

File
Image Analysis
analog image
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REMOVAL OF IMAGE MOTION ABERRATIONS

by



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ABSTRACT

The resolution of aberrated photographic images distorted by arbitrary image motion have been substantially improved using holographic spatial filtering techniques. Examples of imagery corrected for the effects of one and two dimensional motion are presented. Also a novel technique is described for constructing, on film, Fourier transform holograms of functions with large amplitudes without exceeding the conventionally defined linear dynamic range. Noise and distortion due to the photographic media and filter alignment are minimized since the final filter, usually composed of two or more discrete elements, is recorded on a single piece of film.

REMOVAL OF IMAGE MOTION ABERRATIONS

Within the framework of linear systems analysis, O'Neill has shown that a uniformly smeared image can be written as a superposition integral over the object transparency.¹ Within this context a transfer function for the aberration can be defined as the Fourier transform of the integrated smear function that multiplies the transfer function of the non-aberrated optical system. When the non-aberrated impulse response can be considered rotationally symmetric and effectively a delta function, the transfer function for the imaging process is the optical Fourier transform of the aberrated impulse response. Since in this application this is the record of movement of the non-aberrated function, the incoherent transfer function can be obtained by optical Fourier transformation, if the aberrated impulse response can be recorded in a recoverable way on a film transparency. In the experiment described, conditions were arranged to record and recover an impulse response characteristic of an arbitrary image motion. Using a novel Fourier transform hologram filtering technique, it was possible to make significant improvements in the deliberately aberrated imagery.

In general a linear incoherent imaging process can be described by the convolution equation

$$I_{im}(x, y) = \int_{-\infty}^{\infty} I_{ob}(x', y') S(x - x', y - y') dx' dy' \quad (1)$$

where $I_{im}(ob)(x', y')$ refer to the intensity distribution in the object and $S(x - x', y - y')$ is the aberrated impulse response. In terms of the object and image spatial frequency spectra, the Fourier transform of the convolution equation is

$$\tilde{I}_{im}(\omega_x, \omega_y) = \tilde{I}_{ob}(\omega_x, \omega_y) \tau_o(\omega_x, \omega_y) \quad (2)$$

where the tilda (\sim) refers to the Fourier transform operation and $\tau_o(\omega_x, \omega_y)$ the Fourier transform of the impulse response or the incoherent transfer function

and ω_x, ω_y are the appropriate spatial frequency coordinates in the x, y plane (e.g., $\omega_x = \frac{x}{\lambda f}$ in a coherent optical transformation system using lenses of focal length f).

When there is relative motion between the object and the image during the exposure (assumed long compared to shutter opening and closing times) the photographic record is a smeared image described by

$$I_{\text{im}}(x, y) = \int_{-\infty}^{\infty} A(t) \int_{-\infty}^{\infty} I_{\text{ob}}(x' - f(t), y' - g(t)) S(x - x', y - y') dx' dy' dt \quad (3)$$

where $A(t)$ is an exposure function and $f(t)$ and $g(t)$ are linear functions describing the motion. When Fourier transformed Equation (3) becomes

$$\begin{aligned} \tilde{I}_{\text{im}}(\omega_x, \omega_y) &= \tilde{I}_{\text{ob}}(\omega_x, \omega_y) \tau_o(\omega_x, \omega_y) \int_{-\infty}^{\infty} A(t) e^{-2\pi i(\omega_x f(t) + \omega_y g(t))} dt \\ &= I_{\text{ob}}(\omega_x, \omega_y) \tau_m(\omega_x, \omega_y) \end{aligned} \quad (4)$$

where $\tau_m(\omega_x, \omega_y)$ (hereafter called $\tau(\omega_x, \omega_y)$) is the linear stationary effective transfer function in the developed transparency that completely describes the effects of the integrated image motion.

In a uniformly illuminated coherent optical system the coherent transfer function is effectively constant over the paraxial region. When a film transparency of a motion aberrated image is processed so that its specular amplitude transmittance is proportional to the original object intensity distribution, the optical Fourier transform of the coherently trans-illuminated transparency will be given by Equation (4). Since in this discussion $\tau(\omega_x, \omega_y)$ characterizes the effect of image motion on an idealized perfect image, the undistorted image can be recovered if the aberrated image spectra, Equation (4) is multiplied by $(\tau(\omega_x, \omega_y))^{-1}$. In this paper, a technique will be described for realizing a complex amplitude transmittance function that when multiplied by the aberrated

image spectra will yield a substantially improved image. Several examples of restored images are presented to demonstrate the efficiency of the filtering operations.

