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23 January 1975

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RESEARCH AID A(ER) 74-64, Methods of Constructing Economic Indexes from Incomplete Data, December 1974

1. The numbers appearing in Appendix B are outdated estimates used solely for the purpose of testing the procedures developed in this Research Aid. Please do not use them for any substantive purpose.

2. A set of unclassified estimates of GNP and industrial production will be disseminated in a few months.

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A (ER) 74-64
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RESEARCH AID

Methods of Constructing Economic Indexes from Incomplete Data

December 1974

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METHODS OF CONSTRUCTING ECONOMIC INDEXES FROM INCOMPLETE DATA

Introduction

1. In constructing indexes of an economy's performance, economists must sometimes work with incomplete data. A typical problem involves estimating missing values in a time series of industrial output, in order to compute an index of industrial progress over time. In particular, the estimation problem presupposes (1) an **incomplete** set of time series observations on quantities produced by several industrial sectors and (2) a complete set of corresponding unit values of output (for an illustrative set of data, see Table 1). The quantity produced by the jth of n sectors in the tth of m years can be labeled $Q(j,t)$, and the time-constant unit value of the jth sector's output can be named $P(j)$. A time index of total value produced by the n sectors is then

$$I(t) = \frac{\sum_{j=1}^{n} (P(j) \times Q(j,t))}{\sum_{j=1}^{n} (P(j) \times Q(j,b))} \times 100$$

where b is the base year from which the index is computed. The problem is to estimate the index $I(t)$ for all years $t=1, \ldots, m$, given that for each of these years some but not all of the quantity observations $Q(j,t)$ are missing.

Table 1

Hypothetical Data on Industrial Output and Unit Values of Output

Industrial Sector	Unit Value of Output	1961	1962	1963	1964	1965	1966	1967
Ferrous metals	21	95	85	?	?	190	?	?
Fuels	17	118	177	236	295	354	324	?
Electric power	87	23	?	?	?	46	?	56
Chemicals	43	47	56	65	75	84	94	103
Lumber	24	83	103	124	145	166	186	207
Building materials	4	503	402	477	?	729	804	855

2. A procedure for this problem was suggested in 1960 by Kaplan and Moorsteen.[1] Two alternative and equally plausible procedures for the problem are the Field method [2] and the naive method of linear interpolation, in which an analyst merely uses a ruler to connect adjacent quantity values in a sector's time series of output.

[1] Norman M. Kaplan and Richard H. Moorsteen, *Indexes of Soviet Industrial Output,* Vol. I, RAND Corporation, Research Memorandum 2495, May 1960.

[2] R.M. Field III, *A Method of Constructing Index Numbers,* unpublished memorandum, CIA, Office of Economic Research, September 1974.

3. This publication

- explains each of the three methods, with reference to the sample data in Table 1;

- presents computer programs for each method; and

- tabulates Monte Carlo tests of each method on five different sets of Soviet economic data.

Principal Findings

4. In our tests of the three methods, the method of linear interpolation usually produced the smallest errors in estimation. The Field method was second best, and the Kaplan-Moorsteen (KM) placed third. This ranking resulted partly from the fact that we based our tests on time series that usually grew smoothly. The Field method would probably have excelled, had the series followed a cyclical pattern. Further research might profitably be directed toward developing an estimation method better than any of the three considered in this report and toward isolating classes of problems in which each method excels.

Kaplan-Moorsteen Method

5. *Step 1.* Calculate benchmark indexes B(t) for years in which output data are available for each industrial sector. If the data are not complete for at least two years, the method does not apply. Since the sample data in Table 1 are complete for the first and fifth years (1961 and 1965), the benchmark indexes are

$$B(1) = \frac{\sum_{j=1}^{6} (P(j) \times Q(j,1))}{\sum_{j=1}^{6} (P(j) \times Q(j,1))} \times 100 = 100 \text{ and}$$

$$B(5) = \frac{\sum_{j=1}^{6} (P(j) \times Q(j,5))}{\sum_{j=1}^{6} (P(j) \times Q(j,1))} \times 100 = 203.9$$

where we have selected year 1 as the base year.

6. *Step 2.* Calculate average annual rates of growth between each successive pair of benchmark indexes. Since the sample problem contains only two benchmark years (1 and 5) we calculate a single growth rate $\alpha(1,5)$ from

$$(1+\alpha(1,5)) = (B(5)/B(1))^{1/(5-1)} = (203.9/100)^{1/4} = 1.1950$$

7. *Step 3.* Calculate interpolating indexes for each year between each successive pair of benchmark years. To make this calculation for a year between two benchmark years, isolate the sectors whose output observations are complete between the two benchmark years. In the sample problem, the sectors 2, 4, and 5 offer complete observations between the benchmark years 1 and 5. We denote the

set {2, 4, 5} by N(1, 5), and write the interpolating index for the tth year, t=1, 2, . . ., 5, as

$$S(t)=\frac{\sum_{j\epsilon N(1,5)} (P(j)\times Q(j,t))}{\sum_{j\epsilon N(1,5)} (P(j)\times Q(j,b))},$$

where we have selected year 1 as the base year b.

