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REFLECTION OF SPHERICAL WAVES FROM PLANE BOUNDARY OF SEPARATION BETWEEN TWO MEDIA

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[Figures referred to herein are appended.]

INTRODUCTION

This article studies the reflection of spherical sound and electromagnetic waves from a plane boundary of separation between two media. The work of Sommerfeld [1], Weyl [2], Fok [3], Leontovich [4] and others on electromagnetic waves has been mainly connected with the cases of highly reflecting boundaries corresponding to the condition

$$|n|^2 \gg 1, \qquad (1)$$

where n is the index of refraction at the boundary.

This is often written in the form

$$|\varepsilon_1| \gg 1, \qquad (1')$$

where ε_1 is the dielectric constant of the ground. These conditions are fulfilled when the conductivity of the ground is sufficiently great.

With regard to shorter radio waves, cases where ε_1 is small and many cases of propagation in the troposphere where ε_1 is close to unity must be studied.

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In acoustics condition (1) is not satisfied in most practical cases, for instance in sound reflection from the surface of ground or water or from the bottom of the sea.

The problems of electromagnetic and sound waves have not been fully solved up to the present time.

This article will restate in greater detail previously published results of the author's research [10] and show that the chief result obtained by Ott [9] was inaccurate (see end of S2). And, whereas results obtained by Krüger [8] and Ott apply only to angles remote from the angle of full internal reflections, this article will describe the fields at all points of the area in the wave zone.

The results shown facilitate clarification of many problems including cases where n is large or close to unity and establish a criterion for the application of geometrical optics (acoustics) in which a wave issuing from an imaginary transmitter may be substituted for a relected wave.

I. STATEMENT OF THE PROBLEM -- FIRST FORM OF THE SOLUTION

Let us examine the point emitter at a distance z_0 from a plane boundary of separation between two media (Figure 1 . For the electromagnetic instance let us select as transmitter a vertical dipole, for the acoustic instance, a pulsating sphere with an infinitely small radius. After assigning the transmitter frequency in electrodynamics, we shall denote the upper and lower media by the wave numbers k_0 and k_1, k_1 having any imaginary frequency and k_0 a small calculated imaginary frequency.

In acoustics, in addition to the wave numbers, the media will be denoted by the densities ρ_0 and ρ_1 .

We shall denote the field by the scalar function φ, which is the vertical component of a Hertz vector in electromagnetics and the sound potential in acoustics.

The well-known expression for a primary wave, sometimes called the initial excitation, has the form ([3], pages 941, 944):

$$\varphi_0 = \frac{e^{ik_0R_0}}{R_0} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\xi\, d\xi}{\sqrt{\xi^2 - k_0^2}} H_0^{(1)}(\xi r) e^{\pm\sqrt{\xi^2-k_0^2}(z-z_0)}, \qquad (2)$$

where the plus sign is used when $z < z_0$ and the minus sign when $z > z_0$.

Furthermore, $R_0 = \sqrt{r^2 + (z - z_0)^2}$ is the distance from the transmitter to the receiver.

We shall also make use of another expression obtained by substituting $\xi = k_0 \cos\alpha$, where α is a new variable of integration. This will then take the form

$$\varphi_0 = \frac{e^{ik_0R_0}}{R_0} = -\frac{ik_0}{2}\int_{\Gamma_1} H_0^{(1)}(k_0 r\cos\alpha) e^{\mp ik_0(z-z_0)\sin\alpha} \cos\alpha\, d\alpha \quad (3)$$

where the path of integration Γ_1 in the plane α will pass respectively, through points $\pi - i\infty, \pi, \frac{\pi}{2}, 0\ i\infty$ (Figure 2). The latter expression can also be obtained from the well-known resolution of a spherical wave into plane waves (and also [2], and [3], page 943), if integration takes place in it with respect to the azimuth and the angle of incidence ϑ is replaced by the complementary angle $\alpha = \frac{\pi}{2} - \vartheta$ (angles formed by the normal to the wave front from the boundaries of separation.

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It is easy to prove that both the wave equation and the boundary conditions are satisfied, if the reflection and refraction of a wave are put in the form (See [10] for boundary conditions; formula (4) may often be obtained therefrom.)

$$\varphi_r = -\frac{ik_0}{2}\int_{\Gamma_1} f(\alpha)\, H_0^{(1)}(k_0 r\cos\alpha) \times e^{ik_0(z+z_0)\sin\alpha}\cos\alpha\, d\alpha \tag{4}$$

where

$$f(\alpha) = \frac{m\sin\alpha - \sqrt{n^2-\cos^2\alpha}}{m\sin\alpha + \sqrt{n^2-\cos^2\alpha}} \tag{5}$$

and

$$\varphi_d = -\frac{ik_0}{2}\int_{\Gamma_1} g(\alpha)\, H_0^{(1)}(k_1 r\cos\alpha_1)\, e^{-ik_1 z\sin\alpha_1 + ik_0 z_0\sin\alpha}\cos\alpha\, d\alpha \tag{6}$$

where

$$m g(\alpha) = 1 + f(\alpha) \tag{7}$$

and $n = \frac{k_1}{k_0}$ is the index of refraction; $m = n^2$ in electromagnetics and $m = \frac{\rho_1}{\rho_0}$ in acoustics. Moreover, $\sin\alpha_1 = \frac{1}{n}\sqrt{n^2-\cos^2\alpha}$ inasmuch as the sign of the latter radical was so selected that (Im x denotes the imaginary part of x.)

$$Im\sqrt{n^2-\cos^2\alpha} > 0 \tag{8}$$

which is required for the convergence of the integral (6) when $z \to -\infty$.

