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Table of Contents

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PASSAGE OF SIGNAL AND NOISE THROUGH A LIMITER AND DIFFERENTIATING SYSTEM

by

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Full Member of the Society

The mean number of pulses and the law of distribution of pulse height obtained as a result of the passage of a sinusoidal signal accompanied by noise through a limiter and a differentiating system, are determined.

1. In this paper the following problem is considered. The voltage at the input of a system consisting of a limiter and a differentiating system is composed of a sinusoidal voltage (signal) and a voltage of fluctuating character (noise), which obeys the normal law of probability distribution. The spectrum of the fluctuations is concentrated mainly in a relatively narrow frequency band. Within the limits of this same band lies the signal frequency.

At the output of the system, as a result of the limiting and differentiation, pulses are formed, whose height, owing to the presence of noise, is distributed according to a certain law. The mean number of pulses in unit time whose height exceeds an assigned value is determined, together with the law of distribution of pulse height. The assigned values are the effective values of the sinusoidal signal and noise at the input.

It is assumed in the calculation that the voltage at the output of the differentiating system is proportional to the derivative of the voltage at its input. For this reason the following discussion is applicable directly to the practical case, under the condition that the time constant of the actual differentiating system is sufficiently small.

2. The variations with time of the quantities with which we are concerned are illustrated schematically by the curves of Fig.1. A straight line parallel to the

axis of abscissas, marks the level of limiting. The curve of U (the dashed line) represents the time-dependence of the total voltage at the limiter input. The curve of voltage at the limiter output (the heavy continuous line) is formed of segments of the curve U and of rectilinear portions parallel to the time axis (and located at the distance a away from it, that distance being equal to the level of limiting), and of

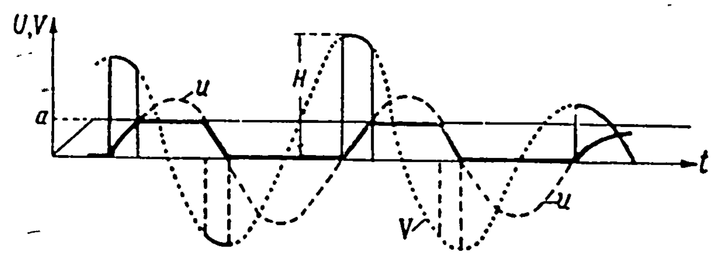

Fig.1

portions coinciding with that axis. The same figure shows the pulses formed as a result of the differentiation of the voltage after its limiting. The pulse height is denoted by H. If there were no limiting (more exactly, if the level of limiting $a \to \infty$), then the voltage at the output of the differentiating system would not be of pulse character, but would vary by a law represented by the curve V (the dotted line). The vertexes of the pulses obtained in the presence of the limiter are formed by segments of the curve V.

3. From here on we shall assume that the spectrum of fluctuation at the input is concentrated, in the main, in a relatively narrow band of frequencies, that is, if $\Delta\Omega$ is the effective width and Ω_0 the mean frequency of the spectrum at input, then $\frac{\Delta\Omega}{\Omega_o} \ll 1$ (in practice, $\Delta\Omega$ is equal to the effective pass-band width of the narrow band filter connected at the input of the device and separating a definite spectral band of the fluctuation voltage). The curve of time-dependence of such voltage, having a narrow spectral band, is approximately sinusoidal in character,

STAT

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that is, it has the form of a distinct sinusoid which frequency is equal to the mean frequency of the spectrum. This fact is known from experiment (cf., for example, reference 1), and is easily justified theoretically. The curve of total voltage at the input, U, has the same character, and is made up of the fluctuation voltage with a narrow spectral band, and the pure sinusoidal signal voltage. The same thing may be asserted with respect to the character of the time dependence of the voltage V (cf.Fig.1) obtained as a result of differentiating the quantity.

Thus the pulse heights H, which would be produced in the presence of limitation (a being finite) are equal to the maximum values (amplitudes) of the approximately sinusoidal quantity V which would be observed at the output of a differentiating network in the absence of limitation ($a = \infty$).

Thus the problem of determining the law of distribution of pulse height H reduces to the linear problem of the distribution of the maxima (amplitudes) of the approximately sinusoidal quantity V, representing the result of the differentiation of the approximately sinusoidal quantity U. Thus, in solving this problem, the limiter is completely eliminated from the discussion. We remark merely that the value of the level at which a is limited obviously affects only the pulse duration.

4. To determine the mean number of maxima of H (that is, the mean number of pulses) exceeding a certain value, let us use at first a method (Bibl.2) directly based on the fact that the curve V has the form of a "sinusoid" of frequency equal to Ω_0, and of slowly varying amplitude and phase.

Let us write the expression for the probability density of the quantity H, in other words, for the probability that the value of the maximum of the curve V lies between H and H + dH. If the signal were absent, then the probability sought would be equal to

$$W_{norm}(H)\,dH = \frac{H}{\sigma_V^2} e^{-\frac{H^2}{2\sigma_V^2}}\,dH, \tag{1}$$

since the latter expression represents, as is commonly known, the distribution law for the envelope of the quantity V (Bibl.3) if V obeys the normal law. The quantity

STAT

σ_V in the formula (Bibl.1) represents the mean-square value of V, $\sigma_V = \sqrt{\overline{V^2}}$. In the presence of the signal, the quantity V is the sum of the fluctuation and sinusoidal voltages, because such a sum is the full voltage U at the input of the differentiating network. We recall that the problem has been reduced to a linear problem. The limiter has been "eliminated" (cf. above, Section 3). That is, if at the input

$$U = U_n + E_s \sin \Omega_0 t, \tag{2}$$

where U_n is the voltage of the fluctuating component and E_s is the signal amplitude, then at the output we shall have

$$V = V_n + H_s \cos \Omega_0 t, \tag{3}$$

where

$$V = \dot{U}, \quad V_n = \dot{U}_n \text{ and } H_s = \Omega_0 E_s \tag{4}$$

(the dot on top denotes differentiation with respect to time).

In the presence of a signal, the law of distribution for the amplitude (envelope) of the value will have the form (Bibl.3, pages 296(21) and 339(50):

$$W_{norm}(H)\, dH = \frac{H}{\sigma_V^2} e^{-\frac{H^2 + H_s^2}{2\sigma_V^2}} I_0\left(\frac{H_s}{\sigma_V^2} H\right) dH, \tag{5}$$

where $I_0(x)$ is the Bessel function of zero order of the imaginary argument. Equation (1) is a special case of eq.(5) for $H_s = 0$.

According to what has been said earlier, eq.(5) represents the law of pulse distribution according to height (cf. Fig.1).

For simplicity, we now introduce the notation:

$$h = \frac{H}{\sigma_V}, \quad h_s = \frac{H_s}{\sigma_V}. \tag{6}$$

Then the probability of the quantity h lying between h and h + dh will be (cf.eq.5) equal to

$$W_h(h)\, dh = h e^{-\frac{1}{2}(h^2 + h_s^2)} I_0(h_s h)\, dh. \tag{5a}$$

The transition from eq.(5) to eq.(5a) may be considered as a selection of the units of measurement of V such that $\overline{V^2} = 1$. We shall likewise term the normed value of h the pulse height.

STAT

5. The probability $P(h_0)$ that the pulse height exceeds the value h_0, i.e., the probability of the inequality $h > h_0$, will be, on the basis of eq.(5a):

$$P(h_0) = e^{-\frac{h_s^2}{2}} \int_{h_0}^{\infty} h\, e^{-\frac{1}{2}h^2} I_0(h_s h)\, dh =$$

$$= 1 - e^{-\frac{h_s^2}{2}} \int_{0}^{h_0} h\, e^{-\frac{1}{2}h^2} I_0(h_s h)\, dh, \qquad (7)$$

since (Bibl.4)

$$\int_0^{\infty} h\, e^{-\frac{1}{2}h^2} I_0(h_s h)\, dh = e^{\frac{h_s^2}{2}}.$$

For $h_0 = 0$, the value of $P(h_0)$ will be equal, as indeed it should, to $P(0) = 1$. (It should be noted that eq.(7) is in agreement, as indeed it should be, with the definition of opposite events).

Let us now denote by $N_1(h_0)$ the mean number of pulses in unit time whose height exceeds an assigned value h_0. Then the mean number of pulses during the period $T = \frac{2\pi}{\Omega_0}$ will be equal to $N_T(h_0) = N_1(h_0)\,T$. During the course of any time interval of duration T, one pulse occurs (since the number of pulses in unit time, equal to the number of maxima of the "sinusoid", is equal to the frequency $\frac{1}{T} = \frac{\Omega_0}{2\pi}$). For this reason the number of pulses, during a period, whose height exceeds a certain value $h_0 > 0$, may be equal either to 1 or to 0. The probability of the former event ($h > h_0$) equals $P(h_0)$, the probability of the latter event ($h < h_0$) equals $1 - P(h_0)$; and therefore the number of pulses $N_T(h_0)$, in time T, whose height exceeds h_0 will, by the definition of mean value, be equal to

$$N_T(h) = 1 \cdot P(h_0) + 0\,[1 - P(h_0)] = P(h_0) \qquad (8)$$

and the mean, related to unit time:

$$N_1(h_0) = \frac{1}{T} P(h_0) = \frac{\Omega_0}{2\pi} P(h_0)$$

and, on the basis of eq.(7):

$$N_1(h_0) = \frac{\Omega_0}{2\pi}\, e^{-\frac{h_s^2}{2}} \int_{h_0}^{\infty} h\, e^{-\frac{h^2}{2}} I_0(h_s h)\, dh \qquad (9)$$

The latter expression, in the special case where $h_s = 0$, takes the form (Bibl.2):

STAT

$$N_1(h_0) = \frac{\Omega_0}{2\pi} e^{-\frac{h_0^2}{2}}. \qquad (9a)$$

The mean-square value σ_V of the quantity $V_n = \dot{U}_n$ equals

$$\sigma_V = \Omega_{II} \sigma_U, \qquad (10)$$

where σ_U is the mean-square value of the fluctuation component U_n of the value of U at the input, i.e., $\sigma_U = \sqrt{\bar{U}_n^2}$, and Ω_{II} is the mean-square value of the spectrum of the quantity U_n, determined by the expression (Bibl.3), page 202):

$$\Omega_{II}^2 = \frac{1}{\sigma_U^2} \int_0^\infty \Omega^2 G_U(\Omega)\, d\Omega,$$

where $G_U(\Omega)$ is the spectral density of the quantity U_n. With a narrow spectral band of the quantity U_n, the quantity Ω_{II} may be put approximately equal to the signal frequency

$$\Omega_{II} = \Omega_0$$

(in the special case, the signal frequency Ω_0 may be exactly equal to Ω_{II}).

On the basis of eq.(10), (6), and (4) we may now write, for the quantity h_s, entering into eq.(9), the following formula:

$$h_c = \frac{H_s}{\sigma_V} = \frac{\Omega_c E_s}{\Omega_{II} \sigma_U} \approx \frac{E_c}{\sigma_U} = \sqrt{2}\, Q. \qquad (11)$$

The latter equation expresses h_s in terms of the quantities determining the signal and noise at the input. Q here denotes the signal-noise ratio at the input:

$$Q = \frac{\frac{E_s}{\sqrt{2}}}{\sigma_U}. \qquad (12)$$

6. In deriving eq.(9) for the mean number of pulses with height exceeding an assigned value, we start out from the assumption that the law of distribution of pulse height is determined by eq.(5a) (or by eq.(5), as the case may be). The truth of this assumption is justified by considerations of a physical nature (Bibl.2). We shall now present in brief another derivation of eq.(9), which is free from this assumption, and we shall then obtain eq.(5a) as a consequence of eq.(9).

For our derivation, we shall make use of the general expression for the mean number of peaks of the stochastic quantity v, exceeding a certain level $v = h_0$. This expression has the form (Bibl.3, p. 329):

$$N_1(h_0) = \int_0^\infty \dot{v} W_{v,\dot{v}}(h_0, \dot{v})\, d\dot{v}, \qquad (13)$$

STAT

where $W_{v,\dot{v}}$ is the two-dimensional probability density of the quantity v under consideration and of its derivative with respect to time, $\dot{v}$. In our case, we are discussing the peaks of the quantity v (cf.supra) exceeding the level h_0. We shall understand the quantities $\dot{v}$ and h_0 to mean, respectively, (cf.eq.(6)

$$v = \frac{V}{\sigma_V}, \qquad h_0 = \frac{H_0}{\sigma_V}. \tag{14}$$

In the case with which we are concerned, the quantity V, or v as the case may be, represents, in consequence of the presence of a signal, a nonstationary stochastic (fluctuation) process, and eq.(13), representing the statistical mean, will depend on time. For this reason, to obtain the total mean (both statistical and with respect to time) we must in general, take the mean of the right side of eq.(13) with respect to time. Then, for the mean number of peaks of the quantity v, exceeding the level $v = h_0$, i.e., in our case, for the mean number of pulses of height exceeding the level $h = h_0$, we shall have the expression

$$N_1(h_0) = \widetilde{\int_0^\infty \dot{v} W_{v,\dot{v}}(h_0 \dot{v}) d\dot{v}}, \tag{15}$$

where the wavy line indicates averaging through time.

7. Let us find at first an expression for the two dimensional probability density $W_{v,\dot{v}}$ of the quantity V and its derivative $\dot{v}$ with respect to time. We have (cf.eq.(3))

$$V = V_n + V_s, \tag{16}$$

where V_n and V_s are, respectively, the fluctuation and sinusoidal components of the quantity V.

$$V_c = H_s \cos \Omega_0 t = \Omega_0 E_s \cos \Omega_0 t. \tag{17}$$

For the derivative $\dot{V}$, we have

$$\dot{V} = \dot{V}_n + \dot{V}_s. \tag{18}$$

The expression for the two-dimensional probability density of the quantities V_n and $\dot{V}_n$, under the normal law of distribution, will have the form (Bibl.3):

$$W_{V_n \dot{V}_n} = \frac{1}{2\pi \Omega'_{II} \sigma_V^2} \exp\left[-\frac{1}{2\sigma_V^2}\left(V_n^2 + \frac{\dot{V}_n^2}{\Omega_{II}^{'2}}\right)\right], \tag{19}$$

where $\sigma_V = \sqrt{\overline{V_n^2}}$ = mean-square value of V_n and Ω_{II}^2 = mean-square frequency, determined

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by the equation

$$\Omega_{II}'^2 = \frac{1}{\sigma_V^2} \int_0^\infty \Omega^2 G_V(\Omega)\, d\Omega.$$

where $G_V(\Omega)$ = spectral density of the quantity V_n.

It follows from eq.(19) that the expression for the two-dimensional probability density of the quantities V and $\dot{V}$ will be

$$W_{V,\dot{V}} = \frac{1}{2\pi\Omega_{II}'\sigma_V^2} \exp\left\{-\frac{1}{2\sigma_V^2}\left[(V - V_s)^2 + \frac{(\dot{V} - \dot{V}_s)^2}{\Omega_{II}'^2}\right]\right\}. \tag{20}$$

The probability density for the quantities v and $\dot{v}$ (cf.eq.(14)) on the basis of eq.(20), will have the form:

$$W_{v,\dot{v}} = \frac{1}{2\pi\Omega_{II}'} \exp\left\{-\frac{1}{2}\left[(v - v_s)^2 + \frac{(\dot{v} - \dot{v}_s)^2}{\Omega_{II}'^2}\right]\right\}. \tag{21}$$

Here the following notation has been introduced:

$$v_s = \frac{V_s}{\sigma_V} = h_s \cos\Omega_0 t, \quad \dot{v}_s = \frac{\dot{V}_s}{\sigma_V} = -\Omega_0 h_s \sin\Omega_0 t, \tag{22}$$

where

$$h_s = \frac{H_s}{\sigma_V}.$$

8. By substituting in eq.(15), for the function $W_{v,\dot{v}}$, its expression by eq.(21), we obtain (cf. also eq.(22))

$$N_1(h_0) = \frac{1}{2\pi\Omega_{II}'} \int_0^\infty \dot{v} \exp\left\{-\frac{1}{2}\left[(h_0 - v_s)^2 + \frac{(\dot{v} - \dot{v}_s)^2}{\Omega_{II}'^2}\right]\right\} d\dot{v} =$$
$$= \frac{1}{2\pi\Omega_{II}'} \int_0^\infty \dot{v} \exp\left\{-\frac{1}{2}\left[(h_0 - h_s\cos\Omega_0 t)^2 + \frac{(\dot{v} + \Omega_0 h_s \sin\Omega_0 t)^2}{\Omega_{II}'^2}\right]\right\} d\dot{v}. \tag{23}$$

Let us now assume, for simplicity, that

$$\Omega_0 = \Omega_{II}'. \tag{24}$$

Then, for the exponent of the integrand expression of eq.(23), we get

$$(h_0 - h_s\cos\Omega_0 t)^2 + \frac{1}{\Omega_{II}'^2}(\dot{v} + \Omega_0 h_s \sin\Omega_0 t)^2 =$$
$$= h_s^2 + h_0^2 + \frac{\dot{v}^2}{\Omega_0^2} - 2h_s\left(h_0\cos\Omega_0 t - \frac{\dot{v}}{\Omega_0}\sin\Omega_0 t\right) =$$
$$= h_s^2 + h_0^2 + \frac{\dot{v}^2}{\Omega_0^2} - 2h_s\sqrt{h_0^2 + \frac{\dot{v}^2}{\Omega_0^2}}\cos(\Omega_0 t + \varphi).$$

We shall not write the expression for the phase φ, since only the fact that does not depend on the time is of substantial importance for us.

Equation (23) is now written in the following way:

STAT

$$N_1(h_0) = \frac{1}{2\pi\Omega_0} e^{-\frac{h_s^2}{2}} \int_0^\infty \dot{v}\, e^{-\frac{1}{2}\left(h_0^2 + \frac{\dot{v}^2}{\Omega_0^2}\right)} e^{h_s \sqrt{h_0^2 + \frac{\dot{v}^2}{\Omega_0^2}} \cos(\Omega_0 t + \varphi)} d\dot{v}$$

or, after the substitution:

$$h_0^2 + \frac{\dot{v}^2}{\Omega_0^2} = h^2,$$

$$N_1(h_0) = \frac{\Omega_0}{2\pi} e^{-\frac{h_s^2}{2}} \int_{h_0}^\infty h\, e^{-\frac{h^2}{2}} \cdot e^{h_s h \cos(\Omega_0 t + \varphi)} dh. \tag{25}$$

On the basis of the well known integral expression for a Bessel function

$$I_0(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos x} dx$$

we have

$$\overline{e^{h_s h \cos(\Omega_0 t + \varphi)}} = \overline{e^{h_s h \cos \Omega_0 t}} = \frac{1}{\pi} \int_0^\pi e^{h_s h \cos x} dx = I_0(h_s h). \tag{26}$$

By virtue of eq.(26), after averaging over time, the integrand in eq.(25) takes the form

$$N_1(h_0) = \frac{\Omega_0}{2\pi} e^{-\frac{h_s^2}{2}} \int_{h_0}^\infty h\, e^{-\frac{h^2}{2}} I_0(h_s h)\, dh. \tag{27}$$

The expression so obtained for the mean number of pulses agrees with that found earlier by a different method (cf.Section 4, eq.(9)).

9. Equation(5a) for the law of pulse-height distribution follows from eq.(9).

Indeed, let n pulses occur in unit time, where n is a random quantity. If W_h is the pulse-height probability density, then the mean number of pulses per unit time exceeding h in height, is equal to

$$N_1(h) = \bar{n} \int_h^\infty W_h(x)\, dx, \tag{28}$$

where $\bar{n}$ = mean number of pulses in unit time, which is obviously equal to $\bar{n} = N_1(0)$.

It follows from eq.(28) that

$$W_h(h) = -\frac{1}{\bar{n}} \frac{dN(h)}{dh} = -\frac{1}{N(0)} \cdot \frac{dN(h)}{dh} \tag{29}$$

and on the basis of eq.(27) (or of eq.(9)), the expression for the wanted pulse-height probability density will be

$$W_h(h) = h\, e^{-\frac{h^2 + h_s^2}{2}} I_0(h_s h), \tag{30}$$

STAT

which agrees with eq.(5a).

10. Equations(30) and (27), or eqs.(5a) and (9), so obtained, represent the solution of the problem posed (cf. also eqs.(11), (12) and (14)).

Thus the law of pulse height distribution is defined by the expression of eq.(30)

$$W_h(h) = h\, e^{-\frac{h^2 + h_s^2}{2}} I_0(h_s h) \qquad \text{(I)}$$

and the mean number of pulses related to unit time and exceeding the value h_0 in height, is equal to

$$\left.\begin{aligned} N_1(h_0) &= \frac{\Omega_0}{2\pi} N_T(h_0) \\ N_T(h_0) &= e^{-\frac{h_s^2}{2}} \int_{h_0}^{\infty} h\, e^{-\frac{h^2}{2}} I_0(h_s h)\, dh = \\ &= 1 - e^{-\frac{h_s^2}{2}} \int_{0}^{h_0} h\, e^{-\frac{h^2}{2}} I_0(h_s h)\, dh \end{aligned}\right| \qquad \text{(II)}$$

(cf.eq.(7)). The quantity $N_T(h_0)$ is equal to the mean number of pulses for the period $T = \frac{2\pi}{\Omega} = \frac{1}{f_0}$, of a height exceeding the value h_0.

At great values of h, or of h_s, at which $h_s h \gg 1$ (cf.eq.(I)), or (cf.(II)), the well known asymptotic expression for a Bessel function may be used:

$$I_0(x) \sim \frac{e^x}{\sqrt{2\pi x}} \quad (x \gg 1).$$

Then the expression for the distribution law of eq.(I) may be represented in the form

$$W_h \approx \frac{1}{\sqrt{2\pi}} \sqrt{\frac{h}{h_s}}\; e^{-\frac{(h-h_s)^2}{2}} \quad (h_s h \gg 1), \qquad \text{(Ia)}$$

while the expression for the mean number of pulses for a period may be expressed by

$$N_T(h_0) = \frac{1}{\sqrt{2\pi}} \int_{h_0}^{\infty} \sqrt{\frac{h}{h_s}}\; e^{-\frac{(h-h_s)^2}{2}}\, dh. \qquad \text{(IIa)}$$

By performing the exchange of the integration variable $x = h - h_s$ in the latter equation, we get

$$N_T(h_0) = \frac{1}{\sqrt{2\pi}} \int_{h_0 - h_s}^{\infty} \sqrt{1 + \frac{x}{h_s}}\; e^{-\frac{x^2}{2}}\, dx. \qquad \text{(II'a)}$$

At $h_0 < h_s$ (i.e., for $h_0 - h_s < 0$), and at sufficiently great values for h_s, the following approximation will hold:

STAT

$$N_T(h_0) \approx \frac{1}{\sqrt{2\pi}} \int_{-(h_s-h_0)}^{\infty} e^{-\frac{x^2}{2}} dx = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_{0}^{h_s-h_0} e^{-\frac{x^2}{2}} dx. \qquad (31)$$

11. Equations (I) and (II) are graphically represented in Figs.2-6.

Figure 2 gives curves for the pulse-height probability density $h = H/\sigma_v$ at various values of h_s, i.e., H_s representing the pulse height in the absence of noise (or in other words, at various values of the signal amplitude E_s at the input, or, more accurately, at various values of the signal-noise ratio at the input (cf.eq.11). The curves are constructed by eq.(1). The curves for $h_s \leq 5$ are constructed by the approximate (asymptotic) eq.(Ia). As h_s increases, i.e., as the signal amplitude increases, the most probable values of the pulse height (i.e., the values of H, corresponding to the maxima of probability density) approach, as indeed was to be expected, the corresponding values $H_s = \Sigma_v h_s$, equal to the pulse height in the absence of noise.

