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SECTION I

INTRODUCTION

This study was carried out in an effort to develop improved methods of rectifying aerial photographs. The main emphasis is on methods of using electronic techniques to achieve rectification of all significant distortions present in a photograph.

The areas studied were (1) the mathematics of rectification with special attention to developing equations best suited to electronic computation techniques, (2) the present methods of rectification and their disadvantages, (3) the components and electronic techniques now available or likely to become available in the near future which would be useful in the electronic rectifier, (4) the most suitable method of mechanizing a rectifier and the operating parameters to be expected.

The goals of the study appear to have been accomplished successfully. Although requiring a substantial amount of engineering development, the design and construction of a high accuracy, high resolution, electronic rectifier appears feasible.

SECTION II

GENERAL DISCUSSION OF PROBLEM

2.1 Aerial Photography

Aerial photographs are widely used as sources of information for military intelligence, mapmaking, road planning, forestry and hydrographic studies, determination of storage pile size or quantities of earth to be moved, and many other applications. Probably the two most important uses are intelligence and mapmaking.

When a photogrammetrist takes aerial photographs, he attempts to obtain a point perspective whose central ray is truly vertical at the point of intersection with the ground. Unfortunately, this is never possible, although under ideal conditions it may be closely approached. Under conditions likely to be encountered by military aerial photographers, it may be impossible to even come close to the desired conditions.

Many factors work against the photogrammetrist in his efforts to achieve a vertical point perspective of the ground. With standard aerial cameras one of the most important factors is lack of verticality of the lens axis due to motion of the aircraft and to lack of an accurate vertical reference. In some cases such as tri-metrogon systems the lens axis is deliberately given a large tilt to provide more ground coverage. Other errors are caused by lens distortion, air refraction, and motion of the aircraft during exposure. These will be discussed more completely in Appendix I. For vertical pictures the entire camera assembly is often mounted on a gyro-stabilized camera mount which attempts to hold the camera vertical as the aircraft rolls and pitches. This method is not entirely satisfactory even for vertical photographs mainly because this equipment is usually large and heavy. This often precludes its use in the confines of the high performance aircraft needed for modern military reconnaissance systems. It also corrects only one of the many errors introduced into the photograph.

In an effort to obtain a more nearly correct vertical point perspective, photogrammetrists rectify, or correct, the photograph. This report discusses improved methods of rectification.

2.2 Rectification

An aerial photograph is a perspective view of the ground similar to what would be seen by a human eye from a single point above the ground.

A map is an orthogonal view in which each detail is indicated as if viewed from directly above it.

A tall object, such as a smokestack, will show as a circle anywhere on a map but will show as a line of definite length on a photograph except at the one point which is vertically below the aircraft.

A vertical photograph is an aerial photograph taken with the principal axis (optical axis) vertical.

The scale of a photograph is the ratio of distances on the photograph to distances on the ground and on a theoretical vertical photograph is constant at any point on the picture.

If a photograph does not have constant scale, it is said to be distorted and should be rectified before use. Rectification may be considered as the process in which a distorted photograph is converted to a vertical, point perspective taken from the original camera station. The scale will be constant at any part of the photograph and will be equal to the focal length of the camera divided by the altitude above the ground.

It is important that the geometric properties of a perspective view are accurately preserved by the rectification process. A rectified photograph is not a map.

A rectified photograph as discussed in this report may be considered as being the equivalent to a photograph taken from the same camera station, through a medium of uniform index of refraction, by a stationary camera having a distortion-free lens, a known focal length, and whose principal axis is truly vertical.

In addition to meeting these conditions, it is desirable to modify the picture to correct for the effects of the earth's curvature. In this report the earth's curvature adjustment will be considered to be included under the term rectification.

2.2.1 Present Methods

The method now used to rectify aerial photographs is to project an image of the photograph onto light-sensitive paper as is done in any standard enlarger. In order to rectify the photograph, the negative, the lens, the easel, or a combination of all three, are tilted.

The projection-type rectifiers suffer from many drawbacks. They only correct for the tilt of the photograph and not for the many other sources of distortion. They are very large and unwieldy and require different projection lenses for each taking lens. As the tilt angle becomes very large, the light rays hitting the paper at glancing incidence are reflected instead of being absorbed where they can expose the print. They are not capable of the high accuracy that is required by advanced photogrammetrists, nor can they yield the full resolution inherent in the original film. In many cases a photograph with large tilt must be rectified in two stages, each with its inherent loss of accuracy, resolution, and time.

2.2.2 Need for New Method of Rectification

Because of the great advances in the state of the photogrammetric art, accuracy and resolution are outstripping the capabilities of the present rectification process.

In recent years electronic techniques have made tremendous advances, especially in the communications and computer fields. Some photogrammetrists have become interested in the great potential gains that lie in a combining of photogrammetry and electronics. It was this line of thinking that led to this study.

Some people have done work in this field, and the results have been good. A few of these will be mentioned for background information.

U. V. Helava of the National Research Council, Canada, has applied an electronic computer to position the plates of a stereo plotter to remove distortion as the plot is made. Others in Canada have made an electronic stereo perception attachment for stereo plotters. This device randomly scans the stereo plates, subjects the output signals from each plate to electronic correlation techniques, and positions the plotter by a servomechanism to plot the contours.

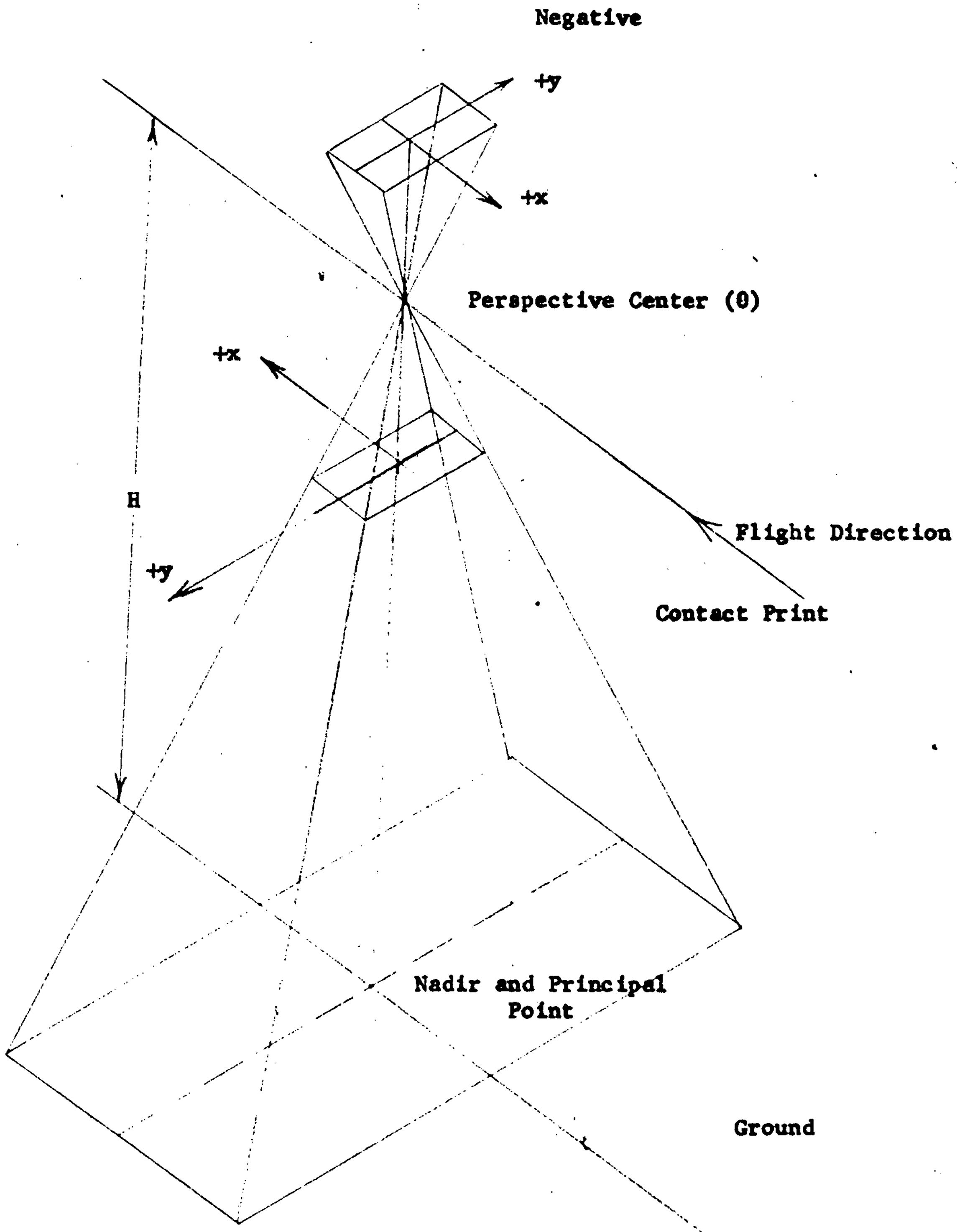
Professor A. McNair of Cornell University uses digital electronic computers to solve analytic aerotriangulation problems, and many people are processing stereo-plotter outputs automatically with digital computers.

The Fairchild Graphics Corporation, Syossett, New York, under a contract with Rome Air Development Center, is building an electronic rectifier to remove tilt from aerial photographs. This unit replaces the conventional projection rectifier with one compact unit with no need for changing lenses, since at a turn of a knob, any focal length lens from 3 to 100 inches may be accommodated. In addition to rectifying, this unit will enlarge or reduce by a factor of .3x to 3x.

The use of electronics in the handling of photogrammetric information is a rapidly growing field. The conventional plotters, rectifiers, and other measuring instruments are optical and mechanical analogs requiring large size and weight and present great manufacturing difficulties.

The use of electronic techniques permits the use of a minimum of mechanical parts, such as lead screws, which can be chosen during design as those which may be easily made to high accuracy.

As more and more complex methods and vehicles are used to obtain aerial photographs, the rectification problem becomes more and more difficult. Reconnaissance aircraft are flying at higher speeds and altitudes. Photographs taken from satellites and missiles will cover larger distances in one exposure, balloons may be used as camera vehicles, and non-photographic imaging systems and special-type cameras may present very distorted photographs. All these will require rectification which can only be done by a new type of rectifier.



GEOMETRY OF VERTICAL PHOTOGRAPH

Figure 1

2.3 Study Philosophy

The aim of this study is to investigate, as thoroughly as possible, the methods that could be used to design and build a versatile electronic rectifier of high accuracy and resolution. The tentative requirements of .01% geometric accuracy, 100 line pairs per millimeter resolution, 15 grey scale tones of dynamic range, and 10-20 minutes' time of operation were established as a goal. These areas were set down because they appeared to be reasonable all-over goals for a device that would considerably advance the state of the art of rectification. If no tentative limits were set except to study for the "best," the study could range far beyond the time and money available. As an example, if several days were taken to rectify a photograph, a unit of a few micron accuracy capability could be designed. This would be an impractical solution since the variables are not known to enough accuracy to justify this precision and the time is not available if they were.

One goal of the study is to have a rectifier which is as versatile as possible and is adaptable to future problems that may arise.

This dictates the input transducer (reading end) and the output transducer (writing end) do not contribute to the geometry of the picture but only shift the reading and writing heads around, supply positional information, image parts of the photograph, or supply density information. All of the geometry must be handled in the computer section. The computer section will rotate coordinate systems, vary coordinates and parameters, and solve all of the geometry equations. Each distortion should be handled in a separate computer block which is easily removable. In this way the unit can be modified and further developed without major design changes.

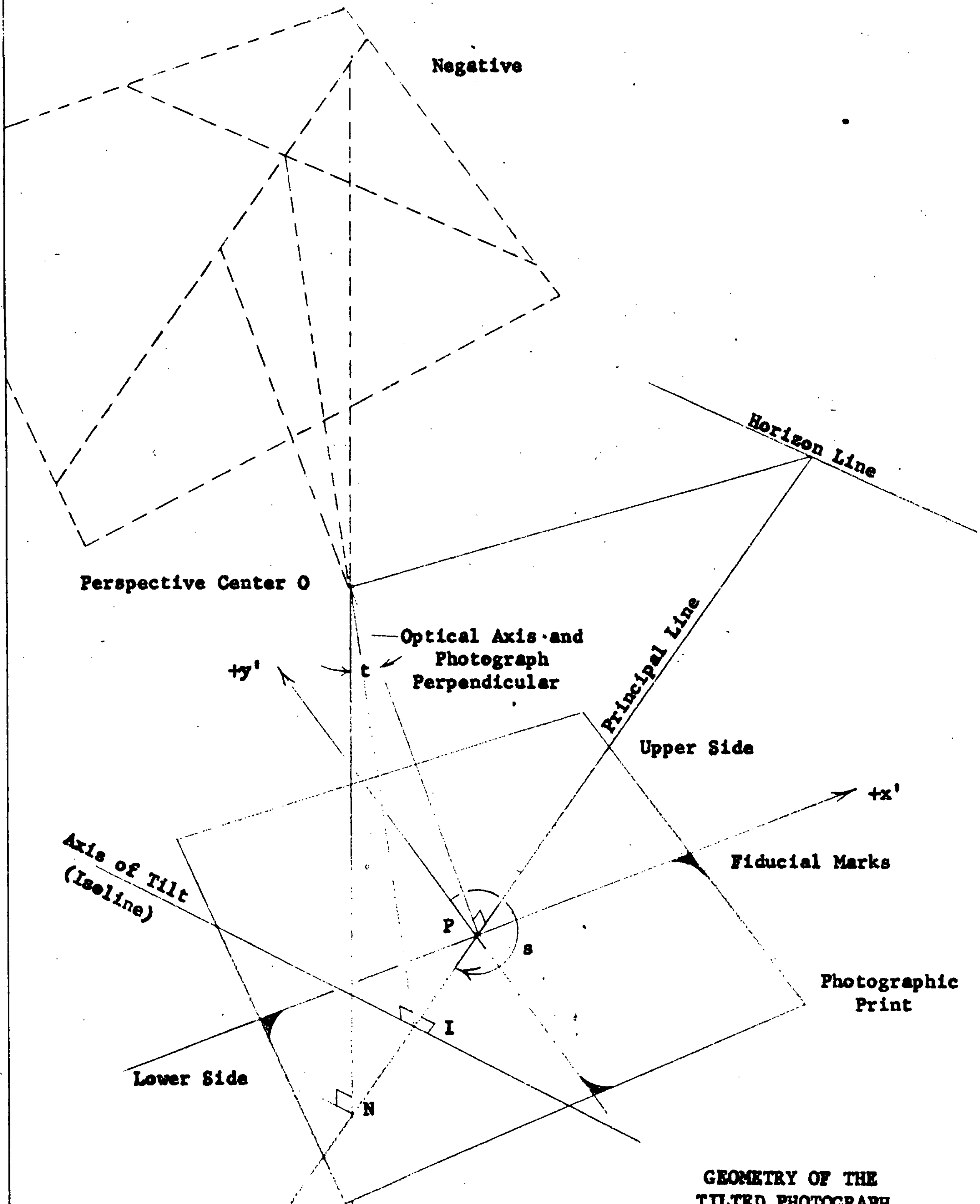


Figure 2

GEOMETRY OF THE TILTED PHOTOGRAPH

Use of this type of system will permit modification, addition, or replacement of computer blocks to solve new problems. If some as yet undesigned camera, with a different geometry than any now in use should be used, it would only be required to design a computer block to correct its pictures, build the block, and plug it in. As an example, it might be desired to map underwater obstructions or reefs near a beach. A computer block to correct for the bending of the light rays as they pass from the water to the air could be constructed and used in the rectifier to correct the geometry of the underwater photograph.

This philosophy was followed throughout the study, and it is recommended that any rectifier built as a result of this study incorporate these principles.

NOTE: Before reading section 3 it is suggested that the reader who is not familiar with photogrammetric terms study the definitions of Paragraph 2.4 and the geometry of Figures 1 and 2.

2.4 Definition of Terms Used in This Report

DISTORTION Any deviation from a point perspective whose central ray is vertical. Distortion results in local variations in the scale of the photograph.

ALTITUDE (H) The vertical distance from the datum plane being photographed to the interior perspective center of the lens.

CRAB The angle between the projection of the longitudinal axis of the aircraft on the ground and its track.

FIDUCIAL MARKS Four index marks which image on the film. The intersection of lines connecting opposite fiducial marks defines the principal point and, in this report, the coordinate origin of the unrectified photograph.

FOCAL LENGTH (f) The perpendicular distance from the film plane to the interior perspective center of the lens. This is often referred to as the calibrated focal length and is so chosen as to distribute the effect of lens distortion over the useful field of the lens. The principal distance is similar to the focal length and is used in place of f when measuring a photograph which has been enlarged or reduced. The principal distance is equal to the product of the focal length of the lens with which the photograph was taken and the enlargement factor. If a photograph taken with a 6-inch focal length lens is enlarged by a factor of two, its principal distance is 12 inches.

ISOCENTER (I) A point defined by the intersection of the planes defined by a tilted photograph, a vertical photograph taken from the same camera station with the same lens, and the plane defined by the principal axis and vertical (see Figure 2).

ISOLINE A line through the isocenter and perpendicular to the principal line. If only tilt is considered, the scale is constant along this line and any line parallel to it.

<u>NADIR (N)</u> The point at which the vertical line through the perspective center pierces the ground or the photograph.

<u>OBLIQUE PHOTOGRAPH</u> A photograph taken with the principal axis intentionally not vertical. Oblique photographs usually include the horizon.

<u>PRINCIPAL POINT (P)</u> The point at which a line perpendicular to the photograph and through the interior perspective center pierces the photograph. On a vertical photograph the nadir and the principal point coincide.

<u>PRINCIPAL AXIS</u> The line connecting the principal point and the interior perspective center.

<u>PRINCIPAL LINE</u> The line, in the plane of the tilted photograph, connecting the nadir with the principal point.

<u>RECTIFICATION</u> The process of making the scale constant at every point on the photograph. See page 3 .

<u>SCALE</u> The ratio of distance on the ground to a corresponding distance on the photograph. The scale is numerically equal to f/H. In this report local scale is used to describe the scale at some point on a distorted photograph as opposed to the over-all scale which may be changed by a simple enlarging process.

<u>SWING (s)</u> The angle at the principal point of a photograph measured clockwise from the +y axis to the principal

line at the nadir. This definition is used throughout this report because it appears to be the most commonly used. The +x axis can also be used as a starting point. In this case the tilt equations, while maintaining the same form, will have some sign changes. Either system can be incorporated into the rectifier.

TILT (t)

The angle at the perspective center between the principal axis (photograph perpendicular) and the plumb line. The direction of tilt is specified by the swing angle. See Figure 2 . In working with oblique photographs, reference is often made to the true depression angle, which is the angle between the true horizon and the principal axis. This system is not used here, but the simple conversion, tilt angle = 90° - T.D.A., may be used to find the tilt.

SECTION III

SOURCES OF DISTORTION

3.1 GENERAL

Any departure of the photograph from a vertical point perspective is termed distortion. There are many sources of distortion, and most of these are present to some degree in every aerial photograph. Some distortions are large enough to require correction in every picture, and others are so small that they can always be neglected. A list of distortion sources is given here, and each will be given a brief discussion. A detailed mathematical analysis of the various sources of distortion is given in Appendix I, and a comparison of magnitudes for typical conditions is given.

Tilt.
Non-planar focal surfaces.
Air refraction.
Lens distortion.
Film distortion.
Motion of film during exposure.
Earth curvature.
Terrain variations.

The distortions discussed below are the most significant.

3.2 Tilt

The distortion due to tilt or lack of verticallity of the principal axis may be seen in Figure 3 as the displacements δ_1 and δ_2. As the tilt angle becomes very large as in oblique photographs, the distortion becomes so large as to approach infinity at the horizon. The distortion

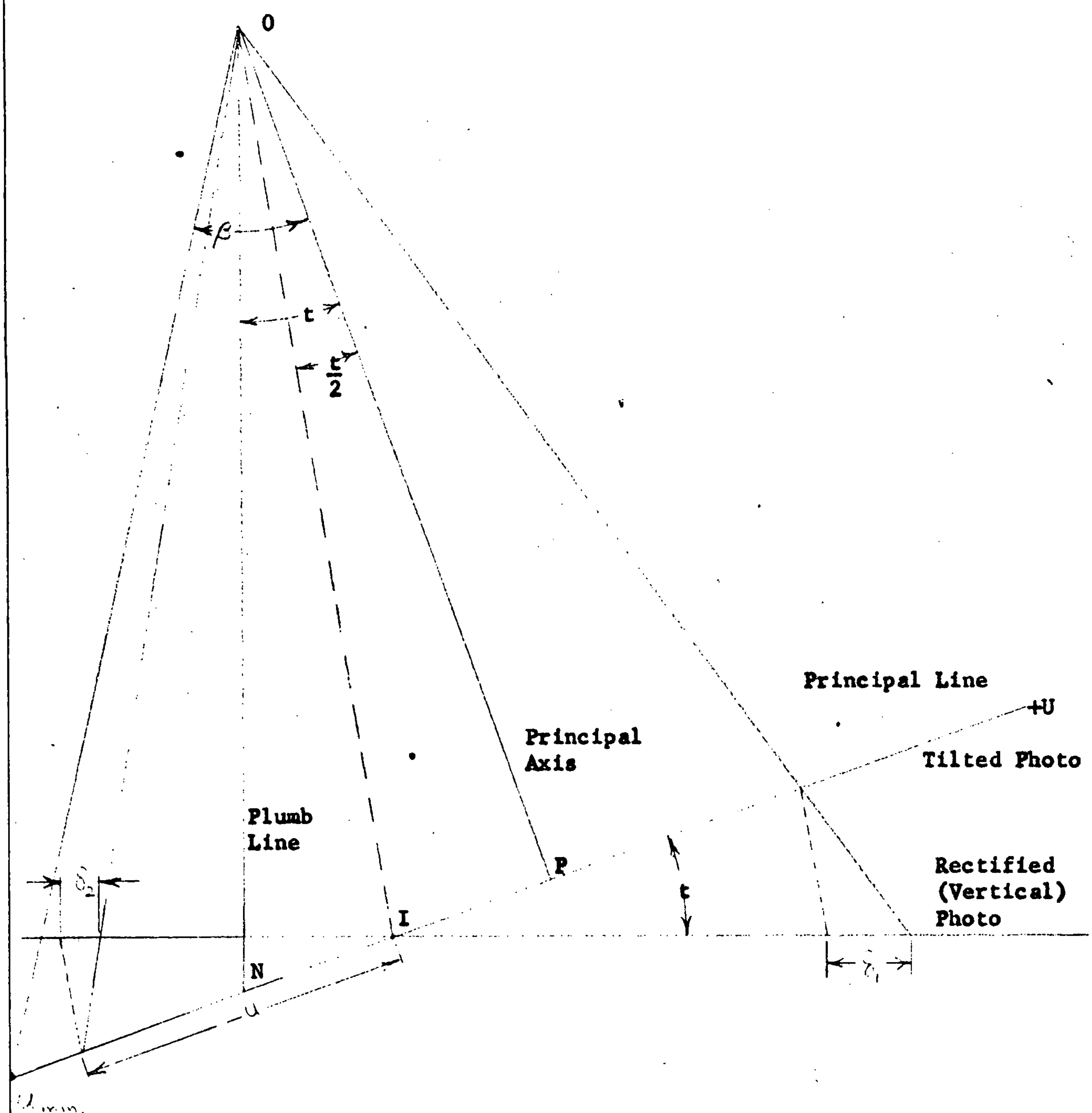


IMAGE DISPLACEMENT DUE TO TILT

Figure 3

due to tilt is very significant and is corrected with projection rectifiers in present photogrammetric practice.

The tilt of a photograph is the maximum angle between the photograph and the horizontal plane. The direction of tilt is given by the swing angle. Thus the tilt and swing of a photograph specify the amount and direction of tilt. In figure 3 the isocenter is the coordinate origin, where the scale of both photographs is the same. The scale at any point is given by $S = \frac{f - u \sin t}{H}$. The scale changes along the principal line but remains constant along any line perpendicular to the principal line. The image displacements due to tilt are shown as δ_1, and δ_2 in the figure and are given by the equation $\delta = \frac{u^2}{\frac{f}{\sin t} - u}$. Note that the rectified photograph becomes larger as u increases positively and smaller as u increases negatively.

This system of coordinates is not optimum for the rectifier but is shown here because it is easier to visualize than the selected system. The tilt corrections are more completely discussed on pg. 87 .

3.3 Non-planar Focal Surfaces

In order to achieve better resolution over a large field, the platens of some cameras are made non-planar, usually as surfaces of revolution about the optical axis. This introduces considerable radial distortion.

A type of camera that is being used frequently does not take a point perspective view but utilizes a rotating prism to image a narrow strip of the ground on the film. The film usually moves at the image velocity and the resulting photograph, which covers a field of 180° hor. to hor. is effectively that which would be imaged by a lens on a cylindrical focal surface. These cameras are usually referred to as panoramic cameras and produce a very distorted image of an extremely wide field. (See Pg. 91 .)

In addition to these pure photographic systems, there are a number of electronic systems, such as radar scanners, infrared scanners, and TV scanners, which produce distorted, wide angular coverage, photographic records. Many of these require rectification for most effective use.

3.4 Air Refraction

The density of the air, and as a result its index of refraction, decreases as the altitude increases. Because of this, a light ray bends as it travels from the ground to the lens, thus giving a radially distorted image. (See Pg. 101 .)

3.5 Lens Distortion

All lenses distort the image somewhat although in modern cartographic lenses this distortion is extremely small. Certain lenses have high distortion as a consequence of some other feature, such as better illumination over a wide field. (See Pg. 118.)

3.6 Film Distortion

Film distortion is due to film shrinkage during handling and storing. Aerial films shrink at a different rate across the roll and along the roll, giving rise to a radial distortion. (See Pg. 116 .)

3.7 Motion of the Film During Exposure

Distortion is caused by translation or rotation of the film during exposure. During the exposure the aircraft is translating along and across the fuselage axis and may be rolling and pitching. These motions give rise to complex distortions of the photograph, especially in panoramic cameras which have relatively long exposure times. (See Pg. 95 .)

3.8 Terrain Variations

Variations in terrain height above or below the datum plane give rise to displacements of the images of the corresponding points on the film. Figure 4 illustrates this situation. If distances are to be determined from the photograph, the point A, whose position on the datum plane is represented by A', should appear on the photograph at a' rather than a. The distance a-a' is called the relief displacement. The distance b-b' is the relief displacement for a point below the datum plane.

Terrain variations above the datum plane cause a positive displacement of the image point, and variations below cause a negative displacement.

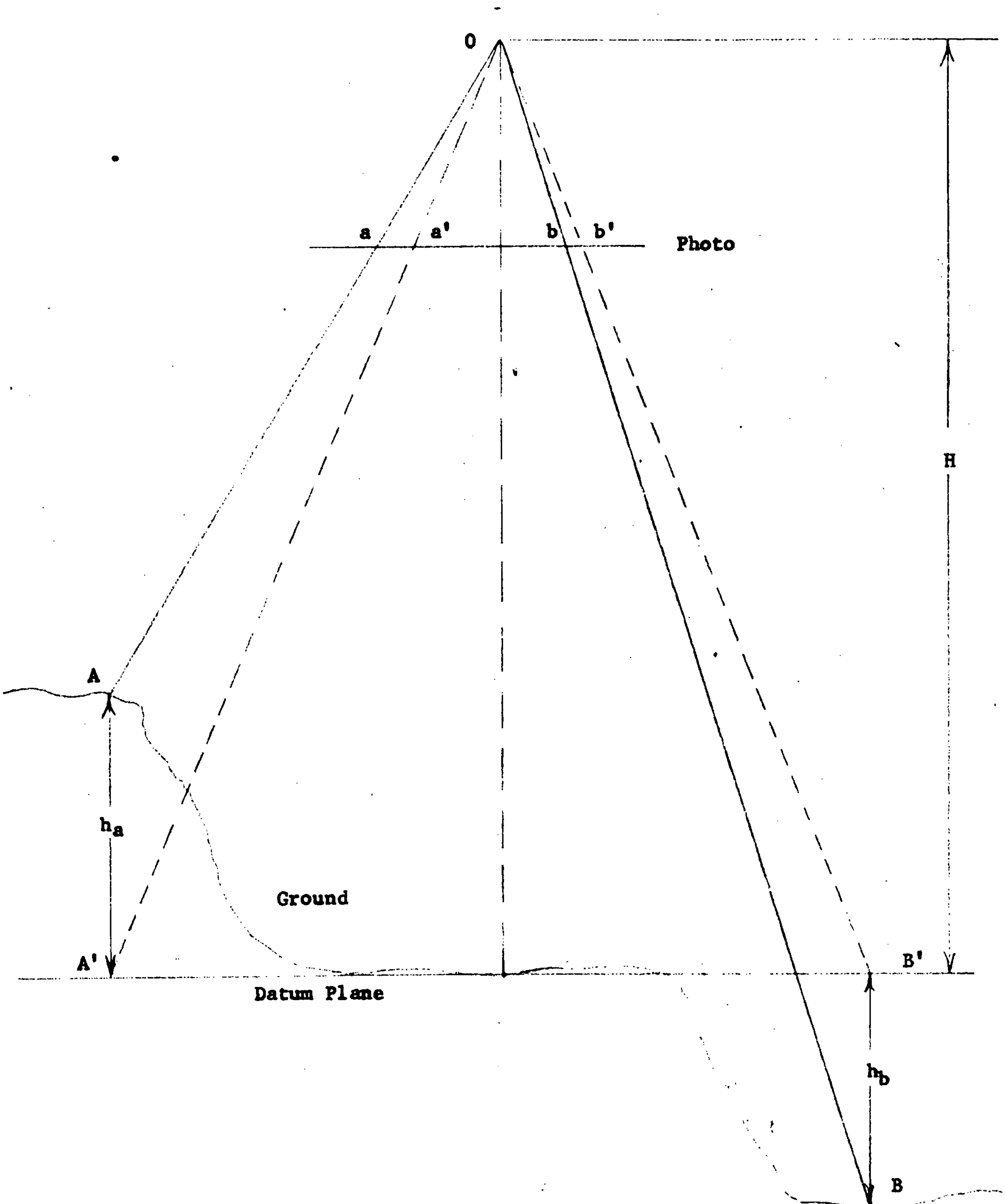
This form of distortion will not be corrected by the rectifier and will not be discussed further in this report.

3.9 Earth Curvature

High altitude, wide angle, aerial photographs suffer from a foreshortening of radial distances from the nadir because of the curvature of the earth.

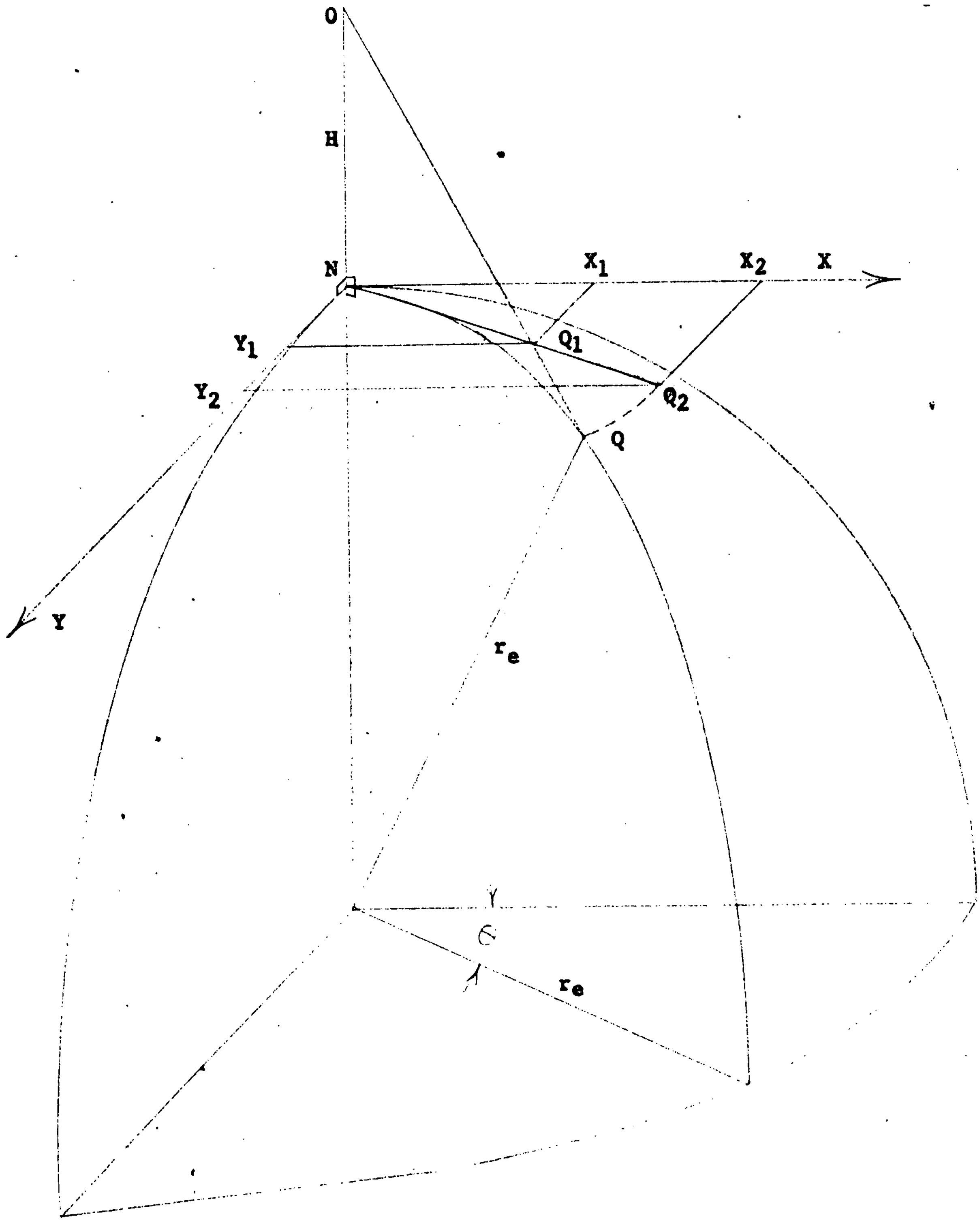
In Figure 5 the point Q appears on the tangent plane to the earth as if it were at Q_1. The actual distance from the nadir is the arc length N-Q, but this appears on the film as the distance $N-Q_1$.

There is no distortionless way of transforming a spherical surface to a plane. The approximation that appears most useful here is to make radial distances from the nadir and azimuth angle correct. The point Q



RELIEF DISPLACEMENT DUE TO ELEVATION
ABOVE OR BELOW DATUM PLANE

Figure 4



EARTH CURVATURE CORRECTION

Figure 5

will be rectified to the position of Q_2 and θ will remain the same. The length $N-Q_2$ is made equal to the arc length N-Q.

This projection is often used for maps and is termed polar azimuthal equidistant projection. In this case the earth's pole is replaced by the nadir.

This is not the only approximation that can be made and is not necessarily the best choice for all systems. Almost any conceivable correction could be made by the electronic rectifier.

Because of the considerable mathematical and computer simplification obtainable, earth curvature and air refraction corrections have been combined in the derivation of Pg. 101 .

All of the distortions mentioned here and in Appendix I can be corrected with the type of rectifier discussed in Sections 5.4 and 5.5 . It may not be desirable to include computer blocks for all of these distortions since the errors due to unknowns, such as inaccuracy of tilt determination or to inaccuracy of the rectifier itself, may exceed the error due to some of the smaller distortions.

3.10 Application to Mapping

The end use of many rectified photographs is the preparation of maps. Since the spherical earth cannot be exactly represented on a plane, the map is deliberately distorted to some projection, such as Mercator, Gnomonic, Lambert conformal, or Polyconic.

After adding the computer blocks required for the desired projection, the electronic rectifier is well suited to converting aerial photographs directly to map projection. The rectifier could also be adapted to converting existing maps to other projections.

