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RESEARCH MEMORANDUM

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STOCKAGE POLICIES FOR MEDIUM AND LOW-COST PARTS

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Assigned to ______________________

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in brief

This memorandum describes a set of practical, systematic procedures for establishing efficient base and depot stock levels for low priced Air Force parts, Categories II and III. The use of such procedures should make it possible to reduce the work load on supply personnel and to increase supply effectiveness by cutting reorder requirements and supply shortages. The procedures can be used in manual data processing at base level, and also with integrated data processing and centrally controlled resupply.

Under the procedures for base stocking outlined in Air Force Supply Manual 67-1 the Order Quantity is based on demand rate and cost category; and the Reorder Level is determined by taking into account demand rate and pipeline time.

The procedures outlined here take into account several additional factors in an effort to reduce overall provisioning costs. The factors include expected mean demand for each item, variability of demand, unit value of item, variability of demand, cost of reorder, cost of holding Operating Stocks, cost of obsolescence in event of program termination, expected shortage cost, and resupply and procurement pipeline times.

Equations are developed for determining efficient stock levels, taking these factors into account. Tables devised from these equations can be used by clerical personnel to determine stock levels without complicated procedures and decision processes; the equations may be used in an integrated data processing system.

The suggested procedures have been compared with those of Supply Manual 67-1 through tests by regular Air Force supply personnel in the RAND Logistics Laboratory. Results bear out the improved efficiency of the new practices suggested here. (Actually, provisioning has for some time been undergoing changes not reflected in "67-1" and to some extent already take into account some of the features of the suggested system.)

Briefings on this study have been presented to AMC and to members of the Air Staff during recent months.

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SUMMARY

This Research Memorandum presents for Air Force consideration some simple and practical proposals for stocking - that is, provisioning and distributing - those items for which detailed intensive management is not appropriate, for example, Cost Category II and III items. These policies are complementary to deferred procurement policies for some Category I items.[1] While either could be implemented alone, together they would provide increased supply effectiveness and economy through buying fewer costly parts and investing some of the saving in base stocks of cheap items. They would entail more management and closer control of the expensive parts, but decreased materiel movement, paper processing and priority resupply of the low-cost parts.

The research underlying this Memorandum leads to the following conclusions:

A. Stockage rules should consider:

1. Expected mean demand for each item
2. Variability of demand
3. Unit value
4. Cost of incurring a reorder
5. Cost of holding the Operating Stocks
6. Cost of expected terminal obsolescence (termination of the program being supported)
7. Expected shortage cost
8. Resupply and procurement pipeline times.

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[1] J. W. Petersen, Savings from Procurement Deferral with Interim Contractor Support: The Case of High Value Airframe Spares, The RAND Corporation Research Memorandum RM-2085, 10 January 1958.

B. The dynamics of weapon-system or other program phase-in or phase-out can be taken into account effectively by

1. Limited depot stockage during the early part of a program, and subsequently stocking the depot to its full Stock Control Level
2. Gathering and analyzing consumption data intensively early in the phase-in and reacting to that information
3. Using a "final buy" calculation during the later stages of the program and, for some of the least costly low-demand items, early in the program; and
4. Using a "terminal buy" calculation at the time when an item is expected to go out of production.

C. With an integrated data processing system, these results can be largely achieved by using the equations developed in the mathematical appendix. With a manual data processing system and local determination of levels, tables based upon the formulas can be used by clerical personnel to set the appropriate levels.

D. The Air Force can increase supply effectiveness, decrease personnel pressures in supply and achieve dollar economies by adopting the policies described and proposed in this Memorandum.

E. Because the proposed policies permit reduced management per line item, their use should free management to manage the more costly and critical items better, or, alternatively, it might permit reducing somewhat base-level manning where (as in hardened missile installations) there is a premium on personnel space.

F. Further research is needed at RAND to extend the scope of the study, and further developmental studies are required, particularly in the Air Force, to derive adequate estimates of cost and other parameters.

G. The rapid development of an integrated data processing system will improve the application of these as well as other supply policies.

This Memorandum is concerned with aircraft parts and other technical items, not necessarily with quartermaster or bulk items. Inasmuch as we do not yet have formal solutions for certain of the reparable items, the policies described cover nonreparables and, when applied to base stockage, parts repaired at the depot. Some significant error would result if parts repaired on the base were stocked at the base in accordance with these rules. Similarly, for determining depot stock levels, parts repaired at the depot should not be stocked as described below. Hence, this Memorandum applies without modification to virtually all Category III items and to the non-recoverable Category II items.

The study covers the question of how much to stock of line items for which the decision has already been made that they be brought into the inventory. It covers, then, the depth of stockage, but not the breadth or range of items that should be stocked. This range problem is an important one, on which work is under way.

In studying this problem, the aim has been to develop policies which would result in near-minimum system cost and still be practical. The approach is to consider each item as independent of the others in the inventory. First the base levels, then the depot levels are determined; for each, the Order Quantity (Q), which establishes the amount of the Operating Stocks, then the Reorder Point (R), which establishes the amount of the Safety and Pipeline Stocks, are determined. Together Q plus R, of course, constitute the Stock Control Level (SCL).

For any given annual demand rate, the size of Q fixes the average frequency with which orders must be placed as well as the size of the Operating Stocks. The more frequent the reorders, the greater the supply workload and the greater its cost. Since Category II and III parts account for as much as 99 percent of the depot-to-base issues, it is desirable to have infrequent orders for each line item. This requires a relatively large Order Quantity, and a relatively large investment in Operating Stocks. Therefore, the higher the price of the item the smaller Q should be. The proposed policies balance the cost of holding an item against the cost of frequent reorders.

The Reorder Point establishes the level of the Safety and Pipeline Stocks which serve to provide protection during the resupply cycle. In determining R, account is taken not only of the demand rate but also of the variability in demand, considering whatever is known about the probability aspects of demand for groups of parts. Pipeline time is also considered — the longer it is, the larger the Safety and Pipeline Stocks should be because of the greater risk of a shortage during the pipeline time. Given the demand rate, the Order Quantity determines how frequently the Safety and Pipeline Stock is dipped into and, therefore, how frequently the risk of a shortage is run.

In addition, the shortage cost, i.e., the expected cost to the Air Force of overcoming a shortage, is considered. The minimum cost of a shortage appears to be the cost of priority actions. To this minimum should be added, in some circumstances, such costs as that of local manufacture, the cost of keeping a higher assembly on hand, or even the cost of maintaining an extra aircraft (or missile) to substitute for an AOCP (or MOCP) caused by a shortage.

The higher the demand, the more frequent the orders, the longer the pipeline time, and the greater the shortage cost, the higher the economical Reorder Point. Against this must be balanced the investment cost. The higher

the unit cost and the higher the cost of keeping items, the lower the Reorder Point should be.

These, then, are the factors considered in determining the Order Quantity and the Reorder Point:

Order Quantity	Reorder Point
Demand Rate	Demand Rate
Unit Cost	Variation in Demand
Holding Cost	Unit Cost
Reorder Cost	Keeping Cost
	Pipeline Time
	Order Quantity

Equations have been developed to take these factors into account with a good approximation of a theoretically ideal solution. From these equations, tables such as that in Figure 1 are developed. Such tables are proposed for use at base level. With them, complex procedures could be avoided, but full advantage could be taken of the relatively complete and efficient decision rules underlying them.

In Figure 1 the demand rate is shown across the top, increasing from left to right. The price is shown increasing from top to bottom. As demand increases, Reorder Points and Order Quantities increase. On the other hand, as the price increases both fall. For the very high-priced items in Category II, the Reorder Points and Order Quantities are very small. Some large Stock Control Levels appear in the upper left corner of the chart. These figures can readily be rounded to dozens or dollars' worth, to unit pack, or any other convenient unit.

In stocking a base initially for a new weapon, the Stock Control Level would be placed at the base. By stocking several years' supply of the cheap low-demand parts, the danger of a shortage is nearly eliminated, and the chance of having to incur the reorder cost during the program is made small.

Unit price		Annual demand: 0-2	2-6	6-12	12-20	72-90	90-110	110-132	182-210	210-240	240-272
.01-.12	R	4	8	11	14	31	35	39	51	55	60
	Q	19	44	79	124	237	262	288	365	391	417
.13-.24	R	2	6	9	13	30	33	37	49	54	58
	Q	16	40	73	75	167	186	204	259	277	295
.25-.49	R	2	5	9	12	28	32	36	48	52	57
	Q	13	35	41	54	118	131	144	183	196	208
4.00-7.99	R	0	2	5	8	23	26	30	41	45	49
	Q	4	7	10	13	30	33	36	46	49	52
8.00-15.99	R	0	2	4	7	21					
	Q	3	5	7	10	21					
125.00-249.99	R	0	0	1	3	15					
	Q	1	2	2	3	5					
250.00-499.99	R	0	0	1	2	14					
	Q	1	1	1	2	4					

5-Year program
30-Day pipeline
50-Dollar shortage cost
R=Reorder Point
Q=Order Quantity

Fig. 1 — Base stock levels

Tables could be prepared at some central point, using either manual or computer methods, and distributed to the bases. Alternatively, such tables could be used by, say, a Weapon System Supply Manager to set the Order Quantities and Reorder Points directly. With integrated data processing, the levels could be determined by the Data Processing Center, following the equations upon which such tables are based.

To be practical, these policies must handle phase-in and phase-out. For common parts the phase of life of any particular weapon is relatively unimportant, but stockage of parts peculiar to a weapon should, of course, reflect its program. For individual bases, where the transition to a new weapon is relatively abrupt, no change in the decision rules is required. As the end of a program approaches at any base, the Order Quantity should be adjusted to support the expected remaining life of the program; a downward adjustment is typically called for.

Given the base stockage rules which handle stock distribution, the depot-stockage and, hence, requirements rules are developed. The same factors are used as in base stockage; but some of the terms have somewhat different meanings in the depot case. For a long stable program the principles of base and depot stockage are very similar, but the dynamics of phase-in and phase-out have much greater impact upon the depot than upon the base.

During the phase-in, the depot must support a growing program with little demand experience for individual parts. For many items the bases will have several years' stocks from the beginning. If the phase-in is gradual and the procurement leadtime is short, it may be possible for the depot to take advantage of this situation and stock little more than its

Reorder Points, waiting until it has had demands from the bases and experience with the program before bringing its stocks up to their Stock Control Levels. Computations have been developed, adjusting depot Order Quantities and Reorder Points as the phase-out approaches, to take account of the termination of production of peculiar parts and the declining demand for them.

The proposed policies appear to provide close to optimal solutions to the system stockage problems. They constitute consistent distribution and requirements rules which are practical within the present data processing system and organization and which can be applied with little modification in an integrated data processing system. They appear to take into account all of the major relevant factors. But, what evidence is there that they will work in fact?

In the first experiment in the Logistics System Laboratory (LP-1) the results of applying these policies were compared with the results of applying a set of policies which Air Force members of the Laboratory staff developed as an approximation to the current best practice in the Air Force. In general, the proposed policies for low and medium value parts compared extremely favorably. This comparison took account of complete base and depot stockage policies, i.e., distribution and requirements in a dynamic situation in which readiness to fight a simulated war was demanded of both systems.

The proposed policies resulted in far fewer AOCP's, slightly fewer ANFE's, and less than 10 per cent as many priority actions as did the alternative system. Further, they cost less in terms of supply workload.

To conclude this summary statement, this Memorandum presents policies for stocking nonrecoverable items at base and depot level. Their application promises substantial economies in dollars and in personnel and, particularly, large improvement in the effectiveness of the supply support of the combat forces.

FOREWORD

This Research Memorandum covers the setting of base and depot levels for items which the activity in question does not repair. The setting of levels for items which the activity in question does repair includes much of the formulation discussed here but requires the application of some additional principles.

The methods described here were developed principally by A. J. Clark and the authors and are extensions, corrections, and simplifications of the procedures described by E. B. Berman and Clark in An Optimal Inventory Policy for a Military Organization (The RAND Corporation Paper P-647, 30 March 1955), and by Clark in A Technique for Optimal Distribution of Available Stocks to Bases (The RAND Corporation Research Memorandum RM-1621, 30 January 1956).

The extension to the short-program case has been simplified by comparison with the results of unpublished studies by H. W. Karr, using a dynamic programing model for procurement decisions.

These policies have formed a major portion of the first project in the RAND Logistics Systems Laboratory (LP-1).

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FIGURES

I. INTRODUCTION

A. Scope and Approach

There is at present a great deal of interest in the Air Force and elsewhere in the defense establishment in improving the management of the costly inventories of spare parts. Hi-Valu and Lo-Valu programs are now under way in the Air Materiel Command, and Headquarters USAF has initiated a Supply Improvement Program. RAND has done research in this general area for some years.[1] The present study, dealing with those items for which detailed line-item management is not appropriate, is complemented by the work on deferred procurement of Hi-Valu items.[2]

Policies for the provisioning and distribution of support for the major missiles must be firmed up in the near future. Similarly, the development of electronic Data Processing Centers in the Air Materiel Command may present an especially good opportunity for introducing new policies over the next few years. Specifically, within the next year the ELECTRO LOGS Project at Oklahoma City Air Materiel Area may be in a position to introduce some of these policies after the completion of operational testing and evaluation of the Inventory-Control phase.

This paper presents simple and practical provisioning and distribution calculations for items which do not justify detailed management, such as Category III and nonrecoverable Category II parts.

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[1] R. B. McNeill, E. B. Berman, A. J. Clark, H. W. Nelson, A Proposal for a New Air Force Supply Procedure, The RAND Corporation Research Memorandum RM-1417, 28 January 1955.

[2] J. W. Petersen, Savings from Procurement Deferral with Interim Contractor Support: The Case of High Value Airframe Spares, The RAND Corporation Research Memorandum RM-2085, 10 January 1958.

The characteristics of a practical method are:

1. It can deal with the uncertain and erratic demands, characteristic of the supply system.

2. It has reasonable data requirements.

3. It is compatible with the present manual and punched-card data-processing practice with local determination of stock levels, but it is also compatible with integrated data-processing and centrally-controlled resupply; it should also provide rules from which the necessary machine applications for an integrated system can be developed.

4. It must provide effective stockage rules under the dynamic conditions of weapons' phasing-in and -out.

Simplicity of computation is achieved by using, at least for the cheapest parts, approximate, rather than rigorously optimal, formulas for determining the appropriate stock levels. Simplicity in operation can be achieved by printing and distributing a small number of tables to each base for the use of supply personnel.

The methods appear to be applicable to the bulk of the parts in Categories II and III; however, there are some exceptions. No attempt is made to treat problems associated with the long service life of some parts, or the special problems associated with dated items. The former may be important in the case of many Hi-Valu parts, but it does not appear to be of major significance for most lower-cost ones. Further, the methods do not apply to quartermaster, bulk or local-manufacture items.

The general approach is to find economical base[1] stock levels on an item-by-item basis. After we find the economical base levels, depot or storage-site levels are set which will provide effective support for the bases.

[1] For purposes of setting stock levels, parts-repair depots and IRAN depots are considered to be the same as bases; they might have different pipeline times and, of course, different demand rates, but they are consuming activities and, in that sense, are like other bases.

The computational methods discussed in this paper may be used to set base and depot stock levels for nonrecoverable line items with a long and stable base program or during the phase-in or the phase-out of a weapon or other end item. The methods are also applicable to reparable items at activities which do not repair the items in question.

The paper is subject to two important limitations. First, it is confined to the question: Given the affirmative decision to stock an item at an activity, how much should be stocked? The determination of what to stock is outside the scope of this paper. Second, the paper does not deal with the stockage of all reparable parts, but only with those parts which are not normally reparable at the activity in question. Thus it deals only with stocking those parts which are consumed when they are used or which, if reparable, go off base for repair. In both these case, demand for a part can be satisfied only with a part from some outside source. In the other case where a part can be repaired on base, a demand for a part can be met either from an off-base or an on-base, i.e., maintenance, source. The ambiguity in the latter case introduces problems which have not been completely worked out.

B. The Problem

The major characteristics of the Air Force supply system are widely known, but it may be well to summarize the relevant ones briefly as a context for the discussion to follow.

The vast majority of the parts in the Air Force supply system are low-cost parts, and the great bulk of the supply activity measured in terms of the numbers of parts issued or consumed is similarly concentrated in the low-cost items.[1] Since there are many hundreds of thousands of such parts in the Air Force inventory,[2] it is not possible - certainly not economical - to devote a great deal of management to the control of each item. On the other hand, these parts must be managed effectively, since it has been the experience of the Air Force that a very large fraction of supply difficulties are caused by inexpensive parts.[3] Most supply activity is accounted for by the cheap parts. On the other hand, the bulk of the investment is in the small fraction of expensive parts.

These characteristics of the supply system are reflected in Tables 1 and 2, which show the base level experience for two quite different aircraft in two widely separated periods. Table 1 shows very detailed B-47 experience at March and McDill Air Force Bases during 1953 and 1954. Table 2 shows F-86D experience at Clovis Air Force Base in 1956. Both tables show base level supply activity for roughly the equivalent of a base-year, broken down by price and issue-rate groupings. Each shows the percent of total active line items, the percent of total transactions and the percent of the value of transactions accounted for by each price-demand group. While the tables differ in detail, their general characteristics are much the same. Both clearly indicate the concentration of line items in the very low-price, very low-demand groups, the concentration of supply activity - as reflected in

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[1] Bernice B. Brown, Characteristics of Demand for Aircraft Spare Parts, The RAND Corporation Report R-292, July 1956.

[2] M. A. Geisler and A. R. Mirkovich, Analysis of Worldwide Data on Aircraft Spare Parts as to Unit Cost, Quantity and Value Issued, and Inventory Value, The RAND Corporation Research Memorandum RM-1481, 6 May 1955.

[3] H. W. Karr, Analysis of B-47 AOCP Experience, The RAND Corporation Research Memorandum RM-1340, 14 September 1954.

Table 1.

Demand Experience for B-47 Aircraft Parts over 1300 Aircraft-Months[1]

Unit Cost (Dollars)		Demand per 100 Aircraft-Months: Under 0.1	0.1-0.9	1.0-9.9	10.0 and Over
600 and over	% Line Items	2.7	0.5	0.3	0.02
	% Quantity Demanded	0.01	0.1	0.4	0.2
	% Dollar Value Demanded	0.4	9.2	39.2	36.7
10 - 600	% Line Items	16.4	2.5	1.1	0.1
	% Quantity Demanded	0.1	0.5	1.6	1.3
	% Dollar Value Demanded	0.2	1.7	7.0	4.8
Under 10	% Line Items	58.9	7.7	6.5	3.0
	% Quantity Demanded	0.1	1.7	12.8	81.3
	% Dollar Value Demanded	0.0	0.1	0.3	0.5

Total Line Items: 26,445; from Master Spare Parts List

Total Quantity Demanded: 635,334 Parts

Total Value Demanded: 20.4 million dollars

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[1]The data were collected at March, MacDill and Fairford Air Force Bases in 1953 and were previously published in M. A. Geisler's Analysis of Base Stockage Policies, The RAND Corporation Research Memorandum 1431, 17 February 1955, p.4.

SOME STATISTICAL CHARACTERISTICS OF F-86H PARTS

8 Months Experience Clovis Air Force Base

Annual Demand Rate / PRICE		< $ 8.00	$ 8.00-124.99	$ 125.00-32,000.00	Totals
1 - 20	% Line Items	32.0	21.1	7.4	60.5
	% Issues	2.9	1.7	0.6	5.2
	% Value of Issues	0.1	0.8	6.2	7.1
20 - 110	% Line Items	17.5	2.4	1.6	21.5
	% Issues	12.6	1.1	1.1	14.8
	% Value of Issues	0.2	0.7	48.9	49.6
110 - 420	% Line Items	10.1	0.4	0.5	11.0
	% Issues	32.1	1.1	1.5	34.7
	% Value of Issues	0.6	0.5	30.1	31.2
420 -	% Line Items	6.7	-	0.2	6.9
	% Issues	43.8	-	1.3	45.1
	% Value of Issues	0.6	-	11.8	12.4
TOTALS	% Line Items	66.3	23.9	9.7	
	% Issues	91.4	3.9	4.5	
	% Value of Issues	1.5	2.0	97.0	

Table 2.

transactions - in the low-price, high-demand group and the concentration of dollar value of transactions in the very few high-value, relatively high-demand cells. All the data available at RAND indicate that these characteristics are typical.[1]

Almost any supply system would function effectively if there were a smooth flow of issues as in Figure 2.[2] Issues would be made until the Reorder Point was reached, then additional stock would be ordered up to the Stock Control Level. Issues would continue during the pipeline time. By the time the new order arrived, the stock on hand would be down to the safety level, which would be needed only if pipelines were interrupted.

With no uncertainties, it might be possible by trial and error to establish adequate Reorder Points and Order Quantities. However, there is uncertainty. Pipeline times vary, so there is no assurance that stocks will arrive when they are expected. More important, demands do not occur in neat order, one per week, or one every month, or anything of the sort. Figure 3 is a more typical representation of an actual sequence of base demand over a period of 35 weeks for a particular item. This is, if anything, a less erratic demand pattern than is typical; yet, in spite of the fact that this part had a mean demand of 1.7 units per week, there were many weeks when it was

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[1] Bernice B. Brown and M. A. Geisler, Analysis of the Demand Patterns for B-47 Airframe Parts at Air Base Level, The RAND Corporation Research Memorandum RM-1297, 27 July 1954; M. A. Geisler and A. R. Mirkovich, Analysis of the Flying Activity and Spare Parts Demand of F-86D Aircraft at Perrin Air Force Base, 1 September 1953 - 28 February 1954, The RAND Corporation Research Memorandum RM-1456, 4 June 1955; idem, Analysis of Worldwide Data on Aircraft Spare Parts as to Unit Cost, Quantity and Value Issued, and Inventory Value, The RAND Corporation Research Memorandum RM-1481, 6 May 1955.

[2] The figure also serves to define some of the key terms which will be used throughout the discussion.

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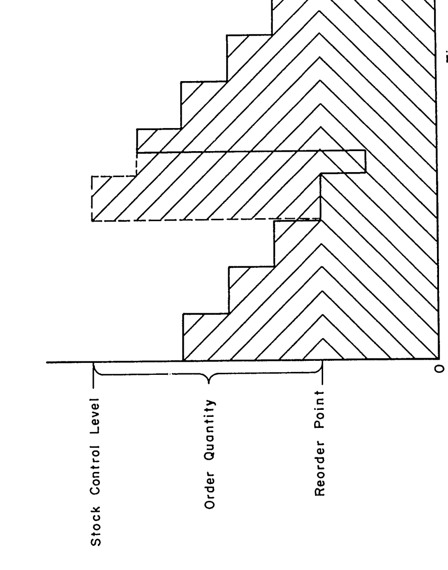

Fig.2 —— Some definitions

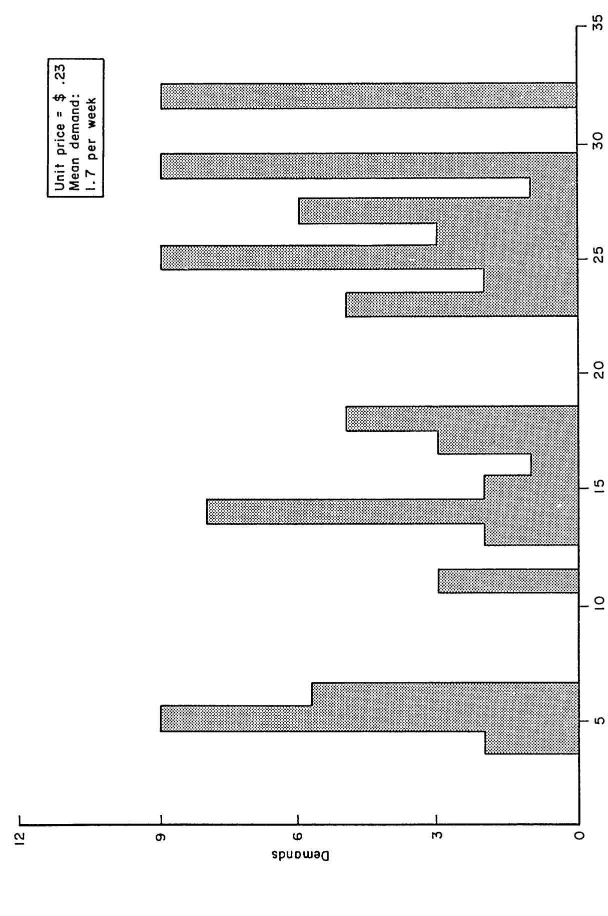

Fig.3 —— Base level demand pattern for a gasket

not demanded at all, and several weeks in which there were demands as high as 9.