REALIZATION OF $\tau(\omega_x, \omega_y)^{-1}$

Numerous techniques have been mentioned to realize good approximations to complex optical filter functions since Tsujiuchi published his excellent results on image restoration using complex transmission masks in 1963.² Only minor improvements have been reported in the usually complicated procedures to construct phase filters by evaporation, thin films, polarization or other elaborate techniques. After coherent illumination sources became readily available, the capability of holograms to record continuous phase and amplitude distributions photographically was seen as a relatively simple method to realize a method of complex optical transmission filters. In 1964, Vander Lugt proposed what is now nearly the standard modified Mach-Zehnder interferometer approach to construct Fourier transform hologram filters for matched and detection filtering.³ Cutrona extended the basic techniques to coding and image restoration in 1965 when the removal of simulated one-directional linear image motion was accomplished by inverse Fourier transform hologram filtering.⁴ At the time, the principal limitation to an optimum realization of an image restoration filter was film linearity. In more recent papers by Lohmann and Werlich and Stroke, Indebetouw and Puech,⁶ imagery aberrated by defocusing and circular image motion respectively, was partially restored by the same basic techniques with approximate film linearity. In this paper a significant refinement in restored image quality is demonstrated when the hologram filter is constructed through an appropriate amplitude function and when both hologram and the amplitude transmission masks are processed linearly* in the conventional sense, which is made possible for some classes of large signals, by the exposure procedure. (The transmission through the amplitude filter is similar to the technique proposed independently by Lohmann and Werlich for code translation.)

The conventional technique to construct a filter proportional to the aberrated transfer function, $\tau(\omega_x, \omega_y)^{-1}$ is to approximate an amplitude filter proportional to $\frac{1}{|\tau(\omega_x, \omega_y)|^2}$ and combine it with a Fourier transform

hologram of the same function $F(\omega_x, \omega_y) = (|k + \tau(\omega_x, \omega_y)|^2)^{-\gamma/2} e^{i\alpha x}$. For $\gamma = -2$ (or for low contrast development) the aberrated object spectra multiplied by the two element filter will be

$$\tilde{I}_O(\omega_x, \omega_y) \tau(\omega_x, \omega_y) \cdot \frac{1}{|\tau(\omega_x, \omega_y)|^2} \cdot \left[k^2 + |\tau(\omega_x, \omega_y)|^2 + k \tau(\omega_x, \omega_y) e^{-i\alpha x} + k \tau^*(\omega_x, \omega_y) e^{i\alpha x} \right] \quad (5)$$

Ideally, the positive exponential term in the combined product will be

$$\tilde{I}_O(\omega_x, \omega_y) \frac{\tau(\omega_x, \omega_y)}{|\tau(\omega_x, \omega_y)|^2} \tau^*(\omega_x, \omega_y) e^{i\alpha \omega_x x} = \tilde{I}_O(\omega_x, \omega_y) e^{i\alpha \omega_x x}$$

which when Fourier transformed will be the unaberrated object distribution separated from the zero order terms by an amount determined by α .

Unfortunately, the image quality resulting with the two element filter has not been very encouraging. Extremely small misalignments between either element and/or the object spectra will produce no correction, and the inherent aberrations present when liquid gating any filter seem aggravated with two films or plates, possibly because of the increased depth or combined grain noise. More serious is the fact that in most cases only a small fraction of $\frac{1}{|\tau(\omega_x, \omega_y)|^2}$ can be recorded linearly. For example, if we consider a linear smear whose transfer function is proportional to $\text{sinc } \omega_x x$, the signal swing between the central and first order maxima is about 4.5:1. To construct a function on film with amplitude transmission $T_A(\omega) = \frac{1}{\text{sinc}^2 \omega_x x}$, the intensity

transmission will be $T_I(\alpha) \frac{1}{\text{sinc}^4 \omega_x x}$ or $D = \log T_I(\alpha) \log \text{sinc}^4 \omega_x x$, (Figure 1a).

To record just the first two maxima of the $\text{sinc}^2 \omega_x x$ function, 22 db film exposure response is required. In the experiment described here, the function to be recorded will be $\frac{1}{|\tau(\omega_x, \omega_y)|}$ which for expressions no more extreme than sinc or

besinc function can be recorded linearly in the conventional sense except in the vicinity of the zeros of the transfer function, (Figure 1b). (Since there is no scene information at these frequencies, and the optical filter is passive, this is to be expected and is no practical consequence.)

