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A REVIEW OF SOVIET CELESTIAL-MECHANICS LITERATURE

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A REVIEW OF SOVIET CELESTIAL-MECHANICS LITERATURE

In this report, about 375 Soviet papers on celestial mechanics are discussed. An attempt was made to keep the review of each paper brief but yet of sufficient length to summarize the author's description of his problem, methods, results, and conclusions. For the majority of the papers, opinions are given on the merit or value of the work.

Some of the work discussed below is quite old, but its inclusion is believed justifiable as a basis for obtaining a coherent picture of the Soviet development of the subject from the pre-Soviet era. Also, the 1956 and 1957 literature could not be fully covered because of the delay in obtaining, studying, and processing into a report this most recent information.

1. Two-Body Problem

a. Fundamental Equations

(1). Kepler's Equation

In elliptic motion, three angles, called anomalies, are treated. They are v, the true anomaly; E, the eccentric anomaly; and M, the mean anomaly. All these anomalies are functions of time, and what is really wanted is the true anomaly, v, that is, the angle between the direction to the perihelion and the radius vector of the body under investigation. In order to derive v as a function of M, which is known if the major semiaxis of the orbit is known, an auxiliary anomaly, E, is introduced. The connection between E and M is:

$$E - e \sin E = M.$$

This is the famous Kepler equation on the solution of which many papers have been written. Despite its simplicity, it cannot be solved analytically and, in practice, the acceptable solution is obtained from tables and successful approximations.

Before computing machines were available, the form by Oppolzer (1885) was commonly used for the solution of Kepler's equation. Oppolzer's solution, however, was given in 55 pages in quarto and, therefore, the method becomes quite unmanageable. Marth's work (1890) on this problem still required 16 pages of tables. Subbotin[733] (1928) greatly facilitated the solution of Kepler's equation by Oppolzer's method when he reduced the manipulation to 2-1/2 pages of tables. Subbotin's work had its value about 25 years ago, but today, the solution of Kepler's equation by successive adjustments, starting with a trial value of E_1, is preferred in using computing machines. In practice, the earlier attempts have left little trace in astronomy.

The same can be said of M. A. Vil'yev's paper[779] (1917) on the calculation of the time anomaly in an ellipse with a large eccentricity, whereby he reduced the determination of the eccentricity to the solution of Barker's equation for parabolic motion.

Later work of Vil'yev[783] (1938), in which he was especially concerned with the solution of fundamental equations of theoretical astronomy and the number of solutions, deserves mention. This work is a critical review of the problem of orbit determination, with some contributions by Vil'yev. These investigations were published after the death of the author, who was one of the most promising workers in celestial mechanics in recent times. In the introduction, Vil'yev presents a historical review, starting with Tycho Brahe and Kepler, and compares carefully the various methods of orbit determination. He concludes that the main problem is the determination or correction of an orbit based on a maximum interval of time, including the effect of perturbations.

After the general basis of the theory of orbit determination from observations is discussed, Vil'yev proceeds to the differential form of the fundamental equations, and to Laplace's method and its development. The method of Laplace is unique insofar as it is valid quite generally, without special assumptions concerning the form of the acting force. Further, it is emphasized that this is also the only method permitting the determination of an orbit directly, including perturbations. The methods of Gauss and Olbers are then compared with the method of Laplace. Here the author employs an ingenious procedure which apparently was not used before. He makes use of the fact that, if the length of the time interval converges to zero, the various methods must necessarily converge asymptotically toward the one of Laplace. This process can be used to test new methods and, in this way, the author finds theoretical inconsistencies in the method of Du-Sejour. Similarly, he finds an inconsistency even in Gauss' method on the basis of four observations, and gives the corrected and completed form of the method of Gauss.

Vil'yev does not say much that is new in his reduction of the fundamental equations to one unknown with the graphical method of solution, but he does analyze the existing procedures and methods very thoroughly and critically.

In his investigation of the number of the solutions in special cases, it is stressed that multiple solutions will be obtained only during a limited interval of time from the observations.

In Chapter IV, the derived forms of the fundamental equations and the general conclusions regarding the number of solutions are considered; In Chapter V, the geometrical method of solution is treated along about the same lines as the method of Charlier, only more completely and more critically.

In Chapter VI, Vil'yev succeeds in the geometrical construction of certain regions as limited by certain surfaces, the equations of which are considered so that the location of the comet or planet in question inside these regions determines the number of possible solutions.

Chapter VII deals with the problem of parabolic orbit determination, and especially with the differential form of the fundamental equations for an arbitrary position of the fundamental circle of the problem.

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Finally, Chapter VIII is concerned with circular orbit determination and the related number of solutions. The Laplacian method, especially, is dealt with, and the circumstances arising from particular constellations, as rigorous opposition or conjunction, are considered in detail. An appendix deals with the general properties of curves satisfying the relation $r_1^m r_2^n = C$ in bipolar coordinates, and this part therefore is not directly related to specific problems of celestial mechanics. A bibliography of 246 titles concludes the work.

Oppolzer's method on the solution of Kepler's equation is not discussed by Dubyago in his textbook[126] (1949) on the determination of orbits. This book is of the same type as Stracke's Bahnbestimmung der Planeten und Kometen; however, Dubyago places more emphasis on machine computations and he also includes the determination of meteor orbits. The standard approach to the problem is adopted, and simplified tables are given for the determination of the difference E - M following Stracke and others, all based eventually on Astrand's tables. Dubyago's book is a rather complete presentation of the subjects belonging under the general title of the book, although Dubyago can be criticized concerning minor inaccuracies in his use of theoretical concepts. In fact, he has been so criticized by another Soviet astronomer, Shchigolev[697].

A somewhat different approach to the problem was adopted by Subbotin[740, 742] (1936 and 1937). He considers the mean, true, and eccentric anomalies in an elliptical orbit, as well as the anomaly of Callandreau, as particular or special cases of a more general anomaly w, which he introduces. The generally anomaly w contains a parameter, α. For $\alpha = 0$, the general anomaly becomes identical to E; for $\alpha = e$ (e = eccentricity), the angle w is the same as v, and a closely related angle, w', is identical to Callandreau's tangential anomaly, C. This work is of interest because it reveals the relative meaning and selection of the various anomalies used in celestial mechanics from a more general point of view. This is a solid (but not outstanding) piece of work, but the generalization does not appear to be anything actually significant.

Interesting from the mathematical point of view, but not representing anything new in celestial mechanics, is Yelenevskaya's[825] (1949) determination of the coefficients of the developments for the undisturbed coordinates of motion in a conic section, if the eccentric anomaly, E, is the independent variable instead of the mean anomaly, M. While Bessel's functions play an important role in the developments depending on M, the corresponding functions as they determine the coefficients of the development depending on multiples of E are derived by her. She competently presents a thorough study of these functions, based on the related concepts and theorems of the general theory of functions, and makes a comparison with the properties of Bessel's functions. A certain similarity is found, and the author comments that this had to be expected, because both types of functions satisfy the same type of differential equation as studied by Fuchs.

Yelenevskaya presents correct results in a well-accomplished manner, but it has to be noted that such developments in

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dependence on the eccentric anomaly, E, are not principally new. She mentions the comparable developments obtained by Hansen, and claims that her form is more simple compared with Hansen's, but she does not mention the general results of this form (depending on E) as given by Brown and Innes. In their specific form, the author's developments are new, but her coefficients must be related to those of the other authors by a system of transformations. Apparently, the author was not aware of the results by Brown and Innes, because they are not mentioned at all.

In Reference 825, Yelenevskaya replaced Bessel's functions and the mean anomaly, M, in the expansion of the coordinates of elliptic motion by a different type of function, valid for the eccentric anomaly, E, as the independent variable. In some other work in 1949(826), she proceeds similarly for the Fourier series development, which applies to hyperbolic motion if the hyperbolic eccentric anomaly is used. The main purpose of the paper is the investigation of the convergence of these developments, where the coefficients are proportional to increasing powers of $\frac{1}{e}$. Depending on the individual value of the eccentricity, e, it will be useful to employ either such developments depending on $\frac{1}{e}$, or a different type of series where the coefficients depend on the increasing powers of $(e - e_o)$, e_o being some proper constant close to e. The radius of convergence is studied for the two different methods and for given values of e.

This is definitely an interesting paper, because it contains some substantial and well-founded results, even though the subject itself is not a very difficult one. These results may certainly be of value for future studies on hyperbolic orbits. The author is well qualified for work in celestial mechanics.

Tables for the solution of Kepler's equation by a computing machine were published in 1935 by Bazhenov(57), who makes no mention of Subbotin's(733) and Vil'yev's(779) work. The method presented is essentially that of Tietjens. The author puts:

$$\theta = E - M \text{ and } x = \operatorname{tg}\theta = \frac{e \sin M = f(x)}{1 - e \cos M},$$

so that:

$$f(x) = \theta \sec\theta - \operatorname{tg}\theta = \operatorname{arc\,tg} x \sqrt{1 + x^2} - x$$
$$= \frac{x^3}{3!} - 11\frac{x^5}{5!} + 309\frac{x^7}{7!} \ldots ,$$

and tabulates f(x). A good approximate solution of Kepler's equation can be obtained by first omitting f(x) in the computation of x from M and e, and the successive approximations including f(x) (taken from the given table) converge very fast.

The method presented here and the table may be very useful for a computer who has decided to solve Kepler's equation in this way. It seems, however, that Bazhenov's results will not save time compared with the method which solves Kepler's equation directly by repeated trial solutions, based on an estimated or approximate value of E. Even if the method of successive direct trials on a desk calculator involves perhaps a greater number of successive approximations, one has the advantage of dealing only with one equation and with no additional formulas or tables. Furthermore, the direct method leaves e^o in the machine all the time and improves sin E in the last significant figures only, while Bazhenov's method requires repeated divisions and other operations. For these reasons, the author's paper seems to be of no practical importance.

(2). Gauss' Equation

This equation occurs in the Gaussian method of determination, when the expression for the second place is obtained in the form:

$$\rho 2 = K_o - \frac{l_o}{r_2^3}. \qquad (1)$$

Here, the unknowns are the geocentric distance, ρ_2, and the heliocentric distance, r_2.

In the triangle sun-earth-body, the angle between the directions from the body to the sun and the earth is denoted z_2. From the triangle comes the relationship:

$$r_2^2 = R_2^2 + 2R_2 \cos \theta_2 \rho_2 + \rho_2^2, \qquad (2)$$

which in connection with (1) can be put in the form:

$$\sin (z_2 - q) = m \sin^4 z_2, \qquad (3)$$

where q and m are parameters compounded of known quantities. This is an equation of Gauss which cannot be solved analytically. Many solutions have been suggested, all based on the application of some sort of expansion into series.

One of the more notable contributions to this problem was made by Banachiewicz[43] in 1917. Banachiewicz (1882-1954) was a Soviet astronomer of Polish origin who published many papers on celestial mechanics. The first part of his work was done at Kazan' (1910-1915) and Dorpat (1915-1918). Later, he settled in Cracow, Poland, where he became best known for the development of special kinds of matrix operators (which he called Cracovians) and their application to the problems of astronomy and geodesy.

Banachiewicz's main paper[43] on the Gauss equation contains the theory of this equation and tables for its resolution for the calculations into seven decimals. These tables, reduced to five decimals, have found their way into the standard collection of astronomical tables for the calculation of orbits, those of Bauschinger-Stracke. This, in itself, is an acknowledgment of Banachiewicz's work.

Investigations of Gauss' equation were carried out also by Vil'yev in two papers[777, 781] (1916 and 1923), but the series obtained by him does not converge rapidly enough. In later work, Vil'yev[783] (1938), as indicated previously, compared the methods of Gauss and Olbers with the method of Laplace.

A. Ya. Orlov[534], in 1915, proposed a method of solution of the Gauss equation which had been used before by Witt.

J. Witkowski (now in Poznan, Poland, and formerly at Odessa) and Balassoglo published tables for the solution of Gauss' equation, but they do not seem to have been used.

(3). Lambert's Equation

Expansion of Stracke's closed form of the Lambert equation with $a = \infty$ results in Euler's equation for parabolic motion. Whereas Euler's equation is of importance in the determination of parabolic or near-parabolic orbits, little use is made of Lambert's equation for elliptical motion because of the slow convergence of the series.

Subbotin(732) (1924) seems to be the only one who attempted to use Lambert's equation for elliptic orbits. He applies a certain transformation to the quantities involved in the Euler-Lambert equation (he connects two different radii vectors with the chord of the arc between the two positions and with the semimajor axis of the orbit) in order to arrive at an expression which is more convenient for elliptical orbits. A number of tables are given for the auxiliary quantities to facilitate the computations. The rest of this paper is devoted to the application of this form of the Euler-Lambert equation for the purposes of orbit determinations and orbit corrections. This method has failed to find much practical application in the U.S.S.R. or elsewhere. Stracke does not discuss it at all, although he gives the titles of Subbotin's papers in his bibliography. Bucerius notes Subbotin's work, but makes no further comment. Dubyago(126) (1949) indicates Subbotin's treatment as a possible new method for the determination of orbits, but he does not discuss it in the body of his book. Also, Bazhenov(66) (1952), in his review of Soviet work, only mentions this work by Subbotin but does not discuss it.

Subbotin did, however, apply his new form to the variation-of-distances method[735] (1929) and to the determination of first orbits[731] (1922). All in all, more modern methods have made Subbotin's new form obsolete. He utilized his new form of the Euler-Lambert equation in an issue of a journal of the Tashkent Astronomical Observatory in 1929[735]. This work might be called the Soviet counterpiece of Bauschinger's Tafeln zur theoretischen Astronomie. The first part contains tables and formulas for interpolation, numerical differentiation, and integration, conversion tables, tables for computation of the parallax, and tables for the computation of the precession by transformation of the rectangular equatorial coordinates. Tables and formulas for elliptical motion (Kepler's equation, etc.), as well as for parabolic and near-parabolic motion, are given in the second part. Last, Subbotin presents the complete formula for the first orbit determination according to Gauss-Encke in a form which is convenient for machine computations, and also Olbers' method for the determination of parabolic orbits, with auxiliary tables.

The derivation of this equation has attracted the attention of many Russian and Soviet astronomers, beginning with M. Khandrikov (1873) and A. Savich. A. N. Krylov[321, 322] (1936) considered Savich's derivation as the best available. Dubyago[126] (1949) has reproduced Savich's derivation of Euler's equation for parabolic motion in his textbook. Krylov[318, 319] (1924 and 1925) has also called attention to the fact that the Euler-Lambert theorem actually goes back to Isaac Newton, who gave it in a purely geometrical form as Lemma X, Book III of his Principia. Lagrange, in his Mecanique Analytique, had already pointed out this fact. Even Bauschinger apparently was not aware of this fact, since in his Bahnbestimmung he said that Newton's relation was graphical and approximate. Krylov proves that Newton's lemma does contain the full theorem of Euler and Lambert.

An unusual derivation of Lambert's theorem was made by the Soviet N. Ye. Zhukovskiy (published in his Complete Works, 1937). Kerglotz noted this derivation in Bahnbestimmung der Planeten und Kometen.

Geometrical demonstrations of the Lambert-Euler theorem were given by Chernyy[103] (1907) and Rak[618] (1925). All Rak did was to use some theorems from analytical geometry; Chernyy's purpose was to clarify some theoretical points which, according to him, were not sufficiently represented by Encke.

b. Multiple Solutions

Soviet papers on this subject are very few; in fact, most are from the era of Russian science.

In his well-known textbook on orbit determination, Th. Oppolzer gives criteria, based on a graphical method of investigation, by which it can be decided whether the general method of orbit determination (without any limiting assumption with respect to the eccentricity) has one or two solutions. Chernyy[101] (1907), in an elementary paper, shows that one can arrive at this decision also by a simple and purely analytical procedure, making use of certain properties of the important fundamental equation of the eighth degree in Oppolzer's book.

Apparently, it was not known to Chernyy that Encke treated this problem in 1854.

Also in 1907, Chernyy(102) refined Oppolzer's equation for the investigation of the existence of multiple solutions for parabolic orbits by the addition of the small terms of the first order. He also derives a criterion for finding the real one of the three possible solutions. Finally, he applies his equations to the case of the Comet 1882 II, which had been dealt with also by Oppolzer.

This paper was of interest and practical value when it was published about 50 years ago and, even today, these results may be useful whenever a computer encounters these problems in connection with an individual orbit determination.

The first orbits computed for the Comet 1910a showed great differences between the various sets of elements which resulted. Chernyy(106), in 1910, found that for the three observations on January 18, 19, and 20, 1910, Olbers' method actually produces three different solutions. The theoretical criterion for the existence of three solutions is fulfilled in this case. The paper made it clear that the large differences in the elements were not caused by errors in the observations, but that they are explained by the nature of the special problem.

Chernyy's paper was of actual interest insofar as it answered the question of the origin of the discrepancies in the elements. Although the author's investigation settled this question, it was based on the existing theory and did not contain any new theoretical results.

Fogel'(203), in 1912, dealt with the development of the heliocentric coordinates x, y, and z in power series depending on the ascending powers of the time interval $(t_2 - t_1)$. Although Kühnert was the first to find closed expressions for the coefficients α and β of $\bar{r}_1$ and of $\frac{d\bar{r}_1}{dt}$, respectively, so that $\bar{r}_2 = \alpha\ \bar{r}_1 + \beta \frac{d\bar{r}_1}{dt}$ in the case of elliptical motion, the author finds expressions for α and β which are valid for any type of conic section. These results had been published before in the Russian language, in 1891, but the earlier publication had remained widely unknown.

This work certainly was of real interest as a generalization of the earlier and more special expressions of Kühnert. Although no actual theoretical difficulties were involved, the paper bears witness to a certain amount of original thinking by the author.

A very short paper by Chernyy(109) (1913) contains errors or erroneous statements. It was written apparently in consequence of a remark by R. Fogel'(203), even though Fogel's name is not mentioned. A later reply (1913) by Fogel'(204), however, makes this clear. Fogel', in his earlier paper(203) (1912), made a short remark concerning the fact that, in Olbers' method for the determination of a parabolic orbit, the condition that the three given orbital points lie in one heliocentric plane is neglected.

Chernyy obviously misconstrued this remark by Fogel', as if it were a criticism of the justification of Olbers' method itself. Trying to refute Fogel's rather correct remark, the author claims to give proof that the heliocentric plane condition is satisfied in Olbers' method. Actually, he makes the mistake, now, of claiming that each one of the three so-called fundamental equations determines the location of the three points in one heliocentric plane; actually, all three equations together are necessary for this determination. He further makes some misstatements concerning the equations which are actually used in the method of Olbers (this is clarified in Fogel's paper[204]).

Of all the papers by Chernyy which have been reviewed, this one is the weakest. It is completely wrong. Since the argument is concerned with relatively simple theoretical concepts, one has to conclude that the abilities of this author were not too much in evidence during the early part of his career. (Chernyy is not known to have published any more papers between 1913 and 1949; one single paper published in 1949[110] seems to be on a much higher level.)

Chernyy[109] claimed that Olbers' method for the determination of a parabolic orbit did not sacrifice the condition that the three given points must lie in one heliocentric plane, as was remarked by Fogel'[203], and Chernyy claimed to prove the satisfaction of the heliocentric-plane condition. Doing this, however, Chernyy considered this heliocentric plane to be determined by any one of the three so-called fundamental equations, while actually, as Fogel' clarifies in Reference 204 (1913), all three equations together are necessary for this determination; Fogel' also points out that Chernyy made erroneous statements concerning the equations which are actually used in Olbers' method.

In a very thoroughly written clarification of the basic misunderstandings in Chernyy's paper[109], and of the related theoretical concepts, Fogel' seems to display a very keen understanding of Olbers' method, while Chernyy's short paper proves nothing except the fact that he had not mastered the basic geometrical and analytic concepts of such a problem.

On the basis of the equation of the sixth degree for the geocentric distance ρ, Fogel[205] (1913) found that only one of the two solutions is positive, and the other one negative. Since the possibility of a third solution was eliminated in the earlier paper by the author's consideration of all three fundamental equations for the heliocentric-plane condition (of which only two are used in the classical method by Olbers), the present paper leads to the conclusion that only one solution is actually found in the case of a rigorous determination.

Olbers' method of orbit determination for parabolic orbits is "incomplete" from the theoretical point of view, insofar as essentially only five instead of six coordinates are used. An investigation by Il'inskiy[233] (1924) served a theoretical purpose alone, namely, the study of the possibility of multiple solutions. He considered the parabolic case as a special case of the general method of Gauss for $e = 1$. He proves that three solutions are impossible in a "theoretically complete" determination of a parabolic orbit, and that the special parabolic case is not different from the elliptical case, where two solutions are possible. The well-known equation by Gauss, which is of the eighth degree in general, is reduced to the seventh degree in the case of the parabola, because the solution which corresponds to the orbit of the earth is eliminated or impossible in this case.

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The paper is an interesting, even though not an outstanding, contribution from the theoretical point of view. Il'inskiy apparently has publications only in the field of orbit determination, but he seems to be well versed in this particular field.

The most recent Soviet paper(768) (1941) on the subject of multiple solutions for the determination of a parabolic orbit was written by Usov and could not be located. This is the only paper known to have been written by this person.

c. Series Development and Convergence Problems

Bazhenov(63) (1949) puts the conditional equations for the convergence of an iterational process into the form of a matrix, and then the characteristic values of the related determinant have to be smaller than certain limits, in order to assure convergence of the iteration process. This is a well-known criterion, which Bazhenov then applies to the methods of orbit determination of Gauss and of Harzer; he finds that, in the average case, both methods of orbit determination will converge about equally well. Therefore, the method of Harzer may be preferred because of its relative simplicity.

Only minor details are actually new in this investigation, as far as the analysis is concerned, and, therefore, this paper should be classified as a routine work without any particular highlights or important results.

Two papers(825, 826) by Yelenevskaya in 1949 pertaining to this subject are discussed in the section on Fundamental Equations.

In a later paper[832] (1953), Yelenevskaya recommends expansions depending on the true anomaly, v, rather than on the mean anomaly, M, if the eccentricity is close to unity. It is shown that this method would more effectively overcome the well-known difficulties which occur near e = 1. While the basic principle of the method is given, she does not complete the method in this paper.

The idea presented by her appears to be as good as some other proposals which have previously been made for dealing with the development of the coordinates in near-parabolic orbits, but, on the other hand, it remains doubtful that the majority of computers will abandon the classical method of Gauss in favor of the one proposed here. Gauss' method still appears as the one with the most simple and convenient features. Furthermore, it should be expected that the author would present her method in a more complete and detailed form, if she herself expects it to be used.

In a theoretical investigation that considered the singular points of the differential equations of the two-body problem on the basis of the analytical theory of functions, Samoylova-Yakhontova[662] (1927) found that all the individual cases of elliptic, parabolic, or hyperbolic motion depend on the same general formula, as far as the contribution of the singular points is concerned. All the singular points are branching points of the first order, which only in the case of straight-line motion fall together in pairs to form branching points of the second order. The radius vector, r, is zero in each singular point. The Riemann surfaces have an infinite number of leaves for

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elliptic or hyperbolic motion, but, although all the singular points of the elliptical case lie on the main leaf, only two such points lie on the main leaf in the case of hyperbolic motion. For this reason, in the case of elliptical motion, the radius of convergence can be fixed by one arbitrary singular point; for hyperbolic motion, only two such points will determine this.

This paper is of real theoretical interest, because the results have a bearing on the convergence of the series which are used in connection with orbit determination, and also because the two-body problem is the simplest case of the n-body problem. Without question, this is a good, although not an outstanding, contribution by a very competent author.

The well-known developments by Leverrier and others went up only to the eighth power of the eccentricity, e, inclusive. Sharaf[687] (1953) found it necessary to proceed to the terms depending on e^9 in her theory for the planet Pluto. For this very specific reason, the general extension to e^9, as contained in this paper, had to be made. The general developments, using the well-known Bessel functions and Cauchy's theorem, are given here for the benefit of anybody who later on may find it necessary to use them in connection with other problems.

This paper is the fruit of an elaborate but rather automatic extension of earlier work in this field. The results are certainly useful, but their derivation was a matter of patience and carefulness, rather than of ingenuity.

d. Graphic Methods

In a rather elementary paper, Bazhenov[52] (1929) dealt with the graphical determination of a first approximation for the solution of the following three equations: (1) Kepler's equation, (2) Gauss' equation, and (3) the equation $\lambda e \operatorname{tg} F - \log \operatorname{tg} (45° + \frac{1}{2}F) = M$. A graphical table is printed for the approximate solution of each of these equations. The more rigorous solution may then be determined, by one or two differential variations of the approximate value, from the equations themselves.

These nomographs can be of some value if one has to solve such an equation for a greater number of points, even though certain tables which were already available, such as the one by Astrand for the solution of Kepler's equation, may serve the same purpose.

Il'inskiy's paper[232] (1924) is based on Fogel's transformation of the classical method by Olbers. Devised is a procedure which permits the determination of a better approximate starting value of $r + r_2$ than $r + r_2 = 2$. The method reduces the number of necessary approximations and also avoids the case of multiple solutions.

This paper undoubtedly was a valuable contribution at the time of its publication. Today, more modern methods or, rather, modifications of Olbers' method, are used by most orbit computers.

In 1954, Il'inskiy[237] transformed the fundamental equations of the problem of determining the orbit from three observations in such a way that one has to solve, in succeeding approximations, for the triangle ratios n_1/n and n_2/n. Besides this innovation, the author presents a graphical method of solution which replaces the trial computations with the drawing of certain diagrams.

It appears that these diagrams have to be of a rather large dimensional scale, if one wants to obtain results of some accuracy. Altogether, it seems doubtful that the author's modifications will be adopted by many orbit computers. The paper is of some interest insofar as it illustrates certain modifications which can be made in the application of the basic theory of orbit determination, but the particular proposals do not seem to lead to any essential progress, as far as the practice of orbit determination is concerned.

e. Construction of Tables

For the purpose of computing special perturbations of the elements of minor planets, the Astronomical Sector of the Latvian Academy of Sciences prepared a set of auxiliary tables(3) (1954); practically, these tables are only an extension of Stracke's well-known tables for the very same purpose. The accuracy has been increased, however, by adding one more decimal for the tabulated quantities, and by making provision for the computation of the perturbations produced by Saturn, as well as of those caused by Jupiter. The necessary coordinates of Jupiter and Saturn have been tabulated for an interval of 20 days for the years from 1930 to 1960.

This is an entirely technical contribution. For computers calculating approximate special perturbations by means of a desk calculator, these tables are certainly a valuable improvement, because they should permit a higher degree of precision than Stracke's tables were able to give. Considering the fact that small computing centers and many individual computers will not be in a position to have automatic machines at their disposal for some time to come, tables such as these still serve a good purpose. The actual value of approximate perturbations as such, of course, is relatively low today, because accurate perturbations can be obtained rather easily with the help of modern automatic computing equipment.

Bazhenov's tables(57) for the solution of Kepler's equation by machines are discussed in the section on Fundamental Equations.

Numerov(509), in 1926, presented a series of extensive auxiliary tables, which are intended to facilitate orbit determinations, as well as the practical application of his own method of computing the disturbed motion of a planet by numerical integration of the special rectangular coordinates. Tables for the computation of the parallax corrections (for 27 observatories), for the coordinates of Jupiter and Saturn, as well as for the determination of the proper interval of integration, are included among the 12 tables compiled by the author.

The main purpose of this publication, to facilitate the practical work of orbit computers using Numerov's extrapolation method, is well served indeed, because, here, the computer finds everything conveniently arranged in one place. Today, the more modern methods used on automatic computing equipment tend to depreciate the value of this work, but it still may be helpful to individuals working with desk calculators.

Proskurin and Mashinskaya(572) (1951) divided the circumference of Jupiter's orbit into 400 intervals of equal length (with respect to the mean anomaly). They then computed the rectangular coordinates x and y, $r^2 = x^2 + y^2$, r^{-3}, and the indirect terms X and Y of the disturbing force to be used in numerical integrations of the motion of a small mass. The coordinates are referred to Jupiter's orbital plane, and the x-axis points towards the perihelion of Jupiter's orbit. The indirect terms X and Y are given not only for the interval w (which corresponds to 1/400 of Jupiter's orbital revolution), but also for the larger intervals 2w and 4w.

This is a simple and purely technical contribution, but the tables obtained may be useful for computations of special perturbations of minor planets and comets. Since the authors give their tabulated data to seven significant figures, a rather high accuracy may be reached in the applications.

A description of Subbotin's formulas and tables(735) for the computation of orbits and ephemerides is given in the section on Fundamental Equations.

Bauschinger's Bahnbestimmung gives the formulas for the computation of the coefficients of the element variations in the most convenient form for logarithmic computations. Subbotin(743) (1937) modified the system of formulas in such a manner that it is convenient for the combined use of computing machines and of logarithms. He also recomputes Schönfeld's table for the auxiliary quantities H and log h for intervals of $0\overset{\circ}{.}1$ of the true anomaly v (Schönfeld's original table gives these quantities in intervals of 10').

Although at the time of its publication this contribution was of some definite value for the determination of near-parabolic orbits, the paper has been superseded in the meantime by modern methods of numerical computation.

f. Bodies of Finite Dimensions

The problem of the motion of a material point under the action of a force producing the acceleration $-\mu_1 r^{-2} - 3\mu_2 r^{-4}$ has been studied by many famous authors. H. Gylden used Jacobi's elliptical functions and found the differential relations between the time, t, and Jacobi's functions. He did not find an integration, and thus the results were not adequate for practical application. V. Strazzeri used Weierstrass's function, p(n), but his method was not very elegant, and it was not practical either. Chernyy(110) (1949) succeeded in an integration of the problem, also using elliptical functions, and found the time, t, as a function of the elliptical integrals of the first, second, and third type, and of the elliptical functions sn w, cn w and dn w, where w is a linear function of the longitude of the moving particle.

The two-body problem is obtained as a limiting case of the new results. Other special cases were investigated in connection with the different possibilities for the roots of the cubic equation which is characteristic for the problem. Rather interesting orbits, such as a lemniscate or a cardoid, are found in certain cases from a consideration of the geometrical character of the orbits. For small values of the modulus, k, of the elliptical integral, the author develops for powers of k, and he obtains the radius vector, r, in the form:

$$r = \frac{p}{1 + k_o + k_1 \cos f + k_2 \cos 2f}.$$

This closely resembles the so-called periplegmatic orbit of Gyldén.

This is a first-quality paper in which significant progress has been made in a special and interesting problem of analytical dynamics.

Duboshin(139) (1931), in a popular article on the history and development of the two-body problem, starts with the time of Laplace and Lagrange and gives a very-well-written account of progress in the theoretical treatment. He leads up to such rather complex cases as the relative motion of two bodies with finite dimensions of the order of their distance. Making reference to the existence of other than Newtonian forces, for example, the physical processes affecting the motion of a comet, the author comes to the conclusion that the two-body problem will still be of interest for some time to come, especially as long as no direct analytical success in the solution of the three-body problem is possible. In this connection, special emphasis is given to the reduction of the perturbation problem in the two-body problem by means of the variation-of-constants method.

From 1936 to 1954, Kondurar' published a series of five papers(297, 299-302) on the motion of two ellipsoids. In the first of these(297) (1936), he considers the special case of two spheroids, wherein the circular equatorial sections coincide with the plane of relative motion of the two bodies. After a detailed investigation of the potential of the mutual attraction of the two bodies, the differential equations of their relative motion are derived. Although all of this work is not new, the author has contributed some original details in the present form of the treatment.

The rest of the paper(297) is devoted to an application of the methods of qualitative analysis of the so-called Moscow School of celestial mechanics, especially of the contact-characteristics method as developed by Moiseyev. Certain restrictions of the possible orbital freedom are studied by this method, and also by means of the zero-velocity curve; the consequences arising from the finite dimensions of the two bodies also are considered. In the final paragraph, the author states that, although the rigorous analytical integration of the problem leads to integrals which are much more complicated than simple elliptic integrals, the qualitative method of contact characteristics is relatively simple, yet demonstrative, as far as the general features of relative motion are concerned.

The analysis presented by Kondurar' seems to be thoroughly correct and competent. The qualitative results, illustrated by eight figures, are of interest. On the other hand, it seems that all these qualitative discussions are not of any help in the actual determination of the orbital trajectory, a problem which apparently can be solved only the "hard" way of rigorous analysis.

Kondurar's second paper(299) was not reviewed.

The third paper(300) (1952) deals with a case which is more general, insofar as ellipsoids with three different axes are considered; but it is more specialized, on the other hand, insofar as it is concerned

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only with the stability of circular motion. After the differential equations of the relative motion of the two ellipsoids have been derived, the special case of circular motion is considered. A certain equation, corresponding to Kepler's third law in the case of two point masses, must be satisfied by the basic quantities and parameters of the problem. The stability of such circular motion is then investigated, using the method and definitions by Lyapunov. Owing to the complexity of the problem, only an approximate study of the characteristic equation and of its roots is made. The author arrives at some results concerning the stability or instability of the motion under certain special assumptions for the basic properties of the two ellipsoids.

This paper of Kondurar' has led to more essential and interesting results than his first paper on a related subject. The mathematical analysis is impressively deep and rather complex. It seems that this author has made an essential contribution to the study of the relative motion of two bodies of finite dimensions, a field which definitely is one of the most difficult ones in celestial mechanics.

In a fourth paper(301) (1952), Kondurar' deals with the same problem as in the first(297), but this time with the help of a more accurate development of the disturbing function. He also makes use of Duboshin's development(162) as employed for the motion of Saturn's satellites under the effect of the planet's figure. The author proves the existence of circular solutions and investigates the stability of these solutions by Lyapunov's method. Stability is found to exist for the scalar value, $\underline{R}$, of the radius of the relative orbit, and for the velocity, $\dot{\underline{R}}$, while the "longitudinal" stability with respect to the angular argument, θ, is not considered and may be nonexistent. The transverse stability is, of course, of main interest. The author then establishes the existence of stable, periodic, near-circular solutions, where stability exists with respect to $\underline{R}$ and $\dot{\underline{R}}$. The angular coordinate, Φ, normally contains a secular term, and only under special circumstances (the coefficient of this secular term must be the form $\frac{p}{q}$) will it be possible to find solutions which also possess longitudinal periodicity and stability.

This is a substantial and highly competent contribution, which reflects well on the competency of this author.

Finally, Kondurar' (302) (1954) treats the special problem where two spheroids are moving within their common plane of symmetry, but, at the same time, rotating around axes which are perpendicular relative to each other. The differential equations of motion are first derived in Lagrange's form, and the methods developed by Duboshin and Lyapunov are then used to investigate the possible periodic solutions and their stability (in the sense of Lyapunov). Circular orbits are found to exist as particular solutions. If the center of inertia of the one spheroid coincides with the origin of the coordinate system, and if the axis of rotation of the second spheroid makes an angle, Ψ, with the direction towards the center of the first spheroid, then it is found that the circular solutions are stable for $\Psi = 0°$ and $\Psi = 180°$, but unstable for $\Psi = 270°$ (only for these four special values of Ψ are circular solutions found to exist). Then proof is given for the existence of an infinite number of periodic solutions which are close to circular solutions. The conditions for the occurrence of such solutions, and the form of the series which represent these solutions, are studied in much detail. It is found that the conditions for stability are, at the same time, sufficient conditions for the existence of periodic solutions.

This paper is definitely on the same high level as the earlier ones. While the first paper (published in 1936) by Kondurar' was only more or less qualitative in its analysis, the following papers were of increasing depth and significance.

In a paper(374) (1950) that could not be located, Magnaradze developed the theory of the potential of an elliptical ring, which may also be of a finite width. He devoted another paper(372) (1950) to the investigation of the convergence of the development for the potential of the mass distribution in certain singular points, which are located on the boundary of the mass configuration itself. The points in question play an analogous part to the points on the boundary of convergence in the region of an analytical function. It is found by the author that the development of the potential is convergent, even in these singular points.

The investigation apparently was influenced or inspired by earlier work by Duboshin(161, 162, 164), even though Duboshin's name is not mentioned. Nevertheless, the author has added his own contributions. Nothing extraordinarily deep is involved, but the author's contribution seems to be the fruit of intimate knowledge of the subject, and of clear and independent thinking on the special problems involved.

Magnaradze[373] (1950), by means of a related dominant function, uses certain analytical inequalities from his previous paper[372] for an estimate of the neglected residual terms in the expansion of the Newtonian potential of an elliptical orbit. The study is limited to the case of $z = 0$, i.e., to field points within the plane of the elliptical mass distribution. The author intended to investigate the more general case of $z \neq 0$ in future publications.

As in the previous papers, this one also apparently has been influenced or inspired by earlier work of Duboshin. Again, however, the author's contribution appears as an original addition to previous knowledge, and he seems to have arrived at his results by independent thinking.

In certain respects, A. A. Orlov's paper[529] (1953) on the motions of a particle under the Newtonian attraction of a spheroid is an extension of a paper by Chernyy[110]. The goal of the paper is to establish, for a force function of the type $U = \frac{\mu 1}{r} + \frac{\mu 3}{r^3}$, periodic solutions for a particle or satellite, when the inclination, i, relative to the equatorial plane of the spheroid is permitted to have any given value. For inclinations close to $i = 0$, the problem has been previously solved in papers by Duboshin and by Brouwer.

The author succeeds in finding periodic solutions for near-circular motion with $i \neq 0$. The developments of the power series for U follows the methods by Lyapunov and Poincaré.

The author deserves credit, first, for giving proof for the existence of such periodic solutions and, second, for determining the analytical expressions for the major terms in the resulting series (in a step-by-step introduction of the higher order terms according to Lyapunov). The paper is an addition to knowledge in the field of periodic solutions.

A. A. Orlov(532), in 1954, then considered an extension of the above results to elliptical orbits. He finds, however, that no rigorous periodic solutions on the basis of fixed elliptical reference orbits are possible in this more general problem. Since the observations, however, reveal the fact that no secular perturbations in the elements a, e, and i of the satellite orbit are produced by the disturbing action of the central spheroid, the author undertook the task of finding developments for the force function which introduce the slow variation of the pericenter of the satellite orbit as superimposed on the basic period of the orbital motion. The developments for the actual motion, by the method of Lyapunov-Poincaré, proceed as power series of a small parameter, α; in turn, α is related to the angular velocity, μ, of the rotating coordinate system, which serves as the reference frame for the coordinates which are of advantage in the present problem. Only the major terms of the theory are evaluated by the successive approximations of increasing order, which are characteristic of Lyapunov's method of development.

The results of this paper are certainly of interest from the practical, as well as from the theoretical, point of view. It seems that the author has competently solved the problem which he wanted to investigate. However, whether Orlov actually "hit the nail on the head", if he is interested in the problem of computing the artificial-satellite orbit, has been questioned.

No new results are contained in A. A. Orlov's paper(531) (1954) on the method of the expansion of the potential of an oblate spheroid of revolution into a series of Legendre polynomials. The derivation of the well-known results, however, is made by a new method of development for the force function of the homogeneous ellipsoid. This is done by certain transformations leading to expressions of the form:

$$\frac{1}{\sqrt{1 - 2hw + h^2}}$$

in the force function, U. Since these expressions can be developed in the form:

$$\sum_{n=1}^{\infty} h^n P_n (W),$$

where the P_n are Legendre's polynomials, U will finally be developed by means of these polynomials. In the first section, a homogeneous spheroid is treated in this manner; in the second section, the same method is applied to a nonhomogeneous ellipsoid formed by shells of coaxial ellipsoids of different densities. In the third section, the first two coefficients of the developments are determined.

As was already mentioned, no new results are presented in this paper. Only the little transformations leading to expressions of the above form in the force function, U, represents the author's own contribution in this paper. No extraordinary effort was necessary in order to find these transformations. Therefore, although the somewhat

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different derivation of well-known results deserves some interest, considered as a whole, this is a rather moderate contribution.

Reyn(641) (1940) investigated the regions of possible and impossible motion for a small point mass moving (without friction) inside a homogeneous spheroidal mass distribution and in the additional gravitational field of a central mass. She also studied the contacts of the trajectories with a family of circles according to Moiseyev's method. The motion is limited to the equatorial plane of the spheroidal mass distribution. Following essentially Chibisov's, Moiseyev's, and Tarasashvili's earlier ideas and methods, the essential characteristics of the problem are considered in detail and the results are illustrated by a number of graphs. Families of trajectories are considered in dependence on the parameters h (energy constant) and P (constant of the integral of areas), and the related characteristics of the apsides, as well as of the associated regions of possible and impossible motion, are determined.

None of the essential elements of this paper is new in itself. The paper is of some interest, however, insofar as a systematic representation of the various features, and some new minor details, are given. Altogether, this paper is useful, because it is more or less comprehensive with regard to this particular problem, but it certainly is not of above-average value.

Shchigolev(694) (1936) presents a mathematical investigation of the surfaces of equal potential which are associated with a rotating ellipsoid formed as the figure of equilibrium of an ideal, incompressible fluid. This rotating body may have three different principal axes, and the characteristic features of the various surfaces of equal potential outside the rotating body are basically determined by the directions and sizes of these three axes, as well as by the rate of rotation and the (homogeneous) density of the fluid. These equipotential surfaces are studied in much detail by means of the analytical expressions for the potential and its derivatives. Of special interest are two singular points on the x-axis and two similar ones on the y-axis, these two axes being coincident with the two principal axes of the ellipsoid which are perpendicular to the axis of rotation. These singular points represent minima of the total potential in the directions of the coordinate axes; while in three dimensions, they represent the point of certain conic surfaces. The equipotential surfaces are classified as functions of the potential and, for their computation, the author reduces the necessary formulas, involving rather complicated elliptical integrals, to functions of the elliptical normal integrals of the first and second type. Tables are given to facilitate numerical evaluations.

This is a rather thorough and complete analysis of the given subject, even though none of the basic elements of this discussion is, by itself, new. Although it is doubtful that all these details are actually of interest in connection with the motion of celestial bodies, some of them may today be useful in studies of the motion of close or artificial satellites, or of cosmogonical problems. In any event, the author deserves credit for a mathematically sound and penetrating analysis.

g. Resistant Medium

Agrest's[7] (1945) treatment of the stability of free solution of the restricted circular problem of three bodies in a resistant medium is presented in the section on Stability.

In one of Duboshin's earliest papers[140] (1932) on motion in a resistant medium, the differential equations for the relative motion of two point masses are augmented by a term for the effect of the resisting medium, which is not supposed to exert a noticeable gravitational action by itself on the two masses. Although no closed integrals are possible, in general, because of the nonconservative nature of the additional force of resistance, certain "quasi" integrals take the place of the well-known energy integral and of the law of areas, and these make it possible to arrive at certain conclusions concerning the character and the form of the orbital trajectories. A number of theorems are obtained on the basis of these quasi integrals, leading to certain conclusions about the limits and characteristics of the motion under various assumptions for the initial or starting conditions.

Only rather elementary considerations are involved in this paper; nevertheless, the well-founded formulations of the various theorems on the general properties of this type of motion are of definite value for further progress in this field. The various conclusions are of general interest, even though no large mental effort was necessary for their derivation.

Also in 1932, Duboshin[141] studied the relative motion of one mass in relation to a second, assuming that the resistant medium has no effect on this second mass at the origin of the coordinate system. The effect of the medium on the motion of the first mass is introduced into the differential equations of motion by the corresponding proper terms, where λ appears as the coefficient of resistance, multiplied by the velocity and by the inverse square of the distance, r, from the center of the coordinates. Proceeding to relations which take the place of the general integrals in the case of the ordinary two-body problem, the author then studies the consequences of his expressions for the trajectories under different starting conditions. He finds that, if a lower limit exists for the distance, r, between the two masses, then later on, $r \longrightarrow \infty$ as $t \longrightarrow \infty$, and that this motion toward $r = \infty$ will be asymptotic with respect to a special value of the angular coordinate, ω. The trajectory in this case is a spiral ending in an asymptote. This orbital possibility is contrasted with the other one, where the moving mass gradually approaches the central mass until it finally collides with it. The velocity with which the collision occurs is finite and of the order $\sqrt{\frac{1}{\lambda}}$.

Although the problem considered is a rather special one, the results are of some interest. On the other hand, results of this kind (considering the qualitative nature of the essential conclusions) had to be expected and, insofar as this is the case, nothing extraordinary emerges from the investigation. Nevertheless, this paper had a positive value at the time it was written.

Duboshin(151) (1936) applies Lyapunov's theory of stability to the study of the motion of a particle that is moving under the

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influence of the attracting force of some central body possessing a gravitating, rotating, and resisting atmosphere. It is assumed that the surfaces of equal density of the body and the atmosphere are similar surfaces of rotation around the axis having the same plane of symmetry.

This is another of Duboshin's works that illustrates the use of Lyapunov's theory of stability in the motion of celestial bodies. It is a well-written paper. The methods used are known to U. S. mathematicians and frequently used by them for the study of the stability of motion.

The following year (1937), the motion of a material point in the field of the Newtonian attraction of a stationary body was considered by Duboshin[153], who assumed that the mass of the stationary body changes with time, and that this body is surrounded by an atmosphere having a spherical structure and a density which varies with time. The gravitational action of the atmosphere is taken into account, as well as its resistance to the motion of the material point. Assuming that the total mass (body and atmosphere) is constant, but that the central body has a slowly changing mass, the elements of the osculation of the motion are examined. The motion here is taken as the disturbed Keplerian motion. A method is given for the determination of the elements of osculation in the form of converging series, arranged in ascending powers of a small parameter. Analytical expressions are given for the first approximation in the case where the density of the atmosphere and the function of the resistance are given in particular forms.

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Then, Duboshin(156) (1939) considered the motion of a particle under the gravitational action of a fixed mass in the center of the coordinate system, and under the effect of a resistant medium with spherical symmetry and with a density inversely proportional to the distance from the center of symmetry. In this case, the disturbing force which can be associated with the disturbing action of the resistant medium is colinear with the tangent of the orbital trajectory, and its direction is opposite to the velocity vector of the particle. The resulting motion takes place in a fixed plane, and the problem can be solved by quadratures leading to so-called pseudointegrals, as compared with the actual integrals of the undisturbed two-body problem.

The first part of this paper is merely a specialization and modification of the author's earlier paper(141), leading to the theorem that the orbit of the particle will never intersect itself. Assuming, then, that the resistant medium has only a limited (spherically symmetric) extension, the author considers the possibilities for "capture" and "ejection" of a particle moving initially in an orbit partly outside and partly inside the medium. Depending on the starting conditions, the capture as well as the ejection may be "relative" (temporary) or "absolute" (permanent); the author arrives at certain qualitative conclusions concerning the conditions under which the one or the other possibility will be realized.

The contribution is of some interest with regard to possible cosmogonical applications. Although the derivations are simple from the mathematical point of view, the results certainly deserve the attention of workers in the fields of such cosmogonical problems.

Zamorev[844] (1936), in his study on the motion of two bodies in a resisting medium, assumed that both masses are moving relative to the resisting medium. In this respect, the problem is more general than the related problem discussed by Duboshin[141], where the central mass is assumed to be at rest, at the origin of the coordinates, relative to the resisting medium. As the author states in the introduction, the present, more general problem cannot be reduced to the more simple one. In the present case, only three general integrals for the motion of the center of gravity can be obtained, instead of the six in the more simple case.

In the first section of his paper, Zamorev presents the differential equations of the problem and the first integrals. The second section contains the determination of certain equations which are needed for the study of the motion: the modified or quasi-energy integral and the equation for the moment of inertia of the system. In the third section, a qualitative analysis of the motion is made on the basis of the quasi-energy integral and of the integrals which exist for the center of gravity. The author finds that the distance between the two bodies remains limited (proper starting conditions being assumed), that the masses will collide only with $t \longrightarrow \infty$, in the general case, and that a collision within a finite time interval will occur only in the special case of a straight-line approach of the two bodies. The fourth section of the paper is devoted to the case of equal masses, i.e., $m_1 = m_2$. In this case, all six integrals exist for the motion of the center of gravity, while the general picture remains unchanged. If relative coordinates are used (the previous investigation employed absolute coordinates referred to a system which was especially convenient in connection with the motion of the center of mass), then the integrals for the law of areas can be obtained, too.

The paper is limited to the derivation of the general framework for studies of orbital motions in a resisting medium. The conclusions, in their generality, have been reached in a relatively simple manner. Although the paper presents correct results, this contribution evidently is not outstanding or too significant.

2. Three-Body Problem

a. Disturbing Function

The regions of convergence of the developments of the disturbing function in dependence on the powers of the eccentricities e and e' of the disturbed and of the disturbing planet are different for the mean, the eccentric, and the true anomaly as independent variables entering the Fourier series. Samoylova-Yakhontova[666] (1939) derived a criterion, in the form of an inequality, for the occurrence of divergence of the development for the reciprocal of the distance, Δ, in the case where the Fourier expressions depend on multiples of the eccentric anomaly. By a comparison with the analogous criteria of Sundman and Banachiewicz for the mean and the true anomaly, respectively, she finds that the best convergence with respect to the eccentricities will be obtained by using the true anomaly, and that the eccentric anomaly occupies an intermediate position in this regard between the mean and the true anomaly. The author's criterion for divergence is first given taking into account only the first power of the two eccentricities involved; the relation is then refined to the second order with respect to e and e' by the addition of the necessary terms.

This investigation is of definite interest from the theoretical point of view, as well as in connection with practical applications; on the other hand, it was not too hard to proceed to these results, after the analogous criteria by Sundman and Banachiewicz existed for the mean and true anomalies.

Other studies by Samoylova-Yakhontova related to this topic are presented in the section on Regularization and Collision.

M. F. Subbotin(747) (1943) contributed a rather ingenious method to improve the convergence of fundamental trigonometric series in celestial mechanics. Its value lies in the generality and flexibility of the method. The method is based on analytical transformations of the complex variable z into the new variable w in the form:

$$\prod_{n=1}^{m} \frac{w - \alpha_n}{1 - \alpha_n w} = \prod_{n=1}^{m} \left(\frac{z - \beta_n}{1 - \beta_n z} \right).$$

By this transformation, convergence of the trigonometric series:

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$$f(H) = \sum_{-\infty}^{+\infty} a_k \cos kH = \sum_{-\infty}^{+\infty} a_k z^k, \; z = e^{iH}, \; (a_{-k} = a_k)$$

is greatly improved, depending on the proper choice of the quantities α_n and β_n. Subbotin shows that the particular transformation by Legendre, which had been used by Brendel for the general perturbations of minor planets, as well as for the major planets, is a special case of his general method, which also incorporates developments that converge in the same ways as those obtained by Gylden's transcendental transformation.

