 File 8265

[redacted]

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[redacted]

January 31, 1966

Dear John:

Re: Invoice #7

Enclosed are two copies for Bill G., one copy for Helen R. and one copy for your file.

Funding ran out on Thursday, January 27, 1966, so the invoice covers time spent up to that date.

We have continued on to finish the LASER METROLOGY report and it is now ready for reproduction. It will be ready to mail in a couple of days.

Regards,

[redacted]

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Approved For Release 2005/06/23 : CIA-RDP78B04770A002900010038-0

STAT [redacted]

January 27, 1966

STAT

Monthly letter progress report - Contract [redacted]

LOG OF ACTIVITIES

Monday, Jan. 3, 1966 through Thursday, Jan. 27, 1966
Continuation of analytical work on Laser Metrology and preparation of reports. (Two Principal Associates, 117 hours Task II, Invoiced at 115.46+ hours.)

Monday, 1/3	1 3/4 day	Monday, 1/10	3/4 day
Tuesday, 1/4	1 3/4 day	Wednesday, 1/12	1/2 day
Wednesday, 1/5	1 ½ day	Friday, 1/14	2 days
Thursday, 1/6	1 3/8 day	Thursday, 1/20	1/4 day
Friday, 1/7	1 Day	Friday, 1/21	1 day
		Monday, 1/24	1/4 day
		Tuesday, 1/25	1/2 day
		Wednesday, 1/26	1 day
		Thursday 1/27	1 day -(Invoiced at 6.46+ hours)

[redacted] STAT

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31 January 1966

Monthly letter progress report, Contract

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Comments on Status

Task I - Item 1 "Special Investigations"

There were no specific requests for visitations this period.

Task II - Item 8 "Laser Metrology"

Since there is an urgent requirement for the results of the analytical investigations on Laser Metrology, effort was concentrated during this period on the completion of the first technical report. The report was completed and is being reviewed and prepared for submission. The recommendations and a summary of the findings are included herewith.

Recommendations - A laser interferometer can be applied to sub-micron accuracy metrology to measure a meter or more by straight-forward engineering design and analysis with only two areas of uncertainty. One area of uncertainty is in achieving a high fringe counting rate and a suitable traversing rate. Since a count reliability of one part in 400 million (6 standard deviations) or better is desired, normal electronic component reliability numbers (1 or 2 standard deviations) are not applicable. It is recommended that a development program be initiated to investigate this area prior to the incorporation of a laser interferometer into proposed measuring engines.

The other area of uncertainty is in vibration control. Since vibration control is intimately related to structural design and machine design, it is recommended that a parallel program of vibration analysis and test be conducted concurrently with new measuring engine designs.

Summary - An analytical investigation was made of the problems associated with use of an interferometer for precision measurement of length. The investigation was oriented toward the

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Comments on Status (Continued)

usage of the helium-neon gas laser for the interferometer light source. The precision criterion was the measurement of lengths up to 1 meter to an accuracy of ¼ micron. Interferometers have been used for many years for the precise measurement of short lengths. Pre-laser light sources permitted precise measurement of lengths up to about 10 cm. Laser light sources permit precise measurement of lengths of at least several meters and perhaps several hundred meters.

Quantitive estimates are presented in this report of the effect on the precision of measurement of: wavelength determination, mirror alignment, atmospheric variations, particles in the beam, traversing speed, polarization, spectral purity and vibration. Spectral purity (i.e., spatial coherence) and mirror alignment are of paramount importance. Only traversing speed presents unresolved problems and vibration of course requires special analysis of detail structure.

The classic Michelson arrangement with minor modifications has proved most practical for metrology. The Fabry-Perot arrangement is well suited only to the measurement of a fixed length.

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Task II Item 8

Technical Report

"LASER METROLOGY"

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[redacted]

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[redacted]

February 1, 1966

Task II Item 8 Technical Report

LASER METROLOGY

Work Statement: Investigate the use of the helium neon gas laser for measuring engine applications. The use of a laser interferometer and fringe counting for measuring length has problems with counting rate and with vibration and thermal gradients interfering with counting. There are certain precautions which must be taken.

This report presents an analysis of the magnitude of potential errors in applying a laser interferometer to a high precision measuring engine.

Submitted by:

[redacted]

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Task II, Item 8, Technical Report "Laser Metrology"

CONTENTS

Task II, Item 8, Technical Report "Laser Metrology"

List of Figures

Approved For Release 2005/06/23 : CIA-RDP78B04770A002900010038-0

1. INTRODUCTION

1.1 Summary

An analytical investigation was made of the problems associated with use of an interferometer for precision measurement of length. The investigation was oriented toward the usage of the helium-neon gas laser for the interferometer light source. The precision criterion was the measurement of lengths up to 1 meter to an accuracy of ¼ micron. Interferometers have been used for many years for the precise measurement of short lengths. Pre-laser light sources permitted precise measurement of lengths up to about 10 cm. Laser light sources permit precise measurement of lengths of at least several meters and perhaps several hundred meters.

Quantitive estimates are presented in this report of the effect on the precision of measurement of: wavelength determination, mirror alignment, atmospheric variations, particles in the beam, traversing speed, polarization, spectral purity and vibration. Spectral purity (i.e., spatial coherence) and mirror alignment are of paramount importance. Only traversing speed presents unresolved problems and vibration of course requires special analysis of detail structure.

The classic Michelson arrangement with minor modifications has proved most practical for metrology. The Fabry-Perot arrangement is well suited only to the measurement of a fixed length.

1.2 Conclusions

There is no doubt as to the technical feasibility of using a helium-neon gas laser interferometer to make measurements of ¼ micron accuracy over a distance of a meter or more. Whether or not satisfactory traversing speeds can be obtained needs to be established by test. It is necessary to recognize the critical elements involved in a laser interferometer and the compensation required. The more important ones are:

Wavelength: The wavelength of the interferometer light beam must be accurately known since the wavelength error is multiplied by the number of fringes counted. The spectra physics laser Model 119 has a highly stable wavelength which can be determined to the required accuracy.

Mirror Alignment: Alignment of the mirror which moves over the length being measured is extremely critical. The moving mirror must be parallel to the virtual position of the fixed mirror within a small fraction of a wavelength. This can be accomplished by servo control of a plane mirror or by using a corner cube reflector.

Atmosphere: Changes in barometric pressure, air temperature and humidity will cause errors in the measured length by causing changes in the wavelength of the interferometer light beam. The atmospheric changes must be measured and in part controlled and corrections applied to the measured length.

Thermal Gradients: It appears that reasonable precautions can prevent thermal gradients in the interferometer light beam from becoming a significant source of error.

Particles in the Beam: Under ordinary laboratory conditions, particles in the interferometer light beam will not affect measuring and extraordinary clean room conditions are not required.

Traversing Speed: To date, interferometer measuring engines have used extremely slow traversing speeds and low counting rates. With the recent advent of high speed reversible counters, it appears to be possible to substantially increase counting rates and traversing speeds. This implies broad-band electronic components with attendant increase in noise levels. Careful investigation is essential to achieving suitable traversing speeds and this aspect requires developmental verification.

Polarization: The coincidence of the planes of polarization of the interferometer light beams is not critical and the requirement is easily met in the usual instrument design.

Spectral Purity: A high order of spectral purity is essential to permit measurement of long lengths. Spectral side bands must be eliminated to produce uniform fringe modulation along the measured length. The [redacted] Model 119 laser has STAT more than adequate spectral purity.

Vibration: Machine vibration can be a serious detriment to satisfactory operation. In a measuring engine particular attention must be paid to the attenuation and the elimination of vibration since the tolerable amplitudes are only a fraction of a wavelength of light. Utilizing high fringe counting rates helps and it appears that by careful design and analysis vibration can be controlled to tolerable levels.

1.3 Recommendations

A laser interferometer can be applied to sub-micron accuracy metrology to measure a meter or more by straightforward engineering design and analysis with only two areas of uncertainty. One area of uncertainty is in achieving a high fringe counting rate and a suitable traversing rate. Since a count reliability of one part in 400 million (6 standard deviations) or better is desired, normal electronic component reliability numbers (1 or 2 standard deviations) are not applicable. It is recommended that a development program be initiated to investigate this area prior to the incorporation of a laser interferometer into proposed measuring engines.

The other area of uncertainty is in vibration control. Since vibration control is intimately related to structural design and machine design, it is recommended that a parallel program of vibration analysis and test be conducted concurrently with new measuring engine designs.

2. BASIC INTERFEROMETRY

2.1 Interference Phenomena

2.1.1 General Description - Interferometry is the utilization of optical interference phenomena for the measurement of length. The interference phenomena are manifested by variations in intensity in the regions in which light beams are superimposed. The variations in intensity are generated by the vector addition and subtraction of the electric vectors of the superimposed light beams. In the measurement of length, one beam traverses a fixed length while the other beam traverses a variable length. The variation in the variable length is the distance to be measured. The arrangement is illustrated in Figure 1. When the moveable mirror is at point A, the electric vectors of the two beams arrive at the photo detector in phase and add to give an intensity maximum. When the moveable mirror is at point B (¼ wavelength from point A), the path length has been changed by ½ wavelength and the electric vectors of the two beams arrive atthe photo detector out of phase and subtract to give an intensity minimum.

As the moveable mirror moves on to point C, which may be many wavelengths from point A, the photo detector will sense a maximum intensity and a minimum intensity for every ½ wavelength of mirror travel. By counting the maxima and minima and multiplying by the wavelength of the light the distance traversed by the mirror can be determined.