In expression (4) it should be noted that z and z_0 appear only as the sum $z + z_0$, and hence the wave reflection is not changed if the transmitter be raised and the receiver lowered, or vice versa, by the same amount.

II. GEOMETRICAL OPTICS AS A FIRST APPROXIMATION

In the simplest case of an absolutely reflecting boundary, when $f(\alpha) = 1$, the reflected wave is reduce to form (3), except that $z + z_0$ replaces $z - z_0$ in the exponent. Hence we shall have

$$\varphi_r = \frac{e^{ik_0 R}}{R},$$

where $R = \sqrt{r^2 + (z+z_0)^2}$ is the distance from the observation point P to the imaginary transmitter Q_1 (Figure 1). Here also the wave reflection will be spherical. It has been generally supposed that this was true only in the case of absolutely reflecting boundaries, but it is evident from (4) that in all cases where the reflection coefficient does not depend on the angle of incidence, that is when $f(\alpha) = C$, the reflected wave will be spherical

$$\varphi_r = C\frac{e^{ik_0 R}}{R}, \tag{9}$$

This case occurs, for example, in acoustics when $c_0 = c_1$, (conditions approximating this may occur when sound is reflected from certain forms of sea bottoms), when the reflection coefficient equals $C = \frac{\rho_1 - \rho_0}{\rho_1 + \rho_0}$.

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$$H_0^{(1)}(x) \sim \sqrt{\frac{2}{\pi x}}\, e^{i\left(x-\frac{\pi}{4}\right)}\left(1+\frac{1}{8ix}+\cdots\right). \tag{10}$$

The reflected wave will now be expressed in the form

$$\varphi_r = \sqrt{\frac{k_0}{2\pi r}}\, e^{i\frac{\pi}{4}} \int \frac{m\sin\alpha - \sqrt{n^2-\cos^2\alpha}}{m\sin\alpha + \sqrt{n^2-\cos^2\alpha}}\, e^{ik_0 R\cos(\alpha-\chi)}\left(1+\frac{1}{8ik_0 r\cos\alpha}\right)\sqrt{\cos\alpha}\, d\alpha, \tag{11}$$

where the complementary angle (Figure 1), will be denoted by χ, and

$$z + z_0 = R\sin\chi,\quad r = R\cos\chi. \tag{12}$$

Calculating the last integral by means of the well-known "pereval" method [literally "pass" method, used by Sommerfeld and others], let us introduce the substitution

$$\cos(\alpha-\chi) = 1 + ix^2 \tag{13}$$

and choose a new form of integration such that the x in it is real and passes through all values from $-\infty$ to $+\infty$. This will be path Γ in Figure 2. In intersects the real axis at the point $\alpha=\chi$ at an angle of 45 degrees. Substituting (13) under the integral in (11) gives the exponent $e^{-k_0 R x^2}$. Because k_0R is large, small values of x in the integral play a greater role than they will later.

Let us consider the singular points in the expression behind the integral sign in (11), since in passing from Γ_1 to Γ we must by-pass them. Because of the presence of the radical $\sqrt{n^2-\cos^2\alpha}$, the function behind the integral sign is double-valued: $\cos\alpha = \pm n$. Two of these, S_1 and S_2, Figure 2, pertain to cases where the imaginary part $\underline{n}$ is damping and small, and the real part is less than unity. All remaining branch points will be obtained from S_1 and S_2 by a parallel translation along the real axis, a distance of an integer times π.

The function behind the integral sign is identical with Riemann's double-layer surface. One layer — the "upper" — will satisfy the condition

$$Im\sqrt{n^2-\cos^2\alpha} > 0, \tag{14}$$

while in the other, the "lower", $Im\sqrt{n^2-\cos^2\alpha}<0$. The sections between them run along the lines $Im\sqrt{n^2-\cos^2\alpha}=0$. One of these sections, coming from the branch point S_1 is shown by a dotted line in Figure 2. The section, starting in S_2, runs symmetrically to it relative to the origin of the coordinates. The initial path of integration Γ_1 passes along the upper layer.

The function behind the integral sign will also have poles at the points

$$\cos\alpha = \pm\sqrt{\frac{m^2-n^2}{m^2-1}}. \tag{15}$$

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It can, however, be demonstrated [9 and 16] that neither of them will lie in that part of the plane α which is included between Γ and Γ_1.

Profiting by the smallness of x, let us expand the whole expression behind the integral sign in (11), except the exponent, in a series in x as far as the third term or term containing the square of x. Due to the smallness of the term $1/8\ .ik_o r.\cos\alpha$, we shall retain only the term independent of x, so that $\cos\alpha$ will be replaced by $\cos\chi$. Let us assume that in (11) $\sin\alpha \equiv \gamma$ and that, at the point x = 0, $\sin\alpha = \sin\chi \equiv \gamma_0$, and let us expand $f(\gamma)$ into a series around γ_0:

$$f(\gamma)=f(\gamma_0)+f'(\gamma_0)(\gamma-\gamma_0)+\frac{1}{2}f''(\gamma_0)(\gamma-\gamma_0)^2+\dots \quad (16)$$

It can now be found that

$$\gamma-\gamma_0=ix^2\sin\chi-x\sqrt{x^2-2i}\cos\chi .$$

It is simple to expand the rest of the expression behind the integral sign into an x series. Substituting the values of the well-known integrals of the function $e^{-k_0Rx^2}$ and $e^{-k_0Rx^2}x^2$ and denoting by φ_r' the integral (11) taken along path Γ, we finally obtain

$$\varphi_r'=\frac{e^{ik_0R}}{R}\left[f(\gamma_0)-\frac{iN}{k_0R}\right] \quad (17)$$

where

$$N=\frac{1}{2}f''(\gamma_0)\cos^2\chi-f'(\gamma_0)\sin\chi . \quad (18)$$

The expression

$$f(\gamma_0)=\frac{m\sin\chi-\sqrt{n^2-\cos^2\chi}}{m\sin\chi+\sqrt{n^2-\cos^2\chi}} \quad (19)$$

is the ordinary coefficient of reflection for a plane wave incident on the boundary at a vertical angle χ. The term $f(\gamma_0)\frac{e^{ik_0R}}{R}$ is the field according to an approximation used in optics. It may also be considered the field of the imaginary transmitter Q_1, which in this case will have a definite characteristic direction. The term N/k_oR is a correction term for geometric optics and vanishes when $\lambda \to 0\ (k_0 \to \infty)$.

