CLASSIFICATION [redacted] **FOR OFFICIAL USE ONLY**

CENTRAL INTELLIGENCE AGENCY
INFORMATION FROM
FOREIGN DOCUMENTS OR RADIO BROADCASTS

REPORT
CD NO.

STAT

COUNTRY	USSR	DATE OF INFORMATION	1949
SUBJECT	Scientific - Electronics, wave propagation, modeling		
HOW PUBLISHED	Monthly periodical	DATE DIST.	7 Jun 1952
WHERE PUBLISHED	Moscow	NO. OF PAGES	10
DATE PUBLISHED	Feb 1951		
LANGUAGE	Russian	SUPPLEMENT TO REPORT NO.	

CENTRAL INTELLIGENCE AGENCY
CLASSIFICATION
Cancelled
Changed
BY AUTH

THIS IS UNEVALUATED INFORMATION

SOURCE *Elektrichestvo*, No 2, 1951, pp 50-55.

STAT

SOLUTION OF SOME BOUNDARY PROBLEMS OF THE ELECTROMAGNETIC FIELD BY MEANS OF TWO-DIMENSIONAL MODELS

Docent N. I. Shteyn, Cand Tech Sci
Moscow Power Eng Inst imeni Molotov
Submitted 14 Dec 1949

[Figures referred to in text are appended.]

(Author's abstract. In this paper, an analogy is brought out between the field equations and the equations characterizing the relationships between voltages and currents in a model. Examples are given of drawing up models for the solution of problems connected with the propagation of electromagnetic waves in wave guides and with the study of the electromagnetic field in cavity resonators).

INTRODUCTION

The accurate solution of some boundary problems involved in the study of processes occurring in the electromagnetic field in a two-dimensional, and especially a three-dimensional space, with certain boundary conditions is very difficult in practice.

The use of the finite difference method for the solution of equations describing these phenomena involves the solution of many equations with many unknowns, a difficult and time-consuming process.

It has been shown that some problems may be solved approximately by the electrical modeling method, using models developed by L. I. Gutenmakher [1, 2], consisting of passive inductive, resistive, and capacitive elements (this problem was discussed in a paper read by the author at the All-Union Conference on Electrical Modeling in 1949).

CLASSIFICATION [redacted]

STATE	X	NAVY	X	NSRB	DISTRIBUTION
ARMY	X	AIR	X	FBI	

FOR OFFICIAL USE ONLY

Sanitized Copy Approved for Release 2011/08/19 : CIA-RDP80-00809A000700070001-1

[redacted]

STAT

The desired components of the field under study can be found by measuring the voltage at the junction points of the model. But the distribution of voltage at the junction points of the model characterizes the scalar field of the space. Therefore, if the model is used to study a vector field, the latter must be resolved into components with respect to the axes of the coordinate system in such a manner that the numbers used to measure the components of the field vectors may be scalar. Consequently, in the solution of an actual vector field problem, one must perform as many cycles of voltage measurements at the junction points as are necessary to determine the components of the field. In solving a number of two-dimensional boundary problems which are frequently of great practical importance (for instance, in the study of magnetic waves of the type H_{on} and H_{onl}), it is necessary to perform these measurements three times, since the vector equations of the field are resolved in this case into three scalar components on the Cartesian coordinate system.

It can be shown, however, that the solution of boundary problems of the type under consideration can be effected by a single cycle of measurements for all components of electric and magnetic field intensities. In this method, the distribution of currents in the separate components of the model must be measured, as well as the voltage distribution at junction points.

Whinnery and Ramo [3], referring to the work of Kron [4], used two-dimensional electric circuits to solve certain problems of electromagnetic wave propagation in wave guides. Examination of the 1943 and 1944 publications on Kron's models shows them to be essentially no different from the models developed by L. I. Gutenmakher and described in a 1940 publication [1].

