CLASSIFICATION SECRET SECRET

50X1-HUM

CENTRAL INTELLIGENCE AGENCY
INFORMATION FROM
FOREIGN DOCUMENTS OR RADIO BROADCASTS

REPORT
CD NO.

COUNTRY	USSR	DATE OF INFORMATION	1947
SUBJECT	Scientific - Radio, microwaves	DATE DIST.	18 Sep 1950
HOW PUBLISHED	Bimonthly periodical		
WHERE PUBLISHED	Moscow	NO. OF PAGES	15
DATE PUBLISHED	Mar/Apr 1947		
LANGUAGE	Russian	SUPPLEMENT TO REPORT NO.	

THIS IS UNEVALUATED INFORMATION

SOURCE Izvestiya Akademii Nauk SSSR, Seriya Fizicheskaya, Vol XI, No 2, 1947, [redacted]

50X1-HUM

RADIATION OF MICROWAVES AND THEIR ABSORPTION IN AIR

V. L. Ginzburg
Physical Inst imeni P. N. Lebedev
Acad Sci USSR and Gor'kiy U

As is known, the problem of building a sufficiently powerful and reliable oscillator for microwaves of a given spectral composition in the wave length range $\lambda = 0.01$-1 cm has not as yet been solved.

Klystrons make it possible to obtain waves with $\lambda \sim 1$ cm, but further decrease in wave length is limited by the necessity for still further decrease in the dimensions of cavity resonators whose volume, generally speaking, is $\sim \lambda^3$; but this decrease also leads to a drop in power. However, combining klystrons with crystal detectors, which permit frequency multiplication because of their nonlinearity, does make it possible to work in the millimeter range [1], but not lower. Single-anode magnetrons permit propagating waves [2] with frequencies less than

$$\omega_0 = \frac{eH}{mc} = 1.76\cdot10^7 H; \quad \lambda_0 = \frac{2\pi c}{\omega_0} = \frac{1.07\cdot10^4}{H}.$$

Even in a very strong field ($H = 3\cdot10^4$ gausses) $\lambda_0 = 3.56$ mm, so that it is impossible to get wave lengths below a few millimeters by this method. Oscillations with higher frequencies can be obtained in magnetrons with split anodes, but even here the practical limit of the wave length is $\lambda = 1$ mm. Thus, the methods of generating radio waves now in use cannot go lower than the millimeter range or even, in general, the centimeter range.

Infrared spectroscopic methods can trace the wave-length spectrum of a mercury-quartz lamp up to $\lambda = 0.04$ cm [3]. The spectrum of a mass radiator extends from $\lambda = 0.01$ cm to centimeter waves [4]. However, in view of the extremely small power and for many other reasons, the usefulness of a mass radiator, even for laboratory purposes, is very limited. (There has, undoubtedly, been a lag in studying the properties of substances in the micro-wave

CLASSIFICATION SECRET SECRET

STATE	X	NAVY	X	NSRB		DISTRIBUTION
ARMY	X	AIR	X	FBI		RDB X

field compared with the technical possibilities of generating these waves. Research on the behavior of superconductors in the millimeter wave range are of great scientific interest [5]. However, just recently [6] measurements have been made with $\lambda = 3$ cm. Meanwhile, it is obviously possible to determine the coefficient of reflection of a superconductor for the whole range which interests us by using a mass radiator; in any case, by employing modern methods such measurements are feasible for $\lambda \lesssim 1$ cm.)

The purpose of this article is to discuss certain unexploited possibilities of generating microwaves ($\lambda \lesssim 1$ cm). Section 1 discusses the problem of exciting molecular spectra lying in the microwave range. The study of microwaves as a result of employing relativistic electrons is treated in Sections 2 and 3. Sections 4 and 5 investigate the possibility of studying microwaves by means of nonrelativistic electrons moving near a dielectric.

Section 1

The rotation spectra of the majority of molecules lie in the range of microwaves; therefore, a "tube" in which the rotation level of the molecules is somehow excited will be a source of radio radiation. The energy radiated by such a tube in a second equals

$$U = \frac{64\pi^4 \nu^4_{nm}}{3c^3} \left|\vec{P}_{nm}\right|^2 N_n \approx 0.78\cdot 10^{-28} \nu^4_{nm} \left|\vec{P}_{nm}\right|^2 N_n \approx 6.3\cdot 10^{13} \frac{\left|\vec{P}_{nm}\right|^2}{\lambda^4_{nm}} N_n, \quad (1)$$

where $\nu_{nm} = \frac{c}{\lambda_{nm}}$ is the radiated frequency; $\left|\vec{p}_{nm}\right|^2$ the square of the matrix element of the dipole moment corresponding to the investigated transition $n \rightarrow m$; and N_n the number of excited molecules (molecules on the level n) [7, 8].

The magnitude $\left|\vec{P}_{nm}\right|^2 \sim p_0^2$, where p_0 is the constant electrical dipole moment of a molecule, for instance in the case of a simple rotator [7], is equal to

$$\left|\vec{P}_{J,J-1}\right|^2 = P_0^2 \frac{J^2}{J(2J+1)}, \quad \nu_{J,J-1} = 2B_0 J = \frac{2h}{8\pi^2 I} J,$$

where $J\frac{h}{2\pi}$ is the moment of the amount of rotor movement, J = 0, 1, 2, 3, ..., for the transition $\Delta J = \pm 1$, that is, n = J and m = J-1; I is the moment of inertia of the rotator. Since $p_0 \sim 10^{-18}$,

$$U \sim \frac{10^{-22} N_n}{\lambda^4} \quad (2)$$

At room temperature, molecules are found on many rotation levels and various transitions take place all the time. Of course, in the presence of thermal equilibrium, the tube with molecular gas radiates as much energy as it absorbs from its surroundings. Therefore, N_n in (1) and (2) must be assumed to be the excess number of molecules compared with the number during thermal equilibrium with the surrounding medium. Because of the expansion of the pressure lines in the tube, it is scarcely possible to obtain a higher-than-atmospheric pressure; under these conditions, increasing the number of molecules on excited levels by means of heating or any other method will hardly permit making N_n greater than 10^{17} per cubic centimeter. Thus, it is reasonable to assume that, for all tubes, $N_n \lesssim 10^{18}$. In this event, $U \sim 10^{-4}$ erg/sec when $\lambda = 1$ cm; $U \sim 1$ erg/sec when $\lambda = 1$ mm and $U \sim 10^4$ ergs/sec when $\lambda = 10^{-2}$ cm = 100 microns.

