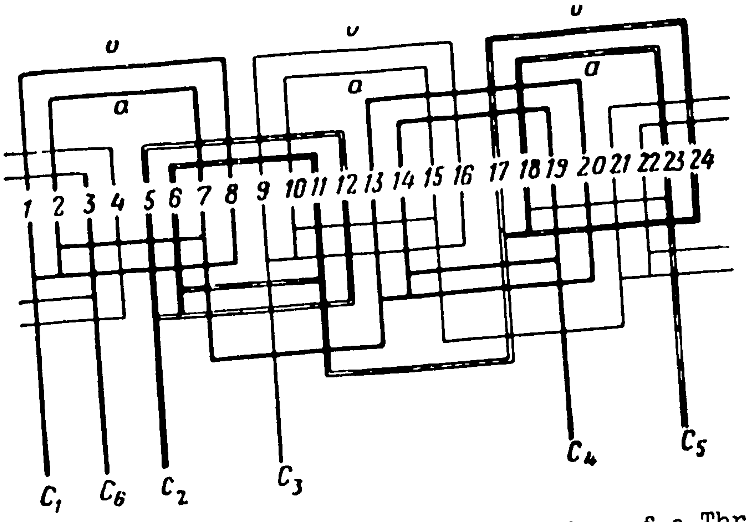


Fig.13 - Schematic Layout of Concentric Coil Winding of a Three-Phase Rotor

The machines included in the first group are divided into two subgroups, based on the principle of laying the turns:

a) With stationary stacks. In these machines, the shuttle performs a complex

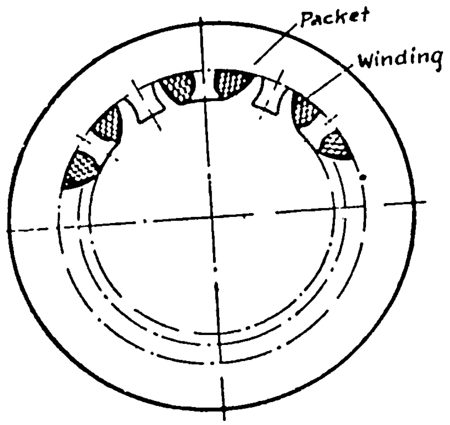


Fig.14 - Coil Winding with Internal Slots of Multipole Electrical Machines (the pitch of the winding is equal to unity).