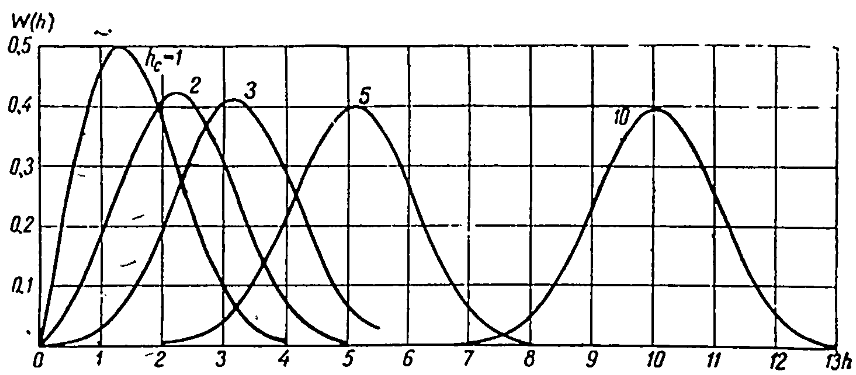

Fig.2

On Fig.3 the curves are plotted (solid lines) for the mean number of pulses exceeding the value h_0 in a period, or in other words, $H_0 = \sigma_v h_0$ is plotted against the ratio $\frac{h_o}{h_c}$, i.e., of the ratio of H_0 to the height H_s, which the pulses would have in the absence of noise ($\frac{h_o}{h_s} = \frac{H_o}{H_s}$). The mean number of pulses during the period represents, in other words, the ratio between the mean number of pulses for any desired time interval in the presence of noise to the mean number of pulses for

STAT

the same time interval in the absence of noise. The curves are constructed by eq.(II) by means of numerical integration. The curves for $h_s \geq 5$ are constructed by the aid of the approximate formula (IIa). At $h_s \to \infty$ ($Q \to \infty$ or $\sigma_U \to 0$), i.e., in the limit, in the absence of noise, the form of the curve for the mean number of pulses N_T plotted against h_0 tends, as was to be expected, to rectangular. Namely

$$N_T(h_0) = \begin{Vmatrix} 1 & at & \frac{h_0}{h_s} < 1, \\ 0 & at & \frac{h_0}{h_s} > 1, \end{Vmatrix} \quad (32)$$

if $h_s \to \infty$. Equation(II) in the limit as $h_s \to \infty$ passes over into eq.(32). Indeed,

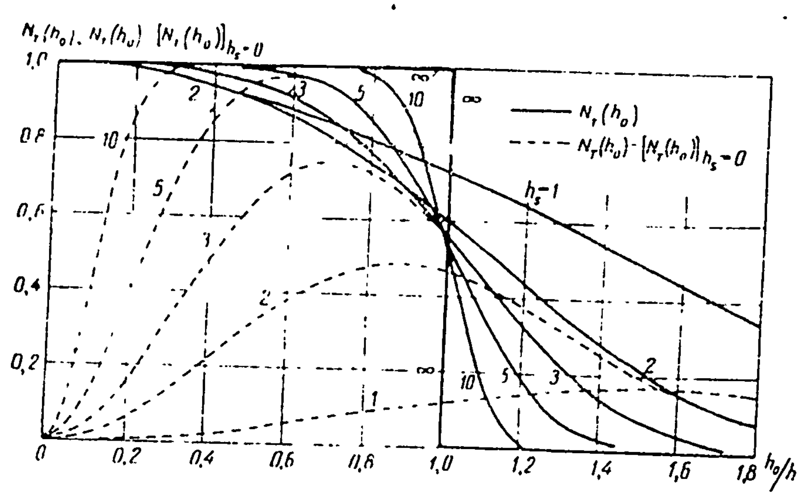

Fig.3

at $\frac{h_o}{h_s} < 1$, i.e., at $h_s - h_0 > 0$, and at $h_s \to \infty$, we have

$$N_T(h_0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = 1.$$

On the other hand, at $\frac{h_o}{h_s} > 1$, i.e., at $h_0 - h_s > 0$ and $h_s \to \infty$, the value of $N_T(h_0)$ approaches 0, as follows directly from the approximate eq.(IIa).

On Fig.3 are also constructed (with dashed lines) curves for the quantity $N_T(h_0) - [N_T(h_0)]h_s = 0$, equal to the difference between the mean value (for the period) of the number of pulses in the presence of the signal on the corresponding quantity in the presence only of noise. This difference is obviously equal to the ratio $\frac{N(h_o) - [N(h_o)]_{h_c = 0}}{[N(h_o)]_{\sigma_U = 0}}$, which, in a certain sense, represents the signal-

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noise ratio at the output. In this connection the fact that these curves have maxima must be taken into account. At large values of the ratio $\frac{h_o}{h_s} = \frac{H_o}{H_s}$, the dashed curves for the difference $N_T(h_O) - [N_T(h_O)]h_s = 0$ tends to coincide with the curves for the quantity $N_T(h_O)$.

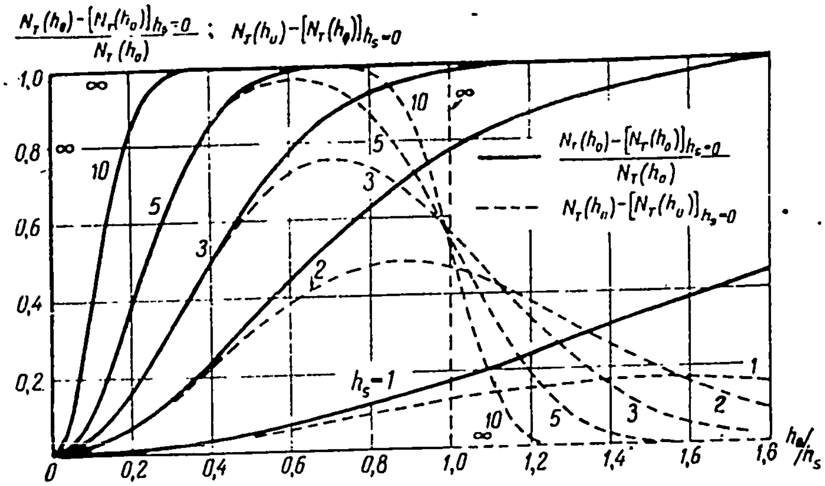

Fig.4

On Fig.5, curves for the same quantities are constructed as in Fig.3, but here h_O is the independent variable. The curves given in Fig.4 (solid lines) for the ratio

$$\frac{N_T(h) - [N_T(h_0)]_{h_s=0}}{N_T(h_0)} = 1 - \frac{[N(h_0)]_{h_s=0}}{N(h_0)}$$

likewise in a certain sense characterize the signal-noise ratio (more exactly, the

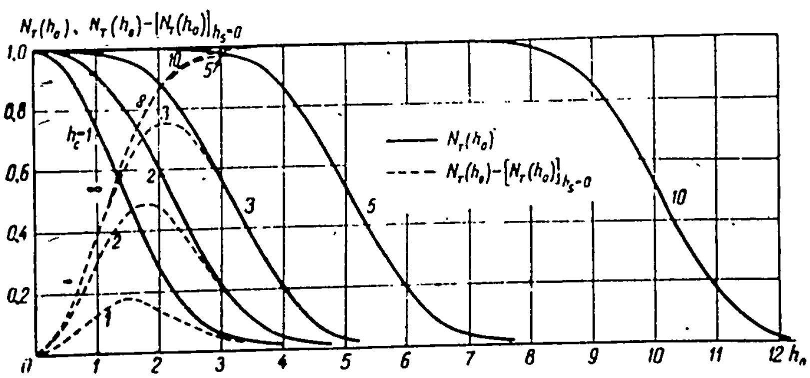

Fig.5

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signal noise ratio at the output). The ratio $\frac{N(h_0)}{[N(h_0)]_{h_c=0}}$, , representing the necessary signal-noise ratio, tends to increase without limit when the level of h_0 increases without limit, i.e., at a signal as weak as may be assigned; the ratio of the mean number of pulses of height exceeding a certain value h_0, in the presence of a signal, to the mean number of such pulses, in the absence of a signal, tends to increase without limit as h_0 increases. Of course, however, the actual number of pulses in both cases does approach zero.

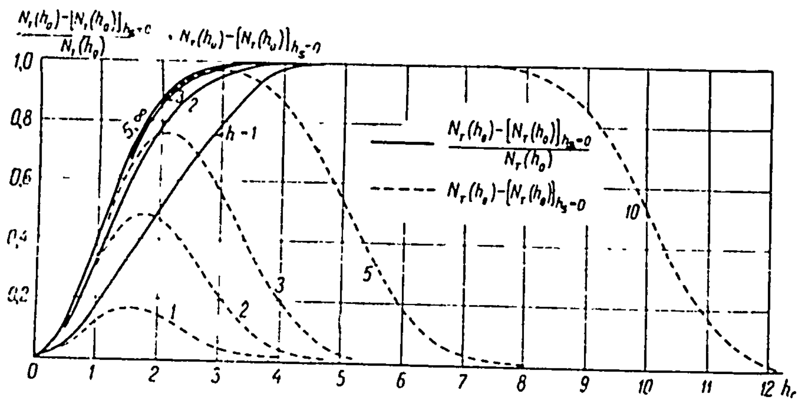

Fig.6

Figure 4 also gives curves (dashed lines) for the difference $N_T(h_0) - [N_T(h_0)]h_s = 0$ which were also represented in Fig.3. These curves at small values of the ratio $\frac{h_o}{h_s}$, coincide with the curves for the quantity $\frac{N_T(h_o) - [N_T(h_o)]_{h_s} = 0}{N_T(h_o}$.

Figure 6 represents the same quantities shown in Fig.4, graphically, as a function of h_0. As large values of h_s, the quantity $\frac{N_T(h_o) - [N_T(h_o)]_{h_s} \equiv 0}{N_T(h_o)}$ tends to coincide with the curve $1 - e^{-\frac{h_o^2}{2}}$ (beginning in practice with values of h_s equal, let us say, to 5).

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RECEPTION OF PULSE SIGNALS BY THE METHOD OF MUTUAL CORRELATION

by

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Full Member of the Society

The ratio of signal to fluctuation noise on the reception of pulse signals by the method of mutual correlation is determined; the ratio so obtained is compared with the corresponding ratio at the output of an ideal band filter.

The purpose of this paper is to analyze the noiseproof features on the reception of pulse signals by the method of mutual correlation (Bibl.1, 2, 3). From the block diagram of the receiving installation shown in Fig.1, it follows that the mixture of the useful signal $U_F(t)$ and the fluctuation noise $x_F(t)$ passing through the input filter, is multiplied by the signal of the local heterodyne $e(t)$. The product of these quantities is averaged by the means of an integrating device. Thus there exists at the output of the correlation receiver a discrete series of consecutive values of $y_T(\tau)$, which may be written in the form:

$$y_T(\tau) = \frac{1}{T}\int_0^T [u_F(t) + x_F(t)]\, e(t+\tau)\, dt = \frac{1}{T}\int_0^T u_F(t)\, e(t+\tau)\, dt + \frac{1}{T}\int_0^T x_F(t)\, e(t+\tau)\, dt = \varphi_{ueT}(\tau) + \varphi_{xeT}(\tau). \quad (1)$$

This quantity is the sum of the short-time functions of mutual correlation of the noise $x_F(t)$ and of the useful signal $u_F(t)$ with the signal of the local heterocyne $e(t)$. The noise at the output of the correlation receiver is represented by a function of mutual correlation between the noise and the local signal $\varphi_{xeT}(\tau)$, and the useful signal is represented by a function of mutual correlation between the useful and local signals $\varphi_{ueT}(\tau)$. Since the interval of averaging T is a finite quantity, $\varphi_{xeT}(\tau)$ and $\varphi_{urT}(\tau)$ are functions of the time, the function $\varphi_{xeT}(\tau)$ as the mean of a random quantity over the time in a finite interval is a stochastic func-

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tion of time and is characterized, as usual, by the mean square of the fluctuation. It follows from this that the signal-noise ratio at the output of the correlation receiver may be determined as the ratio of the square of the short-time function of the mutual correlation of the useful and local signals σ_s^2, calcutated at the instant of existence of the signal, to the mean square of fluctuation of the short-time function of mutual correlation of the noise and the local signal σ_x^2. It will be shown below that the quantity σ_x^2 decreases with increasing T; however, an increase of T leads to a reduction of accuracy in determining the position of the useful signal pulse on the time axis (the duration of the interval of averaging equals the maximum error in determining the instant of occurrence of the useful signal pulse). For this reason, by comparing the signal-noise ratio at the output of the correlation receiver with the corresponding ratio at the output of an ideal band filter, it is necessary to adopt the condition that the error in determining the position of the pulse on the time axis at the output of an ideal band filter shall equal the corresponding error at the output of the correlation receiver.

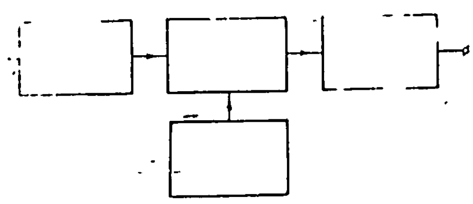

Fig.1

a) Input filter; $\Delta\omega_c$; b) Multiplier; c) Integrating (averaging) unit; d) Generator of local signal e(t)

Let a useful signal u(t), in the form of a segment of a sinusoid of duration and frequency ω_0, act on the input of a receiver with band $\Delta\omega_c$:

$$\left.\begin{aligned} u(t) &= E_c \cos\omega_0 t \quad \text{at} \quad -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ u(t) &= 0 \quad \text{at} \quad t < -\frac{\tau}{2} \text{ and } t > \frac{\tau}{2} \end{aligned}\right\} \qquad (2)$$

and let the fluctuation noise x(t), determined as a certained stationary stochastic process, likewise act on that input.

The transfer ratio of the filter at the receiver input will be:

$$\left.\begin{aligned} \kappa = 1 \text{ at } \quad & \omega_0 - \frac{\Delta\omega_{norm}}{2} \leq \omega \leq \omega_0 + \frac{\Delta\omega_{norm}}{2} \\ \kappa = 0 \text{ at } \quad & \omega < \omega_0 - \frac{\Delta\omega_\kappa}{2} \text{ and } \omega > \omega_0 + \frac{\Delta\omega_{norm}}{2} \end{aligned}\right\} \qquad (3)$$

The local signal, which frequency is always taken equal to the frequency of the local signal,

$$e(t + \tau) = E_e \cos(\omega_0 t + \varphi). \qquad (4)$$

Let us determine the noise at the output of the correlation receiver, i.e., the mean value of the square of the fluctuation of the short-time function of mutual correlation of the noise and the local signal σ_x^2.

It is well known (Bibl.4) that the mean value of the square of the fluctuations on repeated calculation of the mean value of some stochastic quantity $Z(t)$ with time, over a finite interval T, may be expressed in the form:

$$\sigma^2 = \frac{2}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) \{\varphi_{ZZ}(\tau) - [\overline{Z(t)}]^2\} \, d\tau. \qquad (5)$$

In our case $\varphi_{zz}(\tau)$ is a function of the autocorrelation of the product $x_F(t)\ e\ (t + \tau)$; $\bar{Z}(\bar{t})$ is a function of the mutual correlation of noise and local signal, which, by virtue of the statistical independence of noise and local signal, is equal to zero.

The function of autocorrelation of the products $x_F(t)\ e\ (t + \tau)$ may be represented (Bibl.5) as the product of the function of autocorrelation of the noise at the filter output eq.(3), and a function of autocorrelation of the local signal $e(t)$:

$$\varphi_{ZZ}(\tau) = E_e^2 b^2 \frac{\sin \frac{\Delta\omega_{norm}}{2}}{\tau} \cos^2 \omega_0 \tau, \qquad (6)$$

where b^2 is the noise power on unit band*.

Thus

*Here and below we shall consider the value of the power given off across a resistance of 1 ohm.

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$$\sigma_x^2 = E_i^2 b^2 \frac{2}{T}\int_0^T \left(1-\frac{\tau}{T}\right)\frac{\sin\frac{\Delta\omega_c}{2}\tau}{\tau}\cos^2\omega_0\tau\,d\tau. \qquad (7)$$

Integration of this expression gives:

$$\sigma_x^2 = \frac{E_i^2 b^2}{T}\left\{\mathrm{si}\,\frac{\Delta\omega_c T}{2} - \frac{2}{\Delta\omega_K T}\left(1-\cos\frac{\Delta\omega_c T}{2}\right)\right\} + \frac{E_i^2 b^2}{2T}\left\{\mathrm{si}\left[\left(\frac{\Delta\omega_c}{2}-2\omega_0\right)T\right] - \right.$$
$$\left. - \frac{1}{\left(\frac{\Delta\omega_c}{2}-2\omega_0\right)T}\left[1-\cos\left[\left(\frac{\Delta\omega_c}{2}-2\omega_0\right)T\right]\right]\right\} + \frac{E_i^2 b^2}{2T}\left\{\mathrm{si}\left[\left(\frac{\Delta\omega_c}{2}+2\omega_0\right)T\right] - \right.$$
$$\left. - \frac{1}{\left(\frac{\Delta\omega_c}{2}+2\omega_0\right)T}\left[1-\cos\left[\left(\frac{\Delta\omega_c}{2}+2\omega_0\right)T\right]\right]\right\}. \qquad (8)$$

At $\frac{\Delta\omega_c}{2} \ll 2W_0$, the sum of the last two terms of eq.(8) may be neglected, and then

$$\sigma_x^2 = \frac{E_i^2}{2} b^2 \Delta\omega_c\, \Phi\left(\frac{\Delta\omega_c T}{2}\right), \qquad (9)$$

where

$$\Phi\left(\frac{\Delta\omega_c T}{2}\right) = \frac{\left[\mathrm{si}\,\frac{\Delta\omega_c T}{2} - \frac{2}{\Delta\omega_c T}\left(1-\cos\frac{\Delta\omega_c T}{2}\right)\right]}{\frac{\Delta\omega_c T}{2}}. \qquad (10)$$

Figure 2 shows the values of $\Phi\left(\frac{\Delta\omega_c T}{2}\right)$.

The value of the short-time function of the mutual correlation of the useful and local signals $\varphi_{ue}T(\tau)$, calculated at the instant of existence of the useful signal, depends on the duration of the interval of averaging T and on the time shift between the beginning of the calculation $\varphi_{ueT}(\tau)$ and the instant of appearance of the local signal. The value of $\varphi_{ueT}(\tau)$ with equal probability takes on any values between zero and a maximum value $[\varphi_{ueT}(\tau)]_{max\ g}$, corresponding to the coincidence of the integration interval center with the useful signal pulse center. Thus the useful signal can be determined as the mean value of the square of the quantity $\varphi_{ueT}(\tau)$:

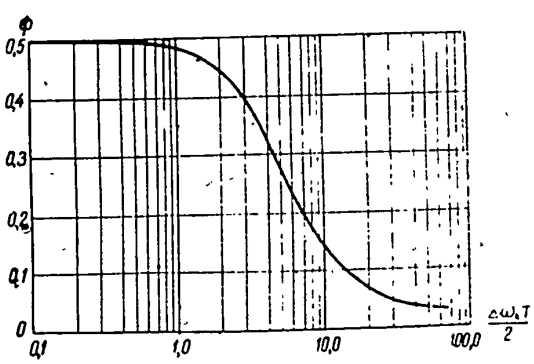

Fig.2

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$$[\varphi_{ucT}(\tau)]_{max} = \frac{2}{T}\int_0^{\frac{T}{2}} u_f(t)\,e(t+\tau)\,dt, \tag{11}$$

where $u_f(t)$ is the useful signal at the filter output. eq.(3).

On substituting the values of $u_f(t)$ and $e(t+\tau)$ in eq.(11) and integrating it, at $\Delta\omega_c\delta \geq 10.2$, with an error not over 10%, we obtain

$$\left.\begin{aligned} [\varphi_{ucT}(\tau)]_{max} &= \frac{E_1E_2}{2} \quad at \quad T \leqslant \delta \\ [\varphi_{ucT}(\tau)]_{max} &= \frac{\delta}{T}\cdot\frac{E_1E_2}{2} \quad at \quad T > \delta \end{aligned}\right\}. \tag{12}$$

At $T < \delta$, the probability density of the quantity $\varphi_{urT}(\tau)$, calculated at the instant of existence of the useful signal, equals

$$W(\varphi) = \frac{2}{E_1E_2}. \tag{13}$$

Thus the mean square value of the quantity $\varphi_{ueT}(\tau)$ is

$$\sigma_c^2 = \int_0^{\varphi_{max}} \varphi^2 W(\varphi)\,d\varphi = \frac{1}{3}\cdot\frac{E_1^2E_2^2}{4}. \tag{14}$$

Let us compare the signal-noise ratio at the output of the correlation receiver with the corresponding ratio at the output of an ideal band filter, with equal errors in the determination of the position of the useful signal pulse along the time axis. The error in determining the position of the pulse on the time axis, after passing through an ideal band filter with a band $\Delta\omega_f$, will be taken as equal to the build-up time of the signal:

$$t_y = \frac{2\pi}{\Delta\omega_f}. \tag{15}$$

The signal noise ratio at the output of an ideal filter with a band $\Delta\omega_f$, at $\Delta\omega_f\delta \geq 10.2$ may be represented with an accuracy within 10%, in the form (Bibl.3):

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$$a_f = \frac{\frac{E_c^2}{2}}{b^2 \Delta\omega_f}. \tag{16}$$

The signal-noise ratio at the output of the correlation receiver (cf.eqs.(9) and (14) is equal to

$$a_c = \frac{\frac{E_c^2}{2}}{b^2 \Delta\omega_c} \cdot \frac{1}{3\Phi\left(\frac{\Delta\omega_c T}{2}\right)}, \tag{17}$$

consequently,

$$\frac{a_c}{a_f} = \frac{2}{3} \cdot \frac{\Delta\omega_f}{\Delta\omega_c} \cdot \frac{1}{2\Phi\left(\frac{\Delta\omega_c T}{2}\right)}. \tag{18}$$

Taking account of the condition that the errors in determining the position of the pulse shall be equal, i.e., putting $T = \frac{2\pi}{\Delta\omega_f}$, we obtain, on expanding eq.(18):

$$\frac{a_c}{a_f} = \frac{0.67\pi}{2\left[\mathrm{si}\,\frac{\Delta\omega_c}{\Delta\omega_f}\pi - \frac{1}{\frac{\Delta\omega_c}{\Delta\omega_f}\pi}\left(1 - \cos\frac{\Delta\omega_c}{\Delta\omega_f}\pi\right)\right]}. \tag{19}$$

Figure 3 shows the relation between $\frac{\alpha_c}{\alpha_f}$ and $\frac{\Delta\omega_c}{\Delta\omega_f}$. It follows from this figure that the expansion of the band at the input of the correlation receiver with unchanged integration time T, or, what is the very same thing, with unchanged error in determining the position of the pulse along the time axis, leads to an insignificant adverse effect (of the order of 30%) on the signal-noise ratio. Taking into account the fact that the signal-noise ratio at the output of an ideal band filter is inversely proportional to the transmittance band (cf.eq.(16), it is easy to reach the conclusion that the advantage with respect to signal-noise ratio at the

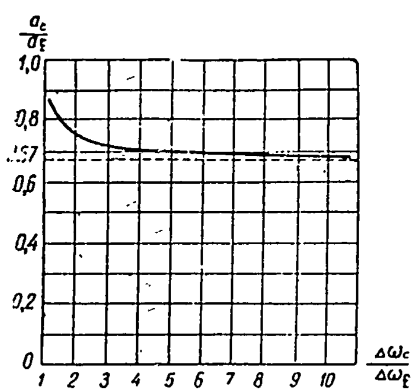

Fig.3

output of the correlation receiver in comparison to the corresponding ratio at the output of an ideal filter with the same band is proportional to the ratio between the band used, $\Delta\omega$, and the optimum band $\Delta\omega_o$:

$$\frac{a_c}{a_f} = 0,67 \frac{\Delta\omega}{\Delta\omega_0}. \quad (20)$$

By the optimum band we mean a band assuring a signal build-up time equal to the allowable error in determining the position of the pulse of the useful signal on the time axis.

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BUILD-UP PROCESSES IN DETECTION OF PULSE SIGNALS

by

A.A.Kulikovskiy,

Full Member of the Society

This paper considers a method of calculating the steady voltage of a pulse signal on the load across the detector load by the aid of the Duhamel integral for the slowly varying constant component. A method, convenient for integration, is proposed for the approximate determination of the DC component of the detector current with a broken-line approximation of the characteristic of the latter. By the aid of this method, simple expressions are found for the steady voltages across the load of plate, diode and cathode detectors in detecting pulses with a rectangular envelope. The application of the expressions so obtained to the calculation of states of detection of pulses with envelopes of other forms is discussed.

Several papers have been published in recent years devoted to the build-up processes in detecting pulse signals (Bibl.1-5). In several works (Bibl.1, 2) the study is performed by the method of fitting differential equations. L.S.Gutkin (Bibl.3, 4) has obtained a solution, by graphic integration, of an equation set up on the basis of the method of slowly varying amplitudes. This same method is used in another paper (Bibl.5). The expressions obtained in these papers are complicated and inconvenient for practical use.