If we introduce the notation $q_0=\sqrt{n^2-\cos^2\chi}=\sqrt{n^2-1+\gamma_0^2}$, calculation of the derivatives will give:

$$N=\frac{m(1-n^2)}{(m\gamma_0+q_0)^3q_0^3}\left[\gamma_0q_0(2n^2+1-\gamma_0^2)-m\gamma_0^4+2m(n^2-1)+3m\gamma_0^2\right]. \quad (20)$$

We shall discuss these results in V., noting only that Ott [9] omitted the second term by mistake in the expression similar to (18).

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III. "SECONDARY" WAVES

Expression (17) is not a complete solution, as integration along the margin of the profile is lacking.

In the case which interests us, where $|n| < 1$ and n is a complex number whose imaginary part is very small let the angle $\chi = \arc\sin \frac{z+z_a}{R}$ be sufficiently large so that in the complex plane α shown in Figure 2, in passing along the contour Γ in the direction shown by arrows, both branch points S_1 and S_2 will remain on the left side. Then the profile will not change by distortion of contour Γ_1 into contour Γ. In fact, if the profile is not bypassed, the dotted part of the path of integration will lie on the lower lobe. But this does not disturb the convergence of our integral.

Let us denote by χ_0 the maximum elevation angle at which a plane wave undergoes full inner reflection. It is known that $\cos\chi_0 = n$, that is, that in S_1 we have $\alpha = \chi_0$. Consequently when $\chi > \chi_0$, that is, at angles at which a plane wave does not have full inner reflection, the reflected wave will be fully expressed by (17) (only if $\chi - \chi_0$ is not small, see IV).

If we now decrease angle χ, contour Γ will shift to the left so that point S_1 will pass through it and be on the right (Figure 3). Now, if the profile be disregarded, the beginning and end of contour Γ will lie on different lobes and it will be impossible to cross over to it from contour Γ_1 lying entirely on the upper lobe. In this case, let us take the integration path shown in Figure 3. It goes from point $\frac{\pi}{2}+\chi-i\infty$ to point $\frac{\pi}{2}-i\infty$, where it meets the profile and goes along one margin of it to S_1, and then back along the other to $\frac{\pi}{2}-i\infty$. Then it intersects the profile and, already at the lower layer, goes to point $\frac{\pi}{2}+\chi-i\infty$, and thence along the normal path Γ, the beginning of which now lies on the lower lobe. Thus it is necessary to add to expression (17) an integral like (11), but taken along the profile margin (contour Γ_2 in Figure 3). Moreover the small term $\frac{1}{8ik_0 r\cos\alpha}$ in parentheses behind the integral sign may be discarded.

It is convenient to divide the integral into two for the two parts of the profile margin, whereupon the expressions behind the integral sign will be distinguished only by the radical

$$q = \sqrt{n^2 - \cos^2\alpha} \quad (21)$$

Reversing the direction of one of them, and combining both expressions, we shall obtain for that part of a reflected wave φ_r (denoted by φ_r''), which is dependent on integration along the profile margin:

$$\varphi_r'' = \sqrt{\frac{k_0}{2\pi r}}\, e^{i\frac{\pi}{4}} \int_L \Phi(\alpha) e^{ik_0 R\cos(\chi-\alpha)} \sqrt{\cos\alpha}\, d\alpha \quad (22)$$

where

$$\Phi(\alpha) = \frac{4mq\sin\alpha}{m^2\sin^2\alpha - q^2}, \quad (23)$$

$$(\mathrm{Im}\, q > 0).$$

and the profile line is denoted by L.

By introducing a new variable of integration $\beta=\alpha-\chi_0$ the exponent under the integral may be written

$$e^{ik_0R\cos(\chi-\alpha)}=e^{ik_0R[\cos(\chi_0-\chi)\cos\beta-\sin(\chi_0-\chi)\sin\beta]}. \tag{24}$$

The right side of this equation can also be written

$$e^{ik_0R\cos(\chi_0-\chi)}e^{-2ik_0R\left[\cos(\chi_0-\chi)\sin^2\frac{\beta}{2}+\sin(\chi_0-\chi)\sin\frac{\beta}{2}\cos\frac{\beta}{2}\right]}. \tag{25}$$

Because k_0r is large, the important expression behind the integral sign involves only small β, of an order of magnitude not greater than

$$|\beta|_{max}\approx\frac{1}{\sqrt{k_0R}} \tag{26}$$

Hence, in converting the remainder of the expression behind the integral sign in (22), we have first of all

$$q^2=\cos^2\chi_0-\cos^2\alpha=(\cos\chi_0-\cos\alpha)(\cos\chi_0+\cos\alpha)=$$
$$=\sin(\alpha+\chi_0)\sin(\alpha-\chi_0)=\sin(2\chi_0+\beta)\sin\beta=\sin 2\chi_0\sin\beta. \tag{27}$$

Here we may discard β as compared with $2\chi_0$, which can always be done when χ_0 is large. When they are small, we have $\chi_0\approx\sqrt{1-n^2}$ and, consequently, in accordance with (26), we must postulate that

$$|\sqrt{k_0R(1-n^2)}|\gg 1. \tag{28}$$

Hence the case where n is very close to unity must be excluded from our study.

