CLASSIFICATION CONFIDENTIAL

CENTRAL INTELLIGENCE AGENCY
INFORMATION FROM
FOREIGN DOCUMENTS OR RADIO BROADCASTS

REPORT
CD NO.

50X1-HUM

COUNTRY	USSR	DATE OF INFORMATION	1948
SUBJECT	Scientific - Physics	DATE DIST.	14 Jun 1949
HOW PUBLISHED	Monthly periodical		
WHERE PUBLISHED	Leningrad	NO. OF PAGES	10
DATE PUBLISHED	Apr 1948	SUPPLEMENT TO REPORT NO.	
LANGUAGE	Russian		

THIS IS UNEVALUATED INFORMATION

SOURCE Zhurnal Tekhnicheskoy Fiziki, Vol XVIII, No 4, 1948. [redacted] Translation requested.)

50X1-HUM

REFLECTION OF SPHERICAL WAVES FROM 'THIN' BOUNDARIES OF SEPARATION

L. Brekhovskikh
Phys Inst Acad Sci USSR
17 November 1947

[All figures are appended]

This is a study of the field of a point radiator emitting sonic or radio waves over a flat boundary of separation in the case where the properties of the media on both sides of the boundary of separation are approximately equal to each other. The study indicates the limitation of applicability of present theories. Rules are found in the fulfillment of which transitional layers can be replaced by boundaries of separation.

INTRODUCTION

We meet the problem of the reflection of spherical waves in studying electromagnetic or sonic radiation on the presence of a boundary surface dividing two media. Much work has been devoted to this problem, especially from two standpoints: (a) the study of the field over strongly reflecting dividing boundaries, corresponding in electromagnetics to the propagation of radio waves over highly conductive ground [1 - 4]; and (b) the study of a field as a function of the various properties of the media [5, 6, 13].

The solutions, however, obtained in these earlier works give infinitely large values for the amplitudes of reflected waves as the index of refraction n approaches unity at the dividing boundary (see the first part of [2], also [5, 6, 13]). Clarification of peculiar characteristics, determination of the limitations of existing theories and the development of formulas, in

CLASSIFICATION CONFIDENTIAL

						DISTRIBUTION				
STATE	X	NAVY	X	NSRB						
ARMY	X	AIR	X	FBI						

CONFIDENTIAL

50X1-HUM

which the index n may approach unity as closely as desirable, are of theoretical and practical interest. Particularly in the study of the propagation of radio waves in the atmosphere, it is often necessary to deal with their reflection from layers which can be regarded as surface boundaries dividing media with dielectric constants approximately equal. The index of refraction for such dividing boundaries can differ from unity only by small fractions of a percent [7]. The reflection of sound from "skip layers" in the sea is such a case. The latter, as we know, represents comparatively fine layers, extending in a horizontal direction, with large gradients of temperature and salinity. The index of refraction, equal to the ratio of the velocities of propagation of sound in water above and below the skip layer, also differs from unity only by small fraction of one percent.

Hereinafter boundaries dividing media with properties closely related to each other will be called thin dividing boundaries.

The formulas obtained by Ryazin [12] for the Hertzian vector are used for this type of boundary, but refer only to the case where the emitter and receiver are at the dividing boundary.

A. Limitations of the Applicability of Existing Theories

The particular situation existing when n approaches unity was noted first by us [13]. Also noted was the fact that the study of reflection by expansion into a series in $\frac{1}{k_0 R}$ (reached by a method used in the work of Sommerfeld [1]; Kruger [5] and Ott [6]) is possible for all angles only when the following condition is satisfied ([13], equation 28):

$$\sqrt{k_0 R\,|n^2-1|} \gg 1, \qquad (1)$$

where R is the distance from the emitter to the receiver. If this condition be not fulfilled, the above-mentioned "pereval" method [literally "pass" method used by Sommerfeld et alii] gives the correct values of the field only for sufficiently large angles χ (Figure 1), satisfying the condition

$$k_0 R \chi^2 \gg 1 . \qquad (2)$$

The inapplicability of this method when n is large and the angles χ are small can be readily seen from graphs which are of no interest here. An important role in reflecting spherical waves is not played by all of the points of the dividing boundary, but only by a certain elliptical zone [14] the area of which tends toward zero in geometric optics, when it is possible to speak of the reflection of a beam from a definite point on the dividing boundary. Consequently the reflected wave at an arbitrary point of space depends on waves passing only along those directions which correspond to straight lines connecting this point with all parts of the effective zone. These will be directions diverging from the direction of the beam reflected according to the laws of geometrical optics at small angles of the order of $\frac{1}{k_0 R}$ or less. Application of the "pass" method is possible if it can be assumed that, within this area of angles, the coefficient of reflection and its derivative with respect to the angle do not change much.