Analogy Between Electromagnetic Field Equations and Model Equations

In the case of isotropic and homogeneous media, assuming no charges and displacement currents, the field equations for magnetic waves of the type H_{on} and H_{onl} are

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{\varepsilon}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi}{c}\sigma E_x, \tag{1}$$

$$\frac{\partial E_x}{\partial z} = \frac{\mu}{c}\frac{\partial H_y}{\partial t}, \tag{2}$$

$$\frac{\partial E_x}{\partial y} = \frac{\mu}{c}\frac{\partial H_z}{\partial t}, \tag{3}$$

$$\operatorname{div}\bar{E} = \frac{\partial E_x}{\partial x} = 0, \tag{4}$$

$$\operatorname{div}\bar{H} = \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0,$$

$$E_y = E_z = H_x = 0; \frac{\partial H_y}{\partial x} = \frac{\partial H_z}{\partial x} = 0. \tag{5}$$

[redacted]

Sanitized Copy Approved for Release 2011/08/19 : CIA-RDP80-00809A000700070001-1

[redacted]

STAT

Transformation of expressions (1) - (5) makes possible the separation of the field components. Moreover, each component is known to satisfy the two-dimensional telegraphic equation of the form:

$$\nabla^2 v = \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \frac{\varepsilon \mu}{c^2} \frac{\partial^2 v}{\partial t^2} + \frac{4 \pi \mu \sigma}{c^2} \frac{\partial v}{\partial t} . \qquad (6)$$

Field equations for magnetic waves of the type H_{m0} and H_{m1} can be written in a similar manner. It must only be remembered that in this instance, the components $E_x = E_z = H_y = 0$. The other field components H_x, H_z, and E_y are independent of the y-axis.

Assuming no losses in the dielectric, the first derivative with respect to time drops out of equation (6), and the latter becomes the wave equation.

Let us consider Figure 1, showing a small portion of the electric model of the space. This portion of the model includes several junctions (0_1, 0_2, etc.). We shall designate the voltages at the junctions as u. The combination of impedances around any given junction point in the circuit of Figure 1 represents the electric model of the corresponding elementary parallelepiped with a volume $\Delta P = \Delta x \cdot \Delta y \cdot \Delta z$ (Figure 2) of the space for which the two-dimensional boundary problem under consideration is to be solved.

On the basis of Kirchhoff's first law, it can be shown by simple transformations that for an arbitrary junction of the model (for example, the point 0_1), the following relation holds

$$\frac{\Delta j_z}{\Delta z} + \frac{\Delta j_y}{\Delta y} = \frac{i_c}{\Delta P} = \frac{u}{z_c} \cdot \frac{1}{\Delta P} . \qquad (7)$$

The respective current densities are:

$$j_y = \frac{i_y}{\Delta x \cdot \Delta z} \text{ and } j_z = \frac{i_z}{\Delta x \cdot \Delta y} .$$

It is easily seen that the voltage drop in the arbitrary n-th element of the model in the direction of the y axis is

$$\Delta_y u = \sum_{k=1}^{n} \Delta_y u_k - \sum_{k=1}^{n-1} \Delta_y u_k = j_y z_y \Delta y , \qquad (8)$$

that is,

$$\frac{\Delta_y u}{\Delta y} = i_y z_y$$

[redacted]

[redacted]

and the voltage drop in the direction of the z axis is

$$\Delta_z u = \sum_{k=1}^{m} \Delta_z u_k - \sum_{k=1}^{m-1} \Delta_z u_k = j_z z_z \Delta z \, ,$$

that is,

$$\frac{\Delta_z u}{\Delta z} = j_z z_z ; \qquad (9)$$

where Z_z and Z_y are the linear impedances along the z and y-axes.

On the basis of Kirchhoff's second law, we can write for an arbitrary mesh of the model in the yz-plane:

$$\Delta_z (\Delta_y u) - \Delta_y (\Delta_z u) = 0. \qquad (10)$$

where $\Delta_z (\Delta_y u)$, for example, represents the voltage drop over a distance Δ_z in the direction of the z axis in the coupling elements of the model located along the y-axis.