We can see that in the range which interests us, where $\lambda > 100$ microns, radiation of microwaves through excitation of molecular spectra cannot lead to powers greater than thousandths of a watt; we can speak only of microwatts, or less, when $\lambda \gtrsim 1$ mm. Such powers are, obviously, of interest only for laboratory purposes. Radiation of microwaves by molecules, in particular, can be used as an ideal frequency standard for centimeter, and shorter, waves where very exact measurements and stabilization of frequency would be difficult. Frequency stability for molecular radiation, or, more precisely, for the frequencies of maximum intensity of a rotational line possessing some width, is ideal. Of course, molecular spectra can be used as a standard not on' by "receiving" the radiation of molecules but also by absorbing them; . r example, if gas is placed in an "endovibrator" [cavity resonator ?], when the oscillation frequency coincides with ν_{nm} for gas molecules, the damping factor of the system increases abruptly.

Section 2

The most interesting and promising method of generating microwaves is based on utilization of fast-moving radiators. As we know, if there is some system (an atom, antenna, or oscillatory electron) radiating (in the system of coordinate axes in which it rests) electro-magnetic waves with a frequency ν_0, a Doppler effect will occur when the system moves and it will radiate frequencies

$$\nu(\theta) = \frac{\nu_0 \sqrt{1-\beta^2}}{1-\beta\cos\theta}, \qquad (3)$$

where $\beta = \frac{v}{c}$; v is the velocity of the system; θ, the angle between $\vec{v}$ and the direction of observation.

When the radiator speed is great, energy will be radiated chiefly in a forward direction, and $\nu \gg \nu_0$.

Form the general viewpoint, the simplest method of using the Doppler effect for frequency "multiplication" is as follows: an electron, moving at high speed, is placed in an electric field which forces it to oscillate in a direction perpendicular to its velocity (we introduce the latter hypothesis only for greater definiteness) [10]. Let the electron move along the y-axis with a velocity $v = \beta c$; moreover, in the direction of the x-axis, let there be an electric field $E = E_0 \cos \omega_0 t$, oscillating the electron in the direction of x-axis (Figure 1). Then, by virtue of the Doppler effect, at an angle θ to the velocity, the electron will radiate frequencies (it is easy to see that the factor $\sqrt{1-\beta^2}$ does not appear in this case):

$$\omega(\theta) = \frac{\omega_0}{1-\beta\cos\theta} \qquad (4)$$

When $\theta = 0$ and $\frac{W}{mc^2} = \frac{1}{\sqrt{1-\beta^2}} \gg 1$, we have

$$\omega(0) = \frac{\omega_0}{1-\beta} \simeq 2\,\omega_0 \left(\frac{W}{mc^2}\right)^2, \quad \lambda(0) = \frac{2\pi c}{\omega(0)} \simeq \frac{\lambda_0}{2}\left(\frac{mc^2}{W}\right)^2. \qquad (5)$$

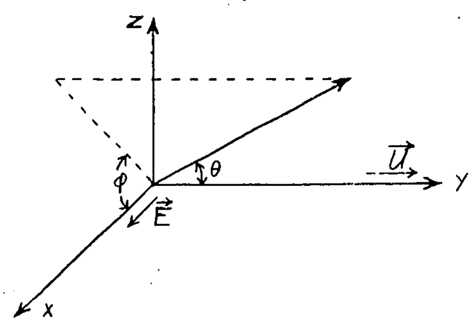

Figure 1

Thus, an electron oscillating in the described manner will "convert" the frequency, increasing it $2\left(\frac{W}{mc^2}\right)^2$ times when $\theta = 0$ (for example, when $\frac{W}{mc^2} = 10$, i.e., $W = 5$ MeV, oscillations with $\lambda = 20$ cm are converted into radiation with $\lambda = 0.1$ cm).

The intensity of radiation in the solid angle $d\Omega$ equals

$$\mathcal{E}(\theta) = \frac{\omega_0^4 p_0^2 \cdot l \cdot \{(1-\beta\cos\theta)^2 - \sin^2\theta\cos^2\varphi(1-\beta^2)\}\, d\Omega}{8\pi\beta c^4(1-\beta\cos\theta)^5}, \tag{6}$$

where $p_0 = ex_0$ is the amplitude of the electrical moment produced by the field E and is equal to the electron charge multiplied by the amplitude of its displacement along the x-axis; l the path traversed by the electron; φ, the angle shown in Figure 1.

It is easy to see that, providing

$$\frac{W}{mc^2} \gg 1, \text{ i.e., } 1-\beta \ll 1, \tag{7}$$

practically all the radiation will be concentrated in small angles, less than

$$\theta_0 \sim \sqrt{2(1-\beta)} \simeq \frac{mc^2}{W}. \tag{8}$$

The total radiated energy under condition (7) equals

$$\mathcal{E} = \frac{\omega_0^4 p_0^2 l}{3c^4}\left(\frac{W}{mc^2}\right)^4 = \frac{1}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{W}{mc^2}\right)^2 l E_0^2, \tag{9}$$

(for the general case,

$$\mathcal{E} = \frac{\omega_0^4 p_0^2 l}{3\beta c^4(1-\beta^2)^2} = \frac{\omega_0^4 p_0^2 l}{3\beta c^4}\left(\frac{W}{mc^2}\right)^4 \qquad (9'))$$

where we find that by virtue of the equation of motion, when $x \ll v = \beta c$, the amplitude of the electron's oscillation is

$$x_0 = \frac{p_0}{e} = -\frac{eE_0}{m\omega_0^2}\left(\frac{mc^2}{W}\right). \tag{10}$$

SECRET

SECRET

50X1-HUM

In order for the above formula to hold good, we must maintain the disparity

$$x_0 \ll \lambda(0), \tag{11}$$

that is, the field E must not be too strong.

