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**TRANSMISSION OF A LAMINAR TRANSFER REGIME INTO TURBULENT ONE  
WITH FREE CONVECTION**

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**Аннотация-** Переход ламинарного режима переноса в турбулентный при свободной конвекции вдоль плоской поверхности и вокруг криволинейной характеризуется разным значением  $ArPr$ , равным в первом случае  $ArPr=2.8 \times 10^8$ , а во втором  $=2 \times 10^7$ . Эти значения сохраняются для переноса тепла и массы.

**Abstract-** Transmission of a laminar regime of transfer into the turbulent with free convection along a plane and round a curvilinear surface is characterized by the different value  $ArPr$  equal in the first case to  $ArPr=2.8 \times 10^8$  and in the second to  $=2 \times 10^7$ . These values of  $ArPr$  are maintained the same both for heat and mass transfer.

**N O M E N C L A T U R E**

- $Ar$ , Archimed number,  $\frac{g l^3}{\nu^2} \cdot \frac{\Delta \rho}{\rho}$
- $Pr$ , Prandtl number,  $\frac{\nu}{\alpha}$
- $Sc$ , Schmidt number,  $\frac{\nu}{k}$
- $Nu$ , Nusselt number,  $\frac{h l}{\lambda}$
- $g$ , gravitational constant,  $m/sec^2$
- $l$ , length (a diameter),  $m$
- $\nu$ , kinetic viscosity,  $m^2/sec$
- $\rho$ , density,  $kg \cdot sec^2/m^3$
- $\Delta \rho$ , difference density,
- $\alpha$ , thermal diffusivity,  $m^2/sec$
- $\lambda$ , thermal conductivity,  $kcal/m \cdot sec \cdot ^\circ C$

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h, heat transfer, kcal/m<sup>2</sup>·s

k, thermal diffusivity, m<sup>2</sup>/sec

S u b s c r i p t s

cr, critical,

con, convection

S u p e r s c r i p t s

lam, laminar,

tur, turbulent.

When calculating and testing it is assumed that with free convection at values  $ArPr \leq 2 \times 10^7$  a regime of a flow is laminar and at values  $ArPr > 2 \times 10^7$ , turbulent [1]. and there is no difference made for free convection round bodies and along plane plates. In all cases  $ArPr = 2 \times 10^7$  and involves volume and surface problems.

Experimental investigation [2] by the interference method of a structure of free flows along a vertical plane and observations for appearing both wave phenomenon and a wave length in a boundary layer of a free flow allowed to determine that the origin of transition of a laminar flow regime into the turbulent for the value  $Pr = 0.71$  takes place at  $Ar_{cr} = 4 \times 10^8$ , e.g. at  $(ArPr)_{cr} = 4 \times 10^8 \times 0.71 = 2.84 \times 10^8$ . This value  $(ArPr)_{cr}$  is approximately 14 times that of  $2 \times 10^7$ .

Naturally the question arises whether the value  $Ar_{cr} = 4 \times 10^8$  is characteristic only for plane problem conformably to which it is experimentally determined or it is general for volume problems as well.

In order to explain this question, consider the equality:

$$Nu_{\text{lam}} = Nu_{\text{tur}}$$

which in an open form for a plane vertical surface may be written

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$$0.67 (\text{ArPr})_{\text{cr}}^{1/4} = 0.135 (\text{ArPr})_{\text{cr}}^{1/3}$$

where 0.67 is taken following [2] and 0.135, following [1].

The solution of this equation gives:

$$(\text{ArPr})_{\text{cr}}^{1/12} = 4.96$$

or

$$(\text{ArPr})_{\text{cr}} = 2.3 \times 10^8$$

At  $\text{Pr} = 0.71$   $\text{Ar}_{\text{cr}} = \frac{2.3 \times 10^8}{0.71} \times 3.23 \times 10^8$  e.g. it has the order of  
 value  $\text{Ar}_{\text{cr exp}} = 4 \times 10^8$

For complete correspondence the numerical value of the coefficient 0.135 is sufficient to be substituted for 0.133 where

$$\text{Ar}_{\text{cr}} = \text{Ar}_{\text{cr exp}} = 4 \times 10^8$$

Proceeding from this equality:

$$0.54 (\text{ArPr})_{\text{cr}}^{1/4} = 0.133 (\text{ArPr})_{\text{cr}}^{1/3}$$

we obtain for spherical bodies that  $(\text{ArPr})_{\text{cr}} = 2 \times 10^7$  [2].