If the rule, "stock 90 days' stock plus the expected number of demands during the pipeline time plus a 15-day safety level," were rigidly applied to the 23-cent gasket, the solid line in Figure 4 would show how the stock on hand at a base would fluctuate over a period of 35 weeks. There are some stockouts. (Stockouts for the cheap items, such as this 23-cent item, have been a major cause of criticism of Air Force stockage policy.)

Further, if that rule were followed, three reorders would have been placed while some 68 parts were demanded. If it costs as little as $5 to process an order, $15 would have been spent on reorders to support $15.64 worth of consumption.

What do these facts mean for stockage policy? To the extent that good management should be directed toward dollar economy, it is desirable to concentrate it on the small fraction of high-value items. It is desirable to buy them in minimal quantities and to control them closely. Since they are few, this may be entirely practical and is, of course, the objective of the Hi-Valu program. However, in view of the great number of less expensive parts, it is impractical to attempt to control them in that fashion, and because so little is invested in them, in relative terms, there is no economic justification for doing so. Further, since a large proportion of the supply activity is accounted for by the cheap parts, it is only by curtailing the administration and management of them that management resources will be freed to control properly the higher-cost items. The low-value parts, however, can and do ground weapons and stop maintenance lines, so they can by no means be ignored.

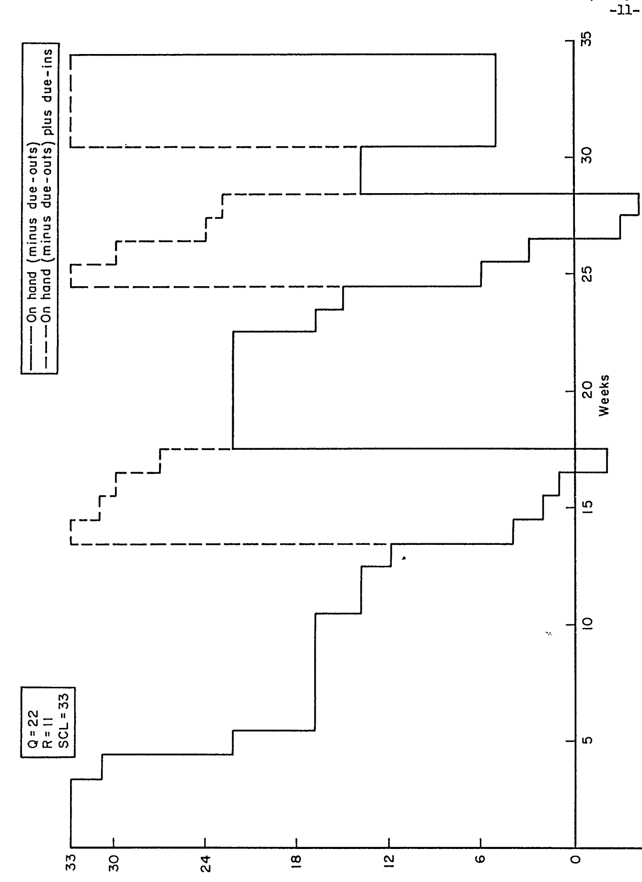

Fig.4 — Pattern of base stocks on hand and due-in
For same gasket (see fig.3)

Sound management, therefore, appears to call for striking a balance among the costs of managing the different kinds of items, the consequence of shortages of them and the costs of investing in and holding them. The study reported here does that. The methods proposed are sufficiently simple to permit their use in the management of the masses of lower-cost items, with considerable improvement in the effectiveness of supply support of the operational units. The policies call for large base stocks and infrequent requisitions of the cheapest parts so as to reduce greatly the risks of shortages of the cheap parts and the costs of processing them. On the other hand, to reduce the investment in the higher-cost items, much smaller stocks and more frequent reorders are appropriate.

The paper is organized as follows:

This introductory chapter describes the organization and scope of the paper and defines the problem of setting efficient stock levels.

In Chapter II, the principles of efficient stockage are stated and used to compute a sample table of economical base stock levels under a stable program. These stock levels are then compared with the base levels which would result from rigid application of the general rules stated in the Air Force Supply Manual.[1]

Chapter III takes up the effect of some dynamic considerations on base stock levels: the setting of quantities for initial-support tables and the

[1] Air Force Manual 67-1. This manual is referred to as "67-1". By the "67-1 system" is meant the rules calling for a reorder of the number of units expected, on the average, to be issued in a period equal to 15 days, plus the length of the routine pipeline time and an operating period of 60 days for Cost Category II items and 90 days for Cost Category III items.

In many areas, the Air Force applies stockage policies which are different from and improvements over the "67-1" system, but the "67-1" system provides a ready bench mark with which the proposed rules may be compared, and, further, for purposes of comparison our set of general rules can best be related to some other set of general rules.

setting of levels as the phase-out of a weapon or program approaches.

Chapter IV, "Depot or Storage-Site Stock Levels", parallels the discussion in Chapters II and III, first taking up the stable-program case and setting levels so that efficient support may be given to bases; and then, the dynamic aspects of determining depot Order Quantities and Reorder Points.

Chapter V describes the Data Requirements of the proposed policies and discusses their sensitivity to inaccuracies in the data.

Chapter VI lists the conclusions which may be drawn from this study.

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II. BASE LEVELS UNDER STABLE CONDITIONS

The present chapter is basic to the whole discussion. It develops the fundamental principles which will be elaborated and modified later on to take account of dynamic and system stockage problems. Specifically, this chapter develops formulas proposed for the computation of base Reorder Points (for determining when to resupply) and base Order Quantities (how much to ship at a time) for a stable program.

Two sets of equations are developed. A rather rigorous set is derived in Appendix I, but most of the discussion in the text of this and the succeeding chapters relates to some approximations to the rigorous equations. The approximations are quite adequate for determining levels for the lower-priced items in Category II and for Category III and are very easy for manual computation. The more exact equations can be used where electronic computing equipment is available. For some of the higher-cost items they give significantly better results than do the approximate equations.[1] These equations still permit automatic computation of levels and, hence, are appropriate for the more expensive parts (including Category I items) in cases where it is decided not to give individual attention (as required, for example, in the deferred-procurement program) to the management of particular line items. As will be shown later, it would also be practical to use tables based on either the approximate or exact equations for very simple manual computation. The choice among these three procedures for actual implementation requires the exercise of judgment based upon the cost and importance of the items in question, the accuracy of the input data available and the computing capacity at hand.

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[1]See Chapter V for a discussion of the sensitivity of system costs to the approximation.

After the formulas are presented, some reasonable assumptions will be made about the environment in which the supply system operates, and the approximation formulas will be used to compute a sample table of base[1] Reorder Points and Order Quantities. The sample table is for illustrative purposes only and is not proposed for actual use. We shall see the way Reorder Points differ for different resupply pipeline times and for different costs associated with a shortage, and we shall compare the sample stock levels with "67-1" levels. Finally, this section will present a very rough comparison of the supply effectiveness and costs of the proposed policies and the "67-1" policies.

A. Approximation Formulas

The definitions to be used are as follows (See Figure 2):

Stock Control Level has the same meaning as the Stock Control Level under the present system.

Reorder Point is the level for determining when to reorder, i.e., when "stock-on-hand-plus-due-in's minus due-out's" is equal to or less than the Reorder Point, an additional order is to be placed.

Order Quantity represents the difference between the Stock Control Level and the Reorder Point.

Operating Stock refers to the "stock-on-hand-or-due-in" over and above the Reorder Point.

Safety and Pipeline Stock is that "stock-on-hand-or-due-in" up to the Reorder Point.

[1]"Base" as used throughout this Memorandum includes the base support activity of a depot.

1. Base Order Quantity

In explaining how to set base levels for any particular line item, we treat first the Order Quantity, then the Reorder Point. A glance at Figure 2 will show that determining these two quantities at each activity for each item solves the stockage problem.

Turning first to the Order Quantity, it is obvious that, if there were no cost or inconvenience associated with ordering again and again, it would be cheapest and simplest to order one unit at a time, thus avoiding the costs of holding the Operating Stocks. On the other hand, if it cost nothing (in money, trouble, or material resources) to hold stock, there would be an incentive to order each item once and for all and to keep vast quantities on hand -- thus avoiding the problems and costs of reordering. Neither extreme situation is true, of course, but looking at both brings out the fact that for any given demand rate the desirable size of order depends on the costs of reordering and the costs of holding the Operating Stocks and, in the approximation, upon nothing else.[1] If each shipment is small, shipments are frequent, and the costs of placing an order are incurred frequently, but only a small amount of Operating Stock is allocated to the base. If each shipment is large, the number of orders -- and hence the yearly reorder costs -- are reduced, but the average Operating Stock is large and so is the cost of holding it. To find the economical Order Quantity for each demand rate, we balance holding cost against reorder costs.

[1]In the rigorous formulation, explicit account is taken of the fact that the Operating Stocks also provide some protection against shortages. See Appendix I.

a. To define reorder costs, compare what happens if a base were to receive two small shipments of an item with what would happen if it were to receive only one large shipment of the same number of units of the same item.. Since at both base and depot level some order costs are independent of the number of units on an order, it costs more to make two small than one large shipment of the same total amount. As illustrated in Figure 5, we call that difference in cost the reorder cost, i.e., the extra distribution cost incurred by each additional order for the base during the year.[1] The reorder costs are those costs which are incurred in the placing and filling of an order, and which do not depend on the number of units ordered or shipped. They consist largely of paperwork and communications costs and some fraction of the stock-picking, packing, transportation and receiving costs.

Figure 6 shows that, given the demand rate at a base, annual[2] cost of reordering declines as the size of each shipment increases. Note that annual cost of reordering declines rapidly as the size of shipment increases from one unit to two units (since the number of shipments is reduced by half), but the cost declines less rapidly for larger shipments,

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[1]For further discussion, see Chapter V, Section B-6.

[2]Costs and demand are referred to as pertaining to a year -- any convenient period would do equally well.

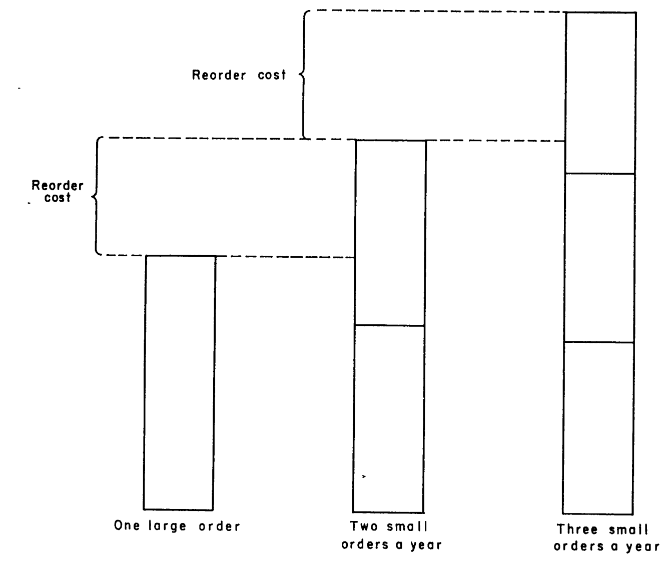

Fig. 5 — Annual distribution costs
(Given total number of units shipped)

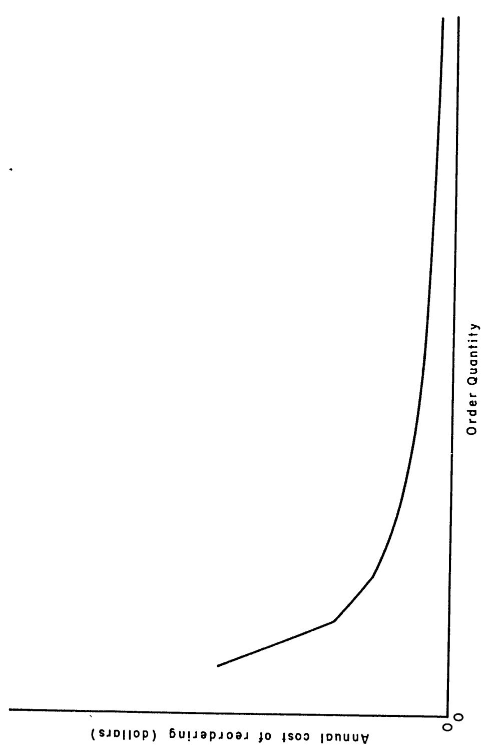

Fig. 6 — Annual cost of reordering
Given demand rate

e.g., if the Order Quantity is increased from nine units to ten, the number of shipments—and hence annual costs—is reduced only 10 per cent.

b. The Operating Stocks must be held until used. The <u>annual cost to hold</u> a unit of stock consists of the unit value of the item multiplied by the unit holding cost. The latter is (1) the physical cost of storing a unit of the item at a base for a year, plus (2) a charge for the capital invested in a unit of the stock.[1] The unit holding cost is expressed as a per cent of the value of the item.

The larger the Order Quantity, the larger the stock held and the greater the costs of holding it. Thus, two kinds of cost are involved: annual cost to reorder, which declines as Q increases, and annual cost to hold, which increases as Q increases, given the unit price of the item and the unit holding cost.

Figure 7 represents annual cost to hold, annual cost of reorders and the sum of the two. Note that annual reorder costs decline rapidly at first as Order Quantity increases; total cost decreases at first and then, as reorder cost levels off, total cost increases. Obviously, it is desirable to select an Order Quantity which minimizes total cost. The economical Order Quantity is that size of order associated with the smallest total of annual cost to reorder plus annual holding cost.

[1] This is a charge for the risk of obsolescence or modification due to engineering changes in the part itself or in the end item or higher assembly of which it is a member, plus a capital charge reflecting the value of money to the Air Force, i.e., the cost of using money to buy this item rather than spending it in some other way. See Chapter V, Section B-4 and Appendix I.

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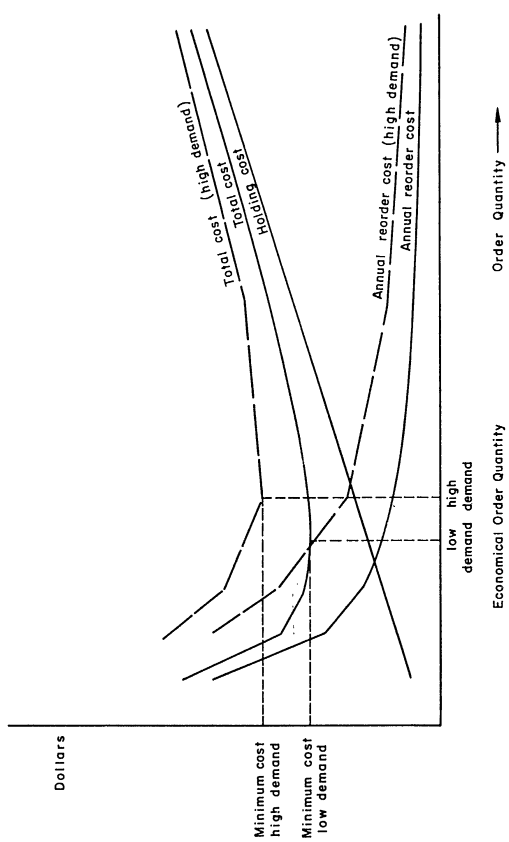

Fig. 7—Reorder cost, holding cost and total cost
The effect of demand rate on the Economical Order Quantity

The demand rate and the unit cost of the item also influence the economical value of Q. If the demand rate for one item is greater than for another, the annual cost to reorder associated with any given size of shipment will also be larger (Figure 7). Thus, the economical Order Quantity will be larger, the greater the demand rate. The higher the unit cost, the more it costs to hold any given number of units; hence, the economical Order Quantity decreases, as illustrated in Figure 8.

In order to use these principles in computing levels we must have a formula;[1] it is

$$(1) \qquad \text{Economical Order Quantity, } Q = \sqrt{\frac{2(\text{Reorder Cost})\ (\text{Annual Demand Rate})}{(\text{Holding Cost})\ (\text{Unit Cost})} + 1}\ .$$

For all practical purposes, this is the equation we would have obtained if, instead of plotting the curves in the last several figures, we had written down their equations and found the mathematical solution for the minimum cost point. The derivation is developed more fully in Appendix I.

Equation 1 shows that the higher the demand rate, the larger the economical Order Quantity, but doubling the demand rate does not double Q; the higher the reorder cost, the larger the economical Order Quantity, but, again, doubling this less than doubles Q; the higher the holding cost or the unit cost, the smaller the economical Order Quantity, but, again,

[1] In the equation, the "1" under the radical is a constant factor which results primarily from the fact that fractional shipments cannot be made. In other respects this equation can be derived from the equation for the optimal number of orders per year which appears in the Air Force Supply Management Handbook, AFM 67-10, Page 149 (March 1, 1956). For the derivation of Equation 1, see the Mathematical Appendix.

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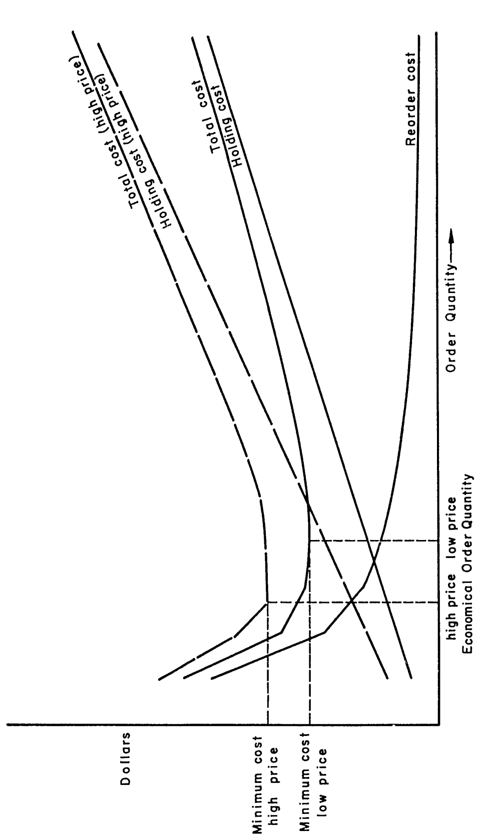

Fig. 8 — How a higher unit price reduces the economical Order Quantity

doubling the holding cost does not cut Q in half. In fact, Q changes as the square root of these factors: a doubling of Q would occur if demand or reorder cost were quadrupled or if unit price or holding cost were quartered. These effects can be seen clearly in Figure 1 or in Table 3 below.

2. Base Reorder Point

The next step is to determine when shipments should be made to a base, i.e., what the base's Reorder Point should be. The purpose of the Reorder Point is to indicate at what inventory level an order should be placed. It marks the amount to be held as Safety and Pipeline Stocks (Figure 2) to avoid shortages while the base is waiting for a shipment. If resupply were instantaneous and the pipeline were never interrupted, or if there were no cost or inconvenience in being without a part while awaiting a shipment, there would be no justification for investing in Safety and Pipeline Stocks. They would be zero. Resupply takes time, however, and it is costly to be without parts when they are demanded. Furthermore, demand is uncertain, and there is no way of predicting with accuracy how many parts will be demanded during the routine pipeline time when the base is awaiting a delivery. Therefore, there are good reasons for having Safety and Pipeline Stocks; the larger those stocks (all else equal), the less is the risk that a shortage will occur. Since it does cost something to keep stocks, the problem in setting the Reorder Point is to stock just enough so that the protection provided against shortages is worth what it costs, and so that keeping additional stocks would cost more than the additional protection would be worth. This is not an easy problem.

To determine the appropriate Reorder Point, we need to know three things:

a. how many shortages a year are avoided by each additional unit in the Safety and Pipeline Stock;

b. how much it is worth to avoid each of these shortages;

c. what the cost is of keeping an additional unit of Safety and Pipeline Stock.

Since stock levels are set before, not after, the fact, only expected values of these quantities can be used.

a. <u>The number of shortages a year avoided by having an additional unit (call it the R^{th} unit) in the Safety and Pipeline Stocks</u> depends on how frequently these stocks are exposed to use and on the probable number of demands during each period of exposure. Figure 9 shows diagrammatically the factors determining how much protection is provided by additional Safety and Pipeline Stocks.

The frequency with which these stocks are exposed to use depends upon the size of the Order Quantity, given the annual demand rate, for they are subject to use whenever the base has exhausted its Operating Stocks and is (presumably) awaiting an order. The expected number of orders[1] a year, of course, equals the expected annual demand rate divided by the Order Quantity, e.g., if the demand rate is 100 per year and 25 are shipped at a time, there will be an average of four shipments a year; but if 50 are shipped at a time, there will be only two shipments a year, etc. The probable number of demands during

[1] The number of orders per year is the reciprocal of the operating period (measured in years). Operating period equals Order Quantity ÷ Annual Demand. The number of orders per year equals Annual Demand ÷ Order Quantity.

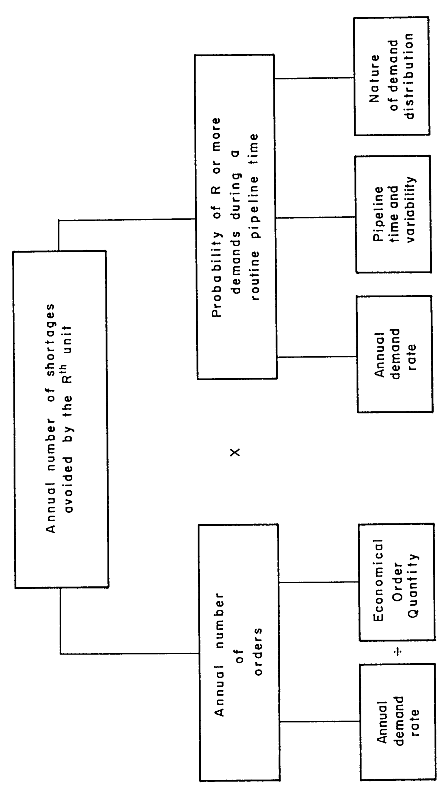

Fig. 9 — Determinants of the protection provided by the R^{th} unit in the Reorder Point

the pipeline time--which is the period of exposure--depends upon the length of the pipeline time, the annual demand rate and the variability of demand.

All else equal, the longer the pipeline time, the greater the number of demands expected while the base is awaiting shipment, and hence the greater the chance that the R^{th} unit will avoid a shortage.

The larger the annual demand rate, the greater the chance that the additional unit will avoid a shortage, because the more demands one can expect during any pipeline period. Usually the greater the variability of demand, i.e., the more erratic is demand, the greater the probability that the R^{th} unit will avoid a shortage.

Variation in the pipeline time has much the same effect as does variation in demand. It can be treated in the same way since we are interested in the probable number of demands during the period while the base is awaiting resupply.

To get some "feel" for how an additional unit in the Safety and Pipeline Stocks avoids a shortage, refer again to Figure 4 showing the stock of the gasket on-hand and due-in. In this case, the Reorder Point is 11 units, and the Order Quantity is 22. At the first reorder, since a total of 14 units were demanded within one routine pipeline time, each of the 11 units in the Safety and Pipeline Stock avoided one shortage because, had the Reorder Point been 10 instead of 11, there would have been three shortages instead of two; if the Reorder Point had been 9 units, there would have been four shortages, etc. On the next order, each unit in the Safety and Pipeline Stock avoided a shortage for similar reasons. But, at the third reorder the seventh, eighth, ninth, tenth, and eleventh units of Safety and Pipeline Stocks avoided no shortages because after the reorder was placed five units were still on

hand and there were no demands during that particular pipeline time. The eleventh unit avoided two shortages in this example and on the average avoided two-thirds of a shortage in each of the three periods of exposure. Thus, the expected number of shortages avoided during a year by the R^{th} unit is equal to the probability that R or more units will be demanded during a pipeline time (for that probability is the fraction of pipeline times in which R or more units will be demanded times the number of reorders per year.)

b. The second quantity which must be known in order to determine the economical Reorder Point for a base is <u>how much it is worth to avoid a shortage</u>.[1] If a shortage would result in priority action, the cost of that action is a lower limit on the amount it is worth to avoid a shortage. In addition, there is the loss in operational capability caused by the shortage.