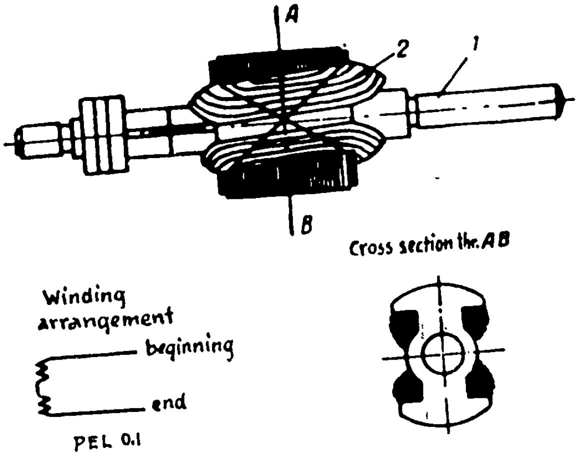


Fig.15 - Winding with Two Wide Open Slanted Slots

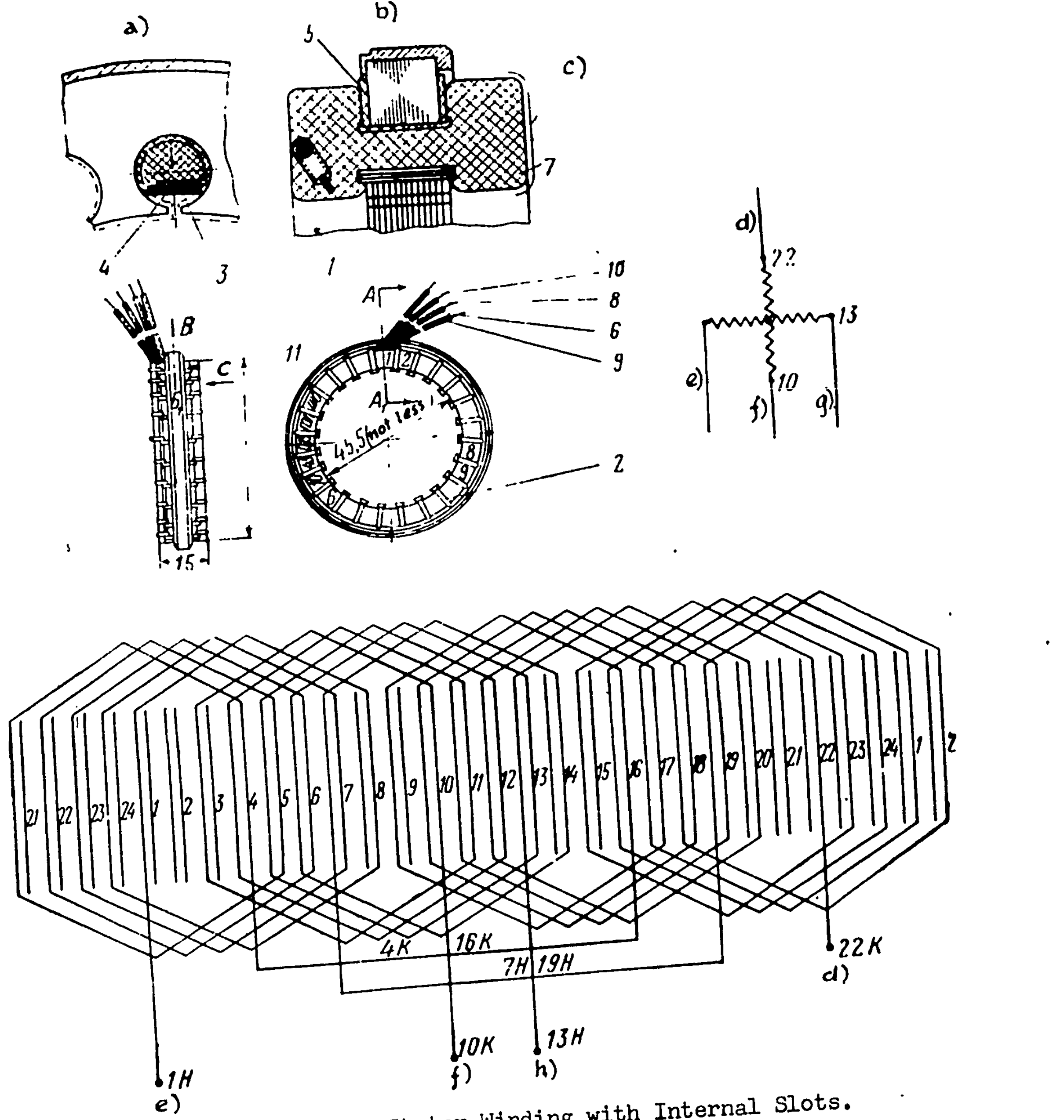


Fig.16 - Two-Phase Stator Winding with Internal Slots.

1 - Insulating tube; 2 - Thread; 3 - Wedge; 4 - Insulating layer; 5 - Plate; 6 - Mounting lead; 7 - Winding conductor; 8, 9, and 10 - Mounting leads; 11 - Name plate. Two windings are staggered by 90°; pitch of winding: 1-8. Each winding - 10 coils of 45 turns. Winding conductor - PELShO wire of 0.15 mm diameter (of copper).

Distribution of windings in slots
1H → 8-24 → 7-23 → 6-22 → 5-21 → 4K +
+ 16K → 9-17 → 10-18 → 11-19 → 12-20 → 13H;
22K → 5-23 → 16-24 → 17-1 → 18-2 → 19H +
+ 7H → 14-6 → 13-5 → 12-4 → 11-3 → 10K.

a) Cross section through BB; b) Cross section through AA; c) Swagged on both sides; d) Green; e) Red; f) Blue; g) Gray; h) Black

[redacted]

motion, while the stator is stationary.

b) With a rocking stack. In these machines, the shuttle has a reciprocating motion, while the stator rocks about its geometric axis.

These subgroups are, in turn, divided into: machines for sectionalized evenly distributed windings; machines for concentric coil windings; and machines for windings with a pitch between the slots equal to unity.

BIBLIOGRAPHY

1. Chumakov,V.P. - Scientific-Investigative Work by the Department of Technology of Aircraft Instrument Making. MATI (1953)
2. Chumakov,V.P. and Nikolayev Ye.H. - Scientific-Investigative Work by the Department of Technology of Aircraft Instrument Making, MATI (1954)
3. Antonov,V.I. - Automation and Mechanization of Winding Operations in Instrument Making, Progressive Technology of Instrument Making. First Edition, Mashgiz (1953)
4. Shabashev,Ya.I. - Mechanization of Winding-Insulating Work. Journal Vestnik Elektropromishlennosti, Nos.5-6 (1940)

STAT

THE WINDING OF SMALL-SIZE ROTORS WITH COILS SPREAD IN THE SLOT

by

Senior Lecturer Ye. N. Nikolayev and Cand. Tech. Sci., Docent V. P. Chumakov

For the winding of small-size rotors of two-pole electric equipment, a machine on which single-chord as well as double-chord windings can be wound, is used.

The operating principle of the machine is similar to that of machines with a rotating stack (see Table 3 of the preceding article). The wire to be wound slides along guide jaws into the slot of the stack (rotor or stator), which, turning together with the jaws, winds the wire upon itself.

The stack is secured in the machine by means of two pairs of guide jaws. This permits manufacture of two-chord windings, without change in direction of rotation of the spindle.

Stacks of diameters from 10 to 50 mm and widths from 4 to 40 mm are wound on the machine. The wire diameter ranges from 0.08 to 0.5 mm. The rotational speed of the spindle is 1500 rpm.

The machine operates at a higher speed than most other machines for slotted windings.

A feature of the machine is the presence of a mechanism which spreads the turns at the front part of the stack to be wound and of a mechanism for automatic shut-off of the machine and brading of the spindle. After winding a given number of turns, a signal from the counter simultaneously disengages the spindle of the machine from the driver and starts the spindle brake. Such a system considerably decreases the angle through which the spindle turns during the time between initial braking and full stop, since in this case the brake only has to overcome the kinetic energy of the spindle and of the parts rigidly connected with it, but not the energy of the drive and the electric motor.

In addition to slot windings, individual sections for subsequent manual arrange-

STAT

ment in the slots of the stack may also be wound on the machine.

The electric-kinematic arrangement of the machine is given in Fig.1.