We present below a study of the build-up processes in plate, diode, and cathode detectors, based on a relation of the type of the Duhamel integral for the DC components, using a broken-line approximation to the coefficients of the expansion. This method leads to approximate expressions that are simple and convenient for practice.

A circuit consisting of the resistance R and the capacitance C, connected in parallel, (Fig.1) serves as the detector load. The time constant RC is taken as many times longer than the period of the detacted oscillations $T = \frac{\omega}{2c}$:

$$RC \gg T. \tag{1}$$

The contact resistance of such a circuit $r(t) = \frac{U_0(t)}{I \cdot 1(t)}$ is equal to

$$r(t) = R\left(1 - e^{-\frac{t}{RC}}\right). \tag{2}$$

The current i(t) in the load circuit of the detector has a certain complicated character. Then the voltage across the load may be determined by the Duhamel integral $U_0(t) = i(t) \cdot r(0) + \int_0^t i(\tau)\, r'(t-\tau)\, d\tau$. (3)

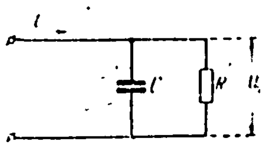

Fig.1

Here r(0) = 0, and the first term may be omitted.

By differentiating eq.(2), we get

$$r'(t-\tau) = \frac{1}{C}\, e^{-\frac{t-\tau}{RC}}. \tag{4}$$

Then eq.(3) takes the form:

$$U_0(t) = \frac{1}{C} \int_0^t e^{-\frac{t}{RC}}\, e^{\frac{\tau}{RC}}\, i(\tau)\, d\tau. \tag{5}$$

With an accuracy to T, we represent the time t as t = kT, where k is an integer. According to this, let us divide the interval of integration (0-t) into partial intervals of duration T:

$$U_0(t) = \frac{1}{C} \sum_{p=0}^{c-1} e^{-\frac{t}{RC}} \int_{pT}^{(p+1)T} e^{\frac{\tau}{RC}}\, i(\tau)\, d\tau. \tag{6}$$

Let us consider the integral entering into the latter expression. On the basis of eq.(1), the exponential factor may be considered constant in the interval of integration, and may be brought outside the sign of the integral. Indeed, by the theorem on the mean,

$$\int_{pT}^{(p+1)T} e^{\frac{\tau}{RC}} i(\tau)\, d\tau = e^{\frac{pT+\xi}{RC}} \int_{pT}^{(p+1)T} i(\tau)\, d\tau = e^{\frac{pT}{RC}} e^{\frac{\xi}{RC}} \int_{pT}^{(p+1)T} i(\tau)\, d\tau, \qquad (7)$$

where $0 < \xi < T$. According to eq.(1), $\frac{\xi}{RC} << 1$, and thus $e^{\frac{\xi}{RC}} \approx 1$.

Then eq.(6) takes the form:

$$U_0(t) = \frac{1}{C} \sum_{p=0}^{\kappa-1} e^{-\frac{t-pT}{RC}} \left[\frac{1}{T} \int_{pT}^{(p+1)T} i(\tau)\, d\tau \right] \cdot T =$$

$$= \frac{1}{C} \sum_{p=1}^{\kappa-1} e^{-\frac{t-pT}{RC}} I_0(pT) T, \qquad (8)$$

where

$$I_0(pT) = \frac{1}{T} \int_{pT}^{(p+1)T} i(x)\, dx \qquad (9)$$

(here the variable of integration, for greater clarity, is denoted by x).

Considering $T << t$, $K >> 1$, let us replace the sum in eq.(8) by the integral, i.e., let us pass to the limit of the sum as $T \to 0$. In this case, the symbol $\sum_{p=0}^{\kappa-1}$ is replaced by $\int^{t}$; pT by τ; T by $d\tau$.

The quantity $I_0(pT)$, must be replaced by $\lim_{T \to 0} I_0(\tau)$.

Let us denote

$$\int_a^{\tau} i(x)\, dx = \eta(\tau), \qquad (10)$$

where a is an arbitrary fixed instant of time. Then

$$I_0(\tau) = \frac{\eta(\tau + T) - \eta(\tau)}{T} = \frac{\Delta\eta}{T} \qquad (11)$$

and

$$\lim_{T \to 0} I_0(\tau) = \lim_{T \to 0} \frac{\Delta\eta}{T} = \frac{d\eta}{d\tau} = \eta'. \qquad (12)$$

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It is obvious that on passage to the integral, the replacement of the quantity $I_0(\tau)$ by the corresponding limit will return us to the original expression. Instead of this, let us approximately maintain the quantity $I_0(\tau)$ under the integral sign without passage to the limit, i.e., instead of the derivative of eq.(12), let us retain the ratio of finite increments of eq.(11).

The condition determining the possibility of such maintenance of the ratio of finite increments instead of the derivative, is the slow variation of I_0 with time. Indeed

$$\eta(\tau + T) = \eta(\tau) + T\eta'(\tau) + \frac{T^2}{2!}\eta''(\tau) + \cdots, \tag{13}$$

then

$$\Delta\eta = \eta(\tau + T) - \eta(\tau) = T\eta'(\tau) + \frac{T^2}{2!}\eta''(\tau) + \cdots \tag{14}$$

and

$$\frac{\Delta\eta}{T} = \eta'(\tau) + \frac{T}{2!}\eta''(\tau) + \cdots \tag{15}$$

Consequently

$$I_0 = \frac{\Delta\eta}{T} \approx \eta' \tag{16}$$

under the conditions that

$$\frac{T}{2}\eta'' \ll \eta'. \tag{17}$$

Let us substitute eq.(16) in eq.(17) and, replacing the expression η'' by I_0', as well as $\frac{T}{2}$ by $\frac{T}{2\pi}$, which strengthens the inequality, we obtain eq.(17) in the form

$$I_0\omega \gg I_0'. \tag{18}$$

Equation (18) is the usual condition of slow variation of the function $I_0(\tau)$.

Under this condition, by replacing the sum in eq.(8) by the integral, we may, by virtue of eq.(16) abstain from passing to the limit of $I_0(\tau)$. Then we finally obtain

$$U_0(t)=\frac{1}{C}\int_0^t e^{-\frac{t-\tau}{RC}} I_0(\tau)\,d\tau=\int_0^t r'(t-\tau)\,I_0(\tau)\,d\tau, \qquad (19)$$

where

$$I_0(\tau)=\frac{1}{T}\int_{\tau}^{\tau+T} i(x)\,dx. \qquad (20)$$

The quantity $I_0(\tau)$ is the DC component, slowly varying with time, of the current $i(\tau)$, calculated for an interval T small in comparison with the time constant RC. Under these conditions, $I_0(\tau)$ may be substituted for the actual current $i(\tau)$ in the Duhamel integral of eq.(19).

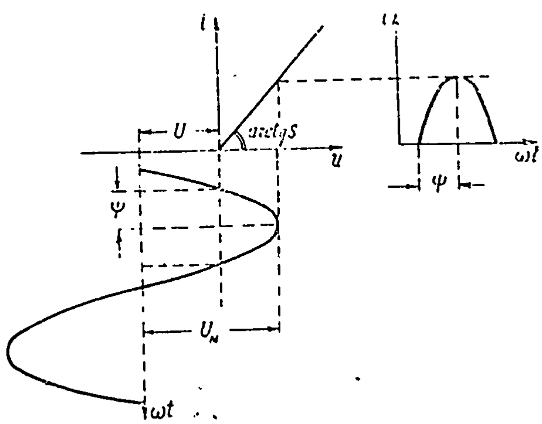

Fig.2

In order to make use of eq.(19), it is necessary to be able to determine the DC component of the current as a function of the voltages characteristic for a given operating state of the detector.

In using the broken-line approximation of the characteristics of the detecting elements for determining I_0, we usually proceed as follows: calculating, at a certain voltage, the cosine of the cut-off angle, we find this angle from the tables, and then from the corresponding graphs we determine, for this angle, the coefficients of the DC component, which already allows us to calculate I_0.

This method, which has given a good account of itself in numerical calculations, but it is inconvenient for the problem under consideration. Here we are required to represent I_0 in the form a function of voltage, as simple as possible, which will be most convenient for the integration of eq.(19).

For the constructions represented in Fig.2, the current, within the limits of the cut-off angle ψ equals

$$i = SU_m(\cos\omega t - \cos\psi). \tag{21}$$

where

$$\cos\psi = -\frac{U}{U_m}. \tag{22}$$

The DC component of the current equals

$$I_0 = \frac{1}{\pi}\int_0^{\psi} i\, d\omega t = SU_m B, \tag{23}$$

where

$$B = \frac{1}{\pi}\left[\sin\psi - \psi\cos\psi\right]. \tag{24}$$

Figure 3 gives a graph of the relation B $(\cos\psi)$. We call attention to the fact that it is convenient to consider $\cos\psi = \frac{U}{U_m}$ as the independent variable instead of ψ.

Plate detectors usually operate at $\psi = \text{const} = \frac{\pi}{2}$ $(U = 0)$, in this case $B = 0.32$ and

$$I_0(t) = 0.32\, SU_m(t). \tag{25}$$

In studying the process of voltage buildup across the load of a diode detector we must have an approximation of the function $B(\cos\psi)$ which shall be as convenient as possible for integration. As the first approximation in the limits $0 < \cos\psi < 1$, the function $B(\cos\psi)$ may be approximated by a straight line in the equation

$$B = 0.25(1 - \cos\psi) \tag{26}$$

(at $U_m < U$,

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Then, for $U_m > U$

$$I_0 = 0.25SU_m(1 - \cos\theta) = 0.25S(U_m - U). \quad (27)$$

From this we obtain a simple expression characterizing the stationary state of the diode detector at any angles of cutoff. In the diode detector in the steady state

$$U = U_0 = I_0 R. \quad (28)$$

Then, from eq.(27), we obtain

$$U_0 = 0.25SR(U_m - U_0). \quad (29)$$

whence the coefficient of detection is obtained as equal to

$$K_d = \frac{U_0}{U_m} = \frac{1}{1 + \frac{4}{SR}}. \quad (30)$$

These same relations also hold true for the cathode detector. In such a detector, the load resistance has such a great value that the initial working point is shifted in the absence of a signal to the lower bend of the cathode current characteristic. When an alternating voltage acts on the diagram, pulses of cathode current are produced, whose DC component I_0 produces the additional voltage drop U_0 across the resistance R. Therefore the state of the diode detector is shown by the same construction shown in Fig.2. The difference between a cathode detector and a diode detector is that the circuit of the alternating component of the current is closed through the source of plate voltage and does not pass through the signal source, thanks to which the input resistance of the detector is close to infinitely great.

Using the relations (19) and (25), let us write the expression for the voltage across the load of a plate detector:

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$$U_0(t) = \frac{1}{C}\int_0^t e^{-\frac{t-\tau}{RC}}\, 0{,}32\, SU_m(\tau)\, d\tau. \qquad (31)$$

In the special case of a rectangular pulse at the input of the detector, $U_m(t) = U_m = \text{const}$ at $0 < t < \tau_u$ (τ_u being the pulse duration) and

$$U_0(t) = 0{,}32\, SU_m \frac{1}{C}\, e^{-\frac{t}{RC}} \int_0^t e^{\frac{\tau}{RC}}\, d\tau = 0{,}32\, SRU_m\left(1 - e^{-\frac{t}{RC}}\right). \qquad (32)$$

Thus the leading edge of the pulse of the detected voltage represents the exponent with a time constant RC approaching the quantity $0.32SRU_m$.

After the action of the pulse on the detector input is completed, the voltage across the load is determined by the discharge of the capacitance C across the resistance R, and is characterized by a falling exponent with the same time constant.

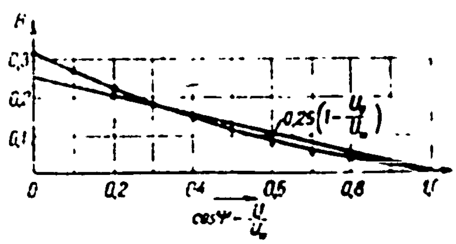

Fig.3

From this the conclusion may be drawn that, from the point of view of the distortions of the envelope of the pulse curve, the detector behaves as a linear device with the transient characteristics

$$h(t) = \frac{U_0(t)}{U_m \cdot 1(t)} = 0{,}32\, SR\left[1 - e^{-\frac{t}{RC}}\right]. \qquad (33)$$

Bearing this in mind, we may determine the input voltage for any form of the envelope curve at the input of the detector, by means of the Duhamel integral for envelopes:

$$U_0(t) = \int_0^t U_m(\tau)\, h(t-\tau)\, d\tau. \qquad (34)$$

In the case of the diode and cathode detectors, $U_0(t) = U(t)$. By using eq.(19) and (27), we obtain an equation determining the build-up of voltage at the output of the detector at any envelope of the curve of input voltage (at $U_m > U_0$):

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$$U_0(t) = \frac{1}{C}\int_0^t e^{-\frac{t}{RC}}\, e^{\frac{\tau}{RC}} 0{,}25\, S\,[U_m(\tau) - U_0(\tau)]\, d\tau. \qquad (35)$$

By differentiating with respect to t, we obtain the linear differential equation:

$$\frac{C}{0{,}25\, S} U_0'(t) + \left(1 + \frac{2}{0{,}25\, SR}\right) U_0(t) = U_m(t). \qquad (36)$$

With a rectangular input pulse having $U_m(t) = U_m$ at $0 < t < \tau_u$, the exponent

$$U_0(t) = K_d U_m \left(1 - e^{-\frac{t}{T_f}}\right), \qquad (37)$$

serves as the solution of this equation,

where K_d is defined by eq.(30).

The time constant of the exponential leading edge of the detected voltage equals

$$T_f = \frac{CR}{1 + 0{,}25\, SR}. \qquad (38)$$

It follows from this that during the process of build-up of the output voltage, the envelope curve of the pulse is distorted by the detector just as it would be distorted by a linear device with the transient characteristics

$$h(t) = K_d \left(1 - e^{-\frac{t}{T_f}}\right). \qquad (39)$$

This allows us to use eq.(34) when the form of the pulse at the input is complex.

For $U_m < U_0$, the tube is blocked, and the voltage at the output of the detectors is determined by the discharge of the capacitance C across the resistance R. It is characterized by an exponential law with the time constant RC.

In determining the form of the pulse at the input of the diode detector, attention should be paid to the shunting influence of the input conductance of the diode detector on the preceding resonant circuit. The value of the input conductance equals $G_{in} = \frac{I_{m1}}{U_m}$, where I_{m1} is the amplitude of the first harmonic of the

For the construction of Fig.2, the current at the limits of the angle of cut-off is determined by eq.(21). Its first harmonic has the amplitude

$$I_{m1} = \frac{2}{\pi}\int_0^{\psi} i(\omega t)\cos\omega t \; d\omega t = SU_m A, \qquad (40)$$

where

$$A = \frac{1}{\pi}(\psi - \sin\psi\cdot\cos\psi). \qquad (41)$$

The ralation A(cos ψ) is represented on Fig.4. In the region $0 \leq \cos\psi \leq 1$, the relation A(cos ψ) is close to linear and may be approximated by the linear function

$$A = \frac{1}{2}(1 - \cos\psi). \qquad (42)$$

Then

$$G_{in} = \frac{I_{m1}}{U_m} = SA = \frac{1}{2}S\left(1 - \frac{U_0}{U_m}\right). \qquad (43)$$

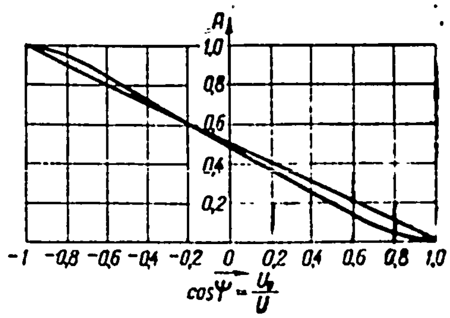

Fig.4

A consideration of the influence of the conductance G_{in} on the resonant amplifier preceding the detector would go beyond the scope of the present work.

Paper Received by the Editors 4 May 1953.

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TRIODE FREQUENCY CONVERTERS FOR METER WAVES

by

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Full Members of the Society

On the basis of the requirements that must be met by the frequency converters in modern AM-FM broadcast radio receivers, the optimum versions for the design of diagrams of triode frequency converters of meter waves are discussed.

1. Introduction

In modern radio broadcast receivers with an ultra-shortwave band for receiving frequency modulated signals, single-grid frequency converters are exclusively used. This is explained by the fact that single-grid converters best satisfy the following basic requirements for modern meter-wave frequency converters:

1) The transfer ratio of the converter must be maximum.
2) The leakage of the heterodyne voltage at the receiver input must be minimum
3) The noise factor of the converter must be minimum.

2. Design Features of Diagrams of Triode Frequency Converters

In diagrams of single-grid frequency-conversion circuits of modern receivers operating at frequencies above 60-80 megacycles, triodes are mainly used. This is explained by the fact that the values of the noise impedances and input admittances are considerably lower in triodes. The former fact is due to the presence of additional noise in pentodes owing to the redistribution of the emission between the plate and screen grid, and the latter to the shorter electron travel time in triodes. Thanks to this fact, at frequencies over 60-80 megacycles, at which the input impedances of pentodes prove to be too low, the use of triode frequency converters

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is found to be more rational than the use of pentode converters. Application of triode frequency converters is particularly advisable when special dual triodes with separated cathode leads are used, allowing a convenient combination of the frequency converter with the UHF stage operating on a circuit with a grounded grid.

It must be noted that at frequencies lower than about 45-60 megacycles, it is more advantageous in many cases to use pentode frequency converters.

Single-grid frequency converters using triodes are constructed at the present time exclusively on the dual bridge scheme. The widespread employment of bridge diagrams is explained by the fact that, when they are properly regulated, they assure the enhancement of the qualitative indexes of the frequency converter.

The first bridge eliminates direct coupling between the signal and heterodyne circuits, which assures both high grade matching of their tuning and sharp reduction of heterodyne voltage leakage across therreceiver input.

The second bridge compensates the negative feedback on intermediate frequency through the plate-grid transfer capacitance, which is very considerable in the triode. With the object of increasing the amplification on intermediate frequency, the parameters of this bridge are so selected as to assure over-compensation of the converter at which the applied positive feedback is somewhat greater than the negative feedback due to the plate-grid transfer capacitance.

A triode frequency converter is usually used in combination with a UHF stage on a triode working in a grounded-grid circuit, thus allowing the entire high frequency tract of the USW band to be constructed on a single special dual triode.

3. The Triode Frequency Converter with Inductive Bridge in the Grid Circuit

Let us consider the diagram of a converter with an inductive bridge in its grid circuit, allowing the use of variable capaciters with a grounded rotor for tuning the UHF circuits and the converter unit (Fig.1).

The plate circuit of the first triode, which is the UHF stage, includes a circuit consisting of the inductance L_3 and the capacitance C_{10} ($C_9 >> C_{10}$). Through

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the bypass capacitor C_3, the amplified signal voltage is fed to the middle point of the feedback coil L_4, which is inductively coupled to a heterodyne circuit $L_5 - C_8$.

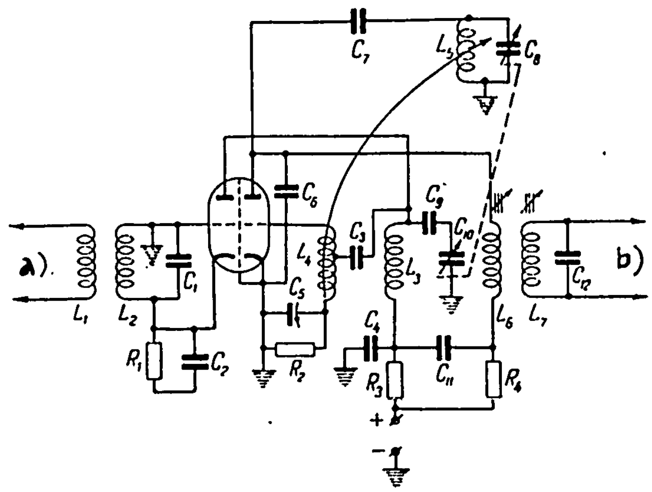

Fig.1

a) To USW Feeder of USW antenna; b) To first IF amplifier stage.

The intermediate frequency voltage obtained as a result of the simultaneous action of the signal and heterodyne oscillations on the frequency-converter grid is separated on a circuit consisting of the inductance L_6 and the capacitances C_6, C_7, and $C_4 (C_{11} >> C_4)$. The capacitor C_6, entering into the capacitance of the intermediate frequency circuit, also serves to pass the high harmonics of the heterodyne directly from the converter plate to the ground. Such a connection reduces the radiation of the heterodyne harmonics, since it reduces their passage into the other circuits of the tract.

To eliminate direct coupling between the signal and heterodyne circuits, a bridge is used, which is formed by the two halves of the inductance L_4 and by the capacitances C_5, C_{gc}, where C_{gc} equals input capacitance of the tube (Fig.2).

Since the input resistance of the converter r and the resistance of the automatic bias R_2 are much larger than the impedances of the bridge arms, the influence of r and R_2 on the bridge balance may be neglected. Then, with the strict symmetry of the halves of the inductance L_4, the condition of balance of the bridge reduces to

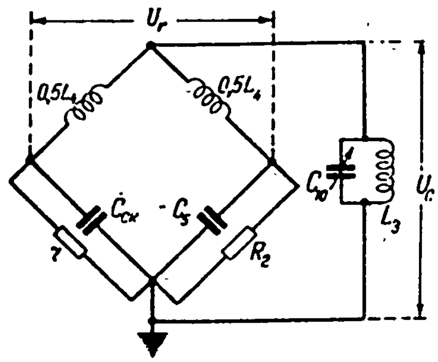

Fig.2

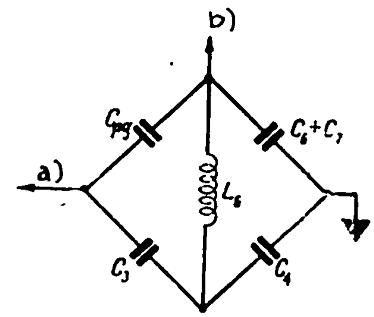

Fig.3

a) To converter grid; b) To converter plate;

the satisfaction of the equality:

$$C_5 = C_{gc}. \qquad (1)$$

The second bridge, serving to compensate the negative feedback through the plate-grid transfer capacitance of the tube C_{pg}, is formed by the capacitances C_{pg}, C_3, C_4, C_6, and C_7 (Fig.3).

The bridge is balanced when the following conditions are satisfied:

$$C_{pg}C_4 = C_3(C_6 + C_7), \quad C_{11} \gg C_1. \qquad (2)$$

If it is desired to increase the intermediate frequency amplification, the so-called overcompensation of the bridge must be accomplished. For this purpose the voltage of the positive feedback taken from the capacitor C_4 is increased, decreasing its capacitance. It must, however, be borne in mind that excessive overcompensation may lead to unstable operation and even to self-excitation of the converter stage.

It is not hard to show that the transfer ratio of the UHF stage is determined by the equation :

$$K=(\mu+1)\frac{R_a}{R_a+R_i}\frac{1}{1-\frac{1}{2}\omega^2 L_4 C_{gc}}. \qquad (3)$$

The factor $\frac{1}{1-\frac{1}{2}\omega^2 L_4 C_{gc}}$ allows for the redistribution of the high-frequency voltage taken from the plate circuit $L_3 - C_{10}$, between the half inductance L_4 and the grid-cathode terminals of the convertor; in this case it is assumed that $r\omega C_{gc} >> 1$, which in practice is always the case.

In eq.(3), μ and R_i are respectively the amplification factor and the internal resistance of the tube, R_a = the resistance of the plate load of the UHF stage.

$$R_p=\frac{ZR}{Z+R},$$

where $Z = \rho Q$ = resonant resistance of circuit (ρ and Q being respectively the wave impedance and the quality factor of the circuit), R = input resistance of converter referred to the terminals of the plate circuit

$$R=r\left[\frac{1}{(r\omega C_{gc})^2}+\left(\frac{1}{2}\omega^2 L_4 C_{gc}-1\right)^2\right]. \qquad (4)$$

R is determined starting out from the system of calculation shown in Fig.2, taking account of the inequality $r\omega C_{gc}>>1$.