Subbotin(748) (1947) applied his method for the improvement of the convergence to the investigation of two special transformations. In both cases, the angular variable, H, in the development

$$\Delta^{-2s} = \sum_{-\infty}^{+\infty} b_n^{2s} \cos nH,$$

where $\Delta^2 = 1 - 2\alpha \cos H + \alpha^2$ (2s is an odd number), is first transformed into a complex variable, z, by means of $z = e^{iH}$. The first special transformation considered in this paper then transforms z bilinearly into w by means of $w = \frac{z - \beta}{1 - \beta z}$; the second one is the transformation by Legendre, which is equivalent to $w^2 = \frac{z(z - \alpha)}{1 - \alpha z}$. The author succeeds in expressing the coefficients of the new developments as functions of the well-known coefficients by Laplace, depending on a smaller ratio, α, than in the case of the original developments by Laplace. This increases the convergence of the series up to a factor of four. Subbotin's transformation contains the one by Legendre as a special case.

This paper is a very interesting contribution to the convergence problem. It is clear and mathematically elegant and, undoubtedly, it is one of the more significant recent contributions to celestial mechanics.

Also in 1947, Subbotin(749) concerned himself with the actual computation and tabulation of the coefficients of the developments which are obtained in the cases of the two transformations. As far as the transformation by Legendre is concerned, this had been used by Brendel, but Brendel had found the necessary coefficients by interpolation processes; in the present paper, they are found analytically and independently. The developments are limited to those of Δ^{-1} and Δ^{-3}, which are needed for the perturbations of the first order.

The coefficients are tabulated in a very convenient form, and the author deserves credit for further facilitating the practical application of these methods, after first contributing essentially to the basic theory in the two previous papers.

N. D. Moiseyev concerned himself with the introduction of "averaging" procedures into the development of the disturbing function. Taking the point of view that the "average perturbations" suffered by the disturbed body will depend on the average relative positions of the disturbing and the disturbed mass, as they affect the analytical expression for the disturbing function, Moiseyev(467, 470)(1945 and 1951)

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considered the various procedures of "averaging" which one may want to introduce into the disturbing function in connection with individual problems. Since the disturbing function depends on trigonometric functions of multiples of the longitudes or mean anomalies of the two planets, his main concern was the elimination of these variables by a proper procedure of averaging. If W_J is the disturbing function produced by the gravitational action of Jupiter, as it influences the motion of a minor planet, and if $\underline{M}$ and M_J are the mean anomalies of the minor planet and of Jupiter, respectively, then in direct application of the related method of Gauss, one has the "two-time average" (according to Gauss) of W_J in the form:

$$[W_J]_G = \frac{1}{4\pi^2} \int_{M=0}^{2\pi} \int_{M_J=0}^{2\pi} W_J \, dM \, dM_J .$$

The author considered averages with respect to only one of the two bodies, as well as averages with respect to both planets (two-time averages), but in each individual case, this was done very similarly by a proper single or double integration and average formation with respect to the basic angular variables. Moiseyev applied his "averaging" procedures first(467) (1945) by assuming circular motion for the disturbing planet (Jupiter), and second(470) (1951), by introducing the "averaged" disturbing function into the differential equations for the variation of the elliptic elements. These investigations of Moiseyev are accomplished by relatively elementary mathematical procedures, but the results are of interest for many problems where a first-order accuracy is sufficient. There is no doubt that a very competent man did this work and presented it in a clear and systematic fashion, although perhaps with too many minor details.

These same remarks apply to Moiseyev's consideration of what he calls the "half-restricted" problem. This problem concerns the motion of two point masses relative to a third which is of much greater mass, which means that the action of the first two on the third can be neglected(471) (1952). By means of the integral of areas, the orbital parameter, p_1, of the one moving mass, m_1, can be eliminated. The author applied his method of "averaging" to the disturbing action, introducing the various possibilities for doing this similar to the way he did in his earlier paper(470). The process of averaging eliminates the corresponding periodic terms from the disturbing function and leads to certain integrals of the simplified differential equations. Of special interest is the elimination of certain periodic terms depending on the mean anomalies, M_1 and M_2, of the two masses in the form of angular arguments $D = k_2M_2 - k_1M_1$; this is similar to the elimination method employed by Delaunay. The author called this "averaging according to Delaunay-Hill".

Moiseyev(472) (1954) demonstrated that his method of "averaging" the disturbing function is applicable not only to the elimination of the terms depending periodically on the mean anomalies and other angular variables, but also to the corresponding effects of the variations of all the orbital elements. He used the canonical form of the differential equations and the related elements. The simplifications resulting from the author's averaging procedures, applied either once (with respect to one planet) or twice (with respect to both planets involved), lead to certain integrals which he calls "empirical integrals".

As in the case of the other papers by the author on the same basic subject, the investigation leads to results which can be used to find the roughly averaged perturbational variations in individual problems. Also, intermediate orbits may be based on the "empirical" integrals, to serve as a closer first-order approximation on which the determination of the finer details of the disturbed motion may be based.

In general, the same comments can be made about Reference 473 (1954) as about the earlier papers. It appears that Moiseyev's desire to exploit all the possible applications of his averaging method led him to indulge in a somewhat monotonous and mechanical repetition of the same principles over and over in these articles. It does not seem so important to investigate all these slight modifications in so much detail. One cannot escape the impression, after reading all these papers on the same basic idea, that his desire was to publish as much as possible, thus somewhat diluting the good impression which the first paper on the subject, which presented the one essential idea, originally made.

Yarov-Yarovoy[(823)] (1954) applied Moiseyev's method of elimination of certain periodic terms from the disturbing function by introducing the "average" effect to the actual orbits of the first 10 minor planets. Although nothing actually new or surprising has resulted from this practical application of Moiseyev's ideas, the paper is nevertheless an interesting example of the possibilities, as well as of the limitations, of these approximating methods.

For each planet, he takes the available osculating element systems, over a considerable length of time, and averages the actual perturbations suffered by the mean anomaly, M, by the perihelion, π, and by the node, Ω, by determining the coefficients s_1, s_2, and s_3 from a least-squares solution of the system of equations:

$$\mu = s_1 M + s_2 \omega + s_3 \bar{\Omega} .$$

Here, M, ω, and $\bar{\Omega}$ are the related values of the mean anomaly, of the angular distance of the perihelion from the node, and of the distance of the node from the longitude of Jupiter at the given moment, respectively. The least-squares solution attempts to make μ a constant, as nearly as possible. The factors s_1, s_2, and s_3 obtained from this solution will then determine the average secular changes or variations of the three angular quantities, M, ω, and $\bar{\Omega}$, or s_1, s_2, and s_3 give the correlation coefficients between the three angular variables. For the 10 planets under investigation, the author finds a pronounced correlation between M and $\bar{\Omega}$, which means that all the perihelia have secular motions in the same direction, counterclockwise. Of course, the quantity μ will not actually come out as a constant, and the degree of correlation between the various angular variants depends on the scattering of the "interpolation anomaly", μ.

Reznikovskiy applies the process of "averaging" to the circular restricted problem[651] (1952) and to the more general case of an

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elliptical orbit(652) (1952). In the first paper(651), the process of "averaging", leading to the elimination of certain arguments in the expressions and developments for the disturbing function, is applied to the disturbed planet's mean anomaly as an "internal variant". The main interest is focused here on the elements e and ω, or eccentricity and longitude of the perihelion; for the mean distance, the integral a = constant is obtained under the basic provisions. Instead of e and ω, however, the corresponding Lagrangean elements h and l are used. The author succeeds, in the developments for the averaged disturbing function in dependence on the osculating elements, in obtaining polynomials of a form which depends on certain functions that had been studied previously by Duboshin and Yelenevskaya. This makes it possible to utilize the related results of these two authors. The developments are studied also for the case of the synodical elements of Lagrange and, here, the results have a more simple form.

The investigation incorporates an interesting series of transformations. Otherwise, the goal of this investigation could probably be reached more easily by means of Brown's method of development of the disturbing function, depending on the true anomaly. In this case, the operators of Brown's method would take the place of the rather elaborate recurrence formulas in the Reznikovskiy's method. Nevertheless, the author has demonstrated ability and imagination.

In the second paper(652), Reznikovskiy uses perturbations in the polar coordinates, instead of the variation of elements, as he did in the earlier paper. The disturbing function is developed in dependence

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on Legendre's polynomials. The author then considers the process of "averaging" with respect to various variants, such as the mean anomaly of the disturbing planet or of the disturbed planet. The essential transformations are the same as in the previous paper. The polynomials previously studied by Yelenevskaya are used again with advantage for the coefficients of the developments, as far as they depend on the eccentricity, $\underline{e}$. One paragraph deals with the so-called two-time averaged problem of Gauss and the purely secular part of the perturbations.

This paper constitutes a considerable extension of Reznikovskiy's previous study, and he deserves credit for the results, which are certainly of some interest. Although nothing of basic significance is involved, papers such as this do enrich the detailed knowledge concerning the many features and aspects of the perturbation problems.

One special process of "averaging" is the replacement of a disturbing planet by a circular or elliptical ring of matter. Some of the papers by Duboshin, Magnaradze, and Tarasashvili are concerned with this problem. Magnaradze's investigations[372-374] (1950) apparently have been influenced or inspired by Duboshin[161, 164, 167], even though Duboshin's name is not mentioned. Nevertheless, Magnaradze's papers also reflect an intimate knowledge and independent thinking on the problems involved.

In the first of three papers[374], Magnaradze developed the theory of the potential of an elliptical ring, which may also be of finite width. The convergence of the development for the potential of the mass distribution in certain singular points, which are located on the boundary of the mass configuration itself, is investigated in Reference 372. The points in question play an analogous part to the points on the boundary of convergence in the convergence region of an analytic function. It is found by the author that the development of the potential is convergent, even in these singular points. Later(373), Magnaradze uses certain analytical inequalities from the previous paper(372) for an estimate of the neglected residual terms. This is done by means of a related dominant function. The study is limited to the case of $z = 0$, i.e., to field points within the plane of the elliptical mass distribution. The author intends to investigate the more general case of $z \neq 0$ in future publications.

For the gravitational action of the various planets in the solar system on a comet, when the latter is outside all the planetary orbits, Tarasashvili(757) (1939) replaced the disturbing planets by circular rings of homogeneous density (problem of Fatou). He then considered the variation of the comet's semiaxis, $\underline{a}$, or of $\frac{1}{a}$, as the comet departs from the solar system. Making use of the fact that a comet departing from the solar system in a hyperbolic orbit will asymptotically approach a certain fixed longitude and latitude in the sky, as seen from the sun, the author bases his conclusions on the dependence of $d(\frac{1}{a})$ on $d\underline{R}$ alone. He finds that the negative $\frac{1}{a}$ will increase as the distance, $\underline{R}$, from the sun increases. Only the unique case where, for an inclined cometary orbit, the rectangular distance, $\underline{z}$, from the invariable plane of the major planets does not increase simultaneously with $\underline{R}$ as the comet departs farther and farther from the center of the solar system is excluded. A numerical example illustrates the fact that a hyperbolic orbit can become parabolic in this way, by means of the asymptotic decrease of a (a is a large negative quantity in the case of a hyperbolic orbit, and the author finds $a \longrightarrow -\infty$). The author mentions that his results agree with the results of the computations by Thraen, Faye, and E. Stroemgren, and that his treatment follows a proposition previously made by E. Stroemgren.

The paper is a good contribution to the interesting problem of moderately hyperbolic (or parabolic) orbits, that is intimately related to the problem of the origin of comets; however, it makes use of rather elementary methods. It is of some interest that the author mentions the circumstance that his paper was a "competitive contribution in connection with the 18th Convention of the (Communist) Party".

Duboshin[163] (1946), after considering the various requirements which should be met by a satisfactory method for the development of the disturbing function, proposes and develops a new form for the development of the reciprocal of the distance, making use of cylindric coordinates. Introducing Legendre's polynomials and Gegenbauer's more general expressions, the necessary expansions are made in a rather elegant and simple manner.

This paper is a good contribution to a special field, even though the theory has not yet been completed to allow actual application to the motion of a celestial body. It is possible that utilization of the new method for a complete theory may create some problems which might partly offset the advantage of this development. Nevertheless, this investigation has its theoretical merits. Of special interest is the investigation of the regions of convergence.

The results obtained by Duboshin(169) (1950) on the expansion of the force functions were well known before, but he derived them in an interesting new way by introducing the negative powers of the "applicate". The "applicate" is an expression used by Soviet mathematicians for the z-coordinate, or the rectangular distance of a given point from a given basic plane. He first considers the attraction of an infinitely narrow ring of matter on a point outside the plane of this ring, and he develops the potential, U, into a series depending on the negative powers of z. He then proceeds to a ring of finite width, and the results can be extended then to a circular disc and, finally, to the case of a body possessing rotational symmetry. An application is made also to the special case of an elliptical ring, used in Gauss's theory of secular perturbations.

Numerov(520) (1935) developed the disturbing forces of the planetary perturbation problem in dependence on four angular arguments in the trigonometrical functions, which are essentially the true anomalies and longitudes of the disturbed and the disturbing planet. The general expressions for the coefficients, depending on the ascending powers of the eccentricities, of the mutual inclination, and of the ratio of the orbital parameters, are given, including the third powers of the small quantities e and i, and the comparable powers of the ratio, α, of the parameters p and p'. It is assumed that α^3 is of the order of the eccentricities and inclinations.

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These general expressions are useful for the establishment of individual theories, even though three of the angular arguments will finally have to be eliminated in favor of one independent variable. Although this elimination is given in the author's later paper[519], it may be desirable from time to time to use the unreduced form of the expressions in the earlier paper; therefore, the equations, as collected in this publication[520], are of permanent value to workers in this field. Nothing essentially new is contained in the theoretical details, but a competent author provided a convenient scheme for the application of the theory to individual problems.

Numerov[519] (1935) continued the subject of general formulas for the development of perturbing forces in the calculation of absolute perturbations in polar coordinates, reducing the developments now to two angular arguments. Since the second of these two arguments is a linear function of the true anomaly of the disturbed planet (which serves as the first argument), the disturbing force is actually obtained as a function of the true anomaly of the disturbed planet as the only independent argument.

Newcomb's original method for the development of the disturbing function, along with later modifications and additions by various authors, is contained in a paper by B. A. Orlov[539] (1936). Apparently, the main purpose of this publication was to make the method available to Soviets in Russian print.

Yelenevskaya[829] (1952) claims that her method for the expansion of the perturbation function in a Fourier series with respect to the inclination is of advantage compared with the methods using the powers of $\sin^2 \frac{1}{2} I$. She states the fact that, in the existing developments of the disturbing function, generally, the inclination as well as the eccentricity have been introduced in the analytical form of their ascending powers. For the inclination, the powers of $\sin^2 \frac{1}{2} I$ have been used, where I is the relative inclination between the planes of the disturbing and of the disturbed planet. The author proposes to develop for I in the trigonometrical form, similar to the treatment of the longitudes or of the difference between the longitudes of the two planets. The developments are made by elaborate but elementary transformations, using the integral expressions for the necessary coefficients, $A_{pqr}^{(k)}$, of the resulting trigonometrical series. As far as the ratio between the mean distances, a and a', enters the developments, the proper coefficients by Laplace are used. The main part of the disturbing function is developed first under the assumption that the orbits of both planets are circular (restricted problem: the mass of the disturbed planet is assumed to be negligible), and then the effect of the eccentricity, e, of the disturbed planet is taken into account by means of Newcomb's method of symbolic operators for these additional variations. Finally, the second part of the disturbing function is more easily developed along the same lines, and she gives a tabulation of the various numerical coefficients which appear in the general expressions for the trigonometrical development of the disturbing function.

The following year[831] (1953), Yelenevskaya used her functions $J_n^*(\alpha, x)$ (see Reference 825 in Series Development and Convergence

Problems), as well as the principle of developing the disturbing function in dependence on trigonometrical expressions of multiples of the inclination, rather than on ascending powers of *i*, for a complete application to the restricted three-body problem. Since the inclination is introduced, the small body has the freedom of three-dimensional motion. Her method, which replaces the well-known Newcomb operators by her new functions $J_n^*(\alpha, x)$, has the advantage that, in combination with the introduction of Fourier series depending on multiples of the inclination, one needs only the Laplace coefficients for $\frac{1}{\Delta}$, but not those for the higher powers of the reciprocal of the distance. This advantage is offset somewhat by the occurrence of additional developments and recurrent formulas for the determination of the coefficients. It is true that the author is able to give an expression for the general term of the developments by her method, but, apparently, she was not aware of the fact that this is also possible in the case of Newcomb's method using operator symbols, as Brown has pointed out. She gives no references to Brown's investigations.

It can be said that this is a valuable paper; its principal merits are the possibility of giving an expression for the general term of the developments and the circumstance that only a limited range of Laplacean coefficients is needed. The actual effort required in applications of the new method may not differ very much from the amount of labor which is necessary in the case of Newcomb's method.

Bazhenov[68] (1955), by more or less elementary procedures, explored the details of one of the typical series, $(1 - 2h \cos z + h^2)^{-\frac{n}{2}}$, in celestial mechanics. The limits are investigated between which the Laplacean coefficients of the development are located, and he succeeds in the derivation of expressions for these limits, which are more narrow than the ones that have been used before. Graphs are given which indicate how far in the development one has to go in order to reach a certain desired accuracy, i.e., how many terms have to be included for a prescribed degree of numerical precision.

The results are obtained by more or less elementary procedures, but the paper, as a whole, is of definite value for the worker in celestial mechanics. It represents a good and competent contribution, exploring the details of one of the series which are so typical in celestial mechanics.

Dirikis[120] (1953), in his work on the determination of the original orbits of long-period comets, derived a much-simplified expression for the upper limit of $\Delta\left(\frac{1}{a}\right)$, where $\Delta\left(\frac{1}{a}\right)$ is the neglected part of the perturbations of the reciprocal of the mean distance of the comet from the sun. Using this upper limit expression, one can estimate the maximum error which is introduced by the discontinuation of the numerical integrations at a certain moment when the comet was relatively far from the sun. In other words, after going backwards in time to a certain distance of the comet from the sun (before it approached the inner region of the solar system), one can conclude that the original value of the mean distance, a, at a time when the comet was at its maximum distance, cannot differ by more than a given, very small amount from the value which has already been obtained by the backwards integration of the comet's motion.

The essential idea of such an estimate of the error in the original value of a or $\frac{1}{a}$ is from E. Stroemgren, and the present paper represents only a rather direct and primitive application of the basic idea, illustrated by some computations for certain assumed actual cases.

b. Periodic Orbits; Absolute Orbits of Chebotarev

Schwarzschild, in his investigations, dealt only with periodic solutions in the orbital plane of Jupiter. Yu. V. Batrakov[45] (1955) investigated the periodic orbits which are valid for the case of three-dimensional motion of the small body in the so-called restricted problem. Characterizing the deviation of the actual periodic orbit from the original reference orbit by the small parameter μ, the author derives an equation of condition between μ and the orbital eccentricity, e, and the inclination, i, of the basic reference orbit. He finds that for circular reference orbits within the plane of Jupiter's orbit, periodic solutions are possible only if $p - q = 3, 5, 7, \ldots$, where $p : q$ is the ratio of commensurability between the daily motions of the minor planet and of Jupiter. In the general problem with finite eccentricities and inclinations, Batrakov finds periodic solutions for the special case $p : q = 3 : 1$, as related to the Hestia group of minor planets. The resulting solutions, however, have either a very large inclination or a very large eccentricity; therefore, these Schwarzschild-type solutions are not practically applicable to actual planets of the Hestia group.

The periodic solutions as studied in this paper have a rotating line of nodes, as compared with the rotating perihelion in the case of

Schwarzschild's own solutions; otherwise, the general features of the results are similar to the ones by Schwarzschild. The result of this investigation is negative, but, nevertheless, the finding of these facts is of interest as an addition to knowledge about periodic solutions.

Also in 1955, Batrakov(46) made a relatively small but interesting contribution concerning the properties of periodic solutions of the third type in the general problem of three bodies. Using Jacobi's relative coordinates and canonic variables according to Delaunay, he investigated the relations and equations of condition between the variations of the various quantities which characterize a periodic solution of the third type. The relation $\Delta T = \nu\mu$ is used for the introduction of the arbitrary variation of the period, T, where μ is a small parameter such that for $\mu = 0$, the initial periodic solution of the undisturbed case is obtained. The author finds the proper relations which are valid between the eccentricity, e, and the inclination, i, depending on the arbitrary value of ΔT.

This paper is of some interest, insofar as ΔT serves as the independent parameter. This permits the study of the consequences of a small change in the period, T, on the character of the periodic solutions.

G. A. Chebotarev's(92) (1950) lengthy paper on the theory of periodic orbits in celestial mechanics is primarily a representation of methods and results which are well known. One gets the impression that the author was concerned mainly with writing a complete "handbook" for Soviet astronomers and mathematicians working in celestial mechanics, in order to make the investigations by Poincaré and Schwarzschild on periodic solutions accessible to those who can read only Russian text.

The report contains also the author's own contributions to this field, which have been published in more detail in separate papers(93, 94, 99). Considering all these facts, the present article has no more value than any summarizing review.

The investigation by Chebotarev(93) (1951) on the application of periodic orbits to the study of the motion of minor planets is relatively simple from the theoretical point of view, because numerical processes are used extensively. On the other hand, it represents a considerable and interesting effort by the author and a successful application of periodic solutions for a first-order theory of certain minor planets. In this work, Chebotarev utilized some periodic orbits established by K. Schwarzschild, Poincaré, and others as intermediary orbits for certain minor planets which are moving in the neighborhood of the ideal periodic solutions.

In the first chapter, the author starts with a short review of the three types of periodic solutions investigated by Poincaré. He gives a table listing possible cases as they would be of interest for the motion of minor planets. Although Poincaré did not introduce secular perturbations of the line of apsides, Schwarzschild's approach makes use of a moving perihelion. Another table, therefore, classifies the periodic solutions according to Schwarzschild. The author states that the treatment by Poincaré and Schwarzschild is too specialized for practical applications. He then considers the numerical investigations

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by which periodic solutions have been found by Darwin and at the Copenhagen Observatory. The author concludes that the most promising approach for practical applications would be to find a Schwarzschild solution which agrees approximately with the elements of the given planet and then to compute first-order perturbations by numerical integration (Stracke's method for the variation of the elliptical elements). Accordingly, he performs such an integration for the special case of the 2 : 3 commensurability with Jupiter's motion, and establishes the periodic orbit by a method of interpolation. A similar method was used for the periodic solutions that were found in the somewhat different studies made at the Copenhagen Observatory. The results, in the form of the elliptical elements a, e, M, and π for Jupiter's longitude λ_1 as the argument, are listed in Table 7 of the paper.

In the second chapter, the author defines the perturbations of the planet (153) Hilda as the changing deviations of the planet from the basic Schwarzschild solution as established in Chapter I. The secular terms in longitude and perihelion are included in the basic orbit and, in the disturbing function, R, only the secular terms are considered (including those terms of long period which become secular because of the rigorous commensurability 2 : 3). For the variations of the elliptical elements, which are transferred into Lagrange's elements h, l, p, and q, a system of linear equations is established, the constant coefficients of which are derived by numerical computations (calculating the disturbing force or the derivatives of the disturbing function as they enter the various equations). The actual ellipticity

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of Jupiter's orbit is introduced by corresponding corrections to the basic solution for the restricted problem. All the computations are first made with the elements of the basic Schwarzschild solution; then, the resulting perturbations are used for a representation of the observations of (153) Hilda from 1875 to 1949 (12 selected positions) and improved elements are derived. The perturbations are then recomputed in a second approximation, and, again, the elements are adjusted for the final representation. The agreement between theory and observation over these 74 years is of the order ± 0ˢ5 in α, with one exception which is -0ˢ8.

In 1951, Chebotarev[94], with the assistance of others, applied his method (cf. Reference 93) to the Hestia type or 1 : 3 commensurability; in the earlier investigation, it was established for the 2 : 3 commensurability of the Hilda type.

After the determination of the basic absolute orbit, the variations from this basic orbit are determined for the planets (46) Hestia and (495) Eulalia. For (46) Hestia, the observations are finally represented within ± 0ˢ19 over an interval of 89 years. For (495) Eulalia, the corresponding residuals are within ± 0ˢ17 over a period of 46 years.

Although considerable numerical work was done for this publication, the paper is only a more or less mechanical application of the method developed earlier by the author. The results demonstrate, however, that the work has been done with all the skill of a competent, able worker in celestial mechanics.

In a later paper[96] (1953), Chebotarev's goal was the representation of the disturbed orbit of the minor planet Hilda as a variation of the nearest periodic orbit. Again, he uses as a basis the results presented in his 1951 paper[93]. He does not attempt to use the results for ephemeris purposes, but, rather, to obtain a general picture of the long-range orbital variations. After he finds the closest periodic solution on the basis of the given initial elements of the planet Hilda, the changing osculating elements of this orbit are established by numerical integration, using the variation-of-constants method according to Störmer. The results of this integration are presented in his Table 1. Then, the deviations of the actual orbit from the periodic orbit are introduced in their general form, including terms of the second order which had been neglected in the author's earlier paper[93]. The coefficients of the differential equations for these variations of the Lyapunov-type depend on the derivatives of the disturbing function; and the "averaged" values of these derivatives, and consequently of the coefficients of the linear differential equations for the orbital variations, are finally computed on the basis of mean elements. The equations (23) of his paper represent the integrated expressions for the disturbed motion of the true planet in the form of trigonometric terms multiplied by exponential functions of time. The resulting osculating elements of Hilda are then computed from these results for three dates in the years 1925, 1938, and 1950, and a comparison is made with the more rigorous numerical integrations obtained by Hirayama and Akiyama for the period from 1875 to 1956. The resulting differences in the various elliptical elements at the three chosen dates are listed in his Table 7. The deviations are quite substantial. Then, Chebotarev drops the terms of higher order in the coefficients of his differential equations for the variations, except in the absolute terms, and repeats the comparison with Hirayama's and Akiyama's results. The deviations are of the same order as before, thus indicating the relative unimportance of the corresponding higher order terms in the coefficients.

In the next paragraph, he reduces the problem to the restricted three-body case, i.e., he assumes a circular orbit for Jupiter. The period of the solution is 8,620.0 days, or approximately two revolutions of Jupiter in its orbit. The basic periodic orbit plus the analytical expressions for the variations of the true, osculating orbit from the basic solution are used now to compute the disturbed values of the elements a, e, and π (it is assumed here that the inclination and the line of the nodes remain essentially undisturbed, as far as their mean values are concerned) in intervals of 100 years, over a total interval of 1,000 years. The Jacobi constant, C, is computed for each set of these disturbed elements, and the relatively small deviations of the values of C from constancy, which is theoretically required, indicate the reasonable soundness of these results. The figures given by the author, which reveal rather large changes in the elements a, e, and π, represent a time interval extending 1,000 years into the future. The author performed the same computations for 1,000 years backward in time, but he mentions that, from the corresponding results, it follows that Hilda in this interval approached Jupiter so closely in a cometary type of orbit as to make the numerical results very uncertain. The test of the results by means of the Jacobi constant, C, became unsatisfactory, and the "backward" results are not listed. In the final paragraph, the author goes back to the more rigorous expressions of the nonrestricted problem and computes the disturbed elements of the planet Hilda for 900 years forward and 700 years backward. The results confirm the general trend of the previous, more approximate results. On the basis of the author's results, the orbit of Hilda has suffered very considerable deformations in a relatively very short interval of time, leading from a cometary orbit toward increasingly "normal" minor planet orbits.

It is believed that the above results should be received rather skeptically. First, many higher order terms are still neglected in the study and, second, the basic assumption of a constant node and inclination might have seriously affected the results. It is well known that most minor planets have considerable secular perturbations of their nodes, and the close approach to Jupiter would most likely have affected the inclination very seriously, too. It is not indicated how the criterion of the Jacobi constant is satisfied by the various sets of elements in the final results, and it seems that a serious lack of rigor pervades the results, owing to noncompliance with this strict theoretical requirement. Still, the investigation, as such, is an interesting attempt at practical use of periodic orbits in connection with the motion of minor planets; final acceptance of the quantitative results, however, should await further progress in this direction.

In another paper(97) (1953), Chebotarev applied his basic method for the general perturbations of the Hilda group to seven individual planets: (153) Hilda, (190) Ismene, (361) Bononia, (499) Venusia, (748) Simeisa, (958) Asplinda, and (1038) Tuckia. The perturbations are obtained as deviations from the proper absolute orbits of the periodic type according to Chebotarev. The author evaluates his resulting analytical expressions, in intervals of 100 years, for about 1,000 years forward and backward in time. He finds very large orbital changes. Some planets, according to these results, had almost cometary orbits about 100 years ago; others would come close to Jupiter a few hundred years from now. Some planets are approaching the Thule type and may thus remain in orbits of the planetary type. Considering the extremely large orbital variations, the author thinks that even the possibility of capture by Jupiter, or of transfer to the status of a Trojan, may not be excluded for some planets of this group.

Certainly, the planets of this group seem to be exposed to rather large perturbations by Jupiter. Considering, however, the simplifications made in the author's basic theory, and the related lack of rigor(96), it remains doubtful whether the present results are sufficiently accurate to represent the real orbital evolution. It appears not to be impossible that a more rigorous theory might reduce the author's "secular" variations to long-period large fluctuations. Although the paper, notwithstanding these reservations, is an interesting contribution, one has to be cautious about final acceptance of these approximate results.

The same analysis, which was applied to seven individual planets of the Hilda group in his 1953 paper(97), is used by Chebotarev(98) (1954) to derive the corresponding expressions for nine more planets: (1162) Larissa, (1180) Rita, (1202) Marina, (1212) Francette, (1268) Libya, (1345) Potomac, (1439) Vogtia, (1512) 1939 FE, and (1529) 1938 BC. Again, the analytical results are evaluated, in intervals of 100 years, for about 1,000 years into the future as well as into the past. In the case of (1202) Marina, this evaluation extends to 2,400 years into the past and 2,000 years into the future, because here the author finds an especially interesting type of oscillating evolution of the orbit. After presenting the individual results, the author reviews them together with the results presented in the earlier paper(97) to arrive at some general conclusions.

The orbits of all 16 planets appear to be very unstable, and planets change from one commensurability ratio to another in relatively short intervals of time. The commensurability 2 : 3, however, presents a barrier, explaining, according to the author, the accumulation of planets near the corresponding mean daily motion of about 450". The form of the individual orbits may change so much that no barrier seems to exist in this group between typical minor-planet orbits and those of a cometary type. The author presents graphs for the 16 individual planets, giving the eccentricity, $\underline{e}$, as a function of the mean motion, $\underline{n}$, during the intervals of time for which he had evaluated his results. It can be seen that the trend of orbital evolution is individual and quite different for different planets. In one additional graph, the individual planets are summarized to indicate the difference between the e(n) distribution of the 16 planets before and after a time interval of 2,000 years. This graph indicates a systematic, average trend towards smaller mean motions and larger eccentricities, as time goes on.

As was remarked about the preceding paper(97), one should be cautious about accepting these evolutional trends as definitely established, because the theory used for the derivation of these results is approximative, and terms are omitted which may become significant after longer intervals of time. Otherwise, however, Chebotarev certainly made the best use of his basic theory in order to obtain some insight into the orbital variations of the planets of a group which is so strongly disturbed by Jupiter. Insofar as the necessary theory was derived by the author in his earlier paper(96), the present contribution is of considerable interest, because these numerical evaluations reveal the related features of the orbital changes in this group of planets.

A discussion of G. N. Duboshin's paper(161) (1945) on periodic motions in the system of Saturn's satellites is given in the section on <u>Planetary Satellites</u>.

E. Klier(278) (1954), a Czechoslovakian, used vectors instead of the classical method of scalar components for the derivation of Lagrange's well-known rigorous solutions of the three-body problem. He further considers the features of the relative motion of m_1 and m_3 relative to m_2 by means of the necessary coordinate transformations (in vectors), comparing the period of revolution with the corresponding value in the two-body problem.

Although the author thinks that he is the first one to use vectors for the derivation of the Lagrangean solutions, this had actually been done before by Milankovich. Furthermore, nothing essentially new is gained by the use of vectors, except for a simplification of the expressions. The paper has the level of a good exercise in celestial mechanics, but not of a research contribution leading to any advance in knowledge.

Chapter I of a rather lengthy paper by G. A. Merman[393] (1952) contains a discussion of the theory of periodic solutions depending on a small parameter, as initiated by Poincaré and further developed by Malkin. Difficulties were encountered by Poincaré when the Hessian of the Hamilton function is equal to zero, which corresponds to the case where the two-body solution is adopted as the generating solution of the problem. A criterion is given for the existence of periodic solutions for the case where the functional determinant of the resolving equations vanishes equal to zero. The author proposes a method which, in order to avoid the difficulty of Poincaré's method, makes use of a generating solution that incorporates the effect of a properly averaged perturbing function, instead of neglecting the perturbing function altogether. Jacobi's method is used to reduce the averaged system to quadratures, so that the variation-of-constants method can be applied to the initial system. All the equations remain in their canonical form.

In Chapter II and III, the averaged system of equations is studied in considerable detail, and proof is given for the existence of periodic solutions in the restricted problem as well as in Hill's problem of satellite orbits. A physical interpretation of the averaged system leads to the concept of the sun's mass being distributed on a circle in the case of Hill's problem. A very detailed investigation of the system is made by means of the qualitative method of singular points. It is shown that, for sufficiently small values of the parameter, the Hessian mentioned above differs from zero. This proves the existence of periodic solutions in Hill's problem. A corresponding investigation is made for the restricted problem, where a simplified equation is used to establish the generating solution. It is shown that, under quite liberal conditions, the Hamilton function of the simplified problem has a Hessian which differs from zero, thus establishing the existence of periodic solutions. The simplified system, which takes into account only the largest terms of the related developments, approximates the averaged system of the problem.

Although the actual astronomical value of the approximation made by Merman in the choice of the approximating system remains doubtful, because the approximation is quite rough, the whole study is of considerable theoretical interest and value. The author definitely contributes a new idea for dealing with certain difficulties in the theory of periodic solutions and demonstrates his thorough familiarity with his chosen subject.

Whittaker's criterion for the existence of periodic orbits, in the modified and generalized form given to it by N. D. Moiseyev, was applied[442] (1937) by the latter to prove the existence of periodic

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solutions about the Lagrangean libration center, L_1, in the special case of the Copenhagen problem, i.e., for two equal finite masses m_1 and m_2. It is shown, in particular, that the theory based on whittaker's criterion leads to the most simple method of proving the existence of these periodic orbits around L_1. In the Copenhagen problem, the point L_1 coincides with the center of mass of m_1 and m_2.

Numerov[512] (1929), assuming a circular orbit for Jupiter, made a preliminary investigation of periodic orbits of Schwarzschild's type for the case of the 2 : 1 commensurability of the mean motions, i.e., for asteroids with periods of revolution which are approximately half as long as Jupiter's period of orbital revolution. The study is limited to motion in Jupiter's own orbital plane. Considering four different starting conditions, representing the moments of opposition and conjunction at the perihelion or aphelion of the asteroid with respect to Jupiter, Numerov makes use of Hill's method of general perturbations (which he presents also in considerable detail) for the analytical determination of the proper relation between the eccentricity and the starting mean motion, so as to assure periodicity of the asteroid's motion.

In the second half of the paper, Numerov uses his own numerical method of special perturbations, characterized by his well-known process of extrapolation, for an actual determination of one periodic orbit for which the approximate starting conditions had been taken from the preceding analytical study. For an adopted eccentricity angle $\Phi = \arc\sin e = 1°$, and a related starting value of 620.″46 for the mean

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motion of the minor planet, the computations are carried out over one complete orbital revolution of Jupiter. It is found that the starting mean motion has to be changed to 620".31 in order to obtain a true periodic orbit.

This paper contains no new theory, but the author's application of Schwarzschild's basic theory to such a specific and practical problem has definite merits. It seems, indeed, that Chebotarev's later contributions to the problem of periodic orbits were inspired by this paper of Numerov.

A. A. Orlov's[526] (1945) investigation of the problem of the existence of periodic orbits for fixed and constant values of the orbital inclination, i, is based essentially on Poincaré's theory, but the parameter, λ , of the solutions is chosen according to Hopf and Perron. The assumption of a fixed inclination enables Orlov to introduce this orbital plane as the fundamental coordinate plane, and a rotating coordinate system is introduced in this plane so as to follow the mean or average motion of the disturbed planet. The true position of the disturbed body oscillates about the fixed mean position, and the existence of periodic solutions is proved. It is found, however, that periodic orbits are possible only for $i = 0°$ and $i = 90°$. In the first of these two cases, the periodic orbit is circular, relative to the nonrotating, fixed, coordinate system. In the second case, where the orbital plane is rectangular to the orbit of the disturbing planet, as represented by Fatou's ring in the present problem, the true position of the disturbed planet describes a small ellipse about the body's mean location, at least as far as the terms up to the sixth order of λ are concerned. The period of the orbit depends on the value of the parameter, λ , which, at the same time, represents one of the constants of integration. The remaining arbitrary constants are the longitude of the node and the time passage through the node.

This is an interesting analysis of a very special problem, the peculiar features of which are determined by the adoption of Fatou's ring for the disturbing planet and by the assumption of a constant inclination. Great theoretical competence and ability are undoubtedly demonstrated by this thorough analysis of the chosen problem, even though the methods employed by the author are not new in themselves.

Orlov(527) (1950) made an extensive study of periodic solutions in space of the restricted three-body problem; the analysis is simplified by the application of Moiseyev's process of "averaging". First, the mean anomaly of the disturbed planet may be eliminated by averaging this planet's position as far as it enters the disturbing function; in the first part of the paper, the author makes use of this "inner variant" for the determination of periodic solutions. Alternatively, the mean anomaly of the disturbing planet may be averaged for its effect on the perturbations; this "exterior-variant" procedure is the basis for the developments in the second part of the paper. The fundamental theory of the periodic solutions of this investigation is essentially Poincare's, but it is modified insofar as the significant parameter for the development is not Poincaré's mass of the disturbing planet, but a proper coefficient, λ , which is defined in connection with a convenient transformation of the coordinates, that of Hopf and Perron. The first chapter of the first part is dedicated to a representation of Poincaré's method of finding the periodic solutions.

Chapter 2 then deals with the proof of the existence of periodic solutions for the case of the inner variant of the averaged problem. Here, two integrals, corresponding to the energy and area integrals, are found to exist. The author determines the starting conditions for which the solutions of the differential equations become periodic. Two essentially different classes of orbits are found. The first class consists of periodic orbits of small osculating eccentricity, but of unrestricted orbital inclination, *i*; the second class is comprised of orbits with finite eccentricities, *e*, and inclinations, *i*, but with a conditional relation between *e* and *i*.

Chapter 3 contains the actual determination of the main terms of the developments, up to a certain order and degree of approximation.

Part II of the paper treats, in an analogous manner, the case of the exterior variant of the averaged problem. Chapter 1 gives the proof for the existence of periodic solutions; in Chapter 2, the actual determination and evaluation are presented. It is of interest that, in the case of the exterior variant, only one class of periodic orbits exists, namely, for small eccentricities but finite inclinations.

This is a very good contribution of considerable interest. The mathematical analysis of the problem is very well done, and the author's competence and ability in theoretical work are demonstrated beyond any doubt. Altogether, it is a solid and substantial addition to knowledge in the field of periodic orbits.

This work was continued by Orlov[528] in 1952, when he considered the more flexible case of a variable inclination with an arbitrary starting value. Otherwise, the analysis essentially follows the lines of the earlier paper. The osculating orbital plane at the time t_o serves as the basic or reference plane, relative to which the rectangular coordinates ξ, η, and ζ are defined. These coordinates are referred to this inclined fundamental plane, and, at the same time, the coordinate system rotates about an axis which goes through the central mass and which is rectangular to the plane of Fatou's ring, representing the disturbing planet (Jupiter). The author proves the existence of nearly circular periodic orbits and determines the larger terms of the corresponding power series depending on the ascending powers of the Hopf-Perron parameter, λ. The integration constants and, thus, the necessary starting conditions are found by means of the conditional equations for the occurrence of periodicity. In the periodic orbits found by the author, the small mass describes a librational motion with respect to a "mean" planet which, in turn, moves uniformly in a circular orbit located in the fundamental plane and centered in the principal mass of the problem. The motion is periodic only relative to the rotating coordinate system.

This paper is of definite value with regard not only to the interesting new results, but also to its excellent mathematical form and elegance. Because of the adoption of Fatou's scheme, this paper deals with an idealized concept as far as astronomical problems are concerned, but the periodic solutions obtained may be suitable as variational orbits in concrete problems of the solar system. The author appears to be far above the average in competence and ability.

Reyn[634] (1937) points out that one of Whittaker's equations for his double integrals, extended over the region which is enclosed by a periodic trajectory, contains an error and also is valid in the given form only for direct periodic motion. The author re-establishes these interesting integrals, with emphasis on their different form for direct and retrograde orbits.

The author certainly deserves credit for correcting and completing Whittaker's results, and for finding the source of the error in Whittaker's original note (1902).

If the inner branch of Moiseyev's so-called osculatrice does not intersect the exterior branch as well as the curve of zero velocity, then, according to the theorem of Whittaker-Moiseyev, a ring-shaped zone can be constructed inside of which retrograde periodic trajectories (referred to the rotating coordinate system) exist. As the required topographic system of curves, Reyn[635] (1938) adopts a certain family of ellipses about the libration center, L_2, and then, by means of the method of contact characteristics, she proves the existence of simple, closed trajectories between two given ellipses of the topographic family. Making use of the apparent shape of the counterglow, as determined by Fesenkov's isophots, and comparing the corresponding region in space about the point L_2 with the results of the preceding qualitative analysis, she arrives at the approximate limits of the range of the Jacobi constant h* which would be compatible with the observed features of the counterglow.

This is a constructive contribution of moderate importance, because it explores, by methods already available, some of the dynamic characteristics which should apply if the counterglow is actually produced by a cloud of particles near the libration center, L_2. The results of this study have to be considered in making any final decision about the admissibility of the Gyldén-Moulton hypothesis on the origin of the counterglow.

c. Criteria of Stability

(1). A. M. Lyapunov's Method

The Russian period of activity on the problem of the stability of motion dates from 1892, when Lyapunov's classical paper, The General Problem of the Stability of Motion, appeared. It reached the West in 1907, when it was published in French in Annales de Toulouse. There is some overlapping between the paper of Lyapunov and Poincaré's work. It is not too much to say that through his careful treatment, which is indeed wholly modern, Lyapunov laid down the basis for the general theory of stability. Before considering his work, it should be said that after Poincaré and Lyapunov, the chapter of mathematics which they opened lay dormant until it was revived by Van der Pol and the modern Soviet school.

Since Soviet work is based mainly upon Lyapunov's efforts, a brief discussion of his work is presented.

The problem taken up by Lyapunov is the following. Consider a dynamic system represented by:

$$\dot{x} = X(x,t), \tag{1}$$

where the dot indicates the differentiation with respect to time, $\underline{x}$ and $\underline{X}$ are n-dimensional vectors, and $X(0,t) = 0$ for all t. Thus, $x = 0$ is a solution, the steady-state solution. It is supposed that $\underline{X}$ is such that the existence and uniqueness of solutions of System 1 hold in a suitable neighborhood of the origin. What is to be said regarding the solutions which start at time t_0 from any point sufficiently close to the origin? In particular, do all or only some of them leave the origin (absolute or conditional instability), or remain quite close (stability), or perhaps actually tend to the origin (asymptotic stability)? The same problems may be considered for a periodic motion, as well as for the origin, but such problems can be reduced to the preceding one.

A special case of System 1 is autonomous systems, i.e., of the form:

$$\dot{x} = X(x). \tag{2}$$

A large part of Lyapunov's original paper is devoted to such systems under the additional restriction that $\underline{X}$ is an analytic function. More explicitly, the system has the form:

$$\dot{x} = Ax + f(x), \tag{3}$$

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where $\underline{A}$ is a constant $n \cdot n$ matrix and the components f_i of $\underline{f}$ are power series of degree at least two in the components x_j of $\underline{x}$. Basically, Lyapunov compares the solutions of (3) with those of the "first approximation":

$$\dot{y} = Ay. \tag{4}$$

(This is usually referred to as Lyapunov's method or his "first method", to distinguish it from his so-called "second method", which involves a positive function known as the "Lyapunov function".)

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the characteristic roots of the matrix $\underline{A}$; that is to say, the roots of the polynomial $|A - \lambda I| = 0$, where $\underline{I}$ is the unit matrix of order $\underline{n}$. Suppose, for simplicity, that the $\underline{n}$ roots λ_n are distinct; this is indeed the general case. Lyapunov showed that if the λ_n all have negative real parts, then $\underline{x}$ will go to zero as $\underline{t}$ goes to infinity, and the system is asymptotically stable. If only $\underline{k}$ of the roots have negative real parts, there is a k-dimensional family of stable solutions and their stability is asymptotic; if all have positive real parts, the system is unstable. Actually, the same stability properties hold even when there are repeated characteristic roots.

Lyapunov also gave a complete treatment of the case when one of the characteristic roots is zero or when there is a pair, $\pm i\lambda$, of pure imaginary roots. In the latter case, with a certain additional condition, he showed that the system admits a one-parameter family of periodic solutions. In 1937, Malkin, an outstanding Soviet mathematician and follower of Lyapunov, treated at length the case of two characteristic roots equal to zero.

Lyapunov also dealt rather fully with System 3 in which the terms of A and the coefficients of the power series f are bounded functions of the time, t. Finally, he made a full study of the stability of periodic solutions of autonomous System 2.

The results described are based on the explicit representation of the solutions by certain series obtained as follows. Introduce a dummy parameter, ε, and look for a solution of System 3 represented by:

$$x = \varepsilon x^1 + \varepsilon^2 x^2 + \ldots . \qquad (5)$$

To obtain the various terms, one substitutes in (3) and identifies like powers of ε. There results a sequence of linear equations for x^1, x^2, . . ., which may be solved one at a time. The system for x^1 is actually System 4. Under reasonable conditions, the series solution with $\varepsilon = 1$ is shown to be convergent.

Whatever Lyapunov obtained by the above-described method of solution is referred to by him and his successors as his "first method". He also introduced a "second method" which is in high favor in the Soviet Union, but which has not received much attention in Western Europe or in the U. S. This method involves the utilization of a certain function, V(x,t), known as Lyapunov's function. The first step in the application of this method to a given problem consists of the construction of the appropriate Lyapunov function for the given system. If V(x,t) is such a function with continuous first partial derivatives near the origin, then along a trajectory of System 1:

$$V = \sum X \cdot \frac{\partial V}{\partial x_i} + \frac{\partial V}{\partial t} . \qquad (6)$$

If one can find a $\underline{V}$ such that, for small $\underline{x}$, $V > 0$ while $\dot{V} < 0$ for $t > \tau$, then the origin is stable for System 1. If $\dot{V} > 0$ under the same conditions, the origin is unstable. Roughly speaking, V = constant represents a family of ovals surrounding the origin that shrink along the trajectories in the case of stability and expand when the system is unstable. Other criteria of this nature were developed by Lyapunov and later by I. G. Malkin(382, 383) and Persidskiy. They were all inspired by the classical condition of stability of equilibrium, according to which the potential energy must be a minimum.

Regarding the second method of Lyapunov, it is important to observe that it does not impose analyticity of the X_i, and thus it has a much larger range of application than his first method. It is true, nevertheless, that in all the applications made by Lyapunov and by most of his successors, analyticity is assumed. Exceptions to this are the works of N. D. Moiseyev(434, 435) (1936). In these works, Moiseyev extends the class of functions which can play the role of Lyapunov functions. He shows that $\frac{dV}{dt}$ does not need to be continuous, but only integrable.

It is interesting to compare the contributions of Poincaré and Lyapunov to differential equations and the theory of motion. Lyapunov is the analyst who treats the problem in detail; Poincaré is much more universal. Lyapunov dealt strictly with the local problem - stability around a given solution - and, in this respect, gave much more precise results and went a great deal further than Poincaré. However, Poincare, in one of his papers, laid down the basis for the treatment of differential equations in the large. He also made very many applications to celestial mechanics. Such applications of Lyapunov's theory were introduced into the literature on celestial mechanics much later, especially by G. N. Duboshin and N. D. Moiseyev.

(2). General Concepts

The papers on the criteria of stability could not be readily arranged in terms of subproblems. The situation is, rather, that each problem depends upon the assumptions, definitions, and criteria which are adopted. For purposes of presentation, however, the papers to be discussed have been divided into two groups: (1) general concepts and (2) astronomical concepts. In general, the discussion of the papers is given chronologically by author.

N. G. Chetayev(111), as early as 1931, proved his ability in dynamics. Making use of the canonical form of the differential equations of motion for a system of n degrees of freedom, and of the related equations by Poincaré for the perturbational variations, and introducing the concept of the local density of a bundle of trajectories with given starting conditions, as well as a certain "function of density", the author finally obtains for the function of density a partial differential equation quite analogous to Schroedinger's wave equation of quantum mechanics. Solutions are possible only for a certain discrete "spectrum" of the constants α_i. From the way the force function,

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W, and the function of density are involved in his "measure of influence of the perturbations", it is found that the stable trajectories (which are least likely to be deflected because a certain maximum condition is fulfilled) are the ones which are determined by the discrete values of the constants α_i. The stable trajectories themselves, therefore, will be distributed as discrete curves in general, and only for a continuous spectrum of α_i could the (phase) space be filled compactly. In further analogy to quantum mechanics, the deviations and possible oscillations of the neighboring trajectories around the stable ones may be interpreted as the "waves" associated with the equation.

Three years later, Chetayev[112] demonstrated his mathematical ingenuity in stating and proving a general theory of nonstability, making use of Lyapunov's basic ideas. If the differential equations of the disturbed motion are of such a form that: (1) for a certain function, V, with an infinitesimally small upper limit, a region exists where $V V' > 0$, and (2) if, in an arbitrarily small part of this region, a subregion around $x_s = 0$ exists with the property that for a certain function, W, one has $W > 0$ inside this subregion and $W = 0$ on its boundary, where $\frac{dW}{dt}$ is also definite within the same subregion, then the undisturbed motion is not stable. Proof of this theorem can be based on geometrical evidence, making use of Poincaré's related concepts and procedures, or it can be obtained analytically by means of Lyapunov's function. Lyapunov's theorems on nonstability are special cases of the more general theorem presented and proved in this paper.