The experimental procedure is to optically Fourier transform the impulse response characteristic of the aberration and record it through the film backing on appropriately prefogged film at $\gamma = +1$, so that the processed amplitude transmission is $\frac{1}{|\tau_2(\omega_x, \omega_y)|}$. After processing, this transparency is replaced into a congruous position and the hologram filter is constructed by exposure through the transparency of the Fourier transform of the impulse response and a reference wave producing a filter function with amplitude transmission

$$\left\{ \left[\frac{1}{|\tau_2(\omega_x, \omega_y)|} \left(k + \tau_1(\omega_x, \omega_y) e^{i\alpha \cdot x} \right) \right]^2 \right\}^{-\gamma/2} \quad (6)$$

If processed to $\gamma = -2$ (by direct reversal process),* the appropriate filter function term becomes

$$\frac{k \tau_1^*(\omega_x, \omega_y)}{|\tau_2(\omega_x, \omega_y)|^2} e^{i\alpha x} = \frac{k' \tau^*(\omega_x, \omega_y)}{|\tau(\omega_x, \omega_y)|^2} e^{i\alpha x} \quad (7)$$

since τ_1 and τ_2 differ only by a constant which can be adjusted to optimize the filter characteristics.

If we examine the construction of the filter for linear unidirectional function on the basis of the amplitude transmission of the elements as in Figure 2, it can be shown that the desired function (Figure 2d) can be recorded linearly in the conventional sense as in Figure 3. The net effect of this technique is to reduce the size of the signal as seen by the film so that a closer approximation to the calculated filter function can be realized with conventional techniques.

DESCRIPTION OF THE EXPERIMENT

To generate a transmission function whose optical Fourier transform is the appropriate incoherent transfer function, a small pinhole, whose dimension establishes the upper limit on recoverable resolution, was placed in the format of an object scene transparency. While being moved in an arbitrary fashion in the object plane, the integrated motion of the modified transparency is recorded on film. The photographic record of the path of the point object is the appropriate impulse response for the motion aberration if the resulting transparency is processed linear in amplitude or to a gamma of -2. If this impulse response is isolated and optically Fourier transformed, the complex amplitude distribution existing in the transform plane is the transfer function characteristic of the aberration. Admittedly this simulation by-passes the difficulty of extracting an impulse response from an arbitrary scene but it is useful to optimize this filtering operation in preliminary stages of the experiment.

In the first experiment the simulation experiment described by Cutrona was duplicated. The impulse response characteristic of a unidirectional linear smear was simulated by a slit 1 mm long and 0.080 mm wide. An inverse Fourier transform filter was constructed as described above using the optical Fourier transform of the small rect function as $\tau(\omega_x, \omega_y)$. Figure 4a is a slightly tapered slit about 2 mm wide which in Figure 4b has been shrunk to a mean width of 0.016 mm or about twice the impulse response width. Figure 4d is a micro-densitometer trace of the filtered output indicating a signal to noise ratio of 7.5:1. An additional reason for performing this experiment is that the physical parameters

of the aberration could be very closely controlled permitting independent optimization of the filter function exposure and processing.

In the more general application to non-symmetric, two-dimensional smears the general procedure is the same but exposure control and filter adjustment becomes very critical. In Figure 6 and 7 a - d, the experimental procedure is illustrated for two different aberrations of a printed page containing a small point to generate the aberrated impulse response (Figure 5). Figures 6 and 7b have been smeared to generate the aberrated imagery with the impulse responses seen magnified in c. After filtering, most of the aberrations have been removed as exemplified in d and e. One characteristic of the data presented here that should be noted is that the entire format could not be restored by a single adjustment of the filter. Optimum filtering in different regions of the format could be obtained by only a small readjustment of the filter. This tendency, due primarily to astigmatism and spherical aberration, can be minimized if all elements, especially the liquid gates in both the construction and filtering operations, are as nearly identical as possible.