8. *Step 4.* Compute average annual rates of growth between each pair of benchmark years. Base the calculation on the interpolating index values for each of the pair of benchmark years in question. In the sample problem, the benchmark interpolating indexes are S(1) and S(5). The interpolating growth rate—$\beta(1,5)$—between the two benchmark years is derived from

$$(1+\beta(1,5))=(S(5)/S(1))^{1/(5-1)}=(226.2/100)^{1/4}=1.226.$$

9. *Step 5.* Compute extrapolation indexes for the most recent pair of benchmark years and for all years beyond the most recent benchmark year. For this calculation we use M(5) to denote the set of sectors offering complete observations beyond the most recent benchmark year, which is year 5. Then the extrapolation index for the tth year, t=1, 5, 6, 7, is

$$E(t)=\frac{\sum_{j\epsilon M(5)} (P(j)\times Q(j,t))}{\sum_{j\epsilon M(5)} (P(j)\times Q(j,b))}.$$

10. *Step 6.* Calculate the extrapolation growth rate—γ—between the most recent pair of benchmark years. In the sample problem, γ is determined from

$$(1+\gamma)=(E(5)/E(1))^{1/(5-1)}=(174.5/100)^{1/4}=1.1493$$

11. *Step 7.* Compute an index of industrial value added for each year spanned by the data on output. For years between the two benchmark years in the sample problem, the value-added index—KM(t)—is

$$KM(t)=\frac{(1+\alpha(1,5))}{(1+\beta(1,5))}\times\frac{S(t)}{S(t-1)}\times KM(t-1),$$

where the initial value KM(0) for this problem is B(1)—the value-added index for the first benchmark year. For years since the most recent benchmark year, the value-added index is

$$KM(t)=\frac{(1+\alpha(1,5))}{(1+\gamma)}\times\frac{S(t)}{S(t-1)}\times KM(t-1),$$

where the initial value KM(0) is B(5)—the index number for the latest benchmark year. This completes the procedure. (For values of the KM index, see Table 2; for a program of steps to compute the index, see Appendix A.)

Field Method

12. *Step 1.* Calculate benchmark indexes—B(t)—for years in which output data are available for each industrial sector. Since the sample data in Table 1

Table 2

Hypothetical Data on Industrial Output and Unit Values of Output and on Corresponding Indexes of Value Added

Industrial Sector	Unit Value of Output	1961	1962	1963	1964	1965	1966	1967
Ferrous metals	21	95	85	?	?	190	?	?
Fuels	17	118	177	236	295	354	324	?
Electric power	87	23	?	?	?	46	?	56
Chemicals	43	47	56	65	75	84	94	103
Lumber	24	83	103	124	145	166	186	207
Building materials	4	503	402	477	?	729	804	855
Kaplan-Moorsteen index		100	127.7	154.3	180.1	203.9	236.4	268.7
Field index		100	115.7	143.8	175.9	203.9	212.2	264.3
Linear interpolation index		100	114.6	143.1	173.7	203.9	219.4	234.0

are complete for the first and fifth years (1961 and 1965), the benchmark indexes are

$$B(1) = \frac{\sum_{j=1}^{6} (P(j) \times Q(j,1))}{\sum_{j=1}^{6} (P(j) \times Q(j,1))} \times 100 = 100 \text{ and}$$

$$B(5) = \frac{\sum_{j=1}^{6} (P(j) \times Q(j,5))}{\sum_{j=1}^{6} (P(j) \times Q(j,1))} \times 100 = 203.9$$

where we have selected year 1 as the base year.

13. *Step 2.* Calculate average annual rates of growth between each successive pair of benchmark indexes. Since the sample problem contains only two benchmark years (1 and 5) we calculate a single growth rate—$\alpha(1,5)$—from

$$(1+\alpha(1,5)) = (B(5)/B(1))^{1/(5-1)} = (203.9/100)^{1/4} = 1.1950$$

14. *Step 3.* Calculate link relatives for each pair of years i and k in which year k is greater than or equal to i. To make this calculation for a pair of years, isolate the sectors with output observations for both years. In the sample problem, sectors 2, 4, 5, and 6 offer observations for the years 1 and 3. We define the composite indicator function $\delta(j, ik)$ as

$$\delta(j, ik) = \begin{cases} 1 & \text{if both } Q(j,i) \text{ and } Q(j,k) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

We write the link relatives—$r(i,k)$—for the pairs of years, (i,k), $i=1, 2, \ldots, 7$, $k \geq i$, as

$$r(i,k) = \frac{\sum_{j=1}^{6} (\delta(j,ik) \times P(j) \times Q(j,k))}{\sum_{j=1}^{6} (\delta(j,ik) \times P(j) \times Q(j,i))}.$$

15. *Step 4.* Calculate the adjusting growth rate $\lambda(i,k)$ between the pair of benchmark years containing the years i and k. In the sample problem, $\lambda(1,3)$ is determined from

$$1+\lambda(1,3)=\left\{\frac{\sum_{j=1}^{6}(\delta(j,ik)\times P(j)\times Q(j,5))}{\sum_{j=1}^{6}(\delta(j,ik)\times P(j)\times Q(j,1))}\right\}^{1/(3-1)}.$$

When extrapolating beyond the last benchmark year or before the first benchmark year, compute the extrapolating growth rate between the last two benchmark years and the first two benchmark years, respectively.

16. *Step 5.* Compute adjusted link relatives—$R(i,k)$—for each pair of years i and k in which year k is greater than or equal to year i. In the sample problem, the adjusted link relative—$R(1,3)$—linking years 1 and 3 is

$$R(1,3)=((1+\alpha(1,5))/(1+\lambda(1,3))^{3-1}))\times r(1,3)=1.449.$$

If the data are not complete for at least two years, we omit step 5 and set $r(i,k)=R(i,k)$.

17. *Step 6.* Compute an index of industrial output for each year spanned by the data on output. We compute the index—$\tilde{Y}(t)$—from a smoothing algorithm. The algorithm minimizes the multiplicative error term between the ratio of the value-added indexes and the adjusted link relatives, subject to the constraint that the index equals the benchmark index for each benchmark year. For the sample problem, the algorithm solves the constrained optimization problem of selecting values for $Y(k)$, $k=1, \ldots, 7$, to minimize

$$\sum_{i=1}^{7}\sum_{k=i}^{7}(\log Y(k)-\log Y(i)-\log R(i,k))^2$$

subject to

$$\log B(5)=\log Y(5)$$
$$\log B(1)=\log Y(1)$$
$$Y(t)\geq 0,\ t=1,2,\ldots,7.$$

The solutions to this problem, namely the indexes $\tilde{Y}(t)$, $t=1, 2, \ldots, 7$, are solutions to a system of simultaneous equations (see Table 3). Solving these equations completes the Field method (for a program of the method, see Appendix B; for results for the sample problem, see Table 2).