This latter cost depends upon the immediacy of the need, the mission effect of a shortage and the cost of possible compensatory action. If the item is needed immediately, the shortage cost will be higher than if the need is discovered during, say, a periodic inspection when the item could as well be installed several days later. Secondly, a shortage of one (of two) landing light would have far less effect on operational capability than would the shortage of a nose wheel. If the effect of the shortage can be reduced temporarily or permanently by local manufacture or cannibalization, the actual shortage cost is likely to be less than if the aircraft is forced to remain without the item. If a shortage can be compensated by using an inexpensive higher assembly, of which there are many stocked on the base anyway, it is far less serious and less costly than would be the case

[1] The shortage cost is discussed more fully in Chapter V, Section B-7.

if the next higher available assembly were the aircraft itself. It appears that the pro-rated cost of the aircraft or other end item sets an upper limit on the shortage cost.

c. The third quantity that must be known to set the economical Reorder Point is the cost of keeping each additional unit of Safety and Pipeline Stock on hand. In general, we can expect that, after the first unit of an item is stocked, the cost of keeping the Safety and Pipeline Stock is increased by roughly the same amount by each additional unit in the Safety and Pipeline Stock. To determine the cost of keeping a unit for Safety and Pipeline Stock, all of the factors used for the determination of unit holding cost for operating stock are needed, viz., physical-storage costs, capital costs, and engineering-obsolescence costs. It was expected that the Operating Stocks would be used up periodically, but there is a good chance that an additional unit of Safety and Pipeline Stock will eventually have to be salvaged or otherwise disposed of at less than the purchase price; for, by having enough stock to achieve a high degree of protection, there is a very good chance that some of the Safety and Pipeline Stock will be on hand at the end of the program. Thus, terminal obsolescence must also be taken into account in determining keeping cost.[1]

Figure 10 illustrates how the three factors are used to determine the Economic Reorder Point. Compare the expected saving of shortage costs from each additional unit of stock with the cost of having that unit. For instance, the expected saving due to the first unit (the first bar in Figure 10) of the Safety and Pipeline Stock is the probability of one or

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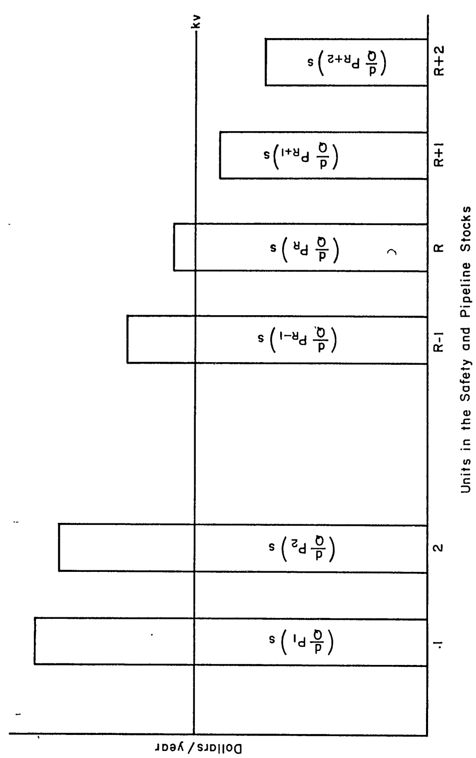
[1]Terminal obsolescence is defined as that obsolescence of parts caused by the phasing-out of the program to which they apply. Keeping cost is discussed more fully below in Chapter V., Section B-5.



$\frac{d}{Q}$ = average number of orders a year
$P_1, P_2, \ldots P_R$ = probability of 1 or more, 2 or more, ... R or more etc., demands during the routine pipeline time
s = shortage cost
k = annual cost of keeping a unit of Safety and Pipeline Stock as a fraction of unit cost
v = Unit cost

Fig. 10 — Determination of the economical Reorder Point

more demands during the routine pipeline time, (P_1), multiplied by the number of orders expected each year, ($\frac{d}{Q}$), multiplied in turn by the shortage cost per unit (s). Then compare this figure with the cost of keeping an additional unit on hand (kv). If the savings are greater than the cost, hold at least one unit in the Safety and Pipeline Stock, make the Reorder Point at least one. Then, look at the expected saving from the second unit. It is the same as the expected saving from the first unit, except that the probability of two or more demands during the routine pipeline time must be used instead of the probability of one or more demands; this, of course, is less than the probability of one or more demands (by the probability that exactly one unit will be demanded), so the net savings are less. If, however, the savings still are greater than the cost of keeping the additional item, it pays to have at least two units. Do this for each successive unit until you come to the R^{th} unit; the expected savings from the R^{th} unit are greater than the cost of keeping the R^{th} unit, so it pays to stock the R^{th} unit, but the expected savings from the $(R+1)^{st}$ unit, i.e., one more unit, are less than the cost of keeping a unit, so stop at R units. This process is summarized by Equation 2:[1]

(2) $$s \frac{d}{Q} P_R > kv > s \frac{d}{Q} P_{R+1}$$, where terms are defined as in Figure 10.

In words:

> A particular Reorder Point, R, is the economical Reorder Point if the expected savings in shortage costs from the last unit in R (the R^{th} unit) are greater than the cost of keeping the R^{th} unit, but the additional cost of keeping one more unit, (the $R+1)^{st}$ unit, is greater than the expected additional savings from that unit.

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[1] For the derivation of Equation 2 see Appendix I.

If the savings from the $(R+1)^{st}$ unit are greater than the keeping cost, then the economical Reorder Point is greater than R. If the savings from the R^{th} unit are less than the keeping cost, then the economical Reorder Point is less than R.

Notice, the lower the cost of keeping an additional unit (k) or the lower the unit price (v), the greater the economical Reorder Point tends to be. The greater the shortage cost, the greater the relative benefits from each unit are; hence, the greater the economical Reorder Point would be. For higher-demand items and for longer pipeline times, the probability of R or more demands would be greater during the pipeline time, and so the economical Reorder Point would be greater. Since an increase in reorder cost increases the Order Quantity and reduces the annual number of orders, the greater the reorder cost, the smaller the Reorder Point, but this is a rather indirect and small effect.

Now we have seen in principle how economical Order Quantities and Reorder Points may be computed for a base. What sort of stock levels result when we apply these principles?

B. Resultant Stock Levels

To evaluate these policies one must know what sort of base stock levels are likely to result from the use of Equation 1 (to compute Order Quantities) and Equation 2 (to compute Reorder Points) and how the levels are affected by changes in the parameters used in their computation.

1. The Base Stockage Table

Table 3 is a sample table of base levels under a particular set of

WORKSHEET

R = Reorder Point
Q = Order Quantity

Table 3
BASE STOCK LEVELS

30-day Pipeline Time
50-dollar Shortage Cost

Unit Price \ Annual Demand at least / but less than		0- 2	2- 6	6- 12	12- 20	20- 30	30- 42	42- 56	56- 72	72- 90	90- 110	110- 132	132- 156	156- 182	182- 210	210- 240	240- 272	272- 306	306- 342	342- 380	380- 420	420- 462	462- 506
$ .01-	R	4	8	11	14	18	21	24	28	31	35	39	43	46	51	55	60	65	70	75	80	85	91
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13-	R	2	6	9	13	16	20	23	26	30	33	37	41	45	49	54	58	63	68	73	78	83	89
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25-	R	2	5	9	12	15	18	22	25	28	32	36	40	44	48	52	57	61	66	71	76	81	87
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50-	R	1	5	8	11	14	17	20	24	27	31	34	38	42	46	50	55	59	64	69	74	79	85
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00-	R	1	4	7	10	13	16	19	22	26	29	33	36	40	44	49	53	57	62	67	72	77	83
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00-	R	0	3	6	9	12	15	18	21	24	28	31	35	39	43	47	51	56	60	65	70	75	81
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00-	R	0	2	5	8	11	14	17	20	23	26	30	33	37	41	45	49	54	58	63	68	73	78
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00-	R	0	2	4	7	10	12	15	18	21	25	28	32	35	39	43	47	52	56	61	66	71	76
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00-	R	0	1	4	6	9	11	14	17	20	23	26	30	34	37	41	45	50	54	59	64	68	74
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00-	R	0	0	2	5	7	10	13	16	19	21	25	28	32	36	39	44	48	52	57	61	66	71
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00-	R	0	0	2	4	7	9	11	14	17	20	23	26	30	33	38	42	46	50	54	59	64	69
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00-	R	0	0	1	3	5	7	10	12	15	18	21	25	28	32	35	40	43	48	52	56	62	66
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00-	R	0	0	1	2	4	6	9	10	14	17	19	23	27	30	34	37	41	46	50	54	59	64
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

hypothetical conditions,[1] viz., a 30-day pipeline time,[2] an expected shortage cost of $50 per unit, a reorder cost of $5 per order, annual holding costs of 20 per cent of unit price (say, 10 per cent storage cost plus 10 per cent capital cost), a five-year program, and negative-binomial demand probability distributions with the variance equal to four times the mean.[3] It shows in a form which can be readily used by clerks, Reorder Points and Order Quantities which might result from the application of the proposed policies for those items which it has been determined will be stocked at the base with unit prices of one cent to $500 and expected annual demand rates from less than 2 up to 506 (monthly demand rates from less than 1/6 to 42).

Note that the annual demand classes used for the table (0-2, 2-6, 6-12, ... 380-420, 420-462, etc.) are not equal in absolute or relative width and that (except for the second) each price class is about twice the width of the preceding class ($0.12-0.24, $0.25-0.49, $0.50-1.00, etc.). This choice of class intervals resulted from an analysis of the effects of

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[1]Sample tables of base levels under some other conditions are presented in Appendix II.

[2]By pipeline time we mean the time from the occurrence of that issue at the base which reduces the stock-on-hand-and-due-in to the Reorder Point until the resulting shipment arrives and is available for issue.

[3]The cost and probability assumptions are believed to be reasonable guesses for many items in the Air Force supply system. But they are guesses and not estimates. Therefore, Table 3 is presented for illustrative purposes only. It is not necessarily recommended for use in either service-testing or implementing the proposed policies.

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errors on system performance.[1] It is designed to minimize the effects of these errors. Sensitivity to error is discussed in Chapter V. Among other things, the discussion shows that it is better to overstate the demand rate than to understate it. Hence, the Reorder Points (R) and Order Quantities (Q) are not computed for the midpoint of each cell but, instead, are computed for a point near the lower end of the price class and near the upper end of the demand class.

2. Sample Computation

For example, the levels for the cell "Demand, 72-90, Price, $4.00-7.99" was computed for an item with a demand of 84 per annum (72 x 1/3 + 90 x 2/3) and a price of $4.80 ($4.00 x 4/5 + $8.00 x 1/5).[2] The computation was then carried out by solving Equations 1 and 2.

$$Q = \sqrt{1 + \frac{2rd}{hv}}, \tag{1}$$

assuming

$r = \$5.00$

$d = 84$

$h = 0.20$

$v = \$4.80$

Hence,

[1]If a cell computed on the basis of a demand rate of 10 is used for an item with a demand rate of 8, an "error" of 2 units is made in using the formulas.

[2]In the $0.01-0.12 price class $0.075 was used. In the cells to the left of the heavy line the adjustment for a short base program which will be described in the next section was used.

$$Q \quad \sqrt{1 + \frac{(2)(5)(84)}{(.20)(4.80)}} = \sqrt{876} = 29.60 = \underline{\underline{30}} \tag{3}$$

We then enter 30 as the Order Quantity in the cell "Demand, 72-90; Price, $4.00 - 7.99." The next step is to find the Reorder Point, R, from Equation 2.

$$\frac{sd}{Q}\left(P_R\right) > kv > \frac{sd}{Q}\left(P_{R+1}\right) \tag{2}$$

Equation 2 may be rewritten:

$$P_R > \frac{kv\,Q}{sd} > P_{R+1} \tag{4}$$

For these values we have:

$k = 0.35$[1]

$v = \$4.80$

$Q = 30$ (derived above)

$s = \$50$

$d = 84$

$P_R = ?$

So that Equation (4) becomes

$$P_R > \frac{(0.35)(4.80)(30)}{(50)(84)} > P_{R+1} \tag{5}$$

Then

$$P_R > 0.012143 > P_{R+1} \tag{6}$$

[1]For an explanation of how to derive k for different expected program lengths see Appendix I.

The Reorder Point, R, is then found by comparing 0.012143 with the values in a table showing the probability of j or more demands per pipeline time. Demands have been assumed to have a negative binomial distribution with variance four times the mean, expected annual demands(d) are 84 and the pipeline time (t) is 0.08 years. Hence, the table is used for a mean of (84)(.08) = 6.92 and variance of (4)(6.92) = 27.68.

Such tables can be computed by an electronic computer. The relevant part of the particular table needed shows:

j (Possible Reorder Point)	P(j) Probability of Exactly j Demands	$P_j = 1 - \sum_{x=0}^{j-1} P(x)$ Probability of j or More Demands
0	0.044811	1.000000
1	.075283	.955189
2	.091468	.879906
.	.	.
.	.	.
.	.	.
20	.005539	.026200
21	.004400	.020660
22	.003486	.016261
23	.002755	.012775
24	.002173	.010020
25	.001711	.007846

By inspection of the probability table we find that:

(7) $$P_{23} = 0.012775 > 0.012143 > P_{24} = 0.010020$$

therefore, the Reorder Point, R_1, is 23 units. Thus, we have found the entries (R = 23, Q = 30) for the cell "Annual Demand: 72-90, Unit Price: $4.00-7.99" in Table 3 .

Table 3 shows that the higher the demand rate, the greater the Reorder Point and the Order Quantity. It also shows that the lower the price, the greater the Reorder Point and the Order Quantity; however, there is no simple relationship between the two. For a demand of 6 to 12 a year (1/2 to 1 a month) the Reorder Point is 11 units for the cheapest item (about one or two years' supply) and the Order Quantity is 70 units (six to thirteen years' supply) -- the Order Quantity is seven times the Reorder Point. For 8-16-dollar items, again those with demands of about 10 per annum, however, the Reorder Point is 4 (four to eight months' supply) and the Order Quantity is 7 (seven to fourteen months' supply) -- the Order Quantity is less than twice as great as the Reorder Point.

For items with a base demand rate of 110 to 132 annually, the relationship is quite different. The Reorder Point drops from 39 units (about four months' supply) for the cheapest items to 19 units (or two months' supply) for the $250-$500 cell and the Order Quantity drops from over two years to about two weeks supply.

With or without integrated data processing, the formulas from which these Reorder Points and Order Quantities were derived can be used to produce tables like Table 3 for use by bases.[1] A moderate number of tables should be sufficient for any one base. It should be noted that total cost is barely affected if the Order Quantity for low-cost Category III items is rounded to some standard unit, e.g., dozens, 25's, gross, etc., or, to the nearest unit pack.

[1] If tables are centrally computed, the more rigorous process described in Appendix I may be more desirable than the approximations provided by Equations 1 and 2.

In the high price ranges the errors introduced by using the approximation equations are relatively large. If computing facilities are available, it would be better to use the more rigorous formulation of Appendix I for parts costing, say, more than $50 each. If tables are prepared centrally, with EDPE, of course, the more rigorous form should be used throughout.

3. Effect of Pipeline Time and Shortage Cost

Table 3 is based upon a 30-day pipeline and a 50-dollar shortage cost. What happens with other pipeline times and other shortage costs? As stated in discussing Equation (2), the larger the shortage cost and the longer the pipeline: the larger the economical Reorder Point. The Order Quantity, of course, is not affected. Appendix II provides five additional tables similar to Table 3 showing all combinations of 4-day pipelines (such as might be realized in the ZI with highly effective air-electronic resupply) and 30-day pipelines, and $50, $500, $5000 shortage costs (which probably bracket the relevant ranges of shortage costs for technical items).

Tables 4 and 5 give particular examples of the effect of different pipeline times and shortage costs upon the Reorder Level. An examination of these summary tables or tables in Appendix II will indicate the sensitivity of the Reorder Level to the pipeline time and its relative insensitivity to the value of the shortage cost. An increase in the pipeline time from 4 to 30 days causes increases in the Reorder Point of several months' stock in most cases. In Table 4, for example, the increases range from about five months' to nearly eleven months' worth of stock (ignoring the upper right-hand cell). In Table 5, the higher demand example, the increases are less dramatic but are still large.

EFFECT OF PIPELINE TIME AND SHORTAGE COST UPON
THE REORDER POINT
(Annual Demand = 10)

Unit Price:	$ .25 (Q = 41)		$5.00 (Q = 10)		$300.00 (Q = 1)	
Pipeline Time:	4 days	30 Days	4 days	30 days	4 days	30 days
Shortage Cost						
s = $50	3	9	1	5	0	1
s = $500	9	16	5	12	2	6
s = $5,000	15	23	11	20	6	13
"67-1" R	1	1	1	1	1	1
("67-1" Q)	(3)	(3)	(3)	(3)	(1)	(1)

Table 4

EFFECT OF PIPELINE TIME AND SHORTAGE COST UPON
THE REORDER POINT
(Annual Demand = 84)

Unit Price:	$ .25 (Q = 118)		$ 5.00 (Q = 30)		$300.00 (Q = 4)	
Pipeline Time:	4 days	30 days	4 days	30 days	4 days	30 days
Shortage Cost						
s = $50	12	28	8	23	3	14
s = 500	19	37	15	32	9	23
s = 5,000	27	46	22	41	16	33
"67-1" R	4	10	4	10	4	10
("67-1" Q)	(20)	(20)	(20)	(20)	(7)	(7)

Table 5

In contrast, the Reorder Point is relatively insensitive to the value of the shortage cost. A one-hundred-fold range in shortage cost is shown in the Appendix and in these brief tables. Yet the largest increase (ignoring the increase from zero in the fifth column) in either Table 4 or 5 is thirteen-fold. In the Appendix tables the same general situation is shown throughout. In low demand items, however, which account for the bulk of the items,[1] the increase is proportionately greater than in the cases of high annual demand.

The fact that the Reorder Point is not very sensitive to the value of the shortage cost does not mean that the shortage cost can be ignored. What it does mean is that taking it into account at all covers a large range of possibilities reasonably well. In contrast, the "67-1" rules, which take no explicit account of shortage cost, provide entirely different Reorder points. These (and the "67-1" Order Quantitites) are shown for comparison in Tables 4 and 5.

Notice that the Order Quantity, which is shown for reference in Tables 4 and 5, is not affected by the differences in pipeline or shortage costs.

[1]See for example the Tables in Appendix III.

C. Resultant Effectiveness and Costs

In Section A of this chapter, the stockage equations which provide effective support at or near least cost were derived. In Section B, the resultant stock levels were described. Of primary interest, however, is the impact of these policies on supply effectiveness and, second, the cost of achieving that effectiveness. Four tables follow which compare base stockage under the "67-1" rules[1] and under the proposed rules with various shortage costs and pipeline times. The comparisons are of limited significance, being based on a single sample of issue data, viz., issue experience for B-47 aircraft spares at March and McDill Air Force Bases in 1953 and 1954.[2] It is assumed that each of the 7,000 items issued during that period, and on which we have unit prices, is stocked at the base and that no other parts are stocked. The "67-1" stock levels and the proposed stock levels are then found, using the assumptions underlying Table 3 (except for pipeline time and shortage cost).

Table 6 shows the comparison with a 30-day pipeline and a 50-dollar shortage cost. The 30-day pipeline is approximately the present routine pipeline in the continental United States. The 50-dollar shortage cost is chosen as an extreme on the low side. This represents a rough guess of the cost of

[1] The "67-1" policies used in computing these tables are the general rules in effect 1 September 1957, viz., stock 15-day safety level plus pipeline stocks plus, for Category III items, 90 days' Operating Stock and for Category II items 60 days' Operating Stock. These rules have been modified since September 1957 and, in any event, were never applied blindly. They are used here merely as a bench-mark.

[2] Laboratory Problem I in the Logistics Systems Laboratory compared these policies with more realistic current policies and closely confirmed the results reported here. This is discussed more fully below.

EFFECTIVENESS AND COST UNDER "67-1" AND
THE PROPOSED POLICIES - ILLUSTRATIVE COMPARISON
- BASE LEVEL -
Shortage cost = $50.00 Pipeline time = 30 days

	Quantity		Annual cost	
	"67-1"	Proposed	"67-1"	Proposed
Average Operating Stocks				
Cat III	$ 20,000	$ 70,000	$ 4,000	$ 14,000
Cat II	80,000	230,000	16,000	46,000
Safety and Pipeline Stocks				
Cat III	20,000	40,000	7,000	14,000
Cat II	230,000	320,000	80,000	112,000
Total	$ 350,000	$ 660,000	$ 107,000	$ 186,000
Routine Reorders	24,000	7,000	$ 120,000	$ 35,000
Out-of-Pocket Costs			$ 227,000	$ 221,000
Annual Shortages	4,000	840	$ 200,000	$ 42,000
Total Costs			$ 427,000	$ 263,000

Table 6.

priority action, which is the minimum action necessary in the event of a shortage of a technical item and makes no allowance for the loss of military effectiveness which might result from a shortage, for example, by having an aircraft, missile, or an essential piece of ground support equipment out of commission for parts.

Under the assumed costs, the proposed policies require holding substantially larger base-level inventories of Cost Categories II and III items than does the "67-1" system. With the cost of holding the Operating Stocks computed at 20 per cent per annum and the annual cost of keeping Safety and Pipeline Stock taken as 35 per cent, the economical policies also require a greater charge for stock than the theoretical "67-1" policy, i.e., $60,000 a year to pay for Operating Stocks versus $20,000 under "67-1," and about $126,000 a year for Safety and Pipeline Stocks versus $87,000 under "67-1," a total of $186,000 a year versus $107,000.

What is the return from these additional expenditures? Rather than the 24,000 routine requisitions needed each year if these 7,000 items are stocked under the theoretical "67-1" policy, only 7,000 routine requisitions are needed under the proposed policies. At $5 per requisition, the 24,000 routine reorders under "67-1" would cost $120,000; the 7,000 routine requisitions under the economical policy would cost only $35,000. A large part of the savings would be in the form of reduced workload requirements for management of Categories II and III in base and depot supply. Thus, the out-of-pocket costs with the economical policy are less: $221,000 a year as against $227,000. This difference is, of course, insignificant. What is important is that the extra investment is roughly offset by the reduced cost of supply operations.

More important, the proposed policies increase supply effectiveness. There is a striking difference in the shortages to be expected under the two systems. Given the assumed probability distribution, we would expect 4000 shortages a year under the "67-1" policies but only 840 under the proposed policies. This difference in the number of shortages is, of course, of major importance in itself. It is achieved at no increase in out-of-pocket costs. Further, if only the costs of priority action (assumed to be $50) are considered, the reduction in shortages is worth $160,000. Thus, the over-all cost of using the proposed policies is almost 40 percent less than the cost using "67-1," strictly applied.

To summarize Table 6: With a 50-dollar shortage cost and a 30-day pipeline, although the proposed policies call for larger stocks of Category II and III items <u>at base level</u> and, hence, a larger annual cost for carrying this investment, the reduction in routine reorders alone just about makes up for this additional investment cost. In addition, there is a very large increase in the supply effectiveness. The number of shortages is cut by nearly 80 percent, with the over-all support costs for these items reduced by nearly 40 percent. Procurement costs are increased, but the cost of the requisitions, both routine and priority, which follow is greatly reduced.

Now, instead of the 50-dollar shortage cost, which takes only the cost of priority action into account and ignores the effects of shortages on base performance, Table 7 reflects a 5000-dollar shortage cost, which is large enough to take account of a substantial chance that a shortage has a considerable impact on operational capability. The cost of holding the Operating Stocks is not affected by the shortage cost and is again $20,000 for the theoretical "67-1" system, and about $60,000 a year for

EFFECTIVENESS AND COST UNDER "67-1" AND
THE PROPOSED POLICIES - ILLUSTRATIVE COMPARISON
- BASE LEVEL -

Shortage cost = $5000.00 Pipeline time = 30 days

	Quantity		Annual Cost	
	"67-1"	Proposed	"67-1"	Proposed
Average Operating Stocks				
Cat III	$ 20,000	$ 70,000	$ 4,000	$ 14,000
Cat II	80,000	230,000	16,000	46,000
Safety and Pipeline Stocks				
Cat III	20,000	115,000	7,000	40,000
Cat II	230,000	1,285,000	80,000	450,000
Total	$350,000	$1,700,000	$107,000	$550,000
Routine Reorders	24,000	7,000	$120,000	$ 35,000
Out-of-Pocket Costs			$227,000	$585,000
Annual Shortages	4,000	8.4	$20,000,000	$ 42,000
Total Costs			$20,227,000	$627,000

Table 7.

the economical system. The amount of Safety and Pipeline Stock does not change for the "67-1" system with a change in the expected shortage cost, because the "67-1" formulas make no adjustment for shortage costs. Hence, the average value of stock-on-hand-or-due-in to the base, with a 30-day pipeline, is still $350,000 for the "67-1" system and has an annual keeping cost of about $110,000 a year. The proposed policies do take shortage cost into account. With a shortage cost of $5000, instead of $50, the proposed policies increase the Safety and Pipeline Stocks by an amount large enough to reduce the expected annual number of shortages from 840 to 8.40. Such a reduction in shortages requires a large increase in base inventories of Category II and III parts: $1,700,000 with a 5000-dollar shortage cost versus $350,000 under the theoretical "67-1" system. Just as there is no change in the Operating Stocks, so there is no change in the number of routine reorders--since shortage cost does not enter the equation for Q. However, compared with the "67-1" system, the relative cost advantage of the economical policies is even greater with a 5000-dollar shortage cost than it is with a 50-dollar shortage cost. This is as one would expect, since it explicitly adjusts to the shortage cost.