SECTION IV

METHODS OF ELECTRONIC RECTIFICATION

4.1 Image Transfer Techniques

There is no practical method known to us with which to rectify an entire photograph to the accuracy we require (.01 %) in one exposure.

While most of the required corrections could be made by some form of analog, such as projection on a sphere for earth curvature correction, these methods are cumbersome and impratical.

Consideration was given to methods of projection printing of small areas of the photograph one at a time and positioning the center of each sub-area in its proper location. A digital computer could be used to calculate the position of each sub-area since the time required for each positioning and exposure would be relatively long.

Each sub-area could be projected at an angle, as is done in conventional rectifiers, to correct the distortions due to large tilt angles. Thus the large distortion sources, such as high tilts and panoramic distortion, would be corrected in each sub-area and small sources of distortion, such as air refraction, would only be corrected by proper positioning of the sub-area.

Consideration was given to the use of image converter tubes to transfer the sub-images. By the use of external electric fields, the image could be warped to an approximation of the desired shape.

So many sub-areas would be required to secure the required accuracy and so many difficulties would prevent shaping the sub-images and achieving proper exposure of each sub-image that all these methods were dropped early in the study.

Because we could find no suitable method of utilizing sub-images it became apparent that the picture would have to be scanned and reproduced in a manner similar to that used in television systems.

This method makes use of picture elements. If a photographic or printed image is examined very closely, it is found to consist of elemental areas of light and dark. These elemental areas are usually integrated by the eye to yield an over-all picture. In a photograph these elements are composed of grains of silver and in a halftone engraving of fine black dots. In the halftone each dot is equally black, but they differ in size or spacing in such a way as to represent the tonal details of the picture. The number of dots used per unit area determines the over-all information content or resolution of the picture.

The method to be used in the electronic rectifier is to break the picture down into a large number of picture elements, each of which will be some tone of grey, and reassemble the elements in a different geometrical orientation to produce a rectified picture.

The elements will be transfered from the reading end of the machine to the writing end by current pulses in an electrical circuit.

An electrical circuit can carry only one item of information at a time; and if many pulses are applied to the circuit at one time, the pulses will lose their identities and will be undistinguishable at the output end.

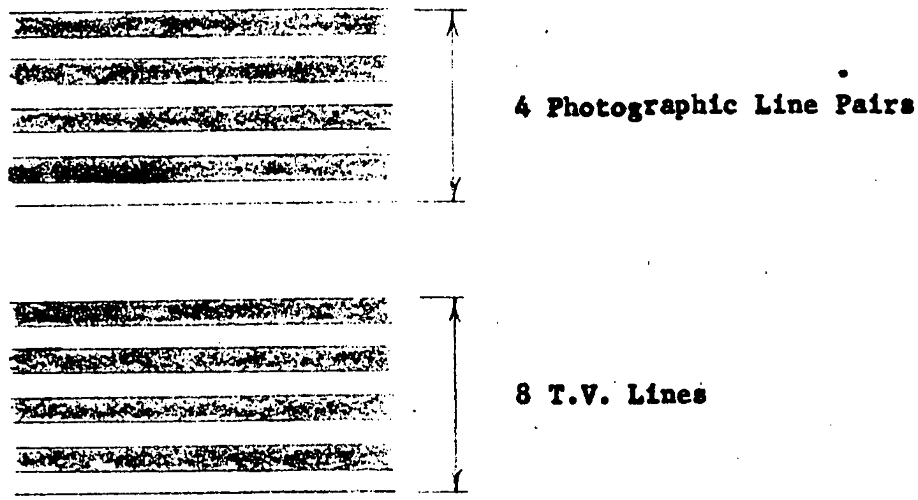
If only a few elements are used to make up the picture, a separate channel can be provided for each; and the entire picture can then be transmitted simultaneously. For a system to transmit a photograph of even very low resolution, the number of channels would be completely prohibitive. The alternative is to employ one channel and transmit the elements in time sequence.

4.2 Scanning

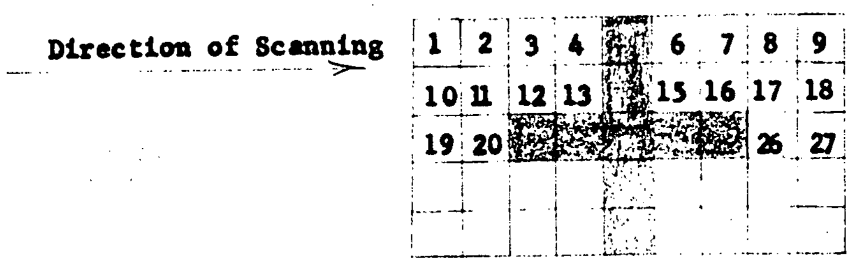
It is necessary to select a scanning method which examines each picture element in some orderly sequence. There are a great number of scanning methods, both electronic and mechanical, utilizing many different scanning patterns, which could be used. The method best suited to the rectifier is known as "uniform linear scanning" and is the scanning method used by all television systems in current use. This method is similar to the action of the human eye when reading. The eye begins at the upper left corner of a page and examines the information while moving to the right. When the right edge of the page is reached, the eye quickly goes back to the left edge and down one line and begins reading again.

A television system works in a similar manner. The picture elements lie in parallel lines. As the reading spot passes over each element, its brightness is converted to an electrical signal and transmitted to the read-out and where it is converted back to a light impulse.

The spot proceeds across each line and then starts on the next line, thus covering each picture element in order as shown by the numbered boxes in Figure 6 B. In a television system the entire frame must be covered 30 times per second to eliminate flicker and give the illusion of motion. We



A. DIFFERENCE BETWEEN PHOTOGRAPHIC AND TELEVISION LINES



B. ORDER OF SCANNING OF PICTURE ELEMENTS

Figure 6

have no such problem since the film stores the information presented to it.

We can take as long as we want to scan each picture and still have a complete photograph when we finish.

Uniform linear scanning is best suited to this problem for several reasons. It offers the most economical (timewise) method of covering rectangular areas and is easy to generate either mechanically or electronically. A spiral scan would be advantageous for the purely radial corrections, such as earth curvature, lens distortion, or air refraction; but its advantage is lost because of the non radial corrections, such as tilt, and because the radial corrections are about two different axes. (Earth curvature and air refraction are radial about the nadir, while lens distortion, film distortion, and platen distortion are radial about the principal axis.) Much of the discussion of line scanning systems follows from television practice and may be referred to as a TV system.

4.3 Resolution

In photography resolution is usually discussed in terms of the number of lines per millimeter and is stated as the maximum number of parallel black and white bars which may just be discerned.

In television or facsimile the line made by one sweep of the spot is called a line. It takes two sweeps of the TV spot to make one black and one white line or one photographic line pair. (See Figure 6 A.) In this report photographic resolution will be discussed as line pairs per mm and TV or line scan resolution as lines per inch. When the word lines is used, it means TV lines.

The grain structure of photographic emulsions has a random orientation, and, therefore, has the same resolution in any direction. This is not true of linear scanning systems which have a different resolution in the horizontal (along the line) and vertical (across the lines) directions.

4.3.1 The vertical resolution or resolution across the scanning lines may be considerably less than the number of scanning lines. In the ideal case the number of lines resolved may equal the number of scanning lines; and in the worst case the resolution may be zero. Consider the situation shown in Figure 7A. The object to be transmitted is a series of black and white rectangles (picture elements) equal in size to the scanning spot. If the scanning spot passes over each picture element, the pattern will be transmitted as shown in (2). If the scanning spot passes over the boundaries between black and white as shown in (3), a continous grey pattern will be transmitted as shown in (4).

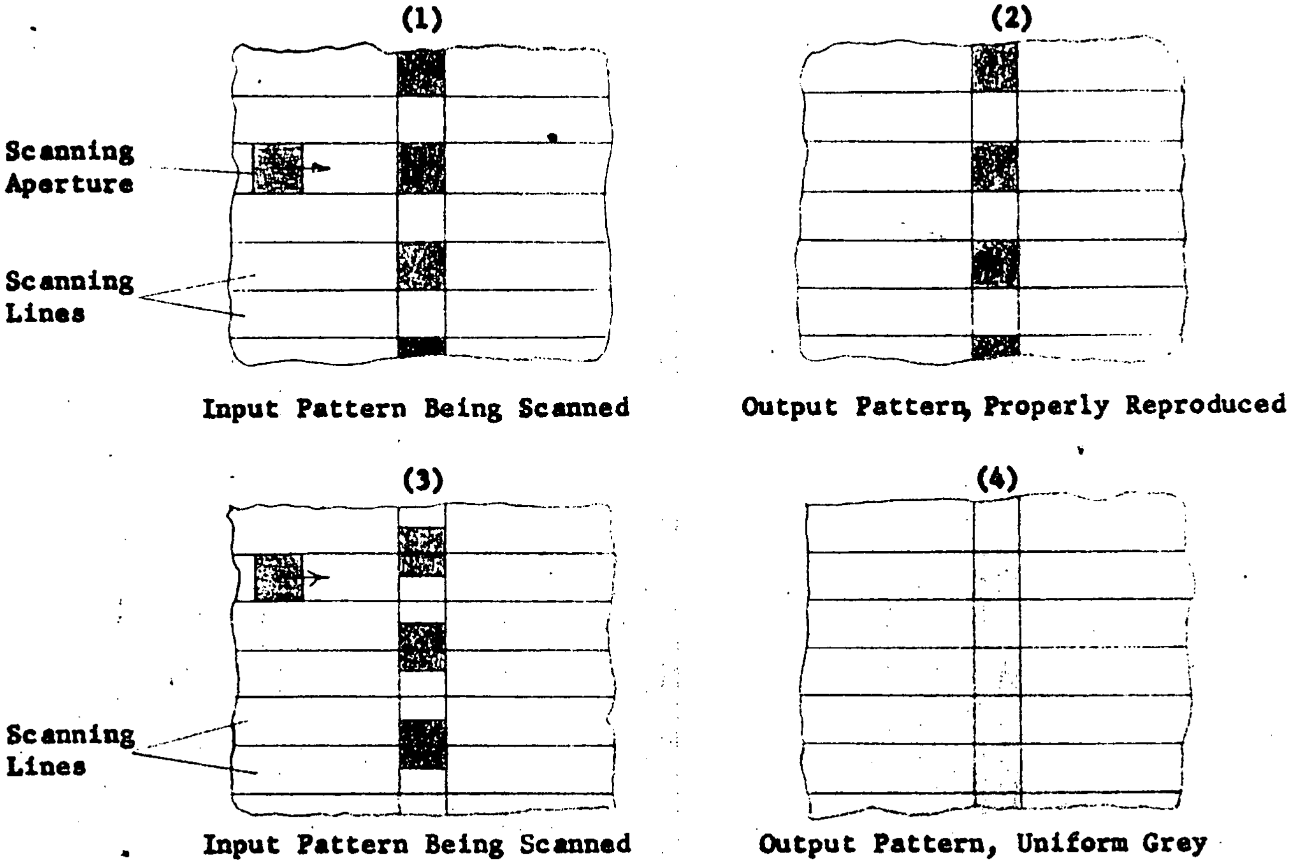
Actually, of course, the picture to be transmitted is not an even geometric pattern but a complex arrangement of various shapes. Experience of the television industry has shown that about 70% of the vertical picture elements are resolved. Wheeler and Loughren[1] computed a theoretical value of 70.7%, which experience has shown to be reasonable. The vertical resolution may be taken as $r_v = KN = .7N$. Where r_v = vertical resolution; N = number of scan lines per inch.

4.3.2 Horizontal Resolution

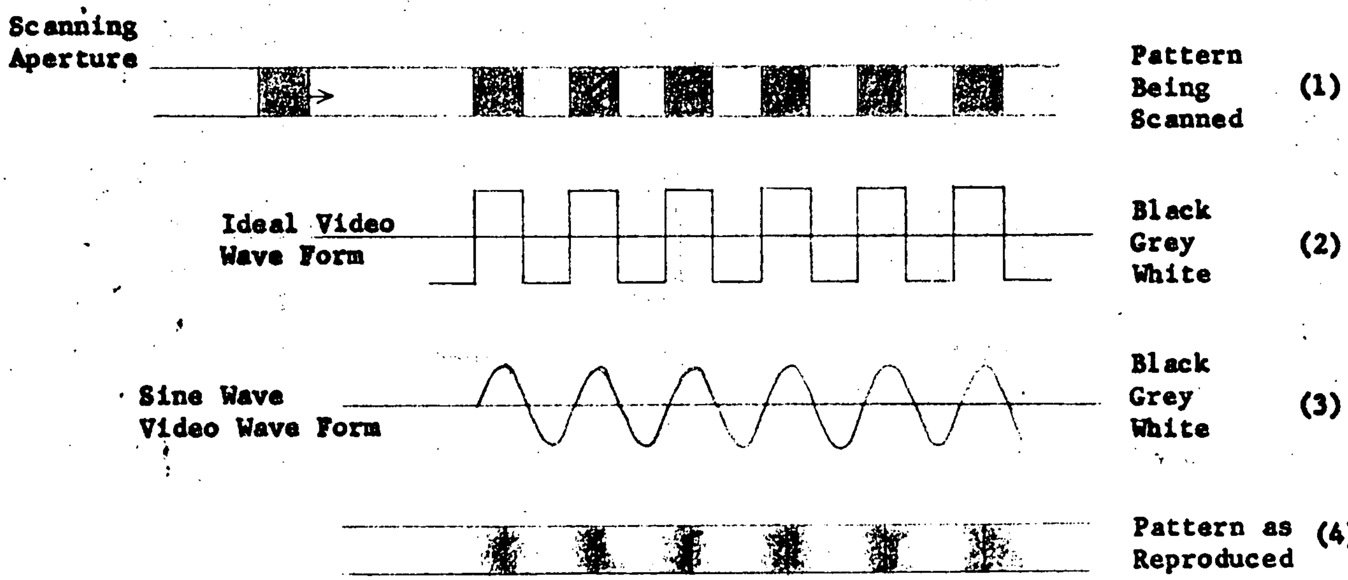
The horizontal resolution, or resolution along the scanning lines, is dependent on the bandwidth of the electronics.

As the scanning spot moves along the line, the density variations of the photograph being scanned are converted to electrical signals. The

[1] Wheeler, H.A., and Loughren, A. V., THE FINE STRUCTURE OF TELEVISION IMAGES, PROC. IRE, 26:540 (May 1938).



A. LOSS OF VERTICAL RESOLUTION BY SCANNING APERTURE OVERLAPPING EDGES OF IMAGE



B. LOSS OF HORIZONTAL RESOLUTION RESULTING FROM INSUFFICIENT BANDWIDTH TO REPRODUCE SQUARE WAVE

Figure 7

number of picture elements that can be resolved per unit time is limited by a number of voltage changes that can occur in this time. Figure 7B shows a row of alternating black and white picture elements of scanning spot size. The ideal video waveform (2) would require an infinite bandwidth to reproduce. The sinusoidal signal (3) will yield seperate discerrible picture elements, although it cannot produce sharply defined elements of uniform tone as shown in (1). The sinusoidal waveform is usually used as the basis of discussion in television practice and reproduces a picture (1) as shown in (4). Assuming a sine wave signal each half cycle can reproduce a black or a white element, thus yielding two picture elements per cycle.

The minimum bandwidth required is equal to 1/2 the number of picture elements to be transmitted per unit time.

$$r_h = \frac{2\,f}{v}$$

Where r_h = horizontal resolution

f = bandwidth

v = velocity of scanning spot

It is desirable that the bandwidth of the video system be greater than the minimum since higher harmonics will result in a closer approach to a square wave and hence sharply defined boundaries between black and white areas. This will improve the acuity of the rectified picture.

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Baldwin's[2] work has shown that the resolution of a scanning system can best be described, not by the horizontal or vertical resolution taken separately, but by the product of these, which is proportional to the total number of resolvable picture elements.

Because we are not used to thinking of resolution in terms of the total number of picture elements per unit area, it is reasonable to take the square root of this number as a figure of merit.

4.3.3 Resolution of Proposed System

The print out drum will rotate at about 3600 RPM or 60 RPS. If we scan a 3" line on the unrectified photograph the scan velocity will be approximately 3 x 60 = 180 I.P.S.

The bandwidth of the video section can be made at least 10^6 C.P.S. using a glow modulator tube as an exposing light source. The horizontal resolution $r_h = \frac{2f}{v} = \frac{2 \times 10^6}{180} = 11{,}000$ lines/inch.

We will scan in the vertical direction at 5,000 scan lines/inch.

The vertical resolution $r = .7N = .7 \times 5000 = 3500$ lines/inch.

The average resolution capability $= \sqrt{11{,}000 \times 3500} = 6{,}200$ lines/inch or 122 line pairs/mm.

It should be understood that this does not mean that the system will resolve a resolution test chart with 122 line pair per mm but it is an

[2] Baldwin, M., THE SUBJECTIVE SHARPNESS OF SIMULATED TELEVISION IMAGES, PROC. IRE, 28(10):458 (October 1940).

indication of the information transfer capability of the system.

The system should reproduce a test chart with somewhat more than 100 line pairs/mm in the horizontal direction and somewhat less than this in the vertical direction.

This capability depends on the manner in which the spot crosses the test chart bars and no absolute figure can be stated.

4.4 Aperture Distortion

The resolution as indicated in paragraph 4.3.3 is reduced by aperture distortion caused by the finite size of the scanning spot. When the spot crosses a picture element boundary, it gradually occupies more and more of the element, giving rise to a gradually increasing signal as shown in Figure 8C. The signal is proportional to the element density only at the instant that the spot is completely superimposed on the picture element. The same effect occurs as the spot leaves the element, thus giving rise to a signal which appears over a distance equal to three times the element size. This leads to a loss of horizontal resolution which in the extreme case could reproduce the pattern of Figure 7B as a continuous grey tone.

The ideal solution would be to make the width of the scanning spot very small relative to the width of the resolution elements. This is unfortunately not technically feasible with the size resolution elements being considered here. The losses due to aperture distortion can be largely corrected by suitable emphasizing the upper end of the frequency band. If a filter whose admittance vs. frequency curve is the inverse of the curve of spot admittance, the aperture distortion will be corrected. This is standard TV practice and presents no special problem.

4.5 Local Scale Variations

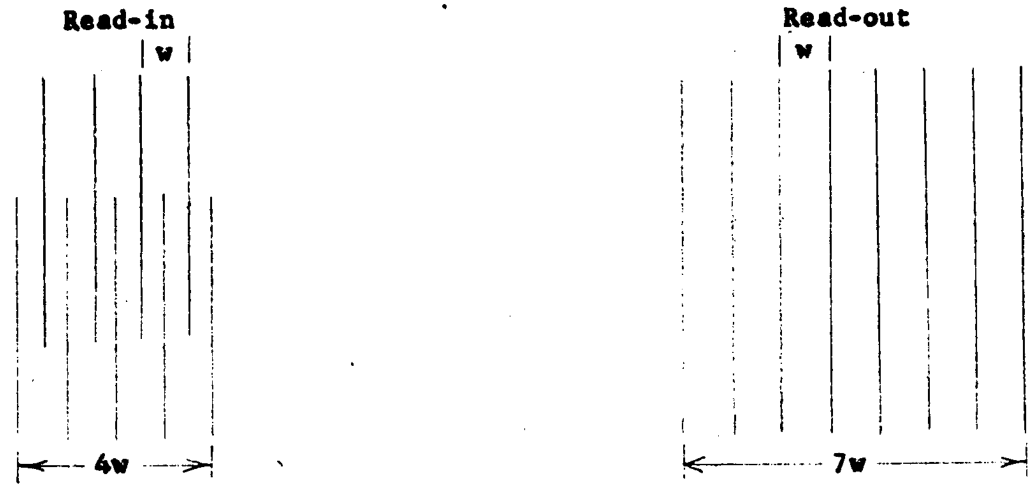
The images on the unrectified film must be stretched out or compressed when placed on the rectified film. There are two ways to accomplish this in a scanning system.

One method is to vary the size of either the scanning spot or the printing spot. If it is desired to increase the scale at some point by a factor of 1 1/2, the size of the read-out spot could be made 1 1/2 times as large as the read-in spot (assuming no over-all enlargement factor). The size of the read-out spot would be varying rapidly as the scale varied during each line scan.

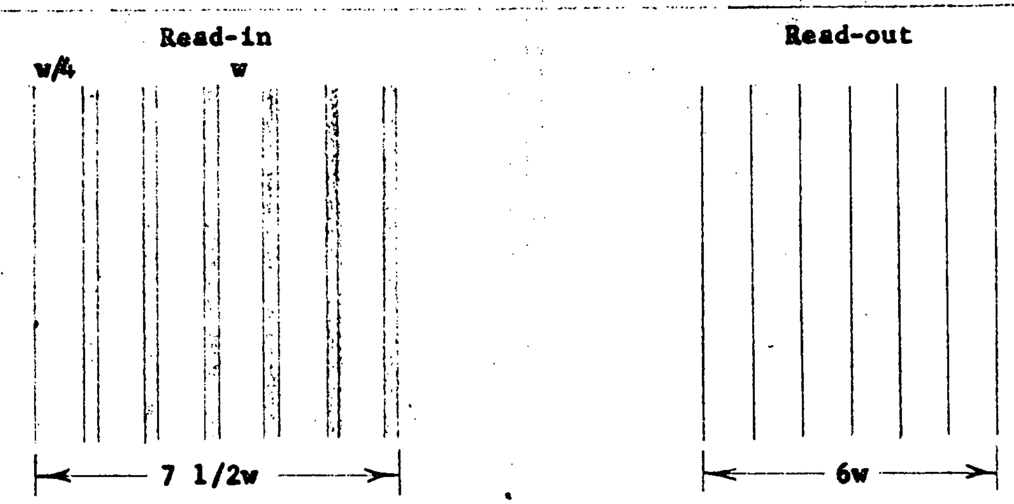
It would not be practical in our case to vary the size of the reading spot because this would cause the resolution of the system to vary as the scale changed and because we are working with the smallest practical spot obtainable.

It appears impractical to vary the size of the read-out spot during the scanning of each line since the only method we know of to do this is with a servo-controlled iris, which would be imaged on the film. The servo system would have to have a frequency response considerably higher than the line repetition rate which, in order to complete the rectification in a reasonable time, must be extremely high.

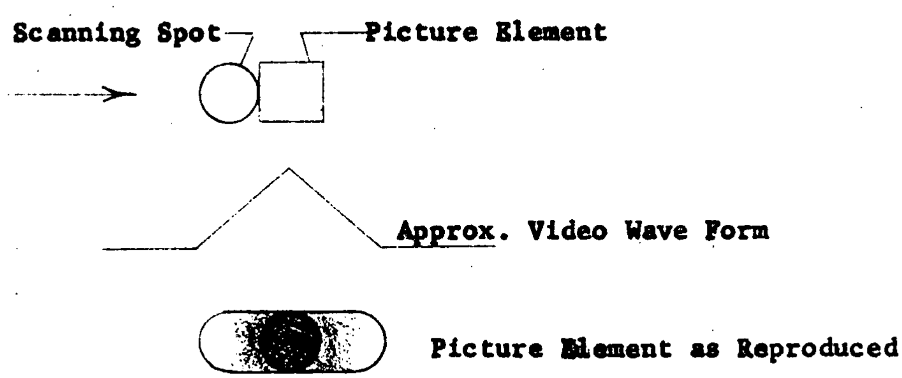
The other method available to us is to overlap or overspace the read-in scan lines and maintain constant read-out line width. This is illustrated in Figure 8. If it is desired to increase the scale at one point by a factor of 1 1/2, each read-in line would overlap 1/2 of the last line while



A. EXPANSION OF SCALE BY OVERLAPPING OF READ-IN SCAN LINES



B. COMPRESSION OF SCALE BY SPACING OF READ-IN SCAN LINES



C. EFFECT OF APERTURE DISTORTION

Figure 8

the information would be printed out with full line spacing. Where the scale must be decreased (read-out compressed), the line spacing will be made greater than the spot size. This results in a loss of the information that lies in the spaces, and the method could not be used if it were not for the fact that the reduction factor never exceeds 25%. In the rectification of a tilted photograph, the information on one side of the isoline (upper side, see Figure 2) must be stretched, and the information on the other side (lower side) must be compressed.

As the tilt increases, both the compression and the stretching requirement increase. When the tilt becomes large enough, the isoline no longer lies on the unrectified photograph, and no further compression is required.

Since the maximum compression that will be required is 25%, this means that the spaces between lines will never exceed 1/4 of the spot diameter. For 5000 line/inch scanning system the maximum theoretical spacing will not exceed .00005. This figure is somewhat meaningless since the spot edges are not sharply defined and the instrument will not be capable of positioning the spot with this accuracy.

The compression required to correct distortions other than that due to tilt have negligible effect on line spacing.

The stretching process does not affect the vertical (across the lines) resolution and the compression process has negligible effect on it.

Variation of line spacing changes the scale in the vertical direction. Scale variations in the horizontal direction result when the read-in spot is made to go slower or faster than read-out spot.

If the read-in spot goes slower, the information that it covers or reads in a given time is spread out over a longer distance on the output film.

Stretching has no effect on horizontal resolution.

If the read-in spot goes faster than the read-out spot, compression results. Compression does not affect the horizontal resolution as long as sufficient bandwidth is provided to cover the higher frequencies that the spot will generate.

The line spacing and velocity variation method of locally changing scale appears to best meet the requirements of this study.

4.6 Component Study

A study of available or potentially available components that would be useful in the electronic rectifier was undertaken. This work was most concerned with three important components: cathode ray tubes to read the film, light modulators to expose the rectified film, and suitable computer components.

4.6.1 Cathode Ray Tubes

Either the input or output end of the rectifier must be capable of high speed, non-linear scanning involving very high accelerations. A cathode ray tube used as a flying spot scanner provides an almost inertialess, very bright, spot of light with which to read the film. The face of the CRT is imaged on the film to be rectified and the spot of light positioned wherever desired. The amount of light seen by a photo-tube on the other side of the film is a measure of the transparency of the picture element illuminated by the spot at that instant. The spot of light can be positioned at much higher velocities and accelerations than can be utilized by other sections of the rectifier and provides a good choice for the read-in end.

Unfortunately, there are several limitations to the use of a CRT for this application. The two primary limitations are spot size and linearity.

To obtain a resolution of 100 lines pairs/mm, we require at least 5000 scan lines per inch. This requires a spot diameter of .0002 inches. This is beyond the capabilities of the tube manufacturers at this time but may be obtainable in the future. The solution to this problem is to project a reduced image of the tube face onto the film. While this reduces the size of the spot, it also reduces the usable size of the tube.

The best linearity that can be expected is about 0.1%, and this requires great care. All of the tubes suitable for this application are magnetically focussed and deflected. To obtain 0.1% linearity, the yokes and focussing coils must be of the highest precision and usually must be custom designed for the application. Since the deflection sensivitity depends on the accelerating voltage, the power supply for this voltage (20,000 to 30,000V) must be regulated at least 0.1%. The tube must even be protected from the effects of the earth's magnetic field. The deflection currents would be obtained in a closed loop system.

In spite of these difficulties, 0.1% linearity appears to be obtainable in a practical system. A number of manufacturers have or can build tubes that would be applicable to this problem.

Dumont has a 5", magnetically focussed and deflected, .001in.spot size tube available from stock at $300 called the K1725-P15 or K1725-P16. In a short time it will be available in 3" and 7" dia.

CBS-Hytron has a 5", magnetically focussed and deflected, 0.7 mil spot size tube available from stock. The cost is about $5,600 at present, but they expect to reduce this considerably. This tube is designated 5BYP5. They also can supply on special order a similar tube with .0005 in. spot size. This tube is soon to become a shelf item and will cost about $2,000. CBS says that they could build for us a 10" dia. tube with 0.5 mil spot size if desired. CBS claims a corrected circuit linearity of 0.1%, but as mentioned previously this is somewhat difficult to obtain.

Litton Industries stated that they would be interested in building a 7" dia. tube and yoke package with 15,000 line resolution and 0.1% linearity. As an example of what could be done with one of these tubes, we can consider the Litton Industries proposal. They feel that a 15,000 lines, 0.1% linearity, 7" diameter tube can built to order for us.

This tube face would be imaged onto the film to be rectified with a magnification of 3/7X. This would yield 5,000 scan lines per inch and a 3" dia. useful area on the film. The maximum excursion of the spot would be 1 1/2 inches, and 0.1% of this is .0015 inches. The contribution of the spot-position error to this system would then be only .0015 inches, which is about the accuracy we require.

From this we see that presently available CRT's are suitable for this job, although the .0015 inch figure is a bit optimistic.

A CRT could be imaged on the unexposed film and used as a read-out device. The scan would be supplied by sawtooth deflecting voltages, which would also be the inputs to the computer. The spot would be intensity modulated to expose the film to the proper density. Unfortunately, the CRT suffers from limited dynamic range because of phosphor limitations and halation effects at the face plate.

A tube which is large enough to cover a reasonable area of the rectified film would be extremely large, and non-linearity would cause very large positional errors on the film. If the film were covered, small area x small area, we would have problems with overlapping of lines with consequent loss of resolution.

Since only one end need have the capability of high accelerations and non-linear scanning and this capability is the only reason for using a CRT, it would be inadvisable to use one at each end.

4.6.2 Light Modulators

The video signal, after modification for the film characteristics and gamma corrections, must be converted from an electrical signal to light. This light, when imaged as a small spot on the film, exposes each picture element to produce the rectified photograph.

The light source must have a reasonably linear light output with variations in signal voltage, must produce sufficient light to expose the film at the extremely short exposure times used. Several available light modulators have been investigated.

4.6.2.1 Glow Modulator

The glow modulator or crater lamp is the simplest and cheapest light modulator available. These cold cathode, gas ionization, light modulating tubes are small, simple, inexpensive, and easily replaced when worn out. A tube which appears to be suitable for application here is the Sylvania R1168. This tube has a .015 inch dia. crater with a light output of .023 CP and brightness of 132 candles/square inch. These values can be increased by overdriving at the expense of losing some of the 150 hour rated average life. The upper frequency limit is one megacycle at which point the output is 3 db down. The device has fairly good linearity of light output vs. input current. The frequency limit can be substantially increased, and the linearity corrected by the use of a closed-loop feedback system, which monitors the actual light output of the tube.

It appears that this tube will meet the rectifier requirements for all films except the very slow, high resolution films.

4.6.2.2 The Ultrasonic Light Modulator

The ultrasonic light modulator, manufactured by the Fairchild Camera and Instrument Company, takes advantage of the defraction of light at ultrasonic wavefronts. The ULM consists of a rectangular glass box, filled with liquid, and having a piezoelectric transducer immersed in the liquid at one end. Collimated light from the source passes through the cell and is then focussed on a stop which prevents it from reaching the film. When a signal is applied to the crystal, ultrasonic wave fronts are caused to travel through the tube at the velocity of sound in the liquid. Defraction at the wavefronts causes light to pass the stop and expose the film. The principles of operation are more thoroughly discussed in a paper by Levi[3]. The ULM has an upper frequency limit of over 10 megacycles, a dynamic range of over 200 to 1 and very good linearity of light transmission vs applied voltage.

One feature of the ULM is both an advantage and a disadvantage. In use a highly demagnified image of the cell is projected onto the film to be exposed. When the signal is applied to the cell, the image of one element moves along the image of the cell. To prevent this from becoming a blur,

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[3] Levi, Leo, HIGH FIDELITY VIDEO RECORDING USING ULTRASONIC LIGHT MODULATION, JOURNAL OF SMPTE, Vol.67, Page 657, October, 1958.

a rotating prism must be placed in the path between the cell and the film. This prism is arranged to compensate for the motion of the element through the cell. The rotating prism is, of course, an added complexity unless such a prism is required to obtain a line scan as is done in the system described in Section 5.5, page 81.

An advantage of this feature is that the element is exposing the film for the entire time it is passing through the cell. With high frequency signals and slow film, this could be of considerable importance. With a device such as a Kerr cell or glow tube, recording at 2 megacycles, the exposure time is about 1/4 microsecond, but with the ULM the exposure time is the time required for a sound wave to travel the length of the cell. This may be in the order of 40 microseconds, which is a gain in exposure time of about 160 times.

Since the light source is external to the ULM, almost any brightness desired may be obtained through the use of a large lamp or even an arc light.

4.6.2.3 The Kerr Cell

The Kerr cell manufactured by Svenska AB Gasaccumulator of Stockholm, Sweden, is a small, transparent, liquid, filled cell with two electrodes immersed in the liquid. Polarized light from an external source is passed between the electrode plates. A high voltage applied to the plates causes rotation of the plane of polarization of the cell; and if enough voltage is applied, extinction takes place.[4]

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4 For a more detailed discussion, see, for example, Robert W. Wood, PHYSICAL OPTICS, The Macmillen Company, New York.

The above-mentioned cell is about 1 1/2 inches in diameter, has a 1mm spacing between plates, and can be modulated up to about 12 megacycles.

One disadvantage to the Kerr cell is the high operating voltage required. A modulating voltage of several thous. volts is needed.

The cell transmission is quite non-linear and would have to be used with a non-linear amplifier or a closed-loop feedback circuit. As much light as desired can be obtained since the light source is external to the cell. The Kerr cell is relatively inexpensive.

4.6.3 Choice of Modulator

Of the three modulators discussed, the glow tube is the most desirable, being very simple and very inexpensive. If the film to be used is too slow to be exposed by the glow tube, the Kerr cell would be the best choice because it is considerable cheaper and less complex than the ULM.

4.6.4 Closed-Loop CRT Spot Positioning Servo

In Section 4.6.1 the CRT was discussed and its advantages as a source of an inertialess spot of light pointed out. The main disadvantage of the CRT is its linearity limitation of about .1% under ideal conditions. For most applications this is very good, but for the rectifier it is unsatisfactory except when only a small image of the tube face is used.

One approach to solving the linearity problem is to use a closed-loop spot positioning servo system. In a system of this type some of the light from the spot would be imaged onto code plates. The code plates, one for horizontal and one for vertical, could be either digital or a simple grid pattern. If a digital or pulse code were the computer output, it could be compared with the present address of the spot and an error signal generated which would drive the spot to the correct address.

Hoover[5] describes such a system as part of an information store. Hoover's device is in operation and can change its address 1 spot position in .8 microseconds. If a closed-loop system were developed for the electronic rectifier, one of the greatest blocks to increased accuracy would be overcome. If a CRT-rotating drum system similar to that described in Section , but with spot position feedback, were constructed using an 18 inch dia., 2500 line/inch, CRT, we could scan the full width of a 9" photograph at 100 line pair/mm resolution. A spot position accuracy of .0005 inches on the photograph could probably be achieved, and the speed of rectification could be increased because the full width of the unrectified photograph would be scanned once for each resolution of the drum.

The design of a suitable spot servo system would require an extensive development program. It is not recommended that this feature be undertaken for the first rectifier which, being a completely new type of system, will require so much development of its own. After an electronic rectifier has been constructed, tested, and evaluated, a decision should be made, based on the results obtained, as to whether or not the higher accuracy which could probably be obtained with a spot position servo would justify the additional cost and complexity.

If it does appear to be justified, considering the inaccuracies in the determination of tilt, altitude, etc., a program could be undertaken to develop a spot position servo which, if possible, would be adaptable to the then existing rectifier.

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[5] Hoover, C. W., Jr., R. E. Staehler and R. W. Ketchledge, FUNDAMENTAL CONCEPTS IN DESIGN OF THE FLYING SPOT STORE, Bell Telephone System, Monograph 3135.

After evaluating this system, a final rectifier could be designed utilizing all the knowledge gained from testing the first model. It does not appear advisable to undertake the development of a spot position servo actuated rectifier as the first design.

4.6.5 Alternate Video Pickup

By replacing the flying spot scanner with a Vidicon tube, we could eliminate the photomultiplier, the condensing optics, and the constant intensity feedback circuitry.

