ON THE STRENGTH OF SHIP BOTTOM PLATING

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NOTATION

a	Frame spacing
b	Breadth of ship
h	Thickness of plate
E	Modulus of elasticity
μ	Poisson's ratio
q	Hydrostatic pressure
$D = \dfrac{Eh^3}{12(1-\mu^2)}$	Rigidity of plate
w, w_0	Deflection of plate, or maximum deflection
m_0	Fixed end moment of 1 cm. wide plate strip
n	An integer
σ_0	Tensile stress
$\lambda^2 = \sigma_0 : \left(\pi^2 \dfrac{D}{a^2 h}\right)$	
$i = \sqrt{-1}$	
σ'	Maximum bending stress
δ_{max}	Greatest absolute value of combined stress
u	Increase in frame spacing due to longitudinal bending of ship
A	Cross sectional area of longitudinal members excluding bottom plating
c, c_0	Distance of C.G. from bottom of keel for cross sectional area of longitudinal members without and with bottom plating
I, I_0	Corresponding inertia moments
$W = I/c$, $W_0 = I_0/c_0$	Corresponding resisting moments
M	Bending moment for longitudinal bending of ship.

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1. The bottom plating is divided by frames and longitudinals into panels. Within such a panel, each plate is loaded in bending by the hydrostatic pressure and simultaneously stressed in tension or compression by longitudinal bending of the (ship) hull. The bending deflection of such a plate is sufficiently large to require its influence on the bending moment to be considered. The basic assumption of Kirchhoff's theory of the rigid plate [1-4]* is, therefore, not fulfilled, and the application of this theory may lead to erroneous results.

In ships' bottoms of usual construction, the panels are rectangles, of which one side (spacing of longitudinal girders) is so long that the bent plate may be regarded as a cylindrical envelope, the generating lines of which are parallel to such long side (longitudinal). This enables the as yet unsolved problem of the bending of thin plates to be equated to the much simpler problem of the bending of a plate strip parallel to the short edge of the plate (Figure 1).

The validity of this procedure is indirectly confirmed by the results of the Kirchhoff theory, according to which, for a rectangular plate fixed at the edge, the maximum bending moment even for an aspect ratio of ≥1, differs only by 0.1% from that for a plate of infinite length.

2. The differential equation of the elastic curve of a uniformly-loaded strip is

$$D\frac{d^2 w}{d x^2} = \tfrac{1}{2} qx\,(a-x) - m_O \qquad \ldots\ldots (1)$$

from which, in known manner, for the freely supported plate:

$$w = \frac{1}{24}\cdot\frac{q\,a^4}{D}\cdot\frac{x}{a}\left[1 - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right] \qquad \ldots\ldots (2)$$

$$w_O = \frac{5}{384}\cdot\frac{q\,a^4}{D} \qquad \ldots\ldots (3)$$

and for a plate fixed at the edges:

$$w = \frac{1}{24}\cdot\frac{q\,a^4}{D}\left(\frac{x}{a}\right)^2\cdot\left(1-\frac{x}{a}\right)^2, \qquad \ldots\ldots (4)$$

$$w_O = \frac{1}{384}\cdot\frac{q\,a^4}{D} \qquad \ldots\ldots (5)$$

These closed expressions [5] are not very suitable for further analysis, and are therefore expanded in trigonometrical series [6]; obtaining then, for the freely-supported plate:

$$w = \frac{4}{\pi^5}\cdot\frac{q\,a^4}{D}\sum_{n=1}^{\infty}\frac{1}{n^5}\sin n\pi\frac{x}{a} \qquad \ldots\ldots (6)$$

and for the plate fixed at the edges:

$$w = \frac{1}{8\pi^4}\cdot\frac{q\,a^4}{D}\sum_{n=1}^{\infty}\frac{1}{n^4}\left(1-\cos 2n\pi\frac{x}{a}\right) \qquad \ldots\ldots (7)$$

Retaining, in equations (6) and (7), only the first terms of the expansions, we get values of w_O, differing only by 0.5% and 1.5% from (3) and (5).