Chetayev has to his credit a textbook[105] (1946; second edition 1955) which is on a considerably higher level than Duboshin's book[172] which is a text for fourth- or fifth-year students. Although Duboshin more or less restricted his work to the presentation of Lyapunov's fundamental theory, Chetayev gives a more comprehensive treatment, and also seems to be more critical as well as creative than was Duboshin. Chetayev's seems to be the finest and most complete and competent textbook on the stability of motion available today.

Chapter I deals with the formulation of the stability problem in general, and with its historical development up to the time of publication. Chapter II treats the general theorems of Lyapunov's "direct" method, involving his so-called V-function. Considered are various aspects of the theory, including, also, the more recent developments. An interesting application is the author's own treatment of the motion of a rigid and heavy body, one point of which is held fixed. Lyapunov's function of this problem is determined here for the first time. Chapter III deals with the stability of equilibrium, with the acting forces possessing a force function. Again, the author contributes some results of his own. This chapter, too, is illustrated by some applications, one of which deals with the stability of equilibrium of a material ellipsoid resting on a plane surface. Some ballistic applications are given, too. Classical theorems, such as those of Kronecker and Poincaré, are contained in this chapter.

Chapter IV, on the basis of matrices, treats the linear differential equations with constant coefficients. The relevant

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classical results are presented, and the author makes use of Hurwitz's theorem concerning the roots of the characteristic equation for the stability of a rigid body. Chapter V contains the classical results concerning the effect of a disturbing force on the status of equilibrium. This chapter is distinguished by Chetayev's presentation of rigorous proofs by means of Lyapunov's theory; most of the earlier authors were satisfied with proofs to the degree of a first approximation. The chapter further deals with binding conditions, with the effect of dissipative forces, with gyroscopic forces, with the stability of a ballistic missile, and with forced oscillations. Chapter VI is devoted to the stability of the first approximation, and the fact is stressed that stability found from the first approximation may turn out to be invalid when a rigorous treatment according to Lyapunov's theory is made. Lyapunov's pertinent theorems are given.

Chapter VII deals with the singular case of one zero root of the characteristic equation (of the first approximation). Here, the rigorous treatment is more difficult, even by Lyapunov's theory, and the author apparently has contributed to progress in this special case. In this chapter, the conditions are investigated under which a stable solution is obtained for the motion of an airplane. Chapter VIII contains Lyapunov's results for the case of two imaginary, conjugate roots of the characteristic equation of the first approximation. In Chapter IX, the author proceeds to the treatment of so-called unsettled motion, i.e., to the case where the coefficients of the differential equations according to Lyapunov are not constant. Most of the results for this more complicated problem had been found by Lyapunov. The problem is treated with the aid of exponential functions, and the important role of the so-called characteristic number with respect to the exponents of these functions is properly demonstrated.

The final chapter, Chapter X, is devoted to periodic motion, and it is made clear that the stability problems related to periodic motions have not been completely solved as yet. Lyapunov's approximate method for the determination of the characteristic equation in this problem is presented in detail.

G. N. Duboshin(148) (1935) gives several methods for the construction of Lyapunov's function, V, for the case where the system of differential equations is of the canonical form. Lyapunov had already pointed out that the trivial solution ($x_1 = 0, x_2 = 0, \ldots, x_n = 0, y_1 = 0, y_2 = 0, \ldots, y_n = 0$) is stable if the characteristic function, H, is a definite function ($dx_i/dt = \partial H/\partial y_i$; $dy_i/dt = -\partial H/\partial x_i$). The author gives additional theorems on the occurrence of stability or nonstability, depending on the definiteness of a new function, W + H, and on the behavior of the related Poisson brackets, (W,H), with respect to $(W,H) + \frac{\partial H}{\partial t}$. The problem is thus reduced to the determination of a suitable definite function, W + H, or of a corresponding function, W.

The five theorems presented increase and facilitate the actual possibilities for the construction of a function which, in the case of canonical differential equations, takes the proper place of Lyapunov's function, V. Since, on the basis of the new theorems, it is possible to decide on stability or nonstability in cases where the function H is of variable sign, while Lyapunov considered only a definite function, H, these theorems represent an addition to the theory of stability.

An interesting exhibition of the actual application of Lyapunov's second method was given by Duboshin in 1935(149). Otherwise, the paper is of no importance, since no theoretical advance is involved in the investigation. The stability or nonstability of the trivial solution $x = 0$, with respect to x and $\dot{x}$, depends on the behavior and limitations of the function $p(t)$. For four already known theorems concerning the stability or nonstability in this problem, the author gives a new demonstration of proof based on Lyapunov's general theorems. In each case, he introduces a convenient Lyapunov function, V, for the purpose of this demonstration.

The classical theory of stability by Lyapunov considers only the effects of momentary and random variations or perturbations on the basis of the related perturbation equations $\frac{dx_s}{dt} = X_s\ (t, x_1, x_2, \ldots, x_n)$. In 1940, Duboshin(157), without much difficulty, extended the theory by introducing a continuously acting perturbing influence, such as the actual planetary perturbations in celestial mechanics. This additional influence is introduced by a corresponding function, R_s, in the form:

$$\frac{dx_s}{dt} = X\ (t, x_1, x_2, \ldots, x_n) + R_s\ (t, x_1, x_2, \ldots, x_n).$$

Duboshin found that the new terms can be rather easily combined with the original terms of Lyapunov, leading to no essential modifications of the basic method.

The trivial solution $x = 0$, $\dot{x} = 0$ of the differential equation $\ddot{x} = X(t, x, \dot{x})$ was considered by Duboshin(158) (1940). He investigated the conditions under which this trivial solution can be either stable or unstable, in the sense of Lyapunov. Eight theorems are presented for various assumptions concerning the character of the function $\underline{X}$, and especially concerning the partial derivatives $X\dot{x}$ (t, 0, 0) and $X\dot{y}$ (t, 0, 0).

In 1952, Duboshin(172) published a textbook on stability of motion for fourth- or fifth-year students in celestial mechanics. Nothing quite like this book is known to exist in the Western literature. The text is devoted mainly to the presentation and explanation of Lyapunov's methods, which had been insufficiently publicized before. The author also extended some of the results by Lyapunov.

The first chapter contains the basic concepts and definitions of stability according to Lyapunov. Chapter 2 deals with the integration of the differential equations of the disturbed motion, using Lyapunov's method. Especially treated are linear differential equations with periodic coefficients. Chapter 3 gives the fundamentals of Lyapunov's so-called second or V-method, where the problem of stability is reduced to the determination of a certain function, $V(t, x_1, x_2, \ldots x_n)$. This chapter contains four important theorems by Lyapunov, the first of which may be considered as a typical example for Lyapunov's methods. If, for the "perturbations" x_s, the differential equations are of the form $\frac{dx_s}{dt} = X\ (t, x_1, x_2, \ldots x_n)$, and if a function, $V\ (t, x_1, x_2, \ldots x_n)$, can be found so that always (after a given moment, t_o)

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$V > 0$ (or always, $V < 0$), and:

$$\dot{V} = \frac{\partial V}{\partial t} + \sum \frac{dV}{dx_s} X_s \leq 0 \text{ (or always } \dot{V} \geq 0),$$

then the motion is stable.

Chapter 4 deals with the case where the X_s are linear forms of the x_s, with constant coefficients; Chapter 5 is devoted to singular cases. Lyapunov's conditions for the existence of periodic solutions for the disturbed motion are given in Chapter 6, with applications to the motion of the neighborhood of the straight-line libration points in the restricted problem of three bodies. The seventh and last chapter considers problems where the coefficients of the x_s in the expressions for the X_s are functions of the time, t. In this case, no systematic procedures for the determination of Lyapunov's function, V, exists, but, in most cases, one tries to use quadratic forms. Under certain conditions, it becomes possible to find the function, V.

I. G. Malkin(383) (1951) considered the case where the differential equations of the disturbed motion, in the sense of Lyapunov, have such a form that the characteristic equation has one pair of purely imaginary roots. In this case, as Lyapunov has shown, one can transform the variables in such a manner that two variables, x and y, can be obtained from two equations involving only the second and higher powers of the additional variables $x_1 \ldots x_n$. The problem is reduced to the determination of the solution of a system of n partial differential equations, in order to find the solutions for $x_1 \ldots x_n$ in the form of power series depending on the first two variables, x and y. The resulting developments, in turn, can be inserted into the differential equations for x and y, in order to study the stab. lity (including the higher order terms) of the whole original system. Although all this had been shown by Lyapunov, the actual integration of the partial differential equations is a very complex and elaborate task. In the present paper, the author presents a method by which the process of successive solutions for the various terms depending on the ascending powers of x and y is reduced to one basic integration, the analytical structure of which repeats itself for the terms of ascending higher order. This striking simplification of the int gration procedures is reached by the simple transformation $x = \cos \Phi$, $y = \sin \Phi$. This reduces the system of partial differential equations to a system of ordinary linear differential equations, the individual terms of which can now be integrated separately. The analytical structure of the coefficients of the higher order is the same as the one of the first-order coefficient.

Malkin clearly has made a rather substantial contribution in this paper; it is surprising only that this simple but significant transformation had not been found before. This is a paper of permanent value, and the author evidently is highly competent in the field of stability problems.

In Lyapunov's second method, it is assumed that the well-known V-function must have a continuous derivative, $\frac{dV}{dt}$. Moiseyev(434) (1936) finds, from a thorough study of Lyapunov's work, that continuity is required only insofar as to guarantee the existence of the integral:

$$V - V_o = \int_{t = t_o}^{t} \frac{dV}{dt}\, dt.$$

This leads Moiseyev to the conclusion that all the theorems of Lyapunov's second method remain valid if it is admitted that the derivative:

$$\frac{dV}{dt} = \sum_{s=1}^{n} \frac{\partial V}{\partial x_s} X_s + \frac{\partial V}{\partial t}$$

may have finite instantaneous changes without a change of sign in such points whose measure, in the point set for which the function $V(x_1, x_2, \ldots x_n, t) = C$ is defined, is equal to zero. The importance of the result lies in the possibility that a larger variety of problems may become accessible to Lyapunov's second method because of this conclusion.

The author's finding proves his thorough familiarity with Lyapunov's ideas, and illustrates his own vision and independent thinking in dealing with such theoretical subjects.

Moiseyev(435) (1936) showed, in an interesting and ingenious manner, how Lyapunov's concept of stability appears as just a special case of an even more abstract system of mathematical concepts which makes use of the theory of point sets. He seems to have mastered the basic mathematical field which he needed for his generalization of the stability problem.

Moiseyev points out that Lyapunov's theory of stability can be generalized and completed in various respects, and Reference 435 contains one example; it considers a set of points within a given sphere in connection with the possible starting conditions of a set of trajectories, which leads to the concept of the probability of stability. In the generalized theory, Lyapunov's class of stable motions appears as a partial set within the class of motions with a probability of stability equal to unity. For Lyapunov's definition of stability, the related or coordinated class of nonstable motions comprises a zero set; from the extended point of view, the corresponding class of nonstability is a set of the measure zero, and the probability is equal to unity for the stability, under its generalized definition.

Reference 445 contains the detailed description and definition of the new concept presented in Reference 435 and of its various aspects, and also a method for the actual computation of probabilities of stability. The investigation is limited to those cases where the right-hand side of the differential equations does not explicitly depend on the time, t. The terminology and the basic elements are those of the theory of point sets. The general theory is illustrated by some selected examples of second-order differential equations. The whole analysis appears as a mathematical generalization of Lyapunov's basic theory of stability.

Reference 445 is a mathematical investigation rather remote from the practical problems of celestial mechanics, and it was published in a mathematical journal, not in an astronomical one. Nevertheless, these generalized aspects of the stability problem contain, in themselves, the special methods normally applied to problems of celestial mechanics, and they are of as much interest to the field as are other mathematical fields (theory of differential equations, theory of functions, etc). Obviously, the writer's mathematical background is impressively strong, even in fields which are not closely related to most problems in celestial mechanics.

Moiseyev[443] (1937) established stability definitions of a generalized kind for a given set of trajectories, the starting points of which are first classified into two point sets with respect to a given one-parameter system of surfaces, $\Phi(x_1, x_2, \ldots x_n) = e$. The concepts of stability and nonstability are defined by means of certain limiting inequalities between the original and the final deviations of the given set of orbital points and a "maximum" limiting surface; the concepts "counterstability" and "noncounterstability" are introduced on the basis of similar conditions involving a "minimum" limiting surface. The applicability of the one or the other set of definitions depends on the relative size of the final deviations of the set of given orbital points as compared with the variations of the surfaces which are involved. On the basis of these definitions, the associated probabilities of stability are then established.

These are highly theoretical considerations, consisting as they do only in setting up very general definitions, rather than any theorems or other conclusions. The author's "third" type of stability differs from Lyapunov's concepts mainly by a greater simplicity, insofar as no analytical passage to the limit is involved here. Although the new concepts are certainly interesting from a systematic and general point of view, and although they also reflect the mathematical ingenuity of the author, it remains to be seen how far they are actually useful for the solution of dynamic problems.

After referring to a 1938 paper[448] in which he established the algorithm for the construction of regions of stability and nonstability in phase space, Moiseyev[449] (1938) makes some additional remarks concerning the importance of such regions of stability and nonstability for the qualitative analysis of dynamic problems; he is also concerned with the possibility of certain generalizations of the method for the construction of the regions. Finally, Moiseyev refers to a paper by Stepanov[728] and says that it is based on the same principles as his investigations[448,449], even though the methods and results do not duplicate each other. The common basic principle is the formulation of the conditions of stability in dependence on the phase coordinates, rather than on the time.

This addition to Reference 448 is not too substantial by itself; the considerations contained in Reference 449, however, complement the whole system of concepts and conclusions developed by the author in a series of contributions. Taken as a whole, these are contributions of substantial and real interest, because they expand a field of knowledge in a certain direction.

In 1939, Moiseyev[455] was concerned with the construction or the finding of compact regions (in the abstract phase space of a generalized system of coordinates) where the trajectories represented by a given system of differential equations for the perturbations, in the sense of Lyapunov, are either stable or unstable. The mathematical treatment of the problem is rather deep; it makes use of so-called "natural coordinates" referred to a frame which is intimately connected with the motion of the particle on the "undisturbed" trajectory. The axes of this natural-coordinate system are determined by the tangent and the two normals of the undisturbed curve, everything being referred to an abstract and generalized basic system of coordinates. Operating with quadratic forms for the function which takes the place of Lyapunov's V-function, Moiseyev arrives at a generalization of Lyapunov's classical method for dealing with the stability problem. The actual, disturbed location of the moving particle is referred to the system of natural coordinates, which is moving with the "undisturbed" particle. Considering the properties of the quadratic form for the V-function, the author finds it possible, in certain cases, to locate certain regions, cut out by a system of surfaces in the generalized phase space, where the motion is either stable or unstable, depending on the analytical criteria.

He expresses the opinion that these results will lend themselves to applications in actual problems of celestial mechanics, and it seems, indeed, that this quite general investigation may be valuable and useful for further progress in this field. The paper definitely is a profound mathematical study of high interest from the "methodical" point of view.

Moiseyev continued and generalized this work in Reference 461. Although Reference 455 dealt with stability of motion in ordinary three-dimensional space, Reference 461 leads to certain theorems which are valid for motion in phase space of $\underline{n}$ dimensions. Still, it is assumed that the time, $\underline{t}$, shall not appear explicitly in the right-hand sides of the differential equations for the perturbations in Lyapunov's sense. Essential in the present study is the introduction of natural or accompanying coordinates, i.e., of coordinates which are intimately connected with the moving point and its trajectory. One of the n coordinates, σ, is taken as an arc on the trajectory itself, and designates the curvilinear distance between the "undisturbed" particle and the rectangular projection of the "disturbed" position of the particle onto the undisturbed trajectory. The remaining (n - 1) coordinates refer to directions which are normal to the trajectory, and these coordinates are not curved.

The new generalized coordinates have the important property that the longitudinal perturbation (in the coordinate σ) can easily be separated from the perturbations in the directions of the (n - 1) orthogonal coordinates. The differential equations of the perturbations in Lyapunov's sense (as the corresponding differences between the undisturbed solution and the disturbed true motion) are transformed from their original form to the system of new coordinates. Actually, two different systems of new generalized coordinates are employed in this investigation. System I has its origin at the location, P_o, of the undisturbed particle, and the (n - 1) coordinates which are normal to the first coordinate, σ, lie in the plane by which the trajectory is orthogonally intersected in P_o. System II is centered in the point P_o', where the disturbed point or position P has its orthogonal projection onto the undisturbed trajectory. System I is used for the discussion of the characteristics of the so-called longitudinal asymptotic stability and of the longitudinal absolute nonstability, and a general criterion concerning the occurrence

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of one or the other property is given with proof. This criterion depends essentially on the sign of the coefficient II_{oo} of the σ term in the differential equations. The expression for II_{oo} as the characteristic for longitudinal stability or nonstability is then considered in its dependence on the coordinates (neglecting the dependence of all these coordinates on the time t), in order to study the regions of compact longitudinal asymptotic stability and of longitudinal absolute nonstability. These regions are separated by the surface $II_{oo}(x_1, x_2, \ldots, x_n) = 0$.

The second system of coordinates is then used for a similar study of stability and nonstability with respect to the system of $(n - 1)$ normal coordinates. This leads to a general theorem concerning transverse nonstability or asymptotic stability. Geometrically, these possibilities correspond to the divergence or asymptotic convergence of neighboring trajectories. The construction of compact regions of transverse asymptotic stability and of transverse absolute nonstability depends essentially on the mutual intersections of $(n - 1)$ surfaces in n-dimensional space. The author then goes back to an ordinary rectangular system of accompanying coordinates and derives the corresponding expressions for the characteristic of transverse nonstability.

Up to this point, the paper has dealt with the trajectories as curves in space, but not with moving points in these trajectories. The author now states and proves the theorem that the simultaneous existence of transverse and longitudinal asymptotic stability is a sufficient condition for orbital stability in Lyapunov's sense. A similar theorem is true for nonstability.

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The main significance of this investigation lies in the establishment of definitions of stability and nonstability which are independent of Lyapunov's function (in Lyapunov's second method), but which are in agreement with respect to the results derived. The author has presented new theoretical concepts which are very interesting, at least from the mathematical point of view. The author said that this paper is of a preliminary character, that these studies would be continued, and that applications to celestial mechanics would be given (perhaps by somebody else now, since the author has died). The value of the theory in the present publication seems to be considerable.

(3). Astronomical Concepts

Previous work of members of the Shternberg Institute on the problem of the Gegenschein phenomenon led Agrest[7] (1945) to the consideration of the mutual interaction of the Gegenschein particles and, thus, to the introduction of a resistant medium into the differential equations of motion for the individual particle. The main result is that the mutual interaction of the particle may actually increase the tendency of the particles to form a Gegenschein cloud in the case of the collinear libration centers. Altogether, however, the results are not conclusive enough.

Agrest investigated the roots of the characteristic equation of the small variations, first, for trajectories in the immediate neighborhood of the straight-line libration centers, and, second, for those in the neighborhood of the triangular libration points. In each case, the motion is nonstable in the sense of Lyapunov, because roots with a positive real part always occur in the present problem; but this result does not exclude the possibility of a conditional stability in a more narrow sense, inasfar as a curve of a certain family or type may be disturbed in such a manner as to remain in the same class of librational or asymptotic motion relative to the corresponding libration center. Comparing the results of the present analysis with the classical ones where no resistant medium is considered, the author finds, for the case of the collinear libration centers, that one two-parameter group of solutions, which is periodic in the classical problem,develops into a class of asymptotic motions (approaching the corresponding libration center with $t \to \infty$) under the effect of the resistant medium. In this particular group of solutions, a tendency of increasing conditional stability under the effect of the resistant medium seems to be indicated, while in other groups or classes of solutions, the probability of stability seems to decrease.

In a paper on the origin of Gegenschein, Duboshin[155] (1938) defines the problem of motion in the neighborhood of the collinear libration points, for which the point L_2 serves as a representative, and develops the force function in powers of the rectangular coordinates for which L_2 is the center point. Whereas informerly the stability or nonstability of periodic orbits was investigated by finding out about the periodicity or nonperiodicity of the "disturbed" motion (as produced by a small initial deviation from the basic periodic orbit), Duboshin obtains his proof for the nonstability of the periodic orbits under consideration by means of an important theorem by Lyapunov concerning the existence of periodic solutions of differential equations in general. He also succeeds in the actual determination of the various ascending terms of the analytical developments for the periodic solution. The new proof of nonstability is of importance, first, because it has been established on the basis of a quite general method and, second, because all earlier discussions of the nonstability of these periodic orbits were based on approximate analytical expressions. The new proof is rigorous, and the convergence of the series which are involved is proved.

This is a really good and substantial contribution, because it makes use of Lyapunov's elegant method for the first rigorous proof of nonstability of the periodic motions near the collinear points.

A good and critical study of the problem of the stability of the ring of Saturn was made by Duboshin[159] (1940). He starts with a short review of previous investigations of the stability of Saturn's ring. The problem was first solved in an approximate manner by Maxwell, who arrived at the result that the rings are stable configurations of small particles or meteoritic dust. Although it is most likely that the rings are formed by small dust particles, Maxwell's proof of stability is not sufficient, because Lyapunov has shown that some type of motion may be found to be stable from a first-order analysis, but that the inclusion of the second-order terms may lead to nonstability in Lyapunov's sense. Bucerius studied the stability problem of Saturn's rings on the basis of a model of two three-dimensional rings, A and B, assuming uniform circular motion for the bulk of the "disturbing" ring particles. He investigated the motion of a single "disturbed" particle under the gravitational action of a flattened Saturn and of the bulk of the ring particles, neglecting the action of the disturbed particle on all the others. In Duboshin's opinion, Bucerius' approach is incomplete because of this neglect, and, furthermore, Bucerius solved the problem with approximating equations. In the present paper, Duboshin uses Bucerius' scheme, but he attacks the problem by means of Lyapunov's first method, including terms of higher order. He also introduces the gravitational action of seven satellites (omitting Japetus and Phoebe because of their great distance), by replacing them with "averaging" rings of matter according to Gauss' scheme for secular perturbations. Otherwise, the author's scheme has the same limitations as the one of Bucerius and, for this reason, the author himself states that the problem still is not solved in a rigorous way. Neglected is not only the direct gravitational interaction of the ring particles, but also the effect of any continuous perturbations in Lyapunov's sense. The effect of friction between the ring particles, on the other hand, has been included. Duboshin finds that under certain conditions, which are satisfied in the present model, stable circular motion of the particles is possible in the equatorial plane of Saturn. The actual particles may oscillate about such ideal circular orbits, but they will not tend to deviate permanently from their original orbits, as long as the disturbing action has the character of momentary impulses and not of a steady force. The author considers three parts, A, B, and C, of the ring, while Bucerius considered only two. Although the general results of this paper confirm the stability of Saturn's rings, the author emphasizes the facts that certain factors still have been neglected and that no final proof of actual stability has been obtained.

In a rather popular summary report, Duboshin[160] (1941) gives the historical development and present status of the question of the stability of the solar system. The variation-of-constants

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method of planetary perturbations is presented with a discussion of the contributions of Laplace and Lagrange to the stability problem. The author then goes on to the later contributions of Poisson, Poincare, and others, leading to the conclusion that the problem has not been solved yet in a general or unlimited way. Stressing the point that all investigations have been made for fictitious mathematical systems of point masses moving in empty space, rather than for the more complicated, real, solar system, the author finally defines the problem as one which has to include the effect of the true figures of the planets, the presence of many small masses and possibly of a resisting medium, etc. In this form, the problem was stated by Chetayev, but, apparently, not even an attempt has been made to attack this complex "true" problem of stability in the solar system.

Duboshin certainly makes it clear that no mathematical proof of the long-range stability of the solar system has been given yet; he points out, however, that this failure of methods does not prove the instability of the solar system either. Altogether, this is a fair account of the development and status of a fascinating problem.

The Roche criterion for the stability of a satellite, in dependence on the distance from the planet, is based on the assumptions of circular motion and of a bound rotation of the satellite with the period of its orbital revolution. Fesenkov[195] (1951) considered a satellite rotating with an angular velocity five times as great as the angular velocity of its orbital motion. From the

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proper modification of Roche's formula, it is seen that the critical distance from the planet becomes more than twice the classical value. In other words, a satellite with fast rotation tends to disintegrate at a larger distance from the planet than a satellite with bound rotation.

Ivanenko, et al.,[238] (1951) make use of the formal analogy between the linear approximation of Einstein's theory of general relativity, on the one hand, and the field of electromagnetic radiation, on the other, for an application to the stability of certain types of astronomical systems. Distinguishing between the "Newtonian" part of the field energy and the radiated gravitational energy which leaves the system, it is found that the radius, R, of the material system under consideration should equal or exceed a certain function of the total mass, M, and of the related temperature, T, of the radiation; the "critical" temperature, T, is obtained if the equal sign is used. In general, this relation, $R \geqslant \frac{24\sqrt{M}}{T}$ cm, is interpreted as an expression of nonstability. For the case of the equal sign, the critical temperature, T, is computed for typical representatives of galactic systems of planetary and of diffuse nebulae and, in spite of the very different masses and dimensions of such different systems, T is always found to be only slightly above absolute zero, or of the order of the temperature of interstellar dust. Finally, the critical T-values are computed for the solar systems, as well as for the various satellite systems in the solar system, and, here, much higher temperatures, of the order of the electron temperatures in gaseous nebulae, are obtained. It is also found that the individual T-values of the various satellite systems (always including the mass of the central planet) depend almost monotonicly on the size of the system, with the exception of the Uranus system where an additional satellite at a larger distance from the planet would be required in order to make this system fit the general trend. By the elimination of T from the theoretical relation R(M,T) and the new empirical relation R(T) in the solar system, the authors arrive at a law, R(M), which is interpreted as a cosmogonic characteristic related to the origin of the solar system.

Although this paper is speculative in many respects, the basic assumptions and the various conclusions form a rather consistent physical picture. From the results of the concrete applications, one may indeed tentatively conclude that the basic concepts are of some significance, i.e., that the theoretical expressions which have been used do actually represent certain systematic empirical trends. Altogether, this is an interesting and perhaps inspiring paper.

After defining a generalized concept of stability according to Jacobi, by relating the trajectories to certain ring-shaped topological regions, Moiseyev[450] (1938) proceeds to the consideration of certain properties of simple periodic orbits. He also illustrates the concepts of simple as well as of generalized stability, in the sense of Jacobi, by related applications to the trivial Kepler ellipse.

This is another paper by Moiseyev that deals with fundamental definitions, and with rather general and elementary conclusions derived on the basis of such definitions; as in the case of most of the related papers by the same author, the subject is of interest from a systematic and general point of view. It is possible that the further elaboration and use of these concepts may prove fruitful in connection with special dynamic problems.

The same comments can be made about another of Moiseyev's papers[465] (1945) in a series on certain properties of the trajectories in the restricted problem of three bodies. He first makes a survey of the various ways in which the characteristic, J, of orbital stability according to Jacobi-Stepanov may be used for explorations of the behavior of trajectories, or of regions in the x,y-plane, with regard to stability or nonstability. Such investigations may be made using the ordinary coordinates or in Birkhoff's phase space. Several of the possible approaches have been made; others are still unexplored. The special procedure is the evaluation of J in all the points of a given curve, for all the trajectories which intersect this given curve under a certain given angle.

The present paper is devoted to the corresponding study of the orthogonal intersections of trajectories with the x- and y-axes in the case of the Copenhagen problem (the two finite masses are equal) by means of the related auxiliary curves $J(x,h) = 0$ and $J(y,h) = 0$; h is the Jacobi constant of the individual trajectory. The resulting curves, $h(x)$ and $h(y)$, are represented and compared with the related curves of zero velocity and with the asymptotes to both curves. The results, in their tabular or graphic form, indicate the points on the coordinate axes where $J = 0$ for a given value of $\underline{h}$ and, accordingly, where between these points the intersecting trajectories are stable or nonstable in the sense of Jacobi. Separate branches of the curves $h(x)$ and $h(y)$ are obtained for direct and retrograde motion of the small particle, according to the occurrence of two signs in the basic analytic expressions.

In another paper in the series <u>On Certain Properties of the Trajectories in the Restricted Problem of Three Bodies</u>, Moiseyev[466] (1945) presents the results of his numerical and graphical exploration of the regions of stability, of nonstability, and of indefinite characteristic with respect to the criteria of the "Anti-Coriolis" or Moiseyev type of stability. The boundaries of the regions of compact stability and of compact nonstability are determined by means of the equations (AC1) = 0 and (AC2) = 0, i.e., by means of the equations which represent the limiting curves of the regions where the two essential functions, (AC1) and (AC2), are either positive or negative. Since, in the special case of the Copenhagen problem (where $m_1 = m_2 = 1$ for the two finite masses), real motion is possible for a certain range of values of the Jacobi constant, $\underline{h}$, depending on the starting conditions, the various types of stability, etc., vary in function of the coordinates, $\underline{x}$ and $\underline{y}$, and of $\underline{h}$. Since the conditional equations for the determination of the boundaries of the regions are too complicated for an analytical determination, the author proceeds by numerical evaluation of the two functions which eventually determine these boundaries. After considering the much simpler expressions for the special cases $x \equiv 0$ and $y \equiv 0$, and after arriving in this manner at relations h(y) and h(x), from which the intersections of the regions of compact stability or nonstability with the coordinate axes can be obtained, he makes a detailed numerical study of the families of curves (AC1) = 0 and (AC2) = 0. The regions of compact stability or nonstability are then obtained by the superimposition of the results obtained for the two families of curves. Six theorems are then presented, expressing the occurrence of stability or nonstability (in the Anti-Coriolis or Moiseyev sense) in certain regions of the x,y-plane, depending on the value of h. The essential results are represented in tabular form, as well as by numerous graphs. After having explored the x,y-plane in this manner for the occurrence of regions of compact stability or nonstability, the author finally applies his numerical results to the periodic orbits of three classes of the Copenhagen problem. First, he considers the classes f and g, representing retrograde and direct orbits around the mass m_2, and finds the range of h-values for which these orbits are stable. Second, he studies the orbits of the l-class, which are described around both finite masses together. The author's results for the three classes of orbits are in agreement with the corresponding results by N. F. Reyn(644) on the basis of Jacobi-Stepanov's stability criterion, and also with those by A. I. Rybakov(658) on the basis of the Zhukovskiy criterion. These orbits are stable, therefore, according to all three criteria of stability. The present method of investigation is more general than the methods of Reyn and Rybakov (who computed the value of the stability coefficient for selected points on the few periodic orbits selected from the whole range of these classes), and it can be used for the study of selected orbits, as well as for the establishment of whole regions of stability for given values of h.

This paper(466) is one of the most valuable contributions by the author; it not only outlines a method of investigation, but also successfully applies this method toward obtaining a deeper insight into the stability problem for periodic orbits of the Copenhagen type. The results are of permanent interest and, as they are augmented by later contributions, they will always be appreciated as the pioneer work in this field of celestial mechanics.

Lindblad's work on the evolution of a rotating system of material particles is of cosmogonical interest, because the results were supposed to be applicable to Saturn's rings, to the planetary system, and even to the galaxy. In 1937, Reyn(632) reviewed this work critically.

Lindblad is criticized by Reyn first for defects in his basic scheme, wherein he neglects the gravitational interaction of the particles, inasfar as the action of the individual moving particles on all the remaining particles is concerned. Lindblad distinguished "passive" and "active" particles with respect to gravitational action; however, the force of resistance of the medium is introduced considering all particles as "passive" paricles whose total attraction is equated to zero.

Second, Reyn criticizes Lindblad for his method. She points out that the theorem by Routh, which was used by Lindblad to prove the stability of the particular case of strictly circular motion in the nonresistant case (where the force of resistance is neglected), is only a particular case of Lyapunov's first theorem in his so-called second method. She proceeds to show that the stability found by Lindblad for this case is not absolute but conditional, in Lyapunov's sense. Further, it is shown that Lindblad did not sufficiently or rigorously prove his results concerning the stability of circular motion in the more general case of a resistant force, and that this problem, in the sense of Lyapunov's strict definition, appears to be unsolvable by Lindblad's method. Finally, the author refers to the work of Duboshin, who actually arrived at strict proofs for Lindblad's most important theorems, using Lyapunov's basic theory.

Reyn's criticism of some work by the well-known Swedish astronomer appears to be justified. Not only did Lindblad commit inconsistencies in his scheme and conclusions, but apparently, he was also unaware of Lyapunov's work on such stability problems.

Taking three orbits of the class l of the Copenhagen problem and three of the class m (the necessary data were obtained from the Copenhagen Observatory), Reyn[656] (1940) computed the values of the coefficient of stability according to Jacobi-Stepanov for about 20 points on each of these orbits. Since the stability coefficient, J, has symmetrical values with respect to the x- and y-axes, it was necessary to complete the computations only for one quarter of the whole periodic orbit. All the individual J-values found by Reyn came out positive, and this means that the orbits under consideration are stable according to the criterion of Jacobi-Stepanov. The results are conclusive, of course, only for the range of Jacobi constants, K, covered by the oribts of this investigation. Since Moiseyev, in his qualitative investigations on orbits in the restricted three-body problem, had found two groups of periodic orbits which he believed to be related to the l- and m-class orbits of the Copenhagen problem, and since he also found stability according to Jacobi for the orbits of his two groups, the author of the present paper concludes that her results lend heavy support to the suspected relation between Moiseyev's groups and the classes l and m of the Copenhagen studies.

This is a really interesting application of the stability criterion of Jacobi-Stepanov to six selected periodic orbits. Although all these criteria of stability have no "absolute" significance, it can nevertheless be seen, from studies such as this one, that they have a definite dynamic meaning. The author profited from the availability of the basic data from Copenhagen; still, she deserves credit for the execution of the further computations without the help of automatic equipment.

Although the nonstability of the Lagrangean straight-line solutions has been proved before for the case of circular relative motion of three finite masses (Lyapunov and Zhukovskiy), as well as for the restricted three - body problem (Duboshin), the case of elliptical motion had not been dealt with. Also, the results by Lyapunov and Zhukovskiy for the circular problem of three finite masses were based on only the first-order terms of Lyapunov's theory. Ryabov(655) (1954) investigated the more general case of elliptical relative motion of three finite bodies. Using Lyapunov's second method in successive approximations, the author proves that the straight-line solutions are strictly nonstable as long as the eccentricity does not exceed a certain limit. The exact value of this limit, which depends on the relative values of the three masses, is estimated.

These results are of purely theoretical value, because, for example, in the solar system, the presence of more than three finite masses makes the strict existence of the Lagrangean solutions impossible. Nevertheless, all these stability or non-stability properties are of considerable fundamental interest, especially for a comparison with the corresponding behavior of the triangular solutions. Although the present paper contains no fundamental advances of any kind, the author deserves credit for a systematic investigation of the nonstability features of the particular problem.

Ruprekht and Vanysek[654] (1955) (Czechoslovakians) criticize the work of Radziyevskiy[616] on the problem of the dispersion of scattered clusters as applied to a general solution of an unrestricted problem of three bodies with Newton-Hook mutual attraction. Radziyevskiy derived his cosmogonical conclusions from a combination of his (correct) mathematical expressions for the combined gravitational action of neighboring stars and of the galaxy as a whole with the related surface of zero velocity. The authors point out that, at such large distances (as assumed) from the galactic center, Hill's surface of zero velocity loses all practical significance for motions near the galactic plane, because the branches of these surfaces are then located far outside this plane. The maximum separations of neighboring stars, as computed by Radziyevskiy, make sense only with respect to the z-coordinate, which is rectangular to the galactic plane, and one is actually not forced to conclude that the dispersion of clusters near the periphery of the galactic system is impossible. The related conclusion by Radziyevskiy, that, in the neighborhood of the sun, dispersion of groups of stars must be much slower than previously thought, does not hold for the same reasons.

The six periodic orbits, selected from the classes l and m of the so-called Copenhagen problem (where the two finite masses of the restricted three-body problem are equal), which had previously been studied by N. F. Reyn[644] with respect to stability in the sense of Jacobi, are taken up again by Rybakov[658] (1950) for

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the application of the different criterion of stability established by Zhukovskiy. As Reyn did, the necessary computations were made for about 20 points on each orbit, on the basis of orbital data supplied by the Copenhagen Observatory. Since Zhukovskiy's definition of the stability coefficient incorporates the opposite sign, as in the case of the Jacobi-Stepanov criterion, stable orbits in the sense of Zhukovskiy must show a negative sign for the related coefficient everywhere on the respective orbit. Indeed, all the computations for the selected three orbits of the Copenhagen l-class, as well as those for the three orbits of the m-class, lead to negative stability coefficients and, therefore, to stability in the sense of Zhukovskiy. Therefore, the results are in agreement with those obtained by N. F. Reyn using the Jacobi-Stepanov criterion of stability, and the various conclusions are quite analogous to those by Reyn. In addition to the results of the computations, Rybakov gives the necessary equations for the application of the Zhukovskiy criterion, in the form established by Moiseyev.

As was remarked about Reyn's earlier investigation, this work is an interesting application of stability criteria to individual periodic orbits. Of course, this is only a numerical evaluation of existing theoretical expressions, and it does not represent any fundamental progress.

Severnyi's paper[683] (1940) on the theory of gravitational instability deals with a problem of theoretical astrophysics, rather than celestial mechanics. Although his paper treats only a very special case of the complex problem of the gravitational instability of a gaseous sphere, it also demonstrates in an interesting manner the universal importance of the theory of stability, in Lyapunov's general sense, for celestial mechanics, as well as for certain astrophysical problems. From this point of view, the paper is of more than special interest.

The gravitational stability, in the sense first introduced by Jeans, of a gaseous sphere is considered for the very special case where the deviations, $\delta\rho$, δp, from the undisturbed values, ρ_o and p_o, of density and pressure (in hydrostatic equilibrium) follow the relation $\delta \log \rho / \delta \log p = d \log \rho_o / d \log p_o$. The theory is carried out rigorously, involving the proper boundary conditions and the theory of the "Eigenwerte". The results are used to arrive at some conclusions concerning the maximum size of stable nebulae and stars. The author admits that these conclusions are very qualitative only. A final section on the definition of gravitational instability indicates the importance of Jacobi's and Lyapunov's basic definitions and theories, even for these astrophysical problems, and reference is made to N. F. Reyn's rigorous definition of gravitational instability as an instability of the undisturbed distribution of densities, in terms of Lyapunov's theory.

Stepanov[728] (1936), in a paper on stability in the sense of Jacobi, first describes Birkhoff's method of dealing with the concept of stability according to Jacobi and with the related orbital variations. Using Birkhoff's results for the variation δn of the normal to the orbit, he then transforms the coefficient, I, of the related differential equation back into a function of the phase coordinates, x, y, and Φ (where Φ is the direction angle of the tangent to the orbit in the given point, x,y). It follows that, depending on the sign of I, the whole x, y, Φ-phase space may be divided into regions of stability or instability, in Jacobi's sense.

Although earlier papers by various authors had already dealt with the more limited problems of transverse or longitudinal stability according to Jacobi, Stepanov deserves credit for proceeding to an explicity "absolute" criterion of stability with respect to x, y, and Φ. This is a good and interesting contribution, the results of which are suited for further use in various problems of celestial mechanics.

Tomson[762], in 1955, applied three different characteristics of orbital stability, as developed by Zhukovskiy, Jacobi-Stepanov, and Moiseyev, to the various cases of Hill's well-known periodic satellite solutions. The values of the characteristics are computed for a number of points along each of Hill's selected periodic orbits, and it is seen that almost all of these orbits are stable according to all the characteristics. Only for the two outermost orbits do any of these characteristics fail to give a conclusive answer with respect to orbital stability. The author then considers the case of isoenergetic orbits and the existence of a kinetic focus. Using the theorem by Sturm-Liouville, he succeeds in finding, on a given orbital curve, certain locations for the kinetic focus in the neighboorhood of which a system of isoenergetic curves will intersect again, after emerging first from a given original intersection point. The various isoenergetic curves may represent the disturbed trajectories resulting from small displacements of the given "undisturbed" trajectory, and it is shown that the disturbed and the undisturbed trajectories will intersect each other repeatedly, as long as the orbits are stable according to the various characteristics of orbital stability.

This is a well-worked-out mathematical research paper. The principles used are not new, for Tomson makes a direct application of a well-known theorem to a particular differential equation; but, since Hill's periodic solutions may be used as intermediate orbits in lunar theory or in the theory of satellite motion, it is interesting and important to know that these orbits are stable, as far as the three criteria of stability are concerned.

Although a detailed analysis of the stability features of Hill's periodic orbits appears in Investigation of a Case of Orbital Stability of the Solutions of a System of Differential Equations(762), Tomson(761) (1955) considers the whole region in a qualitative way, determining regions of compact stability, of compact nonstability, and of conditional stability for each of the three different criteria of stability. An essential part of the paper is the comparison of the results from the different criteria of stability. Although the finding of regions of compact stability, of compact nonstability, or of conditional stability is relatively simple for the criteria of Moiseyev and Zhukovskiy, because the problem is reduced in these cases to the behavior of the signs of two essential quantities in the expression for the coefficient of stability, the problem is very complex in the case of stability according to Jacobi, and only a partial investigation has been made of this case. In all cases, a region of compact stability for a given class of isoenergetic trajectories, starting from one common point, is defined as the region in which the stability coefficient, D*, is negative in all points of all the possible (isoenergetic) trajectories within such a region. Similarly, compact nonstability is associated with D* 0 in all points of the region. In the case of conditional stability, however, the sign of D* can be positive or negative within the related region. In the cases of the criteria of Moiseyev and Zhukovskiy, the borderlines of the different regions are found by solving the proper equations for $D^* = 0$; for stability according to Jacobi, the author limits his study to the intersections of the various regions with the x- and y-axes, and to a qualitative consideration of the immediate neighborhood of the origin of the coordinate system (or of the planet), as well as of the points at very large distances from this origin. The various findings are illustrated by numerous figures and drawings, as well as in the form of numerical tables.

As to the essential results, it is interesting that certain regions may be stable according to one criterion, but nonstable according to one or two of the others. Nevertheless, the following substantial results emerge: (1) If $\underline{h}$ is the energy constant of the Jacobi integral, then for $-\infty < h < h_L$ (where h_L is the Jacobi constant of a particle resting in the nearest Lagrangean point), a ring-shaped region exists around the planet where compact stability is found on the basis of all three criteria of stability; (2) for all $h < 0$, a ring-shaped region around the planet exists which is characterized by compact stability according to Moiseyev, as well as according to Zhukovskiy; (3) for all $h > 0$, a region exists around the planet where no trajectory can have nondisappearing negative D* values in all its points within this region; this region of "endangered stability", or even nonstability, becomes smaller as $h \rightarrow 0$. (4) For $h > +1.24$, certain regions intersecting the y-axis between $y = +1.000$ and $y = +1.710$, as well as between $y = -1.000$ and $y = -1.710$, are regions of "contradictory" status, i.e., regions where the orbits are stable according to one, but unstable according to another, criterion (in Hill's problem, the sun is located on the positive x-axis).

Although the results are quite interesting, the analysis is rather technical, insofar as all the necessary equations had been established by Moiseyev. The author, who seems to be a pupil of Moiseyev, expresses gratitude to Moiseyev for his help in the preparations for this investigation. Still, the author deserves much credit for presenting many interesting facts, such as the stable behavior of close satellites according to all three criteria of stability.

d. Restricted Three-Body Problem

Agrest's[7] (1945) consideration of the mutual interaction of Gegenschein particles, and his introduction of a resistant medium into the differential equations of motion for the individual particle, were discussed in the section on Criteria of Stability.

Chebotarev[99] (1956), with Bozhkova, applied his method (cf. Application of Periodic Orbits to the Study of the Motion of Minor Planets[93]) to the special group of the Trojan planets. The absolute orbit from which the individual perturbational variations are studied is the rigorous triangular solution by Lagrange, where Jupiter and the small body are moving in ellipses in the same orbital plane. For an actual planet, (588) Achilles, the coefficients of the analytical expressions for the variations of the true elliptical elements are computed. The results are used to represent, in two successive approximations, six positions of the planet from 1926 to 1951. The residuals are of the order $\pm$ 0°3.

For the short time interval of 25 years, the agreement between the theory and the observations must be considered rather poor, even though perfection could not be expected from a first-order theory. It seems, also, that the well-known librational motion with a period of about 150 years is not obtained; the analytical expressions evidently do not produce this librational motion, but the authors do not even mention anything relating to this most significant feature of the motion of the Trojan planets. The Trojans represent a very special and difficult problem for any analytical theory; therefore, the poor results may have no bearing on the general quality of the method by Chebotarev. Altogether, this paper appears to be a failure as far as the given theory for the Trojans is concerned. The authors should have realized that such a relatively simple theory must fail in the rather singular and complex Trojan problem.

The distribution of the minor planets in dependence on their Jacobi constants, h, was investigated by A. N. Chibisov[115] (1936). Although a similar study had been made previously by A. Klose, he neglected the influence of the inclinations; also, Chibisov had 241 planets more than Klose at his disposition. The individual h-values as computed by the author from the elements a (mean distance), e (eccentricity), and i (inclination) are still approximate, because the Jacobi integral has additional terms of the order of Jupiter's mass. Furthermore, the application of the restricted problem of three bodies itself is an approximation, because Jupiter's orbit actually is elliptical, not circular. Nevertheless, the neglect of these factors is less important for the "average" h-value of a minor planet than the neglect of the planet's inclination.

Chibisov's frequency distribution of the Jacobi constants of 1,264 planets shows five maxima, one of which is not so clearly pronounced as the other four. Using various criteria for the "statistical stability" or the reality of the five maxima (depending on the choice of the intervals Δh on the time of discovery of the individual planets, etc.), the author finds that all five maxima appear to be statistically stable, i.e., that their real existence seems to be assured. Chibisov then says that Klose, in his earlier study, had a tendency to remove the maxima by means of certain considerations concerning selective observational effects, and that actually this is qualitatively confirmed by the added number of planets in the present investigation; nevertheless, he asserts, the nonhomogeneous distribution with five maxima remains in existence.

Chibisov now uses his more complete material for an investigation of the validity of the following hypotheses: (1) the hypothesis by Klose, namely, that the minor-planet orbits were originally almost uniformly distributed with respect to their mean distances from the sun, and that they moved in nearly circular orbits; and (2) the hypotheses by Olbers and Bobrovnikoff, namely, that the system of minor planets is the result of the destruction or decay of one original planet (Olbers) or of a comet (Bobrovnikoff). The author says that his investigation is only of a preliminary nature. He finds, from a consideration of the possible changes in the elements e and a (for constant h), that it is impossible to reduce the distribution of the mean distances, a, to anything like a uniform arrangement, and that Klose's hypothesis therefore cannot be confirmed. In making the necessary computations, he neglects the inclinations, i, just as Klose did. As to Olbers' hypothesis, a common origin of the asteroids should be indicated by a corresponding concentration of a,e-curves at a certain region, without violation of the condition h = constant for each individual planet, but no possibility of this kind is indicated either. Also, the origin from a common cometary-type orbit is impossible according to the present distribution of the numbered minor planets in the a,e-diagram.

The author deserves credit for determining the frequency distribution of the Jacobi constants for an increased number of planets, and for the increased accuracy obtained as a result of the proper consideration of the orbital inclinations relative to Jupiter's plane of motion. He also has to be credited with making it quite clear that the

Jacobi constants are not disturbed homogeneously, and that the various hypotheses by Klose, Olbers, and Bobrovnikoff are not compatible with the condition h = constant. This is a constructive but not outstanding contribution which consists mainly in the continuation of earlier studies on the basis of increased empirical material. The author may be criticized for neglecting the inclinations, i, in the last and most important part of his paper, especially after he had criticized Klose for computing the h-values without including them.

For this paper, Chibisov needed the individual values of the Jacobi constants for the 1,264 minor planets which were included in the 1934 Minor Planet Volume; he gives these in Catalog of the Values of the Jacobi Constant for the Asteroids of the 1934 List(116) (1936).

T. V. Vodop'yanova(785) (1939) made an investigation of the comets with periods under 100 years similar to the one by Chibisov for the minor planets. The frequency distribution of the Jacobi constants, h, of those comets, and the related distribution of the aphel distances, Q, were studied. Although the data from not more than 70 comets with periodic orbits are small from the statistical point of view, the existence of two major maxima in the h-distribution, and thus of two comet groups, F and G, is well indicated. A third group, H, which comprises the remaining comets, spreads out over a rather wide range of h-values. As to the distribution of the aphel distances, Q, most of the comets belong to the so-called Jupiter group and have Q-values between four and eight astronomical units; three additional groups are associated with the orbits of Saturn, Uranus, and Neptune in a similar manner. Only the comets of the Jupiter group are studied in more detail, and it is found that they include the groups F and G with respect to the distribution of the Jacobi constants, h. One interesting result is that all the cometary orbits are unstable in the sense of Hill's characteristic of the zero-velocity curve, although the vast majority of the minor planets are stable with regard to this criterion (cf. Reference 110). Further, it is found that, with respect to the h-distribution, the system of minor planets appears to be "continued" by the comets, with an overlap of both systems in a certain interval of h-values. Accordingly, a small number of minor planets have "cometlike" orbits, and a small fraction of the comets move in "minor-planet-like" orbits. Although for some comets increases or decreases of their Jacobi constants are indicated from a comparison of their elements in numerous apparitions, altogether, it seems that the constancy of the h-values is well established for the majority of the objects.

No new methodology is contained in this paper; nevertheless, it constitutes a very commendable contribution, because this statistical study clearly reveals the essential features of the known system of periodic comets, as compared with the corresponding characteristics of the minor-planet system. The investigation was made under the general supervision of Moiseyev.

