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DIFFRACTION OF ELECTROMAGNETIC WAVES ON CERTAIN BODIES OF ROTATION

(Diffraktsiya Elektromagnitnykh Voln Na Nekotorykh Telakh Vrashcheniya)

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by

Various Authors

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FOREWORD

The collection contains articles devoted to the strict theory of diffraction of electromagnetic waves on conductive bodies.

The V. A. Fok report explains the mathematical apparatus allowing boundary problems of electro-dynamics in the system of coordinates of a rotational paraboloid to be solved. As an example the author offers a detailed analysis of the electromagnetic field excited within a rotational paraboloid by means of a radiator (emitter) oriented in its focus.

The M. G. Belkina - L. A. Vaynshteyn report discusses the radiation characteristics of vibrators and slots oriented along a conductive sphere. The report offers numerous characteristic graphs plotted according to conventional diffraction series and in accordance with formulas obtained as a result of asymptotic adding of these series.

The M. G. Belkina report offers a solution to the problem concerning a dipole oriented on the axis of an elongated rotational ellipsoid or disk. In this report the radiation characteristics are calculated (for a majority of cases) allowing the effect of an elongated body on a field of a radiator situated near it to be estimated. A new solution is also given to the problem concerning the diffraction of a plane wave on a disk and the numerical results are compared with the approximated theory of diffraction. The radiation characteristics of an unilateral slot on a disk are described.

The book is intended for radio-physicists and radio engineers dealing in supersonic frequencies.

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From the Editor's Office

The theory of diffraction of electromagnetic waves on metal bodies is acquiring greater practical importance in recent years, especially in connection with the development of the centimeter radiowave technique. Radio engineering has brought up a series of new problems differing from the classical diffraction problems in optics. These include problems of diffraction of electromagnetic waves radiated by various antennas on metal bodies situated near by. The articles in this collection are devoted to just such a type of problems.

The V. A. Fok article describes a new mathematical apparatus enabling boundary problems of electrodynamics to be solved in the system of coordinates of a rotational paraboloid. In the second part of this report an important problem is solved concerning a dipole oriented in the focus of the paraboloid. A solution is given in the form of integrals and series with a connection established between the obtained terms and the laws of geometric optics.

The M. G. Belkina and L. A. Vaynshteyn report investigates the electromagnetic field of vibrators and slots oriented on a conductive sphere. For the fields, asymptotic formulas, obtained from ordinary diffraction series by the method of improved asymptotic adding in accordance with the method introduced by V. A. Fok in his transactions on diffraction, were derived. The report contains numerous graphs for the radiation characteristics, formulated in accordance with the diffraction series, as well as in accordance with the asymptotic formulas.

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The M. G. Belkina reports offer a solution to the problems concerning a dipole, oriented on the axis of an elongated rotational ellipsoid or disk. The author calculated (for a majority of cases) the radiation characteristics allowed an opinion to be formed on the effect of an elongated body on a field of a radiator situated near it.

The second report also offers a new solution to the classic problem of diffraction of a plane wave on a disk or circular orifice in a flat screen. The accurate results obtained in these cases are compared with the results offered by the approximate methods of physical optics. The report also describes the characteristics of slot radiation on a disk.

In spite of the fact that several years have already passed since these reports have been published and, that during this time a greater number of reports on this very same subject have appeared, especially reports on diffraction on a rotation ellipsoid and disk, the articles of this collection still bear a scientific interest both in a methodical respect and with respect to the numerical results as well.

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The Theory of Diffraction of a Rotational Paraboloid

by

V. A. Fok

Introduction:

This report consists of two parts. The first part describes the theory of parabolic functions (Par. 1-3) and presents certain analysis of these functions (Par. 4 and 5). Next is given the theory of solutions of Maxwell equations in parabolic coordinates (Par. 6-8). The most important result of the first part is the introduction of the parabolic potentials P and Q allowing the boundary conditions to be formulated without the aid of equations in the final differences which simplifies the solution of all problems connected with the rotational paraboloid. By introducing four auxiliary functions (Par. 7) connected by simple ratios with each other and with the P and Q potentials, it is possible to attain a simplification of terms (expressions) for the field.

Thus, the first part contains the mathematical apparatus necessary for solving diffraction problems of a rotational paraboloid.

The second part is devoted to the problem of dipole radiation in a focus of an absolute-reflecting rotational paraboloid. The primary field of the dipole is expressed through the parabolic potentials of the general theory (Par. 9). The field potential of a reflected wave is expressed first in the form of integrals (Par. 10) and then in the form of series (Par. 11) of two types (with different convergence zones). The expressions

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for the auxiliary functions are also given in the form of series. In the last two paragraphs (12 and 13) a field in the wave zone is discussed and plain expressions corresponding to the approximation of geometric optics (including the first correction members) are given for this field. In particular, we obtain the dependence of the field amplitude upon the distance from the axis.

At first it was intended to include in our report, as part three, the solution, obtained by us in 1944, to the problem of diffraction of a plane wave falling on a paraboloid from without under any given angle. This solution also contains an approximate summation of series and a determination of the field on the surface of the paraboloid in the zone of semi-shadow, which is of importance in connection with our established principle of a local field in the semi-shadow zone. However, in view of the considerable volume of the first two parts of our report we found it advisable not to include this third part. In the role of a brief resume of the content of the third part we refer to our report on the distribution of currents excited by a plane wave on the surface of a conductive body of any given form (1).

Part 1. General Theory

Par. 1. Parabolic Coordinates.

The letters x, y, z will designate the rectangular coordinates and we will write the rotary paraboloid equation in the form of

$$x^2+y^2-2az-a^2=0. \qquad (1.01)$$

With the letter k we designate the absolute value of the wave vector

$$k=\frac{2\pi}{\lambda_0}, \qquad (1.02)$$

(where lambda$_0$ is the wave length) and we shall introduce the following values in the role of variables: first, the angle phi between the surface passing through the axis z and through the given point, and a certain fixed surface passing through the very same axis (e.g., the surface xOz); this angle will be the same as in ordinary cylindrical coordinates; secondly, the parabolic coordinates u, v are connected with the rectangular ones according to the following formula

$$u=k(R+z); \quad v=k(R-z), \qquad (1.03)$$

$$r=\frac{1}{k}\sqrt{uv}; \quad z=\frac{1}{2k}(u-v); \quad R=\frac{1}{2k}(u+v), \qquad (1.04)$$

where

$$r=\sqrt{x^2+y^2}; \quad R=\sqrt{x^2+y^2+z^2}. \qquad (1.05)$$

The rectangular coordinates are expressed through a parabolic according to the formula

$$x=\frac{1}{k}\sqrt{uv}\cos\varphi; \quad y=\frac{1}{k}\sqrt{uv}\sin\varphi; \quad z=\frac{1}{2k}(u-v). \qquad (1.06)$$

The coordinate surfaces u = const and v = const represent a system of mutually-orthogonal paraboloids of rotation. The equation of the paraboloid given (1.01) has the form of

$$v = v_0, \text{ где } v_0 = ka, \qquad (1.07)$$

as can be easily verified by direct substitution of (1.06) in (1.01). The zones, external with respect to the given paraboloid have corresponding values $v > v_0$; the internal zone has corresponding values $v < v_0$. The variable u changes within $0 \leq u <$ infinity.

The square of the linear element in the parabolic coordinates has the form of

$$ds^2 = \frac{1}{k^2}\left(\frac{u+v}{4u}\,du^2 + \frac{u+v}{4v}\,dv^2 + uv\,d\varphi^2\right). \qquad (1.08)$$

As always during the utilization of curvilinear orthogonal coordinates it is convenient to distinguish between the projections of the vector in a given coordinate direction and the covariant components of the vector (the latter are converted as partial derivatives of a certain scalar function in accordance with coordinate parameters). The projections of the physical vector will be enclosed in parentheses, e.g., $(\mathcal{E}_u)$, $(\mathcal{E}_v)$ $(\mathcal{E}_\varphi)$ leaving the designation $\mathcal{E}_u, \mathcal{E}_v, \mathcal{E}_\varphi$ for the covariant components. Then it will be

$$\left.\begin{aligned} (\mathcal{E}_u) &= \frac{k\sqrt{u}}{\sqrt{u+v}}\,\mathcal{E}_u \\ (\mathcal{E}_v) &= \frac{k\sqrt{v}}{\sqrt{u+v}}\,\mathcal{E}_v \\ (\mathcal{E}_\varphi) &= \frac{k}{2\sqrt{uv}}\,\mathcal{E}_\varphi \end{aligned}\right\} \qquad (1.09)$$

and analogously for other vectors.

Par. 2. Parabolic Functions with Continuous Parameter.

The Laplace operator in parabolic coordinates has the form of

$$\Delta\psi = \frac{4k^2}{u+v}\left\{\frac{\partial}{\partial u}\left(u\frac{\partial\psi}{\partial u}\right) + \frac{\partial}{\partial v}\left(v\frac{\partial\psi}{\partial v}\right) + \frac{u+v}{4uv}\frac{\partial^2\psi}{\partial\varphi^2}\right\}. \qquad (2.01)$$

consequently, the equation of oscillations

$$\Delta\psi + k^2\psi = 0 \qquad (2.02)$$

will be written in the form of

$$\frac{\partial}{\partial u}\left(u\frac{\partial\psi}{\partial u}\right) + \frac{\partial}{\partial v}\left(v\frac{\partial\psi}{\partial v}\right) + \frac{1}{4}\left(\frac{1}{u} + \frac{1}{v}\right)\frac{\partial^2\psi}{\partial\varphi^2} + \frac{1}{4}(u+v)\psi = 0. \qquad (2.03)$$

Assuming that

$$\psi = U(u)\,V(v)\,e^{is\varphi} \qquad (2.04)$$

and substituting this expression in (2.03) we become convinced that the variables in equation (2.03) become divided and we obtain for the functions U and V equations

$$u\frac{d^2U}{du^2}+\frac{dU}{du}+\left(\frac{u}{4}-\frac{s^2}{4u}+\frac{t}{2}\right)U=0 \qquad (2.05)$$

$$v\frac{d^2V}{dv^2}+\frac{dV}{dv}+\left(\frac{v}{4}-\frac{s^2}{4v}-\frac{t}{2}\right)V=0 \qquad (2.06)$$

where t is the parameter which was formed during the division of the variables.

In order that solution (2.04) should be a synonymous function of the point in space it is necessary that s be a whole number (whereby in equations (2.05 and 2.06) it can apparently be considered that $s \geqslant 0$). The parameter t can be arbitrarily substantial or a complex number. In many cases, however, it is convenient to introduce integral representations of solutions in which s plays the role of an integration variable. Keeping in mind these cases we will consider U and V as analytical functions of the variable s.

For the functions U and V equations of one and the same type (they differ from each other only by the sign at t) were obtained. The solutions of these equations are thoroughly discussed in literature (e.g., in (2) and (3).

We shall first discuss the solution of equation (2.05), finite at u = 0. Thus, assuming the role of the solution the function will be

$$U=\xi(u,\ s,\ t),$$

where

$$\xi(u,\ s,\ t)=\frac{u^{\frac{s}{2}}e^{-i\frac{u}{2}}}{\Gamma\left(\frac{s+1+it}{2}\right)\Gamma\left(\frac{s+1-it}{2}\right)}\times$$

$$\times\int_0^1 e^{iuz}z^{\frac{s-1+it}{2}}(1-z)^{\frac{s-1-it}{2}}dz. \qquad (2.07)$$

Breaking down the integral into power series we will also obtain

$$\xi(u,\ s,\ t)=e^{-i\frac{u}{2}}\sum_{k=0}^{\infty}i^k\frac{u^{\frac{s}{2}+k}}{k!}\frac{\Gamma\left(\frac{s+1+it}{2}+k\right)}{\Gamma\left(\frac{s+1+it}{2}\right)\Gamma(s+k+1)} \qquad (2.08)$$

or

$$\xi(u,\ s,\ t)=e^{-i\frac{u}{2}}u^{\frac{s}{2}}\frac{1}{\Gamma(s+1)}F\left(\frac{s+1+it}{2},\ \ s+1,\ \ iu\right), \qquad (2.09)$$

where f (alpha, gamma, chi) is the series, compiled in accordance with the law

$$F(\alpha,\ \gamma,\ x)=1+\frac{\alpha}{\gamma}\cdot\frac{x}{1}+\frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{x^2}{1\cdot 2}+\cdots \qquad (2.10)$$

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By changing the integral (2.07) z into 1 - z it can be easily shown that at essential values u, s, t the value xi (u, s, t) will be substantial. It is evident from formula (2.09) that xi (u, s, t) will be an integral transcendental function of t and s.

We will now analyze the second solution of equation (2.03), namely, the solution which at greater u has an asymptotic expression containing a multiple $e^{i\frac{u}{2}}$. Such a solution can be obtained provided we carry out an integration in (2.07) according to z within the boundaries not from 0 to 1 but from 1 + i infinity to 1. The constant multiple factor in front of the integral can, of course, be selected in a different form than in (2.07). Converting the integral by substituting

$$1 - z = \frac{x}{u} e^{-i\frac{\pi}{2}}$$

and by selecting the constant multiple in a proper way the expression

$$\zeta_1(u, s, t) = e^{-\frac{\pi}{4}t - i\frac{s+1}{4}\pi} \cdot u^{-\frac{1}{2}+\frac{it}{2}} \cdot e^{i\frac{u}{2}} \cdot F_{20}, \quad (2.11)$$

may be used in the role of a second solution where

$$F_{20} = \frac{1}{\Gamma\left(\frac{s+1-it}{2}\right)} \int_0^\infty e^{-x} x^{\frac{s-1-it}{2}} \left(1 + i\frac{x}{u}\right)^{\frac{s-1+it}{2}} dx. \quad (2.12)$$

The expression (2.12) can be broken down into an asymptotic series according to powers $\frac{1}{u}$ applicable during u ⟶ infinity, namely, if we write

$$F_{20}\left(\alpha, \beta, \frac{1}{x}\right) = 1 + \frac{\alpha\beta}{1} \cdot \frac{1}{x} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1 \cdot 2} \cdot \frac{1}{x^2} + \ldots, \quad (2.13)$$

then expression (2.12) will be equal

$$F_{20} = F_{20}\left(\frac{1-s-it}{2}, \frac{1+s-it}{2}, -\frac{i}{u}\right). \quad (2.14)$$

The function $zeta_1$ (u, s, t) in contrast to xi (u, s, t) is also complex during substantial u, s, t. Among others, the function satisfies equation (2.05) with substantial coefficients. It is evident, therefrom, that function

$$\zeta_2(u, s, t) = e^{-\frac{\pi}{4}t + i\frac{s+1}{4}\pi} u^{-\frac{1}{2}-\frac{it}{2}} e^{-i\frac{u}{2}} \times$$
$$\times F_{20}\left(\frac{1-s+it}{2}, \frac{1+s+it}{2}, \frac{i}{u}\right), \quad (2.15)$$

obtainable from $zeta_1$ (u, s, t) by changing i into - i will also serve as a solution of this equation. At any given u, s, t the functions $zeta_1$ and $zeta_2$ will be linearly independent integrals of equation (2.05). Just as the functions of parameters s and t the values $zeta_1$ and $zeta_2$ will be, like xi, integral transcendental functions. By changing the sign s we will obtain

$$\left.\begin{aligned} \zeta_1(u, -s, t) &= e^{i\frac{s\pi}{2}} \zeta_1(u, s, t) \\ \zeta_2(u, -s, t) &= e^{-i\frac{s\pi}{2}} \zeta_2(u, s, t). \end{aligned}\right\} \quad (2.16)$$

We will establish the relation between the functions xi, $zeta_1$ and $zeta_2$. We then have

$$\xi(u, s, t) = \frac{\zeta_1(u, s, t)}{\Gamma\left(\frac{s+1+it}{2}\right)} + \frac{\zeta_2(u, s, t)}{\Gamma\left(\frac{s+1-it}{2}\right)} \quad (2.17)$$

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Taking into consideration equation (2.17) and an analogous equation with a reverse sign at a and utilizing the ratios (2.16) it is possible to express

$$\zeta_1(u, s, t) \quad и \quad \zeta_2(u, s, t)$$

through

$$\xi(u, s, t) \quad и \quad \xi(u, -s, t).$$

We will have

$$\zeta_1(u, s, t) = \frac{i\pi}{\sin s\pi} e^{-t\frac{\pi}{2}} \left\{ \frac{e^{-i\frac{s\pi}{2}}}{\Gamma\left(\frac{-s+1-it}{2}\right)} \xi(u, s, t) - \right.$$
$$\left. - \frac{1}{\Gamma\left(\frac{s+1-it}{2}\right)} \xi(u, -s, t) \right\} \qquad (2.18)$$

The value $zeta_2$ will be derived, therefrom, by changing the sign at i. If, under xi we comprehend here the series (2.08) then for $zeta_1$ (u, x, t) formula (2.18) offers a break down according to ascending powers u. When s tends toward a whole number the right side of (2.18) shows a tendency toward the final boundary; but, after converting to the boundary the series for $zeta_1$ will contain logarithmic members.

Between the functions xi (u, s, t) with parameters s, differing by ± 1, and with parameters t, differing by ± 2i, there exist different recurrent ratios of which we will mention the following:

$$2u\frac{\partial}{\partial u}\xi(u, s, t) + \xi(u, s, t) =$$
$$= \frac{s+1+it}{2}\xi(u, s, t-2i) + \frac{s+1-it}{2}\xi(u, s, t+2i). \quad (2.19)$$

$$i(u+t)\xi(u, s, t) =$$
$$= \frac{s+1+it}{2}\xi(u, s, t-2i) - \frac{s+1-it}{2}\xi(u, s, t+2i). \quad (2.20)$$

$$\frac{s+it}{2}\xi(u, s, t-i) + \frac{s-it}{2}\xi(u, s, t+i) = \sqrt{u}\,\xi(u, s-1, t). \quad (2.21)$$

$$\xi(u, s, t-i) - \xi(u, s, t+i) = i\sqrt{u}\,\xi(u, s+1, t). \quad (2.22)$$

It is sometimes convenient to introduce the function

$$\psi(u, s, t) = \Gamma\left(\frac{s+1+it}{2}\right)\xi(u, s, t), \qquad (2.23)$$

into the calculations instead of xi (u, s, t) which will no longer be an integral transcendental function of s and t. Recurrent ratios for psi (u, s, t) can easily be obtained from (2.19)-(2.22). These ratios, however, such as psi (u, s, t) are also satisfied by function $zeta_1$ (u, s, t). Therefore, we will have

$$2u\frac{\partial}{\partial u}\zeta_1(u, s, t) + \zeta_1(u, s, t) =$$
$$= \zeta_1(u, s, t-2i) + \frac{1}{4}(s+1-it)(s-1+it)\zeta_1(u, s, t+2i); \quad (2.24)$$

$$i(u+t)\zeta_1(u,s,t)=$$
$$=\zeta_1(u,s,t-2i)-\frac{1}{4}(s+1-it)(s-1+it)\zeta_1(u,s,t+2i); \qquad (2.25)$$
$$\zeta_1(u,s,t-i)+\frac{1}{2}(s-it)\zeta_1(u,s,t+i)=$$
$$=\sqrt{u}\,\zeta_1(u,s-1,t); \qquad (2.26)$$
$$\zeta_1(u,s,t-i)-\frac{1}{2}(s+it)\zeta_1(u,s,t+i)=$$
$$=i\sqrt{u}\,\zeta_1(u,s+1,t). \qquad (2.27)$$

The recurrent ratios for functions $zeta_2$ (u, s, t) are obtained from (2.24)-(2.27) by changing i to -i.

In order to evaluate the different integrals and series having parabolic functions, it is necessary to have asymptotic expressions for these functions appropriate at greater values $[t]$. The asymptotic expression for the function xi (u, s, t) has the form of

$$\xi(u,s,t)\approx\left(\frac{t}{2}\right)^{-\frac{s}{2}}J_s(\sqrt{2ut}). \qquad (2.28)$$

where j_s is the Bessel function. This expression is appropriate at finite and small u (all the way up to u = 0), at conditions where $[t]\geqslant 1$. For functions $zeta_1$ and $zeta_2$ we will have t in the upper simi-space

$$\zeta_1(u,s,t)=\frac{\pi e^{-i\frac{\pi}{2}}e^{i\frac{s\pi}{4}}}{\Gamma\left(\frac{1-it}{2}\right)}H_s^{(1)}(\sqrt{2ut}). \qquad (2.29)$$

$$\zeta_2(u,s,t)=\frac{1}{2}e^{-i\frac{s\pi}{4}}\Gamma\left(\frac{1-it}{2}\right)\times$$
$$\times\{H_s^{(2)}(\sqrt{2ut})-e^{is\pi-t\pi}H_s^{(1)}(\sqrt{2ut})\} \qquad (2.30)$$

and in the lower semi-space

$$\zeta_1(u,s,t)=\frac{1}{2}e^{i\frac{s\pi}{4}}\Gamma\left(\frac{1+it}{2}\right)\times$$
$$\times\{H_s^{(1)}(\sqrt{2ut})-e^{-is\pi-t\pi}H_s^{(2)}(\sqrt{2ut})\}, \qquad (2.31)$$
$$\zeta_2(u,s,t)=\frac{\pi e^{-i\frac{\pi}{2}}e^{-i\frac{s\pi}{4}}}{\Gamma\left(\frac{1+it}{2}\right)}H_s^{(2)}(\sqrt{2ut}). \qquad (2.32)$$

Here $H_s^{(1)}$ and $H_2^{(2)}$ are the first and second Hankel functions. The values of the functions G $\left(\frac{1-it}{2}\right)$ and G $\left(\frac{1+it}{2}\right)$ can be changed by their asymptotic expressions.

In axial symmetry problems and in such which lead to them, a special role is played by functions with the parameter s equal to aero. For brevity we will write xi (u, t) instead of xi (u, 0, t, and also $zeta_1$ (u, t) instead of $zeta_1$ (u, 0, t) and analogously for other functions.

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Par. 3. Parabolic Functions with Whole Sign

Solving diffraction problems connected with a rotational paraboloid is, for certain purposes, more convenient to represent in the form of integrals and for some purposes in the form of series. The integrals include functions xi, $zeta_1$, $zeta_2$, discussed in the preceding paragraph. The series are oriented according to functions with whole signs for which it is convenient to have special designations even though they are expressed through the preceding ones.

We will write

$$\xi_{ns}(u) = \xi(u, s, -i(2n+s+1)) = \frac{\zeta_1(u, s, -i(2n+s+1))}{\Gamma(n+s+1)} \qquad (3.01)$$

$$\eta_{ns}(u) = (-1)^n n! \zeta_2(u, s, -i(2n+s+1)). \qquad (3.02)$$

Both functions xi_{ns} and eta_{ns} represent solutions for the differential equation

$$u\frac{d^2\xi_{ns}}{du^2} + \frac{d\xi_{ns}}{du} + \left(\frac{u}{4} - \frac{s^2}{4u}\right)\xi_{ns} = i\left(n + \frac{s+1}{2}\right)\xi_{ns}, \qquad (3.03)$$

where $xi_{ns}(u)$ is the regular solution and at u = 0, $eta_{ns}(u)$ has a characteristic. The functions $xi_{ns}(u)$ and $eta_{ns}(u)$ are satisfied with uniformly recurrent ratios which are obtained from (2.19)-(2.22). We have

$$2u\frac{\partial\xi_{ns}}{\partial u} + \xi_{ns} = (n+s+1)\xi_{n+1, s} - n\xi_{n-1, s}, \qquad (3.04)$$

$$(2n+s+1+iu)\xi_{ns} = (n+s+1)\xi_{n+1, s} + n\xi_{n-1, s}. \qquad (3.05)$$

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Further

$$\xi_{ns} - \xi_{n-1, s} = i\sqrt{u}\,\xi_{n-1, s+1}, \qquad (3.06)$$

$$(n+s)\xi_{ns} - n\xi_{n-1, s} = \sqrt{u}\,\xi_{n, s-1}. \qquad (3.07)$$

By combining the preceding ratios we also obtain

$$\left(\frac{\partial}{\partial u} + \frac{i}{2}\right) u^{-\frac{s}{2}} \xi_{ns}(u) = i(n+s+1) u^{-\frac{s+1}{2}} \xi_{n, s+1}(u), \qquad (3.08)$$

$$\left(\frac{\partial}{\partial u} - \frac{i}{2}\right) u^{-\frac{s}{2}} \xi_{ns}(u) = inu^{-\frac{s+1}{2}} \xi_{n-1, s+1}(u). \qquad (3.09)$$

The general expressions for functions $xi_{ns}(u)$ and $eta_{ns}(u)$ have the form of

$$\xi_{ns}(u) = e^{i\frac{u}{2}} \frac{u^{\frac{s}{2}}}{\Gamma(s+1)} F(-n, s+1, -iu), \qquad (3.10)$$

$$\eta_{ns}(u) = -e^{i\frac{s\pi}{2}} \frac{n!}{\Gamma(n+s+1)} e^{-i\frac{u}{2}} u^{-\frac{s}{2}} \times$$

$$\times \int_0^\infty e^{-x} x^{n+s} (x+iu)^{-n-1} dx, \qquad (3.11)$$

where F is the series (2.10) which, in the given case, is reduced to a polynomial of the power n.

When s is a whole number the functions xi_{ns} and eta_{ns} can be expressed with the aid of (3.06) by xi_{n0} and eta_{n0}. The latter are expressed by Laguerre polyromials.

$$L_n(x) = e^x \frac{d^n}{dx^n}(e^{-x}x^n). \qquad (3.12)$$

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and through integral sine and cosine.

Assuming, for the sake of brevity, that

$$\xi_{n0}(u)=\xi_n(u);\quad \eta_{n0}(u)=\eta_n(u), \tag{3.13}$$

we have

$$\xi_n(u)=\frac{1}{n!}e^{i\frac{u}{2}}L_n(-iu), \tag{3.14}$$

$$\eta_n(u)=-\frac{1}{n!}e^{-i\frac{u}{2}}\int_0^\infty e^{-x}\frac{L_n(x)\,dx}{x+iu}. \tag{3.15}$$

Formula (3.15) is obtained from (3.11) by n-multiple integration in parts and utilization of ratio (3.12). The formula for $eta_n(u)$ can be written in the form of

$$\eta_n(u)=-\frac{1}{n!}e^{-i\frac{u}{2}}L_n(-iu)\int_0^\infty\frac{e^{-x}dx}{x+iu}-$$
$$-\frac{1}{n!}e^{-i\frac{u}{2}}\int_0^\infty e^{-x}\frac{L_n(x)-L_n(-iu)}{x+iu}dx. \tag{3.16}$$

In this case the integral in the first member is expressed through integral sine and cosine

$$\int_0^\infty\frac{e^{-x}dx}{x+iu}=e^{iu}\int_u^\infty e^{-iu'}\frac{du'}{u'}=e^{iu}\left(-\mathrm{Ci}(u)+i\,\mathrm{Si}(u)-i\frac{\pi}{2}\right) \tag{3.17}$$

but the integral in the second member represents a polynomial of u.

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The first of the functions $xi_n(u)$, $eta_n(u)$ are equal to

$$\left.\begin{aligned}\xi_0(u)&=e^{i\frac{u}{2}},\\ \xi_1(u)&=e^{i\frac{u}{2}}(1+iu),\\ \xi_2(u)&=e^{i\frac{u}{2}}\left(1+2iu-\frac{1}{2}u^2\right),\\ &\cdots\cdots\cdots\end{aligned}\right\} \tag{3.18}$$

$$\left.\begin{aligned}\eta_0(u)&=e^{i\frac{u}{2}}\left[\mathrm{Ci}(u)-i\,\mathrm{Si}(u)+i\frac{\pi}{2}\right],\\ \eta_1(u)&=(1+iu)\,\eta_0(u)+e^{-i\frac{u}{2}},\\ \eta_2(u)&=\left(1+2iu-\frac{1}{2}u^2\right)\eta_0(u)+e^{-i\frac{u}{2}}\left(\frac{3}{2}+\frac{iu}{2}\right),\\ &\cdots\cdots\cdots\end{aligned}\right\} \tag{3.19}$$

and the remaining are expressed through them by means of recurrent ratios (3.05) which, in this case, acquire the form of

$$\left.\begin{aligned}(2n+1+iu)\,\xi_n(u)&=n\xi_{n-1}(u)+(n+1)\,\xi_{n+1}(u),\\ (2n+1+iu)\,\eta_n(u)&=n\eta_{n-1}(u)+(n+1)\,\eta_{n+1}(u).\end{aligned}\right\} \tag{3.20}$$

The asymptotic expressions for functions $xi_n(u)$, $eta_n(u)$ at greater n values are obtained from the general formulas (2.25)-(2.28).

We have

$$\xi_n(u)=J_0\left((1-i)\sqrt{(2n+1)u}\right), \tag{3.21}$$

$$\eta_n(u)=i\pi H_0^{(2)}\left((1-i)\sqrt{(2n+1)u}\right). \tag{3.22}$$

Hence, it is evident that the functions $xi_n(u)$ increase in modulus and the functions $eta_n(u)$ decrease with the increase in n.

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Par. 4. Break Down of the Point Characteristic According to Parabolic Functions

When solving the problem of a dipole in the focus of a rotational paraboloid it is necessary to know how to break down the expression

$$\Pi = \frac{e^{ikR}}{kR} = \frac{2}{u+v} e^{i\frac{u+v}{2}} \qquad (4.01)$$

according to parabolic functions.

Since this expression does not depend upon psi it becomes clear that its decomposition (break down) will include only functions with the parameter s = 0. Further, keeping in mind the exponential multiple in (4.01), we naturally search for a break down in the form of

$$\frac{2}{u+v} e^{i\frac{u+v}{2}} = \int_{-\infty}^{+\infty} f(t)\zeta_1(u, t)\zeta_1(v, -t)\,dt, \qquad (4.02)$$

where f (t) is the function subject to determination.

Let us now discuss the following representations of functions $zeta_1$ (u, t) and $zeta_2$ (v,-t) [they originate from (2.11) and (2.12)] :

$$e^{\frac{\pi}{2}(t+i)}\Gamma\left(\frac{1-it}{2}\right)e^{-\frac{iu}{2}}\zeta_1(u, t) = \int_0^{\infty} e^{iup} p^{-\frac{1}{2}-\frac{it}{2}}(1+p)^{-\frac{1}{2}+\frac{it}{2}}\,dp; \qquad (4.03)$$

$$e^{\frac{\pi}{2}(-t+i)}\Gamma\left(\frac{1+it}{2}\right)e^{-i\frac{v}{2}}\zeta_1(v, -t) = \int_0^{\infty} e^{ivq} q^{-\frac{1}{2}+\frac{it}{2}}(1+q)^{-\frac{1}{2}-\frac{it}{2}}\,dq. \qquad (4.04)$$

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We will multiply these expressions by each other and integrate by t from - infinity to + infinity. We obtain an integral

$$I = -\int_{-\infty}^{+\infty}\Gamma\left(\frac{1-it}{2}\right)\Gamma\left(\frac{1+it}{2}\right)e^{-i\frac{u+v}{2}}\zeta_1(u, t)\zeta_1(v, -t)\,dt = \int_0^{\infty}\int_0^{\infty} e^{i(up+vq)} f(p, q)\,dp\,dq, \qquad (4.05)$$

where

$$f(p, q) = \frac{1}{\sqrt{pq(1+p)(1+q)}} \cdot \int_{-\infty}^{+\infty} dt \left(\frac{1+p}{1+q}\cdot\frac{q}{p}\right)^{i\frac{t}{2}} \qquad (4.06)$$

On the basis of the characteristics of a non-characteristic Dirac function, expressed by equalities

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ixt}\,dt \qquad (4.07)$$

$$\delta[\varphi(p) - \varphi(q)] = \frac{1}{\sqrt{|\varphi'(p)\varphi'(q)|}}\delta(p-q), \qquad (4.08)$$

the expression (3.06) can be interpreted as

$$f(p, q) = 4\pi\delta(p-q) \qquad (4.09)$$

By substituting this value f (p, q) in (4.05) we obtain an integral which can be conditionally understood as

$$I = 4\pi\int^{\infty} e^{i(u+v)p}\,dp = \frac{4\pi i}{u+v} + 4\pi^2\delta(u+v), \qquad (4.10)$$

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whereby, just as in the case where $u + v > 0$, the member delta $(u + v)$ can be disregarded.

Comparing (4.05) and 4.10) and utilizing the equality

$$\Gamma\left(\frac{1+it}{2}\right)\Gamma\left(\frac{1-it}{2}\right)=\frac{\pi}{\operatorname{ch}\frac{\pi t}{2}}, \qquad (4.11)$$

we can write our result in the form of

$$\frac{i}{2}\int_{-\infty}^{+\infty}\frac{dt}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_1(v,-t)=\frac{2}{u+v}e^{i\frac{u+v}{2}} \qquad (4.12)$$

This result obtained by us in a non-strict way can also be proven in a perfectly strict way. This can be discussed in the following manner. Let us discuss the identity resulting from (2.25)

$$i(u+v)\zeta_1(u,t)\zeta_1(v,-t)=F(t)+F(t+2i), \qquad (4.13)$$

where for the sake of brevity is written

$$F(t)=\zeta_1(u,t-2i)\zeta_1(v,-t)+$$
$$+\frac{1}{4}(1+it)^2\zeta_1(u,t)\zeta_1(v,-t+2i) \qquad (4.14)$$

Utilizing the fact that

$$\operatorname{ch}\frac{\pi t}{2}=-\operatorname{ch}\frac{\pi(t+2i)}{2} \qquad (4.15)$$

we can write

$$\int_{-\infty}^{+\infty}\frac{i(u+v)}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_1(v,-t)\,dt=$$
$$=\int_{-\infty}^{+\infty}\frac{F(t)}{\operatorname{ch}\frac{\pi t}{2}}dt-\int_{-\infty}^{+\infty}\frac{F(t+2i)}{\operatorname{ch}\frac{\pi}{2}(t+2i)}dt=4F(i), \qquad (4.16)$$

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because the integral difference equals the deduction in the point $t = i$. However, according to (3.01) and (3.18) we have

$$F(i)=\zeta_1(u,-i)\zeta_1(v,-i)=\xi_0(u)\xi_0(v)=e^{i\frac{u+v}{2}}. \qquad (4.17)$$

Substituting this expression in (4.16) we again obtain the equality

$$\frac{2}{u+v}e^{i\frac{u+v}{2}}=\frac{i}{2}\int_{-\infty}^{+\infty}\frac{dt}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_1(v,-t). \qquad (4.12)$$

In the integral representation (4.12) the decomposition (break down) is carried out according to functions which turn into infinity on the axis of the paraboloid; nevertheless the entire integral remains finite there.

From the integral representation of the point characteristic it is easy to change over to the representation in the form of series. For this purpose it is sufficient to calculate the integral as a sum of the deductions in points $t = -(2n + 1)i$ or in points $t = (2n + 1)i$. In the first case we obtain a series

$$\frac{2}{u+v}e^{i\frac{u+v}{2}}=2i\sum_{n=0}^{\infty}\xi_n(u)\bar{\eta}_n(v), \qquad (4.18)$$

and in the second case

$$\frac{2}{u+v}e^{i\frac{u+v}{2}}=2i\sum_{n=0}^{\infty}\bar{\eta}_n(u)\xi_n(v). \qquad (4.19)$$

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The convergence zones of these series can easily be established with the aid of asymptotic expressions (3.21) and (3.22). The series (4.18) converges at $u < v$ and the series (4.19) - at $u > v$.

Par. 5. Plane Wave Resolution

We shall now consider a scalar plane wave

$$e^{i\Omega} = e^{ik(x \sin\delta + z\cos\delta)} \qquad (5.01)$$

and will analyze it in accordance with the partial solution of a wave equation found in Par. 2. By expressing the rectangular coordinates x, z according to formula (1.06) through the parabolic coordinates we have

$$\Omega = \frac{1}{2}(u - v)\cos\delta + \sqrt{uv}\sin\delta\cos\varphi. \qquad (5.02)$$

Analyzing the expression (5.01) according to cosines of multiples phi we obtain

$$e^{i\Omega} = e^{\frac{i}{2}(u-v)\cos\delta}\Big\{J_0(\sqrt{uv}\sin\delta) + 2\sum_{s=1}^{\infty} i^s J_s(\sqrt{uv}\sin\delta)\cos s\varphi\Big\} \qquad (5.03)$$

where J_s is the Bessel function.