In order that the variations in intensity of the

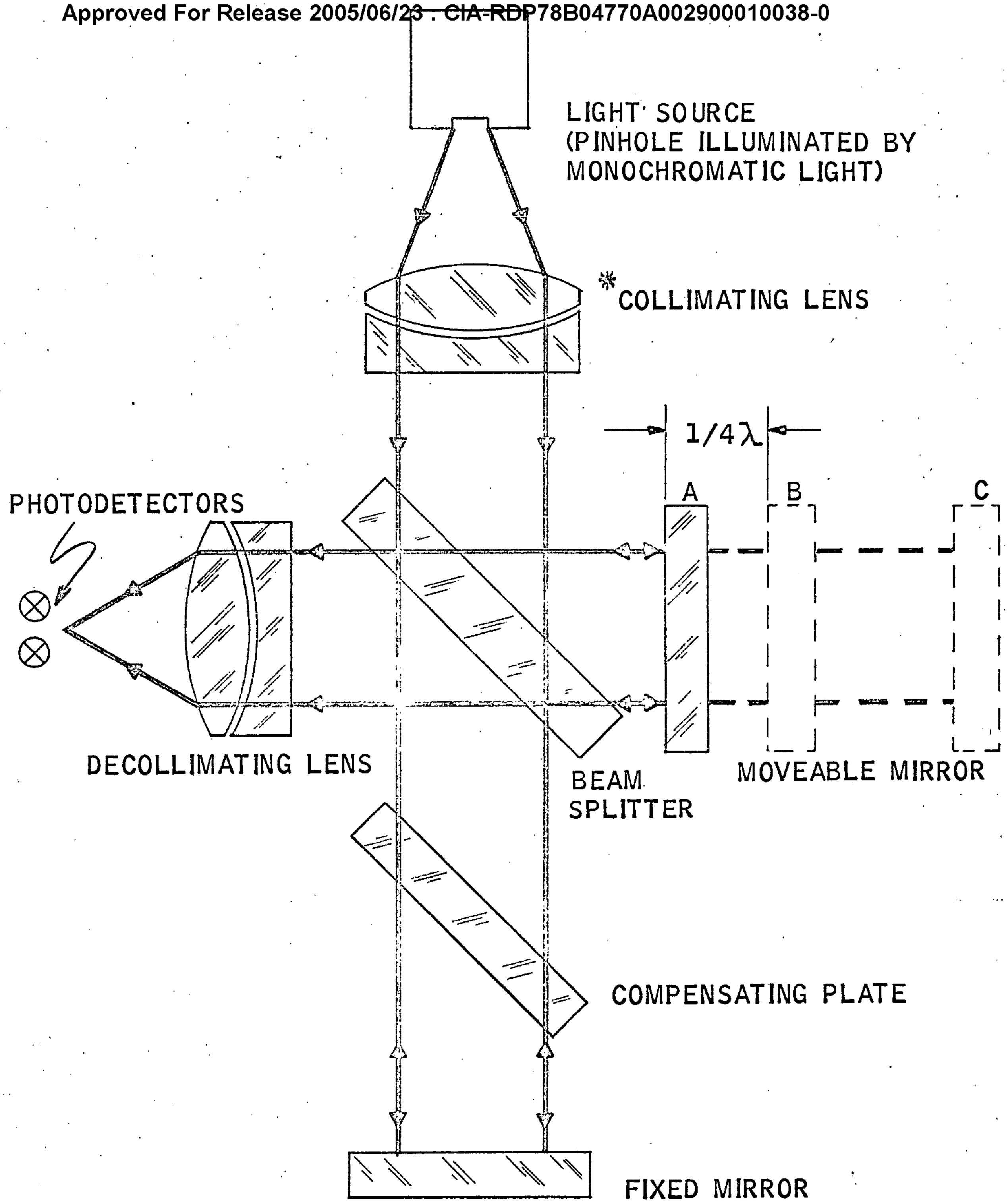

FIG. 1: MICHELSON INTERFEROMETER DIAGRAM

*collimating lens not required for laser light source.

two superimposed beams be clearly and precisely defined and measureable:

a. The electric vectors of the two beams must maintain a fixed phase relationship, i.e., the beams must be coherent.

b. The electric vectors must be rotationally coincident, i.e., identically polarized.

c. The beams must be parallel.

d. The beams must be spectrally pure.

The interference phenomenon of the two superimposed beams will be degraded to the extent that the above requirements are not met. The magnitude of the degradations and the factors producing them will be considered in the following sections.

2.1.2 Coherence Conditions - Coherence conditions for interferometry express the beam to beam periodicity relationships necessary for the occurrence of steady, clearly observable fringes in regions of beam superposition. Two beams are coherent when their electric vectors are each periodic (in space and time) and their periods have a fixed phase relationship. We will discuss Time and Space Coherence separately.

Time Coherence requires a Timewise Constant Phase Relation of the electric vectors of the superimposed beams impinging on the interferometer photo sensor.

It is necessary to have time coherence over the response time of the photo detector and its associated electronics. The photo detector does not detect the interference phenomenon of individual wave fronts. It detects the time integration of the interference phenomena of the multiplicity of wave fronts arriving during the response time of the detector.

We must at this point distinguish between time coherence of a beam relative to itself (which we shall call time self-coherence) and time coherence of one beam relative to another (which we shall call time relative coherence). For most lamps, the emission of light at one instant of time is an atomic event independent of the emission of light at another instant of time. The electric vector phase relationship of the two events is random and the light is time self-incoherent. For a laser however, the resonance lasing action produces light in which the electric vector is maintained in constant phase relationship in time and the light is time self-coherent.

It turns out, however, that time self-coherence of a beam is not required for interferometry.

The usual interferometer used a single light source and a beam splitter to produce two beams traversing separate paths. The beams are recombined at the photo detector to produce the interference phenomenon. It is the relative phase of the electric vectors of the two beams impinging on the photo detectors which must be constant in time.

If the path length is constant and the interferometer geometry rigid, time relative-incoherence cannot occur if the superimposed beams originate from one source.

We conclude therefore that the time self-coherence of the laser is not necessary and by using a single source, time relative-coherence will always be obtained under the conditions being considered. Note however that vibration of a mirror which is not common to both beams (i.e., non-rigid interferometer geometry) can cause time relative-incoherence. The effects of vibration are considered in later sections.

Space Coherence requires a spacewise constant phase relation of the electric vectors of the superimposed beams. It is convenient for our purpose to consider separately coherence across the beam and coherence along the beam in an interferometer.

Coherence across the beam in an interferometer denotes constant phase differences across the area of superposed beams viewed by the interferometer photo sensor. Were there local variances in the phase differences the effective integration over the total area by the photo sensor would produce an effective averaging of local intensities and resultant loss of fringe contrast. Ideal cross beam coherence, which would result from superposition of (at least instantaneously) monochromatic beams with parallel, plane wavefronts, can be closely approached in practical configurations. Coherence along the beam in an interferometer implies superposition of beams of identical periodic variation aside from fixed phase differences in a common propagation direction. The degree to which this coherence, and associated fringe contrast, can be approached is fundamentally limited by the widths of the narrowest utilizable quasi-monochromatic light sources and the optical path length differences of the beams.

The coherence length, L, of a spectral line, λ_o, of spectral width, $\Delta\lambda_o$, is defined by

$$\frac{L}{\lambda_o} = \frac{\lambda_o}{\Delta\lambda_o}$$

and $$L = \frac{\lambda_o^2}{\Delta\lambda_o}$$

At the path length difference, L, the wave length width $\Delta\lambda_o$ produces a spread, λ,

$$\frac{L}{\lambda_o}\Delta\lambda_o = \lambda$$

such that interference fringes are smeared beyond usable contrast. We shall see in a later section that a laser source can increase L by orders of magnitude over the marginal values afforded by pre-laser sources.

2.1.3 Polarization Conditions - Since the interference phenomenon of superimposed beams is produced by the vector addition of the electric vectors, it is optimum that light of the two beams be identically polarized at impingement on the photo detector. The intensity maxima and minima will be degraded to the degree that the electric vectors are not rotationally coincident. The amplitude of the electric vectors $\underline{E}_1$ and $\underline{E}_2$ of the two beams can be expressed as:

$$\underline{E}_1 = \underline{E} \sin\left(\omega t - \frac{2\pi}{\lambda} x\right)$$

$$\underline{E}_2 = \underline{E} \sin\left(\omega t - \frac{2\pi}{\lambda} x - \phi\right)$$

for equal maximum amplitudes, $\underline{E}$, of the two beams.

Where:

ωt = time dependent variation

$\frac{2\pi}{\lambda} = k$ = propagation constant

x = distance along the path of the beams

ϕ = phase angle of the two beams determined by the difference in path lengths

The resultant amplitude vector, $\underline{A}$, is

$$\underline{A} = \underline{E}\sin(\omega t - kx) + \underline{E}\sin(\omega t - kx - \phi)$$

The intensity, I, detected by the photo detector is a scalar quantity proportional to the square of the amplitude vector, $\underline{A}$.

$$I = \underline{A} \bullet \underline{A} = A^2$$

Substituting for $\underline{A}$:

$$I = E^2\left[\sin^2(\omega t - kx) + E^2\sin^2(\omega t - kx - \phi)\right.$$
$$\left. + 2\underline{E} \bullet \underline{E}\sin(\omega t - kx)\sin(\omega t - kx - \phi)\right]$$

Let θ be the angle between the planes of polarization of the two beams. The dot product is then:

$$\underline{E} \bullet \underline{E} = E^2\cos\theta$$

and

$$I = E^2\sin^2(\omega t - kx) + \sin^2(\omega t - kx - \phi)$$
$$+ 2\cos\theta\sin(\omega t - kx)\sin(\omega t - kx - \phi)$$

but $\sin(\omega t - kx - \phi) = \sin(\omega t - kx)\cos\phi - \cos(\omega t - kx)\sin\phi$

and

$$I = E^2\left[\sin^2(\omega t - kx) + \sin^2(\omega t - kx - \phi)\right.$$
$$+ 2\cos\theta\sin^2(\omega t - kx)\cos\phi$$
$$\left. - 2\cos\theta\sin(\omega t - kx)\cos(\omega t - kx)\sin\phi\right]$$

but $\sin(\omega t - kx)\cos(\omega t - kx) = \frac{1}{2}\sin 2(\omega t - kx)$

then

$$I = E^2 \Big[\sin^2(\omega t - kx) + \sin^2(\omega t - kx - \phi)$$
$$+ 2 \cos \theta \cos \phi \sin^2(\omega t - kx)$$
$$- \cos \theta \sin \phi \sin 2(\omega t - kx) \Big]$$

Since the frequency response of the photo detector is very much less than the frequency of light, the intensity detected, $\bar{I}$, is the time average over many cycles. The average value of $\sin^2$ is ½ and is independent of phase angle. The average value of $\sin 2(\omega t - kx)$ is zero. Substituting average values, we get:

$$\bar{I} = E^2 \left[\tfrac{1}{2} + \tfrac{1}{2} + \cos \theta \cos \phi - 0 \right]$$

and

$$\bar{I} = E^2 (1 + \cos \theta \cos \phi)$$

The maximum value occurs for

$\theta = 0, \phi = 0$ or $\theta = \pi, \phi = \pi$ and is:

$$\bar{I}_{max} = E^2 (1 + \cos 0 \cos 0) = 2E^2$$

Also $\bar{I}_{max} = E^2 (1 + \cos \pi \cos \pi) = 2E^2$

The minimum value occurs when

$\theta = 0, \phi = \pi$ or $\theta = \pi, \phi = 0$

$$\bar{I}_{min} = E^2 (1 + \cos 0 \cos \pi) = 0$$

Also $\bar{I}_{min} = E^2 (1 + \cos \pi \cos 0) = 0$

When the angle of polarization, θ, between the two planes of polarization of the beams is 90° then

$$\bar{I}_{90^\circ} = E^2 (1 + \cos\tfrac{\pi}{2} \cos \phi) = E^2$$

and no modulation occurs when ϕ is varied by changing the path length. Cos θ varies less than 1½% for values of $\theta < 10^{\circ}$. Therefore it is not necessary to maintain precise coincidence of polarization of the two beams. In addition, since the polarization of the two beams will normally maintain coincidence unless deliberately changed, the problem can generally be neglected.

2.1.4 Spectrally and Spatially Distributed Sources - It is convenient to analyze and describe interference phenomena on the basis of ideal light sources - geometric points emitting light at single wavelengths.

Real light sources are not ideal in that they always appear to have finite extensions in space and to have spreads in wavelength during the response times ordinarily required for observations of interference. It has been firmly established, however, that light is emitted as a succession of very short duration individually monochromatic wave trains (photons) each of which originates from some extremely small region of space (e.g., an atomic volume), and interacts with matter independently of other wave trains.

The intensity pattern of a real source can then be treated as a summation of ideal, monochromatic, point source intensity patterns, the point sources being distributed over space and wavelength to be equivalent to the real source.

Since for given geometry the interference pattern of an ideal source is in general a continuous function of its wavelength and position, a real source equivalent to a small, smooth range of ideal sources over space and wavelength will produce an interference pattern that may be viewed as the somewhat smeared pattern of its mid-range ideal source.

2.2 Application of Interference to Length Measurement

2.2.1 Classification of Interferometers - Coherent beams in an interferometer arise through division of light from a single primary source either by division of wave front (as in two slit experiments) or division of amplitude (as by a partially reflector mirror). Interference fringes are formed in an interferometer by superposition of two or more beams originating from the same light source. On these bases interferometers are conventionally and conveniently divided into three groups.