Unfortunately no tubes that would meet the requirements are available. A Vidicon that might be considered here is the RCA 6326. This unit has a 600 line resolution and a .62 inch diameter mosaic. To achieve 5000 line/inch resolution, the photograph would have to be imaged on the mosaic at a magnification of 5000/600, or 8.4 times. The largest square that will fit the .62 diameter mosaic is .44 x .44 inches. The largest area of the photograph that could be covered at a time is .44/8.4, or .05 x .05 inches.

The Vidicon has a high inherent signal-to-noise ratio, but for uniform illumination of the mosaic the output current may vary as much as 100% as different areas of the mosaic are scanned. This problem also exists with the CRT because of phosphor variations, but it can be corrected as discussed in Section 5.4 . Another problem of the Vidicon is that the output current varies with the velocity of the scanning electron beam. Since our beam velocity must vary as the scale varies, this will distort the tonal characteristics of the rectified photograph.

The Vidicon has a limited tonal range. Under ordinary conditions about 10 grey scale tones may be expected. Under ideal conditions this could be extended to about 15 tones.

It appears that the flying spot scanner offers a better solution than the Vidicon.

4.6.6 Tonal Range

One of the most important characteristics of a photographic reproduction system is its tonal gradation capabilities.

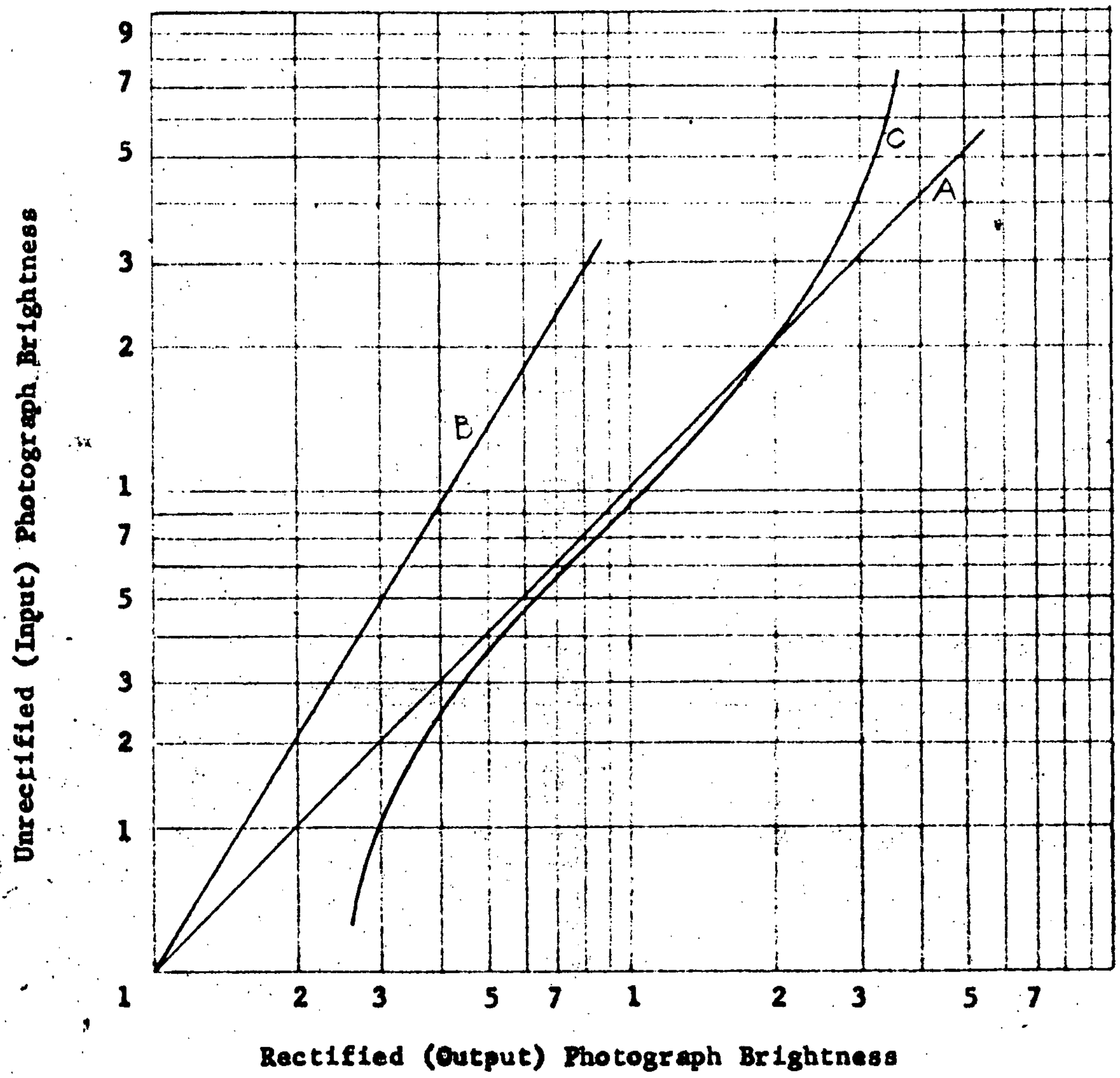
Ideally the system should be capable of a linear brightness transfer characteristic. Figure 9 is a plot of input brightness vs. output brightness or brightness transfer characteristic. This is usually plotted on log-log coordinates since, according to the Weber-Fechner law, the visual sensation induced by brightness varies as the log of the brightness.

Equal distances along the axis represent equal changes in visual sensation. If the curve is straight and has a slope of unity (curve A), the system is distortionless since the contrast ratio, or ratio of maximum to minimum brightness, remains unchanged. A unity slope which does not pass through 0-0 has the effect of brightning or darkening the output without changing its contrast ratio, and it may be compared to a neutral density filter in an optical system. Uniform brightness distortion, or changing slope of the curve (curve B), alters the contrast ratio of output with respect to input and may be compared with changing the "gamma" of a photograph.

If the characteristic is curved (curve C), it has non-uniform brightness distortion.

These curves are often plotted for a photographic emulsion as shown in Figure 10 and are called H and D curves.

The straight line portion of the curve is usually used, thus yielding a uniform brightness characteristic. The slope of this curve is called the "gamma" (γ) of the film. A film with a γ of one maintains the same contrast as the original scene (with the straight line portion of the curve).



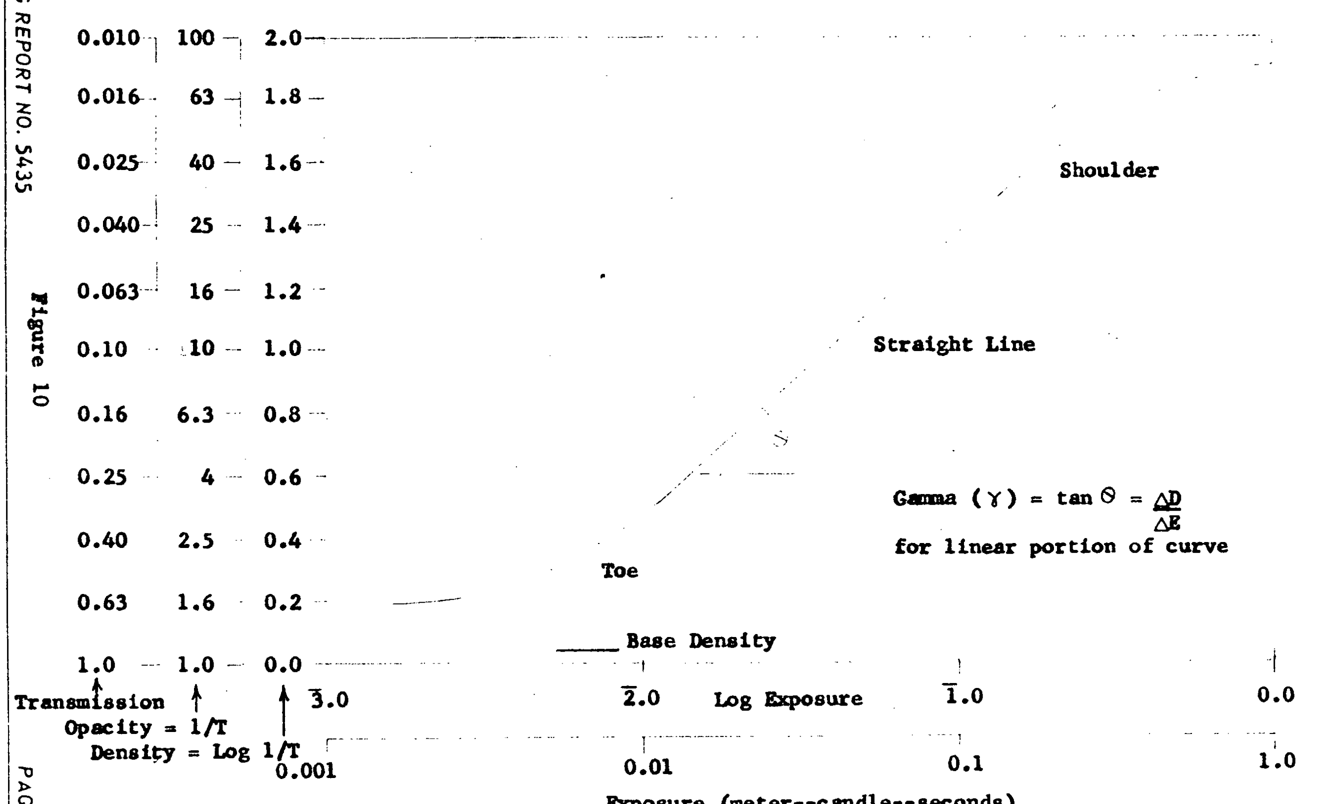
(A) Distortionless
(B) Uniform Brightness Distortion
(C) Non-Uniform Brightness Distortion (Compression of highlights and shadows)

BRIGHTNESS TRANSFER CHARACTERISTIC

Figure 9

Figure 10

From Kodak Films, Kodak Pub. No. F-1
Eastman Kodak Company
Rochester, New York



THE CHARACTERISTIC CURVE

Characteristic curve for hypothetical film showing the relationship between transparency, opacity, density, and exposure.

A γ of one is not always desirable. When photographing a low contrast scene, such as encountered in aerial photography, a γ greater than one enhances the contrast. A few of the fundamental photographic relationships are given here and are shown in Figure 10.

Let I = Intensity of incident light.

i = Intensity of transmitted light

T = Transparency.

O = Opacity.

D = Density

$$T = \frac{i}{I} \quad \text{and} \quad O = \frac{I}{i} \quad \text{then } O \times T = 1$$

Density is defined as

$$D = \log \frac{I}{i} = \log O = -\log T$$

From this it follows that a film with a density of 1 will transmit 1/10 of the incident light, a density of 2 will transmit 1/100 of the incident light, etc.

The unrectified negative will be scanned with a spot of light of constant brightness. The light passing through the negative is proportional to the transparency; and, since the photomultiplier has a linear light input vs. output current characteristic, the output current will be proportional to the transparency of the negative.

If we wish to make a rectified negative from which positive prints can be made, the transparency of the rectified picture must be equal to the transparency of the corresponding picture element on the unrectified picture. Actually, the transparencies will not always be equal since as discussed in Section 4.6.7 we do not always wish to have a linear, one for one, transfer characteristic; but they will be called equal here since that is the zero setting of the contrast controls.

T_1 = Transparency of unrectified photo.

T2 = Transparency of rectified photo.

E = Exposure.

$T_1 = T_2$ for no contrast modification.

$$D_2 = \gamma \log E_2 = \log \frac{1}{T_2} = \log \frac{1}{T_1}$$

$$\frac{1}{T_1} = E^{\gamma} \qquad \text{and } E_2 = \frac{1}{T_1^{1/\gamma}}$$

If we wish to get a positive rectified picture $T_1 = O_2 = \frac{1}{T_2}$

$$\log \frac{1}{T_2} = \log T_1 = \gamma \log E_2$$

$$T_1 = E_2 \qquad \text{and } E_2 = T_1^{1/\gamma}$$

This is equivalent to a contact print; if the γ is one, the exposure required is directly proportional to the transparency of the unrectified picture.

If the γ is taken as the slope of the transfer characteristic of the entire system including the film, we see that for a γ of 1, $E_2 = \frac{K}{T_1}$ for a negative and $E_2 = KT^1$ for a positive. Thus the video amplifier must generate a hyperbolic function of the input or a linear function of the input depending on whether we want a negative or a positive picture. Actually all of the components of the train will not be linear. The film, of course, is non-linear; and the output light transducer is non-linear.

The video amplifier will be tailored to give an over-all linear characteristic which can be modified as desired.

4.6.7 Contrast Modification

It may appear that the rectifier should have a uniform brightness transfer characteristic passing through the origin, thus yielding a rectified photograph with the same brightness and tonal characteristics as the original. This will sometimes be so, although it frequently will not.

For the reason discussed below, the over-all rectifier--film transfer characteristic should be adjustable in slope, intercepts, and degree of uniformity or straightness.

The original may be of very low contrast and require increased contrast (slope greater than 1).

The original may be so contrasty as to beyond the range of available printing papers and require contrast compression. Different areas of the original may have varying contrast. Wide angle obliques may range from good contrast near the vertical to very low contrast near the horizon.

In cases like this, it may be desirable to modify the contrast as different areas of the photograph are scanned as is done in the Log E Tronics system.

The Log E Tronics method is not applicable here because of the small resolution-size picture elements system used. The exposing spot used in Log Etronics covers a similar area to several million of our picture elements, and it is the average transmission of this area that is used to determine the exposure. Such a system could not be used here, although certain approximations might be experimented with.

The scan lines will lie in the general x direction. Panoramic and trimetrogen systems usually include the horizon, and in these cases the horizon will be roughly parallel to the scan lines. Seldom will the aircraft pitch enough to cause the horizon to approach parallelism with the y axis. Since the contrast reduces as the horizon is approached, a system where the exposure is varied as a function of the average transmission of the last 200 or 300 lines would correct for changes along the y axis. This method would not be of any help along each scan line. The value of such a system could best be determined experimentally after completion of a rectifier.

The rectifier should be capable of a linear transfer characteristic which can be modified as desired.

This modification could be achieved with an adaptation of the Fairchild Variable response unit or some similar system.

This device is capable of varying the transfer characteristic of the system in almost any way desired. Among the more important modifications are:

1. Expansion of dark tones.
2. Expansion of high light tones.
3. Expansion of middle tones.
4. Expansion of dark and high light tones (middle tone compression).
5. Step response.
6. Negative response.

Some form of equipment for varying the transfer characteristic of the rectifier should be included in its design.

4.6.8 Accuracy

The tentative specifications call for a geometric accuracy of .01%. This implies that the position of any point on the rectified photograph will be within .01% of the largest dimension of the picture from the theoretical position as indicated by the equations of rectification when solved with the selected input variables, such as tilt or altitude.

It is extremely difficult to specify the accuracy of the rectifier in these terms because the rectified accuracy is so dependent on the local change in scale required.

The photograph to be rectified will be broken into sub-areas, each of which will be scanned by the CRT. The error in locating a point on the unrectified photo will include computer error, CRT linearity error, and platen positioning error. The error in the platen position will be a given uncertainty, and this uncertainty will not change much as different areas of the photo are covered. The error due to the CRT will be a maximum at the edge of the faceplate; but since the image of the CRT may be positioned anywhere on the film, the maximum CRT error may appear anywhere on the film. Thus, the spot positioning error at the read-in end is essentially independent of distance from the center of coordinates.

At the output end the accuracy of position of a point is determined by the accuracy of the drum position transducers and the amount by which the system time lag varies.

If we assume the case of no over-all scale change (no enlargement factor), the accuracy with which we must pick a piece of information from the unrectified photograph in order to achieve a given accuracy of location on the rectified photograph varies with the scale. This is discussed in Appendix 1, page 130 . If there were no errors in the system, we would have to locate a point on the unrectified photo within .00025 inches to have the rectified location correct within .001 inches for the case of the panoramic camera at a point corresponding to a scan prism angle of 60°. This problem is much less significant for near vertical pictures taken with standard cameras. As large local scale changes occur, the accuracy of the rectified picture changes significantly even when the accuracy of locations of points on the input end remain constant.

Because of the difficulty of expressing the over-all system accuracy except for a particular photograph, we have chosen to discuss the accuracy of individual sections of the rectifier. These requirements are:

1. The ability to position the spot on the unrectified photograph to .01% of the diagonal of a 9 x 9 photo. This is .0012 inches.

2. The transducer outputs (computer inputs) on the output end should indicate the position of the writing spot within .01% of the diagonal of the rectified picture.

3. The computer accuracy should be .01%.

4.6.9 Testing of the Completed Rectifier

Testing of a rectifier of the accuracy considered in this report presents a number of problems and must be done with considerable attention to detail.

Resolution tests can be made by reproducing a standard resolution test chart transparency with the computer set for zero geometric change.

Tonal range can be tested using a standard grey scale transparency with the contrast controls set for a 1 to 1 linear characteristic.

Geometric accuracy testing will be somewhat more difficult. Several test plates must be made up by computing the theoretical positions of points of intersection of rectangular grids on the ground and marking these points on transparent plastic sheet such as Chronar with a coordinate comparator. The points must be computed to include the effects of whatever distortions are being checked for that particular test. As a start a simple rectangular pattern would be used and all computer inputs set at zero.

As testing progresses, various distortions can be included in the test plates. For high-tilt testing the test plate would resemble a canadian grid.

After processing, the rectified photographs would be measured on a coordinate comparator and the error from a perfect grid computed. All of the measurements on the test plates and rectified plates must be accurate to better than .0005 inches, and film changes during handling must be known to similar accuracy. Needless to say, reliable accuracy tests must be conducted with great care and will consume considerable time. These test plates will be used for periodic checks to assure the instrument remains in proper calibration.

4.6.10 Film for Recording Rectified Photograph

Since we intend to apply an over-all enlarging factor of at least two to the rectified print the film need only record a maximum resolution of 50 line pairs/mm.

There are a considerable number of films that could be used here.

As an example, consider Kodak Plus -X Aerecon[6] (type 8401) which has a daylight speed of A.S.A. 80 and a high contrast (1000:1) resolution of 90 line pairs/mm. At low contrast (2:1) the resolving power is given as 30 line pairs/mm.

Many films have higher resolution capabilities than these and the rectifier should be tested with several available films before a final decision is made. One advantage of using 8401 is its high speed which would permit use of a glow modulator tube as an exposing light source.

One problem that must be considered in the selection of a film is failure of the reciprocity law at the very short exposure times we are using. Kodak [7] lists the relative speed of Tri-X Aerecon at 10^{-6} seconds exposure time as one half the relative speed at 10^{-2} seconds.

This indicates that twice the energy must be applied to the film for a given exposure. This loss of half the speed must be considered when cal-

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6 Kodak Publication, INSTRUCTIONS FOR USE OF KODAK PLUS x AEROCON FILM, (TYPE 8401)

7 Kodak Publication, Kodak High Speed Films for Short Exposure - Time Applications, 1959

culating the required exposure and, while not a serious problem, will limit the choice of films that can be used with the glow modulator.

Plus-X Aerecon is recommended as an adequate film to use as a design basis. If a better film is found after actual test on the rectifier it could then be used.

SECTION V

MECHANIZATION OF THE RECTIFIER

5.1 General System Block

The very generalized block diagram shown in Figure II is useful to describe the system arrangement and the directions of information flow.

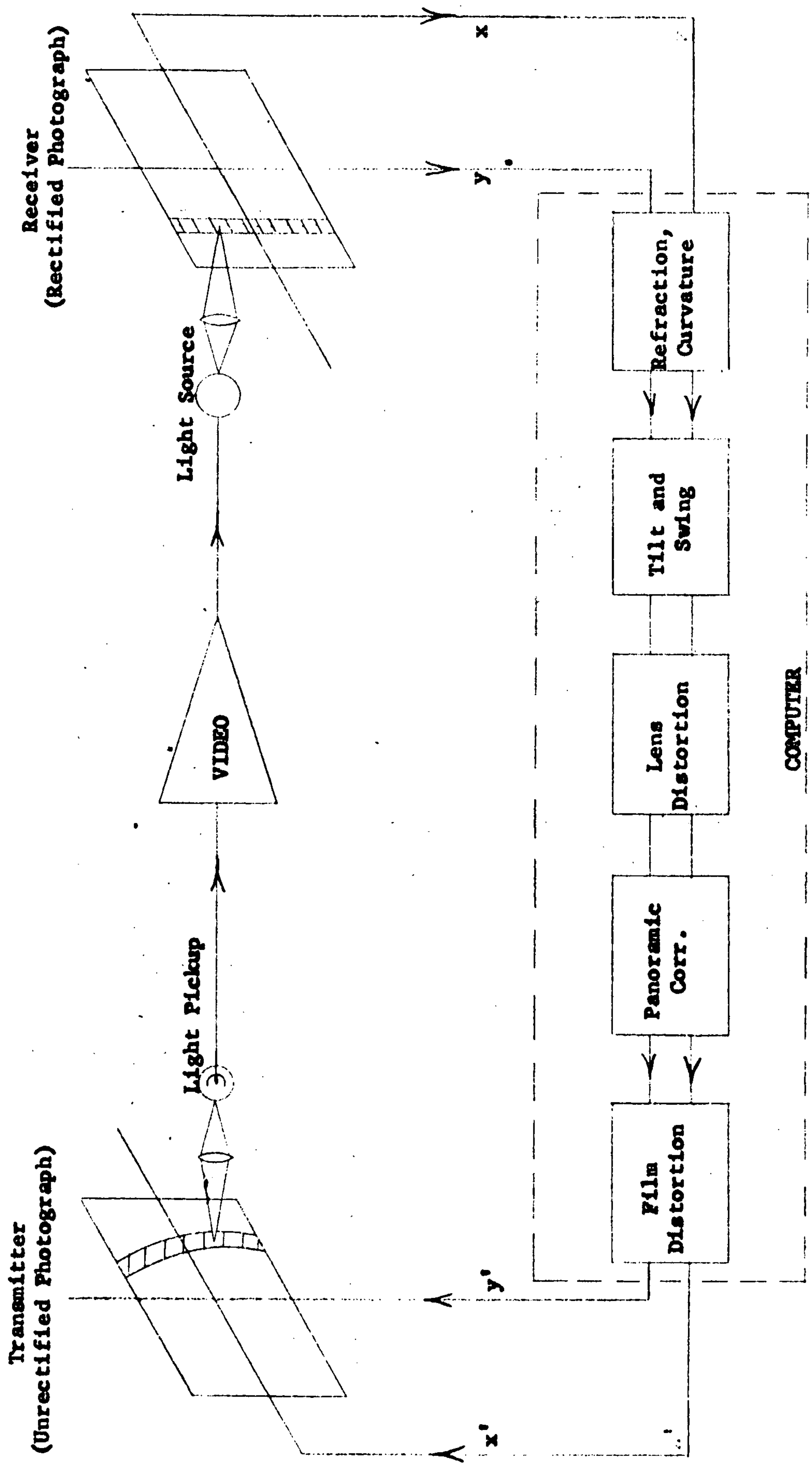
Very basically we must pick a point on one photo, send its coordinates to the computer, which computes the corresponding position on the other picture, and position the other end. The transparency of the unrectified photo is then measured and a proportional amount of light applied to the film at the rectified end. In this manner one picture element is printed.

If this process is carried out at high speed in geometric sequence, we have a scanning process; and the entire picture can be rectified.

In TV and facsimile systems the geometric arrangement of the picture elements remains the same; and by using larger or smaller elements at the output end, the reproduction can be magnified or reduced. The scan velocity is constant at both ends.

In the rectification process, we have no such advantage. In general, a straight line which is scanned at constant velocity at one end corresponds to a curved line which is scanned at continually varying velocity at the other. Because of this, a simple facsimile-type system is inadequate. At least one end must be capable of scanning under the control of the computer.

Only one end requires non-linear scan capability. The input end is the best choice for this for the following reasons:



GENERALIZED BLOCK DIAGRAM SHOWING INFORMATION FLOW

Figure 11

1. The exposure problem is simplified if the film passes the exposing light source at constant velocity since the exposure will then always be proportional to the brightness of the source.

2. The scan method we have chosen results in overlap of lines at times. If lines overlap on the unrectified print there is no problem but if they overlap on the light sensitive film on the output end the film will be exposed twice with consequent loss of information and change in density.

3. When highly magnified the scan lines will be visible and it appears that a uniform pattern of lines will be less distracting to an observer than a pattern of curved, unevenly spaced lines. If we adopt the uniform linear scan for the output end the most obvious and simple system is to wrap the film to be exposed on a rotating drum. The light modulator will move on a lead screw that is driven by the drum. This method results in a simple and very accurate line scan.

The x and y positions are obtained from position pickoffs and constitute the inputs to the computer. Note that the computer inputs are the coordinates of the rectified picture and that the input end must seek the point on the unrectified photograph where the video information to be printed here is located. The methods discussed in detail in section5.3 are based on this concept and the rectification equations are given in this form. (Unrectified as a function of rectified coordinates.)

The equations of rectification are written as if no other distortions were present. This system is the most flexible way to correct a large number of distortions and provide a simple means of introducing future modifications. The only requirement is that the corrections must be done in proper order.

All of the rectification equations except tilt have either the nadir or the principal point as their coordinate origins and the tilt correction is a coordinate rotation about the perspective center from the plumb line to the principal axis. Since our system starts out with rectified coordinates as computer inputs the first step is to apply earth curvature and air refraction corrections. These corrections convert the coordinates from those of a perfect rectified photograph to those of a vertical photograph of a curved earth with air refraction. These coordinates are those of the point from which we would take our video information if there were no tilt, panoramic, lens, or film distortion.

If tilt is present we next go into the tilt coordinate rotation. The outputs of this block are the coordinates of the information, or picture element, on the tilted photograph. The computer is then ready to compute corrections which have the principal axis as a coordinate origin and these are, in order of correction; lens distortion, panoramic or other special camera geometry, and finally film distortion. The final output is the location on the unrectified photograph of the picture element that we wish to print on the rectified film at the original coordinates that went into the computer.

5.2 Ideal System

After developing the general system block an idealized system can be sketched out. This ideal system cannot be built at this time and may never be possible but it is interesting to discuss because it forms a guide to the compromises that will be required of a workable system and shows the need for the additional complexities of the workable system.

The ideal system is shown schematically in fig. 12 . The rectified film is mounted on a rotating drum which drives a lead screw carrying a light modulator to obtain a line scan. As the drum and lead screw rotate the transducess produce voltages proportional to x and y which are fed into the computer. The output of the computer positions the CRT spot to the proper position and the

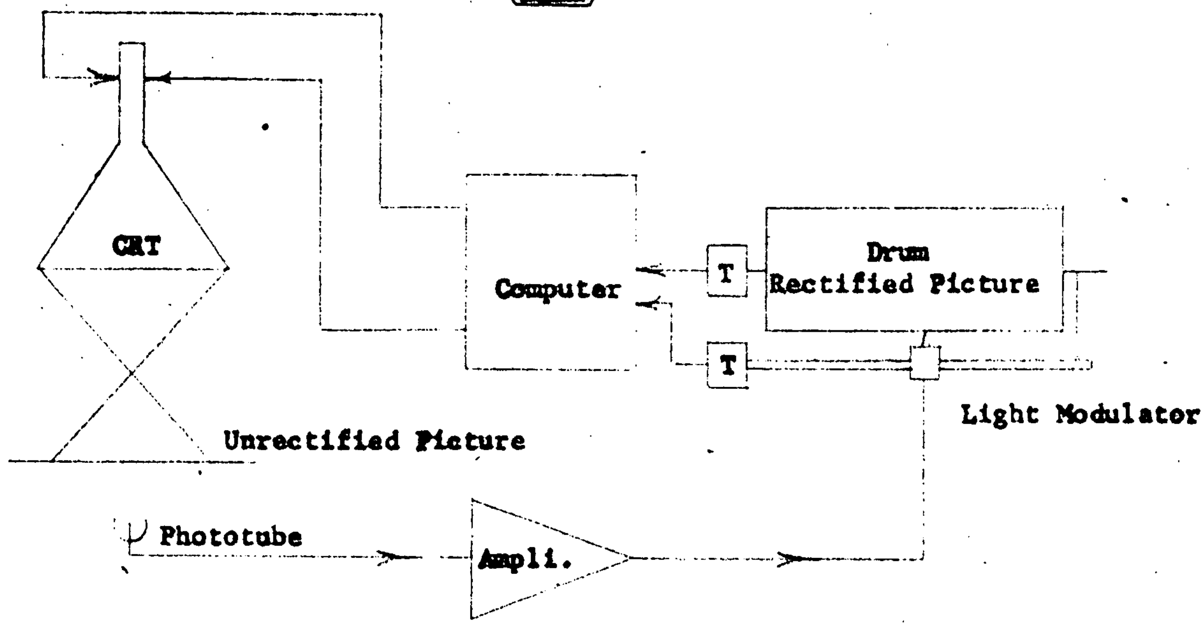


Figure 12. IDEAL SYSTEM

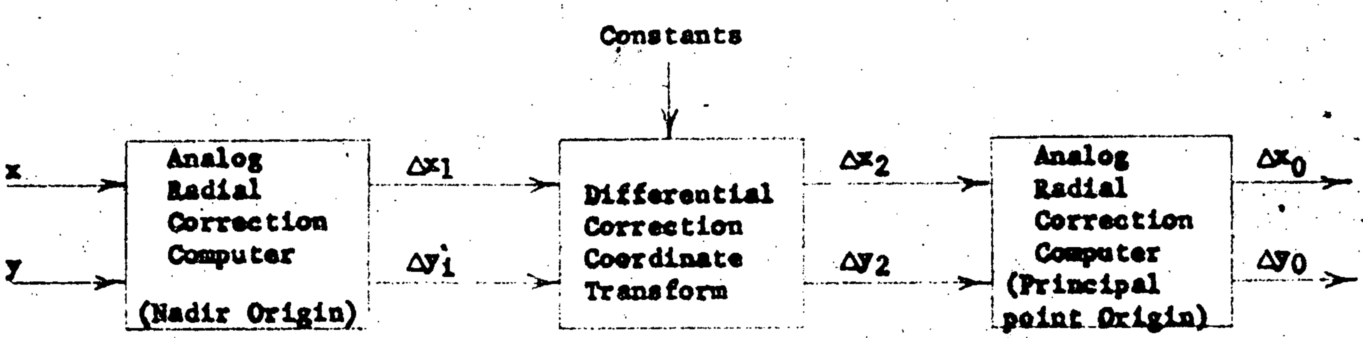


Figure 13. ANALOG DIFFERENTIAL CORRECTION COMPUTER

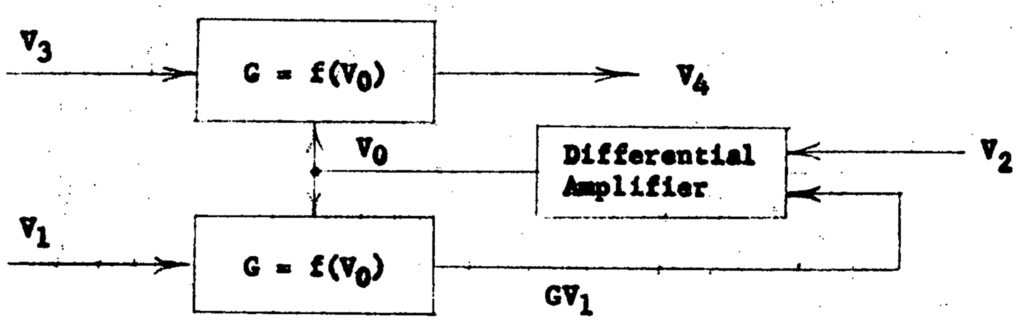


Figure 14. BASIC ANALOG BLOCK $V_4 = \frac{V_2}{V_1} V_3$

video signal would be printed on the film.

Unfortunately this very simple rectifier cannot be built with components that are now available or are likely to become available in the forseeable future. The limiting components are the cathode ray tube and the computer.

A simple, all electronic, anolog computer with an accuracy of at least .01% is required and this is not practical at this time. The cathode ray tube would have to be 24" diameter, 2500 lines per inch, imaged down to cover the diagonal of a 9 x 9 film at 5000 lines/inch.

This tube would have to have a linearity of .01% which is about one order of magnitude better than that now obtainable.

It is possible that as faster digital computers, or more accurate analog computers, become available a spot position servo system like that described in section 4.6.4could be developed and applied to the system just described. This would lead to an electronic rectifier with many ideal features but because of state of the art limitations of the computer and CRT other more complex systems must be used. The approaches described in the next section appear to offer the best solutions to the present problem.

5.3 The Computer

Developing a computer suitable for use with the electronic rectifier presents the designer with many difficult problems.

The problems stem from the need for high accuracy and high speed of computation. We require a computation accuracy of .01% and a solution rate sufficiently high to prevent degradation of the accuracy through interpolation of the solution points.

Electronic analog computers are capable of sufficient speed of operation for our purposes but are limited to about 0.1% accuracy.

Electronic digital computers are available with far more accuracy than we require but are not fast enough.

The Packard-Bell Corporation manufactures special-purpose incremental digital computers which are both fast enough and accurate enough for our purposes. This computer, called TRICE, can be used to solve the equations that involve large scale changes, such as tilt and panoramic distortions, and a parallel analog computer used to solve for the increments that must be added to the digital solution to correct for the small distortions, such as air refraction and film shrinkage.

Extreme accuracy is not required of the analog computer since it only computes small corrections, and fairly large errors in these corrections have small effect on the complete solution.

5.3.1 The High Speed Incremental Digital Computer

A digital incremental computer combines the high iteration speed and accuracy required for the rectifier computations. For example, the "TRICE" Computer, manufactured by Packard-Bell[8], is capable of 100,000 iterations

8. The TRICE--A High Speed Incremental Computer, Mitchell, J. M., and S. Ruhman, 1958 IRE Convention Record, Part 4, Page 206.

per second at a maximum precision of 26 binary digits and sign. The following are the basic building blocks for this computer:

1. Integrator

Transistorized gates, flip-flops, and diodes, as well as 3 circulating, 30 binary digit, delay line registers are used in the construction of the integrator.

Register 1 holds the initial value of y.

Register 2 holds the current value of y.

Register 3 accumulates the integrated area.

The clock repetition rate is 3 megacycles so that the integrator can be iterated 100,00 times per second.

Increments Δx of the integral are given out in terms of one binary digit and a sign.

The basic integrator thus has stored into it initial and current values of y. The inputs are dx and dy and the output is dz = ydx.

2. Multiplier

Multiplication can be performed by the use of two integrators to obtain xdy and ydx, their output being summed to give d(xy). Greater circuit economy is achieved by combining the two integrators into a single "multiplier" unit.

3. Digital Servo

This computing element is used as a nulling device in the solution of equations or as a decision-making element in the generation of discontinuous and non-linear functions.

4. Adder

This unit sums up to six incremental inputs.

These building blocks can be combined in much the same manner as analog computer blocks. For example, Figure 15C shows the generation of d(ln fx) for an input d(fx). The integrator is hooked up so that dz = zdw, or $d(\ln z) = \frac{dz}{z} = dw$, where ln z is the natural logarith of z. The digital servo performs the required decision making function to identify z with fx.

Another example is shown in Figure 15B. Here two integrators solve the simultaneous equations

$$dz = udw$$

$$du = -zdw$$

$$\text{or} \quad z = \sin w$$

$$u = \cos w$$

Stable high frequency sin waves can be generated in this manner.

Figure 16 shows the computer configuration for the solution of the equation $x' = \frac{fx}{\sqrt{f^2 + y^2}}$

$$\text{or} \ln x' = \ln fx - 1/2 \ln (f^2 + y^2)$$

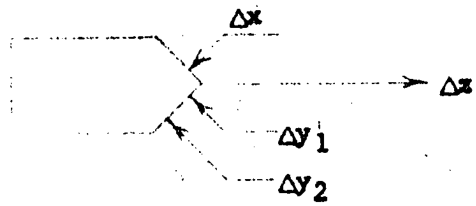
The input dx is multiplied by a constant by means of the multiplier. d(fx) is introduced into the servo-integrator combination to give d(ln fx).

Another integrator has two dy inputs and solves $dz = ydy = 1/2\ d(y^2)$.

