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Technical Memorandum

Aspheric Calculation

8 January 1965
By: STAT

Abstract: A technique for eliminating high order positive spherical aberration is presented. The idea of positive high order spherical is defined, the aspheric equation is discussed, and from the differential rates of changes of aspherics coefficients versus actual ray deviation a matrix is formed by predetermined ray intercepts.

Reducing image curvature very often introduces high order positive spherical aberration. In Figure 1 a doublet is shown where the front positive element has undercorrected spherical. If the negative rear element were not present, the marginal rays of the first element would focus in front of the paraxial focal plane. As it is, these rays strike the negative lens with higher angles of incidence than they would otherwise if undercorrected spherical were not present. The deep curvature of the negative lens in this case reduces image curvature, which is desirable, but it overcorrects the marginal rays, which is undesirable.

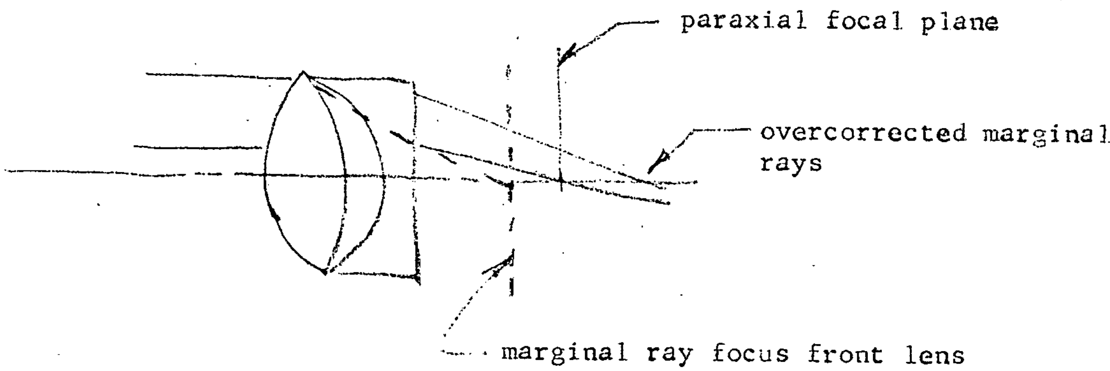

Figure 1

Either the positive or negative lens can be aspherized to recorrect these marginal rays. The assumption is the shapes and glass indexes of the elements cannot be varied to reduce the overcorrected spherical. "Rolling off" a surface on the positive lens or negative lens will introduce power into the marginal rays and bring them to focus at the paraxial focal plane, or any other point in front or behind. It can be seen that the spherical aberration must be overcorrected if the aspherizing is to be easy. Correcting undercorrected marginal spherical means adding negative power to the outer rim which is a much more difficult operation.

The desired shape of the rolled off sphere can be arrived at by adjusting the coefficients of the aspheric equation. This equation is an even powered polynomial written in a form having a spherical expression and deformation terms. This form facilitates starting with a spherical surface equation and adding power curves to it which leave the basic sphere unaltered while changing it above a determined ordinate.

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(1) $$x = \frac{cy^2}{1 + \sqrt{1 - c^2 y^2}}$$

Equation (1) is the expression for a sphere where the center of curvature is located a distance equal to the radius. It is also made single-valued so that any value of the ordinate (y), only one value for x is obtained which is the one closest to the origin. Furthermore, the radius, entered as a reciprocal $c \hat{=} \frac{1}{R}$, can go to infinity without upsetting the computation.