In view of (8) and (9), we see from (10) that when $Z_y = Z_z$

$$\frac{\Delta j_y}{\Delta z} - \frac{\Delta j_z}{\Delta y} = 0. \qquad (11)$$

The voltage distribution in the junction points of the model, when $Z_y = Z_z = Z_o$, is described by the equation (2):

$$\nabla^2 u = \frac{1}{\Delta P} \frac{z_o}{z_c} u , \qquad (12)$$

where $\nabla^2 u$ represents the Laplace operator in the difference form.

By means of simple operations with expressions (7) and (11), it can be shown that distribution of current densities j_y and j_z in the model satisfies the conditions

$$\frac{\Delta^2 j_y}{(\Delta y)^2} + \frac{\Delta^2 j_y}{(\Delta z)^2} = \nabla^2 j_y = \frac{1}{\Delta P} \frac{z_o}{z_c} j_y , \qquad (13)$$

[redacted]

[redacted]

$$\frac{\Delta^2 j_z}{(\Delta y)^2} + \frac{\Delta^2 j_z}{(\Delta z)^2} = \nabla^2 j_z = \frac{1}{\Delta P}\,\frac{z_0}{z_c}\, j_y. \tag{14}$$

Let us assume

$$z_0 = R + Lp \quad \text{and} \quad \frac{1}{\Delta P \cdot z_c} = Cp + \frac{1}{R'},$$

wherein p is the Heaviside operator; R and L are linear parameters of the coupling impedances; and C and R' are specific parameters of the dircharge impedance.

In this instance, equations (12), (13), and (14) assume the following form:

$$\Delta^2 A = LC\frac{\partial^2 A}{\partial t^2} + \left(RC + \frac{L}{R'}\right)\frac{\partial A}{\partial t} + \frac{R}{R'}A \tag{15}$$

The analogy of equations (15) and (6) can be established by comparison. Their difference is merely that the left-hand members of the model equations are presented in the form of finite differences, while the left-hand members of the field equations are shown as partial derivatives. Moreover, the right-hand portion of the model equation contain an additional member $\frac{R}{R'}A$, which drops out when $R = 0$ and $R' = \infty$.

A comparison of equations (7) and (1), or of (11) and (5), shows that distribution of current density j_y and j_z in the model gives the values of the magnetic-field components H_z and H_y in the area of the space under study. Measurements of voltages u at the junctions of the model permit the determination of electric field intensity E_x in the modeled space.

Thus, by means of an electric model consisting of inductances, capacitance, and resistances combined in a definite way, the electromagnetic field of a space of any shape can be studied approximately whenever this study reduces to the solution of problems of the H_{on} and H_{onl} type. This requires first that initial and boundary conditions be imposed on the model corresponding to those of the modeled area, and second that the so-called similarity criteria are equal.

Field components corresponding to waves of the H_{mo} and H_{mol} type can be studied on a similar model. This requires only that the x-coordinate be substituted for the y-coordinate.

Electric waves of types E_{oi} and E_{oil}, or magnetic waves of type H_{oi} and H_{oil}, which can be obtained in cylindrical systems of circular cross section, are characterized by the presence of the following field components:

With waves H_{oi} and H_{oil} there are present field components E_ϕ, H_r, and H_z which are not dependent on the ϕ-axis. The remaining components $E_r = E_z = H_\phi = 0$.

[redacted]

[redacted]

STAT

With waves E_{oi} and E_{oil} there are present components E_r, E_z and H_ϕ, which also are independent of the ϕ-axis. The remaining components $E_\phi = H_r = H_z = 0$.

The proper model for the study of these waves is outwardly similar to the model shown in Figure 1. The difference consists in that here, the coupling impedances z and the discharge impedances z_c are not constant along the r-axis.