After making similar eliminations, expression (22) may be presented in the form

$$\varphi_r''=\frac{8\sqrt{2k_0}\,ne^{\frac{i\pi}{4}}}{m\sqrt{\pi r}\sin\chi_0}\int_0^{\frac{\pi}{2}-\chi_0-i\infty}e^{ik_0R[\cos(\chi_0-\chi)\cos\beta-\sin(\chi_0-\chi)\sin\beta]}\left(tg\frac{\beta}{2}\right)^{\frac{3}{2}}\frac{d\beta}{2\sin\frac{\beta}{2}} \tag{29}$$

As demonstrated by V. A. Fok, the last integral may be expressed by a Weber function [12] (the function of a parabolic cylinder). For this it is necessary to use the equation demonstrated by him (integral (29) may be expressed by a Weber function without recourse to relation (30). If we take the function behind the integral sign in form (25) and substitute $\sin\frac{\beta}{2}=t$, and $\cos\frac{\beta}{2}\approx 1$, the integral obtained coincides with those forming part of the integral representation of Weber's function ([12], page 157). But it is sufficient here and later to use formula (30).)

$$\frac{1}{\Gamma(-n)}\int_0^{\frac{\pi}{2}+\beta_0-i\infty}e^{\frac{i}{2}(\xi^2-\eta^2)\cos\beta-i\xi\eta\sin\beta}\left(i\,tg\frac{\beta}{2}\right)^{-n}\frac{d\beta}{2\sin\frac{\beta}{2}}=D_n(\xi-i\xi)D_n(\eta+i\eta), \tag{30}$$

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where $-\frac{\pi}{2} < \beta_0 < \frac{\pi}{2}$.

Our integral comes to this, if we put

$$\xi = \sqrt{2k_0 R} \cos\frac{\chi_0 - \chi}{2}, \quad \eta = \sqrt{2k_0 R} \sin\frac{\chi_0 - \chi}{2}, \tag{31}$$

and as a result, we shall obtain

$$\varphi_r'' = \frac{4in}{m}\sqrt{\frac{2k_0}{r\sin\chi_0}}\, D_{-\frac{3}{2}}(\xi - i\xi) D_{-\frac{3}{2}}(\eta + i\eta). \tag{32}$$

As $\xi \geqslant 1$, instead of the function $D_{-\frac{3}{2}}(\xi - i\xi)$, it is possible to use its asymptotic representation ([12], page 154)

$$D_{-\frac{3}{2}}(\xi - i\xi) = \frac{e^{\frac{3\pi i}{8} + \frac{i\xi^2}{2}}}{(\sqrt{2}\xi)^{\frac{3}{2}}} \tag{33}$$

Now considering that

$$\frac{1}{2}(\xi^2 - \eta^2) = k_0 R\cos(\chi_0 - \chi) = k_0 R(\cos\chi_0\cos\chi + \sin\chi_0\sin\chi) = k_0\left(nr + \sqrt{1-n^2}(z + z_0)\right) \tag{34}$$

we may write for

$$\varphi_r'' = \frac{in}{mR\left(\frac{k_0 R}{2}\right)^{\frac{1}{4}}} \frac{e^{ik_0(nr + \sqrt{1-n^2}(z+z_0))}}{\left(\sin\chi_0\cos\chi\cos^3\frac{\chi_0 - \chi}{2}\right)^{\frac{1}{2}}} \frac{F(\eta)}{\eta^{\frac{3}{2}}}, \tag{35}$$

where

$$F(\eta) = e^{-\frac{5\pi i}{8}}(2\eta^2)^{\frac{3}{4}} e^{i\frac{\eta^2}{2}} D_{-\frac{3}{2}}(\eta + i\eta). \tag{36}$$

The asymptotic expansion of $F(\eta)$ in $\frac{1}{\eta^2}$ has the form:

$$F(\eta) = 1 - \frac{3\cdot5\cdot7\cdot9}{2\cdot4(8\eta^2)^2} + \frac{3\cdot5\ldots15\cdot17}{2\cdot4\cdot6\cdot8(8\eta^2)^4} - \ldots + \frac{15i}{16\eta^2}\left(1 - \frac{7\cdot9\cdot11\cdot13}{4\cdot6(8\eta^2)^2} + \frac{7\cdot9\ldots19\cdot21}{4\cdot6\cdot8\cdot10(8\eta^2)^2} - \ldots\right). \tag{37}$$

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Let us also expand the series in η;

$$F(\eta)=\sqrt{\pi}e^{\frac{3\pi i}{8}}\eta^{\frac{3}{2}}\left\{\frac{4}{\Gamma\left(\frac{1}{4}\right)}\left[1+\frac{3\eta^{3}}{2}i-\frac{3\cdot 7}{2!\cdot 1\cdot 3}\left(\frac{\eta^{2}}{2}\right)^{2}-\right.\right.$$

$$\left.-\frac{3\cdot 7\cdot 11}{3!\cdot 1\cdot 3\cdot 5}\left(\frac{\eta^{2}}{2}\right)^{3}+\cdots\right]-\frac{\sqrt{2}(1+i)\eta}{\Gamma\left(\frac{3}{4}\right)}\left[+\frac{5}{3}\left(\frac{\eta^{2}}{2}\right)i-\right.$$

$$\left.\left.\frac{5\cdot 9}{2!\cdot 3\cdot 5}\left(\frac{\eta^{2}}{2}\right)^{2}-\frac{5\cdot 9\cdot 13}{3!\cdot 3\cdot 5\cdot 7}\left(\frac{\eta^{2}}{2}\right)^{3}i+\cdots\right]\right. \tag{38}$$

This makes the following tabulation of the function F (η) possible.