If we have not just one electron, but a cluster to which the current J corresponds, during radiation by all the electrons, independently of each other, the energy generated in one second will be

$$U = \mathcal{E} \cdot \frac{J}{c} = 1.82 \cdot 10^{-12} \left(\frac{W}{mc^2}\right)^2 E_0^2 \cdot J \cdot l \text{ ergs/sec} \tag{12}$$

where J is measured in amperes, E_0 in volts per cm, l in cm. When $J = 10^{-2}$ amp, $E_0 = 10^4$ v/cm, $l = 10^2$ cm, and $\frac{W}{mc^2} = 10$, $U = 1.8 \cdot 10^{-2}$ erg/sec; when $J = 10^{-3}$ amp, $E = 10^3$ v/cm, $U = 1.8 \cdot 10^{-5}$ erg/sec.

It is clear from the above examples and formulas that no great powers can be obtained in the incoherent radiation of electrons. To obtain sufficient power, electrons must be bunched. If the dimensions of the bunch d are less that the radiated wave length λ, the electrons forming part of the bunch will radiate coherently, that is, the bunch will radiate as a whole, and the multiple ν will appear in (12); where ν will be the number of electrons in the bunch -- an obvious instance of this fact is pointed out by L. I. Schiff [8]. (If $d \ll \lambda$, the bunch radiates like a "particle" with a charge $e\nu$, and thus the multiple ν^2 will appear in (9). Since $J = e\nu N$, where N is the number of bunchings per second, the multiple ν will be added in (12).) Utilizing bunching permits a limited increase in power with the same average current J. Thus, if $J = e\nu N = 10^{-2}$ amp and $N = 10^7$ bunches fly past in a second, ν will equal $6 \cdot 10^9$ and, in the above example, $U = 1.8 \cdot 10^{-2} \cdot 6 \cdot 10^9 \simeq 10^8$ ergs/sec = 10 w. When $\frac{W}{mc^2} = 50$ (W = 25 MeV), U will equal 250 w under the same conditions; moreover, all radiation will be enclosed in the angle $\theta_0 \sim 1$ degree.

As an example, let us demonstrate the dependence of the radiated energy on the dimensions of the bunch, assuming that the latter is sharply limited and has the form of a parallelepiped with the dimensions a, d, b along the x, y, z axes. In this case, we shall have for the radiation in the direction lying on the plane y, z

$$\mathcal{E} = \mathcal{E}_0 \left(\frac{\sin\left(\frac{\pi b}{\lambda(\theta)} \sin\theta\right)}{\frac{\pi b}{\lambda(\theta)} \sin\theta} \right)^2 \cdot \frac{\sin \frac{\pi d}{\lambda(\theta)} \cos\theta}{\frac{\pi d}{\lambda(\theta)} \cos\theta} \tag{13}$$

where $\mathcal{E}_0$ is the intensity of the bunch radiating as a whole, that is, when $\frac{\pi b}{\lambda} \sin\theta \ll 1$ and $\frac{\pi d}{\lambda} \cos\theta \ll 1$.

If individual bunches are found at greater accurately fixed distances than λ, the radiation of these bunches will also interfere. The intensity in this case will be determined by the formula from the theory of diffraction for several slits. If k bunches are simultaneously present on the path l at a distance $n\lambda(0)$ from each other ($n = 1, 2, 3, \ldots$), the intensity will increase k times compared with the case of incoherency in the radiation of individual bunches. In practice, however, little is gained by dealing with several bunches simultaneously (for example, when $N = 10^7$ and $l = 10^2$, $k = \frac{lN}{c} = \frac{1}{30}$).

SECRET

SECRET

Forming bunches is a special, and obviously, a difficult task. So far as fast electrons are concerned, the development of electron accelerator techniques (the betatron, synchrotron, etc. [8]) permits us to hope that in the near future obtaining particles with $W \sim 5 - 25$ MeV will no longer be a problem.

Section 3

In the system examined above, an electron cluster traversing the path l must either be lost or turned back with the aid of a special device, for example, a magnetic mirror. In many respects, it is more suitable to use, as a source of radiation, an electron moving in a circle in a magnetic field H. In this instance, there is no special need of producing oscillations in the electron about its trajectory, as its peripheral motion is in itself an accelerating process and hence the electron radiates. Recently, this radiation was studied in connection with the theory of electron accelerators [8, 9].

Under condition (7), per unit of path the electron radiates energy

$$\mathcal{E} = \frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2 H^2 \left(\frac{W}{mc^2}\right)^2, \tag{14}$$

and after one rotation it radiates energy

$$\mathcal{E}_R = \frac{4\pi}{3}\cdot\frac{e^2}{R}\left(\frac{W}{mc^2}\right)^4 \tag{15}$$

where $R = \frac{c}{\omega_0} = \frac{W}{eH}$ is the orbit radius.

If the strength of the current equals J, the energy radiated per second will be

$$U = \mathcal{E}\cdot c\cdot\frac{2\pi\cdot J}{e\,\omega_0} = \mathcal{E}\cdot 2\pi R\cdot\frac{J}{e} = \frac{4\pi}{3}\left(\frac{e^2}{mc^2}\right)H\left(\frac{W}{mc^2}\right)^3 J = 3.5\cdot 10^{-3}\cdot H\left(\frac{W}{mc^2}\right)^3 J \text{ ergs per sec}, \tag{16}$$

where, in the last term, J is measured in amperes (H, always in gausses). When $\frac{W}{mc^2} = 10$, $H = 10^4$ gausses and $J = 10^{-2}$ amp, $U = 350$ ergs/sec.

The conversion frequency of electrons along the orbit equals

$$\omega_0 = \frac{eH}{mc}\left(\frac{mc^2}{W}\right) = \frac{c}{R} = \frac{2\pi c}{\lambda_0}; \quad \lambda_0 \simeq \frac{10^4}{H}\left(\frac{W}{mc^2}\right). \tag{17}$$

In the nonrelativistic case $\omega_0 = \frac{eH}{mc}$, and only this frequency ω_0 is radiated. But in a relativistic case all overtones (harmonics) of the frequency ω_0 are also radiated. Moreover, the radiation is concentrated in the plane of the orbit (in the region of the angles determined by formula [8], and the maximum intensity is converted [9] into an overtone of the order of $\left(\frac{W}{mc^2}\right)^3$, i.e., to the frequency

$$\omega_{eff} \sim \omega_0\left(\frac{W}{mc^2}\right)^3, \quad \lambda_{eff} \sim \lambda_0\left(\frac{mc^2}{W}\right)^3 \simeq \frac{10^4}{H}\left(\frac{mc^2}{W}\right)^2. \tag{18}$$

When $\omega < \omega_{eff}$ the intensity of radiation increases slowly (from ω_0 to ω_{eff}), but when $\omega > \omega_{eff}$, the intensity drops sharply.