However, from the well-known expression for the coefficient of reflection of plane waves from a boundary dividing two media:

CONFIDENTIAL

CONFIDENTIAL

50X1-HUM

$$B(\chi)=\frac{m \sin \chi - \sqrt{n^2-\cos^2\chi}}{m \sin \chi + \sqrt{n^2-\cos^2\chi}} \qquad (3)$$

we find that, when n approaches unity and χ is small, the first and second derivatives of $B(\chi)$ may be as large as desired. Here $m = n^2$ in electrodynamics and $m = \frac{\rho_1}{\rho_0}$ (ratio of the densities of the media) in acoustics.

Application of the "pass" method gives the usual formulas of geometrical optics (acoustics) in the first approximation and their corrections in succeeding expressions. When n approaches unity, condition (2) guarantees the smallness of these corrections. Hence it is the condition that ordinary geometric optics describe reflected wave.

B. Radiation of a Point-Emitter Located Above a Thin Dividing Boundary

Figure 1 shows at point (r, z) the radiation reflected from a point-emitter at a height z_0 (which may be zero) from the plane dividing boundary. In electromagnetics, a vertical dipole is selected as an emitter; in acoustics, a pulsating sphere of infinitely small radius. The radiation field will be characterized by the scalar function φ, which corresponds to the vertical component of the Hertzian vector or to sound potential.

It may be demonstrated [9] that for any expression of constant media for a field in the upper medium, we shall have:

$$\varphi=\frac{e^{ik_0R_0}}{R_0}+\varphi_r, \qquad (4)$$

where the first term represents a primary wave, the second a reflected wave. And

$$\varphi_r=\int_0^\infty \frac{mb_0-b_1}{mb_0+b_1}e^{-b_0\zeta}J_0(\xi r)\frac{\xi d\xi}{b_0}. \qquad (5)$$

Here $b_0=\sqrt{\xi^2-k_0^2}$; $b_1=\sqrt{\xi^2-k_1^2}$;

k_0 and k_1 are wave numbers in the upper and lower media, respectively; m has the same value as in (3) and $\zeta = z+z_0$.

Our problem is the study of expression (5) when $n = \frac{k_1}{k_2}$ approaching unity.

Let us expand the expression behind the integral sign (5) into a series in (n^2-1). To this end we introduce the notation

$$\chi=\frac{k_1^2-k_0^2}{b_0^2}=\frac{k_0^2(n^2-1)}{b_0^2} \qquad (6)$$

Then

$$\frac{mb_0-b_1}{mb_0+b_1}=\frac{m-\sqrt{1-\chi}}{m+\sqrt{1-\chi}}=\sum_{s=0}^{\infty}B_s\chi^s. \qquad (7)$$

Substituting this in (5) and taking (6) into account, we obtain for a reflected wave (the radius of convergence of series (7) equals unity. Hence representation of φ_r in form (8) is possible only if in (5) a path of integration is selected such that anywhere in it $|\chi|<1$, that is $|b_0|^2>|k_0^2-k_1^2|$.

We can, for instance, select a path which runs first from $\xi = 0$ along the real axis; then by-passes the point $\xi = k_0$ along a semicircle, lying in the fourth quadrant, with a radius greater than $|\sqrt{k_0^2-k_1^2}|$; and comes out

again at the real axis. Since, in the fourth quadrant, the expression behind the integral sign in (5) has no singular point, transition to such a path of integration is possible. It is assumed, moreover, that in integrals (9), obtained after expansion into a series, the integration path is the real axis, for which reason the values of the integral, of course, do not change.):

CONFIDENTIAL

 50X1-HUM

$$\varphi_r = \sum_{s=0}^{\infty} B_s k_0^{2s}(n^2-1)^s I_s, \tag{8}$$

where

$$I_s = \int_0^{\infty} \frac{e^{-b_0\zeta}}{b_0^{2s+1}} J_0(\xi r)\xi \, d\xi. \tag{9}$$