Thus it is true that numerical values of  $(\text{ArPr})_{\text{cr}}$  are different under the conditions of space and plane-surface convection.

Now let us show on the basis of joint consideration of energy, heat and mass transfer equations that for a laminar regime with free convection along a plane surface the proportionality coefficient in transfer equations for all types of transfer is the same and equal to  $= 0.67$ .

From the energy transfer equation we have:

$$0.667 \text{Re}_{\text{cr}}^{0.5} = 0.037 \text{Re}_{\text{cr}}^{0.8}$$

hence

$$\text{Re}_{\text{cr}} = 16200$$

On the basis of the comparison of turbulent energy and heat transfer equations with free convection:

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$$0.133 (\text{ArPr})_{\text{cr}}^{1/3} = 0.037 \text{Re}_{\text{cr}}^{0.8} = 0.037 (16200)^{0.8}$$

**we find that**

$$(\text{ArPr})_{\text{cr}} = 2.25 \times 10^8$$

**This value  $(\text{ArPr})_{\text{cr}}$  is closer to  $(\text{ArPr})_{\text{cr}} = 2.3 \times 10^8$  determined above.**

**Proceeding from the heat transfer equation:**

$$A (\text{ArPr})_{\text{cr}}^{1/4} = 0.133 (\text{ArPr})_{\text{cr}}^{1/3}$$

**we determine that**

$$A = 0.133 (\text{ArPr})_{\text{cr}}^{1/12} = 0.133 (2.25 \times 10^8)^{1/12} = 0.66 = 0.67$$

**Keeping in mind that experimental investigations (3) on evaporation of some liquids from the vertical surface at  $\text{ArSc} \leq 3 \times 10^8$  determined the value  $A = 0.67$  as well, it is possible to assume the existence of rather a close analogy in transfer mechanisms that allows to write calculation formulas for heat and mass transfer on the basis of hydrotechnical representations.**

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HEAT TRANSFER BETWEEN A GAS AND A SPHERICAL SURFACE  
AT COMBINED ACTION OF FREE AND FORCED CONVECTION .

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**Аннотация**—В критериальном виде даны формулы, подтвержденные экспериментами, для определения теплообмена между газом и шаровой поверхностью в условиях совместного действия свободной и вынужденной конвекции.

**Abstract**—Formulas for determining heat transfer between gas and a spherical surface at combined action of free and forced convection which are confirmed by experiments are given in a criterion form.

Н О М Е Н К Л А Т У Р Е

- Gr, Grashof number,  $\frac{gd^3}{\nu^2} \cdot \frac{\Delta t}{T}$  ;  
 Nu, Russelt number,  $\frac{hd}{x}$   
 Re, Reynolds number,  $\frac{wd}{\nu}$   
 Pr, Prandtl number,  $\frac{\nu}{\alpha}$   
 g, gravitational constant,  $m/sec^2$ ,  
 d, diameter,  $m$ ,  
 $\nu$ , kinetic viscosity,  $m^2/sec$ ,  
 h, heat transfer,  $kcal/h m^2 \cdot ^\circ C$ ,  
 $\alpha$ , thermal diffusivity,  $m^2/sec$ ,  
 x, thermal conductivity,  $kcal/h m \cdot ^\circ C$ ,  
 w, velocity,  $m/sec$ ,  
 T — temperature,  $^\circ K$ ,  
 $\Delta t$ , difference temperature.

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At present great consideration is given to heat and mass transfer at combined action of free and forced convection.

Results of new investigations on heat transfer of spherical particles with small sizes in air under the conditions of both free and forced convection and their combined action are presented in recently published work [1] of Professor N. Juge (University of Tokyo).