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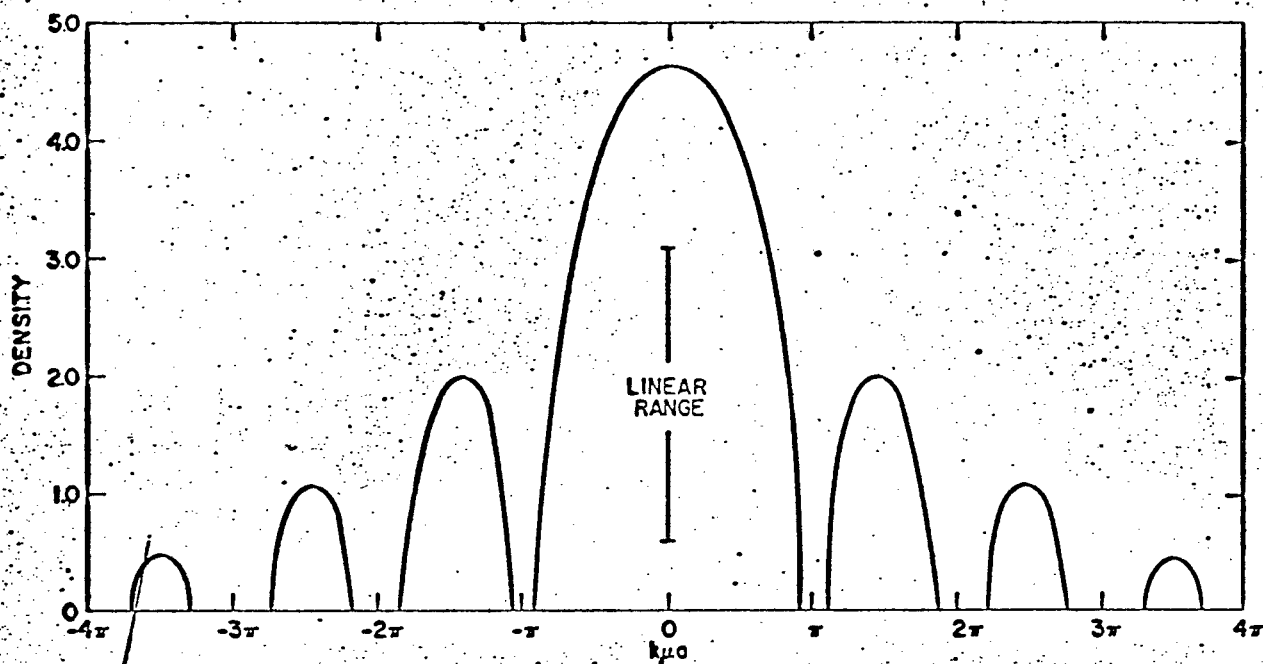


Figure 1a Density Distribution Necessary to Record $T_A(\mu) = 1/(I_0 \text{sinc}^2 k\mu a)$

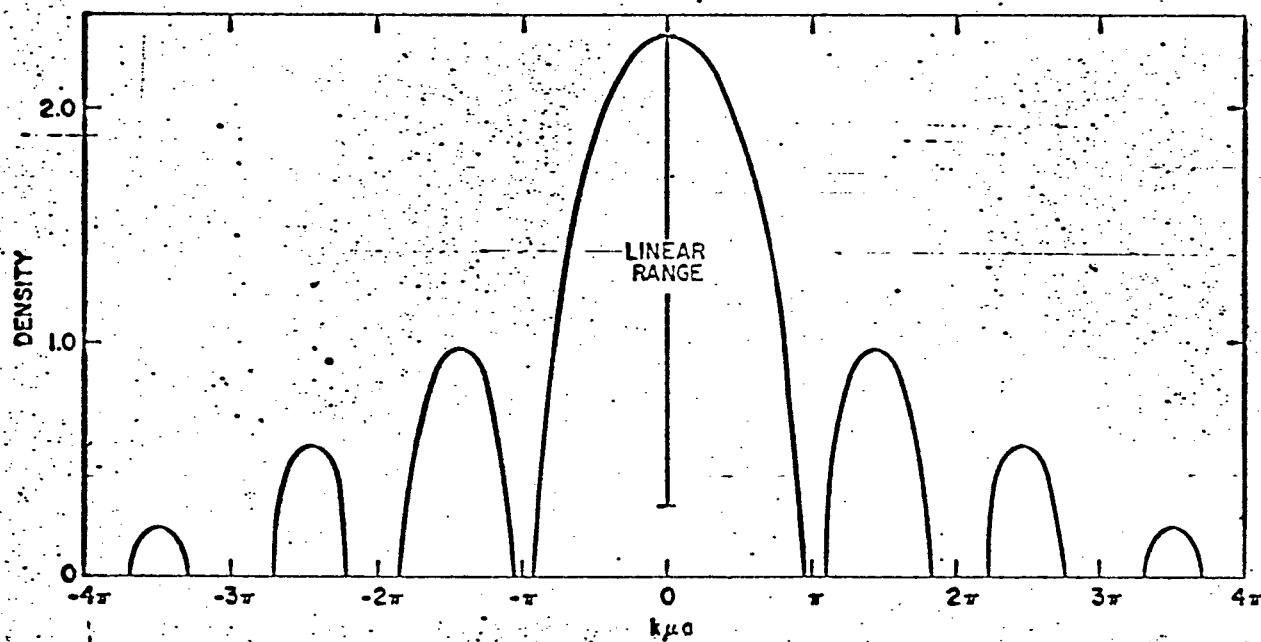


Figure 1b Density Distribution Necessary to Record $T_A(\mu) = 1/(I_0^{1/2} \text{sinc } k\mu a)$