The independent member of this expression should be broken down according to functions of the type (2.04). By writing

$$C_s = e^{\frac{i}{2}(u-v)\cos\delta} J_s(\sqrt{uv}\sin\delta) \qquad (5.04)$$

we should have an equality of the form

$$C_s = \int_{-\infty}^{+\infty} \psi(u, s, t)\,\psi(v, s, -t) f(t)\,dt \qquad (5.05)$$

(the function zeta is not included here so that the expression (5.04) remains finite at u = 0 and at v = 0). The expressions (5.04) and (5.05) should be equal at all v as well as when $v \to 0$. Multiplying both parts of these expressions by $G(s + 1)v^{-\frac{s}{2}}$ and changing over to the point where $v \to 0$ we will obtain

$$\lim_{v\to 0}\Gamma(s+1)v^{-\frac{s}{2}}C_s = e^{\frac{i}{2}u\cos\delta}u^{\frac{s}{2}}\left(\frac{\sin\delta}{2}\right)^s = \int_{-\infty}^{+\infty}\Gamma\left(\frac{s+1-it}{2}\right)\psi(u, s, t) f(t)\,dt. \qquad (5.06)$$

In order to determine f(t), one can utilize formula (2.23) and in (5.06) subsitute the integral expression in (2.07) for xi (u, s, t). By reducing the multiplier $u^{\frac{s}{2}}\ e^{-i\frac{u}{2}}$ we will obtain

$$\left(\frac{\sin\delta}{2}\right)^s e^{\frac{i}{2}u(1+\cos\delta)} = \int_{-\infty}^{+\infty} f(t)\,dt\int_0^1 e^{iuz} z^{\frac{s-1+it}{2}}(1-z)^{\frac{s-1-it}{2}}\,dz. \qquad (5.07)$$

This equation will be satisfied provided f(t) is selected in such a way that

$$\int_{-\infty}^{+\infty} f(t) z^{\frac{s-1+it}{2}}(1-z)^{\frac{s-1-it}{2}}\,dt = \left(\frac{\sin\delta}{2}\right)^s \delta_1\left(z - \frac{1+\cos\delta}{2}\right), \qquad (5.08)$$

where δ_1 (in order to avoid confusion with the angle delta) designates the non-characteristic Dirac function. By deliberating, as in Par. 4, one can easily see that equation (5.08) will be satisfied provided we write

$$f(t)=\frac{1}{2\pi\sin\delta}\left(\operatorname{tg}\frac{\delta}{2}\right)^{it} \qquad (5.09)$$

Substituting this value f(t) in (5.06) we will obtain

$$e^{\frac{i}{2}u\cos\delta}\,u^{\frac{s}{2}}\left(\frac{\sin\delta}{2}\right)^{s}=$$
$$=\frac{1}{2\pi\sin\delta}\int_{-\infty}^{+\infty}\Gamma\left(\frac{s+1-it}{2}\right)\psi(u,s,t)\left(\operatorname{tg}\frac{\delta}{2}\right)^{it}dt. \qquad (5.10)$$

Then by subsituting this expression in (5.05) we obtain a more general equality

$$e^{\frac{i}{2}(u-v)\cos\delta}\,i^{s}J_{s}(\sqrt{uv}\sin\delta)=$$
$$=\frac{1}{2\pi\sin\delta}\int_{-\infty}^{+\infty}\psi(u,s,t)\psi(v,s,-t)\left(\operatorname{tg}\frac{\delta}{2}\right)^{it}dt. \qquad (5.11)$$

The derivation of this equality was not perfectly strict because we applied the Dirac function. The result, however, can be checked by direct calculation. For this purpose it is sufficient to substitute

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the expression in (5.11) for psi by the series (2.08) and integrate by terms, applying the formula

$$\frac{1}{2\pi\sin\delta}\int_{-\infty}^{+\infty}\Gamma\left(\frac{s+1+it}{2}+k\right)\Gamma\left(\frac{s+1-it}{2}+l\right)\left(\operatorname{tg}\frac{\delta}{2}\right)^{it}dt=$$
$$=\Gamma(s+1+k+l)\left(\sin\frac{\delta}{2}\right)^{s+2l}\left(\cos\frac{\delta}{2}\right)^{s+2k} \qquad (5.12)$$

The resulting double series can be transformed into a form coinciding with the left part of (5.11).

We wish to mention that formula (5.11) is correct not only for whole non-negative s, but also for all values s for which the substantial (material) part s + 1 is positive. If Re(s + 1) $<$ 0 then in (5.11) we must select such a way of integration which could be applicable in (5.12).

Our final result - plane wave resolution according to parabolic functions - can be written in the form of

$$e^{i\varrho}=e^{\frac{i}{2}(u-v)\cos\delta+i\sqrt{uv}\sin\delta\cos\varphi}=$$
$$=\frac{1}{2\pi\sin\delta}\int_{-\infty}^{+\infty}\Big\{\psi(u,0,t)\psi(v,0,-t)+$$
$$+2\sum_{s=1}^{\infty}i^{s}\psi(u,s,t)\psi(v,s,-t)\cos s\varphi\Big\}\left(\operatorname{tg}\frac{\delta}{2}\right)^{it}dt. \qquad (5.13)$$

Par. 6. Maxwell Equations and Potentials in Parabolic Coordinates

We will now change over from the scalar wave equation to Maxwell equations. The dependence of all field components upon time is assumed in the form e^{-iwt}, where omega = ck, and in further discussion this multiple is not written out.

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For the coordinate components of the field (see (1.09), the Maxwell equations in parabolic coordinates will be written as follows

$$\left.\begin{aligned} \frac{\partial E_\varphi}{\partial v} - \frac{\partial E_v}{\partial \varphi} &= iuH_u, \\ \frac{\partial E_u}{\partial \varphi} - \frac{\partial E_\varphi}{\partial u} &= ivH_v, \\ \frac{\partial E_v}{\partial u} - \frac{\partial E_u}{\partial v} &= i\frac{u+v}{4uv} H_\varphi; \end{aligned}\right\} \qquad (6.01)$$

$$\left.\begin{aligned} \frac{\partial H_\varphi}{\partial v} - \frac{\partial H_v}{\partial \varphi} &= -iuE_u, \\ \frac{\partial H_u}{\partial \varphi} - \frac{\partial H_\varphi}{\partial u} &= -ivE_v, \\ \frac{\partial H_v}{\partial u} - \frac{\partial H_u}{\partial v} &= -i\frac{u+v}{4uv} E_\varphi. \end{aligned}\right\} \qquad (6.02)$$

The simplicity of solving the problem with boundary conditions on a surface of any given shape and form depends to a very large extent upon the proper selection of potentials or auxiliary functions through which the field is expressed. For rectangular coordinates and a plane surface (surface xy), it is most convenient to express the field through the phi and psi potentials connected with the Hertzian vector in accordance with formulas

$$\left.\begin{aligned} E_x &= \frac{\partial^2 \Psi}{\partial x \partial z} - ik\frac{\partial \Phi}{\partial y}; & H_x &= -\frac{\partial^2 \Phi}{\partial x \partial z} - ik\frac{\partial \Psi}{\partial y}, \\ E_y &= \frac{\partial^2 \Psi}{\partial y \partial z} + ik\frac{\partial \Phi}{\partial x}; & H_y &= -\frac{\partial^2 \Phi}{\partial y \partial z} + ik\frac{\partial \Psi}{\partial x}, \\ E_z &= \frac{\partial^2 \Psi}{\partial z^2} + k^2\Psi; & H_z &= \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \end{aligned}\right\} \qquad (6.03)$$

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For spherical coordinates R, delta, psi the most convenient appear to be the Debye potentials U, V through which the field is expressed according to formulas

$$\left.\begin{aligned} E_R &= k^2RU + \frac{\partial^2 (RU)}{\partial R^2}; & iH_R &= k^2RV + \frac{\partial^2 (RV)}{\partial R}, \\ E_\vartheta &= \frac{\partial^2 (RU)}{\partial R \partial \vartheta} + \frac{k}{\sin\vartheta}\frac{\partial (RV)}{\partial \varphi}; & iH_\vartheta &= \frac{\partial^2 (RV)}{\partial R \partial \vartheta} + \frac{k}{\sin\vartheta}\frac{\partial (RU)}{\partial \varphi}, \\ E_\varphi &= \frac{\partial^2 (RU)}{\partial R \partial \varphi} - k\sin\vartheta\frac{\partial (RV)}{\partial \vartheta}; & iH_\varphi &= \frac{\partial^2 (RV)}{\partial R \partial \varphi} - k\sin\vartheta\frac{\partial (RU)}{\partial \vartheta} \end{aligned}\right\} \qquad (6.04)$$

Both the Phi, Pse potentials as well as the U, V potentials should satisfy the scalar oscillation equation (2.02).

As to the parabolic coordinates it would be most convenient to introduce potentials connected not with the entire field but with its Fourier components along the angle psi.

We will first represent the parabolic components of the field through the Debye potentials. By changing from (6.04) to these components we will have

$$\left.\begin{aligned} E_u &= \frac{1}{4}(u+v)U + \frac{\partial}{\partial u}(MU) - \frac{u+v}{4u}\frac{\partial V}{\partial \varphi}, \\ E_v &= \frac{1}{4}(u+v)U + \frac{\partial}{\partial v}(MU) + \frac{u+v}{4v}\frac{\partial V}{\partial \varphi}, \\ E_\varphi &= \frac{\partial}{\partial \varphi}(MU) + uv\left(\frac{\partial V}{\partial u} - \frac{\partial V}{\partial v}\right) \end{aligned}\right\} \qquad (6.05)$$

where we have written, for the sake of brevity,

$$MU = u\frac{\partial U}{\partial u} + v\frac{\partial U}{\partial v} + U \qquad (6.06)$$

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The value iH_u, iH_v, iH_{psi} are obtained from (6.05) by the rearrangement of letter U and V.

We shall now decompose (break down) the Debye potentials into Fourier series of the form

$$\left.\begin{aligned} U &= \frac{1}{2} U^{\circ} + \sum_{s=1}^{\infty} U^{(s)} \cos s\varphi, \\ V &= \sum_{s=1}^{\infty} V^{(s)} \sin s\varphi \end{aligned}\right\} \qquad (6.07)$$

It can easily be seen that the series for E_u, E_v, H_{psi} will then be oriented according to cosines and the series for H_u, H_v, E_{psi} - according to sines. We will have

$$\left.\begin{aligned} E_u &= \frac{1}{2} E_u^{\circ} + \sum_{s=1}^{\infty} E_u^{(s)} \cos s\varphi; \quad E_v = \ldots; \quad H_\varphi = \ldots, \\ H_u &= \sum_{s=1}^{\infty} H_u^{(s)} \sin s\varphi; \qquad H_v = \ldots; \quad E_\varphi = \ldots \end{aligned}\right\} \qquad (6.08)$$

where the dotted lines (first line) designate the series by cosines; and in the second line - series by sines.

The coefficients of series (6.08) are expressed through $U^{(s)}$, $V^{(s)}$ according to formulas

$$\left.\begin{aligned} E_u^{(s)} &= \frac{1}{4}(u+v)U^{(s)} + \frac{\partial}{\partial u}(MU^{(s)}) - \frac{u+v}{4u} sV^{(s)} \\ E_v^{(s)} &= \frac{1}{4}(u+v)U^{(s)} + \frac{\partial}{\partial v}(MU^{(s)}) + \frac{u+v}{4v} sV^{(s)} \\ E_\varphi^{(s)} &= -sMU^{(s)} + uv\left(\frac{\partial V^{(s)}}{\partial u} - \frac{\partial V^{(s)}}{\partial v}\right) \end{aligned}\right\} \qquad (6.09)$$

$$\left.\begin{aligned} iH_u^{(s)} &= \frac{1}{4}(u+v)V^{(s)} + \frac{\partial}{\partial u}(MV^{(s)}) + \frac{u+v}{4u} sU^{(s)} \\ iH_v^{(s)} &= \frac{1}{4}(u+v)V^{(s)} + \frac{\partial}{\partial v}(MV^{(s)}) - \frac{u+v}{4v} sU^{(s)} \\ iH_\varphi^{(s)} &= sMV^{(s)} + uv\left(\frac{\partial U^{(s)}}{\partial u} - \frac{\partial U^{(s)}}{\partial v}\right) \end{aligned}\right\} \qquad (6.10)$$

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Since the functions U, V satisfy the scalar oscillations equation, their Fourier-components will satisfy equations of the type

$$\left(L_u + L_v - \frac{s^2}{4u} - \frac{s^2}{4v}\right) U^{(s)} = 0, \qquad (6.11)$$

where L_u, L_v designate the operators

$$\left.\begin{aligned} L_u &= u\frac{\partial^2}{\partial u^2} + \frac{\partial}{\partial u} + \frac{1}{4}u \\ L_v &= v\frac{\partial^2}{\partial v^2} + \frac{\partial}{\partial v} + \frac{1}{4}v. \end{aligned}\right\} \qquad (6.12)$$

According to formulas

$$U^{(s)} = (\sqrt{uv})^s P_s^{\circ}; \quad V^{(s)} = (\sqrt{uv})^s Q_s^{\circ} \qquad (6.13)$$

we will introduce the values P_s^o, Q_s^o and these values should satisfy equations

$$\left(L_u + L_v + s\frac{\partial}{\partial u} + s\frac{\partial}{\partial v}\right) P_s^{\circ} = 0 \qquad (6.14)$$

Finally we will write

$$P_s^{\circ} = \frac{1}{u+v}\left(\frac{\partial P_{s-1}}{\partial u} + \frac{\partial P_{s-1}}{\partial v}\right); \quad Q_s^{\circ} = \frac{1}{u+v}\left(\frac{\partial Q_{s-1}}{\partial u} + \frac{\partial Q_{s-1}}{\partial v}\right) \qquad (6.15)$$

It can easily be verified that if P_{s-1}, Q_{s-1} satisfy equations

$$\left[L_u + L_v + (s-1)\left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)\right] P_{s-1} = 0, \qquad (6.16)$$

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then the values P_s^o, Q_s^o will satisfy equations (5.14). In other words, the values P_s, Q_s satisfy the very same equations as P_s^o, Q_s^o.

We will express the Debye potentials and the field through the values P_{s-1}, Q_{s-1}. We have

$$U^{(s)}=\frac{(\sqrt{uv})^s}{u+v}\left(\frac{\partial P_{s-1}}{\partial u}+\frac{\partial P_{s-1}}{\partial v}\right) \qquad (6.17)$$

$$V^{(s)}=\frac{(\sqrt{uv})^s}{u+v}\left(\frac{\partial Q_{s-1}}{\partial u}+\frac{\partial Q_{s-1}}{\partial v}\right) \qquad (6.18)$$

$$MU^{(s)}=(\sqrt{uv})^s\left(\frac{\partial^2 P_{s-1}}{\partial u\partial v}-\frac{1}{4}P_{s-1}\right) \qquad (6.19)$$

By calculating the value $MU^{(s)}$ and utilizing (6.16), for the electric field we will have

$$\left.\begin{aligned}
2uE_u^{(s)}&=(\sqrt{uv})^s\Big\{2\frac{\partial}{\partial v}P^{*}_{s-1}-s\frac{\partial^2 P_{s-1}}{\partial u\partial v}-\\
&-\frac{s}{4}P_{s-1}-\frac{s}{2}\frac{\partial Q_{s-1}}{\partial u}-\frac{s}{2}\frac{\partial Q_{s-1}}{\partial v}\Big\}\\
2vE_v^{(s)}&=(\sqrt{uv})^{(s)}\Big\{-2\frac{\partial}{\partial u}P^{*}_{s-1}-s\frac{\partial^2 P_{s-1}}{\partial u\partial v}-\\
&-\frac{s}{4}P_{s-1}+\frac{s}{2}\frac{\partial Q_{s-1}}{\partial u}+\frac{s}{2}\frac{\partial Q_{s-1}}{\partial v}\Big\}\\
E_\varphi^{(s)}&=(\sqrt{uv})^s\Big\{Q^{*}_{s-1}-\frac{s}{2}\frac{\partial Q_{s-1}}{\partial u}+\\
&+\frac{s}{2}\frac{\partial Q_{s-1}}{\partial v}-s\frac{\partial^2 P_{s-1}}{\partial u\partial v}+\frac{s}{4}P_{s-1}\Big\}
\end{aligned}\right\} \qquad (6.20)$$

Analogous expressions are obtained for the magnetic field

$$\left.\begin{aligned}
2iuH_u^{(s)}&=(\sqrt{uv})^s\Big\{2\frac{\partial}{\partial v}Q^{*}_{s-1}-s\frac{\partial^2 Q_{s-1}}{\partial u\partial v}-\\
&-\frac{s}{4}Q_{s-1}+\frac{s}{2}\frac{\partial P_{s-1}}{\partial u}+\frac{s}{2}\frac{\partial P_{s-1}}{\partial v}\Big\}\\
2ivH_v^{(s)}&=(\sqrt{uv})^s\Big\{-2\frac{\partial}{\partial u}Q^{*}_{s-1}-s\frac{\partial^2 Q_{s-1}}{\partial u\partial v}-\\
&-\frac{s}{4}Q_{s-1}-\frac{s}{2}\frac{\partial P_{s-1}}{\partial u}-\frac{s}{2}\frac{\partial P_{s-1}}{\partial v}\Big\}\\
iH_\varphi^{(s)}&=(\sqrt{uv})^s\Big\{P^{*}_{s-1}-\frac{s}{2}\frac{\partial P_{s-1}}{\partial u}+\\
&+\frac{s}{2}\frac{\partial P_{s-1}}{\partial v}+s\frac{\partial^2 Q_{s-1}}{\partial u\partial v}-\frac{s}{4}Q_{s-1}\Big\}.
\end{aligned}\right\} \qquad (6.21)$$

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For the sake of brevity we introduced into these formulas designations P^o_{s-1}, Q^o_{s-1}, where

$$P^{*}_{s-1}=u\frac{\partial^2 P_{s-1}}{\partial u^2}+s\frac{\partial P_{s-1}}{\partial u}+\frac{1}{4}uP_{s-1}=$$
$$=-v\frac{\partial^2 P_{s-1}}{\partial v^2}-s\frac{\partial P_{s-1}}{\partial v}-\frac{1}{4}vP_{s-1}, \qquad (6.22)$$

and the value Q^o_{s-1} is connected with Q_{s-1} just as P^o_{s-1} is with P_{s-1}. It should be mentioned that the values P^o_{s-1}, Q^o_{s-1} satisfy the very same equations of the (6.16) type as P_{s-1} and Q_{s-1}. Since P_{s-1} and Q_{s-1} have the form of a product of the function u by function v then P^o_{s-1} will be directly proportional to P_{s-1}, and Q^o_{s-1} proportional to Q_{s-1}.

The transformed expressions (6.20) and (6.21) have the advantage over the initial expressions (6.09). (6.10) since they allow a simple formulation of the boundary conditions on the surface of a paraboloid v = const.

The fact is, if we write

$$2\frac{\partial P_{s-1}}{\partial v}+Q_{s-1}=A \qquad (6.23)$$

$$4Q^{*}_{s-1}+2s\frac{\partial Q_{s-1}}{\partial v}+sP_{s-1}=B, \qquad (6.24)$$

the tangential components of the electric field and the normal component of the magnetic field will be equal

$$\left.\begin{aligned}
2uE_u^{(s)}&=(\sqrt{uv})^s\left(u\frac{\partial^2 A}{\partial u^2}+\frac{1}{4}uA+\frac{s}{2}\frac{\partial A}{\partial u}-\frac{1}{4}B\right)\\
E_\varphi^{(s)}&=(\sqrt{uv})^s\left(\frac{1}{4}B-\frac{s}{2}\frac{\partial A}{\partial u}\right)\\
2ivH_v^{(s)}&=(\sqrt{uv})^s\left(-\frac{s}{4}A-\frac{1}{2}\frac{\partial B}{\partial u}\right)
\end{aligned}\right\} \qquad (6.25)$$

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These expressions should convert into zero on the surface of the absolutely conductive paraboloid. For this purpose it is necessary that at $v = v_o$, any u A should be zero and B = 0 or more specific

$$2\frac{\partial P_{s-1}}{\partial v} + Q_{s-1} = 0 \qquad (6.26)$$

$$4Q^{*}_{s-1} + 2s\frac{\partial Q_{s-1}}{\partial v} + sP_{s-1} = 0 \qquad (6.27)$$

It is very essential that the left parts of the boundary conditions (6.26) and (6.27) are obtained from functions P_{s-1}, Q_{s-1} by means of operations which contain neither multiplications by the variable u nor differentiations by this variable*. Therefore, if we should analyze P_{s-1}, Q_{s-1} according to any given u-functions then the left parts of the boundary conditions will represent break downs according to the very same functions, and in order to reduce these functions to zero it is sufficient to bring down the break down coefficients to zero.

If instead of expressions (6.20) and (6.21) we would use expressions (6.09) and (6.10) and seek the break-down coefficients for the functions $U^{(s)}$, $V^{(s)}$ then for these break-down coefficients we would not have obtained algebraic linear equations but linear equations with finite differences which would have complicated the problem a great deal. A still greater complication of the problem would have arisen during the application of the Phi, Psi potentials included in formula (6.03).

* According to (6.22) the value Q^{*}_{s-1} can be obtained from Q_{s-1} by formula

$$Q^{*}_{s-1} = -v\frac{\partial^2 Q_{s-1}}{\partial v^2} - s\frac{\partial Q_{s-1}}{\partial v} - \frac{1}{4}vQ_{s-1}$$

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Thus, the introduction of P_s, Q_s potentials gives us the possibility of avoiding equations with finite differences and the entire complex apparatus necessary for the solution of same.

Par. 7 Transformation of Expressions for the Field

A certain disadvantage of the P_s, Q_s potentials consists in the fact that the expressions (6.20), (6.21) for the field expressed through these potentials have a rather complicated form.

This disadvantage can be eliminated, provided we take into consideration, together with the P_{s-1}, Q_{s-1} potentials, also four auxiliary functions C_{s-1}, D_{s-1}, F_s, G_s through which the field is expressed in a simpler way. These functions possess simple characteristics and are connected with the potentials by simple ratios.

We will write

$$C_{s-1} = P^{*}_{s-1} - \frac{s}{2}Q_{s-1} \qquad (7.01)$$

$$D_{s-1} = Q^{*}_{s-1} + \frac{s}{2}P_{s-1} \qquad (7.02)$$

$$F_s = \frac{\partial^2 P_{s-1}}{\partial u \partial v} + \frac{1}{4}P_{s-1} + \frac{1}{2}\frac{\partial Q_{s-1}}{\partial u} - \frac{1}{2}\frac{\partial Q_{s-1}}{\partial v} \qquad (7.03)$$

$$G_s = \frac{\partial^2 Q_{s-1}}{\partial u \partial v} + \frac{1}{4}Q_{s-1} - \frac{1}{2}\frac{\partial P_{s-1}}{\partial u} + \frac{1}{2}\frac{\partial P_{s-1}}{\partial v} \qquad (7.04)$$

The expressions (6.20) and (6.21) for the field will now be rewritten as follows

$$\left.\begin{aligned} 2uE_u^{(s)} &= (\sqrt{uv})^s\left(2\frac{\partial C_{s-1}}{\partial v} - sF_s\right) \\ 2vE_v^{(s)} &= (\sqrt{uv})^s\left(-2\frac{\partial C_{s-1}}{\partial u} - sF_s\right) \\ E_\varphi^{(s)} &= (\sqrt{uv})^s(D_{s-1} - sF_s) \end{aligned}\right\} \qquad (7.05)$$

$$\left.\begin{aligned} 2iuH_u^{(s)} &= (\sqrt{uv})^s\left(2\frac{\partial D_{s-1}}{\partial v} - sG_s\right) \\ 2ivH_v^{(s)} &= (\sqrt{uv})^s\left(-2\frac{\partial D_{s-1}}{\partial u} - sG_s\right) \\ iH_\varphi^{(s)} &= (\sqrt{uv})^s(C_{s-1} + sG_s). \end{aligned}\right\} \qquad (7.06)$$

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The functions C_s, D_s, F_s, G_s satisfy the very same equation as P_s, Q_s, namely,

$$\left(u\frac{\partial^2}{\partial u^2}+(s+1)\frac{\partial}{\partial u}+\frac{1}{4}u\right)F_s= \\ =-\left(v\frac{\partial^2}{\partial v^2}+(s+1)\frac{\partial}{\partial v}+\frac{1}{4}v\right)F_s. \quad (7.07)$$

It is in itself understood that the functions C_{s-1}, D_{s-1} satisfy the equation of the (7.07) type in which s is changed to s-1. The ratios between functions C_{s-1}, D_{s-1}, P_{s-1}, Q_{s-1} can be written in the form of equalities

$$C_{s-1}+iD_{s-1}= \\ =\left(u\frac{\partial}{\partial u}+s-\frac{iu}{2}\right)\left(\frac{\partial}{\partial u}+\frac{i}{2}\right)(P_{s-1}+iQ_{s-1})= \\ =-\left(v\frac{\partial}{\partial v}+s+\frac{iv}{2}\right)\left(\frac{\partial}{\partial v}-\frac{i}{2}\right)(P_{s-1}+iQ_{s-1}) \quad (7.08)$$

and in the form of analogous equalities obtainable from (7.08) by changing i to -i (just as if the values P, Q, C, D would be substantial). On the other hand, from formulas (7.03) and (7.04) we obtain

$$F_s+iG_s=\left(\frac{\partial}{\partial v}-\frac{i}{2}\right)\left(\frac{\partial}{\partial u}+\frac{i}{2}\right)(P_{s-1}+iQ_{s-1}) \quad (7.09)$$

and an analogous equality obtainable from (7.09) by changing i into -i. Two ratios become evident by comparing (7.08) and (7.09)

$$\left.\begin{aligned}&\left(u\frac{\partial}{\partial u}+s-\frac{iu}{2}\right)(F_s+iG_s)= \\ &\qquad=\left(\frac{\partial}{\partial v}-\frac{i}{2}\right)(C_{s-1}+iD_{s-1}), \\ &\left(v\frac{\partial}{\partial v}+s+\frac{iv}{2}\right)(F_s+iG_s)= \\ &\qquad=-\left(\frac{\partial}{\partial u}+\frac{i}{2}\right)(C_{s-1}+iD_{s-1})\end{aligned}\right\} \quad (7.10)$$

and two other ratios are obtainable from (7.10) by also changing i to -i. By separating (formally) the real and imaginary parts we will obtain

$$\left(u\frac{\partial}{\partial u}+s\right)F_s+\frac{u}{2}G_s=\frac{\partial}{\partial v}C_{s-1}+\frac{1}{2}D_{s-1}, \quad (7.11)$$

$$\left(u\frac{\partial}{\partial u}+s\right)G_s-\frac{u}{2}F_s=\frac{\partial}{\partial v}D_{s-1}-\frac{1}{2}C_{s-1}, \quad (7.12)$$

$$\left(v\frac{\partial}{\partial v}+s\right)F_s-\frac{v}{2}G_s=-\frac{\partial}{\partial u}C_{s-1}+\frac{1}{2}D_{s-1}, \quad (7.13)$$

$$\left(v\frac{\partial}{\partial v}+s\right)G_s+\frac{v}{2}F_s=-\frac{\partial}{\partial u}D_{s-1}-\frac{1}{2}C_{s-1}. \quad (7.14)$$

The auxiliary functions introduced by us make it possible to express the field through ordinary scalar and vector potentials and also through corresponding magnetic potentials. The fact is that the field with parabolic components

$$E_u=E_u^{(s)}\cos s\varphi;\quad E_v=E_v^{(s)}\cos s\varphi;\quad E_\varphi=E_\varphi^{(s)}\sin s\varphi, \quad (7.15)$$

$$H_u=H_u^{(s)}\sin s\varphi;\quad H_v=H_v^{(s)}\sin s\varphi;\quad H_\varphi=H_\varphi^{(s)}\cos s\varphi \quad (7.16)$$

can be represented in the form of

$$\mathbf{E}=ik\mathbf{A}-\operatorname{grad}A_0;\quad \mathbf{H}=\operatorname{curl}\mathbf{A}, \quad (7.17)$$

where

$$\operatorname{div}\mathbf{A}=ikA_0, \quad (7.18)$$

as well as in the form of

$$\mathbf{E}=\operatorname{curl}\mathbf{B};\quad \mathbf{H}=-ik\mathbf{B}+\operatorname{grad}B_0, \quad (7.19)$$

where

$$\operatorname{div}\mathbf{B}=ikB_0, \quad (7.20)$$

whereby the electric potentials are equal

$$\left.\begin{aligned} A_x &= iD_{s-1}(\sqrt{uv})^{s-1}\cos(s-1)\varphi, \\ A_y &= -iD_{s-1}(\sqrt{uv})^{s-1}\sin(s-1)\varphi, \\ A_z &= -iG_s(\sqrt{uv})^{s}\cos s\varphi, \\ A_0 &= -F_s(\sqrt{uv})^{s}\cos s\varphi, \end{aligned}\right\} \quad (7.21)$$

and the magnetic potentials

$$\left.\begin{aligned} B_x &= C_{s-1}(\sqrt{uv})^{s-1}\sin(s-1)\varphi, \\ B_y &= C_{s-1}(\sqrt{uv})^{s-1}\cos(s-1)\varphi, \\ B_z &= -F_s(\sqrt{uv})^{s}\sin s\varphi, \\ B_0 &= -iG_s(\sqrt{uv})^{s}\sin s\varphi \end{aligned}\right\} \quad (7.22)$$

Par. 8 Series for Potentials and for Auxiliary Functions

If the potentials for P_{s-1}, Q_{s-1} are known one can then easily derive analogous series for the values C_{s-1}, D_{s-1}, F_s, G_s from the series arranged according to parabolic functions.

For the sake of brevity we will write

$$\chi_{ns}(u, v) = \frac{\Gamma(n+s+1)}{\Gamma(n+1)}(uv)^{-\frac{s}{2}}\xi_{ns}(u)\xi_{ns}(v) \quad (8.01)$$

We have then, on the basis of the differential equation (3.03),

$$\left(u\frac{\partial}{\partial u} + s + 1 - \frac{iu}{2}\right)\left(\frac{\partial}{\partial u} + \frac{i}{2}\right)\chi_{ns} = i(n+s+1)\chi_{ns}, \quad (8.02)$$

$$\left(u\frac{\partial}{\partial u} + s + 1 + \frac{iu}{2}\right)\left(\frac{\partial}{\partial u} - \frac{i}{2}\right)\chi_{ns} = in\chi_{ns} \quad (8.03)$$

and on the basis of ratios (3.08) and (3.09)

$$\left(\frac{\partial}{\partial v} - \frac{i}{2}\right)\left(\frac{\partial}{\partial u} + \frac{i}{2}\right)\chi_{ns} = (n+s+1)\chi_{n,\,s+1}, \quad (8.04)$$

$$\left(\frac{\partial}{\partial v} + \frac{i}{2}\right)\left(\frac{\partial}{\partial u} - \frac{i}{2}\right)\chi_{ns} = n\chi_{n-1,\,s+1}. \quad (8.05)$$

These ratios will be satisfied by function chi$_{ns}$ provided we change one or both xi$_{ns}$ functions to eta$_{ns}$. Further, under chi$_{ns}$ we will understand one of the four functions obtained in such a way.

Assuming that a $\geqslant$ 1 and assuming that the series for P_{s-1} and Q_{s-1} have the form of

$$P_{s-1} = \sum_n p_n \chi_{n,\,s-1}, \quad (8.06)$$

$$Q_{s-1} = \sum_n q_n \chi_{n,\,s-1}. \quad (8.07)$$

Applying the above formulas we will obtain

$$C_{s-1} + iD_{s-1} = i\sum_n (n+s)(p_n + iq_n)\chi_{n,\,s-1}, \quad (8.08)$$

$$C_{s-1} - iD_{s-1} = i\sum_n n(p_n - iq_n)\chi_{n,\,s-1}, \quad (8.09)$$

and also

$$F_s + iG_s = \sum_p (n+s)(p_n + iq_n)\chi_{n,s}, \quad (8.10)$$

$$F_s - iG_s = \sum_n n(p_n - iq_n)\chi_{n-1,s}. \quad (8.11)$$

The latter formula acquires a specific meaning only at $p_o = iq_o$ because the value $n_{\text{Chi}\, n-1,s}$ is specific only at $n \neq 0$.

Analogous formulas for the auxiliary functions are obtained if the series for the potentials P_s, Q_s are oriented according to functions in complex conjugation with Chi_{ns}.

Assuming we have

$$P_{s-1} = \sum_n p_n \bar{\chi}_{n,s-1}, \qquad (8.12)$$

$$Q_{s-1} = \sum_n q_n \bar{\chi}_{n,s-1}. \qquad (8.13)$$

Then it will be

$$C_{s-1} + iD_{s-1} = -i \sum_n n (p_n + iq_n) \bar{\chi}_{n,s-1}, \qquad (8.14)$$

$$C_{s-1} - iD_{s-1} = -i \sum_n (n+s)(p_n - iq_n) \bar{\chi}_{n,s-1}, \qquad (8.15)$$

and also

$$F_s + iG_s = \sum_n n (p_n + iq_n) \bar{\chi}_{n-1,s}, \qquad (8.16)$$

$$F_s - iG_s = \sum_n (n+s)(p_n - iq_n) \bar{\chi}_{ns} \qquad (8.17)$$

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In conclusion, we shall present formulas for the derivatives of the series arranged according to Chi_{ns} functions. If r and z are cylindrical coordinates then we have

$$\frac{1}{k}\frac{\partial F}{\partial z} = \frac{2}{u+v}\left(u\frac{\partial F}{\partial u} - v\frac{\partial F}{\partial v}\right), \qquad (8.18)$$

$$\frac{1}{k^2 r}\frac{\partial F}{\partial r} = \frac{2}{u+v}\left(\frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}\right) \qquad (8.19)$$

If F is a series

$$F = \sum_{n=0}^{\infty} a_n \chi_{ns}(u, v), \qquad (8.20)$$

then, generally speaking, the right part of (8.18) is a series of the very same form

$$\frac{2}{u+v}\left(u\frac{\partial F}{\partial u} - v\frac{\partial F}{\partial v}\right) = i\sum_{n=0}^{\infty} b_n \chi_{ns}(u, v), \qquad (8.21)$$

and the right part of (8.19) is a series arranged according to $\text{Chi}_{n,s+1}$ functions, which we write in the form

$$\frac{2}{u+v}\left(\frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}\right) = 2\sum_{n=0}^{\infty} c_n \chi_{n,s+1}(u, v). \qquad (8.22)$$

The clause, generally speaking, is necessary because our statement is true without reservations; but, only if Chi_{ns} has the form of (8.01), i.e., if this value is composed of xi_{ns} functions. However, if Chi_{ns} includes one or two eta_{ns} functions then the formulas (8.21) and (8.22) are correct only at a condition where $a_o = 0$.

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In order to find the relation between the a_n and b_n we will set-up an expression $2(u\frac{\partial F}{\partial u} - v\frac{\partial F}{\partial v})$, where F is the (8.20) series, and we will utilize the recurrent formula (3.03); on the other hand, we will multiply the (8.21) series by u + v and utilize formula (3.05). By adjusting both expressions for

$$2\left(u\frac{\partial F}{\partial u} - v\frac{\partial F}{\partial v}\right).$$

we will obtain

$$a_n + a_{n+1} = b_n - b_{n+1}. \qquad (8.23)$$

In an anlogous manner by means of recurrent formulas (3.08) and (3.09) we will obtain

$$a_{n+1} = c_n - c_{n+1}. \qquad (8.24)$$

We can conclude, therefore, that

$$b_{n+1} = c_n + c_{n+1} + \text{const}. \qquad (8.25)$$

If Chi_{ns} contains only xi_{ns} functions then the series (see circle) should coincide and we then have

$$c_n = \sum_{k=n+1}^{\infty} a_k; \; b_n = a_n + 2\sum_{k=n+1}^{\infty} a_k. \qquad (8.26)$$

Analogous transforms can be applied also to integrals, but we decided not to waste any time on this matter.

Part 2. A Dipole in the Focus of a Rotation Paraboloid.

Par. 9 Primary Field from the Dipole

We will consider a dipole, oriented in the beginning (base) of the coordinates and having a moment directed along the axis x (perpendicular to the axis of rotation).

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The magnetic field from the dipole is expressed by formulas

$$H^{\bullet}_x = 0; \; H^{\bullet}_y = \frac{\partial A^{\bullet}_x}{\partial z}; \quad H^{\bullet}_z = -\frac{\partial A^{\bullet}_x}{\partial y} \qquad (9.01)$$

through the vector-potential having only one component different from zero

$$A^{\bullet}_x = A = \frac{iC}{2kR} e^{ikR} = \frac{iC}{u+v} e^{i\frac{u+v}{2}}. \qquad (9.02)$$

where C is a certain constant.