The effective quality factor Q_e, determining the selectivity of the UHF stage, is calculated by the formula:

$$Q_e=Q\frac{R}{R+Z}. \qquad (5)$$

The transfer factor of the convertor K_{con}, when the bridge is in balance in intermediate frequency is determined from the relation:

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$$K_{con} = S_{con}\sqrt{\frac{Z_1 R_{icon}}{Z_1 + R_{icon}} \cdot \frac{Z_2 R_{in}}{Z_2 + R_{in}}} \frac{\eta}{1+\eta^2}, \qquad (6)$$

where S_{con} = transconductance of conversion, equal to 0.25 S_{max}, S_{max} being maximum transconductance of triode; R_{icon} = internal resistance of converter; Z_1 = resonant impedance of plate circuit of converter; Z_2 = resonant impedance of circuit included in bridge circuit of first UHF stage, and, together with the plate circuit, forming the band filter; η = coupling parameter; R_{in} = input resistance of first intermediate-frequency amplification stage.

Equation (6) allows for the shunting action across the circuits of the internal resistance of the converter and the input resistance of the intermediate frequency amplifier, since these quantities in many cases are found to be comparable with the resonant resistances of the band filter circuits.

4. Triode Frequency Converter with Capacitative Bridge in Bridge Circuit

We pass now to a consideration of the second frequency converter hookup with capacitative bridge in the grid circuit of the converter (Fig.4)

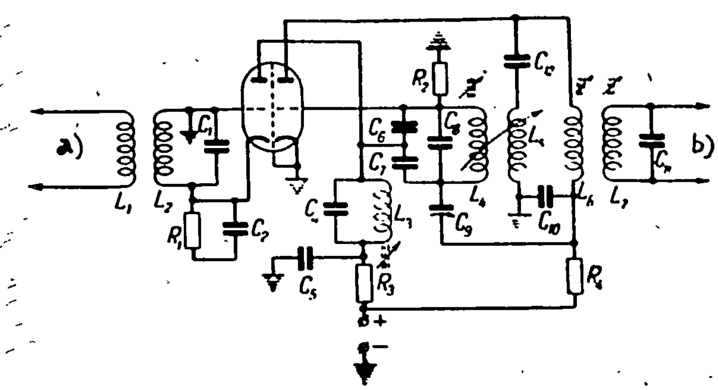

Fig.4

a) To feeder of USW antenna; b) To first stage of intermediate frequency amplifier

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Since in this diagram the heterodyne circuit is insulated with respect to ground, the use of a unit of variable capacitors with grounded rotor as a tuning element is here impossible. For this reason the tuning of the circuits in this diagram must be accomplished by means of a variable inductance unit $L_3 - L_4$.

The signal voltage, amplified by the high frequency amplification stage, taken from the $L_3 - C_4$ circuit, is fed to the middle point of the capacitance divider $C_6 - C_7$ and thus enters the converter grid. Simultaneously the heterodyne voltage taken from the circuit $L_4 - C_8$ is fed to the converter grid. The inductance L_5 is a feedback coil. The intermediate frequency oscillations obtained as a result of the conversion process are separated on the circuit $L_6 - C_{10} - C_{12}$ and are further amplified in the intermediate-frequency amplifier stage.

The signal circuit $L_3 - C_4$ and the heterodyne circuit $L_4 - C_8$ are connected to different diagonals of the bridge, whose arms are formed by the capacitances C_{gc}, C_6, C_7, and C_9 (Fig.5). The bridge is balanced when the following equalities are satisfied

$$C_{gc} C_7 = C_6 C_9$$
$$\left(R_2 \gg r \gg \frac{1}{\omega C_{gc}}\right). \qquad (7)$$

The bridge is tuned by varying the value of the capacitance C_9.

A bridge serving to compensate the negative feedback due to the plate-grid transfer capacitance of the converter is formed by the capacitances C_{pg}, C_9, C_{10}, and C_{12} (Fig.6). The condition of balance of the bridge reduces to the satisfaction of the equality

$$C_{pg} C_{10} = C_9 C_{12}. \qquad (8)$$

By reducing the capacitance C_{10}, the bridge may be overcompensated and the amplification of the converter may thereby be increased.

We present below the principal computational relations for this diagram.

Transfer ratio of the high frequency amplifier stage:

$$K=(\mu+1)\frac{R_a}{R_a+R_i}\,\frac{C_6}{C_6+C_{gc}}. \qquad (9)$$

Here the factor $\frac{C_6}{C_6 + C_{gc}}$ allows for the redistribution of the high frequency voltage taken from the plate circuit $L_3 - C_4$ between the capacitance C_6 and the grid-cathode terminals of the converter. In this case it is assumed that the inequality $\frac{rR_2}{r + R_2} \gg \frac{1}{\omega C_{gc}}$ holds, which in practice is always the case.

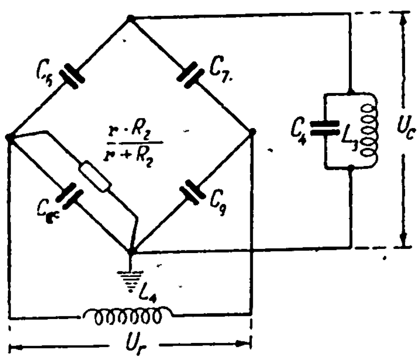

Fig.5

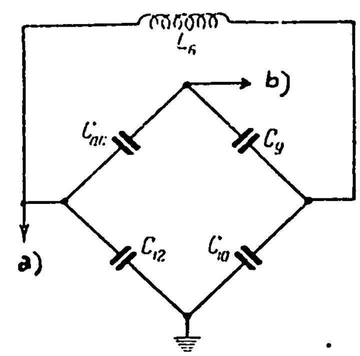

Fig.6

a) To plate of converter; b) To grid of converter

In eq.(9), $R_p = \frac{ZR}{Z + R}$, where $Z = \rho Q$ = resonant impedance of plate circuit of high frequency amplifier stage; $R = r\left(\frac{C_6 + C_{gc}}{C_6}\right)^2$ = input resistance of convertor referred to terminals of circuit.

The resistance R is determined starting out from the computational diagram shown in Fig.5, taking account of the inequality:

$$\frac{rR_2}{r+R_2} \gg \frac{1}{\omega C_{gc}}.$$

The effective quality factor of the plate circuits of the high frequency amplifier stage Q_e:

$$Q_s = Q \frac{R}{R + Z}. \qquad (10)$$

The transfer coefficient of the converter, K_{con}:

$$K_{con} = S_{con} \sqrt{\frac{Z_1 R_{1con}}{Z_1 + R_{1con}} \cdot \frac{Z_2 R_{a.x}}{Z_2 + R_{a.x}}}\, \frac{\eta}{1 + \eta^2}. \qquad (11)$$

5. Experimental Verification of Computational Relations

For the experimental verification of the above presented computational relations, two amplifier-converters were assembled using the circuits shown in Figs.1 and 4. The amplifier-converters operated in the 66-73 megacycle band (in the band assigned for FM USW broadcasting) with a dual triode type 6N3P with the following data:

E_L = 6.3 v	E_p = 150 v	E_g = -2 v
μ = 40	S = 5 ma/v	R_i = 8 kiloohms
C_{gc} = 2.8 μμf	C_{pg} = 1.3 μμf	C_{pc} = 0.35 μμf.*

The computational values of the transfer ratios for the central frequency of the band (70 megacycles) are as follows:

for circuit shown in Fig.1:	K_{HFA} = 14.4; K_{con} = 10.5
for circuit shown in Fig.4:	K_{HRA} = 11.7; K_{con} = 10.5

The measured transferratios for the first diagram were: K_{HFA} = 12, K_{con} = 9.5 and for the second diagram: K_{HFA} = 11, K_{con} = 9.5. There is a certain deviation between the experimental data and the calculated data, which is explained by the difficulty of exactly calculating the wiring capacitances and the values of the input resistances, as well as by the absence of ideal balance in the balanced bridges.

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*With grounded grid.

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6. Conclusion

Both circuits considered yield about the same qualitative indexes, since they are based on the same method, which allows, to the maximum extent, satisfaction of the basic requirements for modern USW frequency converters.

The selection of one of these circuits or the other depends mainly on design considerations, in particular, on the tuning element in the USW band and on the question of its mechanical connection with the tuning elements on the other bands of the receiver. When a block of variable capacitors is used as the tuning element for the USW range, the converter circuit using the inductive bridge should be employed, while when a unit of variable inductances is used for tuning, the capacitative bridge circuit should be used instead.

It must be noted that when modern triodes, having high transconductance characteristics and small interelectrode capacitances ($S = 5-6$ ma/v, $C_{pc} = 0.2-0.25\,\mu\mu$) are used in the high-frequency amplification stage, it is possible to obtain stable amplification at high frequency, of the order of 10-12.

Thanks to so considerable a high-frequency amplification, the converter noise will have practically no effect on the noise factor of the receiver, and this factor will be determined by the noise of the high-frequency amplification stage.

Paper Received by the Editor 16 July 1954.

A METHOD OF STUDYING TRANSIENT PROCESSES IN LINEAR SYSTEMS*

by

R.D.Leytes and L.N.Gutman

Full Members of the Society

In this paper an approximate method of studying transient processes is developed, based on the application of the theory of finite differences to the integral equation under consideration.

It is shown that after the introduction of special coefficients a simple expression is obtained, which relates the input and output voltages of the system. This expression allows the solution of a number of problems relating to transient processes in amplifiers. As an example, a stage with plate correction is investigated.

A method of approximate determination of the transient characteristics directly from the differential equation describing this circuit is presented.

Introduction

The investigation of transient processes in linear systems reduces in most cases either to the direct application of the inversion Svertyvaniya integral, or to the solution of an integral equation of the Volterre type.

The former problem is always solved, but the functions which in practice enter into the equation may prove to be so complex that there are serious difficulties in finding the solution of the integral equation. In addition, cases are possible when certain of the functions are assigned graphically and cannot be expressed with

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*Read before the All-Union Scientific Session of VNORiE [All Union Scientific and Technical Society for Radio Engineering and Telecommunications] imeni A.S.Popov, in May 1953.

a sufficient degree of accuracy in a convenient analytical form. It is therefore very desirable to use approximate methods of investigating transient processes. A large number of studies conducted in various directions, have been devoted to this question.

In a considerable number of studies, the "method of time sequences" (Bibl.1) is used. This method is based on the fact that a continuous function of time with a bounded spectrum is determined by the assignment of a finite series of numbers, connected with the values of this function taken at discrete instants of time. A proof of this theorem was introduced into communications theory by V.A.Kotel'nikov in 1933, and was subsequently (1949) treated by Shannon.

In essence the method of time sequences is a method of the theory of finite differences, applied to the study of transient processes in linear systems; this connection has also been pointed out in the literature (Bibl.2). For this reason, this group may also include works based on the approximate representation of differential equations describing diagrams by equations in finite differences, and also studies in which the methods of numerical solutions of integral equations are employed. (Bibl.3-6).

The approximate method of study of transient processes developed in the present paper may be included in this group of works*. A feature of the method is the convenient computational formulas, which allow the solution of a number of problems connected with transient processes in complicated diagrams.

1. APPLICATION OF THE METHOD OF FINITE DIFFERENCES TO THE DUHAMEL INTEGRAL

1. Derivation of the Fundamental Relation between Output and Input Voltages

The voltage at the output of the system $U(t)$ is connected with the voltage at the input $V(t)$ by the well known relation (Duhamel integral):

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*The present paper was written under the influence of a study (Bibl.7) belonging to the field of meteorology.

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$$U(t)=h(0)V(t)+\int_0^t h'(\xi)V(t-\xi)\,d\xi, \quad (1)$$

where t = time, h(t) = transient characteristic or transient admittance of the system (reaction of the system to a single voltage shock).

For application of the method of finite differences it is convenient to introduce the notation $U(m\tau) = U_m$, $V(m\tau) = V_m$, $h(m\tau) = h_m$, where τ = step of interpolation. Then eq.(1) takes the form

$$U_m=h_0V_m+\int_0^{m\tau} h'(\xi)V(m\tau-\xi)\,d\xi. \quad (2)$$

Linear interpolation is most convenient, for it makes it easy to evaluate the error of this approximate method. Be Newton's formula (Bibl.8)

$$V(m\tau-\xi)=(n+1)V_{m-n}-nV_{m-n-1}+(V_{m-n-1}-V_{m-n})\frac{\xi}{\tau}+r_{m-n}(\xi). \quad (3)$$

Here $n\tau \le \xi \le (n+1)\tau$

$(n = 0, 1, 2, \ldots, m)$, r_{m-n} is the residual term equal to

$$r_{m-n}(\xi)=\frac{1}{2}(n\tau-\xi)(n\tau+\tau-\xi)V''(\xi_{m-n}), \quad (4)$$

consequently,

$$(m-n-1)\tau<\xi_{m-n}<(m-n)\tau;\quad \xi_{m-n}\neq\xi.$$

On substituting eq.(3) in eq.(2), performing integration by parts and elementary transformations, we obtain a formula for determining the output voltage U for an assigned input voltage V:

$$U_m=\sum_{n=0}^{m} M_nV_{m-n}-R_m \quad (m=0,\ 1,\ 2,\ldots), \quad (5)$$

where

$$M_0=H_1;\ M_n=\Delta^2 H_{n-1}\ (0<n\leqslant m), \quad (6)$$

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$$H_n = \frac{1}{\tau}\int_0^{n\tau} h(\xi)\, d\xi, \tag{7}$$

$$R_m = -\sum_{n=0}^{m-1} \int_{n\tau}^{(n+1)\tau} r_{m-n}(\xi)\, h'(\xi)\, d\xi + V_0(\Delta H_m - \overset{\circ}{h}_m). \tag{8}$$

Here we introduce the notation of the theory of finite differences:

$$\Delta H_\kappa = H_{\kappa+1} - H_\kappa;\ \Delta^2 H_\kappa = H_{\kappa+2} - 2H_{\kappa+1} + H_\kappa.$$

The quantities M_n represent the weights with which the values of the input voltage V at various intervals of time from the instant $t = m\tau$ enter into eq.(5).

R_m is the residual term allowing us to evaluate the error of the formula:

$$U_m = \sum_{n=0}^{m} M_n V_{m-n} \tag{9}$$

with the interpolation interval selected.

2. Relation between the Coefficients M_n and the Transient Characteristics of the System

Let us dwell in somewhat greater detail on the physical meaning of the coefficients M_n.

We assume that the transient characteristic of the system h(t) is known or assigned numerically for equidistant values of the argument τ, 2τ, 3τ ... Let us denote h_n = h (n) and let us take as h(t) the linear interpolation between h_n and h_{n+1}. Then, on the basis of the well known "trapezium formula", eq.(7) for H_n is now rewritten as follows

$$H_n = \frac{1}{2}h_0 + h_1 + h_2 + \ldots + h_{n-1} + \frac{1}{2}h_n.$$

On substituting the respective values of H in eq.(6), we get

$$M_0 = \frac{1}{2}(h_0 + h_1);\ M_n = \frac{1}{2}(h_{n+1} - h_{n-1}). \tag{10}$$

In this way the coefficients M_n $(0 < n \leq m)$ are the averaged increments of the transient characteristic h(t) over the time from $(n - 1)\tau$ to $n\tau$ (Fig.1). An exception is the case $n = 0$. The coefficient M_0, if $h_0 \neq 0$, represents the mean value of the transient characteristic for the time from 0 to τ.

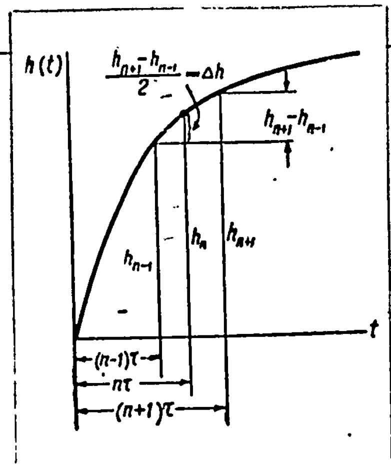

Fig.1

Obviously,

$$\sum_{n=0}^{m} M_n = \frac{h_m + h_{m+1}}{2}.$$

determines the mean value of the transient characteristic for the instant of time $m\tau < t < (m + 1)\tau$. In other words, the coefficients M_n characterize the steepness of the growth of the transient characteristic h(t) in the interval under consideration. It will be shown later that these arguments allow us to draw certain conclusions on the behavior of complicated circuits without a complete calculation of the transient characteristics.

The use of the pulse characteristics B(t) which is connected with the transient characteristics of the system h(t) by the simple relation:

$$B(t) = h'(t).$$

is very convenient.

It is not hard to see that the coefficients M_n are equal to the discrete values of the ordinates of the pulse characteristic B(t), and, consequently, determine it. A direct consideration of eq.(9) confirms this conslusion: the value of the output voltage at some calculated instant of time $t = m\tau$ is equal to the reaction of the system M_n $(n = 0, 1, 2, \ldots, m)$ to the pulses of input voltage with the "amplitudes" V_{m-n} (Fig.2).

Equation (10), determining the relation between the transient and pulse characteristics for discrete values of the argument, may be easily transformed into the

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form:

$$h_n = (-1)^n h_0 + 2(M_{n-1} + M_{n-3} + M_{n-5} + \dots)\ (0 \le n \le m). \qquad (11)$$

In eq.(11), the last summand in the parentheses will be M_0 if n is odd and M_1 if n is even.

Bearing in mind the fact that the initial value of the transient characteristic usually results from the physical meaning of the problem, we reach the conclusion that eq.(11) makes it possible for a known M_n (or in other words, with a known pulse characteristic) to construct the transient characteristic of the diagram under study.

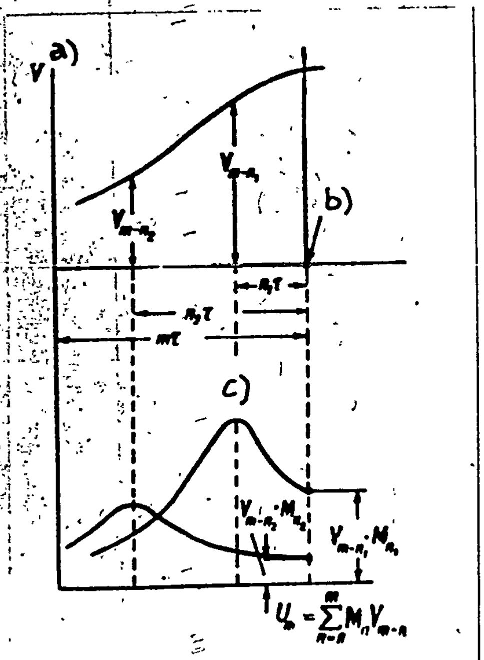

Fig.2

a) Input voltage; b) Calculated instant of time; c) Pulse characteristics of system

If $h_0 = 0$, then eq.(11) is simplified:

$$h_1 = 2M_0;\ h_2 = 2M_1;$$
$$h_3 = 2(M_2 + M_0) \dots (0 < n \le m). \qquad (12)$$

The coefficients M_n may be obtained directly from the differential equations describing the diagram without resorting to solving it (cf. Section VI).

3. Evaluation of Error

Let us evaluate the error of eq.(9). For this purpose we turn to eq.(5) and consider the residual term R_m.

On substituting, in eq.(8), eq.(4) for the error of r_{m-n} (ξ) of the interpolation formula of eq.(3), we obtain, by applying the theorem of the mean to the integral:

$$R_m = -\sum_{n=0}^{m-1} h'(\zeta_n) \int_{n\tau}^{(n+1)\tau} \frac{1}{2}(n\tau - \xi)(n\tau + \tau - \xi) V''(\xi_{m-n})\, d\xi + V_0(\Delta h_m - h_m)$$

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$$n\tau < \zeta_n < (n+1)\tau; \quad \zeta_n \neq \xi$$

or, after integration

$$R_m = \frac{\tau^3}{12} \sum_{n=0}^{m-1} h'(\zeta_n) V''(\xi_{m-n}) + V_0(\Delta h_m - h_m), \qquad (13)$$

where

$$n\tau < \zeta_n < (n+1)\tau; \quad (m-n-1)\tau < \xi_{m-n} < (m-n)\tau.$$

Let us represent H_m, the mean value of the transient admittance during the time from m to (m + 1) , in the form

$$\Delta H_m = \frac{h_{m+1} + h_m}{2}.$$

Then

$$\Delta H_m - h_m = \frac{h_{m+1} - h_m}{2} = \frac{\Delta h_m}{2}.$$

It is likewise obvious that

$$\tau \sum_{n=0}^{m-1} h'(\zeta_n) V''(\xi_{m-n}) \approx \int_0^{m\tau} h'(\xi) V''(m\tau - \xi)\, d\xi.$$

On substituting these expressions in eq.(13), we get

$$R_m \approx \frac{\tau^2}{12} \int_0^{m\tau} h'(\xi) V''(m\tau - \xi)\, d\xi + V_0 \frac{\Delta h_m}{2}.$$

In order that the error of R_m shall not exceed a certain assigned error R, it is necessary that, for any m, the following condition shall be satisfied:

$$\frac{\tau^2}{12} \int_0^{m\tau} h'(\xi) V''(m\tau - \xi)\, d\xi < R - V_0 \frac{\Delta h_m}{2};$$

and this provides grounds for a definition of the interval of interpolation, :

$$\tau \leqslant \sqrt{\frac{12\left(R - V_0 \frac{\Delta h_m}{2}\right)}{\int_0^{m\tau} h'(\xi) V''(m\tau - \xi)\, d\xi}}. \qquad (14)$$

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We note that the integral entering into eq.(14) does not require exact computation; it is sufficient merely to evaluate the quantities $h'(\xi)$ and $V''(m\tau - \xi)$. It is simplest to consider the maximum value of these functions h'_{max} and V''_{max}, and, as $m\tau$, to adopt the instant of calculation $m\tau = t_1$ which is most remote from the instant of connection of the input voltage. Then

$$\int_0^{m\tau} h'(\xi) V''(m\tau - \xi)\, d\xi < t_1 \left| h'_{max} V''_{max} \right|.$$

The maximum value of the increment of the transient admittance Δh_{max} must, likewise, be substituted in eq.(14).

As a result we obtain the inequality for determining the necessary duration of the interval:

$$\tau \leqslant \sqrt{\frac{12\left(R - V_0 \frac{\Delta h_{max}}{2}\right)}{t_1 h'_{max} V''_{max}}}. \qquad (15)$$

We note that before the selection of τ, Δh_{max} cannot be determined. It is therefore convenient to proceed as follows. Assigning the error $R' = R - V_0 \frac{\Delta h_{max}}{2}$ (which is somewhat smaller than the required error of calculation R), let us determine τ; then let us determine $V_0 \frac{\Delta h_{max}}{2}$ and let us calculate the maximum actual error:

$$R_{max} = R' + V_0 \frac{\Delta h_{max}}{2}.$$

If R_{max} is less than the assigned error R, then the duration of the interval has been properly selected; if $R_{max} > R$, then a shorter duration τ must be assigned and Δh_{max} and R_{max} must be recalculated, meeting the condition $R_{max} \leq R$.

4. Example of Calculation

As an illustration of the application of these formulas, we investigate an amplifying stage with plate correction under the action of a bell-shaped pulse at its input.

a) Determination of the Interval of Interpolation

The expression for the transient characteristic of an amplifying stage with plate correction (Bibl.9) is of the form:

$$h(\bar{t}) = 1 - a e^{-\bar{t}} \sin(b\bar{t} + \beta). \qquad (16)$$

Where $\bar{t}$ = generalized time; $a = \frac{2k}{\sqrt{4k - 1}}$, $b = \sqrt{4k - 1}$, $\tan\beta = \frac{\sqrt{4k - 1}}{1 - 2k}$, k = correction parameter.

Figure 3 shows a graph of $h(\bar{t})$ for k = 0.5; in this case a = 1, b = 1, $\beta = \frac{\pi}{2}$.