Types of Interferometers

Wave Front Divided, Dual Beam

Rayleigh (1896) - A simple two slit instrument used to determine refractive indices of gases.

Stellar (Michelson, 1920) - A two slit device used to measure angular diameters of stars by observing overlap characteristics of fringe patterns.

Amplitude Divided, Dual Beam

Jamin (1856) - Used for refractive indices of gases. The Mach-Zehnder modification is extensively used to study air flow in wind tunnels.

Michelson (1881) and Modifications - Outstanding for versatility, simplicity, and stability. Much used in metrology. The Twyman-Green (lens testing) and Kösters (meter comparator) are important modifications.

Amplitude Divided, Multi Beam

Lummer-Gehrcke - Interference effects within a plane parallel plate provide high wavelength resolution of light incident at near grazing angle. Superceded by-

Fabry-Perot - Interference effects between partially reflecting plane, parallel surfaces result from light at near normal incidence. Most versatile of interferometers. Used for metrology of absolute wavelength and meter determinations, highest resolution spectroscopy.

2.2.2 Favored Types for Metrology - Amplitude division (e.g., partially silvered mirror) of light from a primary source is inherently more efficient than wave front division (e.g., narrow slits). Only the most simple and stable amplitude division interferometers of modified Michelson (dual beam) or Fabry-Perot (multi beam) types have been employed in metrology.

Fabry-Perot Etalons are interferometers of fixed length notably employed in several multiples of a unit length to establish meter equivalents in standard wavelengths. Variable length Fabry-Perot interferometers are impractical for measurement of distances over a few millimeters by reason of the difficulty of constructing ways to maintain the plates sufficiently parallel for the multiple reflections of the beams.

Variants of the basic Michelson dual beam interferometer are the only types utilized for measurement of varying lengths. Satisfactory ways for movement of a mirror (single reflection) to distances over a meter have been produced.

With but a single reflection required, a corner reflector can be substituted for the moving mirror with great relaxation in parallelism requirements. If three or more interferometers share the same movable mirror, parallelism of the mirror may be servo controlled. All fringe counting interferometer length mensuration systems commercially available or described in the literature employ Michelson variants with collimated primary beams.

2.2.3 Light Sources - The International Meter is presently defined as 1,650,763.73 wavelengths of the extremely sharp line of approximate wavelength 6056Å emitted by krypton isotope (KR^{86}) lamps refrigerated to liquid nitrogen temperature. While the KR^{86} lamps are suited to primary standard measurements, their complexity makes them undesirable for light sources in general interferometry. Mercury isotope (Hg^{198}) lamps emit a line somewhat less sharp at approximately 5461Å when operated at about 5 °Centigrade.

Until the recent advent of lasers, the Hg^{198} lamp was altogether the best light source for high accuracy interferometry. This was by reason of

simplicity of operation
high intensity (particularly 5461Å)
easy separation of lines (filtering)
spectral purity (.005Å half width for 5461Å)

The volume from which the light originates in the lamp, in common with all its contemporary monochromatic sources, is undesirably extended.

The Helium-Neon Laser, now readily available in practical configurations, excells every non-laser light source in every respect with regard to interferometric application. By reason of its coherent plane wave output (equivalent to a collimated point source) it provides three to five orders of magnitude greater useful light flux than the Hg^{198} lamp. Its single wavelength (no filter required) is stable within the half-width of any pre-laser spectral line. The half-width of this line is several orders of magnitude less than that of any pre-laser line.

2.2.4 <u>Fringe Counting</u> - Interferometric measurement of length implies the counting of fringes. Three methods are used.

For measurement to highest accuracy of a length known to good accuracy (e.g., a standard meter) the number of fringes actually counted can be reduced to a small fraction of the total number of fringes in the length. To illustrate: count the number of fringes in a bar about 1/8 meter long. Count the difference in fringes between this bar and a second bar of about equal length so that together they are about 1/4 meter within a known number of fringes. Using a 1/4 meter bar and a 1/2 meter bar in succession construct an approximate meter bar of a known number of fringes. Finally count the difference in fringes between the constructed and standard meter bars.

When suitable sources of several accurately known wavelengths are utilized, lengths can be determined in terms of wavelengths without actual counting by the method of excess fractions. To illustrate: use two wavelengths λ_0 and $\lambda_1 = \frac{10}{11}\lambda_0$. Starting at zero length, λ_0 and λ_1 have fringe coincidences at every tenth fringe of λ_0. At intermediate integral fringes of λ_0, the excess fractional fringe of λ_1 is just one tenth the least significant integer in the total integral number of λ_0 fringes. Interpolation to non-integral values of λ_0 fringes can be made. If a second wavelength λ_2 be used where $\lambda_2 = \frac{100}{101}\lambda_0$ the second least significant integer in the total integral number of λ_0 fringes can be determined.

In practical cases the ratios of wavelengths are not as simple as above; but the principles nevertheless apply. With sufficient wavelengths λ_0, λ_1,..... λ_n the exact distance in terms of any of the wavelengths can be determined by excess fractions alone. With fewer wavelengths distance can be determined by an auxiliary measurement of less accuracy.

Finally, by measuring fringe intensities at points effectively about ½ fringe apart to determine sense, fringes can be directly counted as they increase or decrease in number by triggering electronic counters that concurrently display the number of fringes moved from a chosen zero or reference position. Count reliability as affected by triggering levels and intensity modulation is discussed in Section 4.2.

3. LENGTH MEASUREMENT LIMITATIONS AND ERROR SOURCES

3.1 General Discussion

The limitation of maximum length continuously measurable by an interferometer is primarily a result of wavelength spread in the light source used. We shall see that use of laser sources extends this limitation by several orders of magnitude.

Serious errors in interferometry can result from a) defects and shifts in the geometry of the interferometer configuration, b) limitations on fringe resolution, and c) light source wavelength magnitude determination.

3.2 Interferometer Geometry

Variations of dimension and alignment that effect the metrological accuracy of an interferometer may be considered in three groups.

Long term shifts can result from wear, c reep, or stress relief of materials. Considerable caution in design and attention to selection of materials are necessary to reduce such shifts to negligible levels. In addition a regular schedule of system calibrations is necessary to verify the long term maintenance of geometry.

Short term shifts which may be considered rapid variations of geometry are treated in a later section on vibration (see 3.3.1).

Thermal variations of dimension and alignment and variations of alignment with length to which the interferometer is extended constitute an intermediate group that will be considered in this section. They are discussed with reference to a basic Michelson interferometer configuration, this being, as noted previously, the only type employed in measurement of variable length.

3.2.1 Rigid Length - The fixed length reference arm of a Michelson type interferometer can be immunized to significant thermal variations by means of a Koster's prism Michelson modification (see Figure 2).

Here the variable optical path length to the movable mirror from the reference plane is greater than the fixed reference path length by 2L, no matter how prism A B C expands or contracts as long as the prism experiences no significant thermal gradients.

3.2.2 Variable Length - The defined axis of measurement of an interferometer must coincide with the actual axis of translation of the reference point onthe movable mirror. If the direction of the actual axis shifts from that established for the axis defined by calibration, error will occur as a unity less cosine of error angle function. This is a weak dependence for small angles.

Let:

L_M = measured distance between two positions of the reference point on the movable mirror

L_D = distance between positions in the direction of defined axis

ψ = angle between axes in radians, assumed small

ΔL = error in measured distances, assumed as distance in direction of defined axis

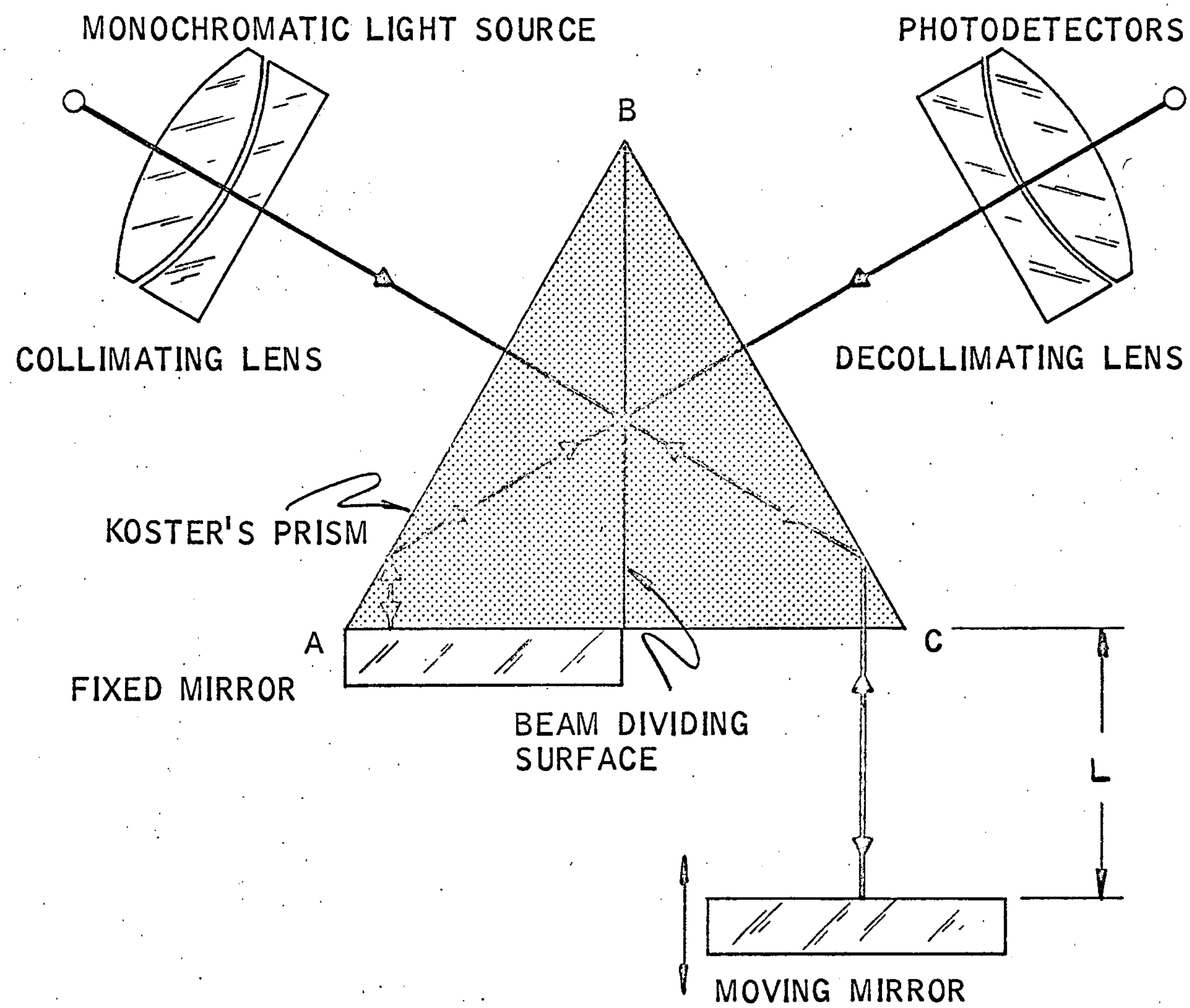

FIG. 2: KOSTER'S PRISM -- MICHELSON MODIFICATION

Then

$$\Delta L = L_M - L_D = L_M(1 - \cos\psi) \simeq L_M \frac{\psi^2}{2}, \text{ for small } \psi$$

and

$$\frac{\Delta L}{L_M} \simeq \frac{\psi^2}{2}$$

Figure 3 illustrates the error magnitude.