For $1.4 million in additional inventories, expected shortages are cut from 4000 a year, under "67-1," to less than 10. If there is a 10 percent chance that a shortage will cause an AOCP and that on the average each AOCP will last four days, 4000 shortages implies 1600 AOCP days, (.10)(4000)(4)=1600. In terms of the number of aircraft available for operations, a reduction in expected shortages from 4000 to 8.4 would be equivalent to adding 4.4 aircraft to the base. This can be thought of roughly as being equivalent to making the administrative support aircraft operationally available. This additional effectiveness is bought for only $1.4 million, or

$\frac{\$1,400,000}{4.4}$ = \$320,000 per aircraft. This provides a clue as to the upper limit on the shortage cost. If, in the judgment of the managers responsible for operations and logistics support, this is too high a price to pay for an additional operational aircraft, the shortage cost (under the assumptions stated above in this paragraph) should be less than \$5000. It is important to repeat two things: first, these calculations are presented solely for purposes of illustration, and, second, the Air Force does not now in fact follow "67-1" policies blindly, so the differences between the proposed policies and present practice might well be smaller than these calculations indicate, but they would certainly be in the direction indicated.[1]

Tables 8 and 9 show the effect of a routine pipeline time of 4 days instead of 30 days. Table 8 is similar to Table 6, except that the pipeline is reduced from 30 days to 4 days. Under both sets of policies, the value of stock required is reduced: Under the "67-1" system it is reduced from about \$350,000 to only \$200,000; under the economical policy, it is reduced from \$660,000 to about \$350,000, a 40-to-50 percent reduction in each case. The Operating Stocks are unchanged by this change in pipeline; the number of routine reorders a year are still 24,000 for "67-1" and 7000 for the economical system. With the 50-dollar shortage cost and the 4-day pipeline, the "67-1" policies (if the 15-day safety level were maintained with the 4-day pipeline) would show slightly fewer shortages than the proposed system.[2] Because of the large reduction in the number of routine

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[1] Base-Depot Model Studies (The RAND Corporation Research Memorandum RM-1803, 1 January 1957), indicates that bases, in fact, hold more stock than the "67-1" system calls for; hence, adoption of the proposed policies would result in a smaller increase in effectiveness than is shown in the tables but would require a smaller addition to base inventories.

[2] The character of the shortage is drastically changed; however, under "67-1" most of the shortages of items carried in stock would be for Category III items; under the proposed policies almost all would be for Category II items.

EFFECTIVENESS AND COST UNDER "67-1" AND
THE PROPOSED POLICIES - ILLUSTRATIVE COMPARISON
- BASE LEVEL -

Shortage cost = \$50.00 Pipeline time = 4 days

	Quantity		Annual cost	
	"67-1"	Proposed	"67-1"	Proposed
Average Operating Stocks				
Cat III	\$ 20,000	\$ 70,000	\$ 4,000	\$ 14,000
Cat II	80,000	230,000	16,000	46,000
Safety and Pipeline Stocks				
Cat III	5,000	10,000	1,750	3,500
Cat II	95,000	40,000	33,250	14,000
Total	\$200,000	\$350,000	\$ 55,000	\$ 77,500
Routine Reorders	24,000	7,000	\$120,000	\$ 35,000
Out-of-Pocket Costs			\$175,000	\$112,500
Annual Shortages	820	840	\$ 41,000	\$ 42,000
Total Costs			\$216,000	\$154,500

Table 8.

EFFECTIVENESS AND COST UNDER "67-1" AND
THE PROPOSED POLICIES - ILLUSTRATIVE COMPARISON
- BASE LEVEL -

Shortage cost = $5000.00 Pipeline time = 4 days

	Quantity		Annual cost	
	"67-1"	Proposed	"67-1"	Proposed
Average Operating Stocks				
Cat III	$ 20,000	$ 70,000	$ 4,000	$ 14,000
Cat II	80,000	230,000	16,000	46,000
Safety and Pipeline Stocks				
Cat III	5,000	50,000	1,750	17,500
Cat II	95,000	660,000	33,250	231,000
Total	$200,000	$1,010,000	$ 55,000	$308,500
Routine Reorders	24,000	7,000	$120,000	$ 35,000
Out-of-Pocket Costs			$175,000	$343,500
Annual Shortages	820	8.4	$4,100,000	$ 42,000
Total Costs			$4,275,000	$385,500

Table 9.

reorders, however, the cost of operating under the proposed system is still about one-third less than the cost of operating under the "67-1" system. It is desirable, under either the "67-1" system or the economical policies, to pay something to reduce the pipeline time. It would be easier to reduce the pipeline times with the proposed policies in use than with the "67-1" rules because the former call for a much smaller number of requisitions.

With a 5000-dollar shortage cost and a 4-day pipeline, shown in Table 9, the "67-1" policy would again have lower costs than with a 30-day pipeline, but, since the proposed policies take shortage costs into account and reduce shortages to an expected 8.4 per year, they would allow savings of over $3.8 million a year, or over 90 per cent. Again, these savings can be thought of as being equivalent to increasing the capability of the base.

D. Summary

This chapter has developed the approximation formulas for economical stockage: Given the decision to stock an item at a base, reorder costs are balanced against holding costs to find the economical Order Quantity for any given line item; and the expected gains from the shortages avoided by having an added unit of Safety and Pipeline Stock are compared with the cost of keeping that unit to determine the Reorder Point. The types of stock levels which result under a long and stable base program from the applicatinn of these principles have been examined, and the theoretical effectiveness and costs of operating under the proposed policies have been compared with those of the "67-1" policy under stable conditions. Application of the proposed policies for Cost Category II and III items would increase base supply effectiveness markedly. Such an application would apparently require an increase in the value of base inventories in these Cost Categories,[1] but once it was installed and working it would also greatly reduce the workload on base supply and on supply personnel at depots or storage sites by reducing the number and frequency of both routine and priority requisitions.

The next chapter will examine how the proposed system operates when account is taken of some of the dynamic aspects of base operation.

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[1]As is developed in Chapter VI, it is not clear that there would be a net increase in the total value of system stocks of Categories II and III.

III. DYNAMIC ADJUSTMENTS TO BASE LEVELS

The preceding Chapter assumed first that base programs (i.e., expected mean demands[1] for the items at a base) would be stable for a long time into the future. But, in fact, weapons phase into and out of the Air Force and individual bases.[2] A particular model or series of aircraft may be assigned to a base for only two or three years (or even less) and there are always some bases which are nearing the phase-out of some weapon.

Before implementing the proposed policies, it is necessary to look into the effect that the phasing-in and -out of a weapon system may have upon the proper levels of base stocks for parts peculiar to that weapon.[3] That is the subject of this Chapter. Specifically, it will outline how quantities for initial-support tables are found and how an approaching phase-out of a base, or the short life of a program at a base, will alter somewhat the economical Reorder Points and Order Quantities.

A. Quantities for Initial Base Support

In this section it will be assumed that bases convert to new weapons at once -- or over very short spans of time. This appears to be realistic, and for the lower-cost parts moderate deviations from this assumption will cause only inconsequential over-stockage in the early months.[4]

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[1]Of course, the erratic deviations from the expected mean demand were considered.

[2]Stockfisch, J. A., Logistics Support During Phase-In of the F-102,(U) The RAND Corporation, Research Memorandum RM-2166, May 2, 1958, p.l. (Secret)

[3]For parts common to many weapons, the program for a particular weapon has little effect. Parts used on only two, or a few, weapons constitute a borderline case requiring further examination.

[4]For the depot, of course, the corresponding assumption cannot be made; in Chapter IV a method of taking account of the fact that demand increases

To stock the first base the best available estimate of mean demand for each item should be used and the base be stocked up to its Stock Control Level for each. The equations or the tables should be used in determining the levels. As will be discussed in the next section, some modification of the formulation in the preceding chapter may be called for for very low-demand, low-price parts.

The problem of how to get a usable estimate of the demand rates for individual line items is a serious one; and it is beyond the scope of this Research Memorandum. When the initial support tables for the first base have to be computed, only sketchy information is available from limited test experience, engineering estimates and experience with similar parts in other applications. Such estimates are subject to wide error. For low-cost items it is desirable to keep the probability of stockouts very small, but for high-cost items it is appropriate to run a greater risk of shortage. Therefore, in determining the initial-support tables for the first base, judgment should be applied to whatever engineering or other estimates are available, "leaning toward the high side" for the cheapest parts and, if anything, tending to underestimate demand for the more expensive items.

As a weapon program grows, after the activiation of the first units, every effort should be made to adjust the initial demand estimate on the basis of actual experience. As subsequent units or bases convert to the new weapon, their initial support tables should be computed, taking account

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during the early stages is explained. In the event that at any base the major weapon is expected to phase in slowly, or to be slow in building up to its normal level of activity, that method could be used for base stocking during the phase-in. For most of the Category III parts, however, such refinement would probably be unnecessary.

of the demand experience at the first few bases converted. The errors in the first demand estimates can be corrected in time to influence provisioning expenditures only if the procurement leadtime is considerably shorter than the period of phase-in of the weapon. If the leadtime is short enough to permit adjustments in procurement to be made on the basis of improved information, moderate over- or under-estimates of demand early in the phase-in will not be very serious.

B. <u>The Phase-Out or Short Program</u>

The formulation (Equation 2) for the Reorder Point takes account of the dynamic aspects of weapon phasing, because terminal obsolescence is explicitly included in the keeping cost.[1]

On the other hand, Equation 1, for the Order Quantity, does not take account of the dynamics, since it is based on the assumption that the base program will last indefinitely. This assumption is satisfactory whenever the base can be expected to order the item several times in the future, in which case no change is required. It is not accurate, however, if the base can be expected to order the item only once or twice during the remainder of the program.

For the lowest-price low-demand parts, the economical initial stock may be so large that no reorder is expected during the life of the program.[2] Thus, for these items the initial stockage should take account of the length of the program. Further, as the end of the program approaches, the routine reorders for more and more parts will be such as to cover, or nearly cover,

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[1]Appendix I, D.

[2]See, for example, the upper left cells in Table 3.

the needs through the expected remainder of the program. In this case, too, the program life should be considered in determining the appropriate Order Quantity.

The consideration of program life has two effects: First, it is desirable to reduce some Order Quantities to avoid excessive stockage for the (remaining) short program. Second, it is desirable in certain other cases to increase the Order Quantity to reduce the chance that additional orders will need to be placed during the remainder of the program.

These points can best be explained with reference to Table 10. In the upper left portion of the table, there are many Order Quantities which equal several years' expected consumption. For example, the third cell in the second column calls for an Order Quantity of from 5 to 14 years' supply. For a one- or two-year program this is excessive. On the other hand, the fifth cell in the seventh column calls for an Order Quantity approximately equal to one year's supply. It may be economical, if an order is needed one year from the end of the program, to order a few more than the 46 units shown so as to reduce the risk of incurring an additional reorder cost, still nearer to the end of the program. Thus, in some cases, as the end of a program approaches, the Order Quantity should be reduced to avoid excessive stockage and excessive terminal obsolescence, and in some other cases increased to avoid the risk of excessive reorders.

Looking at the matter a little more formally: Since demand is typically erratic, a final shipment is the last shipment only with some degree of confidence, not with certainty. The larger the shipment, the greater the probability that it will be the last shipment, and the greater the number of units likely to be left over at the end of the program.

Table 10
Sample Table of Economical Base Order Quantities:
Long Program

Unit Price \ Annual Demand	0-2	2-6	6-12	12-20	20-30	30-42	42-56	56-72	72-90	90-110	110-132	132-156	156-182	182-210	210-240	240-272
$ .01-.12	30	56	82	108	133	159	185	211	237	262	288	314	340	365	391	417
.13-.24	21	39	57	75	94	113	131	149	167	186	204	222	240	259	277	295
.25-.49	15	28	41	54	67	80	92	105	118	131	144	157	170	183	196	208
.50-.99	11	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148
1.00-1.99	8	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104
2.00-3.99	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74
4.00-7.99	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52
8.00-15.99	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37
16.00-31.99	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26
32.00-63.99	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18
64.00-124.99	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13
125.00-249.99	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9
250.00-499.99	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7

In finding the "final" Order Quantity, essentially the same sort of problem must be faced as in setting the economical Reorder Point: each additional unit in the "final" Order Quantity yields some expected savings in future reorder costs by reducing the chance that a reorder will be needed, but each additional unit must be bought and stored at the base, probably until the end of the program.

The savings expected from each additional unit must be compared with the expected cost of adding that unit to base stocks. If savings are less than costs, the unit should not be shipped.[1]

The effects of making the computation of the "final" Order Quantity for items with various unit prices and demand rates are shown on Tables 11 and 12. These tables are all based on the cost assumptions of Table 3.

Table 10 might apply to a very long program or to common parts, e.g., commercial hardware. However, given the general uncertainty of Air Force programs, there is little advantage in using a table for more than, say, a five-year program. In that case, Table 10 should be adjusted to read as Table 11[2] which shows the long-program Order Quantities crossed out and replaced by those appropriate for a five-year program. Note that in Table 11 some of the Order Quantities have been adjusted upward and some downward.

With a one-year program, as in Table 12,[3] the adjustments apply to nearly half of the area shown on these tables. As the phase-out of the

[1] For a more detailed statement, see Appendix I.E.

[2] Table 11 has the same Order Quantities for the range of demands covered as does Table 3 which is also a five-year table.

[3] Table 12 summarizes the adjustments. Each long-program Order Quantity appears in the upper left-hand corner of a cell. "Final" Order Quantities

Table 11. Sample Table of Economical Base Order Quantities: 5-Year Program

Unit Price \ Annual Demand	0–2	2–6	6–12	12–20	20–30	30–42	42–56	56–72	72–90	90–110	110–132	132–156	156–182	182–210	210–240	240–272
$ .01–.12	~~32~~ 19	~~56~~ 44	~~82~~ 79	~~108~~ 124	133	159	185	211	237	262	288	314	340	365	391	417
.13–.24	~~21~~ 16	~~39~~ 40	~~57~~ 73	75	94	113	131	149	167	186	204	222	240	259	277	295
.25–.49	~~15~~ 13	~~28~~ 35	41	54	67	80	92	105	118	131	144	157	170	183	196	208
.50–.99	~~11~~ 10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148
1.00–1.99	~~8~~ 7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104
2.00–3.99	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74
4.00–7.99	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52
8.00–15.99	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37
16.00–31.99	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26
32.00–63.99	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18
64.00–124.99	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13
125.00–249.99	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9
250.00–499.99	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7

Table 12.
Sample Table of Economical Base Order Quantities;
1-Year Program

Unit Price \ Annual Demand	0–2	2–6	6–12	12–20	20–30	30–42	42–56	56–72	72–90	90–110	110–132	132–156	156–182	182–210	210–240	240–272
$ .01–.12	~~30~~ ~~19~~ ~~16~~ 10	~~56~~ ~~24~~ ~~32~~ 18	~~82~~ ~~72~~ ~~56~~ 28	~~108~~ ~~124~~ ~~82~~ 40	~~133~~ ~~187~~ 53	~~159~~ ~~156~~ 68	~~185~~ ~~203~~ 85	~~211~~ ~~255~~ 101	~~237~~ ~~314~~ 122	~~262~~ 141	~~288~~ 172	~~314~~ 199	~~340~~ 229	~~365~~ 260	~~391~~ 294	~~417~~ 329
.13–.24	~~21~~ ~~16~~ ~~13~~ 8	~~39~~ ~~40~~ ~~29~~ 15	~~57~~ ~~73~~ ~~50~~ 25	~~75~~ ~~76~~ 36	~~94~~ ~~130~~ 48	~~113~~ ~~148~~ 63	~~131~~ 79	~~149~~ 96	~~167~~ 117	~~186~~ 140	~~204~~ 165	~~232~~ 192	~~240~~ 221	~~259~~ 252	~~277~~ 285	~~295~~ 320
.25–.49	~~15~~ ~~13~~ ~~11~~ 6	~~28~~ ~~35~~ ~~25~~ 13	~~41~~ ~~45~~ 21	~~54~~ ~~70~~ 32	~~67~~ 44	~~80~~ 58	~~92~~ 73	~~105~~ 92	~~118~~ 113	~~131~~ 135	~~144~~ 160	~~157~~ 186	~~170~~ 215	~~183~~ 245	~~196~~ 278	208
.50–.99	~~11~~ ~~10~~ ~~8~~ 4	~~20~~ ~~21~~ 10	~~29~~ ~~40~~ 18	~~38~~ 28	~~47~~ 39	~~56~~ 53	~~68~~ 67	~~75~~ 85	~~84~~ 105	~~93~~ 126	102	111	120	129	138	148
1.00–1.99	~~8~~ ~~7~~ ~~6~~ 3	~~14~~ ~~17~~ 8	~~20~~ 15	~~27~~ 24	~~33~~ 34	~~40~~ 47	~~46~~ 61	53	59	66	72	79	85	91	98	104
2.00–3.99	~~5~~ ~~4~~ 2	~~10~~ ~~13~~ 5	~~14~~ 12	~~19~~ 20	~~24~~ 29	28	33	37	42	46	51	56	60	65	69	74
4.00–7.99	~~4~~ ~~2~~ 1	~~7~~ ~~8~~ 4	~~10~~ 8	~~13~~ 15	~~17~~ 24	20	23	26	30	33	36	39	43	46	49	52
8.00–15.99	~~3~~ ~~2~~ 1	~~5~~ 2	~~7~~ 6	~~10~~ 11	12	14	16	19	21	23	26	28	30	33	35	37
16.00–31.99	~~2~~ 1	~~4~~ 1	~~5~~ 3	7	8	10	12	13	15	16	18	20	21	23	24	26
32.00–63.99	~~2~~ 1	~~3~~ 1	~~4~~ 3	5	6	7	8	9	10	12	13	14	15	16	17	18
64.00–124.99	~~1~~ 1	~~2~~ 1	3	4	4	5	6	7	8	8	9	10	11	12	12	13
125.00–249.99	1	~~2~~ 1	2	3	3	4	4	5	5	6	7	7	8	8	9	9
250.00–499.99	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7

base becomes imminent (or at that point when the base will no longer find the item useful), the "final" economical Order Quantity ultimately falls to one.

As mentioned above, the computation for the Reorder Point in Chapter 2 took the length of the program into account through the allowance for terminal obsolescence. This allowance is specifically included, even for the stable case with a fairly long program, because an additional unit in the Safety and Pipeline Stock is put at a base primarily for protection and, in general, is not expected to be used. At the end of the program, this additional unit must either be returned to the depot or disposed of. The shorter the program remaining at the base, i.e., if the total program at the base is of short duration or if the making of a replenishment shipment is under consideration late in the base program, the more important will be the terminal-obsolescence factor as a determinant of the cost of keeping a unit of Safety and Pipeline Stock. The Reorder Point, however, remains nearly constant until shortly before the base phases out,[1] especially since the Order Quantity declines as the phase-out approaches. The Reorder Point for all items at the base will become zero shortly before the base phases out; for most of the items, in fact, just before the phase-out it

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are in the other corners: for the five-year program in the upper right; and for the one-year program in the lower right-hand corner. For example, in the cell "Annual Demand 6-12, Unit Price $.13 - .24," the long-program Order Quantity is 57 units; the "final" Order Quantities are 73 for a program of five years' duration and 25 for one year's duration.

[1]This result comes about because the Reorder Point depends, in part, on the product of the keeping cost times the Order Quantity (cf. Equation 2). During the period when the keeping cost is rising rapidly, the Order Quantity is falling; hence, their product remains fairly stable.

would become minus one, i.e., no requisition should be placed until the base has an actual need.

Thus, with the proposed method of determining base stocks, it is possible to take account of the fact that weapons (and other end items) phase into and out of the individual bases, as well as into and out of the Air Force.

In determining the initial stockage for the first bases receiving a particular weapon, only rough guesses of the demand for each part will be available for use with this or any other stockage policy. This is an unavoidable difficulty in Air Force supply. The proposed method, by taking explicit account of the fact that demand is uncertain, provides better results than a method which ignores the uncertainty of demand. Further, every practical effort should be made to take account of the demand experience at the first bases in order to establish better initial stocks at later bases. One of the chief aims and advantages of an integrated data processing system should be to make it possible to take full advantage of early actual experience in this manner. Even before there are data processing centers integrated with both bases and depots, it may be possible to accomplish a great deal through collecting more complete data on supply experience early in the life of a weapon than is the practice later in its life. The details of such a procedure are yet to be worked out.

Since the program for each base changes from time to time, there is no certainty at any given date that the particular weapon at a base will in fact remain there for the time indicated in the current program. Therefore, no effort should be made to make precise adjustments by having, for example, a different set of base tables for each possible program length

from one to six or seven years. A practical rule might be to have one set of "long-program" tables and, perhaps, two sets of "short-program" tables. The former might be based on a five-year program and the latter on a one year program and a six- or three-month program.

Lastly, it should be noted that, for a very large proportion of all line items[1] (i.e. those with very low demand rates and prices), no reorders are expected to occur at all, because the initial Order Quantity constitutes expected life-of-type stockage. In many cases, the final Order Quantities will never be used. Still, they should be computed, for otherwise too large orders would be placed in those cases where, in fact, a reorder is needed.

To summarize, it is necessary to take account of the phase-in and phase-out of individual weapons in stocking the parts peculiar to them. For parts common to several weapons this is, of course, not necessary. Since weapons phase into each base abruptly, no gradual build-up of stocks during the phase-in is desirable. The chief problem at first is to improve the estimate of mean demand.

During the phase-out, or for short programs, it is desirable at each base to take account of the approaching termination of the program. In Equation 2 this is taken care of in computing R, because terminal obsolescence is included in the keeping cost. For computing Q, a new formulation is required (Equation I.36) which explicitly introduces the program life. For purposes of base-level applications in a manual or EAM data processing system, it is appropriate to have two or three sets of tables: one for a long program of, say, five years; one for one year and perhaps one for six or

[1] See Appendix II.

three months of remaining program life. Such tables would show the Reorder Point and the Stock Control Level for each of the program lives specified.

IV. DEPOT OR STORAGE-SITE STOCKAGE

The methods developed so far seek to determine how much stock to put at bases, when to reorder and how much to reorder at a time. Policies for a stable case, in which the usefulness of the item is expected to continue for a long period into the future, and for the dynamic situations of phase-in and phase-out have been discussed. Next comes the problem of depot[1] or storage-site stockage. To set the depot levels, essentially the same formulations as developed in Chapters II and III are applied. But there are some important modifications. This chapter will describe these modifications in general, and then treat the peculiarities of computing the depot Order Quantity during the phase-in and phase-out of a program, and the Reorder Point.

One might legitimately ask: Why stock depots at all; why not locate all stocks at base level? This Memorandum will not explore the pros and cons of the present base-depot supply structure, but two or three pertinent remarks may be made. Basically, depots serve as buffers in the supply system to absorb some of the shocks resulting from program change, demand miscalculation and other uncertainties. They also act as pools of stock which permit taking advantage of the economies of large buys, and which protect the system during the procurement leadtime. If bases were stocked so as to provide individually the same amount of protection for the system,

[1]In this section, whenever "depot" is used, it is meant to include storage sites but to exclude the maintenance and base-support functions carried out at many depots and AMA's. The depot stocks are treated as being a single pool, i.e., no rules are set up for distributing stocks between prime and zonal depots or among storage sites.

the total system stocks would have to be considerably larger than in the case where bases can call upon depots for prompt resupply.

Further, there are some added costs in holding items at the base rather than at the depot level. Probably the greatest of these derives from the fact that with the present data processing and control, systems stocks positioned at a base are not readily and systematically available for use by other bases. With an integrated data processing system, it might be possible to hold the equivalent of depot stocks at base level and to resupply one base from another. Even in this case, unless a substantial portion of base resources were devoted to depot-like activities, there would probably persist a need for depot stockage.