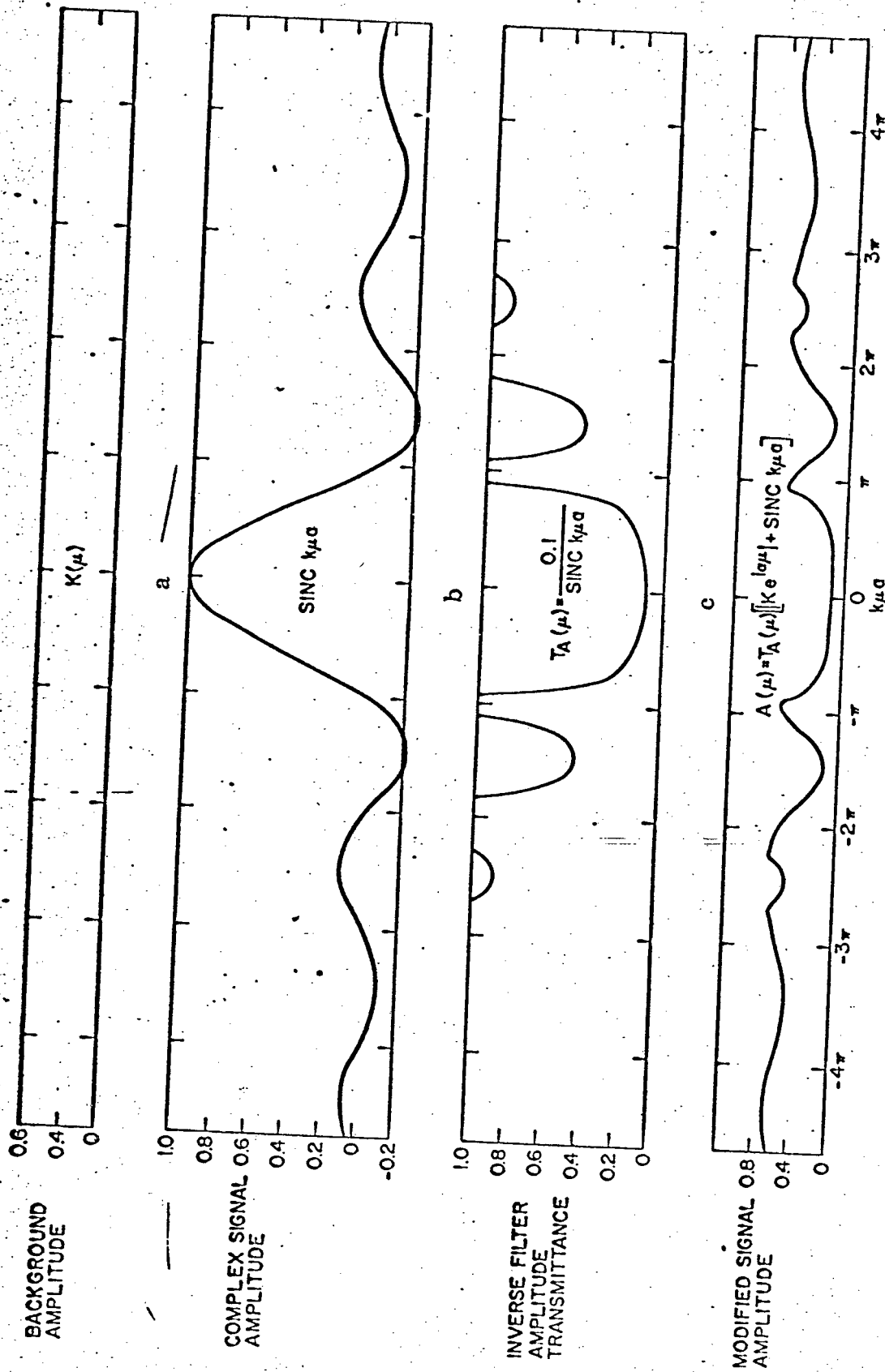


Figure 2 Graphical Description of Hologram Filter Construction for $\tau(\mu) = \int_0^{1/2} \text{sinc } k\mu\sigma$