Linear Interpolation Method

18. *Step 1.* Estimate the missing values in a sector's time series of output by linear interpolation. For sector 1 in the sample problem the years for which the data are available—the node years—are years 1, 2, and 5. We connect the nodes by line segments and estimate the missing observations as shown in the chart. Algebraically, we estimate the missing observation for year 3 in sector 1—$\tilde{Q}(1,3)$—as

$$\tilde{Q}(1,3) = Q(1,2) + (1/(5-2)) \times (Q(1,5) - Q(1,2)) = 120.$$

Table 3

Field Method Smoothing Algorithm for Sample Problem

$$\begin{bmatrix} 12 & -2 & -2 & -2 & -2 & -2 & -2 & 1 & 0 \\ -2 & 12 & -2 & -2 & -2 & -2 & -2 & 0 & 0 \\ -2 & -2 & 12 & -2 & -2 & -2 & -2 & 0 & 0 \\ -2 & -2 & -2 & 12 & -2 & -2 & -2 & 0 & 0 \\ -2 & -2 & -2 & -2 & 12 & -2 & -2 & 0 & 1 \\ -2 & -2 & -2 & -2 & -2 & 12 & -2 & 0 & 0 \\ -2 & -2 & -2 & -2 & -2 & -2 & 12 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & & 0 \end{bmatrix} \times \begin{bmatrix} \log \tilde{Y}_1 \\ \log \tilde{Y}_2 \\ \log \tilde{Y}_3 \\ \log \tilde{Y}_4 \\ \log \tilde{Y}_5 \\ \log \tilde{Y}_6 \\ \log \tilde{Y}_7 \\ \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{bmatrix} =$$

$$\begin{bmatrix} -2(\log R_{12} + \log R_{13} + \log R_{14} + \log R_{15} + \log R_{16} + \log R_{17}) \\ +2(\log R_{12} - \log R_{23} - \log R_{24} - \log R_{25} - \log R_{26} - \log R_{27}) \\ +2(\log R_{13} + \log R_{23} - \log R_{34} - \log R_{35} - \log R_{36} - \log R_{37}) \\ +2(\log R_{14} + \log R_{24} + \log R_{34} - \log R_{45} - \log R_{46} - \log R_{47}) \\ +2(\log R_{15} + \log R_{25} + \log R_{35} + \log R_{45} - \log R_{56} - \log R_{57}) \\ +2(\log R_{16} + \log R_{26} + \log R_{36} + \log R_{46} + \log R_{56} - \log R_{67}) \\ +2(\log R_{17} + \log R_{27} + \log R_{37} + \log R_{47} + \log R_{57} + \log R_{67}) \\ \log B_1 \\ \log B_5 \end{bmatrix}$$

19. *Step 2.* Compute an index of industrial output for each year spanned by the data on output, by using the estimated quantity matrix $\tilde{Q}(j, t)$. Table 4 contains the estimated quantity matrix $\tilde{Q}(j, t)$ for the sample program. The index—$L(t)$—for year 4 in the sample problem is

$$L(4) = \frac{\sum_{j=1}^{6} (P(j) \times Q(4, j))}{\sum_{j=1}^{6} (P(j) \times Q(1, j))} \times 100 = 173.7$$

This completes the process (for a program of the method, see Appendix A; for the method's results for the sample problem, see Table 2).

Table 4

Estimated Hypothetical Data on Industrial Output and Unit Values of Output

Industrial Sector	Unit Value of Output	1961	1962	1963	1964	1965	1966	1967
Ferrous metals	21	95	85	~120 [1]	~155	190	~225	~260
Fuels	17	118	177	236	295	354	324	~294
Electric power	87	23	~28.75	~34.5	~40.25	46	~51	56
Chemicals	43	47	56	65	75	84	94	103
Lumber	24	83	103	124	145	166	186	207
Building materials	4	503	402	477	~603	729	804	855

[1] The superscript (~) denotes an estimated entry in the data matrix.

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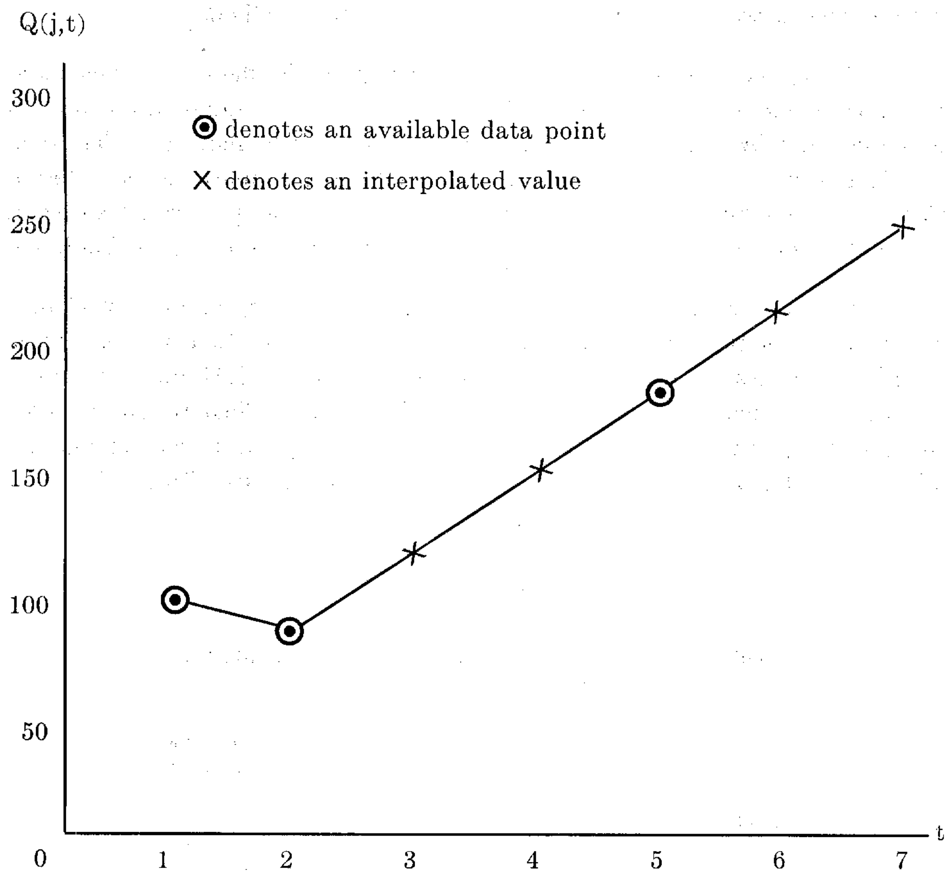
Linear Interpolation Estimations