It can be shown that in a cylindrical coordinate system, the impedances z and z_c are inversely proportional to the r-coordinate. Using the method described previously for a Cartesian coordinate system, we find that in the case of cylindrical coordinate system, the equations of the model are of the form:

$$r\frac{\Delta j_z}{\Delta z} + \frac{\Delta j_r}{\Delta r} = r\frac{u}{z_c}\cdot\frac{1}{\Delta P}, \tag{16}$$

$$r\frac{\Delta_r u}{\Delta r} = j_z z_r, \tag{17}$$

$$\frac{\Delta_z u}{\Delta z} = j_z z_z. \tag{18}$$

By comparing the equations with the known field equations in a cylindrical coordinate system, it can be shown that for H_{oi} and H_{oil} waves, the distribution of current densities in the model, j_z and j_r, and the distribution of voltage values at junction points, u, give the corresponding approximate values of field components H_r, H_z, and E_ϕ.

For waves E_{oi} and E_{oil}, the field components E_r and E_z correspond to the current densities j_z and j_r in the model and the H_ϕ component corresponds to the values of voltage u at the junctions of the model.

Determination of Similarity Criteria

Methods for determining invariants or similarity criteria are well known [1, 2]. The corresponding equations of field and model must be expressed in relative dimensionless units by introducing the so-called fundamental units for all variables and physical constants contained in these equations.

The similarity criteria derived from the field equations and from the model equations must be identical. It is easy to show that when the Cartesian coordinate system is used, identity of corresponding similarity criteria has the form:

$$\left[\frac{\varepsilon_0\mu_0 l_0^2}{c_0^2 t_0^2}\right] = \left[\frac{L_m C_m l_m^2}{t_m^2}\right] \tag{19}$$

[redacted]

[redacted]

STAT

$$\left[\frac{\sigma_0 \mu_0 l_0^2}{c_0^2 t_0}\right] = \left[\frac{L_m l_m^2}{R'_m t_m}\right] \quad for \; R_m = 0. \tag{20}$$

The subscript o refers to physical parameters of the field (original), and m, to the parameters of the model; l is the fundamental length assumed equal for all coordinate axes.

In the electric model, the number of elements in any direction corresponds to length l_m, equal here to h, where

$$h = \Delta x = \Delta y = \Delta z.$$

The greater the number of elements n of the model per unit of length (which corresponds to a decrease of h), the more closely will the distribution of current densities in the components and of voltages at the junctions correspond to the values of the field components at the corresponding points of space. It is convenient to compute the nominal length l_0 in the unit of wave length of the process in the original, regardless of the actual wave length value. In this case, the nominal length l_m in the model per unit wave length l_0.

Identity of the similarity criteria similar to conditions (19) and (20) can also be found for a cylindrical-coordinate system.

<u>Establishing in the Model the Boundary Conditions of the Region of the Field Under Study</u>

The regions of the field under study are usually spaces partially or completely bounded by metallic surfaces (e.g., wave guides, cavity resonators, or coaxial lines) or spaces having some physical characteristics which differ sharply from the corresponding characteristics of the surrounding medium (e.g., dielectric wave guides and dielectric antennas).

Irrespective of the configuration of the region, the electromagnetic field within it is studied by solving Maxwell's equations with consideration for the boundary conditions.

If the metal surface is assumed ideal, the E vector must be perpendicular, and the H vector parallel to the metal surface, i.e., all tangential components of the electric field and all normal components of the magnetic field at any point on the wall surface must be zero. For example, in the case of type H_{on} and H_{onl}, waves $E_{xbound.} = H_{ybound.}$ Thus, to secure the boundary conditions for these wave types in the electric model, all junctions located on the corresponding boundaries of the model must be short-circuited. Thus, we will have

$$u_{bound.} = i_{zbound.} = 0$$

However, it is not particularly difficult to take the finite conductivity of the metal walls into account in the model. In this case, the metal may be considered as another medium having characteristics different from those of the medium forming the bounded region, and the surface of the metal walls is the surface of separation of the two media. A sudden change in the parameters