The covariant parabolic components of the magnetic field are determined from formulas

$$\left.\begin{aligned} 2uH^{\bullet}_u &= -2\sqrt{uv}\frac{\partial A}{\partial v}\sin\varphi, \\ 2vH^{\bullet}_v &= 2\sqrt{uv}\frac{\partial A}{\partial u}\sin\varphi, \\ H^{\bullet}_\varphi &= 2\sqrt{uv}\frac{1}{u+v}\left(u\frac{\partial A}{\partial u} - v\frac{\partial A}{\partial v}\right)\cos\varphi, \end{aligned}\right\} \qquad (9.03)$$

and the components of the electric field are derived from these formulas by applying the Maxwell equations (6.02). Further, we will require both accurate as well as approximate terms for the primary field and we shall, therefore, write them out in full. We have

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$$\left.\begin{aligned}
2uE^*_u &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\cos\varphi\left\{\frac{u-v}{(u+v)^2}+6i\frac{u-v}{(u+v)^3}+\right.\\
&\quad\left.+\frac{4i}{(u+v)^2}-\frac{8}{(u+v)^3}-\frac{12(u-v)}{(u+v)^4}\right\},\\
2vE^*_v &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\cos\varphi\left\{-\frac{u-v}{(u+v)^2}-6i\frac{u-v}{(u+v)^3}+\right.\\
&\quad\left.+\frac{4i}{(u+v)^2}-\frac{8}{(u+v)^3}+\frac{12(u-v)}{(u+v)^4}\right\},\\
E^*_\varphi &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\sin\varphi\left\{\frac{1}{u+v}+\frac{2i}{(u+v)^2}-\frac{4}{(u+v)^3}\right\}
\end{aligned}\right\}\quad(9.04)$$

$$\left.\begin{aligned}
2uH^*_u &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\sin\varphi\left\{\frac{1}{u+v}+\frac{2i}{(u+v)^2}\right\},\\
2vH^*_v &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\sin\varphi\left\{-\frac{1}{u+v}-\frac{2i}{(u+v)^2}\right\},\\
H^*_\varphi &= C\sqrt{uv}\,e^{i\frac{u+v}{2}}\cos\varphi\left\{-\frac{(u-v)}{(u+v)^2}-\frac{2i(u-v)}{(u+v)^3}\right\}
\end{aligned}\right\}\quad(9.05)$$

It is necessary to express the primary field from the dipole (as well as the total field including the reflected wave) through potentials P, Q. The dependence of the total field upon the angle psi will be the same as for the primary field. Assuming that in (6.20) and (6.21) s = 1 and omitting the signs at P and Q we will have

$$\left.\begin{aligned}
2uE_u &= \sqrt{uv}\left\{2\frac{\partial P^*}{\partial v}-\frac{\partial^2 P}{\partial u\partial v}-\frac{1}{4}P-\frac{1}{2}\frac{\partial Q}{\partial u}-\frac{1}{2}\frac{\partial Q}{\partial v}\right\}\cos\varphi,\\
2vE_v &= \sqrt{uv}\left\{-2\frac{\partial P^*}{\partial u}-\frac{\partial^2 P}{\partial u\partial v}-\frac{1}{4}P+\frac{1}{2}\frac{\partial Q}{\partial u}+\frac{1}{2}\frac{\partial Q}{\partial v}\right\}\cos\varphi,\\
E_\varphi &= \sqrt{uv}\left\{Q^*-\frac{1}{2}\frac{\partial Q}{\partial u}+\frac{1}{2}\frac{\partial Q}{\partial v}-\frac{\partial^2 P}{\partial u\partial v}+\frac{1}{4}P\right\}\sin\varphi,
\end{aligned}\right\}\quad(9.06)$$

$$\left.\begin{aligned}
2iuH_u &= \sqrt{uv}\left\{2\frac{\partial Q^*}{\partial v}-\frac{\partial^2 Q}{\partial u\partial v}-\frac{1}{4}Q+\frac{1}{2}\frac{\partial P}{\partial u}+\frac{1}{2}\frac{\partial P}{\partial v}\right\}\sin\varphi,\\
2ivH_v &= \sqrt{uv}\left\{-2\frac{\partial Q^*}{\partial u}-\frac{\partial^2 Q}{\partial u\partial v}-\frac{1}{4}Q-\frac{1}{2}\frac{\partial P}{\partial u}-\frac{1}{2}\frac{\partial P}{\partial v}\right\}\sin\varphi,\\
iH_\varphi &= \sqrt{uv}\left\{P^*-\frac{1}{2}\frac{\partial P}{\partial u}+\frac{1}{2}\frac{\partial P}{\partial v}+\frac{\partial^2 Q}{\partial u\partial v}-\frac{1}{4}Q\right\}\cos\varphi.
\end{aligned}\right\}\quad(9.07)$$

In this case P and Q are the functions not depending upon psi and satisfying the scalar equation of oscillations, which can be written in the form of

$$(L_u+L_v)P=0;\ L_u=u\frac{\partial^2}{\partial u^2}+\frac{\partial}{\partial u}+\frac{1}{4}u. \qquad (9.08)$$

According to general formulas (6.22), the functions P^*, Q^* are connected with P, Q by the ratios

$$P^*=L_uP=-L_vP \qquad (9.09)$$

$$Q^*=L_uQ=-L_vQ. \qquad (9.10)$$

These functions also satisfy the scalar equation of oscillations.

The formulas for the field will become simplified provided we introduce auxiliary functions (7.01) to (7.04). In this case we will write

$$C_0=S;\ D_0=-T \qquad (9.11)$$

and we will retain the designations F_1 and G_1. Thus we assume that

$$S=P^*-\frac{1}{2}Q, \qquad (9.12)$$

$$T=Q^*+\frac{1}{2}P, \qquad (9.13)$$

$$F_1=\frac{\partial^2 P}{\partial u\partial v}+\frac{1}{4}P+\frac{1}{2}\frac{\partial Q}{\partial u}-\frac{1}{2}\frac{\partial Q}{\partial v}, \qquad (9.14)$$

$$G_1=\frac{\partial^2 Q}{\partial u\partial v}+\frac{1}{4}Q-\frac{1}{2}\frac{\partial P}{\partial u}+\frac{1}{2}\frac{\partial P}{\partial v}. \qquad (9.15)$$

With these designations we will have:

$$\left.\begin{aligned}
2uE_u &= \sqrt{uv}\left(2\frac{\partial S}{\partial v}-F_1\right)\cos\varphi\\
2vE_v &= \sqrt{uv}\left(-2\frac{\partial S}{\partial u}-F_1\right)\cos\varphi\\
E_\varphi &= \sqrt{uv}\,(T-F_1)\sin\varphi
\end{aligned}\right\}\quad(9.16)$$

$$\left.\begin{aligned}
2iuH_u &= \sqrt{uv}\left(2\frac{\partial T}{\partial v}-G_1\right)\sin\varphi\\
2ivH_v &= \sqrt{uv}\left(-2\frac{\partial T}{\partial u}-G_1\right)\sin\varphi\\
iH_\varphi &= \sqrt{uv}\,(S+G_1)\cos\varphi.
\end{aligned}\right\}\quad(9.17)$$

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We will find the values $P = P^0$, $Q = Q^0$ which correspond to the field of the free dipole (without reflected wave). It is evident from formulas (9.05) that

$$RH^\circ_R = uH^\circ_u + vH^\circ_v = 0, \tag{9.18}$$

as it should actually be because the magnetic field from the dipole does not possess a radial component. Thus, the sum of the first two expressions (9.07) for the primary field should be equal to zero. This condition can be met by writing $Q^0 = 0$. Regarding the value P^0, it is not difficult to prove that formulas (9.07) will coincide with (9.03) or with (9.05), if we should write

$$P^\circ = \frac{2C}{u+v} e^{i\frac{u+v}{2}} = C\Pi, \tag{9.19}$$

where Π is the point characteristic, discussed in paragraph 4. The auxiliary functions (9.12) to (9.15), corresponding to the found values P^0, Q^0, are equal

$$S^\circ = Ce^{i\frac{u+v}{2}}\left(\frac{2(u-v)}{(u+v)^3} - \frac{i(u-v)}{(u+v)^2}\right), \tag{9.20}$$

$$T^\circ = \frac{C}{u+v} e^{i\frac{u+v}{2}} \tag{9.21}$$

$$F_1^\circ = Ce^{i\frac{u+v}{2}}\left(\frac{4}{(u+v)^3} - \frac{2i}{(u+v)^2}\right), \tag{9.22}$$

$$G_1^\circ = 0. \tag{9.23}$$

The ordinary electric potentials, calculated according to formulas (7.21) with consideration of designations (9.11) appear to be equal (congruent)

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$$A_x^\circ = \frac{iC}{u+v} e^{i\frac{u+v}{2}};\ A_y^\circ = 0;\ A_z^\circ = 0, \tag{9.24}$$

$$-Ce^{i\frac{u+v}{2}}\left(\frac{4}{(u+v)^3} - \frac{2i}{(u+v)^2}\right)\sqrt{uv}\cos\varphi, \tag{9.25}$$

which is in conformity with (9.02).

Par. 10. Expressions for the Field of a Reflected Wave in the Form of Integrals

We shall find expressions for reflected wave potentials in the form of integrals. For the potential P^0 of the primary wave we have, according to (4.12), an integral representation

$$P^\circ = \frac{2C}{u+v} e^{i\frac{u+v}{2}} = \frac{iC}{2}\int_{-\infty}^{+\infty} \frac{dt}{\operatorname{ch}\frac{\pi t}{2}} \zeta_1(u,t)\zeta_1(v,-t). \tag{10.01}$$

The overall field is written in the form of

$$\left.\begin{aligned} P &= P^\circ + P' \\ Q &= Q', \end{aligned}\right\} \tag{10.02}$$

where P' and Q' correspond to the reflected wave. It is evident from considerations of geometric optics that the reflected wave should have a phase multiple

$$e^{ikz} = e^{i\frac{u-v}{2}} \tag{10.03}$$

But this multiple has an asymptotic expression of the function

$$\zeta_1(u,t)\zeta_2(v,-t) = (uv)^{-\frac{1}{2}+\frac{it}{2}} \cdot e^{i\frac{u-v}{2}}(1+\ldots). \tag{10.04}$$

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we will therefore search for P', Q' in the form of integrals

$$P' = \frac{iC}{2}\int_{-\infty}^{+\infty}\frac{p(t)}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_2(v,-t)\,dt, \qquad (10.05)$$

$$Q' = \frac{iC}{2}\int_{-\infty}^{+\infty}\frac{q(t)}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_2(v,-t)\,dt, \qquad (10.06)$$

where the unknown functions p(t), q(t) are subject to determination from boundary conditions. According to the general formulas (6.26) and (6.27), the boundary conditions for the absolutely reflected paraboloid have the form of

$$2\frac{\partial P'}{\partial v} + Q' = -2\frac{\partial P^*}{\partial v} \ (\text{при } v = v_0), \qquad (10.07)$$

$$4Q'^* + 2\frac{\partial Q'}{\partial v} + P' = -P^* \ (\text{при } v = v_0). \qquad (10.08)$$

Here the value Q'^* is equal

$$Q'^* = L_u Q' = -\frac{iC}{4}\int_{-\infty}^{+\infty}\frac{tq(t)}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,t)\zeta_2(v,-t)\,dt. \quad (10.09)$$

By changing into boundary conditions the expressions (10.05), (10.06) and (10.09) for P', Q', Q'^* and assuming that the value v in them is $v = v_0$ then for p(t) and q(t) we will obtain equations

$$\left.\begin{aligned} p(t)\cdot 2\zeta_2' + q(t)\zeta_2 &= -2\zeta_1' \\ p(t)\zeta_2 + q(t)(2\zeta_2' - 2t\zeta_2) &= -\zeta_1, \end{aligned}\right\} \qquad (10.10)$$

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where, for the sake of brevity, we will write

$$\zeta_1 = \zeta_1(v_0, -t); \quad \zeta_1' = \left(\frac{\partial\zeta_1(v,-t)}{\partial v}\right)_{v=v_0} \qquad (10.11)$$

and in an analogous manner for zeta_2 and zeta'_2.

The solution of equations (10.10) gives

$$p(t) = \frac{-\zeta_1\zeta_2 + 4\zeta_1'\zeta_2' - 4t\zeta_1'\zeta_2}{\zeta_2^2 - 4\zeta_2'^2 + 4t\zeta_2'\zeta_2} \qquad (10.12)$$

$$q(t) = \frac{2\zeta_1\zeta_2' - 2\zeta_2\zeta_1'}{\zeta_2^2 - 4\zeta_2'^2 + 4t\zeta_2'\zeta_2} \qquad (10.13)$$

It should be noticed that the numerator of the fraction for q(t) is the Vrenskiy determinant which is equal

$$2\zeta_1\zeta_2' - 2\zeta_2\zeta_1' = -\frac{2i}{v_0}e^{\frac{\pi}{2}t} \qquad (10.14)$$

The substitution of the values p(t), q(t) found in formulas (10.05) and (10.06) offers reflected wave potentials and at the same time also a solution to our problem.

Par. 11. Representation of a Solution in the Form of Series

The reflected wave potentials can also be represented in the form of series. Similar to that which has been done in paragraph 4 for the primary disturbance, these series can be obtained either as a sum of deductions in points t = - (2n + 1)i of the lower semi-space or as a sum of deductions in points t = (2n + 1)i in the upper semi-space. During

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the calculation it is necessary to keep in mind that the general denominator of the functions p(t) and q(t) has a simple root in the point t = - i and has no other roots. Thus, the pole t = - i appears to be double whereas the other poles will be single (simple).

We will first find the deduction in the double pole t = -i.

It is not difficult to conclude from formulas (2.11) to (2.15) that near t = -i it will be

$$\zeta_1(u,\ t) = e^{i\frac{u}{2}}\left\{1 + \left(-\frac{\pi}{4} + \frac{i}{2}\lg u\right)(t+i) + \dots\right\}, \quad (11.01)$$

$$\zeta_2(v, -t) = e^{-i\frac{v}{2}}\left\{1 + \left(\frac{\pi}{4} + \frac{i}{2}\lg v\right)(t+i) + \dots\right\}, \quad (11.02)$$

wherefrom

$$\zeta_1(u,\ t)\,\zeta_2(v,\ -t) = e^{i\frac{u-v}{2}}\left\{1 + \frac{i}{2}(t+i)\lg(uv) + \dots\right\}. \quad (11.03)$$

The common denominator of functions p(t) and q(t) can be written as follows

$$(\zeta_2 - 2i\zeta_2')^2 + 4(t+i)\zeta_2'\zeta_2 = -2i\,e^{-iv_0}(t+i)\times$$
$$\times\left\{1 + (t+i)\left(\frac{\pi}{2} + i\lg v_0 - \frac{1}{v_0} + \frac{i}{2v_0^2}\right) + \dots\right\}. \quad (11.04)$$

The values of the numerators p(t) and q(t) at t = - i are equal according to

$$(2\zeta_1' + i\zeta_1)(2\zeta_2' + i\zeta_2) - 2i(\zeta_1\zeta_2' - \zeta_2\zeta_1') = \frac{2i}{v_0}, \quad (11.05)$$

$$2(\zeta_1\zeta_2' - \zeta_2\zeta_1') = -\frac{2}{v_0}, \quad (11.06)$$

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Consequently, in the proximity of t = -i it will be

$$p(t) = -\frac{1}{t+i}\cdot\frac{e^{iv_0}}{v_0} + p_{00} + \dots \quad (11.07)$$

$$q(t) = -\frac{i}{t+i}\cdot\frac{e^{iv_0}}{v_0} + q_{00} + \dots, \quad (11.08)$$

where p_{00} and q_{00} are constants, the values of which will not be determined by us because they drop out from the expressions for the field. Substituting (11.03), (11.07) and (11.08) by integrals (10.05) and (10.06) we obtain values of the deductions in the point t = -i, namely,

$$P_{00}' = 2iCp_{00}e^{i\frac{u-v}{2}} + \frac{C}{v_0}e^{iv_0}\cdot e^{i\frac{u-v}{2}}\lg(uv), \quad (11.09)$$

$$Q_{00}' = 2iCq_{00}e^{i\frac{u-v}{2}} + \frac{iC}{v_0}e^{iv_0}\cdot e^{i\frac{u-v}{2}}\lg(uv). \quad (11.10)$$

These expressions contain logarithmic terms and do not remain finite along the axis of the paraboloid. However, the values S, T, F_1, G_1 compiled from these terms according to formulas (9.12) to (9.15) do not contain any logarithmic terms (members). Therefore, the field corresponding to terms (members) P'_{00} and Q'_{00} in the potentials will also be finite. For the electric field, for example, we obtain

$$\left.\begin{aligned} 2u(E_u')_0 &= \frac{C}{v_0}e^{iv_0}\cdot e^{i\frac{u-v}{2}}\sqrt{uv}\cos\varphi, \\ 2v(E_v')_0 &= \frac{C}{v_0}e^{iv_0}\cdot e^{i\frac{u-v}{2}}\sqrt{uv}\cos\varphi, \\ (E_\varphi')_0 &= -\frac{C}{v_0}e^{iv_0}\cdot e^{i\frac{u-v}{2}}\sqrt{uv}\sin\varphi, \end{aligned}\right\} \quad (11.11)$$

which corresponds to a plane wave polarized in the direction of axis x.

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The calculation of deductions in poles t = -(2n + 1)i, where n = 1, 2, ... is not connected with any difficulties. Employing formulas (3.01) and 3.02) we obtain the following series for the reflected wave potentials.

$$P' = P'_{\infty} + C\sum_{n=1}^{\infty} p_n^{(1)}\xi_n(u)\bar{\xi}_n(v), \qquad (11.12)$$

$$Q' = Q'_{\infty} + C\sum_{n=1}^{\infty} q_n^{(1)}\xi_n(u)\bar{\xi}_n(v), \qquad (11.13)$$

where

$$p_n^{(1)} = 2i(-1)^n(n!)^2 p[-(2n+1)i], \qquad (11.14)$$

$$q_n^{(1)} = 2i(-1)^n(n!)^2 q[-(2n+1)i]. \qquad (11.15)$$

A calculation of the values p_n, q_n gives

$$p_n^{(1)} = -2i\cdot\frac{\bar{\xi}_n\bar{\eta}_n - 4\bar{\xi}'_n\bar{\eta}'_n - 4i(2n+1)\bar{\xi}_n\bar{\eta}'_n}{\bar{\xi}_n^2 - 4\bar{\xi}_n'^2 - 4i(2n+1)\bar{\xi}_n\bar{\xi}'_n}, \qquad (11.16)$$

$$q_n^{(1)} = -\frac{4i}{v_0}\cdot\frac{1}{\bar{\xi}_n^2 - 4\bar{\xi}_n'^2 - 4i(2n+1)\bar{\xi}_n\bar{\xi}'_n}, \qquad (11.17)$$

where the value v_o appears to be the argument of functions xi_n, eta_n. We like to point out that the denominator of value p_n, q_n can be represented as

$$\bar{\xi}_n^2 - 4\bar{\xi}_n'^2 - 4i(2n+1)\bar{\xi}_n\bar{\xi}'_n = = \frac{4}{v_0}n(n+1)(n\bar{\xi}_{n-1,1}^2 - (n+1)\bar{\xi}_{n,1}^2). \qquad (11.18)$$

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We will now calculate the potentials P', Q' as a sum of deductions in points t = (2n + 1)i. Since all the poles in the upper semi-space are single (simple) then we will have

$$P' = C\sum_{n=0}^{\infty} p_n^{(2)}\bar{\eta}_n(u)\eta_n(v), \qquad (11.19)$$

$$Q' = C\sum_{n=0}^{\infty} q_n^{(2)}\bar{\eta}_n(u)\eta_n(v), \qquad (11.20)$$

where

$$p_n^{(2)} = \frac{2i(-1)^n}{(n!)^2}p[(2n+1)i] \qquad (11.21)$$

$$q_n^{(2)} = \frac{2i(-1)^n}{(n!)^2}q[(2n+1)i] \qquad (11.22)$$

Expressing the values $p_n^{(2)}$, $Q_n^{(2)}$ through the functions $xi_n(v_o)$, eta_n (v_o) we will obtain

$$p_n^{(2)} = -2i\frac{\xi_n\eta_n - 4\xi'_n\eta'_n + 4i(2n+1)\xi'_n\eta_n}{\eta_n^2 - 4\eta_n'^2 + 4i(2n+1)\eta'_n\eta_n}, \qquad (11.23)$$

$$q_n^{(2)} = \frac{4i}{v_0}\cdot\frac{1}{\eta_n^2 - 4\eta_n'^2 + 4i(2n+1)\eta'_n\eta_n} \qquad (11.24)$$

Particularly, at n = 0 it will be

$$p_0^{(2)} = v_0 e^{iv_0}; \quad q_0^{(2)} = -iv_0 e^{iv_0}. \qquad (11.25)$$

We will calculate the field corresponding to the zero members of the break-down (11.19) and (11.20), i.e., to the potentials

$$P_0^{(2)} = Cv_0 e^{iv_0}\bar{\eta}_0(u)\eta_0(v), \qquad (11.26)$$

$$Q_0^{(2)} = -iCv_0 e^{iv_0}\bar{\eta}_0(u)\eta_0(v). \qquad (11.27)$$

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We have

$$S_0^{(2)}=0;\quad T_0^{(2)}=0, \tag{11.28}$$

$$F_{10}^{(2)}=\frac{Cv_0}{uv}\,e^{i\left(v_0+\frac{u-v}{2}\right)}, \tag{11.29}$$

$$G_{10}^{(2)}=-\frac{iCv_0}{uv}\,e^{i\left(v_0+\frac{u-v}{2}\right)} \tag{11.30}$$

The parabolic components of the field are obtained by substituting the values (11.28) to (11.30) by expressions (9.16) and (9.17). This field corresponds to electric potentials with components

$$A_x^{(2)}=0;\quad A_y^{(2)}=0, \tag{11.31}$$

$$A_z^{(2)}=A_0^{(2)}=-C\frac{ax}{r^2}\,e^{ik(a+z)} \tag{11.32}$$

These expressions represent an approach to the actual field under conditions where $u \gg 1$, $uv \gg v_0^2$. In the general case it is necessary to take into consideration further members of the series.

The series for the auxiliary functions S, T, F_1, G_1 are derived from the series for P', Q' by formulas brought forth in paragraph 8.

We will write

$$a_n^{(1)}=(n+1)(p_n^{(1)}+iq_n^{(1)})\quad (n=1,\ 2,\ \ldots), \tag{11.33}$$

$$a_0^{(1)}=2i\,(p_{00}+iq_{00}), \tag{11.34}$$

$$b_n^{(1)}=n\,(p_n^{(1)}-iq_n^{(1)})\quad (n=1,\ 2,\ \ldots), \tag{11.35}$$

$$b_0^{(1)}=\frac{2}{v_0}\,e^{iv_0} \tag{11.36}$$

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We will then have

$$S'+iT'=iC\sum_{n=0}^{\infty}a_n^{(1)}\xi_n(u)\bar{\xi}_n(v), \tag{11.37}$$

$$S'-iT'=iC\sum_{n=0}^{\infty}b_n^{(1)}\xi_n(u)\bar{\xi}_n(v), \tag{11.38}$$

$$F_1'+iG_1'=\frac{C}{\sqrt{uv}}\sum_{n=0}^{\infty}(n+1)\,a_n^{(1)}\xi_{n1}(u)\bar{\xi}_{n1}(v), \tag{11.39}$$

$$F_1'-iG_1'=\frac{C}{\sqrt{uv}}\sum_{n=0}^{\infty}(n+1)\,b_{n+1}^{(1)}\xi_{n1}(u)\bar{\xi}_{n1}(v). \tag{11.40}$$

In an analogous manner one can obtain series arranged according to functions $\overline{\eta}_n(u)\,\eta_n(v)$. We will write

$$a_n^{(2)}=n\,(p_n^{(2)}+iq_n^{(2)}),\quad a_0^{(2)}=0; \tag{11.41}$$

$$b_n^{(2)}=(n+1)(p_n^{(2)}-iq_n^{(2)}),\quad b_0^{(2)}=0 \tag{11.42}$$

and we will obtain

$$S'+iT'=-iC\sum_{n=0}^{\infty}a_n^{(2)}\bar{\eta}_n(u)\,\eta_n(v), \tag{11.43}$$

$$S'-iT'=-iC\sum_{n=0}^{\infty}b_n^{(2)}\bar{\eta}_n(u)\,\eta_n(v), \tag{11.44}$$

$$F_1'+iG_1'=\frac{2Cv_0}{uv}\,e^{iv_0}\cdot e^{i\frac{u-v}{2}}+$$

$$+\frac{C}{\sqrt{uv}}\sum_{n=0}^{\infty}(n+1)\,a_{n+1}^{(2)}\bar{\eta}_{n1}(u)\,\eta_{n1}(v), \tag{11.45}$$

$$F_1'-iG_1'=\frac{C}{\sqrt{uv}}\sum_{n=0}^{\infty}(n+1)\,b_n^{(2)}\bar{\eta}_{n1}(u)\,\eta_{n1}(v). \tag{11.46}$$

Substitution of these expressions in (9.16) and (9.17) gives the reflected wave field.

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It now remains to investigate the convergence of the series obtained. This is not a difficult task provided one utilizes the asymptotic expressions for functions xi_n and eta_n described in paragraph 3.

$$\xi_n(u) = J_0\left((1-i)\sqrt{(2n+1)u}\right) \qquad (11.47)$$

$$\eta_n(u) = i\pi H_0^{(2)}\left((1-i)\sqrt{(2n+1)u}\right) \qquad (11.48)$$

By utilizing these expressions we will obtain, for the coefficients of distant members of our series, approximate values

$$p_n^{(1)} = -i4\pi e^{-2w+2iw}, \qquad (11.49)$$

$$q_n^{(1)} = \frac{4\pi}{2n+1} e^{-2w+2iw}, \qquad (11.50)$$

$$p_n^{(2)} = -\frac{i}{\pi} e^{2w+2iw}; \qquad (11.51)$$

$$q_n^{(2)} = -\frac{i}{\pi(2n+1)} e^{2w+2iw} \qquad (11.52)$$

where, for the sake of brevity, we will write

$$w = \sqrt{(2n+1)v_0}. \qquad (11.53)$$

It is evident therefrom that the series (11.12) and (11.13), as well as other series arranged according to $xi_n(u)\overline{xi}_n(v)$ functions coincide at conditions

$$\sqrt{u} + \sqrt{v} < 2\sqrt{v_0}, \qquad (11.54)$$

and series (11.19) and (11.20) and other series arranged according to $\overline{eta}_n(u)eta_n(v)$ functions coincide at the condition

$$\sqrt{u} + \sqrt{v} > 2\sqrt{v_0}. \qquad (11.55)$$

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The boundary of the convergence zone is the surface determinable by equation

$$R + r = 2a. \qquad (11.56)$$

This is the surface of rotation, the cross-section of which in the plane of symmetry is the parabola with an axis perpendicular to the axis of the paraboloid and with the apex in the point z = 0, r = a, i.e., at the intersection of the focal plane with the paraboloid.

Par. 12. The Field in the Wave Zone

In the case when all the three numbers u, v, v_o are great in comparison to one (1) (large in ratio to one), expressions for the field corresponding to an approximation of geometric optics may be obtained. In our case, P' and Q' will be represented in the form of integrals (10.05) and (10.06). In the case investigated, it is important in these integrals that part of the integration corresponds to finite values t. But, at finite t we can utilize the asymptotic expressions (2.11) and (2.15) for the functions $zeta_1$ and $zeta_2$. Using these expressions we will obtain

$$q(t) = -i v_0^{-it} e^{iv_0} \cdot \left(\frac{1}{t+i} + \frac{it+1}{2v_0} + \ldots\right), \qquad (12.01)$$

$$p(t) = q(t) \cdot (t + \ldots), \qquad (12.02)$$

where dotted lines designate the members of the order $\frac{1}{v_o^2}$. With the very same degree of accuracy we will also have

$$\zeta_1(u, t)\zeta_2(v, -t) = (uv)^{-\frac{1}{2}+\frac{it}{2}} \cdot e^{i\frac{u-v}{2}} \times$$
$$\times \left(1 - \frac{i}{4}(t+i)^2 \frac{u-v}{uv} + \ldots\right). \qquad (12.03)$$

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Substituting these expressions in an integral

$$Q' = \frac{iC}{2}\int_{-\infty}^{+\infty}\frac{q(t)}{\operatorname{ch}\frac{\pi t}{2}}\zeta_1(u,\,t)\,\zeta_2(v,\,-t)\,dt \qquad (12.04)$$

we can write it as follows

$$Q' = \frac{Ce^{i\left(v_0+\frac{u-v}{2}\right)}}{2\sqrt{uv}}\int_{-\infty}^{+\infty}\frac{dt}{\operatorname{ch}\frac{\pi t}{2}}\left(\frac{v_0^2}{uv}\right)^{-\frac{it}{2}}\times$$
$$\times\left\{\frac{1}{t+i}+\frac{1-it}{4}\cdot\frac{u-v}{uv}+\frac{1+it}{2v_0}+\dots\right\}. \qquad (12.05)$$

This integral can be calculated without further disregard with the aid of ratios

$$\int_{-\infty}^{+\infty}\frac{z^{-it}\,dt}{(t+i)\operatorname{ch}\frac{\pi t}{2}} = -\frac{2i}{z}\lg(1+z^2), \qquad (12.06)$$

$$\int_{-\infty}^{+\infty}\frac{(1-it)}{\operatorname{ch}\frac{\pi t}{2}}\cdot z^{-it}\,dt = \frac{8z}{(1+z^2)^2}, \qquad (12.07)$$

$$\int_{-\infty}^{+\infty}\frac{(1+it)}{\operatorname{ch}\frac{\pi t}{2}}\cdot z^{-it}\,dt = \frac{8z^3}{(1+z^2)^2}. \qquad (12.08)$$

As a result, we arrive at the following approximate expression for Q'

$$Q' = Ce^{i\left(v_0+\frac{u-v}{2}\right)}\left\{-\frac{i}{v_0}\lg\left(1+\frac{v_0^2}{uv}\right)+\frac{v_0(u-v+2v_0)}{(v_0^2+uv)^2}\right\}. \qquad (12.09)$$

In a similar way one can also calculate the integral P', but here we can do away with the integration because in our approximation, according to (12.02), p(t) = tq(t) and consequently $P' = -2Q'^*$, where Q'^* has the value (10.09). Calculating P' by formulas

$$P' = -2L_uQ', \qquad (12.10)$$

we will obtain

$$P' = Ce^{i\left(v_0+\frac{u-v}{2}\right)}\left\{-\frac{1}{v_0}\lg\left(1+\frac{v_0^2}{uv}\right)+\frac{2v_0}{v_0^2+uv}+\right.$$
$$\left.+\frac{iv_0}{(v_0^2+uv)^3}\left[(u-v)(uv-3v_0^2)-2v_0^3+6v_0uv\right]\right\}. \qquad (12.11)$$

The auxiliary functions S', T', F_1', G_1' are derived from (12.09) and (12.11) by differentiation. We will have

$$S' = Ce^{i\left(v_0+\frac{u-v}{2}\right)}\left\{\frac{2iv_0^3}{(v_0^2+uv)^3}+\frac{4v_0^3(u-v)(v_0^2-2uv)}{(v_0^2+uv)^4}+\right.$$
$$\left.+\frac{8v_0^2uv(uv-2v_0^2)}{(v_0^2+uv)^4}\right\}, \qquad (12.12)$$

$$T' = 0, \qquad (12.13)$$

$$F_1' = iG_1' = Ce^{i\left(v_0+\frac{u-v}{2}\right)}\left\{\frac{v_0}{v_0^2+uv}-\frac{2iv_0^3(u-v)}{(v_0^2+uv)^3}+\frac{4iv_0^2uv}{(v_0^2+uv)^3}\right\}. \qquad (12.14)$$

The difference $F_1' - iG_1'$ will be of a much higher order of smallness than F_1', namely,

$$F_1' - iG_1' = -Ce^{i\left(v_0+\frac{u-v}{2}\right)}\frac{4v_0^3}{(v_0^2+uv)^3}\sim 0. \qquad (12.15)$$

The substitution of the values of auxiliary functions found in formulas (9.16) and (9.17) offers parabolic components of the reflected wave field. By combining these components with the expressions (9.04) and (9.05) for the primary field we will obtain the overall field. Since the formulas for the parabolic components of the field have a complex form we will not list them but will check only the realization of the boundary conditions. It is evident from condition $E_{psi} = 0$; $H_v = 0$ that on the surface of the paraboloid $v = v_o$ it should be

$$F_1' = T^{\circ} - F_1^{\circ}, \tag{12.16}$$

$$G_1' = -2\frac{\partial T^{\circ}}{\partial u}, \tag{12.17}$$

where T^o and F_1^o pertain to the primary field (formulas (9.20) to (9.23). Here we already took advantage of the fact that $G_1^o = 0$ and $T' = 0$. When $v = v_o$ we have

$$F_1' = iG_1' = Ce^{i\frac{u+v_0}{2}}\left(\frac{1}{u+v_0} + \frac{2i}{(u+v_0)^2}\right). \tag{12.18}$$

Comparing (12.18) with the difference of expressions (9.21) and (9.22) for T^o and F_1^o we will become convinced that the members of the order $1/(u\ v_o)$ and $1/(u + v_o)^2$ in the right and left parts of (12.16) coincide. In this approximation both parts of (12.17) also coincide.

In conclusion we wish to point out that the expressions derived for S', T', F_1', G_1' were obtained in the assumption that the values u, v are high and, consequently, they are substantiated only for greater distances

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from the axis. But these expressions represent holomorphic functions of the coordinates x, y, z also in the vicinity of the axis. They can, therefore, be applied in all these cases where the corrective members are small in comparison with the main ones, including also the axis of the paraboloid.

Par. 13 Rectangular Components of the Reflected Wave Field

In order to calculate the rectangular components of the field it is most convenient to use formulas (7.21) and (7.22) for the electric and magnetic potentials. We will write first the accurate formulas and then take up the approximation discussed in the previous paragraph.

According to (7.21) rectangular components of electric potentials are equal

$$A_x = iT; \quad A_y = 0; \quad A_z = -ikxG_1; \tag{13.01}$$

$$\operatorname{div} \mathbf{A} = ikA_0 = -ik^2xF_1. \tag{13.02}$$

On the other hand, according to (7.22) the magnetic potentials are equal

$$B_x = 0; \quad B_y = S; \quad B_z = -kyF_1; \tag{13.03}$$

$$\operatorname{div} \mathbf{B} = ikB_0 = k^2yG_1. \tag{13.04}$$

It is therefore possible to represent the field in two forms, namely,

$$\left.\begin{aligned} E_x &= \frac{\partial(kxF_1)}{\partial x} - kT = -\frac{\partial(kyF_1)}{\partial y} - \frac{\partial S}{\partial z} \\ E_y &= \frac{\partial(kxF_1)}{\partial y} = \frac{\partial(kyF_1)}{\partial x} \\ E_z &= \frac{\partial(kxF_1)}{\partial z} + k^2xG_1 = \frac{\partial S}{\partial x} \end{aligned}\right\} \tag{13.05}$$

$$\left.\begin{aligned} H_x &= -i\frac{\partial(kxG_1)}{\partial y} = -i\frac{\partial(kyG_1)}{\partial x}, \\ H_y &= i\frac{\partial T}{\partial z} + i\frac{\partial(kxG_1)}{\partial x} = -i\frac{\partial(kyG_1)}{\partial y} - ikS, \\ H_z &= -i\frac{\partial T}{\partial y} = -i\frac{\partial(kyG_1)}{\partial z} + ik^2yF_1. \end{aligned}\right\} \tag{13.06}$$

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Turning to the approximate formulas for the reflected wave we will have $T' = 0$, $F_1' = iG_1'$. The reflected wave field will, therefore, correspond approximately to the vector-potential with the single component A'_z different from zero which, according to (13.01) and (12.14), is equal

$$A'_z = Ce^{ik(a+z)} \cdot x\left\{-\frac{a}{a^2+r^2} - \frac{4ia^2r^2}{k(a^2+r^2)^3} + \frac{4ia^3z}{k^2(a^2+r^2)^3}\right\}. \quad (13.07)$$

The components of the field, perpendicular to the axis of the paraboloid, can be represented in the form

$$E'_x = H'_y = \frac{k}{2r}\frac{\partial(r^2F_1)}{\partial r} + \cos 2\varphi \cdot \frac{k}{2} r \frac{\partial F_1}{\partial r}, \quad (13.08)$$

$$E'_y = -H'_x = \sin 2\varphi \cdot \frac{k}{2} r \frac{\partial F_1}{\partial r}, \quad (13.09)$$

where

$$\frac{k}{2r}\frac{\partial(r^2F_1)}{\partial r} = Ce^{ik(a+z)}\left\{\frac{a^3}{(a^2+r^2)^3} + \frac{4ia^2[r^2(2a^2-r^2)+az(2r^2-a^2)]}{k(a^2+r^2)^4}\right\}, \quad (13.10)$$

$$\frac{kr}{2}\frac{\partial F_1}{\partial r} = Ce^{ik(a+z)}\left\{-\frac{ar^2}{(a^2+r^2)^2} - \frac{4ia^2r^2(2r^2-a^2-3az)}{k(a^2+r^2)^4}\right\}. \quad (13.11)$$

The components, parallel to the axis, can be calculated with the aid of approximate expressions

$$S' = Ce^{ik(a+z)} \cdot \frac{2ia^3}{k(a^2+r^2)^2}; \qquad T' = 0. \quad (13.12)$$

In consequence we obtain

$$E'_z = -Ce^{ik(a+z)} \cdot \frac{8ir\cos\varphi}{k(a^2+r^2)^3}; \; H'_z = 0. \quad (13.13)$$

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For the purpose of demonstration we will write plain expressions for the reflected wave field and we shall confine ourselves to major (primary) terms and express everything through rectangular coordinates

$$\left.\begin{aligned} E'_x &= C\frac{a(a^2+y^2-x^2)}{(a^2+x^2+y^2)^2}e^{ik(a+z)}, \\ E'_y &= -C\cdot\frac{2axy}{(a^2+x^2+y^2)^2}e^{ik(a+z)}, \\ E'_z &= -C\frac{8ia^3x}{k(a^2+x^2+y^2)^3}e^{ik(a+z)}, \end{aligned}\right\} \quad (13.14)$$

$$\left.\begin{aligned} H'_x &= C\frac{2axy}{(a^2+x^2+y^2)^2}e^{ik(a+z)}; \\ H'_y &= C\frac{a(a^2+y^2-x^2)}{(a^2+x^2+y^2)^2}e^{ik(a+z)}; \\ H'_z &= 0. \end{aligned}\right\} \quad (13.15)$$

The written out expressions strictly satisfy equations $\operatorname{div} E' = 0$; $\operatorname{div} H' = 0$ and approximately satisfy the remaining Maxwell equations (together with the primary field) and the boundary conditions.

Let us examine the overall field in the wave zone along the axis of the paraboloid. Component E_x will be different from zero which is equal

$$E_x = -\frac{C}{2R}e^{ikR} + \frac{C}{a}e^{ik(a+z)}, \quad (13.16)$$

where the first member corresponds to the straight line and the second member to the reflected wave. Since $z = R$ the amplitudes will add up at a condition

$$e^{ika} = -1 \quad (13.17)$$

or

$$ka = v_0 = (2m+1)\pi \quad (m - \text{целое}) \quad (13.18)$$ **(m is an integral)**

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and they will be subtracted when

$$ka = v_0 = 2m\pi. \qquad (13.19)$$

The correlation (13.19) indicates that the distance between the focus and apex $\frac{a}{2}$ is equal to the odd (uneven) multiple of one quarter of the wave.