It is easy to understand formula (18) and to obtain it on the basis of the Doppler effect discussed in Section 2. A fast moving electron, moving in a magnetic field, is accelerated in a direction perpendicular to

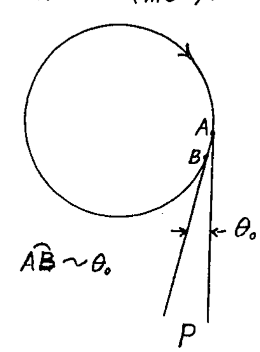

Figure 2

the orbit and radiates in a narrow cone ($\theta \lesssim \theta_0$) in the direction of its instantaneous velocity; from the viewpoint of a motionless observer at the point P (Figure 2), the electron radiates only as long as it is on the segment of the circumference AB, during which time the arc AB $\sim \theta_0$. Thus, the observer sees, so to speak, a "flash" with a duration $\sim \frac{R\theta_0}{c} = \frac{\theta_0}{\omega_0}$, which is repeated every period, i.e., with a frequency $\omega_0/2\pi$. Resolving such a field into a Fourier series, we obtain all the overtones of the frequency ω_0, with the maximum at a frequency $\omega_{eff} \sim \frac{\omega_0}{\theta_0}\left(\frac{W}{mc^2}\right)^2 \sim \omega_0 \left(\frac{W}{mc^2}\right)^3$ where the multiple $\left(\frac{W}{mc^2}\right)^2$ appears by virtue of the fact that the radiator is in motion (see formula (5)). In other words, the role of the frequency ω_0 in (5) is played here by the frequency $\frac{\omega_0}{\theta_0}$, where ω_0 is determined in accordance with (17). When $\frac{W}{mc^2} = 10$, the maximum will take place at the 1,000th overtone; and when $H = 10^4$, $\lambda_{eff} = 10^{-2}$cm. In the example discussed above with U = 350 ergs/sec, for the maximum overtone the energy will reach $U_{eff} \sim 0.3$ erg/sec. Utilization of bunching with $\nu = 6 \cdot 10^9$ gives $U \simeq 200$ kw and $U_{eff} \sim 200$ w, whereby we assume that a bunch coherently radiates waves with $\lambda \gtrsim \lambda_{eff}$.

The advantage of a system with a magnetic field is that, in principle, all the energy transmitted by the electrons can be used for radiation. However, the whole frequency spectrum is obtained, and if one is interested in quasi-monochromatic radiation, then in the example cited the efficiency is $\sim \frac{1}{1,000}$. In rectilinear radiation (Section 2) the power of a cluster equals $10^{-2} \cdot 5 \cdot 10^6 = 5 \cdot 10^4$w, U = 10 w; hence the efficiency $= \frac{1}{5,000}$ (the radiation is immediately quasi-monochromatic). Further, in the case described in Section 2, all radiation is in the forward direction and is distributed in the magnetic field in the whole orbit plane; for $\frac{W}{mc^2} = 10$ in this connection we obtain the multiple $\frac{2\pi}{\theta_0} \sim 60$. Moreover, using a mirror (metal) can change the spectrum and the pattern of the radiator's directivity in the magnetic field, but in the case of a rectilinear radiator the efficiency can be made to rise sharply by turning the cluster back. From preceding statements it is clear that it is impossible to decide a priori what type of radiator will be most advantageous -- the decision depends on both technical and structural problems and on the objectives (whether a narrow or wide spectrum is required, etc.).

Section 4

Generation of microwaves based on utilization of relativistic electrons is closely connected with the problem of constructing electron accelerators and with a number of other unsolved problems. But there is a possibility, at least in principle, of obtaining these results with nonrelativistic electrons [10]. The fact is that if an electron moves, not in a vacuum, but in a medium with a refractive index n, then βn will take the place of β in the formulas, i.e., the relation of the velocity of an electron to the phase velocity of light in the medium is $\frac{c}{n}$. Therefore, instead of (4), let us take the formula for the Doppler effect in a medium (see, for exampke, I. M. Frank [11]):

$$\omega(\theta) = \frac{\omega_0}{|1 - \beta n \cos\theta|} \quad . \tag{19}$$

Further,

$$\omega(\theta) = \frac{\omega_0}{|1 - \beta n|}, \quad \lambda(\theta) = \lambda_0 |1 - \beta n|, \tag{20}$$

i.e., the place of $\left(\frac{W}{mc^2}\right)^2$ is taken by $\frac{1}{2|1-\beta n|}$; the case where $\beta n = 0.995$ or $\beta n = 1,005$ (if $\beta n > 1$, then, beside the radiation discussed in this paragraph, the Cherenkov effect will also take place (see Section 5)) corresponds to our example with $\frac{W}{mc^2} = 10$

Of course, if an electron moves in a medium, it is greatly retarded and traverses only a very short path. For this reason, at first glance it would seem impossible to make use of the Doppler effect in a medium to generate microwaves. This, however, is not true, since, if an electron moves near a medium, not in it, at a distance from it which is far less than the radiated wave length, its radiation will be the same as in motion in the medium (this observation was made by Academician L. I. Mandel'shtam; see his calculations [12]). Thus, an electron can move in a slit between two dielectrics or simply over a dielectric plate, the surface of which coincides with the surface xy in Figure 1. For an electron moving along the axis of a cylindrical channel (in a vacuum channel) with a radius r, when $\theta = 0$, the intensity of radiation will be less than when the channel is absent by the multiple

$$\rho = \frac{1}{\left[I_0\left(\frac{2\pi\sqrt{n^2-1}\cdot r}{\lambda}\right)\right]^2}, \tag{21}$$

where $I_0(x) = J_0(ix)$; J_0 is Bessel's zero function.