If in addition to normal loading the plate strip is also uniformly stressed in axial tension, the differential equation of the elastic curve will be:

$$D\frac{d^2 w}{d x^2} = -\tfrac{1}{2} qx\,(a-x) - m_O + \sigma_O\,h\,w$$

and, as above, for the freely supported plates:

$$w = \frac{4}{\pi^5}\frac{q\,a^4}{D}\sum_{n=1}^{\infty}\frac{1}{n^3\,(n^2+\lambda^2)}\sin n\pi\frac{x}{a} \qquad \ldots\ldots (8)$$

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* For references, see end.

or, for the plate fixed at the edges:

$$w = \frac{1}{8\pi^4} \cdot \frac{q a^4}{D} \sum_{n=1}^{\infty} \frac{1}{n^2 (n^2 + \frac{1}{4}\lambda^2)} \left[1 - \cos 2 n \pi \frac{x}{a}\right] \qquad \ldots\ldots (9)$$

For the fixed end moment in the last case:

$$m_0 = -D\left(\frac{d^2 w}{d x^2}\right) = \frac{q a^2}{2\pi^2} \sum \frac{1}{n^2 + \frac{1}{4}\lambda^2} \qquad \ldots\ldots (10)$$

Putting $x = \frac{1}{2} i \pi \lambda$ in the earlier equation (7):

$$\cot x = \frac{1}{x} + 2x \sum_{n=1}^{\infty} \frac{1}{x^2 - n^2\pi^2} \;;$$

whence

$$\coth \frac{\pi\lambda}{2} = \frac{2}{\pi\lambda} + \frac{\lambda}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{4}\lambda^2}$$

and equation (10) can be written in closed form:

$$m_0 = \frac{q a^2}{\pi^2 \lambda^2} \left[\frac{\pi x}{2} \cdot \coth \frac{\pi \lambda}{2} - 1\right] \qquad \ldots\ldots (10')$$

which will be used presently.

Comparing equations (8) and (9) with (6) and (7) and restricting consideration to the first terms of the expansions, we get from (8) for the freely supported plate:

$$w = \frac{w_0}{1 + \lambda^2} \sin \frac{\pi x}{a} \qquad \ldots\ldots (11)$$

and from (9) for the plate fixed at the edges:

$$w = \frac{1}{2} \cdot \frac{w_0}{1 + \frac{1}{4}\lambda^2} \left[1 - 2 \cos 2\pi \frac{x}{a}\right] \qquad \ldots\ldots (12)$$

It will be seen that positive axial forces (tension) reduce the bending deflection in the ratio of $1:(1 + \lambda^2)$ and $1:(1 + \frac{1}{4}\lambda^2)$ respectively. Negative axial forces (compression, $\lambda^2 < 0$), on the other hand, increase the amount of deflection, the values $\lambda^2 = -1$ and $\frac{1}{4}\lambda^2 = -1$ representing the bulging of a freely supported (Figure 2) and a rigidly held (Figure 3) plate respectively, under axial compression.

3. It has hitherto been assumed that the end cross-sections of the strip are free to move in the X-direction, and that the intensity of the axial forces is known. We shall now examine the case when these motions or displacements are known.

The difference between the length of the bent strip and the frame spacing:

$$\int_0^a \sqrt{1 + \left(\frac{d w}{d x}\right)^2}\, dx - a$$

can be written, neglecting the higher-order terms:

$$\int_0^a \left[1 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] dx - a = \frac{1}{2} \int_0^a \left(\frac{dw}{dx}\right)^2 dx \qquad \ldots\ldots (13)$$

Introducing equation (12), this difference becomes, for a fixed strip:

$$\frac{1}{8}\left[\frac{2\pi w_0}{(1+\frac{1}{4}\lambda^2)\,a}\right]^2 \cdot \int_0^a \sin^2 2\pi\frac{x}{a}\,dx = \frac{1}{a}\left[\frac{\pi w_0}{2\,(1+\frac{1}{4}\lambda^2)}\right]^2 \qquad \text{...... (14)}$$

Were the frame spacing to remain constant, this expression would be identical with the elongation of the strip, viz.:-

$$\frac{1-\mu^2\ h\,a}{E.h} = \frac{1-\mu^2\ a}{E.h}\lambda^2\pi^2\frac{D}{a^2} \qquad \text{...... (15)}$$

If, however, the frame spacing is increased by the longitudinal bending of the strip, by an amount u, then, the elongation must be equated to the sum:

$$u + \frac{1}{a}\left[\frac{\pi w_0}{2\,(1+\frac{1}{4}\lambda^2)}\right]^2 \qquad \text{...... (16)}$$