The movable mirror must be held rigidly parallel throughout its range of motions in order that the fringe pattern be maintained.

In Figure 4 note that:

α = angle between the fixed mirror and the virtual position of movable mirror, an extremely small angle

d = diameter of the beam on the fixed and movable mirror

n = number of fringes occurring across the mirrored beam

λ = wavelength = 0.6329 micron for He-Ne Laser

Then:

$$n = \frac{d \sin\alpha}{\lambda/2} \simeq \frac{2\, d\alpha}{\lambda}$$

For mirrors 2 cm. in diameter to be parallel within 1/5 fringe (about .06 micron) requires

$$\alpha \le \frac{n\lambda}{2d} = \frac{1}{10}\frac{\lambda}{d} = 3.2 \times 10^{-6} \text{ radian} \simeq 0.64 \text{ second of arc}$$

The automatic fringe counting interferometer at the National Bureau of Standards applies servo control to the tip and tilt of the movable mirror to maintain parallelism.

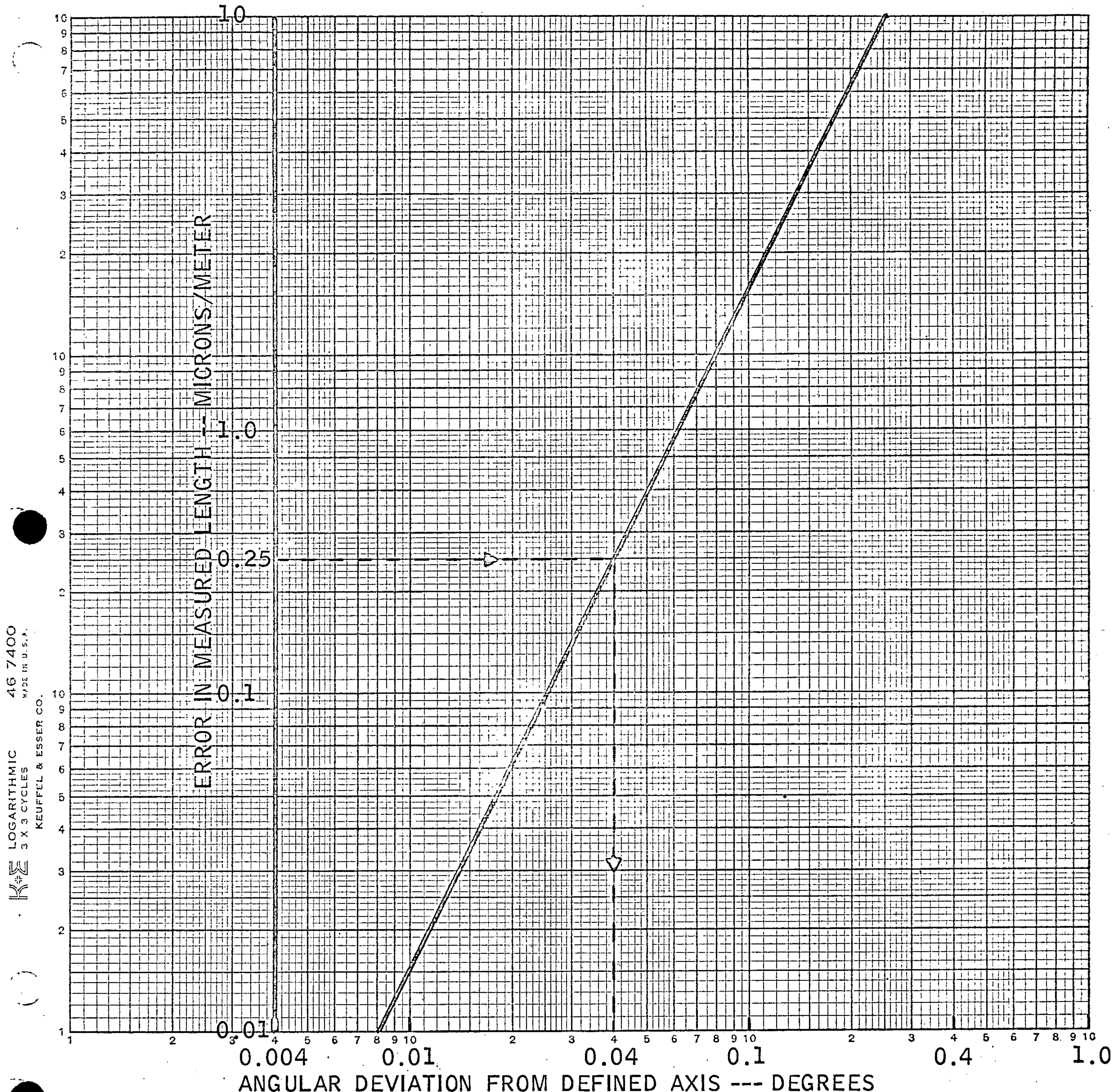

FIG. 3 ERROR IN MEASURED LENGTH DUE TO DEVIATION FROM DEFINED AXIS.

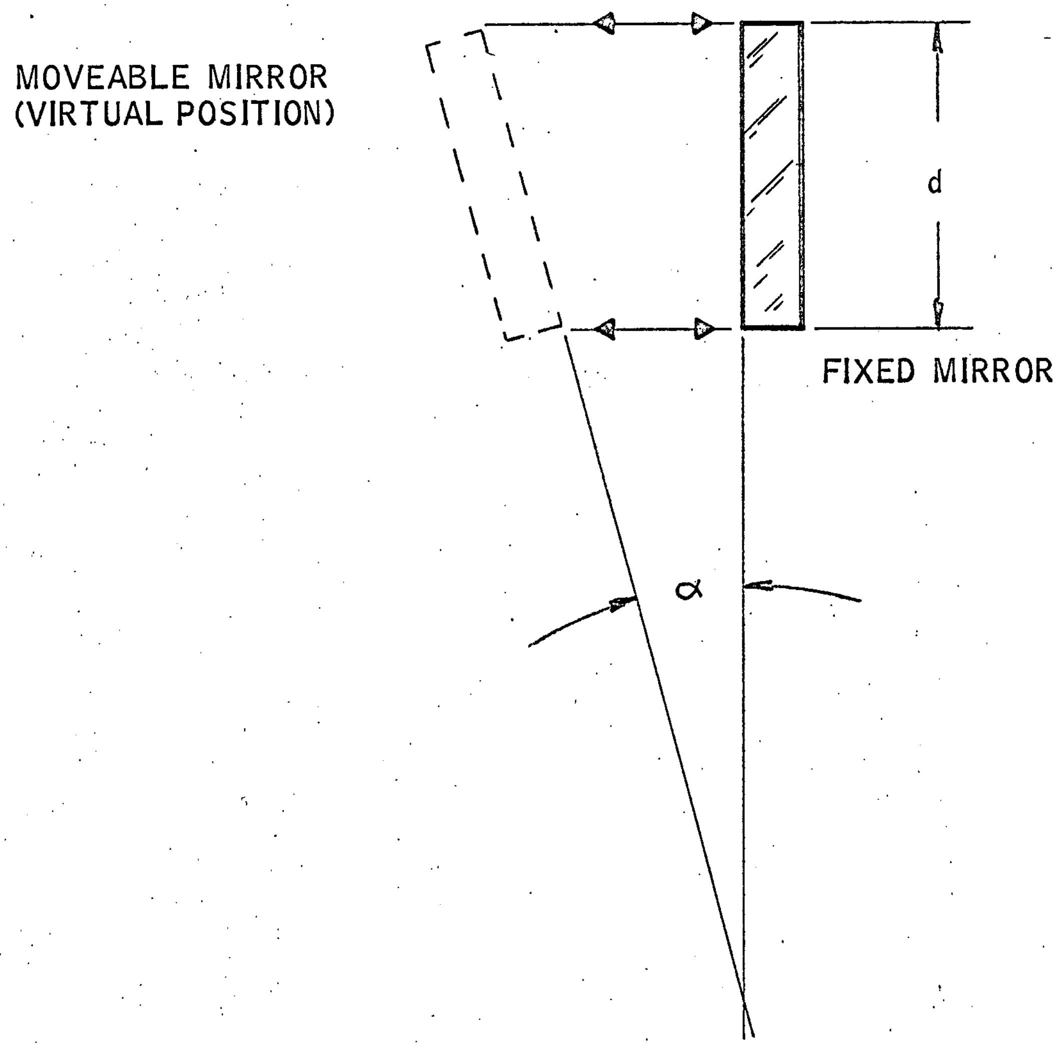

FIG. 4: MIRROR ALIGNMENT.

If a corner reflector is substituted for a plane mirror, beam reflection sufficiently parallel to maintain the fringe pattern is readily obtained despite relatively gross translational or angular deviations of the reflector carriage. However, since the position of the reference point on the carriage can change significantly before the fringe pattern is significantly affected, there must be either independent means of taking account of reference point shift with reflector shift and orientation, or the ways guiding the carriage must be shown to be of such quality as to prohibit significant error from this source.

3.3 Fringe Contrast and Flutter

3.3.1 Vibration - The effect of vibration on a fringe counting interferometer measurement system differs with vibration frequency relative to the cutoff frequency of the counting equipment. If the vibration frequency is well below the counter cut off frequency, the counter will change count with the vibration and the count will not be lost. If the vibration frequency is well above the counter cut off frequency, the counter will not respond and will in effect record the average position. If the vibration frequency is in a region near the counter cut off frequency, the count may or may not be recorded and count errors can be generated. What constitutes "a region near the counter cut off frequency" depends upon the characteristics of the particular electronic components involved in the counter. It is particularly difficult to assess because of the extremely high count reliability desired.

The effect of vibration is intimately related to amplitude as well as frequency. For a vibration amplitude of $\frac{\lambda}{2}$, where λ is the wavelength of the light from the interferometer light source, the fringe will shift from one intensity maxima to an adjacent maxima and will obviously constitute a change in count. If the vibration amplitude is only a fraction of $\frac{\lambda}{2}$, the triggering level of the electronic equipment will determine whether or not a change in count will occur. We have selected $\frac{\lambda}{20}$ as being a reasonably good lower limit. We have assumed that if the vibration amplitude is less than $\frac{\lambda}{20}$, the fringe count will not change. If the vibration amplitude is between $\frac{\lambda}{2}$ and $\frac{\lambda}{20}$, the count may or may not change and an unsteady fringe count will occur. The relation between frequency and amplitude as expressed in acceleration units of g is shown in Figure 5. In Figure 5, the region above and to the left of the $\frac{\lambda}{2}$ amplitude line is one of excessive vibration and fringe count flutter. At low frequencies, the eye or the recording equipment can follow the flutter but at high frequencies, the least count will be unresolvable.

The region below and to the right of the $\frac{\lambda}{20}$ line is the region of acceptable vibration and steady fringe counts will be obtained.