The relationship of ρ and $\delta = \frac{2\pi\sqrt{n^2-1}}{\lambda} r$ is evident from the following table:

δ	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
ρ	1	0.9	0.8	0.37	0.19	0.093	0.042	0.0185	0.0078	0.0033

It is clear from (21) and the table that, when $r \lesssim 0.1 \frac{\lambda}{n}$, the channel has pracically no effect on the intensity.

If an electron moves in a slit with a width h, the intensity of radiation when $h \lesssim 0.1 \frac{\lambda}{n}$ is approximately the same as when there is no slit. When there is one dielectric surface and the electron moves at a distance $h \ll \frac{\lambda}{n}$ from it, the intensity of radiation may be several times less than in motion in a medium (a more precise definition of the proper coefficient requires special calculation). Covering the surface with a semitransparent, conducting layer (whose thickness is considerably less than that of a "skin layer") also weakens the intensity, but the essentials and the order of magnitude do not change.

Let us give the formulas for the intensity relating to the case of movement in a medium (practically the case of a narrow slit). Instead of (6), (9), and (12) we shall have respectively (see I. M. Frank [11]):

$$\mathcal{E}(\theta) = \frac{\omega_0 P_0^2 \cdot n \cdot 2\{(1-\beta n\cos\theta)^2 - \sin^2\theta\cos^2\varphi(1-\beta^2 n^2)\}\,d\Omega}{8\pi c^4\beta\,|1-\beta n\cos\theta|^5} \tag{22}$$

$$\mathcal{E} = \frac{\omega_0 \cdot P_0^2 \cdot l \cdot n}{3c^4\beta(1-\beta^2 n^2)^2} = \frac{1}{3}\left(\frac{e^2}{mc^2}\right)^2 \frac{l \cdot n \cdot E_0^2(1-\beta^2)}{\beta(1-\beta^2 n^2)^2}, \tag{23}$$

$$U = 1.82 \cdot 10^{-12} \cdot \frac{l \cdot n \cdot J \cdot E_0^2(1-\beta^2)}{\beta(1-\beta^2 n^2)^2} \text{ ergs/sec} \tag{24}$$

In (24), J is measured in amperes, E_0 in volts per centimeter. Practically all radiation is concentrated in angles less than the angle

$$\theta_0 \sim \sqrt{2|1-\beta n|} \tag{25}$$

Nowadays there are high-frequency ceramic dielectrics with ε up to 60 and small losses ($tg\ \delta < 0.001$)(3). It is true that these figures pertain to much lower frequencies than those in which we are interested. However, it is difficult to anticipate [anomalous] dispersion of ε and tg δ when $\lambda > 0.001$ cm; thus, we may assume, as we did above, that suitable materials exist with $n = \sqrt{\varepsilon} \sim 7$-8, and without appreciable dispersion. If $n = 7$, $\beta n = 1$ when $V = 5{,}000$ eV for the energy of the electrons; if $n = 4$, $\beta n = 1$ when $V = 16$keV, and if $n = 2$, $\beta n = 1$ when $V = 100$ keV. Thus, frequency conversion with the aid of the suggested use of the Doppler effect in a medium may be feasible at total voltages of some thousands or tens of thousands of volts, but market relativistic effects would require a million volts. Moreover, the radiation intensity in the case under (24) is considerably greater than that under (12). For instance, if as in the example given in Section 2, we assume that in (24) $E_0 = 10^4$, $J = 10^{-2}$ and $\tau = 10^2$, and also assume that $n = 4$ and $\beta n = 0.995$ (for the same frequency change as when $\frac{W}{mc^2} = 10$), U will equal 27 ergs/sec, or 1,500 times more than in formula (24). Nevertheless, as is clear from this figure and from formula (24), the existing increase in power is possible only if bunching is used. When $\nu = 6 \cdot 10^9$ in the example cited, $U = 16$ kw.

If we assume that $E_0 = 10^3$ v/cm -- which is also necessary to satisfy condition (11) -- U will equal 160 w. Let us take it for granted that the presence of a slit reduces the value of U to 16 w (this energy is obtained from the generator which produces the accelerating field E). Now, if we calculate the efficiency, relating the radiated energy to the energy of a cluster, in this example the efficiency will be equal to 1/10, since the energy of a cluster equals $16 \cdot 10^3 \cdot 10^{-2} = 160$ w. It is evident that the efficiency could be made greater than 1/10, and likewise, considerably greater than in the examples in Sections 2 and 3. The dependence of intensity on the dimensions of a bunch was determined by formula (13) (for a bunch in the form of a parallelepiped) substituting $\frac{\lambda}{n}$, the wave length in a medium, for λ. Thus, to enable a bunch to radiate as a whole, when $\theta = 0$, it is necessary to satisfy the condition

$$\frac{\lambda}{n} \gg d. \qquad (26)$$

In practice, the radiation of bunched electrons is coherent for

$$\frac{\lambda}{n} \gtrsim d \left(\text{thus, when } \frac{\lambda}{n} = 2d,\ \mathcal{E} = 0.4\,\mathcal{E}_0\right).$$

To obtain sufficiently monochromatic radiation, there must be little change in the magnitude of βn along the trajectory of the electron and this magnitude must be approximately identical for all electrons in the cluster. The requisite conditions may prove extremely rigorous. For example, if it be required that the width of the spectrum $\Delta\omega$ should not exceed $\frac{\omega}{10}$, the dispersion of electron energy ΔV must not exceed the following value of ΔV_0;

$$\frac{\Delta V_0}{mc^2} = \frac{|1-\beta n|}{10 n^2}. \qquad (27)$$

When $\beta n = 0.995$ and $n = 4$, $\Delta V_0 = 15$ eV, i.e., $\frac{\Delta V_0}{V} = 10^{-3}$. Here we wish to emphasize the fact that when $\beta n < 1$, the electrons are not retarded (in the direction y) by radiation, since the radiation reaction is directed along the x-axis and works against the force of the field E. The application of dielectrics for utilizing the Doppler effect in a medium is, of course, possible not only in the analyzed rectilinear case but also in the case of an electron's motion in a magnetic field (motion on the circumference along a dielectric surface concentric or parallel to the orbit).