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Radiation Characteristics of Spherical Surface Antennas

by

M. G. Belkina and L. A. Vaynshteyn

Introduction:

We come in contact with surface antennas in all those cases where the radiation (emission) or reception of radio waves takes place in the neighborhood of massive conductive bodies. And so, for example, an ordinary oscillator (dipole) mounted on a metal body together with that body forms a complex surface antenna the radiation characteristics of which can be entirely different from the well-known radiation characteristics of an oscillator in free space.

In regarding slot antennas the radiating slot is possible only in the presence of a conductive surface (walls of waveguide or cavity resonator) in which this slot is cut. Consequently, any slot antenna is at the same time also a surface antenna and the form of the surface may exert a strong effect on the radiation characteristic.

The problem concerning the radiation characteristics of surface antennas, particularly slot antennas, was theoretically investigated only in a very low degree. The calculation of radiation characteristics of surface antennas is based on the solution of diffraction problems because the radiation field of a surface antenna is being formed as result of diffraction of electromagnetic waves on a conductive surface.

This report presents a study of the radiation characteristics of spherical surface antennas. Using the classic theory of electromagnetic wave-diffraction on a sphere as a basis and utilizing the general reciprocity theorem, we offer formulas for the radiation characteristics of electric or magnetic dipoles situated near a sphere or on the sphere itself. So in doing we include also the case of a spherical slot antenna because the magnetic dipole on the sphere is electrodynamically equivalent to the slot cut in the sphere. The calculations, according to these formulas, are carried out whenever possible. In other instances these expressions appear to be the basic point for the derivation of approximate formulas by which the radiation characteristics are calculated.

During the analysis of a spherical surface antenna we have the opportunity of explaining, from the quantitative viewpoint, what effect the surface curvature has on the radiation characteristic and we encounter a series of phenomena which are common for the entire class of surface antennas.

Paragraph 1 Diffraction of a Plane Wave on a Sphere

During the study of electromagnetic wave diffraction on a sphere it is customary to introduce the so-called Debya u and v potentials which give the components of the electric and magnetic fields in a spherical system of coordinates in accordance with formulas (see, for example, (1) or (2):

$$\left.\begin{aligned} E_r &= \left(\frac{\partial^2}{\partial r^2} + k^2\right)(ru),\\ E_\theta &= \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}(ru) + ik\frac{1}{\sin\theta}\frac{\partial v}{\partial\varphi},\\ E_\varphi &= \frac{1}{r\sin\theta}\frac{\partial^2}{\partial r\partial\varphi}(ru) - ik\frac{\partial v}{\partial\theta},\\ H_r &= \left(\frac{\partial^2}{\partial r^2} + k^2\right)(rv),\\ H_\theta &= -ik\frac{1}{\sin\theta}\frac{\partial u}{\partial\varphi} + \frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}(rv),\\ H_\varphi &= ik\frac{\partial u}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial^2}{\partial r\partial\varphi}(rv), \end{aligned}\right\} \quad (1)$$

where $k = \frac{\text{omega}}{c} = \frac{2\text{pi}}{\text{lambda}}$ is the wave number in a vacuum (time dependance was taken as $e^{-i\ \text{omega}\ t}$). The u and v functions should serve as solutions for the wave equation

$$\Delta u + k^2 u = 0; \quad \Delta v + k^2 v = 0. \quad (2)$$

The terms (1) and (2) are correct for the vacuum. We can confine ourselves to the study of the fields in vacuum when the diffraction occurs on the sphere which can be considered as ideally conductive. In this case (which we will discuss exclusively) the presence of a sphere of the radius a is considered as boundary conditions:

$$E_\theta = E_\varphi = H_r = 0 \quad \text{при } r = a \quad (3)$$

We will discuss the diffraction of a plane electromagnetic wave falling on a sphere in the direction of the negative axis z and having a field

$$E_x^0 = -H_y^0 = e^{-ikz} \quad (4)$$

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For such a case it is convenient to utilize the P and Q potentials introduced by V. A. Fok (3) paragraph (6) which are connected with the u and v potentials by ratios

$$u = \cos\varphi \frac{\partial P}{\partial \vartheta}, \quad v = -\sin\varphi \frac{\partial Q}{\partial \vartheta}. \qquad (5)$$

The potentials P^0 and Q^0 determinable by the formula

$$P^0 = Q^0 = \frac{1}{ik^2 r} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (-i)^n \psi_n(kr) P_n(\cos\vartheta), \qquad (6)$$

give us the field of the incident wave (4) by virtue of formula

$$e^{-ikz} = e^{-ikr\cos\vartheta} = \sum_{n=0}^{\infty} (2n+1)(-i)^n \psi_n(kr) P_n(\cos\vartheta),$$

The overall diffraction field of an ideally conductive sphere has potentials

$$\left.\begin{aligned} P &= \frac{1}{ik^2 r} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (-i)^n \left[\psi_n(kr) - \frac{\psi_n'(ka)}{\zeta_n'(ka)} \zeta_n(kr)\right] P_n(\cos\vartheta) \\ Q &= \frac{1}{ik^2 r} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (-i)^n \left[\psi_n(kr) - \frac{\psi_n(ka)}{\zeta_n(ka)} \zeta_n(kr)\right] P_n(\cos\vartheta), \end{aligned}\right\} \qquad (7)$$

wherefrom we obtain components

$$\left.\begin{aligned} E_r &= \cos\varphi\, V(\vartheta, r), & H_r &= -\sin\varphi\, \tilde{V}(\vartheta, r), \\ E_\vartheta &= \cos\varphi\, \tilde{V}^{(1)}(\vartheta, r), & H_\vartheta &= -\sin\varphi\, V^{(1)}(\vartheta, r), \\ E_\varphi &= \sin\varphi\, \tilde{V}^{(2)}(\vartheta, r), & H_\varphi &= -\cos\varphi\, V^{(2)}(\vartheta, r), \end{aligned}\right\} \qquad (8)$$

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where

$$\left.\begin{aligned} V(\vartheta, r) &= \left(\frac{\partial^2}{\partial r^2} + k^2\right)\left(r \frac{\partial P}{\partial \vartheta}\right), \\ \tilde{V}^{(1)}(\vartheta, r) &= \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial^2 P}{\partial \vartheta^2}\right) - ik \frac{1}{\sin\vartheta} \frac{\partial Q}{\partial \vartheta}, \\ \tilde{V}^{(2)}(\vartheta, r) &= -\frac{1}{r\sin\vartheta} \frac{\partial}{\partial r}\left(r \frac{\partial P}{\partial \vartheta}\right) + ik \frac{\partial^2 Q}{\partial \vartheta^2}, \\ \tilde{V}(\vartheta, r) &= \left(\frac{\partial^2}{\partial r^2} + k^2\right)\left(r \frac{\partial Q}{\partial \vartheta}\right), \\ V^{(1)}(\vartheta, r) &= -ik \frac{1}{\sin\vartheta} \frac{\partial P}{\partial \vartheta} + \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial^2 Q}{\partial \vartheta^2}\right), \\ V^{(2)}(\vartheta, r) &= -ik \frac{\partial^2 P}{\partial \vartheta^2} + \frac{1}{r\sin\vartheta} \frac{\partial}{\partial r}\left(r \frac{\partial Q}{\partial \vartheta}\right). \end{aligned}\right\} \qquad (9)$$

When determining the functions (9) it is necessary to differentiate the series (7) by the variable theta in segments. The derivatives of the Legendre polynomials P_n according to the angle theta can be best calculated by formulas:

$$\left.\begin{aligned} \frac{dP_n(\cos\vartheta)}{d\vartheta} &= -P_n^{(1)}(\cos\vartheta), \\ \frac{d^2 P_n(\cos\vartheta)}{d\vartheta^2} &= \operatorname{ctg}\vartheta\, P_n^{(1)}(\cos\vartheta) - n(n+1) P_n(\cos\vartheta), \end{aligned}\right\} \qquad (10)$$

where $P_n^{(1)}$ is the associated Legendre function tabulated together with P_n (see [4]); for n = 11, 12..... 20 we calculated these functions by recurrent formulas). The functions psi_n and $zeta_n$ are the spherical Bessel functions determined by means of formulas

$$\psi_n(x) = \sqrt{\frac{\pi x}{2}} J_{n+\frac{1}{2}}(x), \quad \zeta_n(x) = \sqrt{\frac{\pi x}{2}} H^{(1)}_{n+\frac{1}{2}}(x).$$

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These functions are conveniently calculated with the aid of tables [5]. Their derivatives are calculated by formulas

$$\psi_n'(x)=\psi_{n-1}(x)-\frac{n}{x}\psi_n(x).$$

On the surface of the sphere, i. e., when r = a the functions (9) acquire the following values:

$$\tilde{V}(\theta, a)=\tilde{V}^{(1)}(\theta, a)=\tilde{V}^{(2)}(\theta, a)=0, \qquad (11а)$$

$$V(\theta, a)=\frac{1}{a^2}\sum(2n+1)(-i)^n\frac{1}{\zeta_n'(\alpha)}\frac{dP_n(\cos\theta)}{d\theta}; \qquad (11б)$$

$$V^{(1)}(\theta, a)=-\frac{1}{a}\left\{\frac{i}{\sin\theta}\sum\frac{2n+1}{n(n+1)}(-i)^n\frac{1}{\zeta_n'(\alpha)}\frac{dP_n(\cos\theta)}{d\theta}+\right.$$
$$\left.+\sum\frac{2n+1}{n(n+1)}(-i)^n\frac{1}{\zeta_n(\alpha)}\frac{d^2P_n(\cos\theta)}{i\,d\theta^2}\right\}, \qquad (11в)$$

$$V^{(2)}(\theta, a)=-\frac{1}{a}\left\{i\sum\frac{2n+1}{n(n+1)}(-i)^n\frac{1}{\zeta_n'(\alpha)}\frac{d^2P_n(\cos\theta)}{d\theta^2}+\right.$$
$$\left.+\frac{1}{\sin\theta}\sum\frac{2n+1}{n(n+1)}(-i)^n\frac{1}{\zeta_n(\alpha)}\frac{dP_n(\cos\theta)}{d\theta}\right\}. \qquad (11г)$$

In this case the letter alpha designates the frequently encountered dimensionless parameter

$$\alpha=ka=\frac{2\pi a}{\lambda}, \qquad (12)$$

which is equal to the number of waves which pile up on the circumference of the larger circle of the sphere. In a definite scale the series (11) give the distribution of the surface charge and current on an ideally conductive sphere irradiated by the plane wave.

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We wish to point out that thanks to the relations

$$\lim_{\theta\to 0}\frac{d^2P_n}{d\theta^2}=\lim_{\theta\to 0}\frac{1}{\sin\theta}\frac{dP_n}{d\theta}=-\frac{n(n+1)}{2},$$
$$\lim_{\theta\to \pi}\frac{d^2P_n}{d\theta^2}=-\lim_{\theta\to \pi}\frac{1}{\sin\theta}\frac{dP_n}{d\theta}=-\frac{n(n+1)}{2}(-1)^n$$

between the functions (9) at theta = 0 and theta = pi there are relations

$$\tilde{V}^{(1)}(0, r)=-\tilde{V}^{(2)}(0, r), \quad V^{(1)}(0, r)=V^{(2)}(0, r),$$
$$\tilde{V}^{(1)}(\pi, r)=\tilde{V}^{(2)}(\pi, r), \quad V^{(1)}(\pi, r)=-V^{(2)}(\pi, r). \qquad (13)$$

Paragraph 2 Radiation Characteristics of a Spherical Antenna

Formulas (9), (7) and (11) of the preceding paragraph make it possible to obtain rated formulas for the radiation characteristics of electric or magnetic elementary dipoles oriented near the surface of a conductive sphere or on the very surface of the sphere and together with that sphere forming a surface antenna. For this purpose it is necessary to utilize the reciprocity theorem for elementary dipoles. We will formulate, for example, an expression for the radiation field of a radial electric dipole. The plane wave (4) near the sphere can be considered as a part of the spherical wave radiated by the dipole situated on axis z at point B at a greater (in comparison with the radius of the sphere and length of wave) distance from the center of the sphere and having a moment P_x directed along axis x (see Fig. A, page 62). Assuming that in point A with coordinates r, psi = 0, theta exhibits a radial electric dipole with

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These functions are conveniently calculated with the aid of tables [5]. Their derivatives are calculated by formulas

$$\psi_n'(x) = \psi_{n-1}(x) - \frac{n}{x}\psi_n(x).$$

On the surface of the sphere, i. e., when r = a the functions (9) acquire the following values:

$$\tilde{V}(\vartheta, a) = \tilde{V}^{(1)}(\vartheta, a) = \tilde{V}^{(2)}(\vartheta, a) = 0, \qquad (11a)$$

$$V(\vartheta, a) = \frac{1}{\alpha^2}\sum (2n+1)(-i)^n \frac{1}{\zeta_n'(\alpha)} \frac{dP_n(\cos\vartheta)}{d\vartheta}, \qquad (11б)$$

$$V^{(1)}(\vartheta, a) = -\frac{1}{\alpha}\left\{\frac{i}{\sin\vartheta}\sum \frac{2n+1}{n(n+1)}(-i)^n \frac{1}{\zeta_n'(\alpha)} \frac{dP_n(\cos\vartheta)}{d\vartheta} + \right.$$
$$\left. + \sum \frac{2n+1}{n(n+1)}(-i)^n \frac{1}{\zeta_n(\alpha)} \frac{d^2P_n(\cos\vartheta)}{d\vartheta^2}\right\}, \qquad (11в)$$

$$V^{(2)}(\vartheta, a) = -\frac{1}{\alpha}\left\{i\sum \frac{2n+1}{n(n+1)}(-i)^n \frac{1}{\zeta_n'(\alpha)} \frac{d^2P_n(\cos\vartheta)}{d\vartheta^2} + \right.$$
$$\left. + \frac{1}{\sin\vartheta}\sum \frac{2n+1}{n(n+1)}(-i)^n \frac{1}{\zeta_n(\alpha)} \frac{dP_n(\cos\vartheta)}{d\vartheta}\right\}. \qquad (11г)$$

In this case the letter alpha designates the frequently encountered dimensionless parameter

$$\alpha = ka = \frac{2\pi a}{\lambda}, \qquad (12)$$

which is equal to the number of waves which pile up on the circumference of the larger circle of the sphere. In a definite scale the series (11) give the distribution of the surface charge and current on an ideally conductive sphere irradiated by the plane wave.

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We wish to point out that thanks to the relations

$$\lim_{\vartheta\to 0}\frac{d^2P_n}{d\vartheta^2} = \lim_{\vartheta\to 0}\frac{1}{\sin\vartheta}\frac{dP_n}{d\vartheta} = -\frac{n(n+1)}{2},$$
$$\lim_{\vartheta\to \pi}\frac{d^2P_n}{d\vartheta^2} = -\lim_{\vartheta\to \pi}\frac{1}{\sin\vartheta}\frac{dP_n}{d\vartheta} = -\frac{n(n+1)}{2}(-1)^n$$

between the functions (9) at theta = 0 and theta = pi there are relations

$$\tilde{V}^{(1)}(0, r) = -\tilde{V}^{(2)}(0, r), \quad V^{(1)}(0, r) = V^{(2)}(0, r),$$
$$\tilde{V}^{(1)}(\pi, r) = \tilde{V}^{(2)}(\pi, r), \quad V^{(1)}(\pi, r) = -V^{(2)}(\pi, r). \qquad (13)$$

Paragraph 2 Radiation Characteristics of a Spherical Antenna

Formulas (9), (7) and (11) of the preceding paragraph make it possible to obtain rated formulas for the radiation characteristics of electric or magnetic elementary dipoles oriented near the surface of a conductive sphere or on the very surface of the sphere and together with that sphere forming a surface antenna. For this purpose it is necessary to utilize the reciprocity theorem for elementary dipoles. We will formulate, for example, an expression for the radiation field of a radial electric dipole. The plane wave (4) near the sphere can be considered as a part of the spherical wave radiated by the dipole situated on axis z at point B at a greater (in comparison with the radius of the sphere and length of wave) distance from the center of the sphere and having a moment P_x directed along axis x (see Fig. A, page 62). Assuming that in point A with coordinates r, psi = 0, theta exhibits a radial electric dipole with

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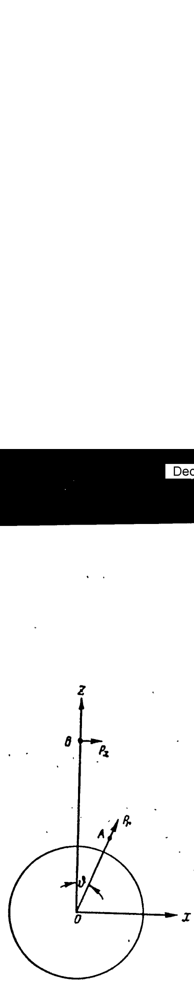

Figure A

such a moment $p_r = p$ that its primary field has a component in point B

$$E_x^0 = \sin\theta.$$

Since the plane wave (4) has a component in point A

$$E_r^0 = e^{-ikr\cos\theta}\sin\theta,$$

the reciprocity theorem gives, for the dipole moments at points A and B,

$$p_x = e^{-ikr\cos\theta}p_r.$$

As result of diffraction on the sphere the field of the plane wave (4), in point A according to (8), has a component

$$E_r = V(\theta, r).$$

According to the reciprocity theorem for overall fields the radial dipole, situated in point A, should produce a field in point B

$$E_x = e^{ikr\cos\theta}V(\theta, r),$$

where theta is the angle between the radii-vectors leading from the center of the sphere toward points A and B. If the axis of the spherical system of coordinates should be drawn through point A, in which the radial electric dipole is situated with the moment p, then its primary field in the wave zone is

$$E_\theta = H_\varphi = -k^2 p\frac{e^{ikR}}{R}\sin\theta, \qquad (14a)$$

where R is the distance from the observation point to point A and the angle theta is read from the direction OA. As result of diffraction on the sphere the overall field has the form of

$$E_\theta = H_\varphi = -k^2 p\frac{e^{ikR}}{R}W(\theta, r), \qquad (146)$$

where "the Weakening Factor" W (theta, r) is connected with the function V (theta, r) [see formulas (9) and (7)] by the ratio

$$W(\theta, r) = e^{ikr\cos\theta}V(\theta, r). \qquad (14в)$$

If the moment p of the electric dipole, oriented in point A, is perpendicular to the radius-vector OA then we guide axis x (beginning of the reading of angles psi) parallel to p (so that $p = p_x$). Then in plane psi = 0, where the primary wave field of this dipole is given by

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$$E_\theta = H_\varphi = k^2 p \frac{e^{ikR}}{R} \cos\theta, \qquad (15a)$$

the overall field of the surface antenna will be

$$E_\theta = H_\varphi = k^2 p \frac{e^{ikR}}{R} \bar{W}^{(1)}(\theta, r), \qquad (15б)$$

whereby

$$\bar{W}^{(1)}(\theta, r) = e^{ikr\cos\theta}\ \bar{V}^{(1)}(\theta, r). \qquad (15в)$$

In the plane where psi is $\frac{pi}{2}$ the primary field of this dipole has the form of

$$E_\varphi = -H_\theta = -k^2 p \frac{e^{ikR}}{R}; \qquad (16a)$$

and the overall field

$$E_\varphi = -H_\theta = -k^2 p \frac{e^{ikR}}{R} \bar{W}^{(2)}(\theta, r), \qquad (16б)$$

where

$$\bar{W}^{(2)}(\theta, r) = e^{ikr\cos\theta}\ \bar{V}^{(2)}(\theta, r). \qquad (16в)$$

We will now investigate a case where, in point A, an elementary magnetic dipole is situated with a moment m. If the dipole is directed radially so that, in the absence of a sphere, its radiation field is

$$H_\theta = -E_\varphi = -k^2 m \frac{e^{ikR}}{R} \sin\theta, \qquad (17a)$$

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then the field of the surface antenna has the form of

$$H_\theta = -E_\varphi = -k^2 m \frac{e^{ikR}}{R} \bar{W}(\theta, r), \qquad (17б)$$

whereby

$$\bar{W}(\theta, r) = e^{ikr\cos\theta}\ \bar{V}(\theta, r). \qquad (17в)$$

However, if the moment of the magnetic dipole is perpendicular to the radius-vector OA, then we, as before, select the direction of the axis x as coinciding with the moment m = m_x and in the plane psi = 0 the primary field in the wave zone containing the dipole is presented by formula

$$H_\theta = -E_\varphi = k^2 m \frac{e^{ikR}}{R} \cos\theta, \qquad (18a)$$

and the diffraction field of the spherical antenna equals

$$H_\theta = -E_\varphi = k^2 m \frac{e^{ikR}}{R} W^{(1)}(\theta, r), \qquad (18б)$$

where

$$W^{(1)}(\theta, r) = e^{ikr\cos\theta}\ V^{(1)}(\theta, r). \qquad (18в)$$

In the plane psi = $\frac{pi}{2}$, perpendicular to the dipole moment, the primary radiation field does not depend upon the angle theta and equals

$$H_\varphi = E_\theta = -k^2 m \frac{e^{ikR}}{R} \qquad (19a)$$

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and the radiation field of the spherical antenna has the form of

$$H_\varphi = E_\theta = -k^2 m \frac{e^{ikR}}{R} W^{(2)}(\theta, r). \qquad (19б)$$

where

$$W^{(2)}(\theta, r) = e^{ikr\cos\theta} V^{(2)}(\theta, r), \qquad (19в)$$

The field of the magnetic dipole above the sphere for any arbitrary value of the angle psi is given by formulas:

$$\left.\begin{aligned} H_\theta &= -E_\varphi = k^2 m \frac{e^{ikR}}{R} W^{(1)}(\theta, r)\cos\varphi, \\ H_\varphi &= E_\theta = -k^2 m \frac{e^{ikR}}{R} W^{(2)}(\theta, r)\sin\varphi, \end{aligned}\right\} \qquad (20)$$

and in a similar manner also for the electric dipole. Formulas (20) are derived directly from formulas (8).

Next, we will calculate the radiation characteristics of elementary dipoles situated on the surface of a sphere, i. e., at r = a. For the sake of brevity we will write:

$$\left.\begin{aligned} W(\theta) &= W(\theta, a) = e^{i\alpha\cos\theta} V(\theta, a), \\ W^{(1)}(\theta) &= W^{(1)}(\theta, a) = e^{i\alpha\cos\theta} V^{(1)}(\theta, a), \\ W^{(2)}(\theta) &= W^{(2)}(\theta, a) = e^{i\alpha\cos\theta} V^{(2)}(\theta, a), \end{aligned}\right\} \qquad (21)$$

where the parameter alpha is determined by formula (12).

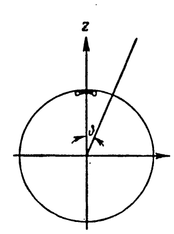

Figure B

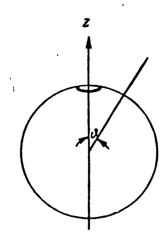

Figure C

It should be mentioned that the functions $W^{(2)}$ (theta) and $W^{(1)}$ (theta) also offer the radiation characteristic of an elementary (dumbbell type) slot (see Fig. B) cut in the sphere because such a slot, as is known, is equivalent to the elementary magnetic dipole situated on the sphere. The function W (theta) gives the radiation characteristic of an annular symmetrical slot (see Fig. C), provided its radius is small in comparison with the wave length and the radius of the sphere; such a "magnetic ring" is equivalent to an elementary radial electric dipole.

Paragraph 3 Threshold Case. Graphs of Radiation Characteristics

The calculation of complex functions (21) is of greatest interest in the theory of spherical surface antennas which is represented in the form of series (11b, c, d).

If

$$\alpha \ll 1, \qquad (22)$$

then these series are approximately reduced to their primary terms and we obtain

$$\left.\begin{aligned} W(\theta) &= 3\sin\theta, \\ W^{(1)}(\theta) &= \tfrac{3}{2}\cos\theta, \\ W^{(2)}(\theta) &= \tfrac{3}{2}. \end{aligned}\right\} \qquad (23)$$

Thus, in a specific case of infinitely long waves, the affect, of the sphere on the radiation characteristic of an elementary electric or magnetic dipole, which is situated on the surface of a sphere, is three times equivalent to the increase in the moment for the radial electric dipole and one and one-half times for the magnetic dipole (elementary slot). From the viewpoint of electro-and magnetostatics this inrease

is connected with the appearance (due to the presence of a sphere) of static "images" of the primary dipole.

In the case when

$$\alpha \gg 1 \qquad (24)$$

we actually deal with geometric optics.

In the proximity of geometric optics and irradiated zone, i. e., at 0 < theta < $\frac{pi}{2}$ we have formulas

$$\left.\begin{aligned} W(\theta) &= 2\sin\theta, \\ W^{(1)}(\theta) &= 2\cos\theta, \\ W^{(2)}(\theta) &= 2, \end{aligned}\right\} \qquad (25a)$$

and in the shaded zone, at $\frac{pi}{2}$ < theta < pi

$$W(\theta) = W^{(1)}(\theta) = W^{(2)}(\theta) = 0. \qquad (25б)$$

5•

The equatorial plane theta = $\frac{pi}{2}$, consequently, appears to be the geometric boundary of light and shade above which the sphere doubles the dipole moments due to the appearance of the "optical" image of the primary source. Below it the sphere completely shades the primary field of dipole radiation. The formation of static and optic images takes place according to different laws; therefore, the coefficients in formulas (23) and (25a) are also different.

Formulas (25) do not take into consideration the diffraction which appears, first of all, in the absence of a distinct boundary between the illuminated and shaded zones, i. e., in the presence of a semi-shaded zone. It is impossible to utilize the series (11) for the derivation of a diffraction chart under condition (24), because, in this case, the series converge very slowly. However, the approximation formulas for functions (21) can be derived by applying the common diffraction theory of a plane wave on convex conductive bodies, developed by V. A. Fok [6, 7]. These formulas are

$$\left.\begin{aligned} W(\theta) &= \sin\theta\, G(\xi), \\ W^{(1)}(\theta) &= \frac{i}{M} F(\xi), \\ W^{(2)}(\theta) &= G(\xi), \end{aligned}\right\} \qquad (26)$$

where M and xi are the dimensionless parameters

$$M = \left(\frac{ka}{2}\right)^{\frac{1}{3}} = \left(\frac{\alpha}{2}\right)^{\frac{1}{3}}, \quad \xi = -M\cos\theta, \qquad (27)$$

and F(xi) and G(xi) are the functions determinable by means of integrals

$$\left.\begin{aligned} F(\xi) &= \frac{1}{\sqrt{\pi}}\, e^{i\frac{\xi^3}{3}} \int_\Gamma \frac{e^{i\xi t}}{w_1(t)}\, dt, \\ G(\xi) &= \frac{1}{\sqrt{\pi}}\, e^{i\frac{\xi^3}{3}} \int_\Gamma \frac{e^{i\xi t}}{w_1'(t)}\, dt, \end{aligned}\right\} \qquad (28)$$

where the contour Γ (G) in the plane of the complex variable t is represented in Fig. D, and w_1 (t) is the complex Eure function (see [6] or [7]). The formulas (26) give a continuous change from light to shadow and in the illuminated zone at greater negative xi they convert into formulas (25a); and in the shade, at greater positive values xi they are practically zero, i. e., they convert into formulas (25b). Since the absolute values of functions F(xi), and G(xi) decrease monotonously during an increase in the parameter xi, the change in the radiation characteristics during the increase of the parameter alpha = ka from zero to infinity may be assumed as taking place according to formulas (26) in the following manner: the radiation in the lower semi-space theta $< \frac{pi}{2}$ weakens monotonously, and in the upper semi-space theta $< \frac{pi}{2}$ the characteristics pass monotonously over from functions (23) to functions (25a); the semi-shaded zone having an angular width of the order $\frac{1}{M}$, decreases.

The fact is that the change in radiation characteristics is more complex as is indicated by the direct calculation according to the diffraction series paragraph 1.

In the drawings 1 to 5 (see at the end of the report) the results of calculating the functions W (theta) for alpha = 1.3....9 are given and in Fig. 10 to 14 and 19 to 23, respectively, the results of calculating the functions $W^{(1)}$ (theta) and $W^{(2)}$ (theta) are given for these very same values of the parameter alpha. The calculations were carried out according to formulas (11) and (21); whereupon, it was necessary in the series (11) to take no less than 2 alpha members: and so, at alpha = 5 we took twelve and at alpha = 10 we took twenty members of the series and, as a result, obtaining an error of not more than two units of the fourth sign (power) after the comma.

Figure D

The graphs show the absolute values and phases (in degrees) of the complex functions W (theta), $W^{(1)}$ (theta) and $W^{(2)}$ (theta) in relation to the angle theta ($0^\circ \leqslant$ theta $\leqslant 180^\circ$) at a given value of the parameter alpha.

For example, the complex function W (theta) should, of course, be called the complex radiation characteristic and its absolute value

[W (theta)] - the amplitude characteristic, the phase arc W (theta) - the phase characteristic of the given antenna. Of greatest interest, certainly, is the amplitude characteristic often called simply, the radiation characteristic.

The graphs mentioned above show the characteristic features of the amplitude radiation characteristics:

1) the appearance of new maximums the total number of which increases with the increase in alpha;

2) the oscillations connected with it which are particularly intensive in the zone of the shadow and where, due to these oscillations, the radiation characteristics acquire a multilobe form; and

3) particularly strong oscillations in the vicinity of the pole theta = pi, where the amplitude characteristic assumes relatively greater values.

As to the phase characteristic, it experiences rapid changes near the minimums of the amplitude characteristic.

We shall analyze more thoroughly the individual series of graphs. The radiation characteristic W (theta) of a radial electric dipole at alpha = 1 is different from the characteristic at alpha = 0 by a small displacement of the maximum into the zone of the shade. When alpha is 2 this maximum was already broken down into two "protuberances". During further increase in the parameter alpha new maxima and minima appear and the number of oscillations increases monotonously occupying approximately an interval $60^\circ <$ theta $< 180^\circ$.

The complex characteristic $W^{(1)}$ (theta) of the magnetic dipole in the meridional plane containing the dipole has, at the very same values of the parameter alpha, a much simpler form. At alpha = 0 it consists of two larger lobes of uniform size - the upper and lower - separated by the zero value of the amplitude characteristic at theta = 90°. With the increase in the alpha parameter the rear ("shaded") lobe, always remaining strictly isolated from the upper one gradually narrows and weakens. With this weak oscillation begin to appear, noticeable especially in the intermediate range of angles between the upper and lower lobes. Because of the presence of these oscillations values alpha > 5 for the lower lobe should, in essence, be considered as the maximum of the oscillations (having a maximum at theta = 180°) in the amplitude characteristic.

The above mentioned criteria are possessed by the function $W^{(2)}$ (theta) in the most distinct form - the complex radiation characteristic of the magnetic dipole are in the meridian plane perpendicular to the dipole. Here the oscillations are quite noticeable over the entire diagram. The oscillatory nature of this function leads to the fact that, in order to plot a graph it is necessary, in many cases, to calculate this function according to the argument theta by 2°. The function $W^{(2)}$ (theta) at alpha is 0, is constant and equal to $\frac{3}{2}$ for all values of the angle theta. During the increase in alpha it acquires a wave-like form and, at a change in theta from 0 to 180°, the oscillation amplitude, in general, increases.

It can be concluded: the value of the parameter alpha = 10 is still insufficiently high in order to allow a clear formulation of the illuminated and "shaded" zones with the monotonous transition between them. In other words, when alpha ⩾ 10 the radiation characteristics are not even qualitatively transmitted by the approximation formulas (26) and the direct utilization of the diffraction series is already impossible. The presence of oscillations from the physical viewpoint remains incomprehensible.

In connection with the statements made above we have two problems closely connected with each other.

1) To explain the origin and physical sense of the oscillations in the radiation characteristics of a spherical surface antenna and;

2) To derive such approximation formulas for the radiation characteristics which would give satisfactory results at alpha ⩾ 10.

The solution to these problems will be given below, in paragraphs 4 and 5.

Paragraph 4 Definition of the Theory of Diffraction on a Sphere at Greater Values of the Parameter Alpha

The potentials P and Q [formula (7)] of the electromagnetic field of a plane wave being diffracted on a sphere, can be written as follows:

$$\left.\begin{aligned} P &= \sum \frac{2\nu}{\nu^2 - \frac{1}{4}} \Phi(\nu) P_{\nu-\frac{1}{2}}(\cos\theta), \\ Q &= \sum \frac{2\nu}{\nu^2 - \frac{1}{4}} \Psi(\nu) P_{\nu-\frac{1}{2}}(\cos\theta). \end{aligned}\right\} \qquad (29)$$

The summation here is carried out in accordance with the nu values obtained

$$\nu = \frac{3}{2}, \frac{5}{2}, \ldots,$$

$$\Phi(\nu) = \frac{1}{k\rho} e^{-i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}} \left[\psi_{\nu-\frac{1}{2}}(\rho) - \frac{\psi'_{\nu-\frac{1}{2}}(\alpha)}{\zeta'_{\nu-\frac{1}{2}}(\alpha)} \zeta_{\nu-\frac{1}{2}}(\rho) \right] \quad (30)$$

$$\Psi(\nu) = \frac{1}{k\rho} e^{-i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}} \left[\psi_{\nu-\frac{1}{2}}(\rho) - \frac{\psi_{\nu-\frac{1}{2}}(\alpha)}{\zeta_{\nu-\frac{1}{2}}(\alpha)} \zeta_{\nu-\frac{1}{2}}(\rho) \right]$$

$$\rho = kr, \quad (31)$$

and the value alpha is determined by the ratio (12).

It can easily be shown that the potentials

$$\hat{P} = i \int_{C_1} \frac{\nu}{\left(\nu^2 - \frac{1}{4}\right)\cos(\pi\nu)} \Phi(\nu) P_{\nu-\frac{1}{2}}(-\cos\vartheta)\, d\nu,$$

$$\hat{Q} = i \int_{C_1} \frac{\nu}{\left(\nu^2 - \frac{1}{4}\right)\cos(\pi\nu)} \Psi(\nu) P_{\nu-\frac{1}{2}}(-\cos\vartheta)\, d\nu, \quad (32)$$

where the contour C_1 comprises the right-half of the material axis (see Fig. E) and the potentials give fields (8) - (9) such as the potentials (29). The fact is that $\hat{P}$ and $\hat{Q}$ differ from P and Q,, respectively, by superfluous items - deductions in the point nu = $\frac{1}{2}$, but the fields of these items, as can easily be proven, are obtained equal to zero.

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Taking advantage of the fact that $P_{\nu - \frac{1}{2}}$, Phi (nu) and Psi (nu) are the even functions of nu, we convert (see, for example, [8] Fig. 68) the course C_1 into C which is parallel to the material axis and above it (see Fig. E). Using the ratio

$$P_{\nu-\frac{1}{2}}(-\cos\theta) = \frac{2i\cos(\pi\nu)}{\pi\sqrt{2\sin\theta}} e^{i\left(\theta\nu-\frac{\pi}{4}\right)} G^*_\nu + e^{i\left(\nu-\frac{1}{2}\right)\pi} P_{\nu-\frac{1}{2}}(\cos\theta)$$

(see [9] par. 2; expression G^*_{ν} was explained in formula (2.12) of this book) we separate $\hat{P}$ and $\hat{Q}$ into components

$$\hat{P} = P' + P''; \quad \hat{Q} = Q' + Q'', \quad (33)$$

where

$$P' = \frac{\sqrt{2}\, e^{-i\frac{\pi}{4}}}{\pi\sqrt{\sin\theta}} \int_{C} \frac{\nu}{\nu^2 - \frac{1}{4}} \Phi(\nu) e^{i\theta\nu} G^*_\nu\, d\nu,$$

$$P'' = -\int_{C} \frac{\nu}{\left(\nu^2 - \frac{1}{4}\right)\cos(\pi\nu)} \Phi(\nu) e^{i\pi\nu} P_{\nu-\frac{1}{2}}(\cos\theta)\, d\nu, \quad (34)$$

and the components Q' and Q" are expressed by such integrals in which Phi (nu) is substituted by Psi (nu).

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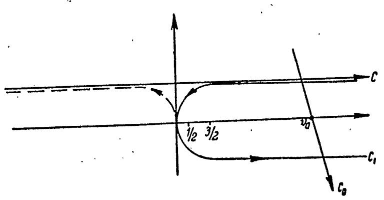

Figure E

Further transformations of integrals are based on the following characteristics of subintegral functions.

1) G*$_{nu}$ is a holomorphic function at Re nu $>$ 0, having the poles on the negative part of the material axis;

2) Zeta$_{nu-\frac{1}{2}}$ and zeta '$_{nu-\frac{1}{2}}$ are integral functions nu, not having zeros in the fourth quadrant;

3) Psi$_{nu-\frac{1}{2}}$ and psi'$_{nu-\frac{1}{2}}$ are integral functions of nu; and

4) Consequently, the functions Phi (nu) and Psi (nu) are holomorphic in the fourth quadrant and meromorphic in the first.