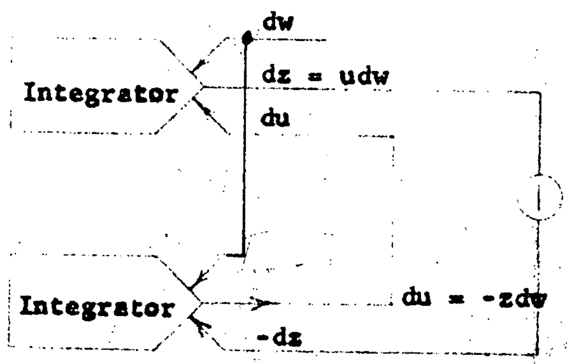
Another servo integrator in turn produces $\ln (f^2 + y^2)$.

A constant multiplier produces $1/2d \ln (f^2 + y^2)$ and the servo subtracts $d(\ln fx) - 1/2d \ln (f^2 + y^2)$.

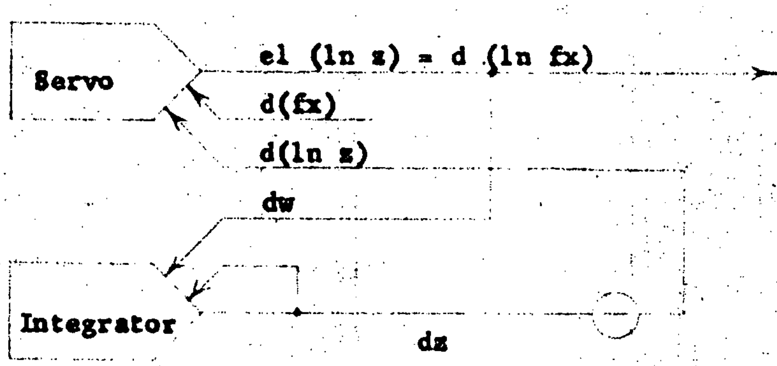
The resulting output d(ln x') is placed into another integrator so that dz = zd(ln x') or dz = dx'.



A. BASIC DIGITAL SERVO

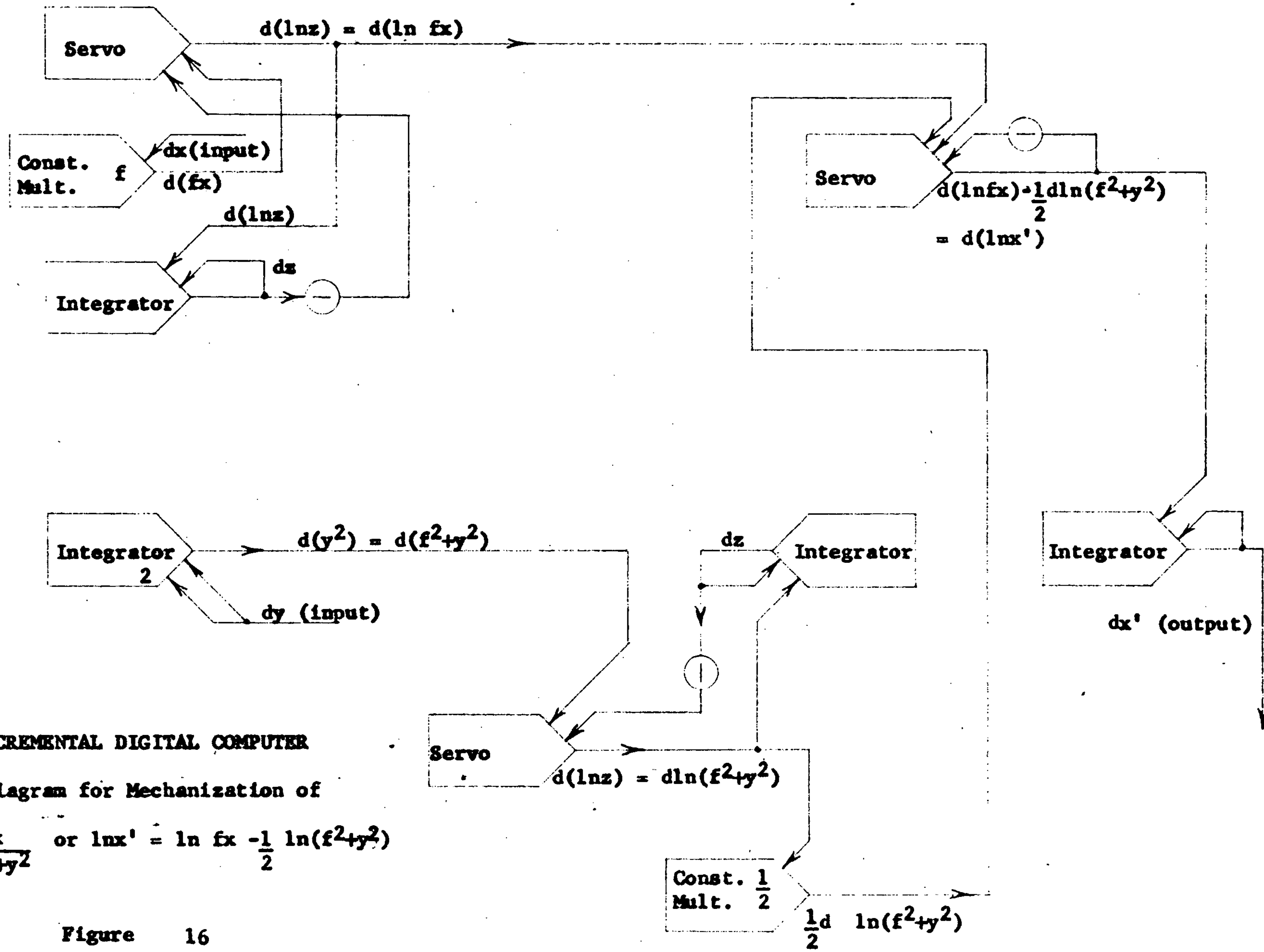


B. SIN-COS GENERATOR



C. INTEGRATOR-SERVO COMBINATIONS

Figure 15



INCREMENTAL DIGITAL COMPUTER

Block Diagram for Mechanization of

$x' = \frac{fx}{f^2+y^2}$ or $\ln x' = \ln fx - \frac{1}{2}\ln(f^2+y^2)$

Figure 16

By similar methods the equations

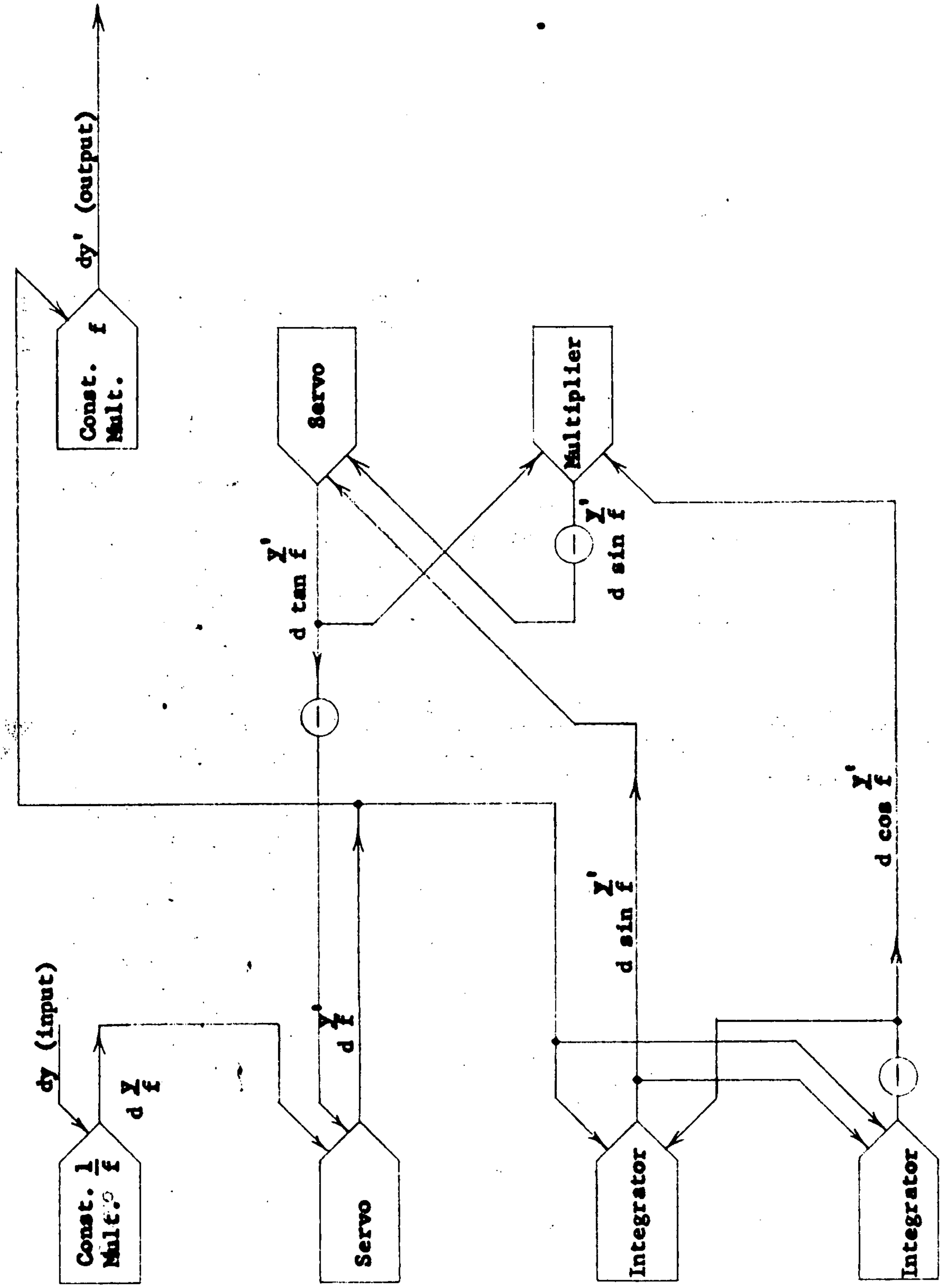
$$y' = f \tan^{-1} \left(\frac{y}{f}\right)$$

$$\frac{x'}{f} = \frac{ax-bf}{-gx-hy+kf}$$

$$\frac{y'}{f} = \frac{-cx+dy+ef}{-gx-hy+kf}$$

are solved by the configurations shown in Figure 17 and 18.

These are the only equations that require solution by the digital computer.



INCREMENTAL DIGITAL COMPUTER

Block Diagram for Mechanization of $y' = f \tan^{-1} \left(\frac{y}{f}\right)$ or $\frac{y}{f} = \tan \frac{y'}{f}$

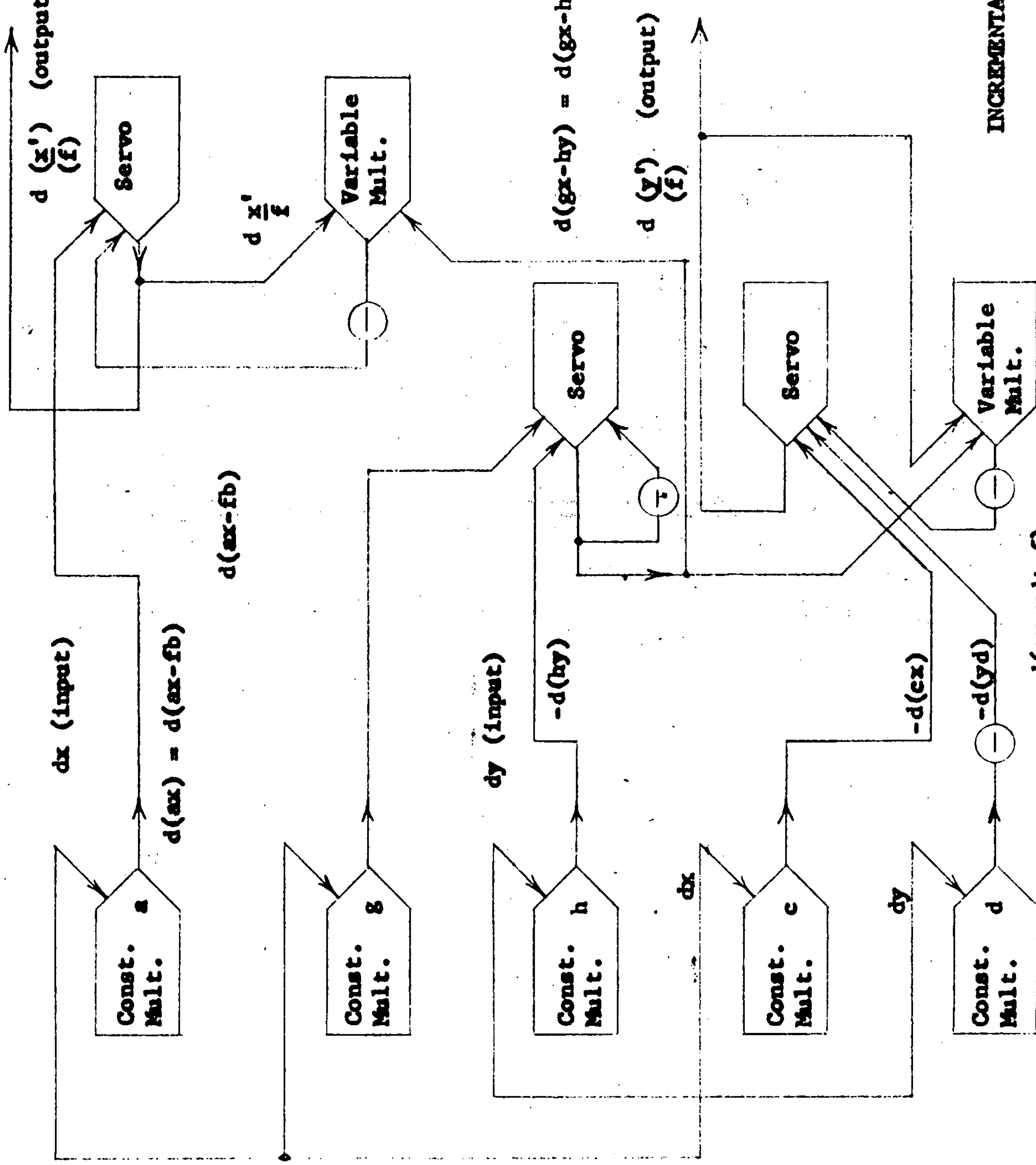
Figure 17

$$\frac{x'}{f} = \frac{ax+bf}{-gx-hy-kf}$$

$$\frac{y'}{f} = \frac{-cx+dy+ef}{-gx-hy+kf}$$

(See page 90)

INCREMENTAL DIGITAL COMPUTER



Block Diagram for Mechanization of Coordinate Transformation Equations

Figure 18

5.3.2 Radial Correction Computer

Several of the distortions to be corrected, such as earth curvature, air refraction, and lens distortion, are radial about either the nadir or the principal point and are small in magnitude. The most practical computer to use here is a differential analog computer. The radial correction computer does not require the accuracy that the coordinate rotation and panoramic computers require since only a small radial correction will be computed and added to the outputs of the digital computers.

The computations will be done in two parallel channels. The digital channel will compute the large magnitude tilt and panoramic corrections, and the analog channel will compute the corrections to be applied to the digital computer outputs to correct them for the radial distortions. The incremental x and y inputs to the digital computer could be converted to analog voltages as inputs to the radial computer, but it appears simpler and less expensive to use a separate set of transducers on the rectified end to provide analog inputs to the radial computer. The analog computer blocks shown in Figures 19 and 20 are arranged as shown in Figure 13 to perform radial corrections about the nadir, rotate them to tilted coordinates, and perform radial corrections about the principal axis.

The analog radial correction computer shown in Figure 19 is applicable to radial corrections about either the nadir or the principal point. In operation the input coordinates x_i, y_i are used to deflect the spot of a CRT. A plate whose transparency increases linearly from the center attenuates the light emitted by the spot. Since the spot position is proportional to the radius $R = \sqrt{x_i^2 + y_i^2}$, the light transmitted is proportional to R and the phototube voltage is proportional to R.

The voltage R is the input to the horizontal deflection circuit of the photoformer circuit. A mask cut in the shape $y = f(x)$, where the function f depends on the increment to be added, is placed in front of the CRT. When the phototube sees no light, the spot is driven up; and when the phototube sees light, the spot is driven down. With proper stabilizing circuitry in the amplifier, the spot will follow the contour of the mask and the vertical deflection voltage will be proportional to the corrected increment $k_1 \Delta R$.

Masks can be cut for any correction or combination of corrections that are radial about the axis being considered. The voltage $k_1 \Delta R$ must now be resolved into its components Δx_i and Δy_i. The basic computer to do this operation is shown in Figure 14 and operates as follows:

Assume two identical amplifiers of variable gain $G = f(V_0)$ where G is always positive. For inputs V_1 and V_2, $V_0 = V_2 - GV_1$.

$$G = f(V_0) = f(V_2 - GV_1)$$

$$V_4 = GV_3$$

If $f(V_0)$ is linear and A is a constant much greater than 1,

$$G = A(V_2 - GV_1)$$

$$G(1 + AV_1) = AV_2$$

$$G = \frac{AV_2}{1 + AV_1} \xrightarrow[A \gg 1]{} \frac{V_2}{V_1}$$

Therefore, $V_4 = \frac{V_2}{V_1} V_3$

Referring again to Figure 19, we see that this scheme is used to obtain Δx_i and Δy_i. One complication is introduced because $k_1 \Delta R$ may go negative. To avoid difficulty here, we introduce the constant k_2 selected so that the input $k_2 - k_1\Delta R$ to the differential amplifier will always be positive.

The solution becomes meaningless at R = 0; and since R will often go to zero, we must gate the output to prevent this solution from being utilized. This presents no problems since the correction is always zero when R = 0.

The solution $x_i(k_2 - k_1\Delta R)$ is added to $\frac{-x_i k_2}{R}$, and the result multiplied by $\frac{1}{k}$ to obtain the required result $\Delta x_i = \frac{x_i \Delta R}{R}$.

This solution for Δx_i will be corrected for earth curvature and air refraction since both of these distortions will be included in the mask. Δx_i must then be modified by the tilt angle coordinate conversion.

The basic coordinate transform equations of page 90 are:

$$\frac{x_0}{f} = \frac{ax + bf}{-gx-hy+kf} \quad \text{and} \quad \frac{y_0}{f} = \frac{-cx+dy+ef}{-gx-hy+kf}$$

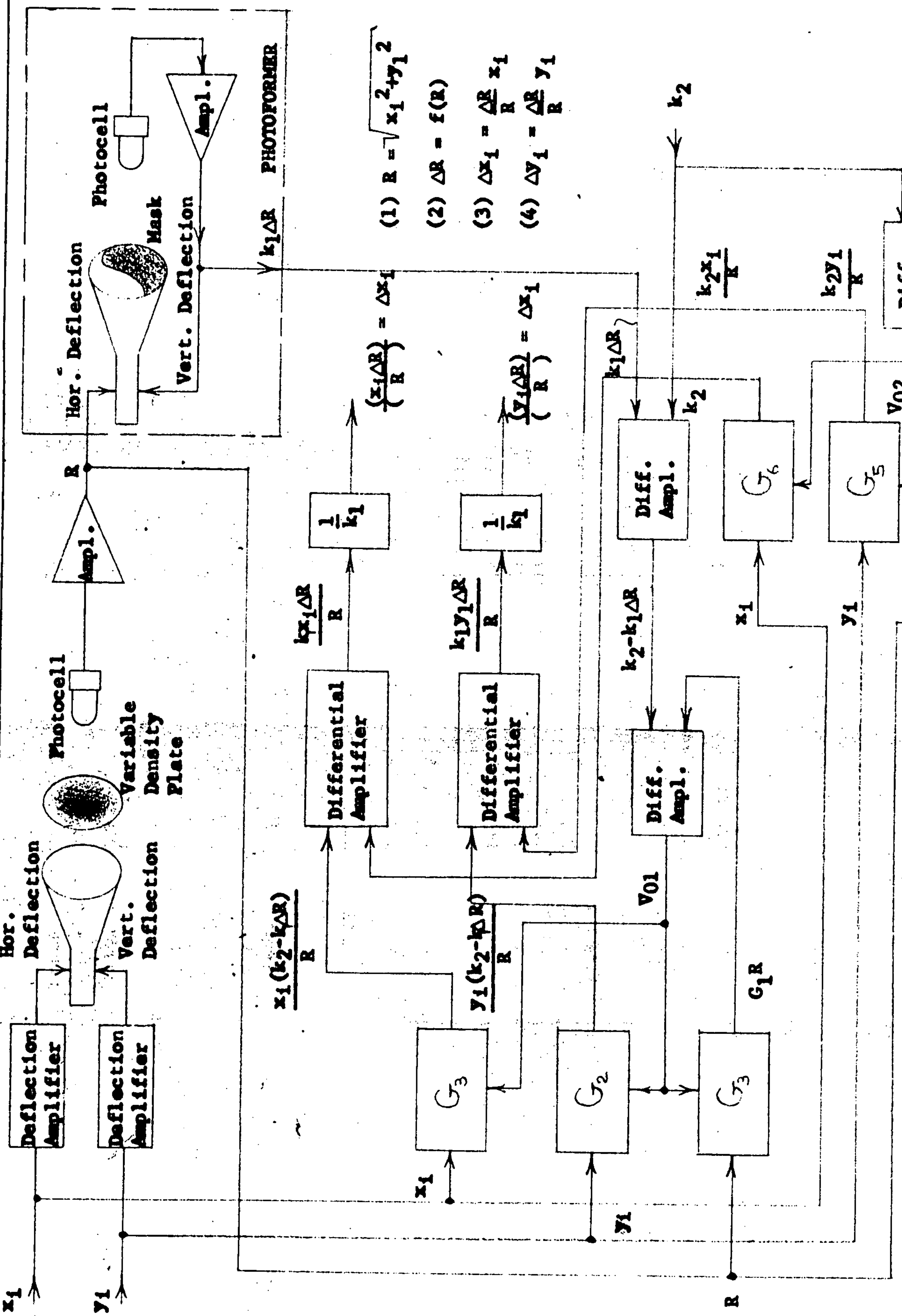
The approximate differential corrections corresponding to the equations are:

$$\frac{\Delta x_0}{f} = \frac{a\Delta x_i}{-gx_i-hy_i+kf} \quad \text{and} \quad \frac{\Delta y_0}{f} = \frac{-c\Delta x_i+d\Delta y_i}{-gx_i-hy_i+kf}$$

Where Δx_i and Δy_i are the differential correction inputs and Δx_0, Δy_0 are the coordinate transformed differential corrections.

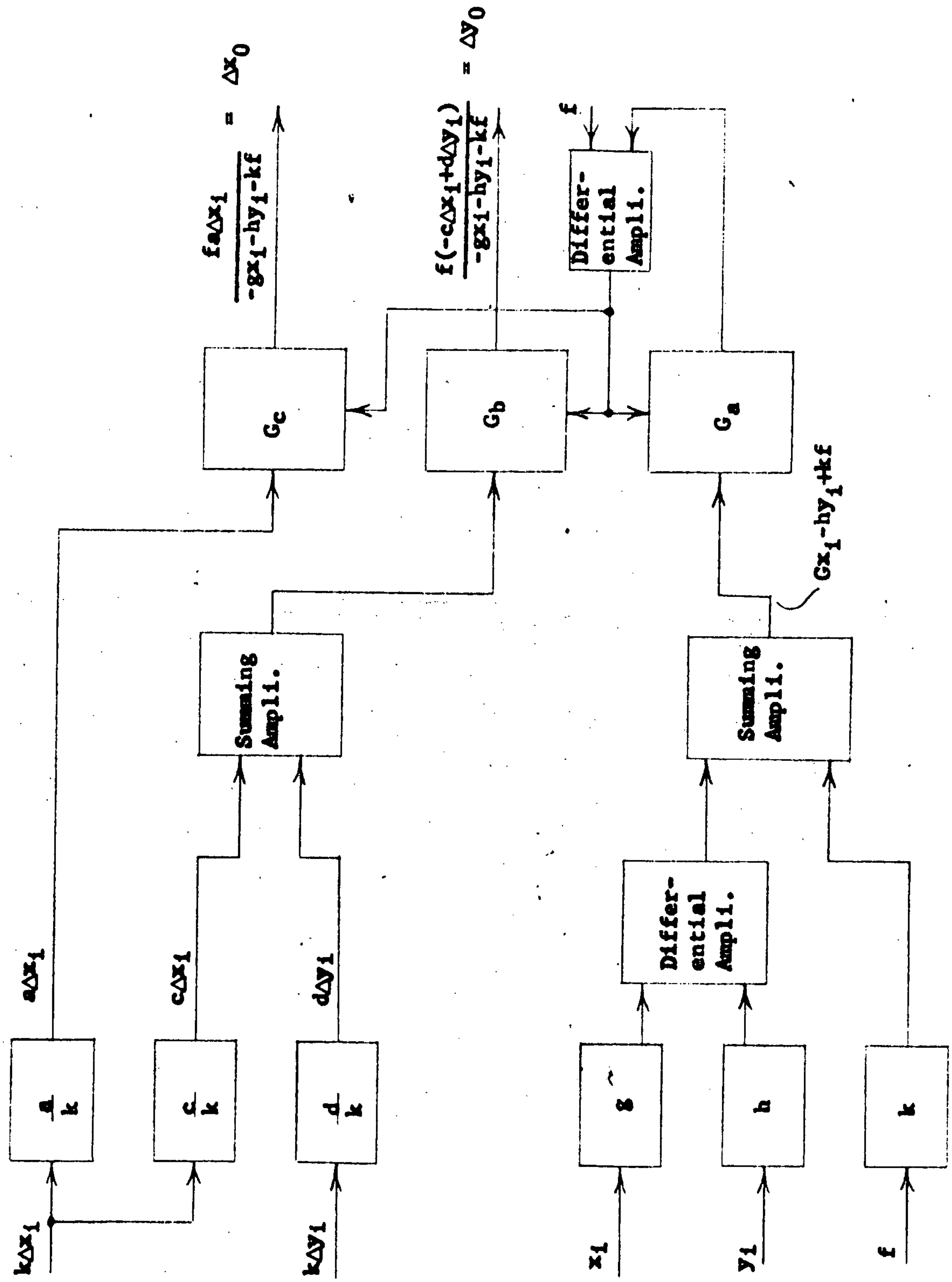
These equations are solved by the computer shown in Figure 20. The outputs of this computer form the inputs to another computer similar to Figure 19 which corrects for distortions about the principal axis.

The final corrections are added to the output of the digital to analog converter and used to deflect the CRT spot.



ANALOG RADIAL CORRECTION COMPUTER BLOCK DIAGRAM USING $\Delta R = f(r)$ PHOTOFORMER MASK

Figure 19



DIFFERENTIAL CORRECTION COORDINATE TRANSFORMER BLOCK DIAGRAM

Figure 20

5.4 System Description - System 1

The over-all block diagram of the rectifier which best appears to meet the requirements of this study is shown in Figure 21. The two computers required are described in paragraphs 5.3.1 and 5.3.2. The system will be described here starting with the rectified end and following in the general direction of information flow.

A sheet of unexposed film is mounted on the drum and held in place by spring-loaded pins or by a vacuum system. The picture to be rectified is mounted on the input platen with the fiducial marks aligned to index marks on the platen.

The variables of the problem, such as altitude and focal length, and the precalculated constants of the coordinate transform equations are fed into the computer; and the proper computer blocks (such as panoramic camera correction) are switched on. The transducer is set at the proper position and the system started. The drum will rotate at 3600 RPM or 60 RPS, thus scanning 60 lines each second. The lead screw will traverse the printing light source along the drum at a constant rate to yield a line scan. The system will apply an over-all enlargment factor of two or three to the photograph being rectified to reduce the further magnification required to exploit the photograph.

Two sets of transducers furnish position information to the computer. One set of transducers produces pulses (Δx and Δy) at fixed intervals as inputs to the incremental digital computer. In the x direction the pulses are produced at maximum rate of 10^5 per second. In the y direction the pulses are produced by a switch on the lead screw at a much lower rate. The x pulses are produced by a photocell which is triggered by light passing through an engraved scale fixed to the drum. A separate scale and photocell will yield a pulse once each revolution to reset the computer, blank the CRT, and return it to zero.

The other set of transducers provides analog voltages in x and y as inputs to the differential correction computer. The computer outputs form the unrectified coordinates of the point to be printed.

The digital computer outputs are 17 bit binary numbers and the outputs of the analog computer are small analog corrections to the digital solution. The digital computer integration rate is 10^5 per second. The spot scans the film at an average rate of 180 in. per second to yield an accurate digital solution every .0018 in. Since the picture elements are only about .0002 in. apart, we must use a smoothing system to interpolate between the digital solutions.

The digital solution for y' is fed into a binary subtraction unit.

The unrectified film platen position is monitored by a linear position transducer whose 17 bit digital output is also fed into the binary subtraction unit.

A Ferranti linear position transducer would probably be used on this axis. This device consists of a grating the length of the platen with a fixed short grating at a slight angle to it. A light shining through the grating is picked up by a photocell. As the gratings move with respect to each other, a Moiré fringe pattern with an approximately sinusoidal distribution is produced as a result of the integrated interference pattern caused by the angular intersection of the individual lines on each grating.

The Moiré fringes are detected by the photosensitive element and the waveforms used to form a digital measuring system. This system is not affected by error or wear in the screws used to move the table and will be accurate to .0002 inches over a maximum usable travel of 26 inches. The maximum speed of one inch per second is completely adequate for this application.

The picture will be covered in strips three inches wide and the full length of the film. A 9 x 9 format must be covered in three passes and a 75 mm format in one pass. The entire three inch width is covered by the CRT, and no servo positioning of the table is required in the x direction. The x table motion is entered by hand after each pass by turning a crank to one of three index points.

As mentioned before, the desired table position in y' is compared with the actual position in the binary subtraction unit. The difference is used to drive a servomotor to position the table. Any difference between actual table position and desired table position is fed to the digital to analog converter. The converter output is changing in steps at the rate of 10^5 steps per second. The output feeds into a smoothing unit containing prediction circuitry which converts the step input to a smooth curve. The output of the smoothing circuit is added to the analog output of the differential correction computer and forms the input to the y' deflection circuit of the CRT. With this system the high speed scan of the CRT is in the x' direction. The platen moves slowly along in y' under the CRT to form a line scan. Any lag in the platen position is taken up by the y' deflection of the spot.

The x' circuitry is similar except that, as mentioned previously, no servo drive is required in x' because the entire 3 inch motion in this direction is supplied by the CRT scan.

The CRT will be about 7-9 inches in diameter, have 15,000 lines resolution across the face, and, with proper yokes, have a linearity of 0.1%. A demagnified image of the tube face will be projected onto the negative to be scanned. This image will be 3 inches in diameter, thus covering a 3 inch wide strip of film at 5,000 lines/inch.

Assuming an 8 inch diameter tube, a linearity of .1% results in a spot uncertainty of .004 inches which, when imaged on the film at a reduction of 8 to 3, will result in a spot error of $.004 \times 3/8 = .0015$ inches. This figure is somewhat optimistic since a .1% linearity is difficult to achieve and a slightly larger tube may have to be used to allow for not being able to use the tube out to the edge.

A phototube can be used to monitor the brightness of the spot. As the spot travels across the tube brightness, changes may occur due to phosphor variations. The velocity of the spot will vary considerably as local scale changes occur.

The brightness of the spot will vary with the velocity and would result in a velocity modulation of the spot if left uncorrected. The output of the phototube is amplified and used in a closed loop to maintain constant brightness of the spot. This system requires that the phosphor decay time be very short since the tube measures all the light emitted by the screen. This causes no problem since the decay time must be shorter than the picture element period of about 10^{-6} second/element. The P-24 phosphor which decays to 10% of original brightness in 1 microsecond would be adequate.

As the scanning spot passes over the negative, an amount of light proportional to the transparency of the film passes through it and, after collection by the condensing system, falls on the photomultiplier tube.

The output of the photomultiplier is amplified and modified as described in paragraphs 4.6.6. and 4.6.7. for proper exposure of the film. The amplified signal is fed to the exposing light source which is monitored by a phototube in a closed loop to eliminate the effects of non-linearity and to extend the usable frequency range of the light source.

The time required to rectify a photograph by this method depends not only on the size of the photograph but on the scale change required.

The output drum speed will be about 3600 RPM or 60 RPS. This will write 60 lines per second regardless of the width of the rectified photograph. A panoramic photograph which has been rectified out to 60° on each side of the horizon is about 8 inches long with no over-all enlargement factor. At 5000 lines per inch, this will require $\frac{8 \times 5000}{60}$ = 670 seconds or 11 minutes. Printing out at an over-all enlargement factor has no effect on the time because the scan lines will be twice as wide.

A 75 mm x 75 mm near vertical photograph will require about 4 minutes. A 9 x 9 photograph must be covered in three 3 inch wide passes since the CRT can only scan 3 inches in the x' direction at one time. This means 27 linear inches must be covered in the y' direction. For a near vertical 9 x 9 format $\frac{5000 \times 27}{60}$ = 2,250 seconds or 37 minutes. This will yield three separate rectified photographs, each of which will represent a 3 x 9 inch strip of the original 9 x 9 photograph.