During motion in a magnetic field, the nature of the radiation will be like that described in Section 3. To obtain microwave radiation in this case, it is possible, as in a magnetron, to create an "electron rotor" revolving with considerable speed between two dielectric plates -- an electron cloud, modulated in density [2].

Section 5

The method of generating microwaves stated in Section 4 is based on utilization of the Doppler effect in a medium. In this case an electron, in order to radiate, must develop oscillations close to its trajectory. It is also possible to use Cherenkov's effect for microwave radiation which facilitates making use of uniformly moving electrons [14].

Super-luminescent electron radiation, or Cherenkov's effect, as we know, means that an electron moving with a constant velocity $\vec{v}$ in a medium with a refractive index n will radiate electromagnetic waves if the following condition is satisfied:

$$v > \frac{c}{n(\omega)}. \qquad (28)$$

Moreover, waves with a frequency ω are radiated only in a direction which, with $\vec{v}$, forms the angle θ_0, determined by the formula:

$$\cos\theta_0 = \frac{1}{\beta n(\omega)}. \qquad (29)$$

The energy radiated by an electron on the path l equals [15]

$$\mathcal{E} = \frac{e^2 l}{c^2} \int_{\beta n \geq 1} \omega \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) d\omega, \qquad (30)$$

where integration takes place in the whole frequency range for which disparity (28) holds good.

Radiation occurs when an electron moves near a medium in a channel or slit, or simply above a dielectric; in practice, as stated in Section 4, the distance between the electron and the dielectric must not be greater than the wave length $\frac{\lambda}{n}$. When there is a channel or slit, the radiation intensity can be determined by formula (30) accurate to a multiple of the order of unity, if the distance between the electron and the dielectric is $\lesssim 0.1\ \frac{\lambda}{n}$ (for a more accurate determination, see article by V. L. Ginzburg and I. M. Frank [12]). For instance, if $n = 7$, $\beta n = 1$ when the electron energy $V = 5{,}000$ eV and when $V = 10^4$ eV, the radiated energy in the frequency interval $\Delta\omega$ is:

$$\Delta\mathcal{E} = \frac{e^2 l}{2c^2}\,\omega\Delta\omega = 1.28\cdot 10^{-40}\, l\cdot\omega\Delta\omega = 10^{-15}\,\mathrm{erg} \qquad (31)$$

where, in converting to a numerical example, it may be assumed that $\omega = 2\cdot 10^{12}$ ($\lambda \simeq$ 1mm), $\Delta\omega = 2\cdot 10^{11}$ and $l = 20$ cm. (We may neglect the dispersion n which is, obviously, perfectly admissible for many materials in the microwave region. If dispersion is great, as a rule losses will increase, which is a disadvantage.) The total energy radiated in the frequency range $\omega \leqslant 2\cdot 10^{12}$ in the case under discussion equals $\mathcal{E} = 5\cdot 10^{-15}$ ergs. When the current $J = 10^{-2}$ amps, if the electrons radiate independently, $\Delta U = 60$ ergs/sec and $U = 300$ ergs/sec. But even in this case the radiation intensity can be greatly increased by making use of bunching. In consequence, the radiated intensity is proportional not to the number of electrons in the bunch ν but to ν^2. In a frequency range where the bunches radiate coherently, the energy radiated per second is:

$$\Delta U = \frac{e^2 l}{2c^2}\,\nu^2 N\omega\Delta\omega = \frac{e\, l}{2c^2}\, J\cdot\nu\cdot\omega\Delta\omega = 8\cdot 10^{-22} J_{amp}\cdot\nu\cdot l\cdot\omega\Delta\omega, \qquad (32)$$

where N is the number of bunches sent out per second and $J = e\nu N$.

In the example cited above with $J = 10^{-2}$ amp, when $\nu = 10^9$, $\Delta U = 6$ kw and $U = 30$ kw. Of course, since the electron energy in this example equals a total of 0.1 kw, the electrons must either be accelerated or, by an increase in the size of the slit, lose a good deal less energy than previously indicated. Nevertheless, it should be emphasized that utilization of bunching may make it possible to apply Cherenkov's effect for intensive microwave radiation; this method may prove simpler than the one described in Section 4, based on applying the Doppler effect in a medium.

On the long-wave side, the radiation spectrum is determined by formula (30), i.e., the intensity is proportional to $\omega \, d\omega$ or $\frac{d\lambda}{\lambda^3}$. On the shortwave side, the spectrum is cut off abruptly, on the one hand, because of the presence of the slit and, on the other, by virtue of the condition of coherency in bunch radiation expressed by the disparity (27). For greater precision it is necessary to use a formula of the (13) type substituting $\frac{\lambda}{n}$ for λ and the angle θ_o, determined by formula (29), for θ.

It is clear from what has been said that the radiation spectrum is continuous but has a clearly expressed maximum when

$$\lambda \sim d \cdot n. \tag{33}$$

From the standpoint of lightening, the conditions imposed on the width of the slit and the size of the bunch d, no advantage is gained by using very large values, $n \sim 7\text{-}8$. Even when $n = 4$, $\beta n = 1$ when $V = 16{,}000$ eV, and consequently, the voltages needed are not large.

As a result of the radiation, the electrons are retarded causing a decrease in the angle of radiation θ_o. Starting from a velocity $\nu = \frac{c}{n}$ or less, radiation will cease. Hence it is clear that for a value of ν substantially larger than $\frac{c}{n}$, the efficiency must be very great (the efficiency equals $\frac{V - V_o}{V}$ where V is the energy of the electrons and V_o is the energy at which $\nu = \frac{c}{n}$).

Section 6

The problem of the generation of microwaves is closely connected with the problem of their absorption in various media. For fluids and solid bodies, it is simple to reduce this problem to an investigation of the complex dielectric constant $\mathcal{E}'$ and its relationship with frequency. Generally speaking, it is very difficult to arrive at a calculation of $\mathcal{E}'$ from the viewpoint of the molecular theory. Investigation of the dispersion of the indexes of absorption and refraction in high-frequency fields, i.e., determining the function $\mathcal{E}'(\omega)$, is a basic problem in the application of microwaves to the study of the properties of a substance. We shall not linger on this topic here.