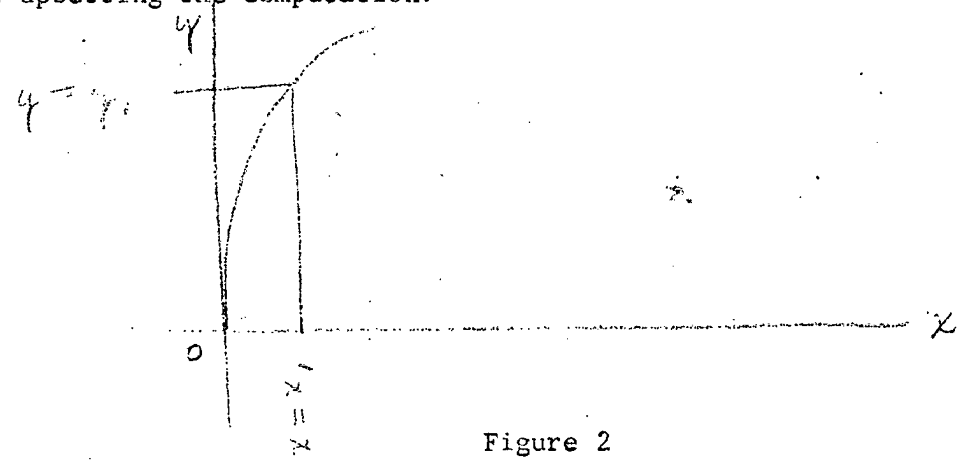

Figure 2

Figure 2 illustrates the geometry. Now if power curves of the type

(2) $y^n = kx$

or to be consistent, written as x a function of y

(3) $x_i = 1/k_i \; y^n$

are added to the expression of a sphere, equation (1), the spherical curve in Figure 2 would be modified according to be magnitude of k and n. That is, if k and n are large, equation (2) has the curve flat through small ordinates and whipping away sharply as y grows large, as shown in Figure 3.

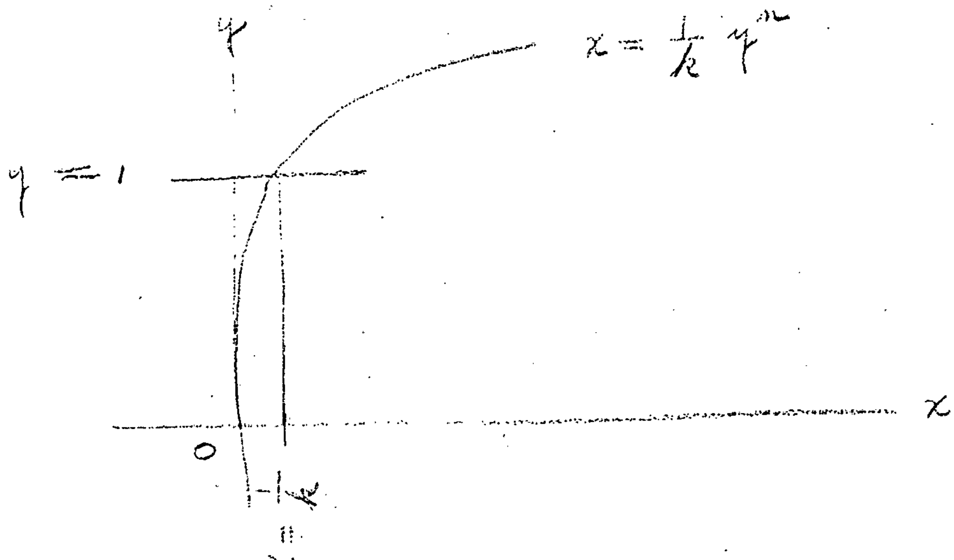

Figure 3

By adjusting k and choosing an appropriate n, a curve can be arrived at which when added to a circle curve alters the circle significantly above a certain height. For any n the value of x at $y \approx 1$ is always $1/k$. For a curve with small x, or sag, contribution, k would have to be large. The magnitude of n determines how rapidly sag increases above $y \approx 1$. It is easy to see that raising y to a large n when $y < 1$ produces small sag and large sag when $y > 1$. To effect a "roll off" on a spherical surface a deformation curve must have negligible sag contribution below a certain ordinate and a predetermined amount above it. Figure 4 illustrates adding such a curve to a sphere. The deformation itself can be the sum of several power curves.