Table of Function F (η)

η^2	$\lvert F(\eta)\rvert$	$\frac{4}{\pi}\arg F$	η^2	$\lvert F(\eta)\rvert$	$\frac{4}{\pi}\arg F$
0 1	0.25	1.12	1.6	0.82	0.48
0.2	0.36	1.00	1.9	0.84	0.43
0.4	0.51	0.85	2.2	0.86	0.39
0.7	0.65	0.71	3.0	0.89	0.30
1.0	0.72	0.61	4.0	0.93	0.25
1.3	0.78	0.53	5.0	0.94	0.22

The "secondary" wave, given by (35), was experimentally observed by Schmidt when a wave was reflected from the boundary dividing two liquids. It is called "Kopfwelle" or "Flankenwelle" in German. A special form of this wave is known as the "Mintrop" wave in seismology where it plays an important part.

In the plane r,z as may be seen from (35), the front of a secondary wave is rectilinear (and will be conical in space, because of the cylindrical symmetrical problem and normal to a straight line running from the imaginary source at an angle χ_0 , as shown in Figure 4 plotted in approximately the same way as by Ott [9]. The lower edge of the front of the secondary wave coincides with the edge of the front of a wave propagating in the lower medium with a velocity c_1 ($c_1 > c_0$). Its upper edge merges with the front of a reflected wave.

Variations in the amplitude of a secondary wave in prolongation along the dividing boundary are determined by the basic function $\lvert F(\eta)\rvert/\eta^{\frac{3}{2}}$. (Figure 5). When η increases, which corresponds to prolongation from the dividing boundary, it increases continuously tending toward a constant magnitude (in our article [10] it was erroneously stated that when $\chi \to \chi_0$, the amplitude of the secondary wave tended toward zero) when $\chi \to \chi_0$ (See IV).

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A secondary wave, just as in the second term of (17), is on the right side of the first term in (17), obtained by an approximation from geometrical optics. It is evident, if only because $\lambda \to 0$ $(k_0 = \frac{2\pi}{\lambda} \to \infty)$,

which corresponds to transference to geometric optics, that the amplitude of a secondary wave in (35) reverts to zero. Nevertheless, as in geometric optics, a secondary wave may be graphically represented by imagining that in addition to ordinary beams reflected from the boundary at angle of elevation χ (beam QOP in Figure 6) there are also beams incident on point P from point Q, which fall on the boundary at the angle of complete inner reflection χ_0, then propagate at the lower medium along the dividing boundary and, finally, are re-emitted in the upper medium (beam QCDP). Here, too, χ must be smaller than χ_0. It is easy to prove that the calculation of the optic length of such beams gives a correct phase value for secondary wave and defines its front. It may also be demonstrated that the beam QCDP satisfies Fermi's principle [sic; Fermat] that the time of its track from Q to P will be less than all other possible beams the paths of which lie partly in the lower medium (see [3], Chapter XII, especially p 528, by S. L. Sobolev) This time will also be less than the time of the track along QOP.

A secondary wave admits of graphic interpretation from the wave view also. When a wave emitted by source Q falls on the dividing boundary at an angle of complete inner reflection, there is developed in the lower medium a wave moving along the boundary and creating a disturbance in it with a spatial period equal to the wave length in the lower medium. This distance conditions the appearance of a new wave in the upper medium. But since the wave length of the latter (λ_0) is less than λ_1 $(\lambda_0 = \lambda_1 \cos\chi_0)$, in order to have periodicity along the boundary with a period λ_1 , it must be so inclined that the normal to its front forms the angle χ_0 with its front. The front of a secondary wave is located exactly in this manner.

The secondary wave is of importance not only as a correction for geometric optics, but also in seismometry. It must obviously appear in the propagation of sound impulses from the sea. It plays an important role in the propagation of radio waves in land transmitters and receivers. In the latter case, primary and reflected waves are generated in the earth and fade very rapidly. A secondary wave remains, which passes into the air from the earth, propagates there and falls back on the earth.

Schmidt's [7] qualitative rules for secondary waves are in agreement with our theoretical results.

Ott's [9] expression for this move is represented by the dotted line in Figure 5, and coincides with the curve when $\eta \gg 1$.

When $n > 1$ (the velocity in the lower medium is less) the secondary wave will occur in the region $\chi \leq \arccos \frac{1}{n}$. The analytical expression for it corresponds with (35) when $F(\eta) = 1$. This is clear since now χ_0 is imaginary and, consequently, χ cannot be near χ_0. Hence here $\eta \gg 1$ always and, hence, in accordance with (37), $F(\eta) = 1$. Putting $n > 1$ in (35) we obtain a wave with a front normal to the dividing boundary, propagating along it with a velocity c_1 and fading exponentially in the direction of the z - axis.

A partial theoretical analysis is also to be found in the work of Jeffries [13].

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IV. THE REFLECTED WAVE AT ANGLES NEAR THE ANGLE OF FULL INNER REFLECTION

To study reflected waves φ_r' (17) in the region of angles near φ_r'', we shall start from expression (11) and discard the term $1/8ik_o r\cdot\cos\alpha$ in parentheses, as it is small. Integration proceeds along Γ (Figure 2). The point $\alpha = \chi$ and the small region $\frac{1}{\sqrt{k_0 R}}$ near it are critical. Near α the radical $q = \sqrt{\cos^2\chi_0 - \cos^2\alpha}$ will be small and hence the expression behind the integral sign may be expanded into a series as far as the second term. Introducing a new variable $\beta = \alpha - \chi_0$ and taking (27) into account, we obtain.

$$\frac{m \sin\alpha - q}{m \sin\alpha + q} \cong 1 - \frac{2}{m \sin\chi_0} \sqrt{\sin 2\chi_0 \sin\beta} = \tag{39}$$

$$= 1 - \frac{2\sqrt{2}n}{m} \sqrt{\frac{\sin\beta}{\sin\chi_0}}.$$

In order to disregard the ratio $\frac{\sin\beta}{\sin\chi_0}$ as compared with unity, it is necessary that

$$|\chi - \chi_0| / \chi_0 \ll 1. \tag{40}$$

The sign of the radical $\sqrt{\sin\beta}$ must be taken as different on the different layers, like the sign of q, inasmuch as when $\chi > \chi_0$ the path Γ on the lower half-plane crosses along the lower layer.