With regard to gases, the absorption of radio waves is valuable and can be studied from the very beginning within the framework of the atomic theory; we shall reduce the calculation of the absorption factor of gas here to an explanation of the nature of absorption for various molecules and accounting for the widening of the absorption lines connected with collisions between molecules. The absorption of gases in the radio-frequency region is explained by the fact that in many cases the molecules may have levels the distance between which correspond precisely to radio frequencies; it is likewise necessary that radiation transitions can take place between these levels. The rotational levels of diatomic and polyatomic molecules are levels of this type, if we do not include the very short waves in the vicinity of the infrared band of the spectrum. Here we find the well investigated so-called inversion spectrum of ammonium NH_3 molecules lying in the region $\lambda = 1.25$ cm [16]. (For recent discussion of a number of other cases, see items 17 and 18 of bibliography.)

In general, it should be borne in mind that the rotational lines, particularly for polyatomic molecules, lie in the microwave range as a rule and not as an exception. It is therefore possible to speak about a special field of molecular spectroscopy -- the radiospectroscopy of molecules. Here also belong possible transitions between sublevels of molecular terms split up in magnetic or electric fields (for example, in the earth's magnetic field [19, 20]) and Zeemen transitions connected with a change in orientation of the magnetic moment of nuclei in a strong magnetic field (see, for example, 21; also see another article by the author in Uspekhi Fizicheskikh Nauk, XXXI, 3, 320, 1947).

The absorption coefficient of a wave with a cyclic frequency ω equals

$$\chi(\omega) = \frac{4\pi^2 e^2 |\vec{x}_{12}|^2 \omega N}{3\hbar c} \cdot \frac{\gamma}{2\pi} \frac{1}{(\omega-\omega_0)^2 + \frac{\gamma^2}{4}} \ cm^{-1}, \qquad (34)$$

where ω_0 is the eigen frequency of transition; $e^2|\vec{x}_{12}|^2$ the square of the modulus for the matrix element of the dipole moment for a given transition; $\frac{\gamma}{2}$ the half-width of the absorption line; N the number of absorbed molecules per cu cm; $\hbar$ Planck's constant divided by 2π. [22]

If the spectrum of incident waves is continuous and the width of the line does not change, the total energy absorbed per cubic centimeter of gas per second will equal

$$S = \int \chi(\omega) I(\omega)\, d\omega = \frac{4\pi^2 e^2 N}{3\hbar c}\, \omega_0 \left| x_{12} \right|^2 I(\omega_0), \qquad (35)$$

where $I(\omega)\, d\omega$ is the flow of radio-wave energy in the frequency interval $d\omega$.

In formulas (34) and (35), it is understood that the statistical weight of the lower (beginning) level and of the upper (final) level is equal to unity and that, for the most part, the induced emission is disregarded. By induced emission we mean the phenomena due to the wave field acting on the excited molecule, causing it to emit additional radiation in the direction of the incident wave. The ratio of the probability of absorption to the probability of induced emission is such that, if the numbers of molecules on the upper and lower levels stand in the same relation to each other as the statistical weights of these levels, the radio waves would not be absorbed at all (see bibliography, 19, 20, and Section 11 of 22).

In fact, on the lower and upper levels, respectively, we find

$$N_1 = C g_1 e^{-E_1/kT} \quad \text{and} \quad N_2 = C g_2 e^{-E_2/kT} \quad \text{molecules,}$$

where C is the normal constant; g the statistical weight; and E the energy of the term.

Computation of the absorption and induced emission at any value of g_1 and g_2 demonstrates that the absorption coefficient observed in experiments may be obtained from (34) by substituting for N

$$\frac{N_1}{g_1} - \frac{N_2}{g_2} = C\left(e^{-E_1/kT} - e^{-E_2/kT}\right) = C e^{-E_1/kT}\left(1 - e^{-(E_2-E_1)/kT}\right). \qquad (36)$$

When $T \sim 300°K$, $kT \sim 4 \cdot 10^{-14}$, and hence, even for $\omega_0 \sim 2 \cdot 10^{12}$ ($\lambda_0 \sim 1$ mm),

$$\frac{E_2 - E_1}{kT} = \frac{\hbar\omega_0}{kT} \sim \frac{1}{20}.$$

Whence it is evident that, in general, in the radio-wave range

$$\frac{N_1}{g_1} - \frac{N_2}{g_2} = \frac{N_1}{g_1} \cdot \frac{\hbar\omega_0}{kT}.$$

Calculating the above and converting to the frequencies $\nu = \frac{\omega}{2\pi}$, we shall finally obtain for χ

$$\chi(\nu) = \frac{8\pi^2 e^2 \sum \left|\vec{x}_{12}\right|^2 \cdot \nu \cdot N_1 \cdot \frac{h\nu_0}{kT} \cdot \Delta\nu}{3hc \cdot g_1 \left\{(\nu-\nu_0)^2 + (\Delta\nu)^2\right\}} \qquad (37)$$

where $\Delta\nu = \frac{\gamma}{4\pi}$ is the half-width of the line [$\chi(\nu_0 + \Delta\nu) = \frac{1}{2}\chi(\nu_0)$] and $\sum \left|\vec{x}_{12}\right|^2$ is the sum of $\left|\vec{x}_{12}\right|^2$ for all degenerate sublevels of the beginning and final levels [22].

In (37) we assume, as we did in (34) that $\nu_0 \gg \Delta\nu$ and $|\nu - \nu_0| \ll \nu_0$. At the maximum of the line when $\nu = \nu_0$,

$$\varkappa_0 = \varkappa(\nu_0) = \frac{8\pi^2 e^2 \sum |\vec{x}_{12}|^2 \cdot \nu_0 \cdot N_1 \cdot \frac{h\nu_0}{kT}}{3hc\, g_1\, \Delta\nu} \tag{38}$$

Formulas (34) -- (38) are directly related to the case of electric dipole transition. In large gas masses, generally speaking, less intensive transitions may prove important. As we know, the ratio of the intensity of quadripole (I_{kd}) and magnetic dipole (I_{md}) transitions to the intensity of electrical dipole transition (I_{ed}), according to the order of magnitude (assuming that the frequencies and other parameters of the transition are identical) are equal to

$$\frac{I_{kd}}{I_{ed}} \sim \left(\frac{a}{\lambda}\right)^2 \sim \frac{10^{-16}}{\lambda^2}, \quad \frac{I_{md}}{I_{ed}} \sim \left(\frac{\hbar}{mca}\right) \sim 10^{-4} - 10^{-5}, \tag{39}$$

where a is the radius of the atom; m the electron mass, and λ the wave length.