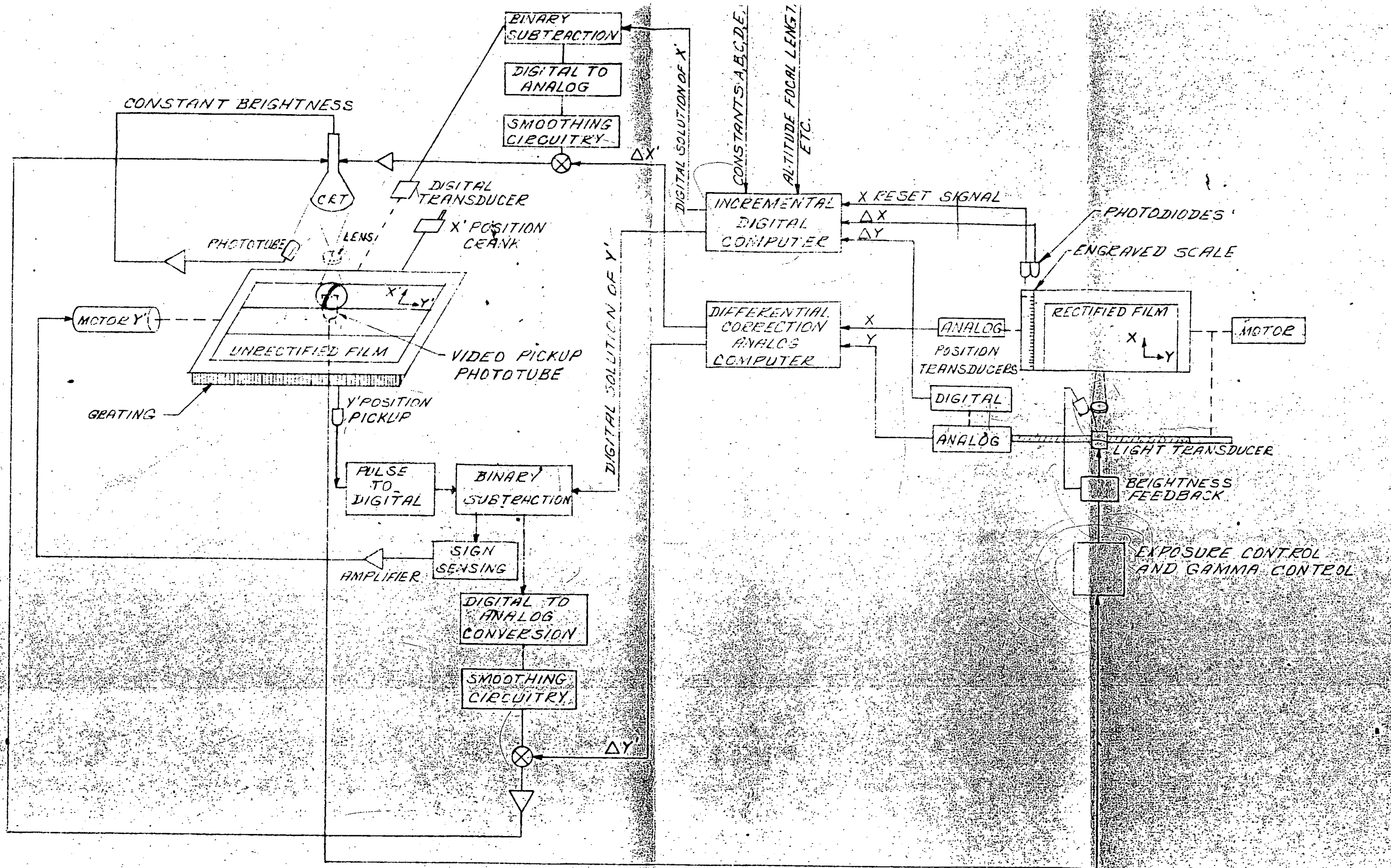
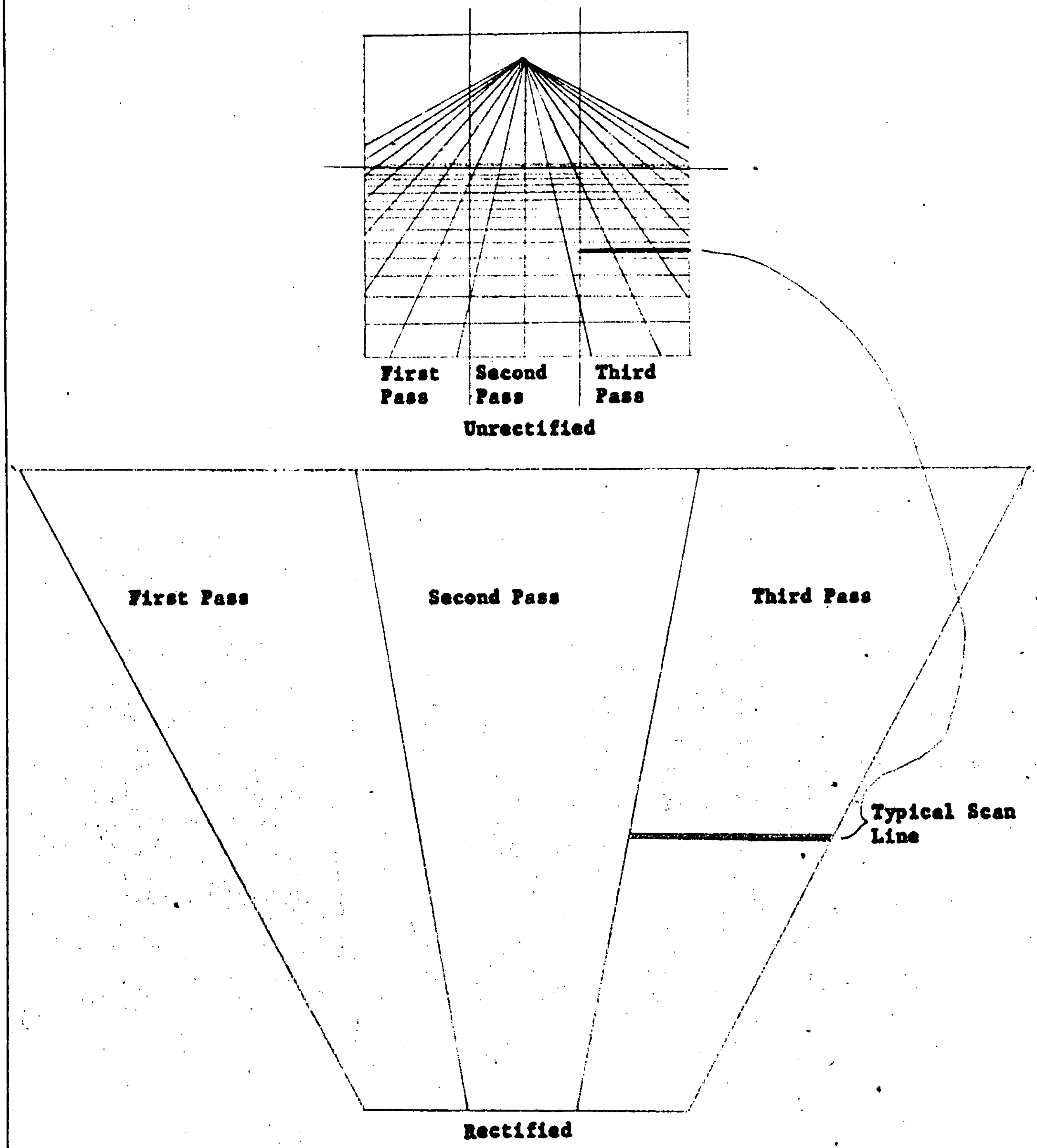


FIGURE 21

BLOCK DIAGRAM
ELECTRONIC RECTIFIER

ENG. REPORT #5435

Sanitized Copy Approved for Release 2010/12/09 : CIA-RDP74B00752R000100160001-8



The upper drawing shows how a 9 x 9" oblique photograph is covered in three passes by the 3" scanning line of the C.R.T. The lower drawing shows the three separate reproductions of the oblique photograph produced by the rectifier. (Not to scale.)

Figure 22 - RECTIFIED OBLIQUE PHOTOGRAPH

5.5 System II

An alternative to System I is to cover the unrectified photograph by scanning it with a very reduced image of a C.R.T. This reduced image would be about 1/8 to 1/4 inch in diameter. The small image would be scanned in lines across the photograph and the C.R.T. spot would scan at high speed across the main scan lines. At the output end the line scan is supplied by a rotating prism. Most of the computer system is quite similar since the basic method of operation is similar.

This system has the advantage of better accuracy of spot positioning on the unrectified film. If a 5-inch diameter tube were optically reduced 20 times to a 1/4 inch diameter image on the film, a linearity of about 1% would be required to achieve a spot position accuracy of .001 inch on the film. The linearity could be held better than this, and, since the table position can be very accurately established, an accuracy of .001 could probably be achieved.

An advantage of System II is that an ordinary C.R.T. could be used in place of the special, high linearity, high resolution, tube required for System 1. A drawback of System II is that the output line length would have to be held to picture element size accuracy to prevent overlapping of elements with consequent banding and loss of resolution.

System I lends itself to future improvement by incorporation of a spot position servo system at some time in the future, while System II, although having better accuracy initially, would be more difficult to modify without major design changes.

It should be noted here that both systems will require heavy machine tool type design in order to maintain accuracy.

There do not appear to be any fundamental limitations to increasing the resolution of either system to approximately 200 line pairs per m.m. This change would result in a longer time of operation. Although four times the number of bits must be processed, the time per rectification would only increase by a factor of about two, since the bandwidth of the video electronics could be increased somewhat. Increasing the resolution of System I would result in a smaller coverage per pass unless a larger C.R.T. can be obtained.

SECTION VI

Recommendations:

It is our recommendation that the rectifier described as System I be constructed to provide a universal rectifier which substantially advances the state of the rectification art, is adaptable to further development, and can be completed with a reasonable amount of engineering development.

This unit will have a resolution capability of about 100 line pairs per m.m., a computational accuracy of .01%, and an information pickoff accuracy of about .003 inches or better. This should result in a rectified accuracy for near vertical photographs of about .02% of the 9 x 9 format. As was discussed in Paragraph 4.6.8, the accuracy of rectification of tilted photographs varies with the amount of local scale change.

The computer blocks to be included and the range of focal lengths and format sizes depend on the needs of the individual customer. An overall enlargement factor of two is recommended because this will result in reasonably sized rectified photographs and will permit use of reasonably fast films such as Plus-X Aerecon and consequent use of a glow modulator tube as an exposing light source.

APPENDIX I

MATHEMATICS OF DISTORTION AND RECTIFICATION

The equations in this section were derived because a search of the literature failed to disclose the equations required. It appears that very little work has been published concerning the mathematics of distortion, especially in the less significant cases, such as air refraction and image motion.

In order to obtain a linear, uniform, scan at the output, the inputs to the computer must be the rectified coordinates of the read-out light source. All of the equations present the unrectified coordinates of a point as a function of its rectified coordinates. Thus, the rectified coordinates are inputs to the computer, and the outputs are the unrectified coordinates, or the coordinates that the scanning spot should take to pick up the proper information for the output end to print.

The equations are derived as if no other distortions were present. Thus, distortion due to tilt is derived as if no air refraction and earth curvature were present. This does not result in any error, since the inputs to the tilt section are the outputs of the earth curvature computer. As explained in Section 5.1, all of the distortions except tilt have either the principal axis or the vertical (nadir) axis as a coordinate origin and tilt may be corrected by a coordinate rotation from one to the other.

The following distortions will be treated in the order listed:

1. Distortion due to tilt and swing.
2. Distortions found in the panoramic camera due to basic method of operation and to aircraft velocity, crab angle, rotation, and image motion compensation (S distortion).

3. Distortion due to earth curvature and air refraction.

4. Distortions due to non-planar focal surfaces.

5. Miscellaneous Distortions.

   a. Distortion due to plane window.
   b. Correction for rapidly moving boundary layer.
   c. Film distortion.
   d. Lens distortion.
   e. Distortion due to prism of lens or windows.

6. Other applications of rectification equations.

   a. Scanning accelerations.
   b. Small areas.
   c. Maximum reduction in scale.

LIST OF SYMBOLS

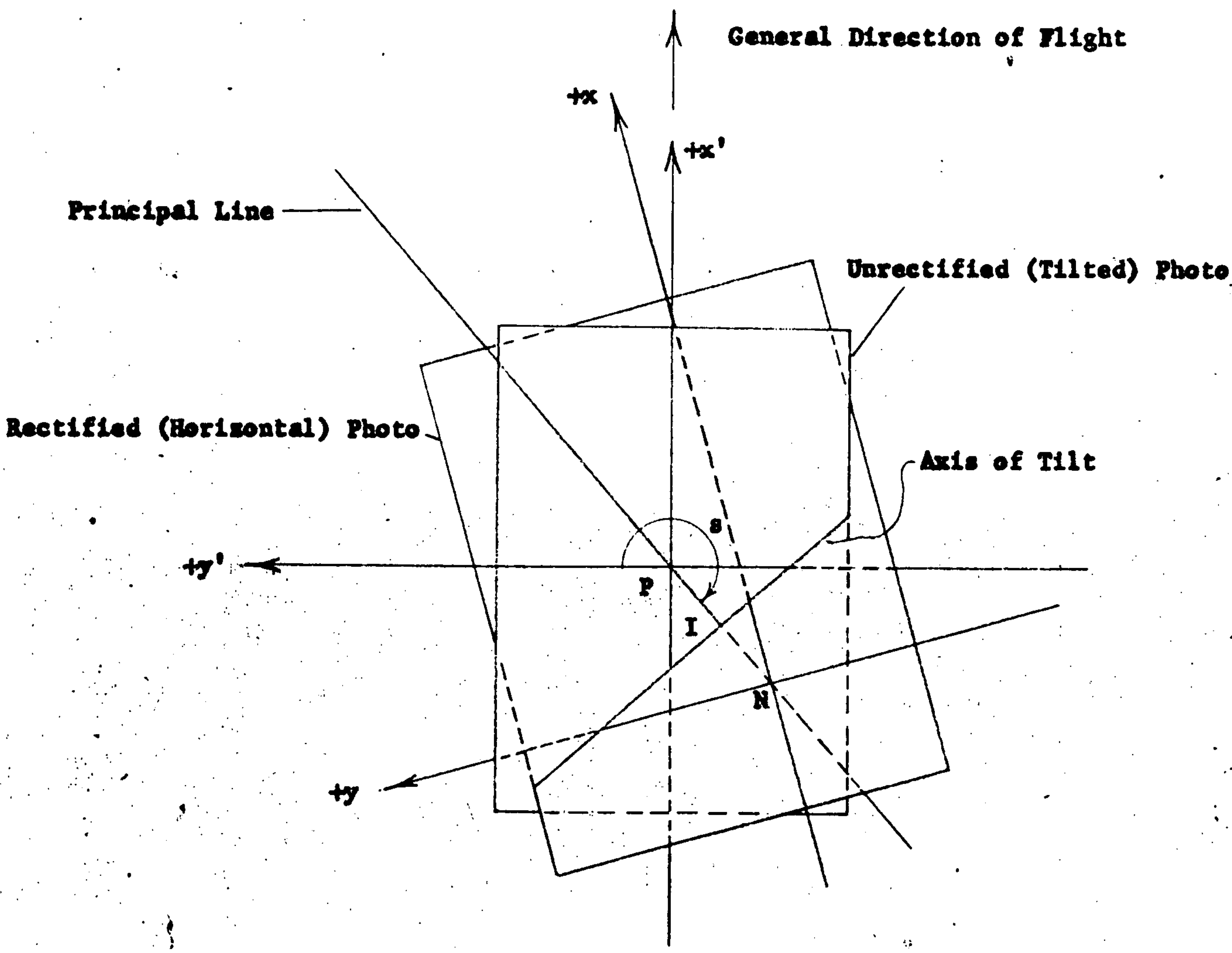
The symbols most frequently used in this report are:

x', y' are coordinates determined by the fiducial marks on photograph to be rectified. The origin of coordinates is at the principal point. The +x' axis is in the general direction of flight, and the +y' axis is in the general direction of the left wingtip.

x, y are coordinates on the rectified photograph. The origin of coordinates is at the nadir.

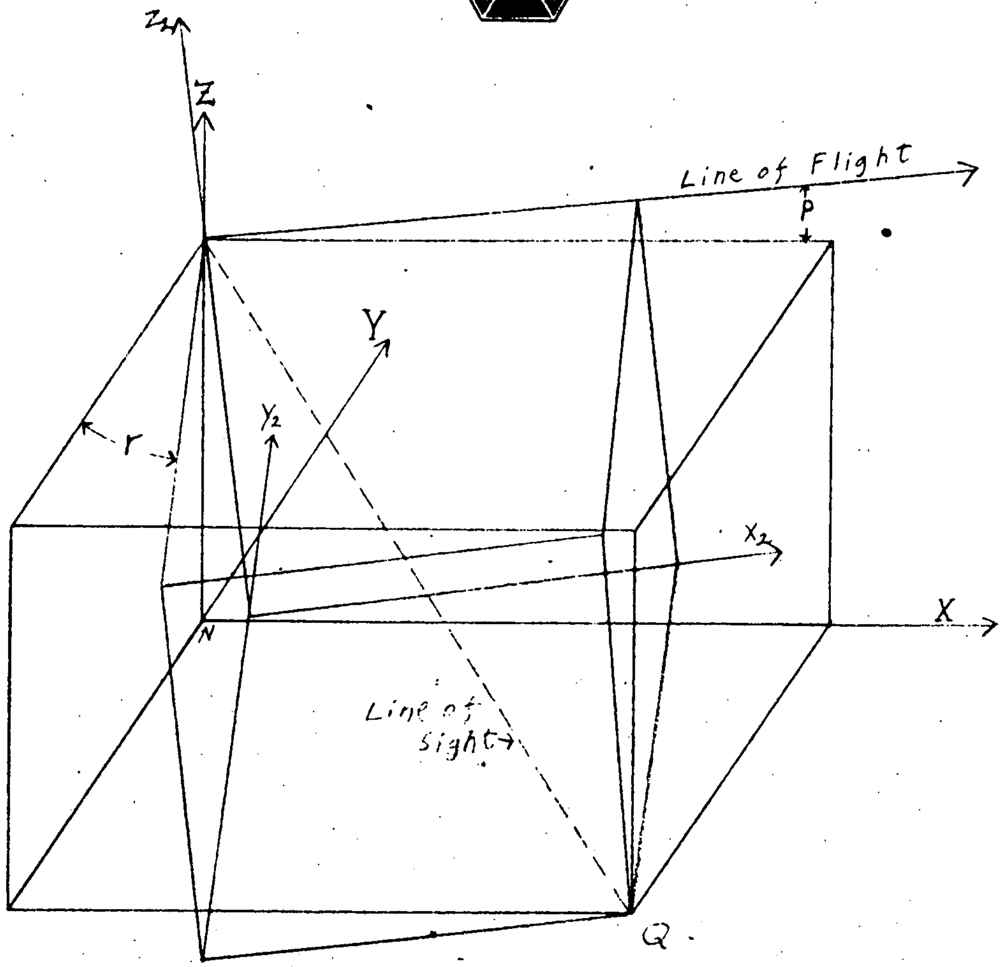
X, Y are coordinates on the ground.

H = altitude
h = height above or below datum plane
t = tilt angle
s = swing angle
p = pitch angle of aircraft
r = roll angle of aircraft
f = focal length of camera
N = nadir point
I = isocenter
P = principal point



COORDINATE SYSTEM OF TILTED AND RECTIFIED PHOTOGRAPHS

Figure 23



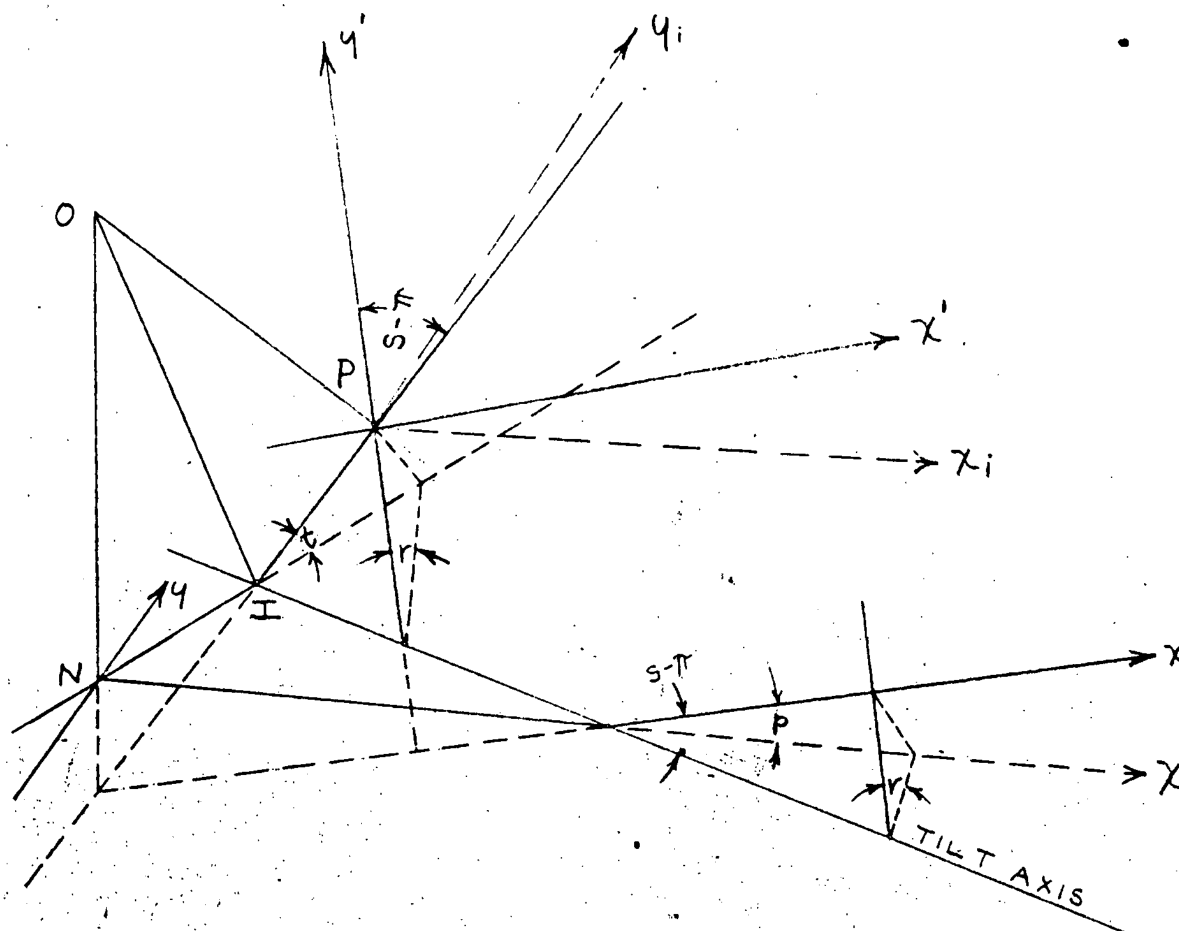
p = pitch angle
r = roll angle
H = Z_1 = altitude
f_1 = camera focal length
X = direction of flight path on ground

X, Y, Z = ground coordinates of observed point Q. (origin at nadir)
X_2, Y_2, Z_2 = transformed coordinates of Q.
X', Y' = unrectified photographic coordinates of Q
X, Y = rectified photographic coordinates of Q

Figure 24 Pitch, Roll Coordinate Transformation

1. Distortions due to tilt and swing

These distortions will first be derived in terms of pitch and roll and then be expressed in terms of tilt and swing.



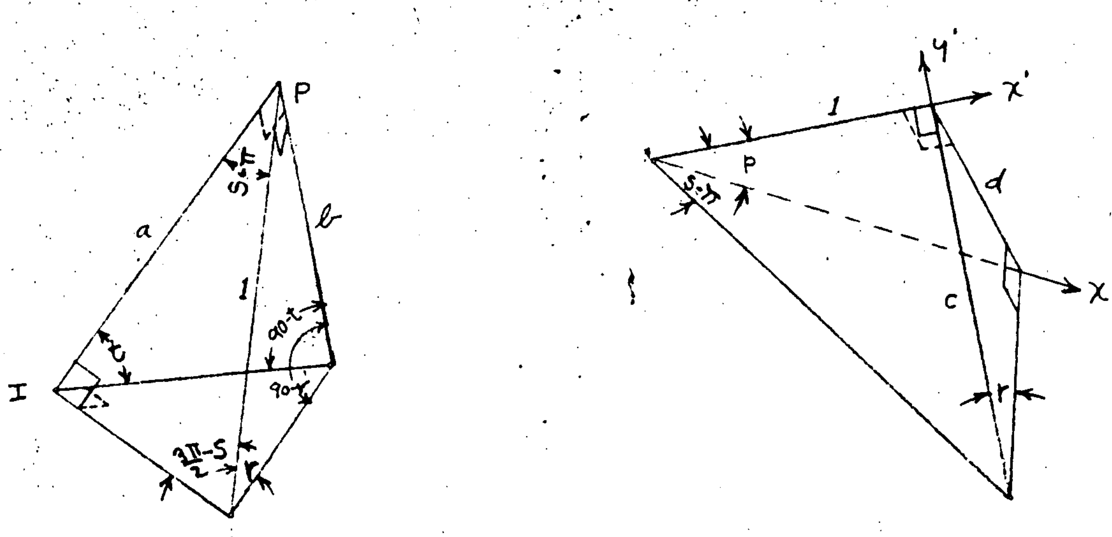


Figure 25. RELATIONSHIP BETWEEN PITCH AND ROLL, AND TILT AND SWING

The coordinate transforms for pitch and roll are:
(See fig. 24)

Pitch

$X_1 = X\cos p - Z\sin p$

$Y_1 = Y$

$Z_1 = X\sin p + Z\cos p$

Roll

$X_2 = X_1$

$Y_2 = Y_1\cos r - Z_1\sin r$

$Z_2 = Y_1\sin r + Z_1\cos r$

so,

$X_2 = X\cos p - Z\sin p$

$Y_2 = Y\cos r - X\sin p\sin r - Z\cos p\sin r$

$Z_2 = Y\sin r + X\sin p\cos r + Z\cos p\cos r$

define $x = \frac{f}{Z_1}X = \frac{f}{H}X$, $y = \frac{f}{H}Y$

also $x' = \frac{f}{Z_2}X_2$, $y' = \frac{f}{Z_2}Y_2$

so,

$$\frac{x'}{f} = \frac{x\cos p - f\sin p}{x\sin p\cos r + y\sin r + f\cos p\cos r}$$

$$\frac{y'}{f} = \frac{-x\sin p\sin r + y\cos r - f\cos p\sin r}{x\sin p\cos r + y\sin r + f\cos p\cos t}$$

To convert these coefficients to terms of tilt and swing we refer to fig. 25 from which we see

$a = \cos(s-\pi) = -\cos s$, $b = \tan r$, $\frac{b}{a} = \tan t = -\frac{\tan r}{\cos s}$

$d = \tan p$, $c = \tan(s-\pi) = \tan s$

$\frac{d}{c} = \sin r = \frac{\tan p}{\tan s}$

$$\cos p = \frac{1}{\sqrt{1+\tan^2 p}} = \frac{1}{\sqrt{1+\sin^2 r\tan^2 s}}$$

$$\sin^2 r = \frac{\tan^2 r}{1+\tan^2 r} = \frac{\cos^2 s\tan^2 t}{1+\cos^2 s\tan^2 t}$$

(1) $$\cos p = \frac{1}{\sqrt{1+\frac{\cos^2 s\tan^2 t\tan^2 s}{1+\cos^2 s\tan^2 t}}} = \sqrt{\cos^2 t + \sin^2 t\cos^2 s} = \sqrt{1-\sin^2 t\sin^2 s}$$

(2) $\sin p = \pm\sqrt{1-\cos^2 p} = \pm\sin t\sin s = -\sin t\sin s$ (from def. of p, t, s)

(3) $$\sin p\sin r = \frac{(-\sin t\sin s)(-\cos s\tan t)}{\sqrt{1+\cos^2 s\tan^2 t}} = \frac{\sin^2 t\sin s\cos s}{\sqrt{1-\sin^2 t\sin^2 s}}$$

(4) $$\cos r = \sqrt{1-\frac{\cos^2 s\tan^2 t}{1+\cos^2 s\tan^2 t}} = \frac{\cos t}{\sqrt{1-\sin^2 t\sin^2 s}}$$

(5) $\cos p \sin r = \dfrac{\cos s \tan t \sqrt{1-\sin^2 t \sin^2 s}}{\sqrt{1+\cos^2 s \tan^2 t}} = -\cos s \sin t$

(6) $\sin p \cos r = \dfrac{-\sin t \sin s \cos t}{\sqrt{1-\sin^2 t \sin^2 s}}$

(7) $\sin r = \dfrac{-\cos s \sin t}{\sqrt{1-\sin^2 s \sin^2 t}}$

(8) $\cos p \cos r = \dfrac{\sqrt{1-\sin^2 s \sin^2 t}\,\cos t}{\sqrt{1-\sin^2 s \sin^2 t}} = \cos t$

Substituting (1)–(8) in the expressions for $\frac{X'}{f}$ and $\frac{Y'}{f}$, we get

$$\frac{X'}{f} = \frac{\sqrt{1-\sin^2 t \sin^2 s}\,X + \sin t \sin s\, f}{-\dfrac{\sin t \sin s \cos t}{\sqrt{1-\sin^2 t \sin^2 s}}X - \dfrac{\sin t \cos s}{\sqrt{1-\sin^2 s \sin^2 t}}Y + \cos t\, f}$$

$$\frac{Y'}{f} = \frac{-\dfrac{\sin^2 t \sin s \cos s}{\sqrt{1-\sin^2 t \sin^2 s}}X + \dfrac{\cos t}{\sqrt{1-\sin^2 t \sin^2 s}}Y + \cos s \sin t\, f}{-\dfrac{\sin t \sin s \cos t}{\sqrt{1-\sin^2 t \sin^2 s}}X - \dfrac{\sin t \cos s}{\sqrt{1-\sin^2 t \sin^2 s}}Y + \cos t\, f}$$

or

$$\frac{X'}{f} = \frac{(1-\sin^2 t \sin^2 s)X + \sin t \sin s (1-\sin^2 t \sin^2 s)^{\frac{1}{2}} f}{-\sin t \sin s \cos t\, X - \sin t \cos s\, Y + \cos t (1-\sin^2 t \sin^2 s)^{\frac{1}{2}} f}$$

$$\frac{Y'}{f} = \frac{-\sin^2 t \sin s \cos s\, X + \cos t\, Y + \cos s \sin t (1-\sin^2 t \sin^2 s)^{\frac{1}{2}} f}{-\sin t \sin s \cos t\, X - \sin t \cos s\, Y + \cos t (1-\sin^2 t \sin^2 s)^{\frac{1}{2}} f}$$

designating
$a = 1-\sin^2 t \sin^2 s$
$b = \sin t \sin s (1-\sin^2 t \sin^2 s)^{\frac{1}{2}}$
$c = \sin^2 t \sin s \cos s$
$d = \cos t$
$e = \cos s \sin t (1-\sin^2 t \sin^2 s)^{\frac{1}{2}}$
$g = \sin t \sin s \cos t$
$h = \sin t \cos s$
$K = \cos t (1-\sin^2 t \sin^2 s)^{\frac{1}{2}}$

these equations take the form

$$\frac{X'}{f} = \frac{aX + bf}{-gX - hY + Kf} = \frac{V_\alpha}{V_\gamma}, \quad \frac{Y'}{f} = \frac{-cX + dY + ef}{-gX - hY + Kf} = \frac{V_\beta}{V_\gamma}$$

a, b, c, d, e, g, h, K are constant for any one photograph.

2. DISTORTIONS OF THE PANORAMIC CAMERA

(a) First, we consider the distortion of the panoramic camera image when the camera is mounted in a level vehicle moving at constant altitude H, ground speed V, and direction X, but rotating about a vertical axis with angular velocity $\Omega = \frac{d\psi}{dt}$. The scan rate of the camera is $\varkappa = \frac{d\theta}{dt}$ and the film image motion compensation velocity is $v_f = \frac{\partial x'}{\partial t}$. Since scan begins at $\theta = -\frac{\pi}{2}$, we have $\theta = -\frac{\pi}{2} + \int_0^t \varkappa\, dt$. The origin of the rectified coordinate system is the nadir when $\theta = 0$. The time, t_n, at which $\theta = 0$ is found from the relation $\int_0^{t_n} \varkappa\, dt = \frac{\pi}{2}$. The azimuthal angle ψ is given by $\psi = \psi_0 + \int_0^t \Omega\, dt$. Thus $\psi_n = \psi_0 + \int_0^{t_n} \Omega\, dt$. Figures 26 and 27 show the relationships among the different earth, aircraft, camera, and unrectified film coordinates. From these figures we see that the following relationships hold, where x_c', y_c' are camera coordinates and x', y' the coordinates on the film itself.

$$(x_1, x_2, y_1, y_2) = \frac{f}{H}(X_1, X_2, Y_1, Y_2)$$

$$\cos\theta = \frac{H}{\sqrt{H^2 + Y_2^2}} = \frac{f}{\sqrt{f^2 + y_2^2}}$$

$$\theta = \frac{y_c'}{f} = \tan^{-1}\frac{Y_2}{H} = \tan^{-1}\frac{y_2}{f}$$

$$x_c' = x_2 \cos\theta = x' + \int_0^t v_f\, dt$$

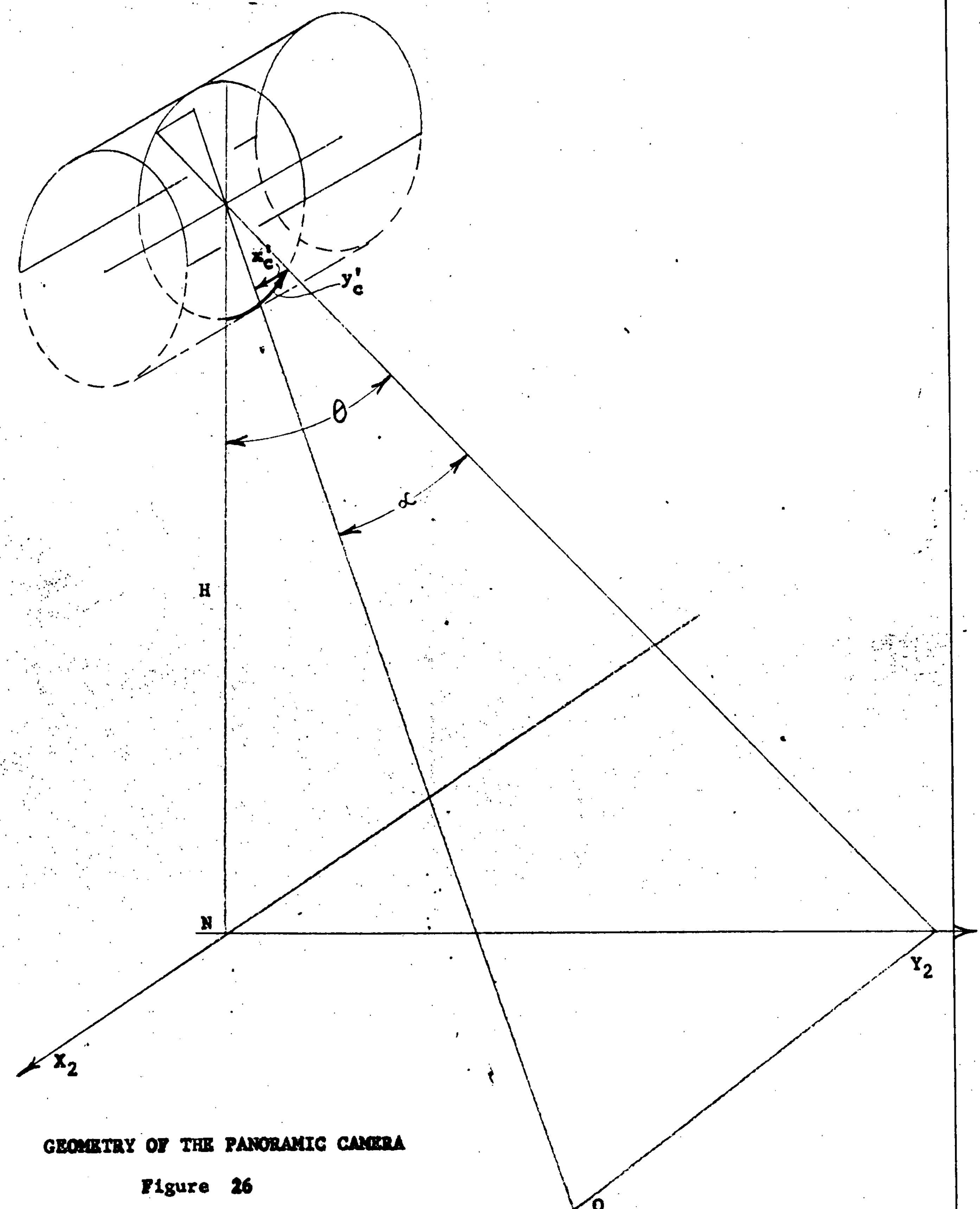
$$y_c' = y'$$

$$x_1 = x\cos\psi_n - y\sin\psi_n$$

$$y_1 = x\sin\psi_n + y\cos\psi_n$$

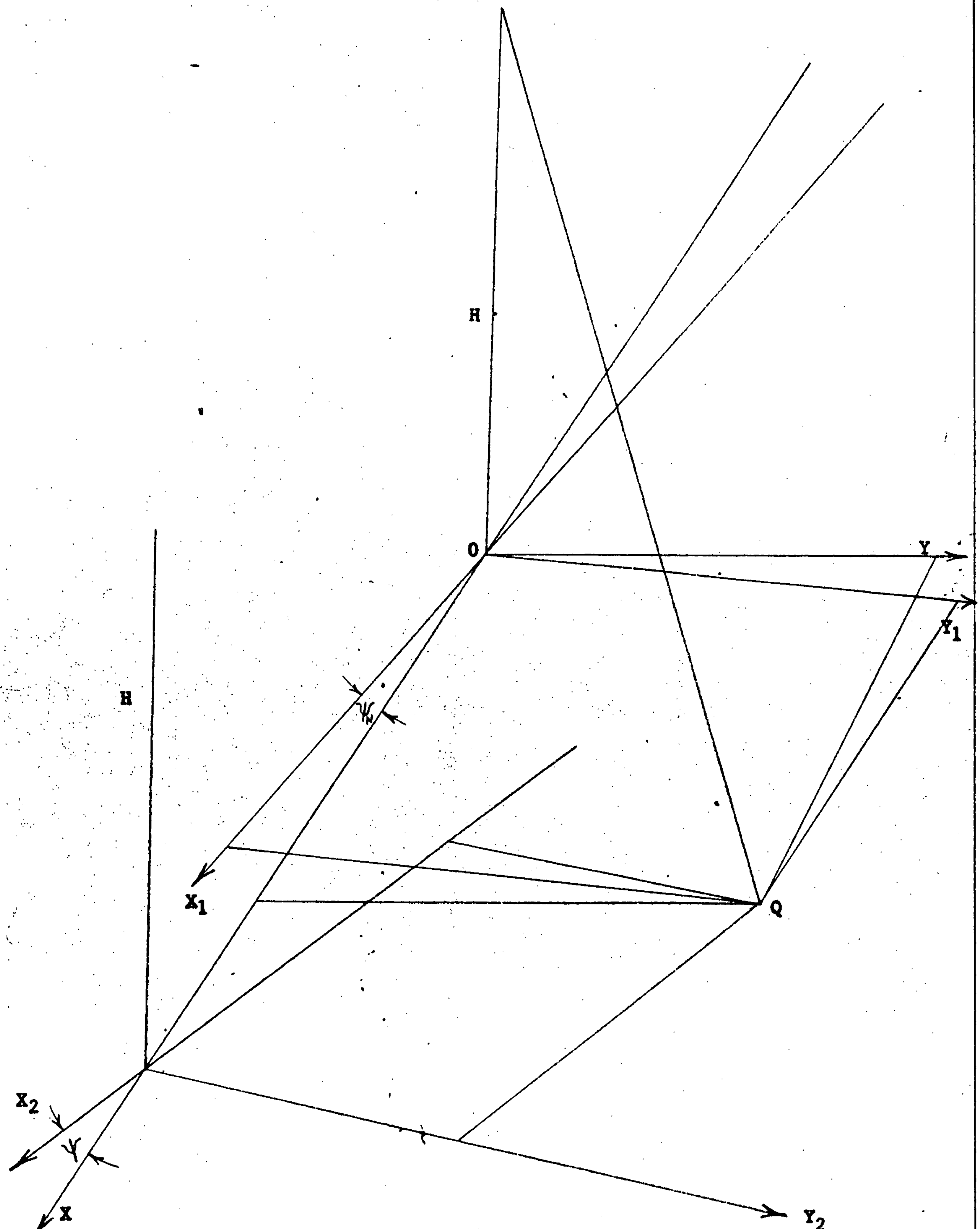
$$x_2 = [x - \frac{fV}{H}(t - t_n)]\cos\psi - y\sin\psi$$

$$y_2 = [x - \frac{fV}{H}(t - t_n)]\sin\psi + y\cos\psi$$



GEOMETRY OF THE PANORAMIC CAMERA

Figure 26



GEOMETRY OF ROTATING, TRANSLATING, SLIT-SCAN CAMERAS

 Figure 27

In the special case of constant rotation and constant scan rate, we have:

$$\Omega = \omega = \text{constant}\;,\quad \kappa = K = \text{constant}$$

$$t - t_n = \frac{\theta}{K}\;,\quad t_n = \frac{\pi}{2K}\;,\quad \psi = \psi_0 - \omega t = \psi_0 + \omega\left(\frac{\theta}{K} + t_n\right) = \psi_0 + \frac{\omega}{K}\left(\theta + \frac{\pi}{2}\right)$$

Let $\varphi_0 = \psi_0 + \frac{\pi\omega}{2K} = \psi_n$. Then:

$$x_1 = x\cos\varphi_0 - y\sin\varphi_0$$

$$y_1 = x\sin\varphi_0 + y\cos\varphi_0$$

$$x_2 = \left(x - \frac{fV\theta}{HK}\right)\cos\left(\varphi_0 + \frac{\omega\theta}{K}\right) - y\sin\left(\varphi_0 + \frac{\omega\theta}{K}\right) = \left(x' + \int_0^t v_f\,dt\right)\sec\theta$$

$$y_2 = \left(x - \frac{fV\theta}{HK}\right)\sin\left(\varphi_0 + \frac{\omega\theta}{K}\right) + y\cos\left(\varphi_0 + \frac{\omega\theta}{K}\right) = f\tan\theta = f\tan\frac{y'}{f}$$

These last two are the equations to be solved for x' and y' to rectify a panoramic picture taken from a constantly rotating craft.