For the coefficients of expansion B_s in (7) we shall have

$$B_0 = \frac{m-1}{m+1}, \quad B_1 = \frac{m}{(m+1)^2}, \quad B_2 = \frac{m(m+3)}{4(m+1)^3} \tag{10}$$

whence the consequent coefficients can be found by means of the recurrent formula

$$(m^2-1)B_s + B_{s-1} = m(B_{s-1})_{m=1}. \tag{11}$$

A study of integral I_s for any s is given in the appendix. Here we are restricting ourselves to the case where (n^2-1) is so small that its second and subsequent powers may be disregarded in (8). Now, taking into account the well-known formula ([3], page 941):

$$\frac{e^{ik_0R}}{R} = \int_0^{\infty} \frac{e^{-b_0\zeta}}{b_0} J_0(\xi r)\xi \, d\xi, \tag{12}$$

we obtain

$$\varphi_r = \frac{m-1}{m+1}\,\frac{e^{ik_0R}}{R} + \frac{m k_0^2(n^2-1)}{(m+1)^2} I_1, \tag{13}$$

where $R = \sqrt{r^2+\zeta^2}$.

As demonstrated in the appendix, the integral I_1 can be represented in the form

$$I_1 = \int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}(\zeta - t)\,dt + A + D\zeta, \tag{14}$$

where

$$R_1 = \sqrt{r^2+t^2},$$

$$A = \frac{i}{k_0} e^{ik_0 r}, \quad D = -i\sqrt{\frac{\pi}{2k_0 r}}\, e^{i(k_0 r - \frac{\pi}{4})} \tag{15}$$

The integral term in (14) must still be studied. Taking the difference of the two integrals, we have obviously

$$\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}(\zeta - t)\,dt = \zeta\int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,dt - \int_0^{\zeta} \frac{e^{ik_0R_1}}{R_1}\,t\,dt. \tag{16}$$

Taking the second integral and substituting the limits gives:

$$\frac{i}{k_0}\left(e^{ik_0R}-e^{ik_0r}\right). \tag{17}$$

To calculate the first interval we substitute variables in accordance with the formula

$$k_0(R_1-r)=\alpha^2, \text{ whence } \frac{k_0 t\,dt}{R_1}=2\alpha\,d\alpha$$

Now

$$\int_0^{\xi}\frac{e^{ik_0R_1}}{R_1}dt=2e^{ik_0r}\int_0^{k_0(R-r)}\frac{e^{i\alpha^2}d\alpha}{\sqrt{\alpha^2+2k_0r}}. \tag{18}$$

We shall assume that $\frac{\xi^2}{r^2}\ll 1$,

Now for all integration paths $\alpha^2\cong\frac{k_0t^2}{r}\ll k_0r$ and as a result the first term under the square root sign in (18) can be discarded.

Therefore, we obtain

$$\int_0^{\xi}\frac{e^{ik_0R_1}}{R_1}dt=\sqrt{\frac{\pi}{k_0r}}\left[C(u)+iS(u)\right]. \tag{19}$$

where

$$u=\sqrt{\frac{2k_0}{\pi}(R-r)}\cong\sqrt{\frac{k_0\xi^2}{\pi r}}, \tag{20}$$

and $C(u)$ and $S(u)$ are Fresnel integrals. Substituting this result in (14) and taking (19) into account, we obtain an expression for I_1 which, by substitution in the expression for a reflected wave (13), gives for the latter:

$$\varphi_r=\frac{e^{ik_0R}}{R}\left\{\frac{m-1}{m+1}-\frac{im(1-n^2)k_0r}{(m+1)^2}\left[1-\pi i u e^{-i\frac{\pi u^2}{2}}\left(C+iS-\frac{1+i}{2}\right)\right]\right\}. \tag{21}$$

In electrodynamics, where $m = n^2$, the first term in the parentheses is k_0r times less than the second and may therefore be omitted. In acoustics we are only interested in the case where $m - 1$ is equal to or less than n^2-1, so that the first term may again be omitted. (When $m-1$ is not a small quantity, there will be no singularity when n approaches unity.) A reflected wave is now written

$$\varphi_r=B\frac{e^{ik_0R}}{R}, \tag{22}$$

where

$$B=\frac{im(n^2-1)k_0r}{(m+1)^2}\left[1-\pi i u e^{-\frac{i\pi u^2}{2}}\left(C+iS-\frac{1+i}{2}\right)\right] \tag{23}$$

In electrodynamics the coefficient $n/(n+1)^2 = n^2/(n^2+1)^2$ can be replaced by $\frac{1}{4}$ (since then n equals 1).