Let us take as the acting EMF, a bell-shaped pulse of maximum value equal to unity. We shall measure the duration of the pulse "in 0.1", and shall understand T to mean the generalized duration of the pulse, i.e., the duration related to the time constant of the plate circuit of the amplifying stage (just as in the introduction of the generalized time t). Then

$$V(\bar{t}) = e^{-9\left(\frac{\bar{t}}{T} - \frac{1}{2}\right)^2}.$$

Let us put T = 2, then

$$V(\bar{t}) = e^{-2,25(\bar{t} - 1)^2}. \qquad (17)$$

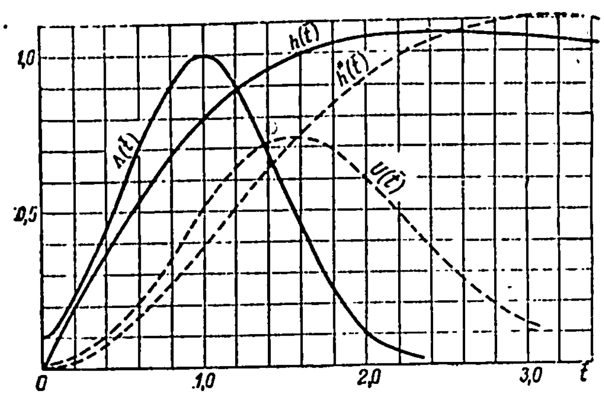

Fig.3

Figure 3 shows the corresponding graph.

The calculation of the input voltage by eq.(5) requires the selection of a certain interval of interpolation τ, determining the error of the computational expression of eq.(9).

Let us make use of eq.(15). It follows from eq.(16) that the maximum value of the transconductance of the transient characteristic $h'(\bar{t})$ is reached at $\bar{t} = 0$, and is equal (with a selected value of the correction parameter k = 0.5) $h'_{max} = 1$.

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The maximum value in absolute value of the second derivative of the input voltage $U''(\bar{t})$ is reached at $\bar{t} = 1$ and is equal to $V''_{max} = 4.5$. The entire process of build-up, as will be seen from the curve of $h(\bar{t})$ of Fig.3, occupies an interval of time of about three units; consequently, $t_1 = 3$. On substituting these values in eq.(15) we obtain

$$\tau \leqslant \sqrt{\frac{12}{3\cdot 4{,}5}R'} = \sqrt{0{,}89R'}.$$

Let us assign the error $R' = 0.05$; then the duration of the interval of interpolation must satisfy the inequality $\tau \leq 0.21$.

Let us take $\tau = 0.2$. In this case $\Delta h_{max} = h'_{max}\tau = 0.2$, and

$$R = R' + V_0\frac{\Delta h_{max}}{2} = 0{,}05 + 0{,}1\,\frac{0{,}2}{2} = 0{,}06.$$

Thus the maximum error of eq.(9) does not exceed 6% in this case. The actual error in determining the output voltage will be considerably less than 6%, since the evaluation adopted by us for the integral

$$\int_0^{m\tau} h'(\xi)\,V''(m\tau-\xi)\,d\xi < t_1\,|h'_{max}\;V''_{max}|$$

is very course, and in this case, where h' and V'' fluctuate very sharply, gives very strongly exaggerated results. However, even with this course evaluation, it is easy to reduce the error to 3%; for this it is sufficient to take $\tau = 0.15$; then $R' = 0.025$, $\Delta h_{max} = 0.15$, and $R \approx 0.03$).

b) <u>Calculation of the Coefficients M_n</u>

Let us substitute the expression for the transient characteristics from eq.(16) in eq.(7), and after integration, we get

$$H_n = n + \frac{a}{\sqrt{1+b^2}\,\tau}\left[e^{-n\tau}\sin(bn\tau+\delta) - \sin\delta\right], \qquad (18)$$

where

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$$\delta = \beta + \gamma;\ \gamma = \mathrm{arctg}\, b.$$

On substituting the values adopted, a = 1, b = 1, $\delta = \frac{3\pi}{4}$ and $\tau = 0.2$, in eq.(18), we obtain a computational formula for H_n in the form:

$$H_n = n + 2{,}5\left[\, e^{-0{,}2n}(\cos 0{,}2n - \sin 0{,}2n) - 1\right].$$

The results of the calculation are presented in the third column of Table 1. In the next column are the results of the calculation of the coefficients M_n by eq.(6).

A consideration of the coefficients M_n shows that the transient characteristic has a peak. The maximum value of the transient characteristic is reached at n = 11 - 12, that is, at generalized time $2.2 < \bar{t} < 2.4$, which corresponds to the curve of the transient characteristic at k = 0.5 shown in Fig.3.

c) Determination of the Output Voltage

The numerical values of the input voltage obtained by eq.(17), related to various instants of time every τ seconds, are given in the fifth column of Table 1. Let us use eq.(9), taking, as the instant of calculation, the time of connecting the voltage at the input; then

$$U_0 = M_0 V_0 = 0{,}009 \cdot 0.100 = 0{,}010.$$

Taking the following instant of time as the calculated instant, we have:

$$U_1 = M_0 V_1 + M_1 V_0 = 0{,}009 \cdot 0{,}237 + 0{,}192\ 0.100 = 0{,}043.$$

Further

$$U_2 = M_0 V_2 + M_1 V_1 + M_2 V_0 = 0{,}103.$$

By continuing this process we obtain the values of the output voltage U_n for all the instants of time with which we are concerned. The results of the calculation are given in Table 1; Fig.3 shows a graph of U(t).

A consideration of the shape of the output pulse shows a considerable elongation of the duration and distortion of the front of the pulse.

II. SOLUTION OF THE PROBLEM OF INVERSION

In this Section we shall consider the determination of the input voltage for a known output voltage. In practice such a formulation of the problem may prove to be necessary (for example in studying the combined operation of pulse generators and shaping circuits): the required shape of the output pulse of the system with a known transient characteristic is given and the shape of the pulse that must be fed to the generator at the input of the system is to be determined.

The mathematical problem reduces down to the solution of the integral equation (1), or, in other words, to the inversion of eq.(5), allowing us to find V(t) from a known U(t).

Let us represent eq.(5) in the expanded form:

$$\left.\begin{aligned} U_0 + R_0 &= M_0 V_0 \\ U_1 + R_1 &= M_1 V_0 + M_0 V_1 \\ &\cdots\cdots\cdots\cdots \\ U_m + R_m &= M_m V_0 + M_{m-1} V_1 + \ldots + M_1 V_{m-1} + M_0 V_m. \end{aligned}\right\} \tag{19}$$

The system of eq.(19) may be regarded as a finite inhomogeneous system of linear equations in the unknowns V_0, V_1, .. V_m. The determinant of this system is equal to

$$D = M_0^{m+1}. \tag{20}$$

Consequently, if $M_0 \neq 0$, then the system of eq.(19) has a solution which may be obtained by using the Kramer formula:

$$V_n = \frac{D_n}{D}. \tag{21}$$

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Here

$$D_n = M_0^{m-n}\begin{vmatrix} M_0 & \cdots & \cdots & & U_0+R_0 \\ M_1 & M_0 & \cdots & & U_1+R_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{n-1} & M_{n-2} & \cdots & M_0 & U_{n-1}+R_{n-1} \\ M_n & M_{n-1} & \cdots & M_1 & U_n+R_n \end{vmatrix}$$

Let us resolve the determinant by elements of the last column:

$$D_n = M_0^{m-n}\Bigg[(U_n+R_n)M_0^n-(U_{n-1}+R_{n-1})M_0^{n-1}M_1+\cdots+ \\ +(-1)^{n+2}(U_0+R_0)\begin{vmatrix} M_1 & M_0 & & \\ \cdots & \cdots & \cdots & \cdots \\ M_{n-1} & M_{n-2} & \cdots & M_0 \\ M_n & M_{n-1} & \cdots & M_1 \end{vmatrix}\Bigg] \tag{22}$$

On substituting eqs.(20) and (22) in eq.(21), we have:

$$V_n = (U_n+R_n)\frac{1}{M_0}-(U_{n-1}+R_{n-1})\frac{M_1}{M_0^2}+\cdots+ \\ +(-1)^{n+2}(U_0+R_0)\frac{1}{M_0^{n+1}}\begin{vmatrix} M_1 & M_0 & & \\ \cdots & \cdots & \cdots & \cdots \\ M_{n-1} & M_{n-2} & \cdots & M_0 \\ M_n & M_{n-1} & \cdots & M_1 \end{vmatrix} \tag{23}$$

Thus we have obtained an inversion formula allowing us, from a known output voltage of the system with an assigned transient characteristic to determine the form of the voltage acting on the input. In order to establish an analogy between the problem of inversion and the direct problem, let us rewrite eq.(23) in a form coinciding with eq.(5):

$$V_n = \sum_{p=0}^{n} \overline{M}_p U_{n-p} - \overline{R}_n;$$

where $\overline{M}_p$ = elements of the inverse matrix

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$$\overline{M}_p = \frac{(-1)^p}{M_0^{p+1}} \begin{vmatrix} M_1 M_0 & & \\ M_{p-1} M_{p-2} & \dots\dots & M_1 M_0 \\ M_p M_{p-1} & \dots\dots & M_2 M_1 \end{vmatrix}; \quad \overline{M}_0 = \frac{1}{M_0}, \qquad (p > 0)$$

and $\bar{R}_n$ = error of inversion

$$\overline{R}_n = -\sum_{p=0}^{n} \overline{M}_p R_{n-p}. \tag{24}$$

In practice it is simpler to calculate $\bar{M}_n$ by the recurrance formula:

$$\overline{M}_p = -\frac{1}{M_0}\sum_{c=0}^{p-1} \overline{M}_c M_{p-c}, \quad \overline{M}_0 = \frac{1}{M_0}.$$

To evaluate the maximum error of the inversion eq.(24), let us consider that n takes its maximum value n = m; on substituting for R_{m-p} the error of the direct problem R, we get

$$|\overline{R}_m| > R\sum_{p=0}^{m} |\overline{M}_p|.$$

Thus, with a given duration of the intervals, i.e., at a given value of the error R of the direct problem, the error of the inverse problem will not exceed $R\sum_{p=0}^{m}|\overline{M}_p|$. If it is necessary that the error of inversion shall not exceed an assigned value $\bar{R}$, then the duration of the interval must be selected from a condition analogous to eq.(15)*:

$$\tau < \sqrt{\frac{12 R'}{t_1 |h'_{max}| \, V''_{max} \left|\sum_{p=0}^{m} |\overline{M}_p|\right|}}; \quad \overline{R}' = \overline{R} - V_0 \frac{\Delta h_{max}}{2} \cdot \sum_{p=0}^{m} |\overline{M}_p|. \tag{25}$$

*If V''_{max} cannot be evaluated from physical reasoning, then it may be assumed without substantial error that this quantity is of the order of unity.

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Hence it is obvious that on solution of the inverse problem, the duration of the interval must be taken less than on solution of the direct problem by a factor of about $\sqrt{\sum_{p=0}^{m} |\overline{M}_p|}$ if we are attempting to maintain the same errors $\overline{R} = R$).

The quantity $\sum_{p=0}^{m} |\overline{M}_p|$ determines the value of the "inverse transient characteristic" for the instant of time $m\tau < t < (m + 1)$.

If $M_0 = 0$ (a system with a lag with respect to time t_3), then it is necessary to shift the origin from which the intervals are measured to the point t_3, in the selection of which $M_0 \neq 0$.

III. DETERMINATION OF TRANSIENT CHARACTERISTIC OF SYSTEM WITH ASSIGNED OUTPUT AND INPUT VOLTAGES

In the experimental study of the transient characteristics of complex circuits not amenable to calculation, we need a generator of pulses of strictly rectangular form, which in practice is not always easy to accomplish. In using the proposed method, we may determine the transient characteristics also for the case when the voltage applied by the generator to the input of the system under study has any arbitrary form. For this purpose we need only the graphic assignment by points of the voltages $V(t)$ and $U(t)$.

The system of eq.(5) may be considered of a finite inhomogeneous system of linear equations in unknown coefficients M_0, M_1, ..., M_m.

By analogy with the solution of the inverse problem, we obtain, on the condition that $V_0 \neq 0$,

$$M_n = \sum_{p=0}^{n} \overline{V}_p U_{n-p}. \tag{26}$$

Where

$$\overline{V}_p = -\frac{1}{V_0} \sum_{c=0}^{p-1} \overline{V}_c V_{p-c}; \quad \overline{V}_0 = \frac{1}{V_0}.$$

Using eq.(26), (11), or (12), we obtain the discrete values of the ordinates of the required transient characteristic.

The error arising in the determination of the coefficients is equal to

$$R_n^{M} = -\sum_{p=0}^{n} \bar{V}_p R_{n-p}.$$

It may be shown that by analogy to eq.(25), it is not difficult to select the duration of the intervals τ_M, on assigning the maximum error R^M:

$$\tau \leqslant \sqrt{\frac{12R^{M'}}{t_1 \left| h'_{max} \; V''_{max} \; \left| \sum_{p=0}^{m} |V_p| \right.\right.}}; \quad R_m^{M'} = R^{M} - V_0 \frac{\Delta h_{max}}{2} \sum_{p=0}^{m} |\bar{V}_p|.$$

The accuracy of the calculations is the higher, the steeper the front of the acting voltage. Indeed, the greater V_0, the smaller $\bar{V}_0$, $\bar{V}_1$, $\bar{V}_2$, ... and $\sum_{p=0}^{m} |\bar{V}_p|$, and, consequently, the smaller the error $|R_m^{M}| = R \sum_{p=0}^{m} |\bar{V}_p|$.

IV. DETERMINATION OF THE INPUT CHARACTERISTIC OF A MULTISTAGE AMPLIFIER

Let us consider the transient processes in a two-stage amplifier, the transient characteristics of each stage of which are known (in other words, the coefficients M_n for the first stage and N_n for the second stage are known).

Let us denote by $V(t)$ the voltage at the input of the first stage, by $U^{(1)}(t)$ the voltage at its output, which is the same as the voltage at the input of the second stage, and by $U^{(2)}(t)$ the voltage at the output of the second stage.

Let us apply eq.(5) to the second stage:

$$U_m^{(2)} = \sum_{n=0}^{m} N_n U_{m-n}^{(1)} - R_m^{(2)}, \tag{27}$$

where

$$U_{m-n}^{(1)} = \sum_{p=0}^{m-n} M_p V_{m-n-p} - R_{m-n}^{(1)}. \tag{28}$$

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Let us substitute eq.(28) in eq.(27) and vary the order of summation; we get

$$U_m^{(1)} = \sum_{n=0}^{m} M_n^* V_{m-n} - R_m^*, \qquad (29)$$

where

$$M_n^* = \sum_{p=0}^{n} N_p M_{n-p}, \qquad (30)$$

$$R_m^* = \sum_{n=0}^{m} N_n R_{m-n}^{(1)} + R_m^{(2)} \qquad (31)$$

Thus the coefficients M_n^* charactering the transient admittance of the two-stage amplifier, may easity be calculated from the known coefficients of the individual stages.

If both are identical, then $M_n = N_n$, and the coefficients of the two-stage amplifier are calculated by the formula:

$$M_n^* = \sum_{p=0}^{n} M_p M_{n-p}. \qquad (32)$$

From the coefficients M_n^* it is easy to calculate the transient characteristic of the two-stage amplifier h_n^*, by using eq.(11) or eq.(12).

By applying eq.(30) the required number of times, we may consider the transient processes in amplifiers with any desired number of stages.

We remark that on considering processes in multistage amplifiers, the duration of the intervals τ must be taken sufficiently small, so as correctly to reflect the peculiarities of the transient characteristic of the intermediate stages.

For a two-stage amplifier with identical stages ($M_n = N_n$) we may consider the maximum errors of the individual stages as equal to $R^{(1)} = R^{(2)} = R$; then the error of the formula will be:

$$U_m^{(2)} = \sum_{n=0}^{m} M_n^* V_{m-n}$$

is equal by eq.(31) to

$$R^* \leqslant R\left(\sum_{n=0}^{m}|M_n| + 1\right)$$

and the duration τ must be selected from the relation:

$$\tau \leqslant \sqrt{\frac{12R^{*\prime}}{t_1|h'_{max}\ V''_{max}|\left(\sum_{n=0}^{m}|M_n| + 1\right)}};$$

$$R^{*\prime} = R - V_0\frac{\Delta h_{max}}{2}\left(\sum_{n=0}^{m}|M_n| + 1\right).$$

If the transient characteristic of one stage of the amplifier has no large peaks, then $\sum_{n=0}^{m}|M_n| \approx 1$ and

$$\tau \leqslant \sqrt{\frac{6(R^* - V_0\Delta h\quad)}{t_1|h'\quad V''|}}.$$

In this way, in studying the action of an EMF on a two-stage amplifier, it is necessary (in order to maintain the same accuracy of calculation as in the analysis of the single-stage amplifier) to shorten the duration of the interval by a factor of about $\sqrt{2}$ by comparison with the τ for the single-stage amplifier.

As an example, we consider a two stage amplifier with plate correction and with identical stages at correction parameter K = 0.5. The duration of the intervals τ is left unchanged: $\tau = 0.2$. The coefficients M_n^* calculated by eq.(32), are entered in Table 1. A consideration of them shows that the transient characteristic must reach its maximum value at $15\tau < \bar{t} < 16\tau$ or $3.0 < \bar{t} < 3.2$, since it is precisely at this value of the generalized time that the coefficients M_n^* change their signs. The maximum steepness of the transient characteristic must occur at $5\tau < \bar{t} < 6\tau$ or $1.0 < \bar{t} < 1.2$.

In the following column of Table 1 are entered the values of the transient characteristic h_n^* of a two stage amplifier calculated by eq.(12). Figure 3 gives a graph of $h^*(\bar{t})$; the peak is equal to 11%, which corresponds to the data in the lit-

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V. Determination of the Transient Characteristics of the Component Elements from the Transient Characteristic of the Entire System

Assume that the transient characteristics of a multistage amplifier with identical stages is known. Required, to determine the transient characteristics of each of the stages. Such a formulation of the problem may prove useful in cases when a multistage amplifier with an assigned transient characteristic is being designed (for example with assigned peak and front steepness) the number of stages is being selected on the basis of the required amplification factor.

Let us consider the "breakdown" of the transient characteristics with coefficients M_n^* into two identical characteristic coefficients M_n.

By eq.(32)

$$M_n^* = \sum_{p=0}^{n} M_p M_{n-p} = 2 M_0 M_n + \sum_{p=1}^{n-1} M_p M_{n-p},$$

whence we obtain the recurrence relation:

$$M_n = \frac{1}{2 M_0}\left(M_n^* - \sum_{p=1}^{n-1} M_p M_{n-p}\right). \qquad (33)$$

In this way, eq.(33) allows us to determine the coefficients (and consequently also the transient characteristics) for each of the two groups of stages making up the complex diagram. Using the same formulas, we may perform a further "breakdown" which will yield the transient characteristics for each of the four groups of stages making up the complex diagram. If the amplifier contains 2^s stages, then the process of "breakdown" must be performed s times; as a result we determine the required transient characteristic of the individual stage.

Let us consider the case when the transient characteristics of the groups of the diagram are different. Here the assignment of only the transient characteristic of the system is no longer sufficient, but we need a certain supplementary condition establishing the relation between the transient characteristics of the groups composing the system (and consequently also between the coefficients of the groups, M_n and N_n). The question is solved most simply in the case where the transient characteristic of one of the groups, is selected from any considerations whatever, for example N_n. Then, from the relation

$$M_n^* = \sum_{p=0}^{n} M_p M_{n-p} = M_n N_0 + \sum_{p=0}^{n-1} M_p N_{n-p}$$

we find

$$M_n = \frac{1}{N_0}\left(M_n^* - \sum_{p=0}^{n-1} M_p N_{n-p}\right). \qquad (34)$$

These same relations allow us to solve the problem of adding a certain number of stages to an amplifier already designed, having known coefficients N_n. For this it is sufficient to assign the transient characteristic M_n^* of the entire new system, and, starting out from eq.(34), to determine the required coefficients M_n of the additional stages. By "breaking down" the coefficients M_n into individual stages, if this is necessary, and making use of the values obtained for the peak and steepness of the front, we may conduct the calculation of the parameters of the supplementary amplifier.

VI. Determination of the Coefficients M_n from the Differential Equation Describing the Diagram

Cases are possible in practice when the differential equation describing the diagram is of high order, and is so complex that its exact solution, which is necessary to determine the transient characteristic of the diagram, is difficult. In these cases the coefficients M_n may be determined directly from the differential equation without having recourse to its solution.

Let us consider as an example the third-order differential equation:

$$y''' + py'' + qy' + ry = f(t).$$

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The equation in finite differences that approximately replaces this equation has the form (Bibl.8):

$$\frac{y_{n+2}-y_{n-2}-2y_{n+1}+2y_{n-1}}{2\tau^3}+p\,\frac{y_{n+1}-2y_n+y_{n-1}}{\tau^2}+q\,\frac{y_{n+1}-y_{n-1}}{2\tau}+ \\ +ry_n=f_n. \qquad (35)$$

Here τ = interval of interpolation, $f_n = f(n\tau)$, $y_K = y(K\tau)$.

Let us group the terms in the difference equation, eq.(35):

$$y_{n+2}\frac{1}{2\tau^3}+y_{n+1}\left(-\frac{1}{\tau^3}+\frac{p}{\tau^2}+\frac{q}{2\tau}\right)+y_n\left(-\frac{2p}{\tau^2}+r\right)+ \\ +y_{n-1}\left(\frac{1}{\tau^3}+\frac{p}{\tau^2}-\frac{q}{2\tau}\right)+y_{n-2}\left(-\frac{1}{2\tau^3}\right)=f_n$$

and, after introducing the notation:

$$\overline{M}_0=\frac{1}{2\tau^3},\quad \overline{M}_1=-\frac{1}{\tau^3}+\frac{p}{\tau^2}+\frac{q}{2\tau},\quad \overline{M}_2=-\frac{2p}{\tau^2}+r, \\ \overline{M}_3=\frac{1}{\tau^3}+\frac{p}{\tau^2}-\frac{q}{2\tau},\quad \overline{M}_4=-\frac{1}{2\tau^3}$$

we get

$$f_n=\sum_{p=0}^{4}\overline{M}_p\,y_{n+2-p}. \qquad (36)$$

The coefficients determining the pulse characteristic (and consequently also the transient characteristic) may be obtained by inversion of eq.(36):

$$y_n=\sum_{p=0}^{n-2}M_p f_{n-2-p},$$

where

$$M_p=-\frac{1}{M_0}\sum_{c=0}^{p-1}M_c\,\overline{M}_{p-c};\quad M_0=\frac{1}{\overline{M}_0}.$$

Equation (35) has meaning only in the case where $y_0 = 0$, i.e., the properties of the system are such that the output voltage (or current) does not appear suddenly.

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It is possible to evaluate the accuracy of the substitution of the differential equation by the difference equation from the formulas given in the literature (Bibl.8).

Conclusion

The method developed in this paper may be applied to the solution of certain more complex problems of electrical engineering and radio engineering, for example, to the study of transient processes in circuits with distributed constants and in systems with parameters depending on time.

The calculations by this method are not more complex, even in the case where the time elapsed since instant of connection is long.

Table 1

n	$t = n\tau$	H_n	[illegible]$_n$	V_n	U_n	[illegible]$^*_n$	h^*_n
0	0	0	0.099	0.100	0.010	0.010	0
1	0.2	0.099	0.192	0.237	0.043	0.038	0.020
2	0.4	0.391	0.175	0.445	0.103	0.072	0.076
3	0.6	0.858	0.152	0.698	0.204	0.098	0.164
4	0.8	1.477	0.127	0.914	0.342	0.114	0.272
5	1.0	2.223	0.102	1.000	0.505	0.122	0.392
6	1.2	3.071	0.078	0.914	0.629	0.122	0.516
7	1.4	3.997	0.060	0.698	0.710	0.116	0.636
8	1.6	4.983	0.035	0.445	0.725	0.104	0.748
9	1.8	6.004	0.027	0.237	0.680	0.089	0.844
10	2.0	7.052	0.014	0.100	0.594	0.074	0.926
11	2.2	8.113	0.005	0.039	0.487	0.057	0.992
12	2.4	9.180	-0.001	0.012	0.379	0.042	1.040
13	2.6	10.245	-0.005	0.003	0.280	0.028	1.076
[illegible]	2.8	11.306	-0.007	0.001	0.195	0.016	1.096

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Table 1 con't

n	t - n	H_n	M_n	V_n	U_n	M_n^*	h_n^*
15	3.0	12.359	-0.008			0.006	1.108
16	3.2	13.404	-0.009			0.001	1.108
17	3.4	14.441	-0.008			-0.007	1.106

In conclusion I consider it my duty to express my thanks to Professor Ya.Z. Tsypkin for a number of valuable suggestions and hints in preparing this paper for the press.

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PROPAGATION OF A PLANE ELECTROMAGNETIC WAVE IN SPACE FILLED WITH PLANE-PARALLEL GRATINGS

by

A.M.Model'

This paper is devoted to a study of the propagation of plane electromagnetic waves in artificial dielectrics. Using the technique of difference equations, formulas are obtained for determining the parameters of artificial dielectrics (phase velocity, refractive index, reflection factor, etc.).