The machine consists of the following basic parts:

Spindle with drive; mechanism for automatic shut-off of the machine; mechanism for spreading the turns; counter for the number of turns, with an attachment for automatic shut-off of the machine; two holders with guide jaws; tailstock; spool holder with tension buckle (not shown in Fig.1); electric motor of the type MSh-11 with a voltage of 26 v and a power of 60 w; pedal with rheostat; countershaft.

Prior to winding, a special jig (38) with two cones is inserted into the aperture of the stack. The jig is used for forming the front parts of the windings. The stack with the jig (38) is clamped in the holders (41) and (31) by means of

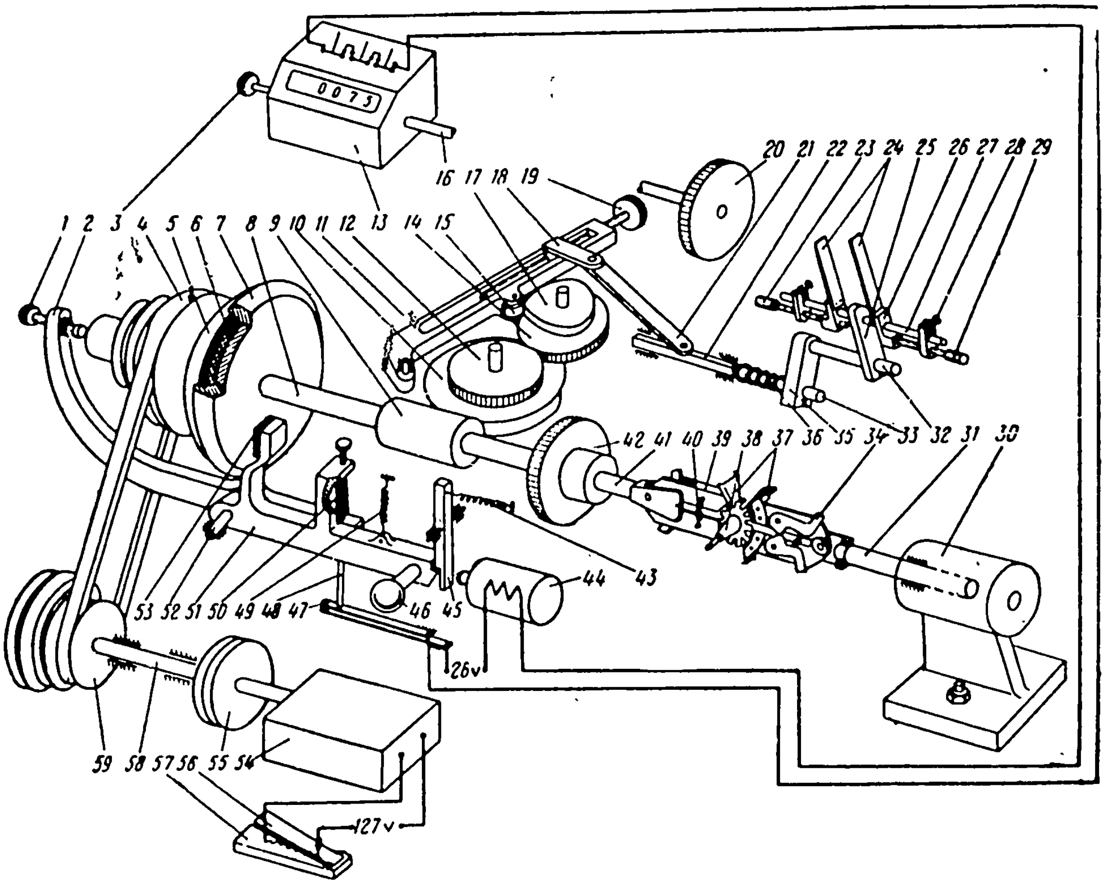


Fig.1 - Electric-Kenematic Arrangement of the Machine.

guide jaws which grip the stack. The edges of the jaws enter the slits of the slots to be wound.

The guide jaws (2) (Fig.2) are attached to the levers (3) and (5) which, on depression of the cams (4) spread apart [(rotating with respect to the axis (6)] and disengage the guide jaws. The guide jaws are held against the stack by the springs (1).

The holder (41) (see Fig.1) is secured in the chuck of the spindle of the machine and turns the packet together with the holder (31), rotating freely in the tailstock (30).

The wire passes through the right-hand guide pieces (24) if the guide jaws of the holder (31) are used. If the guide jaws of the holder (41) are used, the wire is switched to the left guide pieces (24).

Prior to winding the first section, the wire is secured in the ear of the jig (38).

In order to start the electric motor (54), the rheostat is connected by means of the foot pedal (57). The motion from the motor is transmitted to a three-step pulley (5) over the sleeve (55), shaft (58), pulley (59) and drive belt (4). The pulley (5), sitting freely on the spindle (8), may be displaced along the spindle. The pulley is pressed against the friction lining (6) of the disk (7) attached to the spindle (8) by means of the support (1) of the lever (2). When the pulley (5) is forced against the disk (7), the spindle (8) is put into rotation by the shaft of the motor (54).

For starting and automatic stopping of the machine, a mechanism is provided consisting of the electromagnet (44) with the armature (45), lever (51) with the brakeshoe (53) and the supporting screw (48), which closes the contact (47) of the counter circuit.