The integration path in the plane β must cross from $\frac{\pi}{2} - \beta_0 - i\infty$ through 0 to $-\frac{\pi}{2} + \beta_0 + i\infty$. Making use of the smallness of β and expressing the obtained integrals by means of (30) through a Weber function, we shall similarly obtain

$$\varphi_r' = \frac{e^{ik_0 R}}{R} - \frac{2in}{m} \sqrt{\frac{2k_0}{r \sin\chi_0}} D_{-\frac{3}{2}}(\xi - i\xi) \left[\pm D_{-\frac{3}{2}}(\eta \pm i\eta) - iD_{-\frac{3}{2}}(-\eta - i\eta) \right] \tag{41}$$

where the plus sign refers to the case $\chi < \chi_0$ (Figure 3) and the minus sign to the case $\chi > \chi_0$ (Figure 2).

When $\chi > \chi_0$, expression (41) is the complete expression for the field, since there is no secondary wave φ_r'' in this angle region. The linear combination of a Weber function of the order of $-\frac{3}{2}$ in square brackets is joined with the Weber function of the order of $\frac{1}{2}$ [12, page 155]. We now have

$$\varphi_r = \varphi_r' = \frac{e^{ik_0 R}}{R} + \frac{8n}{m} \sqrt{\frac{k_0}{r \sin\chi_0}} e^{\frac{3\pi i}{4}} D_{-\frac{3}{2}}(\xi - i\xi) D_{\frac{1}{2}}(i\eta - \eta). \tag{42}$$

When $\chi < \chi_0$ in (41) the minus sign must be taken and the expression for a secondary wave (32) be added; (42) is again obtained. The latter is now the expression for reflected field φ_r ($\varphi_r = \varphi_r' + \varphi_r''$), applicable to any χ near χ_0.

For analysis with different values of η, through expansion of Weber's function and the use of expression (33), we obtain

$$\varphi_r = e\frac{ik_0R}{R}\left\{1 - \frac{4ne^{\frac{i\pi}{8}}}{m\Gamma\left(\frac{1}{4}\right)\left(\frac{k_0R}{2}\right)^{\frac{1}{4}}}\sqrt{\frac{\pi}{\sin 2\chi_0}}\left[1 + \frac{i\eta^2}{2} + \right.\right.$$

$$+ \frac{3}{2!\cdot 1\cdot 3}\left(\frac{\eta^2}{2}\right)^2 - i\,\frac{3\cdot 7}{3!\cdot 1\cdot 3\cdot 5}\left(\frac{\eta^2}{2}\right)^3 - \cdots \tag{43}$$

$$\left.\left.\frac{(1-i)\Gamma^2\left(\frac{1}{4}\right)\eta}{4\sqrt{\pi}}\left(1 - \frac{i}{3}\frac{\eta^2}{2} - \frac{1\cdot 5}{2!\cdot 3\cdot 5}\left(\frac{\eta^2}{2}\right)^2 + i\,\frac{1\cdot 5\cdot 9}{3!\cdot 3\cdot 5\cdot 7}\left(\frac{\eta^2}{2}\right)^3 + \cdots\right)\right]\right\}$$

When $\chi = \chi_0$, that is $\eta = 0$, there always remain two terms in the outer brackets. The first, unity, equals the coefficient of reflection. Here, too, geometric optics is applicable. The second term, which is its correction, decreases more slowly with increase of k_0R than in (17), actually, like $(k_0R)^{-\frac{1}{4}}$. Half of this term comprises the value of a secondary wave at this point if instead of $F(\eta)/\eta^{\frac{1}{2}}$ in (35) a constant value is substituted to which this function tends when $\eta \to 0$ (Figure 5).

Expansion into a series in $\frac{1}{\eta}$ depends on the sign of η. When $\eta < 0 (\chi > \chi_0)$, we have:

$$\varphi_r = \frac{e^{ik_0R}}{R}\left\{1 - \frac{2\sqrt{\sin 2\chi_0 \sin(\chi - \chi_0)}}{m \sin \chi_0}\left[1 - \frac{1}{2(8\eta^2)} + \right.\right. \tag{44}$$

$$\left.\left. + \frac{1\cdot 3\cdot 5}{2\cdot 4(8\eta^2)^2} + i\,\frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6(8\eta^2)^3} - \cdots\right]\right\},$$

When $\eta^2 \gg 1$, all terms in the square brackets may be disregarded and the two remaining terms in the outside brackets will be the two first terms of the expansion of the usual coefficient of reflection (19) in a series in $q_0 = \sqrt{n^2 - \cos^2\chi} \cong \sqrt{\sin 2\chi_0 \sin(\chi - \chi_0)}$ in hypothesis (40). In this case, (44) amounts to (17). Hence for the validity of the latter, the condition must be satisfied or

$$k_0R(\chi - \chi_0)^2 \gg 1. \tag{45}$$

When $\eta > 0$, the asymptotic representation of $D_{\frac{1}{2}}(\eta - i\eta)$ also has another form and contains two terms, one of which exactly coincides with (44) and the other gives a wave with a conical front (35).