N. D. Moiseyev's papers on the compatibility of osculating orbits are not too impressive. In the first of these, Moiseyev[438] (1936), considering the fact that an osculating orbit, which is rigorously valid only at a certain given moment, t_o, will deviate from the later true motion of a small body more and more as time goes on, gives some theorems concerning the compatibility of such a fixed elliptical orbit with actual motion in certain possible or impossible regions, depending on certain limiting distances from the sun. The main theorem says that a given set of orbital elements determines, in connection with the curve of zero velocity from the Jacobi integral, two limiting circles at the distances $R_m(h)$ and $R_M(h)$ from the sun. The regions inside the smaller and outside the larger circle are regions of possible motion, but, in the region between the two circles, the Jacobi integral (based on the fixed, undisturbed elements) may lead to a negative V^2 and, thus, to incompatible motion. Accordingly, the author states that orbits which are either completely inside the smaller circle or entirely outside the larger circle are compatible with the system of osculating elements, but that the orbits are incompatible if located partly or entirely in the "critical" region between the two circles.

Although these theorems are of a certain qualitative interest, Moiseyev does not give a clear proof for the first and basic theorem. It appears as if the theorems are correct and, apparently, that the author used numerical steps for verifying them, without saying so. Everything should be presented in a clearer and more convincing manner. Altogether, this is not one of the impressive contributions by this author.

In the second paper(439) (1936), Moiseyev considers compatibility with the characteristics of the apsides. For this purpose, the contact characteristics with a heliocentric ramily of circles are

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introduced for direct and for retrograde motion of the small body. Moiseyev constructs the regions of pure perihelion contacts and the corresponding regions of pure aphelion contacts, as well as the regions of "mixed" characteristics. He states certain rather evident theorems for the compatibility or noncompatibility of an osculating orbit (i.e., of a fixed orbit representing the true motion only at one certain moment, t_o) with these regions and contact characteristics. For example, if at any time, the perihelion of the fixed ellipse falls into the region of pure aphelion contacts according to the related characteristic of the restricted three-body problem, then the fixed elliptical orbit is not compatible with the apsides characteristics. Noncompatibility is evident also when a perihelion or aphelion falls into the "mixed" zone of contacts, and when the mean motion of the small body in its heliocentric orbit is not commensurable with Jupiter's daily motion.

Although Moiseyev's definitions and statements indicate certain possibilities in a correct manner, the conclusions are more or less self-evident. It is hard to imagine that anything substantial will ever emerge from considerations such as these. In other words, this subject does not seem to represent a problem worthy of much effort.

Then, Moiseyev[440] (1936) returned to the contents of the first communication, inasfar as he considers again the compatibility of a certain elliptical orbit with Hill's characteristic of zero velocity. This time, three dimensions of space instead of two are permitted for the motion of the small body. Essentially, the investigation amounts to the determination of those regions in the nonrotating, ordinary, coordinate system of planetary astronomy which will not coincide at any time with the regions of impossible motion in the rotating coordinate system, when the surfaces of zero velocity rotate as a rigid structure with respect to the resting coordinate system. Such regions are the so-called regions of unconditional possible motion. Similarly, there are regions of unconditional impossible motion which, at all times, coincide with regions of impossible motion in the rotating coordinate frame; and, finally, regions of conditional orbital motion which, alternatively, coincide with regions of possible and of impossible motion in the rotating system. By a proper combination of the laws of elliptical motion for the undisturbed (two-body) or osculating orbit with the three-dimensional Jacobi integral of the restricted three-body problem, Moiseyev arrives at the necessary expressions for the surface of zero velocity and of its envelope (as the surface rotates in the inertial system) and, thus, at the construction of the regions indicated above. The fixed elliptical orbit, as determined by the osculating original elements, is compatible with Hill's characteristic as long as the square of the relative velocity (small body minus Jupiter) does not become negative.

Again, these considerations are correct and have a certain illustrative interest, but they do not seem to lead to anything except to rather trivial and self-evident statements.

Finally, Moiseyev[458] (1939) made a very detailed application of the compatibility criteria, as developed in his earlier papers, to the special case of circular osculating (or starting) orbits for minor planets inside Jupiter's orbit and within Jupiter's plane of motion.

The isoenergetic trajectories of the plane, restricted, three-body problem, which have the same value of the Jacobi constant, h, as a given circular orbit, and the compatibility of these disturbed trajectories with Hill's characteristic of zero velocity, as well as with the author's own apsides characteristics, are considered. This leads to several conclusions concerning the compatibility of circular motion with these characteristics of the rigorous restricted problem. One essential fact is the existence of certain minimum values for the disturbed eccentricity, so that the actual motion becomes compatible with the requirements which are synonymous with those characteristics or criteria. The results of numerous computations concerning the various relations between the values of h and the elliptical (disturbed) elements a and e are presented in the form of graphs and tables.

Moiseyev mentions many times the preliminary and approximating character of his investigation. Actually, many of his conclusions or statements are rather trivial, and, in spite of the length of the paper, nothing very concrete or substantial seems to emerge. In part, this is an unnecessarily complicated treatment of rather simple circumstances, and the complicated description makes certain trivialities look less trivial than they really are.

Numerov's work[512] (1929) on the periodic solutions of the plane problem assuming a circular orbit for Jupiter is presented in the section on Absolute Orbits of Chebotorev.

The pressure of radiation and the corresponding repulsive force were introduced by V. V. Radziyevskiy[605] (1950) into the otherwise classical concept of the restricted three-body problem. It is shown that the radiation effects are significant not only for very small particles of the size 10^{-5} cm, but that they may be noticeable for particles up to about 1 cm in diameter. If the ratio between the gravitational force and the force of the radiation pressure is expressed by means of a parameter, q, then the shape and the location of the surfaces of zero velocity may be studied in dependence on the Jacobi constant, C, as well as on q. The author made some detailed computations for the Hill surfaces. In the neighborhood of the planetary mass and of the two libration centers, L_1 and L_2, he finds that, for small particles in this region, the characteristic features of the zero-velocity curves (the computations are limited to two-dimensional motion in the planet's orbital plane) depend significantly on the value of q and, thus, on the radiation pressure.

This is a contribution of fundamental theoretical, as well as of cosmological, interest. Although the mathematical deductions are simple once the basic new idea has been introduced, the author certainly deserves credit for conceiving and investigating such an idea, which may prove to be important in certain cosmogonical applications.

Later, Radziyevskiy(611) (1953) considered the motion in space of a particle of very small mass in the gravitational and radiational fields of two finite masses, which are moving in circles relative to their center of mass. The investigation extends his earlier work to the more general case of three dimensions. In this problem, it is possible that the surfaces of zero velocity have seven double points, instead of the five in the regular restricted problem. The additional two points occur when one of the two finite masses acts on the particle with a radiation effect which exceeds the opposite gravitational force. The two new points, L_6 and L_7, lie outside the orbital plane of the two finite masses; they are called the coplanar libration points. Investigated in detail are the so-called libration axes, which are the geometrical locus of the libration centers for particles of varying diameter. Some data are computed concerning the passage of the earth through the libration axes of the system sun-Jupiter-particle. Finally, utilizing the radiation effects discussed in this paper, some speculations are added concerning the explanation of quasi-parabolic orbits of comets.

Although the analysis involved is correct, the entire idea of the possible influence of radiation on the libration centers, etc., seems to be somewhat exaggerated. The theoretical discussion of the combined effects of gravitation and radiation is of definite interest, as long as it is realized that, in general, the effect of radiation will be very moderate, except for particles of the order of cosmic dust. As far as the application of the theory to concrete astronomical problems is concerned, the author's conclusions are rather unclear and nebulous.

A. I. Razdol'skiy(625) (1934) is confused as far as the proper separation of the kinematic and dynamic aspects of the restricted three-body problem is concerned. His main error lies in the interpretation of his computations for the undisturbed motion of some Trojans. After transforming these ephemerides into the rotating coordinate system of the restricted problem, he deals with the transformed orbital curves as if they represented the disturbed motion.

The "results" of this paper have to be discarded. Among other things, Razdol'skiy claims that a Trojan planet close to the libration center L_4 or L_5 could not describe an orbit as represented by one of the two classes of periodic solutions which exist in the neighborhood of these points, but could move only in a spiral resulting from the composition of both periodic motions, according to Charlier. This is not true, of course, and the paths which really cannot be described by the actual particles are the ones corresponding to the erroneous results of this paper.

Two of N. F. Reyn's papers(634, 635) on this subject are discussed in the section on Periodic Orbits. She(636) (1938) uses Whittaker's integral (see Reyn's own completion and correction of Whittaker's results(634)) for the derivation of certain inequalities, by means of which the period of a simple, closed, and regular trajectory can be found approximately without the integration of the equations of motion. The method localizes the trajectory between the two borders of an annular region.

This is an interesting and useful contribution, which may well bear fruit in connection with studies of periodic orbits. The author apparently was thoroughly familiar with her subject.

Whittaker's double integral for the approximate determination of the periods of simple periodic orbits, as corrected and extended by Reyn, was further investigated by her and applied to concrete examples of periodic orbits(640) (1939). She introduces the concept of the "separante" as the curve separating the regions in which the integrand of the essential double integral is either positive or negative. The system of separantes is studied in its dependence on the Jacobi constant, h. It is shown how the introduction of the separante facilitates the derivation of estimates for the orbital periods. The method is practically applied, not only to Hill's satellite problem and to the orbits of the Copenhagen problem, but also to the case of one finite central mass in a rotating coordinate system. Furthermore, the author shows that the periods of certain orbits established by Moiseyev in the case of the Copenhagen problem agree closely with those of similar classes found by Stroemgren, and that this agreement further strengthens the suspected identity of Moiseyev's and Stroemgren's orbital classes.

This is a good and solid contribution from the theoretical, as well as from the practical, point of view. The results are interesting and well presented. Definitely, this is an above-average paper by a competent investigator in this field.

Reyn[637] (1938) introduced a uniformly rotating coordinate system, as in the case of the circular restricted problem of three bodies, but makes an allowance for the elliptical motion of the two true finite masses, m_1 and m_2, by replacing their gravitational action on the small body by the attraction of two material rings. The dimensions of these rings are proportional to the eccentricity, e, of the elliptical orbits of m_1 and m_2, and their centers move in circular orbits relative to the center of mass. The total mass of each ring equals the corresponding mass, m_1 or m_2. The advantage of this scheme is the fact that it admits an integral which corresponds to Jacobi's integral in the circular restricted problem.

The scheme proposed by Reyn may be useful for qualitative studies in the restricted elliptical problem, following the lines of Moiseyev's investigations in the circular restricted problem. The author indicates this possibility and, indeed, it seems that, with such applications in mind, this is a good and constructive contribution.

The ordinary restricted problem of three bodies assumes a circular orbit for the disturbing planet (Jupiter). Reyn(656) (1940) wanted to improve on this approximating assumption by introducing, instead of the "mean Jupiter" of the ordinary restricted problem, the gravitational action of an elliptical distribution of Jupiter's mass spread out on an epicyclic ellipse about the mean Jupiter with a major axis (in the longitudinal direction) of roughly 0.5 astronomical units. Although, in Fatou's scheme, the mass of a disturbing planet is spread out along its heliocentric orbit, Reyn's scheme affects only a limited region in the neighborhood of the mean position of Jupiter. This has the advantage of reducing the range of singularities to a much smaller distance from Jupiter than in Fatou's problem. The form of the elliptical epicycle represents approximately the geometric possibilities for the location of the true Jupiter.

She then studies the related Jacobi integral and the associated curve of zero velocity in Jupiter's orbital plane, especially the differences which are found when comparing the results of the necessary computations with the corresponding features of the regular restricted problem. The differences are most essential, of course, in the neighborhood of Jupiter. The dimension and shape of the curves of zero velocity for large negative values of Jacobi's constant, h, are illustrated for this region in a number of graphs. As to orbital stability, according to Hill, inside the larger oval which encloses the sun, Reyn finds a small region where asteroid motion is stable on the basis of the regular restricted problem, but nonstable according to her scheme.

This paper has a certain theoretical interest; however, without any practical applications, which would actually show a better representation of the motion of a small body than on the basis of the regular restricted problem, it is not possible to say much about the practical value of the author's scheme. The results may be useful, in certain cases, for an estimate of the possible error or uncertainty introduced by using the mean Jupiter instead of the author's version of the problem, but it still remains to be seen how much this new scheme differs from the true motion, when Jupiter's true gravitational action is considered. Nevertheless, the paper shows good intuition and familiarity with the subject.

K. N. Savchenko's(679) consideration of a special case of the restricted three-body problem with variable masses is discussed in the section on Variable Mass.

A comparison of the purely kinematic features of three different concepts or schemes of a three-body problem was made by B. Shchigolev(695) (1940). The first scheme represents the rigorous solution by Lagrange for three masses, m_1, m_2, and m_3, the three masses being located in the three apices of an equilateral triangle of constant dimensions. Assuming that the mass m_3 is very small, and that the

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restricted problem may be defined by considering decreasingly smaller values of m_3 with the limit $m_3 = 0$, either one may deal with the actual finite masses m_1 and m_2 and neglect the gravitational action of m_3 entirely, or one may make allowance for the small value of m_3 by dealing with two fictitious masses, m_1'' and m_2'', corresponding to the actual "center of mass", $M = m_1 + m_2 + m_3$ (see subsequent paper(696) by the same authors). In this way, two different schemes of a restricted problem may be introduced. The author compares the Lagrangean equilateral solutions of these restricted problems with the corresponding motion in the rigorous problem (in which the gravitational action of m_3 on the two finite masses is included, too).

This paper is a simple comparison obtained by forming the differences between the well-known equilateral solutions of the different schemes. One gains the impression that, with too many words and descriptions of rather self-evident facts, Shchigolev tries to make his paper look more substantial than it really is. Nothing of any significance is contained in this publication.

Ten years later, Shchigolev's work on this subject was still not impressive. Then, he(696) (1950) was concerned with the assumption, in the so-called restricted problem of three bodies, that the small body has a "vanishing" mass and produces no gravitational action on the two finite masses. Two schemes are considered for the purposes of this study: the first scheme deals with a restricted problem in which m_1 and m_2 are the real masses of these two bodies, located mathematically and physically in the same positions; in the second scheme, some

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allowance is made for the existence of the third very small mass, m_3, by introducing, instead of m_1 and m_2, the fictitious finite masses $m_1'' = \frac{Mm_1}{m_1 + m_2}$ and $m_2'' = \frac{Mm_2}{m_1 + m_2}$, with $M = m_1 + m_2 + m_3$. In both schemes, the gravitational action of m_3 on the other two masses is neglected.

The author studied the difference between the true motion of the three bodies, that satisfies the rigorous equations of the nonrestricted problem, and the motion resulting from either of the two restricted schemes. The problem is attacked by forming the difference between the complete and the restricted differential equations in each of the two cases, assuming that at a given time, t_o, the starting conditions are represented by all three schemes. Certain approximate conclusions are made for the special cases of selected simple configurations of the three bodies. The general treatment proceeds by developing the differences between the three systems of differential equations in powers of the small parameter $\mu = \frac{m_3}{m_1 + m_2}$. Special attention is given to the circular restricted problem and to the configurations characterized by the straight-line libration centers, L_1, L_2, and L_3, i.e., to the case of librational motion in the vicinity of these points.

Although Shchigolev makes some statements concerning the dependence of the developments on the various powers of μ, no integration of the differential equations, which are obtained as the differences between the equations valid for the three schemes under consideration, is actually obtained in this paper. The most substantial result is the finding that, in the case of the librational motions in the neighborhood of the straight-line Lagrangean points, the actual determination of the differences between the restricted and the nonrestricted motion is reduced to quadratures.

In spite of the rather elaborate analytical expressions, nothing beyond the routine operations used in forming the differences of the various sets of differential equations actually is involved in this paper. The basic concepts do not seem to be too clearly understood by Shchigolev, who indulges rather extensively in the "philosophical" aspects of the problem. The reasoning appears to be somewhat confused or nebulous in the introduction, and in parts of the paper itself. No substantial results are obtained.

(1). Analytical Treatment

V. M. Loseva(363) (1945) dealt with the problem of the passage of a finite mass, m_2, in a hyperbolic (or parabolic) orbit with respect to a fixed mass, m_1, which is accompanied by a satellite of negligible mass, m_3. The study is limited to the interval of time in which the motion of m_3 with respect to m_1 will be noticeably affected by the passing mass m_2, or while m_2 is in the so-called sphere of action with respect to m_1. During the approach of m_2, a rotating coordinate system, determined by the changing distance and variable angular velocity of m_2 relative to m_1, is used for the integration of the corresponding differential equations of motion for the mass m_3. This is facilitated by Nechvil's transformations involving the scale of the coordinate system and a nonuniform "time". Loseva finds that the various particular solutions and libration centers exist in close analogy to the ordinary restricted problem of three bodies.

From here on, the steps taken by Loseva are somewhat nebulous and hard to understand. Introducing average values for the coefficients on the right-hand sides of the differential equations of motion for a small mass, m_3, in the close neighborhood of the various libration centers, she is then able to integrate these equations with constant coefficients and to arrive at periodic solutions, the stability of which is quite analogous to the corresponding cases in the regular restricted problem. The author refers to stability in the sense of Lyapunov, but, actually, by her process of "averaging" and approximation, she departs immediately from any rigorous stability treatment. Her whole procedure, in the discussion of periodic motion and its stability on the basis of a scheme which is limited in time and involves doubtful and unclear approximations, does not make much sense at all. This is a rather poor paper.

G. A. Merman's paper(393) treating periodic solutions in the restricted problem of three bodies and in Hill's problem is discussed in the section on Periodic Orbits.

In one of the first papers of the so-called "Moscow School", which is also a fundamental paper, because it gives the related definitions and develops a terminology, Moiseyev(429) (1935) considered orbital trajectories from the viewpoint of differential geometry. He had some predecessors in this field, but he goes essentially further than they did.

The properties of the curve which is the geometrical locus of the center of curvature are studied as they relate to the actual trajectory. In the case of the restricted problem of three bodies, for example, the center of curvature is located on a conic section relative to the given point on the actual trajectory. The radius, ρ, of curvature depends on the orbital velocity, $\underline{v}$, the total force, $\underline{F}$, the rate of rotation, $\underline{n}$, of the rotating coordinate system, and the angle, γ, between the normal to trajectory and the direction of $\underline{F}$ in the form $\rho = \frac{v^2}{-2nv + F\cos\gamma}$. This expression can be positive, negative, or zero, depending on the values of the various quantities. If one puts $D = F^2 - 4n^2v^2$, then for $D < 0$, the above expression represents an ellipse, and the center of curvature will be located somewhere on this ellipse, the value of ρ depending on the given angle γ between the normal to $\underline{v}$ and $\underline{F}$. In this case, one also has $\rho < 0$, which means that the real point describes an orbit which is curved around the center of curvature in the negative sense, or in a clockwise direction. This is true for all possible values of γ in connection with $D < 0$. For $D = 0$, the geometrical locus of the center of curvature is a parabola, again with $\rho < 0$. However, if the direction of $\underline{F}$ concides with the direction of the normal to the orbit, or for $\gamma = 0$, one gets $\rho = \infty$. For $D > 0$, finally, the locus for the center of curvature is a hyperbola with $\rho > 0$ on one branch and $\rho < 0$ on the other. $\rho = \infty$ occurs, too, in the directions which are normal to the asymptotes of the hyperbola. In this case, the actual trajectory has an inflection point at the given moment.

From the properties of the curve which represents the center of curvature, certain conclusions can be drawn concerning the nature of the related trajectory. The relation between the three variables, ρ , ν , and v, may be represented as a surface in three-dimensional space. These "characteristic surfaces" can be associated with the integrals of the differential equations. If the differential equations produce the same characteristic surface as certain given equations, then these latter equations are the integrals of the problem.

After a review of some earlier work by the U. S. author Kasner, Moiseyev proceeds to Darwin's characteristic curve. This is the curve $D = 0$ in the case of the restricted problem of three bodies, where $D = F^2 - 4 n^2 v^2$ is reduced to the form $D = U'_x + U'_y - 8 n^2 (U + h)$ by means of the Jacobi integral; h is the Jacobi constant. Darwin did not completely investigate the properties of this curve, and the author undertakes the task in the present paper. The curve $D = 0$ separates the elliptical type of curve for the center of curvature from the hyperbolic type. In the restricted problem, one has $D < 0$ at large distances from the center of the coordinates, and $D > 0$ in those regions which are close to either one of the finite masses. Darwin's curve, $D = 0$, consists of closed branches, and the only points where they may intersect with Hill's curves of zero velocity are the five libration points. If the values of h for particles resting in these points are designated by h_1, then, for $h \neq h_1$, Hill's curve of zero velocity is always inside of Darwin's curve.

Next, Moiseyev proceeds to some theorems concerning the general properties of trajectories in the case of a rotating coordinate system. These theorems are concerned with the direction of the radius, ρ, of curvature as compared with, or referred to, the direction of force $\underline{F}$, and with similar qualitative features. The author's geometrical results are in agreement with the behavior of the actual trajectories which Darwin and E. Stroemgren determined by numerical integrations. It should be emphasized, however, that Moiseyev's statements concerning the behavior of trajectories are essentially negative ones, telling mainly what a certain trajectory cannot look like. The method enables Moiseyev to find certain errors in drawings of orbits published by Charlier in his well-known work on celestial mechanics. The investigation is further concerned with the possibilities of cusps and loops in a trajectory.

These results have been discussed in detail, because they are typical of the contents of the whole paper, and because they form the basis for further, more complicated orbital characteristics, the introduction of which is illustrated in the paper by many drawings. The author studies the behavior of the trajectories in the different regions into which the plane is divided by the curves of Hill and Darwin. He finds, for instance, that trajectories between these two characteristic curves will be forced to leave this region as soon as they have experienced a tangential contact with one of the "equipotential curves" on which U = constant. With the investigation of the various possibilities for the contact of a given trajectory with individual curves of given family $f(x,y) = c$, the author has arrived at the main subject of the paper. He gives criteria for the occurrence of interior and exterior contacts with the curves of a given family of functions. Of special interest are the contacts with the curves U = constant. In this connection, the so-called "inflection curve", as the geometrical locus of the inflection points of a family of curves, is of interest, and is applied to the special family U = constant. The properties of this curve are studied, especially as far as they are related to the properties of a given trajectory. Similarly, the geometrical locus of those points where the individual curves of a given family f(x,y) = c are tangential to the direction of the force, F, is called the "distributice". All these concepts are used for the purpose of qualitative classification and for the formulation of certain theorems concerning the properties of the trajectories in certain specific problems. The "contact characteristics" are applied to the restricted problem of three bodies for the related equipotential curves U = constant. The method makes it possible, for instance, to decide whether the contacts of the trajectory with U = constant are interior or exterior in a given region. The contacts with a family of circles are studied; this special application may be useful in connection with the pericenter and the apocenter of a given orbital trajectory. The "characteristics" may be used, also, for considering the possibility or impossibility of motion in a certain region under certain given conditions.

Finally, Moiseyev applies his methods to the generalized coordinates of Poincaré and Birkhoff and arrives at a geometrical interpretation of Lyapunov's definition of orbital stability. He claims that Lyapunov's theory is only a special case of the general theory of contact characteristics. It is hard to see, however, how one could have arrived at Lyapunov's theory in this geometrical and rather complex way.

The author's work is an ingenious and very interesting contribution and, certainly, it is the result of a deep and highly competent effort. Although the actual results generally take the form of negative statements, they are undoubtedly valuable for checking on trajectories which have been determined in some direct way. This paper has already proved fruitful by inspiring many further applications and investigations.

A series of corrections to the contents of the above paper(429) is given in Reference 459. These errors were contained primarily in the descriptions of the figures and in the description of certain features of Darwin's curve.

Reyn(638) (1939) extended the results of Moiseyev to the case where the two finite masses of the restricted three-body problem have an elliptical motion relative to each other. This is done with the help of the special coordinates first introduced by Nechvil. The contact characteristic of a given trajectory with respect to a topographical system of curves is then reduced to a form which depends essentially on the location of the so-called curve of Hadamard. The analytical expression for this curve contains the coordinates and the first and second derivatives of the function $f(x,y) = c$, which determines the topographical system of curves. In the present problem, Hadamard's curve depends also on the angular variable, θ, which determines the periodically changing distance between the two finite masses. The results in the present investigation are obtained from those of the circular restricted problem, essentially by the proper consideration of the two extreme or limiting cases, where the distance between the finite masses is either a maximum or a minimum. In this way, the influence of the eccentricity, e, of the relative orbit of the two finite masses on Moiseyev's qualitative analysis of the contact problem is found. The corresponding results are derived in some detail for two different systems of topographical curves. One section of the paper is concerned with the regions of exterior or interior contact (as determined by Hadamard's curve) with respect to a family of circles centered at the mass center of the whole system. Another section is devoted to the corresponding contact characteristics with respect to a system of "quasi-equipotential curves".

Although the essential "spade work" for this investigation could be taken from Moiseyev's and Nechvil's previous studies, it has to be granted that the author has made an interesting extension to the closely related properties of the elliptical case. Altogether, this work by Reyn is a competent analytical contribution of good quality.

Whittaker intuitively formulated a theorem concerning the existence of periodic solutions for trajectories inside a region between certain limiting curves, but he did not give a rigorous proof. Moiseyev[454] (1939) not only furnishes this proof, but gives it for a more general kind of problem.

The limiting curves of such a "Whittaker region" are defined by the fact that the contact characteristics of the possible trajectories of the given problem with respect to these limiting curves have opposite (and unchanging) signs along these curves, so that the contact of the trajectory with the curve would always be exterior (or interior). Using the concepts and procedures of his general method of contact characteristics (cf. Reference 429), the author makes a thorough study of the problem. The main result of the paper is contained in the proof of the following theorem: If there is considered such a family of isoenergetic trajectories in the restricted problem of three bodies, that no singular points occur within a certain region which is a "Whittaker region" according to the above definition, and if no common points with the curve of zero velocity, as well as with Hadamard's curve, occur, and if the contact characteristic is of opposite sign at the two limiting curves of the region, then there exists inside this region a trajectory (going through the "vertex" in the case where the region is of the corresponding type) which represents a periodic orbit in the case where the region is a ring of finite width. The proof rests on the condition that all the functions involved are analytical. The limiting curve of the region may be composed by a sequence of different analytical functions (curves), but, for a more general type of limits, the proof would not be valid any more.

Without any question, the present paper is a significant contribution. It enriches theoretical knowledge about the qualitative features and possibilities of trajectories in the restricted problem of three bodies, and gives proof of a theorem which was formulated, but not proven, by such an authority as Whittaker.

Moiseyev's third paper[455] in the series on certain general methods of the qualitative analysis of forms of motion in problems of celestial mechanics is discussed in the section on <u>Criteria of Stability</u>.

Moiseyev's previous papers (453-455, 461) (1939 and 1940) dealt mainly with questions of the existence of trajectories in certain given regions with orbital stability in Lyapunov's sense, and with related problems. Moiseyev also devoted a paper (462) (1940) in this series to the characteristics of contact of a given trajectory with a given topographical system of curves, including isoenergetic trajectories of the same type. In Section 1, the general properties or features of contact are studied by means of conform transformations leading to contacts of the transformed trajectories with the straight lines f = constant ($z = x + i\,y$, $w = \sigma + i\,f$, and $z = \Psi\,(w)$ is the analytical expression for the transformation). The sign of $\ddot{f}$ of the trajectory then decides whether the contact is exterior or interior (concave or convex).

Section 2 deals with the contact characteristics with respect to the curves $f + \delta f$ = constant, which differ from the curves f = constant by the small deformations δf, where $\delta f = \delta f(\sigma)$ leads to the simplest expressions in the case where σ is the length of the arc on the "nonvariationed" or original curve f = constant.

Section 3 gives a method for the reduction of the so-called variation of the contact characteristic (or of $\delta \ddot{f} = \ddot{f} - \ddot{f}_o$) to a binomial form. This reduced form depends on one arbitrary parameter, k. For k = 0, one obtains $\ddot{f}$ in the form of Jacobi-Stepanov; for $k = \frac{1}{2}$, a form in agreement with the stability coefficient according to Zhukovskiy emerges; and for $k = \frac{3}{2}$, the form of Moiseyev's or of the "anti-Coriolis" case is found as something new.

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In Section 4, the special case of the trajectory where $f = f_o$ is treated. Again, results are derived for $k = 0, \frac{1}{2}$, and $\frac{3}{2}$.

In Section 5, the so-called "quasi curvature" and the related center of quasi curvature are introduced on the basis of the various coefficients in the expression for $\ddot{f}$ (according to Jacobi-Stepanov, Zhukovskiy, and Moiseyev).

Section 6 shows that the various types of orbital stability can be considered as special cases of a more generally defined stability.

Section 7 deals with the regions of the so-called compact orbital stability, and, finally, Section 8 deals with the possibility of the "localization" of trajectories, essentially by means of Whittaker's zones.

As in most of his papers, Moiseyev's style and terminology are somewhat awkward and sometimes unnecessarily complicated, but the essential contents are quite interesting and a valuable contribution to progress in the field of celestial mechanics. Undoubtedly, Moiseyev was one of the most gifted Soviet astronomers in this field. There is no flaw in his developments, even though they are sometimes made "the hard way" with respect to analytical simplicity.

A geometrical derivation of the first-order terms of the differential relations, by which the relative curvature of two dynamic trajectories in the restricted three-body problem is determined, was given by Moiseyev(430) (1936). He also calls attention to the fact that some authors have used a too-narrow concept of the "stability according to Jacobi" when discussing stability questions, or a concept which actually incorporates only the so-called "transverse" stability, instead of the necessary two-dimensional features.

Although the author arrives at the correct analytical relations, his way of arriving at them is unnecessarily complex and elaborate. He made no use of certain rather obvious relations, which would tend to transform his final equations into a shorter and simpler form. The well-known author demonstrates that he arrives at the right answers, however, even when unnecessary hurdles are climbed; but as far as mathematical elegance is concerned, this work is not too impressive. He deserves some credit, however, for introducing a variation of the Jacobi constant, $\underline{h}$, in connection with the variation of the trajectory, while Hill, Jacobi, and others considered only isoenergetic variations.

Certain characteristics in the restricted three-body problem were the theme of another series of papers by Moiseyev. The first[460] (1936) of these contains a relatively simple application of the characteristic curve of Darwin to the motion in the neighborhood of the libration center L_4. Whereas Darwin had mentioned only certain properties of his curve, the author demonstrates that Darwin's curve in the given case is approximately an ellipse which is concentric and coaxial with the related ellipse representing approximately Hill's curve of zero velocity. After a transformation of the equations to "normal" coordinates, the author studies the contact characteristic with respect to a family of circles which have their center at L_4, and to trajectories which are a family of isoenergetic curves. He finds that, in the case of an exterior contact, the trajectory cannot remain within a certain region of the plane. Finally, he considers the contact of the trajectories with a family of ellipsoids which is enveloping L_4. Investigating the different regions into which the characteristic curves divide the orbital possibilities, certain orbits are found which are not stable according to Lyapunov's theory of stability.

As is typical for the method of contact characteristics, Moiseyev arrives at certain conclusions concerning what the orbits cannot do or look like. The paper is a simple yet interesting application of the general principles developed in his earlier paper(429). Altogether, this paper is a contribution of moderate yet positive value, illustrating more than extending previous knowledge about the actual subject of motion in the neighborhood of the libration center L_4.

Moiseyev(437) (1936) applied his method (cf. Reference 429) to the so-called Copenhagen problem. In this special case of the restricted problem of three bodies, for which numerous periodic solutions have been established at the Copenhagen Observatory by the way of numerical integrations, the two finite masses are equal. Moiseyev's contact characteristics are systematically applied to the problem in the sections of the paper concerned with: (1) the differential equations of motion in the Copenhagen problem and Hill's curve of zero velocity; (2) characteristics of the family of trajectories (Hill's and Darwin's curves); (3) contact characteristics of the trajectories and of a family of circles around the center of the system; (4) auxiliary curves - the distributice and Hadamard's curves; (5) the properties of the geometrical locus of the contacts with the family of circles; (6) the contact characteristic as a function of the Jacobi constant, h; and (7) the characteristic of the contacts with the equipotential curves. Many illustrations are given to help in understanding the sometimes complicated curves and regions, which are of interest in connection with the qualitative properties of the trajectories.

This paper, too, represents the result of a considerable effort, even though the application to this special problem makes the whole procedure somewhat mechanical. Although all these geometrical-dynamic relations are certainly of some interest, the fact that the method is limited to a compilation of negative statements concerning the behavior of trajectories makes it hard to see how the method can be useful in the actual solution of given dynamic problems.

Moiseyev[441] (1936) continues with the contact characteristic of an isoenergetic family of trajectories (Jacobi constant h = constant) with a family of heliocentric circles, thus arriving at certain qualitative conclusions concerning the regions of purely exterior and purely interior contact. For the detailed study of the branches of the curve $\ddot{R}(h) = 0$, in their dependence on h, extensive computations for selected values of the solar distance, R, and of the polar angle, θ, have been made, and the resulting values of $h(R, \theta)$ have been tabulated. The osculatrice and Hadamard's curve (cf. Reference 429) are also used as auxiliary curves in the present study. The results are discussed in detail and illustrated by numerous graphs. Still, the author considered this to be a preliminary investigation and mentions that the singular points, especially, require further study.

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As was remarked in connection with the author's earlier papers, no direct progress in the analytical treatment of a dynamic problem is made by the author's contribution; it is also true, however, that studies such as this enable one to arrive at a good general picture of certain orbital possibilities or limitations. From this point of view, this is a commendable contribution, even though everything is obtained rather automatically by the proper application of the basic method as previously outlined.

Moiseyev[456] (1939) then made extensive use of a modification of a theorem that had been proved in his earlier paper (cf. Reference 454). According to the modified theorem, one can prove the existence of a periodic orbit in the region between two curves of a given topographic family by showing that one, and only one, closed osculatrice (for an isoenergetic family of trajectories) is located within the same region. The first paragraph introduces two new auxiliary curves which are convenient for the application of the basic theorem to concrete cases. These curves are the so-called indicatrice of osculations and the indicatrice of the zero velocities. The first curve is the geometrical locus of the contacts of the osculatrice with the curves of the topographic system; the second represents the geometrical locus of the contacts of the curves of the topographic family with the curve of zero velocity. By means of the two curves, it is possible to construct the stripe-shaped region which contains only one closed osculatrice.

In the second paragraph, the method is used to study the simple problem of periodic orbits in a rotating coordinate system in the field of only one gravitating mass in the center of the system. Two classes of retrograde synodic motion and one of direct synodic motion are found to exist as periodic orbits. The third paragraph deals with the restricted three-body problem, in particular with orbits around the planetary mass. The existence of retrograde, as well as of direct, periodic orbits is proved, but the author doubts that a complete identity exists between his direct orbits and the corresponding orbits found by Hill. Moiseyev shows that his classes of periodic orbits possess stability according to Jacobi; he remarks that Hill's solutions can be identical to his orbits only insofar as they are stable in Jacobi's sense. The fourth and last paragraph brings the application of the author's basic theorem to the Copenhagen problem with equal finite masses. The existence of periodic orbits of two classes, which, in their essential features, are similar to the classes l and m of Stroemgren, is established. The various results are illustrated by a number of drawings. The author stated his intention of exploring the various problems further.

This is one of the more significant contributions from this competent and gifted author. The further detailed studies promised by the author were later made by Reyn and Tomson. Although the method is essentially an application and further development of the concept of Whittaker's zones, Moiseyev deserves much credit for devising and using a practical method based on Whittaker's idea.

In the fifth paper, Moiseyev[457] (1939) begins by remarking that Poincaré's method or theory permits the establishment of the existence of a periodic trajectory through a given starting position, but does not permit the localization of the whole orbit. The author's method, on the other hand, which is based on Whittaker's zones and on the author's theorem "B" in the preceding paper(456), makes it possible to find a region, more or less narrow as the case may be, in which the whole orbit is located. The method is applied to orbital motions around the sun, using a family of concentric circles (centered in the sun) as the topographic system of curves, which permits the establishment or construction of circular rings containing the various periodic orbits under consideration. Paragraph 1 deals with the equations of motion and with the osculatrice of a family of isoenergetic trajectories with respect to the system of concentric circles. It is found that the osculatrice, as the geometric locus of the contacts of higher order, has four closed branches. These four branches correspond to the four classes of periodic orbits, as discussed in the final Paragraph 4. Paragraph 2 introduces the indicatrice of the osculations, and Paragraph 3 the indicatrice of the curve of zero velocity, because these two curves serve as auxiliary curves, as in Reference 456. The form of these curves is illustrated by means of some graphs.

Paragraph 4 contains the determination and localization of four different classes of periodic orbits around the sun, characterized by their range of values for Jacobi's constant, $\underline{h}$, and by their various other principal features. The first class of periodic orbits is retrograde in the rotating, as well as in the fixed, coordinate system. It is related to parts of Stroemgren's classes $\underline{f}$ and $\underline{h}$ and to Darwin's retrograde class of satellites. The second class of simple periodic orbits,

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which has direct motion, is located inside Jupiter's orbit and is of importance for the system of asteroids. This class is related to a class of Hill, to parts of Stroemgren's classes g and i, and to the class B of Darwin. The third class comprises simple periodic orbits of retrograde synodic motion outside Jupiter's orbit and thus, it is described around both finite masses of the problem. Direct sidereal motion is possible in this class for a certain range of h-values. This class is also of some interest with regard to asteroids; it is analogous to Stroemgren's class l. The fourth class is retrograde, too, and analogous to Stroemgren's class m. Concluding, the author finds that the orbits in all four classes are stable in the sense of the generalized criterion of Jacobi.

This is a good and constructive contribution to the literature in celestial mechanics. It demonstrates, at the same time, the fruitfulness of the author's basic ideas and methods.

The sixth part of this work by Moiseyev[465] (1945) is discussed in the section on the Criteria of Stability. He arrives at some interesting results using rather elementary means.

Two other papers[466,467] by Moiseyev that deal with certain characteristics of the trajectories in the restricted problem of three bodies were published in 1945; one[466] is discussed in the section on the Criteria of Stability, and the other[467] in the section on the Disturbing Function.

Moiseyev[431] (1936) applied his method of contact characteristics to the study of the contacts of a family of isoenergetic trajectories with a system of surfaces,

$\sqrt{2(U+h)}\,(x \sin \Phi - y \cos \Phi) + n\,(x^2 + y^2) = f$, in Birkhoff's phase space (x, y, Φ). First, the condition $\dot{f} = 0$ leads to the result that contacts are possible only on two planes, one of which is determined by $y = 0$, while the other goes through the Lagrangean libration centers L_4 and L_5. From the consideration of $\ddot{f}$, it follows, then, that the geometrical locus of the contacts of higher order degenerates to a straight line parallel to the $O\Phi$-axis. In the x, y-plane, this straight line is reduced by its projection to the point where the connection of the points L_4 and L_5 intersects the x-axis. The essential result of the whole study is contained in the theorem that certain regions in the x, y-plane, characterized by the ratio of the distances of their points from the two finite masses and by their location with regard to the x-axis, are anepicyclic regions, i.e., no ordinary trajectories can be contained completely in either one of these limited regions. This leads to the additional theorem that no periodic orbit exists in the restricted three-body problem which does not intersect at least one of the two straight lines mentioned above.

This is one of the more substantial and interesting papers by Moiseyev. The qualitative results emerging from this study are of theoretical significance and should be helpful for any studies on the subject of periodic orbits.

Moiseyev(433) (1936) extended his earlier study (cf. Reference 431), which deals with the ordinary restricted problem where Jupiter's orbit is assumed to be circular, to the more general case of elliptical motion of the two finite masses relative to their center of mass. Using Nechvil's coordinates and considering the small body's motion with respect to Poincaré's phase space $(x, y, \dot{x}, \dot{y})$, he arrives at theorems quite analogous to those established in the earlier paper. Certain anepicyclic regions exist in which no trajectories can be contained in their entirety, and all regular trajectories, in Hadamard's sense, must intersect at least one of the two straight lines determined by the Lagrangean points L_1, L_2, L_3 or L_4, L_5, respectively, an infinite number of times.

This study, too, is of definite theoretical interest, and the results may also be useful for further studies on periodic orbits. Certainly, this is a worth-while contribution.

In 1936, Moiseyev[436] considered the plane restricted problem, using Birkhoff's phase space $(x\ y, \Phi)$. He assumes that at a given starting moment, t_o, this space is filled with phase points distributed with uniform density, ν. From the incompressibility condition, it follows, then, that this density distribution will remain invariant at all later moments, $t > t_o$. The investigation is concerned with a certain property of the stream motion of Birkhoff's phase fluid with respect to a one-parameter topographic family of cylindrical surfaces, $f(x, y) = c$. A certain differential quantity is defined as the measure of the difference between the number of passages through the points of exterior and interior contact with respect to an infinitesimally narrow cylindrical ring, as determined by the differential <u>dc</u>.

This is a purely theoretical and very abstract discussion; it is hard to see how this may lend itself to concrete applications, although the author closes with some reference to the problem of the distribution of the orbital elements of comets and meteorites.

Moiseyev's[442] (1937) treatment of periodic trajectories about the point of libration in the Copenhagen problem is discussed in the section on Periodic Orbits.

In his consideration of the Gyldén-Moulton hypothesis of the origin of Gegenschein, Moiseyev[446] (1938) starts with the differential equations of motion for an isoenergetic family of particles in the three-dimensional restricted problem of three bodies. The form he uses amounts to a generalization of Birkhoff's system. In the related phase space, a fluid governed by these equations would be incompressible, and the density in phase space would remain uniform at all times, if it were uniform for $t = t_o$. The author then determines the expression for the corresponding density distribution, D (x, y, z), in ordinary coordinates of the restricted problem, considering for the Jacobi constant, h, a certain interval, $h_m \leq h \leq h_M$. The resulting density distribution, D (x, y, z) = constant, is always identical to the proper surfaces where the potential U (x, y, z) = constant. The adoption of a stationary and uniform distribution of the particles in the phase space is synonymous with the assumption that, in the ordinary rotating coordinate system (x, y, z), all possible directions of the individual velocities have the same probability. On the basis of these rather reasonable assumptions, it is then shown that the theoretical surfaces of equal density cannot be reconciled with the figure of Gegenschein which is indicated by photometric observations. Without the introduction of additional forces, the

Gyldén-Moulton hypothesis concerning the origin of the Gegenschein is incompatible with the dynamic hypothesis of a stationary distribution considered in this paper.

This is a very interesting and substantial contribution, and the results are important as further evidence against the correctness of the Gyldén-Moulton hypothesis for Gegenschein.

Working on the same subject, I. P. Tarasashvili(755) (1938) adopted a coordinate system which rotates with the (circular) motion of the earth around the sun, but which has its origin in the Lagrangean libration point, L_2, near the earth. Developing the force function, U, into a power series and retaining only terms of an order not higher than the second, the author computes a certain number of points on the elliptical curve which represents the intersection of Hill's surface of zero velocity with a plane which is perpendicular to the direction earth-sun, and which contains the point L_2. The surface of zero velocity in these computations is the one which is associated with a certain Jacobi constant, h^*, which in turn, represents the largest individual h-value of the particles in a given cluster. From the size of the intersection of Hill's zero-velocity hyperboloid with the η, ζ -plane through L_2, and from the distance of L_2 from the earth, the author then computed the angle α for the related apparent diameter of the cluster of particles, as seen from the earth. Comparing the results with the observed isophotic curve of Gegenschein, the author finds the related range of values for the Jacobi constant, h^*, which must be associated with particles of the cluster.

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This is a constructive paper; it relates the observed form and size of Gegenschein with the required range of the h*-values for the particles of the corresponding Gyldén-Moulton cluster. Although the results do not solve the question of the validity of the hypothesis by Gyldén and Moulton concerning the origin of Gegenschein, the numerical data obtained may be useful in further investigations of the dynamic possibilities.

In the introduction to a paper on the contemporary status of qualitative mechanics, Moiseyev[453] (1939) mentioned the fact that the qualitative method, developed by the "Moscow School" and largely a result of his own efforts, has begun to play an increasingly important role in celestial mechanics. The method of contact characteristics has been used for work on the problem of orbital stability.

Section 1 of this paper deals with the mechanical problems which have been accessible to the new methods of qualitative analysis, and to the proper classification of these problems. In Section 2, a historical review of the work related to Hill's characteristic curve of zero velocity and to the general problem of the possibility or impossibility of motion in a given region is given. Among the Soviet authors, Moiseyev, Reyn, and Chibisov are mentioned as contributors, and a great number of contributions, all using the same method, have come from the Moscow group. Section 3 reports on Moiseyev's extended study and use of the characteristic curve of Darwin to investigate the properties of the trajectories in the restricted problem of three bodies, especially in the neighborhood of the triangular points. Section 4 is devoted to the characteristic of Jacobi-Stepanov and deals with regions of stability according to Jacobi, using Birkhoff's generalized coordinates. Three more sections of the paper are concerned with the problems of stability. The result is that stability according to Lyapunov guarantees stability according to various other concepts of stability, but not vice versa. In Section 8, the work on contact characteristics is reviewed. In Section 9, the special case of a time-dependent force function, U(t), is considered; it is remarked that the related investigation by Wilkens on the limiting curves and their envelopes in the restricted problem (Seeliger-Festschrift, 1924) is in error.

Moiseyev[463] (1940) considered, in the rotating coordinate system of the three-dimensional restricted problem of three bodies, two topographical systems of surfaces, the intersections of which represent a system of curves in space depending on two parameters. The angle Ψ is introduced as the angle between a given trajectory and the tangent of the curve intersecting the trajectory in a given point. Going back now to the one given system of surfaces, the author considers the multitude of possible contacts between these surfaces and the system of trajectories and, from this multitude, he selects the contacts which are related to a certain given angle, Ψ, of the given trajectory with respect to the tangent of the curve which has been mentioned above. If $f(x, y, z) = f$ is the system of surfaces, then the selected group of contacts is associated with the contact characteristic $\ddot{f}(x, y, z, h, \Psi)$, where <u>h</u> is Jacobi's constant. Depending on the sign of $\ddot{f}$, the contact will be interior ($\ddot{f} < 0$), exterior ($\ddot{f} > 0$), or possibly of higher order ($\ddot{f} = 0$).

This paper describes the general idea or definition of these "monotypical" contacts between trajectories and a given topographical system of surfaces; it contains nothing concrete or substantial that would allow the actual utilization of the new concept. Although the presentation of this general idea has some merits, the mathematical form of the author's contribution could be simplified by the proper use of vectors and matrices. The author makes use of the mathematical language of the beginning of this century, rather than of its modern form.

Two papers by Moiseyev relating to this subject are discussed in the section on the Disturbing Function; one[470] considers the restricted elliptical problem, and the other[471] the semirestricted problem.

A series of papers by A. A. Orlov[526-528], that is discussed under Periodic Orbits, also relates to the topic at hand.

An error in Whittaker's integral for periodic solutions of the restricted three-body problem was pointed out by N. F. Reyn[634] in 1937. Another error, this one by Nechvil in his paper On a New Form of the Differential Equation for the Restricted Elliptic Problem (1926), was also pointed out by Reyn[642] (1940).

Utilizing certain assumptions regarding the density distribution of the gravitating medium which has spherical symmetry with respect to the one finite mass, M_o, Reyn[631] (1936) investigated the location of the five libration points and the value of the Jacobi constant for material points resting in the various libration centers, as well as the location and shape of the surfaces of zero velocity. Numerous theorems are given concerning the relative location of the libration points, the related Jacobi constants, and the dependence of these points on the value of the second finite mass, μ. The surfaces of zero velocity are studied in their dependence on the Jacobi constant, C, and the mass, μ. Some of the theorems concerning the relative location of the straight-line libration points are valid also (after proper modification) for the ordinary restricted problem (without an additional gravitating medium).

In spite of the correctness of the theorems and of the analytical derivations, this paper is essentially technical; its major content is a detailed transformation of certain well-secured implicit truths into a more explicit form. Nothing basically new is involved in this presentation.

Later, Reyn[639] (1939) dealt with a generalization of the subject of her previous paper[631]. In the earlier paper, she assumed that the period of the orbital revolution and, thus, the rotational velocity of the basic coordinate system of the restricted problem were affected by the presence of the gravitating medium, as if the total mass, $M(r) = M_o + m(r)$, were concentrated in M_o, or as if $m(r)$ were intimately connected with M_o (as an atmosphere). The present investigation is based on the assumption that the angular velocity, n, of the two finite masses, M and μ, depends on the ratio $\frac{M(R)}{M} = \lambda$ by the relation $n^2 = \frac{M(R) + \lambda\mu}{R^3}$. Here, R is the constant distance between the masses M and μ. It is shown that for values of λ not too different from unity ($\lambda = 1$ corresponds to the more special case which the author considered in her earlier paper), the general features and characteristics are not significantly different from those for $\lambda = 1$, at least as far as the existence and location of the straight-line libration points, L_1, L_2, and L_3, are concerned. The points L_4 and L_5, however, suffer certain systematic shifts on the periphery of a circle with radius R about the mass M. The location of the surfaces of zero velocity, as indicated by their intersections with the x, y-plane, as well as with the x, z- and y, z-planes, is determined in its dependence on the singular points L_1 to L_5. The values of the Jacobi constants, C, of particles resting in these libration points are compared, as in the earlier paper. All the results are quite analogous to the ones for the original problem, as long as λ is not very different from unity.

Essentially different features emerge for $\lambda \leq 0.125$, but, since $\lambda = 0.125$ means that n^2 amounts to only one-eighth of the value obtained from a straightforward application of Kepler's third law on the basis of the two masses, M (R) and μ, it seems that such λ -values are very remote from concrete astronomical problems. For the same reason, values essentially larger than $\lambda = 1$ do not seem to make much sense either.

The author's study is of a certain interest, in that it is a more flexible generalization of the problem presented in her earlier paper(631). Perhaps the introduction of the parameter λ may prove useful in connection with related cosmogonical problems.

Moiseyev and Vzorova made use of the contact-characteristics method in connection with Hill's satellite problem. The paper by A. I. Vzorova(803) (1940) is an auxiliary investigation for an analytical study by Reyn(643) (1940).