Specializing still further to the case of no rotation and constant scan rate, we have with $v_f = \frac{fV}{H}\cos\theta$

$$\omega = 0\;,\quad \varphi_0 = \psi_0 \qquad \text{so that}$$

$$x_2 = \left(x - \frac{fV\theta}{HK}\right)\cos\psi_0 - y\sin\psi_0 = \left(x' + \int_0^t v_f\,dt\right)\sec\theta = \left(x' + \frac{Vf}{HK}[1 + \sin\theta]\right)\sec\theta$$

$$y_2 = \left(x - \frac{fV\theta}{HK}\right)\sin\psi_0 + y\cos\psi_0 = f\tan\frac{y'}{f}$$

For very small values of ψ_0

$$\left(\begin{array}{l} \sin\psi_0 = \psi_0\;,\ \cos\psi_0 = 1\;,\ \frac{fV\theta}{HK}\psi_0 \ll x\psi_0 \\ \theta = \tan^{-1}\frac{y}{f} + \frac{fx\,\psi_0}{f^2 + y^2} \end{array}\right)$$

these equations can be solved immediately for x' and y' in terms of x and y and used for rectification. If ψ_0 is not so small, a more complicated iterative procedure may be used.

If we are interested in the error due to crab angle ψ_0, we must compare x' and y' with x_1, y_1 instead of x_2, y_2 to avoid introducing the translation error which is included is the so-called S-distortion. The magnitude of both these distortions will be found.

Putting $\psi_0 = 0$ and solving for x' in the last equations, gives us the normal panoramic rectification equations

$$x' = x\cos\theta - \frac{fV}{HK}(1+\sin\theta - \theta\cos\theta)$$

$$y' = f\theta = f\tan^{-1}\frac{y}{f}$$

so the S-distortion error is

$$\frac{\Delta x'}{x'} = \frac{fV(1+\sin\theta-\theta\cos\theta)}{HKx'}$$

Putting the crab-angle equations in terms of x_1, y_1, we get

$$x_1 = \frac{fV\theta}{HK}\cos\psi_0 + \left(x' + \frac{fV}{HK}[1+\sin\theta]\right)\sec\theta$$

$$y_1 = \frac{fV\theta}{HK}\sin\psi_0 + f\tan\theta.$$

from which we see that the errors in ground coordinates are

$$\frac{\Delta x}{x} = \frac{fV\theta\cos\theta\,(\cos\psi_0 - 1)}{HK\left(x' + \frac{fV}{HK}[1+\sin\theta]\right)} \cong \frac{fV\theta\cos\theta(\cos\psi_0 - 1)}{HKx'}.$$

$$\frac{\Delta y}{y} = \frac{V\theta\sin\psi_0}{HK\tan\theta}$$

A case for which the error is relatively large for both of these is given by the conditions:

$H = 6$ miles $= 31680$ ft.

$V = \frac{1}{6}$ miles per sec. $= 600$ m.p.h

$K = 4\pi$ rad./sec (Scan time $\frac{1}{4}$ sec.)

$\theta = \frac{\pi}{4}$ rad. $= 45°$

$\tan\psi_0 = \frac{1}{6}$, $\sin\psi_0 = .1645$, $\cos\psi_0 = .987$

$x' = \frac{75}{2}$ mm. $= \frac{3}{2}$" $= \frac{1}{8}$ ft.

$f = 3" = \frac{1}{4}$ ft.

Under these conditions:

$$\left(\frac{\Delta X'}{X'}\right)_{S\text{-}dist} = \frac{3}{6\cdot 6\cdot 4\pi\cdot\frac{3}{2}}\left(1 + .707\left[1-\frac{\pi}{4}\right]\right) = 0.50\%$$

$$\left(\frac{\Delta X}{X}\right)_{crab} = \frac{3\pi\ .707(.013)}{\frac{3}{2}\,6\cdot 4\cdot 4\pi\cdot 6} = 0.004\%$$

$$\left(\frac{\Delta Y}{Y}\right)_{crab} = \frac{\pi(.1645)}{6\cdot 4\cdot 4\pi\cdot 6} = .03\%$$

(b) We consider next the distortions produced in the panoramic camera by tilt and swing in the photograph. We first do this in terms of pitch and roll and then express the results in terms of tilt and swing.

Referring to Figure 28, we see that the following relations hold:

$$\theta = -\frac{\pi}{2} + Kt \quad , \; V_g = \text{ground track velocity} = V\cos p$$

$$X_N = -\frac{V_g\pi}{2K} + V_g t = V_g\frac{\theta}{K}$$

$$X_i = X - X_N$$

$$Z_i = H - \frac{V_g\pi}{2K}\sin p + V_g t\sin p = H + \frac{V_g\,\theta\sin p}{K}$$

$$Y_i = Y$$

$$x_i = f\frac{X_i}{Z_i} \qquad \text{define} \qquad x_1 = f\frac{X_2}{Z_2}$$

$$y_i = f\frac{Y_i}{Z_i} \qquad\qquad y_1 = f\frac{Y_2}{Z_2}$$

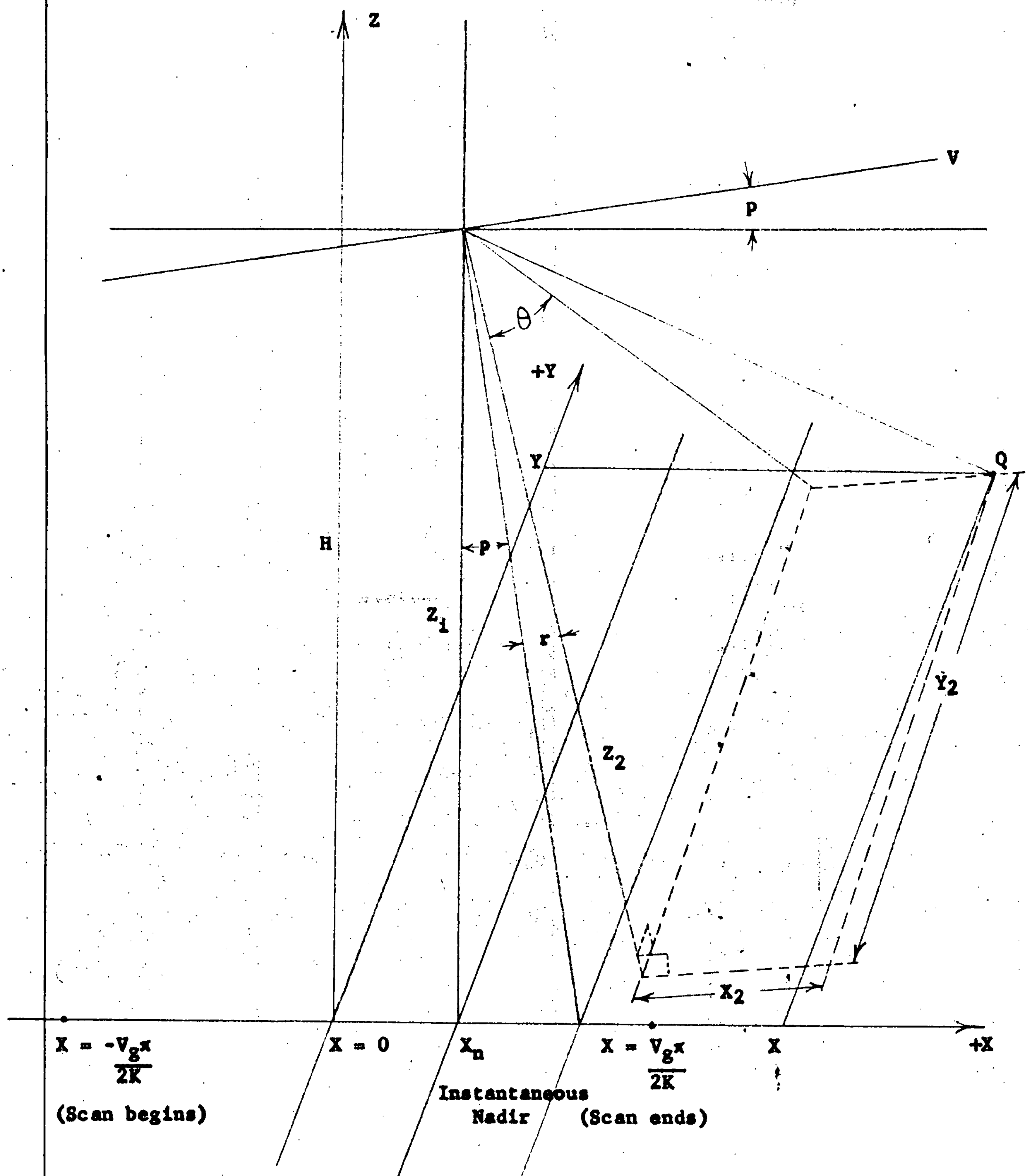
$$\theta = \tan^{-1}\frac{Y_2}{Z_2} = \tan^{-1}\frac{y_1}{f}$$

The relationship between the (2)-system and the (i)-system is the same as that between the (x_2, etc.)-system and the (X, etc.)-system in the previous treatment of tilt and swing. And we have:

$$X_2 = X_i\cos p - Z_i\sin p$$

$$Y_2 = Y_i\cos r - X_i\sin p\sin r - Z_i\cos p\sin r$$

$$Z_2 = Y_i\sin r + X_i\sin p\cos r + Z_i\cos p\cos r$$



GEOMETRY OF MOVING, TILTED, PANORAMIC CAMERA

Figure 28

The transformation equations for a tilted panoramic photograph can now be written down as follows:

$$y' = f\theta$$

$$x' = \underbrace{f\frac{x_2}{z_2}\cos\theta}_{\text{static panoramic}} + \frac{V_g f}{z_2 K}\overbrace{(1+\underbrace{\sin\theta}_{\text{film displacement}} - \theta\cos\theta)}^{\text{aircraft motion}}$$

The difficulty with getting these equations into a rectifiable form is that θ not only occurs directly in the equations but also x_2, z_2 are functions of θ while θ is a complicated function of itself. The situation is not so bad, however.

$$\theta = \tan^{-1}\frac{y_2}{z_2} = \tan^{-1}\left[\frac{Y_i\cos r - X_i\sin p\sin r - Z_i\cos p\sin r}{Y_i\sin r + X_i\sin p\cos r + Z_i\cos p\cos r}\right]$$

$$= \tan^{-1}\left[\frac{Y\cos r - (X - \frac{V_g\theta}{K})\sin p\sin r - (H + \frac{V_g\theta}{K}\sin p)\cos p\sin r}{Y\sin r + (X - \frac{V_g\theta}{K})\sin p\cos r + (H + \frac{V_g\theta}{K}\sin p)\cos p\cos r}\right]$$

$$= \tan^{-1}\left[\frac{y\cos r - x\sin p\sin r - f\cos p\sin r + \frac{fV_g\theta}{HK}\sin p(1-\cos p)\sin r}{y\sin r + x\sin p\cos r + f\cos p\cos r - \frac{fV_g\theta}{HK}\sin p(1-\cos p)\cos r}\right]$$

Defining ε as $\frac{fV_g\theta}{HK}\sin p(1-\cos p)$ we have $\theta = \tan^{-1}\left(\frac{A+\varepsilon\sin r}{B-\varepsilon\cos r}\right)$ where $\theta_s = \tan^{-1}\frac{A}{B}$ defines θ_s for the stationary panoramic, tilted photograph.

Assuming ε to be small so that higher powers can be neglected and expanding θ in a Taylor's series about $\frac{A}{B}$ we have:

$$\theta = \tan^{-1}\left[\left(\frac{A}{B} + \frac{\varepsilon}{B}\sin r\right)\left(1 - \frac{\varepsilon}{B}\cos r\right)^{-1}\right] = \tan^{-1}\left[\left(\frac{A}{B} + \frac{\varepsilon}{B}\sin r\right)\left(1 + \frac{\varepsilon}{B}\cos r\right)\right]$$

$$= \tan^{-1}\left[\frac{A}{B} + \frac{\varepsilon}{B}\left(\sin r + \frac{A}{B}\cos r\right)\right]$$

$$= \tan^{-1}\frac{A}{B} + \frac{\epsilon(A\cos r + B\sin r)}{A^2+B^2} = \tan^{-1}\frac{A}{B} + \frac{f V_g \theta}{HK}\sin p(1-\cos p)\frac{(A\cos r + B\sin r)}{A^2+B^2}$$

solving for θ ,

$$\theta = \left[1 + \frac{f V_g \sin p(1-\cos p)(A\cos r + B\sin r)}{HK(A^2+B^2)}\right]\tan^{-1}\frac{A}{B}$$

In a typical case:

$X = 4,\ Y = 6,\ f = 3,\ V_g = \frac{1}{6}$ m.p.s. $H = 6$ mi., $K = 4\pi$ rad./sec.

$\sin p = \sin r = .2,\ \cos p = \cos r = .98$

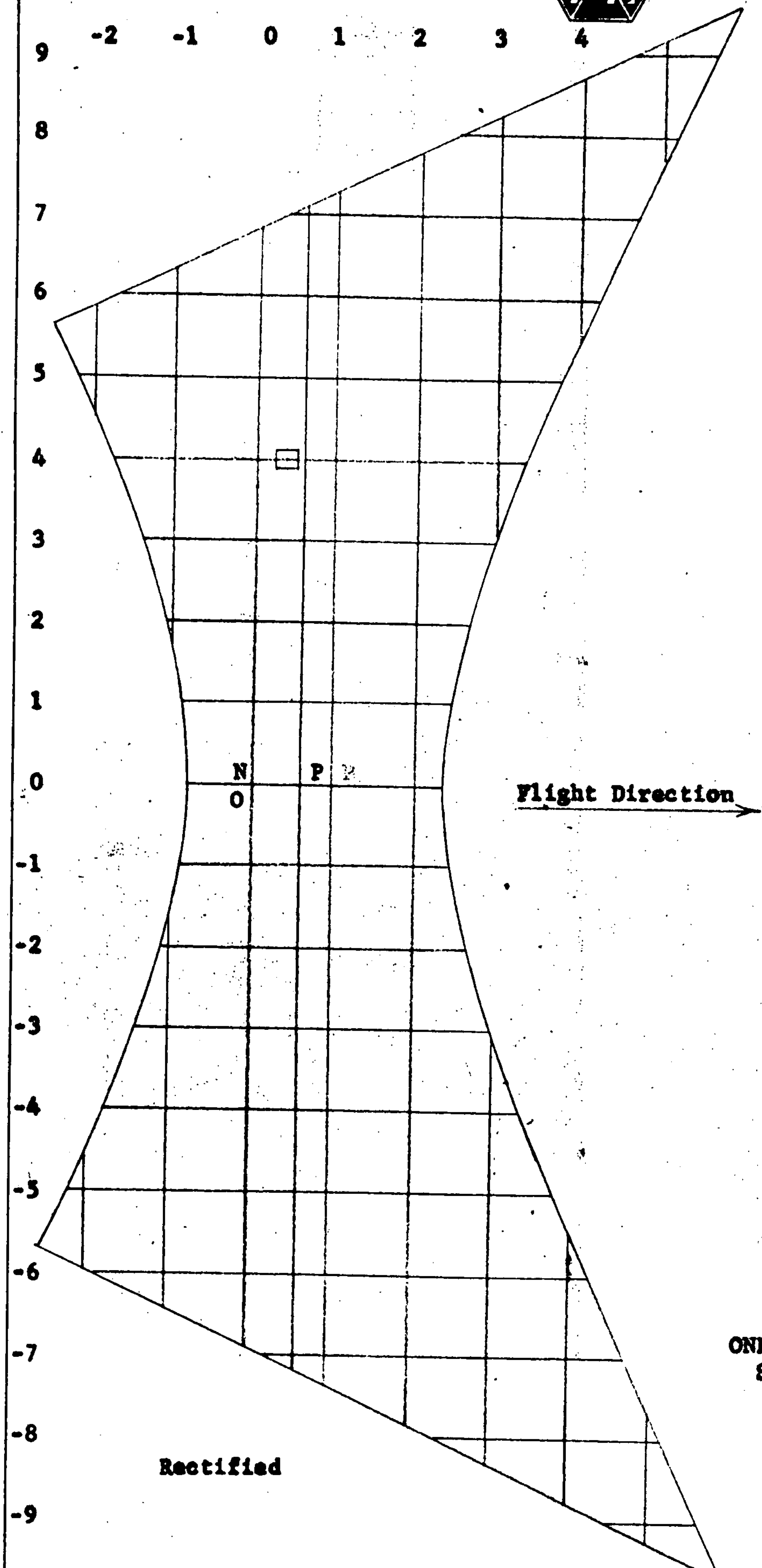
$A = 6(.98) - 4(.04) - 3(.2)(.98) = 5.13$

$B = 6(.2) + 4(.2)(.98) + 3(.98)^2 = 4.86$

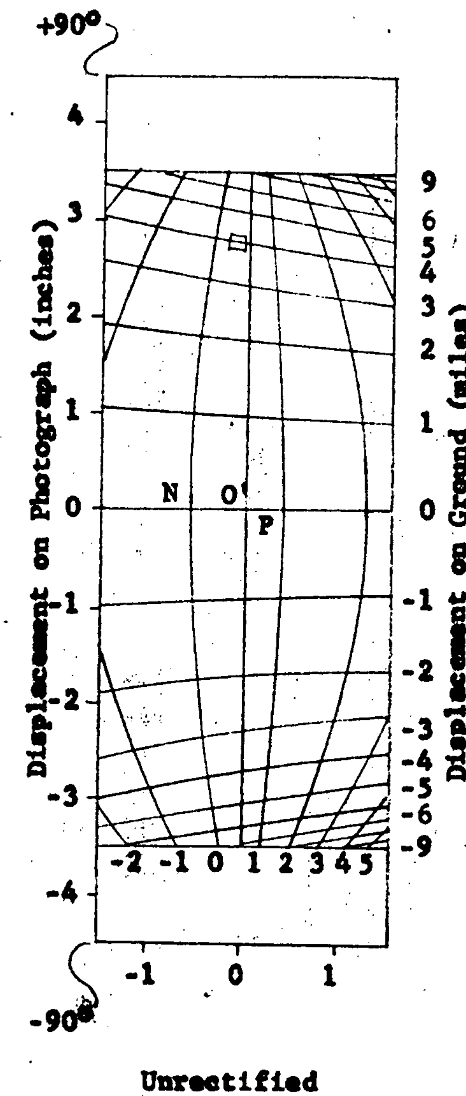
$$\theta = \left[1 + \frac{3(.2)(.02)(5.13[.96] + 4.86[.2])}{6\cdot 6\cdot 4\pi\,(28.6 + 23.6)}\right]\tan^{-1}1.06$$

$$= \left[1 + 3.2\cdot 10^{-6}\right]\tan^{-1}1.06$$

So we see that for all practical purposes $\theta = \tan^{-1}\frac{A}{B}$ **and the transformation equations can be used in the given form for rectification. An example of a rectified panoramic transformation is given in Figure 29.**



H = 15,840 ft.
Tilt = 11 1/2°
Swing = 270°
(11 1/2° pitch, 0° roll)
No earth curvature



ONE MILE SQUARE GRID ON GROUND
SHOWN AS TAKEN BY 3" F. L.
PANORAMIC CAMERA

1/2 Scale

Figure 29

3. DISTORTIONS CORRECTED ABOUT THE NADIR (Earth Curvature and Air Refraction)

According to Fermat's principle, the fundamental principle of geometrical optics, the path taken by a light ray between two points is the path of "quickest arrival." Expressed mathematically

(1) $$\delta \int_{P_1}^{P_2} n\,ds = 0$$

where ds is an increment of the ray path and n is the refractive index in the region of ds. Euler's differential equations[8] solve (1) and from them can be derived the laws of refraction and reflection and the solution to our problem.

If we use the system of spherical coordinates shown in the figure, we have, assuming $n(r, \varphi, \theta) = n(r)$

$$ds = \sqrt{dr^2 + r^2 d\varphi^2} = \sqrt{r_\varphi^2 + r^2}\, d\varphi$$

$$n = n(r)$$

(2) $$\delta \int_{P_1}^{P_2} n\,ds = \delta \int_{\Phi}^{0} n\sqrt{r_\varphi^2 + r^2}\, d\varphi = 0 = \delta \int_{\Phi}^{0} F\, d\varphi$$

Euler's equation for this problem is

(3) $$\frac{d}{d\varphi}\left(F - r_\varphi \frac{\partial F}{\partial r_\varphi}\right) = 0$$

or

$$F - r_\varphi \frac{\partial F}{\partial r_\varphi} = n\sqrt{r^2 + r_\varphi^2} - \frac{n\, r_\varphi^2}{\sqrt{r^2 + r_\varphi^2}} = C$$

At $\varphi = 0$, $r = r_p = r_e + H$, $r_\varphi = \left(\frac{dr}{d\varphi}\right)_p = r_p \cot\psi$ or $C^2 = r_p^2 n_p^2 \sin^2\psi$

so:

(4) $$r_\varphi = \frac{n r^2}{n_p r_p \sin\psi} \sqrt{1 - \frac{n_p^2 \sin^2\psi\, r_p^2}{n^2 r^2}}$$

or

(5) $$\int_{\Phi}^{0} d\varphi = -\Phi = -\int_{r_e}^{r_p} \frac{\frac{n_p}{n}\,\frac{r_p \sin\psi}{r}\,\frac{dr}{r}}{\sqrt{1 - \left(\frac{n_p}{n}\right)^2 \left(\frac{\sin\psi\, r_p}{r}\right)^2}}$$

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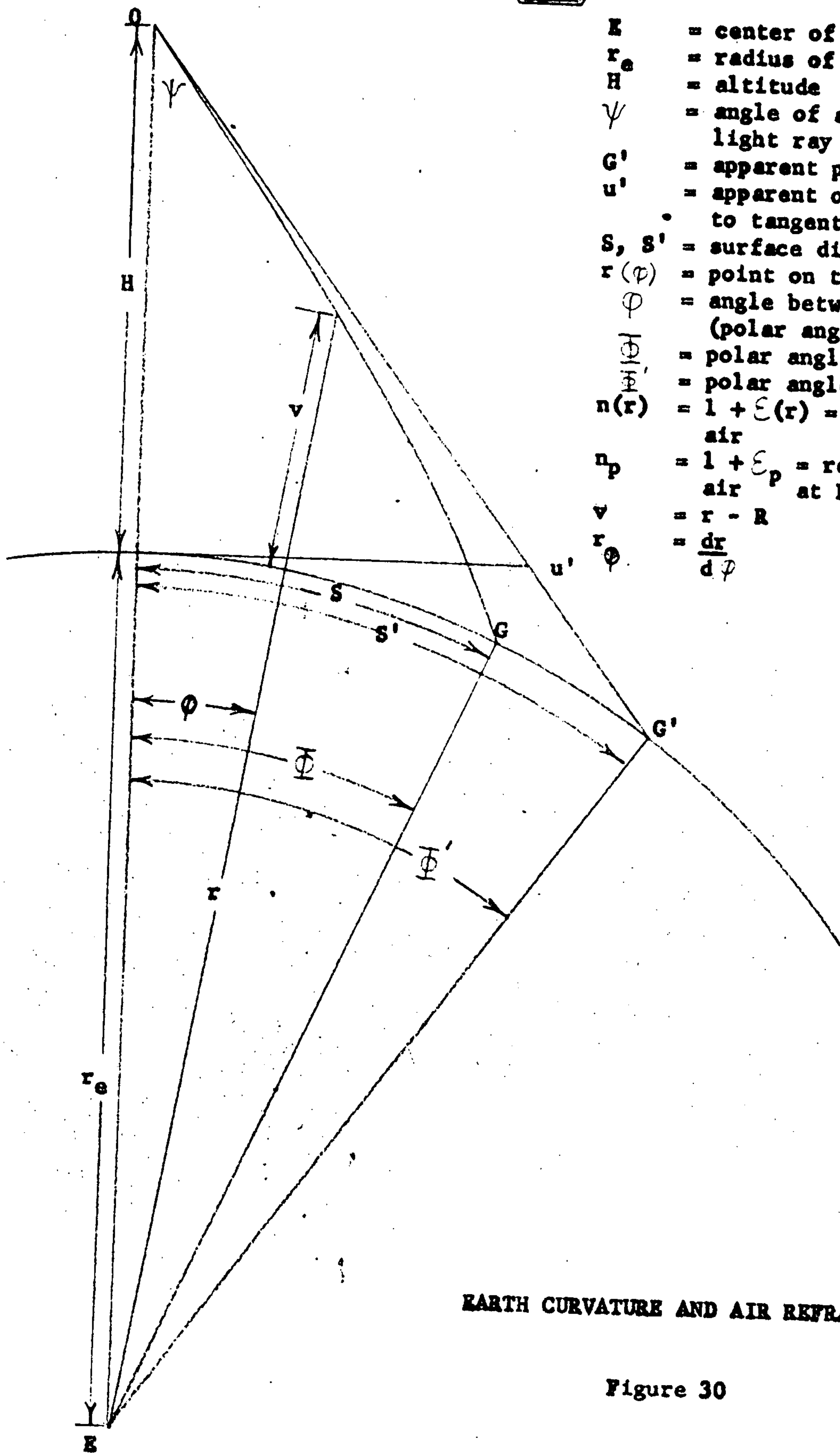
8 See, for instance, Margenau, H. and Murphy, G., "The mathematics of Physics and Chemistry," D. Van Nostrand Co., New York, 1943, Pages 195, 196, 199.



- E = center of Earth
- r_e = radius of Earth
- H = altitude
- ψ = angle of arrival of refracted light ray GP
- G' = apparent point of origin of GP
- u' = apparent origin of GP referred to tangent plane
- S, S' = surface distances to G, G'
- $r(\varphi)$ = point on trajectory of ray GP
- φ = angle between nadir and r (polar angle)
- Φ = polar angle of G
- Φ' = polar angle of G'
- $n(r) = 1 + \mathcal{E}(r)$ = refractive index of air
- $n_p = 1 + \mathcal{E}_p$ = refractive index of air at P
- $v = r - R$
- $r_\varphi = \dfrac{dr}{d\varphi}$

EARTH CURVATURE AND AIR REFRACTION

Figure 30

Letting $D = \frac{r_p \sin\psi}{r}$, we have

(6) $$\Phi = \int_{r_e}^{r_p} \frac{\frac{n_p}{n} D \frac{dr}{r}}{\sqrt{1-\left(\frac{n_p}{n}\right)^2 D^2}}$$

since $\frac{n_p}{n} = \frac{1+\varepsilon_p}{1+\varepsilon} = 1-\left(\frac{\varepsilon-\varepsilon_p}{1+\varepsilon}\right) = 1-\theta$ (definition of θ)

then

(7) $$\Phi = \int_{r_e}^{r_p} \frac{(1-\theta) D \frac{dr}{r}}{\sqrt{1-D^2(1-\theta)^2}} = \int_{r_e}^{r_p} \frac{(1-\theta) D \frac{dr}{r}}{\sqrt{1-D^2}\sqrt{1+\frac{D^2(2\theta-\theta^2)}{(1-D^2)}}}$$

Expanding $\left[1+\frac{D^2(2\theta-\theta^2)}{(1-D^2)}\right]^{-\frac{1}{2}}$ by the binomial theorem, we get

(8) $$\Phi = \int_{r_e}^{r_p} \frac{(1-\theta)D}{(1-D^2)^{\frac{1}{2}}}\left[\sum_{i=1}^{\infty} \frac{(-)^{i-1}(2i-2)!}{2^{(2i-2)}(i-1)!^2}\left\{\frac{D^2(2\theta-\theta^2)}{(1-D^2)}\right\}^{i-1}\right]\frac{dr}{r}$$

$$= \int_{r_e}^{r_p} \frac{D}{(1-D^2)^{\frac{1}{2}}}\frac{dr}{r} + \int_{r_e}^{r_p} \frac{(1-\theta)D}{(1-D^2)^{\frac{1}{2}}}\sum_{i=2}^{\infty} \frac{(-)^{i-1}(2i-2)!}{2^{(2i-2)}(i-1)!^2}\left\{\frac{D^2(2\theta-\theta^2)}{(1-D^2)}\right\}^{i-1}\frac{dr}{r}$$

$$-\int_{r_e}^{r_p} \frac{\theta D}{(1-D^2)^{\frac{1}{2}}}\frac{dr}{r}$$

Since for n = constant = n_p, the ray path becomes a straight line, we see that equation (6) becomes

(9) $$\Phi' = \int_{r_e}^{r_p} \frac{D}{(1-D^2)^{\frac{1}{2}}}\frac{dr}{r}.$$

If we change the variables to $v = r - r_e$, $dv = dr$, $r_p = r_e + H$, $\delta = \frac{v}{r_e}$, $\delta_p = \frac{H}{r_e}$

Also, we see from the figure by the law of sines

(10) $$\frac{r_e}{\sin\psi} = \frac{r_e + H}{\sin(\psi + \Phi')}$$

Solving (10) for Φ', we have

(11) $$\Phi' = \sin^{-1}\left\{\sin\psi\cos\psi\left[1+\delta_p - \left\{1-\delta_p(2+\delta_p)\tan^2\psi\right\}^{\frac{1}{2}}\right]\right\}$$

Putting the results of (9) and (11) in (8), we have, since $\tan\psi = \frac{u'}{H}$, the exact expression relating points on a photograph, u', to actual earth surface distance, S, as influenced by earth curvature and altitude-dependent variations in atmospheric refractive index.

$$(12)\quad S = r_e \sin^{-1}\left\{\sin\psi\cos\psi\left[1+\delta_p-\left\{1-\delta_p(2+\delta_p)\tan^2\psi\right\}^{\frac{1}{2}}\right]\right\} - \int_0^H \frac{D}{(1+\delta)(1-D^2)^{\frac{1}{2}}}\left[\theta-(1-\theta)\sum_{i=2}^{\infty}\frac{(-)^{i-1}(2i-2)!}{2^{(2i-2)}(i-1)!\,i!}\left\{\frac{D^2(2\theta-\theta^2)}{(1-D^2)}\right\}^{i-1}\right]dv$$

Since ε, ε_p are very small compared to 1, $\theta = 1-\frac{(\varepsilon-\varepsilon_p)}{1+\varepsilon}$ can be expressed to great accuracy by a few terms of $\theta = 1-(\varepsilon-\varepsilon_p)(1-\varepsilon+\varepsilon^2-\varepsilon^3\cdots)$. Taking terms up to the third powers of ε, ε_p in the integrand of (12), we get to great accuracy:

$$(13)\quad S = r_e \sin^{-1}\left\{\sin\psi\cos\psi\left[1+\delta_p-\left\{1-\delta_p(2+\delta_p)\tan^2\psi\right\}^{\frac{1}{2}}\right]\right\} - \int_0^H \frac{D(\varepsilon-\varepsilon_p)}{(1+\delta)(1-D^2)^{3/2}}\left[1-\left(\varepsilon+\tfrac{3}{2}[\varepsilon-\varepsilon_p]\frac{D^2}{1-D^2}\right)+\left(\varepsilon^2+3\varepsilon[\varepsilon-\varepsilon_p]\frac{D^2}{(1-D^2)}+\tfrac{1}{2}(\varepsilon-\varepsilon_p)^2\frac{D^2}{(1-D^2)}\left[1+\frac{5D^2}{1-D^2}\right]\right)\right]dv$$

If H is small compared to r_e - say H is less than 400 miles, the first term in (13)--pure curvature term--and the integrand of (13) can be expanded in powers of ε, δ_p. If this is done, the final form of the expression for S will be

$$(14)\quad \frac{S}{H} = u'\left[1+C_1\left(\frac{u'}{H}\right)^2+C_2\left(\frac{u'}{H}\right)^4+\cdots C_m\left(\frac{u'}{H}\right)^{2m}\cdots\right]$$

where the C's are functions of the altitude, H, only. If H is not small compared with r_e, it would be best to express ε in powers of v. A few terms should express ε to the desired degree of accuracy. The integrand of (13) could then be reduced to integrals of the form

$$\int z^m(1+z^2)^{-\frac{n}{2}}dz$$

These can be found directly in tables of indefinite integrals. The first term on the right of (13) must now be used "as is" and the integral in (13) integrates to a complicated function of $\frac{v'}{H}$. The resulting expression, however, would hold at any distance from the earth.