Tests of the Three Methods

20. We first tested the KM, Field, and linear interpolation methods by selecting a complete set of observations on Soviet GNP reported by sector of origin. We deleted randomly 5% of the data and then applied each of the three methods. By comparing the sum of squared errors in estimation produced by each method, we ranked the methods, assigning the first rank to the method that produced the smallest error in estimating the true index of value added. We next deleted 10% of the data and repeated the ranking. Then we deleted 15%, then 20%, 25%, and 30%. This sequence of deletions and rankings constituted one cycle of our tests. After completing one cycle, we began another until we completed 60 tests (or 10 cycles) against the data.

21. In the 60 tests on the Soviet GNP data, the KM method could not be applied in 24 cases, since these cases violated at least one of the assumptions that KM made about data availability. Of the 36 times that the KM method could be applied—and thus compared with the Field and linear interpolation methods—the KM method performed best only twice. It ranked second 10 times, and worst 24 times.

22. The Field and linear interpolation methods could be compared on each of the 60 tests against the Soviet GNP data. In these tests linear interpolation ranked first 47 times, versus 11 firsts for the Field method and 2 firsts for KM (see Table 5). In other words, linear interpolation performed best in 78% of the times that it was tested against the two alternative methods.

23. Having completed 60 tests on the Soviet GNP data, we considered another complete set of data—outputs by Soviet services industries—and ran another 60 tests. We then ran 60 tests against each of three more complete data bases (see Table 5). In total, we tested the methods 300 times. Linear interpolation performed best 74% of its trials. The KM method performed worst in 65% of the times it was compared with the other two methods (see Table 6).

24. By changing the percentage of observations deleted, we did not change the general result that linear interpolation performed better than the other two methods. When 5% of the observations were deleted, the KM method ranked first more often than the Field Method. At 10% sparsity, Field registered more firsts than KM, and, at higher levels of sparsity, KM applied so seldom that we could not establish for it a statistically significant rank (see Table 7).

Table 5

Ranks of Index Construction Methods in Tests on Different Data Bases

	Rank [1]	Linear Interpolation	Field	Kaplan-Moorsteen
Soviet GNP				
	1	47	11	2
	2	12	38	10
	3	1	11	24
	None	....	....	24
Soviet services				
	1	40	9	11
	2	18	37	5
	3	2	14	16
	None	....	....	28
Soviet industry				
	1	47	13	....
	2	13	42	5
	3	....	5	25
	None	....	....	30
Soviet crop				
	1	39	20	1
	2	20	37	3
	3	1	3	27
	None	....	....	29
Soviet livestock				
	1	48	6	6
	2	10	31	19
	3	2	23	21
	None	....	....	14

[1] First rank is assigned to the method that performs best.

Table 6

Overall Ranks of Index Construction Methods

Rank [1]	Linear Interpolation	Field	Kaplan-Moorsteen
1	221	59	20
2	73	185	42
3	6	56	113
None	....	....	125

[1] First rank is assigned to the method that performs best.

Table 7

Performance of Index Construction Methods with Different Percentages of Observations Deleted

	Rank [1]	Linear Interpolation	Field	Kaplan-Moorsteen
5% deleted				
	1	37	3	10
	2	11	21	18
	3	2	26	22
10% deleted				
	1	36	12	2
	2	14	24	12
	3	....	14	32
	None	....	....	4
15% deleted				
	1	38	10	2
	2	11	35	4
	3	1	5	28
	None	....	....	16
20% deleted				
	1	37	13	....
	2	12	33	5
	3	1	4	14
	None	....	....	31
25% deleted				
	1	36	9	5
	2	13	36	1
	3	1	5	13
	None	....	....	31
30% deleted				
	1	37	12	1
	2	12	36	2
	3	1	2	4
	None	....	....	43

[1] First rank is assigned to the method that performs best.

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APPENDIX A

PROGRAMS TO COMPUTE INDEXES OF VALUE ADDED, AND TO TEST THE INDEXES

1. The APL[3] program named MISSING computes the KM, Field, and linear interpolation indexes (for a listing of this program and the subroutines it calls, see Tables 8-18). When executed, the program calls for all the data it needs. In response to the program's call for a price vector, the user types in the list of values added by the industrial sectors, beginning with the first sector. To satisfy the program's request for a quantities matrix, the user enters a table of industrial outputs, in the form of Table 1 in this report.

2. The program MISSING1 does Monte Carlo tests of the three methods (see Table 8). Given a complete set of observations on sectoral outputs, the program deletes randomly a specified percentage of the observations, and then applies the three methods.

---

[3] APL stands for *A Programming Language*. For documentation, see L. Gilman and A. Rose, *APL: An Interactive Approach*, second edition, John Wiley and Sons, Inc., New York, 1974.