Since Whittaker's circular zones for the localization of periodic orbits are not practical in the case of the rather eccentric inner orbits of Hill's satellite problem (for large values of the Jacobi constant, h), Vzorova[803] made use of a family of ellipses instead of circles for the investigation of the contact characteristics of the seven innermost periodic orbits of the Hill problem. The author adopts the value $e = 0.2920$ for the eccentricity of all the concentric and coaxial ellipses which have their common center in the center of the planetary mass; this value represents approximately the average of the ellipses by which the seven individual orbits may be approximated. Actually, this mean value of e was obtained by averaging the individual values of e^2, because the analytical equation for the family of ellipses depends on e^2 rather than on e. Using Moiseyev's method of qualitative analysis, the author makes the necessary computations for the construction of the various characteristic curves, i.e., of the so-called osculatrice, the distributice, and Hadamard's curve, in order to find the regions of interior and exterior contact of the possible periodic trajectories with respect to the concentric family of ellipses. The results, which have been computed for equidistant values of the angular coordinate, θ , are graphically represented in their dependence on h.

This is a rather technical paper, the results of which have been obtained by the proper application of an existing method of analysis to a well-known problem; it does not contain any theoretical advances. Nevertheless, this well-executed example is a useful demonstration of the practical application of the method of contact characteristics.

Reyn(643) utilizes a topographical system of concentric ellipses; here, the various branches of the so-called indicatrix, as the geometrical location of the points of contact of the osculatrice (for h = constant) with the given family of ellipses (for an eccentricity e = constant), are finally derived in the form of analytical developments. The third power of the planetocentric distance, ρ , is developed as a function of the planetocentric longitude, θ , with coefficients depending on the ascending powers of e^2. The reciprocal of ρ^3 is introduced, instead of ρ^3 itself, for the application to large distances, in order to avoid singularities in the fundamental equation by which the indicatrix is determined. It is found that the indicatrix consists essentially of three branches, two of which are reduced to circles if only terms of the order e^2 are retained in the coefficients. These two branches determine the parameters of the two concentric ellipses of the given topographic family between which the osculating orbits of the satellite, for the given value of h, should be located. The two branches of the indicatrix are closed curves. After establishing analytically the existence of the closed branches of the indicatrix in this manner, Reyn further shows that, for retrograde motion, the third branch of the indicatrix may asymptotically go to infinity at $\theta = 60°$. The analytical study is then complemented by a numerical application for $e^2 = 0.085262$, which corresponds to Vzorova's computations (cf. Reference 803). The author's analytical expressions are used to compute the ρ -values of the indicatrix for equidistant values of the longitude, θ , in intervals of 10°. These computations give the final values by a process of iteration. The resulting points of the indicatrix confirm the previous analytical results concerning the existence and the characteristic features of the two branches.

This is a really interesting, mathematically clever, and ingenious study of the subject. The author certainly proves her great ability in this field of celestial mechanics.

The stability of six periodic orbits in the Copenhagen problem, in the sense of Jacobi and in the sense of Zhukovskiy, was investigated by Reyn[644] (1940) and by A. I. Ryakov[658] (1950), respectively. These papers are discussed in the section on Criteria of Stability.

Yu. A. Ryabov[656] (1952) used Lyapunov's method for the integration of differential equations involving power series depending on a small parameter to prove the existence of periodic solutions for the motion of a small body in the neighborhood of the libration centers L_4 and L_5. The same method was used for the actual determination of the coefficients in the developments which represent the solution of the problem. It is shown that the developments are convergent for small values of the parameter and, thus, for the neighborhood of the points L_4 and L_5; the actual determination of a radius of convergence was left for future investigations.

The existence of the periodic solutions has been known and proved for a considerable time; nevertheless, this paper gives an independent proof and determination based on an entirely different method than had been used before. The investigation is mathematically elegant and undoubtedly represents a constructive contribution to celestial mechanics.

T. V. Vodop'yanova(784) (1936) applied two qualitative characteristics, (1) the characteristic of Hill ($V^2 > 0$ in the rotating coordinate system of the restricted problem of three bodies), and (2) the contact characteristic of Moiseyev with respect to a family of heliocentric circles (for the perihelia and aphelia of minor-planet orbits), to the system of minor planets, i.e., to the selected group of planets with small inclinations for which the plane problem may be used. The fact that Jupiter's orbit is not circular but elliptical is taken into account in an approximate manner. The author makes a survey of the actual orbits of the minor planets and finds that most of the osculating orbits under consideration are fully compatible with the two above-mentioned characteristics, i.e., the corresponding minor planet would be able to move in such an orbit at all times (taking into account the variation of Jupiter's distance from the sun) without violating either the condition $V^2 > 0$ or the limitations which are found for the perihelion and aphelion distance from the sun. The author pays special attention to the so-called critical planets, which have been observed in only one or two oppositions; in this group, she finds quite a percentage of osculating orbits which would not be compatible with the two characteristics at all times. The author states that, from this fact, one could have predicted, in many cases, that the planet would probably be "lost". The actual reason for this incompatibility with the two characteristics probably lies in the fact that the elements of these critical planets are very poorly determined as a result of an insufficient material of observations.

This paper is of interest as a practical application of certain qualitative methods developed by the "Moscow School".

e. Regularization and Collision

Although Sundman had proved, in the case of simple collisions and of the nondisappearance of all constants of a real velocity, that in the neighborhood of a collision the coordinates and relative distances can be developed in powers of $(t - t_1)^{\frac{1}{3}}$, Markov(387) (1927) attempted the numerical determination of the first coefficients in these convergent series. For moments not very distant from the instant, t_1, of the collision, the author determines the distances of the third mass, which does not participate in the collision, from the invariable plane of the three bodies. It is found that, in the general case, the development for this distance begins with the power $(t - t_1)^{\frac{10}{3}}$, i.e., with no powers lower than this. The author also gives a complete classification of all the possibility of deviations from the so-called "law of ten-thirds".

The author deserves credit for a thorough examination of the subject chosen for this investigation; he has made an interesting contribution to the theory of collisions.

Poroshin(566) (1945) considered one central mass, S, coinciding with the origin of the coordinate system and a homogeneous circular

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distribution, J, of matter at the distance a from S. In Chapter I, the author studies the trajectories of a third small mass, P, after its radial ejection from the central mass, S. The force function is developed first with respect to ascending powers of r/a, where r is the increasing distance of P from S. Although the two integrals of the problem, i.e., the energy integral and the integral of areas, permit the reduction of the problem to quadratures, it is necessary to regularize the differential equations in order to eliminate the singularities which are associated with $r = 0$. The regularization is possible by means of Moiseyev's so-called parabolic coordinates, and by means of an additional transformation of the time. The regularized equations of motion are then integrated, and the transformed coordinates are obtained as power series with respect to the transformed "time", τ. It follows, as was to be expected, that for motion in the plane of the circular mass distribution, J, the trajectories are straight lines in radial directions.

In Chapter II, the investigation is extended from the plane problem to trajectories in three dimensions, again beginning in the central mass, S. It follows that the projections of the trajectories onto the fundamental plane of J are straight lines, as in the plane problem; with respect to the z-coordinate, however, the trajectories are curved. In proper generalization of the methods employed in Chapter I, the coordinates are transformed and determined in power series depending on the transformed "time", τ. The developments in both chapters are valid only for a certain interval of time, or up to a limited central distance, r, because the series will not converge at large values of r or τ.

Reversing the direction of the trajectories, Poroshin then proceeds from ejection orbits to orbits of free fall, investigating, especially, the conditions under which a trajectory, starting at a given position in three-dimensional space, will end (in a straightforward hit) in S. In the last paragraph, the author goes back to regular coordinates (x,y,z) and regular time, t, to obtain the normal coordinates as functions of the time. The coordinates are found as power series with respect to $t^{3/2}$ and the author actually determines the first two coefficients in these developments in their dependence on the constants of integration.

This is not an outstandingly important contribution. The subject, as such, has been explored very thoroughly and competently, however, and the presentation has features of mathematical elegance. Altogether, this is a well-done investigation of the characteristics of a well-defined, not-too-difficult, problem.

For the computation of the general perturbations of a minor planet, in a special case previously studied by Brendel, Samoylova-Yakhontova(663) (1929) introduces an independent variable, $\mu = A\int\frac{dt}{\Delta}$, which regularizes the problem for collisions with Jupiter. It is shown that the convergence of the series is greatly improved by the adoption of this new independent variable. The method is applied, then, to the Jupiter perturbations of the planet Thule, to obtain a comparison with the corresponding results obtained by Brendel on the basis of his new method.

This is one of the really valuable and interesting recent contributions to celestial mechanics. The results are of interest from the purely theoretical point of view, and they are also important for the practical problem of obtaining improved convergence for the general perturbations of minor planets.

Samoylova-Yakhontova had shown that the introduction of the independent variable $u = A\int\frac{dt}{\Delta}$, which regularizes the simple shocks with Jupiter, improves the convergence of the developments for the perturbations (cf. Reference 663). Then, she[664] (1931) considered the case of large eccentricities, which can lead to close approaches of the minor planet to the sun, and introduced a new independent variable, $u = A\int\frac{dt}{r\Delta}$, in order to regularize any simple shocks with the sun, as well as with Jupiter. The new variable is introduced into Brendel's new method of general perturbations by a process similar to the one used in the previous paper. The investigation is limited by the assumption that the minor planet moves in the orbital plane of Jupiter, but that Jupiter's orbit (or the orbit of any other disturbing planet) is elliptical.

This paper is a valuable addition to the previous one; the new or extended method will improve the convergence for a wider range of conditions than before.

The earlier papers by Samoylova-Yakhontova were on the improvement of the convergence of the series developments, in the theory of planetary perturbations, by means of a regularizing variable (cf. References 662-664). Next, she[665] (1936) demonstrated, in a simple example, the fact that the convergence of the developments depending on the powers of the ratio, α, of the mean distances, in the case of the

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group methods of planetary perturbations, is also strengthened by the introduction of the regularizing variable $u = \int_0^{\Phi} \frac{d\Phi}{\sqrt{1 - K^2 \sin^2 \Phi}}$, where Φ is an auxiliary variable depending on the longitude of the disturbed and disturbing planets, or on the ratio, μ , of the mean motions. It is assumed that both planets move in circular orbits, as far as the undisturbed motion of the minor planet is concerned, and within the same orbital plane. The differential equations of motion according to Hansen-Bohlin are integrated by Bohlin's method. It is found that Hansen's function, W, and the perturbations $n\delta z$ and ν are represented by fast-converging series depending on the variable u, with the coefficients decreasing as in $(2n)^{\lambda} q^n$ or $(2n + 1)^{\lambda} q^{n + 1/2}$ for increasing values of n. Here, the parameter q is of the order of $\frac{1}{4}\alpha$ and λ is a positive integer. Bohlin's method of integration avoids developments into trigonometrical series, which otherwise would be necessary in connection with the introduction of the new variable. For $n\delta z$, a comparison with the corresponding results from Bohlin's tables is given.

This is another paper of real theoretical value and considerable interest. The author has contributed significantly to this special problem, i.e., to the improvement of the convergence of the series developments in celestial mechanics by means of regularizing transformations.

f. Approximate Theory for Minor Planets

The construction of exact theories of minor planets by the standard methods of Hansen, Hill, Encke, etc., is extremely difficult, in view of the often large values of the eccentricities and inclinations. It can be done only for a few planets. For the majority of the minor planets, the main problem is to compute quickly the necessary perturbations, so as to be able to identify the planets at subsequent apparitions. This is done either by numerical integration, or by some simplification of standard methods. The approximate methods of Bohlin and of Brendel are used extensively in the U.S.S.R.

(1). Bohlin's Method

Bohlin (1896), starting with Hansen's differential equation, introduces a parameter, $W = 1 - \frac{\mu}{\mu_o}$, where μ is the ratio of the mean daily motions of Jupiter and of the minor planet, and μ_o is a rational simple fraction not greatly different from μ. The fraction μ_o defines the exact commensurability with respect to which many minor planets can be assigned to "groups". For the planet Pomona, for instance, the mean daily motion is 852".83, and for Jupiter, it is 299".13. Consequently, $\mu = 0.351$, or very close to $\frac{1}{3}$. The fraction $\frac{1}{3}$ defines the Hestia group, for which μ_o is assumed to be exactly $\frac{1}{3}$, and $\frac{\mu}{\mu_o}$ for this case is 1.05225, and $W = -0.05225$.

The perturbation function is expanded by Bohlin in terms of W, eccentricity, and the mutual inclination between the orbits of Jupiter and of the minor planet. Terms depending on W are the same for the whole group of planets within certain limits on either side of commensurability, and they can be computed once and for all. Terms depending on eccentricity and inclination differ from planet to planet.

The problem, then, is to simplify calculations for each individual planet by the use of tables, and to represent perturbations as fractions of certain coefficients and of the difference between multiples of E, the eccentric anomaly of the minor planet, and $A = g + \mu_0 e \sin E$, where g is the mean anomaly of Jupiter.

Samoylova-Yakhontova(668) (1947) made an analysis of Bohlin's method and found some errors in his expansion of power series in terms of the second order of the parameter W. This error affects all published tables, but it is not particularly serious. Furthermore, she(665) (1936) proposed the introduction into the formulas of another variable, $u = A \int \frac{dt}{rr'\Delta}$, where A is a constant, r and r' are radii vectors of Jupiter and the minor planet, and Δ is their mutual distance. It is shown that, with the proper choice of the constant, the fraction u can be expressed as an elliptical integral of the first kind, which results in a more rapid convergence of the trigonometric series used by Bohlin and, consequently, in greater precision of the computations with less effort.

The perturbations of minor planets by Saturn are small, and the tables of perturbations are consequently simple. Tables for the commensurability $\frac{1}{5}$ (n 602") were published by Osten, and for the commensurability $\frac{1}{7}$ (n 843") by Block. Komendantov(295) simplified these tables and extended them to cover the ranges of n from 440" to 800", and from 700" to 1100".

In practice, the Soviets use Bohlin's method only for the Flora and Minerva groups.

(a). Hestia Group

Tables of general perturbations for the Hestia Group ($\mu_o = \frac{1}{3}$) were published by Radynskiy[604] (1935) for the purpose of simplifying Bohlin's tables. Radynskiy's tables, however, were still considered to be too complicated, and a condensation of them was published by Zheverzheyev[848] (1950). These tables are apparently not used at all in the U.S.S.R., and perturbations for this group are not computed by Brendel's method.

(b). Minerva Group

The Soviets have done the most work on the Minerva group with a commensurability of $\frac{2}{5}$. It was found that Bohlin's method can be applied successfully for the range in the mean daily motions between 690" and 800", except in the region between 740" and 755" near the exact resonance (474".821). At present, the limit of application includes 55 planets.

Komendantov[294] published a collection of tables for the Minerva group with detailed instructions for computations. The original tables by Wilson for the Minerva group are supposed to be precise within 1". A precision of representation of only 1' is claimed by Komendantov. The test of these tables was made by Komendantov[291] (1935) himself for the case of minor planet (308) Polyxo. It took only 26 hours to compute all periodic Jupiter terms with a precision of 1", and the secular Jupiter and Saturn terms with a precision of 0".01. The resulting departures of observed data from computed for the period 1891 to 1933, that is, for 42 years, do not exceed 31".

An interesting case is the motion of minor planet (430) Hybris, which has a daily motion of 739".5, that is, nearly exact commensurability; the coefficient of the term with the argument 2E-5A (commensurability $\frac{2}{5}$) is about 5°. The departures in the period 1922 to 1951 (Shmakova[704] (1755) are unusually large, but do not exceed 12'.)

Komendantov[296] (1936) also published a still more condensed table of perturbation for this group, retaining only periodic terms greater than 0'.5 and secular terms greater than 0".5. The precision of representation is of the order of 0°.1, and computation of perturbation requires only a few hours.

(c). Flora Group

Tables based on Bohlin's method were computed by Villemarque for commensurability $\frac{2}{7}$ (n = 1047") and published in the Annals of the Zo-Se Observatory. These tables are used in the U.S.S.R. for 25 minor planets with the mean daily motion between 1,000" and 1,100". The planets are: 113, 228, 281, 291, 296, 317, 326, 336, 341, 345, 364, 367, 370, 376, 391, 422, 440, 443, 453, 540, 641, 700, 711, 736, and 770.

Tables of perturbations for planets (689) Zita, (853) Nansenia, (857) Glazenapia, (393) Isbega, and (1078) Mentha of the Flora group were computed at the Institute of Theoretical Astronomy (Varzar[773] (1952) and Bozhkova[84] (1951).

(2). Brendel's Method

Brendel's method, which is based ultimately on the ideas of the Pulkovo astronomer Gyldén, does not claim great precision. Perturbations are computed with the precision 0°01, and the resulting precision of representation is of the order 20' to 30'.

The greatest usefulness of this method is for minor planets of the Hestia group (commensurability $\frac{1}{3}$; resonance 897"). Brendel published (1913) tables for perturbations from Jupiter for 100 planets of this group; his collaborator, Boda (1921), published tables for 108 planets, splitting the group into two subgroups with the mean daily motion more and less than 897" (range: 845" to 958"). The secular perturbations of the node and longitude of the perihelion for these two subgroups are appreciably different. Further refinements in this problem were made by K. Schitte in 1936.

As far as Soviet work is concerned, there is no evidence in the Soviet literature of any improvements or modifications of Brendel's method. In fact, not a single reference to this method has been found beyond the general statement (in the Ephemerides for 1956) that perturbations for 160 planets of the Hestia group are computed by Brendel's method. Apparently, the Soviets have simply taken over the work of Brendel and his successors without making any changes.

According to Samoylova-Yakhontova(670) (1950), for the planets of the Juno group (mean daily motions of 800" to 850", that is, adjoining and somewhat overlapping the Minerva and Hestia groups),

S. G. Makover has computed an ephemeris with the precision of 0°001. This work apparently has not been published. On the face of it, Makover's work seems to be a radical departure of the Soviets from their practice of merely adopting already well-developed methods either wholly, as in the case of Brendel's method, or with some modifications, as in the case of Bohlin's method.

g. Planetary Theory

Halphen's method for the computation of the secular perturbations is mathematically elegant and very convenient for numerical application, especially in connection with computing machines, but it has not been used to any extent for a long time. The reason for this apparently is to be found in the existence of quite a number of serious errors in the original method by Halphen, which was but one chapter in a three-volume work on elliptical functions. Any potential users of Halphen's theory probably found that they could not arrive at satisfactory results by using his formulas.

Goryachev[221] (1937) has carefully studied the method and found several errors in Halphen's investigation. One of these probably occurred more or less accidentally in the arguments of the hypergeometrical series; in certain integrations, however, Halphen went only from 0 to 2 ω, instead of from 0 to 4 ω. The formula for the gravitational potential of a ring was also in error in Halphen's presentation. Goryachev has corrected these and other minor errors and gives the final formulas; he also presents convenient auxiliary tables for the practical applications of the method. In the first part of this publication, he gives the related method by Callandreau, where an error of sign is also corrected. Finally, Halphen's method is applied by Goryachev to the secular perturbations of the minor planet Ceres, as produced by the planets from Mercury to Neptune; he compares the results with the corresponding ones from Hill's and Callandreau's methods.

Nothing really original is contained in this representation of Halphen's corrected method, but the author has performed a valuable service to celestial mechanics by this work, which makes possible the practical use of a basically good method.

Shkroyev(700) (1938) applied Halphen's method, as corrected by Goryachev(221), to the minor planet (624) Hector. This application is limited, furthermore, to the perturbations produced by Saturn, which are relatively small; the Jupiter perturbations are not computed by this method. For a Trojan such as (624) Hector, moving close to Jupiter's own orbit, Halphen's method, or any method of secular perturbations by Jupiter, may, of course, be inadequate because of the librational motion of the minor planet and the 1 : 1 commensurability of the mean motions of the two planets involved. The paper also contains a collection of all the formulas which are needed for the application of Halphen's method.

Even though this paper is only a numerical evaluation of Halphen's method in one particular case, it apparently represents one of the first actual applications of the method as revised by Goryachev. Because of the clear arrangement of all the results and of the necessary equations, it may well serve as a guide for future users of Halphen's method. For these reasons, this appears to be a quite useful and good paper.

In a critical and interesting examination of an existing situation, Mikhal'skiy[417] (1933) considered the secular part of the longitude perturbations. Here, in addition to the perturbations. Here, in addition to the perturbation $\delta \varepsilon_o = rt$, he also introduces a term $\delta \rho_o = \tau t = \int_o^t \delta n_o \, dt$, where δn_o denotes the constant part of the perturbation of the mean daily motion, n, for the epoch t_o. The author gives an approximate expression for τ and an equation relating the values a_2 and a_o of the mean distance or half major axis to each other when n_2 is the observable average value of the true mean motion, and n_o the undisturbed motion for the epoch t_o. The term τt does not appear in the classical planetary theories; instead, a constant, a_o, is used which differs from the value $a_2 = \sqrt[3]{\frac{f(1 + m)}{n_2^2}}$ by a constant perturbation of the half major axis, a.

Mikhal'skiy[419] (1934) developed his idea that the constant terms of the general perturbations, as obtained by the method of the variation of the arbitrary constants, should be dealt with explicitly in the expressions for the perturbations, instead of by absorbing them in the constants of the basic orbit. If nt is replaced by $\int_o^t n \, dt$, then it is unnecessary to introduce the element ε and its perturbations in place of the perturbations in the longitude of the epoch. The numerical values of the long-period perturbations come out smaller if the constant terms are introduced.

Although Mikhal'skiy can claim certain points in favor of his proposed procedure, his related criticism of Tisserand and others certainly goes far beyond the actual facts. It is not necessary, as the author claims, to deal with the constant terms in his way; actually, one can proceed in either way. So far as the value of an individual theory is concerned, nothing of real importance can be gained by "splitting hairs" about the treatment of the integration constants. The paper remains of some interest, however, inasfar as it considers the various possibilities as such.

Numerov's calculation[515] (1934) of first-order absolute perturbations in polar coordinates is essentially only a presentation of Hill's well-known method of perturbations in Russian, perhaps in order to familiarize Soviet readers with Hill's method. The author also mentions his intention of publishing tables later, in order to facilitate the application of the method. The notations differ from the original ones by Hill.

If special perturbations of the rectangular coordinates of a minor planet are computed by means of Numerov's extrapolation method, the contributions produced by the inner planets Mercury, Venus, Earth, and Mars are troublesome, because of the small divisors, r_1^2, in the corresponding terms of the differential equations. In order to circumvent these difficulties, Numerov and Moshkova[522] (1936) proposed the computation of general perturbations by the inner planets with reference to a transformed system of coordinates, which eliminates the indirect term in the disturbing function. After arriving at the new differential equations in the transformed coordinates, the integration follows the principal lines of Hill's method; different notations are used, however, and a modified disturbing force function is applied. The angular arguments and the integration constants are also modified, so as to fit the basic assumptions of the present application of Hill's method. The results, in the form of general tables, are given as functions of the essential elements of the disturbing and disturbed planets, in order to facilitate application to individual problems. As an example, the general perturbations of (3) Juno by Mercury are tabulated.

Nothing essentially new from the theoretical viewpoint is contained in this paper; however, from the viewpoint of orbit computers, the preparation of such a detailed scheme for the determination of general perturbations by the four inner planets was very much worthwhile. Even today, with automatic computing equipment widely available, the rigorous computation of special perturbations produced by the inner planets is a serious problem which requires smaller integration intervals than are sometimes practicable; therefore, it will frequently be preferable to use such general perturbations as are developed in this paper. This is a good, solid contribution of permanent value.

The final results of the development of the general perturbations by Hill's method are contained in Reference 523. This work was carried out at the former Astronomical Institute. The purpose of this paper obviously was to facilitate the application of Hill's method of general perturbations to minor planets inside the orbit of the disturbing planet. The necessary formulas and developments are arranged and grouped in such a manner as to make the numerical computations as convenient as possible. In Tables I and II, the terms of the first order in the three polar coordinates are listed in their general form; in Table III, the general expressions of the various coefficients are given. In order to evaluate the tables for a given minor planet and a given disturbing planet, one has to insert into these expressions the corresponding values for the eccentricities, for the ratio, α , of the mean distances, for the mass, m, of the disturbing planet, and for the relative inclination. The theory is intended to be approximate and, therefore, the expressions are limited to the third powers of the eccentricities and of the inclination, and to the twelfth power of α . The remark is made that these rather general developments are intended to serve as the basis for tables of the perturbations of planets in the group with mean motions between 800" and 850".

Except for the special arrangements and details in the evaluation, this is generally a technical application of Hill's basic theory. A very useful purpose has been served by the preparation of these tabulated expressions; in this way, the authors have considerably facilitated the actual application of a mathematical theory to problems of astronomical practice.

Orlov(530), in 1953, wanted to obtain the rectangular-coordinate perturbations in the form of power series depending on a small parameter, α. For $\alpha = 0$, the coordinates of the two-body motion would be obtained. The problem is considered here in its generality, and it is shown how everything can be reduced to a (considerable) number of quadratures. If

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$\underline{n}$ is the highest power of α to be considered, one actually has to deal with 3n systems of differential equations which must be solved for consecutive values of the power of α. A part of the analysis closely resembles the derivation of the conditional equations in the problem of orbit correction, because, for the purposes of this paper, the same variational relations between the coordinates and the constants of motion are utilized.

This study has a certain theoretical interest, because the fundamental principles of the general problem are well and clearly outlined by the author. In practical cases, a more direct and special approach to such a problem is usually preferred; nevertheless, a paper such as the present one has its merits, because it contributes to a better mathematical understanding of the developments.

Proskurin[573], in 1952, used Hill's method to establish an accurate general theory of the first order for the motion of the minor planet (1) Ceres. Special attention is given to the determination of the constants of integration, which are referred to osculating elements, as well as to mean elements. For the secular perturbations, the method of Halphen, which is a modification of Gauss' method, is used in order to obtain a higher accuracy for these terms than is possible by Hill's method.

An essential part of this paper contains the detailed comparison of Proskurin's results with earlier theories established by various authors. Elaborate transformations are necessary; these are made by the author in order to make the different theories comparable to each other in spite of their sometimes quite different analytical structures. Proskurin's theory for Ceres gives the perturbations by all the major planets from Mercury to Neptune (for Mercury he gives the secular terms only), and a comparison with the earlier theories is made separately for each of the disturbing planets, as far as such earlier perturbations were available. Apparently, the new theory is of high precision and quality, but, so far, the terms of the first order only have been derived. The computations were made partly on punched-card machines.

This elaborate theory appears to have been worked out with great skill and competence, and with considerable care, as demonstrated by the numerous comparison and check operations. The work constitutes a valuable contribution to the field of planetary perturbations.

With Mashinskaya, Proskurin[574] (1953) used his first-order theory[573] for Ceres to represent normal places for the years from 1801 to 1946. The 19 positions are finally represented with mean errors of $\Delta\alpha \cos\delta = \pm 97''$ and $\Delta\delta = \pm 36''$. This is slightly better than the results of an earlier theory, for which mean residuals of the order of $\Delta\alpha \cos\delta = \pm 112''$ and $\Delta\delta = \pm 52''$ had been reported by Komendantov; nevertheless, these mean errors are considerable. The authors themselves say that they demonstrate the considerable size of the neglected perturbations of the second order.

This paper contains nothing but a rather automatic application of earlier results to the observations of Ceres; it cannot be classified as being of any importance from the theoretical point of view. It is a paper which is of practical interest (demonstrating, in this case, the limited accuracy of a first-order theory), but of no general significance. These evaluations may have been made by a student under the supervision of Proskurin.

In the introduction to his paper on the possibility of a theory of motion of the Trojans, which is based on the assumption of nearness to the centers of libration, Ryabov[657] (1956) made a survey of the various theories (mainly those of Wilkens, Thüring, and Brown) and of the practical applications which Brouwer and Eckert have made of Brown's theory. He wants to show the need for better theories that are not based on the assumption of a small librational deviation from the Lagrangean points L_4 and L_5, and that consider the short-period terms on an equal basis with the long-period ones.

In Section 1, Ryabov introduces the true elliptical orbit of Jupiter (instead of the circular approximation which is used in most of the present theories), using Nechvil's coordinates. The differential equations of motion are formed, using Jupiter's true anomaly, v', as the independent variable, and, by a proper transformation, the corresponding triangular libration point is chosen as the center of the coordinate system.

In Section 2, a method of solution is proposed which develops the coordinates into a power series depending on the powers of Jupiter's orbital eccentricity, e'. The author presents some arguments supporting the assumed convergence of the series for a long interval of time, but he does not give any actual proof for this convergence.

In Section 3, the author derives the well-known typical features of the different periodic solutions in the close neighborhood of the libration centers, using Nechvil's coordinates as adopted in the preceding sections.

In Section 4, the author criticizes the existing theories for not properly representing the true combination of long-period and short-period solutions or terms. Since in Section 3 he had shown that the amplitudes of the long-period and of the short-period solutions are comparable in size, he finds that this principal difficulty has not been mastered by the existing theories. He admits, however, that the librations or oscillations obtained in the form of the various periodic solutions form the basic elements of the true motion.

In Section 5, he finds the approach made by Heinrich and Linders, using canonical variables, to be the most promising one. This represents a return to the variation-of-constants method in a modified form. In this connection, he also finds that Lyapunov's method of solution is a suitable one.

Section 6 once more emphasizes the point that the theories which are based on the assumed nearness of the Trojans to the Lagrangean libration centers are not well suited to represent the motion of the actual Trojans.

Ryabov has reviewed the theories of the Trojans critically. The facts concerning the weaknesses of the present theories, which he emphasizes, have generally been known before, and he does not replace the the theories by something better. He says that certain investigations by

Nechvil, Heinrich, and Linders contain results which should be utilized for a better theory of the Trojans, but does not proceed to utilize them. Although this paper points out the main weaknesses and shortcomings of the theories which are available, it contains little beyond this.

After a short review of previous work on the motion of Pluto, especially of the investigations by Roure on the basis of the Hill-Brouwer method, as modified by Andoyer, Sharaf[688] (1955) developed her own first-order theory by means of Newcomb's method. The first chapter deals with the development of the disturbing function, and the essential equations are a direct application of Newcomb's method of operators. Punched-card machines were used for the computation of the coefficients of the disturbing function and of its derivatives. The operators are given in a form which was most convenient for the Soviet type of punched-card machine. For the derivatives of the coefficients of Laplace, the method of Innes was preferred to the one of Newcomb, because of higher accuracy.

In Chapter II, the coordinate perturbations and the related constants of integration are given in their general form; in Chapters III, IV, and V, these developments are applied to the computation of the first-order perturbations by Jupiter, Saturn, and Uranus. The perturbations produced by Neptune were obtained by numerical integration, because of the possible close approaches between Neptune and Pluto. This numerical integration was done by Cowell's method in rectangular coordinates, but the results were then transformed into perturbations of the elements and then of the polar coordinates of Pluto, in order to obtain them in the same form as the general perturbations by Jupiter, Saturn, and Uranus.

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The final Chapter VII contains the orbit correction for Pluto on the basis of the new theory and of the original elements by Brouwer. For this purpose, 24 normal positions from 1914 to 1951 were formed by means of the various ephemerides which were available. The differential correction was done by the method of Eckert and Brouwer, modified according to Samoylova-Yakhontova.

An appendix contains the complete symbolical development of the disturbing function according to Newcomb. The purpose of this appendix apparently is to popularize Newcomb's method, as well as to serve as a guide for future applications of this general method. The related coefficients have been recomputed by the author and tabulated for convenience. The author, in doing this work, found some errors in Newcomb's corresponding expressions, and the corrected basic expressions are reprinted at the end of this publication.

So far as theory is concerned, nothing new is contained in this elaborate, but rather technical, investigation; the completion of such a detailed theory, however, is always an intricate and demanding task. Apparently, the author has done her work very carefully and rigorously, and she deserves credit for the results, which can be expected to be useful for further work on the motion of Pluto. The author appears to be thoroughly familiar with the theoretical foundations of her subject.

Vil'yev(775) (1916) computed the general perturbations of (55) Pandora by Hansen's method, in order to familiarize himself with this method, which he called the best one available for the absolute perturbations of minor planets. Accurate and also approximate general perturbations of this minor planet had been computed previously by various authors; the present author thus was able to check his own computations. No comparison with observations is made.

After some historical remarks on the orbit of Ceres, Vil'yev(776) (1916) again used Hansen's method to compute the general perturbations of Ceres by Jupiter and Mars; for the perturbations by Saturn, a somewhat modified and simplified version of Hansen's method was used. The results, as far as Saturn is concerned, were checked by means of Block's tables (on the basis of Bohlin's method).

These results could have been useful at the time of their publication; today, they are only of historical interest. Nevertheless, Proskurin(573), in his most recent analytical theory of Ceres, used Vil'yev's results for the Mars perturbations as a check of his own results.

Voronov(788) (1935) refers to Hansen's and Samter's previous results for the general perturbations of the planet (13) Egeria. He says that, although Samter improved the representation of the observations by revising Hansen's theory and adding second-order terms, which Hansen had neglected, there are still systematic differences between observation and theory that might be reduced by computing the main terms of the third order, and by a revision of the terms of Hansen which had not been recomputed by Samter. The author says that he has done all this work; up to terms of the third order, he gives the tabulated coefficients produced by Jupiter, Saturn, and Mars. Also tabulated is a comparison with the earlier results of Hansen and Samter, presented in the form of the differences between the corresponding coefficients.

Later in 1935, Voronov[790] claimed to present the comparison of his previous accurate theory of (13) Egeria (cf. Reference 788) with 27 normal places from 1850 until 1914. He also included the results of a final orbit correction, including systematic corrections to the system of Boss' (P.G.C.) star catalogue and corrections for the masses of Jupiter and Saturn.

Although this work looks rather good, Reference 788 is actually erroneous. In the Pulkovo Circular Nr 17 (1936), Voronov stated:

> "... my paper on the minor planet (13) Egeria, published in Nr 16 of the Pulkovo Observatory Circular (the improvements of the orbit and comparison with observations) has been written by me in a state of great mental fatigue, approaching the state of a nervous breakdown, and that all its results, as it has been proved by my Pulkovo colleagues, are erroneous and should not be taken into consideration".

h. Planetary Satellites

(1). Satellites of Jupiter.

According to Proskurin[576] (1955), the Institute of Theoretical Astronomy is engaged in a study of the following satellites of Jupiter: Jupiter VI - inequalities depending on

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the sun have been evaluated; Jupiter VII - inequalities depending on the sun have been evaluated; Jupiter VIII - numerical theory has been established; and Jupiter X - analytical theory has been started.

(a). Jupiter V

Apparently, there has been no recent work on Jupiter V in the U.S.S.R. From 1928 to 1930, N. M. Mikhal'skiy published four papers on Jupiter V[412-415]. In the last of these papers[415] (1930), Mikhal'skiy applied the method of the variation of constants to the determination of the perturbations of Jupiter V. The entire paper is relatively simple and elementary, and it is of no importance for further work on Jupiter V.

Use was made of a very much abbreviated expression for the disturbing function, in order to study the perturbations of the eccentricity and of the longitude of the perijovium (Bayev[38] (1938)). Bayev reduced the problem to quadratures and showed that the results are in general agreement with those of Mikhal'skiy[415]. Considering the very simple and inaccurate procedure, Bayev's work can at best be considered an illustrative example of Mikhal'skiy's work. The fact that the perijovium and the eccentricity are oscillating between certain limits was known before; there was no need for such a roughly approximating determination of these variations.

The observational material treated by Mikhal'skiy's theory is included in de Sitter's work, and there is no point in considering Jupiter V further in this report.

(b). Jupiter VI

The only Soviet paper noted that deals directly with Jupiter VI is by Proskurin[576] (1955). Previous work of F. E. Ross, based on observations from 1904 to 1906, is analyzed. Ross applied Delaunay's lunar theory to Jupiter VI. Proskurin finds many errors and inconsistencies in Ross' application of Delaunay's theory. Bobone's later work, which was based on Ross' results, may be criticized similarly.

Proskurin follows Ross in adapting Delaunay's theory for Jupiter VI; he computes tables for the coefficients in Delaunay's expansions for $L - l - \pi$, U, and $\frac{a}{r}$. Tables for the expansion in longitude ($V = L - l - \pi$) and latitude (U), and the ratio of the major semiaxes, $\frac{a}{r}$, are constructed in several steps. The final form is $V = \sum A \sin (iD + j\,l + j'\,l' + kF)$; $U = \sum B \sin (iD + j\,l + j'\,l' + kF)$; and $\frac{a}{r} = \sum C \cos (iD + j\,l + j'\,l' + kF)$, where D, l, l', and F have the meaning usual in lunar theory.

Proskurin's tables then, with argument i, j, j', and k, contain:

208 terms for A
197 terms for B
96 terms for C.

Proskurin's tables contain 501 terms; only 65 terms had been used by Ross and Bobone. This leads to considerable differences between the coordinates as computed from Proskurin's theory, on the one hand, and the Ross-Bobone theory, on the other; the differences exceed many times the probable accuracy of ±3" which Ross had claimed for his tables.

The real precision of Proskurin's new results must be judged on the basis of the actual representation of observations over a longer period of time. Such an application has not been made yet, but, in any event, Proskurin's work seems to be a valuable refinement of earlier theories.

According to Proskurin, further Soviet work on Jupiter VI will compare his theory with observations, improve the elements, and determine more exactly the motion of the perijove and the node.

(c). Jupiter VII

The only Soviet paper dealing directly with Jupiter VII is by Tokmalayeva[760]. This 160-page-long paper represents the only known work by this author.

She first gives a short review of the discovery and history of Jupiter VII, citing the previous work done by Ross and Bobone. Influenced by Proskurin's work on Jupiter VIII, the author undertook the present theory for Jupiter VII, which was developed along the lines of the lunar theory of Hill-Brown.

In Chapter I, all details of the general method are presented, beginning with Euler's introduction of a rotating coordinate system and giving a complete description of the method of Hill and Brown that was chosen. In Chapter II, the main terms are derived for Jupiter VII, including all the solar perturbations up to the third order. Only the planetary perturbations by Saturn, etc., are omitted. Chapter III contains the determination of the higher order terms. These have been derived by Delaunay's method, because of the convenient availability of literal developments for these perturbations. Chapter IV gives the final results for Jupiter VII and the constants of integration; in Chapter V, the disturbed coordinates and positions are computed for just two single observations in 1935 and 1941. The differences (observation minus calculation) are about equal in size to those of Bobone's theory. Without a more extensive comparison with observations, and without a final orbit correction, however, no definite conclusions seem to be possible concerning the actual inner accuracy of the new theory.

This new analytical theory should be better than the previous ones by Ross and Bobone, because the earlier theories (developed by Delaunay's method) were not so extensive. Final judgment about the new theory must be postponed until after the necessary adjustment to the observational material has been made. It seems, however, that the very elaborate and complicated, more or less technical, task of a more rigorous theory of Jupiter VII has been dealt with carefully and competently.

(d). Jupiter VIII

The eighth satellite of Jupiter represents a challenge to astronomers that has not yet been successfully met. The variation in its elements is so great that no general theory appears to be possible. By numerical integration, it is possible to represent roughly the position of the satellite, but ephemerides calculated for the future are invariably of very low precision. The Soviets are paying a considerable amount of attention to Jupiter VIII, much more than to any other planetary satellite.

Jupiter VIII was discovered by Melotte in 1908. The efforts to construct an emphemeris precise enough to keep it under observation resulted in the development of Cowell's method of numerical integration. In a review paper by Proskurin[571] (1950), summarizing the work done on Jupiter VIII up to about 1948, the author reviewed the work of various workers in a thorough manner.

The pioneer work of Cowell and Crommelin was considered in all details by Proskurin, and he points out two numerical errors in this work, as well as the fact that the adopted coordinates of Jupiter were not precise enough. Proskurin then reviewed the attempt by Trousset to establish an analytical theory along the lines of Hill's lunar theory, and the various modifications introduced by Brown, Boyer, and others. On the basis of $m = \frac{n'}{n} = -0.172$ for the ratio of the mean daily motions of the sun and of Jupiter VIII, Trousset established the annual motion of the node to 4°.5 and of the perijove to 0°.6; on the whole, however, his theory failed to represent the observations satisfactorily.

The main difficulty with the Trousset theory is the magnitude of the parameter m. For the terrestrial moon it is only 0.08, and yet the Hill-Brown lunar theory is extremely complex. For Jupiter VIII, additional complications are the much greater values of the eccentricity, e, and inclination, i, than for the terrestrial moon. Even though expansion in power series of e and i does not present special difficulties, the magnitude of m is such that the direct application of the Hill-Brown theory to Jupiter VIII would seem to be rather difficult.

Proskurin(569) (1950), in a paper which is one of the most valuable recent contributions to the theory of the motion of Jupiter VIII, applied the Hill-Brown method to the perturbations of Jupiter VIII and established the rather complicated and elaborate analytical developments up to terms of the third order with respect to e and i. Proskurin deserves credit for proving in this way the practical applicability of the Hill-Brown method to Jupiter VIII, and for providing a scheme for the actual computation of the various coefficients. So far as is known, he has not used the results to represent the observations of Jupiter VIII; therefore, final judgment must still be withheld as to the actual precision of his theory. The so-called parallactic terms, and the terms depending on the eccentricity of Jupiter's orbit, were not included in Proskurin's investigation.

In 1877, Hill published his lunar theory with $m = 0.08084893$, but he could not prove the convergence of the power series in m. This was done by Lyapunov, in 1896, for $m \leq \frac{1}{7}$. Lyapunov's result remained unknown, however, and Wintner, in 1929, had to prove it again, but only for $m \leq \frac{1}{12}$. Merman(391) (1952) extended the investigations of Lyapunov to prove convergence for $m \leq 0.18$, and this work includes the case of Jupiter's eighth satellite.

The work by Lyapunov represented remarkable progress compared with the treatment by Hill; the determination of the coefficients avoided the use of an infinite number of equations and was based on a very ingenious new mathematical approach. Merman not only made the original work of Lyapunov more accessible, which was highly desirable, but also established a wider range of convergence. Merman's publication is a first-class contribution to celestial mechanics.

I. Jackson's effort (1912) to reconcile the computed and observed positions of Jupiter VIII, which Proskurin[571] states was rather sloppy, did not go beyond the calculational aspect of the problem. In 1917, Jackson's ephemeris already showed departures on the order of 20' from the observed position of the satellite. Jupiter VIII was observed for the last time at Yerkes in 1923, and was lost thereafter until 1930.

Jupiter VIII was recovered in November, 1930, at Mt. Wilson on the basis of numerical work by the Soviet astronomer N. F. Boyeva[74, 75]. In 1929, she applied Numerov's extrapolation method to Jupiter VIII, using initial rectangular coordinates in 1915 as obtained by Cowell, Crommelin, and Davidson. The resulting ephemeris for the end of 1930 was calculated and sent to U. S. observatories, which led to the recovery of Jupiter VIII at Mt. Wilson. This was seemingly a spectacular triumph of Numerov's method, which, previous to Boyeva's work, had been applied only to the motion of minor planets. The triumph of Numerov's method was rather short lived, however. By 1934, Boyeva's extended ephemeris showed departures of the order of 42^s and 240". Herz continued the integration, but the departures increased to $+2^m.0$ and +23' in July, 1938.

An attempt to develop an analytical theory of Jupiter VIII was made by E. W. Brown, and later by Brown and D. Brouwer. The Brown-Brouwer theory was thoroughly analyzed by Proskurin[568], and the conclusions were repeated in another paper(571) (1950). Proskurin found that the observations for 1908 to 1938 depart up to $2^{m}.5$ in right ascension and 32' in declination from the Brown-Brouwer theory; this is partly explained, she says, by the wrong values of the constants m, e, and i adopted by Brouwer. With improved values for m, e, and i, Proskurin finds that it is possible to reduce the departures to maximum values $0^{m}.66$ and 11!2 for the period 1908 to 1938. Even then, the difference between observation and computation increases to $1^{m}.05$ in right ascension by 1942, and the departure increases rapidly with time. Therefore, Proskurin concludes that the uncertainty in the adopted constants is not the main reason for the unsatisfactory representation of the observations. The trouble seems to be in the slow convergence of the series by Brown-Brouwer. He concludes that the Brown-Brouwer series is useless for practical purposes. Even with the present insufficient number of terms, it is necessary, for instance, to solve an equation (equivalent to Kepler's equation) containing 189 terms by at least three successive approximations, in order to obtain the main argument, V (jovicentric longitude).

Beginning in 1948, D. K. Kulikov(325) (1950), at the Institute of Theoretical Astronomy, analyzed all work done by the methods of numerical integration and finally adopted the method of quadratures with a few modifications. New in Kulikov's paper is his treatment of the problem of orbit improvement, which includes the effect of the related differential changes in perturbations. This is done on the basis of small variations in the starting data for the numerical integrations, leading to related variations in the integrated coordinates. A correction of Jupiter's mass is included among the unknowns for the differential correction, but, on the basis of the observational material from 1930 to 1946, as used in Kulikov's solution, this correction does not produce a significant result for Jupiter's mass. Kulikov succeeded in representing observations of Jupiter VIII for the period 1908 to 1948 with greater precision than was done by H. R. Grosch in 1948 (Grosch based his orbit on observations for 1930 to 1946). Kulikov presented the observations with an accuracy of ±15"; Grosch reached $\Delta = \pm 47''$.

Following the investigation by Kulikov, an extensive program toward the development of an analytical theory of Jupiter VIII was evidently initiated at the Institute of Theoretical Astronomy. Very little of this work has been published, as far as is known, except for a summary by Proskurin(571). The following methods have been considered at the Institute of Theoretical Astronomy:

(1) The method of Hill-Brown, as used in lunar theory

(2) The method of Brown, as developed for Jupiter VIII

(3) Andoyer's modification of the Hill-Brown lunar theory

(4) The method of Gyldén-Zhdanov for intermediary orbits.

Proskurin's paper(570) (1950) on the stability of motion of Jupiter VIII is less significant than some of his other papers. He also reviewed the work of G. Kobb, Jackson, and Moulton(571) (1950). Proskurin computed the average value of the Jacobi constant, C, of Jupiter VIII in the restricted three-body problem sun-Jupiter-Jupiter VIII (neglecting the actual deviations from the idealizing assumptions of this classical model) and found that the value of C is such that no closed surface of zero velocity exists. Therefore, the Jacobi constant, C, of Jupiter VIII puts no limitation on the possible motion of the satellite, and an escape from Jupiter's sphere of action, and even from the solar system, would be compatible with the actual value of C. Moulton (1914), however, had found that the variational orbit of Jupiter VIII is stable. Actually, Proskurin's work does not constitute a contradiction of Moulton, because different definitions of the concept of stability are involved. The Jacobi-Hill criterion is a sufficient, but not a necessary, criterion for orbital stability, i.e., it assures stability if Hill's surface of zero velocity is closed, but it does not assure instability if such a closed surface does not exist.

(2). Satellites of Saturn

Whereas Soviet work on Jupiter's satellites is concentrated at the Institute of Theoretical Astronomy, their work on Saturnian satellites is carried out mainly at the Shternberg State Astronomical Institute by G. N. Duboshin and his collaborators. From 1940 until at least 1953, Duboshin spent considerable time on the analytical theory of the satellites of Saturn. Previously, between 1932 and 1940, Duboshin spent most of his time studying the theory of motion of a material point in a gravitational field and in a resisting medium.

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The analytical theory of the Saturnian satellites differs from the classical theories in two respects. It is constructed on the basis of Lyapunov's methods, which, according to the Soviets, make it possible to obtain convergent series representing the saturnocentric coordinates of the satellites, considering the effect of the attraction of the ellipsoid planet and its rings, the perturbative effect of other satellites, and the perturbative effect of the sun. The second difference lies in Duboshin's rejection of the application of Keplerian coordinates, and his preferred use of ordinary rectangular or cylindrical coordinates.

Duboshin also treats certain problems from the theory of potential, in order to obtain possibly better calculations of the forces affecting a satellite of Saturn.

In a 1945 paper(161), Duboshin presented investigations that are of considerable theoretical interest, even though the application had not been completed to a degree whereby the results could be useful for ephemeris purposes. He intended to represent the motion of any given satellite of Saturn on the basis of certain periodic solutions of a simplified problem. This simplification is obtained by replacing each of the disturbing satellites by a homogeneous distribution of matter along a circular orbit, assuming zero inclination relative to the equatorial plane of Saturn for any one of the first seven satellites. For Japetus and Phoebe, however, the inclinations are considered by a special method of treatment. For each of the disturbed satellites, the gravitational action of the following bodies is taken into account:

(1) The homogeneous rotational ellipsoid of Saturn.

(2) A homogeneous ring of matter in Saturn's equatorial plane, representing the actual rings of Saturn. This ring has a given finite width.

(3) Eight circular rings of infinitely small width, representing the disturbing satellites.

(4) The sun, moving in a fixed elliptical orbit corresponding to the mean orbital elements of Saturn's orbit.

It has to be expected, of course, that the periodic solutions of the problem will have periods not very different from the undisturbed period of revolution of the individual satellite.

Analytical expressions for the various potentials are derived in the first chapter(161), and the gravitational potential of Saturn's rings is treated here in a rather original way. Proceeding to rings of infinite width, the potentials for the seven inner disturbing satellites are then obtained by the same method. For the disturbing action of the sun, only one term is retained in the disturbing function. For each disturbing satellite, with the exception of Japetus and Phoebe, the potential, $\underline{U}$, is divided into two parts, U' and U", so that $U = U' + U''$, where $U'(\rho, z)$ is an even function with respect to the coordinate $\underline{z}$, while U" contains the remainder of $\underline{U}$. ρ is the projection of the jovicentric distance, $\underline{r}$, of the disturbed satellite onto the equatorial plane; $\underline{z}$ is the rectangular distance from this plane, so that $r^2 = \rho^2 + z^2$.

In the second chapter, the first approximation of the problem is derived, i.e., periodic solutions are established for the

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motion as governed by U' along. Considering small deviations, x, of the true value of ρ from a constant value, a, or $\rho = a + x$, the author proves the stability of the solutions for $z = 0$ which are variations superimposed on a basic circular orbit with a radius a. The solutions are found by the method of Lyapunov, i.e., they have the form of power series developed for ascending powers of a small arbitrary constant, with coefficients which are trigonometric polynomials with respect to a linear function of the time. Elliptical integrals are used extensively in the determination of these trigonometric polynomials. The determination of the coefficients, from their relations with the constant, σ, of integration, follows Lyapunov in a very ingenious way.