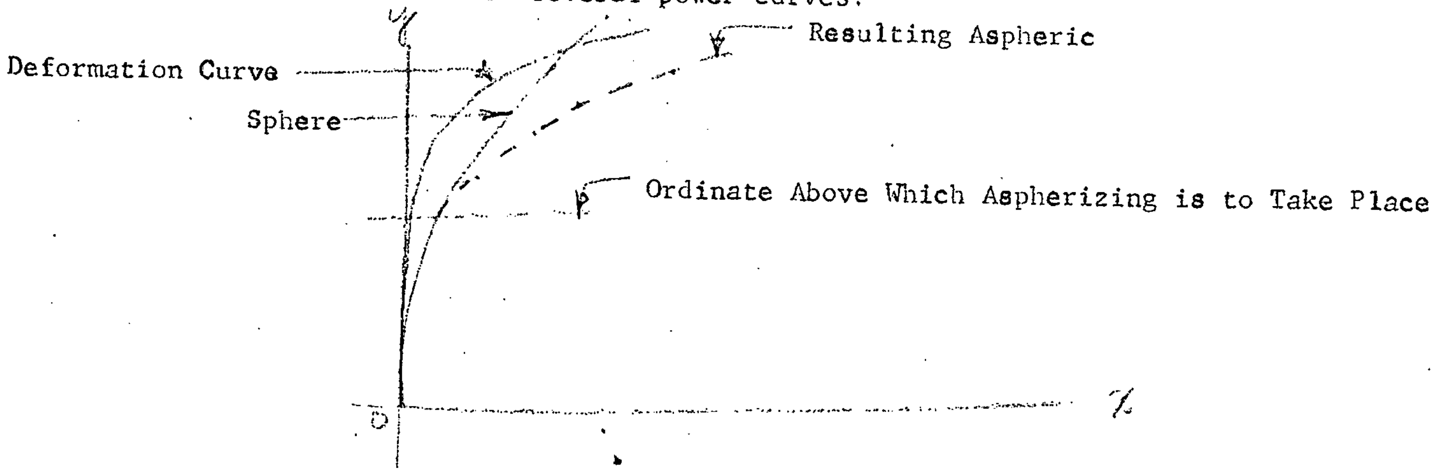

Figure 4

The method for calculating the deformation curve is a practical approach to a solution, easy to understand, and accurate. A surface is chosen for spherizing and it is known from the ray trace plots exactly at what height deformation is to begin. Figure 5 is a plot of on-axis spherical aberration where lateral ray intercept height on an image plane is plotted against fraction of entrance pupil height.

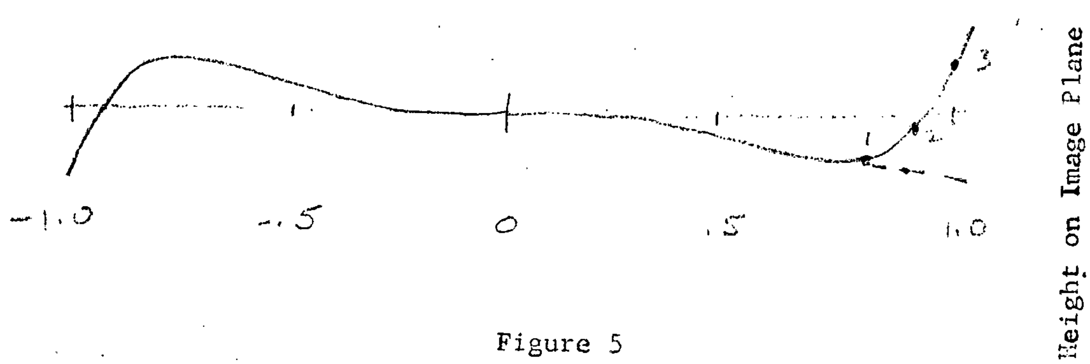

Figure 5

Fraction Entrance Pupil

The equation for a general surface of revolution is in the form of equation (4).

(4) $$x = \frac{cy^2}{1 + \sqrt{1 - c^2y^2}} + ay^4 + by^6 + cy^8 + dy^{10}$$

The three ray intercepts 1, 2, 3, in Figure 5 are to be brought into the dotted portion of the curve. A small change is made in coefficient b and the three rays 1, 2, 3 traced. The assumption is that for small changes in the coefficients b, c, d, there is a proportional change in the ray intercepts. In other words, the region of linearity is assumed. Analytically,

$$\Delta b \propto \Delta h_1$$

or

$$\Delta b = k_1 \; \Delta h_1$$

(5) likewise

$$\Delta b = k_2 \; \Delta h_2$$

$$\Delta b = k_3 \; \Delta h_3$$

Similarly, differential changes are recorded for c and d

(6) $$\Delta c = \ell_1 \; \Delta h_1$$

$$\Delta c = \ell_2 \; \Delta h_2$$

$$\Delta c = \ell_3 \; \Delta h_3$$

(7) $$\Delta d = m_1 \; \Delta h_1$$

$$\Delta d = m_2 \; \Delta h_2$$

$$\Delta d = m_3 \; \Delta h_3$$

A change in Δh_{1T} is desired for ray 1, Δh_{2T} for ray 2, and Δh_{3T} for ray 3. Putting the equations into form, a 3 x 3 matrix is obtained.

(8) $$\Delta b \frac{1}{k_1} + \Delta c \frac{1}{\ell_1} + \Delta d \frac{1}{m_1} = \Delta h_{1T}$$

(9) $$\Delta b \frac{1}{k_2} + \Delta c \frac{1}{\ell_2} + \Delta d \frac{1}{m_2} = \Delta h_{2T}$$

(10) $$\Delta b \frac{1}{k_3} + \Delta c \frac{1}{\ell_3} + \Delta d \frac{1}{m_3} = \Delta h_{3T}$$

If the changes needed are large the calculations go outside the region of linearity and the exact changes Δh_{1T}, Δh_{2T}, and Δh_{3T} are not obtained. It is then necessary to repeat the process, solving for Δb, Δc, and Δd, each time until solution is reached.