Table 8

Program To Compute the Kaplan-Moorsteen, Field, and Linear Interpolation Indexes

```
        ∇ MISSING[⎕]∇
    ∇   MISSING;BY;BM;IM;EI;PBM;KM;FJ;QF;FKI;P;Q;N;M;A;
        ALRM;B;BW;G;INT;VC;LRIVM;INV1;INV2;INV3;INV4
[1]     'ENTER THE PRICE VECTOR'
[2]     P←⎕
[3]     'ENTER THE QUANTITY MATRIX'
[4]     Q←⎕
[5]     BNCHMK
[6]     KAPMOOR
[7]     LINKREL
[8]     FIELD
[9]     LININT1
[10]    LININT2
[11]    'BENCHMARK YEARS';BY
[12]    →((ρBY)≤1)/L1
[13]    'BENCHMARK INDEXES';BM
[14]    'INTERPOLATION INDEXES';IM
[15]    'EXTRAPOLATION INDEXES';EI
[16]    'PRE-BENCHMARK INDEXES';PBM
[17]    'KAPLAN-MOORSTEEN INDEXES';KM
[18]  L1:'FIELD INDEXES';FJ
[19]    'LINEAR INTERPOLATION INDEXES';FKI
    ∇
```

Table 9

Program To Test the Kaplan-Moorsteen, Field, and Linear Interpolation Indexes

```
        ∇ MISSING1[⎕]∇
    ∇   MISSING1;INV1;INV2;INV3;BY;BM;IM;EI;KM;FJ;QF;FKI;
        P;Q;N;M;A;ALRM;B;BW;INT;INV4;VC;LRIVM;Z;X;SP;FKE;
        FJE;EFKE;EFJE;KME;EFKME
[1]     'ENTER THE PRICE VECTOR'
[2]     P←⎕
[3]     'ENTER THE QUANTITY MATRIX'
[4]     Z←⎕
[5]     N←ρP
[6]     M←ρZ[;1]
[7]     'ENTER THE NUMBER OF OBSERVATIONS TO BE
        DELETED'
[8]     X←⎕
[9]     SIMULATE
[10]    Q←YD
[11]    BNCHMK
[12]    KAPMOOR
[13]    LINKREL
[14]    FIELD
[15]    LININT1
[16]    LININT2
[17]    ACTIND
[18]    SQRER
[19]    'BENCIIMARK YEARS';BY
[20]    →((ρBY)≤1)/L1
[21]    'BENCHMARK INDEXES';BM
[22]    'KAPLAN-MOORSTEEN INDEXES';KM
[23]  L1:'FIELD INDEXES';FJ
[24]    'LINEAR INTERPOLATION INDEXES';FKI
[25]    'ACTUAL INDEX';AI
[26]    →((ρBY)≤1)/L2
[27]    'KAPLAN-MOORSTEEN ERROR ';KME
[28]  L2:'FIELD ERROR';FJE
[29]    'LINEAR INTERPOLATION ERROR';FKE
[30]    SP←(X÷(M×N))×100
[31]    'PERCENT SPARSITY';SP
[32]    G
    ∇
```

Table 10

Subroutine To Delete Observations from a Complete Matrix

```
      ∇ SIMULATE[□]∇
    ∇   SIMULATE;DR;DC;MC
[1]     ⍝DELETE ELEMENTS IN QUANTITY MATRIX
[2]     YD←Z
[3]     MC←0
[4]   L1:MC←MC+1
[5]     →(MC>X)/0
[6]     DR←?M
[7]     DC←?N
[8]     YD[DR;DC]←0
[9]     →L1
    ∇
```

Table 11

Subroutine to Compute the Actual Indexes

```
      ∇ ACTIND[□]∇
    ∇   ACTIND;AIN;IZ
[1]     ⍝COMPUTE ACTUAL INDEXES
[2]     AI←⍳0
[3]     AIN←100
[4]     IZ←1
[5]   LOOP1:AI←AI,AIN
[6]     IZ←IZ+1
[7]     →(IZ>M)/L2
[8]     AIN←((+/P×⌽Z[IZ;]) ÷ (+/P×⌽Z[(IZ-1);]))×AIN
[9]     →LOOP1
[10]  L2:→(((ρ(BY)) = 0) = 0)/L3
[11]    AI←(AI÷AI[BY[1]])×100
[12]    →0
[13]  L3:AI←(AI÷AI[M])×100
    ∇
```

Table 12

Subroutine To Compute Benchmark Indexes and Growth Rates

```
        ∇ BNCHMK[□]∇
      ∇ BNCHMK;AB;BN;MC;VA;VD;VB;INV;INP;INT;INT1;INT2;
        INT3;IND;HA;HD;HB;PI;EIN;ES;IZ;IR;IS;IX;IT
[1]     ⍝DETERMINE BENCHMARK YEARS
[2]     N←⍴P
[3]     M←⍴Q[;1]
[4]     BW←0
[5]     BN←100
[6]     MC←0
[7]     A←⍳0
[8]     BM←⍳0
[9]     BY←⍳0
[10]  LOOP1:MC←MC+1
[11]    →(MC>M)/0
[12]    VA←Q[MC;]
[13]    VD←((VA≠0)/⍳⍴VA)
[14]    VB←VA[VD]
[15]    →((N−(⍴VB))≠0)/LOOP1
[16]    BW←BW+1
[17]    BY←BY,MC
[18]    ⍝ COMPUTE BENCHMARK INDEX
[19]    BN←((+/(P×Q[BY[BW];]))÷(+/(P×Q[BY[1];])))×100
[20]    BM←BM,BN
[21]    ⍝ COMPUTE BENCHMARK GROWTH RATE
[22]    →(2>⍴BM)/LOOP1
[23]    AB←(BM[BW]÷BM[(BW−1)])★(1÷(BY[BW]−BY[(BW−1)]))
[24]    A←A,AB
[25]    →LOOP1
      ∇
```

Table 13

Subroutine To Compute Kaplan-Moorsteen Indexes

```
        ∇ KAPMOOR[□]∇
      ∇ KAPMOOR;IN;IW;IX;IT;IS;ZQ;ZY;BR;IR;IS;INV;HA;HD;HE;
        HB;PI;IZ;IN;BI;KMI;INT1;INT2;IND;INT3;ES;EIN;BX;EZ;
        INP;PS;PBI;KMJ
[1]     ⍝DETERMINE INTERPOLATION SERIES
[2]     →((⍴BY)≤1)/0
[3]     IW←0
[4]     IN←100
[5]     IX←0
[6]     IT←BY[1]
[7]     KMI←100
[8]     ZQ←⍳(BY[1]−1)
[9]     ZY←ZQ×0
[10]    KM←ZQ×0
[11]    KM←KM,KMI
[12]    EI←⍳0
```