When studying the integrals P' and Q' the contour C can be substituted with contour C_o which intersects the materials (real) axis in the point nu$_o$ from top to bottom (see Fig. E). On this contour the basic part of integration appears to be the vicinity of the point

nu$_o$ = alpha

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where, at condition

$$\alpha \gg 1 \qquad (35)$$

one can apply the asymptotic expressions (compare [9] par. 5)

$$\left.\begin{aligned}
\zeta_{\nu-\frac{1}{2}}(\alpha) &= -i\sqrt{M}\,w_1(t),\\
\zeta'_{\nu-\frac{1}{2}}(\alpha) &= i\frac{1}{\sqrt{M}}w_1'(t),\\
\zeta_{\nu-\frac{1}{2}}(\rho) &= -i\sqrt{Mm}\,w_1(mt-my),\\
\zeta'_{\nu-\frac{1}{2}}(\rho) &= i\frac{1}{\sqrt{Mm}}w_1'(mt-my),\\
\psi_{\nu-\frac{1}{2}}(\alpha) &= \sqrt{M}v(t),\\
\psi'_{\nu-\frac{1}{2}}(\alpha) &= -\frac{1}{\sqrt{M}}v'(t),
\end{aligned}\right\} \qquad (36)$$

$$\left.\begin{aligned}
\psi_{\nu-\frac{1}{2}}(\rho) &= \sqrt{Mm}\,v'(mt-my),\\
\psi'_{\nu-\frac{1}{2}}(\rho) &= -\frac{1}{\sqrt{Mm}}v'(mt-my),
\end{aligned}\right\} \qquad (36)$$

where the nondimensional values m, M and y are as follows

$$m=\left(\frac{a}{r}\right)^{\frac{1}{3}}=\left(\frac{\alpha}{\rho}\right)^{\frac{1}{3}}, \qquad (37)$$

$$M=\left(\frac{ka}{2}\right)^{\frac{1}{3}}=\left(\frac{\alpha}{2}\right)^{\frac{1}{3}}, \qquad (38)$$

$$y=\frac{k(r-a)}{M}=\frac{\rho-\alpha}{M}, \qquad (39)$$

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and the variable t is connected with the variable nu by the relation

$$\nu = \alpha + Mt. \qquad (40)$$

The functions Phi (nu) and Psi (nu) can, consequently, be written as follows

$$\left.\begin{aligned} \Phi(\nu) &= \frac{1}{k\rho} e^{-i\left(\alpha+\frac{1}{2}\right)\frac{\pi}{2}} e^{-i\frac{\pi}{2}Mt} \sqrt{Mm} \times \\ &\times \left\{ v(mt-my) - \frac{v'(t)}{w_1'(t)} w_1(mt-my) \right\}, \\ \Psi(\nu) &= \frac{1}{k\rho} e^{-i\left(\alpha+\frac{1}{2}\right)\frac{\pi}{2}} e^{-i\frac{\pi}{2}Mt} \sqrt{Mm} \times \\ &\times \left\{ v(mt-my) - \frac{v(t)}{w_1(t)} w_1(mt-my) \right\}. \end{aligned}\right\} \qquad (41)$$

If we are interested in the points oriented at such distances from the surface of the sphere r = a, which are small in comparison with the radius of the sphere then, in formulas (41) it may be written as

m = 1

In the case of surface antennas the primary radiators (emitters) are usually situated near the surface and the latter condition can be considered as fulfilled.

If it is also taken into account that

$$v(t) = \frac{w_1(t) - w_2(t)}{2i},$$

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and the function G*nu is applied, the asymptotic representation (see [9] formula (2.09))

$$G_\nu^* = \sqrt{\frac{\pi}{\nu}} = \sqrt{\frac{\pi}{\alpha}}, \qquad (42)$$

then P' and Q' will be represented as follows

$$\left.\begin{aligned} kP' &= \frac{1}{i\alpha^2} \frac{e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}}{\sqrt{\sin\theta}} V_1(\xi, y), \\ kQ' &= \frac{1}{i\alpha^2} \frac{e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}}{\sqrt{\sin\theta}} V_2(\xi, y), \end{aligned}\right\} \qquad (43)$$

where

$$\left.\begin{aligned} V_1(\xi, y) &= \frac{i}{2\sqrt{\pi}} \int_\Gamma e^{i\xi t} \left\{ w_2(t-y) - \frac{w_2'(t)}{w_1'(t)} w_1(t-y) \right\} dt, \\ V_2(\xi, y) &= \frac{i}{2\sqrt{\pi}} \int_\Gamma e^{i\xi t} \left\{ w_2(t-y) - \frac{w_2(t)}{w_1(t)} w_1(t-y) \right\} dt, \end{aligned}\right\} \qquad (44)$$

and xi designates

$$\xi = M\left(\theta - \frac{\pi}{2}\right). \qquad (45)$$

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The functions V_1 and V_2, determinable by integrals (44), are partial cases (exceptional cases) of the universal function V_1 (xi, y, q) introduced by V. A. Fok [see [6] formula (4.22)], namely,

$$V_1(\xi, y) = V_1(\xi, y, 0); \quad V_2(\xi, y) = V_1(\xi, y, \infty) \tag{46}$$

When xi $>$ 0 the functions V_1 and V_2 can be represented in form of series by deduction

$$\left.\begin{aligned} V_1(\xi, y) &= i2\sqrt{\pi}\sum_{s=1}^{\infty} e^{i\xi t_s'} \frac{w_1(t_s'-y)}{t_s'[w_1(t_s')]^2}, \\ V_2(\xi, y) &= -i2\sqrt{\pi}\sum_{s=1}^{\infty} e^{i\xi t_s^0} \frac{w_1(t_s^0-y)}{[w_1'(t_s^0)]^2}, \end{aligned}\right\} \tag{47}$$

where t^o_s are the roots of the equation w(t) = 0, and t'_s are the roots of the equation w'(t) = 0 (see [9] par. 7).

Under condition (35) these formulas give potentials and fields in the zone of semi-shade and these should be supplemented in such a manner as to obtain expressions for fields in any given point separated from the surface of the sphere by a distance which is small in comparison to the radius of the sphere.

In order to investigate the field in a deep shadow it is most convenient to begin with the integrals (32), where the contour C_1 was substituted with contour C (running in opposite direction). The poles of the subintegral functions Phi (nu) and Psi (nu) are zeros of functions

zeta$'_{nu - 1/2}$ (alpha) and zeta$_{nu - 1/2}$ (alpha) which we designate by nu'_s and nu^o_s, respectively. These zeros can be calculated by formulas

$$\nu_s' = \alpha + Mt_s'; \quad \nu_s^0 = \alpha + Mt_s^0, \tag{48}$$

provided (compare [9], formula (5.18) t'_s and t^o_s satisfy the condition

$$\frac{1}{60M^2}|t_s|^{\frac{5}{2}} < 1. \tag{49}$$

In the vicinity of the poles satisfying this inequality, the functions Phi (nu) and Psi (nu) can be calculated by formulas (41). Since one can apply the asymptotic formula for the Legendre functions near such poles

$$P_{\nu-\frac{1}{2}}(-\cos\theta) = P_{\nu-\frac{1}{2}}(\cos(\pi-\theta)) = \sqrt{\frac{\pi-\theta}{\sin\theta}}\, J_0(\nu(\pi-\theta)), \tag{50}$$

which is suitable all the way up to theta = pi then, at an additional condition,

$$\frac{1}{2M^2}|t_s| \ll 1 \tag{51}$$

we obtain expressions in the form of deduction series for $\hat{P}$ and $\hat{Q}$

$$\left.\begin{aligned} k\hat{P} &= \\ &= \frac{\pi e^{-i\frac{\pi}{4}}}{M^{\frac{9}{2}}} e^{i\alpha\frac{\pi}{2}} \sqrt{\frac{\pi-\theta}{\sin\theta}} \sum_{s=1}^{\infty} e^{iM\frac{\pi}{2}t_s'} J_0(\nu_s'(\pi-\theta)) \frac{w_1(t_s'-y)}{t_s'[w_1(t_s')]^2}, \\ k\hat{Q} &= \\ &= -\frac{\pi e^{-i\frac{\pi}{4}}}{M^{\frac{9}{2}}} e^{i\alpha\frac{\pi}{2}} \sqrt{\frac{\pi-\theta}{\sin\theta}} \sum_{s=1}^{\infty} e^{iM\frac{\pi}{2}t_s^0} J_0(\nu_s^0(\pi-\theta)) \frac{w_1(t_s^0-y)}{[w_1'(t_s^0)]^2}. \end{aligned}\right\} \tag{52}$$

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At a condition

$$\alpha(\pi-\vartheta) \gg 1 \tag{53}$$

the Bessel function can be substituted by its own asymptotic expression

$$J_0(\nu(\pi-\vartheta)) = \sqrt{\frac{1}{2\pi\nu(\pi-\vartheta)}}\left\{e^{i\nu(\vartheta-\pi)+i\frac{\pi}{4}} + e^{-i\nu(\vartheta-\pi)-i\frac{\pi}{4}}\right\}, \tag{54}$$

after which each of the series (52) will be brought to a linear combination of series of the type (47), i.e., to functions V_1 and V_2, namely,

$$\left.\begin{aligned} k\hat{P} &= \frac{1}{i\alpha^2}\frac{1}{\sqrt{\sin\vartheta}}\left\{e^{i\alpha\left(\vartheta-\frac{\pi}{2}\right)}V_1(\xi, y) - ie^{i\alpha\left(\frac{3\pi}{2}-\vartheta\right)}V_1(\xi_1, y),\right. \\ k\hat{Q} &= \frac{1}{i\alpha^2}\frac{1}{\sqrt{\sin\vartheta}}\left\{e^{i\alpha\left(\vartheta-\frac{\pi}{2}\right)}V_2(\xi, y) - ie^{i\alpha\left(\frac{3\pi}{2}-\vartheta\right)}V_2(\xi_1, y),\right. \end{aligned}\right\} \tag{55}$$

where

$$\xi_1 = M\left(\frac{3\pi}{2}-\vartheta\right). \tag{56}$$

However, if we fulfill the condition

$$M(\pi-\vartheta) \ll 1, \tag{57}$$

then we can write

$$\begin{aligned} J_0(\nu_s'(\pi-\vartheta)) &= J_0(\nu_1'(\pi-\vartheta)); \\ J_0(\nu_s^0(\pi-\vartheta)) &= J_0(\nu_1^0(\pi-\vartheta)) \end{aligned} \tag{58}$$

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and sustain these functions beyond the sign of summation, after which the expressions (52) for $\hat{P}$ and $\hat{Q}$ acquire the form of

$$\left.\begin{aligned} k\hat{P} &= \frac{1}{i\alpha^2}2\sqrt{\pi}e^{-i\frac{\pi}{4}}M^{\frac{3}{2}}\sqrt{\frac{\pi-\vartheta}{\sin\vartheta}}\times \\ &\times J_0(\nu_1'(\pi-\vartheta))e^{i\alpha\frac{\pi}{2}}V_1\left(M\frac{\pi}{2}, y\right), \\ k\hat{Q} &= \frac{1}{i\alpha^2}2\sqrt{\pi}e^{-i\frac{\pi}{4}}M^{\frac{3}{2}}\sqrt{\frac{\pi-\vartheta}{\sin\vartheta}}\times \\ &\times J_0(\nu_1^0(\pi-\vartheta))e^{i\alpha\frac{\pi}{2}}V_2\left(M\frac{\pi}{2}, y\right). \end{aligned}\right\} \tag{59}$$

During the derivation of formulas (52) it was assumed that conditions (49) and (51) are fulfilled for all essential (material) members of the series obtained. These conditions actually take place because in the series obtained several of the first members (for which $|t_s| \sim 1$) are usually essential, and the number M is high. Only on the boundary with the illuminated zone do the series for functions $V_1(xi,y)$ and $V_2(xi,y)$ begin showing a poor convergence. But, in this case, it becomes necessary to utilize the integral representations (44), and then formulas (43) again lead us to the expressions (55).

The basic greater parameters alpha and M of our diffraction problem are connected by the relation (38); consequently, the conditions (53) and (57) do not contradict each other so that formulas (55) and (59) have a common zone of applicability. Thus formulas (55) and (59) in their combination envelope the entire shaded zone.

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The physical sense of formulas (52) to (59) consists in the facts that in the zone of the shade the propagation (with attenuation) of complex waves take place along the meridians in a direction running from the geometric boundary of the shade (See Fig. F, p. 80)

$$r \sin\theta = a, \quad \frac{\pi}{2} < \theta < \pi \qquad (60)$$

to the axis

$$\theta = \pi, \quad r \geqslant a. \qquad (61)$$

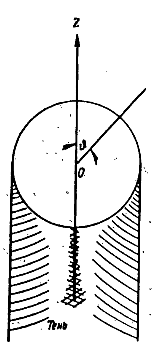

Fig. F.

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These are the first components of formulas (55) coinciding with P' and Q' according to formula (43). The complex waves on the axis (61) become focused and in order to calculate the potentials and fields near the axis theta = pi and on the axis itself, it is necessary to apply formulas (59). After focusing, the waves diverge again propagating (with attenuation) along the meridians toward the boundary of shade (60). The waves propagating in this direction correspond to the second components in formulas (55).

Thus, the break down of potentials, according to formula (33), corresponds to the separation (formation) of a field of direct waves which arrived at the given point of the shaded zone from its geometric boundary by the shortest way over the meridian. This field is determined by components P' and Q'. The components P" and Q" offer a field of waves which passed through the polar axis (61), i.e., passed at a given point over the very same meridian but from the opposite dirrection; since these waves have covered a much longer route, they normally appear weak in comparison with the direct waves. Only in the vicinity of axis (61), i.e., at theta = pi, do we have xi $\approx$ xi_1 $\approx$ M $\frac{pi}{2}$, and both waves are of one and the same order.

Near the geometric boundary of shade, at theta $\approx \frac{pi}{2}$, the field of direct waves is considerably stronger than the field of "polar" waves. This justifies the disregarding of P" and Q" type components in the theory of diffraction of radio waves around the surface of the earth where the

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parameter alpha acquires unusually high values and the investigation is, therefore, limited usually by the zone of the semi-shade. In our own case the components of the P'' and Q'' type cannot be disregarded; these components give precisely (see following paragraph) that complex structure of the radiation characteristics which was already mentioned before.

We will now investigate the fields in the illuminated zone. First of all, we will calculate P'' and Q'' by deductions in the very same points ν'_s and ν^0_s:

$$
\left.\begin{aligned}
kP'' &= \frac{\pi e^{i\frac{\pi}{4}}}{M^{\frac{9}{2}}}\, e^{i\alpha\frac{3\pi}{2}} \sqrt{\frac{\theta}{\sin\theta}} \sum_{s=1}^{\infty} e^{iM\frac{3\pi}{2}t'_s} J_0(\nu'_s\theta)\, \frac{w_1(t'_s - y)}{t'_s\,[w_1(t'_s)]^2}, \\
kQ'' &= \frac{\pi e^{i\frac{\pi}{4}}}{M^{\frac{9}{2}}}\, e^{i\alpha\frac{3\pi}{2}} \sqrt{\frac{\theta}{\sin\theta}} \sum_{s=1}^{\infty} e^{iM\frac{3\pi}{2}t''_s} J_0(\nu^0_s\theta)\, \frac{w_1(t''_s - y)}{[w'_1(t''_s)]^2},
\end{aligned}\right| \qquad (62)
$$

and we substitute the Lengendre function with its asymptotic expression

$$
P_{\nu-\frac{1}{2}}(\cos\theta) \approx \sqrt{\frac{\theta}{\sin\theta}}\, J_0(\nu\theta), \qquad (63)
$$

At the condition

$$
\alpha\theta \gg 1 \qquad (64)
$$

the series (62) can be written as follows

$$
\left.\begin{aligned}
kP'' &= \frac{1}{\alpha^2}\frac{1}{\sqrt{\sin\theta}}\left[e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)} V_1(\xi_1, y) - i e^{i\alpha\left(\frac{3\pi}{2}+\theta\right)} V_1(\xi_2, y)\right], \\
kQ' &= \frac{1}{\alpha^2}\frac{1}{\sqrt{\sin\theta}}\left[e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)} V_2(\xi_1, y) - i e^{i\alpha\left(\frac{3\pi}{2}+\theta\right)} V_2(\xi_2, y)\right].
\end{aligned}\right\} \qquad (65)
$$

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where

$$
\xi_2 = M\left(\frac{3\pi}{2} + \theta\right). \qquad (66)
$$

If we have a fulfilled condition

$$
M\theta \ll 1, \qquad (67)
$$

then P" and Q" can be calculated by formulas

$$
\left.\begin{aligned}
kP &= \\
&= \frac{1}{a^2}\, 2\sqrt{\pi}\, e^{-i\frac{\pi}{4}} M^{\frac{1}{2}} \sqrt{\frac{\theta}{\sin\theta}}\, J_0(\nu'_1\theta)\, e^{i\alpha\frac{3\pi}{2}}\, V_1\left(M\frac{3\pi}{2}, y\right), \\
kQ &= \\
&= \frac{1}{a^2}\, 2\sqrt{\pi}\, e^{-i\frac{\pi}{4}} M^{\frac{3}{2}} \sqrt{\frac{\theta}{\sin\theta}}\, J_0(\nu^0_1\theta)\, e^{i\alpha\frac{3\pi}{2}}\, V_2\left(M\frac{3\pi}{2}, y\right).
\end{aligned}\right\} \qquad (68)
$$

Formulas (65) and (68), including the entire illuminated zone, show that the additional field obtainable from potentials P" and Q" consists of two complex waves which completed (in opposite directions) "an element around the world" journey and have fallen (were caught) in the given point of the irradiated space, having passed over the meridian through the axis of the shade (61). On the "axis of the light"

$$
\theta = 0 \quad r > a \qquad (69)
$$

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all such waves traveling over the meridians become focused (formula (68)). Since these waves have covered more than one half of the circumference they experienced a strong weakening and give a small correction to the fields obtainable from P' and Q'.

In the illuminated zone the potentials P' and Q' produce the basic "direct" field originating during the falling of a plane wave on an ideally reflecting sphere. At a certain distance from the shade boundary (60) the electromagnetic field, determinable by P' and Q', should transform into a field determinable by the law of geometric optics. Near the shade boundary, in the semi-shade zone, the potential P' and Q' are determined by formulas (43). It can be shown that, during the departure from the shade boundary into the illuminated zone, the diffraction formulas derived for the semi-shade convert into negative formulas (formulas of geometric optics). This conversion was investigated by V. A. Fok (6) in a general form but we will not analyze it here.

Paragraph 5 – Calculating Formulas for Radiation Characteristics W, $W^{(1)}$, and $W^{(2)}$ at Greater Values of the Parameter Alpha

The formulas listed by us in the preceding paragraph allow the radiation characteristics of a spherical surface antenna to be calculated at greater alpha values. During the differentiation of the P and Q potentials it is necessary to retain only the main members.

We will write first the asymptotic expressions for complex characteristics W (theta), $W^{(1)}$ (theta) and $W^{(2)}$ (theta) in the shade zone, i. e., at $\frac{\pi}{2} < \theta < \pi$.

$$\left.\begin{aligned} W(\theta) &= \frac{e^{i\alpha\cos\theta}}{\sqrt{\sin\theta}}\left[e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}g(\xi)+i\,e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)}g(\xi_1)\right], \\ W(\theta) &= -2\sqrt{\pi}M^{\frac{3}{2}}e^{i\left(\alpha+\frac{1}{2}\right)\frac{\pi}{2}}e^{i\alpha\cos\theta}\sqrt{\frac{\pi-\theta}{\sin\theta}}\times \\ &\times J_1(\nu_1'(\pi-\theta))\,g\left(M\frac{\pi}{2}\right), \end{aligned}\right\} \quad (70)$$

where function g(xi) is determined as

$$g(\xi) = V_1(\xi,\ 0) = \frac{1}{\sqrt{\pi}}\int\frac{e^{i\xi t}}{w_1'(t)}\,dt. \quad (71)$$

For $W^{(1)}$ and $W^{(2)}$ we have more complex formulas

$$\left.\begin{aligned} W^{(1)}(\theta) &= U^{(1)}(\theta) - \frac{i}{\alpha\sin\theta}W(\theta), \\ W^{(2)}(\theta) &= U^{(2)}(\theta) - \frac{1}{\alpha M\sin\theta}U(\theta), \end{aligned}\right\} \quad (72)$$

where the auxiliary functions $U^{(1)}$, $U^{(2)}$ and U are determined by expressions

$$\left.\begin{aligned} U^{(1)}(\theta) &= \frac{i\,e^{i\alpha\cos\theta}}{M\sqrt{\sin\theta}}\left[e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}f(\xi)-i\,e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)}f(\xi_1)\right], \\ U^{(1)}(\theta) &= -2\sqrt{\pi}M^{\frac{1}{2}}e^{i\left(\alpha+\frac{1}{2}\right)\frac{\pi}{2}}e^{i\alpha\cos\theta}\sqrt{\frac{\pi-\theta}{\sin\theta}}\times \\ &\times J_0''(\nu_1^0(\pi-\theta))\,f\left(M\frac{\pi}{2}\right), \end{aligned}\right\} \quad (73)$$

$$\left.\begin{aligned} U^{(2)}(\theta) &= \frac{e^{i\alpha\cos\theta}}{\sqrt{\sin\theta}}\left[e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}g(\xi)-i\,e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)}g(\xi_1)\right], \\ U^{(2)}(\theta) &= -2\sqrt{\pi}M^{\frac{3}{2}}e^{i\left(\alpha-\frac{1}{2}\right)\frac{\pi}{2}}e^{i\alpha\cos\theta}\sqrt{\frac{\pi-\theta}{\sin\theta}}\times \\ & J_0''(\nu_1'(\pi-\theta))\,g\left(M\frac{\pi}{2}\right), \end{aligned}\right\} \quad (74)$$

$$\left.\begin{aligned} U(\theta) &= \frac{e^{i\alpha\cos\theta}}{\sqrt{\sin\theta}}\left[e^{i\alpha\left(\theta-\frac{\pi}{2}\right)}f(\xi)+i\,e^{i\alpha\left(\frac{3\pi}{2}-\theta\right)}f(\xi_1)\right], \\ U(\theta) &= -2\sqrt{\pi}M^{\frac{1}{2}}e^{i\left(\alpha+\frac{1}{2}\right)\frac{\pi}{2}}e^{i\alpha\cos\theta}\sqrt{\frac{\pi-\theta}{\sin\theta}}\times \\ & J_1(\nu_1^0(\pi-\theta))\,f\left(M\frac{\pi}{2}\right), \end{aligned}\right\} \quad (75)$$

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and the function f(xi) is equal

$$f(\xi) = \frac{dV_2(\xi, 0)}{dy} = \frac{1}{\sqrt{\pi}} \int_{\Gamma} \frac{e^{\xi t}}{w_1(t)} dt. \qquad (76)$$

At greater positive xi values the function f (xi) tends toward zero much faster than the g(xi) function. This justifies the inclusion, in the right side of the first of formulas (72) of the component $-\frac{1}{\alpha \sin \theta}$ W (theta): even though at xi ∼ 1 this component by its order of magnitude in M^2 times smaller than the first one, but, they gradually become equal farther in the shade and in the vicinity of the "dark pole" theta = 0, the second component, as is shown by calculations, appears to be the main one. In contrast to this, the component $-\frac{1}{\alpha M \sin \theta}$ U (theta) is of no material importance during the calculation of $W^{(2)}$ functions and is written out for symmetry. Formulas (72) lead to a relation (compare formulas (13):

$$W^{(1)}(\pi) = -W^{(2)}(\pi). \qquad (77)$$

We will now investigate the illuminated zone 0 < theta < $\frac{\pi}{2}$. Here, in accordance with formulas (33).

$$\left.\begin{aligned} W(\theta) &= W' + W'' \\ W^{(1)}(\theta) &= W^{(1)'} + W^{(1)''} \\ W^{(2)}(\theta) &= W^{(2)'} + W^{(2)''} \end{aligned}\right\} \qquad (78)$$

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For W", $W^{(1)"}$ and $W^{(2)"}$ it is possible to write, with the aid of formulas (65), the following approximate expressions:

$$\left.\begin{aligned} W'' &= i\,\frac{e^{i\alpha\left(\cos\theta + \frac{3\pi}{2} - \theta\right)}}{\sqrt{\sin\theta}}\, g(\xi_1), \\ W^{(1)''} &= \frac{e^{i\alpha\left(\cos\theta + \frac{3\pi}{2} - \theta\right)}}{M\sqrt{\sin\theta}}\, f(\xi_1), \\ W^{(2)''} &= -i\,\frac{e^{i\alpha\left(\cos\theta + \frac{3\pi}{2} - \theta\right)}}{\sqrt{\sin\theta}}\, g(\xi_1), \end{aligned}\right\} \qquad (79)$$

which offer a slight correction to the basic terms, but only at the vicinity of the shade boundary. According to formulas (65) the expressions in (79) will also include components proportional to f (xi_2) and g (xi_2), but these components, as well as the focusing of "round the world" waves near theta = 0 [formulas (68)], can be totally disregarded.

For the basic components in conformity with the Fok formulas (see above Paragraph 3, formulas (26) to (28) it is possible to write expressions

$$W' = \sin\theta\, G(\xi'), \quad W^{(1)'} = \frac{i}{M} F(\xi'), \quad W^{(2)'} = G(\xi'), \qquad (80)$$

where

$$\xi' = -M\cos\theta < 0, \qquad (81)$$

and the function F (xi) and G (xi) are determined by ratios

$$F(\xi) = e^{i\frac{\xi^3}{3}} f(\xi); \quad G(\xi) = e^{i\frac{\xi^3}{3}} g(\xi). \qquad (82)$$

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Near the equator, i. e., at theta $\approx \frac{pi}{2}$, formulas (79) and (80) give a smooth congruence with the radiation characteristics in the shade zone, for example, with formula (70). In the illuminated zone, at a distance from the equator, the variable xi' acquires greater negative values and for functions F and G we use the approximate formulas

$$F(\xi') \approx 2i\xi'; \quad G(\xi') = 2. \qquad (83)$$

Consequently, formulas (80) automatically produce a conversion toward geometric optics [formulas (25a)] . We want to call attention to the fact that formulas (80) include the variable xi' (81) and not the variable xi (45). The physical cause for this is that the "direct" field in the illuminated zone is created directly by the incident plane wave the phase of which is proportional to cos theta (or z), whereas in the shade zone (just as in the case of round-the-world waves in the irradiated hemisphere) the propagation takes place over the arc of the meridian and the corresponding dimensionless variables (45), (56) and (66) are proportional to the angles.

Due to the fact that in the previous paragraph we took into consideration the waves which passed through the "dark" pole theta = pi [in the formulas of this paragraph they have corresponding components proportional to f (xi_1) or g (xi_1)] , we obtained the possibility of offering a qualitative explanation of the different radiation characteristics discovered in paragraph 3. Namely, the oscillations in the shade zone which become amplified during approach to the dark pole, are explained by the interference of two waves propagating (with attenuation) in opposide directions and having equal amplitudes on the dark pole. The general

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increase in amplitude near the direction theta = pi is caused by focusing the waves propagating over the meridians. The increase in the number of oscillations with the increase in the parameter alpha takes place simply because the waves propagate in first approximation with the speed of light and in this way the meridians pile up a greater number of standing waves.

To what extent do the formulas which are given allow the radiation characteristics of spherical antennas to be calculated at greater values of the parameter alpha? In order to answer this question we compare (in Figure 28 to 30) the radiation characteristics for alpha = 10 according to accurate formulas of paragraph 1 and 2 (these curves are marked by the letter T) and by the asymptotic formulas quoted in this paragraph (these curves are marked by the letter A). In figure 28 to 30 the functions W (theta), $W^{(1)}$ (theta) and $W^{(2)}$ (theta) show that the asymptotic formulas for the value alpha = 10 give a perfect representation of the nature of the functions interesting us. Even from a quantitative viewpoint the asymptotic formulas offer a satisfactory result: the divergence between accurate and asymptotic curves do not exceed 15-20%.

The dotted line in Figure 29 also indicates the function $U^{(1)}$ (theta), i. e., [$U^{(1)}$ (theta)] and arc $U^{(1)}$ (theta). All calculations by asymptotic formulas were carried out with the aid of f (xi) and g (xi) function tables, and Bessel function tables of the complex variable [11] .

Using Figures 28 to 30 as a basis we can freely adapt the asymptotic formulas for the calculation of radiation characteristics of spherical

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antennas at alpha $>$ 10. The error produced by the asymptotic formulas, in their order of magnitude is equal to $\frac{1}{M_2}$ and, consequently, upon an increase in alpha it decreases quite rapidly.

Figures 6 to 9 show the radiation characteristics W (theta) for alpha = 15, 25, 50 and 100. In Figures 15 to 18 the radiation characteristics $W^{(1)}$ (theta), and in Figures 24 to 27 the radiation characteristics $W^{(2)}$ (theta) are shown for the very same alpha values.

At values as large as these of the alpha parameter we already clearly see that a change takes place in the radiation characteristics at alpha $\longrightarrow$ infinity: the conversion from light to shade becomes smooth and the zone, where oscillations and focusing take place, is forcibly displaced toward the dark pole and the oscillations become smaller and more frequent.

The broken curves in these diagrams are the "mean lines" of the oscillating radiation characteristics. The mean lines represent the absolute value and phase of function W', $W^{(1)'}$ and $W^{(2)'}$, respectively, and in the illuminated hemisphere these functions are determined by formulas (8). In the dark hemisphere they are derived from (43)-type formulas. These functions appear to be the "basic" components in the asymptotic expressions for the radiation characteristics, and the "additional" components are derived as result of the waves which bypassed the sphere through the dark pole. Without this additional component the oscillations are, obviously, not obtained but only a smooth mean line indicating the existence of focusing at theta $\longrightarrow$ pi. This mean line

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is presented by a dotted line.

The curves for the radiation characteristics, plotted at conditions (53) and (57), should interlock at

$$\text{pi} - \text{theta} \sim \frac{1}{M^2}$$

On the graphs, plotted in accordance with asymptotic formulas, the short sections of the broken curve give a continuation of the curves beyond the limits of their applicability and, in this way, show, the degree of the interlocking curve.

Paragraph 6 Resonances in a Spherical Antenna. Radiation Impendance

The graphs discussed above show the relation between the field of radiation of a spherical surface antenna and the angle theta at fixed values of the parameter alpha. It is also interesting to explain the relation between the parameter alpha, which is proportional to the operating frequency, and the radiation in a given direction.

The dependence of the absolute value of the function W upon the alpha parameter is shown in Figure 31. According to formulas (14) this function gives the radiation field of a spherical antenna excitable by the radial electric dipole situated on its surface. This dependence is depicted during the fixing of values of the angle theta, which are equal to 30°, 60°, 90°, 120°, 150°, whereby for the values 0 $<$ alpha $<$ 10 the calculations were carried out by the accurate formulas of paragraph 1 and 2, and by the asymptotic formulas of paragraph 5, for the values 10 $<$ alpha $<$ 15. Because of the difference in the calculation methods,

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the curves show a certain jump at alpha = 10 indicating the degree of approximation offered by the asymptotic formulas.

If we should examine the curve for any one given value theta we will find that it has the nature of a resonance curve - during an increase in alpha the resonance maxima are replaced by minima and (as it should have been expected from the physical considerations) the oscillations gradually become smoother and the resonance characteristics taper down to nothing. However, by making a comparison between the curves for various values of the angle theta we see that these resonance characteristics virtually are almost entirely illusory because the maxima and minima for different angles theta are oriented at different values alpha. The exception is constituted only by the first maximum oriented on all curves in approximately one and the same point, at alpha values of about alpha = 1. If this maximum should not be taken into consideration then it must be acknowledged that the curves in figure 31 do not indicate the resonance characteristics but the presence of interference.

This interference can be easily comprehended physically provided the sphere is considered as a receiving antenna on which a plane wave is falling. The fluctuations in the radiation amplitude characteristics (alpha = const. theta changes) and in the curves in figure 31 (theta = const., alpha changes) are due to one and the same cause, i. e., interference of the "direct" diffraction wave and the wave which bypassed the sphere through the dark pole. The nodes and antinodes of the "semi-standing" wave are displaced toward the pole theta = pi and the curves in 31 are formed when alpha increases.

For a final explanation of the problem regarding resonances it is necessary to calculate the total power P radiated by the spherical antenna or still better the ratio

$$\Gamma = \frac{P}{P_0}, \qquad (84)$$

where P_o is the total power radiated by the primary radiator in the absence of the sphere. If the primary radiator is an electric dipole then is can be written

$$\Gamma = \frac{R}{R_0}, \qquad (85)$$

where R_o is the radiation impedance of the dipole in free space and R is its radiation impedance in the presence of a sphere. If the primary radiator is a slot (as in figures B and C) then

$$\Gamma = \frac{G_0}{G}, \qquad (86)$$

where G_o is the radiation conductivity (bilateral) of the slot, cut in an infinite plane, and G is its radiation conductivity on the sphere.

If the sphere is excited by the radial electric dipole, situated on its surface, then

$$\Gamma = \frac{\int_0^\pi |W(\theta)|^2 \sin\theta \, d\theta}{\int_0^\pi \sin^3\theta \, d\theta} = \frac{3}{4} \int_0^\pi |V(\theta, \alpha)|^2 \sin\theta \, d\theta \qquad (87)$$

and by substituting the series (11b) instead of V (theta, alpha) we obtain

$$\Gamma = \frac{3}{\alpha^4} \sum_{n=1}^{\infty} \frac{n\left(n+\frac{1}{2}\right)(n+1)}{|\zeta_n'(\alpha)|^2} \qquad (88)$$

The results obtained in calculating the value Γ (G) by formula (88) are shown in figure 32. We see, that the curve has only one clearly expressed resonance maximum, namely, the first one. The remaining resonances are only slightly indicated by slight inflexions of the curves. When evaluating the curve in figure 32 we should keep in mind that the frequencies of the natural electric oscillations of the sphere (see [10], p. 489) correspond to the values alpha, equalling

$$\text{alpha}_1 = 0.86 - i\,0.50$$

$$\text{alpha}_2 = 1.81 - i\,0.70$$

$$\text{alpha}_3 = 2.77 - i\,0.83$$

The fact is that the first resonance on figure 32 takes place approximately at alpha = 0.86 and the two following are noticed at alpha $\approx$ 1.81 alpha $\approx$ 2.77.

When the sphere is excited by a magnetic dipole oriented on its surface then the electric dipoles, as well as the magnetic waves also become excited. Therefore, the value Γ can be represented in the form

$$\Gamma = \Gamma^{(1)} + \Gamma^{(2)}, \qquad (89)$$

where

$$\left.\begin{aligned} \Gamma^{(1)} &= \frac{3}{2\pi}\sum_{n=1}^{\infty}\frac{n+\frac{1}{2}}{|\zeta_n(\alpha)|^2} \\ \Gamma^{(2)} &= \frac{3}{2\pi}\sum_{n=1}^{\infty}\frac{n+\frac{1}{2}}{|\zeta_n'(\alpha)|^2} \end{aligned}\right\} \qquad (90)$$

The function Γ^1 and Γ^2 as well as their sum Γ are presented in figure 33 in relation to the variable alpha. Wee see that the oscillations in the radiation conductivity of the slot depend upon the STAT ments

Γ^2 the series of which are as series (88) which has $|\text{zeta}'_n(\text{alpha})|^2$ in the denominater. The resonance maxima at the slot are oriented approximately at a point where their presence appears at the electric dipole (figure 32). In this way the resonances at frequencies corresponding to the natural magnetic oscillations of the sphere do not take place at all. This is explained by the fact that the natural magnetic oscillations have attenuation that is too high (compare [10]).

When alpha $\longrightarrow$ infinity the value Γ in both instances shows a tendency toward 2. Physically, this can be easily understood since, as is known, the radiation impedance of a vertical dipole on a surface (which can be considered as a boundary case of a sphere at alpha = infinity) is doubled in comparison with the impedance in free space and the radiation conductivity of the slot should tend toward the "unilateral" conductivity of the slot cut in the surface, i.e., toward $\frac{1}{2}G_0$.

Paragraph 7. Radiation Characteristics of a Spherical Antenna Excitable by Complex Primary Sources.

During a theoretical investigation of the radiation characteristics of a spherical surface antenna we studied a sphere excitable by simple primary radiators - elementary electric or magnetic dipole.

Such an arrangement of the problem was due to the fact that we were interested primarily in how the sphere affects the radiation field and this can be explained quite well but only if the primary field has a simple form.

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In the case of nonelementary primary radiators, an entire series of radiation characteristics may appear not as a result of diffraction on the sphere but rather as a result of the complex nature of the primary irradiation.

In spite of all that the entire complex primary radiator can be broken down into a combination of ordinary ones and in this way (by addition or integration) the radiation characteristic of a spherical surface antenna may also be determined. By an addition of this type it is necessary to know not only the amplitude but also the phase characteristic. From these considerations not only the absolute value but also the phase of functions W, $W^{(1)}$ and $W^{(2)}$ are given in Figure 1 to 30. These drawings, for example, allow to find the radiation characteristics of a sphere excitable by a system of slots.

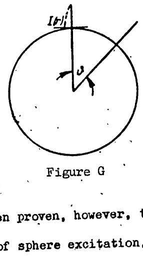

Figure G

Our results have often proven, however, to be directly applicable to real cases of sphere excitation. So not an elementary but a half-wave slot is cut in the sphere then at alpha values that are not too small the radiation characteristic should not be

noticeably affected by this change. It is exactly the same if the sphere is excited not by the elementary dipole but by a quarter-wave vibrator arranged on the sphere in a radial direction in such a manner that, together with its reflection, it forms a half-wave vibrator and the radiation characteristic of such a system should not differ much from the function W (theta).