The region between the two lines $\frac{\lambda}{2}$ and $\frac{\lambda}{20}$ constitutes unacceptable vibration causing unsteady fringe counting and generating counting error. Note that at very low frequencies,

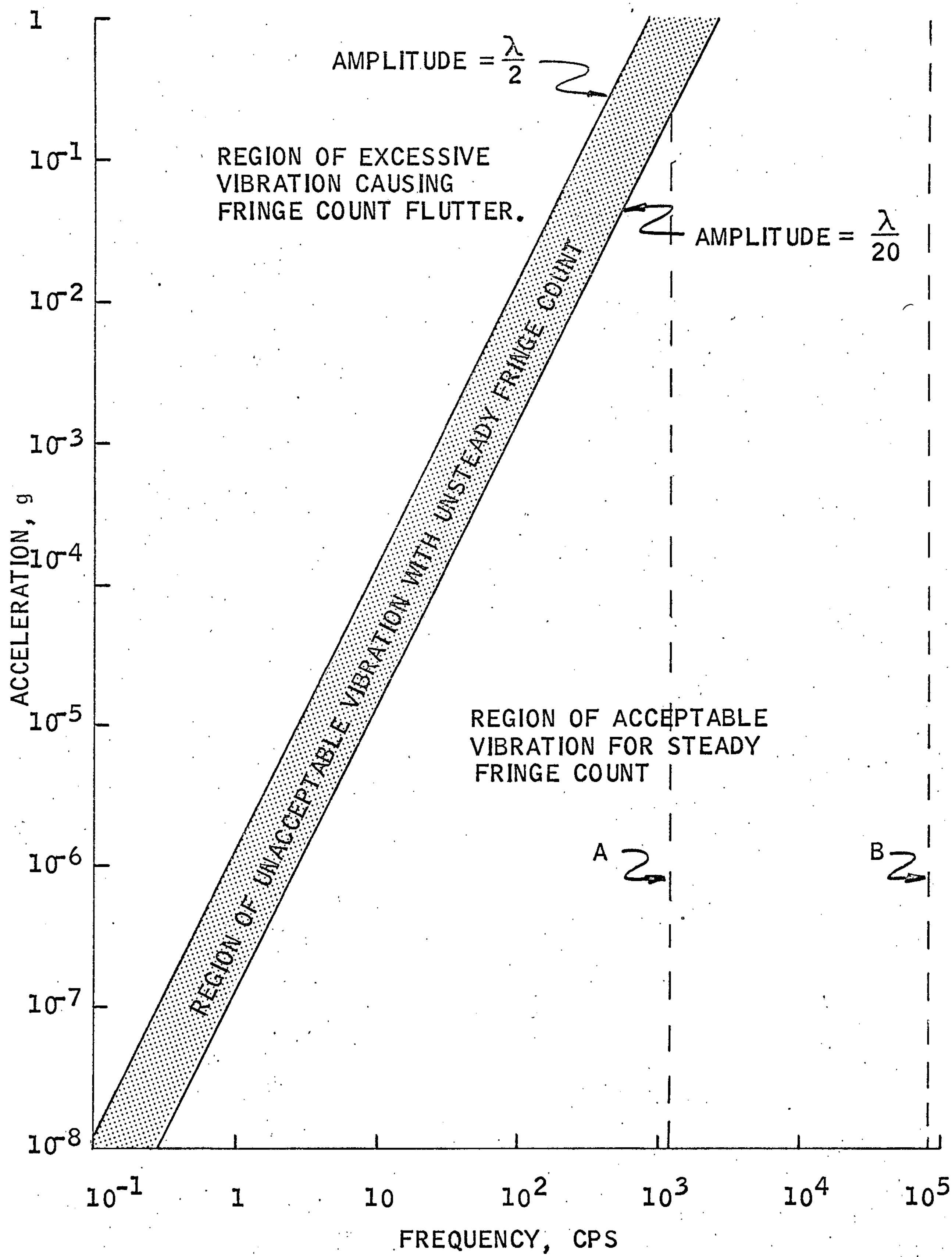

FIG. 5: EFFECT OF VIBRATION ON FRINGE COUNTING.

such as 1/10 cps to 10 cps, very small vibration levels, such as 10^{-8} g to 10^{-5} g, can seriously affect the count. At the higher frequencies, 1,000 cps requires vibration levels over 1/10 g and 100,000 cps requires vibration levels over 100 g to affect the fringe count. It is unlikely that these high vibration levels will be encountered. If we assume for example that in normal operation no vibration levels over 1 g will be encountered, then any counting rate above 2700 cps will be adequate. The counter will count all vibrations below 2700 cps with amplitude above $\frac{\lambda}{2}$. All vibrations above 2700 cps will have an amplitude less than $\frac{\lambda}{20}$ and will not affect the count.

The Bureau of Standards laser interferometer counts at 1,200 cps which is shown at A in Figure 5. Their vibration levels must therefore be less than 1/5 g. The Cutler Hammer laser interferometer counts at about 80,000 cps which is shown at B in Figure 5 and no normal vibration levels can exceed its count rate.

Low frequency vibration of amplitude between $\frac{\lambda}{2}$ and $\frac{\lambda}{20}$ is the region of concern. The exact boundaries of the region are not $\frac{\lambda}{2}$ and $\frac{\lambda}{20}$ but are determined by count triggering levels. Whether or not a count is lost is determined by the hysteresis of the counter. Since the hysteresis can be made small but cannot be zero, there is some small but finite probability that a vibration amplitude will occur which will cause loss of a count. As the counting rate is increased to accommodate faster traversing rates, the probability of a critical vibration occurring and a count being lost is increased. Further analysis of count reliability should be coupled with experimental measurements of appropriate counters and is beyond the scope of the present investigation. Therefore, further work must be reserved for later consideration.

In Figure 6, some vibration levels are superimposed on the regions of Figure 5. It is easy to see from consideration of Figure 6 why it is desirable to obtain the greatest possible attenuation of vibration and why the natural frequency and damping of structural members must be carefully considered.

Vibration A in Figure 6 illustrates the maximum allowable mirror motion due to a structure with 5% damping excited at 10^{-5} g. Note that 35 cps is the minimum allowable natural frequency of the structure. Lower frequencies will cause the peak amplitude to penetrate the $\frac{\lambda}{20}$ line. Since 5% damping is the maximum to be expected in bolted and riveted structures, it is essential that structural members and components have natural frequencies above 35 cps. This can be readily achieved in good structural design. Note that excitation at 10^{-3} g would require natural frequencies of 350 cps which are very difficult to obtain.

Vibration B in Figure 6 illustrates the maximum allowable mirror motion due to a structure with ½% damping excited at 10^{-5} g. Note that 100 cps is the minimum allowable natural frequency of the structure. ½% damping is probably the minimum to be encountered in general and is associated with pure elastic materials such as quartz or glass. Even steel

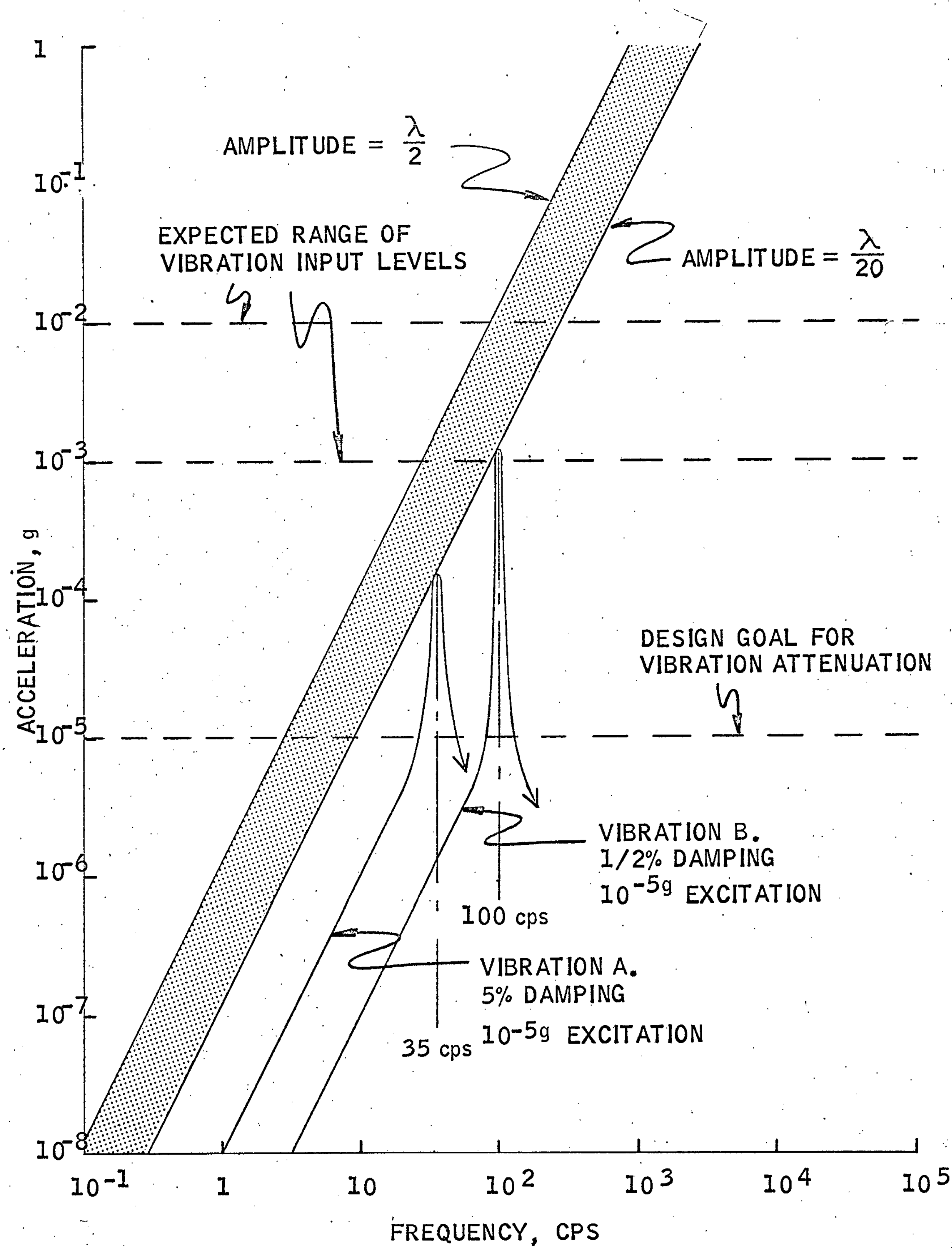

FIG. 6: ALLOWABLE VIBRATION LEVELS

has a somewhat higher internal damping factor. The monolithic structures in a measuring engine such as the base and the platen should therefore have natural frequencies over 100 cps. If there is attenuation between the vibrating structure and the mirror, or if considerable damping is deliberately added, the natural frequency limits can be relaxed.

3.3.2 Spectral Purity - Spectral line sources are never ideally monochromatic. With line splitting effects (Stark, Zeeman) absent and line broadening effects (Doppler, pressure, resonance) negligible a spectrum line has yet a finite natural width with a typical gaussian distribution of intensity.

In Figure 7, $\Delta\lambda_0$ is the half (intensity) breadth of the line centered on wavelength λ_0. The Heisenberg Uncertainty Principle, basic in quantum mechanics, relates the uncertainties in energy and life-time of an excited state of an atom or molecule by:

$$t\,\Delta E_o \simeq \frac{h}{2\pi} \qquad (1)$$

Also we know that:

$$E_o = h\nu_o \quad \text{and} \quad \lambda_o\nu_o = c$$

where:

t = half-life for spontaneous decay from the excited state to the ground state, seconds.