As an indication of orders of magnitude, we will consider terms up to the third power in δ, δ_p and up to the second power in $\varepsilon, \varepsilon_p$. The first term of (13) becomes

$$
\begin{aligned}
s' = r_e \Phi' &= r_e \sin^{-1}\left\{\sin\psi\cos\psi\left[1+\delta_p - \left\{1-\delta_p(2+\delta_p)\tan^2\psi\right\}^{\frac{1}{2}}\right]\right\} \\
&= r_e \sin^{-1}\left\{\sin\psi\cos\psi\left[1+\delta_p - \left\{1-\delta_p\tan^2\psi(2+\delta_p) - \frac{\delta_p^2}{8}\tan^4\psi(2+\delta_p)^2 - \frac{\delta_p^3}{16}\tan^6\psi(2+\delta_p)^3 - \frac{5}{8}\delta_p^4\tan^8\psi\right\}\right]\right\} \\
(15) \quad &= r_e \sin^{-1}\left\{\delta_p\sin\psi\cos\psi\left[1+\tan^2\psi + \frac{\delta_p}{2}(1+\tan^2\psi)\tan^2\psi + \frac{\delta_p^2}{2}(1+\tan^2\psi)\tan^4\psi + \frac{\delta_p^3}{8}(1+\tan^2\psi)(1+5\tan^2\psi)\tan^4\psi\right]\right\} \\
&= r_e \sin^{-1}\left\{\delta_p\tan\psi\left[1 + \frac{\delta_p}{2}\tan^2\psi + \frac{\delta_p^2}{2}\tan^4\psi + \frac{\delta_p^3}{8}\tan^4\psi(1+5\tan^2\psi)\right]\right\} = r_e\sin^{-1}\left\{\delta_p\tan\psi[\;\;]\right\}
\end{aligned}
$$

Using the power series expansion of $\sin^{-1}x$,

$$
\begin{aligned}
s' &= r_e\left\{\delta_p\tan\psi[\;\;] + \frac{\delta_p^3}{6}\tan^3\psi\left(1+\frac{\delta_p}{2}\tan^2\psi\right)^3\right\} \\
(16) \quad &= H\tan\psi\left[1+\frac{\delta_p}{2}\tan^2\psi + \frac{\delta_p^2}{6}\tan^2\psi(1+3\tan^2\psi) + \frac{\delta_p^3}{8}\tan^4\psi(3+5\tan^2\psi)\right]
\end{aligned}
$$

In order to evaluate the integrand of (13), we note that

$$D = \frac{r_e}{r}\sin\psi = \left(\frac{1+\delta_p}{1+\delta}\right)\sin\psi$$

$$(17) \quad 1-D^2 = 1-\sin^2\psi\frac{(1+\delta_p)^2}{(1+\delta)^2} = \frac{\cos^2\psi}{(1+\delta)^2}\left[1-\delta_p(2+\delta_p)\tan^2\psi + \delta(2+\delta)\sec^2\psi\right]$$

$$\frac{D}{(1-D^2)^{\frac{1}{2}}} = \tan\psi(1+\delta_p)\left[1-\delta_p(2+\delta_p)\tan^2\psi + \delta(2+\delta)\sec^2\psi\right]^{-\frac{1}{2}}$$

$$\frac{D}{(1+\delta)(1-D^2)^{\frac{3}{2}}} = \tan\psi\sec^2\psi(1+\delta)(1+\delta_p)\left[1-\delta_p(2+\delta_p)\tan^2\psi + \delta(2+\delta)\sec^2\psi\right]^{-\frac{3}{2}}$$

So, to the desired order, the integral in (13) is

$$
\begin{aligned}
&\int_0^H \tan\psi\sec^2\psi(\varepsilon-\varepsilon_p)\Big[1-(2+3\tan^2\psi)\delta + (1+3\tan^2\psi)\delta_p + \frac{3}{2}\sec^2\psi(2+5\tan^2\psi)\delta^2 \\
&\qquad + \frac{3}{2}\tan^2\psi(3+5\tan^2\psi)\delta_p^2 - (2+15\sec^2\psi\tan^2\psi)\delta\delta_p\Big]\times\left[1-(\varepsilon+\tfrac{3}{2}[\varepsilon-\varepsilon_p]\tan^2\psi\right]dv \\
(18) \quad &= H\tan\psi\sec^2\psi\frac{1}{H}\int_0^H(\varepsilon-\varepsilon_p)\,dv - H\tan\psi\sec^2\psi\left[\frac{1}{H}\int_0^H\varepsilon(\varepsilon-\varepsilon_p)\,dv + \frac{3}{2}\tan^2\psi\frac{1}{H}\int_0^H(\varepsilon-\varepsilon_p)^2\,dv\right] \\
&+ H\tan\psi\sec^2\psi\Big[(1+3\tan^2\psi)\delta_p\frac{1}{H}\int_0^H(\varepsilon-\varepsilon_p)\,dv - (2+3\tan^2\psi)\delta_p\frac{1}{H^2}\int_0^H(\varepsilon-\varepsilon_p)v\,dv \\
&\qquad + \frac{3}{2}\sec^2\psi(2+5\tan^2\psi)\delta_p^2\frac{1}{H^3}\int_0^H(\varepsilon-\varepsilon_p)v^2\,dv + \frac{3}{2}\tan^2\psi(3+5\tan^2\psi)\delta_p^2\frac{1}{H}\int_0^H(\varepsilon-\varepsilon_p)\,dv \\
&\qquad - (2+15\sec^2\psi\tan^2\psi)\delta_p^2\frac{1}{H^2}\int_0^H(\varepsilon-\varepsilon_p)v\,dv\Big]
\end{aligned}
$$

If we define the five dimensionless functions of H:

$$(19)\quad C_1 = \frac{1}{H}\int_0^H (\varepsilon-\varepsilon_p)\,dv \;,\quad C_2 = \frac{1}{H}\int_0^H (\varepsilon-\varepsilon_p)^2\,dv \;,\quad C_2' = \frac{1}{H}\int_0^H \varepsilon(\varepsilon-\varepsilon_p)\,dv$$

$$C_{11} = \frac{1}{H^2}\int_0^H (\varepsilon-\varepsilon_p)v\,dv \;.\quad C_{12} = \frac{1}{H^3}\int_0^H (\varepsilon-\varepsilon_p)v^2\,dv$$

we can express (18) as

$$\int_0^H \tan\psi\sec^2\psi(\varepsilon-\varepsilon_p)[\qquad]\times[\qquad]\,dv =$$

$$H\tan\psi\sec^2\psi\left[\begin{matrix} C_1-(C_2'+\tfrac{3}{2}\tan^2\psi C_2)+(1+3\tan^2\psi)\varepsilon_p C_1-(2+3\tan^2\psi)\varepsilon_p C_{11} \\ +\tfrac{3}{2}\sec^2\psi(2+5\tan^2\psi)\varepsilon_p^2 C_{12}+\tfrac{3}{2}\tan^2\psi(3+5\tan^2\psi)\varepsilon_p^2 C_1 \\ -(2+15\sec^2\psi\tan^2\psi)\varepsilon_p^2 C_{11} \end{matrix}\right]$$

Letting $w' = \frac{u'}{H} = \tan\psi$, we may now give the complete expression for S for the second-third order appxoimation:

$$S = Hw'\left[1+\frac{w'^2}{2}\varepsilon_p+\frac{w'^2}{c}(1+3w'^2)\varepsilon_p^2+\frac{w'^4}{c}(3+5w'^2)\varepsilon_p^3\right]$$

$$(20)\qquad -Hw'\left[(1+w'^2)\left(C_1-\{C_2'+\tfrac{3}{2}w'^2C_2\}\right)\right]$$

$$-Hw'\left[(1+w'^2)\left(\{1+3w'^2\}\varepsilon_pC_1-\{2+3w'^2\}\varepsilon_pC_{11}+\tfrac{3}{2}\{1+w'^2\}\{2+5w'^2\}\varepsilon_p^2C_{12}\begin{matrix}+\tfrac{3}{2}w'^2\{3+5w'^2\}\varepsilon_p^2C_1 \\ -\{2+15(1+w'^2)w'^2\}\varepsilon_p^2C_{11}\end{matrix}\right)\right]$$

It will be noted that S is given in terms of the w' or photograph coordinates. One can solve for w' in terms of S by the usual iterative process employed when an approximate value is known initially.

The first line of (20) is pure curvature effects, the second, pure refraction effects, and the third line is mixed curvature and refraction effects. To approximate the C's, we can assume the dispersion, ε, is proportional to the density and the density varies exponentially with altitude. If we express ε as

$$\varepsilon = \varepsilon_o e^{-\ln\left(\frac{d_o}{d_1}\right)\frac{v}{H_1}}$$

where ε_o is the refractive index at $v = 0$

d_o " " density " " "

d_1 " " " " $v = H_1$

The C's are then found in the following way:

Letting $b = -\frac{1}{H}\ln\frac{d_0}{d_1}$ and defining $B = e^{-bH}$, $A = -\frac{1}{\ln B} = \frac{1}{Hb}$

we have

$$HC_1 = \int_0^H (\varepsilon - \varepsilon_p)\,dv = \varepsilon_0 \int_0^H \left(e^{-bv} - \left(\frac{d_1}{d_0}\right)^{\frac{H}{H_1}}\right) dv = -\varepsilon_0\left[\frac{e^{-bv}}{b} + v\left(\frac{d_1}{d_0}\right)^{\frac{H}{H_1}}\right]_0^H$$

$$= -\varepsilon_0\left[\frac{\left\{\left(\frac{d_1}{d_0}\right)^{\frac{H}{H_1}} - 1\right\}H_1}{\ln\left(\frac{d_0}{d_1}\right)} + H\left(\frac{d_1}{d_0}\right)^{\frac{H}{H_1}}\right] = H\varepsilon_0\left[A(1-B) - B\right]$$

$$HC_2' = \int_0^H \varepsilon(\varepsilon - \varepsilon_p)\,dv = \varepsilon_0^2 \int_0^H \left(e^{-2bv} - e^{-bH}e^{-bv}\right) dv$$

$$= \varepsilon_0^2\left[\frac{e^{-2bv}}{-2b} + \frac{e^{-bH}e^{-bv}}{b}\right]_0^H = H\varepsilon_0^2 \frac{A}{2}(1-B)^2$$

$$HC_2 = \int_0^H (\varepsilon - \varepsilon_p)^2\,dv = \varepsilon_0^2 \int_0^H \left(e^{-2bv} - 2e^{-bH}e^{-bv} + e^{-2bH}\right) dv$$

$$= \varepsilon_0^2\left[\frac{e^{-2bv}}{-2b} + \frac{2e^{-bH}e^{-bv}}{b} + v e^{-2bH}\right]_0^H = H\varepsilon_0^2\left[\frac{A}{2}(1-B)^2 - B\left\{A(1-B) - B\right\}\right] = H(C_2' - H\varepsilon_0 B C_1)$$

$$H^2C_{11} = \int_0^H (\varepsilon - \varepsilon_p)v\,dv = \varepsilon_0 \int_0^H \left(v e^{-bv} - v e^{-bH}\right) dv = -\varepsilon_0\left[e^{-bv}\left(\frac{v}{b} + \frac{1}{b^2}\right) + \frac{v^2}{2}e^{-bH}\right]_0^H$$

$$= H^2\left(AC_1 - \frac{\varepsilon_0 B}{2}\right)$$

$$H^3C_{12} = \int_0^H (\varepsilon - \varepsilon_p)v^2\,dv = \varepsilon_0 \int_0^H \left(v^2 e^{-bv} - v^2 e^{-bH}\right) dv = \varepsilon_0\left[e^{-bv}\left(\frac{v^2}{-b} - \frac{2v}{b^2} - \frac{2}{b^3}\right) - \frac{v^3}{3}e^{-bH}\right]_0^H$$

$$= H^3\left(2AC_{11} - \tfrac{1}{3}\varepsilon_0 B\right)$$

Summarizing, the constants are:

$$C_1 = \varepsilon_0\left[A(1-B) - B\right]$$

$$C_2' = \varepsilon_0^2 \frac{A}{2}(1-B)^2$$

$$C_2 = C_2' - \varepsilon_0 B C_1$$

$$C_{11} = AC_1 - \tfrac{1}{2}\varepsilon_0 B$$

$$C_{12} = 2AC_{11} - \tfrac{1}{3}\varepsilon_0 B$$

At very high altitudes

$$C_1 \rightarrow \varepsilon_0 A$$

$$C_2 \rightarrow \varepsilon_0^2 \frac{A}{2}$$

$$C_2' \rightarrow \varepsilon_0^2 \frac{A}{2}$$

$$C_{11} \rightarrow \varepsilon_0 A^2$$

$$C_{12} \rightarrow 2\varepsilon_0 A^3$$

The refractive index function was chosen to match the ICAO standard atmosphere at $H = 0$, and $H_1 = 3 \times 10^4$ ft. This gives the function

[illegible] (See footnote 9)

To see if our expression for the variation of refractive index with altitude is accurate enough, we evaluate the constant C_1 for a height of 3×10^4 ft. by Simpson's rule--a very accurate approximation formula if the intervals are taken small enough. The ICAO standard atmosphere is used with interpolated values to reduce the size of the intervals. The very accurate interpolating formula $d = \sqrt{d_1 d_2}$ is used where d is the density at altitude midway between the altitudes at which the densities are d_1 and d_2. This formula is a consequence of the [illegible] expression. C_1 calculated in this way is 8.02×10^{-5} for $H = 3 \times 10^4$ ft. Calculated by the formula [illegible] we get $C_1 = 7.68 \times 10^{-5}$ -- a difference of 4.4%.

Since the largest correction for refraction is .32%, the maximum error caused by using this approximation would be .013% -- an allowable error under the extreme condition of $\tan \gamma = 6$ ($\gamma = 80.5°$).

It will be noted from the tables that even three correction terms for earth curvature may not be enough to achieve sufficient accuracy at some altitudes. From these tables we can also see that at these extreme conditions the terms N_2 and M_{12} can be neglected, which simplifies the equations. For smaller values of $\tan \gamma$, other terms could be neglected. We also consider the question of whether temperature variations near the ground caused by solar heating may appreciably distort the picture. Using data obtained experimentally of a rather extreme case and published on page 166 of Geodesy by Brigadier G. Bomford (Oxford, Clarendon Press, 1952), example (i), the experimental

9 The magnitudes of the C's and the relative magnitudes of the different correcting terms are given in Table 1.

H(ft)	tan ψ	C_1	C_2	C_2'	C_{11}	C_{12}	E_1	E_2	E_3	ΣE_i	E_e	N_1	N_2	M_{11}	M_{11}'	M_{12}	M_{12}'	M_{12}''
$3\cdot10^4$	6	$7.678\cdot10^{-5}$	$.866\cdot10^{-8}$	$1.706\cdot10^{-8}$	$2.335\cdot10^{-5}$	$1.102\cdot10^{-5}$	$2.700\cdot10^{-2}$	$.147\cdot10^{-2}$	$.100\cdot10^{-3}$			$28.41\cdot10^{-4}$	$1.79\cdot10^{-5}$	$4.42\cdot10^{-4}$	$1.37\cdot10^{-4}$	$8.55\cdot10^{-6}$	$5.8\cdot10^{-5}$	$3.58\cdot10^{-5}$
6	6	8.701	1.554	1.610	2.377	1.052	5.400	.5886	.800	1.0607	1.0608	32.2		10.03				
9	6	7.867	1.182	1.303	1.9008	.7781	8.100	1.325	2.712			29.1		13.62				
12	6	6.720	1.008	1.046	1.421	.5320	10.80	2.350	6.404			24.87		15.5				
15	6	5.693	$8.455\cdot10^{-9}$	$8.579\cdot10^{-9}$	1.051	.3561	13.50	3.6788	12.507	1.1843	1.1927	21.04		16.38	3.08	$6.88\cdot10^{-5}$	$1.07\cdot10^{-3}$	$4.0\cdot10^{-4}$
30	5.5	2.972	4.348	4.348	$3.021\cdot10^{-6}$	$6.096\cdot10^{-7}$	22.70	10.42	59.597			9.29		12.25	1.26	2.84	1.35	2.8
45	4.5	1.984	2.902	2.902	1.345	1.823	22.80	11.73	60.9			4.21					.62	
60	4.0	1.488	2.176	2.176	$7.564\cdot10^{-7}$	$7.692\cdot10^{-8}$	24.00	11.75	71.71			2.53						
90	3.3	$9.918\cdot10^{-6}$	1.451	1.451	3.362	2.279	24.50	12.40	78.42			1.18		1.72	$6.00\cdot10^{-6}$	$5.15\cdot10^{-7}$	.207	$1.45\cdot10^{-5}$
120	2.8	7.439	1.084	1.064	1.891	$9.615\cdot10^{-9}$	23.50	11.66	69.96			0.66						
150	2.5	5.951	$8.706\cdot10^{-10}$	$8.706\cdot10^{-10}$	1.210	4.923	23.50	11.58	70.37			0.43						
180	2.2	4.959	7.255	7.255	$8.405\cdot10^{-8}$	2.850	22.00	10.13	55.81			0.29						
210	2.0	4.251	6.219	6.219	4.175	1.794	21.00	9.56	53.32			0.21		$2.78\cdot10^{-5}$	$4.35\cdot10^{-7}$	$1.5\cdot10^{-6}$	$2.98\cdot10^{-5}$	$9.5\cdot10^{-7}$

$\tan^{-1} 6 = 80.5°$ For higher altitudes $\tan\psi < \frac{1}{\sqrt{2\delta_p}}$ So ψ falls well below the horizon

$$S/H = \omega'(1 + E_1 + E_2 + E_3 + N_1 + N_2 + M_{11} + M_{11}' + M_{12} + M_{12}' + M_{12}'') \cong \omega'(1 + E_1 + E_2 + E_3 + N_1 + M_{11} + M_{11}' + M_{12}' + M_{12}'')$$

$$E_1 = \omega'^2 \frac{\delta_p}{2} \qquad E_2 = \omega'^2(1+3\omega'^2)\frac{\delta_p^2}{6} \qquad E_3 = \omega'^4(3+5\omega'^2)\frac{\delta_p^3}{8} \qquad \Sigma E_i = 1 + E_1 + E_2 + E_3$$

$$E_e \doteq \frac{\sin^{-1}}{\delta_p \omega'}\left\{\frac{\omega'}{1+\omega'^2}\left[1+\delta_p - \sqrt{1-\delta_p(2+\delta_p)\omega'^2}\right]\right\}$$ exact expression for earth curvature

$$N_1 = (1+\omega'^2)C_1 \qquad N_2 = (1+\omega'^2)(C_2' + \tfrac{3}{2}\omega'^2 C_2)$$

$$M_{11} = (1+\omega'^2)(1+3\omega'^2)C_1\delta_p \qquad M_{11}' = (1+\omega'^2)(2+3\omega'^2)C_{11}\delta_p$$

$$M_{12} = \tfrac{3}{2}(1+\omega'^2)^2(2+5\omega'^2)C_{12}\delta_p^2, \quad M_{12}' = \tfrac{3}{2}\omega'^2(1+\omega'^2)(3+5\omega'^2)C_1\delta_p^2 \qquad M_{12}'' = 1+\omega'^2\left[2+15\omega'^2(1+\omega'^2)\right]C_{11}\delta_p^2$$

RELATIVE MAGNITUDES OF THE DIFFERENT TERMS OF THE EARTH CURVATURE-AIR REFRACTION EQUATION

TABLE 1

equation:

$$d = d_0 \, e^{-\alpha(H-H_0)}\left(1 - [illegible] \ln \frac{H}{H_0}\right)$$

may be derived. For the one or two hundred feet above the ground for which this relation holds, the approximation $d = [illegible]\left(1 - 10^{-[illegible]}(H-H_0) - [illegible] \ln \frac{H}{H_0}\right)$ is good.

The error caused by this effect in the constant C_1 at $H = 3 \times 10^4$ feet where the effect is assumed to die out at $H = H_T = 100$ feet is

$$\Delta C_1 = \frac{[illegible]}{2T_0 H}\int_{H_0}^{H_T} \ln \frac{H}{H_0}\, dH = \frac{[illegible]}{2T_0 H}\left[H_T\left(\ln \frac{H_T}{H_0} - 1\right) + H_0\right]$$

Assuming $T_0 = 300^\circ K$; $H_0 = 1$ ft. ; $[illegible] = 3 \times 10^{-4}$, $H_T = 100$ ft.

$$\Delta C_1 = \frac{3 \times 10^{-4}}{600 (3 \times 10^4)}\left[[illegible](3.6) - 1\right] \simeq .5 \times 10^{-8}$$

As C_1 is about 8×10^{-5}, we see that this effect is entirely negligible at angles up to the 80° we have been considering previously.

4. DISTORTIONS CORRECTED ABOUT THE PRINCIPAL POINT

(a) Non-Planar Focal Surface

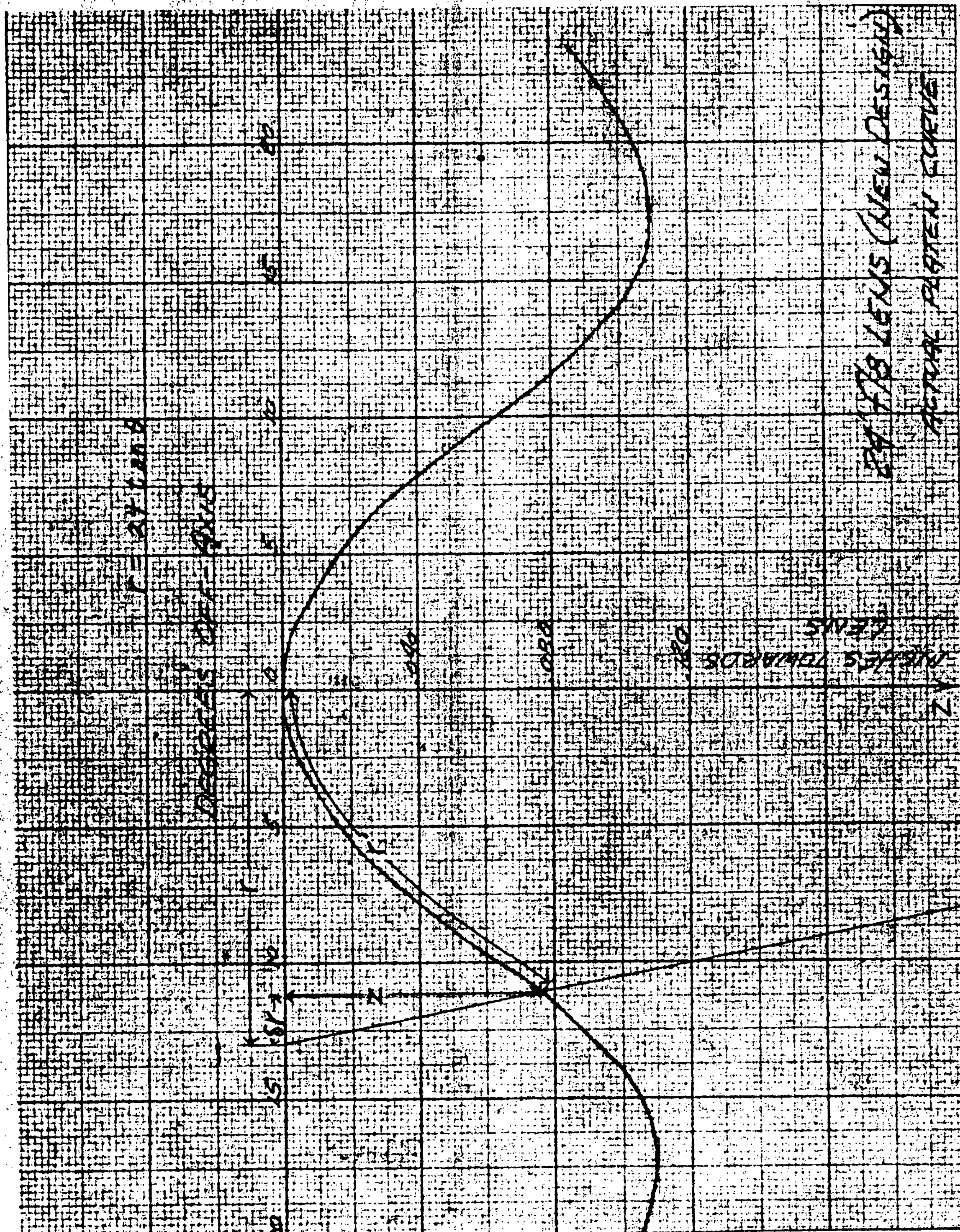
An example of radially dependent curvature of the focal surface is shown in Figure 31. If the film has not been appreciably stretched in fitting it to the platen, the arc length r' is derived in the following way:

<u>Correction for distortion due to curvature of focal surface of a lens.</u>

From Figure 31

$$r' = \int_0^{[illegible]} \frac{ds}{dr}\, dr = \int_0^{[illegible]} \sqrt{1 + \left(\frac{dz}{dr}\right)^2}\, dr \qquad [illegible]$$

$$= \int_0^{r(1-[illegible])} \sqrt{1 + \left(\frac{dz}{dr}\right)^2}\, dr$$



DISTORTION FOR REPRESENTATIVE NON-PLANAR PLATEN

If $Z, \frac{dZ}{dr} \ll 1$ **, then**

$$r' \simeq \int_0^{r(1-\frac{Z}{f})} \left[1 + \tfrac{1}{2}\left(\frac{dZ}{dr}\right)^2\right] dr \simeq r - \frac{Z}{f} r + \frac{1}{2}\int_0^r \left(\frac{dZ}{dr}\right)^2 dr$$

In this particular example

$$Z \simeq .00355\, r^2 (in) \qquad (0° < \theta < 10°) \quad , \quad f = 24\ in$$

$$\frac{dZ}{dr} = 2(.00355)\, r \quad , \quad r = 24 \tan 10° = 4.23''$$

$$r' = r\left(1 - \frac{.0635}{24}\right) + \frac{1}{2}\int_0^r 4(.00355)^2 r^2 dr = r\left[1 - .00264 + \frac{2}{3}(.00355)^2 r^2\right]$$

$$= r(1 - .00267 + .00015) = r(1 - .00252)$$

Corresponding to .25% error

For θ **= 15°, we approximate**

$$Z = .00355\, r^2 \qquad (0° < \theta < 10°)$$

$$Z = .0182\, r \qquad (10° < \theta < 15°)$$

r = 6.43", Z = .105"

$$\frac{1}{2}\int_0^r \left(\frac{dZ}{dr}\right)^2 dr = .00063 + \frac{1}{2}\int_{4.23}^{6.43} (.0182)^2 dr = .00063 + .000365 = .000995$$

$$r' = r\left(1 - \frac{.105}{24} + \frac{.000995}{6.43}\right) = r(1 - .00437 + .000155) = r(1 - .00422)$$

Corresponding to .42% error

5. MISCELLANEOUS DISTORTIONS

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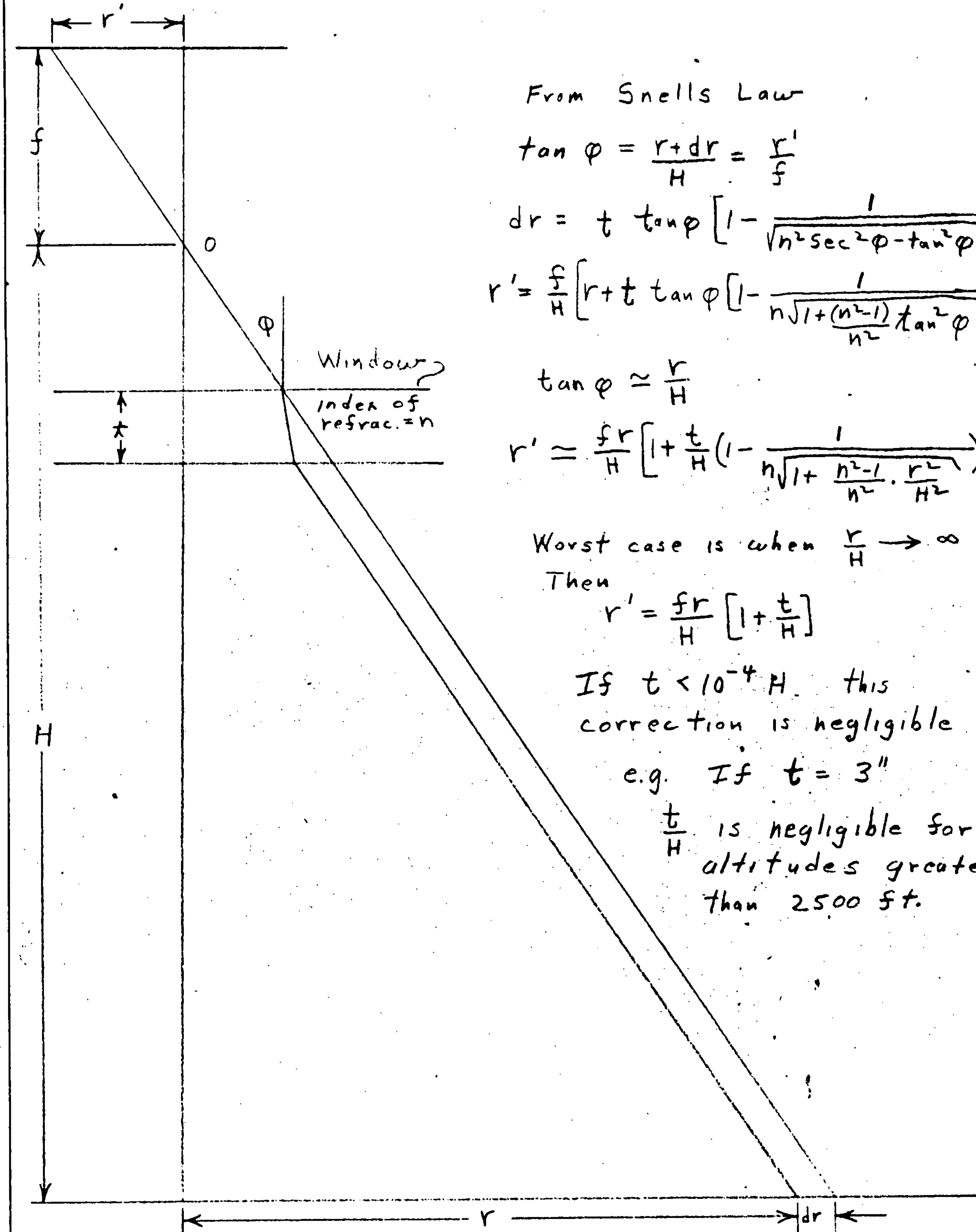

From Snells Law

$$\tan\varphi = \frac{r+dr}{H} = \frac{r'}{f}$$

$$dr = t\tan\varphi\left[1 - \frac{1}{\sqrt{n^2\sec^2\varphi - \tan^2\varphi}}\right]$$

$$r' = \frac{f}{H}\left[r + t\tan\varphi\left[1 - \frac{1}{n\sqrt{1+\frac{(n^2-1)}{n^2}\tan^2\varphi}}\right]\right]$$

$$\tan\varphi \simeq \frac{r}{H}$$

$$r' \simeq \frac{fr}{H}\left[1 + \frac{t}{H}\left(1 - \frac{1}{n\sqrt{1+\frac{n^2-1}{n^2}\cdot\frac{r^2}{H^2}}}\right)\right]$$

Worst case is when $\frac{r}{H} \to \infty$

Then

$$r' = \frac{fr}{H}\left[1 + \frac{t}{H}\right]$$

If $t < 10^{-4} H$ this correction is negligible

e.g. If $t = 3''$

$\frac{t}{H}$ is negligible for altitudes greater than 2500 ft.

DISTORTION DUE TO PLANE WINDOW

Figure 32

CORRECTION FOR RAPIDLY MOVING BOUNDARY LAYER

In Liepmann's Paper[10] on the deflection of a light ray by the boundary layer of air moving past a rapidly moving aircraft, it was considered that the velocity distribution was a function of the normal distance from the skin only; and this distribution was in plane parallel layers. It was considered that if φ_∞ was the angle at which the ray enters the boundary layer and φ_δ the angle at which the ray leaves the layer, then Snell's law holds so that

$$\frac{\sin \varphi_\delta}{\sin \varphi_\infty} = \frac{n_\infty}{n_\delta} = \frac{1 + K\rho_\infty}{1 + K\rho_\delta}$$

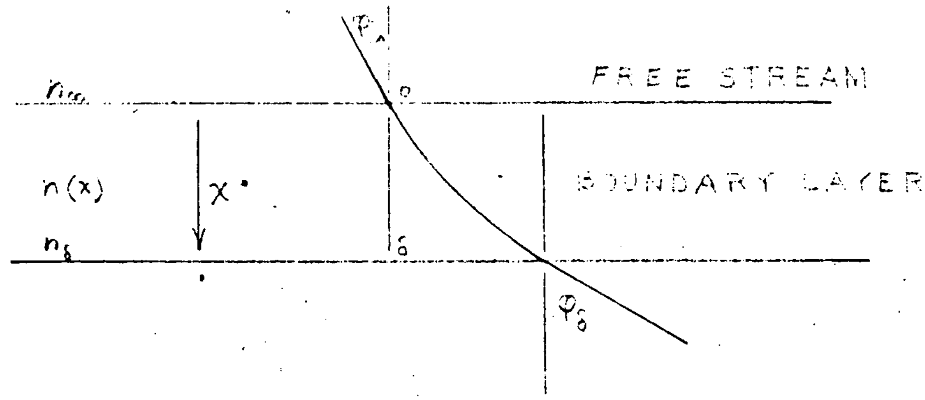

Figure 33. REFRACTION BY BOUNDARY LAYER

From aerodynamic considerations and the perfect gas law, Liepmann says

$$\frac{\rho_\delta}{\rho_\infty} = \left[1 + \frac{\gamma - 1}{2} \alpha M_\infty^2\right]^{-1}$$

where γ is the ratio of specific heats, α is a constant related to the Prandtl number, and M_∞ is the free stream Mach number. From this he calculates the angle of deflection $\varepsilon = \varphi_\delta - \varphi_\infty$ and plots $\frac{\varepsilon}{\tan \varphi_\infty}$ as a function of M_∞ and altitude.

---

[10] Liepmann, H. W., "Deflection and Diffusion of a Light Ray Passing Through a Boundary Layer," Douglas Aircraft Company, Inc., Report SM-14397, 16 May 1952.

However, if in addition the ray goes through a plane glass plate in order to enter the craft, the final direction of the ray will depend only on the free stream refractive index and that inside the aircraft. If the inside temperature and pressure are equal to that of the free stream, there will be no angular deflection.

C. FILM DISTORTION

From the instant of exposure the film begins changing its size and shape. This distortion is due to many causes, the most important of which are humidity changes, temperature changes, changes during processing, and changes due to long-term storage.

If the changes in size are uniform along and across the roll or sheet, the distortion is only a change in scale and is equivalent to a different focal length or altitude. Unfortunately, most films have different coefficients of expansion across and along the film, thus giving rise to a true distortion.

All of these distortions can be corrected by the electronic rectifier if the magnitudes are known. The correction is quite simple since the x and y coordinates of the rectifier correspond to length and width of the film. All that is required is that the over-all multiplying factors for the picture be entered into the computer. All x and y coordinates will be multiplied by these factors during the rectification. There are interesting possibilities for correcting for distortion of the rectified film before it is printed. It will usually not be necessary to correct for temperature and humidity changes of the rectified picture since it will probably be used at the same ambient conditions in which it was rectified, but the processing shrinkage can be programmed into the computer so that the rectified picture will be correct after processing.