In the third chapter, the author presents the development of the potential of Saturn's ellipsoidal figure, the corresponding developments for the nonperiodic part of the solar action, and the special methods for dealing with Japetus and Phoebe. The nonspherical figure of Saturn is neglected for the action on Japetus and Phoebe. The existence of stability, in the sense of Lyapunov's definition, is then demonstrated for the basic circular solutions in the present problem. The analytical developments are completely elaborated for each of the nine satellites. Numerical application of these developments, however, is for $z = 0$ only, thus leading to expressions for the function ρ in the case of each satellite. It can be seen that the related periods, T, are indeed only slightly different from the well-known periods of revolution with respect to Saturn, and that the parameter, σ, of the power series is identical to the differences between the disturbed and the undisturbed values of a. This numerical application is an approximative one, and the author states that improved basic elements and a higher numerical accuracy would be necessary in order to obtain expressions that would be adequate for ephemeris purposes. He says that the main purpose of this paper[161] is to demonstrate the existence and usefulness of periodic solutions based on the simplifications which have been made.

In the fourth chapter, the second approximation of the theory is made by introducing the part U" which had been omitted before. So far as the action of the sun is concerned, no periodic solution exists as long as the satellite distance from Saturn is not very small compared with the distance from the sun, or as long as the relative periods are not commensurable. Only a nonperiodic solution is found here for the solar part, but, nevertheless, this result for the solar perturbations is useful.

From the purely theoretical point of view, considerable credit must be given to Duboshin for his analytical work in this paper, and also for taking up again the earlier contributions of Lyapunov. These contributions were made many decades ago, but, in spite of their importance, they have never been sufficiently known to other workers in the field.

In a systematic way, G. N. Duboshin[162] (1945) proceeded from the potential of a disturbing point mass to a homogeneous distribution of matter in a circular orbit (used later for the secular perturbations produced by the disturbing satellites); to the potential of a circular ring of finite width; and to cylindrical rings and other configurations of matter. These may be used then to build up the oblate figure of Saturn and the actual rings of the planet. The potential of any body of rotational symmetry and comparably complex configurations can be obtained in the same way. Although the results for the gravitational potential of such bodies were known before, the author derives them in a new and rather ingenious manner.

This paper differs from earlier ones by the same author mainly in that it permits any given inclinations for the orbits of the disturbing and of the disturbed satellites.

In the first section, the reciprocal of the distance between the disturbing and the disturbed bodies is developed. The author makes use of Gegenbauer's polynomials, which may be considered to be a generalization or extension of Legendre's well-known polynomials to the case where the integer, , in the generating function $G^{\alpha}(\sigma) = \frac{1}{(1 - 2\nu\sigma + \sigma^2)^{\alpha + \frac{1}{2}}}$ is different from zero. For his present purposes, the author extends the original theory of Gegenbauer for these polynomials and gives some new formulas for the determination of the coefficients. In the present developments, Gegenbauer's polynomials appear multiplied with functions of the time, because the corresponding factors depend on the changing position of the disturbing body.

In the second section, the more general results of the first section are used for the development of the force functions of the different configurations which have been mentioned above. In the third section, the force function is developed for the action of a material point which is moving in a circular or elliptical orbit. In the fourth section, all the previous results are used to construct the force functions for the actual problem of Saturn's satellites; the author introduces the rotational ellipsoid of Saturn's body, rings of three-dimensional extensions for the actual rings of Saturn, one-dimensional rings for the secular perturbations produced by the disturbing satellites (for the action of Japetus and Phoebe, these rings have an inclination relative to the equatorial plane; the other satellites, as disturbing bodies, are assumed to move within the equatorial plane), and a fixed ellipse for the motion of the sun.

Duboshin gives special attention to the question of the convergence of his series; he proves that all the developments used in this investigation converge sufficiently within limits which are wider than the actual variations of the corresponding parameters. A special investigation of the convergence is made for the developments in the case of the satellite Titan.

As in the case of quite a number of other papers by Duboshin, this one is of considerable value and interest for its analytical treatment of the motion of Saturn's satellites. Undoubtedly, Duboshin is one of the foremost authorities in this field at the present time; his results have influenced the activities of other Soviet astronomers working in celestial mechanics.

In a more recent investigation, Duboshin(167) (1950) considered motions which are not periodic; he also wanted to allow for the actual complexities of the problem by using four arbitrary parameters or small constants in the power developments, instead of the one previously employed by him. Again, the disturbing satellites are replaced by the gravitational action of infinitely narrow, homogeneous, and circular rings of matter, thus introducing only the main part of the secular perturbations by the satellites. A part of the action of the sun, the action of Saturn's rings, and the action of Saturn's ellipsoidal figure are introduced, as in Reference 161 (1945). Following Lyapunov's method of development, the decreasingly small terms x_1, x_2, x_3, ... are introduced in a rather ingenious manner by solving the differential equations first for x_1, then by introducing the result for x_1, together with the terms of the second power, into the following solution for x_2, etc. Proof of the stability of the solutions is given then, following Lyapunov's method. Interestingly enough, the author's method of solution involves the appearance of terms which are purely secular as soon as he proceeds to x_4. Since stability, however, has been proven before, the author concludes that the existence of these terms does not actually endanger the convergence of the solution, but that the various terms multiplied by the time, _t_, will offset each other.

Duboshin then proceeds to consider the problem of convergence in connection with small variations of the starting conditions or constants of the problem. He finds that the developments permit analytical continuation with respect to the variations of the integration constants, and that they will converge within a definite interval of possible conditions.

This paper, like the earlier one on periodic solutions in connection with Saturn's satellites, is of considerable theoretical interest. These results will bear fruit in practical applications sooner or later; they constitute, therefore, an appreciable contribution to celestial mechanics. The methods, which may be new, focus attention on certain much earlier ideas of Lyapunov, ideas which are important, but which had been forgotten because of insufficient publication of Lyapunov's work.

The latest paper by Duboshin(174) (1953) on this subject starts with a general discussion of the different disturbing forces which have been considered in accurate analytical theories for the motion of satellites. As to higher order terms, which have been neglected in the existing theories of satellites, Duboshin is concerned mainly with the indirect terms produced by the oblateness of the planet which represents either Jupiter or Saturn. He finds that the influence of the planet's oblateness on the relative orbit of the sun, on the other (disturbing) satellites, and on the planetary perturbations has generally been neglected without an examination of the order of the resulting effects on the motion of the disturbed satellite. The author's discussion of these various effects reveals the fact that they may be neglected, with the exception of the indirect perturbations which are related to the effect of the oblateness on the orbits of the disturbing satellites. He says that terms for the indirect perturbations are comparable to other small terms which are not neglected in the theories of satellites; he derives the necessary equations for the inclusion of these terms in the analytical developments for the disturbed satellite. He also claims that the lunar theory should include the effect of the earth's oblateness on the orbit of the sun as a disturbing body in the problem.

The effects discussed in this paper are very small and may be noticeable only after considerable intervals of time. Their consideration and critical discussion, nevertheless, are very much worth while and make this paper a contribution to the field of satellite theories.

(3). Satellites of Mars

The Soviets have apparently begun to study the satellites of Mars only recently. To date, they have only reviewed existing theories.

Kosachevskiy(303) (1954) discussed the task which still has to be undertaken in order to obtain satisfactory theories for these close companions of Mars. He claims that the existing theories may suffice for ephemeris purposes, but that, even in this respect, improvement is desirable. The difficulties are connected with the various parameters which affect the motions of these satellites and which are not sufficiently known today. Also, the masses of the two satellites have to be determined.

Kosachevskiy plans to attempt a more rigorous theory, utilizing the various investigations which have been made at Moscow in connection with related problems of celestial mechanics. Evidently, the announcement of this intended work on the satellites of Mars is one reason for the publication of this paper, which is nothing more than a review. However, the announcement gives more evidence of the vigor with which Soviet astronomers try to make scientific advances in areas where a challenge or a promise seems to exist.

i. Numerical Integration and Special Perturbations

Numerov's paper(497) (1923) on A New Method of Determination of Orbits and Computation of Ephemerides Taking Into Account Perturbations has become a classic. In it, he introduces his special coordinates for convenient interpolation in the numerical integration of the disturbed rectangular coordinates of a planet. The sixth-order differences can be neglected if his method of interpolation is used. The author presents his own method after discussing the corresponding procedures by Cowell and Störmer. The author's method, which is well known today to all workers in the field of celestial mechanics, is based on Gibbs' formulas and takes into account the effect of Jupiter's perturbations. The method is applied to the determination of the orbital elements, including the effect of perturbations, as well as the computation of ephemerides. The method is illustrated by a detailed example, computed by N. Komendantov, for the planet (1) Ceres. Some tables for ~~the~~ auxiliary quantities involved in the method are included.

Without any question, this has become one of the most fruitful publications in the field. Today, the problem of convenient interpolation is not quite so essential as it was in 1923, because of the availability of high-speed computing machines, and Cowell's method is preferred in many cases to Numerov's. One disadvantage of Numerov's method is the fact that his special coordinates cannot be used for the computation or recomputation of the disturbing forces, and that one has to go back to the true coordinates for this purpose anyway. Nevertheless, Numerov's contribution will always be considered as one of the more significant ones in the field of numerical methods in celestial mechanics.

Numerov(499,502) (1924) next presented a method for the extrapolation of the perturbations, which are expressed as variations of his now well-known special coordinates. The method of extrapolation is similar to the method of Encke. Auxiliary tables and an application of the method to the planet (62) Erato are given, in order to facilitate the use of the new procedures. Numerov(499) (1924) reported results practically identical to those published in the Monthly Notices of the Royal Astronomical Society(502). Apparently, the Soviet version was intended to publicize his method in the U.S.S.R.

These contributions can be considered to be auxiliary papers for the practical utilization of Numerov's special coordinates in numerical integrations representing the disturbed motion of a minor planet or comet. At the time of their publication, these papers certainly helped to accelerate the general trend towards modern computing techniques, although the availability of automatic computing equipment today has depreciated somewhat the value of Numerov's special coordinates.

In Numerov's method of numerical integration using special coordinates, the effect of the sixth- and higher order differences is neglected in the process of extrapolation; in comparison, Cowell's method neglects the fourth- and higher order differences. Numerov(503) (1925) investigated the influence of these neglected higher order differences, considering them to be produced by a fictitious disturbing planet. For this study, he develops an analytical theory of the related perturbations of the polar coordinates, following Hill's method. The resulting expressions are converted into perturbations of the rectangular coordinates, to facilitate their application for the purpose of this study. It is found that the influence of the neglected sixth-order differences may be considerable, especially near perihelion, if the interval of the integrations is too large. Although this influence of the sixth-order differences can be rendered harmless by the proper choice of the interval, the other possibility is to improve the integrations either by successive approximations, or by using the analytical expressions derived in this paper. For a given example, the differences between the accurate orbit and the approximate one (which neglects the sixth and higher differences) have been computed in both ways, i.e., numerically and analytically, and a reasonable agreement is found. This example, at the same time, provides a numerical test of the analytical expressions.

Since certain differences are neglected in the application of the author's method of special perturbations, it was certainly worth while and interesting to investigate the size of the possible errors thus introduced. Therefore, this is a valuable contribution, which makes it possible to estimate the errors of the approximate perturbations.

Since it was originally published in 1923, the Numerov method of extrapolation has been altered considerably in some of its parts. The modifications were made in order to meet the requirements of practical applications more conveniently. Numerov[508] presented the complete formulas of the revised method in 1926, but without giving their derivations. The main purpose of the later paper was to have the final form of the method available in one place, to aid further practical applications. A detailed example of such application is given for the planet (319) Leona.

Insofar as the author's previous results are compiled here merely to allow more convenient use of the method, this paper, by itself, does not represent a new contribution. Publication of this compilation was very commendable, however, because the author's method had become widely accepted and appreciated, and because, for orbit computers, it was very inconvenient to use a number of successive articles in order to find the equations in their latest and most appropriate form.

By adding two additional terms, Komendantov[288] (1931) increased the accuracy of an equation given by Numerov for the interpolation of the disturbed planetary coordinates. The new formula is illustrated by its application in two examples. This formula increases the accuracy of Numerov's method of extrapolation and avoids the necessity of computing the higher differences (as did Numerov's more approximate formula).

Komendantov had used Numerov's extrapolation method for the orbit of the minor planet (1) Ceres, taking into account the perturbations produced by Jupiter. Then, he[283] (1924) utilized additional observations, after recomputing the disturbed coordinates as affected by Saturn, as well as by Jupiter, in order to derive improved elements based on 16 normal places during three successive oppositions from 1920 to 1923. The final representation of these normals has the mean errors $\pm 0^{s}.10$ and $\pm 0''.9$ in α and δ, respectively. The author concludes that, if the sixth derivatives are included in the computations, the extrapolation method is adequate for a rigorous ephemeris, and that three oppositions are required in order to obtain elements which are sufficiently accurate for this purpose.

This paper is only a routine application of an existing method to an individual planet, but, from the final results it is obvious that the computations were done with great care and ability.

Komendantov[286] (1930) made use of Numerov's method of extrapolation for the computation of the disturbed coordinates of the Trojan planet (588) Achilles. The integration includes the disturbing actions of Jupiter and Saturn, and the computations extend over two heliocentric revolutions of the planet. The author found that, for the relatively slow orbital motion of a Trojan, the extrapolation method is particularly convenient and useful. Several corrections of the orbital elements, and recomputation or proper revision of the perturbations, are made, until the observations in nine oppositions from 1906 to 1929 are represented with the high accuracy of $\pm 3''.0$. The author also presents some general considerations concerning the computation of the disturbed motion of a minor planet, about the accumulation of errors in such numerical integrations, and about other related questions.

This is a technical contribution insofar as it presents the results of extensive computations; actually, the level of the paper is rather high, because considerable attention is given to the discussion of the practical problems which are important for the successful execution of such work. The computations, themselves, obviously have been done with the utmost care and have been brought to the highest possible level of perfection. From the discussion of the probable errors of the results, it can be seen that the author thoroughly mastered the theory and practice of perturbations and of orbit determination.

A method of orbit correction by Komendantov(290) (1934) was especially constructed for use in connection with Numerov's extrapolation method for the computation of special perturbations. The quantities to be determined from the solution of the equation of conditions are the six corrections to the rectangular heliocentric coordinates at two successive times, t_1 and t_2, as separated by the time interval, w, of the numerical integrations. The corrections (Δx_1, Δx_2, Δy_1,

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Δy_2, Δz_1, and Δz_2) can be utilized directly to obtain an improved series of special coordinates for the whole interval covered by the numerical integrations.

The method is useful for orbit corrections on the basis of perturbations computed by Numerov's method, and the equations of condition appear to be of a relatively simple form. This is an original contribution, but nothing extraordinary, and the results are of interest only in connection with the special method developed by Numerov for the computation of planetary perturbations by numerical integration.

Postoyev(567) (1926) gave a detailed presentation of Numerov's new method, including all the improvements which had been made since the method was first published in 1923. The computation of the elements of the minor planet (283) Emma is given in considerable detail as an illustration of the method. All the equations and schemes of computation are given in a form which assumes the use of desk computing machines.

The paper, of course, does not represent any scientific progress by itself. Nevertheless, at the time of its publication, it probably was useful to orbit computers as a manual on the practical application of Numerov's method.

In a theoretical discussion of the errors which are committed by omitting the differential quotients higher than the fourth order, in the extrapolation method for the rectangular coordinates, Tyakht(766) (1937) treated the neglected effects as the disturbing effect of a fictitious planet and integrated the related approximate differential equations. In this manner, he arrived at certain formulas for the influence of the neglected derivatives up to the tenth order. Tables are given in order to facilitate the application of the formulas. By making use of these results, one can avoid the process of successive approximations in connection with the extrapolation method.

This discussion is of some interest from the theoretical point of view, insofar as the results are helpful in estimating the size of the effects under consideration. It remains doubtful, though, that the results will be of any practical value. If one wants to compute approximate perturbations, one is inclined to discard such corrections completely. To obtain accurate perturbations, computers are not inclined to bother splitting the perturbations into two parts of different order; they prefer, instead, to improve the complete values of the perturbations by a process of successive approximations, especially since automatic computing equipment has become available. Nevertheless, the investigation remains of some interest from the theoretical viewpoint, especially in connection with the basic principles of the extrapolation method.

Numerov[513] (1929) used his well-known method of extrapolation computations for the disturbed orbital motion of the Trojan planet (659) Nestor. The computations cover the time interval from 1908 to 1928, and the interval of the integrations is 320 days. In the extrapolation process, the terms of the sixth and of the eighth order are taken into account by means of the undisturbed elements. The author also gives a convenient scheme for the interpolation of the special coordinates for $n = \frac{1}{2}$. The ephemeris for 1928, computed on the basis of the extrapolated coordinates, differs by amounts of the order of $0^m.3$ in α and 1' in δ from a rigorous ephemeris by Struisky.

This is only a computational exercise, but it presents an interesting application and test of Numerov's extrapolation method. The results must be considered to be very satisfactory, since they have been obtained by an approximate method, using an extremely large interval. It is evident that the author's method is very economical for planets such as the Trojans.

In his paper on the improvement of the initial six coordinates in the method of extrapolation, Numerov(514) (1931) uses the differential relations between the dynamically consistent, small variations, $d\bar{x}$, $d\bar{y}$, and $d\bar{z}$, of the two initial positions and of a later position, $\bar{x}$, $\bar{y}$, $\bar{z}$, to form the equations of condition by means of which the two initial positions may be improved, if the corrections for later positions are known. The method has the advantages that no elements are involved, and that the effects of the perturbations are readily included. In doing this, the author has proposed an interesting and convenient method, which has been devised to meet the requirements of the extrapolation method for the computation of perturbations. The method is elegant and has the features of modern methods of numerical computation.

Boyeva(74,75) (1933 and 1935) applied Numerov's extrapolation method to Jupiter VIII. On the basis of her 1929 work using this method, the satellite was recovered in 1930.

The formulas which were ordinarily used at the Institute of Theoretical Astronomy, Leningrad, in computing the osculating elements on the basis of the disturbed rectangular coordinates, were given by Komendantov(289) (1932). Although differential corrections to the initial set of coordinates, instead of element corrections, can be determined, and the use of elliptical elements actually can be avoided when computing the disturbed orbit of a planet by Numerov's extrapolation method, the final results, at least, are usually given and published in the form of osculating elements. The formulas presented by the author are convenient to use with machines and numerical tables. The author does not claim that these equations are new; he merely wants to provide orbit computers with the formulas which proved to be the most practical and convenient ones in the work at the Institute of Theoretical Astronomy.

D. K. Kulikov's paper(325) (1950) on numerical methods of celestial mechanics, which are used to study the motion of Jupiter VIII, is discussed in the section on <u>Planetary Satellites</u>.

The principal purpose of an investigation by Mikhal'skiy(416,418) (1932 and 1933) was to derive and present the equations necessary for the computation of the coefficients of the corrections to the masses of Jupiter and Saturn in the equations of condition. The author mentions that the necessary equations had not been given before in textbooks, even though they had been used before by individual workers in the field.

Every competent astronomer can arrive at the necessary formulas without too much trouble, even without reference to such textbooks, but it is certainly worth while to have them derived and printed some place in the literature, particularly for students approaching these problems for the first time. The author deserves credit for providing the necessary equations, which are valid and needed in the case where the planetary perturbations are those of the elliptical elements. In this case, it is possible to utilize the coefficients of the element corrections directly in the equations of condition for the computation of the mass coefficients; in the case of coordinate perturbations, for instance, one would have to use a different method. Altogether, this is a useful, although not extraordinary, contribution.

The motion of the minor planet (659) Nestor, a member of the Trojan group, was determined by Mikhal'skiy(418) (1933) in the form of accurate special perturbations by the eight major planets from Mercury to Neptune. Eighteen normal places from the years 1908 to 1931 were used for a final differential correction of the elements, including two corrections to the masses of Jupiter and Saturn. Assigning weights of from one to three to the various normals, the author obtained the final mean error of $\pm 4\overset{''}{.}3$ for the unit of weight; the individual residuals (observation minus calculation) go up to about $\pm 9''$ for certain normal places. For the reciprocal of the mass of Jupiter, the author finds 1047 ± 0.87, and for Saturn, 3534 ± 20.3.

This is a computational investigation involving determinations of two important planetary masses. The mean or probable errors

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of the results, however, are too large, and the results, therefore, are of no significance. This is clear also from the rather poor representation of the normal places. The mean error of any satisfactory normal places of minor planets should not exceed one or two seconds of arc, unless the observations or the computations are poor. Although the author evidently is theoretically competent and knew what he was doing, it is doubtful that the numerical computations had actually reached the required stage of refinement and convergence.

Proskurin's(572) (1951) heliocentric orbital coordinates for Jupiter are discussed in the section on the Construction of Tables.

Runge's method for the integration of differential equations makes use of the Taylor series developments, discarding terms higher than a certain order. The integral is obtained as a combination of the values of the integrand for certain values of the argument. In general, the evaluation of the (approximate) solution is rather elaborate and, therefore, Runge's method did not find many applications in celestial mechanics. Reznikovskiy and Shchigolev(653) (1953) pointed out, however, that the method may be used with advantage for starting a numerical integration, as well as in certain cases of large perturbations. Their paper amounts to a presentation of Runge's method for differential equations of the first and of the second order, in a form which is most convenient for the problem of celestial mechanics. The form of the solution depends on the highest power to be retained in the developments with respect to the interval, h, of the numerical integration; the authors derive the proper expressions for the two cases where either the third or the fourth power of h is the highest power to be considered.

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In this paper, a well-known mathematical method is merely adapted and presented for a proposed use in celestial mechanics. The presentation is well done, but nothing basically new is given.

Subbotin(737) (1933) reviewed the methods of numerical integration, in order to compare their effectiveness as far as the rapid convergence of the successive approximations is concerned. Starting on the basis of the formulas of Cowell and Störmer, he separates the main equation for the double summation into two parts. Thus, the main part can be extrapolated with high accuracy, while any successive corrections will have to be applied primarily to the separated second part. He then praises highly the method of Tietjen-Oppolzer for polar coordinates, because a second approximation is frequently unnecessary in this method. Numerov's special-coordinates method is considered as a modification of the method by Tietjen-Oppolzer; the author is in favor of the original method by Tietjen-Oppolzer, because, here, the typical features of a simple quadrature prevail and guarantee a slow accumulation of errors.

This is a good paper, not so much because of the new modifications presented by the author, but because of the critical comparison made of the essential features of the various methods. The author demonstrates good judgment and high competence.

Subbotin(738) wrote a popular report on the progress and status of celestial mechanics in 1933; the various methods for the numerical integration of differential equations were given emphasis. The methods of Darwin, Stroemgren, Cowell, Numerov, and Tietjen-Oppolzer receive special attention. The author says that the methods of numerical integration are being used now (1933), more than in the past, because their use has been facilitated by the availability of modern equipment (desk calculators).

A popular article such as this one does not contribute anything new; however, it serves a good purpose by making a wider circle of people familiar with the basic features of numerical integrations in celestial mechanics. Certainly, the author is a very competent person for writing such an account.

j. Capture in the Three-Body Problem

A pioneer paper in the field of the capture problem was written by O. Yu. Shmidt(708) in 1947. The paper is extremely important because it opened an entirely new field of investigation and inspired analytical investigations by other Soviets concerning the possibilities of capture. Although Shmidt started on the basis of results by Öpik, Lyttleton, and Chandrasekhar, these Western authors had not realized that the simple reversal of time would lead to significant progress in the problem of capture. They had different purposes in mind. Credit for the considerable recent progress in the capture problem goes to the Soviet authors, even though the present (and first) paper on this subject employed only well-known numerical techniques and procedures. If it were not for the language barrier (no English abstract is given), this paper might perhaps have inspired parallel efforts in Western countries. The lack of the Western-language abstracts, although it delays world-wide recognition of new results by Soviet authors, does, on the other hand, "protect" them from early Western competition in a newly opened field.

In his introduction, Shmidt states that the idea of capture has frequently been considered in cosmogonical investigations (for example, in his own work on the origin of visual double stars), but that no proof for the possibility of capture had ever been given. On the contrary, Chazy's papers (1929 and 1932) had concluded that capture was impossible in the case of three bodies moving originally in hyperbolic orbits with respect to each other. The author then states that, although Chazy's considerations are correct for the case where the energy constant, h, fulfills the condition $h < 0$, Chazy's conclusions were based on analogies with the case $h > 0$, and no rigorous proof for the impossibility of capture in the case of $h > 0$ had ever been given. Actually, Shmidt says, the investigations by Öpik, Lyttleton, and Chandrasekhar, on the instability and possible dissolution of certain double stars, constitute evidence for the possibility of capture, because the corresponding results follow automatically if the direction of time is reversed.

The author then presents a numerical integration of the motion of three equal masses, two of which are moving in elliptical orbits relative to each other, while the third body approaches in a hyperbola and closely passes one of the first two masses. The result, which is presented graphically in the paper, is the separation of the double-star pair and hyperbolic motion of each mass relative to each of the two others. For the reversed direction of time, then, this numerical investigation established an actual case of capture for the given, very special, starting conditions. The rest of the paper is more or less intuitive and includes considerations of the possibility of capture for a variety of starting conditions, and the (small) probability of capture.

J. Chazy, in his various papers published between 1918 and 1932, was concerned with the problem of capture; by topological considerations, he thought he had proven that under no circumstances could three bodies, originally moving in hyperbolic orbits relative to each other, ever end up with elliptical motion of two of these bodies relative to each other. Gazaryan[209] (1953) points out that there are certain weaknesses in the topological arguments used by Chazy, and he comes to the conclusion that, actually, Chazy did not prove the impossibility of capture for the case that the energy constant, $\underline{h}$, is positive.

This is an important theoretical paper, the result of which removes the obvious contradiction between the theorem of Chazy and the numerical verification of an actual case of capture by Shmidt.

After Shmidt's work appeared, Khil'mi began to publish a series of papers on capture in the n-body and three-body problems. In the first of these[265], in 1948, Khil'mi gives four theorems. The first of these says that the three relative distances will increase beyond any finite limit for $t \to +\infty$, provided that, at a given time, t_0, the values of the coordinates and velocities satisfy certain given inequalities. The proof for this theorem is given in another paper(267) (1950), but this proof is not rigorously correct, as the author later stated(272) (1951). Proof of the other three theorems also was given in later investigations or by numerical results. According to the second theorem, if certain inequalities are satisfied, the distance between two of the three masses will remain smaller than a certain upper limit, R, while the third mass will depart to unlimited distance values relative to the other two for $t \to +\infty$. The third theorem states that capture is possible in the three-body problem. This is the most important result in this paper, because it is contrary to Chazy's earlier claims that capture is impossible. Chazy's reasoning was not rigorous, however, as was shown by Gazaryan (cf. Reference 266). Khil'mi then finds that his first two theorems are satisfied when they are applied to the numerical integrations which Shmidt had obtained for an actual case of capture.

In the fourth theorem, Khil'mi says that the "measure" of the possible starting conditions leading to capture is different from zero, and that, therefore, the related probability of capture has a value different from zero.

Three years after his first paper appeared, Khil'mi(272) (1951) presented two more theorems. The first one states that, if the constant of the energy integral, H, is greater than zero, and if three certain inequalities are satisfied by the relative positions and velocities (using the coordinates of Jacobi) at a given time, t_0, then one of the three bodies will monotonicly increase its distance from the center of gravity of the other two masses with $t \to +\infty$, while the distance between the other two masses will not increase beyond a certain given limit, R. After proving this theorem, Khil'mi proceeds to the second theorem, which he proves by reduction of the proof to the proof of the first theorem. The second theorem states that, if $H > 0$ and if, at the time t_0, two certain inequalities involving the starting positions and velocities and two constants, R^* and ε, with $\varepsilon < R^*$, are satisfied, then one of the three bodies will increase its distance from the other two without limit, while the other two masses will not depart from each other beyond the limit R^*. Both theorems are important and useful as criteria for the permanency of any case of capture, such as the one established by Shmidt on the basis of numerical integrations.

This paper replaces an earlier one by the same author. The proof given in the earlier paper(267), which dealt with a more general case of n-bodies, was not correct, as the author mentions in the present paper. He had been informed by Merman about the lack of validity of his proof. A valid proof now is presented for the case of three bodies, but no actual proof exists so far for the more general theorem of n-bodies.

The first criteria for the determination of the nature of the relative orbits of three bodies before and after a close approach, so as to decide about the occurrence of capture on the basis of numerical integrations, as in the famous first example by Shmidt, were given by Khil'mi. The criteria of Khil'mi required the availability of data for the numerical integration for a rather long interval of time. Merman(394) (1953) presented new criteria, of a sufficient but not necessary type, which, if fulfilled, establish the occurrence of capture. These new criteria are more efficient than the earlier ones of Khil'mi, because they require data for the numerical integrations which become available after a comparatively short interval of time.

This is a rather valuable contribution by a distinguished writer. The new criteria should be of great benefit for many future investigations which employ the tool of numerical integrations, one of the most effective ones in the complicated capture problem.

Merman(396) (1953), in his paper on the sufficient conditions for capture in a restricted hyperbolic problem of three bodies, gives the definition of capture according to Chazy for the case of a double approach. Capture has occurred if, before the approach, all three bodies had hyperbolic orbits relative to each other, but if, after the common approach, the distance of the small mass from the one finite mass, m_1, remains under a certain upper limit for $t \rightarrow +\infty$ while the motion of the small mass and of m_1 relative to m_2 remains hyperbolic. The author mentions the numerical verification by Shmidt of a case of actual capture.

In the first section, Merman investigates the differential equations of the problem and certain simple, yet important, inequalities. If, for example, one of the masses describes a hyperbolic orbit relative to a second mass, the absolute amount of the radius vector, r (after the approach and for $t \rightarrow +\infty$), always lies between

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two certain limits which themselves are linear functions of the time, $\underline{t}$, depending also on the given initial values of $\underline{r}$ and of the velocity vector, $\dot{r}$, at the time t_0. Certain more complicated inequalities are connected with the energy and area integrals.

In Section 2, Merman states and proves not less than 15 different criteria, in the form of inequalities; each of these, if fulfilled, gives proof of the hyperbolic-elliptical character of the given orbital motion. These are the sufficient criteria for hyperbolic-elliptical motion, such as would be realized by the small body after the approach to m_1 and m_2 if capture took place. All these criteria refer to the occurrence of hyperbolic-elliptical motion after the approach (for $t \rightarrow +\infty$), whereas hyperbolic-hyperbolic motion is assumed for $t \rightarrow -\infty$.

In Section 3, comparable criteria are given for the hyperbolic nature of the final motion for $t \rightarrow +\infty$. Capture has not occurred if any of these criteria apply to the actual motion after the approach. An estimation of error is made in Section 4, considering certain simplifications which have been made in the derivation of the various criteria. Section 5, finally, contains the resulting conditions for the occurrence of capture; these are divided into necessary and sufficient conditions.

This paper undoubtedly is a brilliant contribution; significant progress has been made with relatively elementary means. This work certainly supersedes the various papers by Chazy, who was thought to have proved that capture is not possible at all under certain conditions. Merman's investigation now specifies the actual requirements for capture.

Merman(397) (1953) next applied the criteria which he developed in his previous paper (cf. Reference 396). The earlier relations are modified for this purpose by introducing the hyperbolic eccentric anomaly, H, instead of the time, t, as the independent variable of the problem. For equal finite masses, i.e., for $m_1 = m_2 = 1$, and for a given hyperbolic orbit of these two masses relative to each other, the motion of the small body is found by numerical integration (Cowell's method) over a certain length of time before and after the approach.

For two given moments, before and after the approach (or before and after capture), the author applies his process of successive approximations for the determination of the various quantities which are characteristic and significant for his final and rigorous criteria. For the given example (the starting conditions of which undoubtedly had been found only after previous numerical experimenting with varying initial data), the approximate criteria, as well as the rigorous ones, confirm the occurrence of hyperbolic-elliptical motion after the approach, and capture, therefore, has actually occurred.

Compared with the author's previous basic paper(396), the present one is a more technical presentation; still, the presentation of actual examples of capture is a matter of great interest. Therefore, this paper, too, has to be counted as one of the really important contributions to celestial mechanics.

Next, Merman extended[399] (1954) his earliest results (cf. Reference 396) to the case where the two finite masses, m_1 and m_2, describe a parabolic orbit relative to each other, instead of a hyperbola. According to the much earlier papers by Chazy, no capture of the small mass should be possible in such a case, if the small body moves hyperbolically with respect to both finite masses before the common approach. It had been shown by Gazaryan (cf. Reference 209), however, that Chazy's conclusions were wrong, because they were not reached in a sufficiently rigorous way.

In the first section of the present paper[399], the basic differential equations, as well as the important inequalities, are given; this differs from the corresponding section in Merman's previous paper[396] only insofar as the introduction of a parabola, instead of a hyperbola, for the relative motion of m_1 and m_2 is concerned. Similarly, the other four sections of the paper contain the corresponding modifications of the same sections in the original paper and need no further comment. The final results are the necessary and sufficient conditions for the occurrence of capture.

This investigation is a more or less technical extension of previous work to a special and different case. After an important advance onto new ground has been made, however, full "occupation" of this new ground has to be undertaken, too; therefore, this secondary paper is also a valuable contribution, even though the essential efforts were made in a previous paper.

Merman(401) (1955) then treated the more general case where all three bodies have finite masses. Assuming that two of the masses have a relatively small velocity with respect to each other, and that the third mass is at a larger distance or that its distance from the first two masses increases fast enough, the author gives six certain conditions (in the form of inequalities) which, if fulfilled by the given starting data, establish proof for the hyperbolic nature of the motion for $t \rightarrow +\infty$.

The author's deductions are very clear and rigorous from the mathematical point of view, making this an important as well as an elegant contribution.

Shmidt's findings gave cause for the re-examination of Chazy's derivations, and Merman(398) (1954) reinvestigated the principal theorems. Making use of abstract mathematical concepts, such as the measure of a given set of points in 12-dimensional phase space, Merman arrives at a certain refinement of the basic elements which are involved in Chazy's theorems, but this does not help him much in arriving at concrete results. Certain theorems, dealing with the probability of the occurrence of certain developments, are restated in the new and more rigorous form, but nothing really concrete takes the place of the theorems of Chazy that was not actually proved in Chazy's papers. Some results in the present ~~paper take~~ the form "<u>if</u> the corresponding theorem by Chazy is correct, then the following is also true...", but, apparently, even the refined mathematical approach is not sufficient to clear the basic problems theoretically to any great extent.

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Obviously, the mathematical difficulties in the treatment of the capture problem are enormous, and the author cannot be blamed for the limited extent of his positive results. His investigation is deep and competent, and the general level of this contribution is very high.

In his Dynamic Systems, Birkhoff presented seven theorems concerning the problem of three bodies. These theorems deal with the existence of hyperbolic-elliptical motion under certain conditions, and with problems such as have been studied by Merman. Birkhoff's work has appeared in a Soviet translation, and Merman states that some of the formulations and proofs by Birkhoff are not sufficiently precise and clear. For two of the above-mentioned seven theorems of Birkhoff (Nrs 6 and 7), Merman[404] (1955) presents a revised and refined formulation, together with the necessary proof. He also gives a corollary to Birkhoff's theorem Nr 2.

Merman is an authority on these problems and undoubtedly has pointed out some weak points in the related work by Birkhoff. He does not mention the word "error", but apparently, Birkhoff's reasoning was not rigorously correct in connection with these two theorems.

Proskurin was in charge of some numerical studies made at the Institute of Theoretical Astronomy after Shmidt's first demonstration of the possibility of capture. He[575] (1953) gave the results of the numerical integration, which essentially duplicates the one by Shmidt. Shmidt did not publish his results in a detailed, step-by-step integration. Also, it was found that Pariyskiy, who did some of the numerical work for Shmidt, had made a slight error at one place in the integration. The present results reconfirm the essential result by Shmidt, that capture occurs for the given starting conditions of the three bodies involved. The fact of capture is not verified by means of the theoretical criteria of Khil'mi and Merman in this paper, but, evidently, this verification was taken for granted because of all the studies which had already been made on this problem. The main purpose of the paper was to publish the numerical data for each step of the numerical integrations, at least for the time interval during which the essential orbital changes take place.

This is a technical paper, because it presents the detailed results of numerical integrations in the form of tables and graphically, but, in the rather new and unexplored field of capture, this numerical approach is very important indeed. Therefore, this is an interesting contribution which provides workers in this field with the detailed features of capture in one special case.

In 1955, Proskurin(577) presented a numerical example of a case of orbital motion where a very small mass, m_3, approaches the elliptical two-body orbit of the finite masses, m_1 and m_2, in a hyperbolic orbit, and where the close approach changes this hyperbolic motion, relative to m_1 and m_2, into an elliptical one. Actually, it is shown only that the ellipticity of the changed orbit seems to be assured for about 10^4 years. This result is obtained on the basis of the assumption that m_1 and m_2 are the sun and Jupiter, and of the special starting conditions for the motion of the zero mass, m_3. These conditions are such that the numerical integrations could be made for an interval of 50 years.

This is an interesting numerical study, obviously inspired by the earlier investigations of Shmidt and Merman. As in other similar examples, the margin between capture and noncapture apparently is so narrow that, on the basis of the necessarily limited accuracy of such an integration, the author cannot conclude more than that capture seems to be assured for a certain interval of time. The question of permanent capture remains open.

At the suggestion of Proskurin, Khrapovitskaya[276] (1953) carried out a capture study that essentially parallels the numerical integrations carried out by Shmidt. Whereas Shmidt assumed the masses of the three bodies involved to be equal, Khrapovitskaya assumes that the one mass, which initially moves elliptically relative to the second one, has the mass zero. The numerical integration shows that the third, finite mass, approaching in a hyperbolic orbit, still captures the first mass, as in the case of Shmidt's famous first example. Merman's criterion is used to verify the fact that capture has occurred. The most important result of the study is the finding that, whereas capture is possible in connection with an exchange of energy between the masses involved, as was claimed by Khil'mi, merely a change in the magnitude of the relative velocities seems to matter also.

This is a study which leads to certain results of general importance, even though the paper is technical, insofar as it gives nothing more than the results of certain numerical integrations. The capture problem meets so many theoretical difficulties that numerical studies, such as the one by Shmidt and the present one, are of real significance. This is a contribution which deserves attention in connection with further exploration of this field.

Sitnikov(714) (1952) considered a special case in the capture problem where the starting positions and velocities of three equal masses depend on only three parameters. He shows that Khil'mi's theorem for capture applies if certain inequalities are fulfilled by the three basic parameters. Using a system of three masses such that these conditions for capture are met, and also an auxiliary mass system with slightly different starting values, the author demonstrates the possibility of capture for a finite variety of starting conditions.

Although the paper represented an original contribution at the time of its publication, the subject is too specialized compared with the actual problems, which depend on more than three parameters. Therefore, these special results cannot be expected to bear fruit in connection with further investigations of the capture problem.

Fesenkov(191) (1946), in his paper on the possibility of capture at close passages of attracting bodies, was not concerned with the rather exceptional conditions under which captures might occur, for example, in the case studied by Shmidt. Rather, he considers astronomical problems, such as dust particles moving close to the earth and more or less parallel to the orbital motion of the earth, a comet passing close to Jupiter, and similar situations. He assumes that the concepts of the restricted problem of three bodies are applicable and obtains a certain relation between two different sets of orbital elements, a, e, and i, of the small body: the one set of elements is referred to the mass center of the two finite masses (or approximately to the sun); the second one refers to the finite planetary mass. Making a reasonable, but not very clearly defined, assumption about the value of the Jacobi constant of the small body, the author demonstrates that, for example, the half-major axis of the jovicentric elements (at the time when Jupiter is passed by the small mass) is a great quantity compared with the dimension of the system and, therefore, that capture is impossible.

A certain lack of rigor is connected with the rather loose assumptions concerning the starting orbit of the small body. The author has nevertheless shown that, under fairly reasonable assumptions for the Jacobi constant, capture appears to be impossible for the kind of problems considered in this study. The results are of interest for cosmogonical problems, because they indicate that, without the introduction of mass variations or other additional forces (friction in a nebulous medium, etc.), the occurrence of capture (or of escape) cannot be expected.

Kochina(280) (1954) presented a very interesting example, obtained by numerical integration, of a case where a small body of mass $m_0 = 0$ at first moves elliptically with respect to a finite mass, m_2, but is separated from m_2 by the effect of the close passage of another finite mass, m_1, which has a parabolic motion with respect to m_2. After being released from the gravitational hold of m_2, the small mass, m_0, moves approximately parallel to the parabolic orbit of m_1, if both motions are referred to the mass m_2. She found that the osculating elements of m_o with respect to m_1 are elliptical. The major axis of this osculating ellipse evidently increases with $t \rightarrow + \infty$, so that m_o never actually crosses the orbit of m_1 (relative to m_2), but remains finite for all finite values of the time, t.

This example is of special interest with regard to the earlier investigations by Chazy, according to which an orbital change of the kind established here should have been impossible. No actual contradiction exists any more, though, because it was shown that Chazy's conclusions were partly erroneous (cf. Reference 209). The present investigation is a very interesting and important numerical verification of the analytical progress which has been made by Gazaryan and Merman.

3. Calculation Procedures

Yanzhul did some pioneer work on the application of punched-card equipment to scientific calculations, as did Comrie and Eckert in the West. In a descriptive paper, Yanzhul(834) discussed in some detail the punched-card computing machines available in the U.S.S.R. in 1939. In addition to the pictures and descriptions of the various machines, the principal part of the paper deals with the application of these machines to astronomical problems. The author considered applications that already had been made, as well as potential ones. Credit is given to the fact that the first machines were imported from the U. S., but much emphasis is put on the development of similar machines in the

U.S.S.R. The author claims that a new "tabulator", then under construction in the U.S.S.R., would be better and more versatile than the best U. S. tabulators. The new tabulator was to include a punching unit, so that the results could be punched on cards at the same time they were printed. The author stresses the use which Soviet astronomers would not fail to make of the improved machines developed in the U.S.S.R.

Much propaganda is contained in this technical report, of course, but, at the same time, it contains an account of the computing machines that were actually available and used in the U.S.S.R. in 1939. The report praises Soviet work in the field of computing machinery mostly in terms of plans and potential accomplishments, and not so much with regard to accomplished facts. Nevertheless, it becomes clear that the importance of these technical developments was fully realized in Soviet scientific circles at this relatively early time.

In celestial mechanics, the multiplication of two trigonometric series is a frequently occurring problem. Kulikov(325), in 1949, described the proper use of the tabulator, multiplier, and reproducer, in order to make the most efficient use of punched-card equipment for the solution of this problem. The product series includes the common factor 2, which is a certain disadvantage, because, after n consecutive multiplications, the resulting series has the factor 2^n.

This paper indicates a strong interest on the part of Soviet astronomers in utilizing automatic computing machinery for the purposes and problems of astronomy. This contribution would certainly have helped to advance the application of modern computing machines to the field of celestial mechanics in the U.S.S.R. It should be noted that, here, the application of machines to theoretical developments is pursued, in contrast to the more frequent use of such equipment for numerical integrations.

A detailed description of the computation by Numerov's method of the special perturbations of minor planets, as performed on punched-card machines at the Institute of Theoretical Astronomy at Leningrad, was published by Samoylova-Yakhontova in 1952[671]. The integrations by means of machines, which are evidently very similar to the IBM Type 601 Multiplier, are discussed at length; tables are given containing Jupiter's rectangular coordinates and various quantities which are needed in the application of this method of perturbations. Many of these quantities are available in various earlier publications or books; such material is not normally reprinted in new publications in the U. S. merely as a matter of convenience.

Evidently, the so-called reduction in Numerov's method is neglected in these machine computations, and the interval of the integrations is chosen in such a way as to permit this neglect for a certain degree of accuracy. A special table gives the proper interval for a given planet. For a considerable number of minor planets, the residuals (observation minus calculation) are tabulated, thus indicating the quality of the perturbations which have been included in the representation of these observations. Obviously, the results are satisfactory for a method of approximate perturbations, but a more rigorous method would be needed for an accurate representation of the actual motion of a minor planet.

This publication is a technical description of an application of a well-known method. It does not represent anything new or significant.

The existence of equipment similar to the IBM Type 601 Multiplier at the Institute of Theoretical Astronomy was further verified by a paper by Bokhan[83] (1952). The author gives the equations and the computational scheme which are being used at the Institute of Theoretical Astronomy in applying Numerov's method of special perturbations to the performance of integrations on punched-card machines. The procedure differs from the one described by P. Herget not only by the use of Numerov's special coordinates, but also by the determination of the "second sums" from the integration scheme in preference to the direct summation of the $\Delta''x$, $\Delta''y$, and $\Delta''z$.

The paper is of a technical nature; it reports the adoption of well-known methods to practical use in connection with punched-card machines.

The most detailed and most recent account of equipment at the Institute of Theoretical Astronomy was published by Kulikov[330] in 1953. In the introduction, he stresses the great significance of mechanization for scientific computing, and gives a short review of the development of mechanical computing equipment, especially in the U.S.S.R. It is emphasized that the first large-scale applications of such equipments were made in astronomy. The Institute of Theoretical Astronomy in Leningrad began to experiment with punched-card equipment in 1937 in connection with the preparation of various ephemerides in the Soviet Astronomical Almanac for the year 1941. The results of this work were so satisfactory that the idea of a computational center within the Institute of Theoretical Astronomy was advanced.

Of interest is the statement that the war interrupted the development of such mechanical computing procedures in the U.S.S.R. In the U. S., and perhaps in other Western countries also, such developments were accelerated during the war. In 1947, the Institute of Theoretical Astronomy took up mechanization again, doing computations for the Soviet Astronomical Almanac, as well as for minor planet ephemerides. The author says that this work progressed in close cooperation with a factory in Leningrad.

In 1948, the Institute of Theoretical Astronomy definitely adopted punched-card computing for the preparation of the annual coordinates of the sun, of the major planets, of the moon, and of minor planets, and also for the computation of star ephemerides. It is mentioned also that individual large-scale investigations on the figure of the earth, analytical theories of Pluto and Ceres, and numerical integrations of the disturbed motion of minor planets have been performed by means of punched-card equipment. A special computing laboratory of the Institute of Theoretical Astronomy is in charge of this type of work.

In Part 1, which is the most extensive, the author gives a very detailed description of the punched-card equipment at the Institute of Theoretical Astronomy. The principal features of the equipment are almost identical to those of corresponding machines in the U. S., as far as tabulator, reproducer, sorter, and keypunch are concerned. The setup is completed by a multiplier, which clearly is of an antiquated type, as far as it may be compared with standard calculators in the U. S. Obviously, in 1953, the Institute of Theoretical Astronomy was using only multipliers with left-hand and right-hand counters of the type used in the U. S. before the IBM 602-A Calculating Punch became standard equipment. Except for the multipliers, the setup at the Institute of Theoretical Astronomy in 1953 was about equal to comparable installations in minor computing centers of the West. No electronic computing equipment is mentioned in this paper.

The detailed description is illustrated by the following pictures and schematic figures:

(1) Keypunch

(2) Tabulator Type D 11 (Several types of tabulators, with different capacities, etc., apparently were in existence in the U.S.S.R. Only Types T 5 and D 11 are discussed in the paper.)

(3) Punched cards

(4) Operation of the read brushes

(5) Operation of the counters (of the tabulator)

(6) Structure of the counter units

(7) Structure of the "impulsators"

(8) Structure of the selectors (of the same type as on U. S. tabulators)

(9) Scheme of selector-operation

(10) Scheme of the printing mechanism

(11) Plugboards of the tabulator and multiplier

(12) Sorter

(13) Scheme of sorting operation

(14) Reproducer

(15) Scheme of reproducer operation

(16) Multiplier

(17) Scheme of counter arrangement in the multiplier.

The selectors are arranged in groups in the tabulator and are used extensively to make the machine flexible for many operations. For subtraction, complements of 9 instead of 10 are used, just as on U. S. machines of this type. Eight 11-position counters and seven printing units are provided. The average speed of the tabulator is 120 operations per minute. Numbers are sometimes multiplied on the tabulator by means of Yanzhul's method. The sorter, which is very similar to U. S. types, is able to sort about 450 cards per minute. The reproducer also corresponds to U. S. equipment in all its functions and features. At the Institute of Theoretical Astronomy, this machine is used extensively for the reproduction of the values of (trigonometric) functions from table cards, for further interpolation on the multiplier. The reproducer can be connected with the tabulator. The speed of the reproducer is about 100 cards per minute.

The multiplier consists essentially of the same basic elements as the tabulator, and its speed is the lowest of all the machines under discussion. Depending on the number of significant figures, about 12 to 15 multiplications per minute are possible. The multiplier normally has five counters, two of which are used for the two factors of a product and two for developing results; one auxiliary counter serves in the transfer from one counter to another one. Tables of reciprocal values are used for division.

Of all the equipment mentioned, this multiplier clearly is the one machine which would have been an "antique" in 1953 in the U. S.

The second part of this paper describes the solution of certain selected problems by means of the equipment described in Part 1. The computation of the various terms of astronomical nutation serves as an example. In order to deal more conveniently with the different quadrants for the angular arguments of trigonometric functions, the circle is divided into 400 parts, instead of 360 degrees. Since the combination and summation of many products are rather slow, considering the fact that the multiplier can deal only with one product at a time, an ingenious but rather elaborate reduction of the multiplication process to a system of related summations was devised by Yanzhul. This increases the number of operations considerably, and it seems doubtful that much time can be saved, but the method permits a more automatic treatment of extended sums of products, such as in the formation of normal equations from the equations of condition. The actual formation of such normal equations by means of the tabulator serves as another example. Much time is also needed for the preparation of the given material for Yanzhul's method, and the size and order of the different quantities require careful arrangements. Obviously, such a method is of value only in connection with rather primitive multiplication techniques, and it is a monstrosity since the advent of such flexible multipliers as the IBM Type 602-A.

This paper is of interest because it very clearly reveals the state of automatic computing at a major Soviet scientific institution, the Institute of Theoretical Astronomy at Leningrad. Obviously, in 1953, the technical development of these machines had not yet reached the same level as in the West, or, at least, any progress in the U.S.S.R. apparently had not yet led to the production of better standard equipment. The other point of interest, however, is the extended and intelligent use of the available equipment, which also led to the development of special methods, such as Yanzhul's, in order to make the most of the automatic equipment at hand. The fact that no electronic equipment is described or mentioned does not mean, of course, that nothing is being done in this direction in the U.S.S.R. Obviously, this report limits itself deliberately to the routine application of standard equipment over a series of years, up to 1953, and to completed work, rather than to new ventures of any kind.