If the bottom plating is removed in one frame space over the whole breadth of the ship (Figure 4), and replaced by uniformly distributed forces (intensity $\sigma_0\,h$), then

$$u = (1-\mu^2)\,\frac{a}{E}\left[-\sigma_0\,b\,h\left(\frac{1}{A}+\frac{c^2}{I}\right)+\frac{M_c}{I}\right] =$$

$$= (1-\mu^2)\,\frac{a}{E}\left[-\lambda^2\pi^2 D\frac{h}{a^2}\left(\frac{1}{A}+\frac{c^2}{I}\right)+\frac{M_c}{I}\right]$$

Substituting this value in equation (16) and then comparing the latter with equation (15), we get:

$$(1-\mu^2)\,\pi^2\lambda^2\frac{D}{a\,h}\left[1+b\,h\left(\frac{1}{A}+\frac{c^2}{I}\right)\right] =$$

$$= (1-\mu^2)\,\frac{a}{E}\,\frac{M_c}{I}+\frac{1}{a}\left[\frac{\pi w_0}{2\,(1+\frac{1}{4}\lambda^2)}\right]^2$$

or substituting for w_0 the value according to equation (5), the equation:

$$\frac{\lambda^2}{4}\left[1+b\,h\left(\frac{1}{A}+\frac{c^2}{I}\right)\right] = \frac{3\,(1-\mu^2)}{\pi^2}\left(\frac{a}{h}\right)^2\frac{M_c}{E\,I} +$$

$$+ \frac{3\,(1-\mu^2)}{4096}\left(\frac{q}{E}\right)^2\left(\frac{a}{h}\right)^2\frac{1}{(1+\frac{1}{4}\lambda^2)^2} \qquad \text{...... (17)}$$

from which $\frac{1}{4}\lambda^2$ can be determined.

Considering that:

$$c - c_0 = c\,\frac{b\,h}{A+b\,h},$$

$$I_0 - I = b\,h\,c^2 - (c-c_0^2)(A+b\,h) = b\,h\,c^2\,\frac{A}{A+b\,h};$$

$$W_0 = \frac{I\,(A+b\,h)+A\,b\,h\,c^2}{c\,A} = W\left[1+b\,h\left(\frac{1}{A}+\frac{c^2}{I}\right)\right]$$

we can write equation (17) in the following form:

$$\frac{\lambda^2}{4} = \frac{3\,(1-\mu^2)}{\pi^2 E}\left(\frac{a}{h}\right)^2\frac{M}{W_0}+\frac{3\,(1-\mu^2)}{4096}\,\frac{h}{W_0}\left(\frac{q}{E}\right)^2\left(\frac{a}{h}\right)^8\frac{1}{(1+\frac{1}{4}\lambda^2)^2} \qquad \text{...... (18)}$$

If $\lambda^2 > 0$, i.e. if the plating is stressed in tension (in the wave hollow), the second term on the right side of the equation, corresponding to the tension due to hydrostatic pressure is usually small compared with the first term. It is thus possible to write (at any rate, in first approximation):

$$\frac{\lambda^2}{4} = \frac{3\,(1-\mu^2)}{\pi^2 E} \left(\frac{a}{h}\right) \frac{M}{w_O} \qquad \text{......} \; (19)$$

or:

$$\sigma_O = \frac{M}{w_O} \qquad \text{......} \; (20)$$

The second approximation can then be calculated directly, by substituting the value of (19) in the right hand side of equation (18); which, however, will very seldom be necessary.

If $\lambda^2 < 0$ (on the wave crest), the last multiplier on the right hand side of equation (18) > 1, but $< 16/9$, as will presently be shown. The second term on the right side thus remains small, also in this case, and the calculation can also, in this case, be performed in the manner shown above.

In considering the bending of the plate strip it has hitherto been assumed that its end cross-sections are fixed, which follows from the symmetry of the loading (Figure 5).

This should not be assumed, however, for the case of bulging; the plate must then rather be regarded as freely supported; i.e.

$$\lambda^2 > -1, \text{ and } (1 + \tfrac{1}{4}\lambda^2)^{-2} < (1 - \tfrac{1}{4})^2 = 16/9.$$

4. To illustrate the method better, an example will be taken which has been treated by Timoshenko [6] according to Bubnov's method [5] excepting that the frame spacing has been reduced from 40 inches to 30 inches, in better agreement with real conditions.