Introduction

At the present time in antenna technology artificial dielectrics are used, formed by rows of parallel gratings of various structures. These gratings may be made of flat metal bands, of flat metal discs, of parallel wires, in the form of flat metal sheets with openings, etc.

The strict analysis of the propagations of a plane wave through such media involves great mathematical difficulties. At the present time there exists partial and approximate methods of analysis, which are applicable, however, only to gratings of a definite type.

The present paper set forth an approximate methodology* of analysis of the propagation of a plane wave in artificial dielectrics formed of a finite or infinite number of parallel gratings. The methodology of the analysis does not depend on the geometrical form of the elements making up the grating.

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*The analysis reduces to the solution of a second-order difference equation. Analogous equations described the propagation of waves along transmission lines with periodically located inhomogeneities (Bibl.1, 2).

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1. Derivation of the Fundamental Equation

Let us first consider a single grating on which a plane wave impinges. The direction of propagation of the wave is normal to the surface of the grating. The incident wave excites currents in the grating which create a secondary field that is propagated on both sides of the grating.

Let us assume that the secondary field, like the primary one, consists of a plane wave. This also includes the inaccuracy of the analysis. In actuality this structure of the secondary field is more complex than a plane wave. However, if the distances between the individual elements of the grating are small, then the secondary field may be considered approximately plane even at small distances from it. The ratio of the amplitude of the secondary wave to the amplitude of the primary wave will be termed the reflection factor:

$$p = |p| e^{\pm i\varphi}, \qquad (1)$$

where $|p|$ = modulus of reflection factor;

φ = its argument.

Starting out from the energetic relations, it is easy to show that the modulus and argument of the reflection factor are connected by the following relation:

$$-\cos\varphi = |p|. \qquad (2)$$

Since $|p|$ varies from 0 to 1, φ will vary from $\frac{\pi}{2}$ to π. From eq.(1) and (2) we get

$$p = -\cos\varphi\, e^{\pm i\varphi} = -\cos^2\varphi \mp i\cos\varphi\sin\varphi, \qquad (3)$$

where

$$\frac{\pi}{2} \leqslant \varphi \leqslant \pi.$$

Let us consider a medium formed by many parallel gratings equidistant from each

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other (Fig.1). Let us select any three adjacent gratings, for example, the (K - 1)-th, the K-th and the (K + 1)-th.

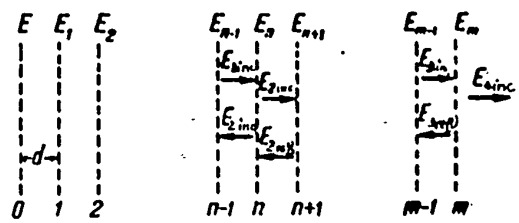

Fig.1

When a plane wave in the space between the (K - 1)-th and K-th gratings falls on the system of gratings, two plane waves will exist. One of them will be propagated from left to right ($E_{1\ inc}$), the other from right to left ($E_{1\ refl}$). In the space between the K-th and (K + 1)-th gratings, two waves will also exist, $E_{2\ inc}$ and $E_{2\ refl}$. Let us set up the equations connecting the amplitudes of the incident and reflected waves with the amplitudes of the fields around the gratings under consideration. In the cross section of the K-th grating, the following relations will obtain:

$$\left.\begin{aligned} E_{1\,inc} + E_{1\,refl} = E_K \\ E_{2\,inc} + E_{2\,refl} = E_K \end{aligned}\right\} \qquad (4)$$

In the cross sections of the (K-1)-th and (K + 1)-th gratings the following relations will obtain:

$$\left.\begin{aligned} E_{1\,inc}\,e^{-i\alpha d} + E_{1\,refl}\,e^{+i\alpha d} = E_{K-1} \\ E_{2\,inc}\,e^{i\alpha d} + E_{2\,refl}\,e^{-i\alpha d} = E_{K+1} \end{aligned}\right\} \qquad (4)$$

where d = distance between adjacent gratings, E_{K-1}, E_K and E_{K+1} = amplitudes of fields around the (K - 1)-th, K-th and (K + 1)-th gratings.

By solving simultaneously the system of eq.(4), we get:

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$$E_{1\,inc} = \frac{E_K e^{i\alpha d} - E_{K-1}}{i2\sin\alpha d}, \qquad E_{2\,inc} = \frac{E_{K+1} - E_K e^{-i\alpha d}}{i2\sin\alpha d}.$$
$$E_{1\,refl} = \frac{E_{K-1} - E_K e^{-i\alpha d}}{i2\sin\alpha d}, \qquad E_{2\,refl} = \frac{E_K e^{i\alpha d} - E_{K+1}}{i2\sin\alpha d}. \qquad (5)$$

In addition to eq.(4), the following relation also holds between the incident and reflected waves on both sides of the K-th grating:

$$E_{1\,refl} = pE_{1\,inc} + E_{2\,refl}(p+1). \qquad (6)$$

On substituting eq.(5) in eq.(6), we get

$$E_{K-1} - 2\left(\cos\alpha d + i\frac{p}{p+1}\sin\alpha d\right)E_K + E_{K+1} = 0. \qquad (7)$$

Equation (7) is a second-order difference equation determining the propagation of a plane wave in the medium formed by the plane gratings. In the case when the medium is formed by a finite number of gratings, boundary conditions must be added to eq.(7).

In deriving the equation defining the boundary conditions, the same technique is used as in deriving the fundamental difference equation (7). The distribution of fields on both sides of the extreme grating is considered. On one side (Fig.1) of this grating, in the space between it and the (m - 1)-th grating, two waves exist, the incident ($E_{3\,inc}$) and the reflected ($E_{3\,refl}$); on the other side of this grating only a single wave exists, the incident one ($E_{4\,inc}$). On setting up for this case equations analogous to the relations of eqs.(4), (6) and (7), we obtain

$$E_{m-1} = E_m\left(\cos\alpha d + i\frac{p-1}{p+1}\sin\alpha d\right), \qquad (8)$$

where (m + 1) = number of gratings forming medium;

E = amplitude of wave at first grating.

2. Study of the Equation Obtained

It is well known that the solution of a difference equation is an expression:

$$E_\kappa = A e^{\kappa\gamma} + B e^{-\kappa\gamma}, \tag{9}$$

where γ is defined by the equation

$$\gamma = \operatorname{arcch}\left(\cos\alpha d + i\frac{p}{p+1}\sin\alpha d\right), \tag{10}$$

A and B are determined by the boundary conditions.

Let us consider the propagation of a plane wave in an unbounded medium formed by plane gratings. In this case, only a single plane wave, propagated in a single direction, will exist in the medium. The solution is written in the form:

$$E_\kappa = A e^{\kappa\gamma}. \tag{11}$$

In those cases where γ is a pure imaginary quantity, the plane wave will be propagated without damping, and the series of parallel plane gratings may be considered as an artificial dielectric with the phase velocity v and the refractive index n:

$$v = c\frac{i\alpha d}{|\gamma|}, \tag{12}$$

where c = speed of light in vacuo;

d = distance between adjacent gratings;

$\alpha = \frac{2\pi}{\lambda}$, where λ = the wave length.

The refractive index

$$n = \frac{|\gamma|}{i\alpha d}. \tag{13}$$

Let us consider the conditions under which γ is a pure imaginary quantity. On substituting eq.(3) in eq.(10), we obtain, after transformations

$$\gamma = \operatorname{arcch}(\cos\alpha d \mp \operatorname{ctg}\varphi\sin\alpha d). \tag{14}$$

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It follows from eq.(14) that γ will be imaginary if the following inequality obtains:

$$-1 < \cos\alpha d \mp \operatorname{ctg}\varphi \sin\alpha d \leqslant 1. \tag{15}$$

Figures 2 and 3 give graphs of the relation between the index of refraction of an artificial dielectric and the reflection factor of a plane wave from a single

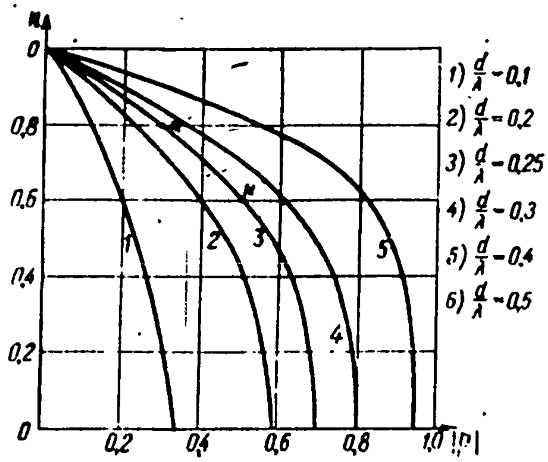

Fig.2

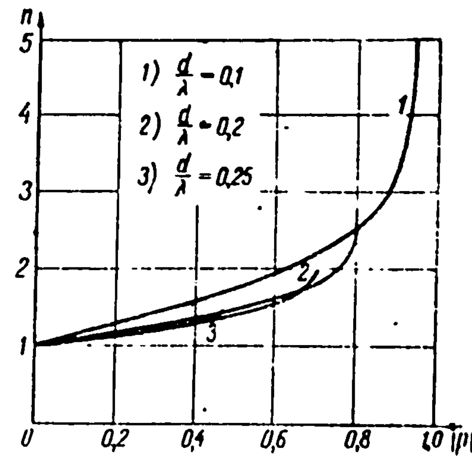

Fig.3

grating for various distances between the gratings. The graphs are calculated by eqs.(13), (10), and (14). The following conclusions may be drawn from the graphs of Figs.2 and 3:

1. The index of refraction of an artificial dielectric formed by plane gratings is completely determined by the reflection factor of a plane wave from a single grating and from the distance between two adjacent gratings. By selecting the reflection factor and the distance, we may obtain any desired value for the refractive index of an artificial dielectric.

2. The closer together the gratings are spaced and the denser the grating itself is (i.e., the higher the reflection factor is from it), the more the refractive index will differ from unity.

3. From the consideration of the graphs for a dielectric with refractive index

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less than unity, it is clear that with increasing wavelength, the refractive index decreases, that is, the phase velocity increases. This is connected with the fact that, with increasing wavelength, the value of the reflection factor (p) increases.

4. For artificial media with lowered phase velocity, that is, for media with $n > 1$, with increasing wavelength, the refractive index increases.

The band properties of artificial dielectrics may be elucidated by a study of the graphs of Figs.2 and 3. It follows in particular from these graphs that artificial dielectric with reduced phase velocity possess wider band properties.

3. The Reflection Factors

The expressions for the reflection factor of a plane wave from the surface of separation air-artificial dielectric, and from the surface of separation artificial dielectric-air, are determined from the boundary conditions. Let us first consider the case of the reflection of a plane wave from the boundary air-artificial dielectric.

Figure 4 shows the edge of an artificial dielectric. In the left half-space two waves, $E_{1\ inc}$ and $E_{1\ refl}$, exist. To the right of the origin of coordinates, in the interval between the first and second gratings, two waves, $E_{2\ inc}$ and $E_{2\ refl}$, likewise exist. For these four waves the following system of equations may be set up:

$$\left.\begin{aligned} &E_{1\,inc} + E_{1\,refl} = E_0 \\ &E_{2\,inc} + E_{2\,refl} = E_0 \\ &E_{2\,inc}\,e^{i\alpha d} + E_{2\,refl}\,e^{-i\alpha d} = E_0 e^{\gamma} \\ &E_{1\,refl} = pE_{1\,inc} + E_{2\,refl}(p+1) \end{aligned}\right\} \tag{16}$$

from which the ratio $\frac{E_{1\ refl}}{E_{1\ inc}}$ may be determined, that is, the reflection factor of a plane wave from the artificial dielectric

$$P_1 = \frac{E_{1\,refl}}{E_{1\,inc}} = -\frac{i\,2p\sin\alpha d + (p+1)(e^{i\alpha d} - e^{\gamma})}{i\,2\sin\alpha d - (p+1)(e^{i\alpha d} - e^{\gamma})}. \tag{17}$$

Let us consider the second case. Let the left half-space be filled with an

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artificial dielectric, and let air be at the right of the origin of coordinates (Fig.5).

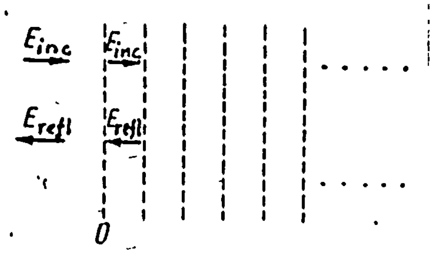

Fig.4

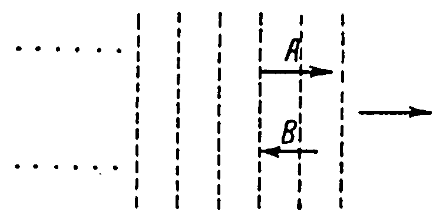

Fig.5

Let us return to eq.(8). From it we determine the ratio between the amplitude of the incident and reflected waves:

$$P_2 = \frac{B e^{-\gamma m}}{A e^{\gamma m}} = \frac{e^{\gamma} - e^{i\alpha d}}{e^{i\alpha d} - e^{-\gamma}}. \quad (18)$$

Let us now consider the case of reflection of an artificial dielectric of finite thickness consisting of m + 1 gratings. Using simultaneously the boundary conditions at the leading and trailing edges of the artificial dielectric, we may obtain the formula:

$$P_2 = p - \frac{(p+1)\, p\, e^{i\alpha d}}{(p+1)\, e^{i\alpha d} - e^{\gamma} + \frac{\mathrm{sh}\,\gamma}{\mathrm{sh}\,\gamma m}\, e^{\gamma m}}. \quad (19)$$

4. The Coefficient of Transmittance

Let us introduce a formula for the coefficient of transmittance t characterizing the transmission of a plane wave through a finite number of plane gratings. We shall consider it equal to the ratio between the amplitude of the plane wave passing through the gratings and the amplitude of the plane wave incident on the first grating

$$t = \frac{E_{trans}}{E_{inc}}. \quad (20)$$

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It is easy to see that

$$E_{trans} = E_m = A e^{\gamma m} + B e^{-\gamma m} = A \frac{2 \operatorname{sh} \gamma}{e^{i \alpha d} - e^{\gamma}} \qquad (21)$$

and

$$E_{inc} = \frac{E_0}{1 + p_3} = \frac{A}{1 + p_3} \cdot \frac{(e^{i \alpha d} - e^{\gamma}) + e^{2\gamma m} (e^{-\gamma} - e^{i \alpha d})}{e^{i \alpha d} - e^{\gamma}} . \qquad (22)$$

On substituting eqs.(21) and (22) in eq.(20), we obtain, after transformations:

$$t = (1 + p_3) \frac{\operatorname{sh} \gamma}{\operatorname{sh} (m + 1) \gamma - e^{i \alpha d} \operatorname{sh} m \gamma} . \qquad (23)$$

4. Conclusion

The formulas presented for the values of γ, n, P_1, P_2, P_3 and t allow the approximate determination of the parameters of various artificial dielectrics made of a series of plane gratings. For this purpose it is sufficient to know only the reflection factor from a single plane grating. This method of analysis considerably facilitates the study of artificial dielectrics. The reflection from a single grating may be determined either by calculation or by experiment. For some types of gratings, for example, those consisting of cylindrical wires, this problem has been solved.

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COMPARATIVE EVALUATION OF COMMUNICATIONS CHANNELS USING DIFFERENT SYSTEMS OF MODULATION, WITH RESPECT TO THEIR TRAFFIC-CARRYING CAPACITY

by

A.G.Zyuko,

Full Member of the Society

This paper gives an analysis of the fundamental relation of statistical communications theory. On the basis of this relation formulas are derived for determining the capacity of a channel in two cases: a) the signal is limited in amplitude; and b) the signal is limited in mean power. The existing systems of modulation are compared with respect to relative traffic-carrying capacity (efficiency).

1. Introduction

In connection with the rapid development of radio engineering, the problem of the traffic-carrying capacity of communications channels takes on great practical importance. In itself, this problem is not new. It was first formulated by V.A.Kotel'nikov as long ago as 1933. We also find certain questions of this problem considered in papers of Nyquist and Hartley.

In Kotel'nikov's paper (Bibl.1), the fundamental theorem of modern communications theory is proved: any continuous signal consisting of frequencies from f_1 to f_2 may be transmitted continuously with any desired accuracy by the aid of numbers following each other at intervals of $\frac{1}{2(f_2 - f_1)}$ sec.

In 1938, this problem was developed further by D.V.Ageyev, who pointed out the substantial influence of noise on the traffic-carrying capacity of a channel. In the 1930's and the beginning of 1940's the problem of the traffic-carrying capacity of channels was not widely studied, since our knowledge of the properties of signals and noises, and of the methods of transmission and reception was still inadequate.

at that time. Studies of these questions, and, in the first place, of the questions of noise-proof communication, made it necessary to allow for the statistical properties of the signal and the noise. The Soviet authors V.I.Siforov, V.A.Kotel'nikov, V.I.Bunimovich, and others, have studied these properties and their quantitative characteristics.

The theory of potential noise immunity, developed by V.A.Kotel'nikov (Bibl.2) allowed the existing methods of reception to be evaluated and the existing methods of transmission to be compared with respect to their sensitivity to noise. The question of the potential possibilities for improving systems of transmission and communications lines as a whole, however, still remained unsolved.

The discovery of new systems of modulation (FM, PM) and studies in the field of noise sensitivity, showed that the traffic-carrying capacity of a communications channel is determined not only by the signal frequency band and the transmission time, but also by the signal-noise ratio (in the general case, by the statistical properties of the signal and the noise). Modern communications theory, which it would be correct to call the statistical theory of communications, has thus established this most general relation between the fundamental quantities that characterize a communications channel. These general relations allow evaluation of the existing systems of transmission and allow indicating the possibilities not only for the improvement, but also for the development of new methods of transmission and reception assuring the maximum immunity from noise. In the present paper we shall consider the fundamental relations of statistical communications theory and shall compare the existing communications systems with respect to their traffic-carrying capacity.

2. Fundamental Relations

It may be shown (Bibl.4b, 8) that the traffic-carrying capacity of a channel, in the general case, is defined by the expression*:

$$Q = Q(A) - Q_x(A), \qquad (1)$$

where

$$Q(A) = -2T \int_{(G)} \int p(a) \lg p(a)\, da\, df, \qquad (2)$$

$$Q_x(A) = -2T \iint_{(V)} \int p(x) p_x(a) \lg p_x(a)\, da\, dx\, df, \qquad (3)$$

p(a) = probability density of transmitted signal A(t);

p(x) = probability density of received signal X(t) (total oscillations of signal A(t) and of noise N(t));

$P_x(a)$ the a priori probability distribution of the signal;

T = time during which the channel is used to transmit the signal A(t).

The domains of integration (G) and (V) are determined by the limits of variation of signal and noise amplitudes, and by the pass band of the channel.

The physical meaning of eq.(1) is rather obvious. It shows that the traffic-carrying capacity of the channel, Q, in transmission of communications by the aid of signals A(t) in the presence of noise N(t) in the channel, is equal to the difference between the quantity of information Q(A) which can be transmitted by the aid of the signals A(t) and the information $Q_x(A)$ which is lost in the channel owing to the presence of noise. Equation (1) may be written in another equivalent form (Bibl.4):

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*In the papers cited (Bibl.4b, 8), it was postulated, in deriving this ralation, that Q is a function of the a priori probability P(a) and of the a posteriori probability $P_x(a)$ of the signal, and that the quantitative measure of information satisfies the condition of additivity.

$$Q = Q(X) - Q_A(X), \tag{4}$$

where $Q(X)$ and $Q_A(X)$ are expressed by relations analogous to eqs.(2) and (3). If $A(t)$ and $N(t)$ are independent, and $X(t) = A(t) + N(t)$, then $P_a(x) = p(n)$ and $Q_A(x) = Q(N)$, where $p(n)$ is the probability density of the noise $N(t)$. Then, by eq.(4),

$$Q = Q(X) - Q(N), \tag{5}$$

where

$$Q(X) = -2T \iint_{(\Omega_x)} p(x) \lg p(x)\, dx df, \tag{6}$$

$$Q(N) = -2T \iint_{(\Omega_n)} p(n) \lg p(n)\, dn df, \tag{7}$$

$p(x)$ is the probability density of the received signal $X(t)$.

Equations (1) and (5) for the traffic-carrying capacity of the channel are derived under very general assumptions as to the properties of the signal and noise; they may therefore be our original formulas in the analysis of both an ideal and a real system of communications.

The function $Q(A)$ is determined by the type of transmitted signals, and therefore characterizes the transmission system. The function $Q_x(A)$ for a given transmission system characterizes the method of reception. Consequently the function Q characterizes the communications channel as a whole, with respect to the transmission of signals in the presence of noise.

3. Conditions of Ideal Reception

A receiver which assures the minimum value of the function $Q_x(A)$ and, consequently, the maximum value of the traffic-carrying capacity of the channel, Q, will be termed an ideal receiver for an assigned method of transmission. We shall show that this condition coincides with the condition of ideal reception given by V.A.Kotel'nikov (Bibl.2). The latter, as is commonly known, reduces to the condition that the ideal receiver must reproduce a communication corresponding to that transmitted signal A(t) for which the quantity $\overline{[X(t) - A(t)]^2}$ shall have the minimum value.

According to the Bayes formula

$$p_X(A) = \frac{p(A)\, p_A(X)}{p(X)}.$$

For an assigned signal X,

$$p_X(A) = \kappa p(A)\, p_A(X),$$

where K is constant that does not depend on A.

In the case of fluctuation noise (Bibl.2)

$$p_A(X) = \frac{1}{(2\pi N)^{n/2}}\, e^{-\frac{1}{2N}\overline{[X(t) - A(t)]^2}},$$

where N = mean noise power, n = 2TF, F = pass band of receiver.

Then

$$p_X(A) = \frac{\kappa p(A)}{(2\pi N)^{n/2}}\, e^{-\frac{1}{2N}\overline{[X(t) - A(t)]^2}}.$$

By eq.(3), the minimum value of $Q_X(A)$ corresponds to the maximum value of $p_X(A)$ or, according to the latter formula, to the maximum value of the expression $\overline{[X(t) - A(t)]^2}$.

4. The Ideal Transmission System

We shall term a transmission system ideal if its signals yield a maximum value of the function Q(A), and, consequently, assure the maximum value of the traffic

STAT

178

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carrying capacity of the channel with an assigned method of reception.

Using eq.(3) for Q(A), we may determine the probability distribution of p(a) and the energy distribution of the signal along the spectrum at which Q(A) has the maximum value.

The solution of this extremely simple problem in variations shows that if the signal is amplitude-limited (Bibl.8), then, in an ideal system, the transmitted signal must have a uniform probability distribution in an assigned interval of variation of amplitude:

$$p(a)=\frac{1}{2U_c}; \quad -U_c<a<+U_c;$$
$$p(a)=0; \quad -U_c>a>+U_c \tag{8}$$

and a spectral amplitude distribution uniform in the pass band of the channel F*:

$$U_c(f)=U_c=\text{const.} \tag{9}$$

When the signal is limited in mean power (Bibl.42), then in the ideal system it must have a normal probability distribution and the distribution of power P(f) along the spectrum in the pass band of the channel must be uniform:

$$p(a)=\frac{1}{\sqrt{2\pi\,\bar{a^2}}}\,e^{-\frac{a^2}{2\bar{a^2}}}, \tag{10}$$

$$P(f)=\text{const.} \tag{11}$$

5. The Channel Capacity

A communication system whose traffic-carrying capacity is maximum, is called

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*It is assumed that the channel has an ideal Pi-shaped frequency characteristic and a linear phase characteristic. In this case it is also considered that the frequency band occupied by the signal is equal to the pass band of the receiver and is equal to F.

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ideal. This maximum traffic-carrying capacity is usually called (Bibl.4, 5) the channel capacity, and is defined as

$$C = \max [Q]. \tag{12}$$

where the maximum is taken for all possible values of p(a), if Q is defined by eq.(1), or for all possible values of p(x), if Q is defined by eq.(5).

Since Q(N) does not depend on p(x), then, by eq.(5) and eq.(12),

$$C = \max [Q(X)] - Q(N). \tag{13}$$

If the signal is amplitude-limited, then on the basis of eqs.(6), (7), (8), and (9)*

$$\max [Q(X)] = 2TF \lg 2(U_s + U_n),$$
$$Q(N) = 2TF \lg 2U_n,$$

where U_s = signal amplitude;

U_f = effective value of noise.