Table 13

Subroutine To Compute Kaplan-Moorsteen Indexes (Continued)

```
[13]     IN←100
[14]     B←⍳0
[15]   LOOP2:IW←IW+1
[16]     →((IW+1)>⍴BY)/LOOP8
[17]     BR←BY[(IW),IW+1]
[18]     IR←0
[19]     IS←⍳0
[20]     INV1←⍳BR[2]
[21]     INV2←⍳(BR[1]−1)
[22]     INV3←(BR[2])⍴0
[23]     INV4←INV1~INV3
[24]     INV←(INV4≠0)/⍳⍴INV4
[25]   LOOP3:IR←IR+1
[26]     →(IR≥N)/LOOP4
[27]     HA←Q[INV;IR]
[28]     HD←((HA≠0)/⍳⍴HA)
[29]     HB←HA[HD]
[30]     →(((⍴INV)−(⍴HB))≠0)/LOOP3
[31]     IS←IS,IR
[32]     →LOOP3
[33]     ⍝COMPUTE INTERPOLATION INDEX
[34]   LOOP4:PI←P[IS]
[35]     IZ←BR[1]
[36]     →(BR[1]>BY[1])/L30
[37]   L31:INC←0×(⍳(IZ−1))
[38]     IM←INC
[39]     IM←IM,IN
[40]   LOOP5:IZ←IZ+1
[41]     →(IZ>BR[2])/LOOP6
         ∇ KAPMOOR[⎕42]∇
[42]     IN←((+/PI×⌽Q[IZ;IS])÷(+/PI×⌽Q[(IZ−1);IS]))×IN
[43]     IM←IM,IN
[44]     →LOOP5
[45]     ⍝DETERMINE INTERP GROWTH RATES
[46]   LOOP6:BI←(IM[BR[2]]÷IM[BR[1]])⋆(1÷(BR[2]−BR[1]))
[47]     B←B,BI
[48]     ⍝DETERMINE K−M INDEXES
[49]     IX←IX+1
[50]   LOOP7:IT←IT+1
[51]     KMI←((A[IX])÷(B[IX]))×(IM[IT]÷IM[(IT−1)])×KMI
[52]     KM←KM,KMI
[53]     →(IT≥BR[2])/LOOP2
[54]     →LOOP7
[55]     ⍝DETERMINE EXTRAPOLATION SERIES
[56]   LOOP8:INT1←⍳M
[57]     INT2←⍳(IT−1)
[58]     IND←M⍴0
[59]     IND[INT2]←INT2
[60]     INT3←INT1~IND
[61]     INT←(INT3≠0)/⍳⍴INT3
```

Table 13

Subroutine To Compute Kaplan-Moorsteen Indexes (Continued)

```
[62]      IR←0
[63]      ES←⍳0
[64]    LOOP9:IR←IR+1
[65]      →(IR>N)/LOOP10
[66]      HA←Q[INT;IR]
[67]      HD←((HA≠0)/⍳ρHA)
[68]      HB←HA[HD]
[69]      →(((ρINT)−(ρHB))≠0)/LOOP9
[70]      ES←ES,IR
[71]      →LOOP9
[72]      ⍝DETERMINE EXTRAPOLATING INDEX
[73]    LOOP10:PI←P[ES]
[74]      EIN←IM[BY[1]]
[75]      EI←EI,EIN
[76]      EIN←((+/PI×⌽Q[BY[BW];ES])÷(+/PI×⌽Q[BY[1];ES]))×EIN
[77]      IZ←BR[2]
[78]      EI←EI,EIN
[79]    LOOP11:IZ←IZ+1
[80]      →(IZ>M)/LOOP20
[81]      EIN←((+/PI×⌽Q[IZ;ES])÷(+/PI×⌽Q[(IZ−1);ES]))×EIN
[82]      EI←EI,EIN
[83]      →LOOP11
          ∇ KAPMOOR[⎕84]∇
[84]      ⍝DETERMINE K−M INDEXES
[85]      ⍝DETERMINE EXTRAP GROWTH RATE
[86]    LOOP20:BX←(EI[2]÷EI[1])⋆(1÷(BR[2]−BR[1]))
[87]      IZ←BY[BW]
[88]      EZ←2
[89]    LOOP21:EZ←EZ+1
[90]      IZ←IZ+1
[91]      →(IZ>M)/LOOP12
[92]      KMI←((A[IX])÷(BX))×(EI[EZ]÷EI[EZ−1])×KMI
[93]      KM←KM,KMI
[94]      →LOOP21
[95]      ⍝ DETERMINE PRE-BENCHMARK SERIES
[96]    LOOP12:INP←⍳BY[1]
[97]      IR←0
[98]      PS←⍳0
[99]      IZ←1
[100]   LOOP13:IR←IR+1
[101]     →(IR≥N)/LOOP14
[102]     HA←Q[INP;IR]
[103]     HD←((HA≠0)/⍳ρHA)
[104]     HB←HA[HD]
[105]     →(INP≠ρHB)/LOOP13
[106]     PS←PS,IR
[107]     →LOOP13
[108]     ⍝DETERMINE P−B INDEX
[109]   LOOP14:PI←P[PS]
[110]     PBI←100
```