According to these experiments heat transfer from spherical bodies with free convection for the region  $1 < Gr < 10^5$  is described by the equation  $Nu_{free} = 2 + 0.392 Gr^{1/4}$  and with forced convection for the region  $10 < Re < 1800$  by the equation  $Nu_{forced} = 2(1 + 0.246 \sqrt{Re})$ .

At combined action of free and forced convection analytical relations are not discussed.

Analysis of results on free convection following Juge shows that they do not coincide with the already known relations proposed by Mikheev [2]. As is known, the form of these relations is as follows:

$$Nu_{free} = 1.18 (GrPr)^{1/8} \quad \text{for } 10^{-3} \leq GrPr \leq 500$$

$$Nu_{free} = 0.54 (GrPr)^{1/4} \quad \text{for } 500 \leq GrPr \leq 2 \times 10^7$$

It is not difficult to convince that for boundary values of the range  $1 \leq Gr \leq 10^5$  inadequacy in values of  $Nu_{free}$  according to the results of both authors reaches 100 per cent at  $Gr = 1$  ( $Nu_{free}$  to Juge twice as greater than  $Nu_{free}$ , following Mikheev) and only at  $Gr \rightarrow 10^5$  they converge. Thus, the principal divergence takes place at not high values of  $Gr$ .

It should be noted that already in the region of considerable values of  $Gr/Re^2$  the results on heat transfer at combined action of forced and free convection obtained by Juge do not agree with those of analogous experimental investigations following Izakhevsky [3]. Results of some experiments of Juge borrr50X1-HUMM his



paper are plotted in Fig. 1. Results of the series of Leitch's experiments ( $Gr = 35$  and  $Gr = 1500$ ) are shown by a dash line in Fig. 1 as well.

Extremely vast experimental material on free convection generalized by Mikheev makes us think that the relations proposed by him are rather cogent. Therefore, it is possible to assume that the divergence between data of Mikheev and Juge at exceeding values of  $Nu$  in data of the latter is caused by some methodical neglect in experiments with free convection.

Such conclusion is based on the fact that measurement methods and all measuring apparatus applied by Juge ensured proper accuracy since the results of experimental investigations on forced convection seem to coincide with the data of classical experiments of Prandtl [4] but at an increased value of the lower limit  $Re = 10$ .

Proceeding from the above mentioned one may draw a conclusion that self-movement of an air medium in the room where an experiment takes place is the only reason which causes heat transfer intensification with free convection and, consequently, increased values of  $Nu$  in Juge's experiments. At the same time there is no doubt that any noticeable motion of air in the room could be found and prevented. It makes us think that if air movement took place, then at any rate it was negligible by its absolute value. Thus, the question arises to what extent negligible self-movement of air may influence results of an experiment.

In order to develop this question, let us consider the results of generalizing investigations on heat transfer at combination of free and forced convection which were carried out by the author of the present paper [5].

According to these data two force zones should be distinguished

$$\begin{array}{l}
 \text{I} \left\{ \begin{array}{l} \frac{Gr}{Re^2} \approx \left\{ \frac{3.7}{\sqrt{Re} Pr^{1/4}} + 1.02 Pr^{1/12} \right\}^4 \\ \frac{Gr}{Re^2} \approx \left\{ \frac{1.7}{Re^{1/4} Pr^{1/8}} + 0.67 Re^{1/4} Pr^{5/24} \right\}^8 \end{array} \right. \quad \text{at} \quad 500 \leq GrPr \leq 2 \times 10^7 \\
 \\
 \text{II} \left\{ \begin{array}{l} \frac{Gr}{Re^2} \approx \left\{ \frac{3.7}{\sqrt{Re} Pr^{1/4}} + 1.02 Pr^{1/12} \right\}^4 \end{array} \right. \quad \text{at} \quad 500 \leq GrPr \leq 2 \times 10^7
 \end{array}$$

In a heat transfer process inertial forces prevail in the first of these zones and the gravitational in the second.