It is here assumed that the received signal $X = U_s + U_f$, that the noise varies from $-U_f$ to $+U_f$, and has in this interval a uniform probability distribution $p(n) = \frac{1}{2U_f}$. Consequently, in this case

$$C = 2TF \lg \left(\frac{U_s}{U_n} + 1 \right). \tag{14}$$

An analogous formula defines the quanity of information H which can be reproduced at the output of the receiver of an ideal system

$$H = 2TF_m \lg \left(\frac{U_{ms}}{U_{mn}} + 1 \right), \tag{15}$$

where

*The derivation of the formulas for this case is given in an earlier paper by the author (Bibl.8).

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F_m = pass band of receiver at low frequency;

U_{ms} = amplitude of signal at receiver output;

U_{mn} = effective value of noise at receiver output.

Equations (14) and (15) were obtained by S.N.Losyakov (Bibl.3) by another method, under the same assumptions as to the statistical properties of the signal and the noise.

If the signal is limited in mean power, then, as shown by Shannon, by eqs.(6), (7), (10, and (11).

$$\max[Q(X)] = 2TF \lg \sqrt{2\pi e \overline{X^2}} = 2TF \lg \sqrt{2\pi e (P+N)},$$

where P = mean signal power;

N = mean noise power,

$$Q(N) = 2TF \lg \sqrt{2\pi e N}$$

and then

$$C = TF \lg \left(\frac{P}{N} + 1\right). \quad (16)$$

Accordingly

$$H = TF_m \lg \left(\frac{P_m}{N_m} + 1\right), \quad (17)$$

where P_m = mean signal power at receiver output;

N_m = mean noise power at receiver output.

It is assumed here that the noise has a normal probability distribution for its amplitudes and a uniform spectral distribution of power.

Thus, depending on what limitations are imposed on the signal and on the noise, we obtain, on the basis of the general relation (1), by eq.(14) for C and eq.(15) for H, in which the ratio of the signal and noise voltages enter, or by eqs.(16) and (17), in which the ratios of signal to noise power enter.

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The actual signals in the transmission of speech and music, as is well known, have an amplitude distribution close to normal. The spectral distribution of signal power, as a rule, is not uniform over the entire pass band of the channel. It may be shown (Bibl.8) on the basis of the same general relation of eq.(5), that in this case the real capacity of the channel is determined by the following well known expressions (Bibl.5):

$$C_p = T\int_{f_1}^{f_2} \lg\left[\frac{P(f)}{N(f)} + 1\right] df, \tag{18}$$

or, introducing the coefficient δ (Bibl.8), depending on P(f) and N(f),

$$C_p = TF \lg \delta \left(\frac{P}{N} + 1\right), \tag{19}$$

where

$$P = \int_{f_1}^{f_2} P(f)\, df \quad \text{и} \quad N = \int_{f_1}^{f_2} N(f)\, df.$$

Accordingly the quantity of information that can be reproduced at the output of the receiver of a real system is determined by the expression:

$$H_p = TF_m \lg \delta_m \left(\frac{P_m}{N_m} + 1\right). \tag{20}$$

The expression of eq.(18) shows that the real capacity of a channel is completely determined by the statistical spectra of signal and noise. It also shows that the degree of filling of the pass band of the channel by the components of the signal-power spectrum characterizes the degree of approximation of a given system to the ideal: the greater the filling, the greater the actual capacity of the channel (Bibl.8). At the limit, when P(f) = const (ideal signal) and N(f) = const (fluctuation noise), C_p will have the maximum value determined by eq.(16).

If N(f) ≠ const, then the condition of the maximum for C_p will be the condition P(f) + N(f) = const (Bibl.5).

6. The Effectiveness of Radio Communication with Various Modulation Systems

Statistical communications theory establishes the fact that in an ideal system the channel capacity may be made equal to the quantity of information to be transmitted (H = C). This equality can be attained by means of a proper selection of the coding method. In a real system, H is always less than C.

The margin of unutilized capacity $\Delta C = C - H$ is determined by the assigned quality of the communication, which, in the simplest case, is expressed by the ratio of signal to noise at the receiver output. The higher the quality of communication required, the larger must ΔC be in a given system. Let us call the ratio between the quantity of information reproduced at the receiver output and the channel capacity, the efficiency of the communication system (Bibl.6, 8). By eq.(19) and (20)

$$\eta = \frac{F_m \lg \delta_m \left(\frac{P_m}{N_m} + 1\right)}{F \lg \delta \left(\frac{P}{N} + 1\right)}. \qquad (21)$$

The value of the efficiency η differs for different communication systems and characterizes the degree of utilization of the channel capacity with a given method of transmission.

Table 1

System of Modulation	$\frac{S}{\sigma}$	η	Remarks
Single Side-Band Transmission System (SSB)	1	1	$\alpha = 1$
Amplitude Modulation (AM)	$\frac{1}{1+\frac{M^2}{2}}$	$\frac{\lg(S+1)}{2\lg\left[1+\left(1+\frac{M^2}{2}\right)S\right]}$	M = Modulation Factor
Frequency Modulation (FM)	$\frac{3}{8}(\alpha-1)^3$	$\frac{\lg(S+1)}{\alpha\lg\left[1+\frac{8}{3}\cdot\frac{1}{(\alpha-1)^3}S\right]}$	$\alpha = 2m + 1$ m = Modulation Index

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Table 1 cont.

Time Pulse Modulation (TPM)	$\frac{\alpha^3}{126}$	$\frac{\lg (S+1)}{\alpha \lg \left(1+\frac{126}{\alpha^3} S\right)}$		
Duration Pulse Modulation (DPM) a) Two-Sided b) One-Sided	$\frac{\alpha^3}{72+16{,}7\alpha}$ $\frac{\alpha^3}{288+33{,}4\alpha}$	$\frac{\lg (S+1)}{\alpha \lg \left(1+\frac{72+16{,}7\alpha}{\alpha^3} S\right)}$ $\frac{\lg (S+1)}{\alpha \lg \left(1+\frac{288+33{,}4\alpha}{\alpha^3} S\right)}$		
Coded Pulse Modulation (CPM)	$10 \lg S - 2{,}2$	$\frac{2 \lg 2}{\lg (1+4{,}55 \lg S)}$		

The most perfect communications system must evidently be considered one with the maximum efficiency at an assigned quality of communication. In comparing systems, we shall put the coefficients δ and δ_m equal to unity. Introducing the notation $\alpha = \frac{F}{m}$, $S = \frac{P_m}{N_m}$ and $\sigma = \frac{P}{N}$, we obtain the following simplified expression for

$$\eta = \frac{\lg (S+1)}{\alpha \lg (\sigma+1)}. \qquad (22)$$

The results of an analysis of various systems and modulation performed by the author on the basis of eq.(22), are presented in the table in which the formulas for η, like those for $\frac{S}{\sigma}$, hold good for signal-noise ratios above the threshold level. The expressions for $\frac{S}{\sigma}$ with TPM and DPM are obtained on the basis of the well known formulas (Bibl.7) in which we set $F_i = 3F_m$, where F_i is the pulse repetition frequency.

Figure 1 gives the curves of communication efficiency $\eta = f(\alpha)$ for S = 60 db. Figure 2 gives curves of the necessary signal power $\frac{P}{N_o F_m} = \varphi(\alpha)$ for S = 60 db. Here $\frac{P}{N_o F_m} = \sigma\alpha$, where N_o = mean noise power in unit band. The curves of Figs.1 and 2 are constructed according to the formulas of Table 1.

For amplitude modulation under these conditions, $\eta = 0.5$ and $\frac{P}{N_o F_m} = 62$ db.

An analysis of the expressions obtained for η and of the corresponding curves of Figs.1 and 2 shows that the TPM and DPM are of practically equal efficiency. In these systems the efficiency is somewhat lower than in FM; its value decreases with

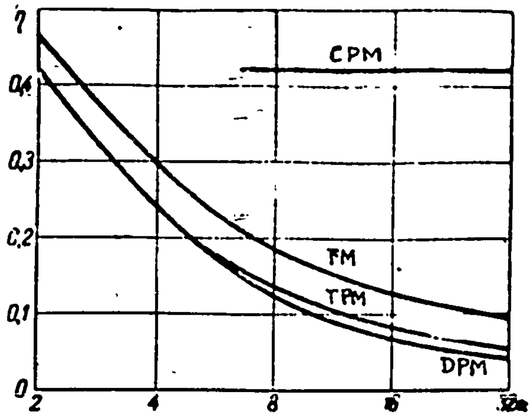

Fig.1

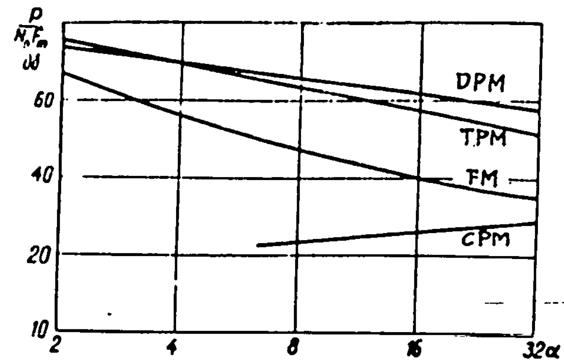

Fig.2

increase in the necessary width of the channel band at S = const, and depends only slightly on S at α = const. With an assigned communications quality, higher signal power is required in a DPM system than in the TPM system.

The FM system is better than the TPM and DPM systems, both in communication efficiency and in the utilization of signal power.

The CPM system is the most advanced of all existing modulation systems. It has the greatest efficiency and assures an assigned signal-noise ratio at the receiver output at a considerably lower signal level of the input than with the other forms of modulation.

The SSB and FM systems have high efficiency, but these systems give no advantage in the value of the signal-noise ratio at the receiver output, and do not allow this advantage to be increased by widening the channel frequency band. Under the same conditions, with AM, that is, under conditions when an assigned communications quality must be secured with a channel band width equal to $2F = 2F_m$, all of the modulation systems we have considered are practically of equal value. They have about the same efficiency and about the same necessary signal power. Under these conditions, the SSB and AM systems have the best characteristics. The conclusions obtained in this Section are in rather good agreement with the results of S.N. Losyakov's work (Bibl.3).

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INCREASING THE EFFICIENCY OF REACTANCE TUBES

by

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This paper considers a method of increasing the efficiency of the reactance tubes used in frequency modulation. It is shown that an increase in the efficiency of the reactance tubes allows the stability of the center frequency to be increased. The results of experiments are presented.

1. Introduction

To accomplish frequency modulation, reactance tubes, which vary the parameters of the self-oscillator circuit, are widely used in practice. A major shortcoming of this method is the low stability of the central frequency.

To improve frequency stability, an automatic frequency control is used, the reliability and efficiency of which increases with the weakening of the destabilizing factors. More specifically, it would be desirable, in order to weaken the influence of the capacitances of the wiring and the tubes, to reduce the wave impedance of the auto-oscillator circuit, i.e., to increase its capacitance. In this case, however, the frequency deviation produced by the reactance tubes is narrowed and therefore, in practice, as a rule, circuits with high wave impedance are used. Under such conditions, to assure normal operation of the system of automatic frequency control, the reactance tubes must be calculated for an elevated frequency deviation, as a result of which their destabilizing action increases. The methods described below allow these difficulties to be overcome.

2. Increasing the Efficiency of the Reactance Tubes.

In the push-pull diagram usually employed (Fig.1) (Bibl.2), the necessary phase

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shift of the grid voltage U_g and of the plate current I_h in phase with it, of the reactance tubes* with respect to the plate voltage U_p, is provided by the phase-shifting dividers R_1C_1 and C_2R_2.

The admittance of the first reactance tube equals:

$$Y_1 = \frac{I_{a1}}{U_a} = \frac{S_1 U_c}{U_a} = S_1 \frac{-i x_{c1}}{R_1 - i x_{c1}} = S_1 \frac{-i\beta_1}{1 - i\beta_1} = S_1 \left(\frac{\beta_1^2}{1+\beta_1^2} - \frac{i\beta_1}{1+\beta_1^2} \right). \qquad (1)$$

By analogy, for the second reactance tube:

$$Y_2 = \frac{I_{a2}}{U_a} = S_2 \frac{R_2}{R_2 - i x_{c2}} = S_2 \left(\frac{\beta_2^2}{1+\beta_2^2} + i \frac{\beta_2}{1+\beta_2^2} \right). \qquad (2)$$

The imaginary part of Y_1 is negative, that is, has the character of a capacitative conductance:

$$I_m(Y_1) = -S_1 \frac{\beta_1}{1+\beta_1^2} = -b_c = -\Delta(\omega C) \qquad (3)$$

and, accordingly,

$$I_m(Y_2) = S_2 \frac{\beta_2}{1+\beta_2^2} = b_L = \Delta\left(\frac{1}{\omega L}\right) \qquad (4)$$

has the character of an inductive conductance.

By differentiating the expression $\ln \omega = \ln \frac{1}{\sqrt{LC}} = -\frac{1}{2} \ln C + \frac{1}{2} \ln \frac{1}{L}$ for the small increments ΔC and $\Delta\left(\frac{1}{L}\right)$, we obtain the expression for the relative frequency deviation:

$$\frac{\Delta\omega}{\omega} = -\frac{1}{2}\frac{\Delta C}{C} + \frac{1}{2}\frac{\Delta\left(\frac{1}{L}\right)}{\frac{1}{L}} = -\frac{1}{2}\frac{\Delta(\omega C)}{\omega C} + \frac{1}{2}\frac{\Delta\left(\frac{1}{\omega L}\right)}{\frac{1}{\omega L}} =$$
$$= \frac{\rho}{2}\left(-b_c + b_L\right), \qquad (5)$$

where $\rho = \frac{1}{\omega C} = \omega L$ = wave impedance of circuit.

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*Pentodes are usually used as the reactance tubes, and the reaction of the plate may be disregarded, considering $I_p = SU_c$.

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In accordance with eq.(5), we shall take the term efficiency of a reactance tube to mean its equivalent reactive conductance (b_g or b_L). With increase of the latter, at an assigned frequency deviation $\frac{\Delta\omega}{\omega}$, the wave impedance of the circuit ρ may be decreased with the object of increasing the frequency stability.

We shall confine ourselves to the consideration of a symmetrical diagram, i.e., we shall put in eqs.(1) to (4);

$$\beta_1 = \frac{x_{c1}}{R_1} = b_2 = \frac{R_2}{x_{c2}} = \beta \qquad (6)$$

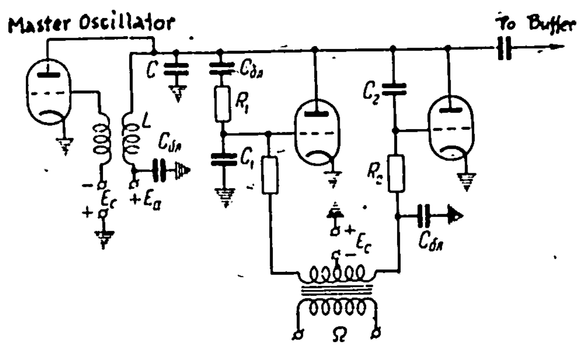

Fig.1

and in the absence of a modulating voltage $S_1 = S_2$, consequently, $b_g = b_L$.

In this case, as will be seen from eq.(5), the synphase action of the modulating voltage on the reactance tube, due to the inconstancy of the feed voltages, will not produce a frequency modulation, since conductances b_g and b_L vary by equal quantities. This constitutes the well known advantage of the push-pull circuit for connecting reactance tubes over the single-ended circuit.

With the antiphase action of the modulating voltage, the frequency deviation, according to the voltages of eq.(5) and eq.(6), will be equal to

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\rho S(|b_c| + |b_L|) = \rho S\frac{\beta}{1+\beta^2}. \qquad (7)$$

To eliminate the considerable parasitic amplitude modulation, $\beta << 1$ is used in the ordinary diagram.

If the parasitic amplitude modulation is not limited, then, as will be seen from eq.(7), in the ordinary diagram, the limiting frequency deviation (at $\beta = 1$) equals

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\, pS. \qquad (8)$$

To obtain high efficiency the ratio $\frac{U_g}{U_e}$ must, according to eq.(1), be increased.

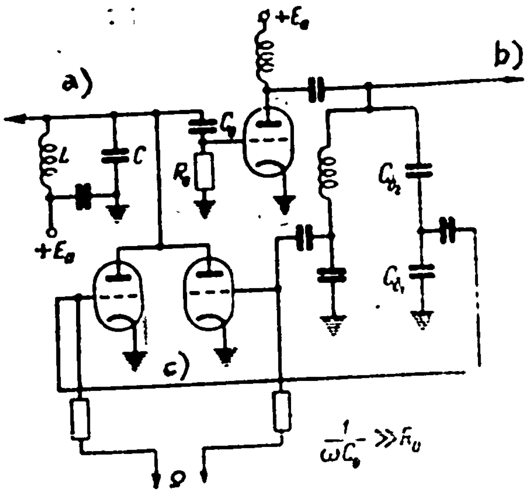

Fig.2

a) Master oscillator circuit; b) To frequency multipliers; c) Reactance tube

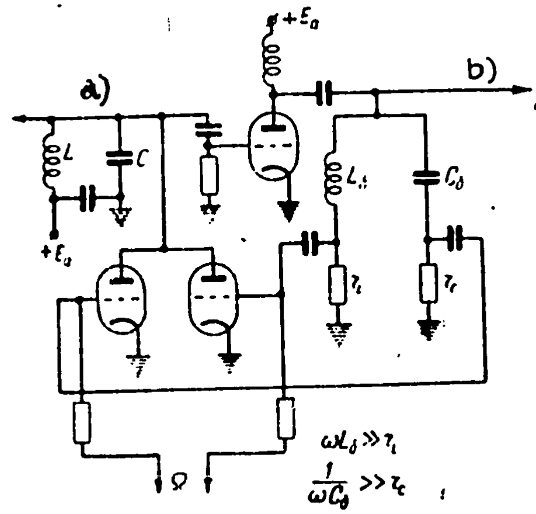

Fig.3

a) Master oscillator circuit; b) To frequency multipliers

This may be accomplished by introducing phase-shifting elements in the circuit. As such an amplifier it is expedient to use a buffer stage in the circuit (Bibl.1). In this case, two versions are possible.

In the first version (Fig.2) the voltage of the master oscillator ZG is at first subjected to a 90° phase shift in the divider C_0R_0, and is then amplified in the buffer stage. This circuit allows an increase of Y by a factor of $K_y\,q$, where $q = \frac{x_{cb1}}{x_{cb1} + x_{cb2}} = \frac{C_{b2}}{C_{b1} + C_{b2}}$ – division factor of the capacitative divider C_{b1}, C_{b2}, and K_y = amplification factor of the buffer stage.

In concrete cases, it is possible to secure K_y = 50 – 100 and $q = \frac{1}{2} - \frac{1}{3}$, i.e., to get a gain by a factor of $K_y\,q$ = 15 – 50.

In another version (Fig.3), the excitation on the buffer stage is imposed without phase shift; the latter is effected in the units L_br_L and C_br_g, which are at the

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same time branches of the buffer stage circuit. In this case the gain obtained will be by a factor of K_y, i.e., 2 to 3 times greater than in the preceding case.

We remark that in the circuit of Fig.2, the voltage on the plate of the buffer stage is only $\frac{1}{q} = 2 - 3$ times greater than the voltage on the grids of the reactance tubes, while in the diagram of Fig.3 it is $| \quad | = | \quad |$ times as great. For the normal operation of reactance tubes, the high frequency voltage on their grids, as a rule, must be small (less than 1 - 2 v)*. For this reason, in the circuit of Fig.2, the voltage on the plate of the buffer stage tube is likewise low, and in a number of cases does not provide the required excitation voltage for the following stage (which ordinarily operates in a state of frequency multiplication), as a result of which, an additional stage of amplification may be necessary.

Since the circuit of Fig.2, which is inferior to that of Fig.3 in efficiency and value of the output voltage, is in all other respects merely equivalent to it, the circuit of Fig.3 is preferable. On the basis of these considerations, we shall, in future, discuss the latter circuit. The theoretical researches presented below show that the introduction of an amplifier in the circuit of phase shifting units does not disturb the stability of operation of the circuit and in practice causes no additional distortions in modulation.

3. Practical Results

For the experimental verification of the method of increasing the efficiency of reactance tubes, an exciter with frequency modulation was assembled. The use of 6Zh4 tubes proved to be most advisable for the master oscillator of the buffer stage and for the reactance tubes connected according to the circuit of Fig.3. The master oscillator was figured for a frequency of 5.4 megacycles. The capacitance of the

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*We have in mind the most widely used state of operation of reactance tubes without a cutoff point and with low grid voltages. Such a state allows us to effect frequency modulation with insignificant nonlinear distortions.

circuits, owing to the increase in the efficiency of the reactance tube, could be taken higher (of the order of 800 $\mu\mu f$, which, at a low output capacitance of the auto generator tube (3 - 5 $\mu\mu f$), assured high stability of the central frequency (on changing the tube of the autogenerator and on deformations of the assembly).

The considerable amplification factor of the buffer stage allowed the connection of its grid to part of the autogenerator circuit, as a result of which the destabilizing action of the input capacitance of the tube of the buffer stage was reduced by a factor of n^2, where n is the coupling factor between the buffer stage grid and the master oscillator plate. In this case we took $n \approx 3.2$, so that the influence of the input capacitance and the buffer stage were reduced by a factor of about 10 and became negligibly small.

The resistances r_g and r_L (Fig.3) were of the order of 5 - 10 ohms, and the high frequency voltage from each of them was fed simultaneously to the grid circuit of one reactance tube and to the cathode circuit of the other. This doubled the efficiency of the reactance tube. The low value of the resistances r_g and r_L eliminates in practice the influence of the capacitances of the reactance tubes and the wiring on them, and assures a stable phase shift.

Owing to the high efficiency of the reactance tubes, it proved possible to connect their plates to the same part of the autogenerator circuit to which the grid of the buffer stage was connected. This allowed us practically to eliminate the instability of the central frequency due to instability of the output self-capacitances of the reactance tubes, and also to decrease the destabilizing action of the variation of their transconductance. In spite of the small coupling between the reactance tubes and the autogenerator circuit, the maximum frequency deviation was 3 to 5 times as great as the 100% modulation level, which assured the necessary margin for the automatic frequency control (the latter was selected at a lower level owing to the general increase of the stability of the center frequency of the autogenerator).

We present below a table characterizing the increase of the stability of the central frequency of this exciter by comparison with the exciters formerly used (Bibl.3).

Name of Index	Former Exciter	New Exciter
1. Equivalent Admittance of Reactance Tubes (Converted to Terms of Capacitance), μμf	5	250
2. Deviation of Admittance of Reactance Tubes Necessary for 100 Percent Modulation (Converted to Terms of Capacitance), μμf	0.27	42.5
3. Relative Frequency Drift ($\Delta f/f$) with a 5% Variation of the Self-Capacitances of All Tubes	6.10^{-3}	$1.5.10^{-4}$
4. Relative Variation of Frequency on Departure of the Mean Transconductance of One Reactance Tube with Respect to the Other by 10%	6.10^{-4}	2.10^{-4}

Without allowing for the advantages due to the use of the modern tubes, the gain by reason of the use of the new circuit (Fig.3) over that of the circuit of Fig.1, amounts to:

a) in efficiency of the reactance tubes, a factor of 25 ($K_y = 25$);

b) in the deviation of the admittance of the reactance tubes necessary for 100% modulation, on account of the increased efficiency of the reactance tubes, a factor of 50, and on account of the more complete utilization of the characteristics of the reactance tubes for modulation, by a factor of 3. The latter circumstance decreases, by the same factor, the relative variation of frequency owing to instability of the transconductance of the reactance tubes;

c) in the frequency drift $\frac{\Delta f}{f}$ on variation of the self-capacitances of all tubes, by a factor of 10 - 20 (4 - 5 times on account of the selection of a higher capacity of the autogenerator circuit and 3 - 4 times on account of connecting the

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reactance tube and the tube of the buffer stage to the part of the oscillatory circuit).