E_o = energy of the photon, ergs

λ_o = wavelength of the photon, cm

ν_o = frequency of the photon, cps

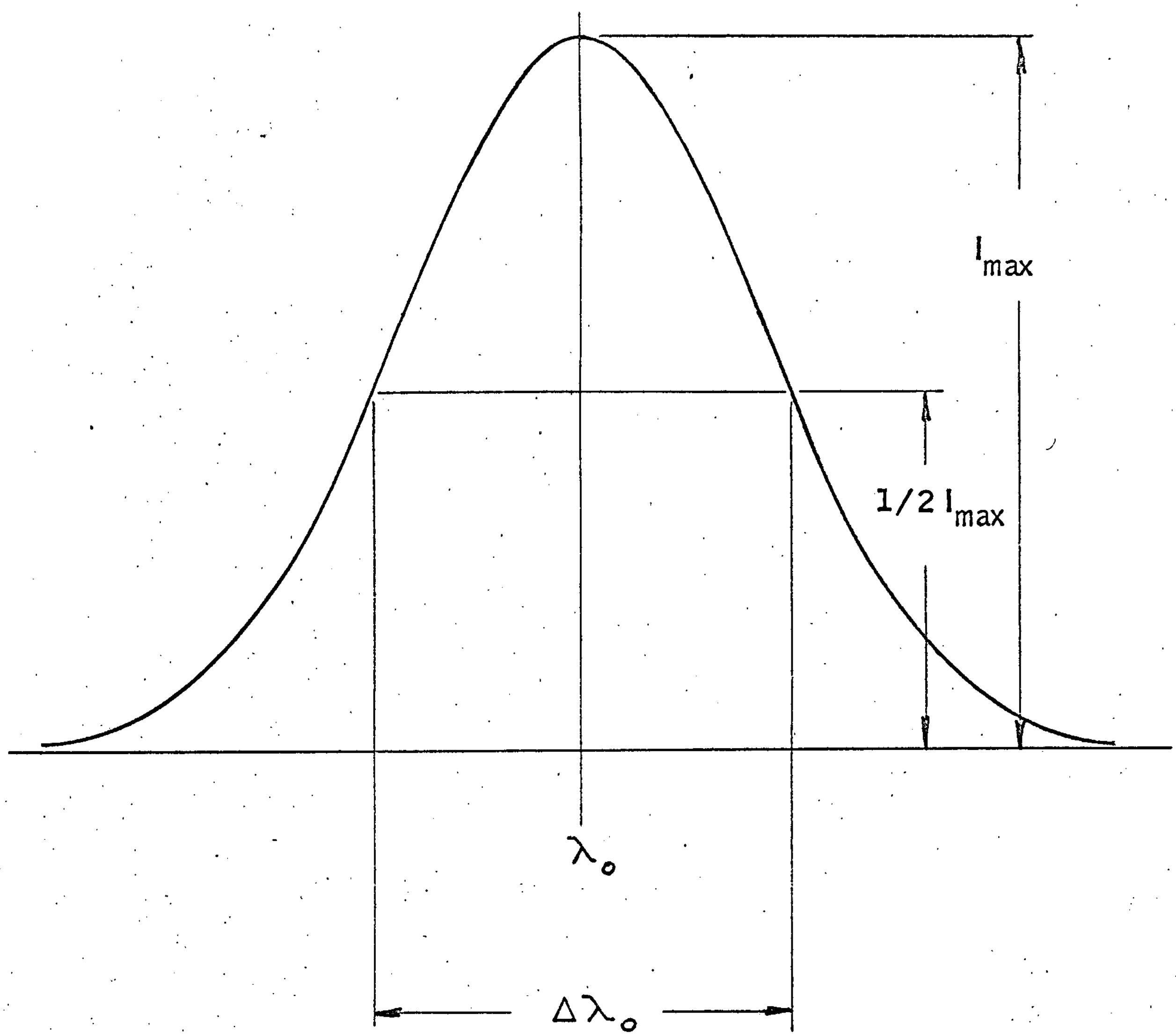

FIG. 7: SPECTRAL LINE WIDTH DISTRIBUTION.

ΔE_o = half-breadth around E_o, of the distribution of photon energies radiated in spontaneous decays to ground state, ergs

h = Planck's constant = 6.624×10^{-27} erg sec

c = velocity of light = 2.998×10^{8} m/sec

Differentiating, we get

$$\Delta E_o = h\,\Delta\nu_o \quad \text{and} \quad \Delta\nu_o = \frac{-c\,\Delta\lambda}{\lambda_o^2}$$

by substitution

$$t\,\Delta E_o = th\,\Delta\nu_o = \frac{h}{2\pi}$$

$$\text{and } t\,\Delta\nu_o = \frac{1}{2\pi} = \frac{-tc\,\Delta\lambda_o}{\lambda_o^2}$$

Thus:

$$\Delta\lambda_o = \frac{-\lambda_o^2}{2\pi Ct} \tag{2}$$

For given wavelength, then, the half-breadth of a spontaneously emitted (i.e, non-laser) spectral line is inversely proportional to the half-life of an excited state that decays to ground to produce it.

For the neighborhood of .5 micron wavelengths

$$\frac{\lambda_o}{\Delta\lambda_o} \simeq 4 \times 10^{15}\ t \text{ seconds}$$

The half-life times of excited states are ordinarily no longer than about 10^{-8} seconds which corresponds to

$$\lambda_0 \simeq 40 \times 10^{6}\ \Delta\lambda_0$$

and indicates that interference fringes in an interferometer will be smeared over substantially a whole wavelength when the length being measured reaches about ± 40 million wavelengths. Thus normal spectral source half-widths limit usable contrast fringes to the order of ± ½ meter measured length.

Metastable excited states have longer than normal lifetimes for spontaneous decay, in some cases seconds, minutes, and even hours. Such longer lifetimes correspond to lines of higher spectral purity (shorter half breadth) but lower intensity. Spontaneous metastable line intensities are generally too low for practical use in interferometry.

Metastable states are, however, the sources of extremely pure, high intensity spectral lines produced by lasers. A metastable state serves as a laser reservoir when atoms are excited or "pumped" into the state and then stimulated to emit photons within subsequent times short compared to the spontaneous half-life of the state. Since stimulated emission is a resonant process involving repeated interactions that occur most strongly at the center of the wavelength band that the metastable state emits spontaneously, the purity of laser lines is greater than spontaneous emission half-widths alone indicate. Gas laser sources provide lines of such purity that fringe contrast is not significantly degraded at tens to hundreds of meters measurement distance. ($\lambda_o / \Delta\lambda_o > 10^{14}$ has been measured for the He-Ne CW Laser).

3.3.3 Particles in Beam - Particles as large as 25 micron may be passed by air conditioner filters and float in the interferometer beams. To facilitate assessing the possible effects of such particles, we assume independent, single scattering without wavelength shifts.

Independent scattering occurs when the scattering particles are sufficiently far from one another with random orientations that intensities scattered by various particles are additive. A mutual distance of 3 radii or greater between particles is usually a sufficient condition. In a dense fog mutual distances are some 20 particle radii. The assumption of independent scattering for normal conditions in the interferometer beam is obviously well justified.

Single scattering holds when the intensity scattered is proportional to the number of scattering particles. Single scattering holds from beam entrance into a particle cloud to such a distance that beam intensity is significantly reduced below the entrant value, conventionally by about 10% of the entrant value. Since even 1% loss of intensity in a beam length of about a meter would imply abnormally smoky or dirty atmosphere, single scattering is assured.

The scattering of intensity out of an interferometer beam has little significance for 1% total intensity loss or less. But forward scattering <u>along</u> the beam might conceivably produce a phase shift equivalent to a significant variation in the index of refraction along the beam path.

However, if we except such unusual anomalies as oriented, lens-like particles, the fraction of total scattered amplitude that is singly scattered effectively parallel to a sharply aperatured beam is very small indeed (Ref., Van de Hulst, <u>Light Scattering by Small Particles</u>). Thus if 1% of the total amplitude were singly scattered only a small fraction of this scattered amplitude could effectively rejoin the beam and there could therefore be no significant reduction in fringe contrast.

We conclude therefore that under reasonably clean laboratory conditions, particles in an interferometer beam will not affect mensuration and no extraordinary precautions need to be taken to assure an adequate clean room.

3.4 Wavelength of Light

3.4.1 Wavelength Error - The velocity of light in air is affected by the temperature, pressure and humidity of the air and also by the presence of contaminants such as CO_2 or ozone. We will now examine the magnitude of the effects of air temperature, barometric pressure, and water vapor (humidity).

To measure length, L, we count the number, N, of wavelengths, λ, occuring.

$$L = N\lambda$$

Differentiating with respect to λ gives the error in length dL due to the error in wavelength, $d\lambda$.

$$dL = Nd\lambda$$

The error ratio is:

$$\frac{dL}{L} = \frac{Nd\lambda}{N\lambda} = \frac{d\lambda}{\lambda}$$

but

$$\lambda = \frac{\lambda_0}{n}$$

where λ = wavelength in air

λ_0 = wavelength in vacuum

n = refractive index in air

and

$$d\lambda = \frac{-\lambda_o\, dn}{n^2}$$

$$\frac{d\lambda}{\lambda} = \frac{\frac{-\lambda_0 dn}{n^2}}{\frac{\lambda_0}{n}} = -\frac{dn}{n}$$

The partial derivatives of n may be taken for temperature, T, pressure, P, and Humidity, H.

$$dn = \frac{\partial n}{\partial T} dT + \frac{\partial n}{\partial P} dP + \frac{\partial n}{\partial H} dH$$

where

dT = air temperature change in $^{\circ}C$.

dP = barometric pressure change in mm of Hg.

dH = water vapor pressure change in mm of Hg.

Thus the measurement error is:

$$\frac{dL}{L} = \frac{d\lambda}{\lambda} = -dn/n = \frac{-1}{n} \left[\left(\frac{\partial n}{\partial T}\right) dT + \frac{\partial n}{\partial P} dP + \frac{\partial n}{\partial H} dH \right]$$

The partial derivatives as obtained from the U.S. Bureau of Standards in Washington, D.C. are:

Air Temperature: $\partial n / \partial T = -9.28 \times 10^{-7} / ^{\circ}C$

Barometric Pressure: $\partial n / \partial P = +3.57 \times 10^{-7}$/mm Hg

Humidity: $\partial n / \partial H = -0.57 \times 10^{-7}$/mm Hg.

The index of refraction of air, n, may be found for a given set of conditions.

For $\lambda = 6329.9$ Å.

$T = 20\ ^{\circ}C$

$P = 760$ mm Hg

$H = 0$ mm Hg (dry air)

$n = 1 + 2713.30 \times 10^{-7}$

To find the relative error, the partial derivatives must be divided by n. Since

$$1/n = 1/1.00027133 = 0.99999+$$

we consider the 1/n factor negligible as a correction of the partial derivatives. We can thus express the measurement error introduced by atmospheric effects in microns per meter (or parts per million, ppm) directly as:

Air Temperature: -0.928 ppm/^{o}C

Barometric Pressure: +0.357 ppm/mm Hg

Humidity: -0.057 ppm/mm Hg.

The coefficients are illustrated in the graph, Figure 8.

The significance of the algebraic signs may be illustrated by considering air temperature. As air temperature increases, the index of refraction of the air decreases and the wavelength of the light increases. Thus the wavelength count for a given distance will be too small and the error will be negative as shown on Figure 8.

The relative importance of the atmospheric effects is illustrated in Figure 9. For the purposes of the illustration, it was assumed that humidity can be controlled to $\pm$10%, that air temperature can be controlled to $\pm$1 ^{o}C and that barometric pressure is uncontrolled and can vary $\pm$ 1" Hg. From the figure it can be seen that the humidity correction can be neglected, the air temperature correction is marginally significant and the barometric correction is essential. If the measurement of a given length is accomplished in a few minutes, then the barometric changes occurring in the course of a measurement are probably non-significant.