Table 13

Subroutine To Compute Kaplan-Moorsteen Indexes (Continued)

```
[111]     PBM←ι0
[112]   LOOP15:IZ←IZ+1
[113]     PBM←PBM,PBI
[114]     →(IZ>BY[1])/LOOP16
[115]     PBI←((+/PI×⌽Q[IZ;PS])÷(+/PI×⌽Q[(IZ−1);PS]))×PBI
[116]     →LOOP15
[117]     ⍝COMPUTE K−M INDEX
[118]   LOOP16:→(BY[1]=1)/0
[119]     IT←1
[120]     PBM←(PBM÷PBM[BY[1]])×100
[121]     BP←(PBM[BY[1]]÷PBM[1])★(1÷(BY[1]−1))
[122]     KMJ←100
[123]     KMK←ι0
[124]     KMK←KMK,KMJ
[125]   LOOP17:IT←IT+1
[126]     →(IT>BY[1])/LOOP18
          ∇ KAPMOOR[⎕127]∇
[127]     KMJ←((A[1])÷(BP))×(PBM[IT]÷PBM[(IT−1)])×KMJ
[128]     KMK←KMK,KMJ
[129]     →LOOP17
[130]   LOOP18:KMK←(KMK÷KMK[BY[1]])×100
[131]     INJ←ι(BY[1])
[132]     KM[INJ]←KMK
[133]     →0
[134]   L30:IN←KM[BY[1]]
[135]     IN←((+/PI×⌽Q[BY[IW];IS])÷(+/PI×⌽Q[BY[1];IS]))×IN
[136]     →L31
      ∇
```

Table 14

Subroutine To Compute the Link Relatives

```
          ∇ LINKREL[⎕]∇
      ∇   LINKREL;ALR1;IX;IW;ALRV;LRIV;IZ;ALR;ZQ;ZY;LR1;LS;
          IR;HA;HD;HB;PI;R1X;RIZ;LR;LR2;LRJ;G
[1]       IX←0
[2]       IZ←IX
[3]       ALRV←ι0
[4]       IW←0
[5]       LRIV←ι0
[6]     LOOP6:→(IW≥⍴A)/LOOP8
[7]       IW←IW+1
[8]       →LOOP7
[9]       ⍝DETERMINE LR SERIES
[10]    LOOP1:IX←IX+1
[11]      →(IX>M)/LOOP5
[12]      IZ←IX
[13]      ALR←ι0
```

Table 14

Subroutine To Compute the Link Relatives (Continued)

```
[14]      ZQ←⍳(IX-1)
[15]      ZY←ZQ×0
[16]      ALR←ZY
[17]      ALR←ALR,0
[18]      LRJ←⍳0
[19]      LRJ←ZY
[20]      LRJ←LRJ,0
[21]    LOOP2:IZ←IZ+1
[22]      →(IZ>M)/LOOP1
[23]      LS←⍳0
[24]      IR←0
[25]    LOOP3:IR←IR+1
[26]      →(IR>N)/LOOP4
[27]      HA←Q[IX,IZ;IR]
[28]      HD←((HA≠0)/⍳ρHA)
[29]      HB←HA[HD]
[30]      →(((ρ(HB))-2)≠0)/LOOP3
          ∇ LINKREL[⎕31]∇
[31]      LS←LS,IR
[32]      →LOOP3
[33]      ⍝COMPUTE LINK RELATIVE
[34]    LOOP4:PI←P[LS]
[35]      RIX←(+/PI×⍉Q[IX;LS])
[36]      RIZ←(+/PI×⍉Q[IZ;LS])
[37]      LR←RIZ÷RIX
[38]      LR2←⍟LR
[39]      LRJ←LRJ,LR2
[40]      →((ρ(BY))≤1)/LOOP10
[41]      ⍝COMPUTE ADJUSTED LINK RELATIVE
[42]    LOOP7:→(IW+1>ρA)/LOOP8
[43]      →(IZ>BY[IW+1])/LOOP6
[44]    LOOP8:→(IZ>M)/LOOP1
[45]      →(IX=0)/LOOP1
[46]      G←(((+/(P[LS]×⍉Q[(BY[(IW+1)]);LS]))÷(+/(P[LS]×
          ⍉Q[(BY[IW]);LS]))))⋆(1÷(BY[(IW+1)]-BY[IW]))
[47]      ALR1←⍟(LR×((A[IW]÷G)⋆(IZ-IX)))
[48]      ALR←ALR,ALR1
[49]      →(IZ≥M)/LOOP5
[50]      →LOOP2
[51]    LOOP5:ALRV←ALRV,ALR
[52]      IW←1
[53]      →(((ρ(ALRV))-(M×M))≥0)/LOOP9
[54]      →LOOP1
[55]    LOOP10:→(IZ<M)/LOOP2
[56]      LRIV←LRIV,LRJ
[57]      →(((ρ(LRIV))-(M×M))≥0)/LOOP11
[58]      →LOOP1
[59]    LOOP11:LRIVM←(M,M)ρLRIV
[60]      →0
[61]    LOOP9:ALRM←(M,M)ρALRV
    ∇
```