Calculation formulas are proposed for each of these zones at combined action of free and forced convection for the first zone:

$$Nu = Nu_{\text{forced}} \left[ 1 + c \frac{Re + Re_0}{Re} \left( \frac{Gr}{Re^2} \right)^{1/4} \right], \quad (1)$$

where  $c = 0.1$  following Jaga's experiments;  $c = 0.15$  as to Lyalikovsky (Fig. 2);

for the second zone:

$$Nu = Nu_{\text{free}} \left[ 1 + \left( \frac{Re^2}{Gr} \right)^{1/5} \right], \quad (2)$$

where

$$Nu_{\text{forced}} = 2 (1 + 0.276 \sqrt{Re} Pr^{1/3}),$$

$$Nu_{\text{free}} = 1.18 (GrPr)^{1/8} \quad \text{or} \quad Nu_{\text{free}} = 0.54 (GrPr)^{1/4}$$

$Re_0$  is determined at  $Nu_{\text{free}} = Nu_{\text{forced}}$  depending upon the value of  $GrPr$ .

From the viewpoint of determining the influence of movement of a surrounding medium on heat transfer in an experimental room heat transfer at the smallest value of  $Gr$  is of great interest. For this purpose when  $Nu_{\text{free}} = Nu_{\text{forced}}$  for  $Gr_{\text{min}} = 400$  the value of  $Re_0$  is determined by the equation:

$$1.18 (400 \times 0.71)^{1/8} = 2 (1 + 0.276 \sqrt{Re_0} \cdot 0.71^{1/3}),$$

and  $D = 6.3 \times 10^{-3} \text{ m}$  )  
 and  $v = 1.9 \times 10^{-5} \text{ m}^2/\text{sec}$

$$\bar{w} = \frac{Re_0 v}{D} = \frac{0.67 \times 1.9 \times 10^{-5}}{6.3 \times 10^{-3}} = 2.10^{-3} \text{ m/sec} = 2 \text{ mm/sec}$$

It means that heat transfer of a sphere with free convection at  $Gr = 400$  is numerically equal to that in air with extremely low movement. Hence it follows that the increased value of  $Nu$  obtained by  $Juge$  at  $Gr = 400$  is really determined by the influence of self-movement of air in the room and a heat transfer process itself proceeds when the influence of air movement prevail

The order of a value of this movement is determined by the equation:

$$Nu_{free}^{Juge} = 2 + 0.392 Gr^{1/4} = 2(1 + 0.276 \sqrt{Re} 0.71^{1/3}) \left[ 1 + 0.1 \frac{Re + 0.67}{Re} \left( \frac{400}{Re^2} \right)^{1/4} \right]$$

from which it follows that  $Re = 4.50$ . And actual movement is defined by the value:

$$\bar{w} = \frac{4.5 \times 1.9 \times 10^{-5}}{6.3 \times 10^{-3}} = 0.014 \text{ m/sec} = 1.4 \text{ cm/sec}$$

Evidently, the presence and influence of such movement were not assumed.

Thus, the order of a value of air movement in the room when testing was really negligible and, consequently, could not cause preliminary fears for the results of experiments. Meanwhile, if it is valid for bodies of surfaces with great sizes, then such movement considerably influenced experimental values of  $Nu$  by virtue of highly small sizes of spherical particles.

Having the value of  $Re = 4.50$  conforming to self-movement of air in the room it is shown that at  $Gr = 1800$  heat transfer proceeded when the influence of free convection prevailed.

For this air it should be noted that:

$$\frac{Gr}{Re^2} > \left( \frac{3.7}{\sqrt{Re} Pr^{1/4}} + 1.02 Pr^{1/12} \right)^4$$

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Indeed, introducing values of  $Gr = 1800$ ,  $Re = 4.5$  and

$Pr = 0.71$  it is easy to define that

$$\frac{Gr}{Re^2} = \frac{1800}{4.5^2} = 89 > \left( \frac{3.7}{\sqrt{4.5} \cdot 0.71^{1/4}} + 1.02 \times 0.99 \right)^4 = 70$$

Now let us calculate the values of  $Nu$  with simultaneous existence of  $Gr$  and  $Re$  for values:  $Gr = 400$  and  $Re = 4.5$ ;  $Gr = 1800$  and  $Re = 4.5$ .

According to the above mentioned the first combination corresponds to the prevailing influence of forced convection while the second, to free convection.