Before starting the machine, the handle (46) must be pushed down; during this process, the lug at the right end of the lever (51) clicks into the space behind the

STAT

[redacted]

step of the armature (45) of the motor (44). The spring (50) presses against the lever (2) causing the support (1) of the lever (2) to force the pulley (5) against the disk (7). The small brakeshoe (53) is disengaged from the disk (7) and releases

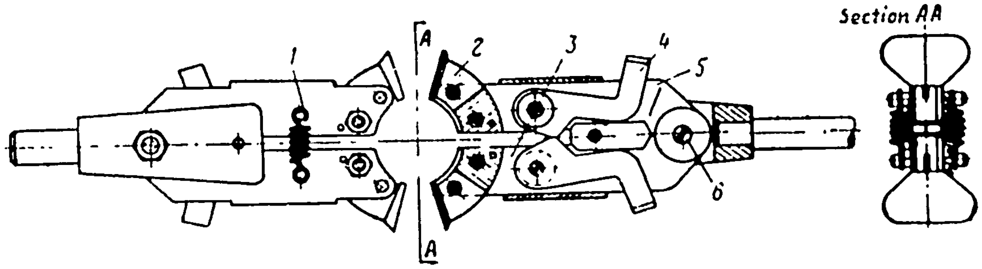


Fig.2 - Holding the Winding Machine with Two Pairs of Guide Jaws

the spindle from the braking position. The screw (48), in the process, closes the contact (47). This contact is installed for disconnecting the current of the electromagnet (44) after the machine is shut off and for preventing start-up of the machine without canceling the counter reading and resetting it to zero.

The motion is transmitted to the counter (13) by means of the gears (42) and (20) [in Fig.1, the counter and gear (20) are purposely raised out of the plane]. In addition to the usual mechanism, the counter is equipped with a special electric contact device set for a given number of turns to be wound; on reaching this number, i.e., after finishing the winding of the section, the contacts close and current flows through the windings of the electromagnet (44).

When the electromagnet attracts the armature (45), the right end of the lever (51) is freed of the retaining armature (45), and the lever (51) starts rotating with respect to the axis (52), under the action of the spring (49). Upon rotation of the lever, the brakeshoe (53) pushes against the disk (7), quickly stopping the spindle. The support (1) moves away from the pulley (5), thus cutting the connection of the spindle with the pulley (5) and with the driver.

The spreader mechanism works in the following manner: The cam (17), displacing the lever (11) of the spreader mechanism, is put into rotation via the spindle of the machine through the worm (9 and 10) and gear (12 and 14) drives. The displacement of the lever (11) is transmitted to the spreading guides (24) over the thrust bearing (18), the traction rod (21), and the slide (22).

The spring (36), via the lever (21), forces the pulley (15) against the cam (17). The travel of the slide (21) to which the guide pieces (24) are connected, is determined by means of the screw (19) which shifts the thrust member (18).

The distance between the guide pieces is regulated by the screw (23) and (29).

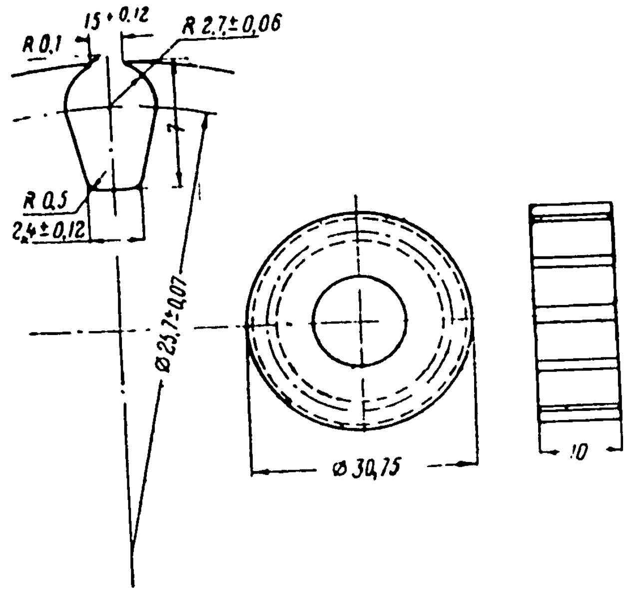


Fig.3 - Stator Stack

The machine is started by depressing the foot pedal (56). The rotational speed of the spindle is also regulated by means of the same pedal (the pedal is connected to a rheostat included in the feed circuit of the electric motor).

The design of the described machine, in our opinion, has many advantages and can be recommended for the winding of small-size rotors ans stators with internal slots, without further modification.

To check the advantages of winding with a spreader, as compared with the usual winding without a spreader, various experiments were made.

The object of the experiments was a stator, whose stack is shown in Fig.3.

Winding Data of the Stator

Copper wire	PEM-1 (with metal-vinyl insulation), diameter 0.15 mm
Pitch of winding along slots	1 - 6
Number of turns in section	75
Resistance of section I	9.5 ohms ± 10%

Resistance between phases	38 ± 4 ohms
Nominal static tension of wire during winding	175 g
Stroke of spreader	5 mm
One double stroke of the spreader . . .	at 50 revolutions of the spindle
Conical jigs of two types	with cone of 16° and diameter of 14 mm (at end of stack) with cone of 14° and diameter of 12 mm (at end of stack)

Experimental Methods

Two series of experiments were performed:

a) Experimental specimens of stators were wound in succession with spreaders and without spreaders, and with different conical jigs. Then, the ohmic resistance between the winding phase was measured, and visual inspection of the inside and of the front part of the windings was made. Tables 1 and 2 show the mean arithmetic results of several measurements.