In any event, one of the aims in developing the present policies is to provide rules which will be practical in providing effective and economical support, not only with an integrated electronic data-processing system, but also with the present system and, in addition, during the period of transition to the new system. With the present and at least the near-term future supply systems, it is necessary to develop rules for depot as well as base stockage.

A. The Equations

In simplest terms, the policy proposed for depot stockage is: Use the Order Quantity and Reorder Point rules developed for base stockage with the necessary modifications for application to depot conditions. Although efficient stockage of depots is based fundamentally on the equations developed in the preceding sections, there are some important differences. First, as will be developed subsequently, the dynamics of weapon life have a much more marked effect upon depot- than upon base-stockage rules. Second,

the interpretation of the parameters in the equations is somewhat different for depot purposes from that appropriate for bases.

The equations are repeated here for convenience.

(1) $$Q = \sqrt{1 + \frac{2rd}{hv}}$$, and

(2) $$\frac{sd}{Q} P_R > kv > \frac{sd}{Q} P_{R+1} ;$$

or, in words:

(2) A particular Reorder Point, R, is the economical Reorder Point if the expected savings in shortage costs (s) from the last unit in R are greater than the cost of keeping that unit, but the additional cost of keeping one more unit (the R+1st) is greater than the expected additional savings from having that unit.

For purposes of depot stockage, the parameters are interpreted as follows:

1. The appropriate r, reorder cost, is not the depot-to-base reorder cost but rather the manufacturer-to-depot procurement reorder cost, including both the Air Force administrative costs and the manufacturers' setup costs, if applicable. (The setup costs may, of course, increase markedly if and when the item goes out of production.)
2. The unit cost, v, is the same in both cases. Should the item go out of production, the unit cost may, like r, increase.
3. The holding cost, h, again expressed as percent of the unit cost, includes the same factors as the base-level holding cost, but there may be measurable differences between the holding cost at the base and at the depot level.
4. Similarly, the keeping cost, k, is qualitatively the same in both cases, but significant known differences in the base and depot values should again be exploited.

5. The leadtime, over which the probability of a stockout is required for Equation 2 is not the depot-to-base resupply time but is the procurement leadtime, consisting of the Air Force administrative leadtime and, if applicable, the manufacturers' setup time. Both will tend, like r and v, to increase when the item goes out of production.

6. The depot shortage cost, s, is also considerably different from the base shortage cost. By a depot shortage is meant the inability to meet all or any part of a base requisition.

The consequences of a depot shortage are quite different, depending on whether the bases typically stock the item in question. First, whenever there is a demand on a depot for an item not stocked at base, there is an existing or anticipated shortage at the base. In this case, a depot shortage will extend the base shortage, perhaps a great deal, and the depot will have to take expedited action. If the item is stocked at base, a depot shortage does not necessarily result in a base shortage; its only consequence is a greater probability of base shortages (and any resulting expediting action by the depot).

In computing the depot shortage cost, attribute to the depot the expected base shortage cost with shipment delayed. Subtract from this the expected shortage cost at the base, when the depot makes prompt shipment, because these shortage costs would be incurred even if there were no depot shortage. Add the cost of any expedited action the depot would undertake. That is:

Depot shortage cost	=	Expected total of base shortage costs with shipment delayed	−	Expected total of base shortage costs, even if depot makes routine shipment	+	Cost of expediting action by depot

7. Demand, d, appears in both equations. In depot stockage, the appropriate "demand" is the (expected) system consumption rate for nonreparable parts and the system condemnation rate for parts (if any) which are reparable only at the base level. However, as is discussed below, a complication is introduced for the peculiar parts by the dynamics of a weapon's life.

The depot stock levels appropriate for different kinds of parts differ significantly. For parts peculiar to a particular end item, depot stockage is influenced significantly by the dynamics of the phase-in and phase-out of that end item. However, for parts common to several weapons or other end items these dynamic aspects of particular end items can be ignored and the equations used without modification, with the terms defined as above. In general, the equations could be used in this way not only for common parts but also for any other parts which are used to support a very long stable program, for example, the B-47, the C-124, or, possibly, some of the standard radio and radar sets, for the next few years.

Similarly, the equations could be used in a straightforward fashion wherever management does not think it worthwhile to make the modifications necessary to take account of the dynamic problems.

The number of line items, then, to which the equations can be applied without modification is large and, probably, substantial economies in supply workload can be achieved by applying these policies at the depot level as well as at the base level.

B. Depot Stockage Under Dynamic Conditions

Depot stockage rules for peculiar parts should reflect the dynamics of the program being supported. As mentioned in the preceding section, the values of r and v and of the pipeline time tend to increase substantially and abruptly as the individual items go out of production. Also, the demand rate to be used in computing depot levels will change from year to year as the particular program grows or wanes. This can, of course, give rise to some rather complicated problems in cases where the Order Quantity would provide several years' support. Although these problems would require elaborate computations for a theoretically ideal solution, it is necessary to develop solutions for the less expensive items which provide reasonably good answers without excessive expenditure of dollars or management time per line item. Some rules are presented and described below which appear to meet these criteria. Further research and, particularly, further development in implementing them are desirable and will almost certainly result in improvement; as of now they seem to be practical and a considerable improvement over present methods.

At the beginning of the program, the depot should stock up to its Stock Control Level those parts for which it is expected that the bases will make demands upon the depot early in the program. For most of the line items, however, the initial base stocks will provide several years' support.[1] Hence, for the first year or more, the basic requirement for depot stocks will be to take advantage of bulk buys in some cases only and, more important, to provide protection during the procurement leadtime and to provide protection against gross errors in estimating demand. Therefore, logically, the depots need stock only up to their Reorder Point.[2] If direct shipment to

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[1]See Table 3 and Appendix III.

[2]This statement will be expanded and modified below.

the early bases is economical, relatively little depot stockage is required at first for the low-cost, low-demand items. In one sense, then, the first problem is to determine the depot Reorder Point, but since that depends on the Order Quantity, the latter will be discussed first.

The following discussion is rather involved because there are several contingencies which must be covered as alternative situations. Readers who do not wish to follow through the detail of the next two sections, may skip to Section E where the main points are summarized.

C. Depot Order Quantities

1. The General Procedure: In the case of common parts there are relatively few problems of stocking depots according to the principles developed in this study, and the procedures outlined in Section A above are appropriate. But for parts peculiar to particular weapons or other end items the holding cost, reorder cost and particularly the annual demand rate may change very markedly during the program. These changes give rise to some difficulties.

Since the computed Order Quantity depends upon the annual demand rate and since the annual demand rate can be expected to change during the operating history of the end item, there is a problem of determining the appropriate demand rate to use in computing Q for each phase in the program. Once the rate has been determined, an initial estimate of the Order Quantity is to be computed. That estimate may subsequently have to be modified through a final Order Quantity computation which takes account of the remaining life of the program and a computation to take account of the fact that when the individual part goes out of production, both r and v tend to rise.

2. Determining the Appropriate Demand Rate: Before even the "initial estimate" can be calculated, however, the "appropriate demand rate" must be selected. Theoretically this is a complex problem, but it can be resolved practically by applying some reasonable if arbitrary rule.[1]

Although system demand for most parts appears to follow the general rise and fall of the programs to which they apply, demand can be predicted only very crudely at the beginning of a program. Since the value of Q depends upon the demand rate and since demand tends to rise as the program increases, if the Order Quantity were computed on the basis of the demand expected very early, say, during the first year of a program, the resulting Q might be too small. As a result, the reorder cost would be incurred too many times and the depot Reorder Point would be too large. On the other hand, determining Q on the basis of too late (and hence too large) a demand rate, would cause errors in the other direction. The first step is to try to understand the problem and its consequences.

For the lowest value, lowest demand parts the Order Quantity is several times the annual demand rate used in computing it. Therefore, there can be inconsistency in the expected operating period on the basis of the demand rate used in computing Q and the operating period realized, simply because the expected annual demand rate changes as the program grows (or declines). In addition there are, of course, the uncertainties associated with the variance of demand around the mean.

During the phase-in, the annual demand rate is expected to rise from year to year. If for computing Q demand as of some date very early in the program is used, Q will be smaller, and (as is discussed in the next section)

[1]It should be emphasized that more research is needed to determine very good rules.

so will R in some cases, than they would be if the expected demand later in the phase-in were used. The significance of such under- or over-estimates depends very much upon whether the depot is stocked up to its Stock Control Level or only up to its Reorder Point (plus, perhaps, some additional protection). During the phase-out, an Order Quantity computed on the basis of one particular year's demand rate might be far too great on the basis of the demands expected in the succeeding years. Without a rather elaborate iterative process it would be only a coincidence if the "correct" demand rate, i.e., the expected demand at the "correct" time in the program, were chosen for computing Q.

The discussion of the effects of using a particular demand rate is complex out of proportion to its importance and certainly out of proportion to the simple rules which seem adequate for use in early implementation. The complexity arises from the fact that many alternative situations must be covered in the discussion, and unless they are mentioned it is impossible to see either the need for considering the problem at all or the reasonableness of the rules suggested. In the paragraphs which follow the effects of underestimating demand -- which is a potential problem during the phase-in -- will be discussed. Then the problem of overestimating demand, which is more likely to occur in the phase-out, is covered.

a. Let us look at errors of underestimation of demand[1]- first,

[1]For purposes of this immediate discussion demand will be said to be "underestimated" when the annual demand rate used in computing levels is less than the demand rate is expected to be at the date when the Operating Stocks (so computed) would be used up if during the operating period the average annual demand in fact equalled the demand rate used in the computation. Demand will be said to be "overestimated" in the opposite case, namely, when the demand rate is greater than the demand rate is expected to be at the date at which the Operating Stocks would be used up if during the operating period the average annual demand in fact equalled the demand rate used in the computation.

in the case where at the beginning of the program[1] depots are to be stocked up to their Stock Control Level. Suppose that the expected demand rate for the mid-point of the first year is used for determining Q; suppose also that on that basis a depot Operating Stock apparently adequate for two years is obtained.[2] Then, as will be discussed below, the Reorder Point would be set on the basis of the demand expected at the end of two years. However, since the program is growing, the total demand through the two years that the Operating Stocks are supposed to cover will in fact be more than that used in computing Q, and stocks will run out before the predicted two years have elapsed. Then, at the time when it will actually be needed, the Safety and Pipeline Stock will be too large, having been computed on the basis of demand at the end of two years; hence, there will be too large an investment in the Safety and Pipeline Stocks. Further, too many depot reorders will have to be placed and excessive reorder costs incurred.

The consequences of these inaccuracies are not as serious as might appear from the complexity of tracing them out. The case where the depot is to stock up to its Stock Control Level very early in the program and

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[1]If the depot were stocked initially up to its Stock Control Level only for those items for which the bases were expected to hit their Reorder Points within three months, this would cover those items with an expected base-level annual demand of $800 or more. If the cut-off were six months, this would cover items with an annual base-level demand of $200 or more. Assuming 10 bases in the system these represent items with an annual mature program expected demand of $8,000 and $2,000 respectively. An examination of the tables in Appendix III indicates that the great majority of the line items have base-level annual demands of less than $200 and, of course, even more have less than $800 per year.

[2]With a reorder cost of $200 and a holding cost of 20 per cent, this would result for a part with an expected system demand of 100 per year and a price of $5 or for all parts with annual value of system demand of $500.

where the demand rate used in computing Q is "too small" can arise only when the computed operating period is substantially longer than that for which the Operating Stocks will in fact last. Long operating periods, say of more than a year, occur only in the cases of parts for which the value of issues (demand times unit cost) is low.[1] On the other hand, the depot should not normally stock up to its full Stock Control Level except for items for which an early demand from the bases can be reasonably expected, that is, for those items for which the base operating period is small, say less than six months. These are items for which the annual value of base-level demand is expected to be relatively large, more than $200 per base under the assumptions of Table 3. The band of line items for which this problem could arise, therefore, is very small. Further, although reorder costs are too high when the Order Quantity is too low, there is a partially offsetting reduction in holding costs.

Since during the very early part of the program the estimates of demand are very inaccurate, there is actually some advantage in keeping depot stocks low in case demand per weapon-month, or per flying hour, has been overestimated. On the other hand, there is little risk of serious trouble in the event that demand has been underestimated, because the production leadtime should be relatively short, and, as will be discussed below, there are ways of providing additional depot protection during the phase-in.

An examination of several arithmetic examples indicates that, even leaving aside the possibility of errors in predicting the mean demand, erring

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[1]With a reorder cost of $200 and a holding cost of 20 percent, the depot operating period exceeds two years only in cases where the annual value of system demand is less than $500.

on the side of using "too early" a demand estimate leads to very small extra costs over the optimal policies. The additional advantage of having an opportunity to obtain some demand experience before the major buys are made means that, on balance, it is probably wise to compute the Order Quantity on the basis of demand early in the program and then to recompute it each time the Reorder Point is reached, or (with an integrated data-processing system) whenever it has been determined that the demand is markedly different from the original prediction. It seems reasonable, therefore, to take initially the annual demand as of some date early in the program, such as the production leadtime plus one year from the date of the computation for parts costing less than $10 and plus six months for parts costing more than that.

In the case where the depot is not to be stocked up to its Stock Control Level at the beginning of the program, underestimating demand in this sense can be avoided altogether. The demand expected at the time when bases begin to make demands upon the depot should be used.

b. If at some point in the program, the operating period is long enough to extend beyond the peak of the program, the <u>danger of overstating demand</u> becomes more important. Account should be taken of the fact that the demand per calendar-month will decrease, even if the demand per aircraft- or missile-month or per flying-hour should remain constant. To illustrate the point, suppose that the depot hits its Reorder Point for some particular item during the peak year of the program. Suppose further that the price of the item and the other factors in Equation 1 are such that Q equals three years' supply[1] at the peak demand rate. If the program were to taper

[1] Assuming a depot reorder cost of $200 and a holding cost of 20 per cent, such an operating period of two years or more would occur for items having an annual value of demand of $500 system-wide.

off fast from the peak, if it were, for example, to be reduced by one-half the peak strength in each of the succeeding two years, the Operating Stocks computed in this way would be clearly excessive. The same situation could, of course, arise at other points in the program and for the very cheap, very low-demand parts it could, in fact, arise before the peak period.

This problem is managed by introducing a "final buy" rule, which is discussed below.

3. <u>Adjusting for the Rise in Price and Reorder Cost as the End of Production Occurs</u>: It is desirable that the Air Force obtain information from the contractor about the pending termination of production of many items in time for the Air Force to buy at the "present" price and "present" reorder cost. With this information, the Order Quantity should be recomputed one procurement leadtime before the termination of production. The problem is to determine whether to buy "now" to avoid (or, more precisely, to increase the chance of avoiding) the higher costs of buying later. This implies balancing the risk of having to buy later, incurring the higher reorder cost and (possibly higher) price, against the cost of holding an additional unit in the Operating Stocks from now until the time when the new order would have to be placed.

The procedure is as follows: Using the pre-termination price and the post-termination reorder cost, compute Q. If this value of Q equals or exceeds two-thirds the total expected demand for the rest of the program, compute a "final buy" quantity, also using the post-termination values of v and r, and use it. If this Order Quantity is less than two-thirds the total demand for the remainder of the program, use it as the Order Quantity. For most items it appears that the final buy will in fact be made at or before the termination of production of the items in question.

It may not always be possible to get the information necessary to determine the time of termination of all items, or the relevant cost factors when they do in fact go out of production. In many cases, it is not necessary to have such information, because relatively early in the program a life-of-type buy will be called for, even on the basis of the lower in-production reorder cost. In cases where the information would be desirable but is not available, the appropriate manager must simply exercise judgment as to whether to continue using whatever the current unit cost and reorder cost are or to introduce some arbitrary rule, such as recomputing one year before the termination of the production of the end item on the assumption that reorder costs and prices will rise by some estimated amount. Certainly, for Category II parts at least, such termination information should be required in any integrated data processing system.

4. The Final Buy Quantity: If at any time during the life of the program being supported the Order Quantity exceeds two-thirds the expected total demand for the remainder of the program, a Final Buy computation should be made. The occasion for this has been discussed briefly in the preceding subsections. There are some important differences between the computation of the base Final Order Quantity (Q_F) and the depot Final Buy Quantity (Q_B).

First, in determining whether to make the Final Buy computation, the Order Quantity should be compared not with the product of the current annual demand rate times the years remaining in the program, as in the base case; but it should be compared with the sum of the expected annual demands throughout the remainder of the program, taking account of the fact that

the expected annual demand changes during the rise and fall of the program.[1] This, of course, arises from the difference between phasing a weapon into and out of the system and converting individual bases.

As will be developed in the next section, the demand rate to be used in computing R is based upon the expected system demand rate at the time when the depot Safety and Pipeline Stocks are expected to be used. The depot Final Buy Quantity provides essentially remaining life-of-type Operating Stocks and consequently, the depot Reorder Point is not expected to be reached at all.[2] To put it another way, the depot Reorder Point is not expected to be reached until the system demand has fallen to zero, hence the Reorder Point would tend to be zero.[3] This introduces an entirely new element into the Final Order Quantity computation.

The Final Order Quantity for the base is derived from equation (I.36) of Appendix I,

$$(I.36) \qquad r'P_{Q_{F-1}} > h' > r'P_{Q_F}$$

where: Q_F = the economical Final Order Quantity;

---

[1]Whereas the relevant comparison for the base is $Q \gtrless \frac{2}{3}\,dn$, where n = the number of years remaining in the program; the relevant comparison for the depot case is

$$Q \gtrless \frac{2}{3} \sum_{t}^{n} d.$$

[2]There is, of course, some probability that the depot Operating Stocks will be used up and the Reorder Point reached. The probability is greater the higher the cost of the item, all else equal.

[3]There appears to be no particularly useful way of treating the expected mean demand as zero and computing a Reorder Point to take account of the positive deviations from such a mean.

h' = the discounted cost of holding one unit from now until the end of the program;

r' = the discounted cost of a shipment of one unit at the end of the program;

P_{Q_F} = the probability of Q_F or more demands between now and the end of the program.

Three modifications in this are required for the computation of the Final Buy Quantity. The depot would logically have no Safety and Pipeline Stocks if it were holding life-of-type Operating Stocks. Hence, any demand upon the depot greater than Q_B would, if no stocks could be shifted from other tasks, cause a depot shortage as well as a new procurement. Further, unless some other action can be taken, the shortage will last throughout the procurement leadtime, which near the end of the program will typically be relatively long since peculiar items will by that time normally be out of production. Consequently, (I.36) should be rewritten as[1]

$$(4) \qquad (s' + r')P_{Q_{B-1}} > h' > (s' + r')P_{Q_B}$$

where: s' = the discounted depot shortage cost taking account of the fact that the shortage would last throughout the post-termination leadtime.

Q_B = the Final Buy Quantity

P_{Q_B} = the probability of Q_B or more demands between now and the end of the program.

In interpreting equation (4) the reorder cost should always be taken as the post-termination reorder cost since Q_B would normally be exhausted only very late in the program. The unit cost, used in determining h' (Appendix I), should be the pre-termination price if the Final Buy is in fact

[1] A small error is introduced in this formulation since the shortage cost would be incurred only if the demands in the rest of the program exceed Q_B while the reorder costs are incurred if demand exceeds Q_{B-1}.

to be made before the termination of production. Otherwise, it should, of course, be the post-termination price.

D. Depot Reorder Points

Now that the determination of the depot Order Quantity under various conditions has been described, we can discuss the depot Reorder Point computation. The general policy to be applied in the determination of depot Reorder Points is to use the equation (2) used for the determination of base Reorder Points, with those modifications necessary to make it fit the depot problem. The differences in the interpretation of the factors in the equation have been outlined above. There remain only three central problems, all related to the dynamics of the program of the end items being supported. They are: (1) the determination of what demand rate to use in the computation of R; (2) the effect of the termination of production of the item upon R; and (3) the appropriate R to use in the event that, at any point in the program, bases have life-of-type Operating Stocks.

1. Determining the Appropriate Demand Rate: As in the case of computing Q, there is a problem of deciding what demand rate to use in computing R. The size of R depends upon the demand rate used; in a growing program, it will be smaller if demand expected in the first year should be used than if demand for the third or fourth year were used. Later in the program, demand rate at the peak of the program would call for a larger R than would demand near the end of the phase-out. Of course, the demand rate to use is the expected demand rate at the time when the depot is expected to reach its Reorder Point.

a. For those parts where the depot will stock up to the Stock Control Level at the beginning of the program, the important date is that date at which the depot Order Quantity is expected to be exhausted, as that is the time when the Safety and Pipeline Stocks probably will be needed. In this case the depot operating period determines the appropriate system demand rate for determining the depot Reorder Point. In other words, using the program information for the date at which the depot Order Quantity is expected to be consumed, estimate the system demand rate and from it derive the depot Reorder Point.

For those parts which the depot will not stock up to its Stock Control Level early in the phase-in, the relevant date in determining R is that at which demands on the depot may first be expected from the bases. Therefore, taking the program level expected at the time when the first base to be phased in[1] is expected to reach its Reorder Point, estimate system demand for the item in question. Compute the depot Reorder Point on the basis of that demand estimate. In this way, the depot Reorder Point is made consistent with the expected system demand at the time when orders will probably be placed against the depot.

For some items this procedure will result in depot Reorder Points based upon expected demand rates late in the program.[2] Since, at the time of initial stockage, demand late in the program is most uncertain, this proposal must be examined more closely. Taking two extreme situations:

[1]If for any reason some operating base other than the first is expected to use up its Operating Stocks earlier than the first, the relevant date is, of course, the date at which that base is expected to hit its Reorder Point. This does not appear to be a case of major practical importance.

[2]The natural tendency to provide ample base stockage for a new weapon will cause this to be true in more cases than are indicated in the Tables or in Equation (1), because this tendency has the same effect as an overestimate of base demand rates.

(1) Suppose, first, that the first base is expected to require resupply at the peak of the program. Then, the depot Reorder Point would be computed on the basis of the highest system demand expected for the entire program. At first glance this may seem to be a source of serious overstockage. More careful examination indicates that, unless demand per program element is badly overestimated in determining R, this is not apt to be the case for three reasons. First this situation typically will arise only for the low-value, low-demand items, so any overstockage is not likely to be serious in dollar terms. Second, for such items the Reorder Point usually constitutes only a small fraction of the Stock Control Level. Third, if there is a demand against the depot at that time, part of the (large) Safety and Pipeline Stocks will be used to fill the base order or orders and a new Order Quantity and Reorder Point will be computed.

The real problems arise only if demand per program element has been seriously overestimated. During the phase-in, it should be possible to adjust the Reorder Point to reflect the demand actually experienced during the early part of the phase-in. With the present manual data processing system this could be done only imperfectly but it is not impossible. It appears to be highly desirable to increase the data-gathering at the first bases issuing any new weapon and, absent even that, it might be possible to catch the worst errors from the Stock Balance and Consumption Reports.

For a program phasing in over three years, the occurrence of the first demand against the depot at the program peak could arise (under the assumptions of Table 3) for items with an annual value of issues of $5.50. This group of parts accounts for only a small fraction of the dollars invested in inventory but for a modest proportion of the line items.

(2) Take as the second extreme the case where bases are initially stocked with life-of-type stocks; this procedure (of setting R by rising the demand expected when a base first orders on the depot) would, of course, result logically in zero depot Reorder Points -- the bases would be expected to reach their Reorder Points when system demand should be zero.

Early in the program, when demand estimates are very unreliable and when there are few bases from which the part might be drawn if demand were far higher than estimated, it is clearly risky to operate with no depot stocks. On the other hand, there is at least as good a chance that future demands will have been overestimated as underestimated, so there is some advantage in limiting depot stocks. There are many ways of meeting this situation (which, of course, will arise only in the case of very inexpensive items[1]). One simple way is to bring into the depot at the beginning of the program the full amount of the Stock Control Level for one or two of the bases which will receive the weapon late in the phase-in. In this way, the system has some degree of protection should the demands at the first few bases prove so much greater than expected that they must be resupplied early. At the same time, should the demand prediction turn out to be roughly correct or overestimated, the depot can ship out to the later bases the stocks brought in for them and no extra procurement need be incurred. Further, if information that demand has been greatly overestimated is made available to the appropriate manager early in the phase-in, he may be able to curtail the

[1] Cf. Table 3.

orders outstanding and use the depot stocks, which were originally earmarked for, say, the last two bases to be phased in, to support several bases earlier in the phase-in.