All of the temperature and humidity conditions of the film during exposure may not be known. It is, therefore, desirable to have accurate reference marks, such as the distance between fiducial marks, on each exposure to aid in finding the correction factors for the computer.

As an example, let us assume that a photo is taken on Kodak Aerographic film and is to be rectified onto the same film. If film were exposed in the aircraft at 10°F and 10% R.H. and rectified at the Kodak recommended conditions of 70°F and 50% R.H., the following changes in film size will occur:

The rectified photo will be used at 70°F and 50% R.H. so it is desired to correct for the temperature and humidity changes in the original and for the processing shrinkage in both the original and rectified photos.

Humidity[11]

50% - 10% = 40% R.H. change.

Length (y) correction = $40 \times 8.5 \times 10^{-5}$ = 3.4%
Width (x) correction = $40 \times 9.0 \times 10^{-5}$ = 3.6%

Temperature

70° - 10° = 60°F temperature change

Length (y) correction = $60 \times 4.2 \times 10^{-5}$ = .25%
Width (x) correction = $60 \times 4.4 \times 10^{-5}$ = .26%

Processing Shrinkage

Length (y) correction .05%
Width (x) correction .06%

The rectified photo must be reduced in size by the humidity and temperature corrections and increased in size by the processing shrinkage correction. The processing correction must be applied twice, once for the shrinkage of the original and once for the shrinkage of the reproduction.

---

[11] Values of film distortion factors are taken from "Kodak Materials for Aerial Photography," 4th Ed., Page 9, Eastman Kodak Co.

Over-All Corrections

Length (y) corrections = .9966 x .9975 x 1.0005 x 1.0005 = .9951
Width (x) corrections = .9964 x .9974 x 1.0006 x 1.0006 = .9950

This process can be extended to cover all of the listed distortions of both original and rectified films. If desired, long-term distortion could be included so that the rectified photograph would be distortion free after a year's storage.

It is recommended that Dupont Chronar base films be used for the rectified photograph since this base has excellent temperature and humidity coefficients, good optical clarity, high strength, and since it contains no solvents or plasticizers, has good long-term aging characteristics.

John Centa[12] gives the following coefficients for Chronar base:

Humidity coefficnent: $1.0 - 2.0 \times 10^{-5}$ in/in/% R.H.
Thermal coefficient: 2.0×10^{-5} in/in/°F.

He also states that accelerated and normal aging tests show no indication of base change or deterioration.

In an instrument of the precision described in this report, every effort should be made to prevent degradation of the results from external sources. Chronar base films will aid in attaining this goal, and their use is highly recommended.

In the worst possible cases the distortion due to film changes will probably never exceed 1%, and .1% is probably a typical figure. All of these distortions in either the original or the rectified photo can be corrected with an electronic rectifier.

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12 Centa, J. M., "Performance Characteristics of 'Chronar' Polyester Photographic Tilm Base," PHOTOGRAMMETRIC ENG., Vol. 2, No. 4, Sept. 1955, Page 539.

D. LENS DISTORTION

Lens distortion may, under some conditions, cause serious errors in the geometry of the photograph.

Aerial lenses very from extremely low distortion lenses, such as the Wild Avigon, to lenses with very high distortion, such as the Zeiss Pleon. The Pleon is a very wide angle (136°) lens which is designed with a large amount of negative distortion in order to obtain better edge illumination.

The two important types of lens distortion are: radial or linear distortion, which is a linear displacement of the image point radially toward or away from the principal point. The positive direction is taken as being away from the center (See Figure 36).

Tangential distortion is a displacement of the image perpendicular to radial lines from the center of the field. Tangential distortion causes a straight line through the center of the field to image as a curved line.

Improper centering of the element causes bent axis distortion. This is equivalent to a small wedge in front of the lens and is discussed on page .

Although the best mapping lenses have distortions so low as to be negligible (the Wild Aviotar is claimed to have a maximum radial distortion of 5μ or .005 mm), it may not always be possible to use such a lens. It appears that lens designers could design lenses with better resolution if they could let the distortion increase. In view of this, it is desirable to have the ability to correct for lens distortion in the rectifier.

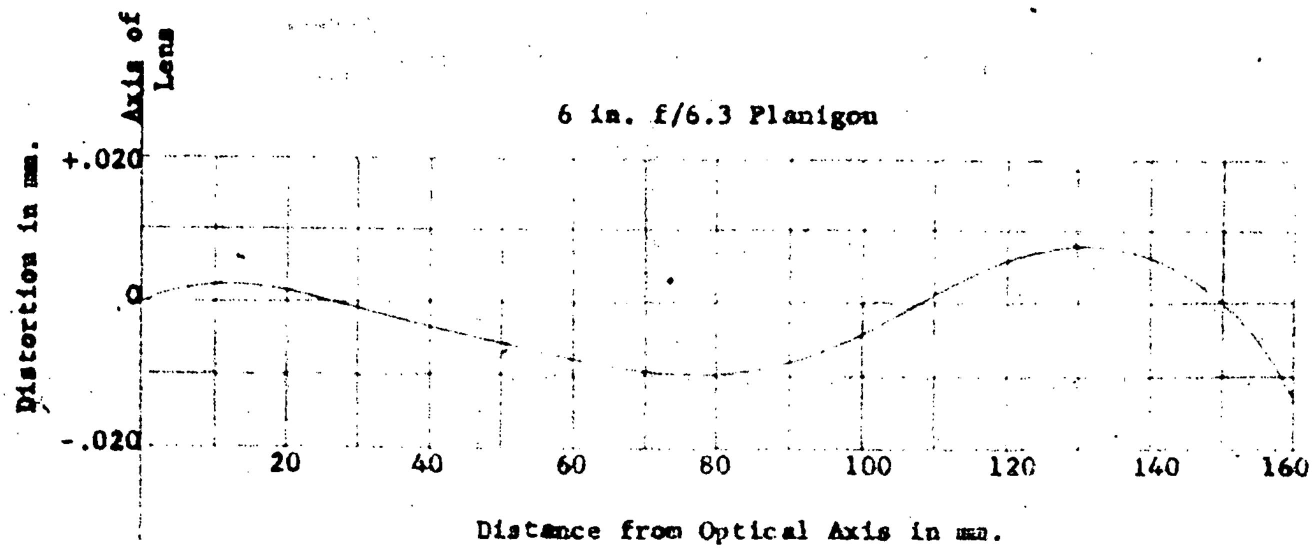
Figure 34 shows the radial distortion curves of two mapping lenses, the Planigon and the Metrogon, and of the wide angle Pleon.

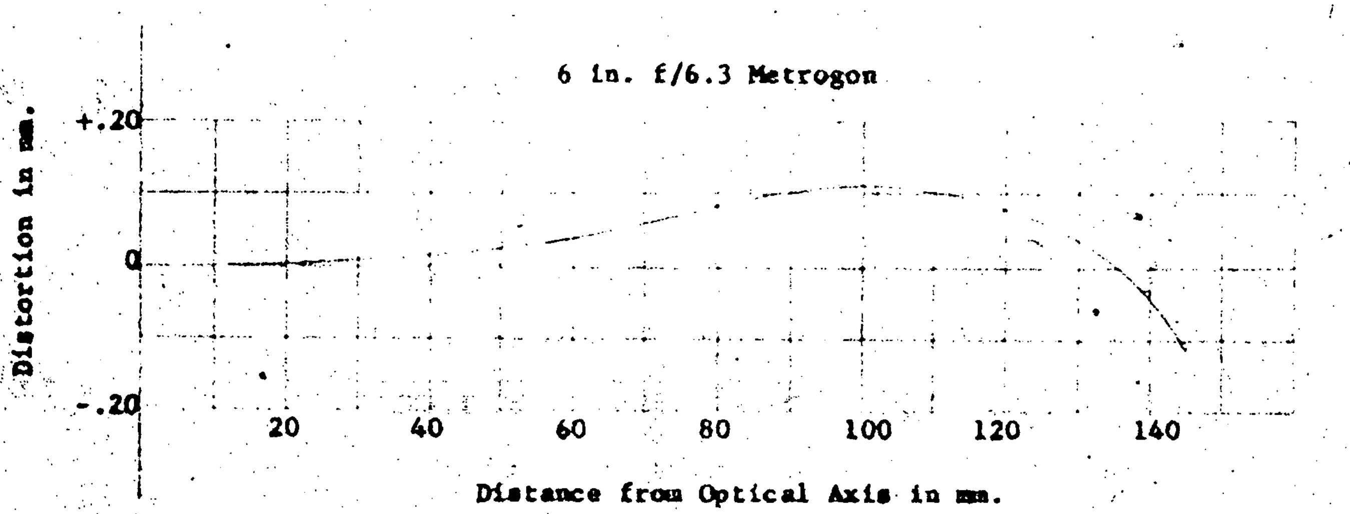
Other examples of magnitudes of distortion that may be expected may be found in military specifications. Mil-L-4325A(ASG) is for a 36-inch, f/8, 9 x 18 format, aerial reconnaissance and spotting lens. This spec calls for a distortion not exceeding 10 mm. Correction during rectification would greatly improve the accuracy of photos made with this lens though it obviously would not be used intentionally as a mapping lens since it has a distortion of about 4%.

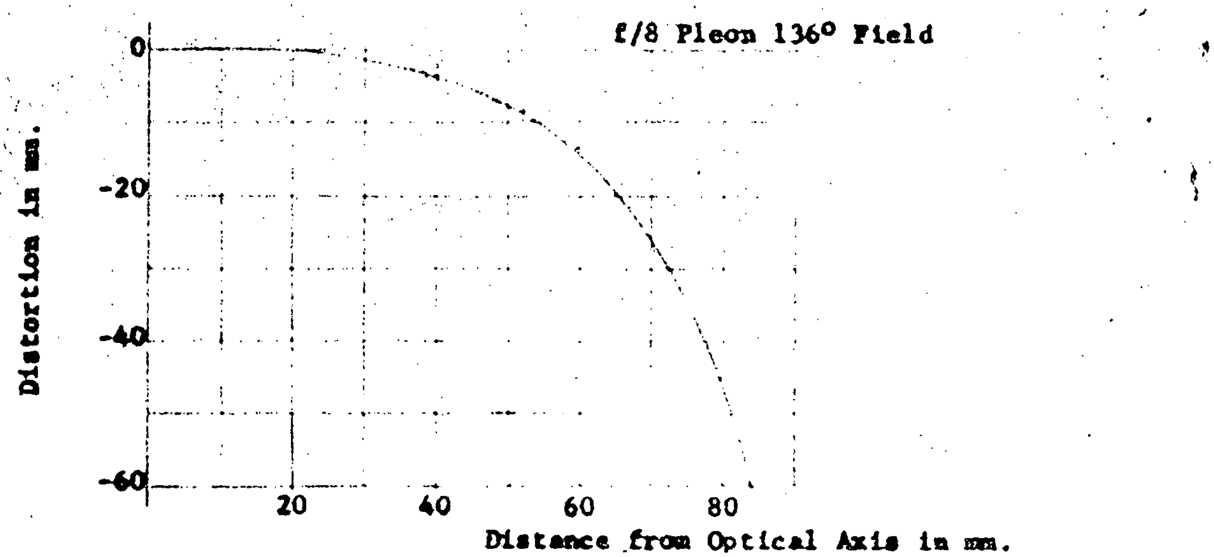
Mil-L-7367B(ASG) is for a 6-inch, f/6.3, 9 x 9 format cartographic lens. The maximum tangential distortion is .02 mm, and the maximum radial distortion is -.17 mm at 45°. This results in a radial positional error of .11% and a tangential error of .013%. Obviously, the radial distortion, as is usually the case, is the more troublesome.

Distortion characteristics of lenses are usually given as curves of distortion vs. radial distance as shown in Figure 34. Correction of distortion in the electronic rectifier is quite simple for radial distortion but is considerably more complex for tangential distortion. Although feasible, it is probably not advisable to correct for tangential distortion since it is so low in good mapping lenses.

As a typical example of a good mapping lens, the Planigon is duscussed here. Mil-L-6637B(ASG) covers a Planigon aerial cartographic lens 6 inches, f/6.3 for a 9 x 9 format. This spec calls for a maximum tangential distortion of .008 mm and a maximum radial distortion of .012 mm. The maximum tangential distortion usually occurs at the maximum radius $D_T = \frac{.008}{152} = .0052\%$. The maximum radial distortion occurs at 130 mm radius (Figure 34) $D_R = \frac{.012}{130} =$ .0092%







DISTORTION CURVES OF SOME REPRESENTATIVE AERIAL LENSES

Figure 34

A rectifier operating at an accuracy of .01% could not improve these figures.

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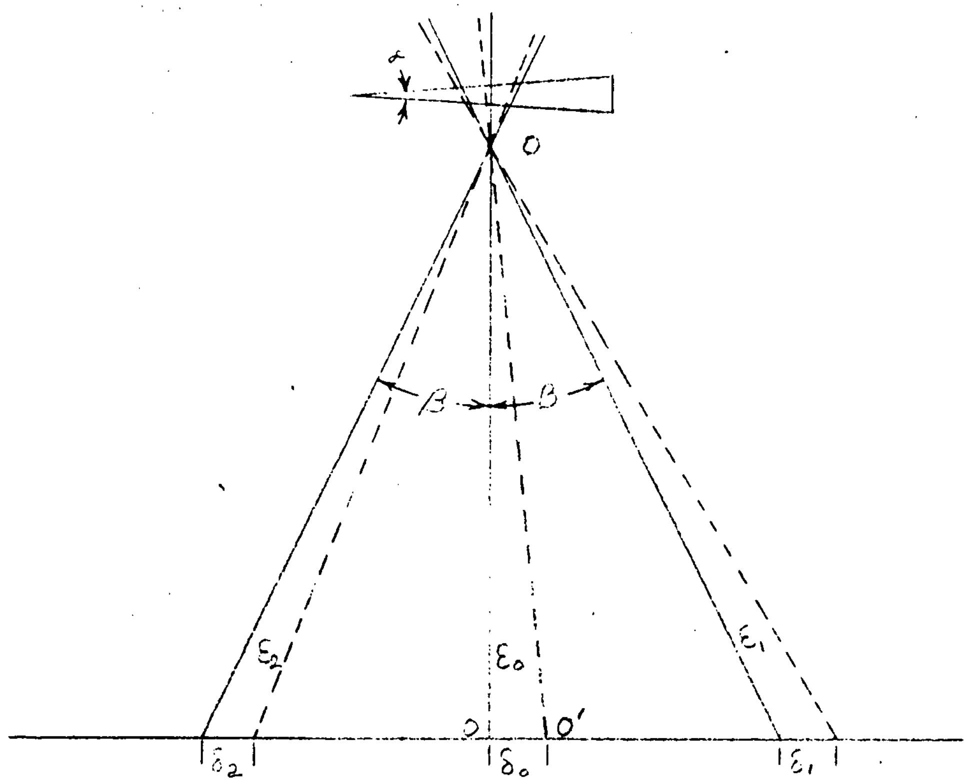
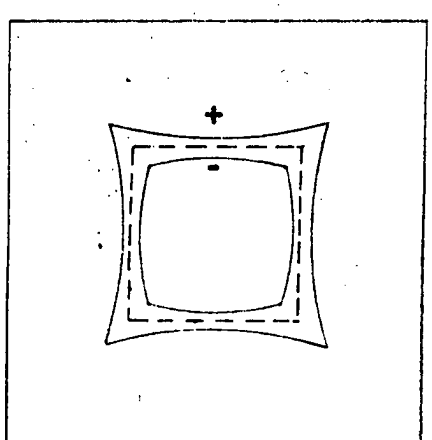
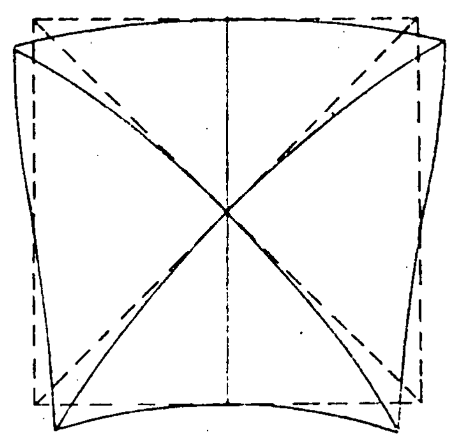


Figure 35. DISTORTION FROM PRISM IN WINDOW OR LENS



Radial Distortion



Tangential Distortion

Figure 36. TYPES OF LENS DISTORTION

<u>**Distortion due to prism of lens or windows**</u>. (See Fig. 35)

If the prism angle is α and α is small, then quite closely

$$\varepsilon_1 \simeq \varepsilon_2 = \varepsilon = \alpha\left[\frac{(n^2 - \sin^2\beta)^{1/2}}{\cos\beta} - 1\right], \quad \varepsilon_0 = \alpha(n-1)$$ [13]

The distortion is worst at large angles β, and since the largest may be around 60°-80°, we have for a bad case, $(\beta \simeq 75°)$

$\cos\beta \simeq .26$, $\sin\beta \simeq .97$, $n \simeq 1.5$ so

$\varepsilon_0 = \frac{\alpha}{2}$, $\varepsilon = \alpha\left[\frac{(2.25 - .94)^{1/2}}{.26} - 1\right] = 3.77\alpha$, $\delta_0 = f\varepsilon_0 = \frac{f\alpha}{2}$

This results in an error

$$\delta = \frac{f\varepsilon}{\cos^2\beta} = \frac{3.77f\alpha}{.06} = 63f\alpha$$

So if

$\delta - \delta_0 = 10^{-3}$ in., $f = 6$ in., then $\alpha = \frac{10^{-3}}{6 \cdot 62.5} \simeq 2.67 \cdot 10^{-6}$ rad or $\alpha \simeq \frac{1}{2}$ sec.

For $\beta = 45°$, $\sin\beta = \cos\beta = \frac{1}{\sqrt{2}}$ and $\varepsilon = \alpha\left[(4.5-1)^{1/2} - 1\right] = .87\alpha$

$\delta = \frac{f\varepsilon}{\cos^2\beta} = 2f \cdot .87\alpha = 1.74f\alpha$. So $\delta - \delta_0 = 1.24f\alpha = 10^{-3}$ in. For $f = 6''$

$\alpha = \frac{10^{-3}}{7.44} = 1.34 \cdot 10^{-4}$ rad. $\simeq 28$ sec.

6. **OTHER APPLICATIONS OF RECTIFICATION EQUATIONS**

(a) One scheme of rectification consists of a read-out drum rotating at uniform speed with the read-out printing spot traveling at uniform speed parallel to the axis of the drum so that it travels one line width in one rotation. The computer would position the read-in drum to the proper spot for pick-up. In order to get an idea of the velocities and accelerations required of the read-in drum for this scheme, we must calculate the partial derivatives involved in the equations:

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[13] Washer, F. E., "The Effect of Prism on the Location of the Principal Point," PHOTOGRAMMETRIC ENGINEERING, Vol. 23, June 57, Page 520.

$$X' = X'(X, Y)$$

$$Y' = Y'(X, Y)$$

$$\frac{dX'}{dt} = V'_x = \frac{\partial X'}{\partial X} V_x + \frac{\partial X'}{\partial Y} V_y$$

$$\frac{d^2X'}{dt^2} = a'_x = \frac{\partial X'}{\partial X} a_x + \frac{\partial X'}{\partial Y} a_y + \left(V_x \frac{\partial}{\partial X} + V_y \frac{\partial}{\partial Y}\right)^2 X'$$

$$= \frac{\partial X'}{\partial X} a_x + \frac{\partial X'}{\partial Y} a_y + V_x^2 \frac{\partial^2 X'}{\partial X^2} + 2 V_x V_y \frac{\partial^2 X'}{\partial X \partial Y} + V_y^2 \frac{\partial^2 X'}{\partial Y^2}$$

Similarly,

$$V'_y = \frac{\partial Y'}{\partial X} V_x + \frac{\partial Y'}{\partial Y} V_y$$

$$a'_y = \frac{\partial Y'}{\partial X} a_x + \frac{\partial Y'}{\partial Y} a_y + \frac{\partial^2 Y'}{\partial X^2} V_x^2 + 2\frac{\partial^2 Y'}{\partial X \partial Y} V_x V_y + \frac{\partial^2 Y'}{\partial Y^2} V_y^2$$

It is likely that the tilted panoramic photograph will be one of the most difficult to follow by this system, so we calculate the appropriate deviations as follows using the transformation for a stationary camera as we are mainly concerned with the large distortions.

We have

$$Y' = f \tan^{-1} \frac{(-cx + dy + ef)_2}{(-gx - hy + kf)_1} = f\theta$$

$$X' = \frac{f(ax + bf)}{\sqrt{(\ )_1^2 + (\ )_2^2}}$$

$$(\ )_1^2 + (\ )_2^2 = (c^2+g^2)x^2 + (d^2+h^2)y^2 + (e^2+k^2)f^2 - 2(cd-gh)xy - 2(ce+gk)xf + 2(de-hk)yf$$

Let

$j = \sin t \sin s$

$a = 1 - j^2$

$b = j\sqrt{a}$

$c = j \sin t \cos s$

$d = \cos t$

$e = \sin t \cos s \sqrt{a}$

$g = j \cos t$

$h = \sin t \cos s$

$k = \cos t \sqrt{a}$

Then

$$c^2+g^2 = j^2(\sin^2 t\cos^2 s + \cos^2 t) = j^2 a$$
$$d^2+h^2 = (\sin^2 t\cos^2 s + \cos^2 t) = a$$
$$e^2+k^2 = a(\sin^2 t\cos^2 s + \cos^2 t) = a^2$$
$$cd - gh = 0$$
$$ce + gk = j\sqrt{a}(\sin^2 t\cos^2 s + \cos^2 t) = ja\sqrt{a}$$
$$de - hk = 0$$

So

$$x' = \frac{f(\sqrt{a}x + jf)}{\sqrt{j^2x^2+y^2+af^2-2j\sqrt{a}fx}} = \frac{f(\sqrt{a}x+jf)}{\sqrt{(jx-\sqrt{a}f)^2+y^2}} = \frac{fv}{\sqrt{u^2+y^2}}$$

$$\frac{\partial x'}{\partial x} = \frac{f}{(u^2+y^2)}\left[(u^2+y^2)^{\frac{1}{2}}\frac{\partial v}{\partial x} - v\frac{\partial (u^2+y^2)^{\frac{1}{2}}}{\partial x}\right] = \frac{f}{(u^2+y^2)}\left[\sqrt{a}(u^2+y^2)^{\frac{1}{2}} - \frac{juv}{(u^2+y^2)^{\frac{1}{2}}}\right]$$

$$= \frac{f}{(u^2+y^2)^{\frac{3}{2}}}\left[\sqrt{a}(u^2+y^2) - juv\right] = \frac{f}{(u^2+y^2)^{\frac{3}{2}}}(\sqrt{a}y^2 - uf)$$

$$\frac{\partial x'}{\partial y} = -\frac{fvy}{(u^2+y^2)^{\frac{3}{2}}}$$

$$\frac{\partial^2 x'}{\partial x^2} = \frac{f}{(u^2+y^2)^3}\left[-jf(u^2+y^2)^{\frac{3}{2}} - \tfrac{3}{2}(\sqrt{a}y^2-uf)(u^2+y^2)^{\frac{1}{2}}2ju\right] = \frac{jf(2u^2f - y^2[3\sqrt{a}u+f])}{(u^2+y^2)^{\frac{5}{2}}}$$

$$\frac{\partial^2 x'}{\partial y^2} = \frac{-fv}{(u^2+y^2)^3}\left[(u^2+y^2)^{\frac{3}{2}} - 3y^2(u^2+y^2)^{\frac{1}{2}}\right] = \frac{-fv}{(u^2+y^2)^{\frac{5}{2}}}(u^2-2y^2)$$

$$\frac{\partial^2 x'}{\partial x\partial y} = \frac{-fy}{(u^2+y^2)^3}\left[\sqrt{a}(u^2+y^2)^{\frac{3}{2}} - 3juv(u^2+y^2)^{\frac{1}{2}}\right] = -\frac{fy\left[\sqrt{a}(u^2+y^2) - 3juv\right]}{(u^2+y^2)^{\frac{5}{2}}}$$

$$y' = f\tan^{-1}\left[\frac{(\ )_2}{(\ )_1}\right]$$

$$\frac{\partial y'}{\partial x} = f\,\frac{\dfrac{-c(\ )_1 + g(\ )_2}{(\ )_1^2}}{1 + \dfrac{(\ )_2^2}{(\ )_1^2}} = \frac{f(cgx + chy - ckf - cgx + dgy + gef)}{(\ )_1^2 + (\ )_2^2}$$

$$= \frac{jfy}{u^2+y^2}$$

$$\frac{\partial y'}{\partial y} = \frac{f[d(\ )_1 + h(\ )_2]}{(\ )_1^2 + (\ )_2^2} = \frac{f(-gdx - hdy + kdf - chx + hdy + hef)}{a(u^2+y^2)}$$

$$= \frac{-fu}{u^2+y^2}$$

$$\frac{\partial^2 y'}{\partial x^2} = \frac{-2fj^2uy}{(u^2+y^2)^2}$$

$$\frac{\partial^2 y'}{\partial y^2} = \frac{2fuy}{(u^2+y^2)^2}$$

$$\frac{\partial^2 y'}{\partial x\partial y} = \frac{fj(u^2+y^2-2y^2)}{(u^2+y^2)^2} = \frac{fj(u^2-y^2)}{(u^2+y^2)^2}$$

Summarizing these functions and derivatives, we have with

$$u = jx - \sqrt{a}\,f, \quad v = \sqrt{a}\,x + jf, \quad j = \sin\tau \sin s, \quad a = 1 - j^2$$

$$x' = \frac{fv}{(u^2+y^2)^{\frac{1}{2}}}$$

$$\frac{\partial x'}{\partial x} = \frac{-f(uf - \sqrt{a}\,y^2)}{(u^2+y^2)^{3/2}}$$

$$\frac{\partial x'}{\partial y} = \frac{-fvy}{(u^2+y^2)^{3/2}}$$

$$\frac{\partial^2 x'}{\partial x^2} = \frac{jf[2u^2 f - y^2(3\sqrt{a}\,u + f)]}{(u^2+y^2)^{\frac{5}{2}}}$$

$$\frac{\partial^2 x'}{\partial x \partial y} = \frac{-fy[(u^2+y^2)\sqrt{a} - 3juv]}{(u^2+y^2)^{\frac{5}{2}}}$$

$$\frac{\partial^2 x'}{\partial y^2} = \frac{-fv(u^2 - 2y^2)}{(u^2+y^2)^{\frac{5}{2}}}$$

$$y' = f\tan^{-1}\left[\frac{(-cx + dy + ef)}{(-gx - hy + kf)}\right] = f\theta$$

$$\frac{\partial y'}{\partial x} = \frac{jfy}{(u^2+y^2)}$$

$$\frac{\partial y'}{\partial y} = \frac{-fu}{(u^2+y^2)}$$

$$\frac{\partial^2 y'}{\partial x^2} = \frac{-2j^2 fuy}{(u^2+y^2)^2}$$

$$\frac{\partial^2 y'}{\partial x \partial y} = \frac{jf(u^2-y^2)}{(u^2+y^2)^2}$$

$$\frac{\partial^2 y'}{\partial y^2} = \frac{2fuy}{(u^2+y^2)^2}$$

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For the case of print-out at a uniform velocity in the x direction (uniform rotational velocity of the print-out drum), we have, if we assume $V_y << V_x$,

$V_x = R w$ where R and w are the radius and angular velocity of the print-out cylinder.

Also $V_y = a_x = a_y = 0$ so,

$$a_{x'} = R^2 w^2 \frac{\partial^2 x'}{\partial X^2} = \frac{[illegible]\, R^2 w^2 [2u^2 f - y^2 (\ldots u + \ldots)]}{(u^2+y^2)^{\frac{5}{2}}}$$

$$a_{y'} = R^2 w^2 \frac{\partial^2 y'}{\partial X^2} = \frac{-2 j^2 R^2 f u y}{(u^2+y^2)^2}$$

For a representative case, R = 4", w = 180 rad/sec. (1800 rpm) for pure pitch, (S = 270°); p = t, j = -sin p, a = $\cos^2 p$, so that if

$f = 3''$, $\sin p = .2$, $\cos p = .98$, $a = .96$ and

$u = jX - \sqrt{a} f \approx -2, -4$

Calling u about -3, we have

$$a_{x'} = \frac{-.2(3)(4 \times 180)^2 [2 \cdot 27 - j^2(-9+3)]}{(9+y^2)^{\frac{5}{2}}} = \frac{-1.86 \cdot 10^{[illegible]}}{(9+y^2)^{\frac{3}{2}}}$$

$$a_{y'} = \frac{2(.04)(7.2)^2 9 \cdot 10^4 y}{(9+y^2)^2} = \frac{3.73 \times 10^5 y}{(9+y^2)^2}$$

From this we see that $a_{x'}$ is greatest at y = 0 when.

$a_{x'} = -6.9 \times 10^4$ in./sec^2 = $-179g$

When y = 4.

$$a_{y'} = \frac{4(3.73) \cdot 10^5}{25^2} = 2.4 \cdot 10^3 = 6.2g$$

$y = 6$

$$a_{y'} = \frac{6(3.73) \cdot 10^5}{45^2} = 1.1 \cdot 10^3 = 2.86g$$

$y = 2$

$$a_{y'} = \frac{2(3.73) \cdot 10^5}{13^2} = 4.4 \cdot 10^3 = 11.4g$$

These accelerations show the infeasibility of positioning the read-out drum in this way.

(b) Another scheme of rectification is to print-out by having the flying spot cover the rectified photograph by small squares centered on equally spaced index points. The points on the unrectified print corresponding to these index points are computed by a digital computer from the equations of rectification. The position of the pick-up scanning beam is then determined by an analog computer from the Taylor's series expansion of the equations of rectification about the index point. A question that arises in this connection is how small must the squares be in order to use only the linear terms of the series expansion. Again, the tilted panoramic camera probably offers the most stringent conditions.

The general Taylor's series expansions are:

$$\Delta x' = x'(x+\delta, y+\epsilon) - x'(x,y) = \frac{\partial x'}{\partial x}\delta + \frac{\partial x'}{\partial y}\epsilon + \frac{1}{2}\left(\delta\frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial y}\right)^2 + \cdots + \frac{1}{m!}\left(\delta\frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial y}\right)^m x'$$

$$\Delta y' = \sum_{1}^{\infty}{}_m \frac{1}{m!}\left(\delta\frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial y}\right)^m y'$$

For small values of ϵ and δ and away from the points where x = 0 or y = 0, the second term should approximate the error in using the linear term only. Letting $\delta = \epsilon$ we have as the errors in $\Delta x'$, $\Delta y'$:

$$E(\Delta x') = \frac{\delta^2}{2}\left(\frac{\partial^2 x'}{\partial x^2} + 2\frac{\partial^2 x'}{\partial x\partial y} + \frac{\partial^2 x'}{\partial y^2}\right) = \frac{\delta^2 f}{2(u^2+y^2)^{\frac{5}{2}}}\Big[2jfu^2 - jy^2(3\sqrt{a}\,u+f) - 2\sqrt{a}\,y(u^2+y^2) + [\text{illegible}]\,ju\,v\,y - [\text{illegible}]\,u^2 + 2\,v\,y^2\Big]$$

$$= \frac{\delta^2 f}{2(u^2+y^2)^{\frac{5}{2}}}\left\{u^2(2jf + 4\sqrt{a}\,y - [\text{illegible}]) + y^2(v[1+a] - 2\sqrt{a}[y+ju])\right\}$$

$$E(\Delta y') = \frac{\delta^2 f}{(u^2+y^2)^2}\left[-j^2uy + ju^2 - jy^2 + uy\right] = \frac{\delta^2 f}{(u^2+y^2)^2}\left[a\,uy + j(u^2-y^2)\right]$$

For $u = -3$, $y = 4$, $j = -.2$, $v = -.3$ we have

$$E(\Delta x') = \frac{\delta^2 1.5}{5^5}\left[9(-1.2 + 15.7 - [\text{illegible}]) + 16([\text{illegible}]) - [\text{illegible}](.3) - [\text{illegible}]\right] = [\text{illegible}].0125\,\delta^2$$

$$E(\Delta y') = \frac{\delta^2 3}{625}\left[-.96([\text{illegible}])\ [\text{illegible}]\right] = [\text{illegible}].0455\,\delta^2$$

These errors must be less than 10^{-3}" so approximately $.05\delta^2 = 10^{-3}$ or $\delta^2 = 2.10^{-2}$ so $\delta = .14$", and it appears that the size of the print-out squares must be of the order of 1/8" or less to neglect second-order terms in the Taylor's series expansion.

Figure 29 shows the relation between the rectified and unrectified photographs in this case and also the relation between a 1/8" square in the rectified plane and its counterpart in the unrectified plane.

(c) Accuracy of positioning considerations.

If a certain percentage error of positioning is made in the rectified photograph, the corresponding tolerance is different and usually much more stringent for the unrectified photograph. The relationship, for instance, between the errors $\delta y'$ in the y' position of the unrectified photograph and δy in the y position of the rectified photograph for the panoramic transformation of a level camera is

$$\delta y' = \frac{\delta y}{1 + \left(\frac{y}{f}\right)^2} = \delta y \cos^2\theta$$

For y = ± 6, this shows that the positional accuracy required for the read-in system must be five times that used for the read-out system and ten times this for y = 9.

Referring to Figure 3, the scale of an image on a tilted photograph is given by the expression [illegible], where u is the distance from the axis of tilt. Comparing this with an untilted photograph, we have as the ratio of the scales,

[illegible]

This ratio is smallest when u has the largest negative value. For a camera of angle β, the edge of the film is $f\tan\beta$ distant from the principal point. Therefore, the largest negative value of u is

$$u_{min} = -\left(f\tan\beta - f\tan\tfrac{t}{2}\right)$$

and the ratio is

$$\frac{S}{S_t} = \frac{1}{1+\left(\tan\beta-\tan\frac{t}{2}\right)\sin t} = \frac{1}{D}$$

The value of tilt for which this is an extreme if found by differentiating this expression or its inverse, putting the derivative equal to zero and solving for t. Doing this, we get

$$\frac{dD}{dt} = \left(\tan\beta-\tan\tfrac{t}{2}\right)\cos t - \tfrac{1}{2}\sin t\,\sec^2\tfrac{t}{2} = 0$$

$$\tan\beta-\tan\tfrac{t}{2} = \tfrac{1}{2}\tan t\,\sec^2\tfrac{t}{2} = \tan\tfrac{t}{2}\,\frac{\left(1+\tan^2\frac{t}{2}\right)}{1-\tan^2\frac{t}{2}}$$

or

$$\left(\tan\beta-\tan\tfrac{t}{2}\right)\left(1-\tan^2\tfrac{t}{2}\right) = \tan\tfrac{t}{2}\left(1+\tan^2\tfrac{t}{2}\right)$$

and

$$\tan\beta\left(1-\tan^2\tfrac{t}{2}\right) = 2\tan\tfrac{t}{2}$$

so

$$\tan^2\tfrac{t}{2} + \frac{2}{\tan\beta}\tan\tfrac{t}{2} - 1 = 0$$

$$\tan\tfrac{t}{2} = -\frac{1}{\tan\beta} + \sqrt{\frac{1}{\tan^2\beta}+1} = \frac{1}{\tan\beta}\left(\left(1+\tan^2\beta\right)^{\frac{1}{2}}-1\right)$$

$$= \tan\tfrac{\beta}{2}$$

$$\therefore\ t = \beta$$

and we see that the maximum decrease of scale comes when the tilt is equal to the half-field of view of the camera. This scale decrease is

$$\frac{1}{1+\sin\beta\left(\tan\beta-\left(\frac{1-\cos\beta}{\sin\beta}\right)\right)} = \cos\beta$$

So $\frac{S}{S_t} = \cos\beta$ or about .707 for a 90° lens, which is the widest angle lens of cartographic quality likely to be used.