3.4.2 Wavelength Determination - In order to calibrate the measuring device the basic wavelength must be

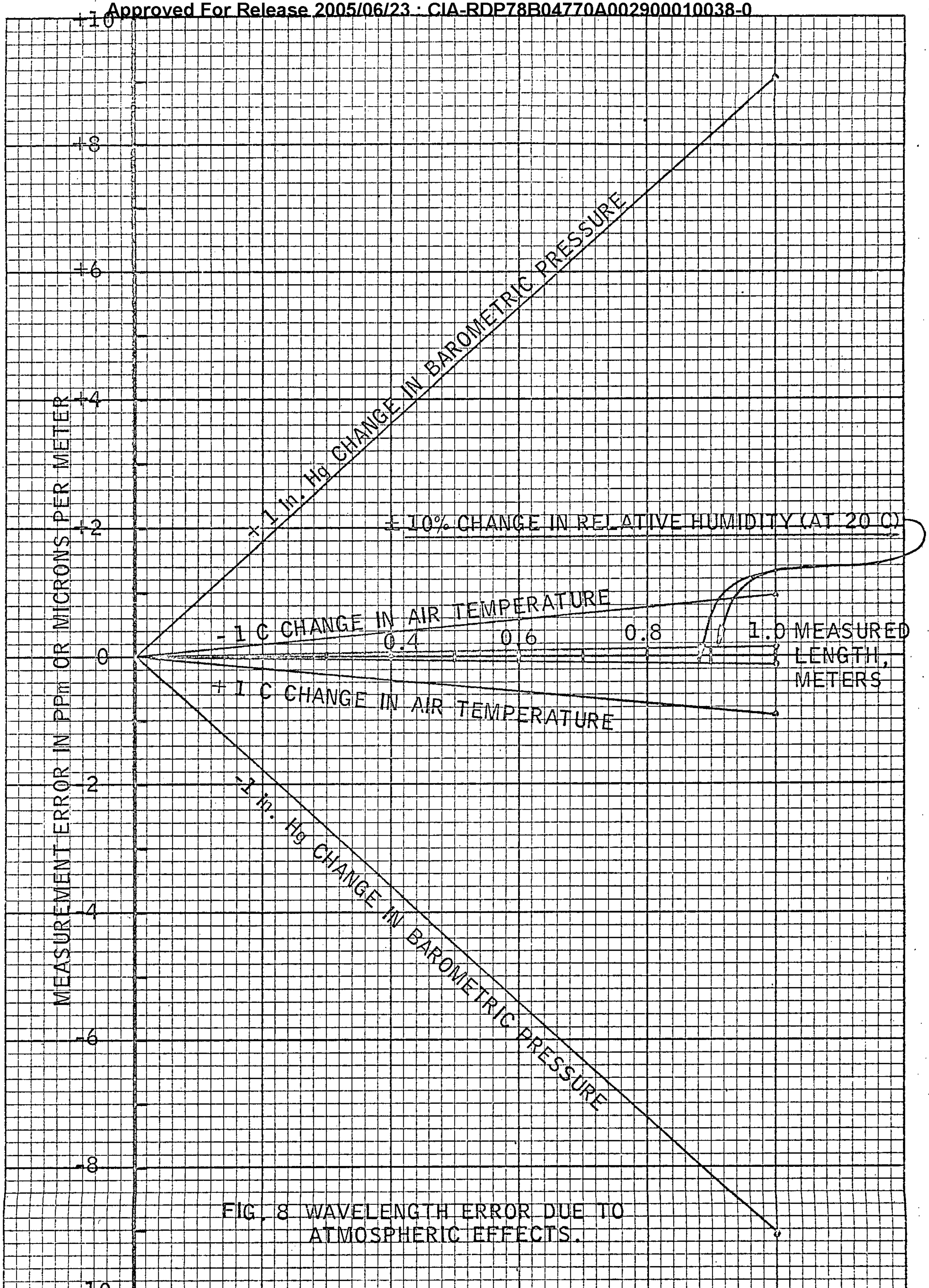

FIG. 8 WAVELENGTH ERROR DUE TO ATMOSPHERIC EFFECTS.

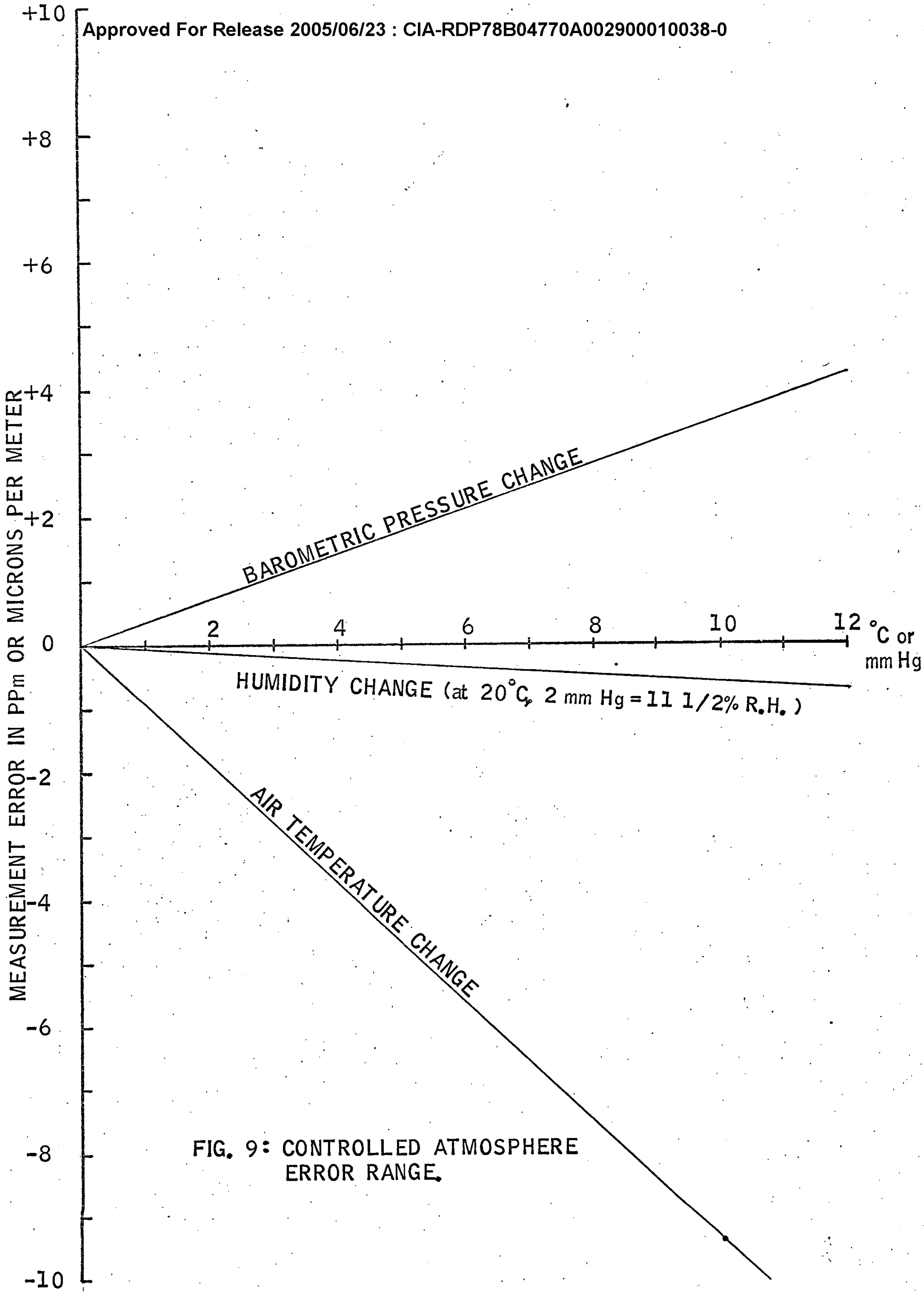

FIG. 9: CONTROLLED ATMOSPHERE ERROR RANGE.

accurately known. The wavelength of the light from the Spectro-physics Model 119 Helium Neon continuous gas laser was measured by the U.S. Bureau of Standards on two lasers and found to be

$$\lambda_0 = 6329.9147 \pm .0003 \text{ Å}$$

as corrected to vacuum. The above uncertainty of $\pm$ .000,000,03 micron is equivalent to .03 ppm or microns per meter and is a negligible uncertainty for measuring accuracy to $\pm$0.25 micron. According to the performance specifications of the model 119 laser, the variation in the basic wavelength of the laser beam is considerably less than the measurement uncertainty. The laser cavity is tuned so that the beam centers on the neon emission line with a maximum deviation of 1 Mc/day with servo control. The frequency is approximately:

$$\lambda_0 f = C_o$$

where

$$\lambda_0 = 0.6329 \times 10^{-6} \text{ meters}$$

$$f = \text{frequency, cps}$$

$$C_o = 2.998 \times 10^8 \text{ meters/sec velocity of light}$$

$$f = C_o / \lambda_0 = 2.998 \times 10^8 / 0.6329 \times 10^{-6}$$

$$= 4.74 \times 10^{14} \text{ cps}$$

$$= 4.74 \times 10^8 \text{ mc}$$

The basic wavelength is therefore controlled to 1 part in 474×10^6 which is $\frac{1}{474}$ ppm = 0.0021 microns/meter. The basic wavelength deviation is negligible.

4. RAPID LENGTH MEASUREMENT-ADDITIONAL ERRORS

When a counting interferometer system is required to traverse lengths being measured at significant velocities the higher frequency response and counting rate necessary may result in susceptibility to noise and fluctuation errors in addition to errors previously considered. It is convenient to consider separately such error factors with regard to 1) fringe intensity and 2) the sensing and counting of fringes.

4.1 Fringe Intensity Fluctuations

4.1.1 Quantum Noise - The quantized nature of light, whereby the energy of a light beam is evidenced by temporally and spatially localized transfers of individual photon energies, sets a fundamental limit to the rate at which interference fringes can be counted with a given average energy incident on the photoreceptor. It is quite possible to so reduçe the source intensity and increase the fringe passage rate of an interferometer that the photoreceptor receives not a single photon during its short transit times over some regions in which fringemaxima would be observed under normal conditions.

Since the temporal and spatial occurrence of photon interaction events on the sensitive surface of a receiver are random there is always a statistical probability, be it ever so small, that a fringe count will be gained or lost no matter how great the beam intensity of the interferometer and how long the transit time over a fringe region. Definition of a photon noise limit on a counting rate must then include a value for the maximum acceptable probability for gaining or losing one fringe count.

A rigorous calculation of photon noise limit on counting rate would involve extensive computations due to the nature of the statistical distribution functions required. Such calculations will be deferred or avoided depending upon whether sufficient information can be otherwise obtained to determine if the desired counting rates are proximate to photon noise limits.

Approximations for photon noise limits have been made for the NBS and Cutler-Hammer Counting Interferometer Systems. These approximations are viewed as suggestive. The arguments follow.