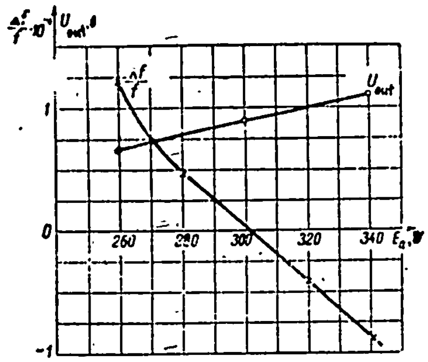

Fig.4

The total maximum instability of the central frequency of the exciter (without automatic frequency control) due to changes in the capacitances of the tubes was reduced by tens of times and did not go beyond the limits of $(\frac{\Delta f}{f})_{max} = 2 - 8 \times 10^{-4}$. The temperature factor remained dominant, and to eliminate its influence the master oscillator and the reactance tubes were placed in a thermostat.

We give below the experimental relations between the frequency drift $\frac{\Delta f}{f}$ and the amplitude of the autogenerator (U_{out}) with a variation of the plate voltage E_p (Fig.4) and with variation of the filament voltage U_f (Fig.5). In the former case, the oscillations broke off at E_p = 50 v; without the reactance tubes, the frequency drift at $E_p = \pm 10\%$ was 5×10^{-6}. In the latter case, without the reactance tubes, the frequency drift at $\Delta U_f = \pm 20\%$ was less than 2×10^{-6}.

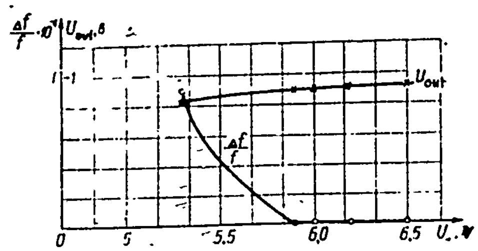

Fig.5

In the presence of the automatic frequency control (based on the use of a frequency divider and a phase detector) (Bibl.4) reliable frequency stability was secured, of the order of stability of the standard quartz oscillator.

With respect to the remaining qualitative indexes (nonlinear distortions, parasitic amplitude modulation, background, etc) the exciter likewise met the requirements of high grade radio broadcasting.

4. Theoretical Study of the Circuits

In practice, certain advantages of the push-pull connection of reactance tubes are not fully realized, owing to the scatter of their parameters. In the presence of automatic frequency control, when the frequency drift of the autogenerator is corrected by blocking one reactance tube and unblocking the other, the tubes operate under substantially different conditions. Thus, for the most objective evaluation of the operation of the diagram its least favorable version, the single ended connection of the reactance tubes, must be analyzed.

Such a circuit is presented in Fig.6. Taking apart the grid circuit of the reactance tubes RL, as shown on this figure, let us investigate the diagram. Considering the voltage U_{in} at the grid of the reactance tube the input voltage, and the voltage U_{out}, taken from the circuit of the buffer stage, the output voltage, we may consider the diagram as a two-stage amplifier. In this case, one of the stages serves as a reactance tube with a load consisting of the admittance of the circuit and the autogenerator tube, and the other as a buffer amplifier, the voltage from which is withdrawn through a divider consisting of the capacitance of its circuit C and the resistance r_g.

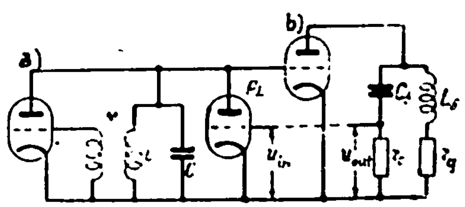

Fig.6

A) Oscillator; b) Buffer stage

Determining the complex amplification factor

$$K = \frac{U_{out}}{U_{in}} = \frac{A + iB}{C + iD}, \tag{9}$$

the condition of dynamic equilibrium of the system in the presence of oscillation K = + 1 may be written in the form of two equations:

$$A = C, \quad B = D. \tag{10}$$

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These equations allow us to find the frequency of oscillation, and on allowing for the nonlinear properties of the diagram, we also find the amplitude of the oscillations as a function of the bias on the grid (or the transconductance) of the reactance tube.

Under normal conditions of operation of the circuit, the self-oscillations are maintained on account of the negative admittance of the autogenerator tube, compensating the admittance of the LC circuit. Since the voltages on the grid and plate of this tube are of opposite phase with a high degree of accuracy (over a wide frequency band), it may be considered that its admittance contains no reactive component and that the generated frequency ω corresponds to the natural frequency of the circuit.

The reactance tube, owing to feedback in its grid, may introduce both active and reactive components into the circuit*. The dynamic equilibrium, however, is restored owing to the fact that the reactive component is compensated on account of the variation of the generated frequency and of the detuning of the autogenerator circuit, while the active component is compensated on account of the variation of the admittance of the autogenerator tube Y_g, which arises due to the variation of the amplitude of the oscillations. The dependence of Y_g on the amplitude of the oscillation may be found experimentally or by the graph-analytical method from the characteristics of the tubes, and in this way it is sufficient instead of the amplitude to determine the value of the admittance Y_s. For many practical cases, within the normal limits of variation of amplitude, the latter is connected with Y_g by a roughly linear relation, so that the character of variation of Y_g also corresponds to the character of variation of the amplitude of the oscillations. In what follows we shall make use of this fact and shall characterize the parasitic

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*The admittance of the tube due to its internal resistance (plate reaction) we shall neglect, considering that a pentode is used as the reactance tube.

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amplitude modulation not as the relative variation of the amplitude of oscillations, but as the relative variation of the admittance of the autogenerator $\frac{\Delta Y_g}{Y_g}$.

Taking account of the fact that at great deviations of the frequency ω from the tuning frequency of the circuits, $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L_b C_b}}$ (Fig.6), their equivalent resistances are low, and the amplification factors of the tubes become less than unity, we may come to the conclusion that in this case the condition of equilibrium $K = +1$ cannot be satisfied. This allows us to limit our consideration to cases of small deviations of frequency $(\frac{\Delta\omega}{\omega_o} << 1)$, and for the equivalent resistances of the circuits we shall make use of the approximate formula:

$$Z_K = \frac{R_K}{1 + i\frac{2\Delta\omega}{\omega_0}Q}, \qquad (11)$$

where R_K = equivalent resistance of circuit at resonant frequency, $\Delta\omega = \omega - \omega_o$, Q = quality factor of circuit.

For the buffer stage circuit, accordingly,

$$Z_{bc} = \frac{R_{bc}}{1 + i\frac{2\Delta\omega}{\omega_0}Q_b}. \qquad (12)$$

The amplification factor (neglecting the plate reaction) is

$$K_b = S_b Z_{bc} = \frac{S_b R_{bc}}{1 + i\frac{2\Delta\omega}{\omega}Q_b} = \frac{K_{0b}}{1 + i\frac{2\Delta\omega}{\omega_0}Q_b}, \qquad (13)$$

where Q_b = quality factor of buffer stage circuit, S_b = transconductance of its tube.

The load of the plate circuit of the reactance tube Z_{rt} is determined by the conductance of the circuit (Y_{Kg}) and of the autogenerator tube (Y_g):

$$Z_{rt} = \frac{1}{Y} = \frac{1}{Y_{kg} - Y_g} = \frac{1}{\frac{1 + i\frac{2\Delta\omega}{\omega_0}Q}{R_K} - Y_g} = \frac{R_K}{1 + i\frac{2\Delta\omega}{\omega_0}Q_g - Y_g R_K}. \qquad (14)$$

The amplification factor of the reactance tube

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$$K_{rt} = SZ_{rt} = \frac{SR_\kappa}{1 - Y_g R_\kappa + i\frac{2\Delta\omega}{\omega_0}Q_g} = \frac{S\rho Q_g}{1 - Y_g R_\kappa + i\frac{2\Delta\omega}{\omega_0}Q_g}, \tag{15}$$

where Q and ρ are respectively the quality factor and circuit characteristic of the autogenerator circuit, and S = transconductance of reactance tube.

The transfer ratio of the phase-shifting network C_b, r_g equals

$$p = \frac{r_c}{r_c - i x_{cb}} = \frac{i\beta}{1 + i\beta}, \tag{16}$$

where

$$\beta = \frac{r_c}{x_{cb}} = \frac{r_L}{x_{Lb}} = \frac{1}{2Q_b}. \tag{17}$$

This quantity may be considered constant, since the frequency variations considered are small.

On the basis of eqs.(13), (15), and (16), eq.(9) may now be written in the explicit form:

$$K = K_{rt}K_b p = S\rho Q_g K_{eb}\frac{i\beta}{\left(1 - Y_g R_\kappa + i\frac{2\Delta\omega}{\omega_0}Q_g\right)\left(1 + i\frac{2\Delta\omega}{\omega_0}Q_g\right)\left(1 + i\beta\right)}. \tag{18}$$

Simultaneous solution of eqs.(10) allows us to obtain the expressions:

$$\frac{\Delta\omega}{\omega_0} = \frac{\frac{1}{2}\frac{\beta}{1+\beta^2}S\rho K_{eb}}{1 + 4\frac{1-\beta^2}{1+\beta^2}\left(\frac{\Delta\omega}{\omega_0}\right)^2 Q_b^2 + 4\frac{\beta}{1+\beta^2}\left(\frac{\Delta\omega}{\omega_0}\right)Q_b + \frac{1}{2}\frac{\beta}{1+\beta^2}S\rho K_{eb}}; \tag{19}$$

$$Y_g R_\kappa = \frac{1 - \left(\frac{\Delta\omega}{\omega_0}\right)^2 4Q_gQ_b - 2\frac{\Delta\omega}{\omega_0}\beta(Q_b + Q_g)}{1 - \frac{\Delta\omega}{\omega_0}}. \tag{20}$$

The latter of these equations determine the stability of the system in amplitude and parasitic modulation.

Stability in amplitude is evidently disturbed when $Y_g R_K$ becomes close to zero, since this in practice means the appearance, under the action of the negative conductance of the reactance tube, of high oscillatory voltages blocking the autogenerator tube.

The dominating term determining the relation of $Y_g R_K$ on the frequency at the limiting allowable frequency deviations, is the second term of the numerator, and therefore, from the condition $Y_g R_K > 0$, we get:

$$\frac{\Delta\omega}{\omega_0} < \frac{1}{2}\sqrt{\frac{1}{Q_g Q_b}}, \qquad (21)$$

that is, with respect to stability in amplitude, the frequency deviation must not exceed half the width of the mean geometrical pass band of the autogenerator and buffer stage circuit. At radio broadcasting frequency modulation, the normal maximum relative frequency deviation is usually equal to $\frac{\Delta\omega}{\omega_o} = (1 - 1.5) \times 10^{-3}$, $Q_g = 100 - 150$, $Q_b = 10 - 20$. For this reason the condition of eq.(21) is satisfied even when the normal frequency deviations are exceeded by a factor of 5 - 10. The parasitic amplitude modulation, defined as the difference between $Y_g R_K$ at normal frequency deviation and $Y_g R_K$ at $\Delta\omega = 0$, constitutes, as will be clear from eq.(20), a quantity of the order of 1% of $Y_g R_K$ at $\Delta\omega = 0$.

Stability in frequency and frequency modulation are both determined by eq.(19). The relation of the frequency deviation $\frac{\Delta\omega}{\omega_o}$ to the transconductance of the reactance tube is given, generally, by a cubic equation. But it is not hard to see that, since the parameter of the diagram are so selected that the value of the numerator in eq.(19) is a small quantity, $\frac{\Delta\omega}{\omega_o}$ will also be small. In this case, all terms in the denominator of eq.(19) may be neglected in comparison with unity and we may find $\frac{\Delta\omega}{\omega_o}$ (in first approximation) as follows:

$$\left(\frac{\Delta\omega}{\omega_0}\right)_1 = \frac{1}{2}\,\frac{\beta}{1+\beta^2}\,S\,\rho\,K_{IIb} = \alpha S. \qquad (22)$$

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For the ordinary single-ended circuit without amplifier in the phase-shifting network we have, by eqs.(5) and (6), the formula

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{2}\,\frac{\beta}{1+\beta^2}\,S\,\rho,$$

which differs from eq.(22) in the absence of the factor K_{ob}, equal to the amplification factor of the buffer stage at resonance. The presence of the factor K_{ob} allows us, at one and the same frequency deviation, to appropriately reduce ρ, that is, to increase the capacitance of the autogenerator circuit.

Under the condition of normal radio broadcasting frequency modulation, according to the figures given above, the correction to the terms of the denominator of eq.(19), which are omitted in eq.(22) and determine the error of eq.(22), amount to tenths of one percent, that is, they are negligibly small. The nonlinear distortions are absent from the first approximation, since, by eq.(22), the frequency deviations have a linear relation with the transconductance, through the factor of proportionality $\alpha = \frac{1}{2}\,\frac{\beta}{1+\beta^2}\,\rho\,K_{ob}$.

To evaluate the nonlinear distortions, let us find the second approximation of $\frac{\Delta\omega}{\omega_0}$, by substituting the first approximation of $\frac{\Delta\omega}{\omega_0}$ in the denominator of eq.(19):

$$\left(\frac{\Delta\omega}{\omega_0}\right)_2 = \frac{\alpha S}{1+4\,\frac{1-\beta^2}{1+\beta^2}\,\alpha^2 S^2 Q_b^2 + 4\,\frac{\beta}{1+\beta^2}\,\alpha S Q_b + \alpha S}. \qquad (23)$$

Taking eq.(17) into account, and neglecting β^2 in comparison with unity, we have, with an accuracy to small quantities of higher orders:

$$\left(\frac{\Delta\omega}{\omega_0}\right)_2 = \frac{\alpha S}{1+4\alpha^2 S^2 Q_b^2 + 3\alpha S} = \alpha S\left(1-4\alpha^2 S^2 Q_b^2 - 3\alpha S\right) =$$
$$= \alpha S - 3\alpha^2 S^2 - 4\alpha^3 S^3 Q_b^2. \qquad (24)$$

To determine the nonlinear distortion factor, we put, in eq.(22):

$$\left(\frac{\Delta\omega}{\omega}\right)_1 = \alpha S = \varepsilon_0 \cos\Omega t, \qquad (25)$$

after which eq.(24) yields:

$$\left(\frac{\Delta Q}{\omega}\right)_s = \varepsilon_0 \cos\Omega t - 3\varepsilon_0^2\cos^2\Omega t - 4\varepsilon_0^3 Q_b^2\cos^3\Omega t = \varepsilon_0(1-3\varepsilon_0^2Q_b^2)\cos\Omega t - \frac{3}{2}\varepsilon_0^2\cos 2\Omega t - \varepsilon_0^3 Q_b^2\cos 3\Omega t - \frac{3}{2}\varepsilon_0^2. \quad (26)$$

Whence the nonlinear distortion factor equals:

$$K_f \approx \sqrt{\frac{9}{4}\varepsilon_0^2 + \varepsilon_0^4 Q_b^4} = \frac{3}{2}\varepsilon_0\sqrt{1+\frac{4}{9}\varepsilon_0^2 Q_b^4}.$$

where ε_0 has the meaning of the amplitude with respect to frequency deviation.

According to this formula and to the above presented figures for ε_0 and Q_b at radio broadcasting frequency modulation, the nonlinear distortions constitute fractions of 1%, i.e., are negligibly small.

What has been set forth above allows us to draw the conclusion that the introduction of an amplifier in the circuit of the phase-shifting elements in radio broadcasting frequency modulation does not lead to instability of the oscillations in amplitude or frequency, to a marked parasitic amplitude modulation, nor to nonlinear distortions.

In conclusion I consider it my duty to express my profound thanks to Professor Z.I.Model' for a number of valuable suggestions, hints and critical remarks on the subject of this work, and also for the help given during its course.

Paper Received by the Editors 25 March 1955.

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AUTHOR'S REVIEW

BAND RESONANT SYSTEMS WITH CONSTANT PASS BAND

by

I.M.Simontov

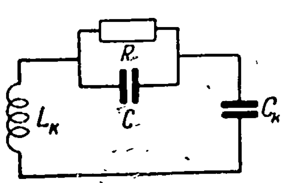
In band resonance systems intended to effect frequency selectivity, it is desirable to have a constant pass band over the entire tuning range. In this case, maximum selectivity is attained over the entire tuning range. Usually with increased tuning frequency, the band $(2\Delta f)_{0.7} = \delta(\omega)f_0$ increases, since the damping of the circuit $\delta(\omega)$ varies only slightly. It is possible to compensate the band by introducing a frequency-dependent damping, positive or negative.

In the former case, the band is compensated as a result of its widening. An RC element may serve as the frequency-dependent damping (cf.Fig.). It is simplest to find the values of R and C by using a method proposed by G.V.Braude for the correction of amplitude-frequency characteristics.

The band at any frequency range is determined by the equation:

$$(2\Delta f)_i = \frac{\delta(\omega_i)\,\omega_i}{2\pi} + \frac{R}{2\pi L(1+\omega_i^2\tau^2)}.$$

By equating to zero the first and second derivatives of the band with respect to frequency, we find the quantities R and $C = \frac{\tau}{R}$. With a linear dependence of the damping on the frequency $\delta(\omega) = a\omega + \delta$, we get:



$$R = \frac{(2a\omega_* + \delta)(1+\omega_*^2\tau^2)^2 L}{2\tau^2\omega_*},\quad \tau = \frac{1}{\omega_*}\sqrt{\frac{\gamma-1}{3\gamma-1}},$$

$$\gamma = \frac{1+\omega_*^2\tau^2}{1-3\omega_*^2\tau^2} = \frac{2a\omega_*+\delta}{2a\omega_*},$$

where ω_* = frequency corresponding to the external value of the band.

If $\delta(\omega) = \delta$, then $\tau = \frac{1}{\sqrt{3}\,\omega_*}$; $R = \frac{8}{3}\delta\omega_* L$; $C = \frac{\sqrt{3}}{8\delta L\omega_*^2}$.

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In this case, the variability of the band in the tuning range will be equal to:

$$\beta_l = \frac{(2\Delta f)_l}{(2\Delta f)_*} = \frac{1}{3}\left(\kappa_l + \frac{8}{3+\kappa_l}\right),$$

where

$$\kappa_l = \frac{f_l}{f_*}.$$

If f_* is the central frequency of the range, then at $K_d = \frac{f_{max}}{f_{min}} \approx 3.K_i$ varies from 0.5 to 1.5, and the variability of the band proves to be slight.

Negative frequency-dependent damping may be introduced in the circuit by the aid of a combined feedback (positive and negative).

A number of authors have investigated systems with combined feedback (in what follows we shall use the abbreviation CF), operating at a fixed frequency (Bibl.1, 2, 3, 4, 5). The physical prerequisite that determines the possibility of accomplishing band circuits with a constant pass band, consists in the fact that on retuning of the amplifier from the low frequency region to a higher frequency region, the EMF of the positive feedback increases, compensating the increase of the band on account of the reduction in the equivalent attenuation of the diagram. The determination of the feedback parameters $m = \frac{M_{coup}}{L}$ and R_K may be performed if the following conditions are adopted: a) on introduction of the feedbacks, the amplification at maximum frequency of the range remains constant; b) the same band is observed at the extreme frequencies of the range.

In this case we get:

$$R_\kappa = \frac{\kappa_d \frac{\delta_2}{\delta_1} - 1}{S\left(1 - \frac{\delta_2}{\delta_1} \cdot \frac{1}{\kappa_d}\right)};$$

$$m = \frac{R_\kappa}{SZ_0}.$$

Here δ_1 and δ_2 = attenuation of the system without CF at the frequencies f_{min} and f_{max} respectively; $Z_{0\ max}$ = resonant resistance of circuits at frequency f_{max}; S = transconductance.

If $\delta_1 = \delta_2$, then $R_k = \frac{k_d}{S}$, while $m = \frac{1}{Z_{omin}}$.

The variability of the band in this case will be equal to:

$$\beta_i = \frac{(2\Delta f)_i}{(2\Delta f)_{1,2}} = \left(1 - \frac{\kappa_i - 1}{\kappa_d}\right)\kappa_i,$$

where

$$\kappa_i = \frac{f_i}{f_{min}} = 1 \div \kappa_d.$$

The band proves to be maximum at the central frequency of the range. The relation between the maximum variation of the band and the band of the circuit is determined by the equation $\beta_{max} = 0.25\ (K_d + \frac{1}{K_d}) + 0.5$ is reflected in the following table:

K_d	1.5	2.0	3.0	5.0	6.0	7.0	8.0	10.0
β_{max}	1.03	1.12	1.33	1.80	1.94	2.28	2.53	3.03

It must be borne in mind that, in both cases, at $\delta(\omega)$ = const, an additional attenuation is introduced into the circuit, whose variation is inversely proportional to the frequency, and an additional non-uniformity of amplification equal to K_d is observed.

By using "elongated" antenna and plate circutis, it becomes possible to attain equal amplification at the ends of the band. In this case the elongation factor of the antenna circuit must be selected starting out from the relation $f_p \approx f_{min}$

If the "elongation" factor of the plate circuit $K_p = \frac{f_p}{f_{min}} = 0.5 - 0.7$, then the nonuniformity of amplification in the range at $K_d \approx 3$ will be respectively

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1.25-2.0. On using circuits with magnetic tuning in the band circuits (according to the method of direct connection of the oscillatory circuit) the compensation of the amplification takes place simultaneously with the compensation of the band. In a combined feedback circuit, in this case, a constant part must be taken off the inductance of the circuit by means of which constant part the positive feedback will be supplied. Otherwise, in retuning the amplifier, the quantity $m = \frac{M_{out}}{L}$ will vary.

In conclusion I remark that systems with combined feedback allow narrow-band range filters to be realized.

Paper Received by the Editors 21 December 1953. Authors Review Received 10 April 1954.

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NEW BOOKS

O.B.Lur'ye. Video Frequency Amplifiers. Publishing House "Sovetskoye radio". Moscow, 1955, 280 pp. + 5 plates; price 9.35 rubles.

The basic questions of theory and design of video frequency amplifiers used in television and pulse technology are considered. The frequency and time method of analysis and calculation of video-frequency amplifiers are described and the most important properties of the transient characteristics are analyzed. Calculations of the most widely used circuits are given. Indirect methods of transmitting low frequencies are set forth, together with the method of calculating anti-noise systems for correction of television amplifiers. The last chapters of the book are devoted to manual and automatic gain control, and the the design features, tuning and testing of video-frequency amplifiers.

Academy of Sciences USSR. Laboratory for development of scientific problems of wire communications. Collection of scientific papers on wire communications. Issue 3. Professor G.V.Dobrovol'skiy, Editor. Academy of Sciences USSR, Moscow, 1954, 206 pages. Price 9.75 rubles.

This collection contains the following papers: G.V.Dobrovol'skiy: Finding the Frequency characteristics of reduced conductance assuring an assigned transient process, and: Influence of phase distortion on steady processes in communications channels; E.V.Zelyakh and S.L.Epshteyn, Electrical filters from line segments; I.I.Grodnev: Parameters of influence and protection from noise in shielded cable circuits; V.I.Kovalenkov: Primary parameters of a bundle of wires; and: On the question of the synthesis of computational diagrams; I.T.Turbovich: Principles of measurement of phase distortions in television main lines, and: An instrument for measuring the group time of propagation in television main lines; V.M. Shteyn, Determination of the quality of structural lengths of coaxial cable; A.D.Kharkevich:

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Accessibility of selector of automatic telephone exchange on connection of connecting lines in its contact field; I.T.Turbovich and V.G.Solomonov: On the error of measurement of oscillations of the frequency characteristic of the modulus of the transfer ratio by the method of frequency modulation.

V.V.Solodovnikov, Yu.I.Topchiyev, G.V.Krutikova. Frequency method of constructing transient processes, with appendix of tables and nomograms. Gostekhteoretizdat, Moscow, 1955. 195 p., price 12.40 rubles.

This book states the theoretical principles and the essential nature of the method of trapezial frequency characteristics, used in the theory of automatic regulation for the construction of transient processes. The method is based on the concept of frequency characteristics which may be found not only from the differential equation of the system but also from experiment.

This book gives examples of the construction of transient processes for various systems (aircraft with autopilot, electro-hydraulic servo system, etc.). To facilitate the use of the method and make it more convenient, the book gives tables of the h_x functions which are far more complete than in works published previously and provide the required accuracy of construction.

Technique of transmitting the results of measurements by radio. Manual of translations on radiotelemetry. Voyenizdat, Moscow, 1955, 150 pp, price 6 rubles.

This book gives certain information on the theory of radio telemetry and also describes concrete types of systems of transmitting measurements by radio, using frequencies and time-division of channels with various methods of modulation.

The book is intended for persons interested in questions of radio telemetry.

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