C. Discussion of the Results Obtained

Expression (22) represents the usual spherical wave reflected with a coefficient of reflection B. It may be shown that when $u^2 \gg 1$ this B coincides with the coefficient of reflection of plane waves (3), if the latter also is expanded into a series of powers of $(n^2 - 1)$ and equals approximately the first term in the expansion. This is to be expected in view of the condition $u^2 \gg 1$, which may also be written

$$\frac{k_0 \zeta^2}{R} \gg 1, \tag{24}$$

equivalent to condition (2) which when fulfilled should be true for geometric optics.

Introducing $\chi = \frac{1}{2}\pi u^2$ instead of u and using the asymptotic expansion of Fresnel [8] integrals in powers of $\frac{1}{\chi}$, we shall have

$$C + iS - \frac{1+i}{2} = -\frac{1}{\sqrt{2\pi\chi}}\left(i + \frac{1}{2\chi} - \frac{1 \cdot 3}{(2\chi)^2}\, i + \ldots\right).$$

Putting this in (23), and considering that $\sin \chi = \zeta/R$ and disregarding small quantities, we shall obtain for B

$$B_1 = \frac{n(1-n^2)}{(n+1)^2 \sin^2 \chi}, \tag{25}$$

the same expression as found by expanding (3) in powers of $(n^2 - 1)$, to as far as the first term, and discarding the term $\frac{n-1}{n+1}$, for reasons indicated above.

When $u \ll 1$ in (23), all terms in the square brackets may be disregarded, which will again leave the coefficient of B in a simple form.

It is interesting to study the ratio $\frac{B}{B_0}$ as u runs through all possible values.

In accordance with (23) and (25) we have

$$\frac{B}{B_0} = -i\pi u^2 \left[1 - \pi i u e^{-i\frac{\pi u^2}{2}}\left(C + iS - \frac{1+i}{2}\right)\right] \tag{26}$$

Figure 2 gives the graph of this ratio as a function of u, with logarithmic scales on the axes. As u increases, $\left|\frac{B}{B_0}\right|$ increases continuously. At first this growth follows the law for u^2, which corresponds to a straight line on the given scale. When u is large, $\left|\frac{B}{B_0}\right|$ tends toward unity, which is the condition for the applicability of geometrical optics; that is, the coefficients of reflection of a spherical and a plane wave coincide.

This result refers to the case of the fairly fine transitional layer treated in 4 above. For cases of thick transitional layers only the reflection of plane waves has been calculated [7], but it is possible that condition (24) may also be used in determining these cases.

Condition (24) cannot be used for example, for the case of radio waves near a thin dividing boundary 400 meters above ground and 50 kilometers distant, when $\frac{k_0 \zeta^2}{R}$ equals 10 for $\lambda = 2$ meters and 2 for $\lambda = 10$ meters.

In the latter case, the field of reflection of a wave must be calculated by formulas (22) and (23).

In hydroacoustics, when the depth of the skip layer is 30 meters and distance R equals 2 kilometers, a reflected wave cannot be calculated according to geometrical acoustics when $\lambda \geqslant 1$ meter.

For practical conditions it is interesting to study the propagation of sound and radio waves in a layer limited on one side by a thin dividing boundary. We have previously shown [9] that in the most general case a field in a layer may be developed for an infinite number of waves reaching the receiver after a number of reflections from the boundaries of the layer. In this case, the amplitude of the wave experiencing k reflections from a thin dividing boundary will have the coefficient $(n^2 - 1)^k$; whence it follows that, in our approximation, it will be necessary to calculate the following six waves which are not reflected or are reflected only once from this boundary: (1) primary; (2) reflected from the usual dividing boundary; (3) reflected from a thin reflecting boundary; (4) reflected first from the usual and next from the thin boundary; (5) the same in reverse order; and (6) reflected twice from the usual and once from the thin dividing boundary.