Zagrebin[(841)] (1953) gave a detailed description of the manner in which the lunar ephemeris has been computed on punched cards at the Institute of Theoretical Astronomy on the basis of Brown's tables. The various terms were computed individually and then summed up. It was found convenient to arrange the computations for intervals of three years, because this corresponds to about 2,000 half days, or to the

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approximately 2,000 cards contained in one drawer. The computations were tested by means of difference checks involving differences up to the sixth order. The half-day results were interpolated for an hourly ephemeris with the help of Bessel's interpolation formula. In the trigonometric functions, a circle divided into 400 parts, instead of into 360 degrees, was used for greater convenience. Although the computations reported here cover only a limited number of years, it is planned to continue them into the future. According to the author's remarks, the corrections provided by Woolard to Brown's tables, as well as the various recommendations of Clemence, Porter, and Sadler concerning the lunar ephemeris, are going to be considered in this future work. The new concept of ephemeris time is recognized, too.

This is another case of duplication of effort between the East and the West. Although this Soviet paper came out somewhat earlier than the recent ephemeris volume published jointly by the Nautical Almanac Offices of the U. S. and of the U. K. (Improved Lunar Ephemeris 1952-1959, Washington, 1954), the Institute of Theoretical Astronomy must have been aware that this volume was being prepared in the West. A comparison of the efforts is favorable to the West, however, because the Western ephemeris was computed on electronic machines, whereas the Institute of Theoretical Astronomy used punched-card machines with mechanical counters.

4. Capture in the n-Body Problem

Khil'mi[266] (1950) criticized the classical form and derivation of the so-called virial theorem, which is important for many problems in stellar dynamics. If, with respect to the center of gravity of a system of n masses, the total moment of inertia is introduced by $I^2 = \sum_{i=1}^{n} m_i r_i^2$, then the assumption of statistical equilibrium is usually expressed in the analytical form $\frac{d^2 I^2}{dt^2} = 0$; this leads to the important result that $U = 2W$, where U is the force function and W the kinetic energy.

Khil'mi stresses the point that $\frac{d^2 I^2}{dt^2} = 0$ is only a statistical relation, and that, actually, oscillations will occur at least around any average value, A, of I^2. Therefore, the classical conclusion $U = 2W$ is not rigorous, but only approximate.

The author himself proves two theorems in this investigation. The first says that if, for $t \to +\infty$, all the individual distances, r_i, from the common center of gravity are limited by a given value $R > 0$, and if no collisions occur, then, for any given value $\varepsilon > 0$, and for any special time $T > 0$, another moment, $t' > T$, can be found for which $|U(t') + 2H| < \varepsilon$, where H is the constant from the energy integral $W = U + H$. The second theorem says that if, for given constants A and η, and for any given time, $> T$, the inequality $|U - A| < \eta$ is fulfilled, then, at any time $t > T$, $|2W - U| < 2\eta$. This second theorem has to take the place of the classical virial theorem in any investigations concerned with the equilibrium or stability of stellar systems.

This is a very important theoretical paper, because it clarifies the actual limits of a theorem which, if employed too generously, can lead to erroneous results in problems of stellar dynamics.

Using the relative coordinates of Jacobi, and making certain assumptions with respect to the minimum distances and minimum relative velocities in a system of n bodies, Khil'mi(267) (1950) proves that not all the relative distances will remain limited with $t \longrightarrow +\infty$, provided that certain inequalities are satisfied at the time t_o.

This is an important investigation which uses new and unconventional methods. In a 1951 paper, however, Khil'mi(272) admitted that the proof given in the previous paper(267) is not rigorously correct and gave proof of his theorem for the case of three bodies. For the general case of n bodies, however, no actual proof has been given yet. According to Khil'mi's remarks in the later paper, Merman had notified him by letter concerning the insufficient or erroneous proof of the theorem. Although the actual error in the conclusions is not pointed out, it is probably contained in the last equations in the transition from t to $\tau > t$.

Also in 1950, Khil'mi(268) presented a proof for the theorem, for the case of a certain system of three inequalities, satisfied at the time t_o by the coordinates and velocities of n bodies, which establishes the division of the whole system into two groups. He claims the following significant characteristics for the groups: In one group, all the relative distances increase beyond a finite limit with $t \longrightarrow +\infty$; in the second group, not all the distances are able to increase without limits. If the latter group consists of only two masses, then they form a stable subsystem within the total system of n masses.

This paper is actually an extension of Reference 267; therefore, the proof for the present theorem is not valid, considering the invalidity of the proof for the theorem in Reference 267, which was announced by the author in a later paper(272).

In light of his earlier investigations on the possibilities of stability or dissipation in the general problem of n bodies, Khil'mi(269) (1950) made a critical and partly philosophical examination of the questions of statistical equilibrium and stability, because, in many studies of stellar dynamics, the assumption of equilibrium has been introduced without proper justification.

The general problems of stellar dynamics have not been solved yet either by celestial mechanics or by statistical methods. The author says that actual stellar systems are not in equilibrium, or at least will not reach equilibrium so fast as sometimes is assumed, and that the gravitational field cannot be neglected in such investigations. An extensive analysis of the n-body problem is deemed necessary, and this paper is intended to be a first step in this direction.

The author claims that Chazy's work hindered progress for some time, because his erroneous conclusions concerning the alleged impossibility of capture in the three-body problem were taken for granted because of his authority in the field. The problem of capture, which in recent years has been solved as far as the actual possibility of such orbital changes is concerned, is intimately related to the problems of stability or dissipation in systems of three and more bodies. The author states two fundamental tasks which have to be undertaken:

(1) An investigation of the necessary and sufficient conditions for stability, for semistability, and for dissipation in a system of n point masses; and (2) An investigation of the possibilities for changes from dissipative behavior for $t \longrightarrow -\infty$ to semidissipative behavior for $t \longrightarrow +\infty$, or vice versa. A semidissipative system is defined as one which can be divided into two subsystems, so that one subsystem is dissipative and the other one nondissipative.

As to the first of these two problems, it is treated by Khil'mi in another paper(268), wherein he established three conditions, in the form of certain inequalities which, if fulfilled, establish the semi-dissipative character of the system. The second problem has been solved, in the form of the capture problem, only for three bodies.

Khil'mi(269) then proceeds to topological considerations, using the so-called phase space, and defines a "stationary" system as one in which the distribution of coordinates and velocities is invariable. Then, the potential, U, is independent of the time. It is also explained that, in general, one cannot expect that stellar systems will approach "stationary" status very fast. The "stationary" status will also depend on the starting conditions of the system, and, therefore, it is of cosmogonical significance.

In his paper on the evolution of a system of gravitational bodies by nonelastic collisions, Khil'mi(271) (1951) considered a system of small particles, such as may form meteoric dust or the original solar nebula. The particles are assumed to be inflexible and to suffer nonelastic collisions. In such collisions, the kinetic energy, T, would necessarily decrease by discrete amounts, because of the related losses to heat energy. Between any collisions, however, the system will behave according to the laws of dynamics. The author says that the problem of the evolution of such a system has been dealt with by statistical methods in the U.S.S.R., but that it should be investigated by the methods of celestial mechanics, too, because the gravitational forces will be very important.

When collisions occur, the two particles involved may separate again, but they may also either break up into several additional particles, or stick together to form one larger particle. The total number, n, of particles, therefore, will not be constant. The total force function is divided by the author into two parts, U and U^*. U represents the part corresponding to the gravitational potential of n point masses; U^* introduces the energy difference which has to be added because of the finite sizes of the individual particles. In the event of a collision, the sum $U + U^*$ will remain constant at the moment of the collision, but U^* must increase if the two particles unite to form one new particle, and it must decrease if the particles break up into more than the original two. U must accordingly change, in order to keep $U + U^* =$ constant. Although these considerations apply to the moment of the collision, the law $T - (U + U^*) = H^*$, with H^* representing the constant of the energy integral, applies to the time intervals between collisions. Since T decreases in consequence of the collisions, H^* must accordingly decrease after each collision. The decrease in H^*, produced by the collisions, might be reflected in orbital changes

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in either one of the two following ways, or most probably in both ways at the same time: (1) The motions may be affected by a general decrease of T and, thus, of the velocities in the system; or (2) the sum (U + U*) may increase by orbital changes. As to the second possibility, an increase in U would be related to a systematic decrease of the relative distance; an increase in U* has to be interpreted as the consequence of the formation of larger and larger particles or bodies from smaller ones. This picture is equivalent to a gradual contraction and condensation of a given original system or nebula. The author proves that, at the same time, the rotational momentum of the increasing condensations would increase in such a way as to produce direct rotation. The excess orbital momentum would be transformed into the direct rotational momentum of the resulting "planets".

This is a rather interesting and important cosmogonical paper, the results of which have to be considered in theories of the origin and development of the planetary system. Undoubtedly, Khil'mi has made an original contribution to this field.

In his book(270) (1951), Khil'mi summarized most of his work on n bodies in celestial mechanics and cosmogony. He says that the cosmogonical processes cannot be reduced to the problems described in analytical dynamics, because the nonelastic collisions of the particles result in the transformation of part of the mechanical energy into heat and a redistribution of energy. Also, the number of material particles does not remain constant. The author points out that, from the standpoint of analytical dynamics, the described process is equivalent to

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the discrete change of the initial conditions. But, the cosmogonical processes, if the distances between the particles are large enough, are also strongly connected with capture or dissipation under the influence of gravitational forces.

In the introductory chapters in this book, Khil'mi gives the well-known results concerning n bodies, and a set of definitions and theorems about the properties of point sets in phase space. He mentions the incorrect conclusions of Poincaré, which resulted from the inappropriate use of a theorem of Liouville.

The kernel of the work is, undoubtedly, the author's own results concerning the capture or dissipation of systems. Khil'mi's sufficient conditions for capture, semidissipation, dissipation, or stability have the form of inequalities for the initial position and velocity vectors. Some of these theorems were published previously in short articles.

The author's cosmogonical conclusion is that the decrease of mechanical energy, as a result of nonelastic collisions, leads to the concentration of matter. If the kinetic energy is decreasing too, then bodies, possessing direct rotation about the axis, will come into existence. These results are on a very high level, and this small book gives a good account of Khil'mi's work and ideas. The last chapter of this work is "philosophical", however. It is pure propaganda, and nobody really can tell the extent to which these "philosophical ideas" represent the author's own opinions, and to which they were dictated by the instinct of self-preservation.

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5. Motion of Bodies of Variable Mass

The differential equations of the motion of bodies with variable mass were first published by the Russian I. V. Meshcherskiy, in the 1890's. He was the originator of, and the chief contributor to, the mechanics of moving bodies of variable mass. He was active in this area until his death in the 1930's. Since that time, his researches have been continued by A. A. Kosmodem'yanskiy from the powered-systems point of view, and by G. N. Duboshin from the celestial-mechanics viewpoint. Since Meshcherskiy was the originator of such an important subject, his works are briefly reviewed here.

Meshcherskiy's collected works(406) were republished in 1952 under the editorship of Kosmodem'yanskiy, who states in the preface that the "application and extension of the scientific researches of I. V. Meshcherskiy will be a rewarding task for Soviet scholars who have devoted their creative ability to the new branch of engineering, i.e., that of rocket engineering". A collection of Meshcherskiy's works represents what may rightly be called the analytical bases of the theory of motion of rockets and of related problems where a variable mass is involved.

In the introduction, the editor mentions various problems from the history of mechanics in which a variable mass is involved, or in which the concept of a variable mass may profitably be used to put the problem into a mathematical form. He then describes Meshcherskiy's life and work. The fact is stated that several results, originally established by Meshcherskiy, were found again by Levi-Civita roughly 30 years later.

The earliest of Meshcherskiy's papers in the collection is from the year 1893 and deals with a variable mass in the case of the two-body problem, assuming that the variable mass, μ , depends on the time, t, in the form $\mu = \frac{1}{a + \alpha t}$. It is proved that this problem can be solved by quadratures after some proper transformations of the coordinates and of the time, and that, by the means of similar transformations, even the more general case where $\mu_i = f_i (a + \alpha t)^{-s-3}$ can be reduced to the ordinary n-body problem (k = 1, . . ., n).

The most extensive paper by Meshcherskiy is his dissertation (1897). It deals with the dynamics of a point with a variable mass in a quite general and comprehensive way. Interestingly enough, the author had difficulties, in 1897, in having his investigation accepted as his dissertation, because it was argued that the subject was too hypothetical and too remote from actual dynamic problems.

The author derives his fundamental differential equations, which are of the form $m \frac{d^2x}{dt^2} = X + \frac{dm}{dt} (\alpha - \dot{x})$; similar expressions are valid for the y and z coordinates. Here, x, y, and z are rectangular coordinates, referred to an absolute system, of the variable mass m; α , β , and γ are the absolute-velocity components of the small particles which are being lost from, or are being added to, the mass, m, so that $\alpha - \dot{x}$ is the relative velocity (in the x coordinate) of m and of the mass loss or mass gain, dm; X stands for the x component of any acting force which influences the motion of m; and t is the time.

The problem is generalized in Chapter II for the case of additional binding conditions in the problem. Here, also, the case $\alpha = \beta = \gamma = 0$ receives special attention, and the author actually was the first to arrive at the then-valid equations $\frac{d(m\dot{x})}{dt} = X$ (with similar expressions in y and z), which are generally known in the West under the name of Levi-Civita, who derived them independently much later than did Meshcherskiy. For the case of a central force, the author also derives generalized integrals of areas, as well as the corresponding energy integral. Several special problems, which permit a reduction to forms of known, classical problems, are treated in much detail.

Chapter III is especially concerned with motion on a straight line, and Chapter IV with the small oscillations of a pendulum of variable mass. The resulting differential equations are of the Riccati type and admit no direct solution in the general case. In the special case, where the mass is a linear function of the time, the equation is reduced to Bessel's type and can be solved by means of the related functions of the first and second kind.

The inverse problem, where the motion is known and the mass variation, which would produce the observed motion, is sought, is given in Chapter V. If the law of the mass variation is also known, then the task may be the determination of the law of resistance, if the motion takes place in a resistant medium.

Chapter VI deals with the motion of a variable mass in the gravitational field of the earth, and Chapter VII with motion in the field of a central force. In all these problems, the author starts on the basis of his fundamental equations and considers many possible subcases.

At the end of this important book, additional essays are dedicated to some more or less general problems of analytical dynamics in which a variable mass is involved. The important core of all these essays, however, is contained in the one which has been reviewed here in some detail. Some papers by Meshcherskiy, dealing with a variable mass in the two-body problem, were published in Astronomische Nachrichten half a century ago.

Without any doubt, Meshcherskiy's contributions were the pioneer work in the variable-mass problem. He treated the basic characteristics of this problem very thoroughly and with full competence, arriving at the fundamental theorems, as well as at many results and conclusions for special applications in celestial mechanics and also in connection with various problems in other fields of "regular" mechanics. The essays describe some of the most important fundamental advances in this problem in recent times.

Kosmodem'yanskiy gave the analytical foundations of the dynamics of a rigid body of variable mass in 1951[304]. All the important equations are given in the vectorial form. The basic concept is that of a rigid body whose mass varies as a result of the addition or removal of particles at the surface of the body, without interference with the rigid nature of the rest of the body. The most important difference, compared with the dynamics of a rigid body of constant mass, lies in the facts that the location of the mass center inside the body will change as the total mass varies, and that, therefore, the translatory motion of the body cannot be described by the corresponding orbital motion of its variable center of mass.

The most important basic ideas and theorems had already been given by Meshcherskiy, and, in this respect, the merits of the present paper lie in the more comprehensive and systematic treatment, and in the addition of some valuable new results or theorems. One example of the extension of the theory by Kosmodem'yanskiy is his theorem concerning the total momentum of the body. Another new theorem is derived, in the form of the proper differential equation of the second order, for the motion of the center of mass. The treatment of an angular momentum of the body of variable mass, as referred to coordinate systems which either are at rest or are moving with the body, is not essentially new, but the author deserves credit for his concise treatment of the concepts which are involved, and for the systematic way in which he proceeds.

Paragraph 5 contains the theorem concerning the angular momentum, and also deals with some special cases. One of these is the case where the principal axes of inertia remained fixed relative to the mass elements of the rigid body; in this case, the well-known equations by Euler are valid in their classical form, except that they depend on the variable mass and the reactive force, which is related to the variability of the mass by Meshcherskiy's fundamental equation.

In Paragraph 6, the theorem for the angular momentum is presented as referred to a moving coordinate system, and Paragraph 7 contains the theorem for the kinetic energy in its differential form.

In Paragraph 8, the motion of the body or rigid system of variable mass is referred to a system of generalized coordinates, and special cases are considered in detail. Again, the first steps were done by Meshcherskiy, but the author goes into the subject somewhat deeper and more systematically, aided by the greater convenience of the modern vectorial form of analysis.

The ninth and last paragraph deals with the canonical form of the general equations, building again on the foundations laid by Meshcherskiy. A strictly canonical form is obtained only if the external forces have a potential, and if the absolute velocity of the departing or added particles (which produce the variability of the total mass of the rigid body) is zero.

In addition to the comments already made in connection with the review of the various paragraphs, it seems correct to say that this is a fundamental contribution of considerable value. Much credit should go to the author, not only for his own and rather important additions to the theory, but also for the construction of a systematic and very elegant representation of the complete theory from the old and the new material. One gains the impression that the author presently is a most outstanding scientist in the Soviet Union in this particular field.

Kosmodem'yanskiy published a collection of lectures which he presented at a military school[305] (1951). After an introduction and a historical and philosophical review, mentioning the names of Meshcherskiy, Oberth, Zander, Goddard, and many others, eight lectures are presented:

(1) The first deals with the law of the conservation of momentum. Here, Kosmodem'yanskiy mainly gives the fundamental theorems of Meshcherskiy in a more condensed and clearer form than in Meshcherskiy's original publications. He applies the theory to such special problems as can be solved by quadratures, and the main emphasis is on the determination of the extremum or optimum solution, so as to reach the technical goal with a minimum of energy or fuel (for a rocket). Some special cases of assumed laws of mass variation are considered in detail.

(2) This lecture deals with the vertical ascent of a rocket in a homogeneous gravitational field. Investigated are the velocity, v(t), and the altitude, h(t), as functions of the variation of mass, and the problem of reaching a maximum altitude with a minimum of fuel is solved. A relatively simple solution is possible, even when the resistant medium of the air is taken into account.

(3) Here, the author deals with the horizontal motion of a variable mass, first on a supporting plane and then in the air. After introducing a homogeneous field of gravity, the author finally considers the case of a variable field at higher altitudes above the surface of the earth. In this case, the effect of a resistant medium is neglected.

(4) In this lecture, Meshcherskiy's fundamental equations are presented in the vectorial form. Essentially, this is a translation of Meshcherskiy's original investigation into the more convenient vectorial form.

(5) The author is concerned, in this lecture, with the motion of of an airplane driven by an engine which takes in air at the front and then ejects it from the rear. In this case, the analytical treatment operates not only with a mass loss and the corresponding $\frac{dM_1}{dt}$, but also with a mass increase, represented by $\frac{dM_2}{dt}$.

(6) This lecture is devoted to the so-called inverse problem of dynamics, where the force field and the trajectory are known, and where the necessary mass variation of the projectile has to be found. The solution of this problem is easily reached by separation of the variables, and the result had been found by Meshcherskiy. Kosmodem'yanskiy further investigates certain special problems in detail, for example, the case of vertical motion with a constant velocity. Following Meshcherskiy, he also introduces the so-called natural coordinates which are associated with the moving body of variable mass. The essential difference from Meshcherskiy's treatment lies in the use of vectors. The two-body problem with variable mass is also considered in this lecture.

(7) Here, the general laws of the dynamics of a point of variable mass are derived, and motion in the field of a central force is studied. Again, the author essentially only transcribes Meshcherskiy's findings into the vectorial language. He also gives a special case, treated by McMillan in 1925. Finally, he investigates the energy efficiency of rockets.

(8) This last lecture is identical to a paper(304) by this author that was reviewed above.

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Lectures 1 through 7 have the character of a textbook written especially for students of the military school where these lectures were given. Nothing essentially new is contributed by the author in these first seven lectures. Lecture 8[304] contains the author's main contributions to the field.

a. Two-Body Problem

The celestial-mechanics aspect of the variable-mass problem has been studied by a very capable Soviet astronomer, G. N. Duboshin. Between 1925 and 1932, he published a series of papers dealing with the motion of a particle in the field of another whose mass is variable. In the first of these works[134] (1925), the general mathematical approach to the problem is presented. Special applications are treated in the later articles. The characteristic feature of Duboshin's treatment is the transformation of the coordinates and of the time into other variables, so as to restore the differential equations of motion to the "undisturbed" form of the two-body problem, after they are referred to the new polar coordinates and to the transformed time, τ. The burden of the problem is thus shifted from the differential equations of motion to the relation which connects the new variables with the assumed law for the variation of the central mass, M. If $M = M_o f(t)$, and for $M_o = 1$, the problem is finally reduced to the solution of an integral-differential equation for the determination of a function, $f(\omega)$, which, in turn, leads to the determination of the radius vector, $r(\omega)$, where the angular variable, ω, instead of the transformed time τ, serves as the independent variable.

In contrast to investigations by other authors, Duboshin makes no use of the variation-of-parameters method. He prefers, instead, to introduce the effects of the variability of into the coordinates of the particle by the above method.

In a continuation of the 1925 article, Duboshin(135) (1927) was concerned mainly with the solution of the integral-differential equation which is characteristic for his treatment of the problem. Although his conclusions were not entirely new, the rather general method of their derivation is interesting and can be considered as a contribution to progress in this field of celestial mechanics.

Duboshin uses a method which is analogous to the one of Picard, employing successive approximations for the quantities to be found. He then assumes that the function Φ, if $f'(t) = \Phi[F(\omega)]$, in the given equation can be developed for ascending powers of a small parameter, μ. In this case, all the functions involved can be represented as analytical functions of μ. The resulting developments are studied for certain points of special interest: the behavior of the line of the apsides, and the question of the existence of an asymptote for the motion of the particle when $t \longrightarrow +\infty$. The author finds that, in the case of an unlimited decrease of the central mass, M, the orbit of the particle will expand in the form of a spiral until a certain time, when asymptotic motion towards $r = \infty$ will set in (terminating, then, the completion of an actual revolution).

Duboshin further demonstrated the usefulness of his general method in 1928(136). He made the special assumption that $M = f(t) = e^{-kt}$ for the dependence of the central mass, M, on the time, t. He then proceeds with the corresponding solution of the integral-differential equation of the problem, determining the successive terms by a series of simple differentiations. He finds that the developments will converge within a certain interval of time. For the investigation of the form of the orbits, the author limits himself to the first two terms in the somewhat complicated developments. The first term (or rather the first part) constitutes the basic elliptical motion; the second part represents the perturbations which are caused by the change of the mass, M. The case of hyperbolic motion is also considered, and the author finds here that the actual, disturbed motion tends to approach the existing asymptote faster than would the undisturbed motion. From a study of the so-called logarithmic type, the author finds that the line of apsides will rotate, in general, about the central mass, M, and that the occurrence of asymptotic motion (terminating the completion of a revolution of the particle) will depend on the precise law for the decrease of the mass, M, and on the starting conditions. In the case of a circular original orbit, the circumstances of the orbital development are quite similar to the ones in the originally elliptical case.

Other methods of treatment can be used in individual cases, and have been used by numerous well-known authors in connection with special problems of variable masses, but Duboshin's method seems to be valuable mainly because of its generality, and because of the resulting possibilities for the systematic classification of various cases.

Duboshin mentions that his integral-differential equation is rather complicated in most cases, and is inconvenient for many practical applications. He then presented(137) (1929) what he calls a "new method for the solution of the problem".

The method employed is the variation of constants, applied to the osculating elliptical elements. Apparently, the author was not aware of the fact that this method had been used by E. Stroemgren as early as 1903 for the study of the same problem in a continuation of earlier investigations by Gyldén and Lehmann-Filhés. Duboshin, therefore, was not the first to use the variation of the elliptical elements in connection with a variable central mass. Furthermore (in the opinion of the writers), he made a mistake in his own application of the method, or, rather, in the proper definition of the osculating elements which are involved. As did Stroemgren, he divides the gravitational action of the central mass into two parts, a constant and a variable. The constant part, depending on the initial mass value, $m_o = 1$, of the central body, would cause the particle to move in a fixed ellipse; the small mass variation, $\Delta m\ (t)$, causes the true or osculating orbit to deviate from the undisturbed or fixed ellipse by varying differences in the four orbital elements.

Duboshin apparently overlooked the fact that his method of computation leads to element perturbations which are still referred to the mass $m_o = 1$ at the center of the coordinate system. In other words, he introduced the disturbing effect of the mass decrease into the right-hand side of the differential equations, but forgot to reduce the left-hand side to the changed mass, $m_o + \Delta m(t)$. In consequence of this error, the author arrives at the closed "integral" $p = a(1 - e^2) = constant$, where p, a, and e are the osculating values of the parameter, of the mean distance, and of the eccentricity of the orbit, respectively. This integral, which is wrong, is then used throughout the rest of the paper as one of the most essential equations. The correct relation, which Lehmann-Filhés had already found, is $p(1 + \Delta m) = constant$ (for $m_o = 1$), and this cannot be used to reduce the number of unknowns, because $\Delta m(t)$ is involved.

The error committed by Duboshin is one which may occur rather easily, if one applies the principles of the variation-of-constants method more or less automatically. That the proper definition of osculating elements depends also on the use of the true or "osculating" mass value of the central mass is not so self-evident, perhaps, as the other features of the method.

Surprising is the circumstance that the author did not know about the earlier and correct results, and that his result, p = constant, was not puzzling to him. It has been found by others that one may have, approximately, e = constant in the case of two variable masses (or of one variable central mass), but that a will vary with Δm, thus making p variable, too, in agreement with the early result by Lehmann-Filhés.

At the same time that he was working on the problem of two bodies, one of which has a variable mass, Duboshin also considered the problem of two bodies, both with variable mass(138,142), which is not principally different from the case where one mass alone is variable.

Duboshin first extended(138) (1930) further some of his earlier results and definitions, which seem to be of special importance for the systematic treatment and classification of whole groups of orbits as they may be related to the variability of masses.

The author states that only one type of function, f(t), for the variability of the mass, M, so that $M = M_0 f(t)$, has yielded to a complete, closed integration of the trajectory. This is the case, studied by McMillan, where:

$$f(t) = \frac{1}{\sqrt{1 + 2\alpha t + \beta^2 t^2}}.$$

This case, incidentally, is verified by Jeans' mass-decrease law, $\frac{dM}{dt} = -\alpha M^3$, where the loss of mass is caused by the radiation of a star. In this special case, the above function f(t) applies for $\beta = 0$.

Duboshin's concern here is with the qualitative analysis of the completely general problem without special assumption regarding the form of f(t). Most important with respect to the trajectories is the question if and when asymptotic motion, approaching a certain direction toward $r = \infty$, will set in. The author finds that the condition $\lim_{t\to\infty} f'(t) = 0$, provided that all the functions which are involved are analytical, is necessary for the occurrence of such asymptotic motion, but not sufficient. On the other hand, if this condition is not satisfied, no asymptotic motion is possible. If the condition

$\lim_{t\to\infty} f'(t) = 0$ is satisfied, then the starting conditions can be chosen in such a way that the distance between the two masses increases indefinitely with t (Theorem Nr 1). The second theorem says that, for $\lim_{t\to\infty} f'(t) \neq 0$, the distance between the two masses remains limited for all possible starting conditions. After proving this theorem, the author proves two theorems by Armellini by reducing the proofs to the ones of his own theorems.

He continues with a study of the different types of motion. The first or hyperbolic type is at best represented by the undisturbed case of rigorous hyperbolic motion. The second or logarithmic type is verified in orbits where the one mass makes at least one complete revolution relative to the other mass and then goes towards $r = \infty$ asymptotically. The third or cyclical type is one where the orbit takes the form of a spiral, without "degenerating" into asymptotic motion. The fourth or spiral type, finally, is the type where the one mass is spiraling inward, approaching closer and closer to the other mass, without the occurrence of a collision.

The definitions "asymptotic" and "periodic", with respect to the behavior of the distance between the two masses, are introduced. In this sense, a trajectory is asymptotic if it approaches, by a spiraling motion, closer and closer to a certain fixed orbit. On the other hand, a trajectory is periodic, according to this definition, if the radius vector, r, oscillates between certain limits: $r_1 < r < r_2$. Evidently, periodic orbits in the established sense of this word are a special case of this more general definition.

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Finally, the following three theorems are proved: (1) If at all times $r_1 < r < r_2$, then the masses themselves vary only within certain limits, or $M_1 < M < M_2$; (2) if the trajectory is asymptotic with respect to a circle, then the total mass of the system converges toward a certain fixed value $\neq 0$; and (3) if the trajectory represents a strictly periodic motion, then the sum of the masses is a periodic function of the time.

Later, Duboshin[142] (1932) gave proof for the following theorem by relatively elementary means: If the mass of the two bodies is a monotonic function of the time and converges, for $t \rightarrow \infty$, towards a well-defined limiting value, then it will always be possible to find orbital trajectories (within a region in the plane of the two bodies) which will converge toward a limiting elliptical orbit, the elements of which are determined by the initial orbital conditions.

Savchenko[678], in 1935, treated the absolute and relative motion of two bodies of variable mass on the basis of Meshcherskiy's fundamental equations for the dynamics of variable masses. The orbital possibilities for the motion of the center of gravity (or of inertia), and for the relative motion of the two point masses, are studied in detail. The resulting variations of the polar coordinates and of the time of revolution are obtained as functions of the masses, $m_1(t)$, and of the related differential quotients $\frac{dm_1}{dt}$ and $\frac{dm_2}{dt}$. For an integration, the functions $m_1(t)$ and $m_2(t)$ have to be known, of course, and the author has studied the resulting trajectories for various simple laws of mass variation.

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This is a very clear presentation of the theory of the motion of two bodies. Although the results were known before, the work is still of value because of the systematic treatment, and because the author uses nothing except the basic principles of physics, namely, Newtonian mechanics, and elementary calculus. His direct approach to the problem and the mathematical details may have made the work longer than it would have needed to be if he had used vector notation, but it has made it more accessible to persons not trained in higher mathematics. It is the author's privilege to use the methods he chooses. The very title of the paper indicates that he wanted to keep his presentation on an elementary level.

Savchenko's(680) (1938) first considerations of the special case of the motion of two bodies, where the ratio of the two masses changes with time, are somewhat lengthy or complicated. This is because he used only elementary mathematics, such as calculus and ordinary differential equations. In his final results, Savchenko obtains the radius vectors, r_1 and r_2, and the angular arguments, θ_1 and θ_2, as functions of the masses, $m_1(t)$ and $m_2(t)$, thus obtaining r_1 and θ_1 as functions of t. These are the parametric equations of the orbit of the mass m_1. If the parameter t is eliminated from these two equations, then it becomes obvious that r_1 is a function of θ_1 and depends on the value of the angular argument. The same reasoning applies to r_2 and θ_2. The author calls attention to the fact that the orbits of the two bodies are not closed curves, but spirals.

Nothing outstanding resulted from Stepanov's[726] (1930) sound but not-deep work on the form of the trajectory of a particle in the case of a Newtonian attraction with variable mass. After referring to Duboshin's reports[134-136], Stepanov proceeds to demonstrate that, as long as one is interested only in the form of the trajectory, some results can be obtained by elementary methods. These results refer to the existence of upper and lower limits of the relative distance, *r*, between the two masses, as related to corresponding limits for the variation of the central mass, *M*, of the system. The author proves one theorem, which is of interest with respect to the manifold possibilities which exist for the trajectory if the problem is of an unrestricted generality. It is shown that the law of mass variation can always be chosen in such a way that the particle will move with respect to the central mass, *M*, on any given curve, provided that this curve is concave relative to *M*, and that a definite value exists for the curvature at each point of this curve. This demonstrates the fact that, in its completely general form, the problem of the form of the trajectory is too arbitrary to be of actual interest. The author concludes his paper with some considerations of the special case where the mass, *M*, suffers periodic variations, and of the other special case where *M* decreases monotonically. He discusses the possibilities for the related orbital changes.

By relatively elementary means, Eigenson[182] (1933) investigated the main features of the relation between the variable central mass, *M*, and the distance r(t) of a small particle in the field of *M* for various assumed types of motion. Not considering the cosmogonical application to a whole system of many variable masses, the author's detailed investigation of the actual two-body case is of definite interest, insofar as it studies the possible relations between M and r, depending on certain assumed mass laws, $M(t)$. Although, in the case of near-circular motion, Jeans' well-known relation Mr = constant holds between r and M, as long as the radial component of the orbital motion is small compared with the longitudinal component, this relation of Jeans will not be satisfied for all possible types of motion and for all possible forms of the mass law, $M(t)$. Eigenson is concerned mainly with the type of motion in which the radial component of the velocity is of main importance, and he investigates the compatibility of a generalized law of the form $M^N r$ = constant with relative motion of the radial type. He then applies his results to the hypothesis that the recessional motion of the spiral nebulae is the consequence of mass losses through radiation; he finds that the quantity N, in the law $M^N r$ = constant, would have to be of the order $N = 4 \times 10^4$, in order to arrive at such an explanation for the observed red shifts of the nebulae.

The essential quantity in the analytical study of the consequences of the variability of the masses, as presented by Subbotin[739] (1936), is the reciprocal of the distance, r, between the two masses. In its relation to the combined mass of the two individual masses, m_o and m_1, r^{-1} serves as a convenient means for the investigation of the possible behavior of r as a function of the time, t, for $t \longrightarrow \infty$, under various assumptions for the variability of the masses. This study completes and generalizes previous investigations by Armellini and Duboshin. For certain assumptions concerning behavior of the masses (or the existence of upper and lower limits for the sum of the masses), certain orbital conclusions are derived in a number of theorems.

This contribution to the problems of variable masses augments previous knowledge concerning the possibilities for the trajectories or orbits under various conditions. Subbotin deals with the problem in a highly competent way.

Although the two-body problem with variable masses can be solved only if the combined mass, M, of the two point masses depends on the time, t, in the form $M(t) = \frac{1}{\sqrt{a + bt + ct^2}}$, Batyrev[49] (1941) limited his study of the trajectories to the case where $M(t) = \frac{1}{a + bt}$. The quantities a, b, and c are (arbitrary) constants. The author applies the transformations of the coordinates and of the time that Meshcherskiy already had used, and thus introduces the so-called auxiliary point moving on a conic section while the actual trajectory is spiraling outwards (for $b > 0$). For the various types of conic sections described by the auxiliary point (ellipse, parabola, hyperbola, and straight line), the author derives the expression $r(\Phi)$, which describes the actual relative distance of the two masses as a function of the angular coordinate Φ. These expressions depend on the eccentricity of the auxiliary conic section, as well as on the integration constants which are involved. It is seen that, for the various conic sections described by the auxiliary point, the true relative motion is of the hyperbolic-spiral type. In the special case of straight-line motion, the same straight line determines the auxiliary as well as the true motion.

Batyrev also gives special expressions for the final values of the angular argument, Φ, where asymptotic motion, $r \longrightarrow \infty$ (for $t \longrightarrow +\infty$), takes place in the two-body case. Meshcherskiy had arrived at comparable results in a more implicit and less concise way. Most of the basic concepts and analytical operations in this paper go back to Meshcherskiy.

In a continuation of his work, Batyrev[50] (1949) refers to Duboshin's qualitative analysis of the general two-body problem with variable masses, and then limits his study to the case where the sum of the two masses is $M = \frac{M_0}{1 + \alpha t}$, for $\alpha > 0$. The conic sections described by the auxiliary point are characterized by their elements e and p (eccentricity and parameter), and the true motion, $r(\Phi)$, is studied in dependence on the given conic section for the auxiliary point. The paper goes beyond the earlier one only for the case where the auxiliary curve is a hyperbola; the quantities which determine the asymptotic motion in this case are derived. The corresponding results for an elliptical or parabolic auxiliary-point motion had been given in the earlier paper.

Since an earlier study is extended only to a very moderate degree, this paper does not have much importance by itself. The essential features of this problem had been previously found by others, and the author merits praise only for a more explicit and concise presentation of the subject.

b. Three-Body Problem

Savchenko's[679] (1938) extension of the rigorous equilateral solution of the three-body problem of Lagrange to one special case of mass variability is certainly of considerable interest. The author finds that three bodies of variable mass may permanently form an equilateral triangle of variable dimensions, if the mass ratios of the three bodies relative to each other are not changed by the individual mass variations. The derivation of the results is clear and elegant. This paper may be classified as moderately significant, although there is nothing extraordinary about the method itself, since it follows the lines of the classical problem with constant masses. The result, however, is of definite dynamic interest and perhaps is important for further advances in the variable-mass problem of three bodies.

Batyrev[49] (1941) considered the special case where all three masses, m_i, vary according to the law $m_i = \mu_i (a + bt)^{-1}$ $(i = 1, 2, 3)$, and where they are located at the three corners of an equilateral triangle, corresponding to the case of the Lagrangean solution for constant masses. It is shown that the problem is easily reduced to the classical case for the auxiliary orbits, if one of the three masses is very small and the relative motion of the two finite masses is circular (for their auxiliary points). The author finds that, in this case, the three masses will describe hyperbolic spirals relative to their center of mass, but that they will always maintain their configuration at the corners of an equilateral triangle of varying dimensions.

The existence of the extended-triangular solution for a special case of three variable masses was found three years earlier by Savchenko(679). Batyrev either did not know of this work, or else chose not to make reference to Savchenko's publication. Both authors arrive at essentially the same results by different methods of procedure, but the earlier results by Savchenko do, of course, "depreciate" somewhat the general importance of the present paper.

Savchenko's(677) (1935) derivation of a law of areas for the case of variable masses is clever and original. It is, of course, a generalization of the corresponding law for the motion of bodies with constant mass; it differs from the latter insofar as vectorial velocities appear as products, instead of as sums, as they do in the classical case of constant masses. The paper is based entirely on fundamental physical principles and makes use only of the elementary calculus and ordinary differential equations. The avoidance by the author of vector methods and more sophisticated mathematical notation makes this work accessible to beginning graduate students.

Although Savchenko has studied the problem of the Lagrangean solutions on the basis of the assumption that the absolute velocity of the lost or gained particles is zero, or $\frac{d\,(m\overline{v})}{dt} = \overline{F}$, Orlov(525) (1939) investigated the same problem in the case where the relative velocity of the lost or gained mass is zero, or for the law $m\,\frac{d\overline{v}}{dt} = \overline{F}$. Savchenko(679) found the equilateral solutions to be valid only if the mutual mass ratios, $\frac{m_1}{m_2}$ and $\frac{m_1}{m_3}$ (and $\frac{m_2}{m_3}$), all remain constant. On the basis of Orlov's assumption, Lagrange's equilateral solutions can be extended to the case of variable masses, without any limiting conditions concerning the ratios of the masses to each other. In the case of the Lagrangean straight-line solutions, Orlov finds the conditions $\frac{m_1}{m_2}$ = constant and $\frac{m_1}{m_3}$ = constant to be sufficient for the existence of an extended straight-line solution for variable masses. This does not mean, however, that such solutions are not possible under more general provisions.

Orlov deserves credit for adding substantially to the previous studies on this special subject. Moreover, the basic assumption of relative zero velocity of a mass loss, dm, appears as the more realistic one with regard to cosmogonical applications. This is a valuable contribution by a thoroughly competent author.

Savchenko(676), in 1935, seems to have laid a good foundation for further studies of the details which are associated with a general theory of the potential of variable point masses. His mathematical analysis is not very intricate, but, nevertheless, the investigation has its merits as a clear and systematic approach to the subject. He starts with the consideration of the force field of one variable mass, $m(t)$, and of the related gravitational potential, $V = \Phi(r,t)$, and then systematically develops the analytical expressions for the potential of two variable masses and for a system of n variable masses. He also considers the changing equipotential surfaces of a system of variable masses in their relation to the motion of a unit mass in the variable force field. Of special interest is the result that a closed or periodic orbit of such a unit particle will be possible only if the related work, as performed by the gravitational force of the field on the particle, is different from zero. Excepted is the very unlikely case where the period of a periodic mass variation coincides with the period of the assumed closed orbital motion. The paper is illustrated by eight figures.

6. Figure and Libration of the Moon

In the pages that follow, some Soviet papers on the earth's moon, that were found during the compilation of the bibliography in celestial mechanics, are discussed. The papers deal primarily with libration and figure. The discussion is not to be taken as the fruit of an exhaustive study. Nevertheless, it is believed that the material presented is a representative sampling of Soviet lunar studies, which can be considered to be somewhat relevant to the subject of Soviet work in celestial mechanics.

Heliometric observations of the lunar crater Mösting A, referred to other observable points at the moon's bright limb, began at the Kazan' Observatory in 1895 (A. Krasnov). They were continued by A. Michailovski and, after the transfer of the 4-inch heliometer of Repsold to the Engel'gardt Observatory in 1910, by T. Banachiewicz. After Banachiewicz' departure in 1915, Yakovkin continued this program.

Yakovkin[812] (1928) evaluated and discussed Banachiewicz's five-year series of observations (1910 to 1915) on the basis of Hayn's theory of the physical libration of the moon. The first part of the paper consists of a short presentation of Hayn's theory. The second part contains a discussion of the instrumental and related reductions and corrections. In the third part, the author gives a description of the method of observation and of the necessary reductions, taking into account the unevenness of the moon's limb. The necessary corrections were computed according to Hayn's formulas. The corrected values of the observed distances and position angles are tabulated. Further discussion deals with the derivation of the constants of the physical libration and of the selenographic coordinates of Mösting A. The author arrives at a system of values which is in good agreement with Hayn's corresponding results. For the radius vector, h, of the crater, however, the author finds a value 2".1 larger than Hayn did, and this is an essential difference, considering the mean error of ± 0".5. For the constant f, which is related to the ratio of the moon's principal moments of inertia, the author obtains 0.74, in close agreement with Hayn's value of 0.73. Finally, the author succeeds with an approximate determination of the so-called free libration in longitude.

This is a report primarily on observational procedures and on the evaluation of numerous observations by well-established methods. On the basis of a well-known theory of the moon's libration, the author apparently performed a most thorough and critical investigation. It becomes evident that the author is highly competent and thoroughly familiar with all aspects of the problems involved. The results are outdated, insofar as superseded values of some constants in Hayn's theory have been used as a basis for the author's evaluations, but this fact does not diminish the credit which is due the author.

Later, Yakovkin[813] (1939) published the reduction and evaluation of the observations obtained on the 4-inch heliometer of Repsold at the Engel'gardt Observatory at Kazan' during the years 1916 to 1926. Each observation relates seven or eight well-defined points near the moon's bright limb to the center of the lunar crater Mösting A. The method of observation and the essential elements of the instrumental reductions are discussed in much detail first. The second chapter contains the observations and their reduction and the preliminary results for the constant of the physical libration. The so-called free libration is neglected in this paper, and it is announced that a more complete solution, based on the material from 1916 to 1931, will be published as the second part of this investigation (cf. Reference 814). The reduction of the observations and the least-squares solution for the corrections to the constants of the physical libration are based on the classical theory, which had been brought to perfection by Hayn and which, as the author finds from a comparison of results by various authors, is not in need of any revision. The author mentions, however, that he has discovered a periodic variation of the apparent diameter of the moon, which might be interpreted as the consequence of an asymmetry of the moon's figure.

The work described in this paper is of an observational and computational nature. The rather difficult observations were obtained with greatest care, and the very elaborate reductions were also made as scrupulously as possible. The results apparently are of high quality and in general agreement with other determinations. Altogether, this

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is a very good technical report on work that was done with high skill and the utmost care.

The extension of the evaluation of the lunar observations obtained at Kazan' was published by Yakovkin(814) in 1945; this paper includes all the material from 1916 to 1932. The reduction of the observations and the determination of corrections to the preliminary values of the constants of the moon's physical libration follow the same lines as in the earlier paper (cf. Reference 813). New is an attempt to derive the essential parameters of the so-called free libration, which is very minute compared with the normal or physical libration.

In the introduction to his paper on the physical libration of the moon, Bel'kovich(73) (1949) explains that his paper is a continuation of earlier work by Yakovkin(813,814). Bel'kovich's investigation of the physical libration and of the related figure and rotation of the moon is based on heliometric observations from 1932 to 1942. In the first chapter, the basic theory of Laplace is considered in detail, and previous results from various series of observations by different astronomers are compared, with due consideration of the possible sources of errors. Since Yakovkin found the effect of the libration in latitude on the apparent radius of the moon, the author concludes that Hayn's lunar charts are no longer sufficient for the adequate reduction of the observations. Nevertheless, in Chapter 2, the determination of the various constants of the physical libration, on the basis of 151 observations by Bel'kovich during the interval 1932 to 1942, follows largely the lines of Yakovkin's earlier work.

Chapter 3 deals with the accuracy of the observations and with the best possible derivation of the important quantity f (as related to the ratio of the principal moments of inertia) from three series of observations obtained at Kazan'. The most probable value of f is found to be 0.71. In Chapter 4, dealing with the free libration of the moon, the author arrives at the conclusion that it does not exist at all, because quite different results are obtained from the individual series of observations; if free libration should exist, he believes, the effects are too small to be determined with certainty from the present material. Nevertheless, he derives average values from six series of observations. The fifth and final chapter deals with the asymmetry of the lunar disc. Here, confirmation is given to the findings by Yakovkin concerning the somewhat "thicker" southern hemisphere; in addition, the author finds an east-west asymmetry. In his concluding summary, Bel'kovich states that the concept of a rigid body must now be considered only as an approximating working hypothesis, and that further progress on the libration problem will depend, first of all, on a more accurate determination of the true figure of the moon. It has been learned that the polar diameter of the moon is certainly larger than the equatorial diameter, but the finer details of the problem require further studies.

This appears to be a very critical evaluation of the observational material and a thorough examination of 1949 knowledge about the moon's libration, figure, and rotation. Although one might hesitate to agree at once that the moon has to be considered as a nonrigid body for the interpretation of the observations (considering the probable errors which are involved), the author deserves much credit for his careful investigation. The study follows the lines of well-known theories and methods, but the author demonstrates originality and ingenuity in connection with his derivation of the most probable value of f. Altogether, this appears to be a significant contribution toward a better knowledge of the constants and characteristic properties which are involved in these problems. Bel'kovich is the discoverer of the moon's east-west asymmetry, and he also was the first to recognize the ambiguity or duplicity of the solutions for f.

Nefed'yev[481] (1951) evaluated his 143 heliometric observations of Mösting A between 1938 and 1945. These were used for the determination of the constants of the physical libration by the same method used by the earlier investigators. The residuals (observation minus calculation) were then used for an investigation of the dependence of the moon's radius on the libration. The author confirms the corresponding effect in latitude, in agreement with Yakovkin and Bel'kovich, but the existence of a similar effect in longitude cannot be proved on the basis of this material. Further, the author finds a difference of 0".14 between the eastern and western radii of the moon, thus confirming the earlier discovery by Bel'kovich. Since the resulting value of f came close to the critical value of 0.66, the author recomputes the coefficients of the equations of condition for various equidistant values of f and determines the improved values of f which reduce the square sums of the residuals to a minimum. He finds that $f_1 = 0.60$ and $f_2 = 0.71$ are in

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good agreement with the results of Yakovkin and Koziel. Since there are reasons to consider this value of f_1 as the most likely one for the true f, the solution is repeated again on the basis of $f_1 = 0.60$ for the conditional equations. A final value of $f = 0.57 \pm 0.04$, and related values of the other constants of the problem are obtained in this manner.

Following as it does the previous investigations by Yakovkin, Bel'kovich, and others, the present paper is only another application of the same methods to a given set of observations. As in the case of the other papers published on this subject by members of the Kazan' Observatory, this appears to be a report on work done with the greatest care. More credit is due for the very cautious and refined determination of the important constant f.

Nefed'yev[484] (1955) published a complete report on the evaluation of V. A. Krasnov's observations. He starts with the remark that the evaluation of Krasnov's observations, which were obtained at Kazan' in the years 1895 to 1898, could not be accomplished originally at Kazan', because the Observatory was in no position to do this work before the Revolution of 1917. This probably refers to the circumstance that, at some Russian observatories, only the Director was salaried before the revolution. The author claims that the evaluations made abroad of Kazan' series of observations were not made satisfactorily.

The author evaluates 112 sets of observations, using the method previously described, but Nefed'yev also employs Banachiewicz's Cracovian method for an independent check of his computations. In the first approximation, he solves for f simultaneously with the other constants, and finds f = 0.63 ± 0.05. The interpolation method is used in the second approximation with respect to f, leading to the two solutions 0.60 and 0.71, which agree with the results from other series of observations. The author is of the opinion that the question of which one of the two values for f is the most probable is still open. He also finds that Krasnov's series confirms the effect of the optical libration in latitude on the apparent radius of the moon, and the author states that no doubt should remain concerning the reality of this effect.

This is another good contribution by Nefed'yev. The completion of this work extended to 50 years the complete interval of time during which lunar observations at Kazan' have been made and reduced as homogeneously as is desirable for a reliable determination of the constants of the moon's figure, rotation, and libration. It seems true, indeed, that the only essential contributions to the determination of the physical libration of the moon, and of the related constants of figure and rotation, have been made by the astronomers at the Kazan' Observatory.

The dependence of the apparent radius of the moon on the optical libration in latitude had been found by Yakovkin on the basis of Kazan' observations and an analysis of a series of observations obtained at the meridian circle at Greenwich. Koziel, on the other hand, found no such effect when he evaluated another series of observations, using Banachiewicz's Cracovian method of analysis. At the same time, Koziel obtained the various related results with smaller probable errors when he used the Cracovian method, solving for east-limb and west-limb observations simultaneously. Commission 4 of the International Astronomical Union then expressed a desire for a further test or check of Koziel's finding, according to which no libration effect on the radius of the moon exists.