Table 15

Subroutine To Compute the Field Index

```
          ∇ FIELD[⎕]∇
      ∇   FIELD;FC;FD;ZX;ZI;ZY;ZQ;ZV;ZW;D;FP;FQ;FI;BW;D1;D3;
          D2;C1;C2;C;FA;FB;FE;FF;FG;D4;FH;Z1;FI
[1]       ZX←⍳(M−1)
[2]       ZI←(⍳1)×0
[3]       ZX←ZI,ZX
[4]       ZY←(⍳(M))−ZX
[5]       ZQ←⍳(M−2)
[6]       ZV←ZI,ZQ
[7]       ZW←(⍳(M−1))−ZV
[8]       D←⍳0
[9]       FP←0
[10]      FQ←0
[11]      FI←⍳0
[12]      BW←1
[13]      →((⍴(BY))≤1)/LOOP6
[14]      ⍝ FORM D MATRIX AND C VECTOR
[15]    LOOP7:D1←((⍳(M))∘.=⍳(M))
[16]      D3←((⍳(M))∘.≠⍳(M))
[17]      D2←((+/(2×(ZW)))×D1)−2×D3
[18]      C1←((¯2)×(ALRM))+.×(ZY)
[19]      C2←((2)×(⌽(ALRM)))+.×(ZY)
[20]      C←C1+C2
[21]      ⍝ EXPAND C VECTOR
[22]    LOOP1:FP←FP+1
[23]      →((⍴BY)=0)/LOOP8
[24]      C←C,ALRM[(BY[1]);(BY[(FP)])]
[25]      →(FP≥(⍴(BY)))/LOOP2
[26]      →LOOP1
[27]      ⍝ EXPAND D MATRIX
[28]    LOOP2:FA←⍳(M+(⍴(BY)))
[29]      FB←⍳M
          ∇ FIELD[⎕30]∇
[30]      FE←⍳0
[31]      FF←⍳0
[32]    LOOP3:FQ←FQ+1
[33]      →(FQ>M)/LOOP5
[34]      →(FQ=(BY[(BW)]))/LOOP4
[35]      →LOOP3
[36]    LOOP4:FC←FB×0
[37]      FD←FA×0
[38]      FC[(FQ)]←1
[39]      FD[(FQ)]←1
[40]      FE←FE,FC
[41]      FF←FF,FD
[42]      →(BW=(⍴(BY)))/LOOP3
[43]      BW←BW+1
[44]      →LOOP3
[45]    LOOP5:FG←((⍴(BY)),M)⍴FE
[46]      D4←D2,[1](FG)
[47]      FH←((⍴(BY)),(M+(⍴(BY))))⍴FF
```

Table 15

Subroutine To Compute the Field Index (Continued)

```
[48]    D←D4,[2](⍉(FH))
[49]   L9:Z1←(⌹D)+.×(⍉C)
[50]    FI←Z1[⍳(M)]
[51]    FJ←(100×(⋆FI))
[52]    →((ρ(BY)−0)=0)/L10
[53]    →0
[54]   L10:FJ←(FJ÷FJ[M])×100
[55]    →0
[56]   LOOP6:ALRM←LRIVM
[57]    →LOOP7
[58]   LOOP8:D←D2
[59]    →L9
      ∇
```

Table 16

Subroutine To Replace Missing Observations in a Data Matrix by Linear Interpolation

```
        ∇ LININT1[⎕]∇
      ∇  LININT1;MC;VA;VD;MD;VE;ME;VF;ET;VH;MF;VC
[1]     ⍝DETERMINE MISSING OBSERVATIONS
[2]     MC←0
[3]     QF←⍳0
[4]     QF←Q
[5]    LOOP1:MC←MC+1
[6]     →(MC>N)/0
[7]     VA←Q[;MC]
[8]     VD←((VA≠0)/⍳ρVA)
[9]     →(M=ρVD)/LOOP1
[10]    MD←0
[11]    VE←((VA=0)/⍳ρVA)
[12]    ⍝COMPUTE REPLACEMENTS
[13]   LOOP2:MD←MD+1
[14]    →(MD>ρVE)/LOOP1
[15]    →(VE[MD]≤1)/LOOP4
[16]    →(VE[MD]≥M)/LOOP8
[17]    ME←VE[MD]
[18]   LOOP3:ME←ME+1
[19]    →((ME>M))/LOOP2
[20]    VF←ME∊VE
[21]    →(1=VF)/LOOP3
[22]    QF[VE[MD];MC]←QF[(VE[MD]−1);MC]+(1÷(ME−
        (VE[MD]−1)))×(QF[ME;MC]−QF[(VE[MD]−1);MC])
[23]    →(MD=ρVE)/LOOP1
[24]    →LOOP2
[25]   LOOP4:MF←VE[MD]
[26]    ET←⍳0
[27]   LOOP5:MF←MF+1
[28]    VH←MF∊VE
[29]    →(0=VH)/LOOP6
```

Table 16

Subroutine To Replace Missing Observations in a Data Matrix by Linear Interpolation (Continued)

```
[30]      →LOOP5
[31]    LOOP6:ET←ET,MF
[32]      →(((ρET) − 2) ≥ 0)/LOOP7
[33]      →LOOP5
[34]    LOOP7:QF[VE[MD];MC]←QF[ET[1];MC] + ((1 ÷ (ET[2] −
          ET[1])) × (QF[ET[2];MC] − QF[ET[1];MC]))
[35]      →LOOP2
[36]    LOOP8:QF[VE[MD];MC]←QF[VE[MD] − 1;MC] + ((QF[VE[MD] −
          1;MC] − QF[VE[MD] − 2;MC]))
[37]      →LOOP2
     ∇
```

Table 17

Subroutine To Compute the Linear Interpolation Index

```
          ∇ LININT2[□]∇
     ∇    LININT2;KI;FK;FKJ;FKK
[1]       ⍝ COMPUTE LINEAR INTERPOLATION INDEX
[2]       KI←0
[3]       FKK←⍳0
[4]     LOOP1:KI←KI + 1
[5]       →(KI > M)/LOOP2
[6]       FKJ←((+/(P×QF[KI;])) ÷ (+/(P×QF[1;])))×100
[7]       FKK←FKK,FKJ
[8]       →LOOP1
[9]     LOOP2:→(((ρ(BY)) − 0) = 0)/L3
[10]      FKI←(FKK ÷ (FKK[BY[1]]))×100
[11]      →0
[12]    L3:FKI←(FKK ÷ FKK[M])×100
     ∇
```

Table 18

Subroutine To Compute the Sum of Squared Differences of the Kaplan-Moorsteen, Field, and Linear Interpolation Indexes from the Actual Index

```
          ∇ SQRER[□]∇
     ∇    SQRER
[1]       ⍝COMPUTE SQUARE ERROR AND EFFICIENCY
[2]       →((ρBY) ≤ 1)/LP1
[3]       KME←(+/((KM − AI) ⋆ 2))
[4]       EFKME←1 ÷ KME
[5]     LP1:FJE←(+/((FJ − AI) ⋆ 2))
[6]       EFJE←1 ÷ FJE
[7]       FKE←(+/((FKI − AI) ⋆ 2))
[8]       EFKE←1 ÷ FKE
     ∇
```