Table 1

Resistance (in ohms) between Phases of Stators, Wound with Spreading of Turns

№	Angle of cone of jig in degrees	№ of leads		
		1—5	5—9	9—1
1	16	37,83	40,25	38,45
2	14	35,5	37,25	36,5

Table 2

Resistance (in ohms) between the Phases of Stators, Wound without Spreading of Turns

№	Angle of cone of jig, in degrees	№ of lead		
		1—5	5—9	9—1
1	16	37,56	40,46	38,6
2	14	35	38	37,5

[redacted]

STAT

b) The number of turns wound on a pair of slots, up to the point when the wire stopped entering the slot, was determined. The results of these experiments are summarized in Table 3.

Table 3

Number of turns, Laid in a Pair of Slots

№	Method of winding	№ of sections						Mean number of turns
		1	2	3	4	5	6	
1	Without spreading	150	145	151	145	149	140	147
2	With spread	155	150	161	155	159	150	155

The purpose of the first series of experiments was to clarify the influence of the spreading on the front parts of the windings, while the purpose of the second series of experiments was to clarify the influence of the spreading on the lay of the wire in the slot.

1. The front parts of the windings filled by spreading along the slots, practically do not differ from the windings obtained without spreading.

2. During winding with a spreader, the winding is more closely laid into the slots, but the difference is not very large.

3. It is possible that, for windings in which machine winding is difficult due to a high space factor for the slot, a spreader attachment may prove of some help. Therefore, further experimental work on winding with spreaders, for various types and dimensions of stacks, is recommended.

STAT

MACHINES FOR WINDING ROTORS AND STATORS WITH INTERNAL SLOTS

by

Cand. Tech. Sci. Docent V. P. Chumakov and Senior Lecturer Ye. N. Nikolayev

Two machines for the winding of small-size rotors and stators with internal slots, the machine produced by "Schetmash" and the machine SNYa-3, are included in the group of machines with rotating reel (see preceding Classification of Winding Machines, Table 3).

In these machines, the reel rotates while winding the wire on a stationary rotor or stator.

The wire is directed into the slots of the stack by stationary guides, along which it slides into the slots during the winding process. Such a winding method is useful in winding large rotors and rotors with long shafts. For the winding of such rotors on machines in which the stacks rotate, together with the spindle, chucks of large diameters and of considerable moments of inertia would be needed. The type of work on winding machines required that the machine be started at frequent intervals (every few seconds) and, after a comparatively small number of revolutions of the spindle, be quickly stopped. Under such operating conditions it is desirable to have the moments of inertia of the rotating masses, relative to the axis of rotation, remain as small as possible. Therefore, the best method of winding large rotors is the method where the stack remains stationary. In working on machines with stationary armatures, this process of winding should be further investigated.

On some machines with rotating reels (specifically on the machine SNYa-3), the spool from which the wire is unwound rotates together with the reel. This is done to prevent the wire from twisting in the process of winding. In our opinion, such twisting is no major hazard since it is negligibly slight (one revolution per turn of winding) and does not affect the mechanical or electrical properties of the wire and insulation. Care must be taken, however, that thin wire with excessive slack cannot form loops which, on subsequent tightening will not open but will be pulled closer (forming a "kink"); this weakens the wire and damages the insulation. However, such danger is present in any method of winding.

During the time of operation of the machine, the wire is under tension and the formation of "kinks" during its normal operations is impossible.

Experimental work on the machine "Schetmash" indicates, that minor twisting of the wire in winding armatures on this machine, does not affect the quality of performance of the armature. On the other hand, rotational motion of the spool axis complicates the machine specifically creating great difficulties due to the requirement of keeping the wire under tension in the mechanism.

Machine for Winding Rotors, Developed by the "Schetmash" Plant

Stacks of diameters up to 100 mm and widths up to 120 mm may be wound on the machine. Winding conductors of diameters of 0.08 - 1.2 mm may be used. Stacks with straight and slanted slots can be wound on the machine.

The machine has a reverse which permits manufacture of windings with "two-chord" characteristics. The speed of the spindle of the machine is 800 rpm.

The kinematic diagram of the machine is shown in Fig.1.

The machine can be broken down into the following parts:

1. Drive of the machine;
2. Reversing mechanism;
3. Spindle-braking mechanism;
4. Hollow spindle with flywheel and reel;

These four parts are united into one tailstock assembly in actual construction.

5. Headstock with holder and centers for clamping the stack to be wound;
6. Guide jaws;
7. Counter of turns (revolutions);
8. Spool holder post with tension mechanism.

[redacted]

The body of the counter (6) is mounted to the frame of the machine. The spool-holding post is screwed to a table, at the right of the machine.

The rotor to be wound, together with its shaft (9), is clamped in the headstock (10) between the fixed center (8) and the moving center (11), connected with the spring (12).

The headstock may be moved toward the tailstock or moved away from it, along the bed. The location of the headstock is determined as a function of the diameter of the rotor. The rotor may be in any given position, i.e., the geometric axis of the rotor may be at any given angle, which permits selection of the rotor position most convenient for the winding process.

After positioning the rotor on the centers, the guide jaws are moved toward it. The body (16) with the guide jaws is secured by the cylindrical shank (17), entering the spindle (26) and having freedom of motion along the spindle.