The reflected wave φ_r can now be described for all angles. When condition (45) is satisfied, it is given by (17). On the other hand, if this condition is not satisfied, it means that $k_0R(\chi - \chi_0)^2 \lesssim 1$. We can write the latter as $\frac{\chi - \chi_0}{\chi_0} \lesssim \frac{1}{\chi_0\sqrt{k_0R}}$ or, by virtue of (38), as $\frac{\chi - \chi_0}{\chi_0} \ll 1$. Satisfaction of this condition permits expansion (39), whence expression (41) or (42) may be obtained.

V. DISCUSSION OF RESULTS -- LIMITS TO THE APPLICATION OF GEOMETRICAL OPTICS

In the study of further approximations we shall not find any new wave types like the secondary waves given by the second approximation in the formulae for geometric optics, but we shall give more precise values for the integrals in the theory according to the "pass" path and the profile margin. The former gives the reflected wave φ_r', the latter the secondary wave φ_r''.

It is obviously possible to disregard other approximations if the second approximation [correction term in (17)] is already small, that is,

$$|N| \ll k_0 R. \tag{46}$$

(this assertion has bee corroborated by calculations not given here).

Assuming first that $|n| > 1$ and substituting n^2 for $q_0^2 = n^2 - 1 + \sin^2\chi$, with insignificant simplifications we find that N has the order of magnitude:

$$N \approx \frac{\left(\frac{m}{n}\right)^2}{\left(\frac{m}{n}\sin\chi + 1\right)^3} \tag{47}$$

Or if $\frac{m}{n}\sin\chi < 1$, then simply $N \approx \left(\frac{m}{n}\right)^2$; whereupon condition (46) may be written (when $|n| < 1$ will appear in (47) and (48) instead of $\frac{m}{n}$.).

$$k_0 R \gg \left|\frac{m}{n}\right|^2. \tag{48}$$

I. electrodynamics where m = n^2 this will be

$$\frac{k_0 R}{|n|^2} \gg 1, \text{ that is } \frac{k_0 R}{|\varepsilon_1|} \gg 1, \tag{49}$$

which signifies that the distances must be great. This condition cannot be fulfilled when $|n|$ is large, but it is then possible to use the Weyl-van der Pol formula ([3], p 954; [14], p 105.). From a purely mathematical viewpoint, the similar convergence of a series in $\frac{1}{k_0 R}$, when $|n|$ is large (or $\frac{m}{n} = \frac{\rho_1 c_1}{\rho_0 c_0}$ — in acoustics) depends on one of the parts of the expression behind the integral sign studied above being situated near a point on the path of integration, as a result of which the ordinary "pass" method would be inapplicable. Ott [15] and Fok [16] generalized the "pass" method, taking this circumstance into account and thus were about to obtain the Weyl-van der Pol formula, but we shall not stop there.

Condition (49) is fulfilled in most cases of radio wave propagation. Since secondary waves (fading as rapidly as waves in the earth) can be disregarded when the source and receiver are located in the air, the Hertz vertical vector component of a reflected wave will be fully given by (17). Adding a primary wave, we shall obtain the total value of the above vector in the upper medium, when the transmitter and receiver are raised

$$\varphi = \frac{e^{ik_0R_0}}{R} + \frac{e^{ik_0R}}{R}\left(\frac{n^2\sin\chi - \sqrt{n^2-1}}{n^2\sin\chi + \sqrt{n^2-1}} - \frac{iN}{k_0R}\right), \tag{50}$$

where it is assumed that $\sin^2\chi$ can be disregarded as compared with $n^2 - 1$.

In this approximation of N, we shall have

$$N = \frac{-n^2}{(n^2 \sin\chi + \sqrt{n^2-1})^3}\left[(2n^2+1)\sin\chi + 2n^2\sqrt{n^2-1}\right]. \qquad (51)$$

Components of the electromagnetic field are obtained from (50) by ordinary differentiation. This expression is not new in electromagnetics and is derived in most cases from Wise's series, but as far as we know the expression for the corrected term of N given here has a wider significance than anywhere else.

In addition to the cases in electrodynamics and acoustics mentioned, the "pass" method has a limited application when n is so close to unity that condition (28) is no longer satisfied.

In an approximation for geometric optics (acoustics) the field in the upper medium is given by the sum of the primary wave $\frac{e^{ik_0R_0}}{R_0}$ and the reflected wave $f(\gamma_0)\frac{e^{ik_0R}}{R}$, which may also be represented as originating from an imaginary transmitter Q_1 (Figure 1). This representation is often used in electrodynamics and acoustics. Our results give clear criteria for its application to arbitrary dividing boundaries.

Returning to the case when the source (or receiver) is on the dividing boundary $R_0 = R$, and the condition of the smallness of the corrective term in (17), as compared with the sum of the primary and reflected waves may be written

$$k_0 R\,|1 + f(\gamma_0)| \gg |N|. \qquad (52)$$

Here we do not take account of the correction introduced by the secondary wave, as it does not change the order of magnitude on the right-hand side of the latter condition. Considering that, in accordance with (19),

$$1 + f(\gamma_0) = \frac{2m\sin\chi}{m\sin\chi + \sqrt{n^2-\cos^2\chi}} \qquad (53)$$

and also that $R\sin\chi = z$, condition (52) may be rewritten in the form

$$k_0 z = \frac{1}{2m}\left| N(m\sin\chi + \sqrt{n^2-\cos^2\chi}\right), \qquad (54)$$

When $|n| > 1$, the order of magnitude N is given by (47) and the latter condition becomes

$$k_0 z \gg \left| \frac{\frac{m}{n}}{\left(\frac{m}{n}\sin\chi + 1\right)^2} \right|. \qquad (55)$$