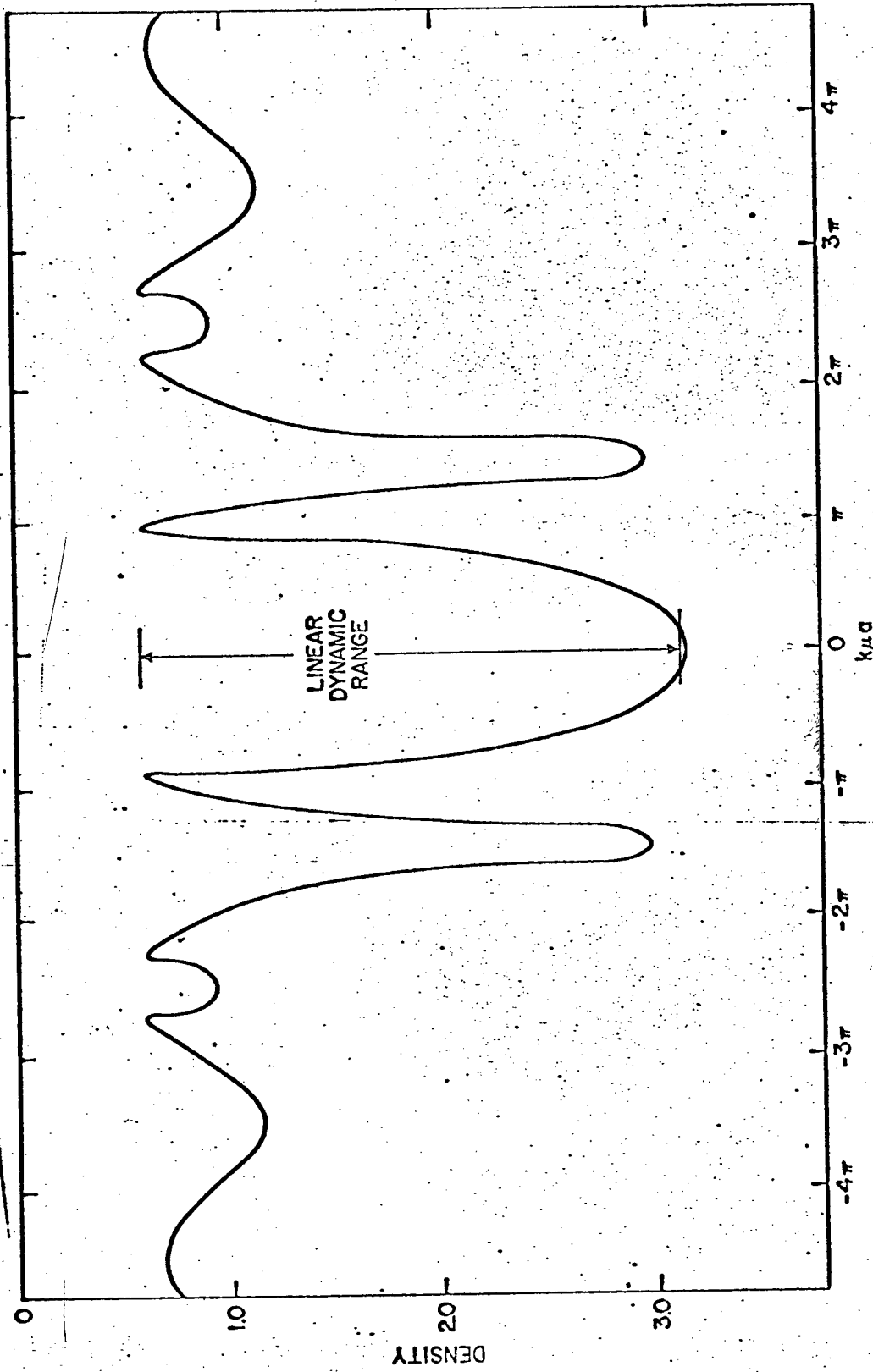


Figure 3 Density Distribution Corresponding to Values Used in Figure 3

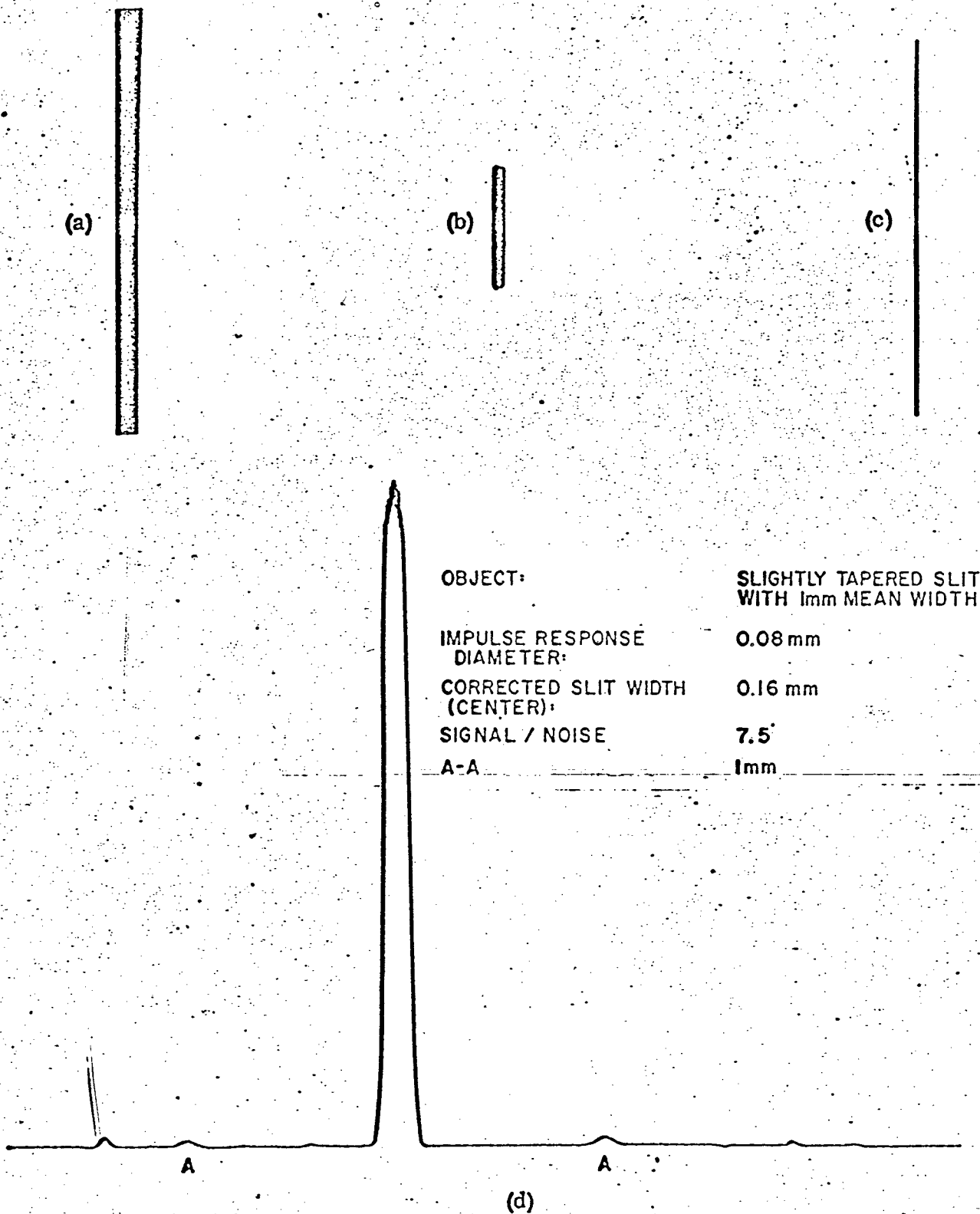


Figure 4 Simulated Smear of Typical Line Source (a), Enlargement of Simulated Impulse Response (b), Corrected Image (c), and Microdensitometer Trace Across Corrected Image (d)

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(a) Nonaberrated Image

(b) Aberrated Impulse Response