Even though the latter assertions are almost apparent it would be desirable to verify them quantitatively on any given example. We shall discuss a case of a quarter-wave vibrator (see figure G) along which a current I (r) is distributed according to the law

$$I(r) = I_0 \cos k(r-a) \quad (\text{при} \quad a < r < b), \qquad (91)$$

Where I_0 is the current amplitude, and b is the radius-vector of the final point of the vibrator (dipole), whereby

$$b = a + \frac{\lambda}{4}. \qquad (92)$$

According to formulas (14) each element of the current I(r)dr of the radial vibrator gives a radiation field

$$E_\theta = H_\varphi = -\frac{i}{c} kI(r)\,dr\,\frac{e^{ikR_0}}{R_0} V(\theta, r), \qquad (93)$$

where, according to the first of the formulas (9), we have

$$V(\theta, r) = \frac{i}{\rho^2} \sum_{n=1}^{\infty} (2n+1)(-i)^n \frac{\zeta_n'(\alpha)\psi_n(\rho) - \psi_n'(\alpha)\zeta_n(\rho)}{\zeta_n'(\alpha)} P_n^{(1)}(\cos\theta). \qquad (94)$$

Here criteria (12) and (31) are introduced and R_o designates the distance of the observation point from the center of the sphere.

The overall radiation field of a quarter-wave vibrator is given by formula

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In spite of all that the entire complex primary radiator can be broken down into a combination of ordinary ones and in this way (by addition or integration) the radiation characteristic of a spherical surface antenna may also be determined. By an addition of this type it is necessary to know not only the amplitude but also the phase characteristic. From these considerations not only the absolute value but also the phase of functions W, $W^{(1)}$ and $W^{(2)}$ are given in Figure 1 to 30. These drawings, for example, allow to find the radiation characteristics of a sphere excitable by a system of slots.

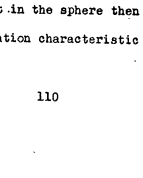

Figure G

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noticeably affected by this change. It is exactly the same if the sphere is excited not by the elementary dipole but by a quarter-wave vibrator arranged on the sphere in a radial direction in such a manner that, together with its reflection, it forms a half-wave vibrator and the radiation characteristic of such a system should not differ much from the function W (theta).

Even though the latter assertions are almost apparent it would be desirable to verify them quantitatively on any given example. We shall discuss a case of a quarter-wave vibrator (see figure G) along which a current I (r) is distributed according to the law

$$I(r)=I_0\cos k(r-a) \quad (\text{при } a<r<b), \qquad (91)$$

Where I_o is the current amplitude, and b is the radius-vector of the final point of the vibrator (dipole), whereby

$$b=a+\frac{\lambda}{4}. \qquad (92)$$

According to formulas (14) each element of the current I(r)dr of the radial vibrator gives a radiation field

$$E_\theta=H_\varphi=-\frac{i}{c}kI(r)\,dr\,\frac{e^{ikR_0}}{R_0}V(\theta,r), \qquad (93)$$

where, according to the first of the formulas (9), we have

$$V(\theta,r)= =\frac{i}{\rho^2}\sum_{n=1}^{\infty}(2n+1)(-i)^n\frac{\zeta_n'(\alpha)\psi_n(\rho)-\psi_n'(\alpha)\zeta_n(\rho)}{\zeta_n'(\alpha)}P_n^{(1)}(\cos\theta). \qquad (94)$$

Here criteria (12) and (31) are introduced and R_o designates the distance of the observation point from the center of the sphere.

The overall radiation field of a quarter-wave vibrator is given by formula

$$E_\theta = H_\varphi = -\frac{i}{c} I_0 \frac{e^{ikR_0}}{R_0} V(\theta), \qquad (95)$$

where

$$V(\theta) = k \int_a^b \cos k(r - a)\, V(\theta, r)\, dr. \qquad (96)$$

Substituting the series (94) in this expression and applying formulas of the type

$$\int \cos(\rho - \alpha)\, \zeta_n(\rho) \frac{d\rho}{\rho^2} = \frac{1}{n(n+1)} \left[\cos(\rho - \alpha)\, \zeta_n'(\rho) + \sin(\rho - \alpha)\, \zeta_n(\rho) \right]$$

we obtain a series for the function $\overline{V}$ (theta)

$$\overline{V}(\theta) = i \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (-i)^n \frac{\zeta_n'(\alpha)\, \psi_n(\beta) - \psi_n'(\alpha)\, \zeta_n(\beta)}{\zeta_n'(\alpha)} P_n^{(1)}(\cos\theta), \qquad (97)$$

where

$$\beta = kb = \alpha + \frac{\pi}{4}. \qquad (98)$$

In conformity with paragraph 2 it is natural to introduce a function

$$\overline{W}(\theta) = e^{i\alpha \cos\theta}\, \overline{V}(\theta), \qquad (99)$$

and the radiation field of a quarter-wave dipole can then be presented as follows

$$E_\theta = H_\varphi = -\frac{i}{c} I_0 \frac{e^{ikR}}{R} \overline{W}(\theta), \qquad (100)$$

where R designates the distance from the vibrator (dipole) base (point r = a, theta = o) to the point of observation.

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Figure 34 shows the results of calculating function $\overline{W}$ (theta) by formulas (97) to (99) at alpha = 5. For the purpose of comparison we plotted a dotted line which also represents the function W (theta) for the elementary dipole (broken curves repeat Figure 3). As is evident, the radiation characteristics $\overline{W}$ (theta) and W (theta) differ from each other only slightly. The arrangement of maxima and minima in these characteristics is perfectly analogous.

If we should turn to the directivity diagrams, with which one usually confines himself during experimental and theoretical studies of antennas, i.e., to reduce both amplitude characteristics to a common maximum, then the difference between both cases will be almost completely obliterated (eliminated).

During an increase in the parameter alpha, the difference between both cases should also decrease because the radiation characteristic of a quarter-wave dipole on a sphere in the illuminated semi-space $0 < \text{theta} < \frac{\text{pi}}{2}$ should approach that characteristic which it has in emptiness (vacuum). The latter is only slightly different from the characteristic of an elementary dipole. In the zone of shade and semi-shade the characteristic should convert into function W (theta), because the "dimensionless height" (39) of a quarter-wave dipole

$$y = \frac{\text{pi}}{2M}$$

tends toward zero during an increase in alpha.

On the basis of paragraph 5 it can be established that at large values of the alpha parameter not only this vibrator but an entire series of composite radiators are similar to elementary dipoles when the

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radiation characteristics are in the semi-shade and shade zones (where radiation is determined by diffraction on the sphere). In free space, as well as in the illuminated zone above the sphere, these composite radiators may have radiation characteristics different from dipole radiators.

Conclusions

The total results obtained allow a complete quantitative representation about the radiations of a spherical surface antenna to be prepared.

The presence of an ideally reflecting spherical surface may bring a radical change into the characteristic of a dipole or slot which excite the surface antenna. If the radius of the sphere is compared with the wavelength then its radiation characteristic has lobes the number of which increases with the reduction in wavelength.

The explanation on the origin of the lobes and other features of the radiation characteristics would be more realistic if the sphere is considered as a receiving antenna on which a plane wave is falling. The lobes can then be interpreted as the interference of a direct diffraction field excited by the incident wave with a "round-the-world" diffraction wave which by-passed through the "dark pole", being a point on the sphere oriented at a distance from its illuminated part. At sufficiently high values of the ratio $\frac{\text{alpha}}{\text{lambda}}$ the radiation characteristic has minute oscillations only in a small section (theta   pi) and acquires a simple form in the remaining part.

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A spherical antenna has almost no resonance characteristics. Exceptions are constituted only by values alpha   1 where resonance occurs near the natural frequency of the basic electric oscillation.

The radiation characteristics of a spherical antenna excited by composite primary sources pertain in their major part to the characteristics of a sphere excited by elementary dipoles.

The phenomena mentioned above should take place also in other more complex (compounds) surface antennas. In view of the fact that these phenomena can, to a large extent, change the directivity diagram (radiation pattern), they must be taken into consideration during practical utilization of antennas operating in conditions of electro-dynamic interaction with massive conductive bodies situated nearby.

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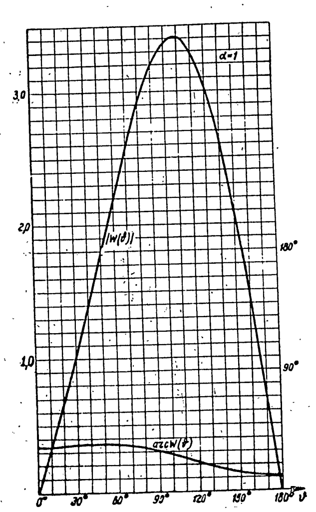

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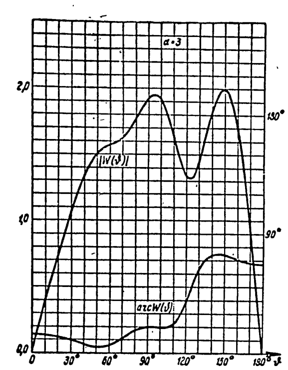

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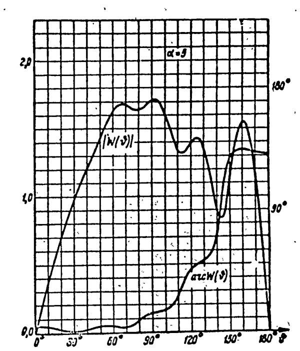

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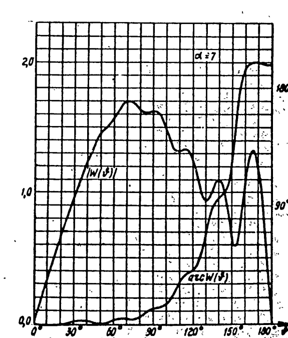

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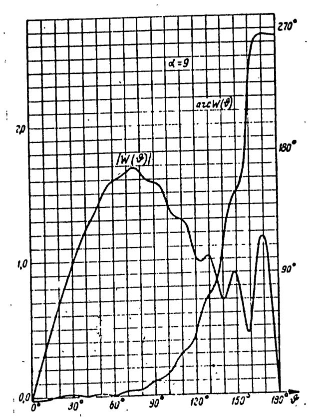

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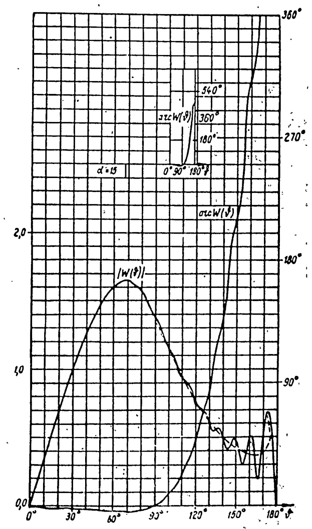

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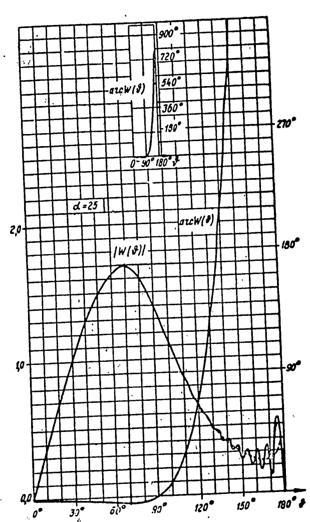

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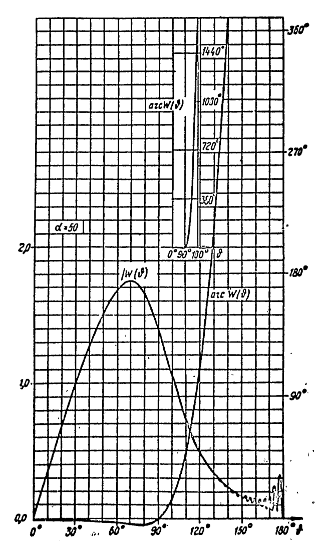

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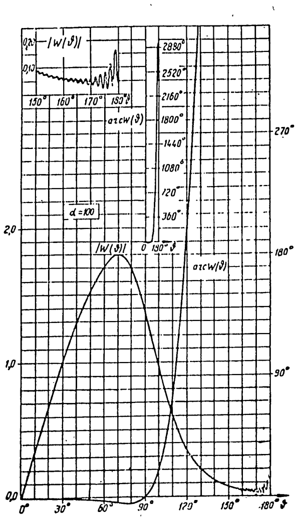

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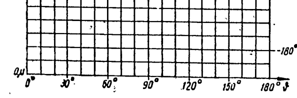

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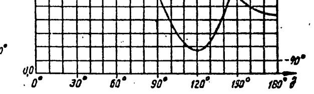

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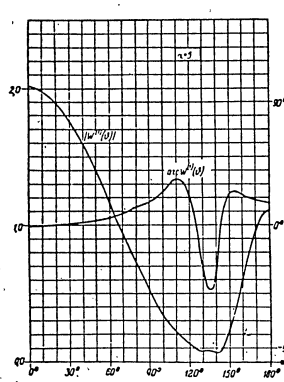

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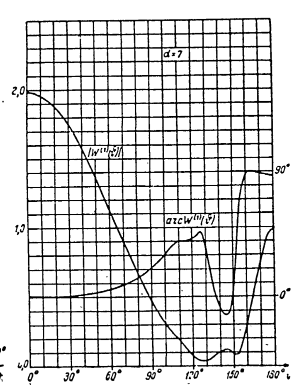

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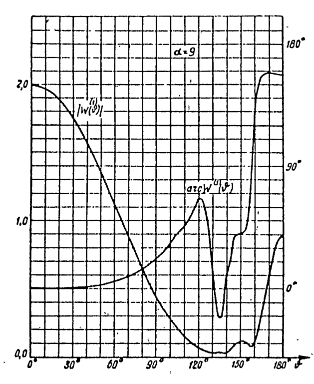

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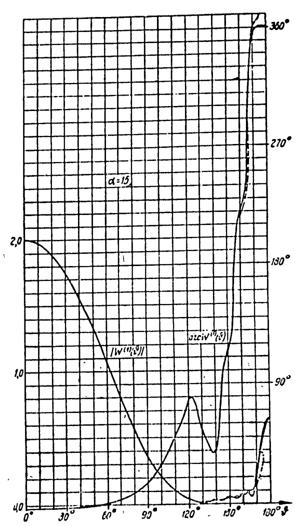

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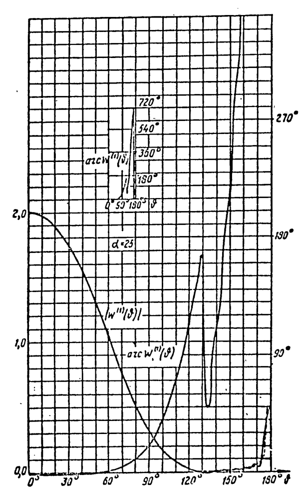

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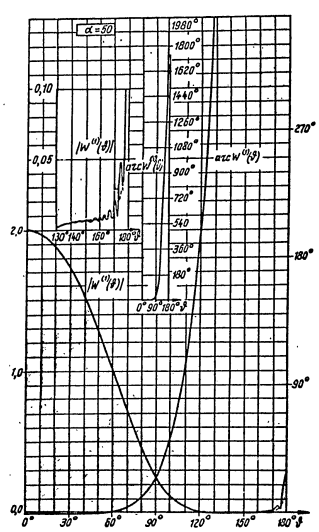

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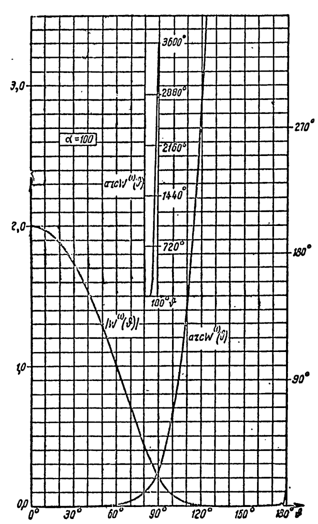

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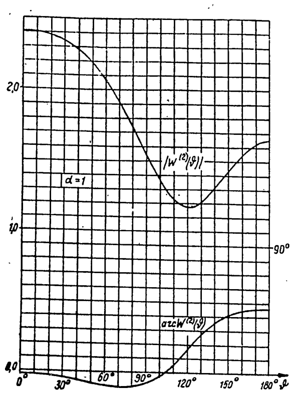

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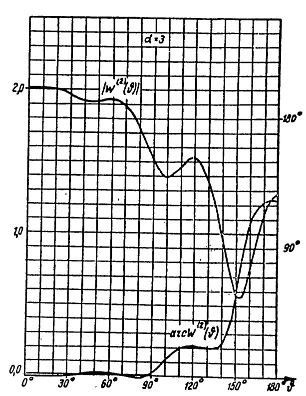

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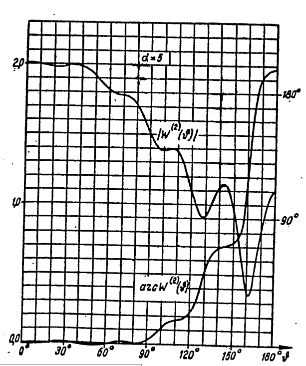

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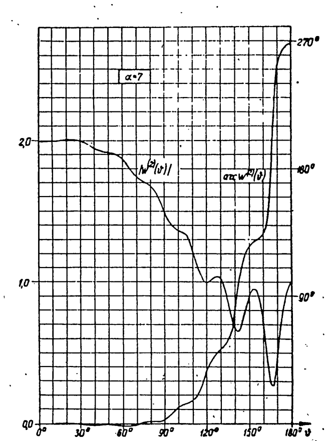

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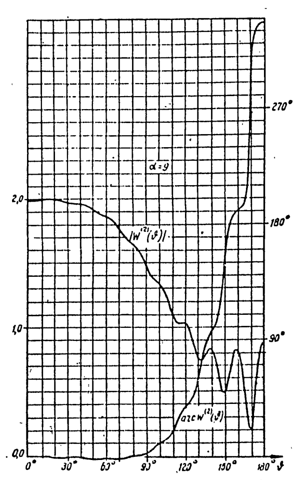

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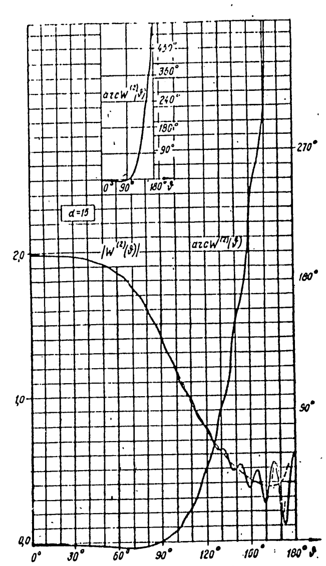

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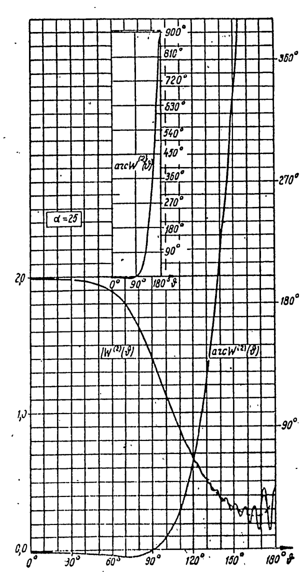

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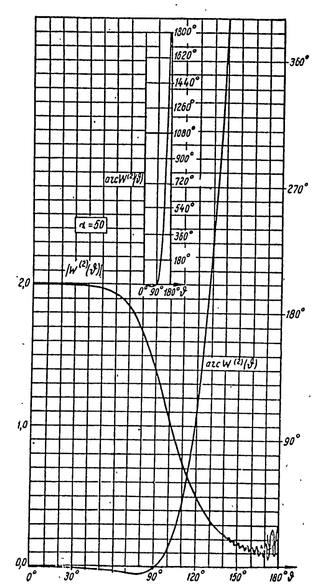

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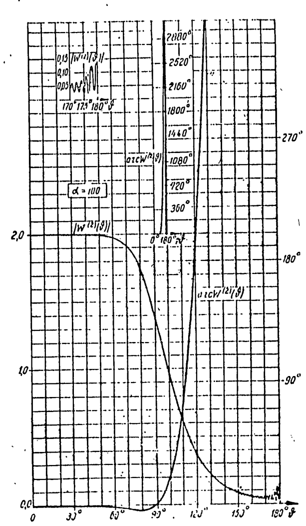

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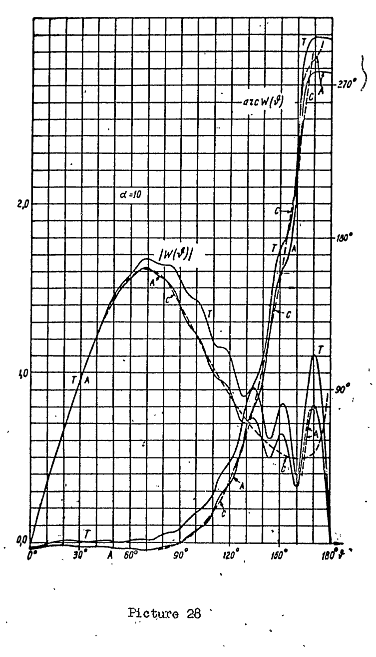

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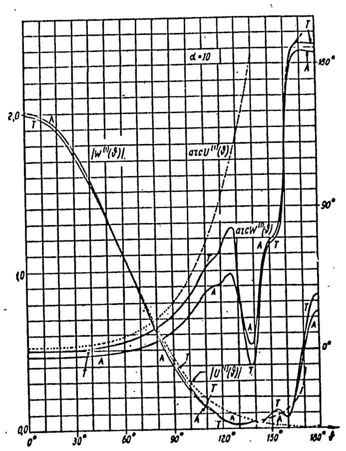

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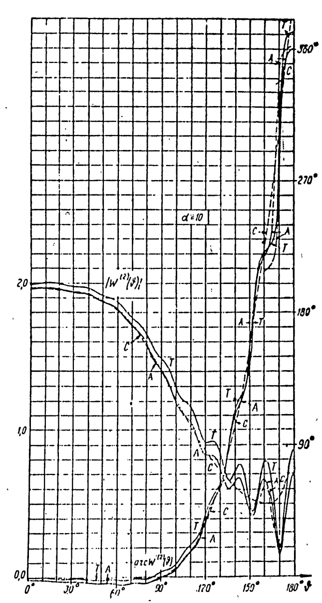

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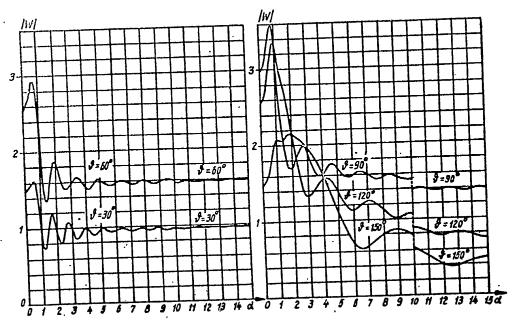

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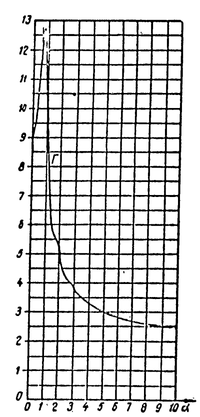

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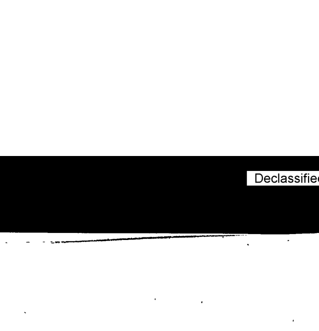

Picture 34

RADIATION CHARACTERISTICS OF AN ELONGATED ROTARY ELLIPSOID
by
M. G. Belkin

Par. 1. Wave Functions in Spheroidal Coordinates.

In this report, the problem of diffraction on an elongated ideally-conductive spheroid, excited by an electric dipole situated in any given point on the spheroid axis and directed along this axis, is discussed.

The problem is solved in a spheroidal system of coordinates xi, eta and phi connected with the Descartes coordinates x, y, z, by the ratios

$$\left.\begin{aligned} x &= f\sqrt{(\xi^2-1)(1-\eta^2)}\cos\varphi, \\ y &= f\sqrt{(\xi^2-1)(1-\eta^2)}\sin\varphi, \\ z &= f\xi\eta \end{aligned}\right\} \qquad (1.1)$$

and obtainable by rotating the elliptical system of coordinates about the greater axis of the ellipse family. The areas in this system of coordinates xi = const are elongated spheroids (elongated rotary ellipsoids). The coordinate xi changes from 1 to infinity and the coordinate eta changes from -1 to +1. The sign 2f designates the distance between the foci of the spheroid family xi = const. In particular, when xi = 1, we obtain an infinitely thin rod with a length of 2f. On the spheroid xi = const the value eta = 0 corresponds to the equator of the spheroid (its intersection with the plane z = 0), and the values eta = ± 1 are its poles oriented on the axis z when z = $\neq$ fxi. The metric coefficients of this system have the form of

$$h_{\xi} = f\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}; \quad h_{\eta} = f\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}; \quad h_{\varphi} = f\sqrt{(\xi^2-1)(1-\eta^2)}. \tag{1.2}$$

If one would look in the spheroidal coordinates for the function Pi (Russian letter P) satisfying the wave equation

$$\Delta\Pi + k^2\Pi = 0 \tag{1.3}$$

in the form of

$$\Pi = R(\xi)\,S(\eta)\,\frac{\cos m\varphi}{\sin m\varphi}, \tag{1.4}$$

then for R(xi) and S(xi) we would obtain equations

$$\left.\begin{aligned} &(\xi^2-1)\frac{d^2R}{d\xi^2} + 2\xi\frac{dR}{d\xi} + \left(\overline{A} + c^2\xi^2 - \frac{m^2}{\xi^2-1}\right)R = 0 \quad (\xi \geqslant 1), \\ &(1-\eta^2)\frac{d^2S}{d\eta^2} - 2\eta\frac{dS}{d\eta} - \left(\overline{A} + c^2\eta^2 + \frac{m^2}{1-\eta^2}\right)S = 0 \quad (-1 \leqslant \eta \leqslant 1), \end{aligned}\right\} \tag{1.5}$$

where the parameter

$$c = kf = \frac{2\pi f}{\lambda} \tag{1.6}$$

is proportional to the ratio in the distance between the foci of the ellipsoid family to the wave length.

Solutions of these problems were investigated in the report (1). We shall write out here these results of this report which will be necessary to us in further investigations.

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The angular functions S(eta) appear to be the functions

$$S^{(1)}_{m;l}(c, \eta) = \sum{}' d^{m,l}_n(c)\, P^m_{n+m}(\eta), \tag{1.7}$$

which are regular in points where eta = $\neq$ 1. Here $P^m_{n} \neq m$ (eta) are the associated Legendre functions, determined in such a manner that if

$$\eta = \cos\vartheta, \tag{1.8}$$

then

$$P^m_n(\cos\vartheta) = \sin^m\vartheta\,\frac{d^m P_n(\cos\vartheta)}{d(\cos\vartheta)^m}, \tag{1.9}$$

where P_n (cos ϑ) is the Legendre polynomial. The coefficients $d^{m,1}_n$ (C) are determined from the trinomial recurrent ratio

$$A^m_{n+2}d^{m,l}_{n+2} + B^{m,l}_n d^{m,l}_n + C^m_{n-2}d^m_{n-2} = 0, \tag{1.10}$$

which binds $d^{m,1}_n$ with the indices n of one and the same evenness. Consequently the coefficients $d^{m,1}_n$ differ from zero only when the indices n and l have one and the same evenness and in this connection a prime at the sign representing the sum in formula (1.7), and indicate everywhere below that the summation is carried out in accordance with indices n $\geqslant$ 0 of the very same evenness as l. The values A^m_n, $B^{m,1}_n$, C^m_n are fixed by formulas

$$\left.\begin{aligned} &A^m_n = \frac{(n+2m)(n+2m-1)}{(2n+2m+1)(2n+2m-1)}, \\ &B^{m,l}_n = \frac{2n^2+2n(2m+1)+2m-1}{(2n+2m+3)(2n+2m-1)} + \\ &\quad + \frac{n(n+2m+1)}{c^2} - \frac{b^m_l}{c^2} = \beta^m_n - \varkappa^m_l, \\ &C^m_n = \frac{(n+2)(n+1)}{(2n+2m+3)(2n+2m+1)}, \end{aligned}\right\} \tag{1.11}$$

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where

$$\varkappa_l^m = \frac{b_l^m}{c^2}, \qquad (1.12)$$

and b_l^m is connected with the constant $\bar{A}$ included in equation (1.5) by the ratio

$$b_l^m = -\bar{A}_l^m - m(m+1). \qquad (1.13)$$

The coefficients $d_n^{m,l}$ are standardized in such a way that

$$\left.\begin{aligned} S^{(1)}_{m,l}(c,\ 0) &= P^m_{l+m}(0) \quad (l - \text{четное}), \\ \left[\frac{d}{d\eta} S^{(1)}_{m,l}(c,\ \eta)\right]_{\eta=0} &= \left[\frac{d}{d\eta} P^m_{l+m}(\eta)\right]_{\eta=0} \quad (l - \text{нечетное}). \end{aligned}\right\} \qquad (1.14)$$

At a fixed value, m, the angular functions $S^{(1)}_{m,l}$ (C, eta) form in the interval (-1,1) an orthogonal system with a norm

$$N_{m,l}(c) = \int_{-1}^{1} [S^{(1)}_{m,l}(c,\ \eta)]^2 d\eta = 2\sum{}' (d_n^{m,l})^2 \frac{(n+2m)!}{(2n+2m+1)!\,n!}. \qquad (1.15)$$

For future reference it is convenient to introduce a designation

$$\sigma_{m,l}(c) = \lim_{\eta \to 1} \frac{S^{(1)}_{m,l}(c,\ \eta)}{(1-\eta^2)^{\frac{m}{2}}} = \frac{1}{2^m m!} \sum{}' d_n^{m,l} \frac{(n+2m)!}{n!}. \qquad (1.16)$$

In particular

$$\sigma_{0,l}(c) = \sum{}' d_n^{0,l} = S^{(1)}_{0,l}(c,\ 1). \qquad (1.17)$$

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The radial functions R(xi) of the wave equation are fixed by terms

$$R^{(1)}_{m,l}(c,\ \xi) = K_{m,l}(\xi^2-1)^{\frac{m}{2}}(c\xi)^{-m} \times \\ \times \sum{}' (-i)^{n-l} \frac{(n+2m)!}{n!} d_n^{m,l} j_{n+m}(c\xi), \qquad (1.18)$$

$$R^{(2)}_{m,l}(c,\ \xi) = K_{m,l}(\xi^2-1)^{\frac{m}{2}}(c\xi)^{-m} \times \\ \times \sum{}' (-i)^{n-l} \frac{(n+2m)!}{n!} d_n^{m,l} n_{n+m}(c\xi), \qquad (1.19)$$

where

$$K_{m,l} = \frac{r c^m}{2^m m!\, \sigma_{m,l}}, \qquad (1.20)$$

$$\left.\begin{aligned} j_k(c\xi) &= \sqrt{\frac{\pi}{2c\xi}}\, J_{k+\frac{1}{2}}(c\xi), \\ n_k(c\xi) &= \sqrt{\frac{\pi}{2c\xi}}\, N_{k+\frac{1}{2}}(c\xi), \end{aligned}\right\} \qquad (1.21)$$

and $J_{k \neq \frac{1}{2}}$ (cxi) and $N_{k \neq \frac{1}{2}}$ (cxi) are the Bessel functions of first and second order.

The functions $R^{(1)}_{m,l}$ (c, xi) and $R^{(2)}_{m,l}$ (c, xi) can also be presented in the form of

$$R^{(1)}_{m,l}(c,\ \xi) = \nu_{m,l} \sum{}'' d_n^{m,l}\, P^m_{n+m}(\xi), \qquad (1.22)$$

$$R^{(2)}_{m,l}(c,\ \xi) = \mu_{m,l} \sum{}'' d_n^{m,l}\, Q^m_{n+m}(\xi). \qquad (1.23)$$

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The double prime at the sign of the sum in formulas (1.22), (1.23) indicates that the summation is carried out from n = - infinity to n = infinity in accordance with indices n of the very same evenness as 1, and the coefficients $nu_{m,l}$ and $mu_{m,l}$ have the form of

$$\nu_{m,l} = \begin{cases} \dfrac{\pi c^{m} d_0^{m,l} (2m)! \left(\frac{l}{2}\right)!}{2^{2m+1}\left(m+\frac{1}{2}\right)\left(\frac{l-1}{2}+m\right)! \sum' d_n^{m,l} \frac{(n+2m)!}{n!}} & (l\text{-четное}) \\[2ex] \dfrac{\pi c^{m+1} d_1^{m,l} (2m+1)! \left(\frac{l-1}{2}\right)!}{2^{2m+3}\left(m+\frac{3}{2}\right)\left(\frac{l}{2}+m\right)! \sum' d_n^{m,l} \frac{(n+2m)!}{n!}} & (l\text{-нечетное}) \end{cases} \quad (1.24)$$

$$\mu_{m,l} = \begin{cases} \dfrac{2c^{m-1}\left(\frac{l-1}{2}+m\right)!}{\left(\frac{l}{2}\right)!\left(m-\frac{3}{2}\right)! d_{-2m}^{m,l} \sum' d_n^{m,l} \frac{(n+2m)!}{n!}} & (l\text{-четное}) \\[2ex] \dfrac{-8c^{m-2}\left(\frac{l}{2}+m\right)!}{\left(\frac{l-1}{2}\right)!\left(m-\frac{5}{2}\right)! d_{1-2m}^{m,l} \sum' d_n^{m,l} \frac{(n+2m)!}{n!}} & (l\text{-нечетное}) \end{cases} \quad (1.25)$$

The associated Legendre functions at xi $\geqslant$ 1 are determined in accordance with Hobson (2).

We wish to mention that during the calculations the representation $R^{(1)}_{m,l}$ is convenient to use in the form of (1.19), and $R^{(2)}_{m,l}$ - in the form of (1.23). It is immediately evident from these terms that $R^{(1)}_{m,l}$ (c, xi) does not have any characteristics whereas $R^{(2)}_{m,l}$ (c, xi) has a characteristic at xi $= \neq \pm 1$.

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The radiation condition on infinity is satisfied by the function

$$R^{(3)}_{m,l}(c,\xi) = R^{(1)}_{m,l}(c,\xi) + iR^{(2)}_{m,l}(c,\xi), \qquad (1.26)$$

which at cxi ⟶ infinity has an asymptotic representation

$$R^{(3)}_{m,l}(c,\xi) = (-i)^{l+m+1}\frac{e^{ic\xi}}{c\xi}. \qquad (1.27)$$

We select the time dependence in the form of $e^{-i \text{ omega } t}$). Finally, the Vronskiy determinant for radial functions $R^{(1)}_{m,l}$ (c, xi) and $R^{(3)}_{m,l}$ (c, xi) equals

$$R^{(1)}_{m,l}(c,\xi)\frac{d}{d\xi}R^{(3)}_{m,l}(c,\xi) - R^{(3)}_{m,l}(c,\xi)\frac{d}{d\xi}R^{(1)}_{m,l}(c,\xi) = \frac{i}{c}\frac{1}{\xi^2-1}. \qquad (1.28)$$

Par. 2. Decomposition of a Plane and Spherical Wave According to Spheroidal Functions.

Next we shall search for diffraction fields and their potentials in the form of decompositions (breakdowns) according to spheroidal wave functions. For this reason we shall first analyze the primary fields in accordance with these functions.

The decomposition of the plane wave

$$e^{-ikz} = e^{-ic\xi\eta} \qquad (2.1)$$

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should be sought in the form of

$$e^{-ikz} = \sum_{l=0}^{\infty} D_l R_{0,l}^{(1)}(c, \xi)\, S_{0,l}^{(1)}(c, \eta), \qquad (2.2)$$

because the function (2.1) does not depend upon φ and has no characteristic at xi = 1. Utilizing the orthogonality of the angular functions, their norm (1.15) as well as the known formula

$$e^{-ic\xi\eta} = \sum_{n=0}^{\infty} (-i)^n (2n+1)\, j_n(c\xi)\, P_n(\eta) \qquad (2.3)$$

(see for example (3), page 895), we will obtain

$$e^{-ikz} = e^{-ic\xi\eta} = 2\sum_{l=0}^{\infty} (-i)^l \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} R_{0,l}^{(1)}(c, \xi)\, S_{0,l}^{(1)}(c, \eta). \qquad (2.4)$$

During the decomposition (breakdown) of a spherical wave

$$\frac{e^{ikR'}}{R'}, \qquad (2.5)$$

originating from point xi = xi_1, eta = 1 ($z = z_1$, $x = y = 0$), it is necessary to distinguish two cases. At xi > xi_1 the decomposition should have the form of

$$\frac{e^{ikR'}}{R'} = \sum_{l=0}^{\infty} D_l'(\xi_1)\, R_{0,l}^{(3)}(c, \xi)\, S_{0,l}^{(1)}(c, \eta) \qquad (2.6)$$

because of the fact that at xi ⟶ infinity the condition of radiation into infinity should be fulfilled. At xi < xi_1 the decomposition (breakdown) should be sought in the form of

$$\frac{e^{ikR'}}{R'} = \sum_{l=0}^{\infty} D_l''(\xi_1)\, R_{0,l}^{(1)}(c, \xi)\, S_{0,l}^{(1)}(c, \eta), \qquad (2.7)$$

because it should be right at xi = 1, where the spherical wave (2.5) has no characteristics. It is necessary to assume in both cases that m = 0 because the term (2.5) does not depend upon φ.