It is evident from (39) that quadripole radiation in the range of radio frequencies is absolutely unimportant (this is also true of higher multipoles) and that only magnetic dipole radiation can play a part. In the latter case, formulas (34) -- (38) are applicable after replacing

$$e^2 |\vec{x}_{12}|^2 \quad by \quad |\vec{\mu}_{12}|^2, \tag{40}$$

where $|\vec{\mu}_{12}|^2$ is the square of the modulus of the matrix element of the magnetic moment.

As we can see from (37) and (40), to find $\varkappa(\nu)$ it is necessary to know ν_0, $|\vec{x}_{12}|^2$ or $|\vec{\mu}_{12}|^2$, g_1, $\Delta\nu$ and N_1. Finding all these values, except $\Delta\nu$, is essentially a problem in molecular spectroscopy [23]. Theoretically, this can also be done for rotational transitions by bringing in a few experimental data. The value of $\Delta\nu$ for a gas which is not too dense is determined by Laurent's impact expansion mechanism and equals

$$\Delta\nu = \frac{1}{2\pi\tau} = \frac{Z}{2\pi}, \tag{41}$$

where τ is the time of the free path of a molecule and Z the number of its collisions per second.

Moreover, in (41) it is impossible to take, as is usually done, the value of τ used in gas kinetics. Instead we have to speak of a certain effective free path time. The half-width of $\Delta\nu$ can, and must, be determined experimentally. But, in accordance with its order of magnitude, τ in (41) is equal to its value of gas kinetics; for example, in the case of air, for a pressure of 1 atm $\tau \sim 10^{-10}$ and $\Delta\nu \sim 2 \cdot 10^9$ $\left(\frac{\Delta\nu}{c} \sim 0.1\ \mathrm{cm}^{-1}\right)$.

Let us note that if $\frac{\Delta\nu}{\nu_0} \gtrsim 1$, the cited formulae will no longer be accurate and must be somewhat modified [24].

[Here follows a detailed discussion on the absorption of microwaves in air, especially by water vapor and oxygen, summarized as follows:

[If we disregard nonresonant absorption and factors which are of secondary importance in absorption by isotopic molecules, we may say that when $\lambda > 0.2$ cm air absorbs waves at $\lambda \sim 0.5$ cm and $\lambda \sim 1.3$ cm. Waves with $\lambda \sim 0.5$ cm are absorbed e times (that is, their intensity drops e times) in a distance of approximately 300 m (at maximum absorption); waves with $\lambda \sim 1.3$ cm are absorbed by water vapor and their intensity diminishes e times in a distance of approximately 20 km when the water content is about one percent by volume (at maximum absorption, when $\lambda = 1.34$ cm). Moreover, the absorption line of water is very wide, and the absorption connected with it is of great importance in a rather wide range of wave lengths [26, 27].]

BIBLIOGRAPHY

1. R. Beringer, Phys Rev, 70, 53, 1946.

2. L. Brillouin, Phys Rev, 63, 127, 1943, Collected Works "Magnetron," Moscow-Leningrad, 1946.

3. K. Shefer and F. Matossi, "Infrared Spectra," Section 13, Moscow-Leningrad, 1935.

4. A. A. Glagoleva-Arkad'yeva, Trudy GEEI, Vol II, 1924. DAN SSSR, XXXII, 540, 1941; XLV, 10, 1945.

5. V. L. Ginzburg, "Superconductivity," Sections 6, 11, 25, Iz Ak Nauk SSSR, 1946.

6. F. Bitter et. al., Phys Rev, 70, 97, 1946.

7. R. Kronig, "Band Spectra and Molecular Structure," Khar'kov, 1935.

8. L. I. Schiff, Rev Sci Instruments, 17, 6, 1946.

9. L. Artsimovich and I. Pomeranchuk, ZhETF, 16, 370, 1946.

10. V. L. Ginzburg, DAN SSSR, "Radiation of an Electron Moving near a Dielectric" (in printing).

11. I. M. Frank, Iz Ak Nauk SSSR, Seriya Fiz, 6, 3, 1942.

12. V. L. Ginzburg and I. M. Frank, DAN SSSR, "Radiation of an Electron and an Atom Moving in a Channel" (in printing).

13. B. M. Vul and G. I. Skanavi, Iz Ak Nauk SSSR, Seriya Fiz, 8, 194, 1944.

14. V. L. Ginzburg, DAN SSSR, "Application of Cherenkov's Effect to Radio Wave Radiation" (in printing).

15. I. Ye. Tamm and I. M. Frank, DAN SSSR, 14, 108, 1937.

16. W. E. Good, Phys Rev, 69, 539, 1946; 70, 213, 1946.

17. W. D. Hershberger, Journ Appl Phys, 17, 495, 1946.

18. R. M. Heiner, G. W. King, and P. C. Gross, Phys Rev, 70, 108, 1946.

19. V. L. Ginzburg, DAN SSSR, 35, 302, 1942.

20. V. L. Ginzburg, Iz Ak Nauk SSSR, Seriya Fiz, 7, 96, 1943.

21. E. M. Purcell, H. C. Torrey, and R. V. Pound, Phys Rev, 69, 37, 1946.

22. V. Geytler, "Quantum Theory of Radiation," Sections 12, 13, GTTI, 1940.

23. G. Herzberg, "Molecular Spectra," 1939.

24. J. H. van Vleck and V. F. Weisskopf, Rev Mod Phys, 17, 227, 1945.

25. D. M. Dennison, Rev Mod Phys, 12, 175, 1940.

26. R. H. Dicke, R. Berginer, R. L. Kyhl, and A. B. Vane, Phys Rev, 70, 340, 1946; short report; Phys Rev, 69, 694, 1946.

27. G. E. Becker and S. H. Autler, Phys Rev, 70, 300, 1946; short report: Phys Rev, 69, 694, 1946.

28. R. Schlapp, Phys Rev, 51, 342, 1937.

- E N D -