On rotating the spindle, the shank remains stationary since it is held, together with the guide jaws, by the stationary rotor with the shaft (9). The conductor to be wound is transmitted from the spool (27) through the pulley (28) into the hollow spindle (26); through the pulley (19), whose axis is secured to the spindle, it is conveyed to the pulley (13) on the end of the reel (14).

The end of the conductor being unwound from the pulley (13), at the beginning of the winding process, is secured to the shaft of the rotor. The reel (14) is attached to the flywheel (18) which is rigidly connected with the spindle and rotates with it, while winding the conductor into the slots of the rotor.

The starting and stopping of the machine is performed manually. To start the machine, the handle (33) is pushed down. During this time, the pulleys (31) and (32) travel upward and tighten the V-belt (3); the pulley (34) has rotation transmitted to it from the pulley (2), located on the shaft of the electric motor (1). The pulley (2) continues to turn when the machine stops, since the prime mover is not shut off at that time.

[redacted]

STAT

The shaft (5), rigidly connected with the pulley (34), also carries rigidly connected gear (4) from which, over the gear (20), rotation is transmitted to the hollow spindle (26).

Shaft (5) rotates in one direction, while the direction of rotation of the spindle with the reel changes (for the case of "two-chord" winding). Therefore, the machine has a reversing mechanism. When the handle (21) of the reversing mechanism occupies the position shown in Fig.1, the gear (4) is connected to the gear (20) over the gear (36) so that the direction of rotation of the spindle coincides with the direction of rotation of the shaft (5).

If the handle (21) is moved "away from the operator" (i.e., to the left, looking at it from the side of the headstock), then the gears (36) and (20) are disen- gaged so that motion will be transmitted to the gear (20) through the gear (35). In this position of the reversing mechanism, the gears (20) and (4) will rotate in dif- ferent directions. Since the number of teeth of the gears (4) and (20) are the same, the gear ratio between the shaft (5) and the spindle is equal to unity. Therefore, the counter (6), connected to the shaft (5), indicates the number of revolutions of the spindle, i.e., the number of turns wound. The counter in this machine is placed in the kinematic linkage in fron of the reversing mechanism, so that the counter shaft rotates only in one direction, regardless of the direction of rotation of the spinu f the machine.

As soon as the reel lays the required number of turns, the machine is stopped by moving the handle (33) upward. During this time, the pulley (31) and (32) are lowered; this causes the tension on the belt (3) to decrease and the pulleys (34) and (2) to disengage.

On further raising of the handle (33), the braking mechanism is actuated and the spindle of the machine is stopped rapidly.

The braking mechanism is constructed in the following manner:

The brake pulley (23), against which the spring (25) presses the brakeshoes (22)

[redacted]