In order to determine the coefficients D_1', we will assume that the values xi and cxi in the ratio (2.6) tend toward infinity. Then

$$R = \sqrt{r^2 + z^2} = f\sqrt{\xi^2 - \eta^2 + 1} \approx f\xi, \qquad (2.8)$$

$$R' = \sqrt{r^2 + (z - z_1)^2} \approx R - \frac{zz_1}{R} = R - z_1 \cos\vartheta, \qquad (2.9)$$

where R, ϑ, φ are spherical, and r, φ, z are cylindrical coordinates of the observation point. By converting the ratio (2.7) by means of formulas (2.8), (2.9) and (1.27), we can write:

$$\frac{e^{ikR}}{R} e^{-ic\xi_1\cos\vartheta} = \frac{e^{ikR}}{kR} \sum_{l=0}^{\infty} (-i)^{l+1} D_l'(\xi_1)\, S_{0,l}^{(1)}(c, \cos\vartheta) \qquad (2.10)$$

and finally, by comparing (2.10) with formula (2.4), in which eta = cos ϑ, we will obtain

$$D_l'(\xi_1) = 2ik \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} R_{0,l}^{(1)}(c, \xi_1). \qquad (2.11)$$

Thus for xi $>$ xi_1, the decomposition of the spherical wave has the form of

$$\frac{e^{ikR'}}{R'} = 2ik \sum_{l=0}^{\infty} \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} R_{0,l}^{(1)}(c, \xi_1) R_{0,l}^{(3)}(c, \xi) S_{0,l}^{(1)}(c, \eta). \qquad (2.12)$$

As we turn to the determination of the coefficients $D_1''(x_1)$, we wish to mention that at xi $=$ xi_1 and eta $\neq$ 1 the spherical wave (2.5) does not have a characteristic and the term (2.7) should coincide with (2.12). This condition, apparently, is fulfilled provided we write

$$D_l''(\xi_1) = 2ik \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} R_{0,l}^{(3)}(c, \xi_1), \qquad (2.13)$$

i. e., if for xi $<$ xi_1

$$\frac{e^{ikR'}}{R'} = 2ik \sum_{l=0}^{\infty} \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} R_{0,l}^{(3)}(c, \xi_1) R_{0,l}^{(1)}(c, \xi) S_{0,l}^{(1)}(c, \eta). \qquad (2.14)$$

For the purpose of verification we shall calculate also the fluctuation of the derivative in xi from the function $\frac{e^{ikR'}}{R'}$ during the passing through the surface of the spheroid xi $=$ xi_1. Applying the ratio (1.28) we will obtain

$$\left[\frac{\partial}{\partial \xi} \frac{e^{ikR'}}{R'}\right]_{\xi=+\xi_1} - \left[\frac{\partial}{\partial \xi} \frac{e^{ikR'}}{R'}\right]_{\xi=-\xi_1} = -\frac{2}{f(\xi_1^2 - 1)} \psi(\eta), \qquad (2.15)$$

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where

$$\psi(\eta) = \sum_{l=0}^{\infty} \frac{S_{0,l}^{(1)}(c, 1)}{N_{0,l}} S_{0,l}^{(1)}(c, \eta) = 2\delta(\eta - 1), \qquad (2.16)$$

because at any given value 1

$$\int_{1} \psi(\eta) S_{0,l}^{(1)}(c, \eta)\, d\eta = S_{0,l}^{(1)}(c, 1) \qquad (2.17)$$

and such decomposition coefficients in the complete orthogonal system $S_{0,1}^{(1)}$ (c, eta) are contained in the double delta-function at the point where eta $=$ 1.

Thus, $\frac{d}{dxi}$ $\frac{e^{ikR'}}{R'}$, and it indicates that also the normal derivative, differing from the xi derivative by the multiple

$$\frac{1}{h} = \frac{1}{f} \sqrt{\frac{\xi^2 - 1}{\xi^2 - \eta^2}}, \qquad (2.18)$$

when passing through the surface of the spheroid xi $=$ xi_1 is continuous everywhere except for point eta $=$ 1, where the normal derivative experiences a drop having the nature of a delta-function, a fact which was to be expected from physical considerations.

Finally, we will obtain the decomposition (breakdown) for the magnetic field of the vertical electric dipole

$$H_\varphi^0 = -k^2 p \frac{e^{ikR'}}{R'} \left(1 - \frac{1}{ikR'}\right) \sin\vartheta, \qquad (2.19)$$

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oriented in the point where xi = xi_1, eta = 1 and directed along the axis of the spheroidal system of coordinates (along the axis z). Since $H^o_r = H^o_\vartheta = 0$, indicating that

$$H^0_x = -H^0_\varphi \sin\varphi, \qquad (2.20)$$

and the Descartes component H^o_x satisfies the wave equation, then H^o_{phi} should be decomposed (broken down) according to spheroidal functions with an azimuthal index m = 1, whereby for xi > xi_1 it should be

$$-\frac{e^{ikR'}}{R'}\left(1 - \frac{1}{ikR'}\right)\sin\vartheta = \sum_{l=0}^{\infty} \tilde{D}_l(\xi_1)\, R^{(3)}_{1,l}(c, \xi)\, S^{(1)}_{1,l}(c, \eta). \qquad (2.21)$$

Assuming that in the term (2.3)

$$\eta = \cos\theta, \qquad (2.22)$$

then, by differentiating it by ϑ and consequent comparison with the term (2.21), in which it is assumed that xi ⟶ infinity, cxi ⟶ infinity, we will obtain the correlation

$$\sum_{l=0}^{\infty}(-i)^l \tilde{D}_l(\xi_1)\, S^{(1)}_{1,l}(c, \eta) = \frac{1}{i f \xi_1}\sum_{n=0}^{\infty}(-i)^n(2n+1) J_n(c\xi_1)\, P^1_n(\eta). \qquad (2.23)$$

Hence, by applying formulas (1.18), (1.20) and (1.16) we can determine the coefficients D_l.

The decomposition of the magnetic field H^o_{phi} for the case where xi > xi_1 is then obtained in the form of

$$H^0_\varphi = k^2 p \frac{4}{f\sqrt{\xi_1^2 - 1}} \sum_{l=0}^{\infty} \frac{\sigma_{1,l}}{N_{1,l}} R^{(1)}_{1,l}(c, \xi_1)\, R^{(3)}_{1,l}(c, \xi)\, S^{(1)}_{1,l}(c, \eta). \qquad (2.24)$$

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From the very same consideration as mentioned above, at xi < xi_1 one can write

$$H^0_\varphi = k^2 p \frac{4}{f\sqrt{\xi_1^2 - 1}} \sum_{l=0}^{\infty} \frac{\sigma_{1,l}}{N_{1,l}} R^{(3)}_{1,l}(c, \xi_1)\, R^{(1)}_{1,l}(c, \xi)\, S^{(1)}_{1,l}(c, \eta). \qquad (2.25)$$

Par. 3. Radiation Characteristics of a Spheroidal Surface Antenna.

We will consider a problem regarding symmetrical excitation of an ideally conductive spheroid xi = xi_0 with an elementary electric dipole oriented on the axis of the spheroid at the point xi = xi_1, eta = 1 and having moment p directed along the axis (see Fig. 1).

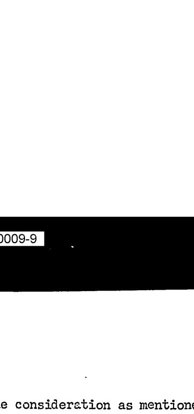

Figure 1

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The primary field of this dipole is given by formulas (2.19), (2.24) and (2.25). The secondary field H^1 should satisfy the condition of radiation into infinity, and in addition at $\xi = \xi_1$ this field does not have a characteristic. Therefore the decomposition of this field everywhere has the form of

$$H^1_\varphi = k^2 p \frac{4}{i\sqrt{\xi_1^2 - 1}} \sum_{l=0}^{\infty} A_l(\xi_1) R^{(3)}_{1,l}(c, \xi) S^{(1)}_{1,l}(c, \eta). \quad (3.1)$$

Since the magnetic field has only the H_φ component, the boundary condition will be

$$E_\eta \big|_{\xi=\xi_0} = -\frac{1}{ic\sqrt{\xi^2 - \eta^2}} \frac{\partial}{\partial \xi}\left[\sqrt{\xi^2 - 1}\, H_\varphi\right]_{\xi=\xi_0} = 0. \quad (3.2)$$

By applying it in the sum of fields (2.25) and (3.1), we obtain

$$A_l(\xi_1) = \left\{ -\frac{\sigma_{1,l}}{N_{1,l}} \frac{R^{(3)}_{1,l}(c, \xi_1) \frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(1)}_{1,l}(c, \xi)\right]}{\frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(3)}_{1,l}(c, \xi)\right]} \right\}_{\xi=\xi_0} \quad (3.3)$$

The complete field at $\xi > \xi_1$ is given by the sum of fields (2.24) and (3.1), consequently

$$H_\varphi = k^2 p \frac{4}{i\sqrt{\xi_1^2 - 1}} \sum_{l=0}^{\infty} F_l(\xi_1, \xi_0) R^{(3)}_{1,l}(c, \xi) S^{(1)}_{1,l}(c, \eta), \quad (3.4)$$

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where

$$F_l(\xi_1, \xi_0) = \\ = \frac{\sigma_{1,l}}{N_{1,l}} \left\{ R^{(1)}_{1,l}(c, \xi_1) - \frac{R^{(3)}_{1,l}(c, \xi_1) \frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(1)}_{1,l}(c, \xi)\right]}{\frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(3)}_{1,l}(c, \xi)\right]} \right\}_{\xi=\xi_0} \quad (3.5)$$

In particular, when the dipole is oriented on the very spheroid, i. e., if $\xi_1 = \xi_0$ then, as a result of (1.28), the formulas become simplified and we have

$$F_l(\xi_0, \xi_0) = \frac{i}{c\sqrt{\xi_0^2 - 1}} \frac{\sigma_{1,l}}{N_{1,l}\left\{\frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(3)}_{1,l}(c, \xi)\right]\right\}_{\xi=\xi_0}}. \quad (3.6)$$

Thus, at $\xi_1 = \xi_0$

$$H_\varphi = k^2 p \frac{4l}{ic(\xi_0^2 - 1)} \sum_{l=0}^{\infty} \frac{\sigma_{1,l} R^{(3)}_{1,l}(c, \xi) S^{(1)}_{1,l}(c, \eta)}{N_{1,l}\left\{\frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(3)}_{1,l}(c, \xi)\right]\right\}_{\xi=\xi_0}}. \quad (3.7)$$

Finally, in the wave zone, where the function $R^{(3)}_{1,l}$ can be substituted by its asymptotic representative (1.27) we can write

$$H_\varphi = -k^2 p \frac{e^{ikR}}{R} V(\vartheta), \quad (3.8)$$

where at $\xi_1 = \xi_0$ the function $V(\vartheta)$ is given by the term

$$V(\vartheta) = \frac{4i}{c^2(\xi_0^2 - 1)} \sum_{l=0}^{\infty} \frac{(-i)^l \sigma_{1,l} S^{(1)}_{1,l}(c, \cos\vartheta)}{N_{1,l}\left\{\frac{d}{d\xi}\left[\sqrt{\xi^2 - 1}\, R^{(3)}_{1,l}(c, \xi)\right]\right\}_{\xi=\xi_0}}. \quad (3.9)$$

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The function V (ϑ) we also call the (complex) radiation characteristic of an elongated spheroidal surface antenna.

For the case where c $\ll$ 1, i. e., for the case of very long waves, formula (3.9) acquires the form of

$$V(\vartheta)=\frac{\sin\vartheta}{(\xi_0^2-1)\left(\frac{\xi_0}{2}\ln\frac{\xi_0+1}{\xi_0-1}-1\right)}=g(\xi_0)\sin\vartheta, \qquad (3.10)$$

i. e., it represents a radiation characteristic of a dipole in a free space (sinusoid), multiplied by the coefficient g (xi_0), determinable by the form of the spheroid.

Par. 4. Results of calculations.

The radiation characteristic of an elongated spheroidal antenna depends upon two parameters: c, proportional to the ratio of the focus distance of the spheroid to the wave length of the radiator (emitter)

$$c=kf=\frac{2\pi f}{\lambda} \qquad (4.1)$$

and xi_0 bound with the ratio of the spheroid semi-axes a and b (alpha > b) by formula

$$\frac{1}{\xi_0}=\sqrt{1-\left(\frac{b}{a}\right)^2}. \qquad (4.2)$$

[redacted]

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These parameters are included in all formulas and should therefore be accepted as basic. However, during the evaluation of calculation results together with the basic parameters, we will also always have in mind the parameters ka and kb, connected with c and xi_0 formulas

$$\left.\begin{array}{l} ka=c\xi_0, \\ kb=c\sqrt{\xi_0^2-1}. \end{array}\right\} \qquad (4.3)$$

These parameters are interesting in the fact that, if f $\rightarrow$ 0, and $xi_0 \rightarrow$ infinity, the product fxi_0 remains constant, i. e., if the spheroid passes over into the sphere of radius rho = fxi_0, both parameters (4.3) convert into krho, which is an ordinary parameter of the sphere.

The calculations were carried out in accordance with formula (3.9) for all pairs (sets) of parameters

$$c=0{,}9801;\ 3;\ 5;\ 7 \qquad (4.4)$$

and

$$\xi_0=1{,}000801;\ 1{,}005037;\ 1{,}02;\ 1{,}154700;\ 1{,}341641 \qquad (4.5)$$

or respectively

$$\frac{a}{b}=25;\ 10;\ 5{,}07;\ 2;\ 1{,}5. \qquad (4.6)$$

Furthermore, according to formula (3.10), we obtained the characteristics for c = 0 and for the very same values of the parameter xi_0 (or $\frac{a}{b}$).

The calculation results are given in Fig. 2, wherein the polar system of coordinates represents the directivity diagrams, i. e., the [redacted]tion characteristic of a spheroidal surface [redacted]e maximum. All diagrams oriented on one line

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correspond to one and the same value of parameter c, marked on the left. All directivity diagrams oriented in the column under the ellipsoid correspond to the form of the rotation ellipsoid represented in the upper line where the ratio of its semiaxes $\frac{a}{b}$ as well as the value of the parameter xi_0 are also indicated.

Near each curve the corresponding values of the parameters ka and kb are also indicated.

The radiation characteristic is obtained from the directivity diagrams by multiplying by the value max $|V(\vartheta)|$. An idea about the characteristic can be obtained, by simultaneously studying Fig. 2 and the graph showing the dependence of max $|V(\vartheta)|$ upon the parameter c during the fixing of ratios of the spheroid semiaxes $\frac{a}{b}$ (Fig. 3, a) or upon the ratio of the semiaxes $\frac{a}{b}$ during the fixing of values of the parameter c (Fig. 3, b). It is immediately evident from Fig. 3, b that, at any given value of the parameter c, the amplitude maximum of the field radiated by the antenna increases sharply during an increase in the ratio of the semiaxes $\frac{a}{b}$.

A study of Fig. 2 shows above all that the number of lobes increases with the increase in the parameter ka = cxi_0. An analogous phenomenon was observed also in the case of a spherical antenna during the increase in parameter alpha = krho, where rho is the radius of the sphere (see [4] Fig. 1 - 5). Together with the curves for $\frac{a}{b}$ = 1.5 are also given (by dotted line) the directivity diagrams for the sphere, corresponding to the parameter alpha = krho = c (ditto Fig. 1-4).

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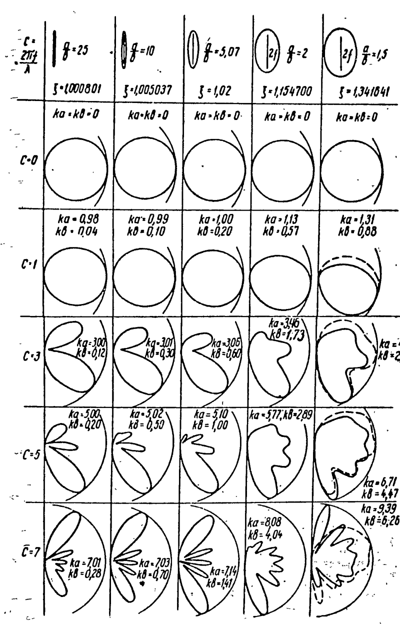

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Figure 2

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Thus, we compare an ellipsoid having a focus space 2f and a sphere of the radius f. As is evident from a comparison of the curves, oriented on one line, Fig. 2 (c = const), with the reduction in the elongation of the ellipsoid, its directivity diagram approaches the diagram of the sphere, which could have been expected from physical considerations. This approximation however depends upon the parameter c (the greater the parameter, c, the slower the approximation). And so, for c = 7 and $\frac{a}{b} = 1.5$, the characteristics of the sphere and ellipsoid have almost nothing in common, while at $c \leqslant 5$ and an identical ratio of semiaxes, the characteristics of the sphere and ellipsoid are qualitatively quite close to each other.

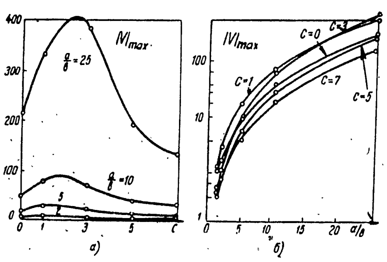

Figure 3

A characteristic feature of an elongated spheroidal antenna appears to be a strong radiation backward*. In this case, each consecutive lobe in the entire series of characteristics appears to be stronger than the preceding one and the lobes are separated from each other by deep minima, which reminds one of the characteristics of a traveling wave antenna. Particularly clearly expressed is this phenomenon for c = 5 and also at c = 1, when the radiation diagram is little different from the sinusoid (c = 0); the directivity phenomenon in the rear semi-space in the case of $\frac{a}{b} = 1.5$ is already clearly noticeable.

For the purpose of comparison we formulated the characteristics of a traveling wave antenna, consisting of a section of thin wire with a length equal to the greater axis of the spheroid. It was assumed thereat that the current wave propagates along this wire with the speed of light in the direction of the negative axis z (i. e., from the pole in which the exciting dipole is situated to the opposite end of the spheroid).

*From the viewpoint of geometric optics, the entire space is divided into two semi-spaces: the illuminated ($0 < \vartheta < \frac{\pi}{2}$) and shaded ($\frac{\pi}{2} < \vartheta < \pi$) (see Fig. 4). The backwards direction is called the direction leading toward the shaded semispace.

Figure 4

This simple model offers a perfectly identical arrangement of lobes as the spheroidal antenna with the exception that its lobes increase more rapidly with the increase in the angle ϑ.

For these cases we also computed the radiation characteristics of a standing current wave, under the assumption that in the section of wire mentioned above $-a < z < a$, the traveling wave of the current is reflected with the coefficient -1 from the end $z = -a$. The characteristic of such an antenna is symmetrical relative to the equatorial plane $\vartheta = \frac{\pi}{2}$.

A comparison of the characteristics for the values $c = 3$ and $c = 5$ is shown in Figures 5 and 6 which also bring the directivity diagrams of the spheroid for the ratios $\frac{a}{b} = 25$; 10; 5.07 and the diagrams of the traveling (Russian abbrev. Beg) as well as standing (Russian abbrev. St) waves of the current. A study of the drawings shows that the curves for the spheroid occupy an intermediate position between the curves for the traveling and standing wave, whereby an increase in the elongation of the spheroid is followed by a change in the directivity

diagrams tending toward the traveling wave. We wish to mention that the parameter, ka, for the given value, c, remains practically unchanged ($ka = 3.00 - 3.06$ for $c = 3$ and $kc = 5.00 - 5.10$ for $c = 5$), whereas the parameter kb changes considerably ($kb = 0.12 - 0.60$ for $c = 3$ and $kb = 0.20 - 1.00$ for $c = 5$). Thus, during a reduction in the parameter, kb, the directivity diagram draws closer to the diagram of the standing wave, and, during an increase in kb, approaches the diagram of the traveling wave.

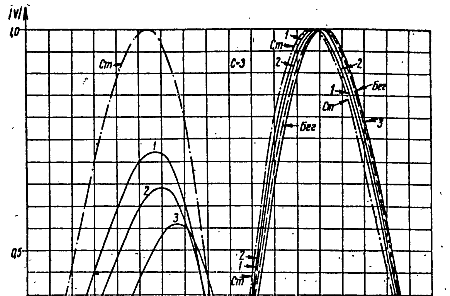

Figure 5

The indicated analogy between the directivity diagrams of the spheroid on one hand and the standing or traveling wave on the other hand does not extend to the phase characteristics (which we will not discuss further here). Namely, if we should calculate the radiation from a semi-standing current wave, then, by proper selection of the standing wave coefficient, it will be possible to attain an accurate conformity with the amplitude charateristics of the spheroid, as given, e. g., in Figures 5 and 6; however, as shown by calculations, the phase characteristics of such a semi-standing wave are much different from the phase characteristics of the spheroid.

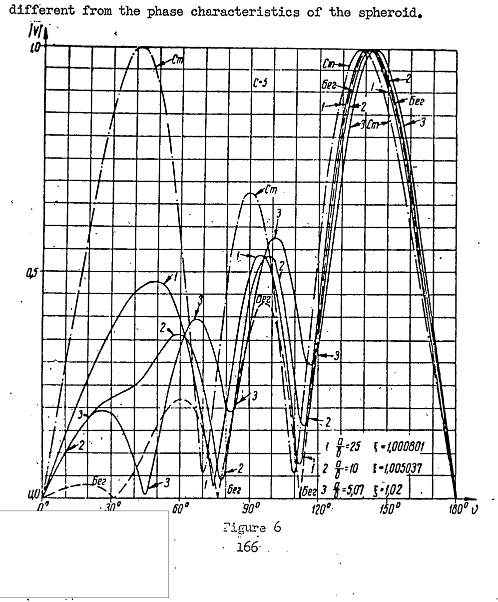

Figure 6

In the case where c = 7, the radiation characteristics have a somewhat special character sometimes very reminiscent of the characteristics of the standing wave ($\frac{a}{b}$ = 10; 5.07). It appears that, together with the previous picture of radiation into the backward semispace, we have here also the screening characteristics of the ellipsoid, i. e., a darkening of the rear semispace. We wish to mention that the latter effect in pure form practically does not appear for the cases investigated by us. An exception is the curve for c = 7 and $\frac{a}{b}$ = 1.5 (lower right angle), for which the parameters ka = 9.39 and kb = 6.26 appear to be the very highest of all these investigated by us.

An idea about the picture corresponding to the very high values of these parameters is offered by the V. A. Fok theory (5). This theory allows one to calculate the field originating during the incidence of a plane wave on any arbitrary convex body provided the radius of curvature, rho, of the surface section of the body is great in the plane of wave incidence in comparison with the wave length and distance from the body on which the field is investigated.

According to the principle of reciprocity, the field in the distant zone in direction ϑ is (for our case) equivalent to the field in the pole of the ellipsoid produced by the plane wave falling at an angle $\pi - \vartheta$.

According to V. A. Fok, the field in the zone of light can be, under the conditions enumerated above, sufficiently well transmitted by geometric optics, and in the field of semi-shade (directions close to $\vartheta = 90^\circ$) it decreases rapidly and smoothly whereby the rate of reduction depends only upon the parameter, krho.

The radius of curvature of the rotation ellipsoid in its pole is rho $= \frac{b^2}{a}$ and it can be considered as changing but little up to direction ϑ of the order of 120° whereupon, i. e., already in the shaded zone, it begins to vary considerably. But at a greater krho, the field in the zone of the shade is very small and is of no interest.

Thus, we can say that, at very great values of the parameter krho $= k \frac{b^2}{a}$, the field of the elongated spheroid in directions ϑ, not exceeding approximately 120°, is the same as the field of the sphere with a radius rho $= \frac{b^2}{a}$. This field was calculated in a previous report of this collection, where Figures 6 - 9 show the radiation characteristics of the sphere for values of parameters alpha = krho = 15, 25, 50, and 100. In this report, on Figure 28 there is shown how the curve calculated in accordance with the V. A. Fok theory does not reproduce the nature of reduction of the radiation characteristic and gives only the mean line of the oscillating curve. It is evident therefrom that even for the very greatest of the investigated values of the parameter krho $= k \frac{b^2}{a} = 4.2$ ($c = 7$, $\frac{a}{b} = 1.5$), the picture obtainable by the V. A. Fok theory for very great krho values, is still considered as unrealizable. In order to obtain such a picture, even

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if only in general outlines, it is necessary to reach values of the parameter krho ~ 10, for which even at $\frac{a}{b} = 1.5$ it is necessary to take $c \approx 16$. At such a value of the parameter c in the (3.9) series, it would have been necessary to calculate approximately 40 members terms.

Upon completion of this report, reports have appeared by (6) and (7) devoted to the symmetrical problem for a rotation ellipsoid. Calculation results are available only in the second report, namely, for the electric dipole oriented in the very pole of the ellipsoid but for a much narrower zone of parameter changes. Wherever the results of report (7) and these of our own article can be compared, they do coincide.

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DIFFRACTION OF ELECTROMAGNETIC WAVES BY A DISK

by
M. G. Belkina

Par. 1 Radiation Characteristics of an Oblate Spheroid and Disk during Their Excitation with a Vertical Electric Dipole.

The solution of the problem concerning the diffraction on an oblate rotation ellipsoid can be obtained in the following manner.

We will analyze a system of coordinates xi, eta, phi, connected with the Descartes system by ratios

$$\left.\begin{aligned} x &= f\sqrt{(\xi^2+1)(1-\eta^2)}\cos\varphi, \\ y &= f\sqrt{(\xi^2+1)(1-\eta^2)}\sin\varphi, \\ z &= f\xi\eta. \end{aligned}\right\} \qquad (1.1)$$

This system is obtained by rotating the elliptical system of coordinates about the small axis of a family of ellipses, and we shall call it the oblate spheroidal system of coordinates. In this case xi varies from 0 to infinity, and eta varies from -1 to $\neq$ 1, the coordinate surface xi = const is represented by an oblate spheroid, xi = 0 is an infinitely thin disk with the radius f, and 2f is the distance between the foci of the meridional cross section of the spheroid family. The value eta = 0 corresponds to the equators of the spheroids, and eta = $\neq$ 1 corresponds to their poles.

Substitution

$$\left.\begin{aligned} \xi &\to -i\xi', \\ \eta &\to \eta', \\ f &\to if' \end{aligned}\right\} \qquad (1.2)$$

formally transforms the system (1.1) into an elongated spheroidal system of coordinates (see (1), Par. 1)

$$\left.\begin{aligned} x &= f'\sqrt{(\xi'^2-1)(1-\eta'^2)}\cos\varphi, \\ y &= f'\sqrt{(\xi'^2-1)(1-\eta'^2)}\sin\varphi, \\ z &= f'\xi'\eta', \end{aligned}\right\} \qquad (1.3)$$

and the wave equation in the system (1.1) - into the wave equation in system (1.3). In this case the parameter c = kf, included in the wave equation, transforms into ic'. Consequently, any solution of the wave equation in an elongated spheroidal system of coordinates (1.3), and any formula equitable in that system convert into a solution of the wave equation in the oblate spheroidal system (1.1) or into a formula equitable in this latter during the substitution

$$\left.\begin{aligned} \xi &\to i\xi \\ \eta &\to \eta \\ c &\to -ic \end{aligned}\right\} \qquad (1.4)$$

Further we will make use of all the formulas of report (1), by changing (1.4) in these formulas. This change will be implied in references.

And so, by changing (1.4) in formula (3.9) of report (1), we obtain the radiation characteristics of an oblate spheroid xi = xi_0, excitable by a vertical electric dipole oriented in the pole of the spheroid (Fig. 1).

$$V(\vartheta) = \frac{4i}{c^2(\xi_0^2+1)} \sum_{l=0}^{\infty} \frac{(-i)^l \sigma_{1,l}(-ic)\, S_{1,l}^{(1)}(-ic, \cos\vartheta)}{N_{1,l}(-ic) \left\{ \frac{d}{d\xi} \left[\sqrt{\xi^2+1}\, R_{1,l}^{(3)}(-ic, i\xi) \right] \right\}_{\xi=\xi_0}} \qquad (1.5)$$

When $\xi_0 \longrightarrow 0$ the oblate spheroid changes into a disk with radius f. By changing to the limit at $\xi_0 \longrightarrow 0$ in formula (1.5), we obtain the radiation characteristics of an infinitely thin and ideally conductive disk, excitable by a vertical electric dipole oriented in its center (Fig. 2)

$$V(\vartheta) = \frac{4i}{c^2}\sum_{l=0}^{\infty} \frac{(-i)^l \sigma_{1,l}(-ic)\, S_{1,l}^{(1)}(-ic,\eta)}{N_{1,l}(-ic)\left[\frac{d}{d\xi} R_{1,l}^{(3)}(-ic, i\xi)\right]_{\xi=0}}. \quad (1.6)$$

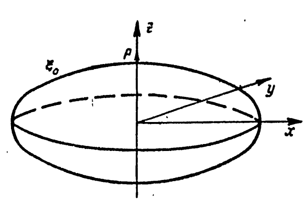

Figure 1

It can be shown that at any given value of the parameter c for the vertical electric dipole on the disk takes place

$$V\left(\frac{\pi}{2}\right) = 1. \quad (1.7)$$

Physically this is explained by the fact that in the direction of $\vartheta = \frac{\pi}{2}$ the currents on the disk, because of their symmetry, do not radiate.

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At $c \ll 1$ for the oblate spheroidal antenna, we obtain from formula (3.10) of report (1)

$$V(\vartheta) = \frac{\sin\vartheta}{(\xi_0^2+1)(-\xi_0 \operatorname{arc\,ctg} \xi_0 + 1)} = g(i\xi_0)\sin\vartheta. \quad (1.8)$$

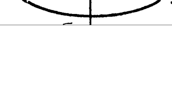
If $\xi_0 \longrightarrow 0$, then

$$g(i0) = 1, \quad (1.9)$$

and for the disk at $c \ll 1$ takes place

$$V(\vartheta) = \sin\vartheta. \quad (1.10)$$

Consequently, the radiation characteristic at $c \ll 1$ for the oblate spheroid and for the disk (as well as for the elongated spheroid, see (1) par. 3), represents a sine curve (sinusoid), i. e., the radiation characteristic of the dipole in free space, multiplied by the coefficient which depends upon the eccentricity of the spheroid and is equal to one for the disk. Thus, in a special (extreme) case of infinitely long waves, the presence of the disk does not affect the radiation characteristic of the vertical electric dipole oriented in the center of the disk.



We made calculations for values of the parameter c = 1, 3, 5 according to formula (1.6). The results of these calculations are given in Fig. 3, whereby for each value, c, the amplitude radiation characteristic

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of the disk $|V(\vartheta)|$ is given (continuous lines). The dotted lines represent the radiation characteristic of a sphere, excitable by a vertical electric dipole, oriented on the surface of the sphere, for values of the parameter alpha = krho = c, where the radius of the sphere rho is taken as equal to the radius of the disk f (see (2), Figures 1 – 3).

Figure 3

It is evident from Fig. 3 that, for c = 1, the radiation characteristic of the dipole on the disk is practically the same as in the case of the dipole in free space (or on the disk at c = 0). When c = 3, the radiation characteristic is already divided into two lobes whereupon the rear lobe is approximately one-third the size of the forward lobe. When c = 5, the radiation characteristic also consists of two lobes. But the shading phenomenon here is already quite clearly expressed, and a smooth conversion from the illuminated into the shaded zone takes place.

Comparing with the radiation characteristics of a sphere, we see that for alpha = 3, the sphere does not quite produce a shading, and STAT

for alpha = 5, this phenomonon is expressed very poorly. The smooth conversion (without oscillations) from the illuminated zone into the shaded appears in the sphere only at values alpha $\sim$ 15 (|2|, Fig. 6).

Thus, at one and the very same (investigated by us here) values of the parameters c and alpha, proportional to the ratio of the body radius to the wave length, the disk gives a considerably greater shading than the sphere. It should be mentioned however, that the problem of comparative effectiveness of radiation shading by the disk and sphere at greater values of c and alpha, requires additional investigation.

The oblate spheroid should occupy an intermediate position between the disk and sphere. The calculation of its radiation characteristics during excitation with an axial electric dipole (formula 1.5) does not represent any great difficulties.

Par. 2 – Horizontal Magnetic Dipole on a Disk.

The conductive surface can become excited not only with the aid of an elementary electric dipole but also with the aid of a slot or system of slots. It is therefore interesting to explain the type of characteristic of an elongated or oblate spheroid excitable by an elementary slot, cut, for example, near the apex eta = 1 of the spheroid. This elementary slot can be considered as an elementary magnetic dipole having a moment

$$m = m_x. \qquad (2.1)$$

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directed along the axis x and oriented on the axis z at the point xi = xi$_1$, eta = 1 at xi$_1$ = xi$_0$ (xi = xi$_0$ is, as above, the equation of the surface of the spheroid).

However, a solution of this problem for an arbitrary xi$_0$, as well as a solution of the problem concerning the diffraction of a plane electromagnetic wave on an ideally conductive spheroid, has not been obtained up to this time.

It is possible only to solve the problem for xi$_0$ = 0, i. e., the problem concerning the diffraction on an ideally conductive disk. It is assumed in this case (see Fig. 4) that the disk is excited by the magnetic dipole (2.1) oriented on the point xi = xi$_1$, eta = 1 (z = z$_1$ = fxi$_1$, X = Y = 0).

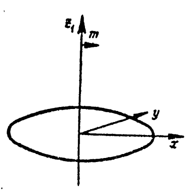

Figure 4

We have confined ourselves to this arrangement of the magnetic dipole because within the boundaries produced by the dipole a case of an elementary slot on the disk is derived (xi$_1$ = 0) and plane wave normally falling on the disk (xi$_1$ = infinity), which interested us.

In the role of potentials we select first the components Π_x and Π_z of the Hertzian magnetic vector Π, bound with the fields E and H by formulas

$$\left.\begin{aligned} E &= ik \operatorname{rot} \Pi \\ H &= \operatorname{grad} \operatorname{div} \Pi + k^2 \Pi \end{aligned}\right\} \qquad (2.2)$$

Since the components of the Hertzian vector of the primary field of the magnetic dipole are

$$\left.\begin{aligned} \Pi_x^0 &= m \frac{e^{ikR'}}{R'} \\ \Pi_y^0 &= \Pi_z^0 = 0 \end{aligned}\right\} \qquad (2.3)$$

where

$$R' = \sqrt{r^2 + (z - z_1)^2}, \qquad (2.4)$$

and r is the radius vector of the observation point in the cylindrical system of coordinates then the Hertzian vector for the entire field has the form of

$$\left.\begin{aligned} \Pi_x &= \Pi_x^0 + \Pi_x^1, \\ \Pi_y &= 0, \\ \Pi_z &= \Pi_z^1. \end{aligned}\right\} \qquad (2.5)$$

In this case the index ^{1} designates the potentials of the secondary field formed by the currents originating (generated) on the disk.

Inasmuch as the component Π_x^0 does not depend on phi then the secondary field, by virtue of the disk symmetry, should also not depend on phi, i. e.,

$$\Pi_x = \Pi_x^0 + \Pi_x^1 = \Phi^0(r, z) + \Phi^1(r, z) = \Phi(r, z). \qquad (2.6)$$

The component Π_z will be sought in the form of

$$\Pi_z = \Psi(r, z)\cos\varphi. \quad (2.7)$$

The magnetic Hertzian vector Π in the cylindrical coordinates will then have components

$$\Pi_r = \Phi\cos\varphi, \quad \Pi_\varphi = -\Phi\sin\varphi, \quad \Pi_z = \Psi\cos\varphi, \quad (2.8)$$

and the field will be expressed through Φ and Psi in the following manner

$$\left.\begin{aligned} E_r &= ik\sin\varphi\left[\frac{\partial\Phi}{\partial z} - \frac{1}{r}\Psi\right]; & H_r &= \cos\varphi\left[\frac{\partial\chi}{\partial r} + k^2\Phi\right], \\ E_\varphi &= ik\cos\varphi\left[\frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial r}\right]; & H_\varphi &= -\sin\varphi\left[\frac{\chi}{r} + k^2\Phi\right], \\ E_z &= -ik\sin\varphi\frac{\partial\Phi}{\partial r}; & H_z &= \cos\varphi\left[\frac{\partial\chi}{\partial z} + k^2\Psi\right], \end{aligned}\right\} \quad (2.9)$$

where, for the purpose of brevity, we designated

$$\chi = \frac{\partial\Phi}{\partial r} + \frac{\partial\Psi}{\partial z}. \quad (2.10)$$

The boundary conditions

$$E_r = E_\varphi = 0 \quad (r < f, \ z = 0) \quad (2.11)$$

for Phi and Psi will be written as follows

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial z} &= \frac{1}{r}\Psi, \\ \frac{\partial\Phi}{\partial z} &= \frac{\partial\Psi}{\partial r} \end{aligned}\right\} \quad (r < f, \ z = 0), \quad (2.12)$$

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whence it is derived that on the disk

$$\frac{\partial\Psi}{\partial r} = \frac{1}{r}\Psi. \quad (2.13)$$

By integrating equation (2.13) and by utilizing any of the given ratios (2.12) we will obtain the boundary conditions in the form of

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial z} &= C \\ \Psi &= Cr \end{aligned}\right\} \quad (r < f, \ z = 0); \quad (2.14)$$

where C is a certain constant, subject to determination from additional conditions. Thus in a cylindrical system of coordinates it is possible to obtain simple boundary conditions for the potential Phi and Psi, which we will now seek in the form of decompositions in accordance with spheroidal functions.