(c) Resultant Aberrated Image

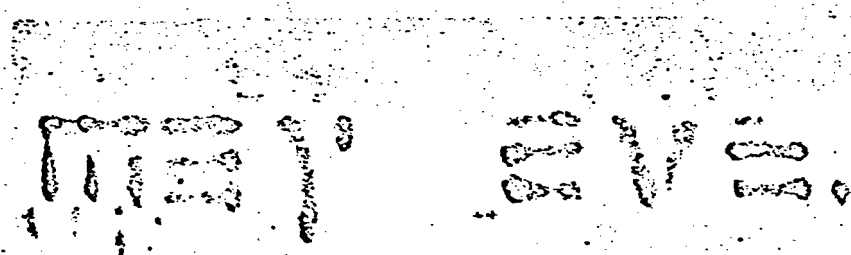
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(d) Filtered Image



(e) Enlargement of Smear



(f) Enlargement of Filtered Image

Figure 6 Printed Page with "L" Smear

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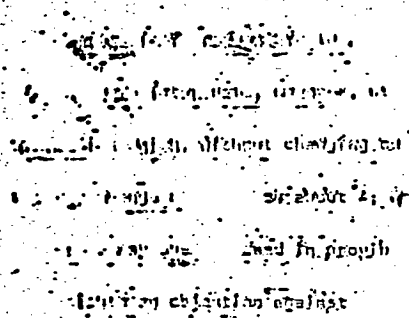


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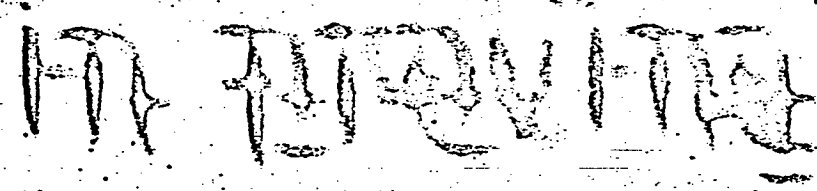
(a) Nonaberrated Image