Let:

P_R = Average light flux incident on photoreceptor

C_R = Quantum efficient of photoreceptor

h = Planck's Constant = 6.6×10^{-27} erg-sec.

c = Velocity of light = 3×10^{8} meters/sec

λ = Wavelength of incident light, meters

The energy of a photon is

hC/λ ergs

The average number of photons detected per second by the photoreceptor is

$$N = (P_R \lambda C_R / hC) \text{ photons/second} \quad (1)$$

Let:

F_c = rate at which fringes are counted, fringes/second

$1/F_c$ = time for passage of one fringe

$1/2F_c$ time fringe maximum region or minimum region is viewed

N_f = number of photons detected per fringe

$\overline{N}_f$ = average number of photons detected per fringe = N/F_c

$\Delta = N_f - \overline{N}_f$ = fluctuation in number of photons detected during passage of a fringe

$\delta = \Delta/N_f = (N_f - \overline{N}_f/N)F_c$ = relative fluctuation in number of photons detected during passage of one fringe

σ = standard deviation of the population of δ's for all fringes counted.

The Poisson Function properly describes the statistical distribution of random events such as detections of radioactive disintegrations, shot effect electrons in vacuum tubes, and photon detections. Using the Normal Distribution approximation to the Poisson Distribution (Ref. Lindsay, Physical Statistics, page 29)

$$\sigma \simeq \sqrt{2Fc/N} \qquad (2)$$

Substituting the equivalence for N determined above:

$$\sigma \simeq \sqrt{2hcFc/P_R \lambda C_R} \qquad (3)$$

Fringes are counted by applying the receptor output voltage to an electronic trigger that has a hysteresis between the voltage levels at which it shifts upward and downward. This hysteresis reduces the probability of noise and fluctuation errors. At best (hysteresis $\simeq \frac{1}{2}$ maximum pulse height) a critical fluctuation $\delta_c = 1$ will cause a fringe count error (note that this fluctuation is $\simeq \frac{1}{2}$ pulse height).

Since a critical fluctuation might occur in either the maximum or minimum region of the approximately 2 million fringes counted in traversing a 20" platen, a critical or greater fluctuation probability of one in

$$2 \times 2 \times 10^6 \times 100 = 4 \times 10^8$$

corresponds to a probability of one miscount (~ 0.3 micron error) per 100 platen traverses. This probability is approximately that of the occurrence of a six standard deviation or greater fluctuation. We therefore set

$$\sigma_c = 1 = 6\sigma$$

or,

$$\sigma = 1/6.$$

Equation (3) becomes

$$1/6 = \sqrt{2hcFc/P_R \lambda C_R}$$

or

$$F_c = \frac{P_R \lambda C_R}{72\ hC} \qquad (4)$$

For both the NBS counting interferometer (considering the system with a Hg^{198} lamp rather than the recently installed laser) and the Cutler-Hammer counting interferometer setting

$$\lambda = 0.5 \text{ micron}$$

is a sufficient approximation for present purposes.

The NBS system has a reported average light flux at each of the four photomultipliers of

$$P_R = 2.25 \times 10^{-5} \text{ microwatts}$$

The Cutler-Hammer system uses a 100 microwatt laser. Assume an average light flux to each of the two photomultipliers of

$$P_R = 10 \text{ microwatts}$$

For both systems assume a typical 10% photonefficiency for the photomultipliers.

$$C_R = 0.1$$

Evaluating Equation (4) above:

For NBS -

$$F_c = C_R/72 \times \lambda/C \times P_R/h$$

$$= 1/720 \times 0.5 \text{ micron}/3 \times 10^{10} \text{ cm/sec} \times 10^4 \text{ micron/cm} \times 2.25 \times 10^{-11} \text{ watts} \times 10^7 \frac{\text{erg/sec}}{\text{watt}}/6.6 \times 10^{-27} \text{ erg-sec}$$

$$= 1 \times .5 \times 2.25 \times 10^{-4}/720 \times 3 \times 10^{14} \times 6.6 \times 10^{-27}$$

$$= 78{,}000 \text{ cps}$$

For Cutler-Hammer -

$$F_c = 78{,}000 \text{ cps} \frac{10 \text{ microwatts}}{2.25 \times 10^{-5} \text{ microwatts}}$$

$$= 35 \times 10^9 \text{ cps}$$

$$= 35{,}000 \text{ megacycles/sec}$$

The maximum fringe counting rates used in the two systems are -

NBS 1200 fringes/second

C-H 80,000 fringes/second (1 inch/sec)

The ratios

F_c/max. fringe count rate

are then for the two systems

NBS 78,000/1,200 = 65

C-H $35 \times 10^9/80{,}000 = 438{,}000$

Since the Culter-Hammer system has a ratio

$$438{,}000/65 \simeq 6{,}750$$

times greater than the successfully-operated NBS system, it appears that the Cutler-Hammer System is far from photon noise limited at its present maximum slewing rate of 1 inch per second. The above result strongly suggests that He-Ne laser illuminated interferometers can reach at least 10 inches per second slewing rates without photon-noise problems.

4.1.2 Laser Noise - Data on Spectra Physics Model 125 helium-neon CW laser gives

Amplitude Stability, Short Term: <1% peak to peak
10 to 100,000 cps

This suggests one standard deviation fluctuation under

$$1\% \times \tfrac{1}{2} \times \frac{\sqrt{2}}{2} \simeq .35\%$$

in a bandwidth of 0 to 100 kc and no greater than about 1% in a bandwidth of 0 to 300 kc.

The fringe counting system can quite easily be desensitized to ten times this fluctuation - a most rare event indeed.

4.2.1 Photosensors - Before lasers photomultipliers were the choice for interferometer fringe sensing by virtue of highest sensitivity and frequency response with excellent signal to noise ratio. With high laser light flux, silicon phototransistors appear as possible alternate photosensors of lower cost, simpler circuitry, and higher stability. But for present purposes we are content to show that with laser sources photomultipliers introduce no additonal errors through increasing slewing rates up to as great as 3 inches per second.

The NBS Fringe Counting Interferometer achieved reliable counting rates of 1,200 fringes per second with a Hg^{198} lamp and an uncooled 10 stage photomultiplier. The average signal current being far above the dark current the photomultiplier signal to noise ratio was

$$S/N \propto \sqrt{F/\Delta f}$$

F = light flux on cathode

Δf = frequency bandwidth

(Ref. RCA Phototube Manual PT-60)

For 3 inches per second slewing rate a fringe counting rate of 300,000 fringes per second is required. If Δf is increased by a factor

300,000/1,200 = 250, then F must be increased by the same factor if S/N is not to deteriorate.

Short of the limitations of electron transit times (order of 10^{-8} seconds) photomultiplier flat response bandwidths are generally limited in their anode output circuit so that, approximately

$$\Delta f \propto F \text{ average}$$

Since F can be increased by a factor greater than 1000 by using a laser, and Δf be increased by a factor of only 250 -

$$\sqrt{1000/250} = 2$$

the signal noise ratio can actually be improved from an established satisfactory value as the response is extended to allow 3"/sec slewing.

4.2.2 Digital and Counting Circuits - Voltage level triggers and digital circuitry to perform reversible fringe counting at rates to 5 megacycles per second are stock items. The requirement of a 300 kcps rate for 3 inches per second slewing is therefore readily met.

Typical sources of such circuitry are:

Cambion Corp.	Cambridge, Mass.
Computer Logic Corp.	Los Angeles, Calif.
Digital Equip. Corp.	Maynard, Mass.
Scientific Data Systems	Santa Monica, Calif.
Siliconix Inc.	Sunnyvale, Calif.
Engineered Electronics Co.	Santa Ana, Calif.

5. TABULATION OF PERFORMANCE OF EXISTING MEASURING INTERFEROMETER SYSTEMS

	Opto Mechanisms	NBS Hg^{198}	NBS Laser	Cutler Hammer
1. Interferometer Type	Michelson	Michelson	Michelson	Michelson
2. Light Source	Hg^{198} Lamp	Hg^{198} Lamp	He-Ne Gas Laser Spectra-Physics Model 119	He-Ne Laser Specially Developed
3. Operating Range of Length Measurements	$\pm$ 5 in.	$\pm 5\frac{1}{2}$ in.	1 meter	100 in.
4. Traversing Speed	Not quoted	1/3 mm/sec	1/3 mm/sec	1 in/sec
5. Counting Rate	500 KC	1200 cps	1000 cps	Not quoted (80 KC min)
6. Least Count	.27 micron .14 micron .07 micron	.003 micron	.003 micron	0.75 micron (30 micro inches)
7. Type of Reflector	Corner Reflector	Plane mirror servo controlled alignment	Plane mirror servo controlled alignment	Corner reflector
8. Measurement Display	Fringe count on 1" Nixie tubes	Digital count to .01 fringe	Digital count to .01 fringe	inches 7 digit
9. Number of Axes	1	1	1	1
10. Approximate Price	$10,744	Not available	Not available	$36,485

6. COMMENTS ON MEASURING METHODS

A number of measuring methods have been used in or recommended for measuring engines such as

a. Moire fringe counters

b. Linear phasolver

c. DIG and MILLIDIG

d. Hg^{198} Interferometer

e. Laser Interferometer

and there may be others. All but the interferometers depend upon utilizing a pattern. For the Moire fringe counter, the pattern is a pair of scribed gratings, slightly skewed. The formation of maximum and minimum intensity fringes is detected by photocells as one grating is moved with respect to the other. The accuracy of the device is dependent on the accuracy of scribing the grating. Accuracy is improved by detecting the fringes of many scribed lines so that error is dependent on average not individual scribing accuracy. It is difficult and costly to scribe long gratings accurately. To date, these devices have often had electronic circuit problems probably caused by poor electronic design and their speed has been severely limited by counting rates.

For the linear phasolver the patterns are metallic plated sinusoids and metallic plated bars on glass. The capacitive coupling between the patterns is measured as one pattern is moved with respect to the other. Again accuracy is gained by measuring over many pattern sinusoidal cycles so that error is dependent on the average accuracy of the pattern and not on the accuracy of a single cycle. The technique has been proved reliable and dependable in field use in the measurement of shaft rotation. The linear phasolver has a major advantage in that position measurement is uniquely determined at each position and is not dependent on counting fringes or patterns during traverse. The electronics are well designed, stable and reliable but expensive. Once the master patterns have been made, duplicates can be readily produced. Position determination requires a few milliseconds or less which makes the device suitable for measuring during rapid traverses. The DIG and MILLIDIG devices use a linear bar pattern on glass covering the full length to be measured and a rotating bar pattern in the reading head. The rotating pattern interpolates the position of the head between the coarsely spaced linear bars. There is no averaging over many bars thus measuring accuracy is dependent on location accuracy of individual bars. The position determination requires 1/10 second which makes the device too slow to determine intermediate positions during rapid traverses.

The DIG and MILLIDIG also uniquely determine position for each measurement without counting bars during traverse.

The Hg^{198} Interferometer has been successfully used over short distances, up to about 14 centimeters. Beyond that, the fringe counting becomes unreliable. The Laser Interferometer has been successfully used over longer distances, up to a meter and it appears it can readily be used over distances of many meters. For short distance, the Hg^{198} interferometer is the more appropriate choice since the light source costs a few hundred dollars as compared to over $5,000.00 for the laser light source.

The interferometers have a major advantage over the other devices in not depending on manufactured patterns. Their basic measuring accuracy is dependent on the wavelength of light which can be accurately determined. Ultimately, least counts of a millimicron or less could be achieved.

The major disadvantage of interferometers (in common with Moire fringe counters) is that measuring is accomplished by counting while traversing the distance to be measured. The interferometers in use traverse slowly and count slowly. With the recent advent of high speed reversible counters slow traverse speed limitation may no longer hold. However, attaining high traversing speeds and high counting rates must be considered a development program.

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