There remains the problem of determining the Reorder Point later in the program. Once demands are made against the depot by the bases, the depot should be stocked up to the Stock Control Level, and the earlier description of how the depot Reorder Point should be determined, using expected system demand at the end of the depot operating period, applies except that the termination of production should be taken into account.

2. Termination of Production: Once the item goes out of production, the Reorder Point should be high enough to provide adequate protection through the longer procurement leadtime, any increase in price will have a (probably slight) offsetting tendency. The increase in reorder cost will have only an indirect effect through increasing Q, and hence slightly decreasing R. One procurement leadtime before the termination of production of the item, the Reorder Point should be computed, using the projected post-production leadtime, price and Q determined with the post-procurement values. The appropriate demand rate is the rate expected to be in effect one operating period after the termination of production. R and Q computed in this way provide the Stock Control Level for the period after the end of production. If the stocks on hand in the depot one procurement leadtime before the end of production are less than the new Reorder Point (plus expected depot issues prior to the termination of production), a buy equal to the difference between the new Stock Control Level and the expected depot assets at the end of production should be made at the in-production prices.

3. Depot R's and Base Q_F's: One contingency remains to be discussed. What should be the depot R after the initial phase-in if the bases are holding

life-of-type Operating Stocks? The logic of the argument of this chapter calls for a zero Reorder Point under these conditions and, possibly, no depot Operating Stocks either. This is too risky. Early in the program the system could be adequately protected as discussed above by the stocks held at the depot for bases phasing in late. During the phase-out, some assets could be returned to the depot to constitute a system reserve. There remains the problem of the mid-range of a program. Here it appears that the appropriate policy might be to hold back from some of the bases their full life-of-type Operating Stocks or to rely on redistribution between bases to cover individual base shortages. This is an area in which management will have to exercise judgment until a practical and reasonably rigorous solution is worked out.

E. Summary of Depot Stockage Policies

In general, the stockage rules developed in Chapters II and III for base stockage are taken over and modified to develop depot stockage rules. Some redefinition of terms is needed to make the formulations fit the depot situation, but the chief problems of developing depot rules are associated with the dynamic aspects of the growth and decline of weapon system and other end-item programs.

Because of the dominance of the dynamic aspects of depot stockage, no simple static comparison between the present and proposed policies such as was presented for the base situation in Tables 6-9 would be of value. The overall effect of applying the proposed policies at the depot level would appear to involve reduced stocks of the relatively high-cost, high-demand items, because of the weight given to the cost of holding and keeping them in the system.

Early in the program, at least, there should be a similar reduction of depot stocks of the great bulk of low-cost, low-demand items. This follows from the fact that the proposed policies explicitly take account of the fact that the bases should be stocked with up to several years' expected consumption of such items. Early in the program the depot stocks of such items should be very small and, assuming that it is feasible to have direct shipment to the bases, should remain small for many of these items during most of the phase-in.

Later in the program the depot stocks of such items would theoretically be greater than in the present system and the frequency of reordering them from the manufacturers would be correspondingly decreased. As individual items go out of production, a terminal buy is called for and its net effects as compared to the terminal buys in the present practice are impossible to determine at this point.

In order for the system to operate at maximum effectiveness, there is a real need for better information coming from the earliest operational locations of a new weapon or other end item to the managers, because much of the economy of depot stockage depends upon being able to react in a period of a few months.

The stockage of common parts at the depot levels presents no great problems because of the fact that the dynamics of a particular program are not apt to influence significantly the consumption of common items. All that is required in this case is that the equation developed for base stockage be interpreted to reflect the depot situation. Thus, the reorder cost is the procurement reorder cost, the relevant pipeline time is the procurement leadtime, the shortage cost is the cost of overcoming a shortage at the depot and the pertinent demand rate is the system consumption or condemnation rate.

For purposes of illustration, the following three tables (13-15) trace out without elaboration or qualification the major steps, questions and decisions for depot stockage of items peculiar to particular end items.

The appropriate initial depot stockage (Table 13) for an item will depend upon the depth to which the bases are initially stocked, and secondarily upon the size of the depot Order Quantity and the total expected consumption of the item during the program. If the stocks at the first bases are expected to last for a relatively long period, say in excess of six months, the depot need only stock up to its Reorder Point plus some additional protection such as the Stock Control Levels of one or two bases. If the base stocks are initially so small that the depot can expect to have demands placed against it in the first few months of the program, the depot should stock up to its Stock Control Level initially. In the unlikely event that the bases might have short operating periods but the depot Order Quantity is large relative to the expected issues throughout the program, the depot should stock only a Final Buy Quantity.

Later in the program when the base receives an order or request to ship to a base, Table 14, it should normally make the issue unless doing so will cause a shortage at the depot. If the issue carries the depot's stocks to or below the Reorder Point, the depot should compute its own Order Quantity using the system demand expected in the relatively near future. If the item can still be obtained at the pre-termination price and reorder cost, Q should be computed using those values of v and r; if not, the post-termination values should be used. Once Q is computed, a determination should be made as to whether to make a Final Buy computation. If so, that is the amount to be stocked. It should be noted that the depot Final Buy

Table 13

Initial Depot Stockage

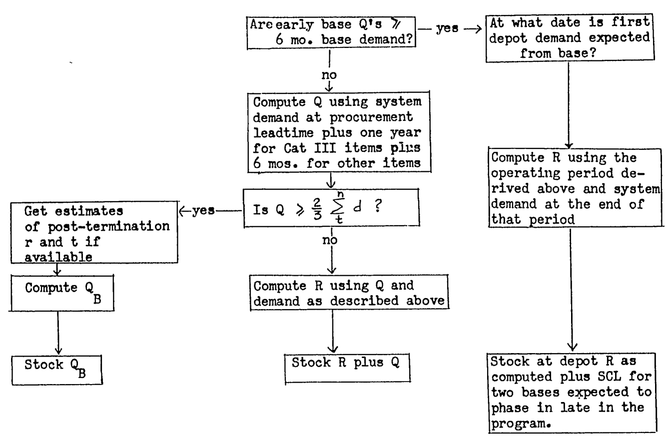
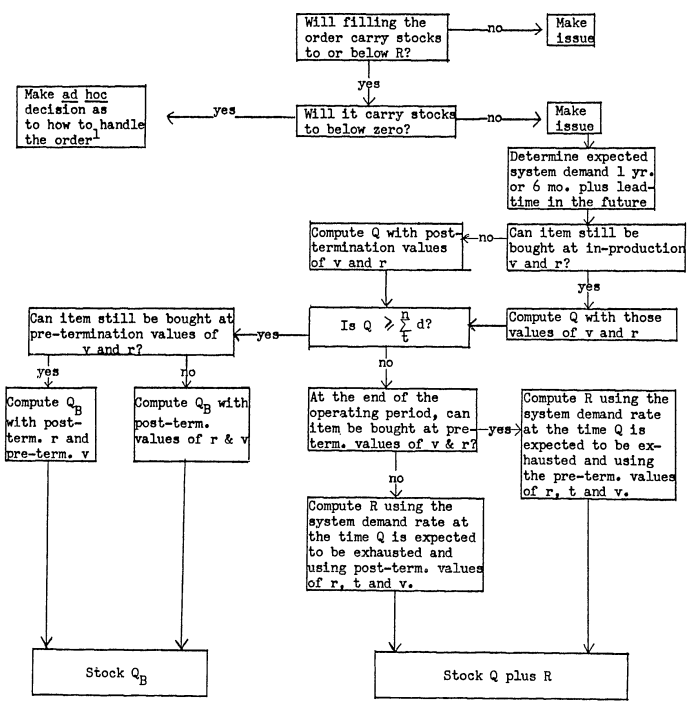


Table 14

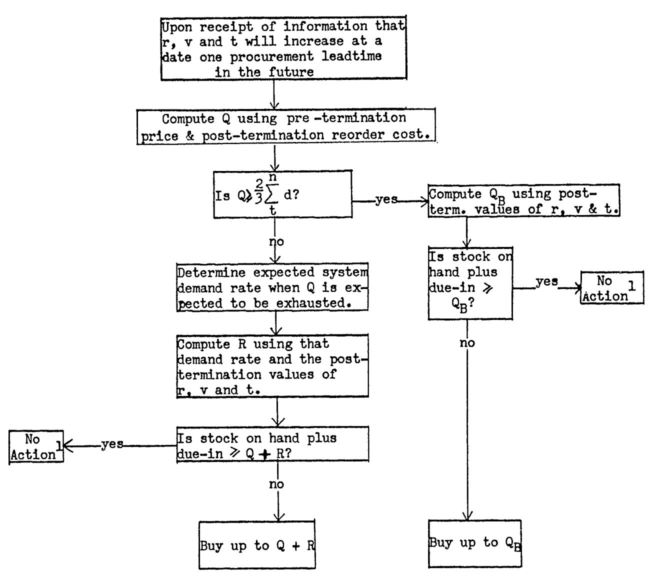
Depot Reaction to a Base Order



[1]Falls outside the scope of this Research Memorandum

Table 15

Terminal Buy



[1]Criteria for disposal are outside the scope of this Research Memorandum.

computation takes account of the risk of incurring a depot shortage. If a Final Buy is not appropriate, a new Reorder Point is to be computed with the appropriate values of r, t and v and the depot should bring its stock on hand and due-in up to the new Stock Control Level.

At least for the higher value Category II items, but if possible for all items, it is desirable to recompute the levels shortly before the item is scheduled to go out of production so that the Air Force may buy at the pre-termination prices and reorder costs. The post-termination values of r, t and v should be used in computing R and Q and, if appropriate, in computing Q_B. The depot stocks should be brought up to the sum of R and Q or to Q_B as appropriate; Table 15 illustrates the steps involved.

V. DATA REQUIREMENTS

Any inventory policy requires data if it is to operate. The purpose of this chapter is to describe the data needs of the proposed policies and to discuss the sensitivity of the policies to inaccuracies in the data. By understanding the latter a reasonable judgement can be made about the effort that should be devoted to improving the accuracy of each kind of data.

It is not intended to discuss systematically here how to make the required estimates, nor will the sources of the data in the Air Force, the information flows or the data manipulations be treated. At present, joint efforts to determine some of the parameter values are under way by personnel from RAND, from the AMC Directorates of Plans and Programs and of Supply with some assistance from Comptroller personnel. It appears that adequate data and estimates can be obtained without excessive cost or delay. The policies will operate far more effectively with an integrated electronic data-processing system than with the present predominantly manual and mechanical system. However, even with present data processing the proposed policies should result in considerably more effective and economical stockage than do the present practices.

For the purposes of applying the equations, the data requirements may be classified as follows:

1. Identifying and program information
2. Demand Data
3. Pipeline Time
4. Holding cost
5. Keeping cost
6. Reorder cost
7. Shortage cost

These are the parameters required for solving Equations 1 and 2 and for setting the "final" Order Quantity. Explicit information on demand and on pipeline times is required by the "67-1" system also. The "67-1" policy parameters carry implicit assumptions about the other variables.

A. Sensitivity

Before discussing further the kinds of data required, it is worth considering the accuracy with which the various parameters need be determined. None of them can be determined with perfect accuracy. Demand and shortage cost are particularly hard to estimate. Even unit cost, which at first glance seems very straightforward, is not, for the cost that we would like to have is the value of the item to the Air Force at the time a stockage decision is being made. The cost information available is usually the price at which the item was last purchased. Since the price may have risen or fallen since then, the last price is only an estimate of the cost of purchasing the item now. Further, for items in short supply the actual value to the Air Force may be a great deal more than the current purchase price, and for items on which disposal action is appropriate the value to the Air Force and on the market may be much less than the purchase cost new. This illustration is presented here merely to show that at best any value for the parameters will be an estimate.

Also, it is relatively easy to get good estimates of some parameters and much harder to get good ones of others. One of the common problems in applying improved management techniques is, in fact, that the costs of getting the necessary information may exceed the value of the improvements. One should not spend more effort and money in estimating any particular parameter than it is worth.

Tables 16-19 summarize sensitivity information to provide some guidance on the importance of accuracy of the different variables. Tables 16 and 18 use a 30-day pipeline time and Tables 17 and 19 a 4-day pipeline time. Tables 16 and 17 cover a moderately low demand case: an annual demand of ten units per base year. The tables are based upon the distribution of demands and prices shown in Table I of Appendix III. They show the percentage increase in cost resulting from using a wrong estimate of each parameter in setting base levels. In computing the tables values were assumed as true values for all the parameters in equations (1) and (2); then R, Q and the total system cost were determined with the approximate equations, given those values. The results and the percentage increase in cost over the exact equations appear at the top and bottom respectively of each table. R and Q were then determined, with one parameter "estimated" at twice or half its true value. The costs of using this R and Q in the face of the "true" parameter value were determined and compared with the cost realized with R and Q computed using the true value. The percentage deviation of the "erroneous" cost from the true cost is shown in each table.[1]

The upper portion of each table shows those errors which tend to increase stockage above what it should be, and the lower portions show those which tend to reduce stockage. Thus, an over-estimate of demand and an under-estimate of price are both in the upper portions. In nearly all the cases shown, errors causing over-stockage are less costly than are the corresponding errors resulting in under-stockage.

[1]Since the comparison is made using the approximate formulas in a few cases, where the error in the estimate results in levels closer to those which would have been obtained with the accurate equations, the use of an erroneous value actually decreases cost.

Table 16. Sensitivity Table (using a Five Year Program and a Correct Yearly Demand of 10)

30 Day Pipeline

The Correct Input Data	The Solution of the Approximation Formulas using Correct Input Data.		
Unit Value = v	$ .30	$ 4.80	$300.00
Cost Per Year	$ 3.74	$23.79	$245.87
Erroneous Input Data	**The Solution with Input Errors Which Cause an Increase in the Average Stock Level. % Increase in the Cost Per Year**		
Yearly Demands = 2d	8.6	8.7	9.8
Reorder Cost = 2r	6.7	-4.1	-14.6
Unit Value = ½v	7.8	-1.1	- 3.9
Shortage Cost = 2s	1.6	2.9	2.4
Erroneous Input Data	**The Solution with Input Errors Which Cause a Decrease in the Average Stock Level % Increase in the Cost per Year**		
Yearly Demands = ½d	20.1	30.6	9.8
Reorder Cost = ½r	6.7	1.3	0
Unit Value = 2v	9.1	2.1	0
Shortage Cost = ½S	2.4	2.6	9.8
Poisson Dist.	25.4	1.4	0
% Increase Over Exact Formula	.5	4.9	36.6

Note: d = 10
r = 5
S = 50

The correct distribution is a negative binomial distribution with a mean of 10 and a variance of 40.

Table 17. Sensitivity Table (Using a Five Year Program and a Correct Yearly Demand of 10)

4 Day Pipeline

The Correct Input Data	The Solution of the Approximation Formulas using Correct Input Data.		
Unit Value = v	$ .30	$ 4.80	$300.00
Cost Per Year	$ 3.06	$14.72	$ 95.00
Erroneous Input Data	**The Solution with Input Errors Which Cause an Increase in the Average Stock Level. % Increase in the Cost per Year**		
Yearly Demands = 2d	24.8	13.4	- 1.8
Reorder Cost = 2r	9.5	- 2.6	- 1.8
Unit Value = ½v	10.1	- 1.0	- 1.8
Shortage Cost = 2s	2.3	4.3	0
Erroneous Input Data	**The Solution with Input Errors Which Cause a Decrease in the Average Stock Level. % Increase in the Cost per Year**		
Yearly Demands = ½d	11.1	23.2	0
Reorder Cost = ½r	7.5	12.8	0
Unit Value = 2v	11.4	23.2	0
Shortage Cost = ½s	1.3	3.8	0
Poisson Dist.	6.2	0	0
% Increase Over Exact Formula	.3	10.1	1.8

Note: d = 10
r = 5
s = 50

The Correct Distribution is a negative binomial distribution with a mean of 10 and a variance of 40.

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Table 18. Sensitivity Table (Using a Five-Year Program and a Correct Yearly Demand of 84)

30 Day Pipeline

The Correct Input Data	The Solution of the Approximation Equations using Correct Input Data.		
Unit Value = v	$ .30	$ 4.80	$ 150.00
Cost per Year	10.56	74.35	1235.65
Erroneous Input Data	**The Solution with Input Errors Which Cause an Increase in the Average Stock Level. % Increase in the cost per Year**		
Yearly Demand = 2d	15.1	19.8	34.7
Reorder Cost = 2r	2.7	- .7	- 7.7
Unit Value = $\frac{1}{2}$v	3.5	.9	- 3.4
Shortage Cost = 2s	.7	2.3	4.3
Erroneous Input Data	**The Solution with Input Errors Which Cause a Decrease in the Average Stock Level. % Increase in the Cost per Year**		
Yearly Demand = $\frac{1}{2}$d	47.9	78.0	47.1
Reorder Cost = $\frac{1}{2}$r	2.7	5.5	4.3
Unit Value = 2v	6.5	9.2	8.6
Shortage Cost = $\frac{1}{2}$s	.9	2.4	4.1
Poisson Dist.	55.1	63.1	15.0
% Increase Over Exact Formula	0	1.0	13.7

Note: d = 84
r = 5
s = 50

The Correct distribution is negative binomial with a mean of 84 and a variance of 336.

Table 19. Sensitivity Table (Using a Five-Year Program and a Correct Yearly Demand of 84)

4 Day Pipeline

The Correct Input Data	The Solution of the Approximation Equations Using Correct Input Data.		
Unit Value = v	$ .30	$ 4.80	$150.00
Cost Per Year	8.75	47.77	518.40
Erroneous Input Data	**The Solution with Input Errors Which Cause an Increase in the Average Stock Level. % Increase in the Cost per Year**		
Yearly Demand = 2d	7.0	8.5	12.5
Reorder Cost = 2r	3.7	.2	-10.0
Unit Value = $\frac{1}{2}$v	4.1	1.2	- 7.0
Shortage Cost = 2s	.5	1.7	7.3
Erroneous Input Data	**The Solution with Input Errors Which Cause a Decrease in the Average Stock Level. % Increase in the Cost per Year**		
Yearly Demand = $\frac{1}{2}$d	17.5	25.8	49.8
Reorder Cost = $\frac{1}{2}$r	6.1	7.3	7.4
Unit Value = 2v	7.4	11.2	13.4
Shortage Cost = $\frac{1}{2}$s	1.5	1.0	9.0
Poisson Dist.	49.9	38.0	9.0
% Increase Over Exact Formula	.1	.7	14.2

Note: d = 84
r = 5
s = 50

The Correct distribution is negative binomial with a mean of 84 and a variance of 336.

B. Description of Data

The significance of sensitivity will be discussed further in connection with the individual kinds of data. The data will be taken up in the order indicated in the enumeration on Page 95.

1. Identifying and Program Information

Unit cost, stock number, noun and other descriptive information, applicability and program information for each item are all necessary if the proposed policies are to be implemented fully. Cost, program and identifying information are required for any reasonable policies. Applicability and program information are desirable both to improve the demand estimation and, in particular, to make the "final order" and other "dynamic" computations. The policies can be used without these data, but to the extent that it is practical to get the information the policies can be used more effectively.

Applicability information is not systematically available on all items now, but in connection with other efforts to continue improving the supply system it is being gathered and disseminated increasingly. The other kinds of information are currently available in usable form.

2. Demand Data

Any system of inventory management is based upon some kind of prediction of demand. Tables 16-19 show that errors in the estimation of demand can be very costly, especially if demand is underestimated. In the examples, estimating demand at half its true rate would result in inventory costs of up to 78 percent greater than they should be. Consequently, it is worthwhile to expend considerable effort in making as good estimates of demand as possible.

Demand, however, is one of the most difficult parameters to estimate. Errors of many hundred percent are not uncommon. Early in a program, at the time of initial provisioning, for example, accurate prediction is impossible. As the program develops every effort should be made to improve the accuracy of the estimates from experience. This is undoubtedly one of the major payoffs to be gained from an integrated data processing system.

As was discussed earlier (Chapter IV), early in a program the depot should not be stocked up to its full Stock Control Level. In this way it is possible to hedge against over-stocking the system. The idea is to bring in depot stocks only up to the Reorder Point (with exceptions discussed above), and when additional information about the actual demand for the part is available to bring the rest of the depot stocks into the system or to reduce the planned stockage for bases coming in later. Unless demand data are accumulated, properly identified to exclude one-time demands and other peculiarities and made available to the appropriate manager early in the program, this way of economizing on initial provisioning cannot be exploited.

At the base level, at present, it is possible to get information on the recent issues, and, using this plus class knowledge and knowledge of program changes, usable estimates of future demands can be made. However, these, too, are crude as is indicated by the fact that typically a very large proportion of the demands upon base supply are for items which are not stocked at that base.[1]

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[1] A substantial amount of research has been undertaken at RAND dealing with the nature of demand. B. Brown, Characteristics of Demand of Aircraft Spare Parts, The RAND Corporation Report R-292, July 1956; T. Goldman, Relationships Between Program Elements and System Demand for Airframe Spare Parts, The RAND Corporation Research Memorandum RM-1858, 24 January 1957; T. Goldman, A Priori Demand Prediction - A Case Study of B-52 Airframe Parts, The RAND Corporation Research Memorandum RM-2088, 10 January 1958.

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An error in the mean demand rate changes both the Order Quantity and the Reorder Point and changes them in the same direction. The sensitivity of these levels to the demand rate is greatest for high-cost, high-demand items and least for low-cost, low-demand items. Good estimates of the demand are more easily made for high-demand items than for low-demand items. In implementing the proposed policies, therefore, data should be gathered more carefully and in greater detail for the higher-cost items than for the lower-cost items. A practice of analyzing demand (and re-computing levels if necessary) each time the stock-on-hand-or-due-in falls to within one unit of the Reorder Point would automatically accomplish this objective; under the proposed policies, the average number of orders a year is nearly proportional to the square root of the dollar value of annual consumption. For example, if $200 worth of an item is consumed annually at a base, the base will reorder twice a year; if $800 worth is consumed, the base will reorder four times a year. Thus, given the demand rate, the higher the unit cost, the more often would levels be revised. An alternative is to compute demand and levels whenever demands exceed or fall short of expectations by more than some predetermined amount.

Since probability considerations influence the Reorder Point, the type of demand distribution which exists for an item is important. Very few items have sufficiently high-demand rates to permit a reliable estimate of demand distribution to be made on an individual item basis. Probably the best way to estimate demand distributions is to study the relatively high-demand items in a particular property class or functional group and to find the kind of demand distribution which fits the data best. In this way it may be possible to find probability distributions applicable to large groups of parts.

Notice that in Table 17 the use of the Poisson distribution, if the true distribution were negative binomial with a variance equal to four times the mean, would introduce only moderate errors except in the lowest price ranges. For the higher demand rate shown in Table 19 the cost is far more sensitive to error in selecting the probability distribution. The greater sensitivity in the case of the high-demand parts is at least partially offset by the fact that it is easier to get a good estimate of demand characteristics for high- than for low-demand parts.

3. Pipeline Time

Both the current and proposed systems require pipeline times; they are nearly as important as demand data in setting Reorder Points which are based on the average demand <u>per pipeline time</u>. The variance in pipeline time is also needed to specify the probability distribution of <u>demands per pipeline time</u>, but computationally this variance can be incorporated into the demand variance, and the comments above on demand variance apply equally to pipeline variance.

For the depot the relevant pipeline time is, of course, the contractor-depot pipeline time including both Air Force and manufacturer administrative leadtime, materials purchase, fabrication, assembly and set-up cost as appropriate, and the transportation, communication and similar times.

Data on pipeline times should be relatively easy to collect.

4. and 5. Holding Costs and Keeping Costs

Holding cost is the sum of the physical holding cost, engineering obsolescence and a capital charge. Keeping cost also includes these same elements. An error in any of these elements will have exactly the same effect as an error of the same proportion in unit cost.[1] The physical holding cost consists of the cost of storing the item in a warehouse, maintaining its static records, inventorying and inspecting it and complying with the non-engineering technical orders pertaining to it. These elements of holding cost should not be particularly difficult to ascertain with satisfactory accuracy.

Engineering obsolescence is the expected cost that will be incurred per year in modifying the item. Statistical estimates of this cost are possible with little difficulty.

The capital charge is another matter. It includes the interest rate but is really the opportunity cost of capital, that is, a measure of the value lost to the Air Force by not spending the price of the item in question in some other way. This is a difficult concept theoretically and further research on it will be needed before completely satisfactory estimates can be obtained.