The following dimensions are given:

$$a/h = 40; \quad W_O = 3670 \cdot 10^6 \text{ cm}^3; \quad W = 1524 \cdot 10^6 \text{ cm}^3; \quad q = 7031 \cdot 10^{-4} \; t/\text{cm}^2;$$

$$E = 2109 \cdot 10^{20} \; t/\text{cm}^2; \quad \mu = 0.3; \quad M = \pm 3625 \cdot 10^6 \text{ cmt.}$$

In the wave hollow (sagging), equation (19) gives the first approximation:

$$\frac{\lambda^2}{4} = \frac{3 \times 0.91 \times 40^2}{\pi^2 \times 2109 \times 10^3} \times \frac{3825}{3670} = 0.2187$$

and equation (18) the correction:

$$\left[\frac{3 \times 0.91^2}{4076} \times \frac{1524}{3670} \left(\frac{7031}{2109}\right)^2 \times 10^{-14} \times 40^8 \right] \times \frac{1}{1.2187^2} = \frac{0.000183}{1.2187^2} = 0.0001 \;\; \text{..} \; (21)$$

Consequently: $\quad \dfrac{\lambda^2}{4} = 0.2188.$

Hence:

$$\frac{\pi\lambda}{2} = \pi\sqrt{0.2188} = 1.470; \quad \coth\frac{\pi\lambda}{2} = \frac{1}{0.900}$$

and, according to equation (10'):

$$\pm\,\sigma' = \frac{6\,M_O}{h^2} = \frac{3 \times 7031 \times 10^{-4}}{2\,\pi^2 \times 0.2188} \times 40^2 \left(\frac{1.470}{0.900} - 1\right) = 0.7807 \times 0.633 = 0.49 \; t/\text{cm}^2.$$

According to equation (20), we can put:

$$\sigma_O = \frac{3825}{3670} = 1.04 \; t/\text{cm}^2$$

and consequently:

$$\sigma_{max} = 1.04 + 0.49 = 1.53 \; t/\text{cm}^2.$$

On the wave crest (hogging), the correction is, instead of equation (11):

$$\frac{0.000183}{(1-0.2187)^2} = 0.0003$$

consequently:

$$\frac{\lambda^2}{4} = -0.2184 ; \quad \frac{\pi\lambda}{2} = i\,\pi\sqrt{0.2184} = 1.468\ i ;$$

$$\coth\frac{\pi\lambda}{2} = -i\cot 1.458 = -\frac{i}{9.73}$$

$$I\sigma' = 0.7807\left(\frac{1.468}{9.73}+1\right) = 0.90\ t/\text{cm}^2$$

and: $$|\sigma|_{max} = 1.04 + 0.90 = 1.94\ t/\text{cm}^2.$$

APPENDIX

5. To facilitate the calculation of σ', the values of

$$K = \frac{3}{\frac{1}{4}\pi^2\lambda^2}\left(\frac{\pi\lambda}{2}\coth\frac{\pi\lambda}{2} - 1\right)$$

are given in the following tables: Table I, for $\sigma_0 > 0$, and Table II for $\sigma_0 < 0$. Equation (10) is then applied in the following form:

$$m_0 = K - \frac{1}{12}\,q\,a^2$$

and: $$\sigma' = K \cdot \frac{q}{2}\left(\frac{a}{h}\right)^2$$

TABLE I. $\lambda^2 > 0$

$\frac{1}{2}\pi\lambda$	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.2	1.4
K	1.000	0.997	0.989	0.977	0.960	0.939	0.939	0.916	0.890
$\frac{1}{2}\pi\lambda$	1.6	1.8	2.0	2.5	3.0	3.5	4.0		
K	0.862	0.834	0.806	0.736	0.672	0.614	0.563		

TABLE II. $\lambda^2 < 0$

$\frac{1}{2}\pi\lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
K	1.001	1.003	1.006	1.011	1.017	1.025	1.034	1.045	1.059
$\frac{1}{2}\pi\lambda$	1.0	1.1	1.2	1.3	1.4	1.5			
K	1.074	1.091	1.111	1.134	1.161	1.192			

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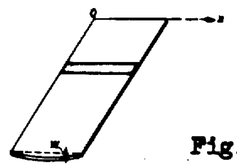

Fig.1

Fig.2

Fig.3

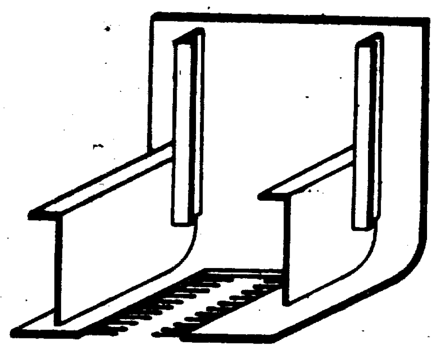

Fig.4

Fig.5