(b) Aberrated
Impulse Re-
sponse

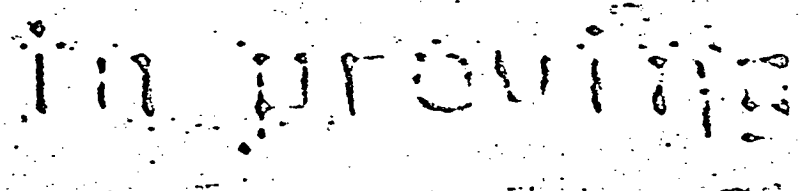
(c) Resultant Aberrated
Image



(d) Filtered Image



(e) Enlargement of Smear



(f) Enlargement of Filtered Image

Figure 7 Printed Page with "Z" Smear

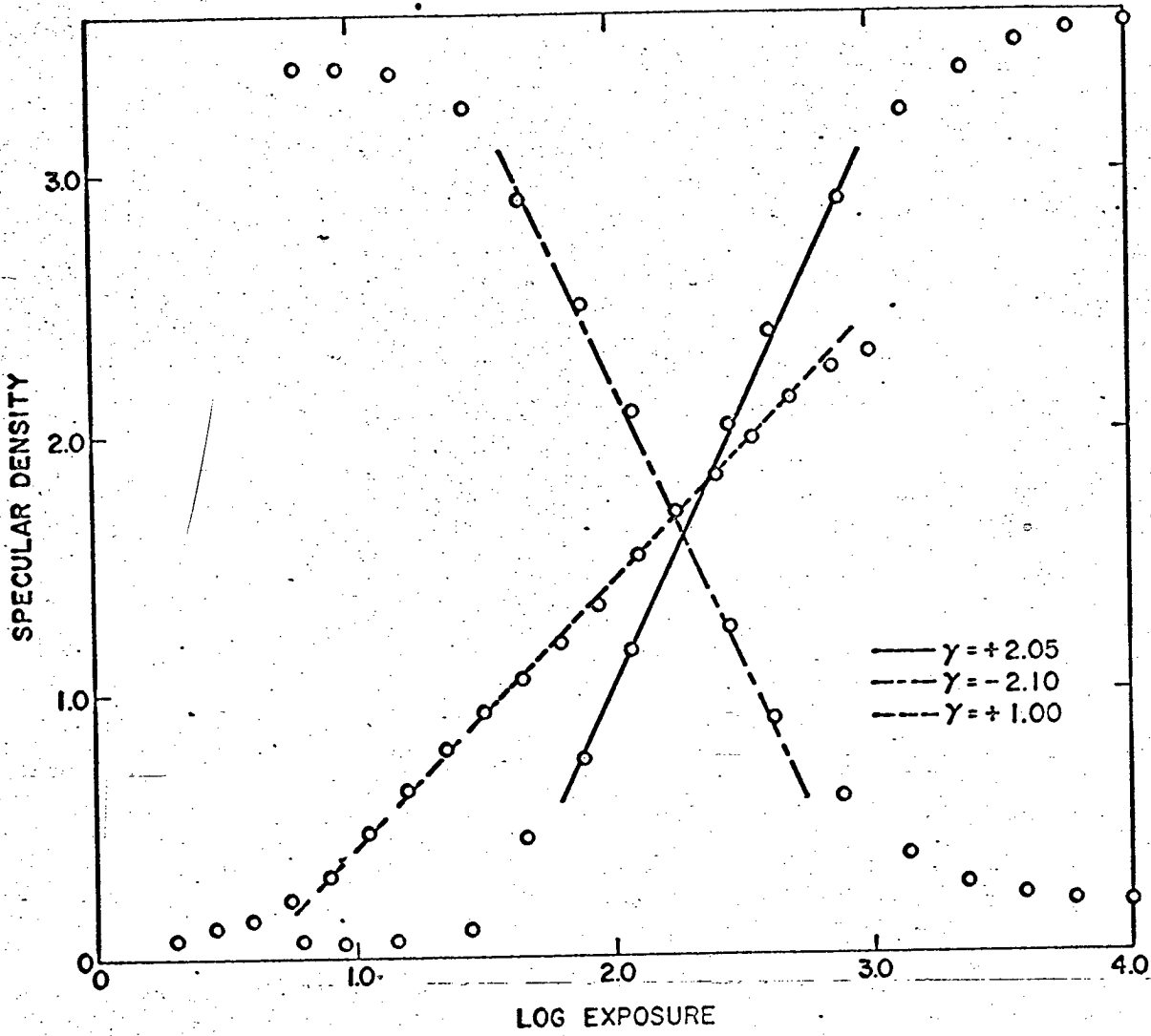


Figure 8 Coherent H and D Curve for Recordak Microfilm, $\gamma = \pm 2, +1$