Since the holding cost is really an expression of the value of money to the Air Force, it is one of the elements in the equations which can be manipulated to apply certain kinds of policy constraints systematically throughout the system. For example, in the event that the budget available at some particular time should not permit stocking the system up to the levels called for by these policies and that it should, consequently, be necessary to reduce stockage, it is not easy to see how one might best do so. By raising the capital charge, Stock Control Levels and Q and R would be reduced systematically in a manner which should cause minimum disruption. Such a step would be less disruptive than an arbitrary decision to reduce all stocks by some specified percent for example.

[1]Hence, holding cost and keeping cost errors are not shown separately in the Tables.

Keeping cost includes holding cost and also depends on the length of time the item will be kept at the base or depot respectively, and on the net salvage price at the end of the base or system program. An error in either of the last two factors will cause an error of the opposite sign in keeping cost per unit. For example, if a program is expected to last for four years but it lasts for five, the keeping cost will be somewhat larger and the Reorder Point somewhat lower than they should be.

Errors in holding costs affect the Order Quantity, and through keeping cost, the Reorder Point. For most items, errors in the obsolescence charge in the keeping cost have only moderate effect on the Reorder Point and none on the Order Quantity. Hence, system performance is less adversely affected by an error in holding or keeping cost than by one in demand rate or pipeline time.

6. Reorder Costs

Reorder costs determine the frequency with which items are to be ordered and have less effect on Reorder Points than do the other factors discussed so far; but they affect the Order Quantity and the total cost significantly, up to 13 % in one case in Table 17.

The costs to be taken into account are all those costs which are incurred whenever a reorder takes place, but which are independent of the size of the reorder. For depot-base resupply they include the paperwork costs at base and depot; the costs of stockpicking, handling, packing and shipping the minimum order (one unit, one unit pack, etc.) at depot; the corresponding costs of receiving, handling and warehousing at base; communications and accounting costs. For the procurement reorder cost, the relevant costs are, again, those incurred whenever a reorder is placed regardless of the size of the order. They include the Air Force and contractor administrative costs, depot receiving, handling, etc. - or in the event of a direct shipment to base, the corresponding base cost - manufacturer's set-up cost if the item is not in production, and the cost of packing and shipping the minimum order. These should be relatively easy to estimate and because of their limited effect on total cost the effort required to determine them adequately should not be very great.

7. Shortage Cost

The shortage cost is the most difficult of the parameters to estimate. Fortunately, system performance, as measured by cost, is fairly insensitive even to errors as large as those shown in Tables 16 - 19. Given the insensitivity of total costs to the shortage cost and the difficulty in obtaining accurate estimates, perhaps a desirable goal might be to find shortage costs within a factor of two or three, e. g., an estimate of $1,000 might be satisfactory so long as the true shortage cost lay between $500 and $2,000.

As mentioned above, the fact that a shortage cost must be taken into account does not imply that one need estimate the cost of losing a war or of not having a critical weapon available in the critical phase of the war or anything of the sort. It implies that it is reasonable to get some estimate of the average cost of overcoming a shortage. The concept is very similar

to the concept of having a specified level of confidence that a demand will, in fact, be met.

One can look at the shortage cost in any one of three ways. First, it can be thought of, as a minimum, as the average cost of the supply and other work needed to overcome a shortage if one occurs. At the base these costs would consist of the cost of searching for the item in maintenance, receiving and elsewhere on the base, the extra cost of premium communication to and transportation from the depot and perhaps most important, the cost of disrupting routine activities. This would provide a minimum estimate of the shortage cost, but in spite of expedited action there is some risk that a weapon would be out of commission because of some part shortages. The shortage cost, therefore, should be greater than the costs of the expediting action. One can think of an upper limit to the shortage cost in the following way. In the worst case if an item is short, a missile (or aircraft) will be MOCP (or AOCP). (In some cases a maintenance facility could be idled.) Obviously, one way to avoid the consequences of such a shortage is to provide the operational unit with an additional weapon to stand-in for the one out of commission. The maximum cost of an expected part shortage would be the cost of providing stand-by weapons as a means of overcoming the effect of a shortage. Of course, providing a component or a black box rather than a stand-by weapon might be a considerably less costly way of overcoming a shortage of most parts. In such cases the cost of keeping on hand the next higher assembly would constitute the shortage penalty for the item in question.

The purpose of the above paragraphs is not to indicate how to estimate the shortage cost but only to show that it is not an impossible concept.

Further research is under way at RAND and in AMC to get usable estimates of shortage cost.

There is a third way of looking at shortage cost. It can be considered not as an objective cost, but as a device whereby a policy decision can be introduced systematically throughout the system or in a part of it, the policy decision being to increase or decrease the risk of a shortage. A decision to decrease the chances of shortages throughout the system, for example, could, if funds were available, be affected by computing stock levels with an increased shortage cost. Similarly, greater protection for high-precedence units than for low-precedence ones could be assured by using differential shortage costs in computing their levels.

Since the shortage cost is not a well-understood concept, let us look at it another way. The establishment of an inventory at all implies that there is some shortage cost. From the level of that inventory, one may estimate the implicit shortage cost.[1] Table 3, showing stock levels, was derived by assuming that demands during a routine pipeline time had negative binomial distributions with variance four times the mean, 20 percent-per-annum holding costs, and a five-year program (35 percent keeping costs.)

Given the same parameters, the "67-1" policies imply the shortage costs shown in Table 20. The implied shortage costs vary from three cents to over $1,300. What is important about Table 20 is not the absolute levels of the implicit shortage costs, which depend on the demand distribution used and on the holding-cost rate, but rather the ratio of the shortage cost for the price on the one hand and the relationship of the shortage cost to the demand rate on the other. With a 30-day pipeline time the implied shortage cost is

[1] An implicit shortage cost is the shortage cost which must prevail if the levels are correct.

SHORTAGE COSTS IMPLIED BY "67-1" POLICIES *

Annual Demand Rate	Unit Cost	30-day Pipeline		
		Reorder Point	Order Quantity	Shortage Cost
1 1/3	$ .10	0	1	$ 0.03 - $ 0.56
1 1/3	5.00	0	1	1.31 - 27.81
1 1/3	250.00	0	1	65.62 -1,390.62
10	.10	1	3	.03 - .06
10	5.00	1	3	1.70 - 3.07
10	250.00	1	1	28.31 - 51.21
103 1/3	.10	13	26	.04 - .05
103 1/3	5.00	13	26	2.22 - 2.69
103 1/3	250.00	13	9	38.36 - 46.48

* Note: The parameter values are the same as those used in Table 3, except for the shortage cost.

Table 20

nearly 30 times as great for a 250-dollar item when the annual demand is 1-1/3 as when that rate is 103-1/3. There is no reason to assume that a single shortage is that much more important for a high-demand item than for a low-demand one.

An inventory system which sets the Reorder Point without considering unit cost has the implicit assumption that shortage cost is proportionate to price, that is, a shortage of a 5-dollar item is treated as being 50 times as serious as the shortage of a 10-cent item, and the shortage of a 250-dollar item is treated as about 25 times as serious as the shortage of a 10-dollar item and 5 times as serious as the shortage of a 50-dollar item, if all three items are stocked. It may well be true that shortages of higher-cost items are more serious than shortages of low-cost items, but the implication that shortage cost is proportional to price appears to be altogether unreasonable.

VI. CONCLUSION

This Memorandum has described a simple and practical method for determining base and depot stock levels in the face of the dynamic and uncertain environment with which the Air Force supply system has to deal. The method takes account of the major variables which must be considered to achieve economical and effective provisioning and distribution under such circumstances.

The logic of the basic method is straightforward. The factors which determine the costs and effectiveness of the supply system are identified and the manner in which they affect cost -- after converting effectiveness into a cost-equivalent through the use of the shortage cost concept -- has been spelled out mathematically. This statement, equation (I.1) of Appendix I, is the keystone of the whole discussion. It has been "solved" to show those values of the Order Quantity and Reorder Point which would provide the minimum system cost. The least cost solution for the Reorder Point is equation (2). Equation (1) for the Order Quantity is an approximation of the rigorous solution in the Appendix (I.5).

The least cost statements which are the subject matter of Chapter II and of the first Section of Chapter IV provide rules for provisioning and distribution where dynamic elements are not of major significance. In depot stockage and where the remaining life of the program at base is short the dynamic elements must be taken into consideration. At the base this means that the approaching termination of the program must be taken into account in determining the Reorder Point through the gradual increase in the keeping cost. This is incorporated in equation (2). For the Order Quantity, adjustments in the equation are required; the expected costs of holding additional operating stocks must be balanced against the expected cost of incurring an additional reorder near the end of the program. This is the subject of Chapter III.

Depot stockage is dominated by its dynamic aspects. Early in a weapon's life the depot can take advantage of the fact that most of the items in base supply will be stocked adequately for relatively long periods and the depot need not stock up to its Stock Control Level until it, in fact, faces demands from the bases. Such a practice would allow the depot stocks to be kept reasonably small and would reduce the risk of gross overprocurement because of serious overestimation of the demand rate.

Depot stockage also must reflect the fact that the costs and lead time involved in procuring items increase markedly when the item goes out of production. This and the problem of phasing-out the weapon from the system as a whole are taken into account through a terminal buy computation.

The corrections for the dynamic aspects of the problem are not rigorously correct; but they do take account of the major factors in the problem, they are reasonably simple to implement and they do not have excessive data requirements. Further they provide close approximations to more elaborate dynamic programming computations which have been developed at RAND.

Consequently the proposed policies appear logically sound; but without actually implementing them, there is no certainty as to how well they will really operate. Some effort has been made to get an empirical evaluation of the policies. The simple comparison in Chapter II with the rigid application of "67-1" policies, indicates that the proposed policies promise considerable improvement. However, since the policies in 67-1 are in fact applied with a good deal of judgment and since those policies have been

changed in recent months, that comparison is not particularly important. Further, it covers only the impact of applying the proposed policies at base.

A much more meaningful comparison has been made in the Logistics Systems Laboratory Project I (LP-1). The full details of that experiment will be developed in other publications. However, because of the fact that it provided a far superior semi-empirical appraisal of these policies than is available in any other way, some major features of the experiment and some relevant results are summarized here.

The proposed policies were a part of a proposed Logistics System called the "1960 System" which also incorporated deferred provisioning and data processing innovations. That system was compared over the simulated life of a weapon with the "1956 System". The latter system was developed by Air Force personnel, who were members of the Laboratory staff, to represent current best practice in the Air Force. Thus the proposed policies were compared not with the rigid application of the manuals but with a serious approximation of current Air Force practice.

The LP-1 comparison had the further advantage of being a system comparison. Bases, parts repair depots, IRAN facilities and storage sites were all stocked in accordance with the policies of the two systems. Maintenance and operational activities were controlled in such a way that the comparisons between the supply systems would not be vitiated by offsetting adjustments in those areas. Both logistics systems were further constrained to stand ready to "fight" a war at any time during the program. In short, every effort was made to make the comparison scientifically sound and operationally significant.

The general results of the comparison are shown in the following charts. Figure 11 compares some measures of the costs of the two systems. The system in-and-out movements provide some measure of the routine supply workload and the priority requisitions, of the expediting workload of the two sets of policies. The proposed Category II and III policies were only 39 and 25 per cent respectively as costly as the "1956" policies. The investment in parts was negligible (less than 1 per cent) greater with the proposed than with the "1956" policies. This is true in spite of the fact that under the proposed policies considerably more, especially of the cheap parts were stocked at the base level.

Figure 12 provides the more important comparison of the policies, on the basis of their supply effectiveness. The "1956" policies resulted in 2.6 times as many Category II and III AOCP-days as did the proposed policies; they showed 15 per cent more ANFE's and more than double the number of stock-out days of the proposed system.

These results do not provide any final "proof" of the superiority of the proposed policies, and certainly they do not prove that these are the best of all possible policies. They are, however, evidence that the policies are superior to present policies.

There is a great deal of interest at all levels in the Air Force in implementing policies such as these. Work is underway with Headquarters AMC and Headquarters USAF on such implementation. In the process of development and implementation in the real environment, a great deal more will be learned. Modifications and perhaps further simplifications of the decision rules will suggest themselves.

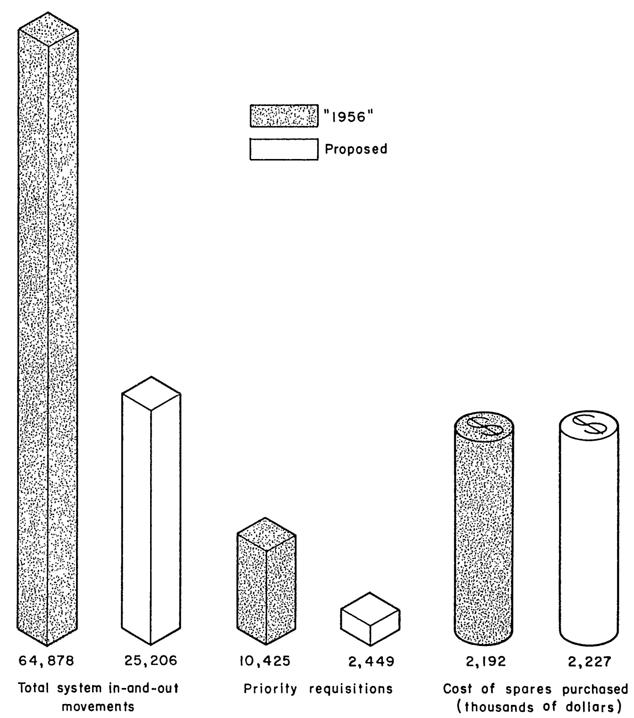

Fig. 11 — Supply cost implications of proposed versus "1956" policies
LP-1 experience for categories II and III (fourteen quarters)

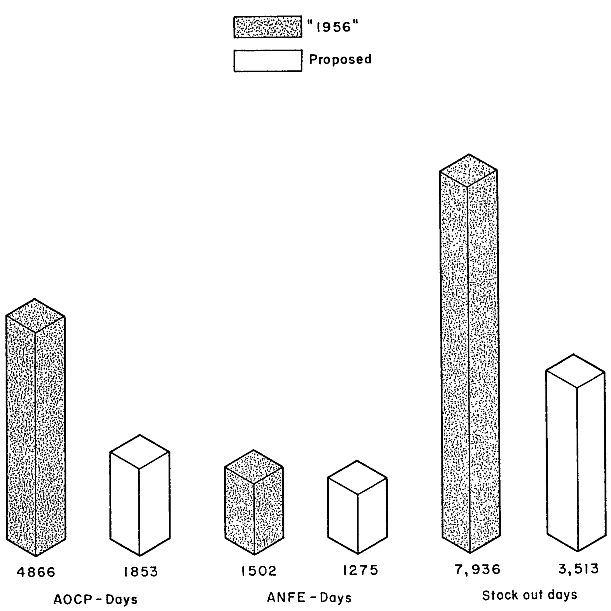

Fig.12 — Supply effectiveness implications of proposed versus "1956" policies

LP-I experience for categories II and III (fourteen quarters)

To summarize, the research underlying this Memorandum leads to the following conclusions:

A. Stockage rules should consider:

1. Expected mean demand for each item,
2. Variability of demand,
3. Unit value,
4. Cost of incurring a reorder,
5. Cost of holding the Operating Stocks,
6. Cost of expected terminal obsolescence (termination of the program being supported),
7. Expected shortage cost,
8. Resupply and procurement pipeline times.

B. The dynamics of weapon-system or other program phase-in or phase-out can be taken into account effectively by

1. Limited depot stockage during the early part of a program, subsequently stocking the depot to its full Stock Control Level;
2. Gathering and analyzing consumption data intensively early in the phase-in and reacting to that information;
3. Using a "final buy" calculation during the later stages of the program and for some of the least costly low-demand items early in the program; and
4. Using a "terminal buy" calculation at the time when an item is expected to go out of production.

C. With an integrated data processing system, these results can be largely achieved by using the equations developed in the mathematical appendix. With a manual data processing system and local determination of levels, tables based upon the formulas can be used by clerical personnel to set the appropriate levels.

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D. The Air Force can increase supply effectiveness, decrease personnel pressures in supply and achieve dollar economies by adopting the policies described and proposed in this Memorandum.

E. Because the proposed policies permit reduced management per line item, their use should free management to manage the more costly and critical items better, or, alternatively, it might permit reducing somewhat base-level manning where — as in hardened-missile installations — there is a premium on personnel space.

F. Further research is needed at RAND to extend the scope of the study, and further developmental studies are required, particularly in the Air Force, to derive adequate estimates of cost and other parameters.

G. The rapid development of an integrated data processing system will improve the application of these as well as other supply policies.

Appendix I

MATHEMATICAL APPENDIX

In Section A of this Appendix the base cost equation for a stable program is stated. From it, the Order Quantity equation and its approximation are derived and the Reorder Point equation is derived: Sections B and C. Section D is a short statement of the derivation of the holding cost, and Section E derives and explains the final Order Quantity calculation.

A. Fundamental Base Cost Equation with a Stable Program

Let

R = Reorder level for an item;

Q = Order quantity;

c(R,Q) = Annual variable cost of operating with a particular R and Q;

d = Average annual demand rate;

v = Unit price;

h = Annual unit holding cost as a fraction of unit price;

k = Annual unit keeping cost as a fraction of unit price;

r = Reorder cost;

s = Unit shortage cost;

$Z_R = \sum_{X=R+1}^{\infty} (X-R)P(X)$ = Expected number of shortages per order placed if the reorder level is R, where

P(X) = probability of exactly X demands during a routine pipeline time.

Then, to a first approximation,

I.1) $$c(R,Q) = hv\left(\frac{Q+1}{2}\right) + kvR + \frac{dr}{Q} + \frac{ds}{Q} Z_R.$$

B. Economical Base Order Quantity

To find the particular Q associated with minimum cost, given R, set

$$\partial c/\partial Q = 0$$

I.2) $$\partial c/\partial Q = \frac{hv}{2} - \frac{d(r+sZ_R)}{Q^2}$$

I.3) $$0 = \frac{hv}{2} - \frac{d(r+sZ_R)}{Q^2}$$

I.4) $$Q^2 = \frac{2d(r+sZ_R)}{hv}$$

I.5) $$Q = \sqrt{\frac{2d(r+sZ_R)}{hv}} .$$

If R is chosen so that $Z_R \approx 0$, I.5 becomes

I.6) $$Q \approx \sqrt{\frac{2dr}{hv}} .$$

The expected annual number of orders,

I.7) $$\frac{d}{Q} = \frac{d}{\sqrt{\frac{2dr}{hv}}}$$

I.8) $$\frac{d}{Q} = \sqrt{\frac{dhv}{2r}} .$$ [1]

[1] See AFM 67-10, p. 149.

This solution (I.8) assumes that C(R,Q) is a continuous function. However, fractional shipments are impossible; therefore, different equations must be used.

Subtract I.1 from

I.9) $$c(R,Q-1) = \frac{hvQ}{2} + kvR + \frac{dr}{Q-1} + \frac{ds}{Q-1} Z_R \quad ,$$

which yields

I.10) $$c(R,Q-1) - c(R,Q) = -\frac{hv}{2} + d(r+sZ_R)\left(\frac{1}{Q-1} - \frac{1}{Q}\right) = -\frac{hv}{2} + \frac{d(r+sZ_R)}{Q^2-Q} \ .$$

Define

I.11) $$= -\frac{hv}{2} + \frac{d(r+sZ_R)}{Q^2-Q} \ .$$

If R is the lowest cost Reorder Level

I.11a)

I.12) $$\frac{2\epsilon + hv}{2} - \frac{d(r+sZ_R)}{Q^2-Q} = 0$$

I.13) $$\frac{Q^2-Q}{2} - \frac{d(r+sZ_R)}{2\epsilon + hv} = 0$$

I.14) $$Q = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2d(r+sZ_R)}{2 \quad +hv}} \quad \frac{1}{2} \quad + \quad \frac{1}{4} \quad + \frac{2d(r+sZ_R)}{hv}$$

Similarly,

I.15) $$c(R,Q) - c(R,Q+1) = -\frac{hv}{2} = \frac{d(r+sZ_R)}{Q^2+Q} = \qquad 0$$

I.16) $$\frac{hv-2\epsilon'}{2} - \frac{d(r+sZ_R)}{Q^2+Q} = 0$$

I.17) $$Q = -\frac{1}{2} + \sqrt{1 + \frac{2d(r+sZ_R)}{hv-2\epsilon'}} > +\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2d(r+sZ_R)}{hv}} \ .$$

Hence:

I.18) $$-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2d(r+sZ_R)}{hv}} < Q < \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2d(r+sZ_R)}{hv}} \ .$$

Therefore, take

I.19) $$Q = \sqrt{\frac{1}{4} + \frac{2d(r+sZ_R)}{hv}} \ .$$

For the vast majority of Cost Category II and III items,

I.20) $$sZ_R > 0 \quad \text{, but}$$

I.21) $$sZ_R << r.$$

Therefore,

1) $$Q = \sqrt{1 + \frac{2dr}{hv}}$$

closely approximates (I.19) for these items.

C. Economic Base Reorder Level

Subtract (I.1) from

I.22) $$c(R-1,Q) = hv\left(\frac{Q+1}{2}\right) + kv(R-1) + \frac{dr}{Q} + \frac{ds}{Q}Z_{R-1} .$$

I.23) $$c(R-1,Q) - c(R,Q,) = -kv + \frac{ds}{Q}(Z_{R-1} - Z_R) ,$$

but

I.24) $$Z_{R-1} = \sum_{X=R}^{\infty} (X-R+1)P(X) = P(R) + 2P(R+1) + 3P(R+2) + \dots .$$

I.25) $$Z_R = \sum_{X=R+1}^{\infty} (X-R)P(X) = P(R+1) + 2P(R+2) + 3P(R+3)+\dots$$

Therefore,

I.26) $$Z_{R-1} - Z_R = P(R) + P(R+1) + P(R+2) + \dots = \sum_{X=R}^{\infty} P(X) .$$

Let

$$P_R = \sum_{X=R}^{\infty} P(X).$$

If Q is the minimum cost Order Quantity

I.27) $$c(R-1,Q) - c(R,Q) = -kv + \frac{ds}{Q} P_R > 0 ,$$

and

I.28) $$P_R > \frac{kvQ}{sd} .$$

Similarly,

I.29) $$c(R,Q) - c(R+1,Q) = -kv + \frac{ds}{Q}P_{R+1} < 0$$

I.30) $$P_{R+1} < \frac{kvQ}{sd} .$$

Hence,

I.31) $$P_R > \frac{kvQ}{sd} > P_{R+1} ,$$

or

2) $$\frac{sd}{Q}P_R > vk > \frac{sd}{Q}P_{R+1} .$$

D. Determination of the Holding Cost Rate and Keeping Cost Rate

If w_o is the probability that an item will become obsolete within the next year because of engineering changes, b is the physical storage cost rate and i is the interest rate, then

I.32) $$h = i + b + w_o \quad .$$

If w_o is small and the item will have zero salvage value, and if n is the length of time (in years) remaining in the base's program, then

I.33) $$k = b + \frac{i}{1-e^{-in}} \quad .$$

If the item has an expected salvage value of v_s, then

I.34) $$k = b + i\,\frac{v_s}{v} + 1 - \frac{v_s}{v}\left(\frac{i}{1-e^{-in}}\right) \quad .$$

Similarly,

I.29) $$c(R,Q) - c(R+1,Q) = -kv + \frac{ds}{Q}P_{R+1} < 0$$

I.30) $$P_{R+1} < \frac{kvQ}{sd} \quad .$$

Hence,

I.31) $$P_R > \frac{kvQ}{sd} > P_{R+1} \quad ,$$

or

2) $$\frac{sd}{Q}P_R > vk > \frac{sd}{Q}P_{R+1} \quad .$$

D. Determination of the Holding Cost Rate and Keeping Cost Rate

If w_o is the probability that an item will become obsolete within the next year because of engineering changes, b is the physical storage cost rate and i is the interest rate, then

I.32) $$h = i + b + w_o \; .$$

If w_o is small and the item will have zero salvage value, and if n is the length of time (in years) remaining in the base's program, then

I.33) $$k = b + \frac{i}{1-e^{-in}} \; .$$

If the item has an expected salvage value of v_s, then

I.34) $$k = b + i\,\frac{v_s}{v} + 1 - \frac{v_s}{v}\left(\frac{i}{1-e^{-in}}\right) .$$

E. Adjusting Q for Short Programs

As discussed in the text it is desirable under some conditions to compute Q taking account of the pending termination of the program which the part in question supports. While there is no rigorously defensible cut off value of Q beyond which the "final" order calculation should be made, an examination of arithmetic examples indicates that whenever

I.35) $$Q \geq \tfrac{2}{3}nd \; ,$$

where n is the time in years to the end of the program, it is cheaper to make one more shipment than to make two.