The fields and boundary conditions in an oblate spheroidal system of coordinates are written in the form of

$$\left.\begin{aligned} E_\varphi &= ik\cos\varphi\left\{\frac{1}{f(\xi^2+\eta^2)}\left[\eta(\xi^2+1)\frac{\partial\Phi}{\partial\xi} + \xi(1-\eta^2)\frac{\partial\Phi}{\partial\eta}\right] - \right. \\ &\quad \left. - \frac{\sqrt{(\xi^2+1)(1-\eta^2)}}{f(\xi^2+\eta^2)}\left[\xi\frac{\partial\Psi}{\partial\xi} - \eta\frac{\partial\Psi}{\partial\eta}\right]\right\} \\ H_\varphi &= -\sin\varphi\left\{\frac{1}{f^2(\xi^2+\eta^2)}\left[\xi\frac{\partial\Phi}{\partial\xi} - \eta\frac{\partial\Phi}{\partial\eta}\right] + \right. \\ &\quad + \frac{1}{f^2(\xi^2+\eta^2)\sqrt{(\xi^2+1)(1-\eta^2)}}\left[\eta(\xi^2+1)\frac{\partial\Psi}{\partial\xi} + \right. \\ &\quad \left.\left. + \xi(1-\eta^2)\frac{\partial\Psi}{\partial\eta}\right] + k^2\Phi\right\} \end{aligned}\right\} \quad (2.15)$$

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial\xi} &= Cf\eta \\ \Psi &= Cf\sqrt{1-\eta^2} \end{aligned}\right\} \quad (\xi = 0). \quad (2.16)$$

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Since the Descartes components of the Hertzian vector satisfy the wave equation, then, by virtue of (2.6) and (2.7), the functions phi^1 (r, z) and psi (r, z) should be decomposed in accordance with the spheroidal functions with azimuthal indices m = 0 and m = 1 respectively. Since, with respect to infinity, they should satisfy the condition of radiation, these decompositions should be sought in the form of

$$\Phi^1(\xi, \eta) = \sum_{l=0}^{\infty} A_l R^{(3)}_{0,l}(-ic, i\xi)\, S^{(1)}_{0,l}(-ic, \eta), \qquad (2.17)$$

$$\Psi(\xi, \eta) = \sum_{l=0}^{\infty} B_l R^{(3)}_{1l,}(-ic, i\xi)\, S^{(1)}_{1,l}(-ic, \eta). \qquad (2.18)$$

As to the potential of the secondary field Phi^0 (xi, eta), then according to formulas (2.12) and (2.14) of report (1) we have Phi^0 (xi, eta) = 2ikmx times

$$\times \sum_{l=0}^{\infty} \frac{S^{(1)}_{0,l}(-ic, 1)}{N_{0,l}(-ic)} R^{(1)}_{0,l}(-ic, i\xi_1)\, R^{(3)}_{0,l}(-ic, i\xi)\, S^{(1)}_{0,l}(-ic, \eta) \qquad (\xi > \xi_1) \qquad (2.19)$$

$$\Phi^0(\xi, \eta) = 2ikmf \times$$
$$\times \sum_{l=0}^{\infty} \frac{S^{(1)}_{0,l}(-ic, 1)}{N_{0,l}(-ic)} R^{(3)}_{0,l}(-ic, i\xi_1)\, R^{(1)}_{0,l}(-ic, i\xi)\, S^{(1)}_{0,l}(-ic, \eta) \qquad (\xi < \xi_1). \qquad (2.20)$$

The coefficients A_1 and B_1 are obtained from the boundary condition (2.16) by utilizing the orthogonality of the angular functions in the form of

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$$\left.\begin{aligned} &A_l(\xi_1) = \\ &= -2ikm \frac{S^{(1)}_{0,l}(-ic, 1)}{N_{0,l}(-ic)} \frac{R^{(3)}_{0,l}(-ic, i\xi_1) \frac{d}{d\xi} R^{(1)}_{0,l}(-ic, i0)}{\frac{d}{d\xi} R^{(3)}_{0,l}(-ic, i0)} + \\ &+ \frac{2}{3} Cf \frac{d^{0,l}_1(-ic)}{N_{0,l}(-ic) \frac{d}{d\xi} R^{(3)}_{0,l}(-ic, i0)} \\ &B_l(\xi_1) = \frac{4}{3} Cf \frac{d^{1,l}_0(-ic)}{N_{1,l}(-ic) R^{(3)}_{1,l}(-ic, i0)}. \end{aligned}\right\} \qquad (2.21)$$

Here and everywhere below it was designated

$$\frac{d}{d\xi} R(-ic, i0) = \left[\frac{d}{d\xi} R(-ic, i\xi)\right]_{\xi=0}. \qquad (2.22)$$

We like to call attention to the fact that $\frac{d}{dxi} R^{(1)}_{0,l}$ (- ic, i0) converts into zero at an even l (see (3), pp. 70-71). Since in this case $d^{0,l}_1$ is different from zero only at an odd l , and $d^{1,l}_0$ - only at an even l , then

$$\left.\begin{aligned} A_l = 0 \quad \text{при } l = 0, 2, \ldots \\ B_l = 0 \quad \text{при } l = 1, 3, \ldots \end{aligned}\right\}. \qquad (2.23)$$

Consequently, the line (2.17) is summarized actually only by the odd, and line (2.18) - only by the even indices l.

In order to determine the constant C, it is necessary that the radial component of the field-free (complete) current on the edges of the disk should convert into zero, i. e., (expressing the current density in CGSM units) in order that

$$J_r = \frac{1}{4\pi}(H^-_\varphi - H^+_\varphi) = 0 \quad \text{при } r = f, \; z = 0 \qquad (2.24)$$

or that ditto

$$H_\varphi\Big|_{\substack{\xi=0 \\ \eta=-0}} - H_\varphi\Big|_{\substack{\xi=0 \\ \eta=+0}} = 0. \qquad (2.25)$$

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By using formulas (2.15), we will derive that the condition (2.55) takes place provided the term

$$-\left[\frac{\partial\Phi(0,\eta)}{\partial\eta}+\frac{\partial\Phi(0,-\eta)}{\partial\eta}\right]+ \\ +\frac{1}{\sqrt{1-\eta^2}}\left[\frac{\partial\Psi(0,\eta)}{\partial\xi}+\frac{\partial\Psi(0,-\eta)}{\partial\xi}\right] \qquad (2.26)$$

at eta ⟶ 0 is an infinitely small value of much higher order than eta. (Under the sign $\frac{\text{delta phi (0, -eta)}}{\text{delta eta}}$ and so on, written here and below, we mean the derivative according to eta, in which instead of eta it was substituted with -eta).

We want to mention that the component H^0_{psi} of the primary magnetic field is a continuous function at z ≠ z_1, particularly

$$H^0_\varphi|_{z=+0}=H^0_\varphi|_{z=-0}, \qquad (2.27)$$

and consequently, the condition (2.25) for the primary field is fulfilled automatically. Consequently in (2.26) under Phi, we can understand the function Phi^1 corresponding to the secondary field and the constant C is determined from the requirement that the value

$$-\sum_{l=1,3,\ldots} A_l R^{(3)}_{0,l}(-ic,\ i0)\left[\frac{d}{d\eta}S^{(1)}_{0,l}(-ic,\ \eta)+ \right. \\ \left. +\frac{d}{d\eta}S^{(1)}_{0,l}(-ic,\ -\eta)\right]+ \\ +\frac{1}{\sqrt{1-\eta^2}}\sum_{l=0,2,\ldots} B_l\frac{d}{d\xi}R^{(3)}_{1,l}(-ic,\ i0)\left[S^{(1)}_{1,l}(-ic,\ \eta)+\right. \\ \left. +S^{(1)}_{1,l}(-ic,\ -\eta)\right] \qquad (2.28)$$

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at eta ⟶ 0 should be infinitely small of much higher order than eta.

Since $\frac{d}{d\ \text{eta}}\ S^{(1)}_{0,l}$ (-ic, eta) at an odd l and $S^{(1)}_{1,l}$ (-ic, eta) at an even l are the even functions of eta (formula 1.7 in report (1)), then they decompose into Maclaurin's series having only even degrees eta. Consequently, the term (2.28) will satisfy our requirement, provided its first member (containing eta in zero degree) of decomposition into the Maclaurin's series converts into zero, i. e., if

$$-\sum_{l=1,3,\ldots} A_l R^{(3)}_{0,l}(-ic,\ i0)\frac{d}{d\eta}S^{(1)}_{0,l}(-ic,\ 0)+ \\ +\sum_{l=0,2,\ldots} B_l\frac{d}{d\xi}R^{(3)}_{1,l}(-ic,\ i0)\,S^{(1)}_{1,l}(-ic,\ 0)=0. \qquad (2.29)$$

Using the ratio (1.28) of report (1) and keeping in mind that $R^{(1)}_{0,l}$ (-ic, i0) at an odd l converts into zero (3, pp. 70-71) we will obtain the constant C in the form of

$$C=\frac{3km}{fc}C_1(\xi_1)=\frac{3m}{f^2}C_1(\xi_1), \qquad (2.30)$$

where

$$C_1(\xi_1)=\frac{\displaystyle\sum_{l=1,3,\ldots}\frac{S^{(1)}_{0,l}(-ic,\ 1)}{N_{0,l}}\,\frac{R^{(3)}_{0,l}(-ic,\ i\xi_1)}{\frac{d}{d\xi}R^{(3)}_{0,l}(-ic,\ i0)}\,\frac{d}{d\eta}S^{(1)}_{0,l}(-ic,\ 0)}{T}, \qquad (2.31)$$

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and

$$T = \sum_{l=1,3,\dots} \frac{d_1^{0,l} R_{0,l}^{(3)}(-ic, i0)}{N_{0,l} \frac{d}{d\xi} R_{0,l}^{(3)}(-ic, i0)} \frac{d}{d\eta} S_{0,l}^{(1)}(-ic, 0) - $$
$$ - 2 \sum_{l=0,2,\dots} \frac{d_0^{1,l} \frac{d}{d\xi} R_{1,l}^{(3)}(-ic, i0)}{N_{1,l} R_{1,l}^{(3)}(-ic, i0)} S_{1,l}^{(1)}(-ic, 0). \quad (2.31a)$$

In this way, the coefficients A_l and B_l of the decompositions (2.17) and (2.18) of the potentials of the secondary field are now completely determined.

In the wave zone, i. e., at xi $\longrightarrow$ infinity and cxi $\longrightarrow$ infinity, the fields (2.15) have the form of

$$\left. \begin{aligned} E_\varphi &= ik \cos\varphi \left[\frac{\eta}{f} \frac{\partial \Phi}{\partial \xi} - \frac{\sqrt{1-\eta^2}}{f} \frac{\partial \Psi}{\partial \xi} \right] \\ H_\varphi &= -k^2 \sin\varphi \Phi. \end{aligned} \right\} \quad (2.32)$$

By substituting in these formulas the decompositions (2.17) and (2.18) for Phi[1] and Psi, to which the asymptotic formula (1.27) of report (1) should also be applied, we will obtain terms in the wave zone for the secondary field excitable by the magnetic dipole (2.1), oriented on the disk in point xi = xi_1, eta = 1.

Par. 3 Diffraction of a Plane Wave on a Disk.

We will assume that the horizontal magnetic dipole oriented on the disk (Fig. 4), stands on axis z and retaining orientation departs

into infinity so that eta_1 = 1, xi_1 $\longrightarrow$ infinity. Another condition necessary will be that cxi_1 $\longrightarrow$ infinity. The Hertzian vector of the primary field (2.3) will then acquire the form of

$$\Pi_x^0 = m \frac{e^{ikz_1}}{z_1} e^{-ikz}, \quad \Pi_y^0 = \Pi_z^0 = 0, \quad (3.1)$$

and consequently in this case the primary field represents a plane wave normally falling on the disk

$$E_y^0 = H_x^0 = k^2 \Pi_x^0 = M e^{-ikz} \quad (3.2)$$

with an amplitude

$$M = k^2 m \frac{e^{ikz_1}}{z_1}. \quad (3.3)$$

In order to obtain the diffraction field of a plane wave, it is necessary to write xi_1 $\longrightarrow$ infinity, cxi_1 $\longrightarrow$ infinity into the terms for $A_1(xi_1)$ and $C_1(xi_1)$ of the previous paragraph. Then, taking into consideration (1.27) in (1), we can write

$$C_1(\infty) = -\frac{e^{ikz_1}}{kz_1} C_2, \quad (3.4)$$

where

$$C_2 = \frac{\displaystyle\sum_{l=1,3,\dots} (-i)^{l-1} \frac{S_{0,l}^{(1)}(-ic, 1)}{N_{0,l} \frac{d}{d\xi} R_{0,l}^{(3)}(-ic, i0)} \frac{d}{d\eta} S_{0,l}^{(1)}(-ic, 0)}{T}, \quad (3.5)$$

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and the value T is determined by formula (2.31a), whereupon according to (2.30) in this case

$$C = -\frac{3m}{fc}\frac{e^{ikz_1}}{z_1}C_2 = -\frac{3}{fck^2}MC_2. \qquad (3.6)$$

$$\left.\begin{aligned} A_l(\varkappa) &= -\frac{2M}{ck^2}a_l = \\ &= -\frac{2M}{ck^2}\frac{(-i)^l cS_{0,l}^{(1)}(-ic,1)\frac{d}{d\xi}R_{0,l}^{(1)}(-ic,i0) + C_2 d_1^{0,l}}{N_{0,l}\frac{d}{d\xi}R_{0,l}^{(3)}(-ic,i0)} \\ B_l(\infty) &= -\frac{4M}{ck^2}C_2 b_l = \\ &= -\frac{4M}{ck^2}\cdot\frac{C\,d_0^{1,l}}{N_{1,l}R_{1,l}^{(3)}(-ic,i0)}. \end{aligned}\right\} \qquad (3.7)$$

By substituting the terms (2.7) in formulas (2.32) we will obtain the secondary field of a plane wave in the wave zone, in the form of

$$\left.\begin{aligned} -H_\vartheta = E_\varphi &= \frac{Mc^2 i}{2}\frac{e^{ikR}}{kR}V^{(1)}(\vartheta)\cos\varphi, \\ E_\vartheta = H_\varphi &= \frac{Mc^2 i}{2}\frac{e^{ikR}}{kR}V^{(2)}(\vartheta)\sin\varphi, \end{aligned}\right\} \qquad (3.8)$$

where

$$\left.\begin{aligned} V^{(1)}(\vartheta) &= -\frac{4i}{c^3}\Big\{\cos\vartheta\sum_{l=1,3,\dots}(-i)^{l+1}a_l S_{0,l}^{(1)}(-ic,\cos\vartheta) + \\ &\quad + \sin\vartheta\cdot 2C_2\sum_{l=0,2,\dots}(-i)^l b_l S_{1,l}^{(1)}(-ic,\cos\vartheta)\Big\}, \\ V^{(2)}(\vartheta) &= -\frac{4i}{c^3}\sum_{l=1,3,\dots}(-i)^{l+1}a_l S_{0,l}^{(1)}(-ic,\cos\vartheta), \end{aligned}\right\} \qquad (3.9)$$

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where the coefficients a_l and b_l are fixed by ratios (2.7). The functions $V^{(1)}(\vartheta)$ and $V^{(2)}(\vartheta)$ are the complex characteristics of the secondary field of the plane wave falling normally on the disk. It should be mentioned that our adopted standardization of these functions, as will be shown below, (see 3.23) warrants at $\vartheta = 0$ the tendency of $V^{(1)}(0)$ and $V^{(2)}(0)$ toward 1, when $c \longrightarrow$ infinity.

When $c \ll 1$, formulas (3.8) acquire the form of

$$\left.\begin{aligned} -H_\vartheta = E_\varphi &= \frac{4f^3}{3\pi}Mk^2\frac{e^{ikR}}{R}\cos\varphi, \\ E_\vartheta = H_\varphi &= \frac{4f^3}{3\pi}Mk^2\frac{e^{ikR}}{R}\cos\vartheta\sin\varphi, \end{aligned}\right\} \qquad (3.10)$$

i. e., for infinitely long waves, the secondary field of the plane incident wave in the wave zone is such as if it would be radiated by an electric dipole with a moment

$$p = \frac{4f^3}{3\pi}M, \qquad (3.11)$$

placed in the beginning of the coordinates and directed along the y-axis.

The currents excitable on the disk by the secondary field of the incident plane wave, are expressed (in the CGSM system) in the following manner.

$$\left.\begin{aligned} j_r &= -i\frac{4c}{3\pi^2}\eta\sin\varphi, \\ j_\varphi &= -i\frac{2c}{3\pi^2}\frac{1+\eta^2}{\eta}\cos\varphi \end{aligned}\right\} \qquad (3.12)$$

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or in cylindrical coordinates

$$\left.\begin{aligned} j_r &= -i\frac{4c}{3\pi^2}\sqrt{1-\left(\frac{r}{f}\right)^2}\sin\varphi, \\ j_\varphi &= -i\frac{4c}{3\pi^2}\frac{1-\frac{1}{2}\left(\frac{r}{f}\right)^2}{\sqrt{1-\left(\frac{r}{f}\right)^2}}\cos\varphi, \end{aligned}\right\} \qquad (3.13)$$

where f is the radius of the disk.

At greater magnitudes of the parameter c, it is possible to obtain approximated formulas for the secondary field in accordance with Huygen's principle. For this it is necessary to take into consideration that the magnetic field on the disk has the very same magnitude as it would have had if, instead of a disk, we would investigate an infinite surface, i. e.,

$$H_x|_{z=0} = 2M. \qquad (3.14)$$

This formula is known to be incorrect on the edges of the disk, but at distances of the order of a wave length from the edge, it can already be considered as equitable. If c = kf is great, i. e., if

$$c = kf = \frac{2\pi f}{\lambda} \gg 1, \qquad (3.15)$$

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takes place then the width of the band, where formula (3.14) is incorrect, is small in comparison with the radius of the disk and in these directions in which the secondary field of the disk is not close to zero "the edge effect" is small in comparison with the basic one which depends upon the area of the disk.

In the hypothesis of (3.14) the current on the disk in the CGSM system is

$$j_y = \frac{M}{2\pi} \qquad (3.16)$$

and a certain potential in the point of observation P_o will have a single component

$$A_y = \int_S j_y \frac{e^{ikR'}}{R'}\, dS, \qquad (3.17)$$

where R' is the distance from the point of observation to the point of integration, and the integral is taken according to the upper slide of the disk.

Taking into consideration that for the wave zone

$$R' = R - r\sin\vartheta\cos(\varphi' - \varphi), \qquad (3.18)$$

where R, ϑ , psi are the spherical coordinates of the observation point P_o, and r and psi' are the cylindrical coordinates of the integration

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point on the disk and by using the known formulas

$$\left.\begin{aligned} J_0(z) &= \frac{1}{2\pi}\int_0^{2\pi} e^{-iz\cos\varphi}\,d\varphi \\ zJ_1(z) &= \int zJ_0(z)\,dz, \end{aligned}\right\} \qquad (3.19)$$

we will obtain

$$A_y = Mf^2\,\frac{e^{ikR}}{R}\,\frac{J_1(c\sin\vartheta)}{c\sin\vartheta} \qquad (3.20)$$

and the approximated formulas for the field will have the form of

$$\left.\begin{aligned} -H_\vartheta = E_\varphi &= M\,\frac{e^{ikR}}{kR}\,\frac{icJ_1(c\sin\vartheta)}{\sin\vartheta}\cos\varphi \\ E_\vartheta = H_\varphi &= M\,\frac{e^{ikR}}{kR}\,\frac{icJ_1(c\sin\vartheta)}{\sin\vartheta}\cos\vartheta\sin\varphi. \end{aligned}\right\} \qquad (3.21)$$

Finally, comparing formulas (3.21 and (3.8) we should write

$$\left.\begin{aligned} V^{(1)}(\vartheta) &= \frac{2}{c}\,\frac{J_1(c\sin\vartheta)}{\sin\vartheta} \\ V^{(2)}(\vartheta) &= \frac{2}{c}\,\frac{J_1(c\sin\vartheta)}{\sin\vartheta}\cos\vartheta. \end{aligned}\right\} \qquad (3.22)$$

It should be mentioned that our selected standardization of functions $V^{(1)}$ and $V^{(2)}$ gives in the case of the Huygen's principle, i. e., at $c \gg 1$, values

$$V^{(1)}(0) = V^{(2)}(0) = 1, \qquad (3.23)$$

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which also justifies its introduction.

The radiation characteristics of the disk which produce the electromagnetic field in H- and E- planes of the incident wave respectively were calculated by us for $c = 1, 3$ and 5 according to formulas (3.9), and for $c = 3$ and $c = 5$ also in accordance with the approximated formulas (3.22). The results of these calculations are given in the Descartes system of coordinates in Fig. 5 - 7, whereby the dotted lines represent curves calculated according to the approximated formulas (3.22). The Descartes system of coordinates is more favorable in this case because it gives a better representation of the behavior of the radiation characteristics in the vicinity of the minimums.

Starting with analysis of characteristics, we wish to mention that all characteristics are symmetrical relative to direction $\vartheta = 90°$, as also should have been the case for the flat and infinitely thin disk; in this case, the characteristic $V^{(2)}(\vartheta)$ converts into zero at $\vartheta = 90°$. The explanation of this circumstance is evident from Fig. 8. The incident wave (3.2) generates, by virtue of the symmetry, such currents on the disk that their component along axis x are able to radiate in direction $\vartheta = 90°$, they are being mutually absorbed and the unabsorbed components according to y along axis y are non-radiating. In the H-plane in direction $\vartheta = 90°$, the radiation is $[\,V^{(1)}(90) \neq 0\,]$.

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Further, everywhere are the ratios

$$V^{(1)}(0) = V^{(2)}(0); \quad V^{(1)}(\pi) = -V^{(2)}(\pi), \qquad (3.24)$$

which is immediately evident from (3.9) since at $\vartheta = 0$ and $\vartheta = \pi$, the second item in $V^{(1)}(\vartheta)$ disappears. In directions $\vartheta = 0$ and $\vartheta = \pi$, the functions $V^{(1)}(\vartheta)$ and $V^{(2)}(\vartheta)$ have maxima and an increase in the parameter c brings these maxima close to unity.

The approximate functions (dotted line) are always equal to one at $\vartheta = 0^\circ$ and $\vartheta = 180^\circ$ and have a course qualitatively very close to the slope of accurate curves. The maximum difference between the approximate and accurate takes place, as it should have been, in the minimums, whereupon the approximate functions in the minimums convert to zero.

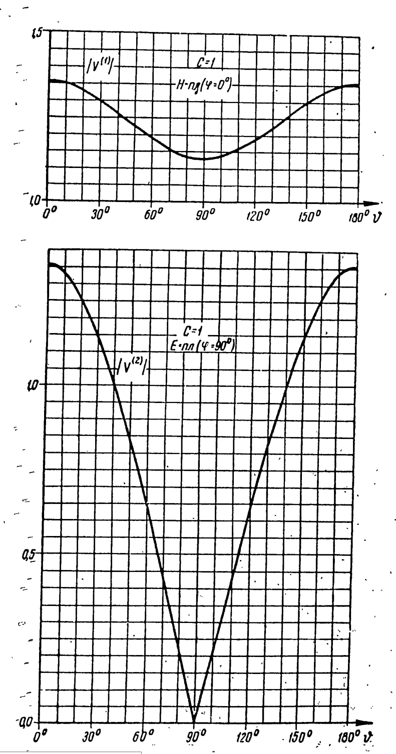

Figure 5

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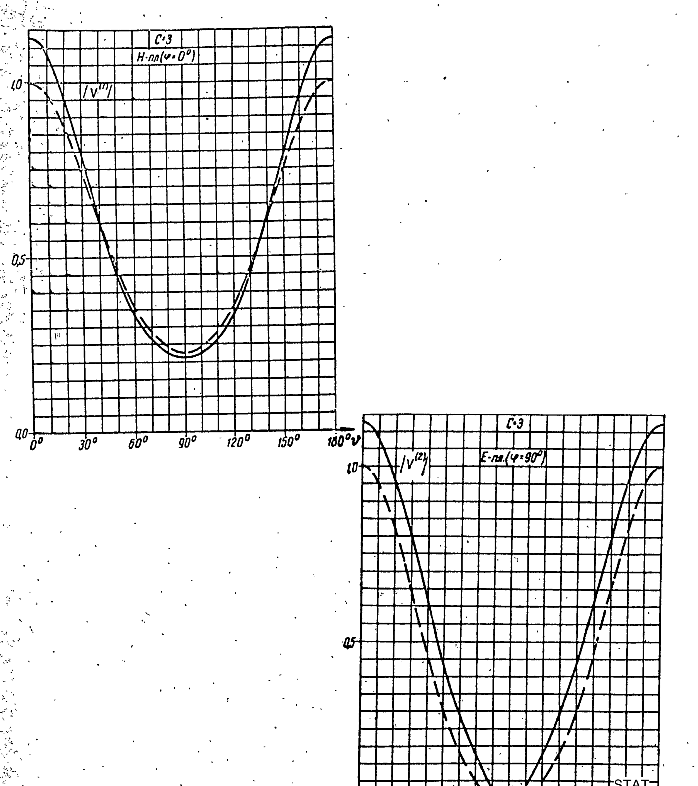

Figure 6

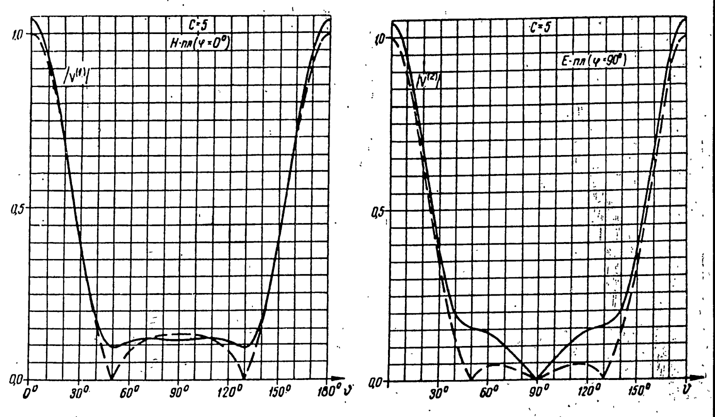

Figure 7

We wish to mention that an increase in parameter c will be followed by a sinking in the minimums of the accurate curves. The divergence in the calculated values of characteristics computed in accordance with accurate and approximate formulas in the vicinity of maxima at c = 3 does not exceed 15%, and at c = 5, does not exceed 5%.

Figure 8

It is also interesting to compare the results obtained during the calculation of the diffusion coefficient alpha (calculated according to accurate and approximate formulas)

$$\alpha = R\frac{|E|}{|E^{\circ}|}, \qquad (3.25)$$

where $|E^{o}|$ is the amplitude of the plane incident wave, and $|E|$ is the amplitude of the reflected field in direction opposite to the incident wave (in our case $\vartheta = 0$) and effective zone of diffusion sigma

$$\sigma = 4\pi\alpha^2. \qquad (3.26)$$

We will bring forth the ratios of values alpha and sigma, calculated according to accurate formulas to the values $alpha_1$ and $sigma_1$, where

$$\left.\begin{aligned} \alpha_1 &= \frac{S}{\lambda} = \frac{c^2}{2k} \\ \sigma_1 &= \frac{\pi c^4}{k^2}, \end{aligned}\right\} \qquad (3.27)$$

are obtainable by Huygen's principle for the normal falling of the wave on the disk.

c	$\frac{\alpha}{\alpha_1}$	$\frac{\sigma}{\sigma_1}$
3	1,13	1,28
5	1,04	1,08

As is evident from the table, at c = 5 these ratios are already quite close to unity. Upon a further increase in parameter c, the approximate formulas should yield still more accurate results.

In this way "the Huygen's Principle", applicable in the classical theory of diffraction for the solution of the problem concerning the diffusion on a disk, offers quite satisfactory results for values of the parameter $c \approx 5$.

Par. 4 Radiation Characteristics of a Slot on a Disk.

Let us assume that the dipole with the magnetic moment (2.1) is now oriented on the very disk in its center, i. e., at xi_1 = 0, eta = 1 (Fig. 9), which corresponds to a unilateral elementary slot, slotted on the disk along axis x. In order to obtain a secondary field, in this case it is necessary to write $xi_1 = 0$ in the formulas for A_1 (xi_1) and B_1 (xi_1) of Par. 2. We will then obtain

Figure 9

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$$\left.\begin{aligned} A_l(0) &= -\frac{2km}{c}\,\frac{S_{0,l}^{(1)}(-ic,1) - C_1(0)\,d_1^{0,l}}{N_{0,l}\,\frac{d}{d\xi}\,R_{0,l}^{(3)}(-ic,i0)} \\ B_l(0) &= \frac{4km}{c}\,\frac{C_1(0)\,d_0^{1,l}}{N_{1,l}\,R_{1,l}^{(3)}(-ic,i0)}. \end{aligned}\right\} \quad (4.1)$$

However, the term for tne constant C_1 (0) cannot be obtained in this manner because if we should write $xi_1 = 0$ in formula (2.31) for C_1 (xi_1), then the set obtained in the numerator will be unsuitable for calculation. The method for effective determination of the constant $C_1(0)$ will be described briefly below.

The secondary field in the wave zone (see 2.32, 2.17, 2.18) has the form of

$$\left.\begin{aligned} -H_\vartheta^1 = E_\varphi^1 &= -k^2m\,\frac{e^{ikR}}{R}\,V_1^{(1)}(\vartheta)\cos\varphi, \\ E_\vartheta^1 = H_\varphi^1 &= -k^2m\,\frac{e^{ikR}}{R}\,V_1^{(2)}(\vartheta)\sin\varphi, \end{aligned}\right\} \quad (4.2)$$

where

$$\left.\begin{aligned} V_1^{(1)}(\vartheta) &= -\frac{2}{c}\Big\{\cos\vartheta\sum_{l=1,3\ldots}\alpha_l S_{0,l}^{(1)}(-ic,\cos\vartheta) - \\ &\quad -\sin\vartheta\, 2C_1(0)\sum_{l=0,2\ldots}\beta_l S_{1,l}^{(1)}(-ic,\cos\vartheta),\Big\} \\ V_1^{(2)} &= -\frac{2}{c}\sum_{l=1,3\ldots}\alpha_l S_{0,l}^{(1)}(-ic,\cos\vartheta), \end{aligned}\right\} \quad (4.3)$$

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where

$$\left.\begin{aligned} \alpha_l &= (-i)^{l+1}\,\frac{S_{0,l}^{(1)}(-ic,1) - C_1(0)\,d_1^{0,l}}{N_{0,l}\,\frac{d}{d\xi}\,R_{0,l}^{(3)}(-ic,i0)}, \\ \beta_l &= (-i)^l\,\frac{d_0^{1,l}}{N_{1,l}R_{1,l}^{(3)}(-ic,i0)}. \end{aligned}\right\} \quad (4.3a)$$

The components of the primary field are presented by formulas

$$\left.\begin{aligned} -H_\vartheta^0 = E_\varphi^0 &= -k^2m\,\frac{e^{ikR}}{R}\cos\vartheta\cos\varphi, \\ E_\vartheta^0 = H_\varphi^0 &= -k^2m\,\frac{e^{ikR}}{R}\sin\varphi \end{aligned}\right\} \quad (4.4)$$

and finally the entire field is

$$\left.\begin{aligned} -H_\vartheta = E_\varphi &= -k^2m\,\frac{e^{ikR}}{R}\,V^{(1)}(\vartheta)\cos\varphi, \\ E_\vartheta = H_\varphi &= -k^2m\,\frac{e^{ikR}}{R}\,V^{(2)}(\vartheta)\sin\varphi, \end{aligned}\right\} \quad (4.5)$$

where

$$\left.\begin{aligned} V^{(1)}(\vartheta) &= \cos\vartheta + V_1^{(1)}(\vartheta), \\ V^{(2)}(\vartheta) &= 1 + V_1^{(2)}(\vartheta) \end{aligned}\right\} \quad (4.6)$$

are the complex radiation characteristics of the disk with a unilateral elementary slot cut in its center.

We will now discuss the conclusion of the formula for the constant C_1 (0). We will apply for this purpose the principle of reciprocity which can be written in the form of

$$\mathbf{m}_1\mathbf{H}_2(P_1) = \mathbf{m}_2\mathbf{H}_1(P_2), \quad (4.7)$$

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where m_1 and m_2 are the moment of the dipoles in points P_1 and P_2, creating the fields H_1 and H_2. Under H_1 and H_2, we mean the secondary fields because the ratio (4.7) should also take place for the primary fields.

We will now investigate, in points P_1 (xi = 0, eta = 1) and P_2 (xi = $\overline{xi} \gg 1$, eta = 1) the dipoles with equal moments m, directed along axis x (Fig. 10). The ratio (4.7) will then be written in the form of

$$H_{2,x}(P_1) = H_{1,x}(P_2) \quad (4.8)$$

or

$$H_{2,\varphi}(P_1)\big|_{\vartheta=0,\varphi=\frac{\pi}{2}} = H_{1,\varphi}(P_2)\big|_{\vartheta=0,\varphi=\frac{\pi}{2}}. \quad (4.9)$$

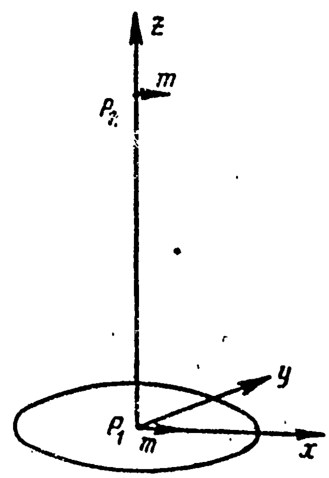

Figure 10

In this case $H_{1,psi}(P_2)\big|_{\vartheta = 0, psi = \frac{\pi}{2}}$ designates the secondary field in the wave zone oriented on the disk of the dipole and is expressed by formulas (4.2) and (4.3) in which it should be written $\vartheta = 0$, psi $= \frac{\pi}{2}$, R = z. Next $H_{2,psi}(P_1)\big|_{\vartheta = 0, psi = \frac{\pi}{2}}$ is the secondary field of the plane wave in the point xi = 0, eta = 1.

By substituting formulas (2.15), (2.17), (2.18) and (3.7) in ratio (4.9) we obtain a term already suitable for calculations

$$C_1(0) = \frac{L}{c^2 \sum\limits_{l=1,3,\ldots} (-i)^{l+1} \dfrac{d_1^{0,l} S_{0,l}^{(1)}(-ic,1)}{N_{0l}\, \dfrac{d}{d\xi} R_{0,l}^{(3)}(-ic,i0)}}, \tag{4.10}$$

where

$$\begin{aligned} L = {} & \sum_{l=1,3,\ldots} (-i)^{l+1} \frac{S_{0,l}^{(1)}(-ic,1) \dfrac{d}{d\eta} S_{0,l}^{(1)}(-ic,1)}{N_{0,l} \dfrac{d}{d\xi} R_{0,l}^{(3)}(-ic,i0)} + \\ & + C_2 \left\{ \sum_{l=1,3,\ldots} \frac{d_1^{0,l} R_{0,l}^{(3)}(-ic,i0) \left[\dfrac{d}{d\eta} S_{0,l}^{(1)}(-ic,1) - c^2 S_{0,l}^{(1)}(-ic,1) \right]}{N_{0,l} \dfrac{d}{d\xi} R_{0,l}^{(3)}(-ic,i0)} - \right. \\ & \left. - 2 \sum_{l=0,2,\ldots} \frac{d_0^{1,l} \dfrac{d}{d\xi} R_{1,l}^{(3)}(-ic,i0)\, \sigma_{1,l}(-ic)}{N_{1,l} R_{1,l}^{(3)}(-ic,i0)} \right\}. \end{aligned} \tag{4.11}$$

At $c \ll 1$ the secondary field of the slot in the distant zone is

$$\left. \begin{aligned} -H_\vartheta^1 = E_\varphi^1 &= k^2 \left(\frac{8\, mc\bar{i}}{3\pi} - \frac{4\, mci}{3\pi} \sin^2\vartheta \right) \frac{e^{ikR}}{R} \cos\varphi \\ E_\vartheta^1 = H_\varphi^1 &= k^2 \frac{8 m\bar{c} i}{3\pi} \frac{e^{ikR}}{R} \cos\vartheta \sin\varphi \end{aligned} \right\}. \tag{4.12}$$

In this way formulas (4.12) show that the secondary field of the unilateral slot on the disk for small values of the parameter c does not represent

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a pure dipole radiation in the wave zone.

The functions $V^{(1)}(\vartheta)$ and $V^{(2)}(\vartheta)$ respectively characterize the field of radiation in the meridional plane containing the slot (psi = 0) and in the meridional plane (psi = $\frac{\pi}{2}$) which is perpendicular to the slot. They were calculated in accordance with formulas (4.6) and (4.3) for values of the parameter c = 1, 3, 5 (Fig. 11).

When c = 1, both amplitude characteristics have two large lobes, with the rear lobe only slightly smaller than the forward one. When c = 3, the shading phenomenon is already clearly expressed and there is a smooth conversion from the illuminated zone to the shaded one. When c = 5, a third lobe appears at the curve $|V^{(2)}(\vartheta)$. The shading phenomenon and the smooth transition from the illuminated zone to the shaded one are preserved for this and other characteristics as well.

A comparison with the radiation characteristics of a sphere, excitable by an elementary slot, at corresponding values of the parameter alpha = k rho (see (2), Fig. 10-12 and 19-21) shows that in this case, there is a specific analogy between the sphere and disk.

However, it should be mentioned that if, during symmetric excitation of the sphere and disk by an electric dipole, the disk produced a considerably higher screening for the investigated values c than the sphere (see Par. 1), the same cannot be said in this particular case. Namely, even though for the value c = 3, the disk

still produces a greater screening effect than does the sphere, for the value c = 5, the difference is nevertheless much smaller.

Figure 11

In conclusion, we wish to state that the strict theory of diffraction of electromagnetic waves on a disk was also investigated in articles (4 - 11), published (with the exception of (4) and (5)) after the given report had already been completed.

It seems to us that our report is nevertheless of current interest because the method employed differs from the one described in the articles mentioned above, and the numerical data greatly supplement the data already available in literature.

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