For this purpose, Nefed'yev(483) (1954) undertook another discussion of Krasnov's lunar observations. These observations, which were obtained at Kazan' in the years 1895 to 1898, had already been evaluated by him (cf. Reference 484), but the east-limb and west-limb observations had been dealt with separately, leading to a confirmation of Yakovkin's libration effect on the radius of the moon. An asymmetry of the moon in the east-west direction, in agreement with Bel'kovich's finding (cf. Reference 73), also was found. Now, Nefed'yev repeated his evaluation of Krasnov's series, this time combining the east-limb and west-limb observations, as Koziel had done; in this manner, the libration effect was eliminated also from the results of Krasnov's series. It is evident, then, that Koziel's failure to find the libration effect was due merely to the unpermissible procedure of solving for all the observations simultaneously. The author also made it clear that Koziel's negative result has nothing to do with any possible defect of the Cracovian method.

This paper deserves much interest, because it clears up the discrepancy between Koziel's results and the earlier results of Yakovkin and others, and because it should remove the last doubt about the reality of the libration effect on the radius of the lunar disc. Although no new theory was involved in this investigation, the author deserves much credit for his good intuition and insight, which led him to the correct interpretation of the existing discrepancies.

After the effect of the optical libration in latitude on the apparent radius of the moon was discovered by Yakovkin and confirmed by others, it became evident that the reductions and computations which had been made of all the meridian-circle observations of the lunar crater Mösting A had not actually provided the orbital path of the moon's center of mass, but only the path of a point oscillating about the true center of mass as the result of the effects of the libration on the radius of the lunar disc. Furthermore, these oscillations are not only of a periodic type, but have an average value different from zero in latitude, because of the systematic difference between the northern and the southern radial extensions of the moon. Yakovkin had already proposed replacing the simple circular shape of the southern limb of the moon with a more refined two-parameter curve, in the reduction of the observations. He[821] (1955) pointed out that there are two different ways to consider these effects of the varying shape of the moon.

The first possibility is to apply corrections to the measured positions of points at the moon's limb, so that the radius of the "reduced" apparent radius becomes independent of the libration. The second method is to compute the differential corrections which have to be applied to the constants of the physical libration, as they have been previously obtained from uncorrected observations, in order to free these constants from the errors which had been introduced by the previous neglect of limb or radius corrections.

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Since quite a number of observational series have already been evaluated on the basis of uncorrected radii, this second method is very desirable for the systematic correction of all the earlier results. These corrections are derived by Yakovkin for the earlier results of Banachiewicz's series of observations from 1910 to 1915. Of principal interest is the relatively large correction, +0.10, to the earlier value of the important dynamic constant f, leading to $f = 0.85 \pm 0.04$ for the larger of the two possible solutions for f. (It still remains an open question whether this value or the smaller one on the other side of the critical value of f is the true one.) Apparently, the value of f derived from the observations is very sensitive with respect to any small variations of the assumed mass center of the moon. It is significant that, on the basis of the improved constants, the residuals of the lunar ephemeris, which in latitude had been of the order of $-0''.3$ to $-0''.5$ before these corrections were applied, are now reduced to an average of zero. In other words, after the introduction of the proper center of mass into the reductions of the meridian-circle observations of the moon, the observed positions of the moon are in perfect agreement with gravitational theory.

This paper is a natural outgrowth of the recent findings by Yakovkin, Bel'kovich, and others concerning the shape of the lunar disc, because, on the basis of the related observational discoveries, it had become clear that the concept of the "center of the moon" needed further refinement. By his discussion of these refinements and of the necessary formulas, and his application of them to one series of observations, the author has further distinguished himself as an essential contributor to this field.

Khabibullin[251] (1954), in his paper on certain simple modifications in the method of determining the physical libration of the moon, starts with historical considerations of the heliometric observations of the moon which have been made at Kazan' using the instrument constructed by Repsold in 1874. Although it became evident from the results of the various series of observations that, as time went on, the accuracy of the results was decreasing somewhat, probably because of the continued wear and tear on the instrument, astronomers at Kazan' have been skeptical about using photographic methods instead. This was mainly due to the fact that, in the determination of position angles by the photographic method, a large error of the zero point or, rather, zero direction is possible. On the other hand, angular differences can be determined very accurately by the photographic method.

The author develops a method, using Cracovians, in which the uncertainty of the zero direction has no bad effect on the results. This method is very similar to Banachiewicz's. Contrary to Koziel's work, the author does not introduce rectangular coordinates but stays with the polar coordinates; he has the opinion that this will improve the accuracy, too, because, even for the heliometric observations, polar coordinates are the direct result of the measurements.

The final test of the author's proposed method will have to be made in practical applications. The author deserves credit, however, for pointing out the practical possibility of good results from a photographic method. This may well be the method of the future in this kind of work, if the general trend towards photographic methods proves applicable in this field, too. The method seems to be suitable for the use of modern computing equipment.

Next, Khabibullin(252) (1955) discussed in detail the difficulties arising in the practical determination of the parameter, f, of the moon's physical libration. These difficulties are caused by the fact that, for certain values of the moments of inertia entering into f, rotation is not stable. The critical value of f is close to 0.662, but the determinations lead to somewhat different values. The author points out that, because of the existence of this singularity, it actually becomes necessary to include higher order terms in the otherwise linear equations of condition, and that this circumstance leads to the result that, in general, two different values of f are obtained from the solutions. The two results are located on different sides of the critical value. On the basis of the existing methods, it was not possible to arrive at a rigorous decision about the "real" value of f. Considering this failure or breakdown of the methods, caused by the nonlinearity of the equations of condition, the author proposes and uses a new method which avoids this trouble. Instead of solving for f itself, the author solves (making use of Lagrange's relative extremum concept) for two unknowns, a_3 and a_4, which are actually functions of f and thus related with each other. Then, the application of the method to the existing series of observations leads to the finding that only one value is obtained for f from each series; for the corresponding second value, the process of solution becomes divergent, and the other f value is discarded. The combined result from all the series is $f = 0.60 \pm 0.02$. The author concludes that Hayn's value, which is still being used in the Berliner Astronomisches Jahrbuch, should be abandoned in favor of the new value.

Khabibullin first presented a good and clear demonstration of the cause of the difficulties in the practical determinations of f; then, he devised a method which actually overcomes these difficulties and leads to one, nonambiguous result for this fundamental quantity. Obviously, this is a rather valuable contribution, the merits of which are to be found in the new mathematical treatment of the conditional equations. Although none of the basic elements of the process of solution is new in itself, the author has to be credited with the proper combination and application of these elements, and with the resulting elimination of an ambiguity which, for a long time, prevented a satisfactory determination of the proper value of f.

Yakovkin(820) (1954) undertook a new discussion of the moon's libration, based on all the available series of observations, and came to the conclusion that the very small free-libration term is real, in spite of the wide scattering of the values for the amplitude at a given date, which had been found from earlier discussions. His new results are obtained by deriving the second approximation for the physical libration, after determining the quantities of the free-libration term; the scatter in the amplitude values from the various series is reduced to about 60°.

Yakovkin's discovery of the dependence of the radius of the moon's disc on the libration in latitude revealed the need for refinements in the reductions of observations of points on the limb of the moon, in order to determine the position of the true center of mass for comparison with the lunar ephemeris. In the lunar maps by Hayn and Weimer, this necessity was not taken into account; the heights of points given in these maps are referred to various normal or niveau surfaces, rather than to one system.

In the preparation of improved maps of the moon's limb, Nefed'yev(485) (1957) undertook the task of deriving the so-called selenoid (corresponding to the geoid in the case of the earth) from six series of heliometric lunar observations obtained at Kazan' during the last 50 years. The new reference surface, which is determined as a gravitational equipotential surface, is linked with about 6,000 heights measured during these 50 years. The equations of condition express the requirement that the reduced measurements are independent of the optical libration, and the additional requirement that the remaining lunar residuals (observation minus ephemeris) in latitude have the character of random errors. From the least-squares solutions determining the selenoid, it is found that the remaining mean error of the individual measured heights on the moon's surface is ± 0.″20.

The author has presented a condensed report on the accomplishment of a rather important task. Although the aspects of his work are mostly technical, the results are important for the reduction of future lunar observations, and, apparently, they have been derived with great care. The observational material was readily available from the consistent series obtained at Kazan' over five decades.

7. Cosmogony

In the first paragraph of his article on the criterion of tidal stability and its application in cosmogony, Fesenkov(196) (1951) treated the tidal stability of a condensation in a rotating diffuse medium, assuming that no collisions between the particles are involved. It is found that a density of $\delta \geq 2.5 \frac{M}{R^3}$, where M is the disturbing or central mass and R the distance of this mass, guarantees the stability of such a condensation. For Roche's well-known problem of a close satellite of fluid consistency, a similar expression for the minimum density of a stable body is derived; a numerical coefficient of 3.52, instead of 2.5 as above, is used. For a more rapidly rotating satellite, this figure is even larger, and, therefore, a rotating satellite requires a larger minimum distance from its primary in order to be stable.

Paragraph 2 introduces the effect of collisions between the particles into the preceding investigation; this leads to an approximate factor of 10, or to $\delta \geq 10 \frac{M}{R^3}$ for stable satellites.

In Paragraph 3, Fesenkov applies the criterion of tidal stability to various cosmogonical problems. Inserting, for example, the known average diameters and mutual distances of the globular clusters into the stability equation, assuming that, in this equation, the sign of equality would approximately characterize the original density of stars per unit volume before the contraction into dense clusters began, Fesenkov finds that masses of the clusters should be of the order of 107,000 to 340,000 solar masses; actually, they are estimated to be between 250,000 and 400,000 solar masses. Taking this rather good agreement as proof of the significance of the criterion of tidal stability for the formation of the globular clusters, Fesenkov also concludes that the size of the galaxy cannot have changed appreciably since these clusters originated, because, otherwise, R would not be of the same order and different results would be obtained. The average distances between individual stars in the neighborhood of the sun are in good agreement with the one computed from $M = 10^{11}$ solar masses and R = 8,000 parsecs. In this case, the application of the criterion of tidal stability leads to $\delta = 10^{-22}$ gr/cm^3, and $r \approx 10^5$ astronomical units for a volume containing one solar mass. The third application deals with the original masses of the planets in the solar system; here, Fesenkov's ideas and results resemble closely G. P. Kuiper's concept (1949) of the origin and development of the solar system.

This is a paper of considerable interest to the field of cosmogony. Kuiper was the first to recognize the importance of tidal stability in the gravitational field of the sun for the formation of the planets of the solar system. Fesenkov's investigation, which does not go so far as Kuiper's work with regard to the solar system, undoubtedly confirms the universal significance of the concept of tidal stability. Although Kuiper systematically developed the detailed consequences for an explanation of the various observed properties of the planetary

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system (not only of the so-called Bode law, as in the present paper), Fesenkov's contribution is especially significant, because it seems to demonstrate the validity of this basic cosmogonic concept for the stellar system as a whole. Apparently, the author was not aware of, or chose to ignore, Kuiper's somewhat earlier work on the planetary system.

Idlis[229] (1952) applied the criterion of tidal stability, in the form given to it by Fesenkov, to the systems of regular satellites of Jupiter, Saturn, and Uranus. It is found that, in each of these systems, the distribution of the mean distances of the satellites from their primaries follows the same general law. In each case, only one parameter enters this law, and this parameter depends essentially on the total mass of the corresponding system. Basically, this distance law is the same as that which is satisfied by the distribution of the major planets in the solar system.

This is another significant contribution, because it strengthens the universal importance of the criterion of tidal stability for the original formation of the planetary and satellite systems. As in Fesenkov's paper, no reference is made to Kuiper's earlier and almost identical results. Although in the history of science there are instances where nearly the same findings were made by different men, at the present time it is rather puzzling that, three years after Kuiper published his basic ideas in the Astrophysical Journal, the Soviet papers contained no reference to Kuiper's work. Notwithstanding the duplication of the essential results, Idlis makes a very thorough evaluation of the satellite data in order to arrive at his results.

Also in 1952, Idlis(230) presented a critical review of three different "laws" that have been offered to explain the distribution of planetary distances from the sun. He shows, in particular, that the law of O. Yu. Shmidt does not satisfactorily represent the planetary distances, regardless of the value chosen for the parameter λ. Only if a different value of λ is chosen for each pair of planets can their mutual distance be represented. This failure of Shmidt's law is demonstrated, also, for the satellites of Saturn by the proper numerical evaluation for the most suitable values of λ. Furthermore, it is pointed out that Shmidt's law lacks any physical foundation and represents only a purely arithmetical construction. Essentially the same criticism is leveled against the law proposed by Gurevich and Lebedinskiy. It is shown that this law also fails to account for the actual distribution of the mean distances of the planets, and that it also lacks any solid physical basis. Fesenkov's explanation of the distances, as a consequence of the significance of the criterion of tidal stability for the formation of the planets and satellites, is verified by Idlis by means of a comparison of theoretical and actual distances. The author praises Fesenkov's law as the only one which is based on sound physical considerations. Again, G. P. Kuiper's theory of the origin of the solar system is not mentioned at all.

Fesenkov and Idlis are developing and accepting the same basic principles that were developed by Kuiper. Idlis' paper is a very sound criticism of certain other contributions; it lends substantial support to Fesenkov's (and thus to Kuiper's) results.

In his paper on the group determination of hypothetical orbits of the nearest stars, Polak[560] (1937) assumes that the orbital motions of the stars in the close neighborhood of the sun can be ascribed approximately to the gravitational action of a central mass, M, concentrated at the distance, R, from the galactic center, and that the stars under consideration move approximately within the galactic plane. The motions of the individual stars are applied to the following theorem: "If meteors moving in a common orbital plane, and in orbits with the same parameter, p, meet in one single intersection point, then the ends of the velocity vectors at the time of the passage through this point lie on a straight line parallel to the radius vector." This theorem is deduced from the integral of areas. Assuming its applicability to the orbits of the stars moving within a small region in the close neighborhood of the sun, Polak is able to show that orbital eccentricities between 0.1 and 0.2 would be most suitable to represent the observed stellar motions.

Polak has applied some fundamental and simple features of orbital motion in the field of a central mass to the local star system. Although such applications to the motions of stars call for caution, it seems that certain characteristics of orbital motion may be transferred from one dynamic system to another, as long as the concept of two-body orbits can be used at all. With the necessary limitations of such methods in mind, Polak's paper is of some interest. The deductions, as such, are simple, and the evaluation is essentially numerical, being based on the velocities of 293 stars with parallaxes of at least 0".05 according to Kohlschütter.

Reyn(630) (1933) considered a central mass, M_o, imbedded in a dust medium (formed by small particles) of density k(r), where r is the distance from the central mass. Inside the dust medium, a point, or condensation, J, of mass μ moves around M_o in a circular orbit so that the scheme is that of the restricted three-body problem applied to a cloud of particles, instead of to a single one. In the first section of Chapter I, the equations of motion for J rotating around M_o are derived, and a pseudo-Jacobi integral is obtained, depending on the mass, M(r), contained in the sphere of radius r around M_o, instead of on M_o alone. The density distribution, k(r), is supposed to be of such a kind that the central core of the nebula is rather massive, compared with the outer regions.

In Section 2, it is shown that five double points of the surfaces of zero velocity exist, as in the classical restricted problem of three bodies, and Section 3 reveals the fact that the structure of these surfaces of zero velocity is quite analogous to the structure of the surfaces of Hill's equation in the classical problem. Reyn concludes that the ovaloid around J, which has a double point with the larger ovaloid around M_o on the line connecting M_o with J, can be defined as a "boundary" surface around the condensation J. If all the particles are initially at rest in the rotating coordinate system, then none of the particles inside this boundary surface will subsequently leave the closed region around J. The author thinks that all this matter may ultimately condense into a planet and its satellites, provided that the original development of such a condensation is possible on physical grounds (which problem is not investigated here). The author limits herself to the computation of the mass which is contained inside the boundary surface, depending on three different density laws. In Chapter II this is done by neglecting the attraction of the nebula on its own particles; in Chapter III, this effect is included, insofar as the nebular mass inside the sphere of radius r is concerned. The author finds that the so-called Schuster law, $k(r) = \frac{k_o}{(1 + \lambda^2 \gamma^2)^{\frac{5}{2}}}$, in dependence on the distance between M_o and J, leads to lower limits of the planetary mass which are in reasonable agreement with the actual present mass distribution in the solar system. The original mass inside the zero-velocity boundary surface is considered to be a lower limit of the final planetary mass, because of the possible addition of extraneous particles whose surfaces of zero velocity are open ones.

Even though it is relatively simple from the mathematical point of view, Reyn's investigation is of interest in connection with cosmogonical problems. Although today, astronomers are inclined to believe that not more than one per cent of the original solar nebula consisted of solid particles, and that the remaining 99 per cent was gas, and, for this reason, the author's scheme appears to be too simple for a successful application to the problems of the origin and development of the planets, her analysis may still be useful as a component part of future, more comprehensive studies.

Radziyevskiy[614] (1953) studied the motion of two material points inside a homogeneous and spherical cloud of dust. He assumes that the density of the cloud is sufficiently small so that no friction hampers the orbital motion of the two material points. The acting forces are determined by the gravitational interaction of the two masses and by the gravitational action of the cloud. The differential equations of motion under these circumstances include the so-called Hook terms. It is shown that the motion of the two point masses can be found by quadratures involving elliptical integrals. The existence of such a solution is demonstrated, but the corresponding elliptical function is not explicitly given.

This is a sound and fruitful contribution, and the results should be useful in connection with certain cosmogonical problems. The deductions themselves are not very difficult, and the author deserves credit mostly for the basic ideas in the paper. Certainly, the author demonstrates his familiarity with the theory, and his competence.

In a later paper, Radziyevskiy(616) (1954) considered the problem of two point masses, m_1 and m_2, moving inside a spherically distributed third mass, m_3. In the case of constant or nearly constant density of the distributed mass m_3, and assuming this density to be so small that the motion of m_1 and m_2 is not affected by any noticeable resistant force, the gravitational action of m_3 on m_1 and m_2 is expressed by Hook's law, which says that this attraction is proportional to the corresponding distance from the center of m_3. A rather trivial theorem - that the mass center, G, of n point masses, moving inside the m_3 distribution, moves in a fixed plane and with constant areal velocity with respect to the center of m_3 - is presented and proved. A second theorem says that the relative motion of m_1 and m_2, in the more limited

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case of three masses, m_1, m_2, and m_3, is independent of the motion of m_3 with respect to m_1 or m_2. Here, m_3 means the center of mass m_3. Less trivial is the third theorem, stating that the relative motion of m_2 with respect to m_1 is determined by a combined relative force, which is the sum of the Newtonian and Hook attractions. A fourth theorem says that the osculating orbital plane of the relative motion of m_1 and m_2 has a translatory motion with respect to m_3. The fifth theorem, finally, states that m_1 and m_2 will have to collide in the special case where the projections of the velocities of m_1 and m_2 (relative to the center of m_3) onto the normal to the direction $m_1 \longrightarrow m_2$ are equal. The author also gives the analytical expression for the energy integral of this problem. All these theoretical results are then applied to the problem of the dissolution of groups of stars in star systems.

Assuming that the periphery of a galactic system rotates approximately as a rigid body, as far as the density distribution is concerned, the maximum distance of two stars, m_1 and m_2, under certain starting conditions is computed. For initial velocities, the author considers a parabolic relative velocity and a velocity 10 times larger. In each case, he finds a permanent separation of the two masses, m_1 and m_2, to be impossible, because an upper limit of moderate size exists for the maximum distance. From this result, it is concluded that, in the neighborhood of the sun, the dissolution of star groups or clusters must be much less frequent or slower than previously thought, because, in this region of the galaxy, the "quasi-elastic" attraction according to Hook's law amounts to roughly 20 per cent of the Newtonian attraction between close individual stars.

The purely mathematical part of this paper is correct and even of definite interest (except for the first, rather trivial theorems). However, the author's application of his work to the dissolution of star groups has been severely criticized by Ruprekht and Vanysek(654). This criticism clearly is justified. Radziyevskiy computed the maximum distances of his two masses, m_1 and m_2, by means of the equation for the surface of zero velocity. However, at such large distances from the center of the galaxy, the related surface of zero velocity has no points in the x,y- or galactic plane, and the stability claimed by Radziyevskiy actually applies to the z coordinate only.

Safronov(659) (1951) investigated the question of whether the planets could have originated from the rapidly rotating sun in accordance with the Laplacian hypothesis. Assuming that the maximum possible mass of a rotating star is the one for which the centrifugal acceleration equals (in the opposite direction) the gravitational acceleration, the author computes a table that gives the ratios of the maximum possible masses and of the related angular momenta of the sun to their present values, dependent on the parameter μ, which measures the departure of the star from uniformity. It is further assumed that the loss of mass follows the mass-luminosity relation $\frac{dM}{dt} = -\gamma M^n$, with $n = 3.9$. If the mass of the sun decreased by five times its present mass in 5×10^9 years, then the present loss would be 4×10^{15} gr/sec, or about 1,000 times the corresponding loss by radiation. Nothing such as this has been observed yet. The table also shows that Neptune could not have formed from original solar mass for any reasonable values of

μ . For a very extreme value of μ , which is physically not acceptable, the relations could formally be satisfied by an original maximum mass 150 times the present solar mass. From all these results, Safronov draws the convincing conclusion that the planets cannot have formed in accordance with the Laplacian hypothesis.

In the remainder of the paper, the author studies the modifications of the previous results that would become necessary in consequence of the spheroidal shape of a rapidly rotating sun, of deviations from the assumed solid-body rotation, and of the loss of mass in the prestellar state. The fundamental character of the results remains unchanged.

Essentially, Safronov has demonstrated something that was known before, namely, that the planets presently possess almost all the rotational momentum in the solar system (98 per cent), whereas the sun carries only two per cent of it. This leads to the failure of the original Laplacian hypothesis. The author deserves credit, however, for the systematic and comprehensive manner in which this is demonstrated.

8. Cosmology of Small Bodies in the Solar System

The problem of the origin and evolution of comets, asteroids, meteors, and meteorites can be considered from the astrophysical point of view, that is, using observational data on the spectra, brightness, structure, etc. of such bodies; it can be treated also from the point of view of celestial mechanics, that is, from a consideration of their orbits. Papers by Soviet astronomers on both aspects of the problem are quite numerous. Here, an attempt has been made to digest only some of the many Soviet papers in which data are treated by the methods of celestial mechanics.

a. Origin of Comets

The problem of the origin of comets (and even of the origin of the entire solar system) is at present in a state of flux. Many papers are being written on this subject, and quickly forgotten. The difficulty is manyfold. First, in view of the established age of the terrestrial crust, i.e., of the order of four billion years, and of the rapidity with which comets disintegrate, they cannot be coeval with the planets, and some other mechanism of their origin must be sought. Second, the present orbits of comets are the result of age-long perturbative action by the planets and, if any conclusions are to be made on the basis of the study of orbits, the influence of these perturbations must be eliminated. Third, comets are usually observed only in a very short interval of their heliocentric path, and computation of the whole orbit (for instance, the position of the aphelion) is, of necessity, an extrapolation. Last, diffuse bodies, as most comets are, cannot be observed with the same accuracy as stars or planets, and conclusions based on the measurement of their positions are vitiated by all sorts of systematic errors.

It is no wonder, then, that the opinions of qualified astronomers differ sharply on these problems. The diversity of opinion among Soviet astronomers is as striking as anywhere else.

(1). Ejection Theory

The ejection theory postulates the continuous ejection of comets from the surface of major planets due to some explusion process. This theory is very old and can be traced back to Lagrange (1814), in whose time the major planets were considered incandescent, more or less like the sun. With present knowledge of the conditions prevailing on the major planets, this theory appears to be unlikely, to say the least. Nevertheless, it was revived and defended by S. K. Vsekhsvyatskiy in a number of papers(796,799-802). Vsekhsvyatskiy starts with his estimate of the age of short-period comets and comes to the conclusion that it cannot be greater than a hundred years or so. This argument has been criticized very severely in the U.S.S.R. and abroad. According to Vsekhsvyatskiy, short-period comets must be continually forming in the solar system, and the place of their formation is the planet Jupiter.

The conditions for this presumed expulsion of matter from the planet Jupiter have never been worked out satisfactorily by Vsekhsvyatskiy. This theory was severely criticized by A. Corlin (Zeitschrift für Astrophysik, 1938, Vol 15, p 239) and others; finally, a detailed criticism was published by A. J. J. Van Woerkom (Bulletin of the Astronomical Institute of The Netherlands, 1948, Vol 10, p 445). The latter criticism apparently resulted in a very definite condemnation of Vsekhsvyatskiy's point of view by the Commission on Comets and Meteors of the Astronomical Council of the Academy of Sciences, U.S.S.R. (Astronomicheskiy Tsirkulyar, Akademii Nauk S.S.S.R., 1950, Nr 101-102).

The resolution reads in part: "The hypothesis of ejection has not been properly developed. It is not shown how the proposed mechanical scheme can explain the observed orbits of comets, and particularly their distribution in respect to periods and inclination to the ecliptic.... The hypothesis of ejection in its present form has no followers and cannot be considered as representing the point of view of all or of the majority of Soviet astronomers".

In spite of this censure, Vsekhsvyatskiy continues to publish articles defending his point of view; however, he has made the satellites of Jupiter, and other planets including Venus, the parents of comets. In his latest paper on the subject(802), he repeats his old arguments, completely ignoring the essay by his own pupil, P. G. Dukhanovskiy(179) (1954), which is a solid piece of research applying the idea of ejection from the surface of Jupiter to 53 short-period comets of the Jupiter group. The problem was to see whether this hypothesis would explain the actually observed distribution of orbital elements of the comets of that group. The result was entirely negative, so these comets could not have originated from the surface of Jupiter. The same argument applies with even greater force to the other planets.

Vsekhsvyatskiy is an astrophysicist who is either unable to understand, or does not take cognizance of, the arguments of celestial mechanics; he bases his entire theory on rather questionable data of astrophysics and the even more questionable distinction between short-period and long-period comets. There is nothing wrong with his

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hypothesis; in fact, the possibility of explosions on surfaces of planets, which result in the formation of small cosmic bodies, has recently been argued on the basis of geophysical data (W. H. Ramsey and others, Monthly Notices, Royal Astronomical Society, 1950, Vol 110, pp 325-338). This work is apparently unknown to Vsekhsvyatskiy, who never mentions it (thus missing a very strong argument in favor of his theory). However, any astronomical theory has to be developed logically and compared with observations; this Vsekhsvyatskiy has not done. This was the point of the criticism of his work by the Astronomical Council, which, in this case, acted as would a scientific body anywhere in the world if it were called upon to pronounce judgment on a piece of investigation. In the West, however, no such official action takes place; a theory would be criticized by other astronomers individually.

(2). Interstellar Theory

According to this theory, which was first proposed by Laplace, comets represent condensations of interstellar media which were picked up by the sun in its journey through space. This theory was considered in great detail by Schiaparelli, Fabry, Von Niessl, and others, and was recently revived by the British astronomer Lyttleton.

The observational check of this theory consists of a comparison of the distribution of the eccentricities of the orbits of comets, but the entire theory depends on the assumed distribution of comets in space, about which little is known.

The Soviet contributions to this theory consist primarily of a series of memoirs by N. D. Moiseyev(425,426). The treatment is strictly mathematical and includes complicated formulas which are of no value whatever for the solution of the problem since the assumed velocity distribution of comets with respect to the sun is quite unlikely. There is practically no attempt to compare the picture so derived with observational data.

More or less along the same lines is the reasoning of O. Yu. Shmidt(709); in his cosmogonical theory, however, comets are a minor item, and the subject of their origin is not elaborated upon to a sufficiently convincing degree. Shmidt's cosmogonical theory is important, not so much for its intrinsic worth, as for the fact that it has been extensively discussed in the U.S.S.R. and has given impetus to other investigations in the problem of n-bodies.

(3). Collision Theory

In this theory, it is assumed that comets are formed by collisions between small bodies in the solar system, which gradually reduce the originally large pieces of matter to small particles. The theory was proposed by S. V. Orlov, but he did not develop it beyond suggesting the possibility of such collisions.

A much more serious approach to the problem was taken by V. G. Fesenkov. The existence of small bodies in the solar system is indisputable (i.e., asteroids, meteors, and meteorites). Fesenkov assumed that they are the result of the explosion of a planet, which used to be between Mars and Jupiter; the problem, then, is to trace the evolution of the debris left by this planet. Fesenkov considered the effect of perturbations of the nearest stars(189,197) and concluded that these perturbations would not explain the observed distribution of the aphelia of comets, and that comets must be considered as members of the solar system. Recently, the same problem was treated in a more refined way by K. A. Shteyns(713) (1955), who arrived at practically identical conclusions. Both these investigations were based on the idea of the motion of a particle about two stationary centers (the sun and the star), a necessary simplification without which the problem would be insoluble, but one which undermines the validity of the solution if the long intervals of time involved are considered. Further, Fesenkov shows that the debris of the explosion of the original planet would be removed to great distances from the sun; it eventually would be forced back to the sun by the perturbations produced by the nearest stars. This theory shows great similarity to the now-accepted theory of the Dutch astronomer Oort, who did not notice Fesenkov's work.

b. Comets and Meteors

(1). Meteoric Radiants

The connection of comets with some meteoric streams was proved by Schiaparelli nearly 100 years ago. Much work on this subject was done by Bredikhin, whose papers have recently been reprinted by the U.S.S.R. Academy of Sciences.

Soviet papers on individual meteoric streams and their radiants are very numerous(27,40,181,309,310,384,793,833,834). Of the more extensive Soviet papers on meteoric radiants, the following are of note. N. N. Sytinskaya(716) (1952) published a catalogue of 827 meteoric radiants and the orbits corresponding to them. Only a few of these radiants can be safely identified with individual comets, but further connection is reported from time to time. Of a similar nature are the catalogues of Muzafarov(479) (1940) and Bakharev(41) (1938).

E. N. Kramer adopted a different point of view. In his extensive paper(310) (1953), he computes the position of the radiants of 280 comets for future identification of the observed radiants of individual comets. This paper represents a large piece of work of obvious value. All known methods of computing the nearest approach between the orbits of the earth and a comet are analyzed; that of A. D. Dubyago(126) (1949) is considered the most precise. However, for a large-scale calculation, Dubyago's method is too cumbersome, and Kramer develops his own, partially graphic, method.

(2). Formation of Meteors

In view of the proved connection of some meteoric streams with comets, the process of ~~the~~ disintegration of cometary nuclei needs to be considered. At the present time, there is available considerable information about the physical processes in comets, which permits some opinion to be formed as to the phenomena and mechanism involved in the disintegration of comets.

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According to Bredikhin's theory, meteors originate from the anomalous tails of comets.

Recently, Babadzhanov(32), in his monograph on the problem of the origin of meteoric streams, presented a very thorough review of the theory of cometary synchrones, with a detailed application of this theory to Comet 1901 I. He considered also the available data on the velocities of particles in the halos surrounding cometary nuclei, and the theory of anomalous tails. He shows that these three classes of observed phenomena are of the same nature and represent a synchronic phenomenon with rather small velocities of ejection of the order of 1 km/sec.

In the second part of the investigation, Babadzhanov considered the orbits of comets and of generated meteors, and applied Moiseyev's theory(467) (1945) of averaging secular perturbations from major planets. The result for Comet 1862 III and the Perseid meteors (for which the data are most reliable) is not very encouraging. The minimum possible velocity of meteors with respect to the nucleus of this comet was found to be about equal to the maximum velocity of the particles in the synchrones. The idea of the origin of meteors from synchrones of comets must therefore be abandoned.

This paper is important from the point of view of the scientific problems considered in it; it also is an example of work carried out on a definite long-range program of meteoric study at the rather small Stalinabad Observatory. The participants in this program (Katasev, Bakharev, Dobrovol'skiy, and Babadzhanov) show some familiarity with the results of astrophysical research and the methods of celestial mechanics.

The problem of the origin of meteors from comets was considered also by A. D. Dubyago(127) (1950). He shows that a swarm of particles forming the nucleus of a comet must have some components of rather large size to survive at all. With frequent collisions, the size of the particles decreases, until some of them are expelled from the nucleus to form meteoric streams. The velocity of expulsion is found to be rather high - of the order of 5 km/sec. This is in agreement with the results of Babadzhanov and of Voroshilov(794), and also with results published in the West. Whipple and Hamid (Harvard College Observatory Reprint Nr 361, 1951) found that, in order to explain the observed connection between Encke's Comet and the Taurid meteor stream, the velocity of separation must be of the order of 3 km/sec.

The problem of the division of comets, which must ultimately result in the formation of meteoric streams, was considered by S. V. Orlov(548). He presented a detailed discussion of the remarkable group of comets with short perihelion distances (Comets 1882 II, 1893 I, and others). In another paper on the general problem of the evolution of comets, S. V. Orlov(550) tried to establish the existence of a number of genetically connected comet groups.

This aspect of the problem was further developed by Vodop'yanova(786), who, in a very substantial piece of work, investigated the possibility of the intersection of cometary orbits. The existence of intersections is a very strong argument in favor of the common origin of groups of comets. On this basis, she established the existence of 27 groups of comets, each group presumably having a common origin.

c. Comets, Meteorites, and Asteroids

The connection between comets and meteors has been established beyond doubt; the suspected connection of comets with meteorites and asteroids is still very much in dispute. So far as meteorites are concerned, the main difficulty lies in the undertainty of their orbits, since their motion near the earth cannot be foreseen and is usually observed by untrained individuals.

The heliocentric paths of individual meteorites are often computed in the U.S.S.R. It has been found that meteorite orbits are very much like those of some asteroids; these results are in complete agreement with Western work. Of more general interest are the results of I. S. Astapovich(22) (1939), who analyzed the orbits of 66 meteorites. He found a connection between the orbits of eight meteorites and Comet 1790 III, which also has four meteoric showers assigned to it. Six other comets also were found to move in the same orbits as six meteorites.

Astapovich's paper represents a monumental work considering the amount of data which had to be analyzed and interpreted. Unfortunately, it contains systematic errors in the determination of meteor velocity (later proved by Wylie), all of which make his conclusion, that most meteorites are of cosmic origin, incorrect. Also, he attributed some meteorite falls to comets on the basis of doubtful observations.

A recent trend is to consider meteorites to be closely linked to asteroids. The heliocentric orbits of the two classes of bodies are very similar, and some astronomers accept the theory that meteorites are simply very small asteroids that happened to come close to the orbit of the earth.

The great Sikhote-Alin iron meteorite of February 12, 1947, was extensively observed in the Soviet Far East. Its heliocentric orbit was calculated by V. G. Fesenkov(199) (1951) and it turned out to be very much like that of the asteroids Apollo and Hermes, which also can approach the earth very closely. The weak point of this result lies in the velocity; an error of 2 or 3 km/sec would alter the shape of the orbit quite substantially. The original mass of this meteorite was estimated by Fesenkov to be 1,000 tons, which makes its diameter of the order of several meters. The Sikhote-Alin meteorite may represent one of those tiny asteroids which are too small to be observed, but whose existence appears to be certain.

Since many short-period comets move in orbits very similar to those of asteroids, and since some comets have very little coma and sometimes even appear to be perfectly stellar, some astronomers have expressed the idea that the two classes of bodies are of the same origin. However, there is a more rigorous test of this idea, based on the properties of the orbits.

Adopting the idea of the restricted three-body problem (sun, Jupiter, and an asteroid of insignificant mass), and assuming the motion of both Jupiter and the asteroid in circular orbits, the equations (in rotating coordinate axes) can be integrated. The expression for the square of the velocity of the asteroid involves the Jacobi constant, h. By a principle of dynamics, the Jacobi constant must preserve its value during the whole time of existence of a celestial body. It follows, then, that if the Jacobi constants for the asteroids and comets are computed, the question of the common origin of such bodies can be put on a more reliable basis.

The computation of Jacobi constants for so many objects is, of course, no easy matter, even with the simplifications introduced by adopting circular orbits and considering perturbations from Jupiter alone. A. Klose, a Latvian astronomer, computed these constants for 1,019 asteroids (Vierteljahrschrift der Astromischen Gesellschaft, 1928, Vol 63, p 333) using some simplifications. Nevertheless, even from Klose's approximate calculations, the Jacobi constants for asteroids are not distributed entirely at random.

An extensive study of the problem was made by A. N. Chibisov(115-117) (1936 and 1939), who computed Jacobi constants for 1,264 asteroids, also using approximating methods. The constants for all but seven asteroids are between the limits (multiplied by 10^7) 805 and 1075. The most conspicuous exceptions are the asteroids Eras (h = 1301.9) and Hidalgo (h = 581.3), both of which move in very eccentric orbits. Among the remaining asteroids whose Jacobi constants fall between the indicated limits, six groups are indicated. Further work by Chibisov deals with the problem of the stability of motion of asteroids.

Chibisov deserves to be criticized for neglecting the inclinations, i, in the last and most important part of his paper(115) (1939), especially after he criticized Klose for computing the h-values without including i.

The Jacobi constants for 70 periodic comets were computed by T. V. Vodop'yanova(785) (1939). These constants for the short-period comets of the Jupiter family range between 639.9 and 859.2; these values only slightly overlap the limits for the asteroids. Only six short-period comets of very different physical characteristics fall within the limits of the Jacobi constants for asteroids and, at that, they are very close to the lower limit (h from 836 to 859). It may be concluded, then, that short-period comets of the Jupiter group and asteroids have different origins.

As far as other comets are concerned, their Jacobi constants have nothing in common with each other or with those of the asteroids.

In this connection, the work of the Japanese astronomer Hirayama, who considered the problem of the structure of the asteroidal ring, should be mentioned. Hirayama introduced the so-called "proper elements", that is, the combination of orbital elements that will not change in time due to perturbations by the planets. Hirayama found five asteroidal families, the members of which must have common origins.

Hirayama's results were refined by the U. S. astronomer D. Brouwer (Astronomical Journal, 1951, Vol 56, p 9), who investigated 1,537 asteroids and found indications of the existence of 28 families. Apparently unknown to Brouwer, N. Shtaude, as early as 1925, had extended

Hirayama's work and found the existence of 20 families(710). Both Brouwer and Shtaude found that the Flora family has four subdivisions, and, in fact, some of Brouwer's results were anticipated by Shtaude.

Further contributions to the problem of the evolution of asteroids were made by G. A. Chebotarev(97,98) (1953 and 1954). He considered the most remote asteroids of the Hilda group (with mean daily motions of 444" to 456", that is, close to a 2:3 ratio of the daily motions of Jupiter and the asteroid). He applied his own theory(96) (1953) of periodic orbits to the problem and computed orbital elements for 16 asteroids of the Hilda group in the time interval about 1,000 years before and after the present epoch. His most interesting conclusions concern the gradual changes of the orbital elements of the asteroids, and the possibility of the close approach of these asteroids to Jupiter and their conversion into satellites of Jupiter.

Certainly, the planets of the Hilda group seem to be exposed to rather large perturbations by Jupiter. Considering, however, the simplifications and lack of rigor in Chebotarev's basic theory, it is doubtful that his results are sufficiently accurate to represent the real orbital evolution. It could very well be that a more rigorous theory might reduce his gradual or "secular" variations to long-period large fluctuations. As far as is known, this is the only attempt that has been made to apply the principles of celestial mechanics to the problem of tracing the evolution of asteroidal orbits during a 2,000-year interval.

d. Repulsive Forces

With very few exceptions, which were introduced by the theory of relativity, the motion of larger bodies in the solar system may be said to follow Newton's gravitational law. However, if the size of the body becomes of the order of one micron, another force comes into action - the repulsive force of light pressure. The existence of light pressure has been proved experimentally and theoretically, and it has been found to play a role in the universe. Since the Milky Way system consists partly of diffuse clouds of dust and gases, the motion of these clouds is controlled, to some extent, by the light pressure of neighboring stars, especially of B-type stars, which are much more luminous than the sun.

Generally, the motion of individual particles subject to light pressure cannot be followed. Cumulative effects must then be treated in these problems, and the approach is therefore essentially different from that used in the computation of the orbit of an individual body. It can also be shown that even in larger bodies, such as asteroids, where the ratio of light pressure to the gravitational force is very small, the cumulative effect of light pressure may be considerable.

(1). Comets

The tails of comets directed more or less away from the sun show at once the existence of some repulsive force emanating from the sun. It is now identified with light pressure but, in the mathematical treatment of the problem, no such identification is necessary. It may be any repulsive force, provided that the ratio, μ, of this repulsive force to the force of gravity acting on the same particle remains constant. Denoting the repulsive force as f, then the ratio $\mu = -f/k^2$, where k^2 is the Gaussian constant of gravitation; μ represents the effective repulsive force in units of gravitational force. Since it is diminished by one unit of the gravitational force, the total repulsive force acting on the particle is obviously $1 - \mu$.

Each particle ejected from the nucleus of a comet will describe an orbit in the neighborhood of the nucleus and eventually find its way into the tail. The cumulative result of the motion of these particles is the head of the comet, consisting of halos, jets, and envelopes, and the tail. The problem, then, is to derive from the outlines of the head and the tail the motion of particles subject to the repulsive and attractive forces of both the sun and the nucleus.

Obviously, complete understanding of the nature of comets cannot result from such a limited statement of the problem. Astrophysical data, such as data on the spectra, brightness, size, etc., of comets, as well as celestial-mechanical data, are needed. As was pointed out elsewhere, the physical processes in comets have their influence on the motion of comets, so that celestial mechanics and astrophysics here are overlapping. In this report, however, attention is limited to the cometary problems that are obviously connected with celestial mechanics.

Russian and Soviet astronomers have, without doubt, contributed more to this problem than has any other national group. The first mathematical treatment of cometary forces was given by Bessel, in 1837, in connection with the appearance of Halley's Comet in 1835. This work remained largely unnoticed until F. A. Bredikhin (1831-1904) published an essay on it in 1862; he later developed Bessel's theory to such an extent that Soviet astronomers refer to the mechanical theory of cometary forms as the Bessel-Bredikhin Theory, and with considerable justification.

Bredikhin spent most of his active life at the Moscow Observatory; for a short time he was Director of the Pulkovo Observatory. He published over 100 papers on the theory of cometary forms, mostly in the Annals of the Moscow Observatory, but he never presented a full exposition of this subject. This was done by his friend and collaborator, R. Jaegermann, in his monumental (over 500 pages) essay Prof. Dr. Th. Bredichin's Untersuchungen über Cometenformen (St. Petersburg, 1903).

Bredikhin's 30 papers on the closely allied subject of the formation of meteors from comets were collected and published by Bredikhin himself under the title Etudes sur l'Origine des Meteors Cosmiques et la Formation de Leur Courants (St. Petersburg, 1903). This book was recently translated into Russian and published by the Academy of Sciences, U.S.S.R., under the title Study of Meteors (1954).

The further development in the U.S.S.R. of the subject of cometary forms is closely connected with the activity of S. V. Orlov, who published these books on the subject:

(1) Mechanical Theory of Cometary Tails, 1928, 83 pp.

(2) Comets, 1935, 196 pp.

(3) The Head of the Comet and Classification of Cometary Forms, 1945, 91 pp.

Hundreds of papers on the mechanical theory of comets have been published by S. V. Orlov and others in the U.S.S.R. Cognizance cannot be taken of all these papers and only the most important ones will be discussed below.

The complexity of the motion of particles in comets is somewhat alleviated by the fact that only the final result of this motion is seen, as in outlines of the head or tail of a comet, or in the gradual recession of condensations in the tail. These condensations cannot be measured very precisely because of their diffuseness, and the formulas can be simplified to meet this situation. From the determination of the velocity of ejection of the particles from the nucleus, it can be shown that the motion of the ejected particles must be very nearly in the plane of the orbit of the nucleus. Therefore, two of the necessary elements, i and Ω , are identical to those of the nuclear orbit, and the problem is then to determine the elements a, e, ω , T, and the unknown quantity, $1 - \mu$.

The first problem is to project the observed positions of the portions of the comet to the plane of the orbit of the nucleus. It is rather surprising that, in view of the already satisfactory methods developed by Bessel, four more methods were produced by

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A. Ya. Orlov (Byulleten' Akademii Nauk, 1909, Nr 4, p 299), S. V. Orlov (Trudy Gosudarstvennyy Astrofizicheskiy Institut, 1928, Vol 4, pp 4, 35), N. D. Moiseyev(421) (1924), and S. K. Vsekhsvyatskiy(795) (1929). None of these methods has any distinct advantages over the others.

It can further be shown that the path of a particle ejected from the nucleus of a comet and subject to the repulsive force of the sun will depend on the value of μ. If this value is exactly zero, the motion will be rectilinear; if it is less than zero, the particle will be moving in the branch of the hyperbola concave to the sun; and if it is greater than zero, the motion will be in the branch of the hyperbola convex to the sun.

All these cases occur in practice, the most interesting and the most usual case being motion in the branch of the hyperbola convex to the sun.

A contribution to this subject was made by A. Ya. Orlov in his extensive monograph (Trudy Astronomicheskoye Observatorii Yur'yeoskoye Universiteta, 1910, Vol 21, p 3). His method of determining the repulsive force of the sun (corresponding to the Laplacian method of determining an ordinary orbit) has been used by Soviet investigators to obtain seemingly reliable values of the force which might be of importance in the interpretation of physical conditions in comets. Modifications of this point of view, proposed by Moiseyev(423,424) (1925) and by Reyn(629) (1930), amounted to the application of the Gaussian method; this proved to be of little use in practice. However, a critique of A. Ya. Orlov's method shows that it is not so reliable as it might seem, and, in fact, the range of solution involving the orbital elements and the value of $1 - \mu$ is many times greater than the probable errors of $1 - \mu$. Nevertheless, A. Ya. Orlov's method is the only one in existence that gives at least approximate values of the repulsive force.

Another method of investigation is to compute the general outlines of the tail and the head of the comet, assuming a certain velocity of ejection and a certain value for the repulsive force of the sun acting on the particles. If all the particles ejected at various times are moving under the same repulsive force, the equation of a curve called the syndyname is obtained, which includes only the first terms of the expansion. If a number of particles ejected at the same time are subject to various values of the repulsive force, the equation of the curve called the synchrone results. Finally, the equation for the outline of the head of the comet, resulting from the constant emission of particles from the nucleus, is obtained. The methods of applying these formulas to the visual or photographic observation of comets have been worked out mainly by S. V. Orlov; they have been used by him and his pupils, in the case of a number of comets, without due caution and a critical attitude toward the results. Certain relationships (such as the occurrence of a multiple value of the repulsive force) were established, but these relationships never have been accepted by the astronomical world at large. It was not sufficiently realized by Orlov that it is impossible now to talk just of the repulsive force, or of particles in general.

It is somewhat remarkable that so many papers on this subject were written by Soviet astronomers. (S. V. Orlov himself accounted for at least 50 papers on the subject of the mechanics of comets; practically all of them appeared in the period 1925 to 1940.) Of the more recent work on this problem, only the investigations of O. V. Dobrovol'skiy (in the publication of the Stalinabad Observatory) deserve notice. A serious attempt to combine astrophysical data with the Bessel-Bredikhin theory of the structure of comets is in evidence in his work.

Even now, the contributions of Soviet astronomers to the problem of the structure of comets are impressive quantitatively, but they have generally adopted other methods of approach, i.e., spectroscopic analysis (Poloskov), photometric data (Bakharev, Vorontsov-Vel'yaminov, and Martynov), or the detailed study of orbits of the nucleus (Dubyago). Apparently, the attempt to derive by mathematical analysis the answer to the problem of comets has been abandoned, at least for the time being. S. V. Orlov did not succeed in establishing a school to continue the traditions of Bredikhin.

(2). Other Bodies

The ratio, μ , of the light pressure to the force of gravity depends on the size of the particle; for particles greater than one micron in diameter, it is negligibly small. This is quite correct for any particular moment of observation, yet the cumulative effect of this very small force during the long period of the existence of the

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solar system may be profound. This is the point of view generally accepted in science; the problem of tracing the influence of light pressure over millions or billions of years, however, is extremely difficult.

In the U.S.S.R., some attention has been paid to this problem, especially from the cosmogonical point of view. V. V. Radziyevskiy, in a series of recent papers, applied this idea directly to the problem of celestial mechanics.

The effect of the braking influence of light pressure on a particle moving around the sun was considered first by J. Poynting, some 50 years ago, and more recently by H. Robertson (Monthly Notes, Royal Astronomical Society, 1921, Vol 97, p 423). It is now known as the Poynting-Robertson effect.

Radziyevskiy(609) (1952) began with the idea that the isotropic radiation postulated by Poynting and Robertson does not occur in nature. The forward and backward surfaces of any material body rotating on its axis and revolving around the sun will exhibit a temperature difference. The result will be an acceleration of the orbital motion of the body in the case of a body rotating in the same sense as it is revolving, and a deceleration if the rotation is in the opposite sense from the revolution. As a result of the conservation of angular momentum, bodies with direct rotation will approach the sun, and bodies with reverse rotation will recede from the sun.

Applied to the motion of asteroids, this means that in two-billion years (the minimum possible age of the solar system), the asteroidal ring must have expanded to such an extent that asteroids with a diameter of 100 km should be in the neighborhood of the earth. Since this is contrary to observation, the conclusion is that the asteroidal ring is of more recent origin, that is, it is a result of some celestial catastrophe, such as the disruption of a planet.

These ideas are further developed by Radziyevskiy(610) (1952) and applied to the problem of Saturn's rings, which consist of small particles (1 cm or so). The effect of light pressure on these particles is calculated, and the age of the rings is deduced to be between 0.7 and 2.1-billion years.

The effect of the shape of the particles on the radiation pressure experienced by them was investigated by Radziyevskiy in another paper(612). It can be shown that, for a perfectly spherical black body of relative surface area σ in cm^2/g, the relationship $\tau = 2 \times 10^7 \frac{R^2}{\sigma}$ exists, where R is the radius of the circular orbit described by the body around the sun, and τ is the length of time, in years, in which the body will fall into the sun. This formula was derived by Robertson and also by Fesenkov(192) (1946). Radziyevskiy considers bodies of different shapes and comes to the conclusion that the value of τ is substantially the same, no matter what the shape of the body. The problem of the disintegration of meteors under light pressure was also treated by Radziyevskiy(615).

In other papers, Radziyevskiy considers(611)(1953) the case of a limited problem of three bodies in a radiational as well as a gravitational field; he applies his results to the concrete case of sun-Jupiter-particle, and even to the problem of the origin of parabolic comets. His analysis seems correct, but his entire idea of the possible influence of radiation on libration centers may be exaggerated.

Fesenkov applied the idea of light pressure to the problems of zodiacal light and small meteoric particles in many papers, including Reference 192. His treatment does not show much originality, and it is not to be compared in scope with Radziyevskiy's.

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