[redacted]

Sanitized Copy Approved for Release 2011/08/19 : CIA-RDP80-00809A000700070001-1

[redacted]

STAT

of the space occurs at the boundary. In a model, this problem is readily solved in the following manner: beginning at a certain definite section, the values of inductances, capacitances, and resistances of the model components and discharge elements are also changed suddenly, in accordance with conditions (19) and (20). The necessary number of elements of the mesh n', which models the metal walls, can be determined from the following considerations. If n is assumed to be the number of components per wave λ, then,

$$n' = n \frac{\delta}{\lambda} \tag{21}$$

wherein δ is the depth of current penetration into the metal.

The boundary conditions are set up on the same principle in the study by means of models of electromagnetic processes occuring in a dielectric wave guide where there are no metal surfaces. This case also reduces to two media differing in some of their characteristics, primarily in the value of permittivity ε.

The second medium here is not metal but air, in which the attenuation of electromagnetic waves radiating through the "walls" of the wave guide is practically absent (attenuation occurs at infinity). Hence, here, as in a number of other instances, we are confronted by the problem of modeling an unbounded space.

There is an analogy between the characteristic impedances of a boundary space and those of a line, since the termination of a line by an impedance equal to its characteristic impedance has the effect of making the line infinitely long. Therefore, it seems that the effect of an unbounded space could be modeled by connecting to certain points of a network modeling electromagnetic processes in a space impedances equal to the characteristic impedance of the network.

Examples of Setting Up Two-Dimensional Models

Figure 3 shows a two-dimensional model of the rectangular wave guide, illustrated in Figure 4. This model is useful for the study of H_{on} type waves, assuming no losses in the dielectric. The boundary conditions here are established in the following manner. All junction points located on the boundary line ab are short-circuited and grounded. Junction points located on line gc are connected to a source of alternating emf whose frequency is computed from the fact that the field and model equations must be invariant. Resistances modeling the free space are connected to junction points along line ef. Line cd is the line of symmetry. The same model may be used to study type H_{mo} and H_{oi} waves. For H_{mo} waves, the y-axis must be replaced by the x-axis. For H_{oi} waves, the y-axis must be replaced by the r-axis, and in addition, the inductance of the model components and the capacity of the discharge elements vary as a function of r along the r-axis. In this case, Figure 3 would be a model of the cylindrical wave guide of circular cross section illustrated in Figure 5. The same applies to Figure 6, which is a model of a wave guide shown in Figure 5 for the study of waves of the E_{oi} type. The models shown in Figure 3 and 6 differ only in that boundary junction points on line ab are not short-circuited in the second case.

Figures 7 and 8 show models of a cylinder of circular cross section for the study of H_{oi1} and E_{oi1} type waves, respectively, assuming no losses in the dielectric. In these models, the inductances and capacitances are also variables in the direction of the r-axis.

[redacted]

Sanitized Copy Approved for Release 2011/08/19 : CIA-RDP80-00809A000700070001-1

[redacted]

Many other examples of models could be cited, but it is assumed that the systems described are sufficient to clarify the principles of this particular modeling method.

In conclusion, the author would like to thank Professor L. I. Gutenmakher for his valuable advice.

BIBLIOGRAPHY

1. Gutenmakher, L. I., "Elektricheskoye modelirovaniye fizicheskikh yavleniy" (Electric Modeling of Physical Phenomena), Elektrichestvo, No 5, 1950

2. Gutenmakher, L. I., Elektricheskiye modeli (Electric Models), publication of the Academy of Sciences USSR, 1949

3. Whinnery, J. R., and S. Ramo, "A New Approach to the Solution of High Frequency Field Problems," Proc. Inst. Radio Eng., No 5, 1944

4. Kron, Gabriel, "Equivalent Circuit of the Field Equations of Maxwell" Proc. Inst. Radio Eng., No 5, 1944

5. Krug, K. A., Osnovy elektrotekhniki (Fundamentals of Electrical Engineering) Vol II, Gosenergoizdat, 1932

[Appended figures follow.]