Where Q is large relative to nd equation (1) does not accurately estimate the savings in the reorder cost from increasing the order quantity incrementally.

Two examples serve to illustrate the point:

In both cases, assume that the base has a five-year program during which it will use the item and that the item has a negative binomial demand probability distribution with variance four times the mean.

I. Expected annual demand = 10 units;
Expected total demand = 50 units.

With the "long-program" assumption of Section II, the average number of orders placed during a five-year period is reduced by 0.13158, i.e., 50/19 - 50/20, if the order quantity is 20 units instead of 19 units.

Actually, the expected number of orders is reduced by 0.13311, i.e.,

$$\sum_{j=0}^{\infty} P(19j) - \sum_{j=0}^{\infty} P(20j)$$

The long-program assumption underestimates the saving in the number of orders, and therefore understates the savings in reorder costs which can be expected by making the order quantity 20, instead of 19. The error in the expected number of orders, however, is 0.13311 - 0.13158 = 0.00153 which is only 1.15 per cent of expected savings. Errors of this magnitude will have very little effect on base stock levels.

II. Expected annual demand = 1-1/3 units;
Expected total demand = 6-2/3 units.

The long-program assumption implies a reduction in the number of orders of 0.01754 if 20 is the order quantity instead of 19.

The actual reduction, however, is only 0.00694. The saving in reorders per item is overestimated by 0.01060, or 151 per cent, which is large enough to cause substantial overstockage.

Furthermore, in Case I, the probability that the item will be ordered at least once more (after this order) is 0.996 or 0.997, so the assumption that the item will be ordered at least once more is a good one. In Case II, however, the probability that the item will be ordered at least once more is only 0.026 or 0.032; and, hence, our second assumption is very bad in Case II.

Thus, a different method of computing the order quantity is needed when the order quantity found from equation (1) is nearly as large as or larger than the total expected demand at the base over the remainder of the program.

To determine the final Order Quantity, Q_F, assume:

1. That, if a reorder is saved by shipping the additional unit, it is saved at the very end of the program; and

2. That the additional unit is certain to be stored at the base until the end of the program.

The total expected savings from adding a unit to a current shipment are the discounted cost of a shipment of one unit at the end of the base's program, multiplied by the probability that the unit will save a reorder. The total costs of shipping the unit now are unit cost plus the discounted cost of storage.

If the expected savings from shipping an extra unit now are greater than the expected costs, the unit should be shipped. If savings are less than costs, the unit should not be shipped.

This statement is summarized by

I.36) $$r'P_{Q_F-1} > h' > r'P_{Q_F}$$

where Q_F = the economical "final" order quantity,

r' = the discounted cost of a shipment of one unit at the end of the program;

h' = the discounted cost of holding one unit from now until the end of the program;

P_{Q_F} = probability of Q_F or more demands from now until the end of the program.

and

I.37) $$r' = (r+v)e^{-in} \quad .$$

Also, if the salvage price is zero,

I.38) $$h' = v[1 + \frac{b}{i}(1-e^{-in})] \quad .$$

Appendix II

SAMPLE BASE TABLES

This Appendix consists of six base tables reflecting different pipeline times and shortage costs, but otherwise based on the same assumptions as Table 3 in the text. By comparing them the reader can get a detailed impression of the effect of differences in these values over the whole array of prices and demand rates. Of course, only the Reorder Point is affected by shortage cost and pipeline time in the approximations upon which these tables are based. Consequently, tables of just Reorder Points rather than tables including the Order Quantity would have been adequate for just this purpose. It was deemed better to show the full tables to permit other comparisons and to show how the relationship between R and Q is affected by different values of the shortage cost and pipeline time.

Comparisons of the effect of differences in the values of the other parameters (other than the frequency distribution and the variance) can be made visually. Thus, using any one table, the effect of different values of d and v can be readily determined by comparing rows or columns. The effect of differences in h and k can be made in the same way, since price is multiplied by h and v. Thus, doubling h has exactly the same effect upon Q as does doubling v. Similarly, a change in k (with h constant) has the same effect upon R as does a change of the same proportion in v. Changes in h have a more complex effect upon R and cannot be read easily from the tables.

Lastly, the Order Quantity increases as the square root of the reorder cost. So the effect of differences in the Reorder Point can be determined by inspection. The effect of changes in r upon R are indirect and, over most of the cells in any of the tables, small.

WORKSHEET
18 (2-50)

R = Reorder Point
Q = Order Quantity

Appendix Table II.1
BASE STOCK LEVELS

30-day Pipeline Time
50-dollar Shortage Cost

Annual Demand: at least but / less than — Unit Price		0– 2	2 6	6 12	12 20	20 30	30 42	42 56	56 72	72 90	90 110	110 132	132 156	156 182	182 210	210 240	240 272	272 306	306 342	342 380	380 420	420 462	462 506
$.01–	R	4	8	11	14	18	21	24	28	31	35	39	43	46	51	55	60	65	70	75	80	85	91
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13–	R	2	6	9	13	16	20	23	26	30	33	37	41	45	49	54	58	63	68	73	78	83	89
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25–	R	2	5	9	12	15	18	22	25	28	32	36	40	44	48	52	57	61	66	71	76	81	87
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50–	R	1	5	8	11	14	17	20	24	27	31	34	38	42	46	50	55	59	64	69	74	79	85
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00–	R	1	4	7	10	13	16	19	22	26	29	33	36	40	44	49	53	57	62	67	72	77	83
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00	R	0	3	6	9	12	15	18	21	24	28	31	35	39	43	47	51	56	60	65	70	75	81
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00–	R	0	2	5	8	11	14	17	20	23	26	30	33	37	41	45	49	54	58	63	68	73	78
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00–	R	0	2	4	7	10	12	15	18	21	25	28	32	35	39	43	47	52	56	61	66	71	76
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00–	R	0	1	4	6	9	11	14	17	20	23	26	30	34	37	41	45	50	54	59	64	68	74
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00	R	0	0	2	5	7	10	13	16	19	21	25	28	32	36	39	44	48	52	57	61	66	71
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00	R	0	0	2	4	7	9	11	14	17	20	23	26	30	33	38	42	46	50	54	59	64	69
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00	R	0	0	1	3	5	7	10	12	15	18	21	25	28	32	35	40	43	48	52	56	62	66
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00	R	0	0	1	2	4	6	9	10	14	17	19	23	27	30	34	37	41	46	50	54	59	64
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

WORKSHEET
18 (2-50)

R = Reorder Point
Q = Order Quantity

Appendix Table II.2
BASE STOCKAGE TABLES

4-day Pipeline Time, 5-year Program
50-dollar Shortage Cost

Annual Demand: at least but / less than — Unit Price		0– 2	2– 6	6– 12	12– 20	20– 30	30– 42	42– 56	56– 72	72– 90	90– 110	110– 132	132– 156	156– 182	182– 210	210– 240	240– 272	272– 306	306– 342	342– 380	380– 420	420– 462	462– 506
$ .01–	R	0	3	5	6	9	11	12	13	14	16	17	18	19	21	22	23	24	25	27	28	29	30
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13–	R	0	2	3	6	8	9	11	12	13	15	16	17	18	19	21	22	23	24	25	27	28	29
.24–	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25–	R	0	1	3	5	7	8	10	11	12	14	15	16	17	18	19	21	22	23	24	25	27	28
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	183	196	208	221	234	247	260	273	286
.50–	R	0	1	2	4	6	7	9	10	11	12	14	15	16	17	18	19	21	22	23	24	25	27
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00–	R	0	0	2	3	5	6	8	9	10	11	13	14	15	16	17	18	19	20	22	23	24	25
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00–	R	0	0	1	3	4	6	7	8	9	10	11	13	14	15	16	17	18	19	20	22	23	24
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00–	R	0	0	1	2	3	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21	23
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00–	R	0	0	0	1	3	4	5	6	7	8	9	10	12	12	14	15	16	17	18	19	20	22
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00–	R	0	0	0	1	2	3	4	5	6	7	8	9	10	11	13	13	14	16	17	18	19	20
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00–	R	0	0	0	0	1	2	3	4	6	6	7	8	9	10	11	12	13	14	15	16	18	19
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00–	R	0	0	0	0	1	2	3	3	4	6	6	7	8	9	10	11	12	13	14	15	[illegible]	17
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	10	17	18
125.00–	R	0	0	0	0	0	1	2	3	4	4	5	6	7	8	9	10	11	12	13	14	15	16
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00–	R	0	0	0	0	0	0	1	2	3	4	4	5	6	7	8	9	10	11	12	13	13	15
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

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WORKSHEET
18 (3-30)

R = Reorder Point
Q = Order Quantity

Appendix Table II.3

BASE STOCKAGE TABLES

30-day Pipeline Time, 5-year Program
500-dollar Shortage Cost

Annual Demand at least but		0-	2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272	306	342	380	420	462
Unit Price / less than		2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272	306	342	380	420	462	506
$ .01-	R	9	14	18	21	25	29	32	36	40	44	48	53	57	62	66	71	76	81	87	92	98	104
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13-	R	8	13	16	21	24	28	31	35	39	43	47	51	55	60	65	70	75	80	85	90	96	102
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25-	R	7	11	16	19	23	26	30	34	37	41	45	50	54	58	63	68	73	78	83	89	94	100
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50-	R	6	11	15	18	22	25	29	32	36	40	44	48	52	57	62	66	71	76	82	87	93	98
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00-	R	5	10	14	17	21	24	28	31	35	39	43	47	51	55	60	65	69	75	80	85	91	96
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00	R	4	9	13	16	19	23	26	30	33	37	41	45	49	54	58	63	68	73	78	83	89	94
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00-	R	3	8	12	15	18	22	25	28	32	36	40	44	48	52	57	61	66	71	76	81	87	92
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00-	R	2	7	11	14	17	20	24	27	31	34	38	42	46	50	55	59	64	69	74	79	85	90
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00-	R	2	6	10	13	16	19	22	26	29	33	37	40	45	49	53	58	62	67	72	77	83	88
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00-	R	1	5	8	12	15	18	21	25	28	31	35	39	43	47	52	56	60	65	70	75	81	86
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00	R	1	4	7	10	14	17	20	23	26	30	34	37	41	45	50	54	59	63	68	74	79	84
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00	R	0	3	6	9	13	15	19	22	25	28	32	36	40	44	48	53	57	62	66	71	77	82
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00	R	0	3	6	8	12	14	17	20	23	27	30	34	38	42	46	50	55	60	64	69	74	80
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

WORKSHEET
18 (3-30)

R = Reorder Point
Q = Order Quantity

Appendix Table II.4

BASE STOCKAGE TABLES

4-day Pipeline Time, 5-year Program
500-dollar Shortage Cost

Annual Demand at least but		0-	2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272	306	342	380	420	462
Unit Price / less than		2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272	306	342	380	420	462	506
$ .01-	R	4	8	11	13	15	17	18	20	21	23	24	26	27	28	30	31	32	33	35	36	37	39
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13-	R	3	8	9	12	14	16	17	19	20	22	23	24	26	27	28	30	31	32	34	35	36	38
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25-	R	2	5	9	11	13	15	16	18	19	21	22	23	25	26	27	28	30	31	32	34	35	36
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50-	R	1	5	8	10	12	14	15	17	18	20	21	22	23	25	26	27	29	30	31	32	34	35
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00-	R	1	4	7	9	11	13	14	16	17	18	20	21	22	24	25	26	27	29	30	31	32	34
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00	R	0	3	6	8	10	12	13	15	16	17	19	20	21	22	24	25	26	27	29	30	31	33
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00-	R	0	3	5	7	9	11	12	14	15	16	18	19	20	21	22	24	25	26	27	29	30	31
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00-	R	0	2	4	6	8	10	11	13	14	15	16	18	19	20	21	22	24	25	26	27	29	30
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00-	R	0	1	4	5	7	9	10	12	13	14	15	17	18	19	20	21	22	24	25	26	27	29
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00-	R	0	1	3	5	6	8	9	11	12	13	14	15	17	18	19	20	21	22	24	25	26	27
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00	R	0	0	2	4	6	7	8	10	11	12	13	14	15	17	18	19	20	21	22	24	25	26
125.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00	R	0	0	2	3	5	6	8	9	10	11	12	13	14	16	17	18	19	20	21	22	24	25
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00	R	0	0	2	2	4	5	6	7	9	10	11	12	13	14	16	16	18	19	20	21	22	24
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

WORKSHEET

R = Reorder Point
Q = Order Quantity

Appendix Table II.5

BASE STOCKAGE TABLES

30-day Pipeline Time, 5-year Program
5000-dollar Shortage Cost

Annual Demand at least but / Unit Price	less than	0- / 2	2 / 6	6 / 12	12 / 20	20 / 30	30 / 42	42 / 56	56 / 72	72 / 90	90 / 110	110 / 132	132 / 156	156 / 182	182 / 210	210 / 240	240 / 272	272 / 306	306 / 342	342 / 380	380 / 420	420 / 462	462 / 506
$ .01-	R	16	21	25	29	33	37	41	45	49	53	58	62	67	72	77	82	87	93	98	104	110	116
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13-	R	14	20	23	28	32	36	40	43	48	52	56	61	65	70	75	80	86	91	97	102	108	115
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25-	R	13	18	23	27	31	34	38	42	46	50	55	59	64	69	74	79	84	89	95	101	107	113
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50-	R	12	17	22	26	30	33	37	41	45	49	53	58	62	67	72	77	82	88	93	99	105	111
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00-	R	11	16	21	25	28	32	36	40	44	48	52	56	61	66	70	76	81	86	92	97	103	109
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00	R	10	15	20	24	27	31	35	38	42	46	51	55	59	64	69	74	79	84	90	95	101	107
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00-	R	8	14	19	23	26	30	33	37	41	45	49	54	58	63	67	72	77	83	88	94	99	105
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00-	R	7	13	18	21	25	28	32	36	40	44	48	52	56	61	66	71	76	81	86	92	97	103
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00-	R	7	12	17	20	24	27	31	35	38	42	46	50	55	59	64	69	74	79	84	90	95	101
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00-	R	5	11	15	19	22	26	30	33	37	41	45	49	53	58	63	67	72	77	83	88	94	100
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00	R	5	10	14	17	22	25	28	32	35	40	44	48	52	56	61	66	71	76	81	86	92	98
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00	R	3	8	13	16	20	23	27	30	34	38	42	46	50	55	59	64	69	74	79	84	90	96
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00	R	2	8	13	15	19	22	26	29	33	37	40	45	49	53	58	62	67	72	77	83	88	94
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

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R = Reorder Point
Q = Order Quantity

Appendix Table II.6

BASE STOCKAGE TABLES

4-day Pipeline Time, 5-year Program
5000-dollar Shortage Cost

Annual Demand at least but / Unit Price	less than	0- / 2	2 / 6	6 / 12	12 / 20	20 / 30	30 / 42	42 / 56	56 / 72	72 / 90	90 / 110	110 / 132	132 / 156	156 / 182	182 / 210	210 / 240	240 / 272	272 / 306	306 / 342	342 / 380	380 / 420	420 / 462	462 / 506
$ .01-	R	10	14	17	19	22	24	26	27	29	30	32	33	35	36	37	39	40	42	43	44	46	47
.12	Q	19	44	79	124	133	140	198	211	237	262	288	314	340	365	391	417	443	469	495	521	546	572
.13-	R	8	13	15	19	21	23	25	26	28	29	31	32	33	35	36	38	39	40	42	43	45	46
.24	Q	16	40	73	75	94	113	131	149	167	186	204	222	240	259	277	295	313	332	350	368	386	405
.25-	R	7	11	15	18	20	22	24	25	27	28	29	31	32	34	35	36	38	39	40	42	43	45
.49	Q	13	35	41	54	67	80	92	105	118	131	144	157	170	133	196	208	221	234	247	260	273	286
.50-	R	6	11	14	17	19	21	22	24	25	27	28	30	31	32	34	35	37	38	39	41	42	44
.99	Q	10	20	29	38	47	56	65	75	84	93	102	111	120	129	138	148	157	166	175	184	193	202
1.00-	R	5	10	13	16	18	20	21	23	24	26	27	29	30	31	33	34	35	37	38	39	41	42
1.99	Q	7	14	20	27	33	40	46	53	59	66	72	79	85	91	98	104	111	117	124	130	137	143
2.00	R	4	9	12	15	17	19	20	22	23	25	26	27	29	30	31	33	34	35	37	38	40	41
3.99	Q	5	10	14	19	24	28	33	37	42	46	51	56	60	65	69	74	78	83	88	92	97	101
4.00-	R	3	8	11	14	16	18	19	21	22	24	25	26	28	29	30	32	33	34	36	37	38	40
7.99	Q	4	7	10	13	17	20	23	26	30	33	36	39	43	46	49	52	55	59	62	65	68	72
8.00-	R	3	7	10	13	15	17	18	20	21	23	24	25	26	28	29	30	32	33	34	36	37	38
15.99	Q	3	5	7	10	12	14	16	19	21	23	26	28	30	33	35	37	39	42	44	46	49	51
16.00-	R	2	6	9	12	14	15	17	19	20	21	23	24	25	27	28	29	30	32	33	35	36	37
31.99	Q	2	4	5	7	8	10	12	13	15	16	18	20	21	23	24	26	28	29	31	32	34	36
32.00-	R	1	5	8	11	13	14	16	18	19	20	22	23	24	25	27	28	29	31	32	33	35	36
63.99	Q	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	20	21	22	23	24	25
64.00	R	1	4	7	9	12	13	15	16	18	19	21	22	23	24	26	27	28	29	31	32	33	35
124.99	Q	1	2	3	4	4	5	6	7	8	8	9	10	11	12	12	13	14	15	16	16	17	18
125.00	R	0	3	6	8	11	12	14	15	17	18	19	21	22	23	24	26	27	28	29	31	32	33
249.99	Q	1	2	2	3	3	4	4	5	5	6	7	7	8	8	9	9	10	10	11	12	12	13
250.00	R	0	3	6	8	10	11	13	14	16	17	18	20	21	22	23	24	26	27	28	30	31	32
499.99	Q	1	1	1	2	2	3	3	4	4	4	5	5	5	6	6	7	7	7	8	8	9	9

Appendix III

The following tables show the relationship between stock-list price and numbers of units issued for B-47 and F-86H spare parts.

The data are classified according to the cells of Table 3, with number of units issued converted to annual rates.

The B-47 data were obtained at March and McDill Air Force Bases during 1953 and 1954. They represent approximately 18 base months' experience.

The F-86H data were obtained from Clovis Air Force Base during 1956 and represent about six base months' experience.

In both cases, items with no issues, on which prices were not available, or which had units of issue other than "each" (e.g., pounds, feet) were excluded.

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Appendix Table III.1
RELATIONSHIP BETWEEN PRICE AND ISSUE RATES OF B-47 SPARE PARTS

Annual Demand Rate: at least, but / less than — Unit Price	2/3 1-1/3	1-1/3 5-1/3	5-1/3 11-1/3	11-1/3 19-1/3	19-1/3 29-1/3	29-1/3 41-1/3	41-1/3 55-1/3	55-1/3 71-1/3	71-1/3 89-1/3	89-1/3 109-1/3	109-1/3 131-1/3	131-1/3 155-1/3	155-1/3 181-1/3	181-1/3 209-1/3	209-1/3 239-1/3	239-1/3 271-1/3	271-1/3 305-1/3	305-1/3 341-1/3	341-1/3 379-1/3	379-1/3 419-1/3	419-1/3 461-1/3	461-1/3 ∞	Total
$ .01- .12	241	330	266	219	109	173	76	144	60	63	55	87	37	51	29	26	28	26	32	20	14	138	2224
.13- .24	125	124	79	48	37	37	17	18	17	13	11	14	6	4	2	3	6	4	1	3		5	574
.25- .49	175	102	71	57	34	31	14	18	14	4	10	9	9	4	3	2		3	1	1		6	568
.50- .99	244	129	62	38	32	23	14	32	9	8	5	12	1	5		3	2	3	1	1		4	628
1.00- 1.99	207	132	48	38	19	18	9	7	6	4	2	1	2	2	1	2				1		1	500
2.00- 3.99	216	124	40	31	24	17	6	8	5		1	3	2	1	1	1	1	3	1				485
4.00- 7.99	179	118	33	33	14	11	11	5	1	2	2	1	1			6	1	1		1			420
8.00- 15.99	158	86	31	25	11	6	1	2		2	1	4	1										328
16.00- 31.99	131	91	30	11	9	4	2	4	1	3	1	2										1	290
32.00- 63.99	118	66	23	14	8	10	1	1	1	2		2					1						247
64.00- 124.99	118	79	20	19	8	6	4	1	2	2		1	1	1		1		1	1				265
125.00- 249.99	84	60	26	11	11	10	3	2		1	1	1	1										211
250.00- 499.99	68	38	21	22	3	2	5	3			1	2			1								166
500.00- 999.99	35	43	6	17	2	6	2	3		2			1							1			118
1,000.00- 1,999.99	28	13	6	8	4	3	1	1		2			1										67
2,000.00- 3,999.99	24	16	13	7	4	1	1	1	1														68
4,000.00- 7,999.99	6	7	5	2	1		1																22
8,000.00-15,999.99	2	2	3	2	1			1															11
16,000.00-31,999.99	1	2		1			1					1											6
32,000.00 and over																							0
Total	2160	1562	783	603	331	358	169	251	117	108	90	140	63	68	37	44	39	41	37	28	14	155	7198

Source: Demand data collected at March and MacDill Air Force Bases. About 18 base-months in 1953 and 1954.

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Appendix Table III.2
RELATIONSHIP BETWEEN PRICE AND ISSUE RATES OF F-86H SPARE PARTS

Annual Demand Rate: more than but / less than — Unit Price	0 2	2 6	6 12	12 20	20 30	30 42	42 56	56 72	72 90	90 110	110 128	128 156	156 182	182 210	210 240	240 272	272 306	306 342	342 380	380 420	420 462	462 ∞	Total
$ .01- .12	15	11	19	18	21	6	16	14	7	28	10	9	5	15	2	8	10	9	7	17	5	85	337
.13- .24	4	6	3	6	9	6	5	2	3	8	2	3		2	1		3		2	1		5	71
.25- .49	7	5	9	7	6	2	4	3	6	2	4	2	1	4	1	1		1				2	67
.50- .99	11	10	22	8	11	2	5	2	2	4		2	1	1	3			1				1	86
1.00- 1.99	16	21	10	10	8	7	6	2	1	2	1		2	1		1	1	2					91
2.00- 3.99	37	35	14	8	14	6	3	4	1	4		1		1	1		1	1	1	2			134
4.00- 7.99	60	38	42	19	12	4	5	3	1	1	3	1	1										190
8.00- 15.99	31	23	16	10	4	3	2	1					1		1	1							93
16.00- 31.99	39	49	41	18	9	4									1								161
32.00- 63.99	24	12	4	1	3	3	2	1			1												51
64.00- 124.99	19	6	10	8	2			1						1									47
125.00- 249.99	10	22	12	8	1		1	1	1						1							1	58
250.00- 499.99	8	7	4	1		1	1						1	1								1	25
500.00- 999.99	11	6	3	2		2	3		1				1		2								31
1,000.00- 1,999.99	2	6	1	1		1	1	1	2		1										1		17
2,000.00- 3,999.99	1	2		1	2	1	1								1								9
4,000.00- 7,999.99																							0
8,000.00-15,999.99								1		1													2
16,000.00-31,999.99	1																						1
32,000.00-63,999.99																							0
64,000.00 and over																							0
Total	296	259	210	126	102	48	55	36	25	50	22	18	13	26	14	11	15	14	10	20	6	95	1471

Source: Demand data collected at Cannon Air Force Base, Clovis, New Mexico, February to June 1956.

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The RAND Corporation, Santa Monica, Calif. RM-1962
USAF Project RAND

STOCKAGE POLICIES FOR MEDIUM- AND LOW-COST PARTS by A. R. Ferguson and L. Fisher; Research Memorandum RM-1962, April 18, 1958, Unclassified. 158 pp. incl. illus.

ASTIA Document No. AD 156001.

A method for determining base and depot stock levels in the face of the dynamic and uncertain environment with which the Air Force supply system has to deal. The method examines the major variables which must be considered to achieve economical and effective provisioning and distribution under such circumstances. The study indicates what stockage rules should include, how the dynamics of weapon-system or other program phase-in or phase-out can be taken into account, how to free management to manage the more costly and critical items better, what further research is needed, and how the application of these as well as other supply policies can be improved.

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