The second wave can be calculated by earlier formulas [13]. The fourth, fifth and sixth can be reduced to the third, if the usual dividing boundary is absolutely reflecting, like the surface of water in hydroacoustics. Here the amplitude can be calculated by means of (22) and (23); (20) will be true for μ in ξ if every path taken by a corresponding beam is projected on the z-axis and if $R = \sqrt{r^2 - \zeta^2}$. Moreover, if the coefficient of reflection B_0 for an absolutely reflecting boundary is $B_0 = -1$, as in the above hydroacoustic case, the amplitudes of the fourth and fifth waves must have the minus sign. When the second boundary is not absolutely reflecting, the problem is more complicated, but may be solved by expansion into a series of powers of $(n^2 - 1)$ and application of the "pass" method.

D. Substituting a Dividing Boundary for a Transitional Layer

In practical cases, where there is generally a transitional layer instead of a thin boundary, the question arises how many such cases can be reduced to that of a single dividing boundary. It is known that this can be done if the thickness of the layer is small in comparison with the wave length. In this connection also, thin boundaries have a singularity, since the corresponding condition proves to be "weaker." To deduce this, one makes use of certain graphs.

Let us examine the reflection of a plane wave incident at an angle χ to a transition layer, with thickness l, which is parallel to the plane $z = 0$. When l equals or exceeds the wave length λ and the difference between the velocities of propagation at the upper and lower boundaries is small, the propagation of a plane wave may be calculated according to geometric optics, so that the function of the wave phase in terms of the coordinate z will be given by $\exp\left[i\int k_z dz\right]$. The extent of the layer in the direction z can be disregarded, if the path of the phase in the layer is small, that is, if

$$\int_0^l k_z d_z \ll 1.$$

We shall intensify this condition by substituting for the magnitude k_z its maximum value in the layer. It can now be written

$$(k_z)_{max}\, l \ll 1. \qquad (27)$$

When k_z changes monotonically, the only case of interest to us, the maximum value of k_z will be reached at one of the boundaries of the layer. Hence the above condition will be equivalent to the two following:

$$(k_z)_0\, l \ll 1 \;\; (k_z)_1\, l \ll 1, \qquad (28)$$

where the indices 0 and 1 refer to the upper and lower media, respectively. Since $(k_z)_0 = \frac{2\pi}{\lambda_0}\sin\chi$ and $(k_z)_1 = \frac{2\pi}{\lambda_1}\sin\chi_1$ where χ_1 is the angle of incidence of a refracted wave, conditions (26) can be rewritten in the form

$$l\sin\chi \ll \lambda_0/2\pi, \;\; l\sin\chi_1 \ll \lambda_1/2\pi \qquad (29)$$

With thin dividing boundaries $\chi_1 \cong \chi, \lambda_0 \cong \lambda_1$, both conditions are, therefore, reduced to one:

$$l\sin\chi \ll \lambda/2\pi \qquad (30)$$

For small angles this condition can be fulfilled when l is equal to or even greater than λ.

In breaking down spherical waves into plane waves ([2] and [3], p 943), it is necessary to consider z-components of the wave vector both as large as desirable and also imaginary. When k_z is large, condition (27) is not fulfilled. But this difficulty may be overcome, if we examine the field in the wave zone ($k_0R \gg 1$). Now, as shown in 2 above, an important part will be played only by those plane waves which have angles of incidence close to the angle χ depicted in Figure 1, which must be substituted in condition (30).

APPENDIX

Integrals (9) can be represented in a somewhat different form. Differentiating I_s 2s times with respect to ζ and taking (12) into account, we obtain

$$\frac{d^{2s} I_s}{d\zeta^{2s}} = \frac{e^{ik_0 R}}{R},$$

where

$$R = \sqrt{r^2 + \zeta^2}.$$

As may be easily proved by differentiation, the latter equation is satisfied by the function

$$I_s = \frac{1}{(2s-1)!}\int_0^\zeta \frac{e^{ik_0R_1}}{R_1}(\zeta - t)^{2s-1}\,dt + c_0^{(s)} + c_1^{(s)}\zeta + \dots + c_{2s-1}^{(s)}\zeta^{2s-1},$$

where $R_1 = \sqrt{r^2 + t^2}$

We obtain $c_0^{(s)}, c_1^{(s)}, \dots$ by differentiating the latter expression once, twice, etc, and then setting $\zeta = 0$ thus:

$$c_0^{(s)} = (I_s)_{\zeta=0}, \quad c_1^{(s)} = \left(\frac{dI_s}{d\zeta}\right)_{\zeta=0}$$