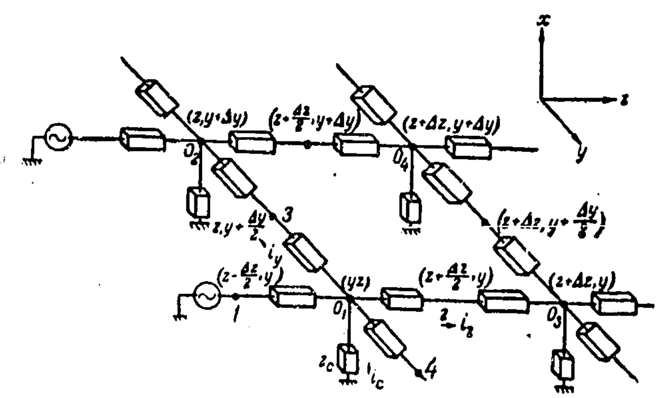

Figure 1. Section of an Electric Model

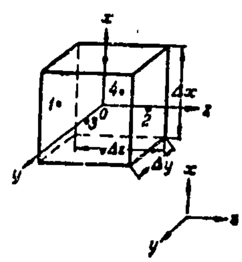

Figure 2. An Elementary Parallelepiped of the Space

[redacted]

[redacted]

STAT

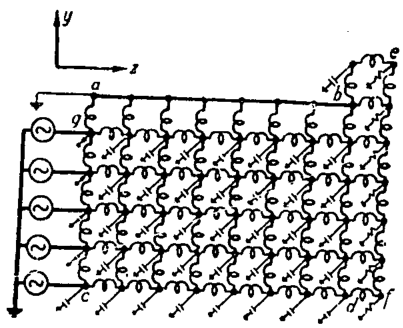

Figure 3. A Two-Dimensional Electric Model of a Wave Guide

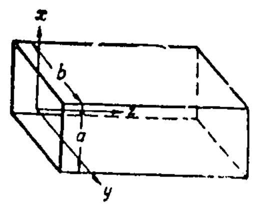

Figure 4. A Rectangular Wave Guide

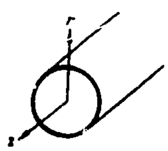

Figure 5. A Cylindrical Wave Guide of Circular Cross Section

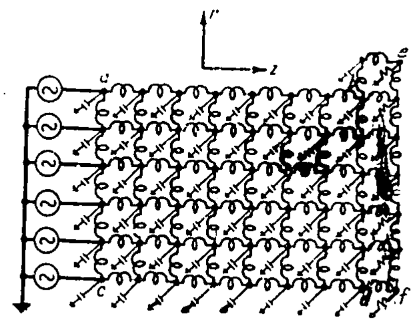

Figure 6. A Two-Dimensional Electric Model of a Cylindrical Wave Guide of Circular Cross Section for the Study of Type E_{0i} Waves

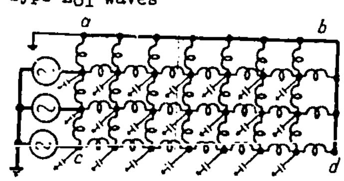

Figure 7. A Two-Dimensional Model of a Cylinder of Circular Cross Section for Study of Type H_{0i1} Waves

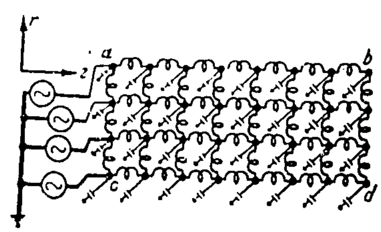

Figure 8. A Two-Dimensional Model of a Cylinder of Circular Cross Section for the Study of Type E_{0i1} Waves

- E N D -

[redacted]