STAT

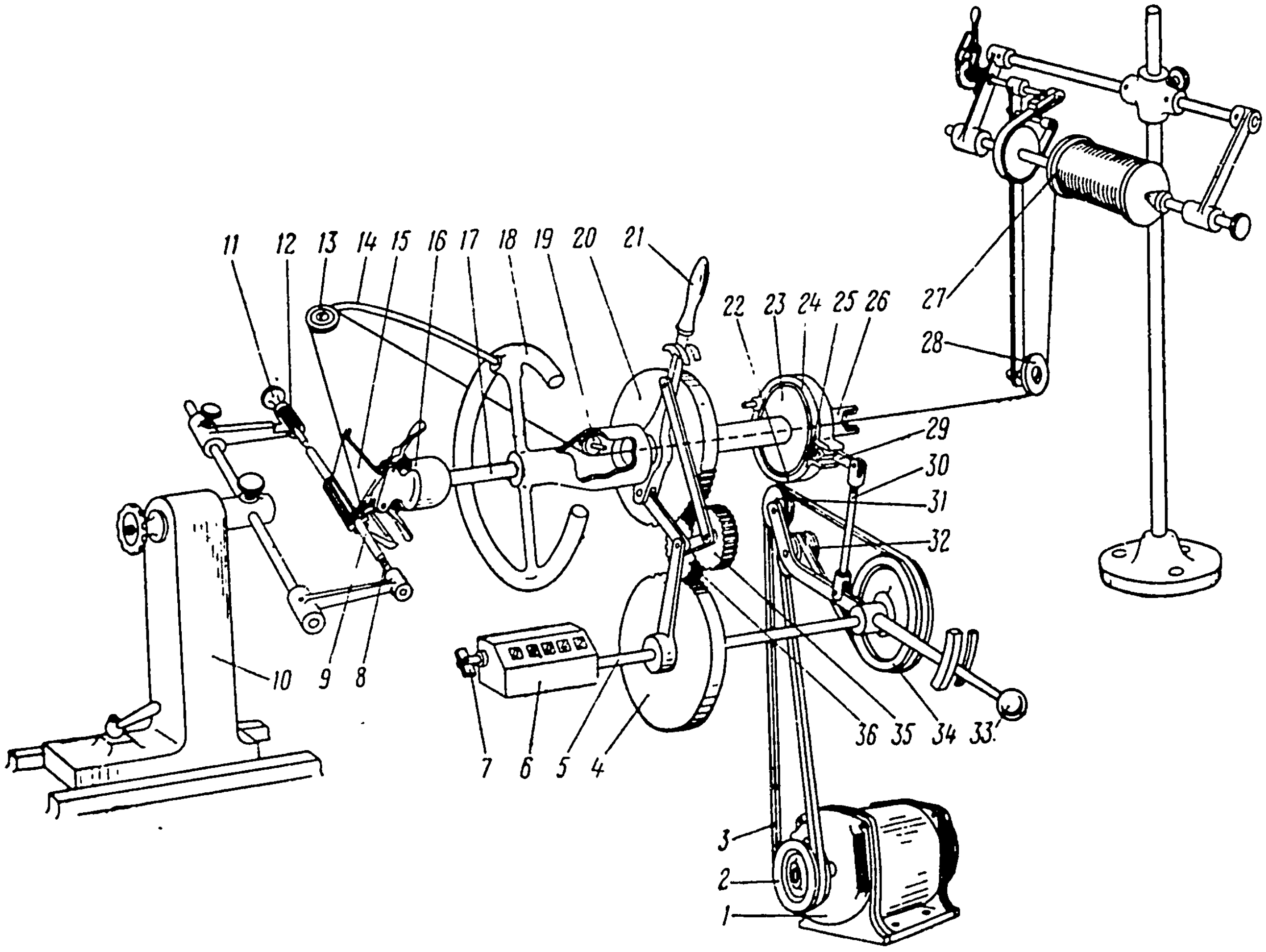


Fig.1 - Kinematic Arrangement of Machine for Winding of Rotor, made by the "Schetmash" Plant

and (24), is secured to the spindle of the machine. On starting the machine, pushing the handle (33) will cause the traction member (30) to turn the square member (29), which spreads the brake blocks and sets the pulley (23) free. The dimensions of the braking mechanism are so calculated that, at the instant of engagement of the pulley (34) over the tightened V-belt (3) with the pulley (2), the brake pulley (23) rides completely free. If this condition is satisfied, it will become obvious; when stopping the machine by raising the handle (33) the connection between the electric motor and the pulley (34) is broken and, on further raising the handle, the member (29) turns until the brake blocks touch the pulley (23).

Figure 1 shows the location of the handle (33) in the lower position, i.e., is with the machine running, when the member (29) has moved the brake blocks (22) and (24) apart. Upon completion of winding one section, the guide jaws are removed from the rotor, the follower is turned on the centers and the guide jaws are attached to a new pair of slots, corresponding to the schematic diagram of the winding. Prior to starting the machine by means of the handle (7), the reading of the counter is canceled, i.e., it is returned to zero.

The machine should be used for winding of large stacks of diameters over 50 mm or of stacks of smaller diameters, but which are elongated, of a width (length) of more than 60 mm.

The speed of the machine (800 rpm) is completely sufficient for the indicated sizes of rotors, since in the case of large rotors the number of turns in a given section is usually not large. An increase in speed of the machine does not result in a noticeable increase in productivity. Under these conditions, the machine time is short in comparison with auxiliary time.

The main drawback of the machine is the absence of a mechanism for automatic shutoff.

After the required number of turns in the section is reached, the machine has to be stopped manually. This requires the operator to watch the counter at all times,

STAT

which complicates the work and causes additional fatigue.

The counter mounted on the machine may be used for automatic shutoff of the machine. Therefore, without changing the design of the machine, a mechanism for automatic shutoff could be added. The operating principle of this mechanism may be the same as for the machine for winding small-size rotors and stators with internal slots (see previously in text).

Machine SNYa-3 for Winding Armatures

The machine SNYa-3 was constructed by engineers V. I. Antonov, N. V. Mironov, and A. V. Salov. An overall view of this machine is shown in Fig.2, and the electrokinematic diagram is given in Fig.3. The machine is meant for winding of small-size rotors and stators with external slots, with single-chord and two-chord characteristics of windings of two-pole motors. The diameter of the stacks to be wound is 12 - 40 mm; the width of the stacks is 10 - 40 mm. The diameter of the wire is 0.08 - 0.5 mm.

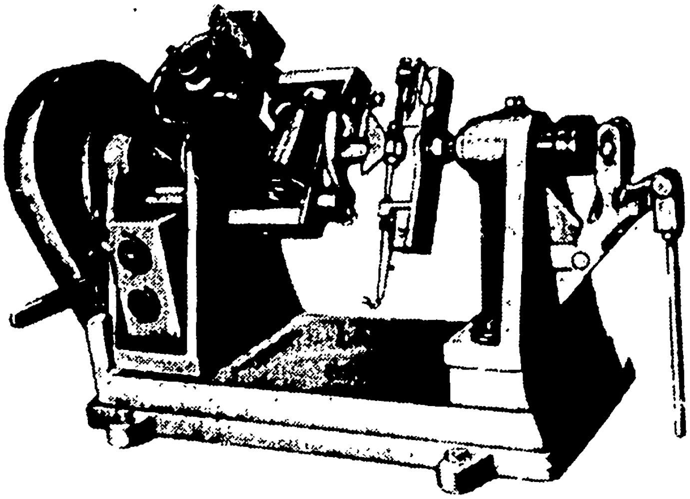


Fig.2 - SNYa-3 Machine Tool for Winding of Armatures

The main difference of the machine SNYa-3 from the machine made by the "Schetmash" plant consists in that, on the machine SNYa-3 the spool from which the conduc-

STAT

tor is unwound rotates together with the traverse, whereas the axis of rotation of the spool in the spool holder of the "Schetmash" machine is rigid. Therefore, in winding armatures on the machine SNYa-3, the wire is not twisted.