In general,

$$c_l^{(s)} = \frac{1}{l!}\left(\frac{d^l I_s}{d\zeta^l}\right)_{\zeta=0} = \frac{(-1)^l}{l!}\int_0 \frac{J_0(\xi r)\xi\,d\xi}{k^{2s-l+1}}.$$

Sanitized Copy Approved for Release 2011/07/18 : CIA-RDP80-00809A000600230675-8

Making use of the formulas for this type of integral given by Watson [10] (see also the work of Fock [11]), we obtain

$$c_l^{(s)} = \frac{\pi}{2}\left(\frac{r}{2k_0}\right)^{s-\frac{l+1}{2}} \frac{(-1)^s e^{\frac{i\pi l}{2}} H_{s-\frac{l-1}{2}}(k_0 r)}{l!\,\Gamma\left(s-\frac{l-1}{2}\right)}.$$

The quantities A and D employed in the text are equal, respectively, to $c_0^{(1)}$ and $c_1^{(1)}$. From the latter formula, replacing the Hankel functions H by their asymptotic forms, we obtain (15).

[Appended figures follow.]

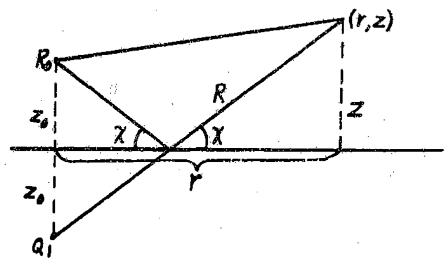

Figure 1. R_0 is the distance from the point of reception (r,z) to the emitter Q; R is the distance from the same point to the imaginary emitter Q_1; χ is the angle between the beam and the dividing boundary.

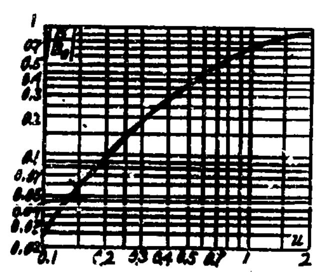

Figure 2. Graph of the ratio B/B_0 as a function of u. B is the coefficient of reflection resulting from the point theory of reflection of spherical waves; B_0 is the coefficient of reflection for a plane wave (geometric optics or acoustics).

Sanitized Copy Approved for Release 2011/07/18 : CIA-RDP80-00809A000600230675-8

50X1-HUM

1. Sommerfeld, A., Ann de Phys, 28, 665, 1909; 81, 1135, 1926.

2. Weyl, H., Ann de Phys, 60, 481, 1919.

3. Frank, F., and Mizes, R., Differential and Integral Equations of Mathematical Physics, ONTI, 1937, Ch XXIII, ed by V. A. Fock.

4. Leontovich, M. A., Izd AN USSR, Phys Ser VIII, 16, 1944.

5. Kruger, M., ZS v Phys, 121, 377, 1943.

6. Ott, H., Ann de Phys, 41, 443, 1942.

7. Vvedenskiy, B. A., Arenberg, A. G., UFN, XXVI, No 1, 1944.

8. Shpil'reyn, Ya N., Tables of Special Functions GTTI, 15, 1933.

9. Brekhovskikh, L., Izd AN USSR, Phys Ser X, 491, 1946.

10. Watson, G., Theory of the Bessel Function, Cambridge par 13, 6, 1922.

11. Fock, V., Ann de Phys, 17, 401, 1933.

12. Ryazin, P. A., The Most Recent Research on Radio Wave Propagation, ed by L. I. Mandel'shtam and N. D. Papaleksa.

13. Brekhovskikh, L., ZhTF, XVIII, No 4.

14. Brekhovskikh, L., UFN, 32, 464, 1947.

- E N D -

Sanitized Copy Approved for Release 2011/07/18 : CIA-RDP80-00809A000600230675-8