If $\left|\frac{m}{n}\right|\sin\chi \lesssim 1$, then

$$k_0 z \gg \left|\frac{m}{n}\right|, \qquad (56)$$

that is,

$$k_0 z \gg \frac{\rho_1 c_1}{\rho_0 c_0} \text{ — in acoustics} \qquad (57)$$

$$k_0 z \gg |n| \text{ — in electrodynamics}$$

Thus, for applicability to geometric optics it is necessary that the elevation of the receiver (transmitter) on the dividing boundary (for the particular case in electrodynamics where $|n| \gg 1$, Vvedenskiy's ([14] p 113) analysis of applicability to geometric optics is correct in a completely different form than ours.) be sufficiently great in comparison with the wave length. With strongly reflecting boundaries, when $\left|\frac{m}{n}\right| \sin\chi \gtrsim 1$, it is possible to use a weaker condition, also derived from (55):

$$k_0 z \gg \left|\frac{n}{m}\right| \frac{1}{\sin^2\chi}, \qquad (58)$$

which gives:

$$k_0 z \gg \left|\frac{\rho_0 c_0}{\rho_1 c_1}\right| \frac{1}{\sin^2\chi} \text{ — in acoustics} \qquad (59)$$

$$k_0 z \gg \frac{1}{|n| \sin^2\chi} \text{ — in electrodynamics.}$$

The latter conditions permit making a limiting transition to the case of absolutely reflecting boundaries, corresponding in acoustics to $\left|\frac{\rho_1 c_1}{\rho_0 c_0}\right| \to \infty$ in electrodynamics to $|n| \to \infty$. In both cases, the right-hand sides in (59) tend toward zero, whence it follows that geometric optics is correct in this case for any z.

Two generalizations of the results obtained are:

a. When $|n| < 1$ in electrodynamics, the conditions will be written like (55), (56) and (58); in acoustics n will appear instead of $\frac{m}{n}$. This is easily obtained from (54) and (20), if it be considered that, keeping the correct order of magnitude, it is now possible to discard n^2 and $\gamma_0^2 = \sin^2\chi$ as compared with unity, and in consequence, for example, $n^2 - \cos^2\chi = n^2 - 1 + \sin^2\chi$ may be replaced by -1 and so forth.

b. In raising the transmitter and receiver, it is sufficient for correctness in geometrical optics to require fulfillment of the above-mentioned conditions, except that it is necessary to replace z by $z + z_0$ (the total elevation of the transmitter and receiver on the dividing boundary). Here $\sin\chi = \frac{z+z_0}{R}$.

Graphs can be made from the study of the reflection of spherical waves given in the preceding paragraphs [18]. It appears, therefore, that of all plane-wave rays given by integral (4), on the boundary, only those rays of which the elevation angles are included in the small region of the order of $\frac{1}{\sqrt{k_0 R}}$ near angle χ play an important part. It was likewise found that not all dividing boundaries are of importance in the reflection of a wave, but only certain "effective zones," elliptical in form.

CONCLUSION

We encounter the problem of spherical wave reflection in the study of the field around a point transmitter of sound or of electromagnetic waves, placed on a plane boundary of separation dividing two media. This article gives a detailed study of a reflected wave in a wave zone by expanding solutions into series in $1/k_0 r$, using the "pass" method [Sommerfeld's method of complex integration]. Thereby practical formulas are obtained for the calculation of a Hertz vector in electromagnetics and of a sound potential in acoustics for any height of elevation of the transmitter and receiver over the dividing boundary. Moreover, a full description is given of a new type of wave (a secondary wave) which plays an important part in many cases. As compared with the published works of Ott and Krüger, a full description is given here of a field which is useful at all points of space in a wave zone. In addition, our article gives a complete analysis of a solution by which criteria are found for applications to geometric optics. It also explains many other important peculiarities of the problem.

I consider it my duty to express my deep gratitude to Academician V. A. Fok for much valuable advice.

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[Appended figures follow.]

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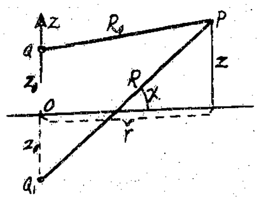


Figure 1

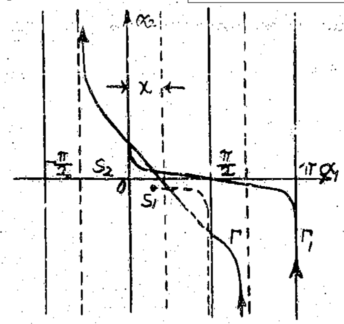


Figure 2

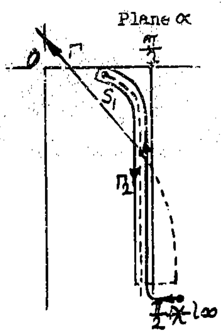


Figure 3

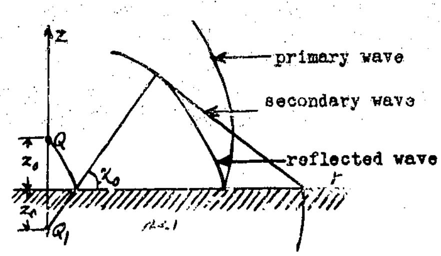


Figure 4

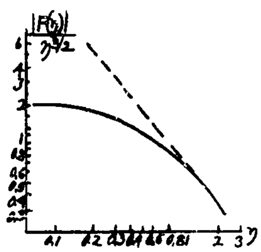
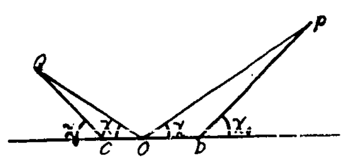


Figure 5



Figure

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