The entire mechanism of the machine may be subdivided into the following parts:

1. Spindle with drive;
2. Chuck with spool holder, braking mechanism, and traverse;
3. Counter with mechanism for automatic starting of the machine;
4. Tailstock with pedal mechanism;
5. Guide jaws;
6. Holder with center for the armature.

Parts 1, 2, 3, 4 and 5 are basically the same as the corresponding parts of the SNYa-4 machine*.

The chuck (2) with the spool holder is attached to the spindle (1) of the machine SNYa-3. The spool (5) together with the cones (4) and (21) is clamped to the chuck on the centers (3) and (19). The brake block (20) is forced against the cylindrical part of the surface of the part (21) by means of the spring (16).

The friction forces between the block (20) and the cone (21) create, with respect to the axis of the centers (3) and (19), a moment required for tightening of the conductor being wound. The compressive force of the block and, consequently, the tractive force on the conductor, is regulated by means of the screw (18).

The frame (22) of the spool holder caries the traverse (6) to whose end the pulley (9) is mounted. The conductor travels across this pulley, going from the spool and being wound on the stationary armature (13).

The clamp (12) with centers in which the shaft of the armature is secured, is usually of the design used in the machine SNYa-4; however, in the machine SNYa-3 the clamp is not attached to the chuck of the spindle of the machine but to the

---

* See V. I. Antonov, Automation and Mechanization of Winding Operations in Instrument Making, Progressive Technology of Instrument Making. Mashgiz (1953)

STAT

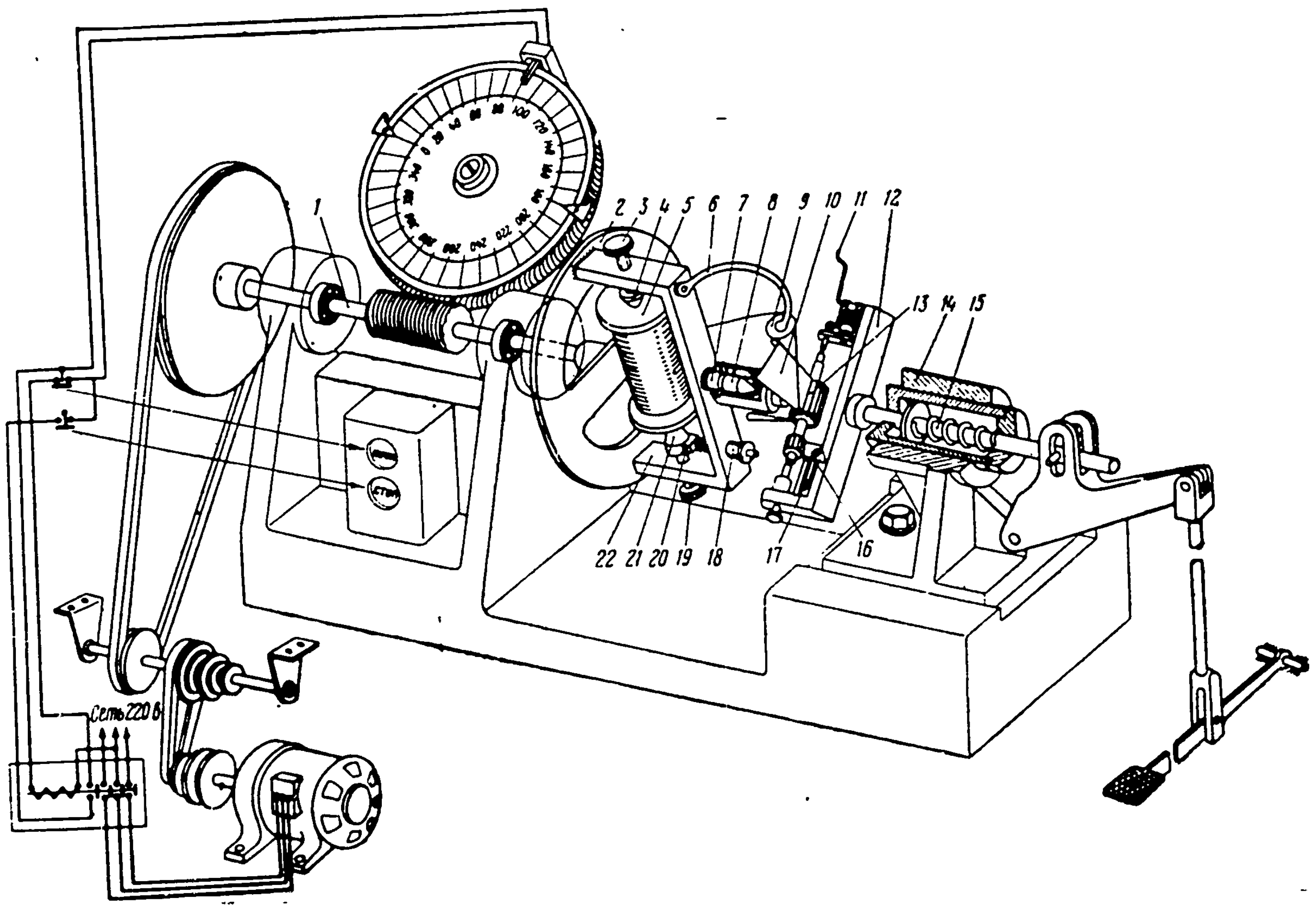


Fig.3 - Electro-Kinematic Arrangement of the Machine SNYa-3

for Winding of Armatures