Hence:

$$Nu_1 = 2 \left( 1 + 0.276 \sqrt{4.5} \cdot 0.71^{1/3} \right) \left[ 1 + 0.1 \frac{4.5 + 0.67}{4.5} \left( \frac{400}{20.25} \right)^{1/4} \right] = 3.75$$

$$Nu_2 = 0.54 (1800 \cdot 0.71)^{1/4} \left[ 1 + \left( \frac{4.5^2}{1800} \right)^{1/5} \right] = 4.55$$

Experimental values following Juge's formula ( $Nu_{free} = 2 + 0.392 Gr^{1/4}$ ):

$$Nu_1 = 2 + 0.392 \times 400^{1/4} = 3.75$$

$$Nu_2 = 2 + 0.392 \times 1800^{1/4} = 4.55$$

satisfy them, respectively.

Fig. 1 presents the results of experiments of Juge in coordinates  $Nu_{forced}^{-2}$  and  $Nu_{free}^{-2}$  in a pictorial form. The origin, equal to  $Nu - 2 = 0$ , physically at all does not conform to  $Nu - 2 = 0$  owing to the influence of self-movement of a surrounding medium in an experimental room. Therefore, in the origin the true value of  $Nu_{forced}^{-2}$  is equal to:

$$2 \left( 1 + 0.276 \sqrt{4.55} \cdot 0.71^{1/3} \right) - 2 = 3.06 - 2 = 1.06$$

but not zero. When on the absciss are located the values of  $Nu_{forced}$  calculated only with respect to a velocity of an artificial flow, and experimental values of  $Nu^{-2}$  located on the ordinate seem to be determined by three phenomena: self-movement of a medium, artificial movement and gravitational flow. Meanwhile, the values of  $Nu^{-2}$  on the ordinate must be functions of only two parameters of  $Gr$  and  $Re$ .

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Thus, Fig. 1 and particularly its left side does not present

Krischer [6] proposed to describe heat transfer at combined action of free and forced convection when they coincide in direction, with some resultant values of  $Re^*$  determined by the expressions:

$$Re^* = Re - \sqrt{1/2 Gr},$$

where  $Re$  is characterized by the velocity of a running flow.

Taking into account the fact that in accordance with Juge's experiments for spherical bodies heat transfer proceeds similarly when direction of both flows coincide and when they are reciprocally normal to each other one may characterize self-movement of air in an experimental room on the basis of the relation mentioned above.

For this aim the following equation is used:

$$Nu = 2 + 0.276 \sqrt{Re^*} Pr^{1/3} = 3.75$$

from which it follows that  $Re^* = 50$ .

And self-movement of a surrounding medium relative to the sphere 6.3 mm in diameter must be characterized by the value:

$$Re = 50 - \sqrt{1/2 \cdot 400} = 36$$

There is no doubt that Juge's experiments on heat transfer in a forced flow with self-movement of air at  $Re = 36$  could not coincide with Prasslins' experiments at  $Re = 10$ . Therefore, the relation proposed by Krischer is apparently inapplicable at small values of  $Re$  and  $Gr$  when viscosity to a great extent influences heat transfer.

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#### C o n c l u s i o n s

(1) Heat transfer equation with free convection for spherical bodies proposed by Juge was obtained when Juge's heat transfer conditions were not observed and, consequently, should not be used.

(2) Formulas (1) and (2) are given by the author for calculating heat transfer processes at combined action of free and forced

By two zones I and II.

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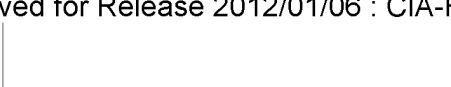


Fig.1

— Juge's experiments,  
--- Lyakhovskiy's experiments.

- $G_p=1800$
- △  $G_p=1540$
- ×  $G_p=1270$
- ⊙  $G_p=1060$
- $G_p=815$
- $G_p=600$
- ▲  $G_p=400$

Fig.2

— calculated curve at  $G_p=1800$  by formula (2)  
and by formula (1) at  $\alpha=0.1$ ,  
--- calculated curve at  $G_p=1500$  by formula (2)  
and by formula (1) at  $\alpha=0.15$ .

- Juge's experiments,
- Lyakhovskiy's experiments.

